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ALTERNATE APOLLO MISSIONS - LIBRATION POINTS

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ABSTRACT

This note discusses the possible application of the Saturn-type vehicles to libration-point missions alternate to the basic design Apollo lunar landing mission. Situated in the earth-moon system are the five libration points at which the third body (space vehicle) will remain stationary relative to the two primary bodies. At least four criteria, namely, velocities in vicinity of libration points, stability of motion, equations of motion, and scientific applications are reviewed.

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I. INTRODUCTION

An immediate objective of alternate Apollo missions will be the utilization of the libration points situated in the earth-moon system for possible applications consideration of the Apollo Applications Program in the area of advanced technology support mission. In 1961 a decision to simplify the basic Apollo mission was made to employ the lunar orbit rendezvous mode for lunar surface exploration operations. This mode provides mission capability with less payload to translunar conditions than other mission modes. Since the current technology for propulsion and vehicle was available at the time of mission mode decision, the lunar orbit rendezvous mode offered maximum probability of achievement of the mission at an early date.

The considerable value of scientific observations of interplanetary phenomena from vantage points in the earth-moon system would be recognized. The space flight program philosophy which guides the evaluation of mission alternatives encompasses the most significant concept of maximizing payload and multi-mission capability and of minimizing program costs. The observational discovery by the Polish astronomer K. Kordylewski of the Cracow Observatory in 1961 of two faint cloud-like satellites in the vicinity of the trailing stable earth-moon libration point marked the first specific step taken by the United States toward casting considerable pervading influence on alternate Apollo mission planning. This provides a possible solar system exploration with some level of confidence attributable to successes in recent technological developments. Kordylewski's additional discovery of a single large cloud-like satellite at the leading stable libration point coupled with earlier sensational observational discovery in 1906 and subsequent sightings of the Trojan asteroids oscillating around the sun-Jupiter triangular libration points provided an additional attest to the augmentative interest. Moreover, considerable speculation has been aroused concerning the stability of motion of a particle around the libration points. Various mathematical demonstrations have been produced to define the stability criteria induced by the phenomenal perturbations. As still unconfirmed yet, Kordylewski's and Simpson's observational reports are apparently the only publications concerning sightings of objects orbiting about the libration points of the earth-moon system (References 2, 3, and 40).

The primary goal of unmanned lunar mission will be to transport an automated spacecraft to the vicinity of stable libration point and transmit the data collected to the earth for assessment of the particle concretion and material composition. This will be an answer to a very fundamental question of whether or not particulate matter can actually remain for increasing periods of time in the vicinity of the libration points. This primary mission will probably precede confirmation of the presence of cloud material by means of the unequivocal photography process during the initial successful NASA experiments. Should the material find be substantiated, further questions should be answered by commencement of evaluating the suitability of the libration points as buoy platforms for astronomical and astrophysical studies and for support of manned lunar operations.

During the past several years, the United States has been conducting theoretical investigations in the various space sciences utilizing the libration point satellites and probes. Therefore, it is essential to have a physical understanding of the libration points. Thus, there are five singular points (or centers), called equilibrium points, Lagrangian points or libration points, long recognized by astronomers as points where the gravitational attraction of the moon exactly balances that of the earth. These points are located in the plane of motion of the two finite bodies. The geometry of the libration points, which are within reach using current Apollo propulsion, is depicted in Figure 1. Two of the libration points at the apex points of the two equilateral triangles formed with the earth-moon line as the base are L_4 at the leading apex and L_5 at the trailing apex. The other three points along the earth-moon axis include one between the massive bodies, L_1, one directly behind the moon, L_2, and one on the opposite side of the earth from the moon, L_3; the actual location is dependent on the mass ratio of the two large masses. At these five libration points the relative velocity is zero. The vector sum of forces, gravitational and centrifugal, is also zero so that the relative acceleration is zero. Therefore, it is theoretically possible for a body to remain at rest indefinitely in the absence of subsequent disturbances with respect to the rotating earth-moon system. The collinear libration points along the earth-moon axis, L_1, L_2, and L_3, are unstable for all mass ratios such that any

