NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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# Octave and One-Third Octave Acoustic Noise Spectrum Analysis


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CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA, CALIFORNIA

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SUBJECT: Errata

Gentlemen:

Please note the following corrections to JPL Technical Report 32-1052, "Octave and One-Third Octave Acoustic Noise Spectrum Analysis," January 15, 1967.

In Table A-1 on page 10, in third column from left and fifth data block down, 103 should read 113 as follows: 88.7-113 and 113-141 (instead of 88.7-103 and 103-141).

In Table C-1 on page 13, make two changes: (1) in the C.G.S. column, fifth data block down, $2.00(10)^{-4}$ dynes/$cm^2$ (instead of cm); and (2), in the in.-lb-sec column, fourth data block down, $1.5471(10)^{-3}$ lb-sec/in.$^3$ (mass/in.$^2$-sec), instead of lb-sec/$ft^3$.

Very truly yours,

~~John~~ Kempton, Manager
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## Abstract

This Report is a compilation of derivations for analyzing acoustic noise spectra. These derivations consist of mathematical techniques for:

1. Combining the decibel levels of either octaves or constant bandwidths into an overall spectrum level;
2. Determining the octave levels in a second octave system when the levels in the first octave are known;
3. Determining one-third octave levels when the octave levels and the decibel-per-octave slope are known.

The Appendixes also contain the following useful tables:

1. Table of the New and Old A.S.A. (American Standards Association) Recommended Octave Designations
2. Table of Decibel Equivalents for Power, Voltage, and Pressure
3. Conversion Table for Some Useful Acoustic Constants

These mathematical techniques and tables provide useful information for the engineer or scientist working in the field of acoustics.

# Octave and One-Third Octave Acoustic Noise Spectrum Analysis

## I. Technique for Combining the Decibel Levels of Either Octaves or Constant Bandwidths into an Overall Sound Pressure Spectrum Level

The following derivation assumes that the energy in all defined bandwidths (octave, constant bandwidth, etc.) results from an effective "coherent" source, and therefore the bandwidth energies add directly to give the total energy. This derivation will be written for the addition of energy in two given bandwidths.[1] By repeating this technique, the total energy (overall spectrum level) of all or part of a specified spectrum may be determined.

Let:

($A$) be defined as the decibel reading of bandwidth 1, and

($B$) be defined as the decibel reading of bandwidth 2 (Note: we assume $B \geq A$)

Then:

$$A = 10 \log_{10} (W_1/W_0) \tag{1}$$

$$B = 10 \log_{10} (W_2/W_0) \tag{2}$$

where:

$W_1$ = energy contained in bandwidth 1

$W_2$ = energy contained in bandwidth 2

$W_0$ = reference energy level

Therefore:

$$W_1 = W_0 10^{A/10} \tag{3}$$

and:

$$W_2 = W_0 10^{B/10} \tag{4}$$

Let ($C$) be defined as the decibel reading of the total energy of bandwidth 1 and 2.

Then:

$$C = 10 \log_{10} \left[ \left( \frac{W_0}{W_0} \right) \{10^{A/10} + 10^{B/10}\} \right] \tag{5}$$

[1] Note: These bandwidths can have identical or different frequency bounds, as long as they meet the requirements stated above.

Let:

$$X \equiv B - A \quad \text{(The difference in the two bandwidth decibel levels)} \tag{6}$$

Then:

$$C = 10 \log_{10} [10^{B/10} \{1 + 10^{-X/10}\}] \tag{7a}$$

Therefore:

$$C = B + 10 \log_{10} \{1 + 10^{-X/10}\} \tag{7b}$$

Let:

$$Y \equiv C - B \quad \text{(The decibel increment that must be added to the higher bandwidth level } (B) \text{ to obtain the value of } (C)\text{)} \tag{8}$$

Therefore:

$$Y\,(\text{db}) = 10 \log_{10} \{1 + 10^{-X/10}\} \tag{9}$$

Equation (9) is the equation which expresses the relationship between the original bandwidth decibel levels and the decibel level of their combination.

Figure 1 is a graph of Eq. (9). Tabular values for Eq. (9) are listed in Table 1 which gives the values of $Y$ for a range of $X$ between 0 db and 15 db.

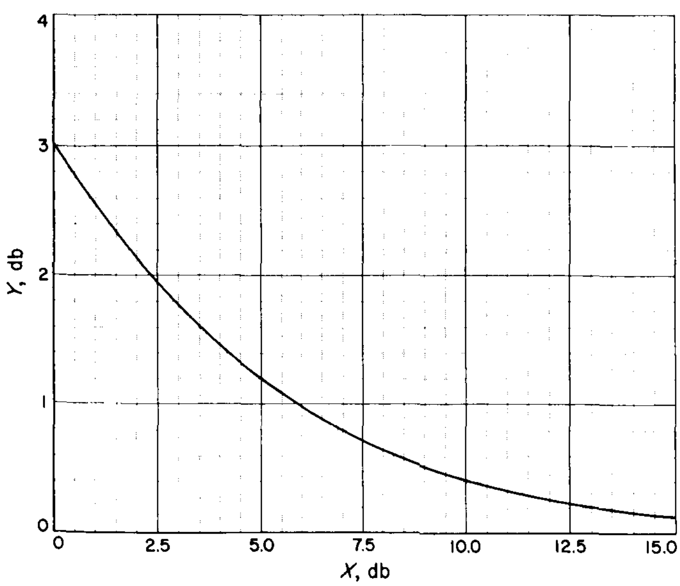


**Fig. 1. Plot of decibel increment (Y) to be added to the higher bandwidth level vs difference (X) in the bandwidth levels (Ref. Eq. 9)**

