

# GASEOUS-HELIUM REQUIREMENTS FOR THE DISCHARGE OF LIQUID HYDROGEN FROM A 1.52-METER- (5-FT-) DIAMETER SPHERICAL TANK 

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# GASEOUS-HELIUM REQUIREMENTS FOR THE DISCHARGE OF LIQUID HYDROGEN 

## FROM A 1.52-METER- (5-FT-) DIAMETER SPHERICAL TANK

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## SUMMARY

An experimental investigation was conducted to determine the effects of various physical parameters on the pressurant gas (gaseous helium) requirements during the pressurization and expulsion of liquid hydrogen from a 1.52 -meter- ( 5 -ft-) diameter spherical tank. The experimental results were compared with results predicted by a previously developed analytical program which was revised and extended for this investigation. Tests were conducted for a range of liquid outflow rates, pressurizing rates, and initial ullage volumes at a nominal operating pressure of $34.47 \times 10^{4}$ newtons per square meter ( 50 psia ) using nominal inlet gas temperatures of 183,267 , and 322 K $\left(329^{\circ}, 480.6^{\circ}\right.$, and $580^{\circ} R$ ). Data were obtained using a hemisphere injector.

The experimental results show that the inlet gas temperature has a strong influence on the actual pressurant gas requirements. There is an average decrease in pressurant gas requirements of 1.3 percent for a $10 \mathrm{~K}\left(18^{\circ} \mathrm{R}\right)$ increase in inlet gas temperature. The analytical program was able to predict the actual pressurant requirements to within a maximum error of 11.5 percent for all cases.

## INTRODUCTION

During the past several years, a great deal of effort has been devoted to the problems associated with the pressurized discharge of a cryogenic liquid from a tank. The main objectives of these efforts have been toward the optimization of a propellant tank pressurization system. One phase of this optimization is a precise determination of pressurant requirements for any given set of operating parameters (i.e., tank pressure, inlet gas temperature, liquid outflow rate, tank size, etc.). This knowledge would allow the design of a system based on only the weight of gas necessary to accomplish the mission.

Several investigators have developed analyses (e.g., refs. 1 and 2) which attempt to predict, according to a selected set of simplifying assumptions, the quantity of pressurant gas required during the pressurized discharge of liquid hydrogen ( $\mathrm{LH}_{2}$ ). Some of these simplifying assumptions may, for certain conditions (i.e., for various injector geometries, tank shapes, and tank sizes), limit the capability of the analysis to predict pressurant requirements accurately. Because of these limitations, the validity of the analytical results have to be largely based on correlations with experimental results.

An experimental investigation was started at Lewis Research Center to determine the effects of various physical parameters on the pressurant gas requirements during the pressurization and expulsion of liquid hydrogen from propellant tanks. The experimental results were also used to extend the capability of the analysis of reference 1 to predict the pressurant requirements for tanks of general size and geometry under various operating conditions. Some results of this program, which used hydrogen gas as the pressurant, are reported in references 3 to 5 . In general, the experimental results reported in references 3 to 5 indicate that the temperature of the inlet gas, the injector geometry, and the tank pressure have a strong influence on the pressurant gas requirements. There was between a 1.2 and 1.4 percent decrease in pressurant requirement for a 10 K $\left(18^{\circ} \mathrm{R}\right)$ increase in the temperature of the inlet gas depending on liquid outflow rate and tank size. These data also indicate that the use of a straight pipe injector could reduce the pressurant requirement by 9 to 35 percent, depending on liquid outflow rate and tank size, compared with that obtained using diffuser-type injectors. It was also shown that pressurant requirements increased in proportion to increased tank pressure level.

The analysis of reference 1 was used to predict the pressurant requirements in references 3 to 5 . It was modified in references 4 and 5 to accommodate a spherical tank geometry and also to include the heat transfer from the gas to the liquid surface (which was neglected in the original analysis). The analysis predicted the actual pressurant $r \geq-$ quirements for the various operating parameters within 12.4 percent with the eyception of the tests performed with the straight pipe injector. The analysis overpredicted the pressurant requirements for the straight pipe injector by as much as 45 percent.

If only pressurant gas weight is considered, it appears advantageous to use hydrogen (because of its low molecular weight) as the pressurant for the expulsion of liquid hydrogen. Since gaseous hydrogen cannot be used as the pressurant for most oxidizers and a dual pressurization system would probably result in an additional payload penalty, it would appear desirable to use a single pressurant gas which is compatible with liquid hydrogen and any oxidizer. To meet this requirement, helium is the only logical choice because it has a low molecular weight and, since it is an inert gas, it is compatible with any fuel-oxidizer combination.

This report presents the results obtained using gaseous helium as the pressurant for the expulsion of liquid hydrogen. The use of helium as a pressurant in the expulsion of
liquid hydrogen adds further complication to both the analytical and experimental programs. The ullage volume for this system contains a binary gas, part of which is condensable and part is not. The analytical model must either (1) account for the possibility of a binary gas in the ullage volume by diffusion and/or evaporation or (2) assume the contribution of hydrogen to the ullage volume to be small enough that the ullage can be considered at 100 percent helium at all times, except at the interface where it is considered to be 100 percent hydrogen. The analysis used in this report is identical to that used in reference 4 with the exception that this analysis treats the ullage volume as if it contained pure helium. (For convenience, the modifications to the analysis of ref. 1 that were used in ref. 4 are presented in appendixes A, B, and C.) Experimentally measured gas concentration gradients in the ullage volume were used to obtain mass and energy balances and to verify the 100 percent helium model used in the analysis.

The primary objective of the test work described herein was to obtain experimental values on the helium pressurant gas requirements during the initial pressurization and expulsion periods for the various operating parameters and to compare these values with predicted ones. A secondary objective was to obtain experimental information on tank wall heating, liquid heating, residual ullage energy, and mass transfer in order to gain an insight into the reasons for any variations in pressurant gas requirements.

The tests were conducted using the same two 1.52 -meter- ( $5-\mathrm{ft}$-) diameter spherical aluminum tanks that were used in reference 4. The pressurant used for this series of tests was gaseous helium. The main test variables were (1) tank wall thickness ( 0.762 and $0.409 \mathrm{~cm} ; 0.30$ and 0.161 in .), (2) nominal inlet gas temperatures of 183,267 , and $322 \mathrm{~K}\left(329^{\circ}, 480.6^{\circ}\right.$, and $\left.580^{\circ} \mathrm{R}\right)$, (3) liquid outflow rates between 0.282 and 0.824 kilogram per second ( 0.623 and $1.817 \mathrm{lb} / \mathrm{sec}$ ), and (4) initial tank ullages of $4,28,55$, and 75 percent. Data were also obtained for pressurization of the tank at various rates $\left(3.24 \times 10^{3}\right.$ to $11.37 \times 10^{3}\left(\mathrm{~N} / \mathrm{m}^{2}\right) / \mathrm{sec} ; 0.47$ to $\left.1.65 \mathrm{psi} / \mathrm{sec}\right)$ from 1 atmosphere to the desired operating level. All tests were performed at a nominal operating tank pressure of $34.47 \times 10^{4}$ newtons per square meter ( 50 psia ).

## SYMBOLS

A area, $\mathrm{m}^{2}\left(\mathrm{ft}^{2}\right)$
b $\quad 1-\frac{Z_{1}}{Z}-2 \frac{\Delta r}{r_{i}}$
C orifice coefficient
$\mathrm{C}_{\mathrm{H}} \quad$ effective perimeter of interior hardware, $\mathrm{m}(\mathrm{ft})$
$\mathrm{C}_{\mathrm{p}} \quad$ specific heat at constant pressure, $\mathrm{J} /(\mathrm{kg})(\mathrm{K}) \quad\left(\mathrm{Btu} /(\mathrm{lb})\left({ }^{\mathrm{O}} \mathrm{R}\right)\right)$
$\mathrm{C}_{\mathrm{v}} \quad$ specific heat at constant volume, $\mathrm{J} /(\mathrm{kg})(\mathrm{K}) \quad\left(\mathrm{Btu} /(\mathrm{lb})\left({ }^{\mathrm{O}} \mathrm{R}\right)\right)$
$C_{w} \quad$ specific heat of tank wall, $J /(\mathrm{kg})(\mathrm{K}) \quad\left(\mathrm{Btu} /(\mathrm{lb})\left({ }^{\circ} \mathrm{R}\right)\right)$
c
D orifice diameter, m ( ft )
d $\quad \frac{Z_{2}}{Z} \frac{\Delta X}{\Delta t} \frac{P^{\prime}-P}{P^{\prime}}-\frac{Z_{1}}{Z} \frac{\Delta X}{\Delta t}$
Gr Grashof number, $\frac{\mathrm{L}^{3} \rho^{2} \mathrm{~g} \beta \Delta \mathrm{~T}}{\mu^{2}}$
g gravity acceleration, $\mathrm{m} / \mathrm{sec}^{2}\left(\mathrm{ft} / \mathrm{sec}^{2}\right)$
H enthalpy, J (Btu)
$\mathrm{h} \quad$ specific enthalpy, $\mathrm{J} / \mathrm{kg}$ (Btu/lb)
$h_{c} \quad$ convective heat-transfer coefficient, $J /\left(\mathrm{m}^{2}\right)(\mathrm{K})(\mathrm{sec}) \quad\left(\mathrm{Btu} /\left(\mathrm{ft}^{2}\right)\left({ }^{\mathrm{O}} \mathrm{R}\right)(\mathrm{sec})\right)$
$\mathrm{k} \quad$ thermal conductivity, $\mathrm{J} /(\mathrm{m})(\mathrm{K})(\mathrm{sec}) \quad\left(\mathrm{Btu} /(\mathrm{ft})\left({ }^{\circ} \mathrm{R}\right)(\mathrm{sec})\right)$
L flow length, m (ft)
2 thickness, m (ft)
M mass, kg (lb)
$\Delta \mathrm{M} \quad$ differential mass, kg (lb)
$\dot{\mathrm{M}} \quad$ mass flow rate, $\mathrm{kg} / \mathrm{sec}(\mathrm{lb} / \mathrm{sec})$
$\overline{\mathrm{M}} \quad$ molecular weight, $\mathrm{kg} /(\mathrm{kg}-$ mole) (lb/(lb-mole))
$\mathrm{M}_{\mathrm{I}} \quad$ ideal pressurant requirement, kg (lb)
$\mathrm{N} \quad$ number of volume segments
$\overline{\mathrm{N}} \quad$ number of data points used in defining average deviation
$N_{1}-N_{z}$ particular volume segments
$\mathrm{Nu} \quad$ Nusselt number, $\mathrm{h}_{\mathrm{c}} \mathrm{L} / \mathrm{k}$
n or i summing index
P
$\Delta \mathrm{P} \quad$ differential pressure, $\mathrm{N} / \mathrm{m}^{2}$ (lb/in. ${ }^{2}$ )
$\Delta P^{*} \quad$ orifice $\Delta P$
$\operatorname{Pr} \quad$ Prandtl number, $\mu \mathbf{C}_{\mathbf{p}} / \mathrm{k}$

Q heat transfer, J (Btu)
$\dot{\mathbf{Q}} \quad$ heat-transfer rate, $\mathrm{J} / \mathrm{sec}$ ( $\mathrm{Btu} / \mathrm{sec}$ )
$\dot{\mathbf{Q}}^{\prime} \quad$ specific heat-transfer rate, $\mathrm{J} /(\mathrm{kg})(\mathrm{sec}) \quad$ ( $\left.\mathrm{Btu} /(\mathrm{lb})(\mathrm{sec})\right)$
$\dot{\mathrm{q}} \quad$ heat-transfer rate pelc unit area, $\mathrm{J} /\left(\mathrm{m}^{2}\right)(\mathrm{sec}) \quad\left(\mathrm{Btu} /\left(\mathrm{ft}^{2}\right)(\mathrm{sec})\right)$
R gas constant, J/(K)(k:g-mole) (Btu/( $\left.{ }^{\mathrm{O}} \mathrm{R}\right)(\mathrm{lb}-\mathrm{mole})$ )
$r$ radius, $m$ ( ft )
$\Delta r$ increment of radius, $m$ ( ft )
$\operatorname{Re}$ Reynolds number, $\mathbb{L} \overline{\mathrm{V}} \rho / \mu$
$\Delta S$ increment of are le:ngth, $m$ ( ft )
T temperat.ure, $\mathbf{K}\left({ }^{\mathbf{O}} \mathbf{R}\right)$
$\Delta T$ differe'ntial temperature, $K\left({ }^{\circ} R\right)$
$\mathrm{T}_{\delta} \quad$ temprarature at the edge of thermal boundary layer, $\mathrm{K}\left({ }^{\mathrm{O}} \mathrm{R}\right)$
$t$ time, sec
$\Delta t \quad$ tirne increment, sec
U jnternal energy, J (Btu)
$\Delta \mathrm{U}$ differential energy, $J$ (Btu)
u specific internal energy, $\mathrm{J} / \mathrm{kg}$ ( $\mathrm{Btu} / \mathrm{lb}$ )
V volume, $\mathrm{m}^{3}\left(\mathrm{ft}^{3}\right)$
$\Delta V$ volume increment, $\mathrm{m}^{3}\left(\mathrm{ft}^{3}\right)$
$\overline{\mathrm{V}} \quad$ velocity, $\mathrm{m} / \mathrm{sec}(\mathrm{ft} / \mathrm{sec}$ )
$v \quad$ specific volume, $\mathrm{m}^{3} / \mathrm{kg}\left(\mathrm{ft}^{3} / \mathrm{lb}\right)$
W work, J (Btu)
$\mathbf{x}$ percent of gas by volume
X n number of net points in the ullage

- $x$ coordinate in direction of tank axis, $m$ ( ft )
$\Delta \mathrm{x}$ space increment, m (ft)
Y expansion factor
y thickness within the boundary layer, m (ft)
Z compressibility factor
z elevation or vertical distance along tank wall, m (ft)

$$
\begin{aligned}
& \alpha \quad 1+\frac{h_{c} \Delta t}{\rho_{\mathrm{w}} l_{\mathrm{w}} C_{\mathrm{w}}} \\
& \left(\frac{2 \mathrm{~h} \mathrm{c}^{\mathrm{RZ} \Delta t}}{\mathrm{rMPC}}\right)\left[1+\left(\frac{\Delta r}{\Delta \mathrm{M}}\right)^{2}\right]^{1 / 2} \\
& \beta \quad \text { coefficient of thermal expansion, } 1 / \mathrm{K}\left(1 /{ }^{\circ} \mathrm{R}\right) \\
& \gamma \quad \text { specific heat ratio } \\
& \delta \quad \text { finite increment, or total boundary layer thickness, } \mathrm{m} \text { ( } \mathrm{ft} \text { ) } \\
& \lambda \quad \text { latent heat of vaporization, } \mathrm{J} / \mathrm{kg} \text { (Btu/lb) } \\
& \mu \quad \text { viscosity, } \mathrm{kg} /(\mathrm{m})(\mathrm{hr}) \quad(\mathrm{lb} /(\mathrm{ft})(\mathrm{hr})) \\
& \rho \quad \text { density, } \mathrm{kg} / \mathrm{m}^{3}\left(\mathrm{lb} / \mathrm{ft}^{3}\right) \\
& \omega \quad\left(\frac{R}{\bar{M}} Z_{1} \frac{\Delta P}{\Delta t}+\frac{R Z C_{H}}{\bar{M} \pi r^{2}} \dot{q}_{H}\right) \frac{\Delta t}{C_{p} P}
\end{aligned}
$$

Subscripts:
A analytical results
ad adiabatic
E experimental results
f final state or condition
G gas added to tank
H internal hardware
He helium
$\mathrm{H}_{2}$ hydrogen
i initial state or condition
$\mathrm{i} \rightarrow \mathrm{f} \quad$ from initial to final state or condition
L liquid
n summing index
o condition prior to ramp
S liquid surface
sat saturation
T total quantity

