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BUCKLING OF STIFFENED CYLINDERS
IN AXIAL COMPRESSION AND BENDING —
A REVIEW OF TEST DATA

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# BUCKLING OF STIFFENED CYLINDERS IN AXIAL COMPRESSION AND BENDING — A REVIEW OF TEST DATA

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#### **SUMMARY**

Test data on stiffened cylinders which failed by general instability under uniform axial compression and/or bending are reviewed, and the adequacy of contemporary methods for predicting buckling are appraised by comparing test data with results obtained from a contemporary buckling theory. Buckling failures of well-constructed cylinders with  $\pm 45^{\circ}$  waffle stiffening were experienced at loads as low as 65 percent of the loads calculated for the cylinders, and other cylinders of different construction failed at loads approaching the 65-percent value. The latter cylinders include both isotropic and corrugated cylinders with ring stiffening, the only other types of construction for which much test data are available. Hence, testing to date has not revealed any type of stiffened cylinder construction that fails by general instability but is immune to low failing loads, and design methods which neglect the disparity between theory and test are likely to be unconservative.

## INTRODUCTION

Several experimental investigations of buckling of stiffened cylinders in axial compression or bending have recently appeared in the literature. (See refs. 1 to 9.) These investigations contain information that is extremely valuable to designers who must rely on such data to obtain correlation factors which correct calculated buckling loads to some safe level. The usefulness of the data is compromised to some extent, however, by lack of a common basis of comparison. The calculations from the various investigations differ in certain respects; these differences and the influences of the differences are not readily apparent from the references. In the present study the data are reviewed and compared with a single buckling theory, and the differences between the calculations made herein and those of the various investigators are discussed.

The review provides an opportunity to assess the adequacy of available data for design purposes; hence, the data are studied and their validity appraised for use in devising a correlation factor. Much of the data had to be rejected as being deficient in

some respect. The reasons for rejection and the adequacy of the remaining data are discussed.

# **SYMBOLS**

The units for physical quantities in this paper are given both in the U.S. Customary Units and in the International System of Units (SI). Factors relating the two systems are given in reference 10; those factors used in the present paper are given in appendix A.

$\mathbf{A}_{\mathbf{n}}$	area of stiffeners in nth set of stiffeners
$b_n$	perpendicular distance between stiffeners in nth set of stiffeners
D <sub>x</sub> ,D <sub>y</sub>	bending stiffness of orthotropic skin in x- and y-direction, respectively
$D_{XY}$	twisting stiffness of orthotropic skin
$\mathbf{E}_{\mathbf{x}}, \mathbf{E}_{\mathbf{y}}$	extensional stiffness of orthotropic skin in x- and y-direction, respectively
E	Young's modulus
$\mathbf{E}_{\mathbf{e}}$	effective value of Young's modulus
$\mathbf{E}_{\mathbf{n}}$	Young's modulus of stiffeners in nth set of stiffeners
$G_n$	shear modulus of stiffeners in nth set of stiffeners
$G_{xy}$	in-plane shear stiffness of orthotropic skin
In	moment of inertia of stiffeners in nth set of stiffeners about centroid of stiffeners
$J_n$	torsion constant of stiffeners in nth set of stiffeners
ı	length of cylinder
$M_X$	bending moment in wall of stiffened cylinder in x-direction
m	number of half-waves in buckle pattern along length of cylinder

$N_X$	normal force in wall of stiffened cylinder in x-direction
$\overline{\mathbf{N}}$	maximum value of axial load in wall of stiffened cylinder at buckling resulting from applied bending and/or compressive loads
$\overline{N}_{O}$	calculated value of $\overline{N}$
$\overline{N}_x, \overline{N}_y$	applied loads in wall of stiffened cylinder at buckling resulting from a uniform axial compressive load and from a uniform radial pressure, respectively
$ ilde{ ilde{N}}_{ extbf{X}}$	maximum value of axial compressive load in wall of stiffened cylinder resulting from bending load on cylinder
n	number of full waves in buckle pattern in circumference of cylinder
$\mathbf{r}$	radius of cylinder
v	displacement of a point in middle surface of orthotropic skin of cylinder in y-direction
w	displacement normal to surface of cylinder
<b>x</b> ,y	axial and circumferential coordinates of cylinder, respectively
$ar{\mathbf{z}}_{\mathbf{n}}$	
	distance from centroid of stiffeners in nth set of stiffeners to centroid of orthotropic skin, positive when stiffeners are on outside of shell
€	
$ar{\epsilon}$ $\epsilon_{ extbf{cr}}$	orthotropic skin, positive when stiffeners are on outside of shell
	orthotropic skin, positive when stiffeners are on outside of shell unit shortening of buckled skin computed strain at local buckling of skin, effect of curvature on buckling being
$\epsilon_{ extbf{cr}}$	orthotropic skin, positive when stiffeners are on outside of shell unit shortening of buckled skin computed strain at local buckling of skin, effect of curvature on buckling being neglected

- $ho_x, 
  ho_y$  radius of gyration of stiffened orthotropic wall about centroid of stiffened wall in x- and y-direction, respectively
- $\phi_{\mathrm{n}}$  angle between stiffeners in nth set of stiffeners and a generator of orthotropic cylinder

## **BUCKLING EQUATIONS**

The buckling equations employed in the present study are generalizations of those presented in references 11 and 12 for cylinders with ring-and-stringer stiffening to include stiffening systems which run at an angle of  $\pm \phi$  to a generator of the cylinder. The generalized equations are given in appendix B as equations (B1) and (B2). The equations have a multiplying factor  $\eta$  on extensional terms which is not included in the corresponding equations of references 11 and 12. The multiplying factor is offered as one means of including a correlation factor in buckling analyses of stiffened cylinders. This correlation factor will be discussed in more detail after computations for individual cylinders have been made with the use of the equations and  $\eta = 1.0$ . Equations (B1) and (B2) are typical of the more advanced buckling equations in use today for computing buckling loads of stiffened cylinders. The equations account for the one-sidedness of stiffening elements whose influence is assumed to be "smeared" over the stiffener spacing; buckling is assumed to occur from a membrane state of stress and deformation.

Specialization of the equations for certain stiffening or wall geometry yields buckling equations which are similar to those used by some of the investigators (refs. 1 to 5) in analyzing their tests; small differences exist but are generally insignificant. Such differences will be discussed in connection with specific calculations presented later.