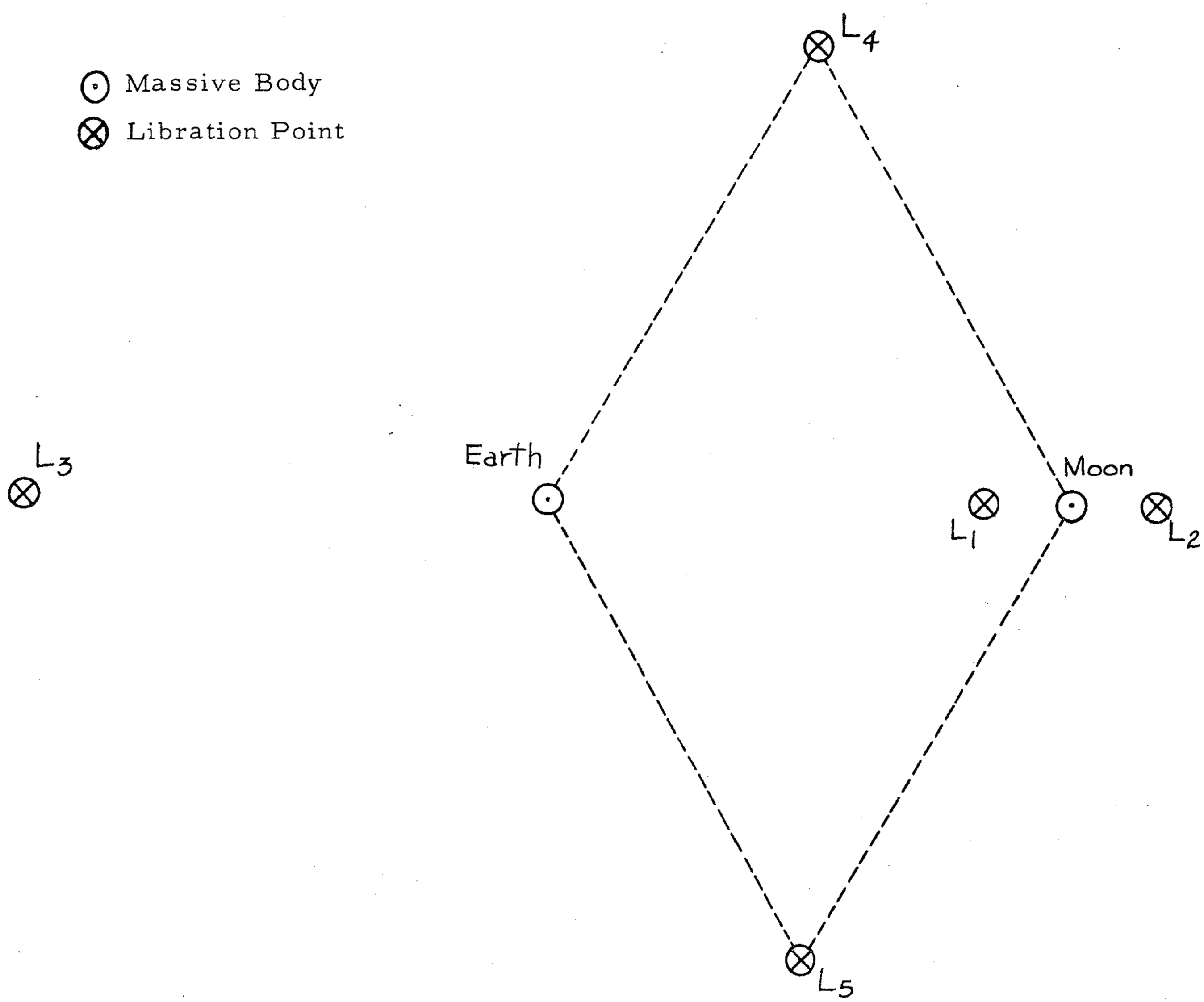

FIGURE 1. Relative Position of the Earth-Moon Libration Points

disturbing force would result in divergent motion away from the point. In contrast, L_4 and L_5 are stable for certain mass ratios and moderate disturbing forces may only produce oscillatory motions in the region of their points. The motion of a space vehicle in the neighborhood of the earth-moon libration points should be analogous to the motion of the asteroids of the Trojan group in the sun-Jupiter-asteroid system.

