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CONFERENCE ON PHOTOGRAPHIC ASTROMETRIC TECHNIQUE

Edited by Heinrich Eichhorn

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16. Abstract

The "Conference on Photographic Astrometric Technique" was held from February 18th through February 20th 1968, in order to discuss the technique used for obtaining astrometric plates, as well as their mensuration and reduction. Because of the applications of photographic astrometrics to satellite geodesy, the conference participants were astronomers as well as geodesists. We hope that this conference has contributed in making astronomers and geodesists more familiar and appreciative of each other's work.

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PREFACE

The "Conference on Photographic Astrometric Technique" was held from February 18th through February 20th 1968, in order to discuss the technique used for obtaining astrometric plates, as well as their mensuration and reduction. Because of the applications of photographic astrometrics to satellite geodesy, the conference participants were astronomers as well as geodesists. We hope that this conference has contributed in making astronomers and geodesists more familiar and appreciative of each other's work.

Thanks are due the many individuals through whose work and cooperation this conference was made possible.

In particular I should like to thank my wife, Mrs. Edelgard Eichhorn-von Wurmb, who transcribed the tapes of the discussions, and generally assisted very efficiently with the work of organizing, holding, and editing the proceedings of the conference.

This volume would not have been published without the very competent and efficient help of Mrs. Dorene L. Malinowski, who not only typed the manuscript, and is responsible for the general layout and production, but who also caught many inaccuracies and little errors that had slipped by the authors and the editor, and who helped in other different ways too numerous to mention.

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THE ORIGIN AND GOALS OF THE AUTOMATED STELLAR PROPER MOTION SURVEY

Willem J. Luyten

University of Minnesota

ABSTRACT

The historical background of the automation of the stellar proper motion survey is traced from its beginnings at Harvard Observatory in 1928 to the taking of the series of plates complementing the National Geographic-Mount Palomar Sky Survey plates taken with the 48 inch Schmidt Telescope at Mount Palomar covering 77% of the sky with 936 plates. The design goal for the automation is the documentation of the proper motions of stars from tenth through twentieth red magnitude. A typical scanning time for a pair of 14 inch star plates is projected to be about 45 minutes with the total processing time estimated at between one and two years for the complete set of plates.

INTRODUCTION

During most of my scientific life I have been interested in stellar motions, especially of faint stars. Virtually the only way of finding these is by comparing two plates taken with the same telescope a number of years apart. Usually this is done with what is known as a blink microscope. In 1928 when I was still at Harvard Observatory, I began a proper motion survey of the southern hemisphere. To complete the actual search of the more than 1000 plates took only six years, but to complete the measurement of all the motions found took more than twenty-five years and it was not until 1962 that we could publish the final catalogue of motions of 85 000 stars.

Meanwhile much better plate material had become available in the form of the Palomar National Geographic Survey done with the 48 inch Schmidt telescope of the Palomar Observatory. The 936 pairs of plates - taken in blue and red - cover 77% of the sky and record images of stars down to $21^{m}.2$ in the blue, and to about $20^{m}.$ in the red. Since the vast majority of faint proper motion stars are red it is obvious that the red plates must be repeated.

In 1958 I began a small pilot program using the oldest plates, then about 9 years old, and this indicated that the optimum interval between plates was between 11 and 13 years - with smaller intervals one would not find all of the significant motions, but once the interval gets too long the number of moving objects found gets so large that the program is no longer manageable. Hence, in 1962 I began in earnest retaking the new plates, and to date more than 700 of the red plates have been repeated.

But almost immediately troubles began to appear. The Palomar-Schmidt red plates show anywhere from 50 000 to 10 000 000 star images per plate - depending on the galactic latitude. To examine even the former pair of plates using the human eye and the human hand takes about 30 hours and usually we find something like between 400 and 500 moving objects on a pair of plates. To measure these - and this can be done by student-assistants - takes at least another 200 to 250 hours. Finally there is another 20 hours or so per plate of paper work before the data are ready for publication. To search the very richest plates in this fashion is probably not possible, but even if it were we estimate that it would probably take at least 300 man-years to finish the job - no one can do this kind of work 40 hours per week.

With the prospect that, if this survey could be completed we would find something like half a million faint stars with large proper motion it is evident that we need to turn to automated and computerized methods. About five years ago Robert Lillestrand of Control Data Corporation and I began thinking seriously about this. In 1964 we sent in a proposal to the National Aeronautics and Space Administration and a contract was awarded to the University of Minnesota with Control Data Corporation as subcontractor as of July 1, 1965. As of today fabrication of the machine is almost completed. What we hope it will do, what it will accomplish follows.

