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A FORTRAN IV PROGRAM TO ESTIMATE THE OFF-DESIGN PERFORMANCE OF RADIAL-INFLOW TURBINES

by Carroll A. Todd and Samuel M. Futral, Jr.

Lewis Research Center
Cleveland, Ohio

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ABSTRACT

This program computes conventional performance parameters at required combinations of speeds and flow rates. Necessary input information consists of flow areas, diameters, blade angles, and an estimate of design point performance. The flow equations and corresponding computer listings are given. A set of example calculations is included.

A FORTRAN IV PROGRAM TO ESTIMATE THE OFF-DESIGN PERFORMANCE OF RADIAL-INFLOW TURBINES

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SUMMARY

A FORTRAN IV computer program for off-design performance of a radial-inflow turbine is presented. The thermodynamic equations used and the corresponding computer listings are given. Use of the program requires as input information the turbine flow areas, diameters, and blade angles. An estimate of design point performance is also necessary. The output consists of conventional performance parameters at specified flow conditions and speeds. A complete set of calculations for an example turbine is included.

INTRODUCTION

A procedure for predicting the off-design performance of radial-inflow turbines was described by Futral and Wasserbauer in reference 1. It is possible to use this procedure for hand calculations, but it is lengthy and contains several iterations. For this reason a digital computer program analog has been used. The computer program used at Lewis Research Center is given here, with the explanations necessary for its use. The thermodynamic equations are also given. These are the equations of reference 1, except for an improvement in the calculation of rotor incidence loss and the calculation of additional performance parameters.

The program has been used in two ways. (1) For an existing turbine, the performance at design operating point may be known, while information on off-design performance is lacking. In this case, the program provides a convenient way to estimate off-design performance without making actual tests.

(2) The program is also useful as a design guide. For a proposed turbine design, the design point performance can usually be closely estimated as part of the design process. However, if the off-design operation is of sufficient importance, modifications in the design may be considered. In this case, the effect of a series as small design

changes on performance may be studied by repeated use of the program.

In addition to the engineering equations and the FORTRAN IV program, this report includes the input and output listings for an example turbine problem.

ENGINEERING ANALYSIS

The analysis consists of a one-dimensional solution of flow conditions along the mean streamline in the turbine, using perfect gas relations throughout. Equations were written in a step-by-step fashion, beginning at the turbine inlet. The analysis is written for subsonic flow only since stator choking is not expected. Symbols used in the engineering equations are given in appendix A, and the equations are listed in appendix B. Figure 1 shows the location of station subscripts in the turbine.

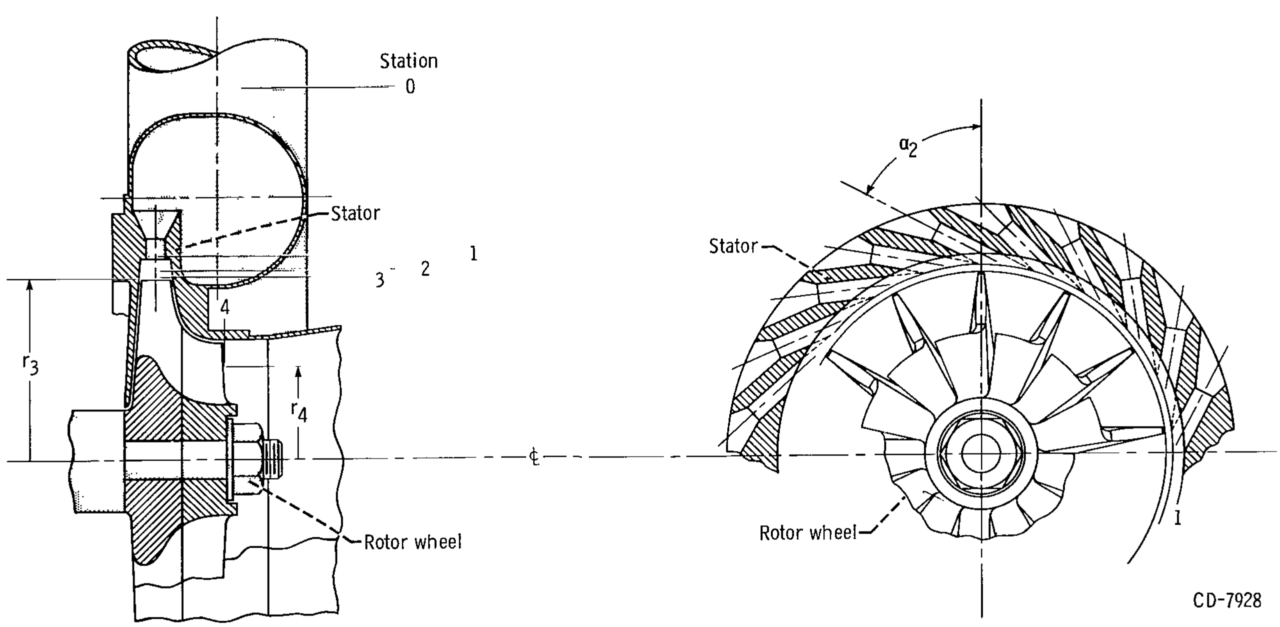

Figure 1. - Turbine stator and rotor.

Required Information

The geometric features of the turbine which must be specified consist of flow areas, blade angles, and major dimensions. The flow areas needed are those at the turbine inlet, stator exit, rotor inlet, and rotor exit. The turbine inlet area is normal to the flow, upstream of the scroll. The stator exit area and the rotor entrance area are surfaces of circular cylinders, with elements parallel to the turbine axis. The rotor exit area is an annular area, perpendicular to the turbine axis. The stator exit area is located just inside the blade row and the rotor inlet and exit areas are calculated at locations just outside the blade rows.

The blade angles at the stator exit and rotor exit must also be specified. These are angles at the mean streamline.

Diameters needed are the stator exit diameter, rotor inlet tip diameter, and the mean diameter at the rotor exit. The axial blade length at the rotor tip is also necessary.

The required inlet conditions of the working fluid are the total temperature and total pressure at the turbine inlet. The gas properties needed are the specific gas constant, the specific heat at constant pressure, and the specific heat ratio.

Input information which applies to the example turbine is as follows:

Turbine inlet area, sq ft	0.12989
Stator inlet area, sq ft	0.04078
Rotor entrance area, sq ft	0.04175
Rotor exit area, sq ft	0.04719
Stator exit diameter, sq ft	0.42650
Rotor inlet diameter, sq ft	0.41417
Stator exit axial dimension, ft	0.03208
Stator exit angle, deg	72.47
Rotor exit angle, deg	-56.85
Number of blades at rotor tip	22
Turbine inlet total pressure, ^{O}R	2060

The thermodynamic properties of the working fluid are as follows:

Specific gas constant	18.437
Specific heat ratio	5/3
Constant pressure specific heat	0.05924

The design point quantities, used in determination of frictional coefficients, are the following:

Mass flow rate, lb/sec	0.7484
Total efficiency	0.897
Wheel speed, rpm	36 000
Total pressure ratio across stator	0.98
Total pressure ratio across turbine	1.7397

Operation of the Program

In solving the equations in appendix B, the computer uses the equations in essentially the order listed. Solutions for the quantities $(V/V_{cr})_0$, $(V/V_{cr})_2$, and $(V/V_{cr})_4$ are accomplished by means of the Newton-Ralphson method since these quantites are implicitly defined.

For operation of the program, the wheel speeds of interest are specified in advance. The initial value for $(V/V_{cr})_1$, the velocity ratio at the stator exit, as well as its increment size are also specified. The final value for this quantity will not be known and is determined by the computer for each wheel speed. For any given wheel speed, repeated increases in the value of $(V/V_{cr})_1$ will eventually result in a flow rate which the turbine cannot pass. Mathematically, this condition is evident in that no solution exists for $(W/W_{cr})_4$, the relative velocity ratio at the rotor exit. When this occurs, the maximum value for $(V/V_{cr})_1$ is computed to five significant figures. The computer prints out a complete set of answers for each value of $(V/V_{cr})_1$ for any given wheel speed. When one wheel speed is finished, the computer goes on to the next speed until all specified speeds have been used.

In this discussion, it is always assumed that the temperature and pressure of the working fluid entering the turbine are constant. This has fulfilled all needs thus far. However, if the program should be required to operate in some other manner, the amount of additional instructions would not be great.

Loss Coefficients

Calculation of rotor incidence loss makes use of the angle φ in equation (B17). This is the optimum incidence angle and is that incidence angle associated with minimum rotor incidence loss. The calculation of φ follows the method of Stanitz (ref. 2), developed to account for "slip" in centrifugal compressors having radial blading.

The loss coefficient k_1 applies to the stator and must be evaluated before the main program is used. The determination of k_1 involves simultaneous solution of equations (B1) to (B5) using the design value of mass flow rate. For this purpose, the total pressure ratio across the stator p'_1/p'_0 is determined from design data. A separate routine is provided for this solution.

The determination of the loss coefficients m and k were presented as a graphical procedure in reference 1. In the computer determination, this is done by means of a search routine. By repeated calculations, the computer finds a pair of values for m and k which yield design values for mass flow rate, efficiency, and pressure ratio. Design speed and inlet conditions are specified.