## Superscripts:

, time index

* evaluation may be performed at beginning or end of time interval

Constants for Beattie-Bridgeman equation:

```
A \(\quad(\mathrm{N})\left(\mathrm{m}^{4}\right) / \mathrm{kg}^{2} \quad\left(\mathrm{ft}^{4} / \mathrm{lb}\right)\)
a \(\quad \mathrm{m}^{3} / \mathrm{kg}\left(\mathrm{ft}^{3} / \mathrm{lb}\right)\)
B \(\quad \mathrm{m}^{3} / \mathrm{kg} \quad\left(\mathrm{ft}^{3} / \mathrm{lb}\right)\)
b \(\quad \mathrm{m}^{3} / \mathrm{kg}\left(\mathrm{ft}^{3} / \mathrm{lb}\right)\)
C \(\quad\left(\mathrm{m}^{3}\right)\left(\mathrm{K}^{3}\right) / \mathrm{kg} \quad\left(\left(\mathrm{ft}^{3}\right)\left({ }^{\mathrm{O}} \mathrm{R}^{3}\right) / \mathrm{lb}\right)\)
\(\bar{M}^{\prime} \quad X_{\mathrm{H}_{2}} \overline{\mathrm{M}}_{\mathrm{H}_{2}}+\mathrm{X}_{\mathrm{He}} \overline{\mathrm{M}}_{\mathrm{He}}\)
\(\epsilon \frac{\mathrm{X}_{\mathrm{H}_{2}} \mathrm{C}_{\mathrm{H}_{2}}+\mathrm{X}_{\mathrm{He}}{ }^{\mathrm{C}_{\mathrm{He}}}}{\mathrm{vT}^{3}}\)
```


## APPARATUS AND INSTRUMENTATION

## Facility

All tests were conducted under vacuum inside a 7.61 -meter- ( $25-\mathrm{ft}-$ ) diameter spherical vacuum chamber (fig. 1) to reduce the external heat leak into the propellant tank to a low value. The vacuum capability of this chamber was approximately $8 \times 10^{-7} \mathrm{~mm} \mathrm{Hg}$. A general schematic of the test tank and associated equipment is shown in figure 2. A heat exchanger and blend valve subsystem capable of delivering gaseous helium at any desired temperature between 167 and $405 \mathrm{~K}\left(301^{\circ}\right.$ and $\left.729^{\circ} \mathrm{R}\right)$ at a maximum flow rate of $9.98 \times 10^{-2}$ kilogram per second ( $0.22 \mathrm{lb} / \mathrm{sec}$ ) was used to control pressurant gas inlet temperature. A ramp generator and control valve were used for controlling the initial rate of pressurization of the propellant tank. A closed-loop pressure control circuit was


Figure 1. - 7.61 -Meter- $(25-\mathrm{ft})$ diameter vacuum chamber.
used to maintain constant tank pressure during the expulsion period. The liquid outflow rate was controlled by remotely operated variable flow valves. The liquid hydrogen outflow from the tank was returned to the storage Dewar.

Liquid outflow rates were measured using a turbine-type flowmeter located in the transfer line. The flowmeter was calibrated (within an estimated uncertainty of $\pm 1 / 2$ percent) with liquid hydrogen over the expected range of flow rates. The calibration was performed at Lewis. Pressurant gas inlet flow rates were determined by the use of an orifice located in the pressurant supply line. Tank, line, and differential pressures were measured with bonded strain-gage-type transducers (estimated uncertainty, $\pm 1 / 4$ percent).

## Test Tanks

The experimental work was conducted in 1.52 -meter- ( $5-\mathrm{ft}-$ ) diameter bare wall spherical aluminum tanks. One tank (fig. 3) had an average wall thickness of 0.762 centimeter ( 0.30 in .). The other tank was identical except the wall was chem-milled down to an average thickness of 0.409 centimeter ( 0.161 in .) 。One stainless steel lid served both tanks. The lid housed the inlet and vent pipes and the electrical connections for all internal tank instrumentation. The lid was 0.457 meter ( 18 in .) in diameter, 3.18 centi-


Figure 2. - General schematic of facility.


Figure 3. - 1.52-Meter-(5-ft-) diameter spherical test tank. Tank wall thickness, 0.762 centimeter ( 0.30 in .).
meters ( 1.25 in. ) thick, and weighed 67.50 kilograms ( 149 lb ). The inner surface of the lid conformed to the contour of the tank wall.

A viewport and television camera were installed on the 0.762 -centimeter( 0.30 -in. -) wall test tank (fig. 4) for the purpose of visually locating the initial liquid level in the tank prior to expulsion, and to enable observation of any physical processes occurring in the tank. Lighting of the tank interior was accomplished by bulbs mounted on the inner surface of the tank wall.

## Pressurant Gas Injector Geometry

A hemisphere injector (fig. 5) was used for all tests reported herein. The hemisphere injector was selected because it injects the pressurant uniformly in all directions into the ullage volume (minimizes ullage gas mixing). The uniform diffusion of pressurant is a basic assumption of the analysis used in this report. The open exit area for the injector was 176.8 square centimeters (27.4 in. ${ }^{2}$ ).


Figure 4. - View port and television camera installation on test tank. Wall thickness, 0.762 centimeter ( 0.30 in .).

## $\longrightarrow$ Denotes pressurant gas flow paths



Figure 5. - Injector geometry for hemisphere injector. Open area, 176.8 square centimeters (27.4 in. 2 ). All dimensions are in centimeters (in.).

## Internal Tank Instrumentation

Temperatures. - Ullage gas temperatures together with gas concentration measurements were used to determine the mass and energy content of the tank ullage. The temperatures must be obtained with sensors capable of accurate measurement of rapid changes in temperature. Internal tank instrumentation is illustrated in figure 6. Location of the vertical and horizontal ullage gas temperature rakes is indicated. The thermopile was the basic temperature measurement technique used in this investigation. The use of thermopiles to measure ullage gas temperature was first developed in reference 6 , and the technique was used with good results in references 3 to 5 . The main advantage of using thermopiles is their fast response time (between 0.2 and 1.0 sec ) in going from saturated liquid to vapor. This time response is approximately an order of magnitude less than carbon or platinum resistance sensors which are generally used in this type of investigation.

A typical three-element thermopile unit and its associated wiring schematic are illustrated in figure 7. The thermopile units were constructed of 0.202 millimeter ( 0.008 in.) chromel-constantan wire. Vertical ullage gas temperature profiles were obtained by stacking the individual thermopile units as shown in figure 7 (b). The support structure was made of laminated thermoplastic to minimize thermal conduction from one measure-


Figure 6. - Test tank instrumentation.


Figure 7. - Thermopile rake.
ment station to another. The spacing between the reference and measuring levels was 2.54 centimeters ( 1.0 in .) for the top 45 thermopiles composing the vertical rake. The 10 units at the bottom of the rake had spacings of 1.27 centimeters ( 0.50 in .) to obtain a more accurate temperature profile of the ullage gas near the liquid surface at the end of an expulsion.

Platinum resistance sensors, which were located at least every tenth station starting from the bottom of the rake, sensed the absolute temperature at their location and provided a reference for the thermopiles above the location.

The horizontal rakes were composed of platinum resistance sensors spaced a maximum of 12.70 centimeters ( 5.00 in .) apart in the radial direction. Two platinum resistance sensors were used at each location to measure liquid and/or gas temperatures for the ranges of 20 to $38.9 \mathrm{~K}\left(36^{\circ}\right.$ to $\left.70^{\circ} \mathrm{R}\right)$ and 38.9 to $277.8 \mathrm{~K}\left(70^{\circ}\right.$ to $\left.500^{\circ} \mathrm{R}\right)$. These dual sensors permitted more accurate measurement of liquid and gas temperatures than could be achieved with one sensor covering the entire range.

The initial static temperature profile near the liquid surface was determined by a fixed interface rake. This rake contained 13 platinum resistance sensors spaced 0.64
centimeter ( 0.25 in .) apart. The range of these sensors was 20 to $38.9 \mathrm{~K}\left(36^{\circ}\right.$ to $70^{\circ} \mathrm{R}$ ). For complete tank expulsions (i.e., 5 percent ullage to 5 percent outage), the initial liquid level was always assured to be on the interface rake in the thick walled tank by monitoring a closed-circuit television system which viewed, through a viewport in the tank, the position of the liquid-gas interface on a scale fixed to the interface rake (fig. 6). For tests conducted at the $28-$, $55-$, and 75 -percent ullage values, the scale was moved lower in the tank and the television camera was refocused. Liquid level for the thin walled tank was determined by hot-wire liquid level sensors located on the interface rake.

Platinum resistance sensors were also used to determine tank wall temperatures at 12 locations and the liquid temperature at the flowmeter. Copper-constantan thermocouples were used to determine tank lid temperatures at 5 locations and the pressurant gas inlet temperature.

Concentrations. - The concentration of helium and hydrogen gas at five positions in the tank (fig. 6) was obtained by a gas sampling and analyzer system. A general schematic of this system is shown in figure 8. The sampling tubes had a 0.157 -centimeter ( 0.062 -in.) outside diameter with a wall thickness of 0.030 centimeter ( 0.012 in .). The analyzing sequencing of the five sampling tubes was done automatically. The sampling operation for one tube was as follows: The sample tube was purged with a negligible flow of helium until the liquid surface passed the entrance of the tube at which time it was included in the sampling sequence of the tubes above it. The gas sample was vented (also


Figure 8. - schematic oí gas analyzer system.
at a negligible rate) to the atmosphere until its particular level was to be analyzed. At this time the flow control vent valve was closed and the sample flow was directed through a variable-leak flow control valve to a constant pressure manifold from where a portion of the sample passed through a 2245 cubic centimeter per hour standard leak to a mass spectrometer. The mass spectrometer then measured the partial pressure of the hydrogen of the gas sample which was used to determine the hydrogen concentration at that level. The manifold and tube were then evacuated and the next level in the sequence was sampled and analyzed. The total time required to sample and analyze each level was approximately 8 seconds.

All measurements except the gas concentrations were recorded on a high-speed digital data system. The measurements were recorded at a rate of $3.125 \times 10^{3}$ channels per second. Each measurement channel was sampled every 0.064 second. The gas concentration measurements were continuously recorded on a direct reading oscillograph.

## PROCEDURE

The spherical test tank was filled from the bottom to approximately a 2 -percent ullage condition. It was then topped off as necessary while the tank lid and peripheral support hardware reached steady-state operating temperatures.

Temperature conditioning of the pressurant gas was then started. Gas flow was established through the heat exchanger loop, through the control valves and orifice arrangement, and then into the tank ullage from where it was vented through the conditioning line to the outside as shown in figure 2. The temperature control circuit shown in figure 2 was used to get the desired pressurant gas temperature level during the flow period. When the pressurant gas temperature conditioning was almost completed, the liquid level in the test tank was adjusted to a desired value by either topping or slow draining.

The pressurant gas flow was then stopped and the test tank was vented in preparation for an expulsion run. The automatic controllers and timers were preset with all the desired run and operating conditions (i.e., tank pressure level, length of ramp period, length of hold period, liquid outflow valve position, start and stop times of the data recording equipment, etc.).

After starting the data recording equipment, the next step of the completely automatic run sequence took electrical calibrations on all pressure transducers. Immediately following this, the test tank was pressurized over a predetermined time period to the nominal operating pressure of $34.47 \times 10^{4}$ newtons per square meter ( 50 psia ). Tank pressure was held constant for about 30 seconds to stabilize internal temperatures. The tank expulsion period was then started. Approximately 90 percent of the total tank volume was expelled at a constant volumetric flow rate. The expulsion period was stopped
when a hot-wire liquid level sensor located at the 95 -percent ullage level indicated the presence of gas. The last step of the automatic run sequence was the stopping of all data recording equipment. The test tank was then vented and refilled with liquid hydrogen for the next expulsion.

Additional ramp pressurization runs, with no expulsion, were made for three different tank ullage levels.

## DATA REDUCTION

The data reduction procedure used in this report is essentially the same that was used in reference 4 with the exception of the treatment for the two component ullage. The complete data reduction procedure is presented in appendix D. The relations derived in appendix D that are used in the presentation of experimental results are now summarized.

## Pressurant Gas Added ( $\mathrm{M}_{\mathrm{G}}$ )

The actual pressurant
adder to the tank ullage was determined from the relation

$$
\begin{equation*}
\mathrm{M}_{\mathrm{G}, \mathrm{i} \rightarrow \mathrm{f}}=\int_{\mathrm{t}_{\mathrm{i}}}^{\mathrm{t}_{\mathrm{f}}} \mathrm{YD}^{2} \mathrm{C} \sqrt{\rho \Delta \mathrm{P}^{*}} \mathrm{dt} \tag{1}
\end{equation*}
$$

## Ideal Pressurant Requirement ( $\mathrm{M}_{\mathrm{I}}$ )

This report uses two different relations to define the ideal pressurant requirement. One is used to determine the ideal requirement for the initial pressurization of the tank and is given by

$$
\begin{equation*}
\mathrm{M}_{\mathrm{I}}=\frac{\overline{\mathrm{M}} \mathrm{P}_{\mathrm{o}} \mathrm{~V}_{\mathrm{o}}}{\mathrm{ZRT}_{\mathrm{G}}}\left[\left(\frac{\mathrm{P}_{\mathrm{f}}}{\mathrm{P}_{\mathrm{o}}}\right)^{1 / \gamma}-1\right] \tag{2}
\end{equation*}
$$

This expression assumes a one component ullage. It also assumes that the pressurizing gas does not mix or exchange heat with the gas in the ullage volume. For the tests conducted herein, the initial ullage volume contained a mixture of hydrogen and helium. However, because of the method used to precondition the inlet gas temperature (described in the PROCEDURE section), the initial ullage volume was predominantly helium.

Therefore, the ideal pressurant requirement used in this report is based on a 100percent helium ullage (i.e., $\bar{M}=4.003, \gamma=1.67$ ), whereas the ideal requirement used in reference 4 is based on 100 -percent hydrogen (i.e., $\bar{M}=2.016, \gamma=1.40$ ). In any case, the ideal requirement is just used as a normalizing factor and does not necessarily indicate a true minimum requirement.

The other relation, which is used to determine the ideal requirement during the expulsion period, is given by

$$
\begin{equation*}
\mathrm{M}_{\mathrm{I}}=\frac{\overline{\mathrm{M}} P \Delta \mathrm{~V}_{\mathrm{U}}}{\mathrm{ZRT}_{\mathrm{G}}} \tag{3}
\end{equation*}
$$

The assumption used to obtain this relation is that the incoming pressurant does not exchange heat or mass with the surroundings.

## Energy Balance

Applying the first law of thermodynamics to the entire tank system (tank + ullage gas + liquid) results in the expression

$$
\underbrace{\int_{t_{i}}^{t_{f}}\left(\dot{M}_{G} h_{G}+\dot{Q}\right) d t}_{\begin{array}{c}
\text { Total energy }  \tag{4}\\
\text { added } \\
\left(\Delta U_{T}\right)
\end{array}}=\underbrace{\int_{t_{i}}^{t_{f}}\left(\dot{M}_{L} h_{L} d t+d U_{L}\right)}_{\begin{array}{c}
\text { Total change in liquid } \\
\text { in tank + liquid } \\
\text { expelled energy } \\
\left(\Delta U_{L}\right)
\end{array}}+\underbrace{\int_{t_{i}}^{t_{\mathrm{f}}} d U_{U}}_{\begin{array}{c}
\text { Total change } \\
\text { in ullage } \\
\text { energy } \\
\left(\Delta U_{U}\right)
\end{array}} \quad+\underbrace{\int_{t_{i}}^{t_{f}} d U_{w}}_{\begin{array}{c}
\text { Total change } \\
\text { in wall } \\
\text { energy }
\end{array}}
$$

Dividing equation (4) by $\Delta \mathrm{U}_{\mathbf{T}}$ gives

$$
\begin{equation*}
1=\frac{\Delta \mathrm{U}_{\mathrm{L}}}{\Delta \mathrm{U}_{\mathrm{T}}}+\frac{\Delta \mathrm{U}_{\mathrm{U}}}{\Delta \mathrm{U}_{\mathrm{T}}}+\frac{\Delta \mathrm{U}_{\mathrm{W}}}{\Delta \mathrm{U}_{\mathrm{T}}} \tag{5}
\end{equation*}
$$

These ratios show the relative distribution of the total energy input into the system. The data for energy distribution presented herein are in the form of these ratios.

