Technical Report No. 32-119

Sound Radiation from a Turbulent Boundary Layer

John Laufer

JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

November 1, 1961

National Aeronautics and Space Administration
Contract No. NASw-6

Technical Report No. 32-119

SOUND RADIATION FROM A TURBULENT BOUNDARY LAYER

John Laufer

JET PROPULSION LABORATORY
California Institute of Technology
Pasadena, California
November 1, 1961

CONTENTS

FIGURES

SUMMARY

If the restriction of incompressibility in the turbulence problem is relaxed, the phenomenon of energy radiation in the form of sound from the turbulent zone arises. In order to calculate this radiated energy, it is shown that new statistical quantities, such as time-space correlation tensors, have to be known within the turbulent zone in addition to the conventional quantities. For the particular case of the turbulent boundary layer, indications are that the intensity of radiation becomes significant only in supersonic flows. Under these conditions, the recent work of Phillips is examined together with some experimental findings of the author. It is shown that the qualitative features of the radiation field (intensity, directionality) as predicted by the theory are consistent with the measurements; however, even for the highest Mach number flow, some of the assumptions of the asymptotic theory are not yet satisfied in the experiments. Finally, the question of turbulence damping due to radiation is discussed, with the result that in the Mach number range covered by the experiments, the energy lost from the boundary layer due to radiation is a small percentage of the work done by the wall shearing stresses.

INTRODUCTION

When the compressibility of the fluid is taken into account, a new aspect of the turbulence problem will arise. In a compressible fluid a disturbance from a source will propagate at a finite speed and will influence the flow field *over a finite distance in a given time.* This means that in calculating the flow properties at a given point and time, it will now be necessary to know the behavior of the disturbance source at a certain earlier time. Thus, the concept of retarded time and retarded potential naturally arises. This fact is reflected in the statistical description of a fluctuating flow field; in order to calculate, for instance, the pressure fluctuations emanating from a turbulent shear field, it will now be necessary to know certain space-time correlation functions within the shear field heretofore not considered.

In order to fix our ideas, we will choose a definite geometry for a turbulent shear flow: the boundary layer. Thus, we have a turbulent fluid streaming over a rigid wall and want to examine the time-dependent pressure field outside of the layer. Within the layer the fluctuations may be described primarily in terms of vorticity and entropy modes and, to a lesser extent, sound modes. Outside the layer the first two modes die out rapidly, so that at a sufficiently large distance from the shear zone (several wavelengths away) one expects to find only fluctuations of the sound mode type present, usually referred to in the literature as aerodynamic noise. We will seek a relation between this sound field and the fluctuations within the boundary layer.

The mathematical tools to handle the radiation have been well developed in the electromagnetic, acoustic and nonstationary supersonic theory. Thus, once the aerodynamic noise problem has been properly formulated and linearized -- and this can be done with reasonable assumptions -- in principle, at least, a solution can be obtained. The main difficulty and the reason for the rather slow progress in this field is the fact that the solution is written in terms of the aforementioned statistical quantities of the turbulence field about which very little if any information is available.

It is interesting to note that one new feature in a compressible turbulence is the fact that the pressure energy radiated away from the turbulent zone represents a new form of energy loss besides the dissipation. The question naturally arises as to whether or not, at sufficiently high Mach number, the radiation can be intense enough to exceed the rate of turbulence production and thus dampen out the turbulence.

The main purpose of this paper is not to give a comprehensive literature survey on the subject, but rather to extract those ideas that seem to be most helpful in understanding the mechanism of radiation in the case of a turbulent boundary layer. Since recent measurements indicate that the intensity of radiation becomes significant mainly in supersonic flows, theories that depend on the assumption $M << 1$ will be merely touched upon; the discussion will concentrate on the supersonic problem.

FORMULATION OF THE PROBLEM

Taking the divergence of the momentum equation and using the continuity equation, one obtains

$$\frac{\partial^2 \rho}{\partial t^2} - \nabla^2 p = \frac{\partial^2 \rho u_i u_j}{\partial x_i \partial x_j} + \frac{\partial^2 \tau_{ij}}{\partial x_i \partial x_j} \tag{1}$$

where τ_{ij} is the viscous stress tensor.