Output Information

A sample of the computer output is shown for the example turbine. To assist in reading this, a list of machine output symbols in terms of engineering symbols is included in the section on program output. The information contained in the machine output may be classified under several headings as follows:

(1) Input information - For each speed, the following quantities are printed out:
- (a) Design speed
- (b) Percent design speed
- (c) Fluid inlet temperature and pressure
- (d) Turbine dimensions and angles
- (e) Fluid constants
- (f) Number of rotor blades (at tip)

(2) Performance parameters
- (a) Efficiences
- (b) Blade-jet speed ratio
- (c) Pressure ratios
- (d) Specific work
- (e) Ideal work

(3) Internal variables - Internal variables are listed for each station throughout the turbine as follows:
- (a) Pressures
- (b) Temperatures
- (c) Velocity components

(4) Additional quantities - Certain ratios have been found useful and are included in the print-out, such as the following:

(a) $wN\epsilon/\delta$
(b) $N/\sqrt{T_0'}$
(c) $w\sqrt{T_0'}/p_{in}'$
(d) Γ/p_{in}'
(e) Losses calculated on a pound mass basis denoted by L with appropriate subscript
(f) Fractional losses denoted by $\Delta\eta$ where the loss has been divided by the ideal work
(g) Losses in efficiency, given in equation (B31) as η_E, η_R, and η_I, which are useful in studying loss distributions in the turbine

THE FORTRAN IV PROGRAM

In this section are presented the FORTRAN IV routines, input and output considerations, and the internal documentation. The program operates in three distinct modes, each depending on the type of information available or the desired output. In mode one operation, the loss coefficients are assumed known. Off-design calculations are made by means of changes in speed and changes in the value of $(V/V_{cr})_1$. This mode furnishes performance results, once the loss coefficients are known. In mode two operation, sets of loss coefficients are entered and calculations are made at design values of speed and $(V/V_{cr})_1$. This mode is used for studying the results of variations in frictional effects. In mode three, the program determines the loss coefficients and produces one complete set of calculations representing design conditions of the turbine. This set of calculations verifies the determination of the loss coefficients.

Program Input

Actual input for the example with card format is presented in this section, followed by the definitions of the terms used on the cards.

TITLE | PROJECT NUMBER | ANALYST | SHEET ____ OF ____

FORTRAN STATEMENT — IDENTIFICATION

```
                         MODE ONE FORMAT

PREF      WREF
.98667    .748
A0        A1        A3        A4        D1        D3        XL        AL1
.055488   .04078    .04175    .047188   .4265     .414166   .032083   72.13
B4        ZBR       P0        T0        U3        U4        G         R
-56.555   22.       3588.12   2060.     780.6838  443.245   1.6667    18.437
CP        VSTART    DELV      VEND
.0592422  .2        .05       .9
MODE
1
XM        XK
2.921401  .0950602
PERCT
120.
110.
100.
 80.
  ~
  ~
 10.
```

TITLE	PROJECT NUMBER	ANALYST	SHEET ____ OF ____

FORTRAN STATEMENT

```
                              MODE TWO FORMAT

PREF      WREF
.98667    .748
AO        A1        A3        A4        D1        D3        XL        AL1
.055488   .04078    .04175    .047188   .4265     .414166   .032083   72.13
B4        ZBR       PO        TO        V3        V4        G         R
-56.55    22.       3588.12   2060.     786.6838  443.245   1.6667    18.437
CP        VSTART    DELV      VEND
.059242   .2        .05       .9
MODE
2
XM        XK
1.        .05
1.5       .05
2.0       .05
2.5       .05
3.        .05
1.        .1
1.5       .1
     ≈
1.        .2
1.5       .2
```

NASA-C-836 (REV. 9-14-59)

CD-6729

TITLE | PROJECT NUMBER | ANALYST | SHEET ____ OF ____

STATEMENT NUMBER | CONT | FORTRAN STATEMENT | IDENTIFICATION

```
                         MODE THREE FORMAT

PREF        WREF
.98667      .798
AO          A1          A3          A4          D1          D3          XL          AL1
.05548      .04078      .04175      .04718      .4265       .414166     .032083     72.13
B4          ZBR         PG          TG          V3          V4          G           R
-56.55      22.         3588.12     2060.       780.6838    443.245     1.6667      18.437
CP          VSTART      DELV        VEND
.059242     .2          .05         .9
MODE
3
PDSGN       ETAD        PE
1.734       .913        22
```

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The definitions of the terms used on the cards are as follows:

Card 1:

PREF	estimate of p_1'/p_0' to determine k_1
WREF	estimate of W to determine k_1

Card 2:

AO	area of turbine inlet, A_0, sq ft
A1	area upstream of stator exit, A_1, sq ft
A3	area upstream of rotor inlet, A_3, sq ft
A4	area downstream of stator exit, A_4, sq ft
D1	diameter upstream of stator exit, D_1, ft
D3	diameter upstream of rotor inlet, D_3, ft
XL	axial distance, l, ft
AL1	absolute gas angle, α_1, deg

Card 3:

B4	free stream gas flow angle, β_4, deg
ZBR	number of rotor blades at tip of rotor, Z
PO	inlet pressure, p_0', lb/sq ft
TO	inlet temperature, T_0', ^{o}R
U3	design speed upstream of rotor inlet, U_3, ft/sec
U4	design speed downstream of rotor exit, U_4, ft/sec
G	ratio of specific heat at constant pressure to specific heat at constant volume, γ
R	specific gas constant, R, ft-lb/(lb)(^{o}R)

Card 4:

CP	specific heat at constant pressure, c_p, Btu/(lb)(^{o}R)
VSTART	starting value of $(V/V_{cr})_1$
DELV	incremental value of $(V/V_{cr})_1$
VEND	end value of $(V/V_{cr})_1$

Card 5:

MODE	if MODE = 1, a performance run varying percent speed	
	if MODE = 2, manual determination of m and k	
	if MODE = 3, automatic determination of m and k	
Card 6:		MODE ONE
XM	loss coefficient	
XK	loss coefficients	
Card 7+:		
PERCT	percent of design speed	
Card 6+:		MODE TWO
XM	loss coefficient	
XK	loss coefficient	
Card 6:		MODE THREE
PDSGN	specified p'_0/p'_4 or p'_0/p'_4	
ETAD	specified η_t or η_s	
P	if p = 1, p'_0/p_4 is specified	
	if p = 2, p'_0/p'_4 is specified	
E	if E = 1, η_s is specified	
	if E = 2, η_t is specified	

Description of the Output

A list of the machine output symbols, internal program variables, and their corresponding engineering symbols is given here. Refer to appendix A for the physical interpretation. Following this are sample output listings showing the results of calculations for modes one, two, and three using the input data given previously. It should be noted that each block of output is for a specific value of $(V/V_{cr})_1$.

Machine Output Symbols and Definitions

ALPHA2	α_2
BETA3	β_3
DEE	$\Delta\eta_E$
DEI	$\Delta\eta_I$
DELTA	δ
DER	$\Delta\eta_R$
DES	$\Delta\eta_S$
EQ-SPEED	N_{eq}
ETA-E	η_E
ETA-I	η_I
ETAR	η_R
ETA-S	η_S
ETA-T	η_t
LE	L_E
LI	L_I
NOT	$N/\sqrt{T'_0}$
NU	ν
PO,/P4	p'_0/p_4
PO,/P4,	p'_0/p'_4

P1,/PO,	p'_1/p'_0
P2,/PO,	p'_2/p'_0
P2,/P1,	p'_2/p'_1
P4/PO,	p_4/p'_0
P4/P4ID	$p''_4/p''_{4,id}$
P4/P4,	p_4/p'_4
P4,/PO,	p'_4/p'_0
PHI	φ
PV/PVCR)O	$\left(\rho V/\rho' V_{cr}\right)_0$
PV/PVCR)1	$\left(\rho V/\rho' V_{cr}\right)_1$
PV/PVCR)2	$\left(\rho V/\rho' V_{cr}\right)_2$
PVX/PVCR)3	$\left(\rho V_x/\rho' V_{cr}\right)_3$
PW/PWCR)4	$\left(\rho W/\rho'' W_{cr}\right)_4$
R2	r_2
RHO)O	ρ'_0
RHO-VCR)O	$\left(\rho' V_{cr}\right)_0$
ROTOR-LOSS	L_R
SMALL-W	w
SPEED	N
SPEED-PAR	wN/δ

STATOR-LOSS	L_S
T,,/T,)3	$(T''/T')_3$
T3,,	T''_3
T4,	T'_4
T4,/T4,,	$(T'/T'')_4$
T4,,/T3,,	T''_4/T''_3
THETA-CR	θ_{cr}
TOP	Γ/p'_0
V3	V_3
V4	V_4
V/VCR)O	$(V/V_{cr})_0$
V/VCR)1	$(V/V_{cr})_1$
V/VCR)2	$(V/V_{cr})_2$
V/VCR)3	$(V/V_{cr})_3$
VCR)O	$V_{cr,0}$
VCR4	$V_{cr,4}$
VU3	$V_{u,3}$
VU4	$V_{u,4}$
VU/VCR)3	$(V_u/V_{cr})_3$
VX3	$V_{x,3}$

VX/VCR)3	$(V_x/V_{cr})_3$
W1	$\Delta h'$
W3	W_3
W4	W_4
W/WCR)3	$(W/W_{cr})_3$
W/WCR)4	$(W/W_{cr})_4$
WCR3	$W_{cr,3}$
WCR4	$W_{cr,4}$
WEQ	w_{eq}
WID	$\Delta h'_{id,s}$
W, ID	$\Delta h'_{id,t}$
WTOP	$w\sqrt{T'_0/p'_0}$