# REVIEW OF TEST DATA

Much of the test data in the literature is not suitable for use in correlating buckling loads obtained from tests with those obtained from equations for predicting general instability failures; calculations are not given for these data in the present review. Early data on cylinders generally conceded to be unrepresentative of contemporary fabrication practices or of modern aerospace structures are not considered. The data of references 13 and 14, for example, now fall into this category; although a few years ago the data were the best available for assessing cylinder buckling behavior. Cylinders with only longitudinal stiffening are likewise not considered. The buckling load of such cylinders is known to depend strongly upon the boundary conditions at the ends of the cylinders, conditions which are not generally known in mathematical terms with any degree of certainty. The precise boundary conditions existing in other types of stiffened cylinders are not known

either, but the sensitivity of buckling load to detail changes in end conditions is not as significant. Hence, tests of axially stiffened cylinders should be subjected to a different scrutiny than tests of other types of stiffened cylinders, and a study of their behavior could well constitute a separate investigation.

Some of the newer data are also deficient. These data are included in the present study to the extent that the deficiencies of the data are discussed, but buckling calculations are not given. This discussion is given after buckling calculations are presented for the cylinders of primary interest in the present study.

The test data analyzed were obtained on cylinders of four basic types of construction: cylinders with  $\pm 45^{O}$  stiffening, cylinders with ring stiffening, corrugated cylinders with ring stiffening, and cylinders with ring-and-stringer stiffening. The data on corrugated ring-stiffened cylinders were obtained in three separate investigations; data on the other three types of construction are limited to single investigations.

# Cylinders With $\pm 45^{O}$ Stiffening

The tests of reference 1 were conducted on  $\pm 45^{O}$  integrally stiffened circular cylinders. Most of the tests were conducted on 96-inch-diameter (2.44-m) cylinders with a mandrel inside the cylinders to control buckle depth and to preserve the cylinders for further testing. Loading was a combination of bending and axial compression so that the in-plane axial compression was a maximum at one generator of the test cylinder and zero at the diametrically opposed generator. Use of the mandrel was largely successful in preventing damage to the cylinders, and two or three tests (the maximum number attempted) were conducted on each cylinder; the maximum compressive stress in successive tests was applied to a generator  $120^{O}$  from that in the previous test.

The cylinders were relatively lightly stiffened; the stiffening members constituted about 25 percent of the mass of the cylinders. Buckling occurred at relatively low stresses; the maximum axial compressive stress was generally less than 15 ksi  $(103 \text{ MN/m}^2)$  and the stiffener stress less than about 5 ksi  $(34 \text{ MN/m}^2)$ .

The results of calculations to predict buckling loads are given in table I. The most significant characteristic is the scatter of test results from 65 to 105 percent of the calculated buckling load. The scatter exists in spite of attempts to minimize scatter with the use of cylinder dimensions in the neighborhood of the maximum stressed generator as input for buckling calculations. Such a scheme would be expected to reduce scatter more than that obtained with the use of a single average set of dimensions for a given cylinder.

The calculated results given in table I differ slightly from those given in reference 1 in presenting the data. The difference is presumably associated with neglect of twisting stiffness of stiffness and neglect of the variation of stress in the circumferential

direction in the analysis of reference 1. Small differences may also exist from geometries used in the two sets of calculations. The present calculations include a small (approximately 1/32-inch (0.8-mm)) fillet in the re-entrant corner between stiffeners and skin; presumably, the calculations of reference 1 did not. In the present calculations, neglect of stiffener twisting stiffness would have decreased the calculated buckling loads of table I by about 3 percent; neglect of the variation in stress distribution would have decreased the calculated buckling load by about  $2\frac{1}{2}$  percent.

# Ring-Stiffened Cylinders

The test specimens of reference 2 consisted of eighteen 14-inch-diameter (35.6-cm) ring-stiffened cylinders machined from steel tubes with an initial wall thickness of 1/4 inch (6.4 mm). Ratios of radius to skin thickness of the finished cylinders varied from approximately 600 to 750; stiffening ratio varied so that 10 to 30 percent of the mass of the cylinders was in the rectangular rings.

The cylinders were tested in axial compression. Several cylinders had small margins against failure by local buckling of the skin between rings, and their failure may have been influenced by the local buckling. Hence, only those cylinders with a calculated margin of at least 15 percent against failure in the local mode are considered herein for evaluation of linear theory in predicting general buckling. Buckling generally occurred at a wall stress of less than 30 ksi  $(207 \text{ MN/m}^2)$ ; the yield stress of the wall material is reported to be about 70 ksi  $(483 \text{ MN/m}^2)$ .

Calculated buckling loads of these test cylinders are given in table II. Buckling loads as observed from tests vary from 67 to 97 percent of those calculated with the use of equation (B1). Buckling was always calculated to occur in an axisymmetric mode; the number of axial half-waves m in the calculated buckle pattern is shown in the table. This number is not very different from the number of rings in the test cylinders. Although this situation suggests that the effect of ring discreteness (not accounted for in eq. (B1)) might be expected to be important for the test cylinders, this question was investigated in reference 2, and only insignificant changes in calculated buckling load were found when ring discreteness was taken into account.

# Ring-Stiffened Corrugated Cylinders

Ring-stiffened corrugated cylinders are the only type of construction on which more than one investigation was conducted and for which buckling calculations are given herein. These investigations are reported in references 3 to 5.

Test specimens of reference 3.- The test specimens of reference 3 consisted of ring-stiffened corrugated cylinders of 7075-T6 aluminum alloy which were tested in axial compression or combined axial compression and bending. Test cylinders of three sizes,

ranging from approximately 50 inches (1.27 m) in diameter to nearly 400 inches (10.16 m) in diameter, were used in the study. The mass of stiffening rings ranged from 20 percent of the mass of the cylinder for the largest specimen to up to 40 percent of the mass of the test cylinder for one of the smaller test cylinders. The cylinders failed at stresses between 35 ksi (241  $MN/m^2$ ) and 45 ksi (310  $MN/m^2$ ). Only those test cylinders with I-section rings are considered here. Results for two cylinders with channel-section rings are also reported in reference 3, but published information is incomplete, and independent calculations could not be made.