The purpose of this report is to exploit the application of the Saturn-type vehicles to libration-point missions alternate to the basic Apollo lunar landing mission for which it is designed. The feasibility study concerning mission analysis for the Saturn IB and Saturn V vehicles will be published as an addendum to this report. In this report, at least four criteria, namely, velocities in vicinity of libration points, stability of motion, equations of motion and scientific applications, are reviewed.

It may be noted that based upon the results of Reference 21, earth-moon libration point missions are found to be within Apollo capability. The entire Apollo Command Module and loaded Lunar Excursion Module, for instance, might be available as payload. The propellant load of the LEM ordinarily might not be needed except for attitude control and stationkeeping or for propulsive excursions away from the CM.

As suggested in Reference 21, a mission to the unstable libration point L_3 seems to offer a rather important, attractive possibility because it presents a similar environment to the moon with respect to the sun but without the gravitational or electromagnetic field influences of the moon and without the interactions between earth and lunar fields. Thus, the vehicle at L_3 would be essentially in a solar orbit at 1 A. U. but remote from the earth (or moon).

Once the Apollo capability to proceed to any libration point has been established, it will be appropriate to inquire into the need for such missions.

II. DISCUSSION

A. Velocities in Vicinity of Libration Points

It appears that one of the most fundamental questions about the alternate Apollo missions to the libration points concerns the relative velocities of the spacecraft and solid particles in the vicinity of the libration points. At least two kinds of solid particles must be distinguished: (1) the meteoroids in general and (2) solid particles which are in orbit around the libration points. The danger represented by meteoroids is about the same or possibly even less at the libration points than during a lunar mission (Reference 44).

The role of particles which are already librating around the libration points, if the presence of material is verified, is quite different. In connection with these bodies, all of the particles and spacecraft will be in retrograde orbits, and consequently the values of relative velocities must be estimated by subtracting the velocity of the particles from that of the spacecraft. Furthermore, the periods of motion of the particles and also of the spacecraft are approximately equal, showing some slight variation with amplitudes. Those particles which are librating in the immediate vicinity of the libration points will have small velocities relative to their respective libration points. Collision of these particles with the spacecraft may present small problems only since the spacecraft, when librating close to the libration point, will also have a small relative velocity with respect to the libration point. As the amplitude increases, the velocities may become rather large (order of 305 m/s at 100,000-mile amplitude). It is necessary to realize that the motion of the spacecraft might be controlled and such extreme amplitudes might not be allowed to occur. But even if this were to happen, the motion of the particles seems to be rather orderly at such large amplitudes and "orderly motion" signifies that the magnitude and direction of their velocities are close to the velocity of the spacecraft librating at the same amplitude. Some rather crude estimate of librational velocities relative to the libration point suggests that these may stay below 3 m/s as long as the amplitude is below two miles.

From an orbital altitude of 100 n. mi., the minimum injection velocities needed to reach the libration points are, respectively,

$$V_{L_1} = 10,901 \text{ m/s}$$

$$V_{L_2} = 10,902 \text{ m/s}$$

$$V_{L_3} = 10,910 \text{ m/s}$$

$$V_{L_4} = 10,911 \text{ m/s}$$

$$V_{L_5} = 10,911 \text{ m/s}$$

Escape velocity at that orbital altitude is 11,052 m/s, which is about the limiting velocity capability of the Apollo booster. Hence, missions to the libration points could require up to some 150 m/s less earth departure velocity than Apollo. At the points, arrival velocity is calculated to vary from 150 m/s to 210 m/s; whereas, the earth-moon system rotational velocity about the barycenter varies from 910 m/s to 1250 m/s. Thus, an impulse is needed to establish the body at the libration point and these impulses have a magnitude given by

