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THE PROPAGATION OF COSMIC RAY NUCLEI IN INTERSTELLAR SPACE AND SOLAR SYSTEM

by

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ABSTRACT

The problem of the propagation of cosmic ray nuclei in interstellar space and solar system has been investigated with special reference to the recent data obtained on the various components of the cosmic radiation, using balloons, satellites and rockets over the years 1961-1965. Three kinds of source spectra were assumed in this analysis, namely power law spectra in total energy per nucleon, kinetic energy per nucleon and in rigidity. The constants and the exponents of these spectra were deduced from the high energy data that is little affected by solar modulation. The effective path length, x , in g/cm^2 traversed by the nuclei, was determined as a function of energy from a study of the variation of charge ratios of nuclei of charge $Z \geq 20$, $Z \geq 10$, $Z = 3$-5 and α-particles; the energy dependent fragmentation parameters needed for this purpose were computed from an empirical relation due to Rudstam. It was found that this path length traversed decreased with decreasing energy, consistent with a model $x = \beta c \rho t = \beta x_e$ where x_e is the high energy value of matter traversed, βc is the velocity of the nucleus, ρ is the average density of matter and t is the age of the nuclei. Two mechanisms of modulation, namely the heliocentric electric field model and the generalized Parker's model were used to study the temporal variations and it was found that the generalized Parker's model would explain the intensity variations fairly well over the period considered for all the components of the cosmic radiation using the source spectrum that is a power law in rigidity. The parameters involved in this model thus deduced for various years were found to be consistent with the values deduced by Parker from general dynamical theory of the solar wind and recent space observations.

INTRODUCTION: The energy spectra of the low energy cosmic ray nuclei observed at the earth depend (a) on the energy spectra of nuclei emanating from the source (b) on the propagation of these nuclei in the interstellar space and (c) on the modulation of the energy spectra in the vicinity of the solar system. These various effects can be studied separately from systematic observations on the energy spectra and intensities of the various components of the cosmic radiation as a function of time. Protons and helium nuclei (neglecting the isotopes of these nuclei which are rare in abundance) have the same energy loss for particles of the same velocity but a charge to mass ratio (Z/A) that differs by a factor of two; this fact could be used to separate rigidity and velocity effects of solar modulation in the case of a diffusion convection type (Parker's type) of modulation or to detect and measure precisely the electric potential in the case of solar modulation by a heliocentric electric field. α-particles and heavy nuclei of charge $Z \geq 3$, except for the rare light nuclei, have approximately the same Z/A ratio of 0.5, but have different energy losses (proportional to Z^2/β^2c^2, where βc is the velocity of the nucleus) for particles of the same velocity; thus their relative intensities are affected in similar ways by solar modulation but depend to a large extent on the diffusion in the interstellar space and source regions. The information about diffusion and the path length traversed in the high energy region was obtained from the measured abundances of rare isotopes (Li, Be, & B) that are almost absent in the universe and hence in the source regions; similar estimates could be made from the abundances in the low energy region. However, such estimates are dependent to a large extent not only on the experimentally measured values of these abundances, but also on the energy dependent fragmentation parameters deduced for these nuclei, the data on which is rather meagre and uncertain. On the other hand, at these low energies, the energy losses suffered by these nuclei increase with increasing charge and thus the modified energy spectrum of heavy nuclei of charge $Z \geq 10$ and very heavy nuclei of charge $Z \geq 20$ observed in the vicinity of the earth could be used to determine the effective path length, provided the spectrum at the source region is known. The values thus deduced would be rather insensitive to the parameters which are better known for heavier nuclei than for the light nuclei.

Systematic studies of the differential energy spectra of protons and helium nuclei were begun in the year 1956 by McDonald (see e.g. Webber, 1962 for a summary) but the spectra have been measured in some comparable energy regions, for the years 1961 and 1963 only. The available experimental information in the energy region less than 1 BeV has recently been summarized by Durgaprasad (1966). Attempts are being made to obtain similar data for the year 1965,(Balasubrahmanyan, et al., 1966, Comstock et al., 1966, Ormes and Webber, Webber et al.,1966) and the spectrum on very heavy nuclei component is being obtained by Waddington and Frier (1966) and Frier et al. (1966). This information is made use of here, to deduce inferences on the three aspects mentioned above, namely the source spectrum, the propagation in the interstellar space and in solar system. We discuss these three aspects below.

SOURCE SPECTRUM: The galactic cosmic ray spectra are normally represented as a power law (a) either in total energy per nucleon, W, the integral intensity of nuclei J_W $(>W)$ being given by $C_W W^{-\gamma}$, (b) in kinetic energy per nucleon, ϵ, the integral intensity J_ϵ $(>\epsilon)$ being given by $C_\epsilon \epsilon^{-\gamma}$ and (c) in rigidity, R, J_R $(>R)$ being given by $C_R R^{-\gamma}$. The total energy W and the kinetic energy ϵ are expressed in GeV/nucleon in the above expressions and the rigidity R in Gv. C_W, C_ϵ, C_R, and γ are constants. These power laws are predicted by various acceleration mechanisms, namely the statistical acceleration process proposed by Fermi (1954), the betatron mechanism of acceleration (Ginsburg and Syrovatsky, 1961) and the acceleration in hydromagnetic shock waves produced during supernova explosions (Colgate and Johnson, 1960 and Colgate and White, 1963). It was also pointed out by Ginzburg and Syrovatsky (1961) that, on the basis of the general arguments concerning the equipartition of the energy in a turbulent magnetized plasma of cosmic dimensions, a power law in kinetic energy could be a fundamental property of such a plasma.

We consider in our analysis the three types of spectra mentioned above and try to see which type of spectrum fits the experimental observations best. For this purpose, we assume in what follows that all the components of the cosmic radiation have the same type of spectrum at the source region, from 50 MeV/nucleon to at least up to about 10^{11} or 10^{12} ev/nucleon, the energy region in which the intensity of the protons and helium nuclei were fairly well measured (See Webber, 1964). We further assume that whatever changes in intensity are observed as a function of energy at a particular instant of time or as a function of time at a particular energy result from the transformations that take place in passage through the interstellar space and the solar system.

We will now proceed to deduce the value of the exponent γ. The integral flux values of cosmic ray nuclei determined for the highest possible known energy, from solar maximum to solar minimum, are near the equator (cut-off rigidity $\sim$16 GV); within the limits of the experimental uncertainties and errors, these values do not show variation. Thus, it is reasonable to assume that the flux values obtained for this energy represent those outside the solar system. These are plotted in Figure 1. The integral flux values of protons and helium nuclei, measured at energies close to 10^{12} eV (Webber, 1964), which are positively least affected by interplanetary medium, are also shown in Figure 1; however, there is some uncertainty in the values of the energy measured here.

From the above figure it can be seen that the protons and α-particle components could be well fitted with a single exponent of γ' $(=\gamma - 1) = 1.35$. This value can be compared with the value of $\gamma_\alpha' = 1.43 \pm 0.10$ (or 1.31, as deduced by Waddington, 1960) obtained for helium nuclei by McDonald (1958) in the energy region 4-15 GeV/nucleon, by comparing the flux values obtained at Texas and Guam. It was pointed out by Singer (1958) and some experimental evidence also exists (see for example Webber, 1966) that the heavy nuclei have a steeper spectrum; however, most of the existing exponents are not inconsistent with a value of 1.35 within two standard deviations. We use this value, in the further analysis. The constants C_ϵ, C_W, and C_R, together with errors for various components of the cosmic radiation will be deduced in the next section using this value and the flux values measured, near the equator. (Daniel and Sreenivasan, 1965, Shapiro et al., 1958).

PROPAGATION IN THE INTERSTELLAR MEDIUM: Various models e.g., the regular model, the diffusion model etc. (Ginzburg and Syrovatsky, 1961) have been postulated for the propagation of cosmic rays in the interstellar space and a model of confinement of these nuclei in the source regions has been given by Kaplon and Skadron (1964); these models predict different variations of the pathlength with energy. We assume here the regular model of diffusion of cosmic rays in which <u>all</u> particles of a <u>unique</u> energy move along a definite path, for example along the magnetic lines of force. We further assume that no acceleration takes place in the interstellar medium; there are various reasons to assume that the Fermi statistical acceleration mechanism cannot operate effectively in the interstellar medium (see e.g., Ginzburg and Syrovatsky, 1961 and Durgaprasad, 1965).

In traversing a path length, xg/cm^2 of interstellar matter, two competing processes occur, namely (a) the loss of energy of particles through ionization and (b) the absorption of nuclei of a given species and their creation from heavier nuclei through fragmentation in nuclear collisions. For nuclei of relativistic energies which we consider at first, the energy loss due to ionization becomes negligible and only the process (b) need be considered. The diffusion of these nuclei in space could be represented by a one-dimensional diffusion equation given by Ginzburg and Syrovatsky (1961) for the regular model,

$$\frac{d J_i(x)}{dx} = -\frac{J_i(x)}{\lambda_i} + \sum_{j \geq i} \frac{P_{ji}}{\lambda_j} J_j(x) \tag{1}$$

where $J_i(x)$ and $J_j(x)$ are the differential intensities of the i and j nuclei at the depth x, λ_i and λ_j are the interaction mean free paths of i and j nuclei and P_{ji} is the fragmentation parameter defining the number of i-nuclei produced in the collision of the primary j-nucleus with that of the medium.

