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STUDIES OF OPTIMUM BODY SHAPES AT HYPERSONIC SPEEDS

by

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SUMMARY

The present study was directed toward the questions that arise in the application of optimum bodies to the design of hypersonic cruise aircraft. The considerations were divided into two parts. The first involved the calculated minimum-drag characteristics of four families of slender bodies for Mach numbers from 2 to 12. The second concerned the experimental evaluation of the effects of body cross-sectional shape on the aerodynamic performance of bodies at a Mach number of 10. The constraints in each study were body length and volume, although the constant values are different in each part of the study.

INTRODUCTION

Hydrogen-fueled hypersonic-cruise aircraft require very large volume bodies, mainly to accommodate the low-density liquid fuel. Such bodies because of their size are especially unattractive from the standpoint of drag, but can be useful in providing significant contributions to the lift of the aircraft. Body profiles that provide minimum drag or maximum lift, or both, are of particular interest to the designer. Attention is generally given to the theoretical minimum wave-drag body profiles. Such profiles have been derived for use at low-supersonic, hypersonic, and low-hypersonic Mach numbers. (See refs. 1 to 5.) The application of these profiles, however, introduces many questions that must be answered. Some of these are: What profile should be used? Should it be one derived for low-supersonic Mach numbers or one derived for hypersonic Mach numbers. Can some base area be permitted, since by this means the wave drag can be reduced for a body of given length and volume? What is the effect of cross-sectional shape? Can the cross-sectional shape be altered appreciably to provide improved lift characteristics of the body without seriously affecting the drag characteristics?

It is the intent of this paper to provide a few answers to these questions. This will be done in two parts. The first part has been prepared at

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the Ames Research Center and deals with the calculated minimum drag characteristics of four families of slender bodies. The second has been prepared at the Langley Research Center and is concerned with the experimental effects of body cross-sectional shape on the aerodynamic performance of bodies.

NOTATION

b	span of body base
C_D	drag coefficient
$C_{D_{min}}$	minimum drag coefficient
C_{D_o}	drag coefficient at zero lift
C_L	lift coefficient
$C_{L(L/D)_{max}}$	lift coefficient for maximum lift-drag ratio
C_{L_α}	lift-curve slope, $\frac{dC_L}{d\alpha}$
C_m	pitching-moment coefficient
h	height of body base
k	body cutoff, $1 - \frac{l}{l_o}$, amount of afterbody portion of fully closed body cut off to provide a body with a base; 0 for a fully closed body and 0.5 for a body with maximum diameter at the base
$\frac{L}{D}$	lift-drag ratio, $\frac{C_L}{C_D}$
$\left(\frac{L}{D}\right)_{max}$	maximum lift-drag ratio, $\left(\frac{C_L}{C_D}\right)_{max}$
l	actual body length
l_o	length of fully closed body, virtual length
M	Mach number
n	exponent of power body equation, $\eta = \xi^n$
R_l	Reynolds number based on body length
r	body radius

r_o maximum body radius

V volume of body

x longitudinal distance along body axis

Z geometric altitude

α angle of attack, deg

η dimensionless radial coordinate of body, $\frac{r}{r_o}$

ξ dimensionless longitudinal coordinate of body, $\frac{x}{l_o}$

RESULTS AND DISCUSSION

Calculated Minimum Drags of Slender Bodies

The total drag characteristics of four families of slender bodies at zero incidence were calculated for Mach numbers from 2 to 12. The length and volume of each of the bodies were held constant such that $V = 0.002655l^3$. Fineness ratios of the bodies ranged from about 12.5 to 14. The study of reference 6 has shown that fineness ratios within this range are the most favorable for hypersonic transport aircraft.

Body families.- Each family is composed of a series of bodies having various base areas formed by cutting off given amounts of the afterbody of a fully closed body. This is illustrated in figure 1. The fully closed body at the top of the figure has no cutoff and is designated $k = 0$. To form the other bodies, the same closed profile was adjusted in diameter and was stretched to such an extent that when 0.1, 0.3, and 0.5 of the virtual lengths were cut off, each remaining body had the same length and volume as the original closed body with no cutoff, but each had different amounts of base area. Four particular profiles were selected to make up the different families under this arrangement of body cutoffs.