In order to eliminate the density (at least from the leading terms), Phillips has rewritten this equation in the form (Ref. 1)

$$\frac{D^2}{Dt^2} \log p - \frac{\partial}{\partial x_i} a^2 \frac{\partial}{\partial x_i} \log p = \gamma \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} + \gamma \frac{D}{Dt} \left(\frac{1}{Cp} \frac{DS}{Dt} \right) - \gamma \frac{\partial}{\partial x_i} \frac{1}{\rho} \frac{\partial \tau_{ij}}{\partial x_j} \tag{2}$$

The last two terms in the equation represent entropy fluctuations and viscous effects. If we restrict ourselves to small fluctuations and to regions not too far from the shear layer, where diffusion effects are not important, these terms may be neglected.

The left-hand side of the equation has the form of a wave equation in which the time derivatives have been replaced by those following the motion, and the propagation velocity is a variable. On the right-hand side, the velocity term is usually referred to in the literature as the pressure-generation term. This nomenclature, however, is somewhat misleading and needs some clarification. The velocity fluctuation should of course be considered as an independent variable together with the pressure. This, unfortunately, renders the problem hopelessly complicated. One could adopt the point of view (see, for instance, Ref. 2) that the velocity field within the shear layer is known from measurements, and therefore the right-hand side may be considered a known forcing function for the wave equation. A much more satisfying approach is that of trying, by a suitable assumption, to decouple the pressure field from the velocity field: provided the Mach number is not very high, one may assume that the velocity fluctuation within the shear layer has predominantly vorticity modes; that is to say, the noise field generated by the turbulent shear layer will contribute only a negligible velocity field within the layer. There is some experimental evidence which indicates that this assumption is reasonable. While the pressure fluctuations in the far field vary an order of magnitude in the Mach number range considered, the velocity fluctuation field in the boundary layer (in an appropriately normalized form) does not change as was shown in the previous paper by Morkovin; similarly the wall pressure fluctuations, measured recently by Kistler, vary only slowly with Mach number (see Fig. 1; $\tilde{p}/\tau_w$ is the ratio of rms pressure fluctuation to wall shearing stress).*

Thus, if the velocity field within the boundary layer is known *a priori*, the problem reduces to solving a wave equation with a known source term. The mathematical difficulty in the solution lies mainly in the fact that the governing partial differential equation has variable coefficients, and some suitable simplification has to be made in order to obtain a solution. In the literature, one finds two approaches which will be discussed below.

a. Low Mach number solution: Lighthill succeeded in reducing the problem to that of classical acoustics by considering flows with $M << 1$ (Ref. 3). Under this circumstance, we can replace the density in the generation term (Eq. 1) by a constant value and neglect a term of the form

$$\frac{1}{a_\infty^2}\,\frac{\partial^2 (a_\infty^2 \rho - p)}{\partial t^2} = \frac{1}{a_\infty^2}\left(\frac{a_\infty^2}{\bar{a}^2} - 1\right)\frac{\partial^2 p}{\partial t^2}$$

under the assumptions that the temperature of the flow field is nearly uniform and the fluctuations are locally isentropic ($\bar{a}$ is the local mean speed of sound; subscript ∞ refers to free-stream conditions). With this simplification we obtain

*The author wishes to express his thanks to Professor Kistler for permission to use these results before publication.

$$\frac{1}{a_\infty^2}\frac{\partial^2 p}{\partial t^2} - \nabla^2 p = \rho_\infty \frac{\partial^2 u_i u_j}{\partial x_i \partial x_j}$$