$$\Delta V_{L_1} = 688 \text{ m/s}$$

$$\Delta V_{L_2} = 1077 \text{ m/s}$$

$$\Delta V_{L_3} = 766 \text{ m/s}$$

$$\Delta V_{L_4} = 804 \text{ m/s}$$

$$\Delta V_{L_5} = 804 \text{ m/s}$$

at the corresponding libration points. The velocity increment for Apollo lunar orbit injection (including a plane change) is about 1067 m/s. Hence, missions to the libration points require anywhere from zero to 381 m/s less arrival velocity increment than Apollo. Since the geometry of a libration point mission, neglecting instability, is invariant with time, the return to earth is a mirror-image of the departure from the earth conditions. Thus, the velocity increment to return to earth also varies from zero to 381 m/s less than the return from lunar orbit. The arrival velocity at earth entry is up to 152 m/s less than lunar return velocity. Mission transit times for optimum energy injections could be in the range of 4.5-6 days on each leg, as is also the case for missions to the moon. However, about a hundred m/s or more above the optimum could reduce transit times to 3-3.5 days which are the nominal Apollo values. Missions to the libration points are thus possible within the Apollo energy capability (Reference 21).

B. Stability of Motion

The study of stability of motions is an extremely complicated problem. The stability of the equilibrium solutions in some circular and elliptic problems of three and four bodies has been previously studied. Before the stability can be investigated, it is appropriate to define stability as well as describe a proper distinction between stability of a libration point itself and stability of periodic solutions about the libration point if they do exist. So, the motion of a spacecraft in the neighborhood of the libration point is stable if it moves in a region bounded by specific minimum and maximum distances from the point for a specified length of time. If the body is displaced slightly from one of the equilibrium positions and does recede indefinitely from that position, the motion is unstable.

To obtain qualitative information about the general nature of the motion of the spacecraft in the vicinities of libration points, more simplified mathematical models must be employed. The eternal problem is in choosing a model so oversimplified that, in order to make it mathematically feasible, it is neither physically or realistically interesting nor mathematically intractable. The best known model for the earth-moon system is undoubtedly the restricted three-body problem. In this problem two massive bodies move in a circular orbit about their center of mass, and a third body of negligible mass (spacecraft) moves about them under their combined attraction. Some of the best known properties of the restricted three-body problem are the five libration points and the zero-velocity surfaces, which confine the motion of the spacecraft to certain regions. A more accurate mathematical model for studying the earth-moon trajectories is the reduced or elliptical problem of three bodies. In this problem the two massive bodies move on ellipses about their center of mass and resemble more closely the motion of the earth and the moon. Research in this problem has dealt principally with the stability of libration points. Zero velocity surfaces cannot be computed in this model.

The perturbation of the sun disrupts the equilibrium of the libration points by introducing a non-homogeneous forcing function in the equation of motion. In this problem, two massive bodies move in circular orbits about their center of mass, or barycenter, which in turn moves in a circular orbit above a third massive body which lies in the same plane as the first two massive bodies. The motion of a small fourth body (spacecraft) which moves under the combined attractions of these three massive bodies resembles the motion of the spacecraft in the earth-moon-sun system. Zero-velocity surfaces cannot be computed and the libration points of the restricted three-body problem are not equilibrium point solutions. Using this model, the qualitative nature of the motion of the spacecraft near these libration points has been radically altered by the presence of the sun. However, if the effect of the sun is to be included in the equations of motion, the choice of a circular orbit for the moon may not be justified since the sun influences the motion of the moon as well as the spacecraft, and circular or elliptical motion of the moon is not possible in actuality. The use of the circular orbit for the moon may cause the indirect effect of the sun on the moon to be neglected in the formulation of the equations of motion. The launch date will probably play an important role in determining how long the spacecraft will remain in the neighborhood of the libration points.

In order to investigate stability, the analysis would require the numerical computation of specific orbits. However, serious difficulties may be involved. If the computation proceeds long enough, round-off and truncation errors become appreciable, and when instability is indicated, the real cause may not be determined. The motion may be unstable if the computational errors could mask the real motion. To avoid this difficulty, an analytical criterion of stability dependent on the differential equations of motion will have to be used. Since it is possible to produce several criteria of stability analytically, another dilemma is encountered; thus, linearization of the equations will be customary. A more comprehensive study of the stability will have to be considered. As the present study continues, many areas will remain to be investigated in the restricted four-body problem; however, generalization of this mathematical model could include the reduced or elliptical four-body problem.