1. The Sears-Haack profile optimized for a given length and volume (refs. 2 and 3) and defined by the equation:

$$\eta = [1 - (1 - 2\xi)^2]^{3/4}$$

2. A parabolic-arc profile defined by the equation:

$$\eta = 4\xi(1 - \xi)$$

3. One of Miele's profiles optimized for a given surface area and volume (ref. 7) and defined by the equation:

$$\eta = 1 - (1 - 2\xi)^{3/2} \qquad [\text{Forebody only, } 0 \leq \xi \leq 0.5]$$

4. The von Kármán profile optimized for a given length and diameter (ref. 1) and defined by either of the equations:

$$\left.\begin{array}{l} \eta = \left[\cos^{-1}(1 - 4\xi) - 2(1 - 4\xi)\sqrt{2\xi(1 - 2\xi)}\right]^{1/2} \\ \text{or} \\ \eta = \pi^{-(1/2)}\sqrt{\theta - \frac{1}{2}\sin 2\theta} \\ \text{where} \\ \xi = \frac{1}{4}(1 - \cos\theta) \end{array}\right\} \begin{array}{c} \text{Forebody only,} \\ 0 \leq \xi \leq 0.5 \end{array}$$

The afterbodies of the last two profiles are undefined by the equations (i.e., for $0.5 < \xi \leq 1.0$), but each has zero profile slope at $\xi = 0.5$. Therefore, each profile was placed back-to-back in order to form a closed basic body.

The contours for the $k = 0.5$ bodies of each family are shown in figure 2. In this figure r/l is plotted to an expanded scale versus x/l. The Sears-Haack contour is the fullest for the smaller values of x/l and has the smallest radius at the base (i.e., at $x/l = 1.0$). A straight-line contour is also shown in this figure for a cone having the same length and volume as the $k = 0.5$ bodies. This cone has the least radii for the lower values of x/l than any of the bodies, and the greatest radius at the base. The contours for the other bodies are generally distributed between the limits of the Sears-Haack and cone profiles. For comparison with the data for the $k = 0.5$ bodies of each family, calculations were also made for a 3/4-power body ($\eta = \xi^{3/4}$) and a cone ($\eta = \xi$) both of which were restricted to the same length and volume as the other bodies.

Minimum drag coefficients.- The calculated minimum-drag coefficients for all the bodies are based on a fictitious wing area equal to $0.07695l^2$ or $4.013V^{2/3}$, and have three components: wave drag, base drag, and skin-friction drag. The wave-drag component was computed by integrating the pressure distribution over the body which was machine calculated by the method of characteristics according to the procedure of reference 8. This procedure, however, was modified to accommodate pointed bodies. The base-drag component was computed by the procedure of Love, as reported in reference 9, extended to hypersonic Mach numbers. The computations of skin friction were made according to the Spaulding-Chi procedure (see ref. 10) and flat-plate skin-friction coefficients corrected to a body of revolution, using Reynolds numbers determined by an assumed length of 300 feet and for the altitudes from the flight profile for hypersonic aircraft given in the following table.

M	2	4	6	8	10	12
Z, ft$\times10^{-3}$	57.5	70	88	101	111.5	119

The effect of body cutoff for each family of bodies is shown in figure 3 for Mach numbers of 2, 4, and 12. In this figure calculated total-drag coefficient is plotted versus body cutoff, k. Drag coefficients for the 3/4-power and cone bodies are plotted along the $k = 0.5$ line. As the Mach number is increased, the minimum drag coefficient corresponds to increasing values of body cutoff. For each Mach number the minimum drag coefficient is associated with the Sears-Haack profile. There is little difference in the drag coefficients for any of the bodies, however, at a Mach number of 12.

Drag coefficients for the Sears-Haack bodies are plotted in figure 4 versus Mach number for constant values of k. The drag coefficients for the cone are also presented for comparison. At Mach numbers from 2 to 5 the drag coefficients for the $k = 0.3$ and 0.5 bodies are undesirably large. Low drag coefficients over the range of Mach numbers from 2 to 12 are provided by the smaller values of body cutoff. At transonic Mach numbers, however, it is likely that no body cutoff would be desirable.