**Table 1. Tabular values for Eq. (9)**

| X (Difference between the levels being added) (db) | Y (Increment between two levels being added) (db) |
|---|---|
| 0.00 | 3.01 |
| 0.50 | 2.77 |
| 1.00 | 2.54 |
| 1.50 | 2.32 |
| 2.00 | 2.12 |
| 2.50 | 1.94 |
| 3.00 | 1.76 |
| 3.50 | 1.60 |
| 4.00 | 1.46 |
| 4.50 | 1.32 |
| 5.00 | 1.19 |
| 5.50 | 1.08 |
| 6.00 | 0.97 |
| 6.50 | 0.88 |
| 7.00 | 0.79 |
| 7.50 | 0.71 |
| 8.00 | 0.64 |
| 8.50 | 0.57 |
| 9.00 | 0.51 |
| 9.50 | 0.46 |
| 10.00 | 0.41 |
| 10.50 | 0.37 |
| 11.00 | 0.33 |
| 11.50 | 0.30 |
| 12.00 | 0.27 |
| 12.50 | 0.24 |
| 13.00 | 0.21 |
| 13.50 | 0.19 |
| 14.00 | 0.17 |
| 14.50 | 0.15 |
| 15.00 | 0.14 |

## II. Technique for Determining the Octave Levels in a Second Octave Bandwidth System When the Levels in the First Octave System Are Known

With both the old and new American Standards Association (ASA) recommended octave bandwidths in wide use today, it is often required to convert from one octave set to the other. The following derivation allows one to convert from any given octave set to a second octave set. This derivation also assumes that the energy in the various bandwidths adds, as in Section I of this Report. To simplify the analysis, it is also assumed that the power spectrum in each given octave bandwidth has uniform distribution. See Fig. 2 for a definition of frequency and power distribution.

This derivation will be written for two consecutive octaves of the first system, with different decibel levels

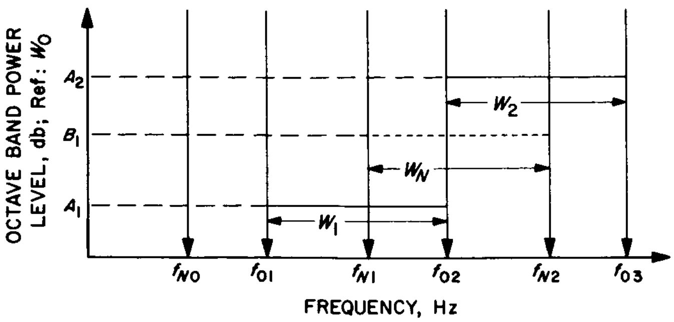


**Fig. 2. Frequency and power distribution**

($A_1$ and $A_2$) and the resulting decibel level ($B_1$) for the octave band of the second system which is contained partly in each of the octave bands of the first system.

The following notation is used:

$A_1$ Sound Pressure Level (S.P.L.) in db of lower octave band of 1st system

$A_2$ Sound Pressure Level (S.P.L.) in db of upper octave band of 1st system

$B_1$ Sound Pressure Level (S.P.L.) in db of octave band of 2nd system, which is contained in the two octave bands of the 1st system

$f_{01}$ Lowest frequency bound (in Hz) of lower octave band of 1st system

$f_{02}$ Upper frequency bound (in Hz) of lower octave band of 1st system; and lower frequency bound (in Hz) of upper octave band of 1st system

$f_{03}$ Upper frequency bound (in Hz) of upper octave band of 1st system

$W_0$ Reference energy level

$f_{N1}$ Lower frequency bound (in Hz) of octave band of the 2nd system

$f_{N2}$ Upper frequency bound (in Hz) of octave band of the 2nd system

Then:

$$A_1 = 10 \log_{10} (W_1/W_0) \tag{10}$$

$$A_2 = 10 \log_{10} (W_2/W_0) \tag{11}$$

where:

$W_1$ = energy contained in the lower bandwidth of the 1st system

$W_2$ = energy contained in the upper bandwidth of the 1st system

$$W_1 = W_0 \, 10^{A_1/10} \tag{12}$$

$$W_2 = W_0 \, 10^{A_2/10} \tag{13}$$

$$B_1 = 10 \log_{10} (W_N/W_0) \tag{14}$$

where:

$W_N$ = the total energy in the bandwidth of the 2nd system with contributions from the two octaves of the 1st system

Since we have assumed flat spectra within each bandwidth:

$$W_N = \left(\frac{f_{02} - f_{N1}}{f_{02} - f_{01}}\right) W_1 + \left(\frac{f_{N2} - f_{02}}{f_{03} - f_{02}}\right) W_2 = \alpha W_1 + \gamma W_2 \tag{15}$$

where:

$$\alpha = \left(\frac{2 f_{01} - f_{N1}}{f_{01}}\right); \text{ since: } f_{02} = 2 f_{01}$$

and:

$$\gamma = \left(\frac{f_{N1} - f_{01}}{f_{01}}\right); \text{ since: } f_{03} = 2 f_{02} \text{ and: } f_{N2} = 2 f_{N1}$$

Therefore:

$$W_N = W_0 \, [\alpha \, 10^{A_1/10} + \gamma \, 10^{A_2/10}] \tag{16a}$$

Let:

$$X = A_2 - A_1 \tag{16b}$$

be the decibel increment, positive, negative or zero, between the upper and lower octave bandwidths.

$$W_N = W_0 \, 10^{A_1/10} \, [\alpha + \gamma \, 10^{X/10}] \tag{17}$$

and:

$$B_1 = 10 \log_{10} \left\{ 10^{A_1/10} \left(\frac{W_0}{W_0}\right) [\alpha + \gamma \, 10^{X/10}] \right\} \tag{18}$$

$$= A_1 + 10 \log_{10} [\alpha + \gamma \, 10^{X/10}]$$

Since:

$$\alpha + \gamma = 1 \qquad (19)$$

therefore:

$$B_1 = A_1 + 10 \log_{10} [\alpha + (1 - \alpha) 10^{X/10}] \qquad (20)$$

In general, for any two consecutive octaves, we may write:

$$B_i = A_i + 10 \log_{10} [\alpha_i + (1 - \alpha_i) 10^{X/10}] \qquad (21)$$

with

$$X \equiv A_{i+1} - A_i$$

(Note: As in Eq. (16b), $X$ may be positive, negative, or zero.) Equation 21 may be put into the same form as in Fig. 1 by the following definition:

$$Y_i = B_i - A_i \qquad (22a)$$

$$Y_i = 10 \log_{10} [\alpha_i + (1 - \alpha_i) 10^{X/10}] \qquad (22b)$$

where $(Y_i)$ is the decibel increment that must be added to the $(A_i)^{th}$ or lower* octave sound intensity level to obtain the level in the included octave of the second octave system.