The problem is thus reduced to finding the pressure fluctuations in a uniform acoustic medium at rest produced by certain types of sources. The solution for the case of the boundary layer was first obtained by Curle (Ref. 4) in a quadrature form in which the integrands contain the Reynolds stresses and pressure forces taken at appropriately retarded times. (Other authors, Refs. 5 and 6, gave the results in somewhat different form.) It follows that the pressure intensity $\overline{p^2}$ will contain the space-time correlations of these quantities. In order to obtain numerical values for $\overline{p^2}$, it is, of course, necessary to know the correlations throughout the shear zone. For the case of a boundary layer of constant thickness, Phillips estimated the intensity of pressure radiation and found it to be negligibly small (Ref. 7). Indeed, measurements of $\overline{p^2}$ in supersonic flows indicate a very rapid decrease in intensity as the Mach number approaches subsonic values.

Thus it appears that for boundary layers the radiation problem becomes interesting mainly in supersonic flows. In this case, however, the mathematical difficulties become much larger. Lighthill's very useful acoustical analogy cannot be extended to higher Mach number flows. The physical problem, of course, becomes more complicated: the sound velocity within the layer will vary appreciably because of the large temperature gradients; the convection velocities of the sources can no longer be neglected and can be subsonic or supersonic with respect to the free stream.

b. Solution for $M \to \infty$: Phillips has studied this problem using Eq. (2) and has succeeded in obtaining a solution for the asymptotic case of $M \to \infty$ (Ref. 1). Although he considered a free shear layer, his theory can be easily adapted to the boundary layer. The main results may be described as follows: the wave-number frequency energy spectrum of the pressure fluctuations at a point outside the boundary layer corresponding to a given wave number k and frequency n is contributed entirely by a certain critical layer within the boundary layer that lies at a distance Y from the wall where $n + k_1 U(Y) = 0$ (k_1 is the x component of the vector k). In other words, one may consider at Y a frozen eddy pattern that is convected downstream with a velocity $U(Y)$ which is supersonic with respect to the free stream (Fig. 2). The pattern thus moves like a wavy wall in a supersonic stream and radiates energy in the form of Mach waves, the direction of the waves depending on the relative velocity between $U(Y)$ and the free stream. The resulting pressure spectrum has the form

$$\frac{\pi(\vec{k})}{\delta \overline{p_\infty^2}} \sim M_\infty^{3/2} \int_0^\infty \frac{\dfrac{d\dfrac{U}{U_\infty}}{d\dfrac{Y}{\delta}} k_1^2 \psi\left(\dfrac{Y}{\delta}\, \vec{k}, n\right)}{\dfrac{a^2}{a_\infty^2}\,[U_\infty - U]} \, d\,\frac{Y}{\delta}$$

where $n = -k_1\, U(Y)$, and $\psi(Y/\delta,\, \vec{k},\, n)$ is the wave-number frequency spectrum of the v' fluctuations in the boundary layer at Y.

Following Phillips, if one now makes the rough approximation that $\psi\,(Y/\delta,\, \vec{k},\, n) \sim \psi\,(\vec{k})\,\theta(\vec{k})$ is independent of Y where $\theta(\vec{k})$ is an integral time scale of the v' spectrum of the order of δ/U_∞, one obtains for the pressure intensity

$$\frac{\overline{p'^2}}{\overline{p}^2} \sim M_\infty^{3/2} \overline{\left(\frac{\partial \dfrac{v'}{U_\infty}}{\partial \dfrac{x}{\delta}}\right)^2} \int_0^\infty \frac{\dfrac{d\dfrac{U}{U_\infty}}{d\dfrac{Y}{\delta}}\, d\left(\dfrac{Y}{\delta}\right)}{\dfrac{a^2}{a_\infty^2}\left[1 - \dfrac{U}{U_\infty}\right]}$$

The following comments may be made about the result:

1. The pressure field outside of the boundary layer is uniform independent of the distance from the plate.

2. The variation of the radiated pressure intensity with Mach number cannot be expressed explicitly. The value of the integrand decreases rapidly with Mach number (roughly as M^2) so that the intensity is expected to vary much slower than $M^{3/2}$.

