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AUTOMATICALLY CONTROLLED AIR SPRING SUSPENSION SYSTEM FOR VIBRATION TESTING

by Grayson V. Dixon and Jerome Pearson Langley Research Center Langley Station, Hampton, Va.



NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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SUMMARY

The description and mathematical analysis of a low-frequency air spring suspension system are presented. The suspension system consists of an air spring with a closed-loop position control of air spring height for supporting a test vehicle at various constant weights from 8000 lb (3629 kg) to 100 000 lb (45 359 kg) or decreasing weights at rates up to 670 lb/sec (304 kg/sec) when water is exhausted from the vehicle tanks to simulate propellant use. The system can maintain the air spring height and vehicle elevation by changing spring pressure and resultant spring stiffness with changes in vehicle weight. A vehicle-spring undamped natural frequency, which is low compared with the vehicle minimum elastic mode frequency, is thereby automatically maintained over the entire weight range of the vehicle. By control of spring height, vibration exciters with displacement limits as small as ± 0.25 in. (± 0.635 cm) are kept within their operating range when they are attached to a vehicle of varying weight.

Response equations are developed for position command and force excitation of the suspension system and the system is also represented by equivalent spring-mass-damper configurations. The alternating components of the forces developed by the spring height control system, for sinusoidal excitation of the vehicle, are shown to be negligible for frequencies above the vehicle-spring undamped natural frequency.

Some test results of the system used in vibration testing of a full-scale launch vehicle are included.

INTRODUCTION

The structural dynamic characteristics of launch vehicles and spacecraft are being studied at the Langley Research Center to assist in the understanding, prediction, and reduction of vibratory responses of space vehicles. One such study is concerned with the transmission of longitudinal vibration through a vertically mounted, full-scale launch vehicle. In addition to testing at various constant vehicle weights, the flight condition of varying weight will be simulated by draining water from the vehicle fuel and oxidizer tanks during the vibration tests. Vibratory force will be applied by vibration exciters having displacement limits as small as ± 0.25 in. (± 0.635 cm).

To implement these studies, a low-frequency suspension system was required to support the test vehicle in a nearly constant mean elevation, regardless of vehicle weight changes, in order to keep the exciter completely operable. To minimize the possibility of affecting the structural dynamic characteristics of the vehicle, the vehicle-spring resonant frequency was required to be approximately one-tenth of the vehicle minimum elastic mode frequency for any vehicle weight.

There are several suspension systems available for supporting a vehicle or spacecraft undergoing vibration tests. For transverse vibration, the vehicle may be suspended by elastic cords (ref. 1) or suspended in a special harness (ref. 2). For longitudinal vibration, the vehicle may be mounted directly on the shaft of an exciter (ref. 3), suspended by cables (ref. 4), or positioned on mechanical springs (ref. 5).

Mechanical springs can be designed to produce a low vehicle-spring resonant frequency compared with the vehicle minimum elastic mode frequency. However, mechanical springs cause changes in vehicle elevation (with changes in vehicle weight) which are unacceptable when compared with the permissible travel of an attached exciter. Adjustment or interchange of mechanical springs, to compensate for weight change, is difficult with full-scale vehicles and impossible if the vehicle weight is rapidly changing with time.

The purpose of this paper is to present the description, mathematical analysis, and dynamic test results of an automatically controlled air spring which maintains its mean height with change in vehicle weight and, also, maintains a nearly constant low undamped natural frequency independently of the vehicle weight. The suspension system can be readily scaled for use with vehicles in a large range of sizes and weights.

SYMBOLS

The physical quantities in this paper are given both in the U.S. Customary Units and in the International System of Units (SI). Factors relating the two systems are given in reference 6.

A effective area of spring, inches² (meters²)

 $A_{\rm H}$ effective area of spring at design height, inches² (meters²)

 A_W constant relating change in spring volume with change in spring height, inches² (meters²)

- F force, pounds (newtons)
- \mathbf{F}_{e} sinusoidal excitation force, pounds force (newtons)
- g acceleration of gravity, 386 inches/second² (9.8 meters/second²)
- G_c gain of control valve, pounds mass/second-volt (kilograms/second-volt)

G_f gain of position sensor, volts/inch (volts/meter)

- G_i integral-plus-proportional controller gain, $\frac{G_0 + 1}{G_0 10\sqrt{2}}$, nondimensional
- G_p proportional controller gain, nondimensional
- G_0 loop gain with type 0 control system, $G_f G_p G_c A_H / k_l K_N$, nondimensional
- h height of spring, inches (meters)
- H design height of spring, inches (meters)

- k spring stiffness, pounds force/inch (newtons/meter)
- k_l spring stiffness resulting from change in spring effective area with change in spring height, $l \bar{p}_g$, pounds force/inch (newtons/meter)

 k_s adiabatic spring stiffness for sealed spring, $\gamma A_H A_W \overline{p} / V_H$ or $\gamma A_H \overline{p} / H$, pounds force/inch (newtons/meter)

- $k_{s}(\omega)$ effective adiabatic spring stiffness for spring with bleed nozzle, $k_{s}(\omega) \approx k_{s}$, pounds force/inch (newtons/meter)
- K_N bleed nozzle coefficient, inches²/second (meters²/second)
- constant relating change in spring effective area with change in spring height, inches (meters)

m	mass of vehicle and adapter jig, pounds-seconds 2 /inch (kilograms-seconds 2 /meter)
р	spring absolute air pressure, pounds force/inch 2 (newtons/meter 2)
p _{atm}	atmospheric pressure, pounds force/inch 2 (newtons/meter ²)
pg	spring gage air pressure, pounds force/inch ² (newtons/meter ²)
R	universal gas constant, inches-pounds force/pounds mass- ^O R (meters-newtons/kilograms- ^O K)
s	Laplace operator
t	time, seconds
Т	spring absolute air temperature, ^O R (^O K)
v	volume of air spring, inches ³ (meters ³)
$\mathbf{v}_{\mathbf{H}}$	volume of air spring at design height, inches ³ (meters ³)
w	weight flow, pounds mass/second (kilograms/second)
Ws	weight of air in spring, pounds mass (kilograms)
Wv	weight of vehicle including adapter jig, pounds mass (kilograms)
γ	ratio of specific heats for air, nondimensional
Δ	incremental change in variable with which it is associated
ζ_N	damping coefficient associated with spring wall and bleed nozzle, $\frac{\lambda_w + \lambda_N}{2\tau_n (k_s + k_l)}$, nondimensional
ζw	damping coefficient associated with spring wall, $\frac{\lambda_W}{2\tau_n(k_S + k_l)}$, nondimensional
ζ0	damping coefficient associated with type 0 control system, $\frac{\lambda_0 + \lambda_W + \lambda_N}{2\tau_0 (k_0 + k_s + k_l)}$, nondimensional
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- ζ_1 damping coefficient associated with type 1 control system, $\frac{\lambda_1 + \lambda_w + \lambda_N}{2\tau_1(k_1 + k_s + k_l)}$
- $\lambda_{\rm N}$ damping factor from bleed nozzle and spring wall, $\lambda_{\rm N} \approx \frac{k_{\rm S}}{\tau_{\rm N} \omega^2}$, pounds force-seconds/inch (newtons-seconds/meter)
- λ_{W} damping factor from spring wall, pounds force-seconds/inch (newtons-seconds/meter)
- λ_0 damping factor with type 0 control system, pounds force-seconds/inch (newtons-seconds/meter)
- λ_1 damping factor with type 1 control system, pounds force-seconds/inch (newtons-seconds/meter)