Table 2 lists the tabulated values of $\alpha_i$ to be used with Eq. (21) or (22b) for the two octave band designations of the American Standards Association (as listed in Table A-1 of Appendix A).

Let us consider the following examples for *Case I* and *Case II:*

*Case I.* Consider where $f_{01}$ refers to the (150 Hz-300 Hz) octave system, and $f_{N1}$ refers to the (177 Hz-354 Hz) octave system.
Therefore, for this special case $\alpha_i = 0.820$; using Eq. (22a) gives

$$Y = 10 \log_{10} [(0.820) + (0.180) 10^{X/10}] \qquad (23)$$

*Case II.* Consider where $f_{01}$ refers to the (177 Hz-354 Hz) octave system, and $f_{N1}$ refers to the (300 Hz-600 Hz) octave system.
Therefore, for this special case $\alpha_i = 0.305$; using Eq. (22a) gives

$$Y = 10 \log_{10} [(0.305) + (0.695) 10^{X/10}] \qquad (24)$$

Equations (23) and (24) are the equations which express the relationship between the octave bandwidth decibel levels of the two octave systems of *Cases I* and *II.*

Table 3 gives values of $(Y)$ for a range of $(X)$ between $-15$ db and $+15$ db for *Cases I* and *II*, with graphical displays in Figs. 3 and 4, respectively.

*Note: The term "lower" has to do only with the octaves' position in the frequency spectrum, and not with relative intensity levels.

**Table 2. Tabular values of $\alpha_i$ to be used with Eq. (21) or (22b)**

| Old to New | | New to Old | |
|---|---|---|---|
| $f_{01}$ (Hz) | $\alpha_i$ | $f_{01}$ (Hz) | $\alpha_i$ |
| 4.7 | 0.787 | 5.7 | 0.322 |
| 9.4 | 0.796 | 11.3 | 0.333 |
| 18.7 | 0.803 | 22.4 | 0.317 |
| 37.5 | 0.813 | 44.5 | 0.310 |
| 75 | 0.817 | 88.7 | 0.306 |
| 150 | 0.820 | 177 | 0.305 |
| 300 | 0.820 | 354 | 0.303 |
| 600 | 0.822 | 707 | 0.303 |
| 1200 | 0.822 | 1414 | 0.304 |
| 2400 | 0.821 | 2830 | 0.304 |
| 4800 | 0.821 | 5600 | 0.298 |
| 9600 | 0.823 | 11300 | 0.288 |
| 19200 | 0.833 | | |

Note: List of $\alpha_i$ for converting between old and new A.S.A. designated octave frequency bands (Ref. Eq. 15, using Table A-1).

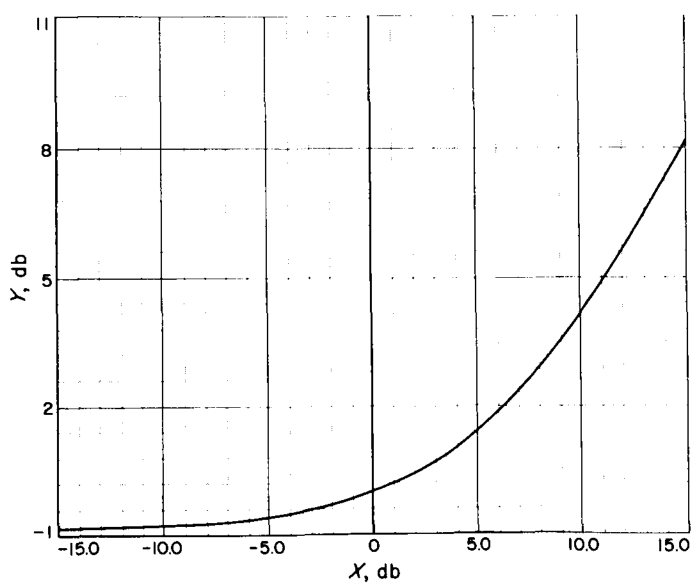


**Fig. 3. Plot of decibel increment (Y) to be added to $A_i^{th}$ sound level vs decibel increment (X) between upper and lower octave bandwidths for $\alpha_i = 0.820$ (Ref. Eq. 23)**

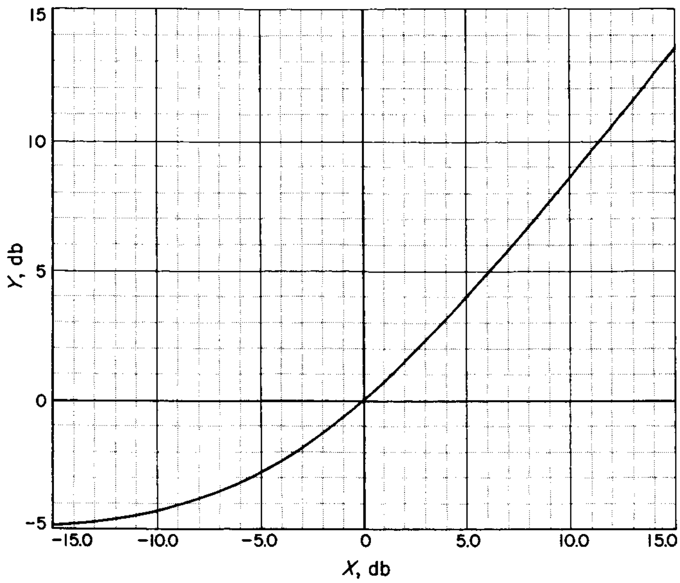


**Fig. 4. Plot of decibel increment (Y) to be added to $A_i$th sound level vs decibel increment (X) between upper and lower octave bandwidths for $\alpha_i = 0.305$ (Ref. Eq. 24)**