The results of calculations to predict buckling of the test cylinders are given in table III. Buckling occurred at loads 88 to 103 percent of those calculated for the test cylinders. These values do not differ much from those given in reference 3. The principal difference is for cylinder 6 which was tested in combined bending and axial compression. Reference 3 reports a discrepancy between theory and test approximately 7 percent greater than that shown in table III. Buckling in reference 3 was calculated with the use of an improvised "bending factor" and a calculation for buckling in uniform axial compression. The buckling load given in table III was obtained by direct calculation for combined bending and compression. In addition, the calculations for table III were made with somewhat different stiffnesses to represent the corrugated skin than were used in reference 3. The stiffnesses given in reference 4 were used in the present calculations — some of those stiffnesses were neglected in the calculations of reference 3, evidently without significant change in calculated results.

Test specimens of reference 4.- The test specimens of reference 4 consist of five ring-stiffened corrugated cylinders of 7075-T6 aluminum alloy in which a single corrugation geometry and ring geometry were used in fabrication of the cylinders. The difference between test cylinders lies in ring spacing which varied from cylinder to cylinder. Failure of the cylinder with the greatest ring spacing was by panel instability, that is, by buckling between rings. The cylinders were 78 inches (1.98 m) in diameter and were tested in bending. The rings contributed from 15 to 35 percent of the mass of the stiffened cylinder for those cylinders which failed by general instability. All cylinders buckled at stresses of less than 35 ksi  $(241 \text{ MN/m}^2)$ .

Results of calculations to predict buckling of the test cylinders are given in table IV. Buckling occurred at loads from 88 to 102 percent of those calculated for the test cylinders with the use of equation (B2). These values are greater than those presented in reference 4, which were based on an incorrect calculation for ring properties. The present calculations make use of the ring geometry shown in figure 2 of reference 4, the geometry of the rings used in construction of the test cylinders.

Test specimens of reference 5.- The test cylinders of reference 5 consist of two ring-stiffened corrugated cylinders of 7075-T6 aluminum alloy, one with rings on the

outside of the corrugated wall and one with rings on the inside. The rings contributed 21 percent of the mass of the cylinders. The cylinders were 78 inches (1.98 m) in diameter and were tested in bending.

Results of calculations to predict buckling of the test cylinders are given in table V. These results differ little from those given in reference 5 in presenting the original data.

# Cylinders With Ring-and-Stringer Stiffening

The test specimens of reference 6 were cylinders with ring-and-stringer stiffening which experienced local buckling of the skin long before general buckling of the shell occurred. The 77-inch-diameter (1.96-m) cylinders, which were tested in bending, were rather heavily stiffened. Stiffening elements accounted for 40 to 50 percent of the mass of the stiffened cylinders; the stringers contributed more than the rings.

The usefulness of the tests of reference 6 in appraising the adequacy of contemporary buckling theories for predicting general instability of stiffened cylinders is tempered by the degree to which some of the stiffnesses of buckled skin can be defined. Many of the stiffnesses are not known with sufficient accuracy to eliminate significant errors in predicted load from this source. However, this type of construction is one of contemporary interest, and therefore the tests are included in the present calculations, but the calculated results are not used later when the correlation of test data with analysis is discussed.

The buckled skin was treated in an approximate manner in the buckling calculations. It was assumed that buckled skin could be treated in the same manner as unbuckled skin if it were assigned an effective value for Young's modulus. The value assigned to modulus is given in figure 1 and was ascertained by comparing buckling results for the tests of reference 6 with those for the rest of the tests considered in the present investigation. Other considerations entailed in obtaining figure 1 are given in appendix C.

Results of calculations to predict general buckling of the test cylinders of reference 6 are given in table VI. The first six cylinders failed by general instability; the remaining cylinder failed by panel instability. The ratios of  $\overline{N}/\overline{N}_O$  for those cylinders which failed by general instability fall within the range of the rest of the data given in tables I to V.

The cylinders of reference 6 were recently analyzed in reference 15 with the use of stiffnesses for buckled skin reported in reference 16 for flat plates. Ratios of  $\overline{N}/\overline{N}_{O}$  between 0.87 and 1.11 were obtained in those calculations, but incorrect values for experimental buckling load  $\overline{N}$  were used in the calculations. Errors in  $\overline{N}$  as large as 25 percent were entailed in deriving  $\overline{N}$  from the applied bending moment on the

cylinders at buckling from neglect of the influence of local skin buckling in changing stress distribution and cylinder bending stiffness.

# Data Unsuitable for Use in This Study

The data from two recent investigations were not used in the present study.

Test specimens of references 7 and 8.- The test specimens of references 7 and 8 consisted of small-diameter (7.6-inch (19.3-cm)) machined cylinders with ring stiffening, grid stiffening, or longitudinal stiffening. The cylinders were subjected to various loadings. The ring-stiffened and grid-stiffened cylinders subjected to bending or axial compression would normally have been of interest in the present study. However, two characteristics of the test cylinders render them unsuitable for evaluating the validity of contemporary analyses in predicting cylinder buckling; hence, results of the investigation are not studied in detail herein.

The first characteristic is the small size of the test cylinders and the associated tolerances on cylinder dimensions. Although references 7 and 8 indicate individual measurements could be made to reasonable accuracy (less than 5-percent error), dimensional variation from one part of a cylinder to another is not discussed; because the specimens were small (r = 3.8 inches (9.7 cm)), rather significant variations in dimensions would be expected. Some idea of thickness variations can be gleaned from the data presented. For instance, table 4.1 of reference 7 indicates that a minimum skin thickness of 0.0060 inches (0.152 mm) was measured on one of the three grid-stiffened cylinders tested in axial compression. Presumably, the measurement was made on cylinder 61, the grid-stiffened cylinder with the thinnest skin. This cylinder had a reported skin thickness (presumably average skin thickness) of 0.0087 inches (0.221 mm) (table 4.4 of ref. 7). If these presumptions are correct, this thickness corresponds to a 30-percent variation from the reported or average thickness. If the 0.0060-inch (0.152-mm) measurement were made on one of the other two grid-stiffened cylinders, an even larger variation would be indicated.

The other undesirable characteristic of the test cylinders is associated with stiff-ener geometry. The stiffening is such that the clear distance between rectangular-shaped stiffeners is about the same as the width of the stiffener. Such stiffeners probably behave more like plates than stiffeners and such behavior should be taken into account in correlating tests with analysis. Unfortunately, analyses which properly treat such stiffeners are unavailable because they are not normally needed in the design of aerospace structures; and the use of available analyses reveals little regarding the adequacy of the analyses in predicting buckling of the stiffened cylinders reported in references 7 and 8.

