A REPRESENTATION OF THE PERIGEE MOTION OF A SATELLITE AS A FUNCTION OF LOCAL TIME

by

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Introduction

Out of the welter of orbital data from a satellite it is frequently useful to be able to isolate and examine the behavior of one phenomena. We have been particularly interested in knowing where perigee is located at any particular time in the lifetime of the OGO-2 satellite, particularly as a function of local time.

There are several reasons for this. First, the orbital plane of a polar satellite moves very slowly with respect to the earth sun line, i.e. the satellite position varies very slowly in local time. This fact is significant when attempting to determine diurnal effects (as to eliminate such effects) from experimental satellite data. Similarly, most geophysical phenomena are latitude dependent and some phenomena may only be observed at low or high latitudes. (For example, the equatorial electrojet phenomena in magnetic field studies is both local time and latitude dependent). It is therefore important for the experimenter to take orbital factors into account in planning his experiment and in analyzing his results.

If a satellite were in orbit about a perfectly spherical earth with no outside perturbations, its perigee would maintain a fixed latitude and the local time of the orbital plane would depend only on the right ascension of the sun. In fact, however, the earth is not spherical and the orbit is subject to many outside perturbations.

As a result of these perturbations, the orbital elements are no longer constant. By considering a first approximation of those orbital changes which have the major effects on the latitude and local time of perigee, we have devised a simple calculation giving results which, for a nine-month period, check to within one degree in both latitude and local time with a more sophisticated computation.

Method

If we consider three coordinate systems (Figure 1) such that X, Y, Z are the geocentric inertial system of some epoch with X along the equinox, X^1, Y^1, Z^1 are in the plane of the orbit with X^1 toward perigee and Z^1 chosen so that its angle with Z is less than 90^0, and X^2, Y^2, Z^2 are a rotation of X, Y, Z so that X^2 lies along the projection of the earth-sun line in the X, Y plane.

Where:

Ω = right ascension of ascending node of the satellite

ω = argument of perigee

i = inclination

RA = right ascension of the mean sun

The transformation from the X^1 system to the X system consists of three rotations 1) through ω, 2) through i, and 3) through Ω. The transformation from the X to the X^2 system is a rotation through RA. The resulting transformation matrix from the X^1 to the X^2 system is then given by:

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \qquad (1)$$

Where:

$$A_{11} = \cos(\omega)\cos(RA - \Omega) + \sin(\omega)\cos(i)\sin(RA - \Omega)$$

$$A_{12} = -\sin(\omega)\cos(RA - \Omega) + \cos(\omega)\cos(i)\sin(RA - \Omega)$$

$$A_{13} = -\sin(i)\sin(RA - \Omega)$$

$$A_{21} = -\cos(\omega)\sin(RA - \Omega) + \sin(\omega)\cos(i)\cos(RA - \Omega)$$

$$A_{22} = \sin(\omega)\sin(RA - \Omega) + \cos(\omega)\cos(i)\cos(RA - \Omega)$$

$$A_{23} = -\sin(i)\cos(RA - \Omega)$$

$$A_{31} = \sin(i)\sin(\omega)$$

$$A_{32} = \sin(i)\cos(\omega)$$

$$A_{33} = \cos(i)$$

Now if a is the earth central distance to the satellite at perigee then in the Y^1 system perigee is at $\begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}$

Therefore, applying the matrix A, perigee in the X^2 system is

$$\begin{bmatrix} a\left[\cos\omega\cos(RA - \Omega) - \sin\omega\cos(i)\sin(RA - \Omega)\right] \\ a\left[-\cos(\omega)\sin(RA - \Omega) + \sin(\omega)\cos(i)\cos(RA - \Omega)\right] \\ a\left[\sin(i)\sin(\omega)\right] \end{bmatrix} \tag{2}$$

Now in the X^2 system the tangent of (local time - 180°) equals the X^2 coordinate of perigee divided by the Z^2 coordinate of perigee.

or

$$\text{TAN (L.T. - 180°)} = \frac{\sin(\omega)\cos i\cos(RA - \Omega) - \cos\omega\sin(RA - \Omega)}{\sin(\omega)\cos i\sin(RA - \Omega) + \cos\omega\cos(RA - \Omega)} \tag{3}$$

$$\text{Also SIN (Latitude)} = \frac{Z^2 \text{ coordinate of perigee}}{a} = \sin i\sin\omega \tag{4}$$

and

$$\text{TAN(Latitude)} = \frac{\sin i\sin\omega}{(1-\sin^2 i\sin^2\omega)^{\frac{1}{2}}} \tag{5}$$