Mode One Sample Output

```
                                   OFF DESIGN PERFORMANCE OF A RADIAL INFLOW TURBINE

TURBINE GEOMETRY  A0    0.129887      A1    0.407800E-01  A3    0.417450E-01  A4    0.471880E-01  D1    0.426500
                  D3    0.414166      L     0.320830E-01  AL1   72.47000      B4    -56.85000     Z     22.00000
INLET CONDITIONS  P0    3600.000      T0    2060.000
DESIGN SPEEDS     U3    780.6838      U4    443.2450
FLUID PROPERTIES  G     1.666700      R     18.43700      CP    0.592420E-01  EPS   0.910949

MODE 1.   PERFORMANCE RUN VARYING PERCENT SPEED
              M    3.592625
              K    0.754039E-01
             K1    0.763553E-01
        PERCENT    100.0000
             U3    780.6838
             U4    443.2450
DESIGN P1./P0.     0.980000
DESIGN SMALL-W     0.748400


    V/VCR)1   0.200000           P2./P0.   0.998079               V3   253.5031            WCR3   1258.994
     VCR)0    1235.911           P2./P1.   1.000023            BETA3   -82.29354             W4   132.7295
     RHO)0    0.947862E-01        ALPHA2   73.31849         T,,/T,)3   1.037702            WCR4   1217.305
 RHO-VCR)0    117.1473                R2   0.212270             T3,,   2137.666             VU4   332.1319
 PV/PVCR)1    0.157008          VU/VCR)3   0.196464         T4,,/T3,,  0.934870        T4,/T4,,   1.016528
   SMALL-W    0.283144          VX/VCR)3   0.589386E-01      W/WCR)3   0.431123         P4,/P0,   0.885221
 PV/PVCR)0    0.186084E-01       V/VCR)3   0.205114      PVX/PVCR)3   0.580112E-01        VCR4   1227.323
  V/VCR)0     0.166108E-01           VU3   242.8122         P4/P4ID   0.918390              V4   339.9744
  V/VCR)2     0.200089                W3   542.7817       PW/PWCR)4   0.108550          P4/P4,   0.952730
 PV/PVCR)2    0.197093               VX3   72.84285         W/WCR)4   0.109036          P4/P0,   0.843376
        W1    1.691284             ETA-E   0.497547            W,ID   5.808875             T4,   2031.472
       WID    8.038581             ETA-T   0.291155           ETA-S   0.210396             WEQ   0.183813
     ETA-I    0.778959                LE   2.308287             DEE   0.287151              LI   2.262151
ROTOR-LOSS    1.689182       STATOR-LOSS   0.928868E-01         DER   0.210134             DES   0.115551E-01
     SPEED    36000.00          EQ-SPEED   29695.14              NU   1.230507        THETA-CR   1.469718
     DELTA    1.701146         SPEED-PAR   5991.953           W1-EQ   1.150754         P0,/P4,   1.129662
       NOT    793.1747              WTOP   0.514045             TOP   0.474460E-01         DEI   0.281412
    P0./P4    1.185710               PHI   -43.96986        P1,/P0,   0.998056           ETA-R   1.066110
       NSP    120.8699


    V/VCR)1   0.250000           P2./P0.   0.996982               V3   316.8903            WCR3   1249.548
     VCR)0    1235.911           P2./P1.   1.000035            BETA3   -79.19788             W4   167.5706
     RHO)0    0.947862E-01        ALPHA2   73.31857         T,,/T,)3   1.022189            WCR4   1207.532
 RHO-VCR)0    117.1473                R2   0.212270             T3,,   2105.710             VU4   302.9650
 PV/PVCR)1    0.244164          VU/VCR)3   0.245580         T4,,/T3,,  0.933882        T4,/T4,,   1.012364
   SMALL-W    0.350528          VX/VCR)3   0.737055E-01      W/WCR)3   0.388769         P4,/P0,   0.847779
 PV/PVCR)0    0.230369E-01       V/VCR)3   0.256402      PVX/PVCR)3   0.718959E-01        VCR4   1214.974
  V/VCR)0     0.230415E-01           VU3   303.5152         P4/P4ID   0.926179              V4   316.5268
  V/VCR)2     0.250111                W3   485.7858       PW/PWCR)4   0.137770          P4/P4,   0.958118
 PV/PVCR)2    0.244267               VX3   91.09343         W/WCR)4   0.138771          P4/P0,   0.812273
        W1    4.100490             ETA-E   0.626467            W,ID   7.800876             T4,   1990.797
       WID    9.739316             ETA-T   0.525645           ETA-S   0.421024             WEQ   0.227558
     ETA-I    0.839339                LE   2.000868             DEE   0.205442              LI   2.073232
ROTOR-LOSS    1.428626       STATOR-LOSS   0.145112             DER   0.146686             DES   0.148996E-01
     SPEED    36000.00          EQ-SPEED   29695.14              NU   1.117917        THETA-CR   1.469718
     DELTA    1.701146         SPEED-PAR   7417.945           W1-EQ   2.789984         P0,/P4,   1.179552
       NOT    793.1747              WTOP   0.636380             TOP   0.142408             DEI   0.212872
    P0./P4    1.231113               PHI   -37.64624        P1,/P0,   0.996947           ETA-R   1.044782
```

Mode Two Sample Output

OFF DESIGN PERFORMANCE OF A RADIAL INFLOW TURBINE

TURBINE GEOMETRY	A0	0.129887	A1	0.407800E-01	A3	0.417450E-01	A4	0.471880E-01	D1	0.426530
	D3	0.414166	L	0.320830E-01	AL1	72.47000	B4	-56.85000	Z	22.00000
INLET CONDITIONS	P0	3600.000	T0	2060.000						
DESIGN SPEEDS	U3	780.6838	U4	443.2450						
FLUID PROPERTIES	G	1.666700	R	18.43700	CP	0.592420E-01	EPS	0.910949		

MODE 2, MANUAL DETERMINATION OF M AND K

M	3.000000
K	0.500000E-01
K1	0.763553E-01
PERCENT	100.0000
U3	780.6838
U4	443.2450
DESIGN P1,/P0,	0.980000
DESIGN SMALL-W	0.748400

V/VCR)1	0.200000	P2,/P0,	0.998695	V3	256.3198	WCR3	1258.537
VCR)0	1235.911	P2,/P1,	1.000640	BETA3	-82.25358	W4	130.5060
RHO)0	0.947662E-01	ALPHA2	73.51138	T,,/T,)3	1.036950	WCR4	1216.832
RHO-VCR)0	117.1473	R2	0.212058	T3,,	2136.116	VU4	333.9932
PV/PVCR)1	0.197006	VU/VCR)3	0.198847	T4,,/T3,,	0.934823	T4,/T4,,	1.016820
SMALL-W	0.283144	VX/VCR)3	0.589232E-01	W/WCR)3	0.428959	P4,/P0,	0. 930118
PV/PVCR)0	0.186084E-01	V/VCR)3	0.207393	PVX/PVCR)3	0.579754E-01	VCR4	1227.023
V/VCR)0	0.186108E-01	VU3	245.7571	P4/P4ID	0.934413	V4	341.5368
V/VCR)2	0.202516	W3	539.8610	PW/PWCR)4	0.106788	P4/P4,	0.952278
PV/PVCR)2	0.199409	VX3	72.82388	W/WCR)4	0.107251	P4/P0,	0.857153
W1	1.750158	ETA-E	0.559111	W,ID	5.030349	T4,	2030.478
W1D	7.296778	ETA-T	0.347920	ETA-S	0.239853	WEQ	0.183813
ETA-I	0.865162	LE	2.329552	DEE	0.319258	LI	2.233187
ROTOR-LOSS	0.924098	STATOR-LOSS	0.630830E-01	DER	0.126645	DES	0.864533E-02
SPEED	36000.00	EQ-SPEED	29695.14	NJ	1.291541	THETA-CR	1.459718
DELTA	1.701146	SPEED-PAR	5991.953	W1-EQ	1.190812	P0,/P4,	1.110965
NUT	793.1747	WTOP	0.514045	TOP	0.490976E-01	DEI	0.306051
P0,/P4	1.166640	PHI	-43.97732	P1,/P0,	0.998056	ETA-R	1.184420
NSP	133.5119						

V/VCR)1	0.250000	P2,/P0,	0.997949	V3	320.4066	WCR3	1248.973
VCR)0	1235.911	P2,/P1,	1.001005	BETA3	-79.12022	W4	165.0909
RHO)0	0.947662E-01	ALPHA2	73.51363	T,,/T,)3	1.021249	WCR4	1206.937
RHO-VCR)0	117.1473	R2	0.212055	T3,,	2103.772	VU4	305.0409
PV/PVCR)1	0.244164	VU/VCR)3	0.248558	T4,,/T3,,	0.933821	T4,/T4,,	1.012692
SMALL-W	0.350528	VX/VCR)3	0.736753E-01	W/WCR)3	0.386049	P4,/P0,	0.850185
PV/PVCR)0	0.230369E-01	V/VCR)3	0.259247	PVX/PVCR)3	0.718263E-01	VCR4	1214.572
V/VCR)0	0.230415E-01	VU3	307.1957	P4/P4ID	0.940376	V4	318.1267
V/VCR)2	0.253144	W3	482.1640	PW/PWCR)4	0.135826	P4/P4,	0.957672
PV/PVCR)2	0.247086	VX3	91.05610	W/WCR)4	0.136785	P4/P0,	0.823775
W1	4.178503	ETA-E	0.680837	W,ID	7.135124	T4,	1989.480
W1D	9.105924	ETA-T	0.585624	ETA-S	0.458877	WEQ	0.227558
ETA-I	0.904346	LE	2.021145	DEE	0.221959	LI	2.035274
ROTOR-LOSS	0.778077	STATOR-LOSS	0.965512E-01	DER	0.854474E-01	DES	0.108228E-01
SPEED	36000.00	EQ-SPEED	29695.14	NJ	1.156144	THETA-CR	1.459718
DELTA	1.701146	SPEED-PAR	7417.945	W1-EQ	2.843064	P0,/P4,	1.162541
NUT	793.1747	WTOP	0.535380	TOP	0.145117	DEI	0.223511
P0,/P4	1.213924	PHI	-37.65760	P1,/P0,	0.996947	ETA-R	1.126307