TABLE I. - MASS BALANCE RESULTS FOR HEMISPHERE INJECTOR
|Tank pressure, $34.47 \times 10^{4} \mathrm{~N} / \mathrm{m}^{2}$. |

| Run | Tank wall thickness, cm | Inlet gas temperature |  | Ramp <br> time, <br> sec | Hold <br> time, sec | Expulsion time, sec | Tank cycle time, sec | Ramp |  |  |  |  |  |  | Expulsion |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | $\left\|\begin{array}{c} \text { Initial } \\ \text { ullage } \\ \text { mass, } \\ \mathrm{kg} \end{array}\right\|$ |  |  |  | Mass <br> added <br> during <br> ramp, <br> kg | $\begin{gathered} \text { Mass } \\ \text { transfer } \\ \text { during } \\ \text { ramp } \\ \text { kg } \end{gathered}$ | Hold |  |  |  | Mass added during expulsion, kg |  | Mass transfer during expulsion, kg | Final <br> ullage <br> mass, <br> kg |
|  |  |  | ${ }^{0} \mathrm{R}$ |  |  |  |  |  |  | Ullage | Mass | Mass | Ullage |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  | mass <br> after <br> ramp, kg | added during hold, kg | transfer during hold, kg | mass <br> after <br> hold, <br> kg | Experimental <br> (a) | Predicted |  |  |
| 83 | 0.762 | 263 | 473 | 15.6 | 22.2 | 391.0 | 428.8 | 0.093 | 0.063 | 0.038 | 0.118 | 0.053 | 0.020 | 0.151 | $3.199 \pm 0.001$ | 3.557 | -0.176 | 3.526 |
| 84 |  | 266 | 479 | 15.7 | 21.6 | 267.3 | 304.6 | . 101 | . 072 | . 036 | . 137 | . 058 | . 016 | . 178 | $2.978 \pm 0.001$ | 3.098 | . 140 | 3.016 |
| 85 |  | 271 | 488 | 16.0 | 21.6 | 136.8 | 174.4 | . 108 | . 073 | . 044 | . 138 | . 062 | . 006 | . 194 | $2.472 \pm 0.002$ | 2.508 | . 457 | 2.209 |
| 88 |  | 184 | 331 | 14.6 | 22.1 | 396.2 | 432.8 | . 082 | . 073 | -. 027 | . 182 | . 062 | . 064 | . 179 | $3.483 \pm 0.002$ | 4.159 | . 324 | 3.338 |
| 92 |  | 170 | 306 | 14.5 | 22.3 | 138.0 | 174.8 | . 108 | . 068 | -. 036 | . 213 | . 050 | . 078 | . 184 | $2.969 \pm 0.002$ | 3.188 | . 159 | 2.994 |
| 94 |  | 324 | 583 | 15.9 | 20.5 | 395.4 | 431.8 | . 090 | . 054 | . 028 | . 117 | . 064 | . 022 | . 158 | $3.024 \pm 0.001$ | 3.439 | -. 213 | 3.395 |
| 95 |  | 311 | 560 | 15.7 | 20.8 | 271.2 | 307.6 | . 087 | . 050 | . 020 | . 117 | . 055 | . 017 | . 156 | $2.779 \pm 0.002$ | 2.834 | -. 071 | 3.006 |
| 97 | v | 335 | 603 | 16.6 | 20.0 | 133.9 | 170.5 | . 101 | . 064 | . 023 | . 142 | . 066 | . 034 | . 173 | $2.235 \pm 0.001$ | 2.175 | . 136 | 2.272 |
| 7 | 0.409 | 287 | 517 | 25.2 | 27.4 | 400.9 | 453.4 | 0.091 | 0.079 | 0.059 | 0.111 | 0.039 | -0.020 | 0.170 | $2.893 \pm 0.001$ | 3.147 | -0.039 | 3.102 |
| 10 |  | 292 | 526 | 25.1 | 27.6 | 140.5 | 193.2 | . 086 | . 073 | . 051 | . 108 | . 033 | -. 022 | . 164 | $2.315 \pm 0.001$ | 2.319 | . 116 | 2.363 |
| 12 |  | 285 | 513 | 24.9 | 27.4 | 340.9 | 393.2 | . 085 | . 071 | . 047 | . 109 | . 037 | -. 010 | . 156 | $2.860 \pm 0.001$ | 2.927 | . 069 | 2.947 |
| 14 |  | 288 | 518 | 24.5 | 28.4 | 284.2 | 337.0 | . 084 | . 068 | . 029 | . 124 | . 032 | . 008 | . 148 | $2.760 \pm 0.001$ | 2.805 | . 139 | 2.769 |
| 15 | $\psi$ | 291 | 524 | 24.8 | 27.7 | 197.2 | 249.7 | . 087 | . 068 | . 041 | . 114 | . 034 | -. 013 | . 161 | $2.514 \pm 0.001$ | 2.556 | . 159 | 2.516 |

[^1]TABLE II. - ENERGY BALANCE RESULTS

| Run | Tank <br> wall <br> thick- <br> ness, <br> cm | Inlet gas <br> temperature |  | Expul- <br> sion <br> time, <br> sec | Expulsion |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Energy added by pressurant gas, J | Energy <br> added by environment, J | Energy gained by tank wall, $\Delta \mathrm{U}_{\mathrm{w}}, \mathrm{J}$ |  | Energy <br> gained by ullage, $\Delta \mathrm{U}_{\mathrm{U}}$, J | Energy gained by liquid,$\Delta U_{L}, J$ |  |
|  |  |  |  |  |  | Experimental <br> (a) | Predicted |  | Experiment | redicted | Experimental <br> (a) | Predicted |
| 83 | 0.762 | 263 | 473 |  | 391.0 | $442.6 \pm 0.2 \times 10^{4}$ | $488.0 \times 10^{4}$ | $28.6 \times 10^{4}$ | $256.5 \times 10^{4}$ | $259.0 \times 10^{4}$ | $105.2 \times 10^{4}$ | $94.4 \pm 43.3 \times 10^{4}$ | $98.3 \times 10^{4}$ |
| 84 |  | 266 | 479 | 267.3 | $414.8 \pm 0.2$ | 431.0 | 19.6 | 228.3 | 232.8 | 118.8 | $93.7 \pm 36.8$ | 67.8 |
| 85 |  | 271 | 488 | 136.8 | $350.2 \pm 0.2$ | 355.7 | 9.9 | 179.9 | 175.7 | 112.5 | $46.7 \pm 21.4$ | 34.7 |
| 88 |  | 184 | 331 | 396.2 | $345.5 \pm 0.2$ | 401.9 | 29.0 | 175.1 | 174.8 | 157.0 | $77.0 \pm 44.5$ | 99.7 |
| 92 |  | 170 | 306 | 138.0 | $278.1 \pm 0.2$ | 296.3 | 10.1 | 108.7 | 106.4 | 125.0 | $86.6 \pm 19.9$ | 34.8 |
| 94 |  | 324 | 583 | 395.4 | $529.0 \pm 0.2$ | 593.6 | 29.0 | 313.5 | 354.9 | 117.3 | $99.2 \pm 39.0$ | 130.0 |
| 95 |  | 311 | 560 | 271.2 | $502.1 \pm 0.3$ | 511.1 | 19.7 | 292.0 | 312.7 | 117.2 | $66.8 \pm 34.6$ | 68.2 |
| 97 | $\dagger$ | 335 | 603 | 133.9 | $411.3 \pm 0.2$ | 401.6 | 9.8 | 204.7 | 227.8 | 100.3 | $142.0 \pm 14.3$ | 33.8 |
| 7 | 0.409 | 287 | 517 | 400.9 | $433.1 \pm 0.2 \times 10^{4}$ | $466.9 \times 10^{4}$ | $30.0 \times 10^{4}$ | $241.2 \times 10^{4}$ | $254.9 \times 10^{4}$ | $115.1 \times 10^{4}$ | $85.6 \pm 28.8 \times 10^{4}$ | $105.6 \times 10^{4}$ |
| 10 |  | 292 | 526 | 140.5 | $355.5 \pm 0.1$ | 356.6 | 10.3 | 171.2 | 157.0 | 109.4 | $89.7 \pm 28.6$ | 36.7 |
| 12 |  | 285 | 513 | 340.9 | $428.7 \pm 0.1$ | 437.9 | 24.9 | 233.8 | 220.9 | 113.1 | 93. $1 \pm 27.6$ | 89.1 |
| 14 |  | 288 | 518 | 284.2 | $417.1 \pm 0.1$ | 423.3 | 20.8 | 223.1 | 210.4 | 114.5 | $119.1 \pm 27.5$ | 74.9 |
| 15 | $\dagger$ | 291 | 524 | 197.2 | $381.1 \pm 0.1$ | 380.9 | 14.4 | 195.1 | 188.2 | 112.3 | $86.9 \pm 22.9$ | 51.8 |

[^2]
## Error Analysis

An analysis was performed to determine the magnitude of probable error which could be present in the integrations of equations (D4), (D13), and (D14). Probable error is defined as follows: There is a 50 -percent probability that the error will be no larger than the value stated. This analysis considered the errors introduced by the inaccuracies of temperature transducers as well as the tank pressure sensor. These calculations were performed for all runs for the expulsion period. The results of this analysis are included with the tabular expulsion data in tables I and II. No error analysis was performed on parameters which were dependent on the measured gas concentration data. The actual uncertainty of determining the gas concentration was unknown although it was estimated to be between $\pm 10$ to $\pm 30$ percent.

## RESULTS AND DISCUSSION

The main parameter used to compare the effectiveness of the various operating parameters on the amount of pressurant gas used is the nondimensional ratio $M_{1} / M_{G}$. The symbol $M_{I}$ is defined as the ideal helium pressurant mass required to pressurize or expel a given volume of liquid at a given inlet gas temperature and tank pressure assuming no heat or mass transfer, and $\mathrm{M}_{\mathrm{G}}$ is the actual pressurant requirement for the same inlet gas temperature and tank pressure。 $A$ high $M_{I} / M_{G}$ ratio means less energy and mass exchange. It does not necessarily mean a lower absolute pressurant requirement $\mathrm{M}_{\mathrm{G}}$ as will be illustrated later in this section.

A value of $M_{I} / M_{G}$ equal to 1 implies that there is no heat transferred to the tank wall or liquid and no mass transfer. This means that for no environmental heating the terms $\Delta \mathrm{U}_{\mathrm{w}} / \Delta \mathrm{U}_{\mathrm{T}}$ and $\Delta \mathrm{U}_{\mathrm{L}} / \Delta \mathrm{U}_{\mathrm{T}}$ in equation (5) of the DATA REDUCTION section are zero and that $\Delta \mathrm{U}_{\mathrm{U}} / \Delta \mathrm{U}_{\mathrm{T}}$ is equal to 1 ; that is, all the energy ( $\Delta \mathrm{U}_{\mathrm{T}}$ ) added to the tank during expulsion appears as an increase in ullage energy ( $\Delta \mathrm{U}_{\mathrm{U}}$ ). Therefore, any value of $\mathrm{M}_{\mathrm{I}} / \mathrm{M}_{\mathrm{G}}$ or $\Delta \mathrm{U}_{\mathrm{U}} / \Delta \mathrm{U}_{\mathrm{T}}$ less than 1 means energy is lost by the ullage system. This loss of ullage energy would then appear as a change in tank wall energy and/or liquid energy; that is, $\Delta \mathrm{U}_{\mathrm{W}} / \Delta \mathrm{U}_{\mathrm{T}}$ and/or $\Delta \mathrm{U}_{\mathrm{L}} / \Delta \mathrm{U}_{\mathrm{T}}$ would be greater than zero.

The discussion of results first presents the effects of the various operating parameters on the ratio $M_{I} / M_{G}$ for the expulsion period only, which was of primary interest, followed by the mass transfer $M_{t} / M_{G}$ results. Then the results of the energy balances are presented in an attempt to point out major reasons for the ratios $M_{I} / M_{G}$ or $\Delta \mathrm{U}_{\mathrm{U}} / \Delta \mathrm{U}_{\mathrm{T}}$ being less than 1 . Finally, a comparison is made between the experimental results and the analytically predicted results to determine the validity of the analytical program. The analytical results are presented in the figures along with the correspond-


Figure 9. - Comparison of ideal pressurant requirement to actual pressurant requirement ratio as function of expulsion time for three inlet gas temperatures. Tank pressure, $34.47 \times 10^{4}$ newtons per square meter ( 50 psia); wall thickness, 0.762 centimeter (0. 30 in. ).
ing experimental results. The comparison between experimental and analytical results are given in terms of an average deviation, which is defined as

$$
\frac{1}{\overline{\mathrm{~N}}} \sum_{\overline{\mathrm{N}}}\left[\begin{array}{c}
\mid \text { Experimental ratio) - (Analytical ratio) } \mid  \tag{27}\\
\text { (Experimental ratio) }
\end{array}\right](100)
$$

where $\overline{\mathrm{N}}$ is the number of data points in a given set of operating conditions (i.e., for a constant inlet gas temperature of $322 \mathrm{~K}\left(580^{\circ} \mathrm{R}\right), \overline{\mathrm{N}}$ would be 3 for the data presented in fig. 9). For convenience, all deviations between the experimental and analytical results are summarized in table III.

The operating parameters (e.g., inlet gas temperature, outflow rate, and tank wall thickness) and major experimental and analytical results are summarized in tables I and II. Table I gives the experimental and analytical mass balance results, while table II gives the corresponding energy balance results.

## Effect of Inlet Gas Temperature

Pressurant requirements. - The effect of inlet gas temperature is shown in figure 9 on the basis of $M_{I} / M_{G}$ for various expulsion times. Expulsion time is the total time

TABLE III. - DEVIATIONS BETWEEN EXPERIMENTAL AND ANALYTICAL
RESULTS FOR HEMISPHERE INJECTORS

| Run | Inlet gas temperature |  | Tank wall thickness, cm | Percent deviation between experimental and analytical results ${ }^{a}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | K | ${ }^{\circ} \mathrm{R}$ |  |  |  |  |
|  |  |  |  | $\mathrm{M}_{\mathrm{I}} / \mathrm{M}_{\mathrm{G}}$ | $\Delta \mathrm{U}_{\mathrm{W}} / \Delta \mathrm{U}_{\mathrm{T}}$ | $\Delta \mathrm{U}_{\mathrm{L}} / \Delta \mathrm{U}_{T}$ |
| 83 | 263 | 473 | 0.762 | +10.25 | -0.91 | -4.00 |
| 84 | 266 | 479 |  | +3.92 | -2.09 | +27.77 |
| 85 | 271 | 488 |  | +1.57 | +2.20 | +26.15 |
| 88 | 184 | 331 |  | +16.38 | 0 | -29.12 |
| 92 | 170 | 306 |  | +6.54 | +2.11 | +59.80 |
| 94 | 324 | 583 |  | +12.21 | -13.16 | -30.89 |
| 95 | 311 | 560 |  | +1.79 | -7.15 | -2.34 |
| 97 | 335 | 603 | $\downarrow$ | -2.40 | -11.31 | +76.26 |
| 7 | 287 | 517 | 0.409 | +7. 82 | -5.75 | -23.24 |
| 10 | 292 | 526 |  | +. 25 | +8.33 | +59.18 |
| 12 | 285 | 513 |  | +2.14 | +5.43 | +4.39 |
| 14 | 288 | 518 |  | +1.48 | +5.88 | +37.13 |
| 15 | 291 | 524 | $\downarrow$ | +. 84 | +3.60 | +40.36 |

${ }^{\text {a }}$ Analysis overpredicts, -; analysis underpredicts, +.
required to expel liquid from a 5-percent ullage to a 95 -percent ullage. Therefore, each data point represents a complete expulsion. For a given inlet gas temperature, there is an increasing pressurant requirement (decreasing $M_{I} / M_{G}$ ) for increasing expulsion times. The longer the pressurant (ullage) gas is exposed to the cold surroundings, the greater the loss in pressurant energy. The ratio $M_{I} / M_{G}$ also decreases for increasing inlet gas temperature (from 0.540 for the $183 \mathrm{~K}\left(329^{\circ} \mathrm{R}\right.$ ) inlet gas temperature at a 138second expulsion to 0.263 for the $322 \mathrm{~K}\left(580^{\circ} \mathrm{R}\right)$ inlet gas temperature at a 395 -second expulsion). This decreasing ratio implies that a larger percentage of the pressurant gas energy is lost to the tank wall and/or to the liquid as the inlet gas temperature is increased. The values of $M_{I} / M_{G}$ obtained here using helium as the pressurant are approximately the same as those obtained in reference 4 using hydrogen as the pressurant for similar inlet gas temperatures and expulsion times. A comparison of the actual pressurant requirements $M_{G}$ for the three inlet gas temperatures for various expulsion times is shown in figure 10. The actual pressurant requirements ( $\mathrm{M}_{\mathrm{G}}$ ) decrease for increasing inlet gas temperature and increase for increasing expulsion times. There is an average decrease of 1.3 percent in $M_{G}$ for a $10 \mathrm{~K}\left(18^{\circ} \mathrm{R}\right)$ increase in inlet gas temperature. The results of reference 4 indicated that there was an average decrease in pressurant gas requirements of 1.4 percent for a $10 \mathrm{~K}\left(18^{\circ} \mathrm{R}\right)$ increase in inlet gas temperature when using hydrogen as the pressurant gas.


