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A CONTROL LAW FOR DOUBLE-GIMBALED CONTROL MOMENT GYROS USED FOR SPACE VEHICLE ATTITUDE CONTROL

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16. ABSTRACT

Space vehicle attitude control, which utilizes control moment gyros (CMG's) to develop the necessary control torques, requires the generation of CMG gimbal rate commands in such a way that the resulting precessional torques on the space vehicle equal the desired control torques; i.e., no torque crosscoupling occurs. Consideration of the combined effect of a pair of double-gimbaled CMG's allows the generation of a no-crosscoupling CMG control law on the basis of easily understandable kinematic relationships. For the control law presented, the only difference between the commanded and the actual control torque exerted on the space station is caused by the difference between the commanded and the actual gimbal rates. The control law is expanded from the application to one CMG pair to the application to three CMG's. Three CMG pairs can then be formed and the desired control torque can be split between them according to their relative control capability. Skylab-A is used as an example for the utilization of the excessive degrees of freedom to better distribute the CMG angular momentum vectors with respect to each other or to their gimbal stops without an effect on the total CMG angular momentum, i.e., without resulting in a net torque on the space vehicle. The general development of the CMG control law assumes arbitrary CMG momentum magnitudes; but it is also shown that the expressions can be simplified if it is assumed that all CMG angular momentum magnitudes are equal. This simplified version is presently used for control of the CMG's on Skylab-A.

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DEFINITION OF SYMBOLS

$\left.\begin{array}{l} i = 1,2,3 \\ j = 1,2,3 \\ k = A,B,C \\ m = 1,3 \end{array}\right\}$ subscripts used in the definitions

$\_$	a bar below a quantity indicates a vector
c	cosine (before greek letter)
$\underline{\dot{e}}_C$	normalized torque command [1/s]
E_{Di}	component of the ith CMG angular momentum vector along the orbit normal [Nms]
$\underline{e}_i$, e_{ij}	unit vector along the ith CMG and its components in vehicle space
$\underline{e}_N$	unit vector along orbit normal (north)
$\underline{e}_{Pk}$, e_{Pk}	normalized cross product of the angular momentum vectors of the CMG's of the kth pair and its magnitude
$\Sigma e_P{}^2$	sum of the e_{Pk}'s squared
$\underline{e}_{Sk}$, e_{Sk}	normalized sum of the kth pair and its magnitude
$e_{i(i-1)}{}'$	$= 1/\sqrt{1 - e_{i(i-1)}{}^2}$
$\underline{e}_T$	$= \underline{e}_1 + \underline{e}_2 + \underline{e}_3$
F_{mi}	angle function $[(\mathrm{rad})^n]$
G_k	variable gain
$\underline{H}$, H	angular momentum vector and its magnitude [Nms]
H_{Dk}	dot product for kth CMG pair $[(\mathrm{Nms})^2]$
$\underline{H}_i$, H_i	angular momentum of the ith CMG and its magnitude [Nms]

DEFINITION OF SYMBOLS (Continued)

H_N	nominal CMG angular momentum magnitude [Nms]
$\underline{H}_{Pk}$, H_{Pk}	cross product of the CMG's of pair k and its magnitude [$(Nms)^2$]
ΣH_P^2	sum of the H_{Pk}'s squared [$(Nms)^2$]
$\underline{H}_{Sk}$, H_{Sk}	sum of the angular momenta of the CMG's of pair k [Nms]
$\dot{\underline{H}}_{Sk}$	angular momentum change of pair k sum [Nm]
$\Delta\underline{H}_{Sk}$	angular momentum difference between initial and final $\underline{H}_{Sk}$ [Nms]
$\underline{H}_T$, H_T	CMG total angular momentum [Nms]
$\dot{\underline{H}}_T$	change of $\underline{H}_T$ [Nm]
K_D, K_R	distribution and rotation gain, respectively [1/s]
n	exponent
s	sine (before greek letter)
S_{kmi}, S_{Tmi}	rotational-sense functions
S_L	limit on all S_{kmi} and S_{Tmi}
t	tangent (before greek letter)
$\underline{T}_C$, T_{Ci}	CMG torque command and its components in vehicle space [Nm]
T_{CAX}	component of torque command perpendicular to the sum of pair A (in $\underline{T}_C$-$\underline{H}_{SA}$-plane) [Nm]
T_{CAP}	component of torque command along sum of pair A [Nm]
$\underline{T}_V$	torque on vehicle caused by CMG's [Nm]
$\underline{u}_P$	unit vector perpendicular to both CMG's of pair A

DEFINITION OF SYMBOLS (Continued)

$\underline{u}_S$	unit vector along pair A sum
$\underline{u}_X$	unit vector perpendicular to pair S sum and $\underline{T}_C$
α	angle between the CMG angular momentum vectors and pair A sum (for the case of $H_i = H_N$) [rad]
α_{iI}, α_{iF}	initial and final angles between ith CMG vector and pair sum [rad], respectively.
$\dot{\alpha}_i$	change of α_i [rad/s]
β	change in direction of $\underline{H}_{SA}$ [rad]
$\underline{\dot{\delta}}_i$	$= [\dot{\delta}_{i1}\ \dot{\delta}_{i2}\ \dot{\delta}_{i3}]^T$ CMG angular velocity caused by gimbal angle rates and its components in vehicle space [rad/s]
$\delta_{1(i)}$, $\delta_{3(i)}$	inner and outer gimbal angles of the ith CMG [rad], respectively
$\dot{\delta}_{m(i)}$	gimbal angle rates [rad/s]
ϵ_{Dk}	rotational rate of kth pair about the vector sum caused by distribution law [rad/s]
ϵ_k	rotational rate of kth pair about the vector sum [rad/s]
ϵ_{kmi}	constituents of ϵ_R of pair k caused by $\delta_{m(i)}$ [rad/s]
ϵ_{Rk}	rotational rate about pair k sum caused by rotation law [rad/s]
ϵ_T	rotational rate of all CMG vectors about the total vector sum [rad/s]
ϵ_{Tmi}	constituents of ϵ_T caused by $\delta_{m(i)}$ [rad/s]
$\underline{\dot{\phi}}_V$	vehicle angular velocity [rad/s]
$\underline{\omega}_i$, ω_{ij}	CMG i angular velocity with respect to the vehicle and its components in vehicle space [rad/s]

DEFINITION OF SYMBOLS (Concluded)

ω_{Pki}	CMG angular velocity used for scissoring [rad/s] (arbitrary H case)
ω_{Pk}	CMG angular velocity used for scissoring (nominal H case) [rad/s]
$\underline{\omega}_{Rk}$	CMG angular velocity caused by R&D laws about pair k sum [rad/s]
$\underline{\omega}_{RT}$	CMG angular velocity caused by R law about total sum [rad/s]
$\underline{\omega}_{Xk}$	CMG angular velocity used for rotating of pair k as a unit [rad/s]

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A CONTROL LAW FOR DOUBLE-GIMBALED CONTROL MOMENT GYROS USED FOR SPACE VEHICLE ATTITUDE CONTROL

SUMMARY

Space vehicle attitude control, which utilizes control moment gyros (CMG's) to develop the necessary control torques, requires the generation of CMG gimbal rate commands in such a way that the resulting precessional torques on the space vehicle equal the desired control torques; i.e., no torque crosscoupling occurs. Consideration of the combined effect of a pair of double-gimbaled CMG's allows the generation of a no-crosscoupling CMG control law on the basis of easily understandable kinematic relationships. For the control law presented, the only difference between the commanded and the actual control torques exerted on the space station is caused by the difference between the commanded and the actual gimbal rates. The control law is expanded from the application to one CMG pair to the application to three CMG's. Three CMG pairs can then be formed and the desired control torque can be split between them according to their relative control capability. Skylab-A is used as an example for the utilization of the excessive degrees of freedom to better distribute the CMG angular momentum vectors with respect to each other or their gimbal stops without an effect on the total CMG angular momentum; i.e., without resulting in a net torque on the space vehicle. The general development of the CMG control law assumes arbitrary CMG momentum magnitudes; but it is also shown that the expressions can be simplified if it is assumed that all CMG angular momentum magnitudes are equal. This simplified version is presently used for control of the CMG's on Skylab-A.

INTRODUCTION

It is desirable for many space vehicles (especially for an orbiting space vehicle like Skylab) [1-4] to have an angular momentum storage device on board to accommodate cyclic angular momentum accumulations. This saves thruster attitude control fuel and simultaneously allows the reduction of the attitude error. Often three double-gimbaled control moment gyros (CMG's) are used. Then the need arises to command six gimbal angle rates to create a control torque on the vehicle which matches the commanded torque.

CMG control laws comtemplated in the past such as the cross product steering law [1,2] resulted in crosscoupling; i.e. the actual torque deviated from the commanded torque in magnitude and direction, even when ideal[1] CMG's were assumed. This report shows that the crosscoupling can be eliminated from the control of the CMG's by a law which considers the CMG's always in pairs, under the assumption that the CMG's are ideal.

For convenience, the control law is broken down into a steering law (which is the control law proper, and the only one to result in a net control torque on the vehicle) and two rotation laws. The conventions and the nomenclature of Skylab-A will be used throughout the development. The fact that failure of a single CMG necessitates two-CMG operations has also been kept in mind throughout the development.

DEVELOPMENT OF A NO-CROSSCOUPLING STEERING LAW FOR A CMG PAIR

Attitude control of a space vehicle is always achieved by application of a control torque $\underline{T}_V$ on the vehicle. For a CMG system with a total angular momentum $\underline{H}_T$ the relationship holds

$$\underline{T}_V = - \dot{\underline{H}}_T \qquad (1)$$

where $\dot{\underline{H}}_T$ is the change rate of $\underline{H}_T$ with respect to inertial space. The problem is therefore how to effect the desired CMG angular momentum change rate $\dot{\underline{H}}_T$.