With the above results, then, we may easily and exactly compute both the latitude and local time of a satellite if we know the values of Ω, ω, i, RA, and a. Normally, however, one is given these values at some epoch, along with values of $\dot{\omega}$ and $\dot{\Omega}$. We have then used the following approximations. If we are given $\dot{\omega}_0$, $\dot{\Omega}_0$, Ω_0, ω_0, i, RA_0, and a at some initial time, t_0, we have assumed i, a, $\dot{\omega}_0$, and $\dot{\Omega}_0$ to remain constant and have computed Ω, ω, and RA at time t using:

$$
\begin{aligned}
\dot{RA} &= 360/365.256 \\
\Omega &= \Omega_0 + \dot{\Omega}_0 (t - t_0) \\
\omega &= \omega_0 + \dot{\omega}_0 (t - t_0) \\
RA &= RA_0 + \dot{RA}_0 (t - t_0)
\end{aligned}
\qquad (6)
$$

and then substituting in (3) and (5) above.

If $\dot{\Omega}$ and $\dot{\omega}$ are unknown, they may be approximated readily.

From (1) page 32:

$$
\begin{aligned}
\dot{\omega} &= \frac{3kN}{a^2(1 - e^2)^2} (2 - 2.5 \sin^2 i) \\
\dot{\Omega} &= \frac{-3kN}{a^2(1 - e^2)^2} \cos(i)
\end{aligned}
\qquad (7)
$$

Where:

$k = .606546$

$N = GM/a^3/2$

G = Universal gravitation constant

M = Mass of the earth

a = Semi-major axis of orbit

e = Eccentricity

Inserting the constants gives

$$\frac{3kN}{a^2(1 - e^2)^2} = \frac{20.8158\ (10^{13})}{a^{3.5}(1 - e^2)^2}$$

where a is in kilometers.

Discussion

Conrath (Reference 1) classifies perturbations as either secular or periodic and states (Pg. 12): "In considering the behavior of the orbit as a whole, the secular perturbations provides the best picture, and the periodic terms can be neglected. However, if the calculations of an ephemeris is desired, the periodic perturbations must be considered." Since we wish to construct a picture of how the orbit is situated with respect to local time, we have considered that a first approximation to the secular terms ($\dot{\omega}$ and $\dot{\Omega}$) is sufficiently accurate. These two variations primarily result from the R^{-3} term in the potential expansion of the earth's gravitational field and, therefore, from the non-sphericity of the earth. From the approximate expressions given in (7) it may be seen that the position of the ascending node regresses in proportion to the cosine of the inclinations, and is therefore quite small for a polar satellite, and that the motion of arguments of perigee depends upon the sign of $(2 - 2.5 \sin^2 i)$. It is thus constant for $i = 63^\circ\ 26'$. Plots of the rate of change of these angles of a function of i may be found in Reference 1, pages 42-48.

For our computations we have used the right ascension of the (fictitious) mean sun (Ref. 4, Pg. 139-141). This brings our computations into agreement with the computation of Universal Time where the mean sun is also used

(Ref. 3, Pg. 139-141, Ref. 4, Pg. 73-76). The actual sun may differ from the mean sun by a maximum of approximately 3° or 12 minutes of local time. In order to compute the "actual" local time the apparent right ascension of the (real) sun should be substituted for our right ascension of the mean sun. The apparent right ascension of the real sun may be found in the American Ephemeris and Nautical Almanac, Pages 18-33; the right ascension of the mean sun is equivalent to the longitude of the mean sun (Ref. 3, Page 140) tabulated on Pg. 50 of the Almanac.

Results

The OGO-2 satellite was launched October 14, 1965 into a low polar orbit with the following characteristics (OGO-2 operations center, Nov. 23, 1965) at epoch October 24th, 0 hrs., 0 min., 0 sec. :

$$a = 7340.5 \text{ km}$$

$$i = 87.359 \text{ degrees}$$

$$RA_O = 208.26 \text{ degrees}$$

$$\Omega_O = 280.49 \text{ degrees}$$

$$\omega_O = 144.211 \text{ degrees}$$

$$\dot{\Omega} = -.2839^{\circ}/\text{day}$$

$$\dot{\omega} = -3.0476^{\circ}/\text{day}$$

Using these figures, we have produced the plot shown in Figure 2. Each dot represents perigee position at zero hours on a particular day with the first day of each month labeled. The black dots indicate perigee in the southern hemisphere and the open dots indicate perigee in the northern hemisphere. The plot is polar with θ representing local time and r representing latitude. A comparison of our results for the OGO-2 satellite with the ORB-1 printout from the GSFC Theory and Analysis Office showed agreement in both latitude and local time within one degree for a period of nine months.