This equation has been used to extrapolate the observed fluxes to the source regions. While making the extrapolation, we have divided the heavy nuclei into the five groups and adopted the following notation suggested by Daniel and Durgaprasad (1962), namely L-nuclei (Li, Be and B), M-nuclei (C, N, O and F), H_3-nuclei (Z = 10-15), H_2-nuclei (Z = 16-19), H_1-nuclei (Z $\geq$ 20), $H_{2,3}$-nuclei

($Z = 10$–19), H-nuclei ($Z \geq 10$) and S-nuclei ($Z \geq 6$). The values of interaction mean free paths of these nuclei have been deduced (Durgaprasad, 1964) assuming a composition of 93% hydrogen and 7% helium for the interstellar medium (Ginzburg and Syrovatsky, 1961); these values for H_1, $H_{2,3}$, M, L, α and P-nuclei are 2.5, 4.3, 6.1, 9.8, 14.6 and 72.0 g/cm^2 respectively. The fragmentation parameters involved in collisions of primary nuclei with protons and helium nuclei of the interstellar space have been deduced assuming that the space consisted mainly of protons. Since we will be using later the parameters that vary in energy, from about 50 MeV/nucleon to ~ 10 BeV/nucleon and since the conclusions arrived at, on the mean pathlength traversed, depend to a certain extent on the parameters thus deduced, we will discuss here briefly the procedure adapted in deducing these values. The parameters for the primary H_1 and $H_{2,3}$ groups of nuclei were obtained as a function of energy using an empirical relation given by Rudstam (1965), for what he calls as CDMD-distribution. This relation gives the cross-sections for the production of isotopes in the inverse process, namely the interaction of isotopes of incident protons and α-particles on various targets. Rudstam (1965) has recently shown that these cross-sections, $\sigma(Z, A)$, for the production of isotopes of charge Z, and mass A in collisions of a proton with energy E with a target material of mass A_t could be predicted reasonably accurately as a function of energy and is given by

$$\sigma(Z,A) = \frac{\sigma' P (d')^{0.66} (A_t)^{-(0.66 e')}}{1.79 \left[e^{PA_t} \left(1 - \frac{2e'}{3PA_t}\right) - 1 + (0.66 e') + \left(\frac{0.66 e'}{PA_t}\right) \right]} \tag{2}$$

$$\exp. \left[PA - R \left| Z - SA + TA^2 \right|^{3/2}\right]$$

where P is a parameter that relates the dependence of the production cross-section to the energy of the incident particle and is given by $P = a' E^{-b'}$ for $E < E_0$ (E_0 has a value about 2 BeV) and $P = c'$ for $E > E_0$, where $a' = 20 \pm 7$, $b' = 0.77 \pm 0.06$ and $c' = 0.056 \pm 0.003$. The parameter R depends on the mass number of the spallation product and is independent of the kind and energy of the projectile; it is given by $R' = d' A^{(-e')}$. S and T are parameters that define the peak of the charge distribution and have values $S = 0.486$ and $T = 0.00038$ and σ' is the inelastic cross-section. The cross-section σ (Z, A) has a maximum value at an energy E that corresponds to a value of P given by $P \sim 1/(A_t - A)$. It was shown by Rudstam (1965) that this relation could predict fairly accurately, the cross-sections for production of nuclides in bombardment by protons varying in energy from $\sim$90 MeV to $\sim$27 BeV, on medium weight elements ($Z \sim 23$-67) and could be extended, down to neon and fluorine, by placing an error of a factor of two on some cross-sections thus deduced. We have used this formula to estimate the fragmentation parameters involved in collisions of primary H_1-nuclei (assumed to be all Fe-nuclei) and $H_{2,3}$-nuclei (replaced by Al-nuclei) as a function of energy, after using proper normalization factors and following the general procedures suggested by Badhwar et al. (1962). The fragmentation parameters thus obtained from these cross-sections are given in Figure 2 and would indicate the energy variation with a fair degree of accuracy (less than a factor of two). The energy dependence of the parameters involved in collisions of M, α and protons are hard to estimate since (a) the Rudstam relation is known not to apply to such low mass values and (b) the measured cross-sections are available only to a few isotopes as a function of energy. For this reason we have used the values suggested in literature (Badhwar et al., 1962, Badhwar and

Daniel, 1963, Honda and Lal, 1960, Durgaprasad, 1964, Fichtel and Reames, 1966c). These are also given in Figure 2. In view of the fact, as will be shown later, that the value of x, derived from various charge ratios agree within errors, it can probably be presumed that the parameters used here are fairly reliable ones.

We have used the high energy parameters given in Figure 2, and a value of 2.5 g/cm^2 for x, the matter traversed (Badhwar et al., 1962) to determine the values of constants C_ϵ, C_W and C_R. These values, with errors, are given in Table 1. In deriving the values for nuclei heavier than α-particles, the following abundance ratios (Badhwar et al., 1962) for the source region have been used, namely

$$H_1 : H_{2,3} : M : \alpha = 0.0104 : 0.0186 : 0.0880 : 1.000$$

It may be remarked here that the ratio obtained for α-particles to protons is 0.20 in the source regions in the case of a source spectrum that is a power law in rigidity and is much larger than the abundance ratios 0.078 (Suess and Urey, 1956) and 0.152 (Cameron, 1959) reported for the universe. This increased abundance in cosmic rays is thus in conformity with the general observation that in cosmic rays the abundance of the heavier elements gradually increases with increasing charge as compared to the universal abundance.

For low energy nuclei, the two processes (a) and (b) discussed above, namely the ionization loss and fragmentation in interstellar space have to be considered. The diffusion equation including these two processes, on the regular diffusion model can be written as (Aizu et al., 1960, Ray, 1960 and Fichtel and Reames, 1966c.)

$$\frac{d}{dx}\left[\omega_i(\epsilon)\, J_i(\epsilon, x)\right] = \frac{J_i(\epsilon, x)}{\lambda_i(\epsilon)}\,\omega_i(\epsilon) + \sum_{K \geq i}\left(\frac{J_k(\epsilon, x)}{\lambda_k(\epsilon)}\, P_{ki}(\epsilon)\,\omega_i(\epsilon)\right) \quad (3)$$

where J_i (ϵ, x) is the differential intensity of i-nuclei in energy/nucleon, ω_i (ϵ) is the energy loss per nucleon, given by, $(d\epsilon/dx)$ for a particular having energy ϵ, x is the matter traversed in g/cm^2, λ_i (E) is the interaction mean free path of i-nucleus in hydrogen and P_{ki} is the fragmentation parameter.

The above equation was used to study the propagation of the energy spectra of various groups of nuclei through interstellar space in steps, small enough such that the variation in terms, ω_i (ϵ) and P_{ki} (ϵ), are less than a few percent. The interstellar space was supposed to consist of neutral hydrogen medium and the range-energy relation and rate of energy loss in hydrogen were calculated using the relations given by Barkas and Berger (1964). The interaction mean free paths used were those given above; the fragmentation parameters used were those given in Figure 2 as discussed above.

Using the above equation and the three types of source spectra, the flux values and the ratios of flux values of the six components (H_1, $H_{2,3}$, M, L, α and protons) were deduced as a function of energy and matter traversed by the nuclei. The spectrum of the H_1-nuclei was calculated first so that the same function would become available for the subsequent species. In the case of protons, there arises an additional low energy secondary contribution due to elastic and inelastic collisions of the primary nuclei with protons and helium nuclei of the interstellar medium. However, if the path length traversed at low energies is less than 3 g/cm^2, (see below), it can be shown using the cross-sections given by Feit and Milford (1965), that this contribution is not appreciable (less than a few percent) for energies $\epsilon \geq 50$ MeV/nucleon, and hence was not considered further. The three types of spectra thus obtained for protons, helium and H-nuclei, after traversal through 0 and 3 g/cm^2 of interstellar hydrogen, are given in Figure 3.

In deriving the estimate on the path length traversed as a function of energy, the comparison of the ratios thus computed is limited here to the four ratios, Γ_{VHH} (of VH-nuclei to H-nuclei), $\Gamma_{VH\alpha}$, $\Gamma_{H\alpha}$ and Γ_{LS} for reasons mentioned previously in the introduction. The first three ratios could be measured fairly precisely and the energy dependence of the parameters used are fairly well-known. The last ratio is used here for comparison purposes, because it would be most sensitive to the path length traversed, if the light nuclei are absent in the source regions, as is normally presumed. However, this ratio is quiet sensitive also to the parameters deduced, the energy dependence of which is not known well. We will try to see whether we can arrive at consistent values of x and energy spectra using these ratios and the three source spectra. The available experimental data are summarized in Figures 4 and 5. Not all these ratios were directly measured by the authors and some are deduced using certain known ratios that are only slightly energy dependent (e.g., $\Gamma_{HM} = 0.30$, Waddington,1964). In cases where the ratios were obtained from integral intensity measurements,these ratios were plotted at energies corresponding to the median energy of the particles, calculated using a value of 1.35 for the exponent of the energy spectrum.

For each value of the measured ratio given in figures 4 and 5, the corresponding value of the matter traversed, x, by the nuclei in g/cm^2 was calculated for the three kinds of spectra assumed for the source region using equation (3) and the parameters given in Figure (2). These values of x, computed using the four measured ratios, are plotted in Figures 6(a), (b) and (c) as a function of energy. If for a particular ratio, at a corresponding energy interval, there is more than one measurement, the mean value of x, thus calculated is plotted in this figure. It can be concluded from this figure, that in all the three cases of source spectra, for energy values of $\epsilon \lesssim 200$ - 300 MeV/nucleon, the matter traversed decreases with decreasing energy. At energies higher than these, it increases slightly and decreases or remains constant with increasing energy.