Mode Three Sample Output

```
                              OFF DESIGN PERFORMANCE OF A RADIAL INFLOW TJRBINE

TURBINE GFOMETRY  AO    0.129887     A1    0.407800E-01  A3    0.417450E-01  A4    0.471880E-01  D1     0.426500
                  D3    0.414166     L     0.320830E-01  AL1    72.47000     B4   -56.85000      Z     22.00000
INLET CONDITIONS  PO    3600.000     TO    2060.000
DESIGN SPEEDS     U3    780.6838     U4    443.2450
FLUID PROPERTIES  G     1.666700     R     18.43700      CP    0.592420E-01  EPS    0.910949

MODE 3.   AUTOMATIC DETERMINATION OF M AND K
               M    3.947835
               K    0.683671E-01
              K1    0.763553E-01
         PERCENT    100.0000
              U3    780.6838
              U4    443.2450
DESIGN P1./PO.      0.980000
DESIGN SMALL-W      0.748400
DESIGN PO./P4.      1.740000
 DESIGN ETA-T       0.913000

    V/VCR)1   C.615818          P2./PO.    0.981934            V3    783.3564          WCR3   1177.725
     VCR)O    1235.911          P2./P1.    1.001973         BETA3   -7.711356           W4    444.0692
     RHO)O    0.947862E-01       ALPHA2     73.38271      T,,/T,)3   0.908058           WCR4   1133.050
 RHO-VCR)O    117.1473               R2    0.212199           T3,,   1870.599           VU4    71.49717
  PV/PVCR)1   0.530352          VU/VCR)3   0.606932      T4,,/T3,,   0.925572       T4,/T4,,  0.974083
    SMALL-W   0.748445          VX/VCR)3   0.182683        W/WCR)3   0.193457      P4,/PO,   0.575019
  PV/PVCR)O   0.491883E-01       V/VCR)3   0.633829     PVX/PVCR)3   0.155864           VCR4   1118.271
   V/VCR)O    0.492331E-01           VU3   750.1139        P4/P4ID   0.965595             V4   253.2051
   V/VCR)2    0.618131               W3    227.8395       PW/PWCR)4  0.369567         P4/P4,   0.968264
  PV/PVCR)2   0.531715               VX3   225.7794        W/WCR)4   0.391924         P4/PO,   0.556771
        W1    22.12419             ETA-E   0.918337           W,ID   24.23218            T4,   1686.502
       WID    25.48582             ETA-T   0.913009          ETA-S   0.868098            WEQ   0.485880
     ETA-I    0.919463                LE   1.280390            DEE   0.502393E-01         LI   0.286848E-01
ROTOR-LOSS    1.342740       STATOR-LOSS   0.901912            DER   0.526858E-01        DES   0.314650E-01
     SPEED    36000.00          EQ-SPEED   29695.14             NU   0.691074       THETA-CR   1.469718
     DELTA    1.701146         SPEED-PAR   15838.75          W1-EQ   15.05336        PO,/P4,   1.739072
       NOT    793.1747              WTOP   1.358794            TOP   1.640600            DEI   0.112552E-02
    PO./P4    1.796072               PHI  -17.28718        P1,/PO,   0.980000          ETA-R   0.969702
       NSP    75.74157
```