3. Within the framework of the theory, directionality of the radiation can also be predicted. It may be shown that as the Mach number of the free stream is increased, a larger portion of the total radiated energy will be concentrated in a given direction, the direction of propagation approaching the perpendicular to the boundary layer as $M \to \infty$.

DISCUSSION

In this case, as in any type of asymptotic solution, one would like to know whether the results of such a solution could be applied to finite values of Mach numbers. In this section we will examine the existing experimental information on sound radiation in the light of Phillips' theory.

In a recent work (Ref. 8) it was shown that in a supersonic wind tunnel the fluctuations in the free stream are mainly pressure or sound waves emanating from the turbulent boundary layers of the four tunnel walls. The sound field intensity was found to be very uniform a few wavelengths outside of the boundary layer. Figure 3 shows the normalized $\overline{v'^2}$ fluctuations near the edge of the boundary layer for several Mach numbers. It is seen that in the free stream the sound field intensity (where v' is proportional to p') is uniform indeed. Furthermore, the nonuniformity in intensity extends farther out of the boundary layer for the low Mach number flow. This is not surprising since, as will be seen later, the nondimensional wave length of the sound field $\lambda/\delta = -(\lambda_x/\delta)\cos\theta$ is larger at lower Mach numbers. (θ is the angle between the normal to the wave front and the flow direction.)

The directional characteristics of the field may also be investigated by measuring the space-time correlation of the pressure fluctuations. Figure 4 shows the result of such measurements for three Mach numbers. The two hot wires were behind each other at a distance Δx apart, indicated in the figure; they were also displaced in the plane perpendicular to the flow direction sufficiently that no mutual interference was observed. It is seen that there exists a particular time delay τ_m for which the correlation is a maximum; or expressing it another way: there exists a preferred velocity $U_c = \Delta x/\tau_m$ with which the fluctuation patterns are convected downstream. Since the measurements are made several wavelengths away from the layer, one can assume the sound waves to be plane. Then the above result implies that the wave fronts have a preferred direction; as a matter of fact, with increasing Mach number more and more of the sound energy is oriented in one particular direction (the maxima of the correlation curves become stronger).

A consistent result is obtained if the spectra of the pressure fluctuation obtained at various Mach numbers are compared (the Re/in. of the tunnel, or approximately $U_\infty\lambda/\nu$ was held constant). Figure 5 shows that by choosing for the convection velocity the values obtained by the correlation method, the spectra exhibit a similarity throughout the wave number range. This implies that the directional characteristics of the sound waves of all wavelengths are the same. The similarity also implies that for a given boundary layer, the wavelengths $\lambda = \lambda_x \cos\theta$ decrease with increasing Mach number.

Figure 6 shows the variation of the convected velocities with Mach number. The upper curve corresponds to those obtained by observing the pressure fluctuations at the wall. As pointed out earlier, these fluctuations are believed to be produced mainly by the vorticity field within the layer, by the large-scale energy-carrying eddies. It is seen that for low speeds the convection velocity is $0.8\ U_\infty$, indeed the same as found by Favre for the large-scale eddies from the space-time velocity correlation measurements (Ref. 9). The lower curve shows the convected velocities of the sound field in the free stream. If one identifies these with some average velocities of sources producing the sound, one may explain the Mach number variation of these convection speeds in terms of Phillips' picture. According to Phillips, within the boundary layer only an inner layer which flows supersonically with respect to the free stream is the effective sound producer. The sound sources are then convected downstream with some average velocity of this layer. Clearly, as the free stream Mach number increases, the relatively supersonic layer thickens, containing higher velocity sources.