The assumption is made that each CMG has a fixed, though arbitrary, angular momentum magnitude, generally different from the magnitudes of the other CMG's. Elimination of the crosscoupling in the CMG steering law requires that the actual angular momentum change is equal to the desired momentum change under the assumption that the commanded and the actual gimbal rates are equal. While one CMG cannot satisfy this condition, it is relatively easy for a pair of CMG's. This will be shown on pair A (CMG's 1 and 2) as an example; in the next section the steering law will be expanded to the other possible pairings. $\underline{H}_1$ and $\underline{H}_2$ are the angular momentum vectors of CMG's 1 and 2, with the initial positions indicated by the subscript I and the final by the

1. A control moment gyro which has no gimbal inertia, whose angular momentum magnitude and direction are known exactly, and which follows the commanded gimbal rates exactly.

subscript F. The desired momentum change is shown in Figure 1 as a momentum difference $\Delta \underline{H}_{SA}$ between the initial pair sum $\underline{H}_{SAI}$ and the final pair sum $\underline{H}_{SAF}$. The angular momentum vectors are all shown lying in the same plane as $\underline{H}_{SAI}$ and $\underline{H}_{SAF}$. This was done for clarity, but it would generally not be the case. On the other hand, $\underline{H}_{SA}$ is not disturbed by a rotation about itself and a rotation is therefore permissible for the development of the momentum change (it is indicative of the fact that one degree of freedom remains, which will be treated later).

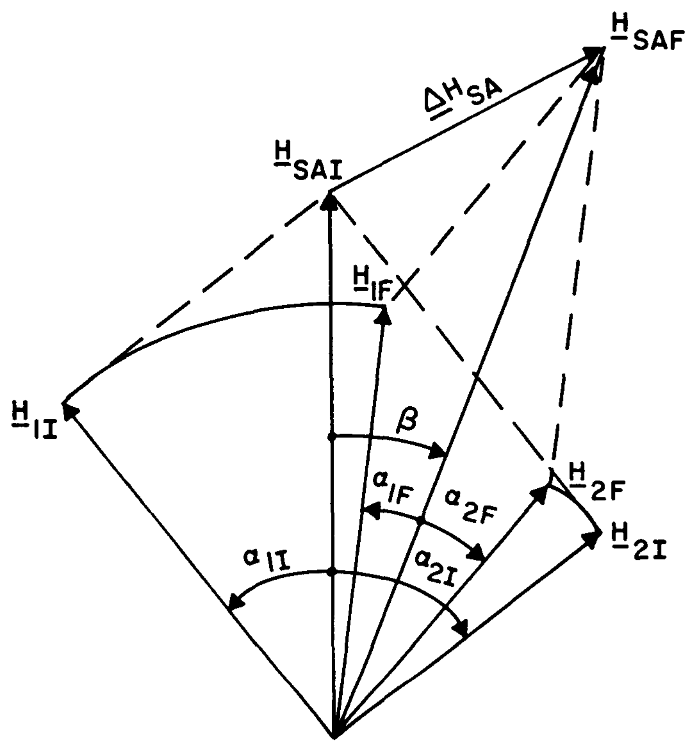

Figure 1. Momentum change.

The change of $\underline{H}_{SA}$ will be broken down into a rotation β of $\underline{H}_{SA}$ (a rotation of the CMG vector pair as a unit) and into a change in magnitude of $\underline{H}_{SA}$ by a change of the angles α_1 and α_2 (scissoring action of the momentum vectors with respect to each other). Of course, both motions occur simultaneously. It might be of interest to note that the sum H_{SA} has not only an upper limit $(H_1 + H_2)$ when the angular momentum vectors are parallel, but also a lower limit $(|H_1 - H_2|)$ when the vectors are antiparallel. The latter becomes important for two-CMG operation.

Before the angular velocities for pair rotation and scissoring are developed, it is convenient to define the following quantities (a bar below a letter indicates a vector; a quantity without a bar indicates either a scalar or a vector magnitude);

$\underline{H}_{SA} \equiv \underline{H}_1 + \underline{H}_2$	[Nms]	pair sum	(2)
$H_{DA} \equiv \underline{H}_1 \cdot \underline{H}_2$	$[(Nms)^2]$	pair dot product	(3)
$\underline{H}_{PA} \equiv \underline{H}_1 \times \underline{H}_2$	$[(Nms)^2]$	pair cross product	(4)
$\underline{T}_{CA} = \dot{\underline{H}}_{SA}$	[Nm]	torque command (equivalent to desired momentum change[2])	(5)
$\underline{u}_S \equiv \underline{H}_{SA}/H_{SA}$		unit vector along pair sum	(6)
$\underline{u}_P \equiv \underline{H}_{PA}/H_{PA}$		unit vector perpendicular to both $\underline{H}_1$ and $\underline{H}_2$	(7)
$\underline{u}_X \equiv (\underline{H}_{SA} \times \underline{T}_{CA})/\lvert\underline{H}_{SA} \times \underline{T}_{CA}\rvert$		unit vector perpendicular to both $\underline{H}_{SA}$ and $\underline{T}_{CA}$	(8)
$T_{CAP} = \underline{u}_S \cdot \underline{T}_{CA} = \dot{H}_{SA}$	[Nm]	component of $\underline{T}_{CA}$ along pair sum	(9)
$T_{CAX} = \lvert\underline{u}_S \times \underline{T}_{CA}\rvert$	[Nm]	component of $\underline{T}_{CA}$ perpendicular to pair sum	(10)

$$\left.\begin{array}{l} i = 1,2,3 \quad (11) \\ j = 1,2,3 \quad (12) \\ k = A,B,C \quad (13) \\ m = 1,3 \quad (14) \end{array}\right\} \text{subscripts used throughout}$$

2. Note that a positive torque command for the CMG's results in a negative torque (reaction) on the vehicle.

The pair rotation will be proportional to T_{CAX} and the necessary angular velocity command $\underline{\omega}_{XA}$ will be along the unit vector $\underline{u}_X$. The effectiveness of $\underline{\omega}_{XA}$ is proportional to H_{SA} and we get

$$\underline{\omega}_{XA} = T_{CAX}\underline{u}_X/H_{SA} \tag{15}$$

or

$$\underline{\omega}_{XA} = (\underline{H}_{SA} \times \underline{T}_{CA})/H_{SA}^{\;2} \quad . \tag{16}$$

Figure 1 will be used as an aid in the development of the angular rate command $\underline{\omega}_{PA1}$ and $\underline{\omega}_{PA2}$ needed for scissoring. They will be proportional to the component $T_{CAP} = \dot{H}_{SA}$. The angular velocity for scissoring will be

$$\underline{\omega}_{PA1} = \dot{\alpha}_1 \underline{u}_p \tag{17}$$

and

$$\underline{\omega}_{PA2} = \dot{\alpha}_2 \underline{u}_p \quad . \tag{18}$$

The following two equations hold

$$H_1 c\alpha_1 + H_2 c\alpha_2 = H_{SA} \tag{19}$$

$$H_1 s\alpha_1 + H_2 s\alpha_2 = 0 \quad . \tag{20}$$

Differentiation yields

$$\begin{bmatrix} -H_1 s\alpha_1 & -H_2 s\alpha_2 \\ H_1 c\alpha_1 & H_2 c\alpha_2 \end{bmatrix} \begin{bmatrix} \dot{\alpha}_1 \\ \dot{\alpha}_2 \end{bmatrix} = \begin{bmatrix} \dot{H}_{SA} \\ 0 \end{bmatrix} \tag{21}$$

or

$$\dot{\alpha}_1 = H_2 c\alpha_2 \dot{H}_{SA}/H_{PA} \tag{22}$$

$$\dot{\alpha}_2 = -H_1 c\alpha_1 \dot{H}_{SA}/H_{PA} \tag{23}$$

with

$$H_{PA} = H_1H_2(s\alpha_2 c\alpha_1 - c\alpha_2 s\alpha_1)$$

$$= H_1H_2 s(\alpha_2 - \alpha_1)$$

$$= |\underline{H}_1 \times \underline{H}_2| \quad . \tag{24}$$

With the relationships

$$H_1 c\alpha_1 = \underline{H}_1 \cdot \underline{u}_S = (H_1^2 + \underline{H}_1 \cdot \underline{H}_2)/H_{SA} \tag{25}$$

$$H_2 c\alpha_2 = \underline{H}_2 \cdot \underline{u}_S = (H_2^2 + \underline{H}_1 \cdot \underline{H}_2)/H_{SA} \tag{26}$$

$$\dot{H}_{SA} = T_{CAP} \tag{27}$$

and equations (22) and (23), equations (17) and (18) become

$$\underline{\omega}_{PA1} = (H_2^2 + H_{DA})(\underline{H}_{SA} \cdot \underline{T}_{CA})\underline{H}_{PA}/(H_{SA}{}^2 H_{PA}{}^2) \tag{28}$$

$$\underline{\omega}_{PA2} = -(H_1^2 + H_{DA})(\underline{H}_{SA} \cdot \underline{T}_{CA})\underline{H}_{PA}/(H_{SA}{}^2 H_{PA}{}^2) \quad . \tag{29}$$

Both CMG's participate in the pair rotation through $\underline{\omega}_{XA}$ [equation (16)] and in the scissoring through $\underline{\omega}_{PA1}$ and $\underline{\omega}_{PA2}$ [equations (28) and (29)] such that the angular velocity commands are

$$\underline{\omega}_1 = \underline{\omega}_{XA} + \underline{\omega}_{PA1} - \dot{\underline{\phi}}_V \tag{30}$$

$$\omega_2 = \underline{\omega}_{XA} + \underline{\omega}_{PA2} - \dot{\underline{\phi}}_V \quad . \tag{31}$$

The angular velocity of the vehicle must be subtracted since $\underline{\omega}_{XA}$, $\underline{\omega}_{PA1}$, and $\underline{\omega}_{PA2}$ are with respect to inertial space (otherwise $\dot{\underline{H}} \neq \underline{T}$), but $\underline{\omega}_1$ and $\underline{\omega}_2$ are with respect to the vehicle).