Test specimens of reference 9.- The test specimens in the investigation of reference 9 consist of integrally stiffened cylinders with longitudinal, grid, or  $\pm 45^{\circ}$  stiffening.

The cylinders were 53 inches (1.35 m) in diameter and were tested in uniform axial compression. The grid-stiffened and the  $\pm 45^{\circ}$  stiffened cylinders would normally have been of interest in the present study. However, stiffening on the cylinders was such that local buckling of the skin between stiffeners occurred prior to general buckling; and, in addition, the cylinders had relatively light stiffening so that local buckling of the skin was influenced by the low deflectional stiffness of the stiffeners normal to the surface of the shell. It is believed that failure was also influenced by the low deflectional stiffness of the stiffeners; that is, the stiffening elements were not heavy enough to provide nodes in the local buckle pattern in the post local-buckling loading regime, and cylinder failure was associated with crippling of stiffener elements from local buckling patterns cutting across stiffeners. Such failures are not amenable to calculation with the use of equations such as equation (B1) even when used in conjunction with effective stiffnesses for buckled skin as were the cylinders of reference 6; therefore, results of detailed calculations are not presented herein.

## DISCUSSION OF RESULTS

Results in tables I to VI are plotted against the parameter  $r/\sqrt{\rho_x\rho_y}$  in figure 2. This parameter has been suggested (ref. 17) as one which can be used to integrate test data on stiffened cylinders with the large mass of data on unstiffened cylinders and thereby render the unstiffened data useful in the design of stiffened cylinders. The search for such a parameter was instigated by the lack of data on stiffened cylinders. Also plotted in figure 2 is a curve taken from reference 18 which represents the lower limit of the bending data or an average of the compression data of the unstiffened cylinders considered in the reference.

The data on stiffened cylinders do not appear to correlate well with the parameter  $r/\sqrt{\rho_{\rm X}\rho_{\rm y}}$ . Particularly conspicuous are the data of reference 5 on ring-stiffened corrugated cylinders; these cylinders buckled at approximately 70 percent of the calculated buckling load even though  $r/\sqrt{\rho_{\rm X}\rho_{\rm y}}$  was small and in the range where considerably higher test loads would be expected on the basis of the parameter  $r/\sqrt{\rho_{\rm X}\rho_{\rm y}}$ .

References 2 and 15 suggest that stiffening ratio (ratio of stiffener area to skin area) should be an important parameter in the correlation between test data and calculation. This parameter immediately comes to mind because the unstiffened cylinder is approached as stiffening ratio is decreased, and the disparity between calculation and test for unstiffened cylinders is generally greater than that indicated herein for stiffened cylinders. However, when the data of tables I to VI were plotted against stiffening ratio, no trends were detected.

The lower limit of the data of figure 2 is suggested for design purposes for lack of a better correlation factor, that is,  $\overline{N}/\overline{N}_{O}=0.65$ . Each basic type of construction represented in figure 2 experienced general instability failures near the 0.65 value except the cylinders with ring-and-stringer stiffening of reference 6, and the cylinders of reference 6 cannot logically be used to establish correlation because their buckling loads were determined with the use of data from the other cylinders represented in figure 2, as explained previously. Hence none of the types of construction for which data are available appear to be immune to the low failing loads. Moreover, none appear to be immune to considerable test scatter. Each type of construction experienced values of  $\overline{N}/\overline{N}_{O}$  which scatter between a value near 0.65 to a value near unity.

The effects of geometric initial imperfections and built-in fabrication or residual stresses probably account for much of the test scatter exhibited in figure 2; of the two, imperfections are perhaps more important than residual stresses. Most of the tests of reference 1 were conducted on nominally identical cylinders, and the tests of reference 2 include some cylinders whose nominal dimensions differ little from those of other cylinders in the series. It is difficult to associate the observed scatter in test results for these tests with anything but initial imperfections or residual stresses. More sophisticated calculations which include prebuckling deformations of geometrically perfect cylinders, or which better describe actual end conditions but which neglect geometric imperfections, would not be expected to reduce the scatter substantially; any calculation which would lower one point would lower the others accordingly. A more sophisticated example calculation (with the use of ref. 19) was made for cylinder 5 of the ring-stiffened cylinders of reference 2 to determine the magnitude of the influence of discrete rings and nonlinear prebuckling deformations on buckling load. The calculation indicated a reduction in buckling load of only 6 percent. Similarly, the calculations made in reference 19 would indicate that inclusion of these effects in the buckling calculations for ring-stiffened corrugated cylinders would have little influence on the results presented in figure 2. Hence, unmeasured geometric imperfections and residual stresses seem to be left as the most likely cause of the scatter in test data exhibited in figure 2.

Considerable attention has recently been given to determining the sensitivity of representative shell structures to loss of stability from geometric initial imperfections. (See refs. 20 and 21, for example.) The study of reference 21 includes ring-stiffened cylinders loaded in axial compression. Its conclusion that ring-stiffened cylinders are sensitive to imperfections is in accordance with the results shown in figure 2. Similar studies have not been made for the other shell structures represented in the figure.

Until studies of the imperfection sensitivity of shells in buckling have progressed to the extent that correlation of measured imperfections with analytical predictions has been ascertained, other, more approximate methods of predicting cylinder buckling, presumably methods which inherently account for the most severe imperfections or residual stresses likely to occur in fabrication, will have to be employed. The use of  $\overline{N}/\overline{N}_O$  = 0.65 for shells representative of those considered in figure 2 is one way of accomplishing this effect.

Another method, which parallels that of reference 18 for unstiffened cylinders, is to use equations (B1) and (B2) with a multiplying factor  $\eta$  of less than unity. This scheme implies that imperfections have a detrimental effect on extensional stiffnesses that results in lower buckling loads. Note that all stiffnesses associated with extension of the cylinder wall, including those associated with shearing in the plane of the cylinder wall, are multiplied by the factor  $\eta$ . Coupling terms are multiplied by  $\sqrt{\eta}$ , and bending terms are left unchanged. Use of equation (B1) in this form results in computed buckling loads for many structures which are proportional to  $\sqrt{\eta}$  and which converge to buckling loads independent of  $\eta$  as shell length is decreased and flat-plate behavior is approached, a trend that correlates well with observed behavior of shell structures.