Figure 10. - Comparison of actual pressurant added as function of expulsion time for three inlet gas temperatures. Tank pressure, $34.47 \times 10^{4}$ newtons per square meter ( 50 psia ); wall thickness, 0.762 centimeter ( 0.30 in .).

Although the absolute value of $\mathrm{M}_{\mathrm{G}}$ decreases for increasing inlet gas temperatures, the ratio $\mathrm{M}_{\mathrm{I}} / \mathrm{M}_{\mathrm{G}}$ at $322 \mathrm{~K}\left(580^{\circ} \mathrm{R}\right)$ is less than the $\mathrm{M}_{\mathrm{I}} / \mathrm{M}_{\mathrm{G}}$ ratio at $183 \mathrm{~K}\left(329^{\circ} \mathrm{R}\right)$ because of the greater decrease in ideal requirements.

The shaded symbols shown in figures 9 and 10 are the results as predicted by the analytical program. The agreement between the analysis and experimental results is better for the 322 and $267 \mathrm{~K}\left(580^{\circ}\right.$ and $480.6^{\circ} \mathrm{R}$ ) inlet gas temperatures (see table III for actual deviations) than for the $183 \mathrm{~K}\left(329^{\circ} \mathrm{R}\right)$ inlet temperature. The reason for the greater deviation for the lower temperature is not clearly understood. The deviation between the analytical and experimental results is also greatest for the longest expulsion time in all cases. This could be attributed to the diffusion of hydrogen into the helium ullage (to satisfy its partial pressure) which is a time-dependent process. The longer the liquid hydrogen is exposed to a helium ullage the more hydrogen will diffuse into the ullage. The limiting condition is when the partial pressure of hydrogen in the ullage is satisfied. The addition of hydrogen into the ullage would therefore reduce the actual helium pressurant requirement. The analysis does not consider this mass transfer and would therefore overpredict the pressurant requirements, especially for the longer expulsion times.

Mass transfer. - The amount of mass transfer was not directly measured experimentally. It was determined indirectly by the use of equation (D8). The accuracy of determining the mass transfer is strongly influenced by the accuracy in which the gas sampling and analysis system determines the helium-hydrogen concentrations in the ullage.

A comparison of the ratio of mass transferred during expulsion to the actual pressurant added to the tank $M_{t} / M_{G}$ is presented in figure 11 for different expulsion times and the three inlet gas temperatures. The experimental results presented in this figure indicate that helium is absorbed into the liquid hydrogen for the short expulsion times for


Figure 11. - Comparison of mass transfer to mass added ratio as function of expulsion time for three inlet gas temperatures. Tank pressure, $34.47 \times 10^{4}$ newtons per square meter ( 50 psia); wall thickness, 0.762 centimeter ( 0.30 in .).
the three inlet gas temperatures. Both the 322 and $267 \mathrm{~K}\left(580^{\circ}\right.$ and $\left.480.6^{\circ} \mathrm{R}\right)$ inlet gas temperatures show a trend toward evaporation of liquid for increased expulsion times. This trend is not apparent for the $183 \mathrm{~K}\left(329^{\circ} \mathrm{R}\right)$ inlet gas temperature.

As mentioned previously, the determination of mass transfer is strongly influenced by the determination of the helium-hydrogen gas concentration in the ullage. Figure 12 compares the ullage gas concentration profiles prior to and after expulsion for a run which indicated absorption (run 85) and a run which indicated evaporation (run 83). Both runs show an increase of hydrogen gas in the ullage over the expulsion period. This fact alone indicates evaporation and/or diffusion. However, the percent of helium near the liquid surface (where most of the ullage mass is concentrated) for run 85 was much less than that of run 83, resulting in a much lower final ullage mass. The use of equation (8) (appendix D) for run 85 results in a value of mass transfer (absorption) of 0.458 kilogram $(1.009 \mathrm{lb})$. Based on a 1.2 -mole-percent solubility of helium in liquid hydrogen (ref. 7) the maximum amount of helium that could go into solution is approximately 2.618 kilogram ( 5.770 lb ). Even though all absorption values were well below this maximum value, the actual mass-transfer values are considered inconclusive because of the rather large uncertainties in determining concentration gradients.

There are no analytical comparisons for the mass transfer because the analysis neglects mass transfer in its development.

Energy remaining in ullage. - The ratios of the energy increase in the ullage over the expulsion period to the total energy added to the system $\Delta \mathrm{U}_{\mathrm{U}} / \Delta \mathrm{U}_{\mathrm{T}}$ for different expulsion times are compared in figure 13 for the three inlet gas temperatures. For all runs, between 21.0 and 43.5 percent of the total energy that was added to the system remains in the ullage after expulsion. In general, the ratio $\Delta U_{U} / \Delta U_{T}$ decreases for increasing inlet gas temperatures for a constant expulsion time. Also, for a given inlet gas temperature, the ratio $\Delta \mathrm{U}_{\mathrm{U}} / \Delta \mathrm{U}_{\mathrm{T}}$ decreases with increasing expulsion time. It should be noted that the absolute value of $\Delta U_{U}$ does not change significantly for the


Figure 12. - Comparison of ullage gas concentration profiles prior to and after expulsion for runs 83 , which indicated evaporation, and 85, which indicated absorption. Tank pressure, $34.47 \times 10^{4}$ newtons per square meter ( 50 psia ); inlet gas temperature: for run $85,271 \mathrm{~K}\left(488^{\circ} \mathrm{R}\right)$; for run $83,263 \mathrm{~K}\left(473^{\circ} \mathrm{R}\right)$.


Figure 13. - Comparison of energy increase in ullage to total energy added ratio as function of expulsion time for three inlet gas temperatures. Tank pressure, $34.47 \times 10^{4}$ newtons per square meter ( 50 psia); wall thickness, 0.762 centimeter ( 0.30 in .).
three operating inlet gas temperatures used during testing. The mean increase in ullage energy for these series of runs (table II) was $117 \times 10^{4}$ joules ( 1106 Btu ) with a standard deviation of $13.7 \times 10^{4}$ joules ( 129 Btu ). Any trends in the ratio $\Delta \mathrm{U}_{\mathrm{U}} / \Delta \mathrm{U}_{\mathrm{T}}$, therefore, depend mainly on variations in the total energy added $\left(\Delta \mathrm{U}_{\mathrm{T}}\right)$ to the system due to variations in energy losses to the tank wall and liquid.

Energy added to tank wall. - The ratios of energy gained by the tank wall to the total energy added to the system $\Delta \mathrm{U}_{\mathrm{w}} / \Delta \mathrm{U}_{\mathrm{T}}$ for different expulsion times are compared in figure 14 for the three inlet gas temperatures. In general, between 37.5 and 56.2 per-


Figure 14. - Comparison of energy gained by wall to total energy added ratio as function of expulsion time for three inlet gas temperatures. Tank pressure, $34.47 \times 10^{4}$ newtons per square meter ( 50 psia); wall thickness, 0.762 centimeter ( 0.30 in .). Curves are faired through experimental data.
cent of the total energy added to the system was gained by the tank wall over the range of conditions. The results of reference 4 showed that between 30 and 58.8 percent of the total energy added to the tank was gained by the tank wall for similar test conditions when using hydrogen as the pressurant. A comparison of results indicate that there is generally a 5 to 7.5 percent increase in the percent of total energy that is lost to the tank wall when using helium as the pressurant. As can be seen in table II and in figure 14, both the absolute value of $\Delta \mathrm{U}_{\mathrm{w}}$ and the ratio $\Delta \mathrm{U}_{\mathrm{w}} / \Delta \mathrm{U}_{\mathrm{T}}$ increase with increasing inlet gas temperature. The increase in $\Delta \mathrm{U}_{\mathrm{w}}$ is due to the larger driving potential $\Delta \mathrm{T}$ for heat transfer between the ullage gas and the tank wall. The total energy added to the system $\left(\Delta \mathrm{U}_{\mathrm{T}}\right)$ does not increase in the same proportion as $\Delta \mathrm{U}_{\mathrm{W}}$ resulting in the increased ratio $\Delta \mathrm{U}_{\mathrm{w}} / \Delta \mathrm{U}_{\mathrm{T}}$.

The agreement between the analysis and experimental value of $\Delta \mathrm{U}_{\mathrm{W}} / \Delta \mathrm{U}_{\mathrm{T}}$ are very good for the 183 and $267 \mathrm{~K}\left(329^{\circ}\right.$ and $480.6^{\circ} \mathrm{R}$ ) inlet gas temperature (within $\pm 2.2$ percent, see table III). However, the agreement for the $322 \mathrm{~K}\left(580^{\circ} \mathrm{R}\right)$ inlet gas is not as good (within $\pm 13.2$ percent).

Energy gained by liquid. - Figure 15 is a comparison of the ratio of energy gained by the liquid to the total energy added to the system $\Delta U_{L} / \Delta U_{T}$ for different expulsion times for the three inlet gas temperatures. In all cases, between 12.8 and 33.7 percent of the total energy added to the system appears as an increase in liquid energy (liquid heating). The results of reference 4 indicated that between 15 and 25 percent of the total energy added to the system appears as an increase in liquid energy for similar test conditions using hydrogen as the pressurant. There is a relatively large amount of scatter


Figure 15. - Comparison of energy gained by liquid to total energy added ratio as function of expulsion time for three inlet gas temperatures. Tank pressure, $34.47 \times 10^{4}$ newtons per square meter ( 50 psia); wall thickness, 0.762 centimeter ( 0.30 in .). Curves are faired through experimental data.
in the data shown in figure 15. This scatter could be the result of the error in experimentally determining $\Delta U_{L^{\prime}}$. The probable error associated with each experimental determination of $\Delta \mathrm{U}_{\mathrm{L}}$ is between 10 and 58 percent (i.e., $14.3 \times 10^{4}$ out of $142.0 \times 10^{4} \mathrm{~J}$ and $44.5 \times 10^{4}$ out of $77.0 \times 10^{4} \mathrm{~J}$, see table II).

The analytical predictions of the ratio $\Delta U_{L} / \Delta U_{T}$ are also presented in figure 15 (see table III for average deviations). The large discrepancy between the analysis and experimental results could also be the result of the error in determining $\Delta U_{L}$.

Temperature distributions. - The results that were discussed previously point out that between 72.4 and 88.6 percent of the total energy that was added to the system is either absorbed by the tank wall or remains in the ullage. The correlation between the analysis and experimental data, therefore, depends largely on the ability of the analysis to predict final wall and ullage gas temperature profiles. These profiles are, in turn, used to determine the increase in wall and ullage energy and the final ullage mass. The ability to predict these temperatures explains the fair agreement between experimental data and analysis reported in references 1 and 3 to 5.

Figure 16 presents comparisons of experimental and analytical ullage gas and wall temperature profiles for the three inlet gas temperatures. Figure 16(a) presents a comparison of experimental and analytical wall and ullage gas temperature profiles at the end of a 396 -second expulsion for an inlet gas temperature of $184 \mathrm{~K}\left(331^{\circ} \mathrm{R}\right)$. For this run the analysis accurately predicted the $\Delta \mathrm{U}_{\mathrm{W}} / \Delta \mathrm{U}_{\mathrm{T}}$ ratio but underpredicted the $\mathrm{M}_{\mathrm{I}} / \mathrm{M}_{\mathrm{G}}$ ratio by 16.38 percent. The experimental gas temperatures shown in figure 16 (a) were obtained from the vertical rake. The horizontal rakes indicate these are average radial temperatures at their respective vertical position. In the absence of any mass transfer, the pressurant mass required for an expulsion could be determined as the difference between the final mass in the ullage and the initial mass prior to expulsion.

One of the inputs to the analytical program is the initial experimental temperature profile prior to expulsion. The analytical program uses this profile together with the temperature-density relation for helium to obtain the initial ullage mass. The initial ullage mass as determined by the analysis is, in all cases, larger than determined experimentally because the experimental values contain hydrogen in the ullage. However, the difference between the analytically and experimentally determined initial ullage mass represents only a small fraction (less than 3 percent) of the final ullage mass. Therefore, the deviation between the analytical and experimental pressurant requirements would largely be the result of the predicted final ullage temperature profile. As seen in figure 16(a) the analytical gas temperatures are slightly lower in the lower 20 percent of the tank but begin to get higher toward the top of the tank. The overprediction of temperatures in the majority of the ullage results in a 16.38 -percent underprediction in the final ullage mass.

The analytically predicted wall temperatures (fig. 16(a)) are very close to those ob-

(b) Expulsion time, 137 seconds; inlet gas temperature, $271 \mathrm{~K}\left(488^{\circ} \mathrm{R}\right)$; run 85.

Figure 16. - Comparison of analytical and experimental gas and wall temperatures. Tank pressure, $34.47 \times 10^{4}$ newtons per square meter ( 50 psia ); wall thickness, 0.762 centimeter ( 0.30 in .).

(c) Expuision time, 134 seconds; inlet gas temperature, $335 \mathrm{~K}\left(603^{\circ} \mathrm{R}\right)$; run 97.

Figure 16. - Concluded.
served experimentally in the lower 47 percent of the tank ullage but begin to get higher toward the top of the tank. Even though the calculated wall temperatures are higher in the upper portion of the tank wall, the analysis predicts the energy gained by the tank wall $\left(\Delta \mathrm{U}_{\mathrm{w}}\right)$ accurately. The effect of overpredicting the temperatures in the upper portion of the tank wall is offset by an underprediction of the temperature of the tank lid (point zero in fig. 16(a)), which is approximately 22 percent of the total tank mass.

Figure $16(\mathrm{~b})$ is a comparison of experimental and calculated wall and ullage gas temperature profiles at the end of a 137-second expulsion for an inlet gas temperature of $271 \mathrm{~K}\left(488^{\circ} \mathrm{R}\right)$. Here the calculated gas temperatures agree very closely with the experimental temperatures in the lower 50 percent of the ullage but begin to get lower toward the top of the tank. However, the predicted pressurant requirement is only 1.57 percent higher (lower $M_{I} / M_{G}$ ) than that observed experimentally.

The calculated wall temperatures agree fairly well with the experimental temperatures except near the top of the tank where they begin to get higher. However, this overprediction is again offset by an underprediction of the lid temperature resulting in an underprediction for wall energy gained of 2.20 percent.

A comparison of experimental and calculated wall and ullage gas temperatures at the end of a 134 -second expulsion for an inlet gas temperature of $335 \mathrm{~K}\left(603^{\circ} \mathrm{R}\right)$ is shown in figure 16 (c). Here again there is good agreement between the calculated and experimental gas temperatures throughout the tank ullage resulting in a good prediction for the pressurant requirements (within 2.40 percent). However, in this case the lower calculated lid temperature does not offset the higher calculated wall temperatures in the upper portion of the tank. And the analysis overpredicts the energy gained by the tank wall by 11.31 percent.

The results of these comparisons indicate that the analytical program is able to predict with reasonable accuracy the pressurant gas requirement and the major energy losses for the test conditions imposed herein. The results also indicate that the analytical assumption of a one component ullage (no mass transfer) is adequate and that what hydrogen gas there is in the ullage has little influence on helium pressurant requirements or energy distributions.

## Effect of Tank Wall Thickness on Pressurant Requirements

The results of reference 3 indicated that between 77 and 93 percent of the total energy lost by the pressurant gas was lost to the tank wall. It was expected, therefore, that tank wall thickness would play an important role in determining the pressurant requirements during the expulsion period. Additional tests were performed using a 1.52-meter- ( 5 -ft-) diameter spherical tank similar to the one already discussed except the tank wall was chem-milled to an average thickness of 0.409 centimeter ( 0.161 in .). Although the chem-milling reduced the wall thickness by 46.3 percent, the total weight of the tank was reduced by only 29.5 percent because of the structural requirement of keeping the same thickness for the tank neck, lid, and girth support.