It is possible to understand this energy variation of the path length on the regular model[1] thus. The matter traversed, x in g/cm^2, in this model is given by $x = \beta c \bar{\rho}\, t$, where βc is the average velocity of the particle and $\bar{\rho}$ and t are the mean density and the age of the particles. If the diffusion of cosmic ray nuclei is such that the product $\bar{\rho} t$ has a constant value for all energies (this is possible, for example, if the bulk of cosmic ray nuclei of all energies are produced at the same time in the past, say, in a violent explosion of our Galaxy (or some other), that would occur with time periods of $\sim 10^7$ years, Burbidge 1963, and traverse, on the average, regions of similar density) then x is given by $x = \beta x_e$ where x_e is the relativistic value of mean path length traversed. We tried to see whether the observed decrease of x with energy is consistent with the above relationship. For this purpose, the value of x in the diffusion equation is replaced by the non-relativistic value that takes into account the β-dependence. The ratios of intensities of four groups of nuclei were recalculated for the three cases of source spectra and for various values of x_e. The values thus obtained for the four ratios, are plotted in figures 4 and 5. The lines A, B and C refer to the power law spectra in rigidity, in kinetic energy per nucleon and in total energy per nucleon respectively and for the value of $x_e = 2.5$ g/cm^2 of hydrogen;

the lines D, E and F are similar spectra obtained for a value of $x_e = 3.5$ g/cm^2 of hydrogen. We have used the two sets of values of x_e because of large uncertainties in the experimental data as can be seen from this figure. One can conclude from this figure, that the observed decrease of the ratios with x is consistent with such an average velocity dependence ($x = \beta x_e$) at low energies.

It should be mentioned that we have considered here the simple regular model. Various other models, especially the three-dimensional model has been considered by others (Ginzburg and Syrovatsky, 1961, Mathiesen and Stenman, 1965, Dahanayake et al., 1964) to explain the dependence of the ratio of L-nuclei as a function of energy. Kaplon and Skadron (1964) found an increase in the path length with decreasing energy, which they explained as due to the preferential confinement of low energy particles by the magnetic fields in the source regions. However, in all these works, the ionization loss of these nuclei has not been considered. Recently, Fichtel and Reames (1966c) found that the experimental data could be explained by assuming a constant path length of 2.8 g/cm^2 of hydrogen over all energy regions and taking into account the fragmentation and ionization loss in the interstellar space. However, a critical examination of the figures they presented reveals that the measured ratios in the low energy region fall below the calculated curves. Recently Comstock et al. (1966) and Waddington and Frier (1966) have suggested, from an analysis of the charge spectrum of nuclei of charges $Z \geq 2$ and $Z \geq 20$, that the path length traversed by the low energy nuclei could be less than the path length traversed by relativistic nuclei. Their conclusion is thus, similar to the one drawn in the present work. We further find that the values of x, deduced by us, from four experimentally measured ratios, consistently indicate a decrease of path length with decreasing energy that could be explained by an average velocity dependence as predicted by the regular model.

PROPAGATION IN THE SOLAR SYSTEM: The spectra thus obtained are modulated to a considerable extent by solar activity; different mechanisms have been postulated by means of which such changes could occur and these have been reviewed from time to time by various authors (for example, Dorman, 1963, Parker, 1966, Dorman and Dorman 1966), but of these, two models seem to be currently in vogue, Parker's model (1963) and the heliocentric electric field model (Ehmert, 1960a, 1960b). The Parker's model with various modifications have been used to represent the intensity variations of protons or helium nuclei or both in restricted energy regions and for restricted time periods (e.g., Fichtel et al., 1966A, Balasubrahmanyan et al., 1965, Gloeckler 1965, etc.) but has never been shown to be valid for all nuclei throughout a vast energy region. The electric field model has been recently examined by Freier and Waddington (1965); they showed that using a total energy spectrum with an exponent of 2.45, that the intensities of both the components, protons and helium nuclei, could be adequately predicted only in a narrow energy interval and that it becomes necessary to postulate modulation by other means, especially at low energies.

We examine here both the mechanisms of modulation using the three types of source spectra given above in an attempt to explain the intensity variations of all components in a broad energy interval (few tens of MeV/nucleon to a few BeV/nucleon).

(a) Heliocentric electric field model: In this model it is assumed that the earth is at a positive potential, V, which is heliocentric and relatively constant out to some distance from the sun; thus positive particles coming from outside the solar system do lose energy (by an amount ZeV/A, per nucleon where Ze and A are the charge and mass of the particles) in traversing through this potential. Also they occupy a different volume in phase space, the density D in phase space being constant. Consequently, if P, M and E are the momentum, relativistic mass and total energy of the particles and J(P) and J(E) the corresponding differential intensities, the following relations hold good:

$$D = \frac{J\,(P)\,M}{P^3} = \frac{J\,(E)}{P^2} = \text{constant} \tag{4}$$

Thus the net reduction in intensity of particles observed in the vicinity of the earth arises because (a) the intensity of these particles correspond to that of a higher energy at the solar system and (b) as seen from equation (4) there will be a net reduction in intensity because they occupy a different volume in phase space. By taking these two effects into consideration, one could write that the differential energy spectra $J_i\,(\epsilon, x, o)$ for i-nuclei of energy ϵ MeV/nucleon, that traversed xg/cm^2 of hydrogen, would be related to the energy spectrum $J_i\,(\epsilon, x, V)$ at the earth at a potential V as

$$J_i\,(\epsilon, x, V) = J_i\,(\epsilon, x, 0) \left[\frac{W^2 - (m_0 c^2)^2}{\left(W + \frac{ZeV}{A}\right)^2 - (m_0 c^2)^2} \right] \tag{5}$$

where W is the total energy expressed per nucleon and $m_0 c^2$ is the rest mass of the nucleon. Using the value of $J_i\,(\epsilon, x, o)$ derived for the solar system in the previous section and the measured values at the earth of the intensities of various nuclei $J_i\,(\epsilon, x, V)$ it thus becomes possible to derive a value of the potential V that would bring the proper reduction in intensity. Since the spectra of protons and helium nuclei were well measured over wide energy range by various authors for the year 1963 (see Durgaprasad, 1966 for a summary of the data and for justification in combining such a data) we tried to use this spectrum for further analysis. The value of the potential has been estimated in all the three cases as a function of energy for the two types of nuclei, protons and helium nuclei. These are given in Table 2. In the case of kinetic energy and rigidity spectra, it has been found that it is not possible to obtain agreement even for a single group of nuclei for the same year with a single value of the potential; further the potentials required to explain the reduction at a discrete energy have values much larger than 1 GV. Whilst the existence of such large potentials in the inner solar system is beyond our comprehension, the fact that a single potential cannot explain the intensity variations of a single component over a wide energy range almost rules out the possibility of the effectiveness of such a modulation mechanism on the source kinetic energy or rigidity spectrum.

In the case of a source spectrum that is a power law in total energy per nucleon, the required potentials are a few hundred MV. However, here again as can be seen from Table 2, a single value of the potential cannot be used to explain the energy spectra of both the components in the energy region considered here. Our conclusion is thus similar to that of Freier and Waddington (1965), that the experimental data could not be explained using this type of modulation alone and the energy spectrum given above and that at low energies, other mechanisms of modulation should be operative. We will now examine the applicability of the second model, namely the generalized Parker model to the present problem.

(b) Parker's model: In the simplified version of this model, the cosmic ray particles diffusing into the inner solar system are scattered out of the solar system by the small scale irregularities of a large scale magnetic field set by the expanding corona. Treating this problem, to a first approximation, as one of isotropic diffusion , Parker (1963) has shown that the differential intensity of particles in the vicinity of the earth, $J_i(\epsilon, x, K, R_o)$ is related to the unmodulated intensity $J_i(\epsilon, x, 0, 0)$ in the vicinity of the solar system as

$$J_i(\epsilon, x, K, R_0) = J_i(\epsilon, x, 0, 0) \exp\left(-\frac{3}{\beta}\int_{r_e}^{\infty} (v/\lambda)\, dr\right) \tag{6}$$

where i refers to the type of the nucleus under consideration, v is the solar wind velocity, βc is particle velocity, r is the distance from the sun and λ is the scattering mean free path. By assuming that (v/λ) is independent of r from r_e to r_0 and zero thereafter equation (6) can be written as

$$J_i(\epsilon, x, K, R_0) = J_i(\epsilon, x, 0, 0) \exp\left(-\frac{1}{\beta} D(R)\right) \tag{7}$$

where

$$D(R) = \frac{3V(r_0 - r_e)}{\lambda(r)}$$

The energy dependence of the depression of the cosmic ray intensity arises from the factor $\beta\lambda(r)$ in the exponent of the above equation. The expression for λ derived depends on the assumed distribution and dimensions of the scattering centers. Parker (1956) and Dorman (1963) have suggested expressions valid for different approximations regarding the distribution of these centers in the vicinity of the solar system. We consider here the simplest of the Parker's models in which he assumed two kinds of scattering centers, namely thin and thick centers. In this case it was shown by Parker (1956) that the mean free path λ is given by

$$\lambda = \text{constant}/(1 + R^2/R_0^2) \tag{8}$$

where $R_0 = 2\ BL/\pi$, L is the size of the magnetic inhomogeneity and B is the magnetic field strength. Thus, the above equation (7) can be written as

$$J_i\ (\epsilon, x, K, R_0) = J_i\ (\epsilon, x, 0, 0)\ \exp\ (-K/\beta(1 + R^2/R_0^2)) \tag{9}$$

where K is a constant that depends on the solar windvelocity, on the distance up to which the effects of solar wind are felt (on its outer boundary) etc. In deriving the above expression, we have assumed that the modulation is of convection-diffusion type and neglected the effects of energy loss by adiabatic deceleration (see Webber, 1966, Lim and Fukui, 1965); however, it was shown by Parker (1966) that the energy loss for a typical cosmic ray particle is only of the order of ten percent.

From the three spectra obtained in the previous section for outside the solar system after traversal through values of matter traversed varying from 0 to 3 g/cm^2, the values of K and R_0 were derived that would fit the experimentally measured intensities for the various years. It was found, here again, that whereas it is almost impossible to get a fit in the case of protons for the source kinetic energy spectra for a single year, it was not possible to get fits for both the components over the entire energy spectrum in the case of total energy spectra. However, in the case of source spectrum that is a power law in rigidity, we were able to obtain reasonably good agreement between the experimentally measured values over the years 1961-1965 and the theoretically predicted curves (see figures 7(a) and (b)), if we take into account the decrease of the path length with energy as obtained in the previous section.