The use of equations (B1) and (B2) with  $\eta < 1.0$  for calculating general instability failures is equivalent to the use of the correlation factor  $\overline{N}/\overline{N}_{O} = 0.65$  for correcting theoretical calculations to the lower limit of test data. The corresponding value of  $\eta$  is  $(0.65)^2$  or 0.42.

The use of equations (B1) and (B2) with  $\eta < 1.0$  for calculating panel instability failures is a more realistic approach than that of using the correlation factor  $\overline{N}/\overline{N}_0 = 0.65$ . Panel instability failures may involve only flat-plate behavior, only cylinder behavior, or any behavior between these extremes. Use of the equations with  $\eta < 1.0$  will provide appropriate corrections to panel instability calculations for each behavior. The correction will be small when plate behavior predominates and will be larger when cylinder behavior predominates. Use of the equations with  $\eta < 1.0$ , therefore, provides a means of extrapolating general instability data to apply to panel instability failures. For the two cylinders of figure 2 which failed by panel instability, use of equations (B1) and (B2) with  $\eta = 0.42$  is equivalent to the use of a correlation factor of  $\overline{N}/\overline{N}_0 \approx 1.0$  for the cylinder of reference 4 and of a factor  $\overline{N}/\overline{N}_0 \approx 0.9$  for the cylinder of reference 6.

Some important types of construction are not represented in the present study because test data are not available. For example, data on conventional ring-and-stringer stiffened cylinders in which skin buckling does not precede general buckling are not available, and data on cylinders with stiffening arrangements entailing either intercostal stiffening or cutouts in stiffening members to allow other stiffening members to pass through are not available. Likewise, data on designs which use floating rings or on designs proportioned to achieve a ratio of low mass and high strength are not available.

#### CONCLUDING REMARKS

Test data on stiffened cylinders in axial compression and/or bending were reviewed, and the adequacy of conventional methods of predicting buckling were appraised by comparing test data with buckling calculations. The buckling equations employed were derived on the assumption that discreteness of stiffening members is not an important consideration and that the cylinders buckled from a membrane state of stress and deformation. Cylinder buckling may take place at loads as low as 65 percent of that derived by such calculations. None of the types of construction for which data are available were without failures approaching the 65-percent value except when failure was by panel instability. The two panel instability failures reported occurred at loads nearer 100 percent of the calculated buckling load. Design procedures which neglect this disparity between buckling calculations and test results for general instability failures are likely to be unconservative.

The number of tests available for appraising design procedures is not large. Less than 30 test cylinders are represented in the available data considered directly applicable for appraising design procedures for general buckling of stiffened cylinders in axial compression and/or bending. Considerable additional data are necessary before it would be possible to develop more rational empirical design procedures than those suggested herein which obtain the design load from the calculated buckling load with the use of a correlation factor independent of geometry. The scarcity of applicable data precludes the use of parametric studies of test data to reveal trends that might lead to a more rational correlation factor.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., October 16, 1969.

# APPENDIX A

# CONVERSION OF U.S. CUSTOMARY UNITS TO SI UNITS

Conversion factors (ref. 10) for the units used in this report are given in the following table:

Physical quantity	U.S. Customary Unit	Conversion factor (*)	SI Unit (**)		
Length Stress, modulus Load per unit length	in. ksi kips/in.	$0.0254$ $6.895 \times 10^{6}$ $1.751 \times 10^{5}$	meters (m) newtons/meter <sup>2</sup> (N/m <sup>2</sup> ) newtons/meter (N/m)		

 $<sup>^*</sup>$ Multiply value given in U.S. Customary Units by conversion factor to obtain equivalent value in SI Units.

<sup>\*\*</sup>Prefixes to indicate multiple of units are as follows:

Prefix	Multiple
mega (M)	106
kilo (k)	10 <sup>3</sup>
centi (c)	10-2
milli (m)	10-3

#### APPENDIX B

# BUCKLING OF SIMPLY SUPPORTED, STIFFENED, ORTHOTROPIC CYLINDERS

A buckling equation for simply supported, stiffened, orthotropic cylinders under uniform biaxial compression may be written as

$$\overline{N}_{x} \left(\frac{m\pi}{l}\right)^{2} + \overline{N}_{y} \left(\frac{n}{r}\right)^{2} = A_{33} + A_{13} \left(\frac{A_{12}A_{23} - A_{13}A_{22}}{A_{11}A_{22} - A_{12}^{2}}\right) + A_{23} \left(\frac{A_{12}A_{13} - A_{11}A_{23}}{A_{11}A_{22} - A_{12}^{2}}\right)$$
(B1)

where

$$\begin{split} \mathbf{A}_{11} &= \eta \mathbf{B}_{11} \left( \frac{\mathbf{m}\pi}{l} \right)^{2} + \eta \mathbf{B}_{33} \left( \frac{\mathbf{n}}{\mathbf{r}} \right)^{2} \\ \mathbf{A}_{12} &= \left( \eta \mathbf{B}_{12} + \eta \mathbf{B}_{33} \right) \left( \frac{\mathbf{m}\pi}{l} \right) \left( \frac{\mathbf{n}}{\mathbf{r}} \right) \\ \mathbf{A}_{13} &= \frac{\eta \mathbf{B}_{12}}{\mathbf{r}} \left( \frac{\mathbf{m}\pi}{l} \right) + \sqrt{\eta} \mathbf{C}_{11} \left( \frac{\mathbf{m}\pi}{l} \right)^{3} + \left( \sqrt{\eta} \mathbf{C}_{12} + 2\sqrt{\eta} \mathbf{C}_{33} \right) \left( \frac{\mathbf{m}\pi}{l} \right) \left( \frac{\mathbf{n}}{\mathbf{r}} \right)^{2} \\ \mathbf{A}_{22} &= \eta \mathbf{B}_{33} \left( \frac{\mathbf{m}\pi}{l} \right)^{2} + \eta \mathbf{B}_{22} \left( \frac{\mathbf{n}}{\mathbf{r}} \right)^{2} \\ \mathbf{A}_{23} &= \frac{\eta \mathbf{B}_{22}}{\mathbf{r}} \left( \frac{\mathbf{n}}{\mathbf{r}} \right) + \left( \sqrt{\eta} \mathbf{C}_{12} + 2\sqrt{\eta} \mathbf{C}_{33} \right) \left( \frac{\mathbf{m}\pi}{l} \right)^{2} \left( \frac{\mathbf{n}}{\mathbf{r}} \right) + \sqrt{\eta} \mathbf{C}_{22} \left( \frac{\mathbf{n}}{\mathbf{r}} \right)^{3} \\ \mathbf{A}_{33} &= \mathbf{D}_{11} \left( \frac{\mathbf{m}\pi}{l} \right)^{4} + 2 \left( \mathbf{D}_{12} + 2\mathbf{D}_{33} \right) \left( \frac{\mathbf{m}\pi}{l} \right)^{2} \left( \frac{\mathbf{n}}{\mathbf{r}} \right)^{2} + \mathbf{D}_{22} \left( \frac{\mathbf{n}}{\mathbf{r}} \right)^{4} \\ &+ \frac{2}{\mathbf{r}} \left( \sqrt{\eta} \mathbf{C}_{12} \right) \left( \frac{\mathbf{m}\pi}{l} \right)^{2} + 2\sqrt{\eta} \frac{\mathbf{C}_{22}}{\mathbf{r}} \left( \frac{\mathbf{n}}{\mathbf{r}} \right)^{2} + \eta \frac{\mathbf{B}_{22}}{\mathbf{r}^{2}} \end{split}$$