From the above discussion of the theoretical and experimental results, one arrives at the following conclusion: Phillips' basic idea – namely, that the sound generation mechanism consists of a moving, spacially random, virtual wavy wall formed by an eddy pattern that is convected supersonically with respect to the free stream – is consistent with the main features of the sound field found experimentally. Such a virtual wall radiates a sound far-field that is homogeneous and has certain directional properties described earlier. However, it seems that the experimental Mach numbers are not high enough to yield the same functional behavior of the sound intensity with Mach number as predicted by the asymptotic theory. The subsonic region (as shown in Fig. 2) even for $M = 5$ extends over half of the boundary layer. This implies that the sound fluctuations produced by the virtual wall will be attenuated in the subsonic region adjacent to this wall as they are radiated out in the far-field, the attenuation being much higher for the large wave numbers. This is believed to be partly the reason that the sound spectrum in the far-field contains much less energy in the high-frequency region than the spectrum of the generating function v' (see dashed line in Fig. 5).

If one now adopts the over-simplified point of view that all the sound is produced in a layer near the wall, the average velocity of which is U_c (the averaging is taken spacially across the layer), one may study the equivalent problem of the sound field produced by a randomly wavy wall moving with a relative Mach number

$$M_r = \frac{U_\infty - U_c}{a_\infty} = \frac{U_r}{a_\infty}$$

From the well-known potential solution, one obtains

$$\frac{\overline{p'^2}}{\gamma^2 \bar{p}^2} = \frac{\overline{v'^2}}{U_r^2} \frac{M_r^4}{M_r^2 - 1} \int \psi(\xi, \eta, \tau)\, d\xi\, d\eta\, d\tau$$

where $\psi(\xi, \eta, \tau)$ is a space-time correlation function. ($\xi = x_1 - x_2$, $\eta = z_1 - z_2$ in the plane of the layer; τ is the time delay $t_1 - t_2$.) Since $M_r = (1 - U_c/U_\infty) M_\infty = 1/\cos\theta$, it is easy to show that the above expression becomes

$$\frac{\overline{p'^2}}{\gamma^2 \bar{p}^2} = \frac{\overline{v'^2}}{U_\infty^2} \frac{M_\infty^2}{\sin^2\theta} \int \psi(\xi, \eta, \tau)\, d\xi\, d\eta\, d\tau$$

In order to estimate the space-time correlation, one has to make some assumptions on the statistical behavior of the wall wavyness. It is reasonable that the fluctuations are correlated only over a certain area, say $L_x L_z$, where L_x and L_z are integral lengths that scale with the boundary layer thickness. Furthermore, the correlation must depend on a time scale corresponding to the average life time of a "bump"; we assume for lack of better information that it scales with δ/U_c. Thus we may write for the space-time correlation

$$\psi(\xi, \eta, \tau) = \frac{L_x L_z}{\delta^2} \frac{\tau U_c}{\delta} \delta_D(\xi)\, \delta_D(\eta)$$

where $\delta_D(\xi)$, $\delta_D(\eta)$ are Dirac delta functions. Finally, since $\overline{v'^2}$ scales with the friction velocity $U_\tau^2 = C_f/2\ U_\infty^2$, we may write

$$\frac{\overline{p'^2}}{\gamma^2 \bar{p}^2} = \frac{\overline{v'^2}}{U_\infty^2} \frac{C_f}{2} \frac{L_x L_z}{\delta^2} \frac{\tau U_\infty}{\delta} \frac{U_c}{U_\infty} \frac{M_\infty^2}{\sin^2\theta}$$

This relation indicates that the pressure intensity varies with the square of the Mach number. In Fig. 7 a comparison is made between the measured rms pressure fluctuations $\tilde{p}$ in the far-field and this relation in which we assumed

$$\frac{\overline{v'^2}}{U_\tau^2} \sim 1, \quad \frac{L_x L_z}{\delta^2} \sim 10^{-2}, \quad \frac{\tau U_\infty}{\delta} \sim 1$$

It is seen that the variation of the pressure fluctuations is much stronger than indicated by the above relation. The explanation may be due to the quantity $\tau U_\infty/\delta$ which, according to Ref. 9, varies rapidly across the boundary layer. In the Mach number range 1.6 to 5, the thickness of the radiating layer increases rapidly, and the increase in $\tilde{p}$ might be partially due to changes in $\tau U_\infty/\delta$.