C. The Equations of Motion

The differential equations governing the motion of a spacecraft of infinitesimal mass that are referred to a space-fixed directional coordinate system with origin at body numbered 1 may be described by

$$\ddot{x}_{k\ell} = -G \left[(m_1 + m_k) \frac{x_{k\ell}}{R_{1k}^3} + \sum_{\substack{i=2 \\ i \neq k}}^{N} m_i \left\{ \frac{x_{k\ell} - x_{i\ell}}{R_{ik}^3} + \frac{x_{i\ell}}{R_{\ell i}^3} \right\} \right] ; \qquad (1)$$

$$(k = 1, \ldots, N; \ \ell = 1, 2, 3),$$

where G is the gravitational constant, m_k is the mass of body k,

$$R_{kj}^2 = \sum_{\ell=1}^{3} (x_{k\ell} - x_{j\ell})^2; \qquad (k = 1, \ldots, N - 1; \ j = k + 1, \ldots, N), \qquad (2)$$

and $x_{k\ell}$ denotes the coordinate axis ℓ of body k. All four bodies are considered to be point masses that move in the same space-fixed plane. These equations describe the motion, in a gravitational force field, of N bodies relative to one of them (m_1). Except in the case N = 2, there are no known solutions of these equations which give qualitative information of the motion for all values of the time for arbitrary initial conditions.

An N-body trajectory program will have the subroutine being used to generate the coefficients in the power series solution for the problem of N-point mass subject to their mutual gravitational attraction. In order to produce a system of second degree, the variables will be introduced in the sense that

$$Q_{kj} = R_{kj}^{-3}; \qquad (k = 1, \ldots, N - 1; \quad j = k + 1, \ldots, N), \qquad (3)$$

since by differentiation the following equation is obtained:

$$R_{kj}\,\dot{Q}_{kj} + 3Q_{kj}\,\dot{R}_{kj} = 0; \quad (k = 1, \ldots, N - 1;\ j = k + 1, \ldots, N). \tag{4}$$

If the coefficients are denoted in the power series expansions as

$$x_{k\ell} = \sum_{\nu=0}^{\infty} x_{\nu k\ell} t^{\nu}, \ \ldots, \ R_{kj} = \sum_{\nu=0}^{\infty} R_{\nu kj} t^{\nu} \tag{5}$$

the Equations (1), (2), and (4) together with the definition (3) produce the following recurrence relations

$$2R_{okj}R_{nkj} = \sum_{\ell=1}^{d} \sum_{\nu=0}^{n} (x_{\nu k\ell} - x_{\nu j\ell})\ (x_{n-\nu,k\ell} - x_{n-\nu,j\ell}) - \sum_{\nu=1}^{n-1} R_{\nu kj}R_{n-\nu,kj}; \tag{6}$$

$$nR_{okj}Q_{nkj} = -3n\ Q_{okj}R_{nkj} - \sum_{\nu=0}^{n-2} (3n - 2\nu)\ Q_{\nu+1,kj}\ R_{n-\nu-1,kj}; \tag{7}$$

$$(n + 1)(n + 2)x_{n+2,k\ell} = (\mu_1 + \mu_k) \sum_{\nu=0}^{n} x_{\nu k\ell}\ Q_{n-\nu,1k}$$

$$+ \sum_{\substack{i=2 \\ i\neq k}}^{N} \mu_i \sum_{\nu=0}^{n} \left[(x_{\nu k\ell} - x_{\nu i\ell})\ Q_{n-\nu,ik} + x_{\nu i\ell}\ Q_{n-\nu,\ell i}\right] \tag{8}$$

where n is the recursion index and $\mu_k = -Gm_k$, $(k = 1, \ldots, N)$.

In some numerical analysis studies incorporating several integration techniques, the N-body trajectory program will include the direct and indirect effects resulting from gravitational potentials of the sun and planets, the eccentricity of the moon's orbit about the earth, solar radiation pressure and the nonspherical effects of the moon and earth. All computations will have use of double precision floating point arithmetic which retains 16 decimal digits (Reference 11).

Upon successful study, a valid and more comprehensive analysis will be provided.

D. Scientific Applications

The existence of libration points in the vast areas of space, which are stable in the sense that artificial objects placed there might remain in the general vicinity with or without the application of modest propulsion, has developed some unique possibilities for mission applications. With a possible minimum of modification compatible with the design, the current Apollo hardware could be fitted with the scientific payloads of experiments or instrumentation to effectively and efficiently accommodate the improved capability of achieving flight missions to any of the libration points.