and

$$\begin{aligned} \mathbf{B}_{11} &= \frac{\mathbf{E}_{\mathbf{X}}}{1 - \mu_{\mathbf{X}}^{\dagger} \mu_{\mathbf{y}}^{\dagger}} + \sum_{\mathbf{n}} \frac{\mathbf{E}_{\mathbf{n}} \mathbf{A}_{\mathbf{n}}}{\mathbf{b}_{\mathbf{n}}} \cos^{4} \phi_{\mathbf{n}} \\ \\ \mathbf{B}_{12} &= \frac{\mu_{\mathbf{y}}^{\dagger} \mathbf{E}_{\mathbf{X}}}{1 - \mu_{\mathbf{y}}^{\dagger} \mu_{\mathbf{y}}^{\dagger}} + \sum_{\mathbf{n}} \frac{\mathbf{E}_{\mathbf{n}} \mathbf{A}_{\mathbf{n}}}{\mathbf{b}_{\mathbf{n}}} \sin^{2} \phi_{\mathbf{n}} \cos^{2} \phi_{\mathbf{n}} \end{aligned}$$

## APPENDIX B

$$B_{22} = \frac{E_y}{1 - \mu_x' \mu_y'} + \sum_n \frac{E_n A_n}{b_n} \sin^4 \phi_n$$

$$B_{33} = G_{xy} + \sum_{n} \frac{E_n A_n}{b_n} \sin^2 \phi_n \cos^2 \phi_n$$

$$C_{11} = \sum_{n} \frac{E_n A_n}{b_n} \, \overline{z}_n \, \cos^4 \phi_n$$

$$C_{12} = \sum_{n} \frac{\mathbf{E}_{n} \mathbf{A}_{n}}{\mathbf{b}_{n}} \, \overline{\mathbf{z}}_{n} \, \sin^{2} \phi_{n} \, \cos^{2} \phi_{n}$$

$$C_{22} = \sum_{n} \frac{E_{n}A_{n}}{b_{n}} \, \overline{z}_{n} \, \sin^{4} \phi_{n}$$

$$C_{33} = \sum_{n} \frac{E_{n}A_{n}}{b_{n}} \, \overline{z}_{n} \, \sin^{2}\!\phi_{n} \, \cos^{2}\!\phi_{n}$$

$$\mathbf{D_{11}} = \frac{\mathbf{D_{x}}}{1 - \mu_{x} \mu_{y}} + \sum_{n} \frac{\mathbf{E_{n} I_{n}}}{\mathbf{b_{n}}} \cos^{4} \phi_{n} + \sum_{n} \frac{\mathbf{G_{n} J_{n}}}{\mathbf{b_{n}}} \sin^{2} \phi_{n} \cos^{2} \phi_{n} + \sum_{n} \frac{\mathbf{E_{n} A_{n}}}{\mathbf{b_{n}}} \overline{\mathbf{z}_{n}^{2}} \cos^{4} \phi_{n}$$

$$\begin{split} \mathbf{D}_{12} &= \frac{\mu_y \mathbf{D}_x}{1 - \mu_x \mu_y} + \sum_n \frac{\mathbf{E}_n \mathbf{I}_n}{\mathbf{b}_n} \sin^2\!\phi_n \, \cos^2\!\phi_n - \sum_n \frac{\mathbf{G}_n \mathbf{J}_n}{\mathbf{b}_n} \sin^2\!\phi_n \, \cos^2\!\phi_n \\ &+ \sum_n \frac{\mathbf{E}_n \mathbf{A}_n}{\mathbf{b}_n} \, \overline{\mathbf{z}}_n^2 \, \sin^2\!\phi_n \, \cos^2\!\phi_n \end{split}$$

$${\rm D}_{22} = \frac{{\rm D}_y}{1 - \mu_x \mu_y} + \sum_n \frac{{\rm E}_n {\rm I}_n}{{\rm b}_n} \sin^4\!\phi_n + \sum_n \frac{{\rm G}_n {\rm J}_n}{{\rm b}_n} \sin^2\!\phi_n \cos^2\!\phi_n + \sum_n \frac{{\rm E}_n {\rm A}_n}{{\rm b}_n} \, \overline{\rm z}_n^2 \, \sin^4\!\phi_n$$

$$\begin{split} \mathrm{D}_{33} &= \frac{\mathrm{D}_{xy}}{2} + \sum_{n} \frac{\mathrm{E}_{n} \mathrm{I}_{n}}{\mathrm{b}_{n}} \, \sin^{2}\!\phi_{n} \, \cos^{2}\!\phi_{n} + \frac{1}{4} \sum_{n} \frac{\mathrm{G}_{n} \mathrm{J}_{n}}{\mathrm{b}_{n}} \! \left(\!\cos^{2}\!\phi_{n} - \sin^{2}\!\phi_{n}\!\right)^{2} \\ &+ \sum_{n} \frac{\mathrm{E}_{n} \mathrm{A}_{n}}{\mathrm{b}_{n}} \, \overline{\mathrm{z}}_{n}^{2} \, \sin^{2}\!\phi_{n} \, \cos^{2}\!\phi_{n} \end{split}$$