**Table 3. Tabular values of Y vs X for Eq. (23) and (24)**

| X | Y | |
|---|---|---|
| (The level difference (db) between the higher and octave bands of the 1st system) | (The decibel increment that must be added to the $A_i$th (or lower) octave level to obtain the level (db) in the included octave of the 2nd octave system) | |
| $= A_{i+1} - A_i$ | *Case I* (Refers to Eq. 23 where $\alpha_i = 0.820$) | *Case II* (Refers to Eq. 24 where $\alpha_i = 0.305$) |
| −15.00 | −0.83 | −4.85 |
| −14.50 | −0.83 | −4.82 |
| −14.00 | −0.82 | −4.78 |
| −13.50 | −0.82 | −4.74 |
| −13.00 | −0.81 | −4.69 |
| −12.50 | −0.81 | −4.63 |
| −12.00 | −0.80 | −4.57 |
| −11.50 | −0.79 | −4.51 |
| −11.00 | −0.79 | −4.43 |
| −10.50 | −0.78 | −4.35 |
| −10.00 | −0.77 | −4.27 |
| − 9.50 | −0.76 | −4.17 |
| − 9.00 | −0.74 | −4.06 |
| − 8.50 | −0.73 | −3.95 |
| − 8.00 | −0.71 | −3.82 |
| − 7.50 | −0.70 | −3.68 |
| − 7.00 | −0.68 | −3.53 |
| − 6.50 | −0.65 | −3.37 |
| − 6.00 | −0.63 | −3.19 |
| − 5.50 | −0.60 | −3.00 |
| − 5.00 | −0.57 | −2.80 |

**Table 3 (contd)**

| X | Y | |
|---|---|---|
| (The level difference (db) between the higher and octave bands of the 1st system) | (The decibel increment that must be added to the $A_i$th (or lower) octave level to obtain the level (db) in the included octave of the 2nd octave system) | |
| $= A_{i+1} - A_i$ | *Case I* (Refers to Eq. 23 where $\alpha_i = 0.820$) | *Case II* (Refers to Eq. 24 where $\alpha_i = 0.305$) |
| − 4.50 | −0.54 | −2.58 |
| − 4.00 | −0.50 | −2.35 |
| − 3.50 | −0.46 | −2.11 |
| − 3.00 | −0.41 | −1.85 |
| − 2.50 | −0.36 | −1.58 |
| − 2.00 | −0.30 | −1.29 |
| − 1.50 | −0.23 | −0.99 |
| − 1.00 | −0.16 | −0.67 |
| − 0.50 | −0.09 | −0.34 |
| 0.00 | 0.00 | 0.00 |
| 0.50 | 0.09 | 0.35 |
| 1.00 | 0.20 | 0.72 |
| 1.50 | 0.31 | 1.09 |
| 2.00 | 0.43 | 1.48 |
| 2.50 | 0.57 | 1.88 |
| 3.00 | 0.72 | 2.28 |
| 3.50 | 0.87 | 2.70 |
| 4.00 | 1.05 | 3.12 |
| 4.50 | 1.23 | 3.55 |
| 5.00 | 1.43 | 3.98 |
| 5.50 | 1.64 | 4.43 |
| 6.00 | 1.87 | 4.87 |
| 6.50 | 2.11 | 5.33 |
| 7.00 | 2.36 | 5.78 |
| 7.50 | 2.63 | 6.25 |
| 8.00 | 2.91 | 6.71 |
| 8.50 | 3.21 | 7.18 |
| 9.00 | 3.52 | 7.65 |
| 9.50 | 3.85 | 8.13 |
| 10.00 | 4.18 | 8.61 |
| 10.50 | 4.53 | 9.09 |
| 11.00 | 4.89 | 9.57 |
| 11.50 | 5.27 | 10.05 |
| 12.00 | 5.65 | 10.54 |
| 12.50 | 6.04 | 11.03 |
| 13.00 | 6.45 | 11.51 |
| 13.50 | 6.86 | 12.00 |
| 14.00 | 7.28 | 12.50 |
| 14.50 | 7.70 | 12.99 |
| 15.00 | 8.14 | 13.48 |

Note: When converting between the old and new A.S.A. octave bands as listed in Table A-1 of Appendix A, the values in Table 3 will have some error, since the values of $\alpha_i$ vary slightly as shown in Table 2, and since Eq. (19) is precisely correct only for perfect octave systems.

## III. Technique for Determining One-Third Octave Levels When the Octave Level and the Decibel-Per-Octave Slope Are Known

The method of determining one-third octave levels uses the following information:

1. The octave level : $A_T$ (db)
2. The decibel-per-octave slope: $X$ (db/octave)

This octave frequency spectrum is described in Fig. 5.

The following derivation assumes that the energies in the various one-third octaves add, as in Sections I and II of this Report.

Let:

$W_T$ = total energy in the three ⅓ octave bands

$W_1$ = the energy in the first ⅓ octave band

$W_2$ = the energy in the second ⅓ octave band

$W_3$ = the energy in the third ⅓ octave band

$A_1$ = the power level, in db, of the first ⅓ octave band

$A_2$ = the power level, in db, of the second ⅓ octave band

$A_3$ = the power level, in db, of the third ⅓ octave band

$A_T$ = the power level, in db, of the octave band which is made up of the three ⅓ octave bands

$W_0$ = the reference energy level

Then:

$$A_T = 10 \log_{10} (W_T/W_0) \qquad (25)$$

$$W_1 + W_2 + W_3 = W_T \qquad (26)$$

$$A_3 - A_2 = A_2 - A_1 = X/3 \qquad (27)$$

$$X/3 = 10 \log_{10} (W_3/W_2) = 10 \log_{10} (W_2/W_1) \qquad (28)$$

Therefore:

$$W_3 = W_2 10^{X/30} \qquad (29a)$$

and

$$W_2 = W_1 10^{X/30} \qquad (29b)$$

Therefore:

$$W_3 = W_1 10^{X/15} \qquad (30)$$

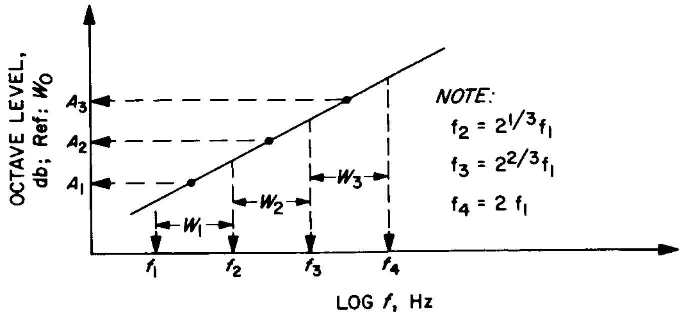


**Fig. 5. Octave frequency spectrum**

and

$$W_T = W_1 (1 + 10^{X/15} + 10^{X/30}) \qquad (31)$$

Let:

$Y_i \equiv A_T - A_i; \quad i = 1, 2, 3$ (The difference, in db levels, between the octave level $A_T$ and a contained one-third octave level $A_i$)

Therefore:

$$Y_1 = 10 \log_{10} (W_T/W_1) = 10 \log_{10} (1 + 10^{X/15} + 10^{X/30}) \qquad (32)$$

and

$$A_1 = A_T - 10 \log_{10} (1 + 10^{X/15} + 10^{X/30}) \qquad (33)$$

which gives the decibel level of the first ⅓ octave band

$$A_2 = A_1 + X/3 \qquad (34a)$$

or

$$A_2 = A_T + X/3 - 10 \log_{10} (1 + 10^{X/15} + 10^{X/30}) \qquad (34b)$$

which gives the decibel level for the second ⅓ octave band, and

$$A_3 = A_2 + X/3 = A_1 + 2X/3 \qquad (35a)$$

or

$$A_3 = A_T + \frac{2X}{3} - 10 \log_{10} (1 + 10^{X/15} + 10^{X/30}) \qquad (35b)$$

which gives the decibel level for the third ⅓ octave band.

We may rewrite Eq. (33), (34b) and (35b):

$$Y_1 = 10 \log_{10} [1 + 10^{X/15} + 10^{X/30}] \quad (36)$$

$$Y_2 = 10 \log_{10} [1 + 10^{X/15} + 10^{X/30}] - X/3 \quad (37)$$

and

$$Y_3 = 10 \log_{10} [1 + 10^{X/15} + 10^{X/30}] - 2X/3 \quad (38)$$

Equations (36), (37) and (38) express the relationship between the overall octave level and the three ⅓ octave levels of which it is comprised.

Tabular values of Eq. (36), (37) and (38) are listed in Table 4 which gives the values of $(Y_i)$ $i = 1, 2, 3$ for a range of $(X)$ between $-30$ db and $+30$ db, with graphical displays for Eq. (36), (37) and (38) in Fig. 6, 7, and 8, respectively.

**Table 4. Tabular values of Eq. (36), (37), and (38)**

| X (Spectrum slope within the octave) (db/octave) | $Y_1$ (Increment that must be subtracted from the octave level to obtain the lowest frequency ⅓ octave level) (db) | $Y_2$ (Increment that must be subtracted from the octave level to obtain the middle frequency ⅓ octave level (db) | $Y_3$ (Increment that must be subtracted from the octave level to obtain the highest frequency ⅓ octave level) (db) |
|---|---|---|---|
| −30 | 0.4532 | 10.4532 | 20.4532 |
| −29 | 0.4908 | 10.1574 | 19.8241 |
| −28 | 0.5315 | 9.8648 | 19.1982 |
| −27 | 0.5757 | 9.5757 | 18.5757 |
| −26 | 0.6236 | 9.2903 | 17.9569 |
| −25 | 0.6756 | 9.0090 | 17.3423 |
| −24 | 0.7321 | 8.7321 | 16.7321 |
| −23 | 0.7933 | 8.4600 | 16.1267 |
| −22 | 0.8598 | 8.1931 | 15.5265 |
| −21 | 0.9319 | 7.9319 | 14.9319 |
| −20 | 1.0101 | 7.6768 | 14.3434 |
| −19 | 1.0949 | 7.4283 | 13.7616 |
| −18 | 1.1869 | 7.1869 | 13.1869 |
| −17 | 1.2865 | 6.9532 | 12.6199 |
| −16 | 1.3945 | 6.7278 | 12.0612 |
| −15 | 1.5113 | 6.5113 | 11.5113 |
| −14 | 1.6377 | 6.3044 | 10.9710 |
| −13 | 1.7743 | 6.1076 | 10.4410 |
| −12 | 1.9218 | 5.9218 | 9.9218 |
| −11 | 2.0808 | 5.7475 | 9.4141 |
| −10 | 2.2521 | 5.5854 | 8.9187 |
| − 9 | 2.4363 | 5.4363 | 8.4363 |
| − 8 | 2.6341 | 5.3007 | 7.9674 |
| − 7 | 2.8461 | 5.1794 | 7.5128 |
| − 6 | 3.0730 | 5.0730 | 7.0730 |
| − 5 | 3.3152 | 4.9819 | 6.6485 |
| − 4 | 3.5733 | 4.9066 | 6.2399 |
| − 3 | 3.8476 | 4.8476 | 5.8476 |
| − 2 | 4.1386 | 4.8053 | 5.4719 |
| − 1 | 4.4464 | 4.7797 | 5.1131 |
| 0 | 4.7712 | 4.7712 | 4.7712 |
| 1 | 5.1131 | 4.7797 | 4.4464 |
| 2 | 5.4719 | 4.8053 | 4.1386 |
| 3 | 5.8476 | 4.8476 | 3.8476 |
| 4 | 6.2399 | 4.9066 | 3.5733 |
| 5 | 6.6485 | 4.9819 | 3.3152 |
| 6 | 7.0730 | 5.0730 | 3.0730 |
| 7 | 7.5128 | 5.1794 | 2.8461 |
| 8 | 7.9674 | 5.3007 | 2.6341 |
| 9 | 8.4363 | 5.4363 | 2.4363 |