Figure 17 presents a comparison of the $M_{I} / M_{G}$ ratio for different expulsion times for the two tank wall thicknesses ( 0.762 and $0.409 \mathrm{~cm} ; 0.30$ and 0.161 in .). Each tank used the hemisphere injector. The average inlet gas temperature for the 0.409centimeter ( 0.161 -in.) tank was $290 \mathrm{~K}\left(\sim 520^{\circ} \mathrm{R}\right)$. The average inlet gas temperature for the 0.762 -centimeter ( 0.30 -in.) tank was $267 \mathrm{~K}\left(\sim 480^{\circ} \mathrm{R}\right.$ ). The 0.409 -centimeter( 0.161 -in. -) thick tank has an average of 6.3 percent less pressurant requirement, higher $M_{I} / M_{G^{j}}$ ratio, than the 0.762 -centimeter- ( 0.30 -in. - ) thick tank for all expulsion times. Similar results were obtained in reference 4 using hydrogen. In all cases, the


Figure 17. - Comparison of ideal pressurant requirement to actual pressurant requirement ratio as function of expulsion time for two wall thicknesses. Tank pressure, $34.47 \times 10^{4}$ newtons per square meter ( 50 psia); iniet gas temperature, $277 \pm 10 \mathrm{~K}$ ( $499 \pm 18^{\circ}$ R).
analysis overpredicts the pressurant requirements (underpredicts the $\mathrm{M}_{\mathrm{I}} / \mathrm{M}_{\mathrm{G}}$ ratio). See table III for deviations between analytical and experimental results.

The 6.3-percent decrease in pressurant requirements was less than expected. However, the test with the 0.409 -centimeter- ( 0.161 -in. -) thick tank had an average 23 K $\left(41^{\circ} \mathrm{R}\right)$ higher inlet gas temperature. Based on the results already obtained on the effect of inlet gas temperature on pressurant requirements (fig. 9), this $23 \mathrm{~K}\left(41^{\circ} \mathrm{R}\right)$ higher inlet temperature would normally decrease the pressurant requirement by approximately 2.9 percent. Thus for equal inlet gas temperatures, a nominal decrease in pressurant requirements of approximately 9.0 percent would be expected for the 0.409 -centimeter-(0.161-in. -) thick tank.

Figure 18 is a comparison of $\Delta \mathrm{U}_{\mathrm{w}} / \Delta \mathrm{U}_{\mathrm{T}}$ for different expulsion times for the two tank wall thicknesses. In this figure the $\Delta U_{W} / \Delta U_{T}$ ratio is broken down into the fractional portion gained by the tank wall and the total fraction gained by the tank wall plus tank lid. (The tank lid represented 30 percent of the total tank weight for the thinner wall tank compared to 22 percent for the thicker tank.) The fraction of the total energy added that is gained by the tank wall is approximately the same for both tank thicknesses (lower curve of fig. 18) and ranges between 0.37 and 0.40 . The values of $\Delta \mathrm{U}_{\mathrm{w}} / \Delta \mathrm{U}_{\mathrm{T}}$ that include the lid indicate a 4 -percent decrease in $\Delta \mathrm{U}_{\mathrm{W}} / \Delta \mathrm{U}_{\mathrm{T}}$ for the thinner wall tank. It is apparent then that the reduction in the total $\Delta \mathrm{U}_{\mathrm{w}} / \Delta \mathrm{U}_{\mathrm{T}}$ is almost entirely due to a reduction in the fraction of the total energy absorbed by the lid for the thinner wall tank. The reason for the thinner tank's reduced lid energy gain was that its initial lid temperature, prior to expulsion, was an average of $41 \mathrm{~K}\left(73.8^{\circ} \mathrm{R}\right)$ higher than the thicker tank's. This higher initial lid temperature reduced the temperature difference between the lid and the ullage gas resulting in less energy gained. The absolute value of the total wall energy gained (table II) by the 0.409 -centimeter- ( 0.161 -in. - ) thick tank was approximately


Figure 18. - Comparison of energy gained by wall to total energy added ratio for two tank wall thicknesses. Tank pressure, $34.47 \times 10^{4}$ newtons per square meter ( 50 psia); inlet gas temperature, $277 \pm 10 \mathrm{~K}$ $\left(499^{\circ} \pm 18^{\circ}\right.$ R).
6.9 percent less than for the 0.762 -centimeter- ( $0.30-\mathrm{in} .-$ ) thick tank.

The absolute values of increase in ullage energy, liquid heating, and mass transfer are approximately the same (within measurement uncertainty) for the two tank wall thicknesses (see tables I and II). Thus, the 6.3 -percent reduction in pressurant requirements when using the 0.409 -centimeter- ( 0.161 -in. - ) thick wall tank is almost entirely due to the 6.9 -percent reduction in tank wall heating ( $\Delta \mathrm{U}_{\mathrm{w}}$ ).

## Pressurant Requirements for Initial Pressurization

The amount of pressurant gas needed to initially pressurize a propellant tank may be important for certain missions, particularly for multiburn missions where the tank is vented after each burn or where the coast period between firings is long enough to enable the ullage gas to collapse.

As stated in the INTRODUCTION, the purpose of this investigation was to determine the capability of the analysis to predict the pressurant requirements during the initial pressurization period as well as the expulsion period. For this purpose, data were collected during the initial pressurization period for various pressurizing rates, inlet gas temperatures, and ullage volumes.

Figure 19 is a comparison of the $M_{I} / M_{G}$ ratio as a function of inlet gas temperature for two ramp rates at an initial ullage volume of approximately 4 percent. The data were taken using the hemisphere injector. At constant inlet gas temperature, the pressurant


Figure 19. - Comparison of ideal pressurant requirement to actual pressurant requirement ratio for ramp period as function of inlet gas temperature for two ramp rates. Ullage volume, 4 percent.
requirements decreased for increased ramp rates. The $M_{I} / M_{G}$ ratio decreases for increasing inlet gas temperature. The reduced pressurant requirements (for the ramp period) for fast ramp rates using low inlet gas temperature is in agreement with the trends during the expulsion period.

The modification of the analysis of reference 1 for the ramp period is discussed in appendix $C$. As can be seen in figure 19, the analysis is not capable of predicting the pressurant requirements accurately during the initial pressurization of the 4 -percent ullage. However, the prediction of the total pressurant requirement (initial pressurization and expulsion) is still adequate because the amount of gas required to pressurize the 4 -percent ullage initially was only 4.0 percent (maximum) of the pressurant requirement during expulsion. The absolute pressurant requirements (both experimental and analytical) for the data presented in figures 19 and 20 are given in table IV.

The comparison of $\mathrm{M}_{\mathrm{I}} / \mathrm{M}_{\mathrm{G}}$ as a function of initial ullage volume for various ramp rates at an inlet gas temperature of approximately $180 \mathrm{~K}\left(324^{\circ} \mathrm{R}\right)$ is shown in figure 20. This figure indicates increased $M_{I} / M_{G}$ for increased ullage volume for any given ramp rate. There is an increased $M_{I} / M_{G}$ for increased ramp rates for a given ullage volume. As can be seen in figure 20, the accuracy of the analytical prediction is not consistent. The actual deviation between analytical and experimental values range between 0 and 20.5 percent. The transient process that occurs during the initial pressurization of the tank is too complex to be described by the present analytical model. The analytical program can, however, at least be used to predict the approximate magnitude of pressurant requirements during the ramp period.


Figure 20. - Comparison of ideal pressurant requirement to actual pressurant requirement ratio for ramp period as function of initial ullage volume for three ramp rates. inlet gas temperature, $180 \mathrm{~K}\left(324^{\circ} \mathrm{R}\right)$. Curves are faired through experimental data.

TABLE IV. - COMPARISON OF EXPERIMENTAL AND ANALYTICAL VALUES OF PRESSURANT GAS REQUIREMENTS FOR RAMP PERIOD

| Run | Initial <br> ullage volume, percent | Ramp rate |  | Mass added |  |  |  | Inlet gas temperature |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | ps | ( $\mathrm{N} / \mathrm{m}^{2}$ )/sec | Experimental |  | Analytical |  |  |  |
|  |  |  |  |  |  | K | ${ }^{\circ} \mathrm{R}$ |  |  |
|  |  |  |  | lb | kg |  |  | lb | kg |
| 84 | 4 | 1.65 | $11.37 \times 10^{3}$ | 0.159 | 0.072 | 0.148 | 0.067 | 265 | 478 |
| 86 |  | . 73 | 5.03 | . 177 | . 080 | . 202 | . 092 | 209 | 377 |
| 88 |  | 1.65 | 11.37 | . 160 | . 073 | . 247 | . 112 | 178 | 320 |
| 95 |  | 1.65 | 11.37 | . 110 | . 050 | . 175 | . 079 | 294 | 530 |
| 96 |  | . 73 | 5.03 | . 101 | . 046 | . 173 | . 078 | 302 | 545 |
| 91 | $\dagger$ | . 86 | 5.92 | . 197 | . 089 | . 247 | . 112 | 177 | 319 |
| 124 | 28 | . 86 | 5.92 | 1.344 | . 610 | 1.179 | . 535 | 181 | 325 |
| 125 | 28 | . 47 | 3.24 | 1.811 | . 821 | 1.701 | . 772 | 181 | 326 |
| 147 | 55 | . 86 | 5.92 | 2.245 | 1.018 | 2.312 | 1.049 | 174 | 313 |
| 148 | 55 | . 73 | 5.03 | 2.360 | 1.070 | 2.389 | 1.084 | 180 | 323 |
| 149 | 55 | . 47 | 3.24 | 2.257 | 1.024 | 2.674 | 1.213 | 182 | 328 |
| 394 | 75 | . 86 | 5.92 | 2.750 | 1.248 | 3.054 | 1.385 | 169 | 304 |
| 395 | 75 | . 73 | 5.03 | 2.618 | 1.188 | 2.350 | 1.066 | 166 | 299 |
| 396 | 75 | . 47 | 3.24 | 2.522 | 1.144 | 2.535 | 1. 150 | 168 | 302 |

## SUMMARY OF RESULTS

Tank pressurization and propellant expulsion tests were conducted in 1.52-meter( $5-\mathrm{ft}-$ ) diameter spherical tanks to (1) determine what factors have the greatest influence on pressurant gas requirements when using helium as the pressurant, and (2) verify the capability of an analysis to predict the helium pressurant requirements during the initial pressurization period as well as the expulsion period. Tests were conducted using three inlet gas temperatures over a range of liquid outflow rates. The results of this investigation are now given.

## Experimental Results

The experimental results indicate an average decrease in helium pressurant gas requirements $\left(\mathrm{M}_{\mathrm{G}}\right)$ of 1.3 percent for a $10 \mathrm{~K}\left(18^{\circ} \mathrm{R}\right)$ increase in inlet gas temperature.

Increased inlet gas temperature decreases the residual mass and slightly decreases the energy remaining in the ullage volume after a given expulsion. Of the total energy added to the tank system, between 37.5 and 56.2 percent was lost to the tank wall (of this between 13 and 20 percent was lost to the tank lid) and between 12.8 and 33.7 percent was lost to the liquid.

The mass transfer values obtained in this investigation are inconclusive because of the uncertainty in measuring the ullage gas concentration gradients. Additional work is necessary to develop better techniques for measuring gas concentration gradients and mass transfer.

Decreasing the tank mass by 29.5 percent (by decreasing the tank wall thickness from 0.762 to $0.409 \mathrm{~cm} ; 0.30$ to 0.161 in .) decreased the pressurant requirement by an average of 6.3 percent ( 9.0 percent when corrected for variations in in ot gas temperature).

The effects of inlet gas temperature and ramp rate on the pressurant gas required during the initial pressurization of the tank were as follows:

1. Increased inlet gas temperature decreased the pressurant requirement $M_{G}$ and mass ratio $M_{I} / M_{G}$ for constant ramp rates.
2. Increasing ramp rate decreased the pressurant requirement and increased the mass ratio for constant inlet gas temperatures.
3. Increased pressurant requirement and increased mass ratio were obtained for larger initial ullage volumes for a constant ramp rate.

The trends shown herein for various inlet gas temperatures, liquid outflow rates, and tank wall thickness are consistent with the results obtained using hydrogen as the pressurant (ref. 4). A comparison of the results obtained for the two pressurants indi-
cate that the mass ratios $M_{I} / M_{G}$ for similar test conditions are approximately the same. It therefore would require approximately twice as much mass to expel liquid hydrogen using helium as the pressurant instead of hydrogen.

## Comparison of Analytical and Experimental Results

The comparison between the analytical and experimental results indicate that, for the range of test conditions used, the analytical program and assumptions are adequate to allow prediction of the pressurant gas requirements during the initial pressurization as well as the expulsion period when using helium as the pressurant. The general results as predicted by the analysis are as follows:

1. The pressurant requirements were predicted to within an average of 4.41 percent for the 322 and $267 \mathrm{~K}\left(580^{\circ}\right.$ and $480.6^{\circ} \mathrm{R}$ ) inlet gas temperatures and within 11.46 percent for the $183 \mathrm{~K}\left(329^{\circ} \mathrm{R}\right)$ inlet gas temperature.
2. Tank wall heating was predicted to within an average of 4.78 percent for all runs.
3. Liquid heating was predicted to within an average of 33.28 percent for all runs.

Lewis Research Center,
National Aeronautics and Space Administration, Cleveland, Ohio, September 22, 1969, 180-31.

## APPENDIX A

## INCORPORATION OF A VARIABLE GEOMETRY TO

## THE ANALYSIS OF REFERENCE 1

The basic analysis used in this report to predict pressurant gas requirements was developed by W. H. Roudebush in reference 1 for a cylindrical tank.

The major assumptions in the analysis of reference 1 are as follows:
(1) The ullage gas is nonviscous.
(2) The ullage gas velocity is parallel to the tank axis and does not vary radially or circumferentially.
(3) The tank pressure does not vary spatially.
(4) The ullage gas temperature does not vary radially or circumferentially.
(5) The tank wall temperature does not vary radially or circumferentially.
(6) There is no axial heat conduction in either the gas or the wall.
(7) There is no mass transfer (condensation or evaporation).
(8) There is no heat transfer from the pressurant gas to the liquid.

Experiments performed at Lewis (ref. 3) confirmed most of these assumptions when using a diffuser type injector such as the one used herein. The experimental results indicated, however, that there is significant heat transfer from the gas to the liquid with resulting mass transfer.

For the purposes of this report, the analysis of reference 1 was modified for application to arbitrary symmetric tank shapes, and an attempt was made to incorporate the heat transfer from the gas to the liquid. The treatment of internal hardware (e.g., tank baffles, instrumentation) was also modified to correspond to the treatment of heat transfer to the tank wall.

The primary equations which deal with the pressurizing gas upon entering the tank are:
(1) The energy equation
(2) The continuity equation
(3) The equation of heat transfer for a point in the tank wall

## ENERGY EQUATION

The form of the energy equation used in the analysis in reference 1 for cylindrical tanks is

$$
\frac{\partial T}{\partial t}=\frac{2 h_{c} \text { ZRT }}{r \bar{M} P C_{p}}\left(T_{w}-T\right)-\bar{V} \frac{\partial T}{\partial x}+\frac{R T Z_{1}}{\bar{M} P C_{p}} \frac{\partial P}{\partial t}+\frac{R T Z \dot{q}_{H} C_{H}}{\pi r^{2} \bar{M} P C_{p}}
$$

Modifying this equation to account for both arbitrary symmetric tank shapes and internal tank heat sinks gives

$$
\begin{equation*}
\frac{\partial T}{\partial t}=\frac{2 h_{c} \mathrm{ZRT}}{r \bar{M} P C_{p}}\left(T_{w}-T\right)\left[1+\left(\frac{d r}{d x}\right)^{2}\right]^{1 / 2}-\frac{\overline{\mathrm{V}} \partial T}{\partial x}+\frac{R^{2} Z_{1}}{\bar{M} P C_{p}} \frac{\partial P}{\partial t}+\frac{\dot{Q}_{H}}{C_{p} M_{H}} \tag{A1}
\end{equation*}
$$

The first term on the right includes the effect of wall curvature. The last term, the energy lost to the internal hardware, is treated as the summation of hardware components: (1) laminated thermoplastic, (2) stainless steel, and (3) copper. For the tanks in this investigation,

$$
\begin{equation*}
\frac{\dot{\mathrm{Q}}_{\mathrm{H}}}{\mathrm{M}_{\mathrm{H}}}=\sum_{\text {Hardware }} \frac{\mathrm{A}_{\mathrm{H}} \mathrm{~h}_{\mathrm{c}}\left(\mathrm{~T}_{\mathrm{H}}-\mathrm{T}_{\mathrm{G}}\right)}{\rho_{\mathrm{H}} \mathrm{~V}_{\mathrm{H}}} \tag{A2}
\end{equation*}
$$

Gluck and Kline, in reference 8, employed the free convection correlation to the pressurant gas (hydrogen, helium) for the pressurized transfer of liquid hydrogen:

$$
\begin{equation*}
\frac{\mathrm{h}_{\mathrm{c}}^{\mathrm{L}}}{\mathrm{k}}=\mathrm{Nu}=0.13(\mathrm{GrPr})^{1 / 3} \tag{A3}
\end{equation*}
$$

This correlation is used herein even though it was developed for cylindrical tanks. Pressurant gas transport properties were evaluated at the mean of the gas and wall temperatures.