#### APPENDIX B

Equation (B1) is a generalization of equation (15) of reference 11 to include stiffeners which run at an angle of  $\pm \phi$  to a generator of the cylinder; the equation is used in the body of the paper to compute buckling loads for comparison with test data reported in the literature. Boundary conditions at the ends of the cylinder are given by

$$\mathbf{w} = \mathbf{M}_{\mathbf{X}} = \mathbf{N}_{\mathbf{X}} = \mathbf{v} = \mathbf{0}$$

and the cylinder is assumed to buckle from a membrane state of stress and deformation. In equation (B1) the stiffness terms  $B_{ij}$  are multiplied by the correlation factor  $\eta$  and coupling terms  $C_{ij}$  by  $\sqrt{\eta}.$  Use of the equation with  $\eta < 1.0$  is one means of correcting buckling calculations to account for the disparity between buckling tests and theory.

Equation (B1) has coupling terms  $C_{12}$  and  $C_{33}$  which have no counterpart in equation (15) of reference 11 which was written for cylinders with stiffening members in longitudinal ( $\phi = 0^{\circ}$ ) and circumferential ( $\phi = 90^{\circ}$ ) directions. These terms appear in buckling equations for layered shells, such as considered in references 22 and 23, as well as in the present problem with a single-layer shell but with stiffeners which can have the directions  $\pm \phi$ .

Some of the test cylinders analyzed were loaded in bending or combined bending and axial compression. Calculations for these cylinders were made with the use of the set of equations given by

$$a_n \left( F_n - \overline{N}_x \right) - \frac{\widetilde{N}_x}{2} \left[ \left( 1 + \delta_{1n} - \delta_{0n} \right) a_{n-1} + a_{n+1} \right] = 0$$
 (B2)

where

$$\mathbf{F}_{n} = \left(\frac{l}{m\pi}\right)^{2} \left[ \mathbf{A}_{33} + \mathbf{A}_{23} \left( \frac{\mathbf{A}_{12}\mathbf{A}_{13} - \mathbf{A}_{11}\mathbf{A}_{23}}{\mathbf{A}_{11}\mathbf{A}_{22} - \mathbf{A}_{12}^{2}} \right) + \mathbf{A}_{13} \left( \frac{\mathbf{A}_{12}\mathbf{A}_{23} - \mathbf{A}_{13}\mathbf{A}_{22}}{\mathbf{A}_{11}\mathbf{A}_{22} - \mathbf{A}_{12}^{2}} \right) \right]$$
 (n = 0,1,2,3,...)

$$\delta_{jn} = 0$$
  $(j \neq n)$ 

and the  $A_{ij}$  are given by equation (B1). Buckling is determined from the nontrivial solution of equation (B2) where the determinant of the coefficients  $a_n$  is set equal to zero. The size of the determinant is taken as large as necessary to achieve the desired accuracy of the buckling load  $\overline{N}_0 = \overline{N}_X + \widetilde{N}_X$ . If  $\widetilde{N}_X$  is zero, equation (B2) simplifies to that given by equation (B1) for a cylinder subjected only to a uniform compressive load  $\overline{N}_X$ . Equation (B2) is given in reference 12 as equation (6).

#### APPENDIX C

#### EFFECTIVE MODULUS OF BUCKLED SKIN

In order to calculate general instability buckling loads of cylinders which experience skin buckling prior to general instability buckling, the reduced stiffness of the buckled skin must be taken into account. One way of accomplishing this is to establish by some means each of the various stiffnesses which enter into the buckling equation, but so far these stiffnesses have not been defined with sufficient accuracy to eliminate significant errors in predictions of buckling load.

An alternate scheme consists of replacing the buckled skin in buckling calculations with an equivalent isotropic skin of reduced modulus. This scheme is employed herein. The value assigned to modulus is given in figure 1 which was constructed by trial and error from computations for buckling load of the cylinders of reference 6 with the use of assumed values for modulus for buckled skin. Ratios of  $\overline{N}/\overline{N}_0$  were sought which were consistent with those obtained in the body of this report and given in tables I to V for the test cylinders of references 1 to 5. The choice of trial values of effective modulus was not completely arbitrary. Values between the secant and tangent moduli for buckled plates in axial compression reported in reference 24 were tried first, and the final curve lies between these values. Sufficient additional trials were made to achieve the desired correlation of  $\overline{N}/\overline{N}_0$ .

Hopefully, the effective modulus given in figure 1 is reasonably independent of stiffener geometry and buckling mode and therefore can be generally applied with little error. A check on this premise was obtained by auxiliary buckling calculations for axially stiffened cylinders whose geometry and buckling mode differ considerably from those of the cylinders of reference 6. Buckling calculations were made for the axially stiffened cylinders of reference 24 with the use of figure 1 and reference 25. Reference 25 which applies to cylinders with clamped support at the ends of the cylinder is the counterpart of equation (B1) for simply supported cylinders. The resulting calculations were compared with calculations made for the cylinders of reference 26 which had closely spaced stringers and therefore experienced no local skin buckling in much the same manner as discussed previously where buckling calculations for the test cylinders of reference 6 were compared with those of the test cylinders of references 1 to 5. The check indicated that effective modulus is not very sensitive to cylinder geometry and buckling mode and that the single curve of figure 1 probably gives a reasonable approximation for effective stiffness for a wide variety of cylinder geometries. Although use of figure 1 in calculating general buckling of cylinders with ring-and-stringer stiffening entails an approximate treatment of buckled skin, the treatment is probably as accurate as state-of-the-art permits at the present time.