## Table 4 (contd)

| X | $Y_1$ | $Y_2$ | $Y_3$ |
|---|---|---|---|
| (Spectrum slope within the octave) (db/octave) | (Increment that must be subtracted from the octave level to obtain the lowest frequency ⅓ octave level) (db) | (Increment that must be subtracted from the octave level to obtain the middle frequency ⅓ octave level) (db) | (Increment that must be subtracted from the octave level to obtain the highest frequency ⅓ octave level) (db) |
| 10 | 8.9187 | 5.5854 | 2.2521 |
| 11 | 9.4141 | 5.7475 | 2.0808 |
| 12 | 9.9218 | 5.9218 | 1.9218 |
| 13 | 10.4410 | 6.1076 | 1.7743 |
| 14 | 10.9710 | 6.3044 | 1.6377 |
| 15 | 11.5113 | 6.5113 | 1.5113 |
| 16 | 12.0612 | 6.7278 | 1.3945 |
| 17 | 12.6199 | 6.9532 | 1.2865 |
| 18 | 13.1869 | 7.1869 | 1.1869 |
| 19 | 13.7616 | 7.4283 | 1.0949 |
| 20 | 14.3434 | 7.6768 | 1.0101 |
| 21 | 14.9319 | 7.9319 | 0.9319 |
| 22 | 15.5265 | 8.1931 | 0.8598 |
| 23 | 16.1267 | 8.4600 | 0.7933 |
| 24 | 16.7321 | 8.7321 | 0.7321 |
| 25 | 17.3423 | 9.0090 | 0.6756 |
| 26 | 17.9569 | 9.2903 | 0.6236 |
| 27 | 18.5757 | 9.5757 | 0.5757 |
| 28 | 19.1982 | 9.8648 | 0.5315 |
| 29 | 19.8241 | 10.1574 | 0.4908 |
| 30 | 20.4532 | 10.4532 | 0.4532 |

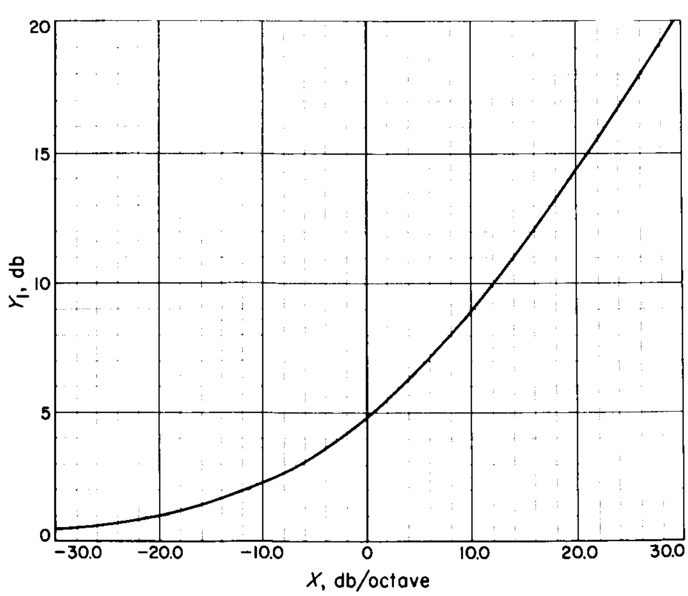


**Fig. 6. Plot of decibel increment ($Y_1$) between first ⅓ octave level and the octave level vs decibel/octave slope (X); Ref. Eq. (36)**

# Appendix A. New and Old A.S.A. Tabular Data for Preferred Octave Designations; and Mathematical Expressions Defining These Bounds and Center Frequencies

**Table A-1. Table of new and old A.S.A. recommended octave designations**

| Old[1] | | New[2] | | | |
|---|---|---|---|---|---|
| Frequency band | Center frequency (geometric mean) | Frequency band | | Center frequency (geometric mean) | |
| (Hz) | (Hz) | ⅓ octave (Hz) | Octave (Hz) | ⅓ octave (Hz) | Octave (Hz) |
| 4.7–9.4 | 6.6 | 5.7–7.1 | | 6.3 | |
| | | 7.1–8.8 | 5.7–11.3 | 8.0 | 8.0 |
| | | 8.8–11.3 | | 10.0 | |
| 9.4–18.7 | 13.2 | 11.3–14.2 | | 12.5 | |
| | | 14.2–17.7 | 11.3–22.4 | 16.0 | 16.0 |
| | | 17.7–22.4 | | 20.0 | |
| 18.7–37.5 | 26.5 | 22.4–28.3 | | 25.0 | |
| | | 28.3–35.4 | 22.4–44.5 | 31.5 | 31.5 |
| | | 35.4–44.5 | | 40.0 | |
| 37.5–75 | 53.0 | 44.5–56.5 | | 50.0 | |
| | | 56.5–70.7 | 44.5–88.7 | 63.0 | 63.0 |
| | | 70.7–88.7 | | 80.0 | |
| 75–150 | 106 | 88.7–103 | | 100 | |
| | | 103–141 | 88.7–177 | 125 | 125 |
| | | 141–177 | | 160 | |
| 150–300 | 212 | 177–224 | | 200 | |
| | | 224–283 | 177–354 | 250 | 250 |
| | | 283–354 | | 315 | |
| 300–600 | 424 | 354–445 | | 400 | |
| | | 445–566 | 354–707 | 500 | 500 |
| | | 566–707 | | 630 | |
| 600–1200 | 848 | 707–887 | | 800 | |
| | | 887–1130 | 707–1414 | 1000 | 1000 |
| | | 1130–1414 | | 1250 | |
| 1200–2400 | 1696 | 1414–1770 | | 1600 | |
| | | 1770–2240 | 1414–2830 | 2000 | 2000 |
| | | 2240–2830 | | 2500 | |
| 2400–4800 | 3392 | 2830–3540 | | 3150 | |
| | | 3540–4450 | 2830–5660 | 4000 | 4000 |
| | | 4450–5660 | | 5000 | |
| 4800–9600 | 6784 | 5660–7070 | | 6300 | |
| | | 7070–8870 | 5660–11300 | 8000 | 8000 |
| | | 8870–11300 | | 10000 | |
| 9600–19200 | 13568 | 11300–14140 | | 12500 | |
| | | 14140–17700 | 11300–22400 | 16000 | 16000 |
| | | 17700–22400 | | 20000 | |
| 19200–38400 | 27136 | 22400–28300 | | 25000 | |
| | | 28300–35400 | 22400–44500 | 31500 | 31500 |
| | | 35400–44500 | | 40000 | |

[1] "Octave-Band Filter Set for the Analysis of Noise and Other Sounds," *American Standards Association*, Specification Z24.10-1953.

[2] "Preferred Frequencies for Acoustical Measurements," *American Standards Association*, Specification S1.6-1960.