Main Program Listing

```
C A FORTRAN IV COMPUTER ROUTINE TO PREDICT OFF DESIGN PERFORMANCE
C OF A RADIAL INFLOW TURBINE
C
      SUB(X,G)=X*(1.-(G-1.)/(G+1.)*X*X)**(1./(G-1.))                    2
      DATA  AC/32.1739/,PI/3.14159/,XJ/778.16/,F1/.01745/
      DATA TOL/.000001/,XNK/9./,XMN/9./
      COMMON /GETK/PREF,WREF,G,AC,A1,AL1,TO,R,PO
     1,PDSGN,CP,G1,G2,G3,ETAD,U3,U4,VU3,B4,T4T3,WCR3,TT3,W3WC3,B3,
     2 PHI,P2PP1P,P1PPOP
      INTEGER PTEST , ETEST
      READ(5,400)PREF,WREF
C
C INPUT TURBINE GEOMETRY-INLET CONDITIONS-SPEEDS-FLUID PROPERTIES
C                                                                       4
    1 READ(5,400) A0,A1,A3,A4,D1,D3,XL,AL1,B4,ZBR ,PO,TO,
     1U3,U4,       G,R,CP,      VSTART,DELV1,VEND                       5
      U31=U3
      U41=U4
      CALL GETK1(XK1,V1VC10)
C                                                                       6
      G1=G+1.
      G2=G-1.
      G3=G2/G1
      EPS=1.4/G*((G1/2.)**(G/G2)/1.8929)                                8
      R1=D1/2.
      R3=D3/2.
C DETERMINE MODE OF OPERATION
C
      READ(5,401)MODE
C MODE=1, IMPLIES A PERFORMANCE RUN VARYING PERCENT SPEED
C MODE=2, IMPLIES INPUTTING VARIOUS VALUES OF M AND K AT DESIGN SPEED
C MODE=3, IMPLIES LETTING THE MACHINE DETERMINE THE VALUE OF M AND K
C                                                                       9
   30 GO TO (2,3,4),MODE
    2 READ(5,400)XM,XK                                                  12
    5 READ(5,400) PERCT                                                 13
      V1VC1=VSTART
      DELV =DELV1
      GO TO 6
    3 READ(5,400) XM,XK                                                 15
      PERCT=100.
      V1VC1=VSTART
      DELV =DELV1
      GO TO 6
    4 READ(5,405) PDSGN,ETAD,PTEST,ETEST                                17
  405 FORMAT(2F10.4,2I1)
      XK=1.E-8
      K=1
      V1VC1=V1VC10
      PERCT=100.
```

```
    6 U3=U31*PERCT/100.
      U4=U41*PERCT/100.
C CASE CONSTANTS
      GO TO 40
   50 VCRO=SQRT(2./G1*AC*TO*G*R)                                              24
      RHOOP=PO/R/TO
      PVCRO=VCRO*RHOOP
C STATOR ANALYSIS
      COSAL1=COS(AL1*F1)                                                      25
      SINAL1=SIN(AL1*F1)
C                                                                             26
   23 CONTINUE
      RHOVC1=SUB(V1VC1,G)
      Q1=2.*XK1*RHOVC1*A1*COSAL1/AO/G1
      Y=V1VC1
      X=Y/2.
      Q1=Q1/(1.-Y*Y)
   10 P1PPOP=(1.-XK1*G3*(X*X+Y*Y)/(1.-G3*Y*Y))**(G/G2)                        29
      SW=P1PPOP*RHOVC1*PVCRO*A1*COSAL1
      RHOVCO=SW/PVCRO/AO
      F=SUB(X,G)-RHOVCO
      Q=(F+RHOVCO)/X
      FP=Q-2.*X**2/G1*Q**(2.-G)+X*Q1*P1PPOP**(2.-G)                           30      31
      X1=X-F/FP
      IF(ABS((X1-X)/X1).LT.TOL) GO TO 9
      X=X1
      GO TO 10
C
C STATOR TO ROTOR ANALYSIS
    9 VOVCO=X1
      V2VC2=(XK1+1.)/(XK+1.)*Y*Y-(XK-XK1)/(XK+1.)*X*X
      V2VC2=SQRT(V2VC2)                                                       37
      RH2VC2=SUB(V2VC2,G)
      P2PPOP=(1.-(G3*XK*(VOVCO**2+V2VC2**2)/(1.-G3*V2VC2**2)))**(G/G2)       38
      P2PP1P=P2PPOP/P1PPOP
      TANAL2=2.*PI*R1*XL*RH2VC2*P2PP1P*SINAL1/(RHOVC1*A1*COSAL1)
      AL2=ATAN(TANAL2)                                                        39
      SINAL2=SIN(AL2)                                                         40
      R2=R1*SINAL1/SINAL2
C
C  ROTOR INLET ANALYSIS
C
      VUVC3=R2/R3*V2VC2*SINAL2
      X=V1VC1/2.
   12 Q=R2/R3*RH2VC2*COS(AL2)                                                 42
      F=Q*(1.-G3*(X*X+VUVC3**2))**(-1./G2)-X                                  43
      FP=2./G1*X*Q**2/(F+X)-1.
      X1=X-F/FP
      IF(ABS((X1-X)/X1).LT.TOL) GO TO 11
      X=X1
      GO TO 12
   11 VXVC3=X1
      V3VC3=SQRT(VXVC3**2+VUVC3**2)                                           49
      VU3=VCRO*VUVC3
      VX3=VCRO*VXVC3
      V3=VCRO*V3VC3
```

```
      W3=(V3**2-2.*U3*VU3+U3**2)**.5                                     50
      COSB3=VX3/W3
      SINB3=(VU3-U3)/W3
      B3=ARSIN(SINB3)                                                    51
      TT3=1.-G3*(2.*U3*VU3-U3**2)/VCRO**2
      T3PP=TO*TT3
      T4T3=1.-(U3**2-U4**2)/(2.*AC*XJ*CP*T3PP)
      W3WC3=VXVC3/(SQRT(TT3)*COSB3)                                      52
      WCR3=W3/W3WC3
      RHXVC3=SUB(V3VC3,G)*VXVC3/V3VC3
      WX3=W3*COSB3
      VU3OPT=U3*(1.-1.98/ZBR)
      WU3OPT=VU3OPT-U3
       PHI=ATAN(WU3OPT/WX3)
C                                                                        53
      IF(MODE.NE.3)GO TO 300
      CALL GFTM(XK,XM,1,ETAD,1)                                          57
  300 CONTINUE
C ROTOR ANALYSIS
C
      X=V1VC1/2.
      Q=RHXVC3*A3/(A4*COS(B4*F1))/(TT3*T4T3)**(G1/G2/2.)                 59    6(
      Q1=W3WC3**2*(XM*XK+SIN(B3-    PHI)**2)/T4T3                        61
      KOWNT=0
   15 Q2=1.-G3*X*X
      KOWNT=KOWNT+1
      Q3=XM*XK*X*X+Q1
       F2=(1.-G3*(Q3    )/Q2)**(G/G2)                                    64
      F3=SUB(X,G)
      F=Q/F2-F3
      F2P=G/G2*F2**(1./G)*(-2.*G3**2*Q3/Q2**2*X-2.*G3*XM*XK*X/Q2)        65
      F3P=-2.*X**2/G1*(F3/X)**(2.-G)+F3/X                                66
      FP=-Q/F2**2*F2P-F3P
      X1=X-F/FP
C
C CHECK FOR CHOKING CONDITIONS
C
      IF((X1.GT.1.).OR.(X1.LT.0.).OR. (KOWNT.GT.300 ))GO TO 13
      IF(ABS((X1-X)/X1).LT.TOL)GO TO 14
      X=X1
      GO TO 15
   14 P4P4ID=F2
       RHWCR4=F3
      W4WC4=X
      WCR4=WCR3*SQRT(T4T3)                                               75
      W4=WCR4*W4WC4
      VU4=U4-W4*ABS(SIN(F1*B4))                                          76
      WU4=W4*SIN(F1*B4)                                                  77
      T4T4=1.+G3*(U4**2+2.*U4*WU4)/WCR4**2
      P4PPOP=P1PPOP*P4P4ID*(TT3*T4T3*T4T4)**(G/G2)*P2PP1P                78
      SINB4=SIN(F1*B4)                                                   79
      COSB4=COS(F1*B4)                                                   80
      VCR4=WCR4*SQRT(T4T4)                                               81
       V4=SQRT(W4**2*COSB4**2+(U4+W4*SINB4)**2)                          82
      P4P4P=(1.-G3*(V4/VCR4)**2)**(G/G2)                                 83
      P4POP=P4PPOP*P4P4P
```

```
T4P=T4T4*T4T3*T3PP
THETCR=(VCRO/1019.46)**2
WK=VU3*U3-VU4*U4
WK=WK/AC/XJ
WK1=CP*(TO-T4P)
WIDP=CP*TO*(1.-P4PPOP**(G2/G))                                        84
WID=CP*TO*(1.-P4POP**(G2/G))                                          85
ETAT=WK/WIDP
ETAS=WK/WID
P3P4P=(1.-G3*(V3VC3**2-(VUVC3**2-(U3/VCRO)**2)**2))**(G/G2)           86
P3P3P=(1.-G3*V3VC3**2)**(G/G2)                                        87
P4PP3P=P4PPOP/P2PPOP
WRIDP=CP*TO*(1.-P4PP3P**(G2/G))-(U3-VU3)**2/(2.*AC*XJ)                88
RCTLOS=XM*XK*(W3**2+W4**2)/(2.*AC*XJ)
VC=VCRO*VOVCO
V2=VCRO*V2VC2
STALOS=XK*(VO**2+V2**2)/2./AC/XJ
 CER=ROTLOS/WID
 CES=STALOS/WID
DEL=PO/2116.22
WEQ=SW*SQRT(THETCR)/DEL*EPS                                           89
WIEQ=WK/THETCR
DEE=V4**2/(2.*AC*XJ*WID)
ETAE=ETAS+DEE
XLE=V4**2/2./XJ/AC
DEI=(W3*SIN(B3-PHI))**2/(2.*AC*XJ*WID)                                90
XLI=DEI*WID
AL2=57.3*AL2
B3=57.3*B3
PHI=57.3*PHI
ETAI=ETAE+DEI
ETAR=ETAI+DEE
XN=60.*U3/PI/D3
ECSPD=XN/SQRT(THETCR)                                                 91
XNU=U3/SQRT(2.*AC*XJ*WID)                                             92
SPPAR=SW*XN*EPS/DEL
PCPP4P=1./P4PPOP
XNOT=XN/SQRT(TO)                                                      93
WTOP=SW*SQRT(TO)*144./PO                                              94
XTOP=SW*WK*3.5616*144./XN/PO*AC*XJ
P4=P4POP*PO
T4=T4P*(1.-G3*(V4/VCR4)**2)
RHO4=P4/R/T4
Q4=SW/RHO4
CC=U3/SQRT(2.*AC*XJ*WIDP)                                             95
H4=XJ*WIDP
XNSP=XN*SQRT(Q4)/H4**.75                                              96     97
SPPAR=SW*XN/DEL
GOD=7537.129*WK*SW/PO/XN*AC*XJ
PCPP4=1./P4POP
IF(MODE.NE.3) GO TO 20
EX=ETAS
IF(ETEST.EQ.2) EX=ETAT
IF(K.EQ.6) GO TO 25
CALL GETM(XK,XM,2,EX,K)                                               106
GO TO 23
```

```
   21 V1VC1=V1VC1+DELV
      IF(V1VC1.GT.VEND) GO TO 22
      GO TO 23
   22 V1VC1=VSTART
      DELV=DELV1
      IF(MODE.EQ.1) GO TO 5
      IF(MODE.EQ.2) GO TO 3
      IF(MODE.EQ.3) GO TO 4
   25 CONTINUE
      WRITE(6,509) XM,XK,XK1,PERCT,U3,U4,PREF,WREF                      123
      IF(PTEST .EQ. 1) WRITE(6,529) PDSGN                               124
      IF(PTEST .EQ. 2) WRITE(6,526) PDSGN                               126
      IF(ETEST .EQ. 1) WRITE(6,527) ETAD                                128
      IF(ETEST .EQ. 2) WRITE(6,528) ETAD                                130
      GO TO 20
C CHOKED
   13 IF(MODE.EQ.3) GO TO 20
      V1VC1=V1VC1-DELV
      DELV=DELV/2.
      IF(DELV.LT.5.E-4) GO TO (5,3),MODE
      GO TO 21
C
C                   OUTPUT FOR SPEED V1VC1
   20 CONTINUE
      WRITE(6,505)                                                      140
  505 FORMAT(1H9)
      WRITE(6,506) V1VC1,P2PPOP,V3,WCR3,VCRO,P2PP1P,B3,W4,RHOOP,
     1AL2,TT3,WCR4,PVCRO,R2,T3PP,VU4,RHOVC1,VUVC3,T4T3,T4T4,SW,
     2VXVC3,W3WC3,P4PPOP,RHOVCO,V3VC3,RHXVC3,VCR4
C                                                                       141
      WRITE(6,507) VOVCO,VU3,P4P4ID,V4,V2VC2,W3,RHWCR4,P4P4P,
     1RH2VC2,VX3,W4WC4,P4POP,WK,ETAE,WIDP,T4P,WID,ETAT,ETAS,WEQ,
     2ETAI,XLF,DEF,XLI,ROTLOS,STALOS,DER,DES,XN,EQSPD,XNU,THETCR
C                                                                       142
      WRITE(6,508) DEL,SPPAR,W1EC,POPP4P,XNOT,WTOP,XTOP,DEI,
     1POPP4,PHI,P1PPOP,ETAR   ,XNSP
C                                                                       143
      GO TO 21
C                 OUTPUT INPUT
   40 WRITE(6,501) A0,A1,A3,A4,D1,D3,XL,AL1,B4,ZBR,     PO,TO,
     1U31,U41,G,R          ,CP   ,EPS                                   145
      IF(MODE.EQ.1) WRITE(6,502)                                        146
      IF(MODE.EQ.2) WRITE(6,503)                                        148
      IF(MODE.EQ.3) GO TO 525
  509 FORMAT(14X,1HM,G15.6/14X,1HK,G15.6/13X,2HK1,G15.6/7X,8H PERCENT,
     1G15.6 /13X,2HU3,G15.6/13X,2HU4,G15.6/1X,14HDESIGN P1,/PO, G15.6 /
     2 1X,14HDESIGN SMALL-W,G15.6  )
      WRITE(6,509) XM,XK,XK1,PERCT,U3,U4,PREF,WREF                      152
      WRITE(6,505)                                                      153
      GO TO 50
  525 WRITE(6,504)                                                      155
      GO TO 50
C                       FORMATS
  529 FORMAT(2X,13HDESIGN PO,/P4  G15.6)
  526 FORMAT(1X,14HDESIGN PO,/P4, G15.6)
  527 FORMAT(2X 13HDESIGN ETA-S  G15.6)
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```
  528 FORMAT(2X 13HDESIGN ETA-T  G15.6)
  400 FORMAT(8E10.5)
  501 FORMAT(1H1,
     1        36X,49HOFF DESIGN PERFORMANCE OF A RADIAL INFLOW TURBINE //
     121H TURBINE GEOMETRY  A0,G16.6,2X,2HA1,G16.6,2X,2HA3,G16.6,2X,
     22HA4,G16.6,2X,2HD1,G16.6 / 19X,2HD3,G16.6,2X,1HL,G17.6,2X,3HAL1,
     3G16.6,1X,2HB4,G16.6,2X,1HZ,G16.6 /21H INLET CONDITIONS  P0,
     4G16.6,2X,2HT0,G16.6 /14H DESIGN SPEEDS,5X,2HU3,G16.6,2X, 2HU4,
     5G16.6/20H FLUID PROPERTIES  G,G17.6,2X,1HR,G17.6,2X,2HCP,G16.6 ,
     62X,3HEPS,G16.6)
  401 FORMAT(I1)
  502 FORMAT( 8HKMODE 1,2X,37HPERFORMANCE RUN VARYING PERCENT SPEED )
  503 FORMAT( 8HKMODE 2,2X,31HMANUAL DETERMINATION OF M AND K )
  504 FORMAT( 8HKMODE 3,2X,34HAUTCMATIC DETERMINATION OF M AND K )
C
  506 FORMAT(12H     V/VCR)1,G15.6,9X,7HP2,/P0,,G15.6,14X,2HV3,G15.6,
     111X,4HWCR3,G15.6/7X,5HVCR)0,G15.6,9X,7HP2,/P1,,G15.6,11X,5HBETA3,
     2G15.6,13X,2HW4,G15.6/8X,5HRHO)0,G15.6,10X,6HALPHA2,G15.6,8X,
     38HT,,/T,)3,G15.6,11X,4HWCR4,G15.6/12H   RHO-VCR)0,G15.6,14X,2HR2,
     4G15.6,12X,4HT3,,,G15.6,12X,3HVU4,G15.6/12H    PV/PVCR)1,G15.6,
     58X,8HVU/VCR)3,G15.6,7X,9HT4,,/T3,,,G15.6,7X,8HT4,/T4,,,G15.6 /
     612H       SMALL-W,G15.6,8X,8HVX/VCR)3,G15.6,9X,7HW/WCR)3,G15.6,6X,
     77HP4,/P0,,G15.6 /12H     PV/PVCR)0,G15.6,9X,7HV/VCR)3,G15.6,6X,
     810HPVX/PVCR)3,G15.6,11X,4HVCR4,G15.6 )
  507 FORMAT(12H     V/VCR)0,G15.6,13X,3HVU3,G15.6,9X,7HP4/P4ID,G15.6,
     113X,2HV4,G15.6 /12H      V/VCR)2,G15.6,14X,2HW3,G15.6,7X,
     29HPW/PWCR)4,G15.6,9X,6HP4/P4,,G15.6 /12H    PV/PVCR)2,G15.6,13X,
     33HVX3,G15.6,9X,7HW/WCR)4,G15.6,9X,6HP4/P0,,G15.6/10X,2HW1,G15.6,
     411X,5HETA-E,G15.6,12X,4HW,ID,G15.6,12X,3HT4,,G15.6/9X,3HWID,G15.6,
     511X,5HETA-T,G15.6,11X,5HETA-S,G15.6,12X,3HWEQ,G15.6 /
     61H 6X,5HETA-I,G15.6,14X,2HLE,G15.6,13X,3HDEE,G15.6,13X,
     72HLI,G15.6/12H  ROTOR-LOSS,G15.6,5X,11HSTATOR-LOSS,G15.6,
     813X,3HDER,G15.6,12X,3HDES,G15.6/1H ,6X,5HSPEED,G15.6,8X,
     X8HEQ-SPEED,
     9G15.6,14X,2HNU,G15.6,7X,8HTHETA-CR,G15.6 )
C
  508 FORMAT(7X,5HDELTA,G15.6,7X,9HSPEED-PAR,G15.6,
     111X,5HW1-EQ,G15.6,8X,7HP0,/P4,,G15.6/9X,3HNOT,G15.6,12X,4HWTOP,
     2G15.6,13X,3HTOP,G15.6,12X,3HDEI,G15.6/
     31H 5X,6HP0,/P4,G15.6,13X,3HPHI,G15.6,9X,7HP1,/P0,,G15.6,
     410X,5HETA-R,G15.6/1H 8X,3HNSP,G15.6)
      END
```