## UÚNTINUITY EQUATION (AREA = $\mathrm{f}(\mathrm{x})$ )

The basic form of the continuity equation for a cylindrical tank is presented in reference 1 (eq. (24)) as

$$
\frac{\partial \overline{\mathrm{V}}}{\partial \mathrm{x}}=\frac{\mathrm{Z}_{1}}{\mathrm{ZT}}\left(\frac{\partial \mathrm{~T}}{\partial \mathrm{t}}+\frac{\overline{\mathrm{V}} \partial \mathrm{~T}}{\partial \mathrm{x}}\right)-\frac{\mathrm{Z}_{2}}{\mathrm{ZP}} \frac{\partial \mathrm{P}}{\partial \mathrm{t}}
$$

The modified form of the continuity equation due to variations in tank radius with distance along the vertical axis becomes

$$
\begin{equation*}
\frac{\partial \overline{\mathrm{V}}}{\partial \mathrm{x}}=\frac{\mathrm{Z}_{1}}{\mathrm{ZT}}\left(\frac{\partial T}{\partial \mathrm{t}}+\frac{\overline{\mathrm{V}} \partial T}{\partial \mathrm{x}}\right)-\frac{\mathrm{Z}_{2}}{\mathrm{Z}_{P}} \frac{\partial P}{\partial \mathrm{t}}-\frac{2 \overline{\mathrm{~V}}}{\mathrm{r}} \frac{\partial \mathrm{r}}{\partial \mathrm{x}} \tag{A4}
\end{equation*}
$$

where $Z_{1}$ and $Z_{2}$ are defined in reference 1 as

$$
\begin{aligned}
& Z_{1}=Z+T\left(\frac{\partial Z}{\partial T}\right)_{P} \\
& Z_{2}=Z-P\left(\frac{\partial Z}{\partial P}\right)_{T}
\end{aligned}
$$

The last term in equation (A4) evolves from the derivation as follows. For the onedimensional expression for continuity,

$$
\frac{\partial}{\partial \mathrm{x}}(\rho \overline{\mathrm{~V}} \mathrm{~A})+\frac{\partial}{\partial \mathrm{t}}(\rho \mathrm{~A})=0
$$

The substitution $\mathrm{A}=\pi \mathrm{r}^{2}$ is made where r is the position radius at location x along the vertical axis:

$$
\frac{\partial}{\partial \mathrm{x}}\left(\rho \overline{\mathrm{~V}} \mathrm{r}^{2}\right)+\frac{\partial}{\partial \mathrm{t}}\left(\rho \mathrm{r}^{2}\right)=0
$$

The expression for density from the equation of state $\rho=\overline{\mathrm{M}} \mathrm{P} / \mathrm{ZRT}$ is substituted:

$$
P \frac{\partial}{\partial \mathrm{x}}\left(\frac{\overline{\mathrm{~V}} \mathrm{r}^{2}}{\mathrm{ZT}}\right)+\mathrm{r}^{2} \frac{\partial}{\partial \mathrm{t}}\left(\frac{\mathrm{P}}{\mathrm{ZT}}\right)=0
$$

The following velocity equation is obtained after performing the partial differentiation and after rearranging terms:

$$
\frac{\partial V}{\partial x}=\left[\frac{1}{T}+\frac{1}{Z}\left(\frac{\partial Z}{\partial T}\right)_{P}\right]\left(\frac{\partial T}{\partial t}+\frac{\bar{V} \partial T}{\partial x}\right)+\left[\frac{1}{Z}\left(\frac{\partial Z}{\partial P}\right)_{T}-\frac{1}{P}\right] \frac{\partial P}{\partial t}-\frac{2 \bar{V}}{r} \frac{\partial r}{\partial x}
$$

When the expressions involving $Z_{1}$ and $Z_{2}$ are substituted in this equation, equation (A4) is obtained.

## TANK WALL HEAT TRANSFER

Reference 1 (eq. (18)) gives the heat-transfer equation which represents the change in wall temperature as a result of the convective process for a cylindrical tank:

$$
\begin{equation*}
\frac{\partial T_{w}}{\partial t}=\frac{h_{c}}{l_{w} \rho_{w} C_{w}}\left(T-T_{w}\right)+\frac{\dot{q}_{w}}{l_{w} \rho_{w} C_{w}} \tag{A5}
\end{equation*}
$$

where $\dot{q}_{W}$ is the rate of heat addition per unit area to the tank wall from outside the tank.
For a small element of volume in the $x$-direction, equation (A5) can be written as

$$
\begin{equation*}
\rho_{w} C_{w} V \frac{\partial t_{W}}{\partial t}=h_{c} A\left(T-T_{w}\right)+\dot{Q}_{w} \tag{A6}
\end{equation*}
$$

For a wall of arbitrary shape, the following is evident from the sketch:


$$
\frac{\mathrm{A}}{\mathrm{~V}}=\frac{2 \pi \mathrm{r} \Delta \mathrm{~S}}{2 \pi \mathrm{r} \Delta \mathrm{r} \Delta \mathrm{x}}=\frac{\Delta \mathrm{S}}{\Delta \mathrm{r} \Delta \mathrm{x}}=\frac{1}{l_{\mathrm{W}}}
$$

Therefore, equation (A5) holds also for this case.
To account for the large mass concentration at the top of the tank, an equivalent $l_{w}$ was used. This $l_{\mathrm{w}}$ was obtained by dividing the mass of the tank lid and flange connections by the surface area at the first net point.

## APPENDIX B

## EQUATIONS OF HEAT AND MASS TRANSFER AT THE GAS-LIQUID INTERFACE

The energy and continuity equations (A1) and (A4) should be modified to incorporate both heat transfer from the ullage gas to the liquid surface and mass transfer into the analysis.

The energy equation should incorporate three additional terms:
(1) The heat-transfer rate from the ullage gas to the liquid interface $\left(\dot{q}_{U \rightarrow S}\right)$
(2) The heat-transfer rate from the interface to the liquid ( $\dot{q}_{S-L}$ )
(3) The energy associated with mass transfer ( $M_{t} \lambda$ ) (evaporation is positive)

These additional terms can be related by performing an energy balance at the gas-liquid interface as done by W . A. Olsen in reference 9 . The resulting energy relation is based on the assumption requiring the interface for a pure system to be at the saturation temperature corresponding to the tank pressure. As shown by the sketch,

the energy balance at the interface is given by

$$
\dot{\mathrm{q}}_{\mathrm{U} \rightarrow \mathrm{~S}}=\dot{\mathrm{q}}_{\mathrm{S} \rightarrow \mathrm{~L}}+\frac{\dot{\mathrm{M}}_{\mathrm{t}} \lambda}{\mathrm{~A}}
$$

In the case where helium is the pressurant gas for liquid hydrogen, it is assumed that there is sufficient evaporation to maintain a hydrogen blanket over the entire liquid surface. The interface can then be assumed to be at the saturation temperature for a pure hydrogen system.

Experimental data indıcat, the energy associated with the mass transfer was relatively small so that the following assumption is made:

$$
\frac{\dot{\mathrm{M}}_{\mathrm{t}} \lambda}{\mathrm{~A}} \cong 0
$$

With the further qualification that the environmental transfer to the liquid is also small, it follows that

$$
\begin{equation*}
\dot{\mathrm{q}}_{\mathrm{U} \rightarrow \mathrm{~S}} \cong \dot{\mathrm{q}}_{\mathrm{S} \rightarrow \mathrm{~L}} \cong \frac{\mathrm{~d}}{\mathrm{dt}}\left(\mathrm{U}_{\mathrm{L}}\right) \tag{B2}
\end{equation*}
$$

The term $\frac{d}{d t}\left(U_{L}\right)$ can be determined from experimental data. The analysis requires that $\dot{\mathrm{q}}_{\mathrm{U} \rightarrow \mathrm{S}}$ must be related to the ullage gas variables. This is done by the following equation which involves defining a heat-transfer coefficient ( $h_{c, L}$ ) and a temperature ( $\mathrm{T}_{\delta}$ ) somewhere in the vapor:

$$
\begin{equation*}
\dot{\mathrm{q}}_{\mathrm{U} \rightarrow \mathrm{~S}} \equiv \mathrm{~h}_{\mathrm{c}, \mathrm{~L}}\left(\mathrm{~T}_{\delta}-\mathrm{T}_{\mathrm{sat}}\right) \tag{B3}
\end{equation*}
$$

The flow process is free convection flow of pressurant gas essentially down the tank wall and then radially inward across the liquid surface.

With regard to $h_{c, L}$, reference 10 develops an equation from boundary layer theory for forced flow across a horizontal, semi-infinite, constant temperature flat plate given by

$$
\begin{equation*}
\mathrm{Nu}=\frac{\mathrm{h}_{\mathbf{c}, \mathrm{L}^{\mathrm{L}}}}{\mathrm{k}}=0.664\left(\frac{\mu \mathrm{C}_{\mathbf{p}}}{\mathrm{k}}\right)^{1 / 3}\left(\frac{\mathrm{~L} \overline{\mathrm{~V}}_{\mathrm{L}} \rho}{\mu}\right)^{1 / 2} \tag{B4}
\end{equation*}
$$

The velocity $\overline{\mathrm{V}}_{\mathrm{L}}$ of the gas across the liquid surface in terms of the gas velocity $\overline{\mathrm{V}}_{\mathrm{G}}$ down a vertical wall is given in reference 11 as

$$
\begin{equation*}
\overline{\mathrm{V}}_{\mathrm{L}}=0.0975 \overline{\mathrm{~V}}_{\mathrm{G}} \tag{B5}
\end{equation*}
$$

where $\overline{\mathrm{V}}_{\mathrm{G}}$, obtained by solving the integrated energy and momentum equations at the wall boundary, is given by

$$
\begin{equation*}
\overline{\mathrm{V}}_{\mathrm{G}}=1.185 \frac{\mu}{\rho \mathrm{Z}} \frac{\mathrm{Gr}^{1 / 2}}{\left(1+0.494 \operatorname{Pr}^{2 / 3}\right)^{1 / 2}} \tag{B6}
\end{equation*}
$$

Combining equations (B5) and (B6) for $\overline{\mathrm{V}}_{\mathrm{L}}$ and substituting into (B4) gives

$$
\begin{equation*}
\mathrm{Nu}=\frac{\mathrm{h}_{\mathrm{c}, \mathrm{~L}^{\mathrm{L}}}}{\mathrm{k}}=\frac{0.226 \operatorname{Pr}^{1 / 3} \mathrm{Gr}^{1 / 4}}{\left(1+0.494 \operatorname{Pr}^{2 / 3}\right)^{1 / 4}}=0.21 \mathrm{Pr}^{1 / 3} \mathrm{Gr}^{1 / 4} \tag{B7}
\end{equation*}
$$

for a value of $\operatorname{Pr}$ between 0.74 and 0.90 . Equation (B7) is similar in form to the empir-
ical relation for free convection flow along vertical planes and cylinders given in reference 12 as

$$
\begin{equation*}
\mathrm{Nu}=\frac{\mathrm{h}_{\mathbf{c}, \mathrm{L}^{\mathrm{L}}}}{\mathrm{k}}=\mathrm{n}(\mathrm{GrPr})^{\mathrm{m}} \tag{B8}
\end{equation*}
$$

Equation (B8) is used herein with exponent $m=1 / 3$ and a multiplier $n=0.14$ because it is simpler and fits the data somewhat better than equation (B7).

At this point, some choice of $\mathrm{T}_{\delta}$ that is consistent with the definition of $\mathrm{h}_{\mathrm{c}, \mathrm{L}}$ and fits the data for $\dot{q}_{U \rightarrow S}\left(i . e ., \frac{d}{d t}\left(U_{L}\right)\right)$ must be made. In reference 13 , liquid hydrogen was pressurized with a low mixing diffuser and no liquid outflow. The adiabatic compression temperature given by $\mathrm{T}_{\mathrm{ad}}=\mathrm{T}_{\mathrm{o}}\left(\mathbf{P} / \mathrm{P}_{\mathrm{o}}\right)^{(\gamma-1) / \gamma}$ was used as the choice for $\mathrm{T}_{\delta}$. This relation gave good agreement between analytical and experimental mass flux results for hydrogen pressurant. For the conditions described in reference 13, appreciable condensation occurred. Reference 13 indicates that as $\mathrm{T}_{\delta}$ increased, there was a tendency toward evaporation, away from the condensation results that occurred when the adiabatic temperature was used. With greater ullage gas mixing (due to diffuser characteristics as well as the liquid outflow process), $\mathrm{T}_{\delta}$ would be expected to be greater than $\mathrm{T}_{\text {ad }}$. For the work described in this report, $\mathrm{T}_{\delta}$ was obtained from the following relation evaluated with experimental data:

$$
\dot{\mathrm{q}}_{\mathrm{U} \rightarrow \mathrm{~S}}=\mathrm{h}_{\mathrm{c}, \mathrm{~L}}\left(\mathrm{~T}_{\delta}-\mathrm{T}_{\mathrm{Sat}}\right)=\frac{\mathrm{d}}{\mathrm{dt}} \mathrm{U}_{\mathrm{L}}
$$

One experimental run, where the expulsion time was 271 seconds (run 95), was used for the determination of $\mathrm{T}_{\delta}$ where $\mathrm{h}_{\mathrm{c}, \mathrm{L}}$ is given by (B8). For this condition $\mathrm{T}_{\delta}$ was determined to be 1.3 times the adiabatic compression temperature or $46.3 \mathrm{~K}\left(83.3^{\circ} \mathrm{R}\right)$ for a tank pressure of $34.47 \times 10^{4}$ newtons per square meter ( 50 psia ). For convenience this value of $\mathrm{T}_{\delta}$ was used for all comparisons since the effect of the interfacial terms in the energy equation was small in the experimental situation.

Using equation (B8), as well as the values for $\mathrm{T}_{\delta}$ and $\mathrm{T}_{\text {sat }}$ discussed previously, gave the final form of the equation used to evaluate the heat transferred from the ullage gas to the liquid interface as follows:

$$
\begin{equation*}
\dot{\mathrm{q}}_{\mathrm{U} \rightarrow \mathrm{~S}}=\frac{\mathrm{k}}{\mathrm{~L}}(0.14)(\mathrm{GrPr})^{1 / 3}\left(46.3 \mathrm{~K}-\mathrm{T}_{\mathrm{Sat}}\right) \tag{B9}
\end{equation*}
$$

In order to incorporate liquid heating to the analysis the term

$$
\begin{equation*}
-\frac{\dot{\mathrm{q}}_{\mathrm{U} \rightarrow \mathrm{~S}} \mathrm{~A}_{\mathrm{L}}}{\mathrm{X}_{\mathrm{n}} \mathrm{~V} \rho} \text { or }-\frac{\mathrm{k}(0.14)(\mathrm{GrPr})^{1 / 3}\left(46.3 \mathrm{~K}-\mathrm{T}_{\mathrm{sat}}\right) \cdot \mathrm{A}_{\mathrm{L}}}{\mathrm{LX} \mathrm{X}_{\mathrm{n}} \mathrm{~V} \rho} \tag{B10}
\end{equation*}
$$

must be added to the right side of equation (A1).