$$\tau_{a}, \tau_{b}$$
 filter time delay, $\tau_{n}\sqrt{2}$, seconds

- $au_{\mathbf{i}}$ controller integral time, seconds
- $\tau_{\rm n}$ time delay associated with vehicle on air spring, $\sqrt{\frac{\rm m}{\rm k_{\rm S}+\rm k_{\rm l}}} = \frac{1}{\omega_{\rm n}}$, seconds

$$\tau_{\rm N}$$
 spring and bleed nozzle time delay, V/K_N γ RT, seconds

$$\tau_{s}$$
 time delay, $\tau_{N}\left(1 + \frac{k_{s}}{k_{l}}\right)$, seconds

- τ_0 time delay associated with type 0 control system, $\sqrt{\frac{m}{k_0 + k_s + k_l}}$, seconds
- τ_1 time delay associated with type 1 control system, $\sqrt{\frac{m}{k_1 + k_s + k_l}}$, seconds
- ω frequency, radian/second
- ω_n undamped natural frequency, radians/second
- $\omega_{\rm u}$ damped natural frequency, radians/second

Subscripts:

С	control valve
N	bleed nozzle
SS	steady-state condition
w	air spring sidewall
0	type 0 control system
1	type 1 control system

A bar above a symbol indicates an average value. Dots with symbols denote time derivatives.

DESCRIPTION OF SUSPENSION SYSTEM

The air spring suspension system and test vehicle with engine removed are shown prior to assembly in figure 1. The air spring consists of two bellows (of the type displayed in fig. 2(a)) which are shown assembled with the adapter jig in figure 2(b). The adapter jig provides upper mounting plates for sealing the bellows and couples the bellows to the gimbal thrust pad of the vehicle. The bottom of the bellows are sealed with mounting plates which are attached to the structural steel stand. Space is provided under the support stand for a vibration exciter which is attached to the adapter jig. Cables provide lateral restraint to the vehicle. The various components of the system are presented schematically in figure 3.

The jig and support stand were designed to have resonant frequencies several times the highest test frequency. The support stand (mounted on a massive foundation) has a maximum deflection of 0.04 in. (0.102 cm) at the maximum vehicle weight of approximately 100 000 lb (45 359 kg).

The air bellows were chosen on the basis of the following requirements:

(1) The air spring must function satisfactorily while supporting a vehicle (in a weight range from 8000 lb (3629 kg) to 100 000 lb (45 359 kg) vibrating at a maximum amplitude of ± 0.1 in. (± 0.254 cm) in the frequency range from 5 cps (5 Hz) to 200 cps (200 Hz).

(2) The undamped natural frequency of the vehicle on the air spring is required to be below 4.5 cps (4.5 Hz) at the minimum vehicle weight and 1.8 cps (1.8 Hz) at the maximum vehicle weight.

(3) A minimum number of bellows, each bellows with the smallest practical area, should be used to minimize the size and weight of the jig.

Since the vehicle-spring resonant frequency could be lowered by the addition of an accumulator, bellows size and maximum load capability became the principal criteria.

The commercially available bellows which most nearly fulfilled the requirements was of 4-ply, 2-convolution, molded-rubber construction and had an unstressed height of 7.25 in. (18.42 cm), a diameter of approximately 22 in. (55.9 cm), and a water volume of 2175 in³ (0.03564 m³) at the unstressed height. Two bellows were required, each with a maximum static load rated by the manufacturer at approximately 50 000 lb (22 680 kg). Weight and size considerations for the jig prevented the use of a tripod arrangement of bellows which would have been the more suitable arrangement from the standpoint of lateral stability.

In order to control the height of the spring, provisions must be made for removing air as well as adding air. A continuous outflow of air from the spring is maintained by a fixed-area bleed nozzle. A solenoid operated valve is used to close the bleed nozzle when the spring is sealed. The height of the spring is sensed by a rectilinear film-type potentiometer mounted on the support stand and powered by a stabilized 100-volt dc supply. The slider of the potentiometer is connected to the adapter jig and its output voltage is proportional to the spring height and vehicle elevation. The spring height voltage is compared with a dc reference voltage corresponding to that of the unstressed spring height, and the difference is the dc error signal to the controller.

The controller operates on the dc error signal and produces a voltage which is filtered and then used as the command to the electropneumatic, servo-positioned cylinder on the control valve. The control valve is a single-seated globe valve which regulates the airflow from a high-pressure source into the air spring.

As a safeguard in the event of air bellows rupture, the system was operated in an enclosed area from which personnel were restricted when the pressure in the spring was above 20 psig (13.8 N/cm²). Relief valves were incorporated to prevent the pressure in the spring from exceeding the manufacturer's pressure rating in the event of a malfunction in the control valve. Mechanical stops were installed on the jig and inside the bellows to limit the maximum travel of the spring to ± 0.5 in. (± 1.27 cm) about its operating height of 7.25 in. (18.42 cm).

ANALYSIS

The equations describing the dynamics of the vehicle and air spring are developed and the control system equations are presented in transfer function form. The suspension system is reduced to a linearized block diagram and to equivalent mass-springdamper configurations for operation at constant weight conditions about the design height of the spring.

Process Equations

The process equations to be developed are considered to be those describing the motion of the spring in response to airflow into the spring through the control valve. For a vehicle resting on the air spring, as shown in figure 3, the equation of motion may be written as

$$\dot{mh} = Ap_{g} + F_{e} - mg - \lambda_{wh}$$
(1)

where m is the mass of the vehicle and adapter jig, h is the height of the spring, A is the effective area of the spring, p_g is the spring gage pressure (a function of h), F_e is the excitation force, g is the acceleration of gravity, and λ_w is the damping factor of the spring sidewalls. Forces proportional to h are included in the term Ap_g .