In order to elucidate some objectives of possible missions envisioned for the libration centers in the earth-moon system, it is interesting to examine several mission potentialities such as (1) observational scientific investigations of the "lunar dust" material subject to collection at the stable points; (2) experimental scientific investigations establishing the feasibility and desirability of using both the stable and unstable points; and (3) comprenensive operational use of both types of points to achieve scientific, engineering or applications objectives not related to the points themselves. In addition, a technological objective should be pursued to further the development of space technology. The scientific objectives, as well as technological, are supporting and will lead to the stated objective of our total space effort, the establishment of U. S. pre-eminence in space.

In the category of observational investigations, and experimental probe or probes should be placed in the vicinity of either stable libration point as an initial step to verify the observations of Kordylewski and other observers. Attempts, both in the United States and abroad at obtaining photographs of the clouds from the surface of the earth have been unsuccessful because most times the clouds' very low surface brightness has posed special difficulties for purpose of interpretation and the time for certain infrequent dates has been critical for which the configuration of the sun, moon, and libration point has the proper orientation relative to the observer's horizon.

It becomes apparent that the occasional sightings of reflected light from the vicinity of two stable libration points suggest the possibility of tenuous clouds of small fragments suspended at each of these points. This material, if it exists, probably has not been contaminated by the earth's atmosphere or affected by collisions with other planetary bodies. The collection of such "virgin" solar-system material would be of considerable scientific interest, especially in view of the relative inaccessibility of the primary bodies such as planets, asteroids, and comets. Also, it would assist in the development of a comprehensive understanding of its composition, environment and history. Information from this source, together with some additional observational data, should be acquired to permit an assessment of whether it would be scientifically of interest to consider an alternate Apollo mission to investigate these libration points further.

The second category of mission, experimental investigations, using both the stable and metastable regions, offers interesting possibilities. Much could be learned about the perturbing influence of the sun and planets on the earth-moon system from the motion of the spacecraft at the libration points. The analysis of the results derived from the orbital motion of the spacecraft could possibly influence the determination of improved measures of the mass of the moon relative to the earth and related astronomical parameters, although other existing methods may appear more accurate. The controlling factor of the stable centers is an advantageous location which leads to the necessity of performing long-term solar flare observations, gravity studies and geophysical experiments. Besides the favorable long-lifetime characteristics of the spacecraft at either libration point, it would be relatively remote from the perturbations due to the earth's magnetic field and other influences on interplanetary phenomena such as the solar wind. Such an exploration probe injected into one of the regions could provide increasing amounts of meaningful, unique scientific data on radiation belts, magnetic fields and particles existing in cislunar space. Additional injection of material into the regions could offer possibilities for studies of light pressure effects and provide some scientific clues as to cometary dynamic mechanisms. These missions could be accomplished with an automated spacecraft. This spacecraft can also serve as an important precursor to manned exploration. In potentially hazardous areas, the spacecraft can provide the information required to determine whether the mission conditions can insure personnel safety. In general, these missions should be considered at the appropriate time for inclusion in the proposed lunar exploration program if a suitable Apollo flight is otherwise justified.

In the third category of the use of the libration points for their special characteristics that can contribute as operational elements of a space system, several possibilities come to mind. The stable location might be considered for communications and navigation deep-space relay functions using Apollo hardware. However, the utility for this purpose would require considerable study on a competitive basis with other approaches. The translunar metastable location could be more promising as a relay location for earth-to-lunar orbit communication, maintaining, by modest propulsion, a position slightly off the earth-moon axis, or alternatively, using a second relay in an earth synchronous satellite. Moreover, this metastable location could constitute an ideal site for radio or possibly optical astronomy, if isolation from the earth could prove more necessary than presently foreseen. It also features special interests for monitoring surface exploration on the far side of the moon and placing a radio telescope on the same side, rather than in earth orbit, to avoid radio noise from the earth. In Reference 23, it has implied that this latter possibility imparts at least three disadvantages when compared to an earth-sun libration point: (1) The radio astronomy observatory would have to be established in a gravitational field; (2) direct communications from earth would not be possible with the telescope landed on the far side of the moon; and (3) a given spacecraft can dispose of considerably less payload on the far side of the moon than it could in a libration point orbit.