We will now try to see whether the values of K and R_0 thus obtained from this data are consistent with values obtained by other means and by different workers. The values of K obtained by us varies between 2 and 3 and is different by a factor of two or three from the values obtained by other workers, (for example, Balasubrahmanyan et al., 1965, Gloeckler, 1965 etc.), from fits made to the experimental data measured mostly in satellites and in restricted energy regions. Parker (1963) suggested from general dynamical theory of the solar corona and wind and other theoretical considerations that the value of K could be close to unity and could vary by a factor of 2 or 3. For example, from this dynamical theory he could estimate approximately the quiet day field strength in the interplanetary space, based on the general 1 gauss field observed near the quiet sun (1952-1954) (Babcock and Babcock, 1955). This field was estimated to be about 3×10^{-5} gauss at the orbit of the earth. The direct observations in space show irregularities in the field (Ness et al., 1964, and Ness and Wilcox, 1964) and we expect from plasma instabilities etc., irregularities on a scale of 10^5 - 10^7 km. (Parker 1958a, 1958b.) The radius of gyration of a 1 GeV proton in a magnetic field of 3×10^{-5} gauss is $\sim 2 \times 10^6$ km so that scattering would be produced principally by irregularities of about this scale. Thus we can conclude that the mean free path (which we assume to be independent of energy) λ is $\sim 10^{11}$ - 10^{12} cm. Assuming a solar wind velocity of a $v = 3 \times 10^7$ cm/sec and the value of K = 2 to 3, the value of $r_0 - r_e$ in equation (7) is obtained as 4 to 6 A.U. This is the distance up to which the solar wind effects are felt and is a reasonable figure for the outer boundary of the solar wind. The value of $R_0 \simeq 1.5$ GV obtained here is also consistent with the expected values of $B = 10^5$ gauss and $L = 10^5 - 10^7$ km. It can further be mentioned here that the value of R_0 would indicate that for particle rigidities $\lesssim 1$ GV, the second term R^2/R_0^2, in the denominator for the expression (equation 7) for mean free path produces a negligible contribution as compared to the first term 1.0, thus suggesting a pure velocity dependent modulation as obtained by others from an analysis mainly of the satellite data (Gloeckler, 1965, Webber, 1966, Balasubrahmanyan et al., 1965).

Thus we conclude that using the simple theory as developed by Parker (1963) and the reasonable parameters that are compatible with recent measurements in space, it becomes possible to explain the intensity variations over a solar cycle with the source spectrum that is a power law in rigidity, and a matter traversal derived from the available information on the intensity of light and heavy nuclei data.

The approach that was used in the present investigation is not a unique one and as was mentioned in the beginning of the section, several approaches to this problem have already been made. However, the usefulness of the present approach lies in the fact that with the simplest assumptions made regarding the nature of the spectrum, for all the components in a wide range of energy region, we are able to explain the intensity variations of the various components during the declining phase of solar cycle.

ACKNOWLEDGMENTS: I am extremely grateful to Dr. L. Meredith and Frank B. McDonald for the encouragement in carrying out this work and to Drs. V. K. Balasubraymanyan, E. Boldt, C. E. Fichtel and D. Reames for discussions, and for various suggestions in going through the manuscript. I am also thankful to the National Academy of Sciences for the award of the Resident Research Associateship.

Table I

Values of C_ϵ, C_W and C_R for protons, alpha particles and heavy nuclei

Nucleus	C_ϵ	C_W	C_R
Protons	5523 ± 745	5953 ± 806	6086 ± 825
Alpha particles	407 ± 35.4	492 ± 42.8	1245 ± 108.3
M-nuclei	35.82 ± 3.12	43.30 ± 3.77	109.56 ± 9.53
$H_{2,3}$ nuclei	7.57 ± 0.66	9.15 ± 0.80	23.16 ± 2.01
H_1-nuclei	4.23 ± 0.37	5.12 ± 0.44	12.95 ± 1.13

Table 2

ϵ	Nuclei	V_W in GV		V_ϵ in GV		V_R in GV	
		A	B	A	B	A	B
70	P	0.77	0.58	9.6	18.7	6.3	3.9
	α	0.77	0.31	7.3	13.8	7.6	2.6
100	P	0.63	0.53	19.8	15.2	4.4	3.3
	α	0.42	0.16	21.5	9.9	4.8	2.2
200	P	0.58	0.56	9.7	8.7	2.4	2.2
	α	0.35	0.16	10.8	7.5	2.2	1.4
300	P	0.63	0.52	7.1	6.7	1.9	1.8
	α	0.37	0.18	8.0	6.2	1.6	1.1
500	P	0.68	0.68	4.9	4.9	1.5	1.5
	α	0.42	0.25	5.9	5.0	1.3	0.9
800	P	0.60	0.62	3.5	3.5	1.1	1.2
	α	0.49	0.28	4.7	4.0	0.9	0.6

ϵ is the kinetic energy of the particle in MeV per nucleon; V_w, V_ϵ and V_R are the potentials calculated that would account for the required reduction in intensities of particles, measured during June-July 1963. (Mt. Washington neutron monitor rate has a value close to 2300.) A and B refer to values of 0g/cm² and 3 g/cm² of hydrogen traversed in interstellar space.

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FIGURE CAPTIONS

Figure 1: Integral intensities of protons and helium nuclei plotted as a function of kinetic energy, ϵ in ev/nucleon (and rigidity in volts), for nuclei having rigidities greater than 15 GV where the effects of solar modulation are presumed to be negligible. The data near 15 GV have been obtained from the work of Daniel and Sreenivasan (1965) and Shapiro et al. (1958); the high energy data shown are obtained from the Webber review article (Webber, 1964).

Figure 2: The variation of the fragmentation parameters plotted as a function of energy. The parameters involving primary H_1 and $H_{2,3}$-nuclei were derived using the Rudstam's relation that would predict the energy variation of these parameters fairly accurately. The parameters involving primary M-nuclei were deduced from Rudstam's relation following the general procedure suggested by Badhwar et al. (1962). The parameters involving α and P-nuclei (protons) used were those suggested in literature. P_{LP} is assumed to be zero and a value of 0.93 was assigned to P_{PP} (Fichtel et al., 1966a).

Figure 3: The differential energy and rigidity spectra of P, α and H-nuclei calculated using equation (3) for the solar system, after passage through matter in the interstellar space. The following notation was adapted in describing the various curves. S_i (x) refers to the S-type of spectrum assumed for the source region for i-nuclei after traversing through x g/cm^2 of matter; S has three types, namely power law spectra in rigidity (R), in total energy per nucleon (W) and in kinetic energy per nucleon (ϵ); three kinds of i-nuclei namely protons, α-particles and H-nuclei, were considered for values of x = 0 and 3 g/cm^2 of matter.

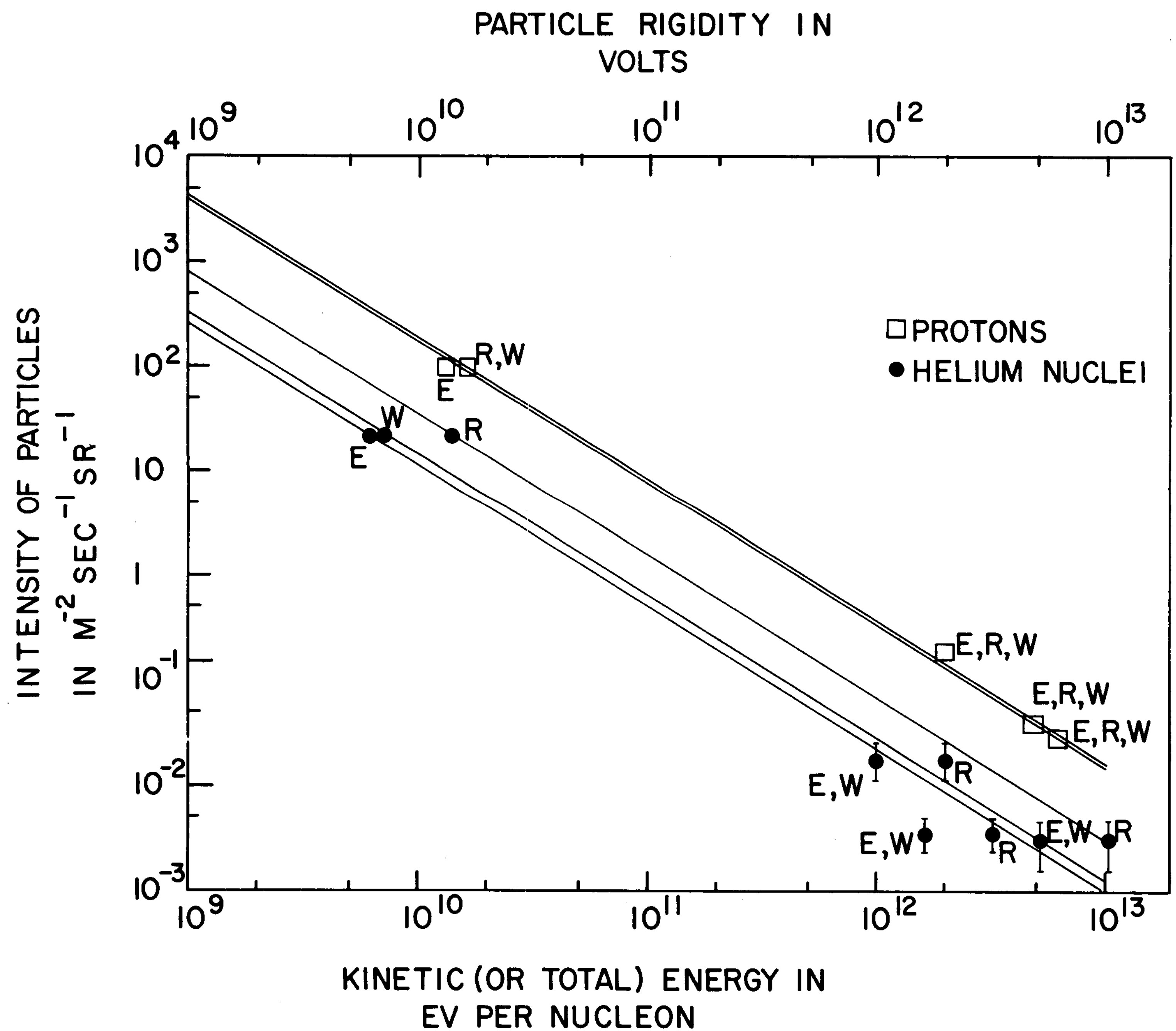
Figure 4: Ratio of intensities of H_1-nuclei to $H_{2,3}$ -nuclei($\Gamma_{H_1,H_{2,3}}$) and H_1-nuclei to α-particles ($\Gamma_{H\,\alpha}$) plotted as a function of kinetic energy, ϵ (Mev/nucleon) of the nucleus. The data shown refer to: (3) Lim and Fukui (1965), (4) Aizu et al. (1960), (6) Webber et al. (1966), (7) Durgaprasad (1966), (8) Koshiba et al. (1963), (9) Fichtel and Reames (1966b) (11) Freier et al. (1966), (19) Daniel and Durgaprasad (1962), (21) Waddington and Freier (1966). The solid lines refer to the calculated values of the ratios obtained using the energy dependent value of x, in the diffusion equation (3), given by the relation $x = \beta x_e$ where βc is the velocity of the nucleus and x_e has values of 2.5 and 3.5 g/cm^2 as explained in the text.