Manufacturer's tests of the flexible bellows type of air spring have determined the following relations for approximately 5 percent variations in h about the spring design or unstressed height H:

$$\begin{array}{c} V_{H} = A_{W}H \\ V = V_{H} + A_{W} \Delta h \\ A = A_{H} - \ell \Delta h \end{array}$$

$$(2)$$

where $V_{\rm H}$ and $A_{\rm H}$ are the spring volume and spring effective area, respectively, at $\overline{h} = H$, $A_{\rm W}$ is a constant relating the change in spring volume with change in spring height, l is a constant relating the change in spring effective area with change in spring height, V is the volume of the spring, and $\Delta h = h - H$. Values of $A_{\rm H}$ and l are determined in a subsequent section.

Equation (1) and the relation for effective area (eq. (2)) are combined to derive, for small variations about $\overline{h} = H$,

$$m \Delta \dot{h} = A_{H} p_{g} - l \bar{p}_{g} \Delta h + F_{e} - mg - \lambda_{W} \Delta \dot{h}$$
(3)

Since $\Delta p_g \approx 0.02\overline{p}_g$ for $\Delta h = \pm 0.1$ in. (± 0.254 cm), p_g has been replaced by \overline{p}_g in the term $l \overline{p}_g \Delta h$.

The polytropic gas law in derivative form for adiabatic conditions may be written

$$\dot{\mathbf{p}}_{g} = \frac{\gamma \mathbf{p} \dot{\mathbf{W}}_{s}}{\mathbf{W}_{s}} - \frac{\gamma \mathbf{p} \dot{\mathbf{V}}}{\mathbf{V}}$$
(4)

The gas relation for the air in the spring is

$$pV = W_{S}RT$$
 (5)

Equations (4) and (5) are combined and for small variations in p, V, and T at $\overline{h} = H$ the result is

$$p_{g} = \int \left(\frac{\gamma \overline{R} \overline{T} \dot{W}_{s}}{V_{H}} - \frac{\gamma \overline{p} A_{W} \Delta \dot{h}}{A_{W} H} \right) dt - p_{atm} - \frac{W_{v}}{A_{H}}$$
(6)

The net rate of change of air in the spring is

$$\dot{W}_{S} = w_{C} - w_{N} \tag{7}$$

where w_c is the weight flow of air into the spring through the control value and w_N is the weight flow out of the spring through the bleed nozzle.

For critical flow pressure conditions at the bleed nozzle and constant spring air temperature, the following relation may be written for w_N :

$$w_{N} = K_{N}p \tag{8}$$

where K_N is the bleed nozzle coefficient discussed in appendix A.

Equations (3), (6), (7), and (8) are referred to as the process equations and have been represented in transfer form in the block diagram of figure 4.

Control System Equations

In order to maintain a constant spring height for testing at various constant vehicle weight conditions, a type 0 control system was used. The principal elements are a position sensor with a gain G_f , a reference set point voltage E_0 , an error detector and amplifier with a gain G_p , a filter consisting of two adjustable first-order time delays τ_a and τ_b , and an airflow control valve with a gain G_c .

To maintain the spring height under the condition of changing vehicle weight, an integral-plus-proportional controller is added to the type 0 control system. The controller has a reference set point voltage E_1 , an error detector and amplifier with a gain G_i , and an integrator with a gain $1/\tau_i$. Position voltage from the type 0 system is fed to the error detector. This combination is referred to as a type 1 control system.

The equations for the type 0 and type 1 control systems have been represented in transfer function form in the block diagram of figure 4. The following relative values of gains and time delays have been derived in appendix B for the control systems with process for constant weight conditions of the vehicle at $\overline{h} = H$ with $W_V = A_H \overline{p}_g$:

$$\begin{aligned} \mathbf{G}_{0} &= \frac{\mathbf{G}_{f}\mathbf{G}_{p}\mathbf{G}_{c}\mathbf{A}_{H}}{\mathbf{K}_{N}} = \frac{0.224\tau_{N}}{\tau_{a}} \left[1 + \left(\frac{\gamma \mathbf{A}_{H}\overline{\mathbf{p}}}{\mathbf{H}} \middle/ \mathcal{l} \, \overline{\mathbf{p}}_{g} \right) \right] - 1 \\ \mathbf{G}_{i} &= \frac{\mathbf{G}_{0} + 1}{\mathbf{G}_{0}10\sqrt{2}} \\ \tau_{i} &= \tau_{n} \end{aligned}$$

where

$$\tau_{\mathbf{N}} = \frac{\mathbf{V}_{\mathbf{H}}}{\mathbf{K}_{\mathbf{N}}\gamma\mathbf{R}\overline{\mathbf{T}}}$$

 $\tau_a = \tau_b = \sqrt{2}\tau_n$

System gains and performance for variable weight conditions are discussed in appendix C.

Mass-Spring-Damper Representation of Suspension System

For a constant weight vehicle operating about $\overline{h} = H$ the response of the suspension system to sinusoidal force excitation and the spring deflection with weight change are determined in appendix B. The system is represented in this section by the following mass-spring-damper configurations:

(1) Spring sealed ($w_c = w_N = 0$)

$$\begin{array}{c|c} & F_{e} \sin \omega t \\ & & \\ \hline m \\ & -\Delta h \\ H \\ & \lambda_{W} \\ \end{array} \\ \begin{array}{c} k_{s} \\ k_{s} \\ k_{s} \\ \end{array} \\ \begin{array}{c} k_{l} \\ k_{l} \\ \end{array} \\ \end{array}$$

$$\mathbf{m} \ \Delta \mathbf{\ddot{h}} = \mathbf{F}_{\mathbf{e}} \ \sin \omega \mathbf{t} - \lambda_{\mathbf{W}} \ \Delta \mathbf{\dot{h}} - (\mathbf{k}_{\mathbf{S}} + \mathbf{k}_{\mathbf{l}}) \Delta \mathbf{h}$$

$$\left(\frac{\Delta h}{\Delta W_{v}}\right)_{ss} = -\frac{1}{k_{s} + k_{l}} \qquad m = \frac{A_{H}\overline{p}_{g}}{g} \qquad k_{l} = l \,\overline{p}_{g} \qquad k_{s} = \frac{\gamma A_{H}\overline{p}}{H}$$