In opposition to the benefits for the stable libration points are the liabilities incurred by attempting to keep the spacecraft aloft at the metastable point, L_2. The penalty is paid by the requirement for station-keeping propulsion propellant expenditures. A station-keeping procedure has been devised by means of active mass expulsion devices controlled by earth-based measurements and commands (Reference 18). In this sense, the spacecraft trajectory would directly pass the moon as a consequence of communications blackout only occasionally because of a rather large nominal motion about which control is exercised. In analyzing the pessimistic results of this investigation, it becomes apparent that the L_2 communications satellite is feasible from a control standpoint. However, further studies of a more quantitative nature will be necessary to determine possible problem areas and actual control strategies.

The concept of the utility of the cislunar libration point L_1 has been proposed as the Lunar Excursion Module (LEM) staging and subsequent rendezvous location in a lunar manned mission (References 33 & 42). A prime characteristic of the study is the less-than-great degree of instability of the collinear libration point L_1 (also L_2 and L_3) with increasing time capability for station-keeping corrective action by means of modest propulsion. The earth-to-moon mission is initiated by placing the spacecraft combine, namely, Command Module (CM) and LEM, into the vicinity of L_1 (approximately 31,000 miles from the moon) with the additional expenditure of 457-610 m/s over and above the earth escape velocity. (Figure 2). After reaching the L_1, the LEM is detached and departs whenever ready for the preselected landing site on the near side of the moon. An initial propulsive effort of V= 152-305 m/s can be added to reduce the time in transit. The CM must be augmented to have the capability of remaining in the L_1 region in a quiescent manned mode and to receive the LEM returning from the moon surface with aid of the homing device. During the operations of descent, stay on the moon and ascent, a line-of-sight connection between the LEM and CM is provided. (However, this may be highly dubious since it would be obtained only for a nearly direct descent, a maneuver which would be more expensive and hazardous than one involving a lunar orbit.) Rendezvous and docking in the vicinity of L_1 are conducted in an environment where the coupling effects between gravitation and spacecraft motion are small, thus possibly simplifying maneuvers. Upon completion of rendezvous, the CM deorbits for return to the earth with the additional expenditure of about 457-610 m/s, thus increasing flexibility for timing of return. By comparison, the lunar orbit rendezvous experiences sufferance from the large orbit plane change requirement. (The proposed cislunar method does increase the time flexibility somewhat for each of these two operations; however, the lunar orbit method provides an opportunity every two hours for both operations of unlimited launch window for the LEM going to and returning from the moon in any case.)

Thus, the cislunar libration mode, obtained at the cost of a substantially greater velocity budget than for the LOR, has some advantages and disadvantages. It is interesting to note that this proposed mode is, in essence, substantially less attractive than LOR for Apollo since approximately the same velocity increments are required from earth orbit to attain either lunar orbit or the cislunar libration point. The same comparison holds for the return flight. Thus, the velocity budget of the two modes will differ primarily for maneuver of the LEM first from the cislunar libration point to the lunar orbit and later the reverse.