Internal Program Variables and Definitions

AO	A_0
A1	A_1
A3	A_3
A4	A_4
AC	acceleration constant
AL1	α_1
AL2	α_2
B3	β_3
B4	β_4
COSAL1	$\cos \alpha_1$
COSB3	$\cos \beta_3$
COSB4	$\cos \beta_4$
CP	c_p (input variable)
D1	D_1
D3	D_3
DEE	$\Delta \eta_E$
DEI	$\Delta \eta_I$
DEL	δ
DELV	incremental value of $\left(V/V_{cr}\right)_1$ (input variable)

DER	$\Delta\eta_R$
DES	$\Delta\eta_s$
DM	Δm
ETAD	design value of efficiency
ETAE	η_E
ETAI	η_I
ETAR	η_R
ETAS	η_s
ETAT	η_t
EPS	ϵ
F	temporary storage
F1	temporary storage
F2	temporary storage
F2P	temporary storage
F3	temporary storage
F3P	temporary storage
FP	temporary storage
G	γ
G1	$\gamma + 1$
G2	$\gamma - 1$
G3	$(\gamma - 1)/(\gamma + 1)$
H4	temporary storage

KMAX	maximum number of k's
MODE	if MODE = 1, performance run varying percent speed if MODE = 2, manual determination of m and k if MODE = 3, automatic determination of m and k (input variables)
PO	p'_0
POPP4	p'_0/p_4
POPP4P	p'_0/p'_4
P1PPOP	p'_1/p'_0
P2PPOP	p'_2/p'_0
P2PP1P	p'_2/p'_1
P3P3P	p_3/p'_3
P3P4P	p_3/p'_4
P4	p_4
P4POP	p_4/p'_0
P4P4ID	$p''_4/p''_{4,id}$
P4P4P	p_4/p'_4
P4PPOP	p'_4/p'_0
P4PP3P	p'_4/p'_3
PDSGN	specified p'_0/p'_4 or p'_0/p_4 (input variable)
PERCT	percent of design speed (input variable)
PHI	φ

PIN	$p_0'/144$
PREF	estimate of p_1'/p_0' to determine k_1 (input variable)
PVCRO	$(\rho' V_{cr})_0$
Q	temporary storage
Q1	temporary storage
Q2	temporary storage
Q3	temporary storage
Q4	temporary storage
R	R (input variable)
R1	r_1
R2	r_2
R3	r_3
RH2VC2	$(\rho V/\rho' V_{cr})_2$
RHOOP	ρ_0'
RHOVCO	$(\rho V/\rho' V_{cr})_0$
RHWCR4	$(\rho W/\rho'' W_{cr})_4$
RHXVC3	$(\rho V_x/\rho' V_{cr})_3$
ROTLOS	L_R
SINAL1	$\sin \alpha_1$
SINAL2	$\sin \alpha_2$
SINB3	$\sin \beta_3$

SPPAR	wN/δ
STALOS	L_S
TO	T'_0
T4	T_4
T4P	T'_4
T3PP	T''_3
T4T3	T''_4/T''_3
T4T4	$(T'/T'')_4$
THETCR	θ_{cr}
TT3	$(T''/T')_3$
U3	U_3 (input variable)
U31	initial U_3
U4	U_4 (input variable)
U41	initial U_4
VO	V_0
V2	V_2
V3	V_3
V4	V_4
VOVCO	$(V/V_{cr})_0$
V1VC1	see input

V2VC2	$(V/V_{cr})_2$
V3VC3	$(V/V_{cr})_3$
VCRO	$V_{cr,0}$
VCR4	$V_{cr,4}$
VEND	end value of $(V/V_{cr})_1$ (input variable)
VSTART	starting value of $(V/V_{cr})_1$ (input variable)
VU3	$V_{u,3}$
VU4	$V_{u,4}$
VU3OPT	eq. (B16b)
VUVC3	$(V_u/V_{cr})_3$
VXVC3	$(V_x/V_{cr})_3$
W3	W_3
W4	W_4
W3WC3	$(W/W_{cr})_3$
WCR3	$W_{cr,3}$
WEQ	w_{eq}
WEQD	see input
WID	$\Delta h'_{id,s}$
WIDP	$\Delta h'_{id,t}$
WREF	estimate of W to determine k_1 (input variable)

WTOP	$w\sqrt{T'_0}/p'_0$
WU3OPT	eq. (B16c)
WU4	$W_{u,4}$
WX3	$W_{x,3}$
X	temporary storage
X1	temporary storage
XJ	constant, 778.16
XK1	k_1
XLE	L_E
XN	N
XNOT	N/T'_0
XNSP	NSP
XNV	ν
XTOP	Γ/p'_{in}
ZBR	Z (input variable)

Subroutine GETK1(XK1:X1)

This routine evaluates the loss coefficient k_1 and a design $(V/V_{cr})_1$ with a specified p'_0/p'_1 and weight flow w where

XK1 loss coefficient k_1

X1 design $(V/V_{cr})_1$

The subroutine is as follows:

```
      SUBROUTINE GETK1(XK1,X1)
      COMMON /GETK/ P,W,G,AO,A1,AL,T,R,PO
      SUB(X,G)=X*(1.-(G-1.)/(G+1.)*X*X)**(1./(G-1.))
      VCRO=(64.34*G*R*T /(G+1.))**.5
      RHOO=PO/R/T
      Q=W/(RHOO*AO*VCRO)
      ROCO=Q
      X=.1
    1 F=SUB(X,G)-Q
      F1=(F+Q)/X
      FP=-2.*X*X/(G+1.)*F1**(2.-G)+F1
      X1=X-F/FP
      IF(ABS((X1-X)/X1).LT.1.E-5) GO TO 2
      X=X1
      GO TO 1
    2 VC=X1
      Q=W/(P*RHOO*VCRO*A1*COS(AL*.01745))
      X=.1
    3 F=SUB(X,G)-Q
      ROC1=F+Q
      F1=(F+Q)/X
      FP=-2.*X*X/(G+1.)*F1**(2.-G)+F1
      X1=X-F/FP
      IF(ABS((X1-X)/X1).LT.1.E-5) GO TO 4
      X=X1
      GO TO 3
    4 XK1=(P**((G-1.)/G)-1.)*(1.-(G-1.)/(G+1.)*X1**2)/
     1((G-1.)/(G+1.)*(VO**2+X1**2))*(-1.)
      RETURN
      END
```

Subroutine GETM(XK, XM, IGO, ETEST, K)

For IGO = 1, this routine computes a value of XM as a function of XK by using equations (B17b) to (B24b). For IGO = 2 this routine varies the value of XK by false positioning until all design specifications are met. The following are defined for this subroutine:

XK	loss coefficient
XM	loss coefficient
IGO	index for mode of operation
ETEST	design value of η_t or η_s
K	indicator used in method of false positioning

The subroutine is as follows:

```
      SUBROUTINE GETM(XK,XM,IGO,ETEST,K)
      COMMON/GETK/PREF,WREF,G,A0,A1,AL1,T0,R,P0,P4PP0Z,CP,G1,G2,G3,
     1ETAT,U3,U4,VU3,B4,T4T3,WCR3,TT3,W3WC3,B3,PHI,P2PP1P,P1PPOP
      IF(IGO.GT.1) GO TO 100
       P4PPOP=1./P4PP0Z
      DHID=CP*T0*(1.-P4PPOP**(G2/G))                                   5
      DH=DHID*ETAT
      VU4=(DH*25033.-VU3*U3)/U4
      VU4=-VU4
      W4=(VU4-U4)/ABS(SIN(.01745*B4))                                  6
      W4=-W4
      WCR4=WCR3*SQRT(T4T3)                                             7
      W4WC4=W4/WCR4
      WU4=W4*SIN(.01745*B4)                                            8
      T4T4=1.+G3*(U4**2+2.*U4*WU4)/WCR4**2
      P4P4ID=P4PPOP/(P2PP1P*P1PPOP*(TT3*T4T3*T4T4)**(G/G2))            9
      T=W3WC3**2/T4T3
      T1=-(P4P4ID**(G2/G)-1.)*(1.-G3*W4WC4**2)/G3                      10
      XMK=(T1-T*(SIN(B3-PHI))**2)/(W4WC4**2+T)                         11
      XM=XMK/XK
      RETURN
  100 GO TO (101,102),K
  101 XK1=XK
      E1=ETEST
      DIF1=ETEST-ETAT
      K=2
      XK=XK+.005
      RETURN
  102 XK2=XK
      E2=ETEST
      DIF2=ETEST-ETAT
      IF(DIF2*DIF1) 106,105,104
  104 E1=E2
      XK1=XK2
      XK=XK+.005
      DIF1=DIF2
      RETURN
  105 XK=XK2
      K=6
      RETURN
  106 XK=-(XK2-XK1)/(DIF2-DIF1)*DIF1+XK1
      K=6
      RETURN
      END
```

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, December 6, 1968,
125-23-02-10-22.