Figure 5: Ratio of intensities of H-nuclei to α-particles ($\Gamma_{H\alpha}$) and L-nuclei to S-nuclei ($\Gamma_{L\,S}$) plotted as a function of kinetic energy, ϵ, (MeV/nucleon), of the nuclei. The data shown refer to: (1) Comstock et al. (1966), (2) Balasubrahmanyan et al. (1966), (3) Lim and Fukui (1965), (4) Aizu et al. (1960), (5) McDonald and Webber (1962), (6) Webber et al. (1966), (7) Durgaprasad (1966), (8) Koshiba et al. (1963), (9) Fichtel and Reames (1966b), (10) Badhwar et al. (1965), (11) Freier et al. (1966), (13) Foster and Debenedetti (1963), (14) Freier et al. (1959), (15) Kerlee et al. (1960), (16) Fichtel et al. (1965), (17) Mathiesen and Stenman (1965), (18) Durgaprasad (1965), (19) Daniel and Durgaprasad (1962), (20) Van Heerden and Judek (1960), (22) Balasubrahmanyan and McDonald (1964), (23) O'Dell et al. (1962). The solid lines refer to the calculated values of these ratios obtained using the energy dependent value of x, (in the diffusion equation (3)), given by the relation, $x = \beta\, x_e$ where βc is the velocity of the nucleus and x_e has values of 2.5 and 3.5 g/cm^2 of hydrogen as explained in the text.

Figure 6: The value of x, the matter traversed by the radiation in interstellar space, plotted as a function of kinetic energy, ϵ , of the particle. The values of x were calculated using the experimentally measured ratios summarized in figures (4) and (5) and using equation (3) (See text). Figures (a), (b), and (c) refer to the three types of spectra namely the power law spectra in rigidity, kinetic energy per nucleon and total energy per nucleon, assumed for the source region.

Figure 7(a): Differential spectrums of protons and helium nuclei measured in 1963 plotted as a function of magnetic rigidity (Mv) of the particle. The curves shown refer to the spectra predicted by Parker's model for values of $K = 2.2$ and $R_o = 1.4$ and x, the interstellar matter traversed. The dotted curve refers to $x = 0$ g/cm^2 and the solid curve to $x = 2$ g/cm^2. The experimental data shown here refer to: Fichtel et al. (1966a). O Protons, 1963, ● Helium nuclei, 1963; Freier and Waddington (1965), □ Protons, 1963, ■ Helium nuclei, 1963, Ormes and Webber (1964), △ Protons, 1963, ▲ Helium nuclei, 1963; Balasubrahmanyan and McDonald (1964) ◇ Protons, 1963, ◆ Helium nuclei, 1963. The Mt. Washington neutron monitor rates corresponding to the respective time periods and the error introduced in combining these data to represent a single spectrum is discussed in detail by Durgaprasad (1966).

Figure 7(b): Differential spectrums of protons and helium nuclei measured in 1961 and 1965 plotted as a function of magnetic rigidity (Mv) of the particle. The curves shown are the spectra predicted by the Parker's model using two values of k and R_o (A: $k=2.2, R_o = 1.2$ and B: $k = 2.4, R_o = 1.7$) and x, the interstellar matter traversed in g/cm^2 of hydrogen. The dotted and solid curves refer to values of $x = 0$ and 2 g/cm^2 of hydrogen respectively. The experimental data shown here refer to: Fichtel et al. (1964) ▽ Protons, 1961, ▼ Helium nuclei, 1961; Balasubrahmanyan et al. (1966), ▷ Protons, 1965, ▶ Helium nuclei, 1965, Ormes and Webber (1966) ◁ Protons, 1965, ◀ Helium nuclei 1965 and Comstock et al. (1966) ⧖ Protons, 1965, and ⧗ Helium nuclei, 1965. The average values of Mt. Washington neutron monitor rates for the time periods 1961 and 1965 are 2148 and 2420 respectively.