EXPANSION OF NO-CROSSCOUPLING STEERING LAW TO THREE CMG's

When three CMG's are operative, they can be paired three ways: $\underline{H}_1$ and $\underline{H}_2$ form pair A, $\underline{H}_2$ and $\underline{H}_3$ form pair B, and $\underline{H}_3$ and H_1 form pair C. Each CMG participates in two pairings and the resulting angular velocities must be added. Basically each pair can produce the commanded torque and a means must be found to split the total command into individual pair commands in such a way that the pair capabilities are considered. Equation (16) shows that $\underline{\omega}_{XA}$ is proportional to $1/H_{SA}$ and equation (28) or (29) shows that the $\underline{\omega}_{RAi}$'s are proportional to $1/H_{PA}$. Since H_{PA} goes to zero when H_{SA} reaches its minimum (zero for $H_1 = H_2$), the splitting or prorating will be done with a function of H_{PA} (prorating must be identical for $\underline{\omega}_{Xk}$ and $\underline{\omega}_{Pki}$ of the same pair). While prorating with H_{Pk} directly will make the $\underline{\omega}$'s insensitive to H_{Pk} when H_{Pk} goes to zero, it is desirable to have the angular velocity commands go to zero too, so that the case of one-CMG-out (i.e., failure of a single CMG) can be accepted without modification. Prorating is therefore done with $H_{Pk}{}^2$.

Before the variable gains used for prorating (splitting) of the torque command are developed, it is convenient to add the following definitions.

$$\underline{H}_{SA} = \underline{H}_1 + \underline{H}_2 \quad [\text{Nms}] \tag{32}$$

$$\underline{H}_{SB} = \underline{H}_2 + \underline{H}_3 \quad [\text{Nms}] \tag{33}$$

$$\underline{H}_{SC} = \underline{H}_3 + \underline{H}_1 \quad [\text{Nms}] \tag{34}$$

$$\underline{H}_{T} = \underline{H}_1 + \underline{H}_2 + \underline{H}_3 \quad [\text{Nms}] \tag{35}$$

$$H_{DA} = \underline{H}_1 \cdot \underline{H}_2 \quad [(\text{NMS})^2] \tag{36}$$

$$H_{DB} = \underline{H}_2 \cdot \underline{H}_3 \quad [(\text{Nms})^2] \tag{37}$$

$$H_{DC} = \underline{H}_3 \cdot \underline{H}_1 \quad [(\text{Nms})^2] \tag{38}$$

$$\underline{H}_{PA} = \underline{H}_1 \times \underline{H}_2 \quad [(\text{Nms})^2] \tag{39}$$

$$\underline{H}_{PB} = \underline{H}_2 \times \underline{H}_3 \qquad [(\text{Nms})^2] \tag{40}$$

$$\underline{H}_{PC} = \underline{H}_3 \times \underline{H}_1 \qquad [(\text{Nms})^2] \tag{41}$$

$$\Sigma H_P^2 = H_{PA}^2 + H_{PB}^2 + H_{PC}^2 \qquad [(\text{Nms})^4] \tag{42}$$

With the above definitions and the preceding discussion, the variable prorating gains become

$$G_A = H_{PA}^2/\Sigma H_P^2 \tag{43}$$

$$G_B = H_{PB}^2/\Sigma H_P^2 \tag{44}$$

$$G_C = H_{PC}^2/\Sigma H_P^2 \tag{45}$$

and the torque commands become

$$\underline{T}_{CA} = G_A \underline{T}_C \tag{46}$$

$$\underline{T}_{CB} = G_B \underline{T}_C \tag{47}$$

$$\underline{T}_{CC} = G_C \underline{T}_C \quad . \tag{48}$$

The angular velocity commands for pair A from equations (16), (28), and (29) are now

$$\begin{aligned} \underline{\omega}_{XA} &= (1/H_{SA}^2)(\underline{H}_{SA} \times \underline{T}_{CA}) \\ &= (G_A/H_{SA}^2)(\underline{H}_{SA} \times \underline{T}_C) \\ &= \left[H_{PA}^2/(H_{SA}^2\, \Sigma H_P^2)\right](\underline{H}_{SA} \times \underline{T}_C) \end{aligned} \tag{49}$$

$$\begin{aligned} \underline{\omega}_{PA1} &= \left[(H_2^2 + H_{DA})(\underline{H}_{SA} \cdot \underline{T}_{CA})/(H_{SA}^2 H_{PA}^2)\right] \underline{H}_{PA} \\ &= \left[G_A (H_2^2 + H_{DA})(\underline{H}_{SA} \cdot \underline{T}_C)/(H_{SA}^2 H_{PA}^2)\right] \underline{H}_{PA} \\ &= \left[(H_2^2 + H_{DA})(\underline{H}_{SA} \cdot \underline{T}_C)/(H_{SA}^2\, \Sigma H_P^2)\right] \underline{H}_{PA} \end{aligned} \tag{50}$$

$$\underline{\omega}_{PA2} = -\left[(H_1^2 + H_{DA})(\underline{H}_{SA} \cdot \underline{T}_C)/(H_{SA}^2\, \Sigma H_P^2)\right]\underline{H}_{PA} \quad . \tag{51}$$

The angular velocity commands for all pairs are then (pair A commands are repeated for completeness)

$$\underline{\omega}_{XA} = \left[H_{PA}^2/(H_{SA}^2\, \Sigma H_P^2)\right](\underline{H}_{SA} \times \underline{T}_C) \tag{52}$$

$$\underline{\omega}_{XB} = \left[H_{PB}^2/(H_{SB}^2\, \Sigma H_P^2)\right](\underline{H}_{SB} \times \underline{T}_C) \tag{53}$$

$$\underline{\omega}_{XC} = \left[H_{PC}^2/(H_{SC}^2\, \Sigma H_P^2)\right](\underline{H}_{SC} \times \underline{T}_C) \tag{54}$$

$$\underline{\omega}_{PA1} = \left[(H_2^2 + H_{DA})(\underline{H}_{SA} \cdot \underline{T}_C)/(H_{SA}^2\, \Sigma H_P^2)\right]\underline{H}_{PA} \tag{55}$$

$$\underline{\omega}_{PB2} = \left[(H_3^2 + H_{DB})(\underline{H}_{SB} \cdot \underline{T}_C)/(H_{SB}^2\, \Sigma H_P^2)\right]\underline{H}_{PB} \tag{56}$$

$$\underline{\omega}_{PC3} = \left[(H_1^2 + H_{DC})(\underline{H}_{SC} \cdot \underline{T}_C)/(H_{SC}^2\, \Sigma H_P^2)\right]\underline{H}_{PC} \tag{57}$$

$$\underline{\omega}_{PA2} = -\left[(H_1^2 + H_{DA})(\underline{H}_{SA} \cdot \underline{T}_C)/(H_{SA}^2\, \Sigma H_P^2)\right]\underline{H}_{PA} \tag{58}$$

$$\underline{\omega}_{PB3} = -\left[(H_2^2 + H_{DB})(\underline{H}_{SB} \cdot \underline{T}_C)/(H_{SB}^2\, \Sigma H_P^2)\right]\underline{H}_{PB} \tag{59}$$

$$\underline{\omega}_{PC1} = -\left[(H_3^2 + H_{DC})(\underline{H}_{SC} \cdot \underline{T}_C)/(H_{SC}^2\, \Sigma H_P^2)\right]\underline{H}_{PC} \quad . \tag{60}$$

The CMG angular velocity commands resulting from the steering law are

$$\underline{\omega}_1 = \underline{\omega}_{XA} + \underline{\omega}_{PA1} + \underline{\omega}_{XC} + \underline{\omega}_{PC1} - \dot{\underline{\phi}}_V \tag{61}$$

$$\underline{\omega}_2 = \underline{\omega}_{XB} + \underline{\omega}_{PB2} + \underline{\omega}_{XA} + \underline{\omega}_{PA2} - \dot{\underline{\phi}}_V \tag{62}$$

$$\underline{\omega}_3 = \underline{\omega}_{XC} + \underline{\omega}_{PC3} + \underline{\omega}_{XB} + \underline{\omega}_{PB3} - \dot{\underline{\phi}}_V \tag{63}$$

Appendix A shows that equations (52) and (60) can be simplified if the angular momenta of the CMG's are equal.

Appendix B shows that the actual torque $\underline{T}$ is equal to the commanded torque $\underline{T}_C$ if the commanded and the actual gimbal rates are equal.

TRANSFORMATION OF A GENERAL RATE INTO GIMBAL RATES

The angular velocity commands [equations (61) to (63)] are generally not perpendicular to the CMG angular momentum vectors and do not depend on the CMG mounting configuration. The gimbal rate commands obviously do depend on the mounting orientation of the individual CMG. The CMG mounting configuration for Skylab-A is used (Figure 2). This configuration is cyclicly permutable and the gimbal rates will be developed for CMG 1 and then permuted for the other two.