A breakdown of the total drag coefficients for the $k = 0.1$ Sears-Haack body into the three components is shown in figure 5, where drag coefficient is plotted versus Mach number. The vertical height of each shaded band corresponds to the magnitude of that component. The skin-friction and wave-drag components are of the same order of magnitude over the range of Mach numbers shown. At a Mach number of 12 the base-drag component is particularly small because of the relatively small base area of this body. For the cone at this Mach number, the base-drag component is greater than the corresponding wave-drag component. This wave-drag component for the cone is only slightly less than the wave-drag component for the $k = 0.1$ Sears-Haack body shown in this figure.

In summary, the calculations show that for Mach numbers from 2 to 12 the Sears-Haack profile provides the lowest total-drag coefficients at zero incidence. Further, it appears that a body with a small base area would provide low total-drag coefficients over this range of Mach numbers. Drag considerations at transonic Mach numbers, however, would dictate to a great extent the allowable base area for an aircraft body.

Effects of Body Cross-Sectional Shape

In this portion of the paper, methods are examined for improving the lift and drag-due-to-lift characteristics at hypersonic speeds of various $k = 0.5$ bodies, that is, bodies having maximum cross-sectional areas at the base. In this study of body cross-sectional shape the geometric constraints of a given length and volume have also been imposed on each body such that $V = 0.016l^3$. Fineness ratios of these bodies ranged from about 5 to 7. Because the fineness ratios are low and the bases of the bodies are large, such bodies are primarily applicable as lifting reentry bodies. In the present paper, however, the experimental results are used to illustrate the aerodynamic trends that may result from changes in the cross-sectional shape of the forward portion of the fuselage of hypersonic cruise aircraft.

Elliptic cross section.- Most analyses to determine optimum profiles of minimum wave-drag bodies at hypersonic speeds have been limited to bodies of circular cross section (see ref. 4). Experimental studies reported in references 11 and 12, however, have shown that bodies of elliptic cross section provide better performance at high supersonic speeds because of improved drag-due-to-lift characteristics. For this reason, Suddath and Oehman investigated profiles of minimum wave drag having elliptic cross section. The results of this investigation, reported in reference 13, indicated that the normalized distribution of cross-sectional area of an optimum body is relatively insensitive to variations in ellipticity, and further, that the wave drag does not change for moderate values of ellipticity. In an effort to explain this insensitivity Miele analyzed the problem using a slender body approximation to the Newtonian pressure relation (ref. 14), and found that a similarity law exists for optimum hypersonic bodies. It was found that the function which describes the optimum longitudinal contour of a body of arbitrary cross section is identical to the function which describes the optimum longitudinal contour of a body of circular cross section.

Experimental studies at a Mach number of 10 have been made both to verify these analytical results and to examine the effects of elliptical cross sections on the performance of power-law bodies and optimum hypersonic body profiles determined under the constraints of a given length and volume. (See ref. 15.) Sears-Haack profiles were not included in these experiments.

A summary of the experimental zero-lift drag characteristics of a series of power-law bodies of circular and elliptic cross section are shown in figure 6 for a Mach number of 10. Also shown are corresponding experimental drag coefficients for the optimum bodies, as determined for a circular cross section in reference 4, and for an elliptic cross section in reference 13. The elliptic bodies shown had major to minor axis ratios of 2.0. The zero-lift-drag coefficients are only the measured foredrag coefficients; that is, the base-drag component is not included. Such drag coefficients have been normalized with respect to the corresponding data for a circular cone. The data for the power-law bodies are indicated by the symbols and are plotted against power-body exponent. The levels of the experimental data for the optimum bodies are indicated by the arrows since these bodies do not correspond to any power-body exponent.

The minimum-drag coefficients for the power-law bodies correspond to an exponent of $n = 2/3$. This is the theoretical value for an optimum profile when the analysis is restricted to power-law bodies. (See ref. 16.) Since these minimum drags correspond to an exponent of 2/3 for both the circular and elliptic cross sections, Miele's similarity law is verified. The values of the zero-lift drag associated with the optimum circular and elliptic body shapes are slightly lower than the minimum values for the power-law bodies. The greater drag for the elliptic bodies is apparently due to the increased skin friction associated with the greater wetted area.