PARTICLE RIGIDITY IN
VOLTS
INTENSITY OF PARTICLES
IN M^{-2} SEC^{-1} SR^{-1}
KINETIC (OR TOTAL) ENERGY IN
EV PER NUCLEON
PROTONS
HELIUM NUCLEI
10^{9}
10^{10}
10^{11}
10^{12}
10^{13}
10^{4}
10^{3}
10^{2}
10
1
10^{-1}
10^{-2}
10^{-3}
E
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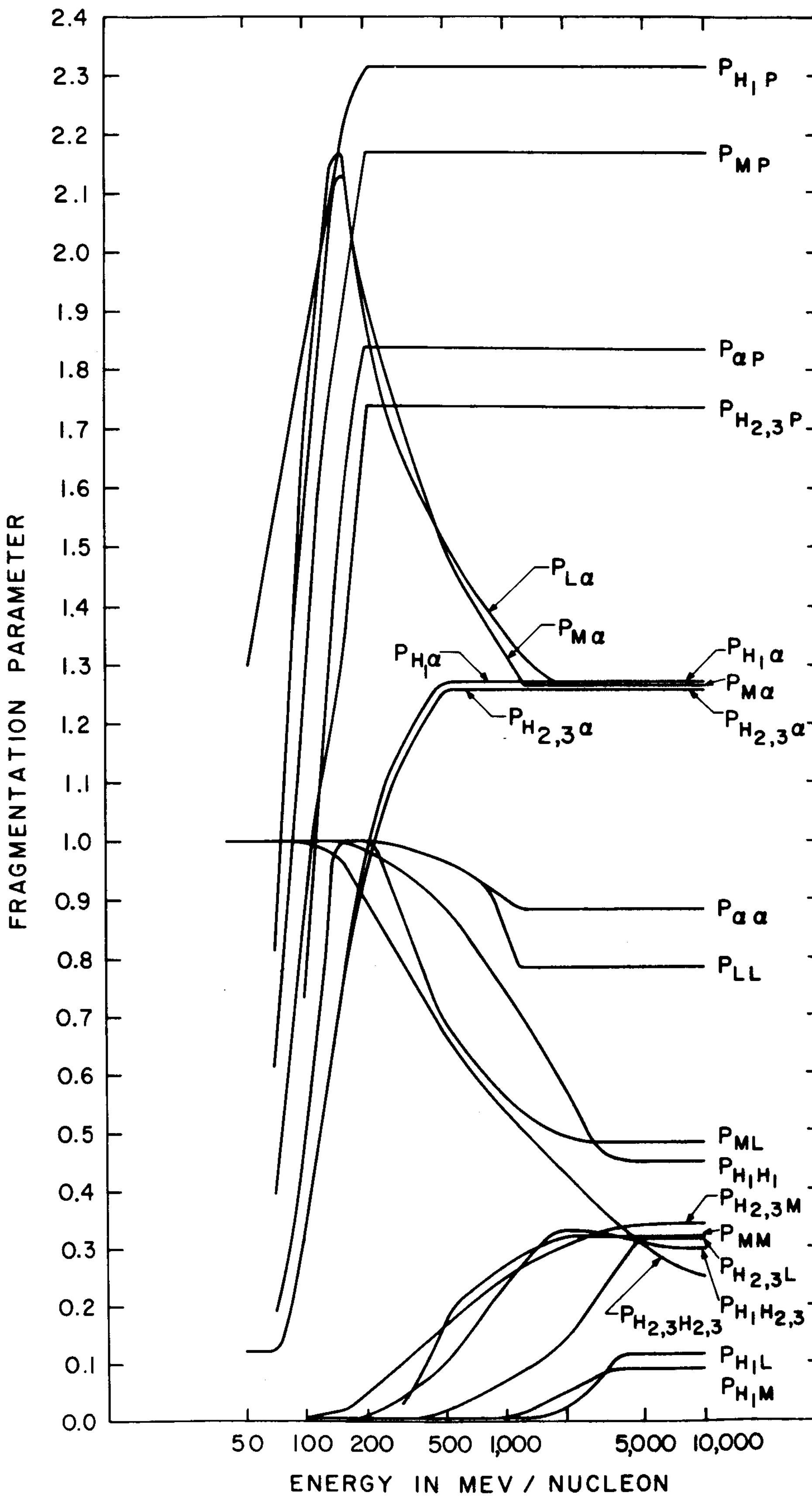
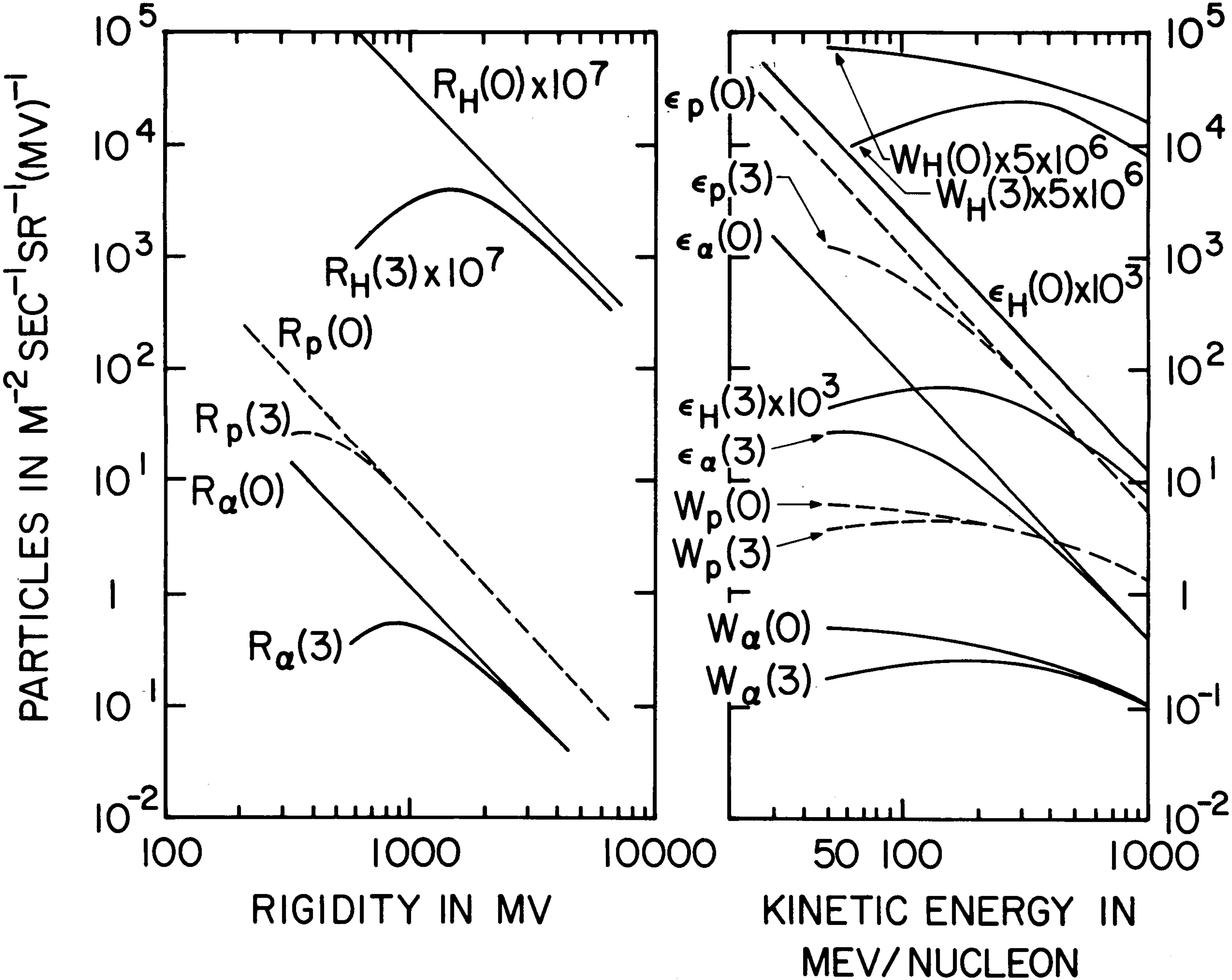
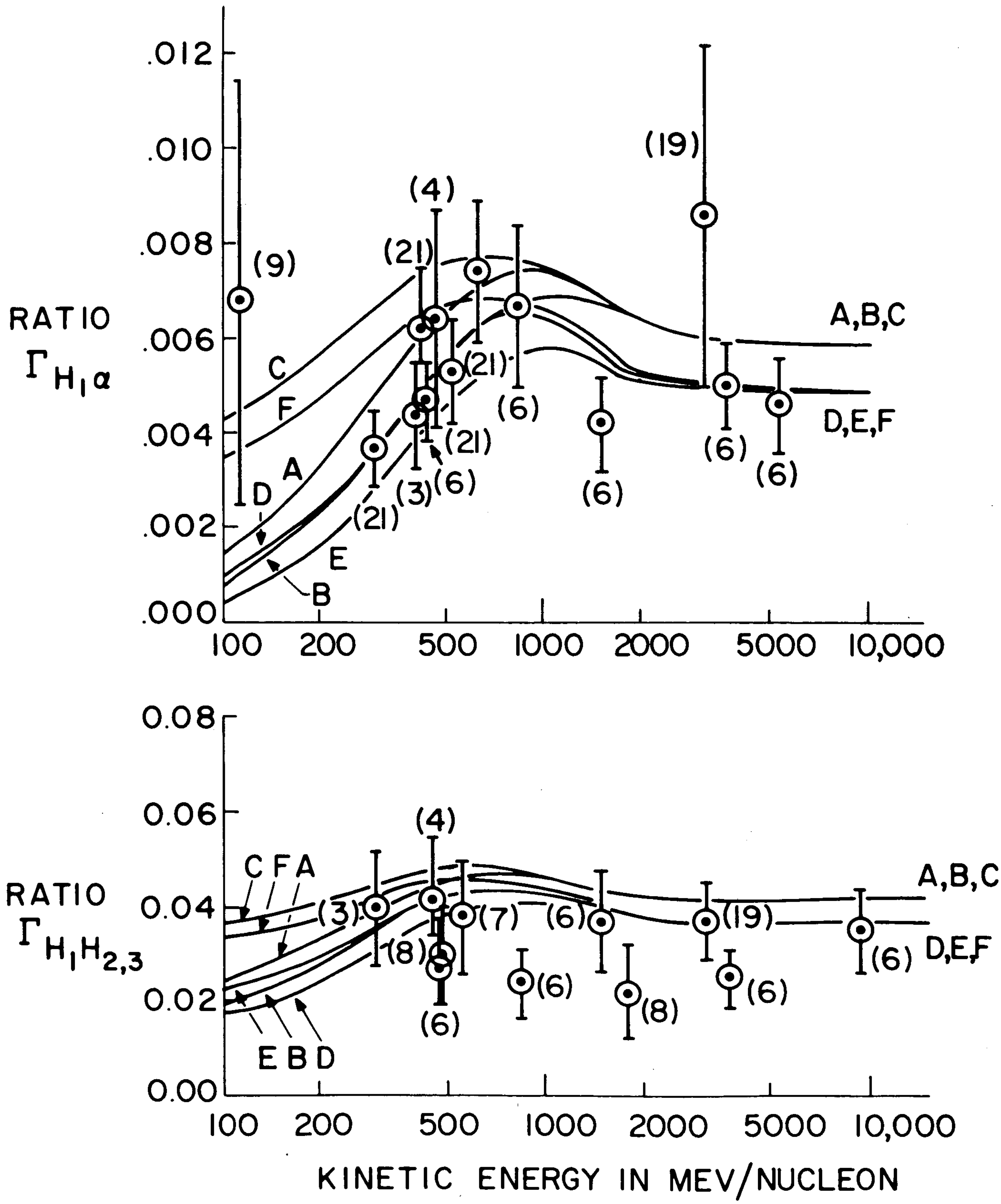
R
R
E,W
R