The momentum change of CMG 1 resulting from the commanded velocity $\underline{\omega_1}$ should also result from $\underline{\dot{\delta}_1}$ (where it is assumed that the actual and the commanded gimbal rates are equal):

$$\underline{\omega_1} \times \underline{H_1} = \dot{\delta}_1 \times \underline{H_1} \qquad (64)$$

or

$$(\underline{\omega_1} - \underline{\dot{\delta}_1}) \times \underline{H_1} = 0 \quad . \qquad (65)$$

Geometric relationships give (s = sin, c = cos)

$$\underline{\dot{\delta}_1} = \begin{bmatrix} +\dot{\delta}_{1(1)} s\delta_{3(1)} \\ +\dot{\delta}_{1(1)} c\delta_{3(1)} \\ -\dot{\delta}_{3(1)} \end{bmatrix} \qquad (66)$$

$$\underline{H_1} = H_1 \begin{bmatrix} +c\delta_{1(1)} c\delta_{3(1)} \\ -c\delta_{1(1)} s\delta_{3(1)} \\ -s\delta_{1(1)} \end{bmatrix} \quad . \qquad (67)$$

With equations (66) and (67), equation (65) results in

$$-\left(\omega_{12} - \dot{\delta}_{1(1)} c\delta_{3(1)}\right) s\delta_{1(1)} + \left(\omega_{13} + \dot{\delta}_{3(1)}\right) c\delta_{1(1)} s\delta_{3(1)} = 0 \qquad (68)$$

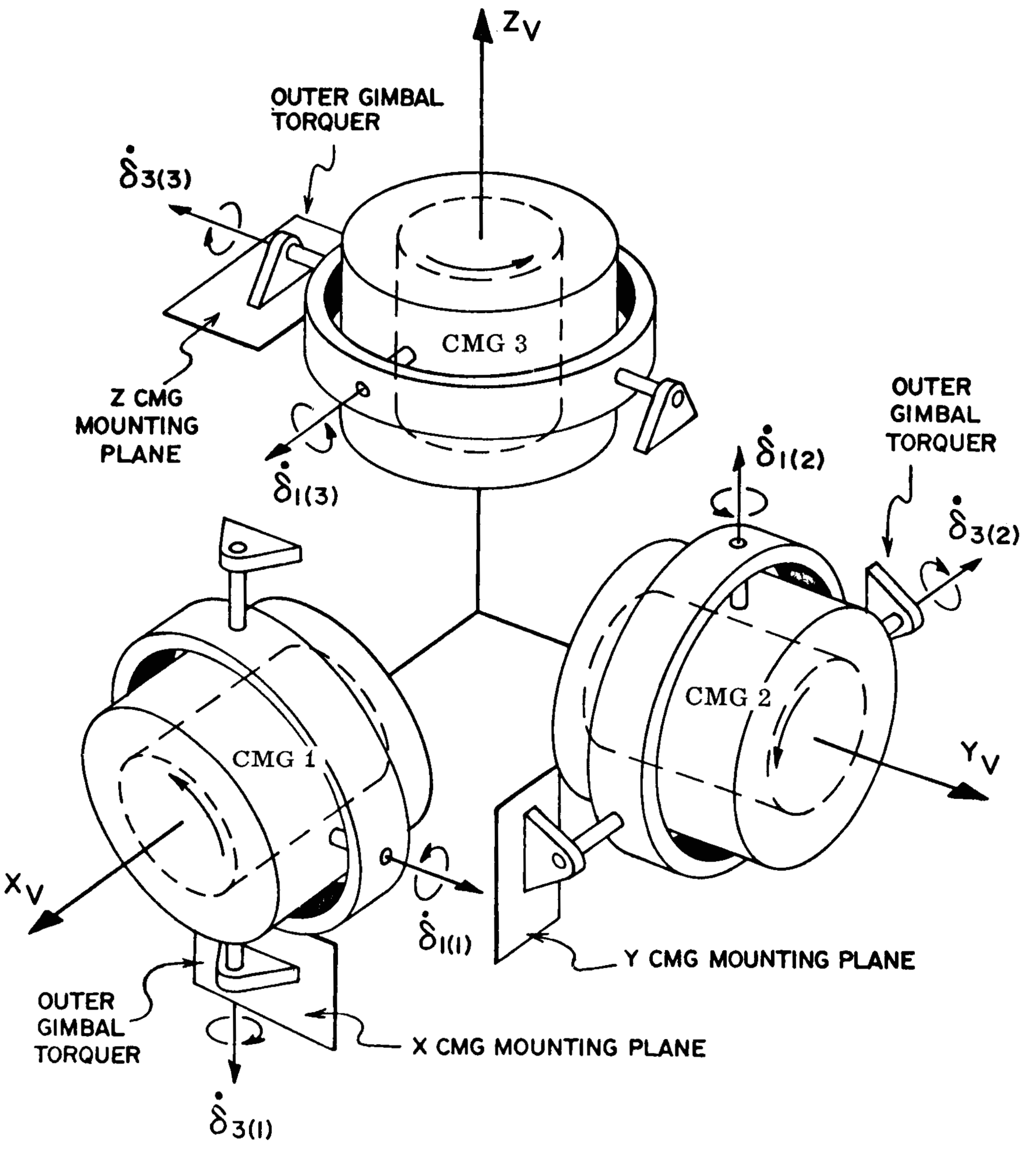

Figure 2. Control moment gyro orientations.

$$\left(\omega_{13} + \dot{\delta}_{3(1)}\right) c\delta_{1(1)} c\delta_{3(1)} + \left(\omega_{11} - \delta_{1(1)} s\delta_{3(1)}\right) s\delta_{3(1)} = 0 \qquad (69)$$

$$- \left(\omega_{11} - \dot{\delta}_{1(1)} s\delta_{3(1)}\right) c\delta_{1(1)} s\delta_{3(1)}$$

$$- \left(\omega_{12} - \delta_{1(1)} c\delta_{3(1)}\right) c\delta_{1(1)} c\delta_{3(1)} = 0 \quad . \qquad (70)$$

Equation (70) yields

$$\dot{\delta}_{1(1)} = \omega_{11} s\delta_{3(1)} + \omega_{12} c\delta_{3(1)} \quad . \qquad (71)$$

This result inserted into equation (68) or (69) yields (t = tan)

$$\dot{\delta}_{3(1)} = -t\delta_{1(1)}\left(\omega_{11} c\delta_{3(1)} - \omega_{12} s\delta_{3(1)}\right) - \omega_{13} \quad . \qquad (72)$$

Putting equations (71) and (72) in matrix form and permuting for CMG's 2 and 3 results in

$$\begin{bmatrix} \dot{\delta}_{1(1)} \\ \dot{\delta}_{3(1)} \end{bmatrix} = \begin{bmatrix} s\delta_{3(1)} & c\delta_{3(1)} & 0 \\ -t\delta_{1(1)} c\delta_{3(1)} & t\delta_{1(1)} s\delta_{3(1)} & -1 \end{bmatrix} \begin{bmatrix} \omega_{11} \\ \omega_{12} \\ \omega_{13} \end{bmatrix} \qquad (73)$$

$$\begin{bmatrix} \dot{\delta}_{1(2)} \\ \dot{\delta}_{3(2)} \end{bmatrix} = \begin{bmatrix} 0 & s\delta_{3(2)} & c\delta_{3(2)} \\ -1 & -t\delta_{1(2)} c\delta_{3(2)} & t\delta_{1(2)} s\delta_{3(2)} \end{bmatrix} \begin{bmatrix} \omega_{21} \\ \omega_{22} \\ \omega_{23} \end{bmatrix} \qquad (74)$$

$$\begin{bmatrix} \dot{\delta}_{1(3)} \\ \dot{\delta}_{3(3)} \end{bmatrix} = \begin{bmatrix} c\delta_{3(3)} & 0 & s\delta_{3(3)} \\ t\delta_{1(3)} s\delta_{3(3)} & -1 & -t\delta_{1(3)} c\delta_{3(3)} \end{bmatrix} \begin{bmatrix} \omega_{31} \\ \omega_{32} \\ \omega_{33} \end{bmatrix} \qquad (75)$$

The direction cosines of the CMG's are usually available and we define $\underline{e}_i$ to be a unit vector along the angular momentum vector of the ith CMG with the following result (for the Skylab-A CMG configuration, Figure 2):

$$\underline{e}_1 = \begin{bmatrix} e_{11} \\ e_{12} \\ e_{13} \end{bmatrix} = \begin{bmatrix} +c\delta_{1(1)} c\delta_{3(1)} \\ -c\delta_{1(1)} s\delta_{3(1)} \\ -s\delta_{1(1)} \end{bmatrix} \tag{76}$$

$$\underline{e}_2 = \begin{bmatrix} e_{21} \\ e_{22} \\ e_{23} \end{bmatrix} = \begin{bmatrix} -s\delta_{1(2)} \\ +c\delta_{1(2)} c\delta_{3(2)} \\ -c\delta_{1(2)} s\delta_{3(2)} \end{bmatrix} \tag{77}$$

$$\underline{e}_3 = \begin{bmatrix} e_{31} \\ e_{32} \\ e_{33} \end{bmatrix} \begin{bmatrix} -c\delta_{1(3)} s\delta_{3(3)} \\ -s\delta_{1(3)} \\ +c\delta_{1(3)} c\delta_{3(3)} \end{bmatrix} . \tag{78}$$

Equations (73), (74), and (75) can now be expressed in terms of the e_{ij}. With the additional definitions of

$$e_{13}' \equiv 1/c\delta_{1(1)} = 1/\sqrt{1 - e_{13}^2} \tag{79}$$

$$e_{21}' \equiv 1/c\delta_{1(2)} = 1/\sqrt{1 - e_{21}^2} \tag{80}$$

$$e_{32}' \equiv 1/c\delta_{1(3)} = 1/\sqrt{1 - e_{32}^2} \tag{81}$$

the result is

$$\dot{\delta}_{1(1)} = e_{13}'(e_{11}\omega_{12} - e_{12}\omega_{11}) \tag{82}$$

$$\dot{\delta}_{1(2)} = e_{21}'(e_{22}\omega_{23} - e_{23}\omega_{22}) \tag{83}$$

$$\dot{\delta}_{1(3)} = e_{32}'(e_{33}\omega_{31} - e_{31}\omega_{22}) \tag{84}$$

$$\dot{\delta}_{3(1)} = (e_{13}')^2 e_{13}(e_{11}\omega_{i1} + e_{12}\omega_{i2}) - \omega_{13} \tag{85}$$

$$\dot{\delta}_{3(2)} = (e_{21}')^2 e_{21}(e_{22}\omega_{22} + e_{23}\omega_{23}) - \omega_{21} \tag{86}$$

$$\dot{\delta}_{3(3)} = (e_{32}')^2 e_{32}(e_{33}\omega_{33} + e_{31}\omega_{31}) - \omega_{32} \tag{87}$$

where it should be remembered that $\underline{\omega}_i = \mathrm{col}(\omega_{i1}, \omega_{i2}, \omega_{i3})$ can be any angular velocity.