Lift and maximum lift-drag ratio characteristics for the same power-law and optimum bodies are shown in figure 7, also for a Mach number of 10. The greatest maximum lift-drag ratios occur for the same power-law bodies having the lowest zero-lift-drag coefficients. A change in cross section from circular to elliptic results in an almost constant incremental increase in maximum lift-drag ratio for any of the bodies. This results from the improved lift characteristics of the elliptic bodies which more than compensate for the increased drag. The maximum lift-drag ratios for the optimum bodies are essentially the same as the corresponding ratios for the 2/3-power bodies.

Other cross sections.- In an attempt to improve further the performance of bodies at hypersonic speeds, additional experimental studies have been made on a series of flat-bottomed bodies which have longitudinal distribution of cross-sectional areas identical with that of the optimum body derived by Eggers, Resnikoff, and Dennis (ref. 4). The results of these studies have been reported in reference 17. The flat-bottomed body profile is illustrated at the top right of figure 8. Directly below are the cross sections at the base of the rectangular, trapezoidal, and triangular bodies. For each body, including a reference body with elliptic cross sections having a major to minor axis ratio of 2, the span of the base, the base area, the length, and the volume have all been held constant, the only variables being body height and cross-sectional shape. Strictly, the significant variable is the angle that the lateral faces make with respect to the flat bottom. It is apparent, however, that this angle varies with body height.

A summary of the results of the experimental studies is shown on the left side of figure 8 for a Mach number of 10. The data have been normalized with respect to the corresponding data for the reference body with elliptic cross sections. Values of $C_{L(L/D)_{max}}$, $C_{D_{min}}$, and $(L/D)_{max}$ are shown as a function of dimensionless base height. It is obvious that $(L/D)_{max}$ increases as the base height is increased. This results from the large increases in lift occurring at $(L/D)_{max}$. There was essentially no change in the measured drag coefficient.

An examination of the pitching-moment characteristics of each of the flat-bottomed configurations indicates unfavorable, that is, negative, pitching moments at zero lift, as would be expected for the camber of these bodies. In order to examine the possibility of providing a favorable C_{m_O} without incurring large penalties in performance, one of the trapezoidal configurations was modified by a reversal in camber by shearing the cross sections. This resulted in a flat topped body with maximum width retained on the lower surface, as illustrated in figure 9. The results of this modification are presented in this figure where pitching-moment coefficient and lift-drag ratio are plotted as a function of lift coefficient. A favorable C_{m_O} and variation of pitching moment with lift were obtained by the camber reversal.

There was little or no corresponding change in $(L/D)_{max}$ or lift at $(L/D)_{max}$, and, in addition, no measurable effect on $C_{D_{min}}$.

In summary, a ꞁ 'udent selection of cross-sectional shape and camber of the forward portion of the fuselage of hypersonic-cruise aircraft may significantly improve the aerodynamic performance of such bodies at hypersonic speeds.

CONCLUDING REMARKS

The present study was concerned with providing some of the answers to questions that arise in the application of optimum bodies to the design of hypersonic-cruise aircraft. This study involved the calculated minimum drag characteristics of four families of slender bodies, and the experimental effects of body cross-sectional shape on the aerodynamic performance of bodies. In both parts of this study the restriction of a constant length and volume have been imposed on the bodies involved, but the constant values are different in each part.

The calculations of the drag characteristics of the families of slender bodies have shown that for Mach numbers from 2 to 12, the Sears-Haack profile provides the lowest total-drag coefficients at zero incidence. It appeared, also, that this profile with a small base area would provide low total-drag coefficients over this range of Mach numbers. The allowable base area for an aircraft body, however, would be dictated largely by drag considerations at transonic Mach numbers.

The experimental results for a Mach number of 10 concern bodies having various cross-sectional shapes. These results indicated that the careful choice of cross-sectional shape and camber for the forward portion of the fuselage of hypersonic-cruise aircraft may significantly improve the aerodynamic performance of such bodies at hypersonic speeds.