APPENDIX A

SYMBOLS

A	area, sq ft
c_p	specific heat at constant pressure, Btu/(lb)(^oR)
D	diameter, ft
g	dimensional constant, 32.1741 ft/sec^2
$\Delta h'$	specific turbine work, Btu/lb
$\Delta h'_{id,s}$	ideal turbine work based on inlet total to exit static pressure ratio, Btu/lb
$\Delta h'_{id,t}$	ideal turbine work based on inlet total to exit total pressure ratio, Btu/lb
J	mechanical equivalent of heat, 778.029 ft-lb/Btu
k_1, k, m	loss coefficients, dimensionless
L	kinetic energy loss, Btu/lb
l	axial distance, ft
N	turbine speed, rpm
NSP	specific speed
p	pressure, lb/sq ft
p_{in}	turbine inlet pressure, psia
R	gas constant, (ft-lb)/(lb)(^oR)
r	radius, ft
T	absolute temperature, ^oR
U	blade speed, ft/sec
V	absolute velocity of gas, ft/sec
W	gas velocity relative to rotor, ft/sec
w	mass flow rate, lb/sec
Z	number of rotor blades at tip of rotor
α	absolute gas angle, angle between absolute velocity vector and meridional plane at mean channel, positive when direction of tangential velocity component and direction of wheel velocity agree and negative when they disagree

β relative gas angle, angle between the velocity vector relative to wheel and meridional plane at mean channel; same sign convention applies as for α

Γ torque, ft-lb

γ ratio of specific heat at constant pressure to specific heat at constant volume

δ ratio of turbine inlet total pressure to U.S. standard atmospheric pressure, $p_0'/2116.22$

ϵ function of γ used in relating parameters to those using air inlet conditions at U.S. standard sea-level conditions

$$\frac{\gamma^*}{\gamma}\left[\left(\frac{\gamma+1}{2}\right)^{\gamma/(\gamma-1)} \Big/ \left(\frac{\gamma^*+1}{2}\right)^{\gamma^*/(\gamma^*-1)}\right]$$

η_E, η_I, η_R modified efficiency figures used in studying loss distribution in turbine, dimensionless

η_s efficiency based on ratio of inlet total to exit static pressure

η_t efficiency based on ratio of inlet total to exit total pressure

θ_{cr} squared ratio of critical velocity at turbine inlet to critical velocity at U.S. standard atmospheric temperature, $\left(V_{cr,0}/1019.46\right)^2$

ν blade-jet speed ratio, $U_3\Big/\sqrt{2gJ\,\Delta h'_{id,s}}$

φ optimum rotor incidence angle, deg

ρ gas density, lb/cu ft

Subscripts:

cr condition corresponding to Mach 1

E exit

eq air equivalent (U.S. standard sea level)

I incidence

id ideal

opt optimum

R rotor

S stator

u tangential component, positive when its direction agrees with that of wheel velocity and negative when it disagrees x meridional component, component in plane containing axis of rotation

0 station at turbine inlet

1 station immediately upstream of stator exit

2 station where flow from stator exit is assumed to occupy entire cylindrical area

3 station immediately upstream of rotor inlet

4 station immediately downstream of rotor exit

Superscripts:

(') absolute total state

('') total state relative to rotor

(*) U.S. standard sea-level conditions (temperature, 518.67° R; pressure, 14.696 psia)

APPENDIX B

THERMODYNAMIC EQUATIONS

The equations in this appendix are listed in the order used by the computer. A complete set of calculations is made for each specified value of $\left(V/V_{cr}\right)_1$ at each speed.

$$\left(\frac{\rho V}{\rho' V_{cr}}\right)_1 = \left(\frac{V}{V_{cr}}\right)_1 \left[1 - \frac{\gamma - 1}{\gamma + 1}\left(\frac{V}{V_{cr}}\right)_1^2\right]^{1/(\gamma-1)} \tag{B1}$$

Equations (B2) to (B5) are to be solved simultaneously for p_1'/p_0':

$$w = \frac{p_1'}{p_0'}\left(\frac{\rho V}{p' V_{cr}}\right)_1 \left(p' V_{cr}\right)_0 A_1 \cos \alpha_1 \tag{B2}$$

$$\left(\frac{\rho V}{\rho' V_{cr}}\right)_0 = \frac{w}{(\rho' V_{cr} A)_0} \tag{B3}$$

Calculate $\left(V/V_{cr}\right)_0$ from

$$\left(\frac{\rho V}{\rho' V_{cr}}\right)_0 = \left(\frac{V}{V_{cr}}\right)_0 \left[1 - \frac{\gamma - 1}{\gamma + 1}\left(\frac{V}{V_{cr}}\right)_0^2\right]^{1/(\gamma-1)} \tag{B4}$$

$$\frac{p_1'}{p_0'} = \left\{1 - \frac{\dfrac{\gamma - 1}{\gamma + 1} k_1 \left[\left(\dfrac{V}{V_{cr}}\right)_0^2 + \left(\dfrac{V}{V_{cr}}\right)_1^2\right]}{1 - \dfrac{\gamma - 1}{\gamma + 1}\left(\dfrac{V}{V_{cr}}\right)_1^2}\right\}^{\gamma/(\gamma-1)} \tag{B5}$$

$$\left(\frac{V}{V_{cr}}\right)_2 = \left[\frac{k_1 + 1}{k + 1}\left(\frac{V}{V_{cr}}\right)_1^2 - \frac{k - k_1}{k + 1}\left(\frac{V}{V_{cr}}\right)_0^2\right]^{1/2} \tag{B6a}$$

$$\left(\frac{\rho V}{\rho' V_{cr}}\right)_2 = \left(\frac{V}{V_{cr}}\right)_2 \left[1 - \frac{\gamma - 1}{\gamma + 1}\left(\frac{V}{V_{cr}}\right)_2^2\right]^{1/(\gamma-1)} \tag{B6b}$$

$$\frac{p_2'}{p_0'} = \left\{1 - \frac{\frac{\gamma - 1}{\gamma + 1} k \left[\left(\frac{V}{V_{cr}}\right)_0^2 + \left(\frac{V}{V_{cr}}\right)_2^2\right]}{1 - \frac{\gamma - 1}{\gamma + 1}\left(\frac{V}{V_{cr}}\right)_2^2}\right\}^{\gamma/(\gamma-1)} \tag{B7}$$

$$\frac{p_2'}{p_1'} = \frac{\frac{p_2'}{p_0'}}{\frac{p_1'}{p_0'}} \tag{B8}$$

$$\alpha_2 = \tan^{-1}\left[\frac{\pi D_1 l \left(\frac{\rho V}{\rho' V_{cr}}\right)_2 \left(\frac{p_2'}{p_1'}\right) \sin \alpha_1}{\left(\frac{\rho V}{\rho' V_{cr}}\right)_1 A_1 \cos \alpha_1}\right] \tag{B9}$$

$$\gamma_2 = \gamma_1 \frac{\sin \alpha_1}{\sin \alpha_2} \tag{B10}$$

$$\left(\frac{V_u}{V_{cr}}\right)_3 = \frac{\gamma_2}{\gamma_3}\left(\frac{V}{V_{cr}}\right)_2 \sin \alpha_2 \tag{B11}$$

Determine $\left(V_x/V_{cr}\right)_3$ from

$$\left(\frac{V_x}{V_{cr}}\right)_3 = \frac{\frac{\gamma_2}{\gamma_3}\left(\frac{\rho V}{\rho' V_{cr}}\right)_2 \cos \alpha_2}{\left\{1 - \frac{\gamma - 1}{\gamma + 1}\left[\left(\frac{V_x}{V_{cr}}\right)_3^2 + \left(\frac{V_u}{V_{cr}}\right)^2\right]\right\}^{1/(\gamma-1)}} \tag{B12}$$

$$\left(\frac{V}{V_{cr}}\right)_3 = \left[\left(\frac{V_x}{V_{cr}}\right)_3^2 + \left(\frac{V_u}{V_{cr}}\right)_3^2\right]^{1/2} \tag{B13a}$$

$$\left(\frac{\rho V}{\rho' V_{cr}}\right)_3 = \left(\frac{V}{V_{cr}}\right)_3 \left[1 - \frac{\gamma - 1}{\gamma + 1}\left(\frac{V}{V_{cr}}\right)_3^2\right]^{1/(\gamma-1)} \tag{B13b}$$

$$V_{u,3} = V_{cr,0}\left(\frac{V_u}{V_{cr}}\right)_3 \tag{B14a}$$

$$V_{x,3} = V_{cr,0}\left(\frac{V_x}{V_{cr}}\right)_3 \tag{B14b}$$

$$V_3 = V_{cr,0}\left(\frac{V}{V_{cr}}\right)_3 \tag{B14c}$$

$$W_3 = \left(V_3^2 - 2U_3V_{u,3} + U_3^2\right)^{1/2} \tag{B14d}$$

$$\cos \beta_3 = \frac{V_{x,3}}{W_3} \tag{B14e}$$

$$\beta_3 = \sin^{-1} \frac{V_{u,3} - U_3}{W_3} \tag{B14f}$$

$$\left(\frac{T''}{T'}\right)_3 = 1 - \frac{\gamma - 1}{\gamma + 1} \left(\frac{2U_3 V_{u,3} - U_3^2}{V_{cr,0}^2}\right) \tag{B15a}$$

$$T_3'' = T_0' \left(\frac{T''}{T'}\right)_3 \tag{B15b}$$

$$\frac{T_4''}{T_3''} = 1 - \frac{U_3^2 - U_4^2}{2gJc_p T_3''} \tag{B15c}$$

$$\left(\frac{W}{W_{cr}}\right)_3 = \frac{\left(\frac{V_x}{V_{cr}}\right)_3}{\left(\frac{T''}{T'}\right)_3^{1/2} \cos \beta_3} \tag{B15d}$$

$$\left(\frac{\rho V_x}{\rho' V_{cr}}\right)_3 = \left(\frac{\rho V}{\rho' V_{cr}}\right)_3 \frac{\left(\frac{V_x}{V_{cr}}\right)_3}{\left(\frac{V}{V_{cr}}\right)_3} \tag{B15e}$$