ROTATION LAWS

Three of the six degrees of freedom of the CMG configuration are used for the generation of a control torque on the vehicle. The remaining three are the rotations of the pairs about their sums (and also a rotation of all three CMG's together about their total sum which is a linear combination of the sum rotations). All these rotations do not result in a momentum change; i.e., no torque is exerted on the vehicle. The rotations have the following form:

$$\underline{\omega}_{RA} = \epsilon_A(\underline{H}_{SA}/H_{SA}) \tag{88}$$

$$\underline{\omega}_{RB} = \epsilon_B(\underline{H}_{SB}/H_{SB}) \tag{89}$$

$$\underline{\omega}_{RC} = \epsilon_C(\underline{H}_{SC}/H_{SC}) \tag{90}$$

$$\underline{\omega}_{RT} = \epsilon_T(\underline{H}_T/H_T) \tag{91}$$

and can be used for some benefit. Contrary to the steering law, there is no unique way to determine the epsilons. One possible solution is given which proved successful in gimbal stop avoidance and in keeping the vectors well separated; the implication being that the CMG's have limited freedom of gimbal movement. All epsilons can be used for gimbal stop avoidance (R-subscript) but only ϵ_A, ϵ_B, and ϵ_C can be used for a proper distribution (momentum vector separation; D-subscript) and they are therefore split into two parts:

$$\epsilon_A = \epsilon_{RA} + \epsilon_{DA} \tag{92}$$

$$\epsilon_B = \epsilon_{RB} + \epsilon_{DB} \tag{93}$$

$$\epsilon_C = \epsilon_{RC} + \epsilon_{DC} \quad . \tag{94}$$

The distribution only applies for the case of three CMG's. No distribution is necessary for the two-CMG case (the vectors are already located at their proper separation and this separation depends upon the sum which cannot be altered).

Gimbal Stop Avoidance (Rotation Law)

Avoidance of the gimbal stops is treated first, and it is referred to as the rotation law (R-subscript) in spite of the fact that the distribution law also uses rotations about the individual momentum vector sums. CMG pair A is used again for the development. Four gimbal angles are affected by ϵ_{RA}; i.e., a compromise is necessary, and it is therefore desirable to make ϵ_{RA} the sum of the individually desirable rotations:

$$\epsilon_{RA} = \epsilon_{A11} + \epsilon_{A31} + \epsilon_{A12} + \epsilon_{A32} \quad . \tag{95}$$

A desirable rotation is such that the gimbal angle magnitude is reduced; i.e., $\delta_{m(i)} \dot{\delta}_{m(i)} < 0$. Therefore each component of ϵ_{RA} was chosen to be of the form

$$\epsilon_{Ami} = - K_R F_{mi} S_{Ami} \tag{96}$$

where K_R is a fixed gain, F_{mi} is an odd gimbal angle function, and S_{Ami} is a modified sign function. The F-functions for the inner gimbal angles are

$$F_{1i} = \left[\delta_{1(i)}\right]^n \tag{97}$$

and for the outer gimbal angles we select

$$F_{3i} = \left[R\left(\delta_{3(i)}\right) - \pi/4\right]^n \quad . \tag{98}$$

The F-functions for the outer gimbal angles had to be modified because the center between the stops is at $+\pi/4$ for Skylab-A, which is the example used throughout this report. The multiplier R is the ratio of inner to outer gimbal freedom giving the gimbal angles equal weight at their stops. The fifth power ($n = 5$) was found by simulation to be appropriate for three-CMG operation where for small gimbal angles the distribution (vector separation) should have preference. For two-CMG operation, however, the first power was found to be more suitable.

The need for the S_{Ami}-functions arises from the fact that the polarity of the gimbal rate depends also on the direction of the pair sum with respect to the individual CMG. To establish the S_{Ami}-functions, a unit vector along $\underline{H}_{SA}$ is used with the gimbal rate equations (82) through (87):

$$S_{A11} = e_{13}'\left[e_{11}(H_{SA2}/H_{SA}) - e_{12}(H_{SA1}/H_{SA})\right]$$

$$= (e_{13}'H_2/H_{SA})(e_{11}e_{22} - e_{12}e_{21}) \tag{99}$$

$$S_{A12} = e_{21}'\left[e_{22}(H_{SA3}/H_{SA}) - e_{23}(H_{SA2}/H_{SA})\right]$$

$$= (e_{21}'H_1/H_{SA})(e_{22}e_{13} - e_{23}e_{12}) \tag{100}$$

$$S_{A31} = (e_{13}')^2 e_{13}\left[e_{11}(H_{SA1}/H_{SA}) + e_{12}(H_{SA2}/H_{SA})\right] - (H_{SA3}/H_{SA})$$

$$= (H_2/H_{SA})\left[(e_{13}')^2 e_{13}(e_{11}e_{21} + e_{12}e_{22}) - e_{23}\right] \tag{101}$$

$$S_{A32} = (e_{21}')^2 e_{21}\left[e_{22}(H_{SA2}/H_{SA}) + e_{23}(H_{SA3}/H_{SA})\right] - (H_{SA1}/H_{SA})$$

$$= (H_1/H_{SA})\left[(e_{21}')^2 e_{21}(e_{22}e_{12} + e_{23}e_{13}) - e_{11}\right] . \tag{102}$$

For pair B and C we get (through cyclic permutation):

$$S_{B12} = (e_{21}' H_3/H_{SB})(e_{22}e_{33} - e_{23}e_{32}) \tag{103}$$

$$S_{C13} = (e_{32}'H_1/H_{SC})(e_{33}e_{11} - e_{31}e_{13}) \tag{104}$$

$$S_{B13} = (e_{32}'H_2/H_{SB})(e_{33}e_{21} \quad e_{31}e_{23}) \tag{105}$$

$$S_{C11} = (e_{13}'H_3/H_{SC})(e_{11}e_{32} - e_{12}e_{31}) \tag{106}$$

$$S_{B32} = (H_3/H_{SB})\left[(e_{21}')^2 e_{21}(e_{22}e_{32} + e_{23}e_{33}) - e_{31}\right] \tag{107}$$

$$S_{C33} = (H_1/H_{SC})\left[(e_{32}')e_{32}(e_{33}e_{13} + e_{31}e_{11}) - e_{12}\right] \tag{108}$$

$$S_{B33} = (H_2/H_{SB})\left[(e_{32}')e_{32}(e_{33}e_{23} + e_{31}e_{21}) - e_{22}\right] \tag{109}$$

$$S_{C31} = (H_3/H_{SC})\left[(e_{13}')e_{13}(e_{11}e_{31} + e_{12}e_{32}) - e_{33}\right] \quad . \tag{110}$$

All gimbal angles are affected by a rotation about the total angular momentum and we select

$$\epsilon_T = \epsilon_{T11} + \epsilon_{T12} + \epsilon_{T13} + \epsilon_{T31} + \epsilon_{T32} + \epsilon_{T33} \quad . \tag{111}$$

Again the selected form is

$$\epsilon_{Tmi} = - K_R F_{mi} S_{Tmi} \quad . \tag{112}$$

The angle functions F_{mi} are unchanged. The S_{Tmi} terms will be developed along the same line as the other S-functions (CMG 1 serves as an example)

$$\begin{aligned} S_{T11} &= e_{13}'\left[e_{11}(H_{T2}/H_T) - e_{12}(H_{T1}/H_T)\right] \\ &= (e_{13}'H_2/H_T)(e_{11}e_{22} - e_{12}e_{21}) \\ &\quad + (e_{13}'H_3/H_T(e_{11}e_{32} - e_{12}e_{31}) \\ &= (H_{SA}S_{A11} + H_{SC}S_{C11})/H_T \end{aligned} \tag{113}$$

$$S_{T31} = (e_{13}')^2 e_{13}\left[e_{11}(H_{T1}/H_T) + e_{12}(H_{T2}/H_T)\right] - (H_{T3}/H_T)$$

$$= (H_2/H_T)\left[(e_{13}')^2 e_{13}(e_{11}e_{21} + e_{12}e_{22}) - e_{23}\right]$$

$$+ (H_3/H_T)\left[(e_{13}')^2 e_{13}(e_{11}e_{31} + e_{12}e_{32}) - e_{33}\right]$$

$$= (H_{SA}S_{A31} + H_{SC}S_{C31})/H_T \quad . \tag{114}$$

Cyclic permutation yields for CMG's 2 and 3

$$S_{T12} = (H_{SB}S_{B12} + H_{SA}S_{A12})/H_T \tag{115}$$

$$S_{T13} = (H_{SC}S_{C13} + H_{SB}S_{B13})/H_T \tag{116}$$

$$S_{T32} = (H_{SB}S_{B32} + H_{SA}S_{A32})/H_T \tag{117}$$

$$S_{T33} = (H_{SC}S_{C33} + H_{SB}S_{B33})/H_T \quad . \tag{118}$$

The S-functions will have an upper limit of S_L. This provides a linear range (besides the sign information) which is needed to avoid limit cycling otherwise introduced by the sign changes. The value of S_L allows selection of the linear range and the loss in gain ($S_L < 1$) can be made up by an increase in K_R.

CMG Vector Separation (Distribution Law)

The bulk of the momentum change occurs quite frequently along a well known axis [5]. If this is the case, one type of distribution law (D-law) can be applied which will separate the CMG vectors by trying to make them contribute equally (in proportion to their magnitudes) to the angular momentum along this axis.