The only conclusion we can draw from the above discussion is that in the interesting region of low supersonic Mach numbers there is no theory yet that describes properly the very fast increase of radiated energy of a boundary layer as the flow Mach number is increased. Additional measurements, especially that of space-time correlations of the v' fluctuation near the wall, are necessary to further clarify the problem.

With reference to the question of turbulence damping mentioned in the Introduction, it is possible, on the basis of the measurements described, to make a rough estimate of the energy loss due to radiation. The sound energy density in a moving medium may be written (Ref. 10)

$$E = \frac{\overline{p'^2}}{\rho a^2} \frac{V_p}{a}$$

where the phase velocity

$$V_p = a + \frac{\vec{U}_\infty \cdot \vec{a}}{a} = a + U_\infty \cos\theta = a(1 + M_\infty \cos\theta)$$

or

$$E = \frac{\overline{p'^2}}{\bar{p}^2} \frac{\rho a^2}{\gamma^2} (1 + M_\infty \cos\theta)$$

The sound energy flux per unit area from the boundary layer

$$\vec{N} = E(\vec{a} + \vec{U}_\infty)$$

$$|N| = \frac{\overline{p'^2}}{p^2} \frac{\rho a^3}{\gamma^2} (1 + M_\infty \cos\theta) \sqrt{M^2 + 1 + 2M_\infty \cos\theta)}$$

or, in a nondimensional form,

$$\frac{N}{\rho U_\infty^3} = \frac{\overline{p'^2}}{\gamma^2 \bar{p}^2} \, \frac{(1 + M_\infty \cos\theta)\sqrt{M^2 + 1 + 2M_\infty \cos\theta}}{M_\infty^3}$$

At $M = 5$ the experiments give (remembering that the hot wire senses the radiation coming from two walls of the wind tunnel):

$$\frac{\overline{p'^2}}{\bar{p}^2} = 1.2 \times 10^{-4} \qquad \cos\theta = -0.45$$

and therefore

$$\frac{N}{\rho U_\infty^3} = 2.8 \times 10^{-6}$$

This energy flux may be compared to the total work done by the wall shearing stress $W = \tau_W U_\infty$. In a nondimensional form

$$\frac{W}{\rho U_\infty^3} = \frac{\tau_W}{\rho U_\infty^2} = \frac{C_f}{2}$$

For the particular example, this value is approximately 3.3×10^{-4}. It is seen that at $M = 5$, the energy lost due to radiation is merely of the order of one percent of the total work done by the wall shearing stress. Thus it is quite clear that in order to resolve the question of complete turbulence damping, the radiation intensity variation with higher Mach numbers would have to be clarified.

REFERENCES

1. Phillips, O. M., "On the Generation of Sound by Supersonic Turbulent Shear Layers," *J. of Fluid Mech.*, Vol. 9, pp. 1-28, 1960.

2. Eckhaus, Wiktor, *Some Remarks on the Theory of Noise Generation by Turbulent Shear Flows*, MIT ASRL TR 92-2, Dec., 1960.

3. Lighthill, M. J., "On the Sound Generated Aerodynamically," *Proc. Roy. Soc.*, A, Vol. 211, pp. 564-587, 1952.

4. Curle, N., "The Influence of Solid Boundaries upon Aerodynamic Sound," *Proc. Roy. Soc.*, A, Vol. 216, p. 505, 1955.

5. Lyamshev, L. M., "Analysis of Acoustic Radiation from a Turbulent Aerodynamic Flow," *Soviet Physics-Acoustics*, Vol. 6, No. 4, Apr.-June, 1961.

6. Ribner, H. S., *A Theory of the Sound from Jets and Other Flows in Terms of Simple Sources*, UTIA Rep. No. 67, July, 1960.

7. Phillips, O. M., "On the Aerodynamic Surface Sound from a Plane Turbulent Boundary Layer," *Proc. Roy. Soc.*, A, Vol. 234, pp. 327-335, 1956.