(2) Spring with constant flow ($w_c = K_N \overline{p} = Constant$)



$$\begin{split} \mathbf{m} \ \Delta \ddot{\mathbf{h}} &= \mathbf{F}_{\mathbf{e}} \ \sin \omega t - \left(\lambda_{\mathbf{N}} + \lambda_{\mathbf{W}}\right) \Delta \dot{\mathbf{h}} - \left[\bar{\mathbf{k}}_{\mathbf{S}}(\omega) + \mathbf{k}_{\overline{\ell}}\right] \Delta \mathbf{h} \\ & \left(\frac{\Delta \mathbf{h}}{\Delta W_{\mathbf{V}}}\right)_{\mathbf{SS}} = -\frac{1}{k_{\overline{\ell}}} \qquad \mathbf{m} = \frac{A_{\mathbf{H}} \overline{\mathbf{p}}_{\mathbf{g}}}{\mathbf{g}} \qquad \mathbf{k}_{\overline{\ell}} = \ell \, \overline{\mathbf{p}}_{\mathbf{g}} \\ & \mathbf{k}_{\mathbf{S}}(\omega) = \frac{\gamma A_{\mathbf{H}} \overline{\mathbf{p}} \tau_{\mathbf{N}} \omega \, \sin\left(\tan^{-1} \tau_{\mathbf{N}} \omega\right)}{\mathbf{H} \left[\left(\tau_{\mathbf{N}} \omega\right)^{2} + 1\right]^{1/2}} \approx \frac{\gamma A_{\mathbf{H}} \overline{\mathbf{p}}}{\mathbf{H}} \\ & \lambda_{\mathbf{N}} = \frac{\gamma A_{\mathbf{H}} \overline{\mathbf{p}} \tau_{\mathbf{N}} \cos\left(\tan^{-1} \tau_{\mathbf{N}} \omega\right)}{\mathbf{H} \left[\left(\tau_{\mathbf{N}} \omega\right)^{2} + 1\right]^{1/2}} \approx \frac{\gamma A_{\mathbf{H}} \overline{\mathbf{p}}}{\mathbf{H} \tau_{\mathbf{N}} \omega^{2}} \qquad \left(\omega \ge \omega_{\mathbf{n}}, \ \frac{\tau_{\mathbf{N}}}{\tau_{\mathbf{n}}} \ge 100, \ \zeta_{\mathbf{N}} << 1\right) \end{split}$$

(3) Spring with type 0 control system (w_c , w_N variable)



$$\begin{split} \mathbf{m} \ \Delta \ddot{\mathbf{h}} &= \mathbf{F}_{\mathbf{e}} \ \sin \omega t - \left(\lambda_{0} + \lambda_{N} + \lambda_{w}\right) \Delta \dot{\mathbf{h}} - \left[\mathbf{k}_{\mathbf{S}}(\omega) + \mathbf{k}_{l} + \mathbf{k}_{0}\right] \Delta \mathbf{h} \\ \left(\frac{\Delta \mathbf{h}}{\Delta W_{v}}\right)_{\mathbf{SS}} &= -\frac{1}{\mathbf{k}_{l} \left(1 + \mathbf{G}_{0}\right)} \qquad \mathbf{m} = \frac{\mathbf{A}_{H} \overline{\mathbf{p}}_{g}}{g} \qquad \mathbf{k}_{l} = l \ \overline{\mathbf{p}}_{g} \\ \mathbf{k}_{\mathbf{S}}(\omega) &\approx \frac{\gamma \mathbf{A}_{H} \overline{\mathbf{p}}}{4\pi} \end{split}$$

$$k_{s}(\omega) \approx \frac{\gamma A_{H} p}{H} \qquad \qquad \left(\omega \ge \omega_{n}, \quad \frac{\tau_{N}}{\tau_{n}} \ge 100, \quad \zeta_{N} << 1 \right)$$
$$\lambda_{N} \approx \frac{\gamma A_{H} \overline{p}}{H \tau_{N} \omega^{2}} \qquad \qquad \left(\omega \ge \omega_{n}, \quad \frac{\tau_{N}}{\tau_{n}} \ge 100, \quad \zeta_{N} << 1 \right)$$

(4) Spring with type 1 control system (w_c , w_N variable)



$$\left| \frac{\Delta h}{\Delta W_{v}} \right|_{ss} = 0 \qquad m = \frac{A_{H}p_{g}}{g} \qquad k_{l} = l \,\overline{p}_{g}$$

$$k_{s}(\omega) \approx \frac{\gamma A_{H}\overline{p}}{H} \qquad \qquad \left(\omega \ge \omega_{n}, \quad \frac{\tau_{N}}{\tau_{n}} \ge 100, \quad \zeta_{N} << 1 \right)$$

$$\lambda_{N} \approx \frac{\gamma A_{H}\overline{p}}{H\tau_{N}\omega^{2}} \qquad \qquad \left(\omega \ge \omega_{n}, \quad \frac{\tau_{N}}{\tau_{n}} \ge 100, \quad \zeta_{N} << 1 \right)$$

The sealed-spring static deflection and dynamic characteristics are analogous to those of a mechanical spring. For this mode the total spring stiffness is the sum of the adiabatic stiffness $k_s = \gamma A_H \overline{p}/H$ and the stiffness caused by the change in spring effective area with change in spring height, $k_l = l \overline{p}_g$. The undamped natural frequency of the vehicle mounted on these parallel springs is

$$\omega_{n} = \frac{1}{\tau_{n}} = \sqrt{\frac{\left(k_{s} + k_{l}\right)g}{m}} = \sqrt{\frac{\left(\frac{A_{H}\overline{p}\gamma}{H} + l\,\overline{p}_{g}\right)g}{A_{H}\overline{p}_{g}}}$$
(9)

Equation (9) demonstrates the ability of the air spring to minimize changes in undamped natural frequency with changes in weight since both spring static load carrying capability and stiffness are functions of pressure. A gradual increase in ω_n results, however, as the pressure ratio $\overline{p}/\overline{p}_g$ increases for a reduction in weight as shown by test results. The range of ω_n may be lowered by increasing the effective volume over that provided by V_H with an accumulator connected to the air bellows. Thus, the suspension system can be scaled for a large range of vehicle sizes and frequency requirements.

The mode employing the uncontrolled spring with flow is not a practical operating condition and is introduced only for the purpose of comparison. The bleed nozzle causes the effective stiffness from the adiabatic spring to become frequency dependent and also adds a frequency dependent damping factor λ_N . With $\tau_N/\tau_n \ge 100$ and $\omega \ge \omega_n$ (where $\lambda_N \ll 1$) the effective adiabatic stiffness becomes $k_s(\omega) \approx A_H \overline{p}/H$ and $\lambda_N \approx k_s/\tau_n \omega^2$ for sinusoidal motion of the spring height.

The modes employing the type 0 and type 1 control systems add spring stiffnesses k_0 and k_1 , respectively, and damping factors λ_0 and λ_1 , respectively, for sinusoidal motions of Δh . These stiffnesses and damping factors are discussed in appendix B and are plotted in figure 5 and figure 6, respectively. A maximum value of k_1 of approximately 5 percent of the air spring stiffness and a maximum value of λ_1 of approximately 1 percent of the system critical damping factor are obtained at $\omega = \omega_n$. As the frequency ratio ω/ω_n increases for sinusoidal excitation, values of k_1 and λ_1 decrease rapidly. The steady-state deflection with type 0 control is a function of the loop gain G_0 , whereas the type 1 control system provides zero steady-state deflection.