Let $\underline{e}_N$ be a unit vector along the bulk of the momentum change (the orbit normal for Skylab); then the angle between $\underline{e}_N$ and the plane formed by a vector pair will be maximized by (for pair A as an example)

$$\epsilon_{DA} = K_D\Big\{\Big[(\underline{H}_1/H_N) - (\underline{H}_1/H_N) \cdot (\underline{H}_{SA}/H_{SA})(\underline{H}_{SA}/H_{SA})\Big]$$

$$- \Big[(\underline{H}_2/H_N) - (\underline{H}_2/H_N) \cdot (\underline{H}_{SA}/H_{SA})(\underline{H}_{SA}/H_{SA})\Big]\Big\} \cdot \underline{e}_N \quad (119)$$

The significance of the various terms is illustrated in Figure 3. K_D is a constant gain and H_N is a nominal momentum used for normalization. Evaluation leads to the intermediate step

$$\epsilon_{DA} = (K_D/H_N)\Big[\underline{H}_1 - \underline{H}_2 - (H_1^2 - H_2^2)(\underline{H}_1 + \underline{H}_2)/H_{SA}^{\,2}\Big] \cdot \underline{e}_N \quad . \quad (120)$$

The third term in the brackets shows that it vanishes when the magnitudes of the angular momentum vectors are equal. Further evaluation yields

$$\epsilon_{DA} = (K_D/H_N H_{SA}^{\,2})\Big[(H_2^{\,2} + H_{DA})\underline{H}_1 - (H_1^{\,2} + H_{DA})\underline{H}_2\Big] \cdot \underline{e}_N \quad (121)$$

with $H_{DA} = \underline{H}_1 \cdot \underline{H}_2$ [cf. equations (28) and (29)]. It should be noted that

$$\underline{H}_{SA} \times \underline{H}_{PA} = (H_2^{\,2} + H_{DA})\underline{H}_1 - (H_1^{\,2} + H_{DA})\underline{H}_2 \quad . \quad (122)$$

With this equality and the use of cyclic permutations we get

$$\epsilon_{DA} = (K_D/H_N H_{SA}^{\,2})(\underline{H}_{SA} \times \underline{H}_{PA}) \cdot \underline{e}_N \quad (123)$$

$$\epsilon_{DB} = (K_D/H_N H_{SB}^{\,2})(\underline{H}_{SB} \times \underline{H}_{PB}) \cdot \underline{e}_N \quad (124)$$

$$\epsilon_{DC} = (K_D/H_N H_{SC}^{\,2})(\underline{H}_{SC} \times \underline{H}_{PC}) \cdot \underline{e}_N \quad . \quad (125)$$

This form results in an isogonal[3] distribution about the total angular momentum vector $\underline{H}_T$ if $\underline{e}_N$ is along $\underline{H}_T$, if all CMG momentum magnitudes are equal, and if there is no rotation law effective ($K_R = 0$). Otherwise, a compromise results between the tendencies to spread the vectors and to reduce the gimbal angles.

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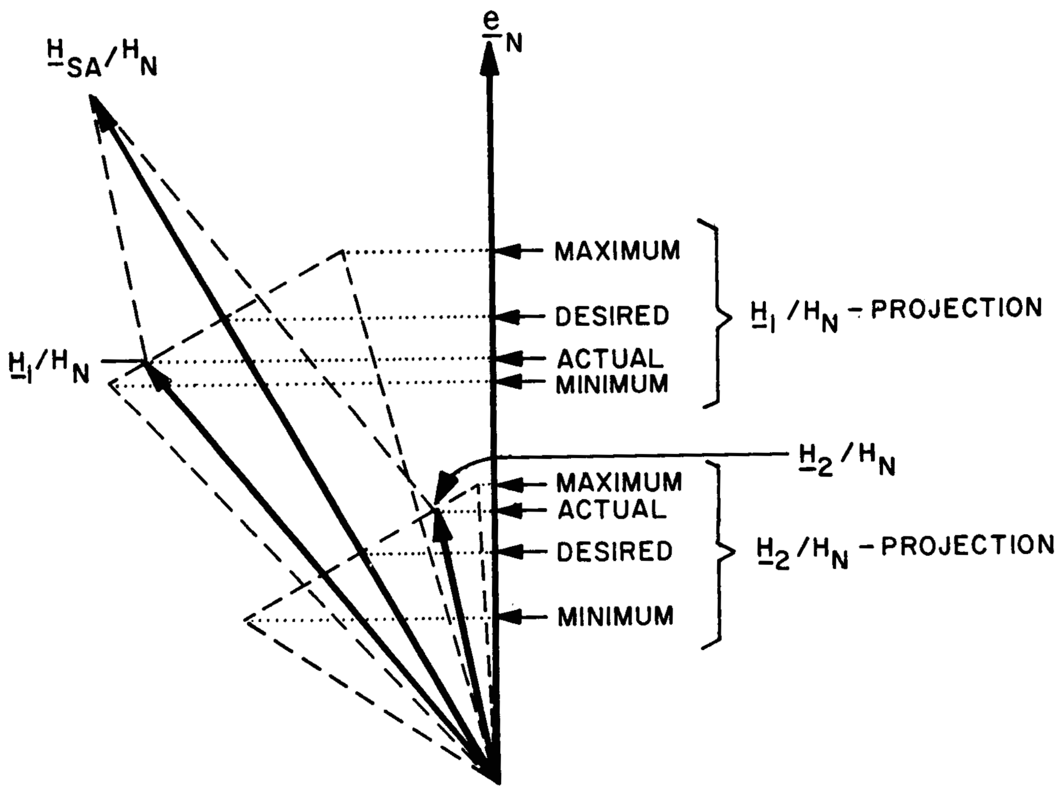
3. A distribution of three control moment gyros of equal momentum magnitude which contribute equally to their total momentum. This distribution results in equal angles between the individual momentum vectors and the total vector and equal angles between the vectors themselves.



ACTUAL PROJECTION $(\underline{H}_i/H_N)\cdot\underline{e}_N$

DESIRED PROJECTION $\left[(\underline{H}_i/H_N)\cdot(\underline{H}_{SA}/H_{SA})\right](\underline{H}_{SA}/H_{SA})\cdot\underline{e}_N$

NOTE: $\underline{H}_{SA}/H_N$ & $\underline{e}_N$ LIE IN THE PAPER PLANE

$\underline{H}_1/H_N$ AND $\underline{H}_2/H_N$ DO NOT LIE IN THE PAPER PLANE, BUT IN A PLANE WITH $\underline{H}_{SA}/H_N$

Figure 3. Distribution.

A positive K_R yields a right-handed and a negative K_R yields a left-handed configuration (looking down on $\underline{e}_N$, right-handed means a sequence of $\underline{H}_1$-$\underline{H}_2$-$\underline{H}_3$). Both configurations are stable. The location of the gimbal stops with respect to $\underline{e}_N$ determines which of the two configurations is preferable.

TOTAL CMG ANGULAR VELOCITY COMMANDS

The CMG angular velocity commands from the various sources can be vectorially added to form the total CMG angular velocity commands:

$$\underline{\omega}_1 = \underline{\omega}_{XA} + \underline{\omega}_{PA1} + \underline{\omega}_{XC} + \underline{\omega}_{PC1} + \underline{\omega}_{RA} + \underline{\omega}_{RC} + \underline{\omega}_{RT} - \underline{\dot{\phi}}_V \tag{126}$$

$$\underline{\omega}_2 = \underline{\omega}_{XB} + \underline{\omega}_{PB2} + \underline{\omega}_{XA} + \underline{\omega}_{PA2} + \underline{\omega}_{RB} + \underline{\omega}_{RA} + \underline{\omega}_{RT} - \underline{\dot{\phi}}_V \tag{127}$$

$$\underline{\omega}_3 = \underline{\omega}_{XC} + \underline{\omega}_{PC3} + \underline{\omega}_{XB} + \underline{\omega}_{PB3} + \underline{\omega}_{RC} + \underline{\omega}_{RB} + \underline{\omega}_{RT} - \underline{\dot{\phi}}_V \tag{128}$$

CONCLUSIONS

A no-crosscoupling control law for double-gimbaled CMG's can be developed using easily understandable kinematic relationships. The control law is based on a CMG pair as the smallest unit, which can give a no-crosscoupling control law; but the law lends itself to easy expansion to the control of any number of CMG's as shown by the expansion to three CMG's. The development is based on the restriction that the CMG's have fixed momentum magnitudes, though the individual magnitudes are not necessarily equal to each other. Unequal magnitudes resulted for the two-CMG case in an upper as well as a lower limit for the total angular momentum. The lower limit approaches zero when the momentum magnitudes become equal, which therefore is a desirable characteristic.

APPENDIX A

CONTROL LAW FOR NOMINAL ANGULAR MOMENTUM MAGNITUDE[4]

If the assumption is made that the angular momentum magnitude of each of the CMG's is equal to the nominal value H_N, simplification and normalizations can be applied. We have

$$\underline{H}_1 = H_N \underline{e}_1 \tag{A1}$$

$$\underline{H}_2 = H_N \underline{e}_2 \tag{A2}$$

$$\underline{H}_3 = H_N \underline{e}_3 \tag{A3}$$

where the $\underline{e}$'s are unit vectors along the CMG's angular momentum [equations (76) to (78)] whose components usually are available from gimbal resolver chains. Using pair A as an example we also have

$$\begin{aligned} \underline{H}_{SA} &= H_N(\underline{e}_1 + \underline{e}_2) \\ &= H_N \underline{e}_{SA} \end{aligned} \tag{A4}$$

$$\begin{aligned} \underline{H}_{PA} &= H_N^2(\underline{e}_1 \times \underline{e}_2) \\ &= H_N^2 \underline{e}_{PA} \quad . \end{aligned} \tag{A5}$$

The relation $H_1 = H_2 = H_N$ leads to $\alpha_2 = -\alpha_1 = \alpha$ and this results in $(i = 1,2)$

$$\begin{aligned} (H_i^2 + H_{DA}) &= (H_N^2 + H_N^2 c2\alpha) \\ &= 2(H_N c\alpha)^2 \\ &= 2(H_{SA}/2)^2 \quad . \end{aligned} \tag{A6}$$

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4. This simplified version is presently used for control of the CMG's on Skylab-A.