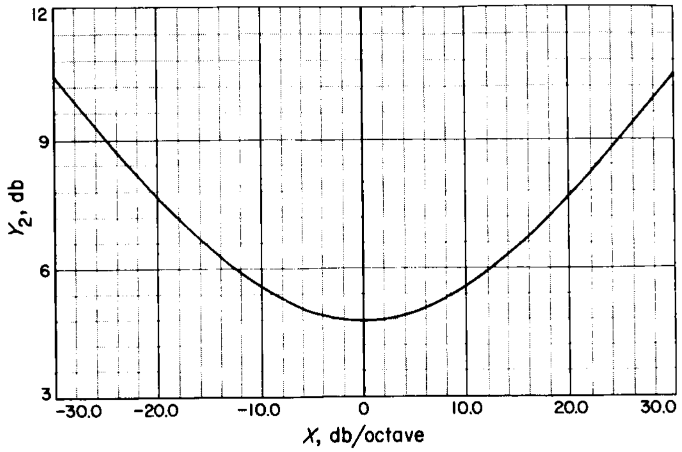


**Fig. 7. Plot of decibel increment ($Y_2$) between second ⅓ octave level and the octave level vs decibel/octave slope (X); Ref. Eq. (37)**

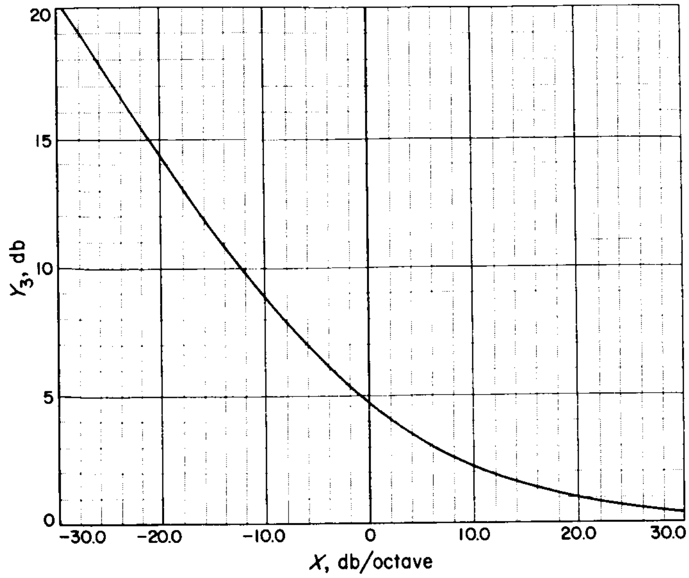


**Fig. 8. Plot of decibel increment ($Y_3$) between third ⅓ octave level and the octave level vs decibel/octave slope (X); Ref. Eq. (38)**

The equations defining an octave system are as follows:

1. The ratio of center frequency $(C.F.)$ of the $(i)^{th}$ to the $(i+N)^{th}$ octave is:

$$\frac{(C.F.)_{i+N}}{(C.F.)_i} = 2^N \tag{A-1}$$

2. The ratio of the bandwidths $(B.W.)$ of the $(i)^{th}$ octave to the $(i+N)^{th}$ octave is:

$$\frac{(B.W.)_{i+N}}{(B.W.)_i} = 2^N \tag{A-2}$$

3. The lower limit frequency $(f_L)$ of a given octave is:

$$f_L = \frac{(C.F.)}{\sqrt{2}} \tag{A-3}$$

4. The upper limit frequency $(f_u)$ of any given octave is:

$$f_u = \sqrt{2}\,(C.F.) \tag{A-4}$$

The equations defining a one-third octave system are as follows:

1. The ratio of center frequency $(C.F.)$ of the $(i)^{th}$ to the $(i+N)^{th}$ one-third octave is:

$$\frac{(C.F.)_{i+N}}{(C.F.)_i} = 2^{N/3} \tag{A-5}$$

2. The ratio of the bandwidth $(B.W.)$ of the $(i)^{th}$ octave to the $(i+N)^{th}$ one-third octave is:

$$\frac{(B.W.)_{i+N}}{(B.W.)_i} = 2^{N/3} \tag{A-6}$$

3. The lower limit frequency $(f_L)$ of any given one-third octave is:

$$f_L = \frac{(C.F.)}{2^{1/6}} \tag{A-7}$$

4. The upper limit frequency $(f_u)$ of any given one-third octave is:

$$f_u = 2^{1/6}\,(C.F.) \tag{A-8}$$

# Appendix B. Decibel-Equivalents Tabular Data

**Table B-1. Table of decibel equivalents for power, voltage, and pressure**

| Power $[\Delta db = 10 \log (W_1/W_2)]$ | | | | Voltage and pressure $\left[\begin{matrix}\Delta db = 20 \log (V_1/V_2) \\ \Delta db = 20 \log (P_1/P_2)\end{matrix}\right]$ | | | |
|---|---|---|---|---|---|---|---|
| $W_1/W_2$ | $\Delta$db | $W_1/W_2$ | $\Delta$db | $\frac{V_1}{V_2}$ or $\frac{P_1}{P_2}$ | $\Delta$db | $\frac{V_1}{V_2}$ or $\frac{P_1}{P_2}$ | $\Delta$db |
| 1.00 | 0 | 0.1 | −10.00 | 1.00 | 0 | 0.1 | −20.00 |
| 1.12 | 0.5 | 0.2 | − 7.00 | 1.06 | 0.5 | 0.2 | −14.00 |
| 1.26 | 1.0 | 0.3 | − 5.23 | 1.12 | 1.0 | 0.3 | −10.46 |
| 1.41 | 1.5 | 0.4 | − 4.00 | 1.19 | 1.5 | 0.4 | − 8.00 |
| 1.59 | 2.0 | 0.5 | − 3.00 | 1.26 | 2.0 | 0.5 | − 6.00 |
| 1.78 | 2.5 | 0.6 | − 2.22 | 1.33 | 2.5 | 0.6 | − 4.44 |
| 2.00 | 3.0 | 0.7 | − 1.55 | 1.41 | 3.0 | 0.7 | − 3.10 |
| 2.24 | 3.5 | 0.8 | − 0.97 | 1.50 | 3.5 | 0.8 | − 1.94 |
| 2.52 | 4.0 | 0.9 | − 0.46 | 1.59 | 4.0 | 0.9 | − 0.92 |
| 2.62 | 4.5 | 1.0 | 0 | 1.68 | 4.5 | 1.0 | 0 |
| 3.16 | 5.0 | 1.5 | 1.78 | 1.78 | 5.0 | 1.5 | 3.52 |
| 3.54 | 5.5 | 2.0 | 3.00 | 1.88 | 5.5 | 2.0 | 6.00 |
| 4.00 | 6.0 | 2.5 | 3.98 | 2.0 | 6.0 | 2.5 | 8.00 |
| 4.46 | 6.5 | 3.0 | 4.77 | 2.12 | 6.5 | 3.0 | 9.54 |
| 5.00 | 7.0 | 3.5 | 5.44 | 2.24 | 7.0 | 3.5 | 10.88 |
| 5.62 | 7.5 | 4.0 | 6.00 | 2.37 | 7.5 | 4.0 | 12.00 |
| 6.30 | 8.0 | 4.5 | 6.54 | 2.52 | 8.0 | 4.5 | 13.07 |
| 7.07 | 8.5 | 5.0 | 7.00 | 2.66 | 8.5 | 5.0 | 14.00 |
| 7.95 | 9.0 | 5.5 | 7.41 | 2.82 | 9.0 | 5.5 | 14.82 |
| 8.90 | 9.5 | 6.0 | 7.78 | 2.98 | 9.5 | 6.0 | 15.56 |
| 10.0 | 10.0 | 6.5 | 8.14 | 3.16 | 10.0 | 6.5 | 16.26 |
| 100.0 | 20.0 | 7.0 | 8.45 | 10.0 | 20.0 | 7.0 | 16.91 |
| 1,000.0 | 30.0 | 7.5 | 8.75 | 31.6 | 30.0 | 7.5 | 17.00 |
| 10,000.0 | 40.0 | 8.0 | 9.00 | 100.0 | 40.0 | 8.0 | 18.06 |
| 100,000.0 | 50.0 | 8.5 | 9.30 | 316.0 | 50.0 | 8.5 | 18.60 |
| 1,000,000.0 | 60.0 | 9.0 | 9.54 | 1,000.0 | 60.0 | 9.0 | 19.10 |
| 10,000,000.0 | 70.0 | 9.5 | 9.78 | 3,160.0 | 70.0 | 9.5 | 19.56 |
| 100,000,000.0 | 80.0 | 10.0 | 10.00 | 10,000.0 | 80.0 | 10.0 | 20.00 |

Note: If $(+\Delta)$db = 10 log $(A/B)$, then $(-\Delta)$db = 10 log $(B/A)$.

# Appendix C. Conversion Tabular Data

The conversion listing of some useful acoustics constants is as follows:

1 in. = 2.54 cm
1 cm = 0.3937 in.
1 ft = 30.48 cm
1 ft = 0.3048 meter
1 cm = $3.281\,(10)^{-2}$ ft
1 meter = 100 cm
1 dyne = $2.248\,(10)^{-6}$ lb

1 slug = 14.6188 Kgm
1 lb = $4.448\,(10)^{5}$ dynes
1 newton = $10^{5}$ dynes
1 dyne/cm$^2$ = 1 $\mu$ bar
1 dyne/cm$^2$ = $14.5\,(10)^{-6}$ lb/in.$^2$
1 lb/in.$^2$ = $6.895\,(10)^{4}$ dynes/cm$^2$

**Table C-1. Conversion table of some useful acoustic constants**

| System Element | C.G.S. | M.K.S. | English ft-lb-sec | English in.-lb-sec |
|---|---|---|---|---|
| $P_a$* standard atmos. pressure | $1.013\,(10)^{6}$ dynes/cm$^2$ | $1.013\,(10)^{5}$ Newtons/M$^2$ | $2.116\,(10)^{3}$ lbs/ft$^2$ | 14.70 lbs/in.$^2$ |
| $\rho$* air density | $1.2250\,(10)^{-3}$ grams/cm$^3$ | 1.2250 Kgm/M$^3$ | $2.3760\,(10)^{-3}$ slugs/ft$^3$ | $1.1453\,(10)^{-7}$ lb-sec$^2$/in.$^4$ (mass/in.$^3$) |
| c* speed of sound in air | $3.4340\,(10)^{4}$ cm/sec | $3.4340\,(10)^{2}$ meters/sec | $1.1266\,(10)^{3}$ ft/sec | $1.3520\,(10)^{4}$ in./sec |
| $\rho c$ (Rayls) impedance | 42.067 grams/cm$^2$-sec | 420.67 Kgm/M$^2$-sec | 2.6734 slugs/ft$^2$-sec | $1.5471\,(10)^{-3}$ lb-sec/ft$^3$ (mass/in.$^2$-sec) |
| $P_0$ "0" db reference pressure level | $2.00\,(10)^{-4}$ dynes/cm | $2.00\,(10)^{-5}$ Newtons/M$^2$ | $4.18\,(10)^{-7}$ lb/ft$^2$ | $2.90\,(10)^{-9}$ lb/in.$^2$ |
| $I_0$ "0" db reference intensity level | $1.000\,(10)^{-11}$ dynes/cm$^2$-sec | $1.000\,(10)^{-12}$ watts/M$^2$ | $6.8470\,(10)^{-11}$ lb/ft-sec | $5.7058\,(10)^{-15}$ lb/in.-sec |
| $W_0$ "0" db reference energy level | $1.00\,(10)^{-6}$ dynes-cm/sec | $1.00\,(10)^{-13}$ watts | $7.37\,(10)^{-14}$ ft-lb/sec | $8.84\,(10)^{-13}$ in.-lb/sec |
| 1.000 horsepower | | 746 watts | 550 ft-lb/sec | |
| $1.341\,(10)^{-3}$ horsepower | | 1.000 watts | 0.737 ft-lb/sec | |
| $1.818\,(10)^{-3}$ horsepower | | 1.356 watts | 1.000 ft-lb/sec | |

*Note: All values of $P_a$, $\rho$, and c are given for "standard" atmospheric conditions.