Analog Simulation

The suspension system, as shown in figure 4, was simulated on an analog computer. For the simulation, however, V_H was made V a variable function of h, H and \overline{p} became the variables h and p in the term $A_H \overline{p}/H$, and \overline{p}_g became the variable p_g in the term $l \overline{p}_g$. The vehicle was simulated by a single lumped mass which could be decreased at a constant rate. The computer solutions confirmed the accuracy of the approximate system responses equations, for position and force upsets, which were determined from the linearized model.

TESTS

In order to determine the effective area of the spring A_H , the vehicle weight was varied while the spring was maintained at H = 7.25 in. (18.42 cm) by adjusting the command of the type 0 control system E_0 . Pressure p_g was measured at various vehicle weights and has been plotted as a function of weight in figure 7. The inverse of the slope of the line determined A_H to be 552 in² (3561 cm²). This value closely agreed with that obtained from the following calculation:

$$A_{\rm H} = 2 \left[\frac{\pi (\rm Effective \ diameter)^2}{4} \right]$$

where the effective diameter is defined in sketch 1.



Sketch 1

To evaluate the constant l, the spring height was varied about H with the type 0 control system at a constant vehicle weight of 40 750 lb (18 484 kg) and the spring pressure was measured. This constant was also evaluated by varying the spring height, with various known vehicle weights, to maintain a constant pressure of 87.2 lb/in² (60.1 N/cm²). From the plots of vehicle weight and spring pressure as a function of spring height in figure 8, values of l of 40.3 in. (102.4 cm) and 40 in. (101.6 cm), respectively, were determined.

The damped natural frequencies ω_u of the suspension system were determined for a vehicle weight range from 8000 lb (3629 kg) to 80 000 lb (36 287 kg) and are plotted in figure 9. Close agreement is seen for values of ω_u and values of ω_n calculated by means of equation (9). For these tests the vehicle was bounced on the sealed spring. It was found that the system is capable of producing a vehicle-spring undamped natural frequency varying from approximately 1.6 cps (1.6 Hz) at 100 000 lb (45 359 kg) vehicle weight to approximately 2.1 cps (2.1 Hz) at 8000 lb (3629 kg) vehicle weight while automatically maintaining the mean spring height and vehicle elevation.

The response of the system to a step change in height command, with the type 0 control system, is shown in figure 10. For this test $\tau_a = \tau_b = \sqrt{2}\tau_n$ and $G_0 \approx \frac{1}{3} \left(\frac{0.224\tau_s}{\tau_a} - 1 \right)$ and is evaluated at the vehicle test weight of 41 500 lb (18 824 kg). A small amplitude vibration at the frequency $\omega \approx \omega_n$ can be observed on the trace. The limit cycle, shown in the enlargement of the error signal, is caused by nonlinearities in the control valve actuator.

Previous testing of air bellows of the type used in this suspension determined a mean sidewall damping factor of 2.35 lb-sec/in. (412 N-sec/m) for a single bellows. The damping factor is essentially independent of pressure in the bellows. A damping factor of 7 lb-sec/in. (1226 N-sec/m) was determined from bounce tests of the vehicle on the sealed spring. This factor is referred to as λ_W and includes the damping from the cables attached to the vehicle. For the tests, λ_N at $\omega = \omega_n$ and λ_0 had computed maximum values of approximately 2.6 lb-sec/in. (455 N-sec/m) and 38 lb-sec/in. (6655 N-sec/m), respectively.

Testing was conducted with a bleed nozzle coefficient $~K_N~$ of $15\times 10^{-5}~in^2/sec$ (9.68 $\times 10^{-4}~cm^2/sec$) and a computed value of $~\tau_N~$ of 60 seconds.

CONCLUDING REMARKS

An air spring suspension system was designed, analyzed, and tested. The suspension system has been used successfully in a full-scale launch-vehicle test program for various constant-weight vehicles in the range from 8000 lb (3629 kg) to 80 000 lb (36 287 kg).

The system is capable of producing a vehicle-spring undamped natural frequency varying from approximately 1.6 cps (1.6 Hz) at 100 000 lb (45 359 kg) vehicle weight to approximately 2.1 cps (2.1 Hz) at 8000 lb (3629 kg) vehicle weight while automatically maintaining the mean spring height and vehicle elevation.

The suspension with closed-loop control of spring height essentially provides a lightly damped air spring support to the vehicle for sinusoidal force excitation of the vehicle in the frequency range $\omega > \omega_n$ (where ω is the frequency and ω_n is the undamped natural frequency). Equivalent spring stiffness (at $\omega = \omega_n$) resulting from the use of the height control is approximately 5 percent of the air spring stiffness.

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In addition to the damping provided by the bleed nozzle and spring sidewalls, a damping factor (at $\omega = \omega_n$) of approximately 1 percent of the system critical damping factor is contributed by the use of a height control. Both spring and damping effects from the control system are rapidly attenuated for frequencies greater than ω_n .

Langley Research Center,

National Aeronautics and Space Administration, Langley Station, Hampton, Va., October 25, 1966, 124-11-05-13-23.

APPENDIX A

DETERMINATION OF BLEED NOZZLE COEFFICIENT

The type of system designed requires a continuous bleed of air to function properly and the minimum amount of air bleed necessary is now derived.

In appendix C, spring deflection Δh with the type 0 control system is shown to be inversely proportional to the loop gain G_0 . The expression for G_0 may be rewritten

$$G_0 = \frac{0.224\tau_s}{\tau_a} - 1 = \frac{0.224V_H}{K_N\gamma R\overline{T}} \left(1 + \frac{k_s}{k_l}\right) - 1$$

Therefore, the ability of the system to hold a constant air spring height is inversely proportional to K_N .

From the expression

$$\zeta_{0} = \frac{\lambda_{w} + \frac{k_{s}\tau_{n}^{2}}{\tau_{N}} + \lambda_{0}}{2\tau_{0}\left(k_{s} + k_{l} + k_{0}\right)}$$

$$\left(\text{where } \frac{k_{s}\tau_{n}^{2}}{\tau_{N}} = \frac{k_{s}\tau_{n}^{2}K_{N}\gamma R\overline{T}}{V_{H}}\right) \text{ it can be seen that the damping term } \frac{k_{s}\tau_{n}^{2}}{\tau_{N}} \text{ is a direct}$$

function of K_N.

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For a decreasing weight, the minimum value of nozzle coefficient necessary for the controlled system to hold spring height may now be determined. Using equations (4) and (7) with $w_c = 0$ and $\dot{h} = 0$ results in

$$\dot{\mathbf{p}}_{g} = -\frac{\gamma \mathbf{R} \overline{\mathbf{T}}}{\mathbf{V}_{H}} \mathbf{K}_{N} \mathbf{p}$$

Letting $A_H \dot{p}_g = -\dot{W}_V$ and solving for K_N gives

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$$K_{N} = \frac{W_{v}V_{H}}{A_{H}\gamma R \overline{T}p}$$
(A1)

The nozzle coefficient K_N is required to have a value not less than that determined from this equation for the minimum value of pressure.