Consequently we have

$$(H_i^2 + H_{DA})/H_{SA}^{\;2} = 1/2 \quad . \tag{A7}$$

Normalized Steering Law

Introduction of the above normalizations and simplifications yields

$$\underline{\omega}_{XA} = \left[e_{PA}^{\;2}/\left(e_{SA}^{\;2}\, \Sigma e_P^{\,2}\right)\right]\left(\underline{e}_{SA} \times \dot{\underline{e}}_C\right) \tag{A8}$$

with

$$\dot{\underline{e}}_C = \underline{T}_C/H_N \tag{A9}$$

and

$$\Sigma e_P^{\;2} = e_{PA}^{\;2} + e_{PB}^{\;2} + e_{PC}^{\;2} \quad . \tag{A10}$$

We now have ($\underline{\omega}_{PA1} = - \underline{\omega}_{PA2} = \underline{\omega}_{PA}$)

$$\underline{\omega}_{PA} = \left[(\underline{e}_{SA} \cdot \dot{\underline{e}}_C)/\left(2\Sigma e_P^{\,2}\right)\right]\underline{e}_{PA} \quad . \tag{A11}$$

With $H_1 = H_2 = H_N$ it is now possible that the sum $\underline{H}_1 + \underline{H}_2$ goes to zero which will result in $\underline{\omega}_{XA}$ being indeterminate since the cross product goes to zero simultaneously. This can be avoided by the relationship

$$\begin{aligned}(e_{PA}/e_{SA})^2 &= (s2\alpha)^2/(2c\alpha)^2 \\ &= s^2\alpha \\ &= 1 - c^2\alpha \\ &= 1 - (e_{SA}/2)^2 \quad .\end{aligned} \tag{A12}$$

With equation (A12), $\underline{\omega}_{XA}$ of equation (A8) becomes

$$\underline{\omega}_{XA} = \left[\left(1 - e_{SA}^{\;2}/4\right)/\Sigma e_P^{\,2}\right](\underline{e}_{SA} \times \dot{\underline{e}}_C) \tag{A13}$$

The pair rate commands for nominal H_N are (pair A commands are repeated for completeness)

$$\underline{\omega}_{XA} = \left[\left(1 - e_{SA}^2/4\right)/\Sigma e_P^2\right](\underline{e}_{SA} \times \dot{\underline{e}}_C) \quad \text{(A14)}$$

$$\underline{\omega}_{XB} = \left[\left(1 - e_{SB}^2/4\right)/\Sigma e_P^2\right](\underline{e}_{SB} \times \dot{\underline{e}}_C) \quad \text{(A15)}$$

$$\underline{\omega}_{XC} = \left[\left(1 - e_{SC}^2/4\right)/\Sigma e_P^2\right](\underline{e}_{SC} \times \dot{\underline{e}}_C) \quad \text{(A16)}$$

$$\underline{\omega}_{PA} = \left[(\underline{e}_{SA} \cdot \dot{\underline{e}}_C)/\left(2\Sigma e_P^2\right)\right] \underline{e}_{PA} \quad \text{(A17)}$$

$$\underline{\omega}_{PB} = \left[(\underline{e}_{SB} \cdot \dot{\underline{e}}_C)/\left(2\Sigma e_P^2\right)\right] \underline{e}_{PB} \quad \text{(A18)}$$

$$\underline{\omega}_{PC} = \left[(\underline{e}_{SC} \cdot \dot{\underline{e}}_C)/\left(2\Sigma e_P^2\right)\right] \underline{e}_{PC} \quad . \quad \text{(A19)}$$

For the nominal H case ($H_1 = H_2 = H_3 = H_N$) the CMG angular velocity commands are

$$\underline{\omega}_1 = \underline{\omega}_{XA} + \underline{\omega}_{PA} + \underline{\omega}_{XC} - \underline{\omega}_{PC} - \dot{\underline{\phi}}_V \quad \text{(A20)}$$

$$\underline{\omega}_2 = \underline{\omega}_{XB} + \underline{\omega}_{PB} + \underline{\omega}_{XA} - \underline{\omega}_{PA} - \dot{\underline{\phi}}_V \quad \text{(A21)}$$

$$\underline{\omega}_3 = \underline{\omega}_{XC} + \underline{\omega}_{PC} + \underline{\omega}_{XB} - \underline{\omega}_{PB} - \dot{\underline{\phi}}_V \quad . \quad \text{(A22)}$$

Normalized Rotation Law

When normalization is introduced, equations (88) to (91) change into

$$\underline{\omega}_{RA} = \epsilon_A (\underline{e}_{SA}/e_{SA}) \quad \text{(A23)}$$

$$\underline{\omega}_{RB} = \epsilon_B (\underline{e}_{SB}/e_{SB}) \quad \text{(A24)}$$

$$\underline{\omega}_{RC} = \epsilon_C (\underline{e}_{SC}/e_{SC}) \quad \text{(A25)}$$

$$\underline{\omega}_{RT} = \epsilon_T (\underline{e}_T/e_T) \quad \text{(A26)}$$

where

$$\underline{e}_T = \underline{e}_1 + \underline{e}_2 + \underline{e}_3 \quad .$$

The S-functions are also affected and change into

$$S_{A11} = (e_{13}'/e_{SA})(e_{11}e_{22} - e_{12}e_{21}) \tag{A27}$$

$$S_{A12} = (e_{21}'/e_{SA})(e_{22}e_{13} - e_{23}e_{12}) \tag{A28}$$

$$S_{B12} = (e_{21}'/e_{SB})(e_{22}e_{33} - e_{23}e_{32}) \tag{A29}$$

$$S_{B13} = (e_{32}'/e_{SB})(e_{33}e_{21} - e_{31}e_{23}) \tag{A30}$$

$$S_{C13} = (e_{32}'/e_{SC})(e_{33}{}^{3}{}_{11} - e_{31}e_{13}) \tag{A31}$$

$$S_{C11} = (e_{13}'/e_{SC})(e_{11}e_{32} - e_{12}e_{31}) \tag{A32}$$

$$S_{T11} = (e_{SA}S_{A11} + e_{SC}S_{C}{}^{11})/e_T \tag{A33}$$

$$S_{T12} = (e_{SB}S_{B12} + e_{SA}S_{A12})/e_T \tag{A34}$$

$$S_{T13} = (e_{SC}S_{C13} + e_{SC}S_{C13})/e_T \tag{A35}$$

$$S_{A31} = \left[(e_{13}')\, e_{13}(e_{11}e_{21} + e_{12}e_{22}) - e_{23}\right]/e_{SA} \tag{A36}$$

$$S_{A32} = \left[(e_{21}')\, e_{21}(e_{22}e_{12} + e_{23}e_{13}) - e_{11}\right]/e_{SA} \tag{A37}$$

$$S_{B32} = \left[(e_{21}')\, e_{21}(e_{22}e_{32} + e_{23}e_{33}) - e_{31}\right]/e_{SB} \tag{A38}$$

$$S_{B33} = \left[(e_{32}')\, e_{32}(e_{33}e_{23} + e_{31}e_{21}) - e_{22}\right]/e_{SB} \tag{A39}$$

$$S_{C33} = \left[(e_{32}')\, e_{32}(e_{33}e_{13} + e_{31}e_{11}) - e_{12}\right]/e_{SC} \tag{A40}$$

$$S_{C31} = \left[(e_{13}')\, e_{13}(e_{11}e_{31} + e_{12}e_{32}) - e_{33}\right]/e_{SC} \tag{A41}$$

$$S_{T31} = (e_{SA}S_{A31} + e_{SC}S_{C31})/e_T \tag{A42}$$

$$S_{T32} = (e_{SB}S_{B32} + e_{SA}S_{A32})/e_T \tag{A43}$$

$$S_{T33} = (e_{SC}S_{C33} + e_{SB}S_{B33})/e_T \quad . \tag{A44}$$

All other equations stay the same [equations (95) to (98), (111) and (112)]

Normalized Distribution Law

Equation (120) of the distribution law allows easy change for the case that $H_i = H_N$ which results in

$$\epsilon_{DA} = K_D(\underline{e}_1 - \underline{e}_2) \cdot \underline{e}_N \tag{A45}$$

$$\epsilon_{DB} = K_D(\underline{e}_2 - \underline{e}_3) \cdot \underline{e}_N \tag{A46}$$

$$\epsilon_{DC} = K_D(\underline{e}_3 - \underline{e}_1) \cdot \underline{e}_N \quad . \tag{A47}$$

APPENDIX B

PROOF OF TORQUE EQUIVALENCE

In the following development the proof is given that the actual torque $\underline{T}$ is equal to the commanded torque under the assumption that the commanded and the actual gimbal rates are equal.

Several identities are needed for the development and are given first.

$$(V_1 \times V_2) \times V_3 = (V_1 \cdot V_3)V_2 - (V_2 \cdot V_3)V_1 \tag{B1}$$

where the V's are arbitrary vectors.

$$\begin{aligned} H_{PA}^2 &= H_1^2 H_2^2 s^2(\alpha_2 - \alpha_1) \\ &= H_1^2 H_2^2 - H_1^2 H_2^2 c^2(\alpha_2 - \alpha_1) \\ &= H_1^2 H_2^2 - H_{DA}^2 \quad . \end{aligned} \tag{B2}$$

The torque applied to the CMG's is (the opposite of this torque is the torque on the vehicle)

$$\underline{T} = (\underline{\omega}_1 + \underline{\dot{\phi}}_V) \times \underline{H}_1 + (\underline{\omega}_2 + \underline{\dot{\phi}}_V) \times \underline{H}_2 + (\underline{\omega}_3 + \underline{\dot{\phi}}_V) \times \underline{H}_3 \quad . \tag{B3}$$

With equations (61) and (63) we get

$$\begin{aligned} T = {} & (\underline{\omega}_{XA} + \underline{\omega}_{PA1} + \underline{\omega}_{XC} + \underline{\omega}_{PC1}) \times \underline{H}_1 \\ & + (\underline{\omega}_{XB} + \underline{\omega}_{PB2} + \underline{\omega}_{XA} + \underline{\omega}_{PA2}) \times \underline{H}_2 \\ & + (\underline{\omega}_{XC} + \underline{\omega}_{PC3} + \underline{\omega}_{XB} + \underline{\omega}_{PB3}) \times \underline{H}_3 \end{aligned} \tag{B4}$$

or

$$
\begin{aligned}
\underline{T} = {} & \underline{\omega}_{XA} \times (\underline{H}_1 + \underline{H}_2) + \underline{\omega}_{XB} \times (\underline{H}_2 + \underline{H}_3) + \underline{\omega}_{XC} \times (\underline{H}_3 + \underline{H}_1) \\
& + \underline{\omega}_{PA1} \times \underline{H}_1 + \underline{\omega}_{PA2} \times \underline{H}_2 + \underline{\omega}_{PB2} \times \underline{H}_2 + \underline{\omega}_{PB3} \times \underline{H}_3 \\
& + \underline{\omega}_{PC3} \times \underline{H}_3 + \underline{\omega}_{PC1} \times \underline{H}_1 \quad .
\end{aligned}
\tag{B5}
$$