It remains to be determined whether or not the results of both parts of this paper will be significantly altered when the body is combined with a wing.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif., 94035, May 16, 1967
126-13-03-01-00-21

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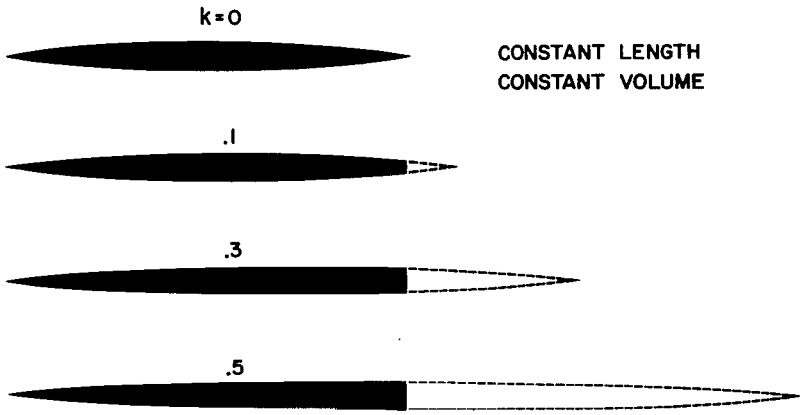

Figure 1.- Illustration of body cutoffs. AAA445-2

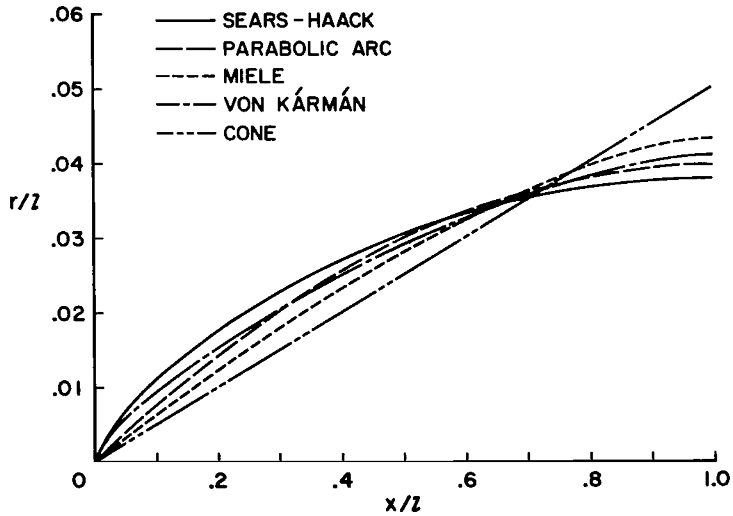

Figure 2.- Comparison of $k = 0.5$ body profiles. AAA445-3

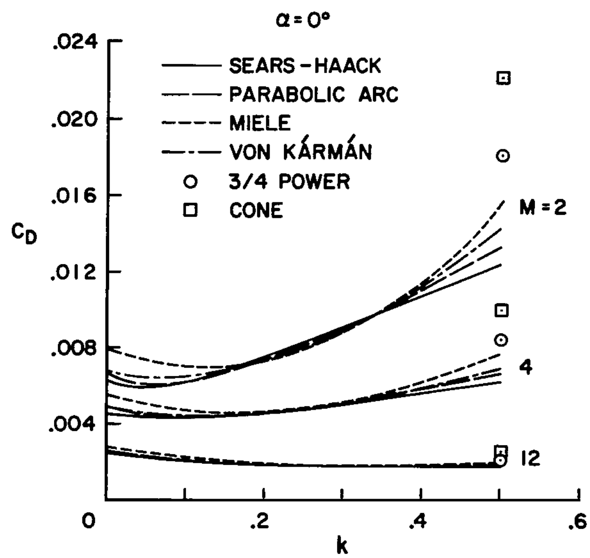

Figure 3.- Effect of body cutoff.