8. Laufer, J., "Aerodynamic Noise in Supersonic Wind Tunnels," *Journal of the Aerospace Sciences*, Vol. 28, No. 9, pp. 685-692, Sept., 1961.

9. Favre, A. J., Gaviglio, J. J., and Dumas, R. J., "Further Space-Time Correlations of Velocity in a Turbulent Boundary Layer," *J. Fluid Mech.*, Vol. 3, Part 4, pp. 344-356, Jan., 1958.

10. Blokhintsev, D. I., *Acoustics of a Nonhomogeneous Moving Medium*, NACA TM 1399, Feb., 1956.

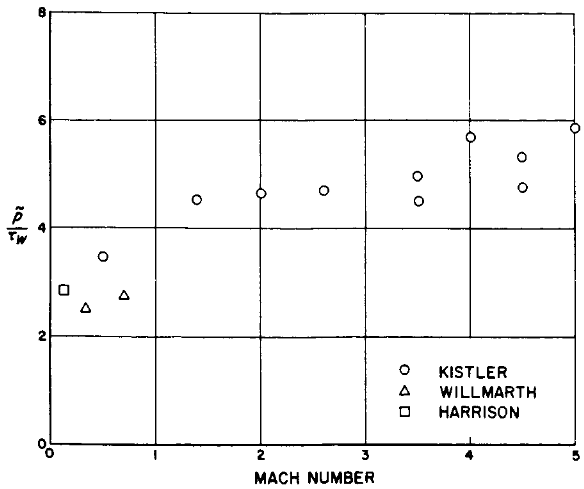

Fig. 1. Wall pressure fluctuation level

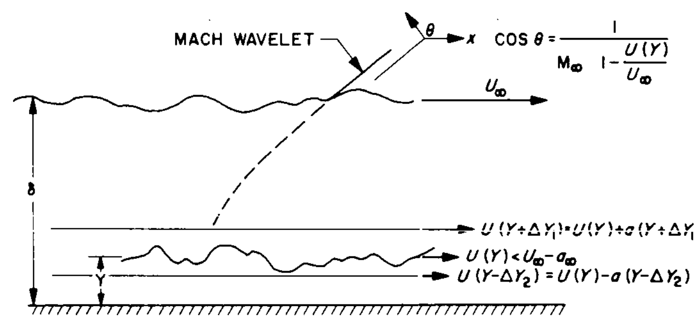