SYSTEM RESPONSE FOR CONSTANT WEIGHT CONDITIONS

Response equations are developed for height command and force excitation to the system for various constant weight conditions of the vehicle, with the restriction that $\tau_N/\tau_n \ge 100$. Equations are determined from figures 4 and 11 for operation about $\bar{h} = H$ with $W_v = A_H \bar{p}_g$.

Response to Height Command

The transfer function for the linearized process with $F_e = 0$ may be written

$$\frac{\Delta h}{\Delta w_{c}} \approx \frac{A_{H}/K_{N}k_{\ell}}{(\tau_{s}s + 1)\left(\tau_{n}^{2}s^{2} + 2\zeta_{N}\tau_{n}s + 1\right)}$$
(B1)

where

$$\begin{split} \tau_{\rm S} &= \tau_{\rm N} \left(1 + \frac{{\rm k_S}}{{\rm k_l}} \right) \\ \tau_{\rm n} &= \left(\frac{{\rm m}}{{\rm k_S} + {\rm k_l}} \right)^{1/2} \\ \tau_{\rm N} &= \frac{{\rm V_H}}{{\rm K_N} \gamma {\rm R} {\rm \overline{T}}} \\ \zeta_{\rm N} &= \frac{\lambda_{\rm N} + \lambda_{\rm W}}{2 \tau_{\rm n} ({\rm k_S} + {\rm k_l})} \\ \lambda_{\rm N} &\approx \frac{{\rm k_S} \tau_{\rm n}^2}{\tau_{\rm N}} \end{split}$$

From equation (B1) and the transfer function for the type 0 control system,

$$\frac{\Delta h}{\Delta E_0} \approx \frac{G_f G_p G_c A_H / K_N k_l}{\left[\left(\tau_a s + 1 \right) \left(\tau_b s + 1 \right) \left(\tau_s s + 1 \right) \left(\tau_n^2 s^2 + 2\zeta_N \tau_n s + 1 \right) + \frac{G_f G_p G_c A_H}{K_N k_l} \right] G_f}$$
(B2)

The filter, consisting of the two first-order time delays $1/\tau_a s + 1$ and $1/\tau_b s + 1$, is used to provide stability to the system with an increase in the allowable value of loop gain in the numerator of equation (B2). The filter time delays τ_a and τ_b are assigned the relative values $\tau_a = \tau_b = \sqrt{2}\tau_n$ discussed in the next section.

By defining
$$G_0 = \frac{G_f G_p G_c A_H}{K_N k_l}$$
 and letting $G_0 = \frac{0.224 \tau_s}{\tau_a} - 1$, with $\tau_a = \tau_b = \sqrt{2} \tau_n$,

the following approximation was found by analog computer evaluation to be valid:

$$\frac{\Delta h}{\Delta E_0} \approx \frac{G_0}{(1 + G_0)(\tau_n s + 1) \left[\left(2.5\sqrt{2}\tau_n \right)^2 s^2 + 1.5 \left(2.5\sqrt{2}\tau_n \right) s + 1 \right] G_f}$$
(B3)

Equation (B3) has been obtained from equation (B2) with $\tau_n^2 s^2 + 2\zeta_N \tau_n s + 1$ considered to be unity.

The type 1 control system is utilized with G_0 , τ_a , and τ_b having the aforementioned assigned relative values. If τ_i is made equal to τ_n and G_i is made equal to $G_0 + 1/10\sqrt{2}G_0$, the following closed-loop transfer function may be written for the system with type 1 control:

$$\frac{\Delta h}{\Delta E_1} \approx \frac{1}{\left(5\sqrt{2}\tau_n s + 1\right)\left[(5\tau_n)^2 s^2 + \sqrt{2}(5\tau_n)s + 1\right]G_f}$$
(B4)

The assigned values of G_0 , G_i , τ_a , and τ_i provide damping coefficients which are in the optimum range from 0.6 to 0.8 in the second-order characteristics of equations (B3) and (B4).

Response to Excitation Force and Weight Change

Response equations of the system to sinusoidal excitation forces F_e on the vehicle and spring deflection for changes in weight of the vehicle ΔW_V are developed by use of figure 11. In this figure F_0 and F_1 are sinusoidal forces which are developed on the vehicle through use of type 0 and type 1 control systems, respectively. The following modes of the suspension are considered:

(1) spring sealed
$$(w_c = w_N = 0)$$

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(2) spring with constant flow $(w_c = K_N \overline{p} = Constant)$

- (3) spring with type 0 control system $(w_c, w_N \text{ variable})$
- (4) spring with type 1 control system $(w_c, w_N \text{ variable})$

Spring sealed.- The spring deflection for sinusoidal variations in F_e for the sealed spring may be written

$$\left(\frac{\Delta h}{F_{e}}\right)_{s=j\omega} = \frac{1}{\left(k_{s} + k_{l}\right)\left[-\tau_{n}^{2}\omega^{2} + 2\zeta_{w}\tau_{n}(j\omega) + 1\right]}$$
(B5)

where

$$\zeta_{\rm W} = \frac{\lambda_{\rm W}}{2\tau_{\rm n} (k_{\rm S} + k_{\rm l})}$$
$$\tau_{\rm n} = \sqrt{\frac{m}{k_{\rm S} + k_{\rm l}}}$$

The spring deflection for changes in vehicle weight is

$$\left(\frac{\Delta h}{\Delta W_{\rm V}}\right)_{\rm SS} = -\frac{1}{k_{\rm S} + k_{\rm l}} \tag{B6}$$

Spring with constant flow.- The spring deflection for sinusoidal variations in F_e for the spring with constant flow may be written

$$\left(\frac{\Delta h}{F_{e}}\right)_{s=j\omega} = \frac{1}{-m\omega^{2} + \frac{\tau_{N}k_{s}(j\omega)}{\tau_{N}(j\omega) + 1} + \lambda_{w}(j\omega) + k_{l}}}$$
(B7)

or

$$\left(\frac{\Delta h}{F_{e}}\right)_{s=j\omega} \approx \frac{1}{\left(k_{s}+k_{l}\right)\left[-\tau_{n}^{2}\omega^{2}+2\zeta_{N}\tau_{n}(j\omega)+1\right]}$$
(B8)

where

$$\zeta_{N} = \frac{\lambda_{W} + \lambda_{N}}{2\tau_{n}(k_{S} + k_{\ell})} \qquad \qquad \begin{pmatrix} \tau_{N} / \tau_{n} \ge 100 \\ \omega \ge \omega_{n} \\ \zeta_{N} < \tau_{N} \end{pmatrix}$$