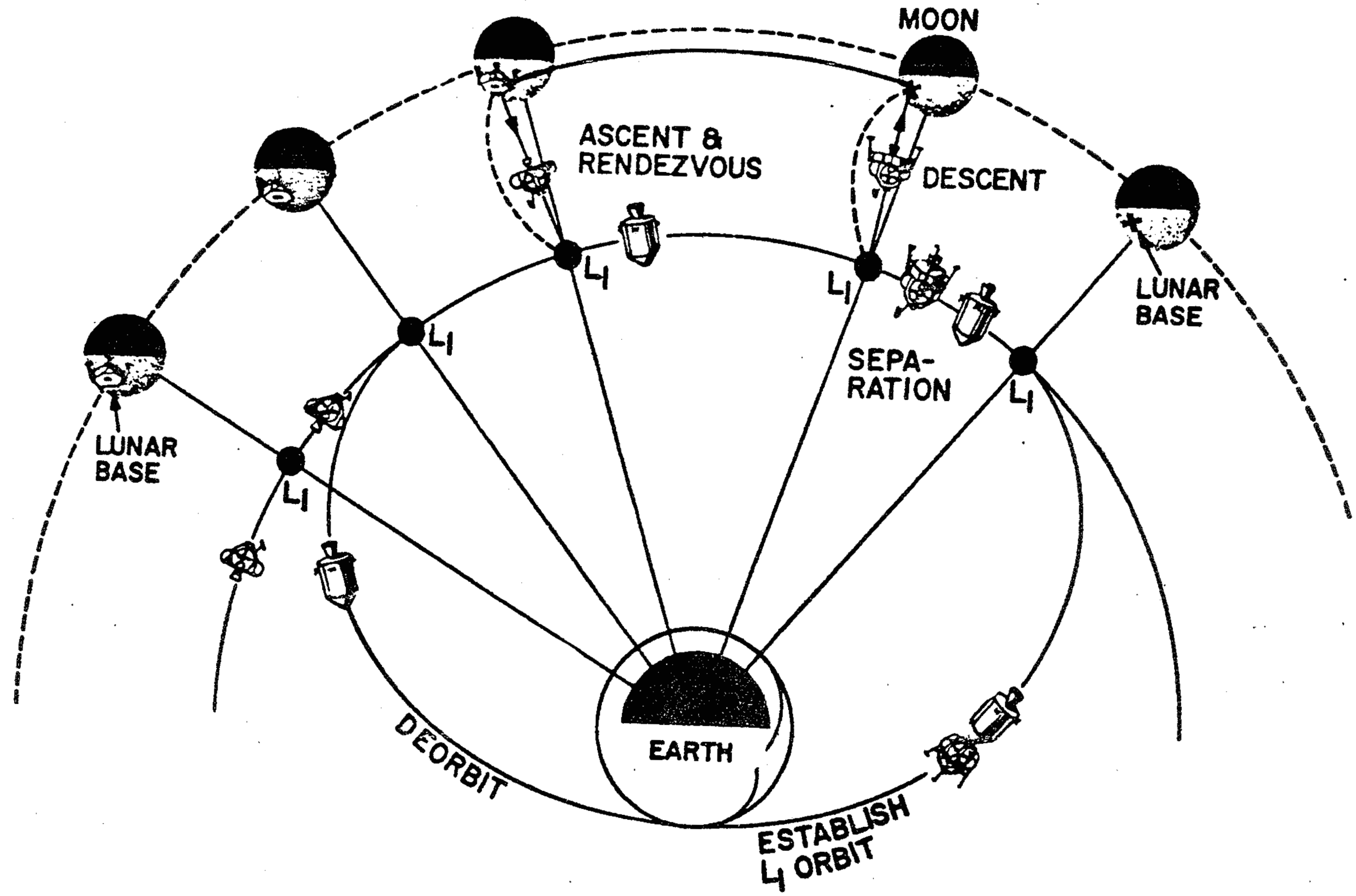

FIGURE 2. Proposed Manned Lunar Operations for the Libration Point L_1 (Reproduced from Reference 42).

Moreover, the cislunar libration point mode would require a substantially longer time in the LEM for abort from the lunar surface because of a solar flare, thus requiring greater shielding weight for the LEM. The total flight time for the proposed mode appears offhand to be two to four days longer than that of the LOR mode. Extended stay of the CM at the cislunar libration point requires continuous navigation and guidance. At this point of unstable equilibrium, temporary navigation, guidance or propulsion failure would result in a departure from the point which could be in any direction in the earth-moon plane and would consequently increase the difficulty of rendezvous. This is in contrast to lunar orbit of the CM during the LEM operation in the LOR mode. This orbit would remain stable during such failure. The modes of unlimited launch window for LEM going to and returning from the moon may not be significant since either mode would use the lunar orbit from which the window is not unlimited. The cislunar mode would save no fuel and lose flexibility by failing to use the lunar orbit. Provisions of full-time communications, as well as optical and radio frequency tracking from the L_1 to the near side of the moon, might require much larger antennas and much greater power from the earth-based equipment.

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ALTERNATE APOLLO MISSION - LIBRATION POINTS

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This document has also been reviewed and approved for technical accuracy.

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