## APPENDIX C

## RAMP ANALYSIS

The amount of pressurant gas needed to pressurize a propellant tank initially may be important for multiburn missions. When the coast period between firings is long, a collapse in ullage pressure may develop. Under these circumstances a significant amount of pressurant may be necessary to repressurize the propellant tank for the next firing.

An application of the work reported in reference 1 is the prediction of the pressurant requirements for the initial pressurization (ramp) and hold periods. A separate computer program, which determines the pressurant as well as energy requirements during the ramp and hold periods, is described herein. The same equations which describe the expulsion period, as outlined in appendix $A$, are also applicable for the ramp and hold periods. However, the ramp period is more difficult to model analytically than the expulsion period, particularly for small ullage volumes where the liquid surface is near the injector outlet. Even though the experimental results indicated relatively large amounts of mass transfer during this period, the incorporation of mass transfer into the ramp analysis was not attempted because of the added complexity and incomplete knowledge of the mass transfer phenomenon. The heat transfer from the ullage to the liquid surface is also neglected in the analytical model for the ramp period.

The analysis used for the ramp period computes the gas temperatures in the ullage at any time during the pressure rise from the gas energy equation. The corresponding gas velocities are computed from the equation of continuity. The iterative method to be described shows how convergence is achieved in the solution of the gas energy and continuity equations. The predicted mass of pressurant is based on an integration of the volume elements in the ullage at the end of the ramp and hold periods assuming a one component ullage ( 100 percent helium). Quantitatively, the entire mass of pressurant requirements for the ramp period was less than the expulsion period by a factor of 30 to 60 when the initial ullage was 4 percent of the tank volume.

## INPUT DATA REQUIREMENTS

For the solution to proceed, a set of boundary and initial conditions are required. These conditions, which are the same for the expulsion as well as the pressurization, are as follows:
(1) At time $t=0$, the values of gas temperature $T$ and wall temperature $T_{w}$ as functions of x , the position within the ullage
(2) On the boundary $x=0$, the value of inlet gas temperature $T$ as a function of time
(3) At the liquid surface, the value of gas temperature $T$, wall temperature $T_{w}$, and velocity $\overline{\mathrm{V}}$ as functions of time (Although movement of the interface has been noted during the ramp pressurization period, no significant effect on the programed output was noted with the value of $\overline{\mathrm{V}}=0$ at the interface.)
(4) Tank pressure $P$, outside heating rate $\dot{q}_{w}$, inside hardware heating rate $\dot{q}_{H}$ as a function of time (Like the other initial conditions, the pressure $P$ as a function of time or ramp pressure curve is defined by a discrete set of points which approximate a smooth curve. In regions of pronounced curvature, more points are needed for accurate definition than for linear portions.)
(5) Constant value of heat-transfer coefficient $h_{c}$, or a correlating equation from which $h_{c}$ may be evaluated at each net point from values of $T, T_{w}$, and $P$
(6) Tank radius as function of axial distance down from the top of the tank
(7) Tank wall material properties: density $\rho_{w}$ and specific heat $C_{w}\left(T_{w}\right)$
(8) Tank wall thickness (average membrane plus weld area thickness) as function of axial distance down from the top of the tank
(9) Pressurizing gas properties: Molecular weight $\bar{M}$, specific heat $C_{p}(T)$, and compressibility factor $\mathrm{Z}(\mathrm{P}, \mathrm{T})$
(10) Initial ullage height, total time of run, the number of net points in the initial ullage space
(11) The initial time step $\Delta t$ used in following the pressure rise as well as establishing the points of computation
(12) If the hold period is to be included in the analysis, then the time for the end of the ramp must be specified

## APPLICATION OF BASIC EQUATIONS

Reference 1 makes the substitution of $T_{w, i}$ from the finite difference form of equation (A6) into the finite difference form of the energy equation. Rearranging gives a quadratic in the gas temperature $T_{i}^{\prime}$ where the prime refers to a step forward in time and the quantities without the prime are evaluated at the previous time step:

$$
\begin{equation*}
\mathrm{T}_{\mathrm{i}}^{\prime 2}+\left[\alpha_{\mathrm{i}}^{*}\left(1+\overline{\mathrm{V}}_{\mathrm{i}}^{*} \frac{\Delta \mathrm{t}}{\Delta \mathrm{x}}-\omega_{\mathrm{i}}^{*}\right)-\mathrm{T}_{\mathrm{w}, \mathrm{i}}-\left(\frac{\dot{\mathrm{q}}_{\mathrm{W}} \Delta \mathrm{t}}{l_{\mathrm{w}} \rho_{\mathrm{w}} \mathrm{C}_{\mathrm{w}}}\right)_{\mathrm{i}}^{*}\right] \mathrm{T}_{\mathrm{i}}^{\prime}-\alpha_{\mathrm{i}}^{*}\left(\overline{\mathrm{~V}}_{\mathrm{i}}^{*} \frac{\Delta \mathrm{t}}{\Delta \mathrm{x}} \mathrm{~T}_{\mathrm{i}-1}^{\prime}+\mathrm{T}_{\mathrm{i}}\right)=0 \tag{C1}
\end{equation*}
$$

The quantity marked with the asterisk may be evaluated either at the beginning or the end of the time interval.

A difficulty can arise when evaluating the gas energy equation expressed by the pre-
vious quadratic. This occurs when the heat transfer takes place from the wall into the ullage gas. For this situation, the solution of the continuity equation provided negative gas velocities which made it impossible for equation ( $\mathbf{C 1}$ ) to converge on the real roots.

At the start of the ramp (immediately after filling the tank), the initial wall temperature distribution in the ullage is higher than the gas temperature distribution. This is brought about since the wall surface above the liquid is exposed to the ambient temperature. But the ullage gas temperature near the liquid interface is close to the saturation temperature at 1 atmosphere.

The technique used when $\mathrm{T}_{\mathrm{w}, \mathrm{i}}>\mathrm{T}_{\mathrm{i}}$ involved a direct substitution. The finite difference form of equation (A4) is

$$
\begin{equation*}
\bar{V}_{i}^{\prime}=\frac{T_{i}^{\prime} \bar{V}_{i+1}^{\prime}-\left(\frac{Z_{1}}{Z}\right)_{i}^{\prime}\left(\frac{\Delta x}{\Delta t}\right)\left(T_{i}^{\prime}-T_{i}\right)+\left(\frac{Z_{2}}{Z}\right)_{i}^{\prime} T_{i}^{\prime}}{P^{\prime}} \frac{\Delta x}{\Delta t}\left(P^{\prime}-P\right) \tag{C2}
\end{equation*}
$$

Combining the equations involving ullage gas temperature $T_{i}^{\prime}$ and velocity $\bar{V}_{i}^{\prime}$, equations (C1) and (C2), gives the following cubic results:

$$
\begin{align*}
& b_{i} T_{i}^{\prime 3}+\left[\left(\frac{Z_{1}}{Z}\right)_{i}^{\prime} T_{i+1}^{\prime}+b_{i} c_{i}+\alpha_{i}^{*} \frac{\Delta t}{\Delta x}\left(\bar{V}_{i+1}^{\prime}+d_{i}\right)\right] T_{i}^{\prime 2} \\
& +\left[c_{i}\left(\frac{\mathrm{Z}_{1}}{\mathrm{Z}}\right)_{\mathrm{i}}^{\prime} \mathrm{T}_{\mathrm{i}+1}^{\prime}+\alpha_{\mathrm{i}}^{*} \frac{\Delta \mathrm{t}}{\Delta \mathrm{x}}\left(\frac{\mathrm{Z}_{1}}{\mathrm{Z}}\right)_{\mathrm{i}}^{\prime} \frac{\Delta \mathrm{x}}{\Delta \mathrm{t}} \mathrm{~T}_{\mathrm{i}}-\alpha_{\mathrm{i}}^{*} \frac{\Delta \mathrm{t}}{\Delta \mathrm{x}} \mathrm{~T}_{\mathrm{i}-1}^{\prime}\left(\overline{\mathrm{V}}_{\mathrm{i}+1}^{\prime}+\mathrm{d}_{\mathrm{i}}\right)-\alpha_{\mathrm{i}}^{*} \mathrm{~T}_{\mathrm{i}} \frac{\Delta \mathrm{t}}{\Delta \mathrm{x}} \mathrm{~b}_{\mathrm{i}}\right] \mathrm{T}_{\mathrm{i}}^{\prime} \\
& -\alpha_{i}^{*} \frac{\Delta t}{\Delta x}\left(\frac{\mathrm{Z}_{1}}{\mathrm{Z}}\right)_{\mathrm{i}}^{\prime} \frac{\Delta \mathrm{x}}{\Delta \mathrm{t}} \mathrm{~T}_{\mathrm{i}} \mathrm{~T}_{\mathrm{i}-1}^{\prime}-\alpha_{\mathrm{i}}^{*} \mathrm{~T}_{\mathrm{i}}\left(\frac{\mathrm{Z}_{1}}{\mathrm{Z}}\right)_{\mathrm{i}}^{\prime} \mathrm{T}_{\mathrm{i}+1}^{\prime}=0 \tag{C3}
\end{align*}
$$

This arrangement of terms eliminates $V_{i}^{\prime}$ and the cubic is solved for the gas temperature $T_{i}^{\prime}$.

## ANALYTICAL PROCEDURE

The analytical procedure uses a variable time increment $\Delta t$ in following the pressure rise. With this technique, the iteration was stable over a range of inlet conditions
and the results were consistent with the recorded data.
First, the velocity distribution is determined in the initial ullage at time $t=0$. The substituted form of equation (A4) (in which the energy equation (A1) is substituted for $\partial T / \partial t)$ is put into finite difference form and solved for the velocity distribution for the first time.

For most of the ramp runs encountered in this investigation, an initial time increment of 1 second proved satisfactory.

## TEMPERATURE CALCULATIONS FROM TOP TO INTERFACE

Since values of $\overline{\mathrm{V}}$ have been obtained at each net point at time $\mathrm{t}=\mathrm{t}_{1}=0$, attention is turned to equation ( C 3 ) which is cubic in $\mathrm{T}_{\mathbf{i}}^{\prime}$. During the iteration, the cubic equation (C3) is solved for the gas temperature $\mathrm{T}_{\mathrm{i}}^{\dot{\prime}}$ starting at the point $\mathrm{N}_{2}$ in figure 21.


Figure 21. - Analytical model. (Coordinate system is positive in downward direction from $X=0$ at $N_{I}$ to $X=n$ at interface $N_{Z}$.)

When this equation is first solved for the ullage temperature distribution, a value for $T_{i+1}^{\prime}$ is not available. A substituted value of $T_{i+1}$ proved to be satisfactory as an initial guess to get convergence. All other quantities in equation (C3) are available from the initial conditions. The value for $\mathrm{T}_{\mathrm{i}-1}^{\prime}$, the temperature at $\mathrm{N}_{1}$, is known as a boundary condition.

The solution for $T_{i}^{\prime}$ at $N_{3}$ follows, and this procedure continues to calculate gas temperatures until the boundary at the interface is reached. The values for the corresponding wall temperatures are calculated using the finite difference form of equation (A6).

## VELOCITY CALCULATIONS FROM INTERFACE TO TOP

Although the ullage temperatures are computed starting at the top (fig. 21), the velocity equation (C2) is used to calculate the ullage gas velocity starting with the point $\mathrm{N}_{\mathrm{y}}$ near the interface. The velocity at the interface $N_{z}$, the boundary value, is zero with no expulsion.

The ullage gas velocity is calculated from point to point until the top of the tank is reached. The new velocities are used in equation (C3) along with previous values of $\mathrm{T}_{\mathrm{i}+1}^{\prime}$ and the temperature distribution is redetermined. This process is continued until convergence is achieved over the entire ullage. The time is advanced to $t_{2}$ and a new set of velocities is determined.

## COMPLETING THE SOLUTION

With the new velocities at time $t_{2}$, we evaluate equation (C3) again starting at point $\mathrm{N}_{2}$ and terminating at the interface. A value for $\mathrm{T}_{\mathrm{i}+1}^{\prime}$ is always available from the previous iteration, although a substituted value of $\mathrm{T}_{\mathrm{i}+1}$ is used as the first value.

The new values for $T_{i}^{\prime}$ at all the points for time $t_{2}$ are used to recompute the velocity distribution. This new set of velocities is then compared with the previous set and convergence is assumed if the deviation is less than half of 1 percent for every velocity in the time set. A time step is then taken to $t_{3}$.

If convergence is not achieved after 40 iterations, the time step is reduced and the iteration process is reinitiated. Generally the reduction in time step becomes necessary only when there is a severe change in the slope of the ramp curve particularly in the early stages of the pressure rise.

For the new time $t_{3}$, the temperature $T_{i}^{\prime}$ in equation (C3) is determined from the converged value using the iterating method. This procedure continues to evaluate the
gas temperatures and velocity distribution in the ullage for each time step taken in following the rate of pressure in the tank.

The initial gas velocity distribution used in solving equation (C3) for each new time $t$ is obtained from the previous time as follows:

$$
\begin{equation*}
\mathrm{V}_{\mathrm{t}, 2}=\mathrm{V}_{\mathrm{t}, 1} \frac{\left(\frac{\Delta \mathrm{P}_{1-2}}{\Delta \mathrm{t}_{1-2}}\right)}{\left(\frac{\Delta \mathrm{P}_{0-1}}{\Delta \mathrm{t}_{0-1}}\right)} \tag{C4}
\end{equation*}
$$

This iterative procedure can be used for a constant pressure representing the hold period. However, for initiating the ramp, an actual pressure rise must be used. A typical example is shown in figure 22.


Figure 22. - Tank pressure as function of time during initial pressurization period for run 96.

## APPENDIX D

## DATA REDUCTION

## PHYSICAL DESCRIPTION OF PROBLEM

An initially vented tank containing two-phase hydrogen was pressurized from 1 atmosphere to a new pressure by adding helium gas. The system was then allowed to stabilize at which time liquid outflow was started. During this expulsion period, pressurant gas (helium at constant temperature) was added to the tank at a rate that maintained a constant tank pressure while expelling the liquid at a desired rate. The amount of pressurant gas used during the expulsion phase is dependent on (1) the volume of liquid displaced with no heat or mass transfer, (2) the heat transfer to the tank wall and liquid, (3) the amount of mass condensed or evaporated, and (4) the amount of helium gas absorbed by the liquid hydrogen.

The main parameter used in the comparisons was the ratio of the ideal pressurant requirement to the actual pressurant requirement. The ideal pressurant was determined under the assumption that the incoming pressurant gas did not exchange energy or mass with the surroundings. Under this assumption, the ideal pressurant required for the initial pressurization of the tank was determined by the adiabatic relation

$$
\begin{equation*}
M_{I}=\frac{\bar{M} P_{o} V_{o}}{Z R T_{G}}\left[\left(\frac{P_{f}}{P_{o}}\right)^{1 / \gamma}-1\right] \tag{D1}
\end{equation*}
$$

The ideal pressurant required for the expulsion period was determined by the relation

$$
\begin{equation*}
\mathrm{M}_{\mathrm{I}}=\frac{\overline{\mathrm{M} P} \Delta \mathrm{~V}_{\mathrm{U}}}{\mathrm{ZRT}_{\mathrm{G}}} \tag{D2}
\end{equation*}
$$

## MASS BALANCE

A mass balance was performed on the ullage volume from an initial time $t_{i}$ to a final time $t_{f}$ as follows:

$$
\begin{equation*}
M_{U, f}=M_{U, i}+M_{G, i \rightarrow f} \pm M_{t, i \rightarrow f} \tag{D3}
\end{equation*}
$$

A discussion of how the terms of equation (D3) were determined follows.

## Pressurant Gas Added ( $\mathrm{M}_{\mathrm{G}, \mathrm{i} \rightarrow \mathrm{f}}$ )

The weight of the actual pressurant gas added from any initial time $t_{i}$ to any final time $t_{f}$ was determined by numerical integration of the gas orifice equation

$$
\begin{equation*}
\mathrm{M}_{\mathrm{G}, \mathrm{i} \rightarrow \mathrm{f}}=\int_{\mathrm{t}_{\mathrm{i}}}^{\mathrm{t}_{\mathrm{f}}} \mathrm{YD}^{2} \mathrm{C} \sqrt{\rho \Delta \mathrm{P}^{*}} \mathrm{dt} \tag{D4}
\end{equation*}
$$

The time necessary to expel $\Delta V_{U}$ of liquid is $\left(t_{f}-t_{i}\right)$.