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TABLE I.- CYLINDERS OF REFERENCE 1

Calindan	r		$\overline{\mathbf{N}}$		$\overline{\overline{N}}_{O}$		$\overline{ m N}/\overline{ m N}_{ m O}$
Cylinder	in.	cm	kips/in.	kN/m	kips/in.	kN/m	14/140
1-1	16.0	40.6	1.920	336	2.028	355	0.947
1-2	48.0	121.9	.855	150	.816	143	1.048
2-1-1	48.0	121.9	.762	133	.839	147	.908
2-1-2	48.0	121.9	.711	125	.820	144	.867
2-1-3	48.0	121.9	.824	144	.824	144	1.000
2-2-1	48.0	121.9	.794	139	.820	144	.968
2-2-2	48.0	121.9	.544	95.3	.832	146	.654
2-2-3	48.0	121.9	.694	122	.870	152	.798
2-3-1	48.0	121.9	.801	140	.988	173	.811
2-3-2	48.0	121.9	.761	133	.811	142	.938
2-4-1	48.0	121.9	.755	132	.875	153	.863
2-4-2	48.0	121.9	.772	135	.785	137	.983

TABLE II.- CYLINDERS OF REFERENCE 2

Cylinder	Number		$\overline{\mathrm{N}}$		$\overline{\overline{\mathrm{N}}}_{\mathrm{O}}$		$\overline{ m N}/\overline{ m N}_{ m O}$
Cymaer	of rings	m	kips/in.	kN/m	kips/in.	kN/m	14/140
5	25	18	0.312	54.6	0.344	60.2	0.907
6	25	18	.232	40.6	.296	51.8	.784
9	25	18	.297	52.0	.348	60.9	.853
10	25	18	.321	56.2	.348	60.9	.922
13	25	19	.275	48.2	.283	49.6	.972
14	25	19	.161	28.2	.239	41.8	.674
15	22	18	.198	34.7	.262	45.9	.756
16	22	17	.305	53.4	.360	63.0	.847
17	21	18	.239	41.8	.286	50.1	.836
18	21	19	.187	32.7	.270	47.3	.693

TABLE III.- CYLINDERS OF REFERENCE 3

C-lindon	r		$\overline{\mathbf{N}}$		$\overline{\mathrm{N}}_{\mathrm{O}}$		$\overline{N}/\overline{N}_{O}$	
Cylinder	in.	cm	kips/in.	kN/m	kips/in.	kN/m	1N/ INO	
2	24.7	62.7	1.12	196	1.28	224	0.875	
3	24.7	62.7	1.44	252	1.53	268	.941	
4	49.4	125.5	2.12	371	2.08	364	1.019	
5	49.4	125.5	2.09	366	2.08	364	1.005	
6	197.6	501.9	8.31	1455	8.11	1420	1.025	

TABLE IV.- CYLINDERS OF REFERENCE 4

Cylindon	Ring spacing		$\overline{\mathbf{N}}$		$\overline{ m N}_{ m O}$		$\overline{N}/\overline{N}_{O}$
Cylinder	in.	cm	kips/in.	kN/m	kips/in.	kN/m	11/11/0
1	20.0	50.8	0.347	60.8	0.340	59.5	1.021
2	13.3	33.8	.484	84.7	.550	96.3	.880
3	10.0	25.4	.600	105.1	.627	109.8	.957
4	8.0	20.3	.653	114.3	.699	122.4	.934
5	4.0	10.2	.883	154.6	.943	165.1	.936

TABLE V.- CYLINDERS OF REFERENCE 5

Cylinder	Ring	$\overline{\mathbf{N}}$		$\overline{N}_0$	$\overline{ m N}/\overline{ m N}_{ m O}$	
Cymnaer	stiffening	kips/in.	kN/m	kips/in.	kN/m	14/140
1	Outside	1.000	175.1	1.424	249.3	0.702
2	Inside	.440	77.0	.611	107.0	.720

TABLE VI.- CYLINDERS OF REFERENCE 6

Cylinder	Ring spacing		$\overline{\overline{N}}$		$\overline{\mathrm{N}}_{\mathrm{O}}$		$\overline{ m N}/\overline{ m N}_{ m O}$
Gymaer	in.	cm	kips/in.	kN/m	kips/in.	kN/m	14/140
1-1	6.0	15.2	0.941	165	1.018	178	0.924
1-2	9.0	22.9	.870	152	.978	171	.890
1-3	12.0	30.5	.800	140	.936	164	.855
2-1	6.0	15.2	.572	100.2	.622	108.9	.920
2-2	9.0	22.9	.521	91.2	.585	102.4	.891
2-3	12.0	30.5	.505	88.4	.563	98.6	.897
2-4	18.0	45.7	.370	64.8	.350	61.3	1.057

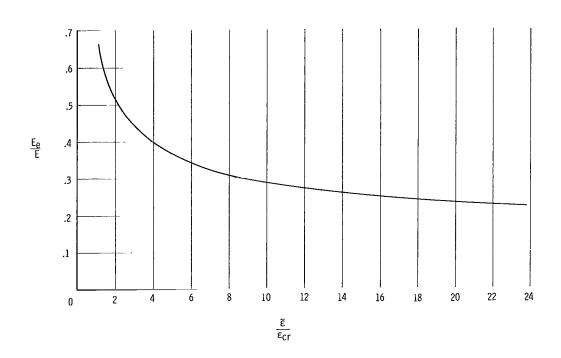


Figure 1.- Effective modulus of buckled skin used in buckling calculations for stiffened cylinders.

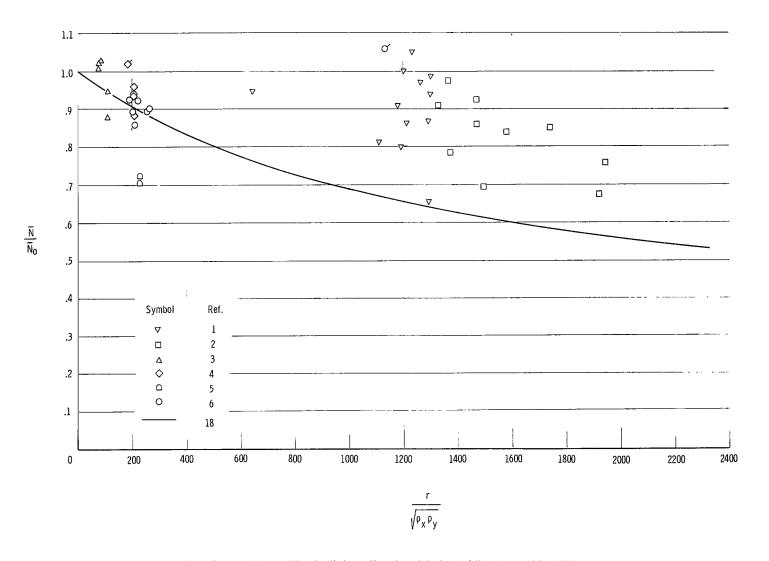


Figure 2.- Test data on stiffened cylinders. Flagged symbols denote failure by panel instability.

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