Summary and Conclusions

We have developed a procedure for a graphic display of one specialized feature of a satellite orbit, the relationship of the perigee of the orbit to latitude and to local time. From such a plot (e.g. Fig. 2) it is a very simple matter to see exactly when in a satellite lifetime it will encounter a particular sunlight configuration. One may also readily see at what latitude the satellite approaches closest to the earth. Obvious applications are predictions of eclipse times and periods of maximum drag. From a geophysical viewpoint, one may predict when satellite data should be examined for effects which are local time and/or latitude dependent.

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References

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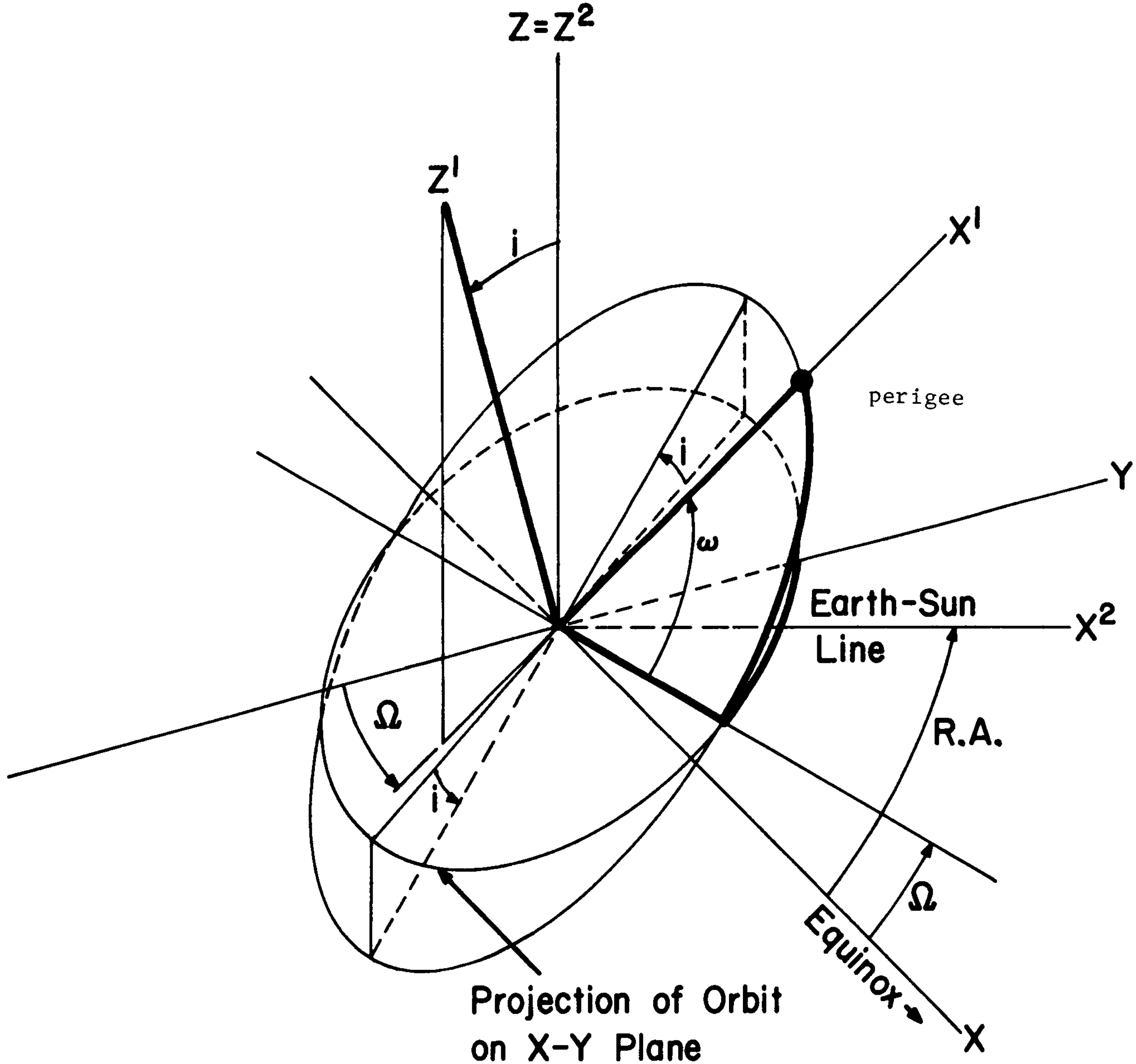
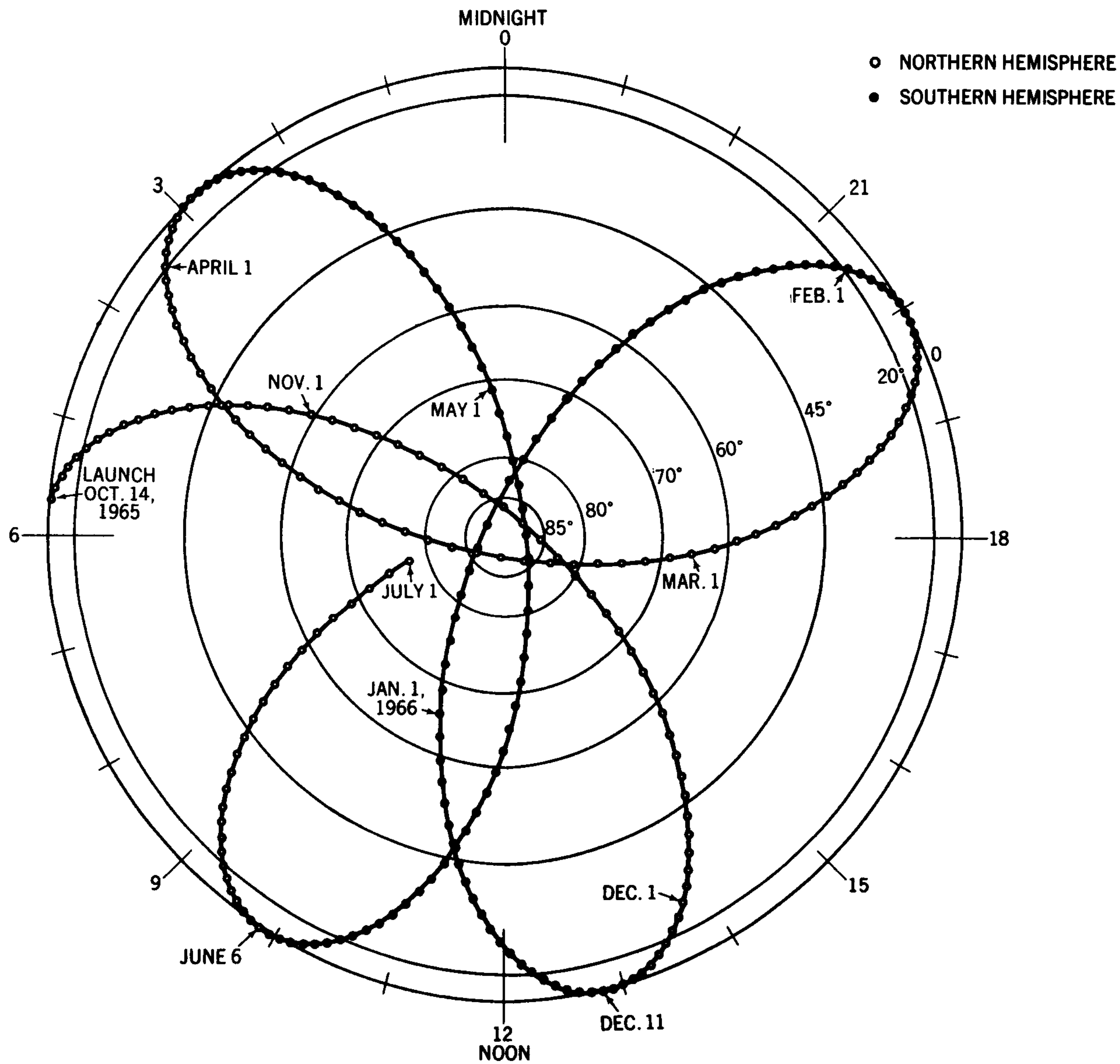

Z=Z²
Z¹
i
X¹
perigee
i
Y
ω
Earth-Sun
Line
X²
Ω
i
R.A.
Ω
Projection of Orbit
on X-Y Plane
Equinox
X

Figure 1



PERIGEE MOTION OF POGO -1 ORBIT
OCT. 14, 1965 - JULY 1, 1966

Figure 2