AAA445-4

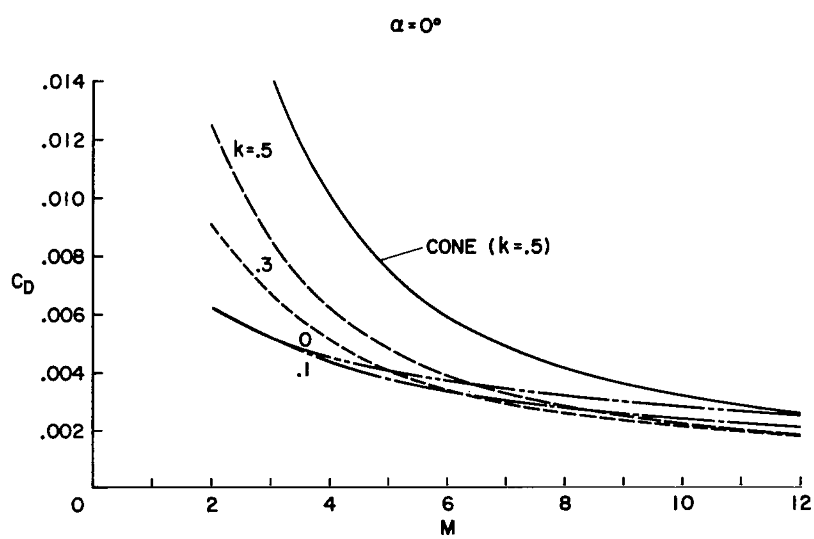

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Figure 4.- Drag characteristics for Sears-Haack bodies.

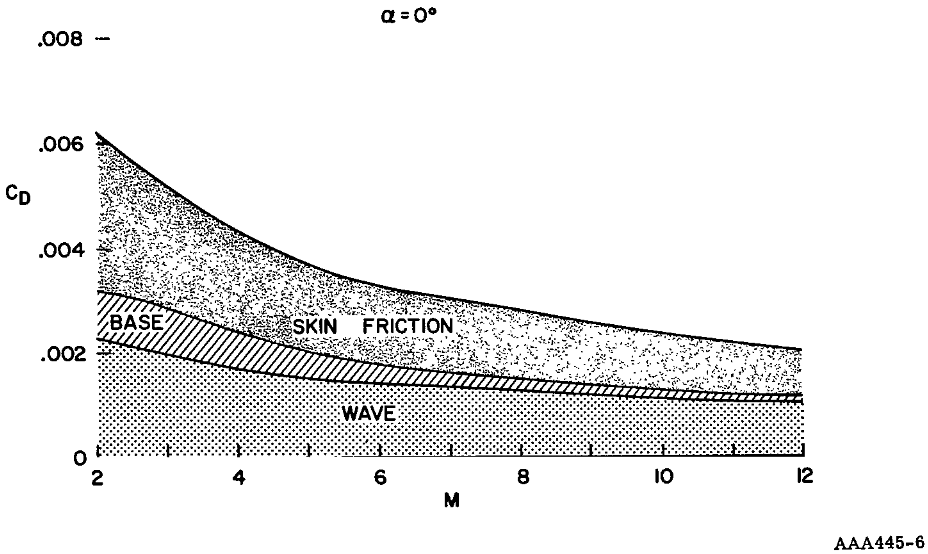


Figure 5.- Drag components for k = 0.1 Sears-Haack body.

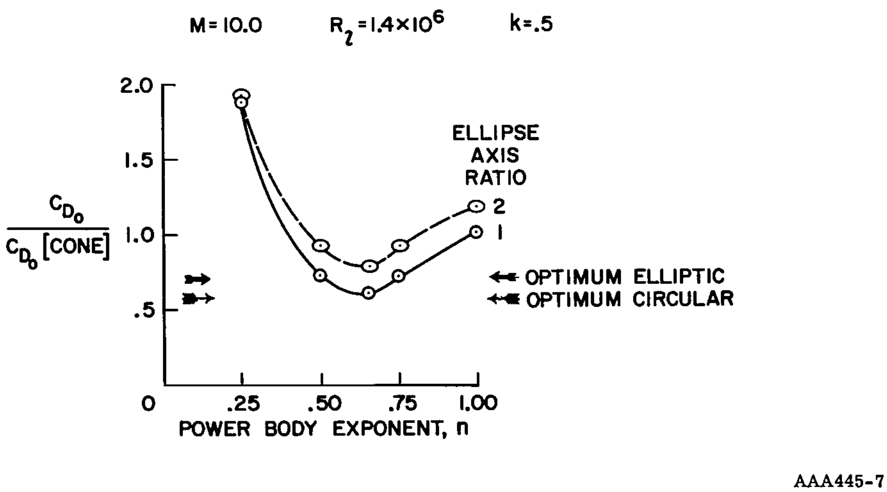


Figure 6.- Drag characteristics; fixed volume and length.