The spring deflection for changes in vehicle weight is

$$\left(\frac{\Delta h}{\Delta W_{\rm V}}\right)_{\rm SS} = -\frac{1}{k_{\ell}} \tag{B9}$$

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For the controlled spring in the frequency range $\omega \ge \omega_n$, F_0 and F_1 add to the vehicle components $k_0 \Delta h$ and $k_1 \Delta h$, respectively, which are in phase with Δh and components λ_0 and λ_1 , respectively, which are in phase with $-\Delta h$. The filter coefficients τ_a and τ_b are assigned the relative values of $\tau_a = \tau_b = \sqrt{2}\tau_n$. At these values λ_0 has an approximately maximum value for a given value of G_0 at the frequency $\omega \approx \omega_n$. Spring stiffnesses k_0 and k_1 and damping factors λ_0 and λ_1 are evaluated as functions of frequency (for $1 \le \omega/\omega_n \le 3$) in figure 5 and figure 6, respectively, by using the following relations:

$$\begin{split} \mathbf{k}_{0} &\approx \frac{\mathbf{k}_{\ell} \, \mathbf{G}_{0} \, \sin\left(2 \, \tan^{-1} \tau_{\mathbf{a}} \omega\right)}{\tau_{N} \omega \left[\left(\tau_{\mathbf{a}} \omega\right)^{2} + 1\right]} \\ &\lambda_{0} \approx -\frac{\mathbf{k}_{\ell} \, \mathbf{G}_{0} \, \cos\left(2 \, \tan^{-1} \tau_{\mathbf{a}} \omega\right)}{\tau_{N} \omega^{2} \left[\left(\tau_{\mathbf{a}} \omega\right)^{2} + 1\right]} \\ \mathbf{k}_{1} &\approx \mathbf{k}_{0} + \frac{\mathbf{G}_{i} \mathbf{k}_{\ell} \, \mathbf{G}_{0} \left[\left(\tau_{i} \omega\right)^{2} + 1\right]^{1/2} \sin\left(2 \, \tan^{-1} \tau_{\mathbf{a}} \omega + \tan^{-1} \tau_{i} \omega\right)}{\tau_{i} \tau_{N} \omega^{2} \left[\left(\tau_{\mathbf{a}} \omega\right)^{2} + 1\right]} \end{split}$$

$$\lambda_1 \approx \lambda_0 - \frac{G_i k_{\ell} G_0 \left[\left(\tau_i \omega \right)^2 + 1 \right]^{1/2} \cos\left(2 \tan^{-1} \tau_a \omega + \tan^{-1} \tau_i \omega \right)}{\tau_i \tau_N \omega^3 \left[\left(\tau_a \omega \right)^2 + 1 \right]}$$

$$\omega \ge \omega_n \qquad G_0 = 0.224 \frac{\tau_s}{\tau_a} - 1$$
$$\frac{\tau_N}{\tau_n} \ge 100 \qquad \tau_n = \frac{\tau_a}{\sqrt{2}} = \tau_i$$
$$\overline{h} = H \qquad G_i = \frac{G_0 + 1}{10\sqrt{2}G_0}$$

Spring with type 0 control system.- The spring deflection with the type 0 control system for sinusoidal variations in F_e is

$$\left(\frac{\Delta h}{F_{e}}\right)_{s=j\omega} \approx \frac{1}{\left(k_{0} + k_{s} + k_{l}\right)\left[-\tau_{0}^{2}\omega^{2} + 2\zeta_{0}\tau_{0}(j\omega) + 1\right]}$$
(B10)

where

$$\begin{aligned} \zeta_{0} &= \frac{\lambda_{0} + \lambda_{N} + \lambda_{w}}{2\tau_{0} \left(k_{0} + k_{s} + k_{l} \right)} \\ \tau_{0} &= \sqrt{\frac{m}{k_{0} + k_{s} + k_{l}}} \end{aligned} \qquad \begin{pmatrix} \tau_{N} \\ \overline{\tau_{n}} \ge 100 \\ \omega \ge \omega_{n} \\ \zeta_{0} << 1 \end{pmatrix} \end{aligned}$$

The spring deflection for changes in vehicle weight is

$$\left(\frac{\Delta h}{\Delta W_{\rm V}}\right)_{\rm SS} = -\frac{1}{k_{\ell}\left(1 + G_0\right)} \tag{B11}$$

Spring with type 1 control system.- The spring deflection with the type 1 control system for sinusoidal variations in $\ F_e$ is

$$\left(\frac{\Delta h}{F_{e}}\right)_{s=j\omega} \approx \frac{1}{\left(k_{1}+k_{s}+k_{l}\right)\left[-\tau_{1}^{2}\omega^{2}+2\zeta_{1}\tau_{1}(j\omega)+1\right]}$$
(B12)

where

$$\zeta_{1} = \frac{\lambda_{1} + \lambda_{N} + \lambda_{W}}{2\tau_{1} \left(k_{1} + k_{s} + k_{l} \right)} \qquad \qquad \left(\begin{array}{c} \frac{\tau_{N}}{\tau_{n}} \ge 100\\ \omega \ge \omega_{n}\\ \zeta_{1} < 1 \end{array} \right)$$

The spring deflection for changes in vehicle weight is

$$\left(\frac{\Delta h}{\Delta W_V}\right)_{SS} = 0 \tag{B13}$$

APPENDIX C

SYSTEM RESPONSE FOR VARIABLE WEIGHT CONDITIONS

Dynamic and steady-state spring deflections are determined for constant rates of reduction in vehicle weight. The type 1 control system is used with a control valve whose gain is proportional to valve flow for gain compensation of the system.

During the propellant flow simulation, water will be exhausted from the vehicle lox and fuel tanks in opposing horizontal streams. The rate of change of water level in the tanks, relative to the vehicle, is considered to be sufficiently small so that the vertical force on the vehicle resulting from the change in momentum may be neglected.

For dynamic testing the vehicle weight range is from 20 percent to 80 percent of maximum vehicle weight for weight reduction rates not to exceed 1 percent of maximum vehicle weight per second. The time delay τ_n varies approximately ± 5 percent from its value at 50 percent of maximum vehicle weight. However, it is assumed to be constant for the determination of the deflection equations. Percentage variation in τ_n is obtained from calculations by using the values of l and A_H determined in the section entitled "TESTS."

The following relative gains and time delays (from appendix B) will be used for dynamic testing:

$$G_0 = \frac{0.224\tau_s}{\tau_a} - 1$$
$$G_i = \frac{G_0 + 1}{G_0 10\sqrt{2}}$$
$$\tau_i = \tau_n$$
$$\tau_a = \tau_b = \sqrt{2}\tau_n$$

where G_0 , G_i , τ_i , and τ_a are evaluated at the 50 percent of maximum weight condition.