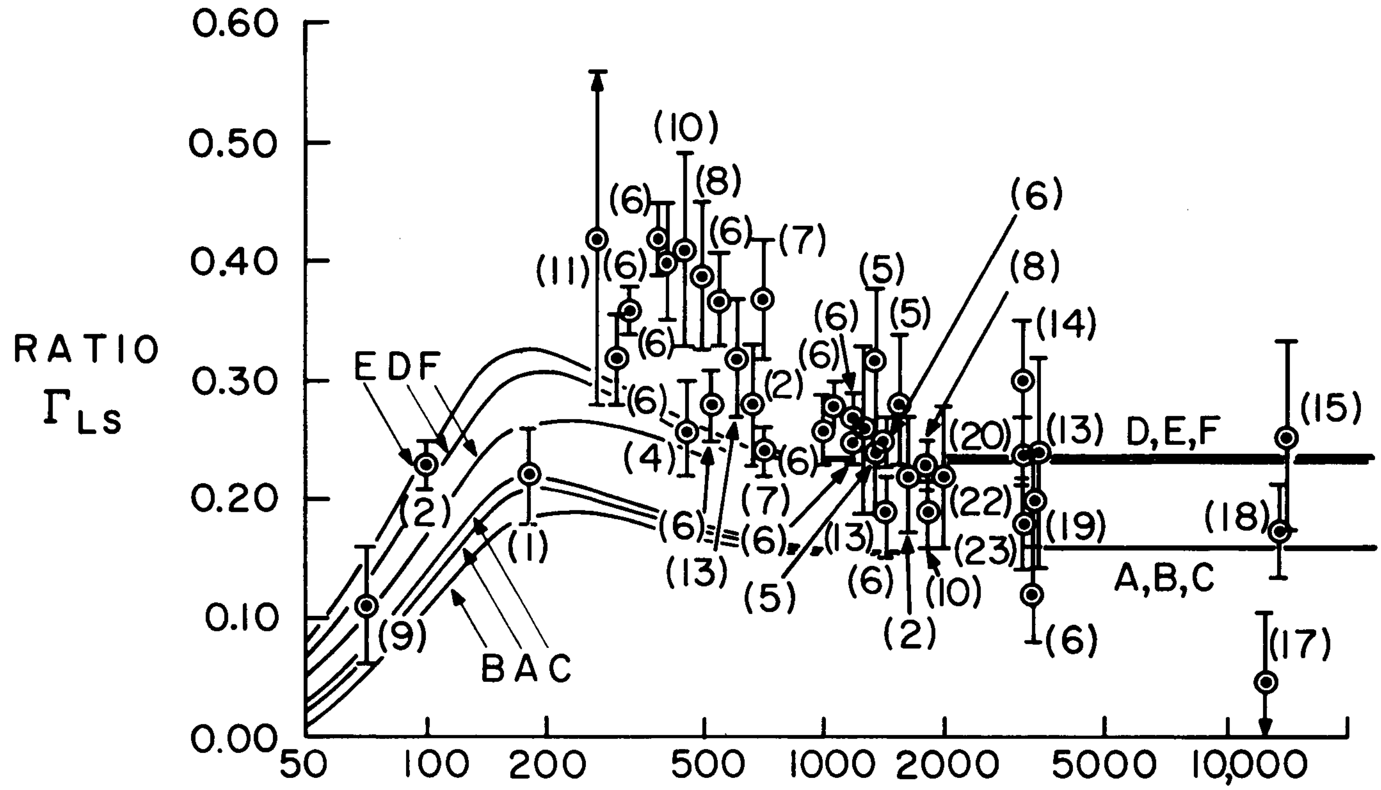


FRAGMENTATION PARAMETER
ENERGY IN MEV / NUCLEON
2.4
2.3
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
50
100
200
500
1,000
5,000
10,000
P_{H_1P}
P_{MP}
$P_{\alpha P}$
$P_{H_{2,3}P}$
$P_{L\alpha}$
$P_{M\alpha}$
$P_{H_1\alpha}$
$P_{H_1\alpha}$
$P_{M\alpha}$
$P_{H_{2,3}\alpha}$
$P_{H_{2,3}\alpha}$
$P_{\alpha\alpha}$
P_{LL}
P_{ML}
$P_{H_1H_1}$
$P_{H_{2,3}M}$
P_{MM}
$P_{H_{2,3}L}$
$P_{H_{2,3}H_{2,3}}$
$P_{H_1H_{2,3}}$
P_{H_1L}
P_{H_1M}

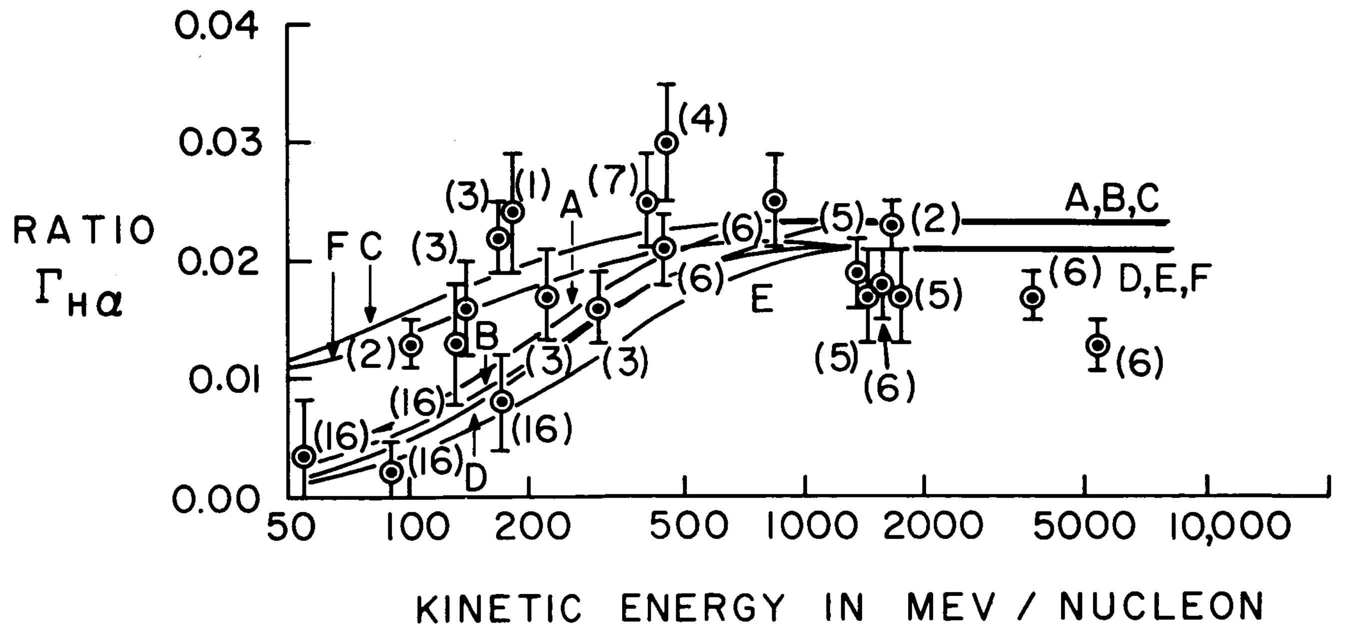

PARTICLES IN M^{-2} SEC^{-1} SR^{-1} (MV)$^{-1}$
10^5
10^4
10^3
10^2
10^1
1
10^{-1}
10^{-2}
$R_H(0) \times 10^7$
$R_H(3) \times 10^7$
$R_p(0)$
$R_p(3)$
$R_\alpha(0)$
$R_\alpha(3)$
100
1000
10000
RIGIDITY IN MV
$\epsilon_p(0)$
$\epsilon_p(3)$
$\epsilon_\alpha(0)$
$\epsilon_H(3) \times 10^3$
$\epsilon_\alpha(3)$
$W_p(0)$
$W_p(3)$
$W_\alpha(0)$
$W_\alpha(3)$
$W_H(0) \times 5 \times 10^6$
$W_H(3) \times 5 \times 10^6$
$\epsilon_H(0) \times 10^3$
50
100
1000
KINETIC ENERGY IN
MEV/NUCLEON



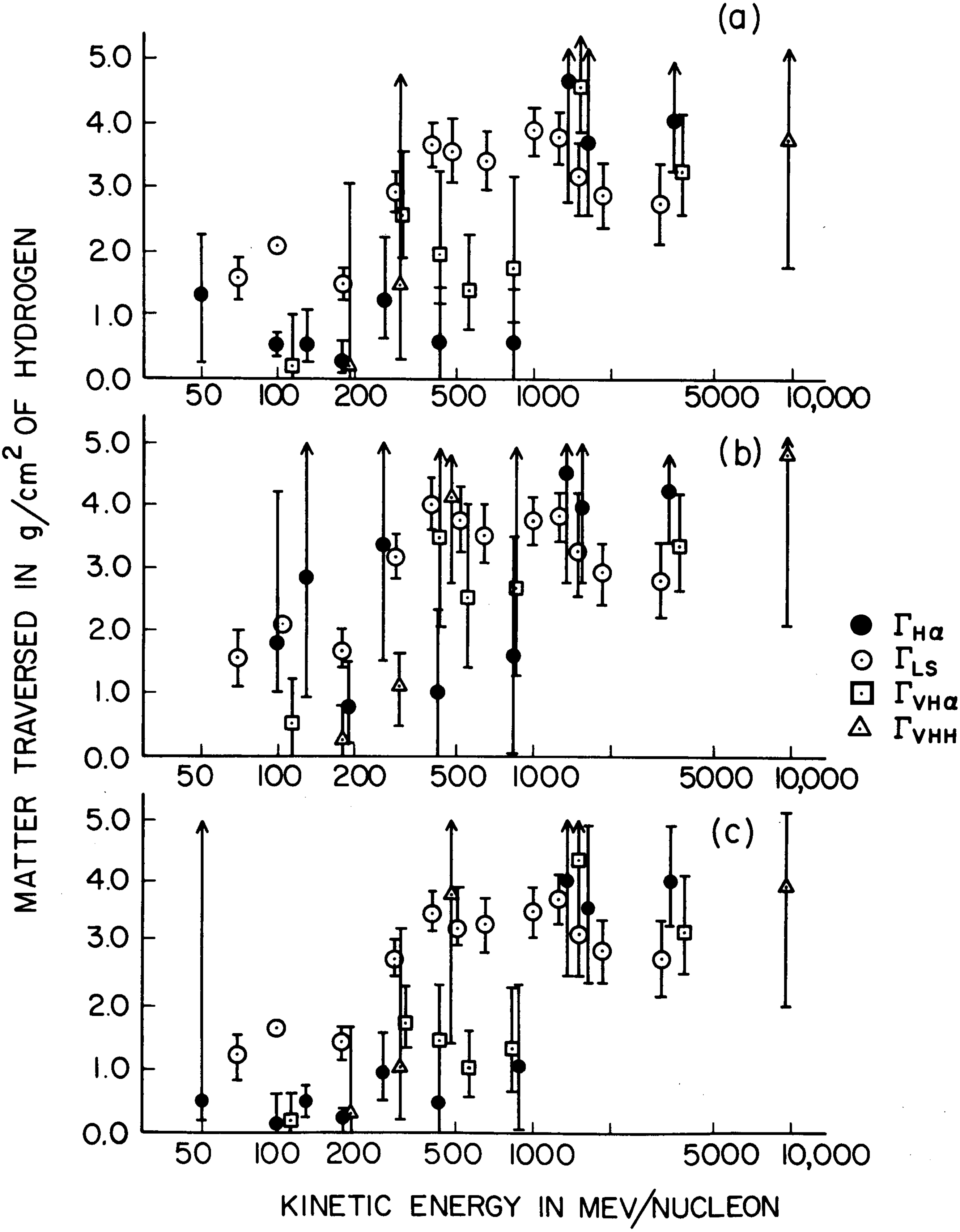
RATIO
$\Gamma_{H_1\alpha}$
.012
.010
.008
.006
.004
.002
.000
(9)
(4)
(21)
(19)
(6)
(3)
C
F
A
D
E
B
A,B,C
D,E,F
100
200
500
1000
2000
5000
10,000
RATIO
$\Gamma_{H_1H_{2,3}}$
0.08
0.06
0.04
0.02
0.00
C F A
E B D
(3)
(4)
(7)
(8)
(6)
(19)
A,B,C
D,E,F
KINETIC ENERGY IN MEV/NUCLEON