Calculate the angle φ as follows:

$$\frac{V_{u,3,opt}}{U_3} = 1 - \frac{1.98}{Z} \tag{B16a}$$

$$V_{u,3,opt} = U_3\left(\frac{V_{u,3,opt}}{U_3}\right) \tag{B16b}$$

$$W_{u,3,opt} = V_{u,3,opt} - U_3 \tag{B16c}$$

$$\varphi = \tan^{-1} \frac{W_{u,3,opt}}{V_{x,3}} \tag{B16d}$$

The following three equations are to be solved simultaneously for $\left(W/W_{cr}\right)_4$ and $p''_4/p''_{4,id}$:

$$\left(\frac{\rho W}{\rho'' W_{cr}}\right)_4 = \frac{\left(\frac{\rho V_x}{\rho' V_{cr}}\right)_3 \frac{A_3}{A_4 \cos \beta_4}}{\frac{p''_4}{p''_{4,id}} \left(\frac{T''}{T'}\right)_3^{(\gamma+1)/2(\gamma-1)} \left(\frac{T''_4}{T''_3}\right)^{(\gamma+1)/2(\gamma-1)}} \tag{B17a}$$

$$\frac{p''_4}{p''_{4,id}} = 1 - \frac{\frac{\gamma - 1}{\gamma + 1}\left\{mk\left(\frac{W}{W_{cr}}\right)_4^2 + \frac{\left(\frac{W}{W_{cr}}\right)_3^2}{\frac{T''_4}{T''_3}}\left[mk + \sin^2\left(\beta_3 - \varphi\right)\right]\right\}^{\gamma/(\gamma-1)}}{1 - \frac{\gamma - 1}{\gamma + 1}\left(\frac{W}{W_{cr}}\right)_4^2} \tag{B17b}$$

$$\left(\frac{\rho W}{\rho'' W_{cr}}\right)_4 = \left(\frac{W}{W_{cr}}\right)_4 \left[1 - \frac{\gamma - 1}{\gamma + 1}\left(\frac{W}{W_{cr}}\right)_4^2\right]^{1/(\gamma-1)} \tag{B17c}$$

$$W_{cr,3} = \frac{W_3}{\left(\frac{W}{W_{cr}}\right)_3} \tag{B18a}$$

$$W_{cr,4} = W_{cr,3}\left(\frac{T''_4}{T''_3}\right)^{1/2} \tag{B18b}$$

$$W_4 = W_{cr,4}\left(\frac{W}{W_{cr}}\right)_4 \tag{B18c}$$

$$\left(\frac{T'}{T''}\right)_4 = 1 + \frac{\gamma - 1}{\gamma + 1}\left(\frac{U_4^2 + 2U_4W_{u,4}}{W_{cr,4}^2}\right) \tag{B18d}$$

$$W_{u,4} = W_4 \sin \beta_4 \qquad (\beta_4 \text{ will be negative}) \tag{B18e}$$

$$\frac{p'_4}{p'_0} = \frac{p'_2}{p'_1}\frac{p'_1}{p'_0}\frac{p''_4}{p''_{4,id}}\left[\left(\frac{T''}{T'}\right)_3\left(\frac{T''_4}{T''_3}\right)\left(\frac{T'}{T''}\right)_4\right]^{\gamma/(\gamma-1)} \tag{B19a}$$

$$V_{cr,4} = W_{cr,4}\left(\frac{T'}{T''}\right)_4^{1/2} \tag{B19b}$$

$$W_4 = W_{cr,4}\left(\frac{W}{W_{cr}}\right)_4 \tag{B19c}$$

$$V_4 = \left[W_4^2 \cos^2\beta_4 + \left(U_4 + W_4 \sin \beta_4\right)^2\right]^{1/2} \tag{B19d}$$

$$\frac{p_4}{p'_4} = \left[1 - \frac{\gamma - 1}{\gamma + 1}\left(\frac{V}{V_{cr}}\right)_4^2\right]^{\gamma/(\gamma-1)} \tag{B20a}$$

$$\frac{p_4}{p_0'} = \frac{p_4'}{p_0'}\frac{p_4}{p_4'} \tag{B20b}$$

$$V_{u,4} = U_4 + W_4 \sin \beta_4 \tag{B21a}$$

$$T_4' = \frac{T_4'}{T_4''}\frac{T_4''}{T_3''} T_3'' \tag{B21b}$$

$$\Delta h' = \frac{V_{u,3}U_3 - V_{u,4}U_4}{gJ} \tag{B22}$$

$$\Delta h_{id,t}' = c_p T_0' \left[1 - \left(\frac{p_4'}{p_0'}\right)^{(\gamma-1)/\gamma}\right] \tag{B23a}$$

$$\Delta h_{id,s}' = c_p T_0' \left[1 - \left(\frac{p_4}{p_0'}\right)^{(\gamma-1)/\gamma}\right] \tag{B23b}$$

$$\eta_t = \frac{\Delta h'}{\Delta h_{id,t}'} \tag{B24a}$$

$$\eta_s = \frac{\Delta h'}{\Delta h_{id,s}'} \tag{B24b}$$

The foregoing equations have given the flow conditions through the turbine and the efficiencies. The following equations give quantities which are useful for special purposes, such as identifying sources of loss in the turbine:

$$L_R = mk\frac{W_3^2 + W_4^2}{2gJ} \qquad \text{(rotor loss)} \tag{B25}$$

$$V_0 = V_{cr,0}\left(\frac{V}{V_{cr}}\right)_0 \tag{B26a}$$

$$V_2 = V_{cr,0}\left(\frac{V}{V_{cr}}\right)_2 \tag{B26b}$$

$$L_S = k\,\frac{V_0^2 + V_2^2}{2gJ} \quad \text{(stator loss)} \tag{B26c}$$

$$\Delta\eta_R = \frac{L_R}{\Delta h'_{id,s}} \quad \text{(fractional rotor loss)} \tag{B27}$$

$$\Delta\eta_S = \frac{L_S}{\Delta h'_{id,s}} \quad \text{(fractional stator loss)} \tag{B28}$$

$$L_E = \frac{V_4^2}{2gJ} \quad \text{(exit loss)} \tag{B29a}$$

$$\Delta\eta_E = \frac{L_E}{\Delta h'_{id,s}} \quad \text{(fractional exit loss)} \tag{B29b}$$

$$L_I = \frac{W_3^2 \sin^2(\beta_3 - \varphi)}{2gJ} \quad \text{(incidence loss)} \tag{B30a}$$

$$\Delta\eta_I = \frac{L_I}{\Delta h'_{id,s}} \quad \text{(fractional incidence loss)} \tag{B30b}$$

$$\left.\begin{aligned}\eta_E &= \eta_S + \Delta\eta_E \\ \eta_R &= \eta_E + \Delta\eta_R \\ \eta_I &= \eta_R + \Delta\eta_I\end{aligned}\right\} \tag{B31}$$

$$\theta_{cr} = \left(\frac{V_{cr,0}}{1019.46}\right)^2 \tag{B32a}$$

$$\epsilon = \frac{1.40}{\gamma}\left[\frac{\left(\frac{\gamma+1}{2}\right)^{\gamma/(\gamma-1)}}{1.8929}\right] \tag{B32b}$$

$$\delta = \frac{p_0'}{2116.22} \tag{B32c}$$

$$w^{eq} = w\frac{\sqrt{\theta_{cr}}}{\delta}\epsilon \quad \text{(equivalent weight flow)} \tag{B33}$$

$$\frac{wN}{\delta}\epsilon \quad \text{(weight flow-speed parameter)} \tag{B34}$$

$$N_{eq} = \frac{N}{\sqrt{\theta_{cr}}} \quad \text{(equivalent speed)} \tag{B35}$$

$$\nu = \frac{U_3}{\sqrt{2gJ\,\Delta h_{id,s}'}} \quad \text{(blade-jet speed ratio)} \tag{B36}$$

$$\Delta h_{eq}' = \frac{\Delta h'}{\theta_{cr}} \quad \text{(equivalent specific work)} \tag{B37}$$

$$NOT = \frac{N}{\sqrt{T_0'}} \tag{B38}$$

$$WTOP = \frac{w\sqrt{T_0'}}{p_{in}'} \tag{B39}$$

where $p_{in}' = p_0'/144$

$$TOP = \frac{\Gamma}{p_{in}'} \tag{B40}$$

where $\Gamma = 89\ 155\ w\ \Delta h'/N$ (in. -lb)

$$NSP = \frac{N\sqrt{Q}}{H^{3/4}} \quad \text{(specific speed)} \tag{B41}$$

where $Q = w/\rho_4$ and $H = J\ \Delta h_{id,t}'$.

For calculation of ρ_4,

$$p_4 = \frac{p_4}{p_0'} p_0'$$

$$T_4 = T_4' \left[1 - \frac{\gamma - 1}{\gamma + 1}\left(\frac{V}{V_{cr}}\right)_4^2\right]$$

$$\rho_4 = \frac{p_4}{RT_4}$$

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