Fig. 2. Schematic diagram of the radiation mechanism

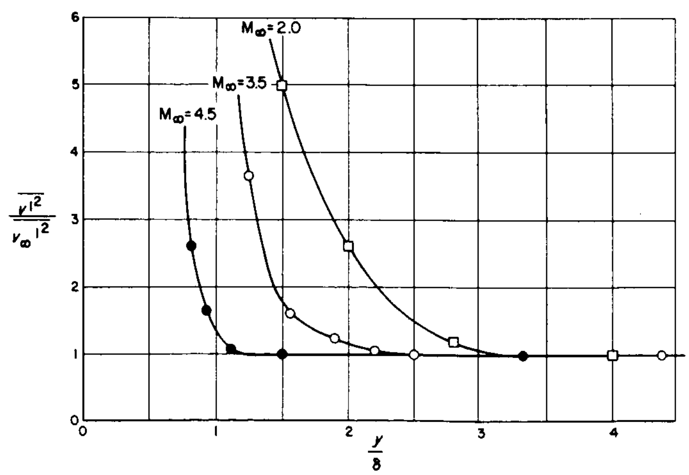

Fig. 3. Fluctuation near the edge of the boundary layer

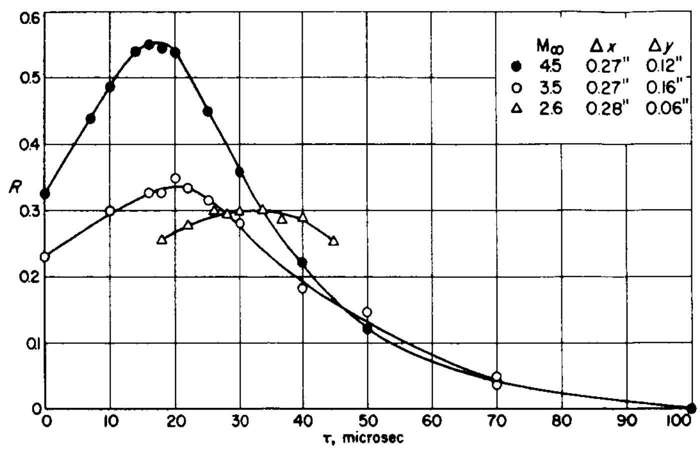

Fig. 4. Space-time correlation in far-field

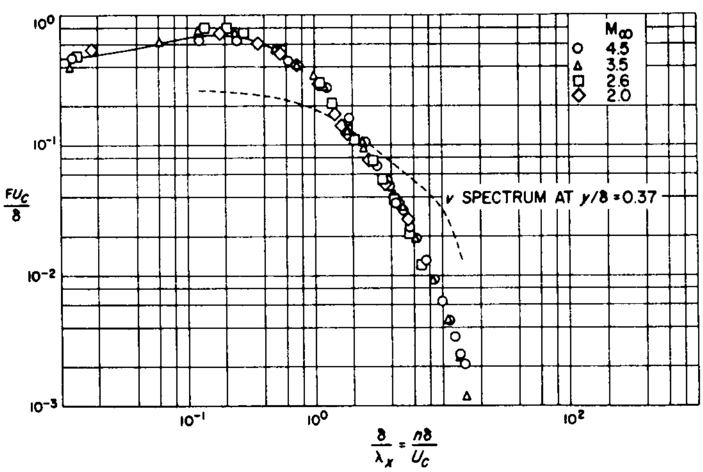

Fig. 5. Energy spectrum of pressure fluctuations

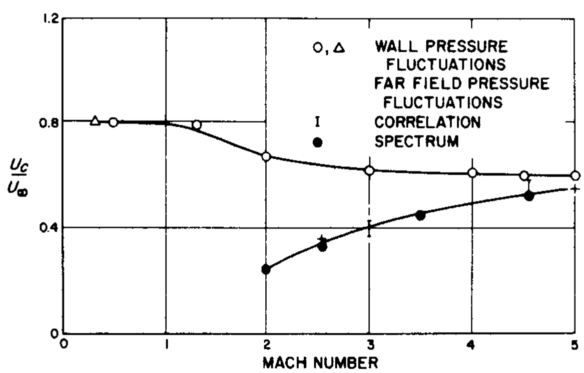

Fig. 6. Convection velocity ratio for pressure fluctuations

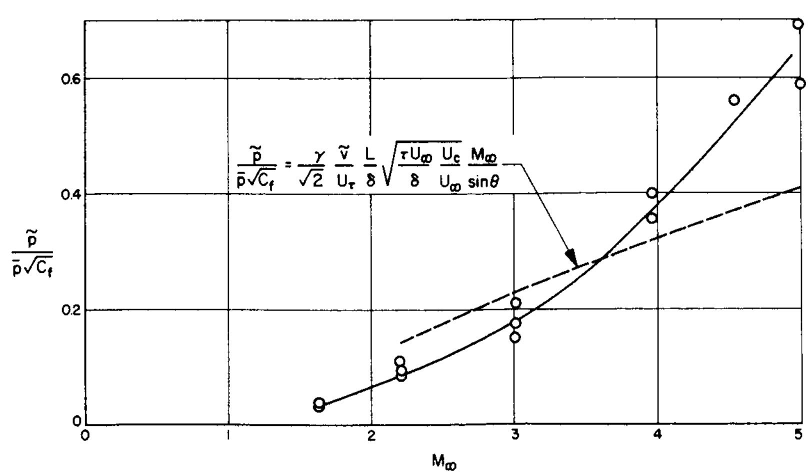

Fig. 7. Far-field pressure fluctuations