Further development with equations (52) to (60) and (B1) yields

$$
\begin{aligned}
\underline{T} = {} & \left[H_{PA}^2 / \left(H_{SA}^2 \Sigma H_P^2\right)\right] (\underline{H}_{SA} \times \underline{T}_C) \times \underline{H}_{SA} \\
& + \left[H_{PB}^2 / \left(H_{SB}^2 \Sigma H_P^2\right)\right] (\underline{H}_{SB} \times \underline{T}_C) \times \underline{H}_{SB} \\
& + \left[H_{PC}^2 / \left(H_{SC}^2 \Sigma H_P^2\right)\right] (\underline{H}_{SC} \times \underline{T}_C) \times \underline{H}_{SC} \\
& + \left[\left(H_2^2 + H_{DA}\right) (\underline{H}_{SA} \cdot \underline{T}_C) / \left(H_{SA}^2 \Sigma H_P^2\right)\right] (\underline{H}_1 \times \underline{H}_2) \times \underline{H}_1 \\
& + \left[\left(H_1^2 + H_{DA}\right) (\underline{H}_{SA} \cdot \underline{T}_C) / \left(H_{SA}^2 \Sigma H_P^2\right)\right] (\underline{H}_2 \times \underline{H}_1) \times \underline{H}_2 \\
& + \left[\left(H_3^2 + H_{DB}\right) (\underline{H}_{SB} \cdot \underline{T}_C) / \left(H_{SB}^2 \Sigma H_P^2\right)\right] (\underline{H}_2 \times \underline{H}_3) \times \underline{H}_2 \\
& + \left[\left(H_2^2 + H_{DB}\right) (\underline{H}_{SB} \cdot \underline{T}_C) / \left(H_{SB}^2 \Sigma H_P^2\right)\right] (\underline{H}_3 \times \underline{H}_2) \times \underline{H}_3 \\
& + \left[\left(H_1^2 + H_{DC}\right) (\underline{H}_{SC} \cdot \underline{T}_C) / \left(H_{SC}^2 \Sigma H_P^2\right)\right] (\underline{H}_3 \times \underline{H}_1) \times \underline{H}_3 \\
& + \left[\left(H_3^2 + H_{DC}\right) (\underline{H}_{SC} \cdot \underline{T}_C) / \left(H_{SC}^2 \Sigma H_P^2\right)\right] (\underline{H}_1 \times \underline{H}_3) \times \underline{H}_1
\end{aligned}
\tag{B6}
$$

$$
\begin{aligned}
\underline{T} = {} & \left[H_{PA}^2 / \left(H_{SA}^2 \Sigma H_P^2\right)\right] \left[H_{SA}^2 \underline{T}_C - (\underline{H}_{SA} \cdot \underline{T}_C) \underline{H}_{SA}\right] \\
& + \left[H_{PB}^2 / \left(H_{SB}^2 \Sigma H_P^2\right)\right] \left[H_{SB}^2 \underline{T}_C - (\underline{H}_{SB} \cdot \underline{T}_C) \underline{H}_{SB}\right] \\
& + \left[H_{PC}^2 / \left(H_{SC}^2 \Sigma H_P^2\right)\right] \left[H_{SC}^2 \underline{T}_C - (\underline{H}_{SC} \cdot \underline{T}_C) \underline{H}_{SC}\right] \\
& + \left[\left(H_2^2 + H_{DA}\right) (\underline{H}_{SA} \cdot \underline{T}_C) / \left(H_{SA}^2 \Sigma H_P^2\right)\right] \left[H_1^2 \underline{H}_2 - H_{DA} \underline{H}_1\right] \\
& + \left[\left(H_1^2 + H_{DA}\right) (\underline{H}_{SA} \cdot \underline{T}_C) / \left(H_{SA}^2 \Sigma H_P^2\right)\right] \left[H_2^2 \underline{H}_1 - H_{DA} \underline{H}_2\right] \\
& + \left[\left(H_3^2 + H_{DB}\right) (\underline{H}_{SB} \cdot \underline{T}_C) / \left(H_{SB}^2 \Sigma H_P^2\right)\right] \left[H_2^2 \underline{H}_3 - H_{DB} \underline{H}_2\right] \\
& + \left[\left(H_2^2 + H_{DB}\right) (\underline{H}_{SB} \cdot \underline{T}_C) / \left(H_{SB}^2 \Sigma H_P^2\right)\right] \left[H_3^2 \underline{H}_2 - H_{DB} \underline{H}_3\right] \\
& + \left[\left(H_1^2 + H_{DC}\right) (\underline{H}_{SC} \cdot \underline{T}_C) / \left(H_{SC}^2 \Sigma H_P^2\right)\right] \left[H_3^2 \underline{H}_1 - H_{DC} \underline{H}_3\right] \\
& + \left[\left(H_3^2 + H_{DC}\right) (\underline{H}_{SC} \cdot \underline{T}_C) / \left(H_{SC}^2 \Sigma H_P^2\right)\right] \left[H_1^2 \underline{H}_3 - H_{DC} \underline{H}_1\right]
\end{aligned}
\tag{B7}
$$

$$
\begin{aligned}
T = {} & \left[\left(H_{PA}^{2} + H_{PB}^{2} + H_{PC}^{2}\right)/\Sigma H_{P}^{2}\right]\underline{T}_{C} \\
& - \left[H_{PA}^{2}/\left(H_{SA}^{2}\Sigma H_{P}^{2}\right)\right]\left(\underline{H}_{SA}\cdot\underline{T}_{C}\right)\underline{H}_{SA} \\
& - \left[H_{PB}^{2}/\left(H_{SB}^{2}\Sigma H_{P}^{2}\right)\right]\left(\underline{H}_{SB}\cdot\underline{T}_{C}\right)\underline{H}_{SB} \\
& - \left[H_{PC}^{2}/\left(H_{SC}^{2}\Sigma H_{P}^{2}\right)\right]\left(\underline{H}_{SC}\cdot\underline{T}_{C}\right)\underline{H}_{SC} \\
& + \Big(H_{2}^{2}H_{1}^{2}\underline{H}_{2} - H_{2}^{2}H_{DA}\underline{H}_{1} + H_{DA}H_{1}^{2}\underline{H}_{2} - H_{DA}^{2}\underline{H}_{1} \\
& + H_{1}^{2}H_{2}^{2}\underline{H}_{1} - H_{1}^{2}H_{DA}\underline{H}_{2} + H_{DA}H_{2}^{2}\underline{H}_{1} - H_{DA}^{2}\underline{H}_{2}\Big)\left(\underline{H}_{SA}\cdot\underline{T}_{C}\right)/\left(H_{SA}^{2}\Sigma H_{P}^{2}\right) \\
& + \Big(H_{3}^{2}H_{2}^{2}\underline{H}_{3} - H_{3}^{2}H_{DB}\underline{H}_{2} + H_{DB}H_{2}^{2}\underline{H}_{3} - H_{DB}^{2}\underline{H}_{2} \\
& + H_{2}^{2}H_{3}^{2}\underline{H}_{2} - H_{2}^{2}H_{DB}\underline{H}_{3} + H_{DB}H_{3}^{2}\underline{H}_{2} - H_{DB}^{2}\underline{H}_{3}\Big)\left(\underline{H}_{SB}\cdot\underline{T}_{C}\right)/\left(H_{SB}^{2}\Sigma H_{P}^{2}\right) \\
& + \Big(H_{1}^{2}H_{3}^{2}\underline{H}_{1} - H_{1}^{2}H_{DC}\underline{H}_{3} + H_{DC}H_{3}^{2}\underline{H}_{1} - H_{DC}^{2}\underline{H}_{3} \\
& + H_{3}^{2}H_{1}^{2}\underline{H}_{3} - H_{3}^{2}H_{DC}\underline{H}_{1} + H_{DC}H_{1}^{2}\underline{H}_{3} \\
& - H_{DC}^{2}\underline{H}_{1}\Big)\left(\underline{H}_{SC}\cdot\underline{T}_{C}\right)/\left(H_{SC}^{2}\Sigma H_{P}^{2}\right) \quad . \qquad \text{(B8)}
\end{aligned}
$$

Equations (42) and (B2) allow the reduction of equation (B8) to

$$
\begin{aligned}
T = {} & \underline{T}_{C} \\
& - \left[H_{PA}^{2}/\left(H_{SA}^{2}\Sigma H_{P}^{2}\right)\right]\left(\underline{H}_{SA}\cdot\underline{T}_{C}\right)\underline{H}_{SA} \\
& - \left[H_{PB}^{2}/\left(H_{SB}^{2}\Sigma H_{P}^{2}\right)\right]\left(\underline{H}_{SB}\cdot\underline{T}_{C}\right)\underline{H}_{SB} \\
& - \left[H_{PC}^{2}/\left(H_{SC}^{2}\Sigma H_{P}^{2}\right)\right]\left(\underline{H}_{SC}\cdot\underline{T}_{C}\right)\underline{H}_{SC} \\
& + \left[H_{PA}^{2}\left(\underline{H}_{SA}\cdot\underline{T}_{C}\right)/\left(H_{SB}^{2}\Sigma H_{P}^{2}\right)\right]\underline{H}_{SA} \\
& + \left[H_{PB}^{2}\left(\underline{H}_{SB}\cdot\underline{T}_{C}\right)/\left(H_{SB}^{2}\Sigma H_{P}^{2}\right)\right]\underline{H}_{SB} \\
& + \left[H_{PC}^{2}\left(\underline{H}_{SC}\cdot\underline{T}_{C}\right)/\left(H_{SC}^{2}\Sigma H_{P}^{2}\right)\right]\underline{H}_{SC} \qquad \text{(B9)}
\end{aligned}
$$

showing that

$$\underline{T} = \underline{T}_C \tag{B10}$$

which was to be proven.

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APPROVAL

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A CONTROL LAW FOR DOUBLE-GIMBALED CONTROL MOMENT GYROS USED FOR SPACE VEHICLE ATTITUDE CONTROL

By Hans F. Kennel

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