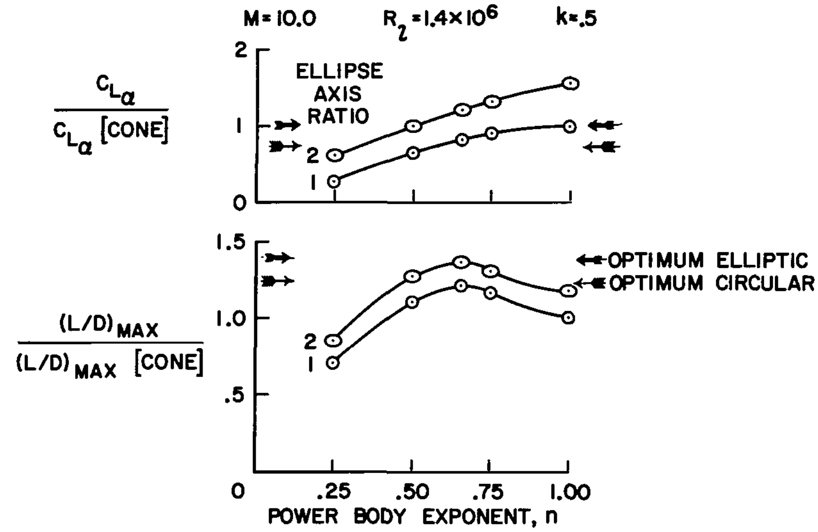


Figure 7.- Lift and performance characteristics; fixed volume and length.

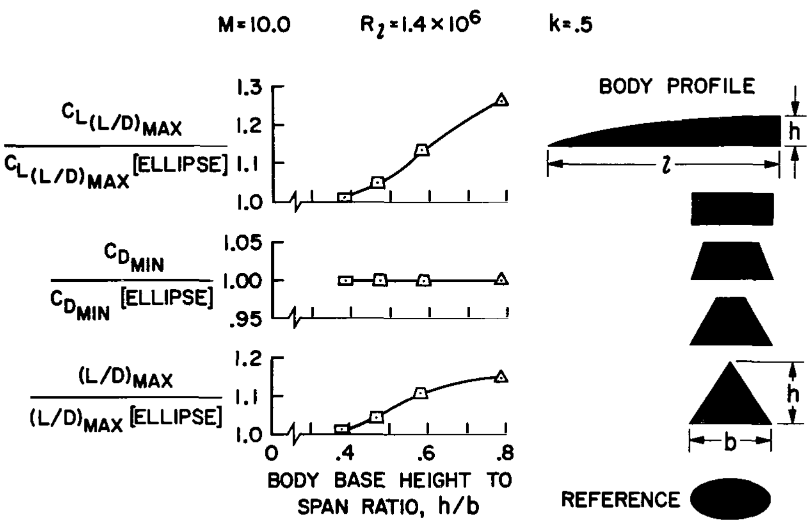
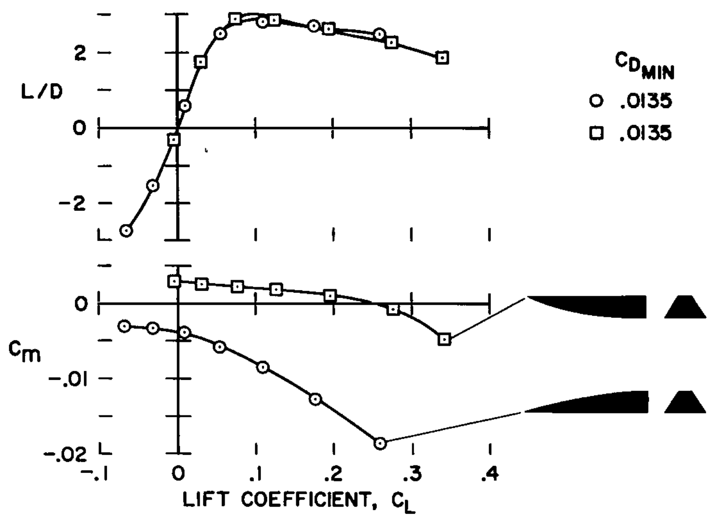


Figure 8.- Minimum wave drag bodies; fixed volume and length, cross section effect.



AAA445-10

Figure 9.- Minimum wave drag bodies; fixed volume and length, camber effect.

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