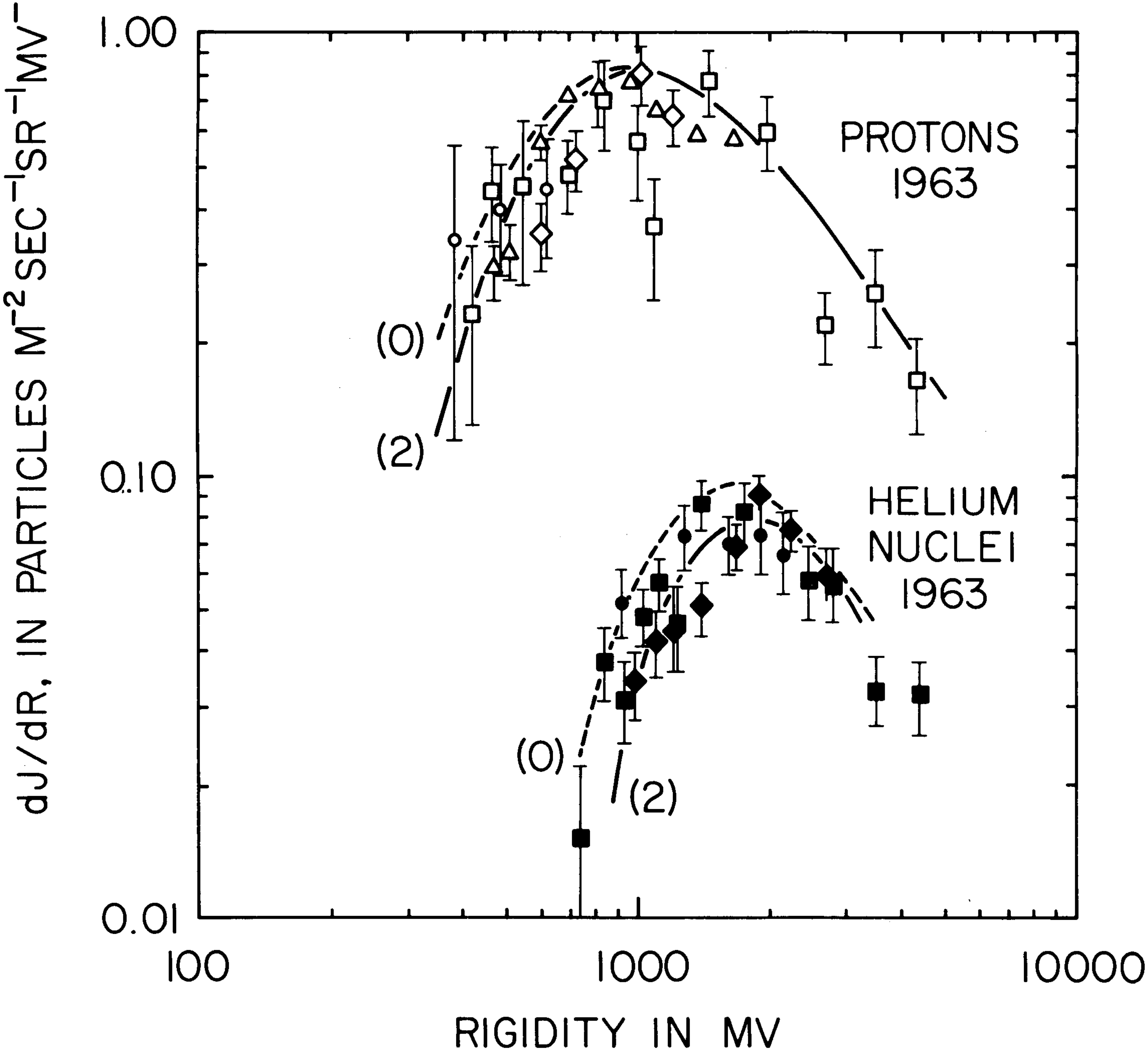
RATIO
Γ_{LS}
0.60
0.50
0.40
0.30
0.20
0.10
0.00
50
100
200
500
1000
2000
5000
10,000
EDF
BAC
D,E,F
A,B,C

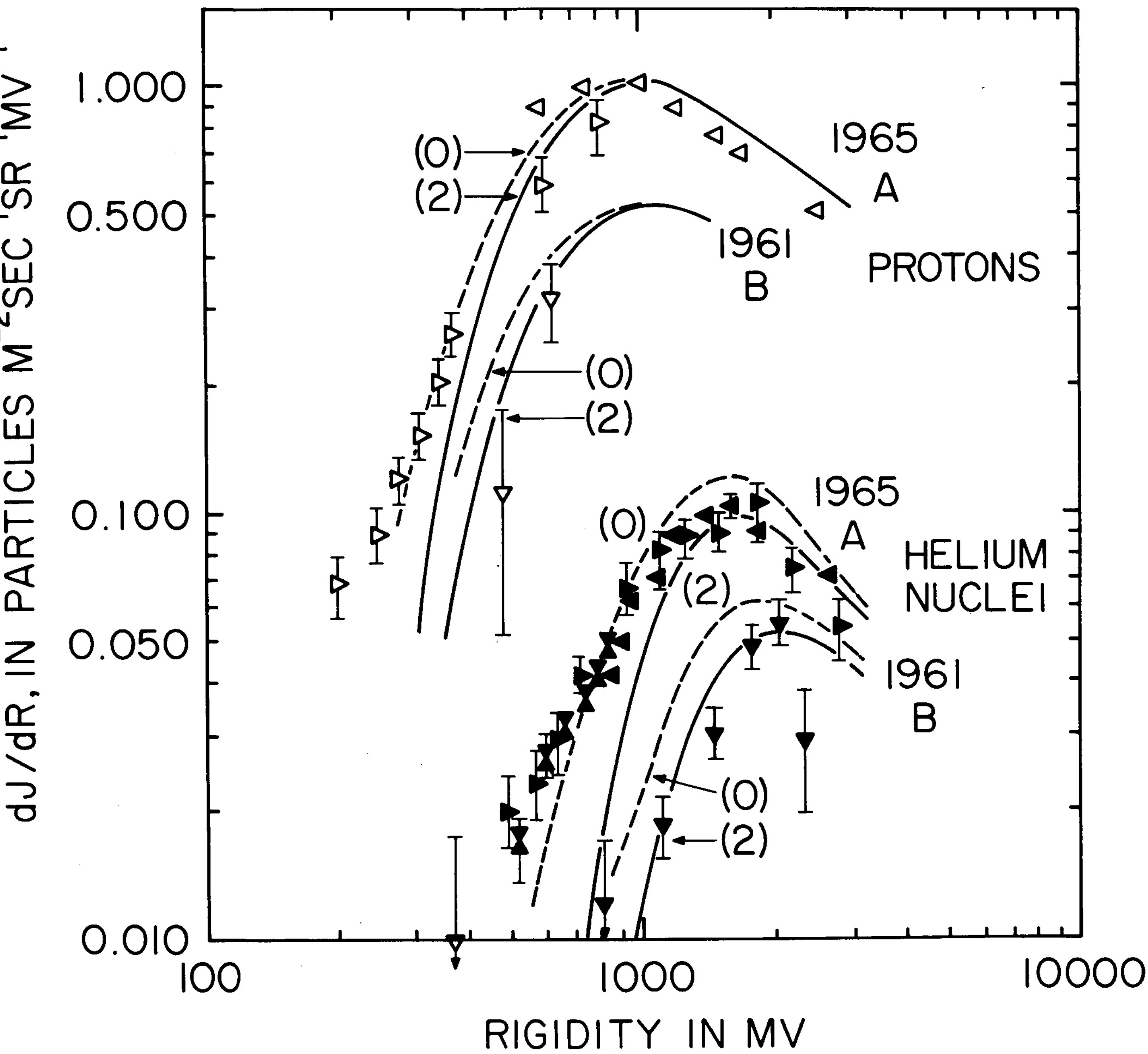


RATIO
$\Gamma_{H\alpha}$
0.04
0.03
0.02
0.01
0.00
50
100
200
500
1000
2000
5000
10,000
FC
A
B
D
E
A,B,C
D,E,F
KINETIC ENERGY IN MEV / NUCLEON



(a)
(b)
(c)
MATTER TRAVERSED IN g/cm^2 OF HYDROGEN
KINETIC ENERGY IN MEV/NUCLEON
0.0
1.0
2.0
3.0
4.0
5.0
50
100
200
500
1000
5000
10,000
$\Gamma_{H\alpha}$
Γ_{LS}
$\Gamma_{VH\alpha}$
Γ_{VHH}


PROTONS
1963
HELIUM
NUCLEI
1963
(0)
(2)
(0)
(2)
1.00
0.10
0.01
100
1000
10000
dJ/dR, IN PARTICLES M^-2 SEC^-1 SR^-1 MV^-1
RIGIDITY IN MV



dJ/dR, IN PARTICLES M^{-2}SEC^{-1}SR^{-1}MV^{-1}
1.000
0.500
0.100
0.050
0.010
100
1000
10000
RIGIDITY IN MV
(0)
(2)
1965
A
1961
B
PROTONS
(0)
(2)
1965
A
HELIUM
NUCLEI
(0)
(2)
1961
B
(0)
(2)