Gain Compensation With Nonlinear Valve

From the process transfer function of equation (B1), the static gain of the process is

$$\frac{A_{\rm H}}{K_{\rm N} l \, \overline{\rm p}_{\rm g}}$$

from which $G_0 = \frac{G_f G_p G_c A_H}{K_N l \, \overline{p}_g}$ is seen to vary inversely with vehicle weight and resulting

spring pressure \overline{p}_g . Significant changes in G_0 and the system stability result from large weight changes. Gain compensation is accomplished by use of a control valve which has a gain G_c that is directly proportional to valve weight flow w_c . Rewriting the definition of G_0 with $G_c \propto w_c$ and $w_c = K_N \overline{p}$ for steady-state conditions gives

$$G_0 \propto \frac{G_f G_p A_H \overline{p}}{l \, \overline{p}_g} \tag{C1}$$

The variation in G_0 , as computed from relation (C1) with the values of l and A_H previously determined, is approximately ± 10 percent of its value at the 50 percent of maximum weight condition.

Spring Deflection With Nonlinear Valve

From figure 11, the spring deflection $\Delta h_{SS}(t)$ caused by the control value with the initial transient neglected is

$$\frac{\Delta h_{ss}(t)}{\dot{W}_{v}} \approx \frac{\tau_{i} \bar{p}_{g}}{k_{l} G_{0} G_{i} \left(\bar{p}_{g} - \frac{\dot{W}_{v}}{A_{H}} t \right)}$$
(C2)

where t > 0. In equation (C2) $k_{\tilde{l}}$ and G_0 (computed with the appropriate value of G_c) are considered to be constant at the 50 percent of maximum weight condition for the approximation. The spring pressure \overline{p}_g is evaluated at the time of initiation of weight reduction, t = 0.

Dynamic Spring Deflection

In addition to the spring deflection from the nonlinear control value a dynamic deflection occurs following initiation and termination of weight reduction. From figure 11 the dynamic deflection following initiation of weight reduction is approximated, for $\zeta >> 1$, by

$$\frac{\Delta h(t)}{\dot{W}_{V}} \approx \frac{\tau_{1}^{3} e^{-\zeta_{1} t/\tau_{1}}}{m} \sin\left(\frac{t}{\tau_{1}}\right) + \frac{4.48\tau_{N} e^{-t/5\sqrt{2}\tau_{n}}}{k_{\ell} \left(G_{0} + 1\right)} \left[1 + 1.03 \sin\left(\frac{t}{5\sqrt{2}\tau_{n}} - 1.33\right)\right]$$
(C3)

APPENDIX C

The maximum value of dynamic deflection, as governed by the last term of equation (C3) occurs approximately at a time $t = 1.36/5\sqrt{2}\tau_n$ after initiation of weight reduction.

The dynamic deflection following termination of weight reduction may be approximated, for $\zeta \ll 1$, by

$$\frac{\Delta h(t)}{\dot{W}_{V}} \approx -\frac{\tau_{1}^{3}e^{-\zeta_{1}t/\tau_{1}}}{m} \sin\left(\frac{t}{\tau_{1}}\right) - \frac{4.48\tau_{N}e^{-t/5\sqrt{2}\tau_{n}}}{k_{l}(G_{0}+1)} \left[1 + 1.03\sin\left(\frac{t}{5\sqrt{2}\tau_{n}} - 1.33\right)\right]$$
(C4)

The maximum dynamic deflection, as governed by the last term of equation (C4) occurs approximately at a time, $t = 1.36/5\sqrt{2}\tau_n$ after termination of weight reduction. The coefficients τ_n and G_0 are evaluated at the 50 percent of maximum vehicle weight condition, whereas τ_1 , ζ_1 , m, and k_{ℓ} are determined for the conditions at the initiation and termination of weight reduction given by equations (C3) and (C4), respectively.

REFERENCES

- Carden, H. D.; and Herr, R. W.: Vibration Studies on a Simplified 1/2-Scale Model of the Nimbus Spacecraft. Shock, Vibration and Associated Environments, Bull. No. 33, Pt. II, U.S. Dept. Defense, Feb. 1964, pp. 57-71.
- Mixson, John S.; Catherine, John J.; and Arman, Ali: Investigation of the Lateral Vibration Characteristics of a 1/5-Scale Model of Saturn SA-1. NASA TN D-1593, 1963.
- 3. Hord, Warner H., Jr.: Environmental Test Program for Ariel I. NASA TN D-2099, 1964.
- Beam, Everette E.: A Comparison of the Flight Evaluation of the Vehicle Bending Data With the Theoretical and Dynamic Test Results for the Saturn I Vehicle. Shock, Vibration and Associated Environments, Bull. No. 33, Pt. II, U.S. Dept. Defense, Feb. 1964, pp. 130-149.
- 5. Albert, R. S.: Agena Tank Modal Test. Rept. No. SS/788/5522 (Contract AF 04(695)-191), Lockheed Missiles & Space Co., July 17, 1964. (Available From DDC as AD 461821.)
- 6. Mechtly, E. A.: The International System of Units Physical Constants and Conversion Factors. NASA SP-7012, 1964.



Figure 1.- Test vehicle, air spring, and support stand.

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(a) Molded rubber air bellows.

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Figure 2.- Air bellows and adapter jig.



(b) Adapter jig.

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Figure 2.- Concluded.



Figure 3.- Schematic of vehicle and suspension system.



Figure 4.- Linearized process and control system for operation about $\overline{h} = H$.



Figure 5.- Spring stiffnesses from control systems. $\overline{h} = H$; $G_0 = 0.224(\tau_s/\tau_a) - 1$; $\tau_n = \tau_a/\sqrt{2} = \tau_i$; $G_i = G_0 + 1/10\sqrt{2}G_0$.



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Figure 6.- Damping factors from control systems. \bar{h} = H; $G_0 = 0.224(\tau_s/\tau_a) - 1$; $\tau_n = \tau_a/\sqrt{2} = \tau_i$; $G_i = G_0 + 1/10\sqrt{2}G_0$.



Figure 7.- Variation of spring pressure with vehicle weight for operation about \overline{h} = H = 7.25 in. (18.42 cm).



Figure 8.- Effective area characteristics of air spring.



Vehicle weight, Wv

Figure 9.- Experimental and calculated suspension system frequencies with air spring sealed and \bar{h} = H = 7.25 in. (18.42 cm).



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Figure 10.- Air spring response to step in command E_0 . $W_V = 41500$ lb (18824 kg).

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(a) Rearrangement of figure 4 with type 0 control system.



(b) Rearrangement of figure 4 with type 1 control system.

Figure 11.- Linearized process with control system for operation about \overline{h} = H.

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