## Ullage Mass

The initial ullage mass $M_{U, i}$ and final ullage mass $M_{U, f}$ were obtained by numerical integration of the particular density profiles:

$$
\begin{array}{ll}
\mathrm{M}_{\mathrm{U}, \mathrm{i}}=\int_{\mathrm{V}_{\mathrm{U}, \mathrm{i}}} \rho \mathrm{dV} \simeq \sum_{\mathrm{n}=1}^{\mathrm{N}_{\mathrm{i}}} \rho_{\mathrm{n}} \mathrm{~V}_{\mathrm{n}} \quad \text { where } \rho=\mathrm{f}\left(\mathrm{~T}, \mathrm{X}_{\mathrm{H}_{2}}, \mathrm{X}_{\mathrm{He}}\right) \\
\mathrm{M}_{\mathrm{U}, \mathrm{f}}=\int_{\mathrm{V}_{\mathrm{U}, \mathrm{f}}} \rho \mathrm{dV} \simeq \sum_{\mathrm{n}=1}^{\mathrm{N}_{\mathrm{f}}} \rho_{\mathrm{n}} \mathrm{~V}_{\mathrm{n}} \quad \text { where } \rho=\mathrm{f}\left(\mathrm{~T}, \mathrm{X}_{\mathrm{H}_{2}}, \mathrm{X}_{\mathrm{He}}\right) \tag{D6}
\end{array}
$$

The internal tank volume was considered as 57 (corresponding to thermopile location) horizontal disk segments. Each of these segments was in turn divided radially into a series of concentric rings, the number of which depended on location of radial temperature sensors and the vertical position of the disk segment being considered. These rings (339 in all) comprised the $\mathrm{V}_{\mathrm{n}}$ 's in the previous calculations. In this manner, vertical temperatures as well as radial temperature gradients could be incorporated into the mass calculations. The position of the liquid level prior to and after expulsion determined the number of gas volume rings ( $\mathrm{N}_{\mathrm{i}}$ and $\mathrm{N}_{\mathrm{f}}$ ) used in the ullage mass calculations. The density of the two component ullage was calculated from the BeattieBridgeman equation of state for a mixture given in reference 14 as

$$
\begin{equation*}
P=\frac{R T(1-\epsilon)}{\bar{M}^{\prime} v^{2}}\left(v+B_{T}\right)-\frac{A_{T}}{v^{2}} \tag{D7}
\end{equation*}
$$

where

$$
\begin{gathered}
\mathrm{A}_{\mathrm{T}}=\left(\mathrm{X}_{\mathrm{H}_{2}} \sqrt{\mathrm{~A}_{\mathrm{H}_{2}}}+\mathrm{X}_{\mathrm{He}} \sqrt{\mathrm{~A}_{\mathrm{He}}}\right)^{2}\left(1-\frac{\mathrm{X}_{\mathrm{H}_{2}{ }^{\mathrm{a}} \mathrm{H}_{2}}+\mathrm{X}_{\mathrm{He}}{ }^{\mathrm{a}} \mathrm{He}}{\mathrm{v}}\right) \\
\mathrm{B}_{\mathrm{T}}=\left(\mathrm{X}_{\mathrm{H}_{2}}{ }^{B_{H_{2}}}+\mathrm{X}_{\mathrm{He}}{ }^{B_{H e}}\right)\left(1-\frac{\left.\mathrm{X}_{\mathrm{H}_{2}{ }^{b_{H_{2}}}+\mathrm{X}_{\mathrm{He}}{ }^{\mathrm{b}_{\mathrm{He}}}}^{\mathrm{v}}\right)}{}\right)
\end{gathered}
$$

and

$$
\epsilon=\frac{\mathrm{X}_{\mathrm{H}_{2}} \mathrm{C}_{\mathrm{H}_{2}}+\mathrm{X}_{\mathrm{He}} \mathrm{C}_{\mathrm{He}}}{\mathrm{vT}^{3}}
$$

Equation (D7) can then be solved for $\frac{1}{\mathrm{v}_{\mathrm{n}}}$ (where $\frac{1}{\mathrm{v}_{\mathrm{n}}}=\rho_{\mathrm{n}}$ ) by knowing the pressure, temperature, and ullage gas concentration for each $\Delta V_{n}$ segment.

## Mass Transfer

The mass transfer was calculated from equation (D3) as a result of knowing $M_{U, f}$, $M_{U, i}$ and $M_{G, i-f}$; that is,

$$
\begin{equation*}
M_{t, i \rightarrow f}=M_{U, i}+M_{G, i \rightarrow f}-M_{U, f} \tag{D8}
\end{equation*}
$$

If $M_{t, i \rightarrow f}$ was a positive quantity, mass was considered leaving the ullage volume (e.g., condensation and/or absorption).

## ENERGY BALANCE

For the thermodynamic system consisting of the entire tank and its contents (tank + ullage gas + liquid), the first law of thermodynamics for an increment of time dt may be written as

$$
\begin{equation*}
d U_{T}=\left(\delta M_{G}\right)\left(u_{G}+P_{G} v_{G}+\frac{\bar{V}_{G}^{2}}{2 g}+z_{G}\right)-\left(\delta M_{L}\right)\left(u_{L}+P_{L} v_{L}+\frac{\overline{\mathrm{V}}_{L}^{2}}{2 g}+z_{L}\right)+\delta Q-\delta W \tag{D9}
\end{equation*}
$$

The kinetic and potential energy terms are small in comparison with the other energy terms and are neglected in this development. If $h=u+P v$ is substituted, equa-
tion (D9) becomes

$$
\begin{equation*}
d U_{T}=\left(\delta M_{G}\right) h_{G}-\left(\delta M_{L}\right) h_{L}+\delta Q-\delta W \tag{D10}
\end{equation*}
$$

For this system, there is no external work done so $\delta \mathrm{W}=0$ and the final form of equation (D9) therefore becomes

$$
\begin{equation*}
\mathrm{dU}_{\mathbf{T}}=\left(\delta \mathrm{M}_{\mathbf{G}}\right) \mathrm{h}_{\mathrm{G}}-\left(\delta \mathrm{M}_{\mathrm{L}}\right) \mathrm{h}_{\mathrm{L}}+\delta \mathbf{Q} \tag{D11}
\end{equation*}
$$

Equation (D11) can be integrated over any time period. The physical interpretation of the quantities in equation (D11) is as follows:

$$
\begin{align*}
& \underbrace{\int_{U_{i}}^{U_{f}} d U_{T}}=\underbrace{\int_{t_{i}}^{t_{f}} \dot{M}_{G} h_{G} d t}-\underbrace{\int_{t_{i}}^{t_{f}} \dot{M}_{L^{\prime}} h_{L} d t}+\underbrace{\int_{t_{i}}^{t_{f}} \dot{Q} d t}  \tag{D12}\\
& \text { Change in Energy input Energy leaving Energy from } \\
& \text { system by pressurant through environment } \\
& \text { energy gas inflow liquid outflow (heat leak from } \\
& \text { (Tank + Gas } \\
& + \text { Liquid) }
\end{align*}
$$

A discussion of how the terms of equation (D12) were evaluated follows.

## Energy Input by Pressurant Gas in Flow

The first term in equation (D12) may be evaluated as follows:

$$
\begin{equation*}
\int_{t_{i}}^{t_{f}} \dot{M}_{G} h_{G} d t \simeq \sum_{n=0}^{n=\frac{t_{f}-t_{i}}{\Delta t}} \dot{M}_{G, n^{h}}{ }_{G, n} \Delta t \tag{D13}
\end{equation*}
$$

The pressurant flow rate $\dot{\mathrm{M}}_{\mathrm{G}}$ was determined from equation (D4). The specific enthalpy of the inlet gas was evaluated at the inlet temperature and pressure at each time increment $\Delta t$.

## Energy Leaving by Liquid Outflow

The energy of the liquid that leaves the system can be evaluated as follows:

$$
\begin{equation*}
\int_{t_{i}}^{t_{f}} \dot{M}_{L} h_{L} d t \simeq \sum_{n=0}^{n=\frac{t_{f}-t_{i}}{\Delta t}} M_{L, n} h_{L, n} \Delta t \tag{D14}
\end{equation*}
$$

The liquid flow rate $\dot{\mathrm{M}}_{\mathrm{L}}$ was determined from the turbine flowmeter. The specific enthalpy of the liquid was evaluated at the outlet temperature.

## Energy Input from Environment

The rate of energy input into the tank from the environment was assumed to be the same for all cases and was determined from a boiloff test. This test indicated a nominal value of $0.732 \times 10^{3}$ joules per second ( $0.694 \mathrm{Btu} / \mathrm{sec}$ ) should be used. This value, which includes heat input by radiation, convection, and conduction through pipes and supports, was in all test cases less than 7.0 percent of the energy added to the tank by the pressurant gas:

$$
\begin{equation*}
\int_{t_{i}}^{t_{f}} \dot{\mathrm{Q}} d t \simeq 0.732 \times 10^{3}\left(\mathrm{t}_{\mathrm{f}}-\mathrm{t}_{\mathrm{i}}\right) \tag{D15}
\end{equation*}
$$

## Change in System Energy

The change in system energy can be separated into three categories: (1) change in ullage energy, (2) change in liquid energy, and (3) change in the wall energy. Stated mathematically,

$$
\begin{equation*}
d U_{T}=d U_{U}+d U_{L}+d U_{W} \tag{D16}
\end{equation*}
$$

## Change in Ullage Energy

The change in the ullage energy over any given time interval $\left(t_{i} \rightarrow t_{f}\right)$ is obtained by subtracting the internal energy of the ullage at time $t_{i}$ from the internal energy at time $\mathrm{t}_{\mathrm{f}}$ :

$$
\begin{equation*}
\int_{U_{t_{i}}}^{U_{t_{f}}} \mathrm{dU}_{U}=\left(\mathrm{U}_{\mathrm{U}}\right)_{\mathrm{t}_{\mathrm{f}}}-\left(\mathrm{U}_{\mathrm{U}}\right)_{\mathrm{t}_{\mathbf{i}}} \tag{D17}
\end{equation*}
$$

Making use of the relation $\mathrm{U}=\mathrm{H}-\mathrm{PV}$ gives

$$
\begin{equation*}
\int_{\mathrm{U}_{\mathrm{t}_{\mathrm{i}}}}^{\mathrm{U}_{\mathrm{t}_{\mathrm{f}}}} \mathrm{dU}_{\mathrm{U}}=\sum_{\mathrm{V}_{\mathrm{f}}} \rho_{\mathrm{U}}\left(\mathrm{~h}-\frac{\mathrm{P}}{\rho_{\mathrm{U}}}\right) \Delta \mathrm{V}_{\mathrm{U}}-\sum_{\mathrm{V}_{\mathbf{i}}} \rho_{\mathrm{U}}\left(\mathrm{~h}-\frac{\mathrm{P}}{\rho_{\mathrm{U}}}\right) \Delta \mathrm{V}_{\mathrm{U}} \tag{D18}
\end{equation*}
$$

The ullage gas density was determined using equation (D7). The ullage gas enthalpy was calculated using the relation

$$
\begin{equation*}
\mathrm{h}=\frac{\mathrm{M}_{\mathrm{H}_{2}}}{\mathrm{M}_{\mathrm{T}}} \mathrm{~h}_{\mathrm{H}_{2}}+\frac{\mathrm{M}_{\mathrm{He}}}{\mathrm{M}_{\mathrm{T}}} \mathrm{~h}_{\mathrm{He}} \tag{D19}
\end{equation*}
$$

Therefore, since the pressure and temperature and ullage gas concentration profiles at times $t_{f}$ and $t_{i}$ were known, the change in ullage energy was evaluated.

## Change in Liquid Energy

The change in energy of the liquid in the tank can be determined in a manner similar to the change in ullage energy:

$$
\begin{equation*}
\int_{U_{t_{i}}}^{\mathrm{U}_{\mathrm{f}}} \mathrm{dU}_{\mathrm{L}}=\left(\mathrm{U}_{\mathrm{L}}\right)_{\mathrm{t}_{\mathrm{f}}}-\left(\mathrm{U}_{\mathrm{L}}\right)_{\mathrm{t}_{\mathrm{i}}} \tag{D20}
\end{equation*}
$$

or

$$
\begin{equation*}
\int_{\mathrm{U}_{\mathrm{t}_{\mathrm{i}}}}^{\mathrm{U}_{\mathrm{t}}}{ }_{\mathrm{dU}}^{\mathrm{L}},=\sum_{\mathrm{V}_{\mathrm{f}}} \rho_{\mathrm{L}}\left(\mathrm{~h}_{\mathrm{L}}-\frac{\mathrm{P}}{\rho_{\mathrm{L}}}\right) \Delta \mathrm{V}_{\mathrm{L}}-\sum_{\mathrm{V}_{\mathrm{i}}} \rho_{\mathrm{L}}\left(\mathrm{~h}_{\mathrm{L}}-\frac{\mathrm{P}}{\rho_{\mathrm{L}}}\right) \Delta \mathrm{V}_{\mathrm{L}} \tag{D21}
\end{equation*}
$$

where the liquid density and enthalpy are functions of pressure and temperature.

## Change in Wall Energy

The change in wall energy was determined by applying the first law of thermodynamics to an element of the wall:

$$
\begin{equation*}
\int_{U_{t_{i}}}^{U_{t_{f}}} d U_{w}=\Delta U_{w}=\Delta M_{w} \int_{T_{1}}^{T_{2}} C_{v} d T \quad \text { where } C_{v}=C_{v}(T) \tag{D22}
\end{equation*}
$$

The total change of the wall is then

$$
\begin{equation*}
\Delta U_{\mathrm{w}, \mathrm{~T}} \cong \sum_{\mathrm{M}_{\mathrm{W}}} \Delta \mathrm{U}_{\mathrm{w}} \cong \sum_{\mathrm{M}_{\mathrm{w}}} \Delta \mathrm{M}_{\mathrm{w}} \int_{\mathrm{T}_{1}}^{\mathrm{T}_{2}} \mathrm{C}_{\mathrm{v}}(\mathrm{~T}) \mathrm{dT} \tag{D23}
\end{equation*}
$$

## Total Energy Change of System

For convenience equation (D16) is substituted into (D12):

$$
\begin{equation*}
\int_{t_{i}}^{t_{f}} \frac{d}{d t}\left[U_{U}+U_{w}+U_{L}\right] d t=\int_{t_{i}}^{t_{f}} \dot{M}_{G} h_{G} d t-\int_{t_{i}}^{t_{f}} \dot{M}_{L} h_{L} d t+\int_{t_{i}}^{t_{f}} \dot{Q} d t \tag{D24}
\end{equation*}
$$

Rearranging terms gives

$$
\begin{align*}
& \underbrace{\int_{t_{i}}^{t_{f}}\left[\dot{M}_{G}{ }^{h} G+\dot{Q}\right] d t}=\underbrace{\int_{t_{i}}^{t_{f}}\left[\dot{M}_{L} h_{L} d t+d U_{L}\right]}+\underbrace{\int_{t_{i}}^{t_{f}} d U_{U}}+\underbrace{\int_{t_{i}}^{t_{f}} d U_{w}}  \tag{D25}\\
& \text { Total energy } \\
& \text { added } \\
& \left(\Delta \mathrm{U}_{\mathrm{T}}\right) \\
& \text { Total change in liquid } \\
& \text { in tank plus liquid } \\
& \text { expelled energy } \\
& \left(\Delta \mathrm{U}_{\mathrm{L}}\right) \\
& \text { Total change } \\
& \text { in ullage } \\
& \text { energy } \\
& \left(\Delta \mathrm{U}_{\mathrm{U}}\right)
\end{align*}
$$

Dividing through by $\Delta \mathrm{U}_{\mathrm{T}}$ gives

$$
\begin{equation*}
1=\frac{\Delta \mathrm{U}_{\mathrm{L}}}{\Delta \mathrm{U}_{\mathrm{T}}}+\frac{\Delta \mathrm{U}_{\mathrm{U}}}{\Delta \mathrm{U}_{\mathrm{T}}}+\frac{\Delta \mathrm{U}_{\mathrm{W}}}{\Delta \mathrm{U}_{\mathrm{T}}} \tag{D26}
\end{equation*}
$$

The data presented herein are in the form of these ratios which show the relative distribution of the total energy input.

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