DESIGN GOALS FOR AUTOMATIC SCANNING

The star images on the pair of star plates are scanned by a rapidly moving spot of light detected by photomultipliers. The positions and widths of the images are recorded to the nearest micron on magnetic tape. The tremendous data flow rate requires the use of a very large, very fast computer to reconstruct the images from the taped data. The positions of the centers of the images for a small (16 × 16 mm) area of one plate are compared with the image centers for the corresponding area on the other star plate. The computer detects any misalignment of the two areas and aligns them to optimum coincidence by translational and rotational coordinate transformation. Non-coincident images are identified, measured, and their coordinates in plate reference and in right ascension and declination are printed by the computer. Depending upon how small the motions we wish to accept and retain, we then set a lower limit to the discrepancy in position between the two plates below which we "throw them back in", e.g., if we wish to obtain a virtually complete list of all motions larger than 0".10 annually - which on our plates with our average interval will amount to a displacement of 18 microns - we might set this lower limit at 15 microns. In the end we thus wind up with a list of objects which have moved more than 15 microns for which we have the X and Y motions, but we also have the X and Y coordinates of the object which the computer will translate into positions in the sky, and, in addition, have a good indication of the diameter of the star image from which the brightness can be derived. The final data output will be on tape as well as on cards.

We hope and expect that this machine will be able to process a pair of plates in 45 minutes on the average. Hence, if all goes well, we should be able to finish the survey in one or two years.

THE AUTOMATION OF THE STELLAR PROPER MOTION SURVEY

James S. Newcomb

Control Data Corporation

ABSTRACT

A microdensitometer, scanning pairs of star plates of identical areas of the sky taken a decade apart, is coupled to a magnetic tape unit through a data acquisition system. This system performs an image density threshold level selection for each plate and a buffer memory system to smooth the data flow as well as analog and digital display controls. The data from the tape are processed

later by a large scale high speed computer which reconstructs the scanned images, discovers, measures, and documents stellar proper motions by precise comparison of corresponding areas of the two star plates to detect those images which have moved at least 0".10 annually in the time interval between the exposures.

INTRODUCTION

The aim of the automation project has been to reduce the significant information on the 14 in. × 14 in. star plates taken with the 48 in. Schmidt telescope at Mount Palomar to measurements in digital form which can be utilized by a computer to discover and measure stellar proper motions. This paper will summarize the implementation of the automation and describe the major elements of the data acquisition system. A laser scanning densitometer and a special purpose computer were built by the Aerospace Research Department of Control Data Corporation to digitize the star plate information.

A block diagram of the system from star plate scanning to computer output is shown in Figure 1. In step number one, signals from the scanning of the star plates and reference reticle flow from the scanner to the detection electronics; digital star plate and spot position signals flow from the scanner to the console The combined signals enter the buffer-core memory complex and are accumulated until a tape-compatible group is present. Then the group of measurements is recorded on the tape. In step number two, the tape is read into the computer; the computer then reconstructs the star images, detects and documents star images which have moved.

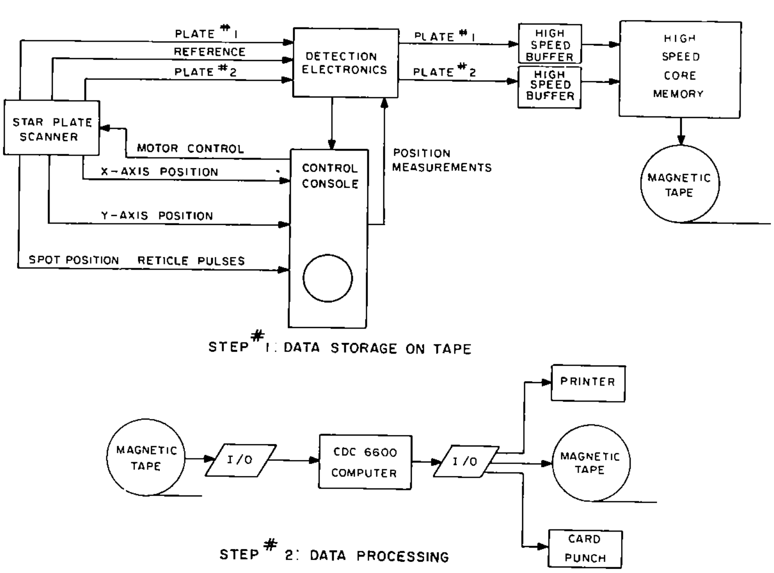

FIGURE 1

Block diagram of Data Reduction System from Star Plate Scanner to Computer Output. The reduction occurs in two separate steps; the measurements of image position and size flow from the scanner through the data control and acquisition system to the digital magnetic tape; the tapes are then read into the computer for the extraction of proper motion information.

THE OPTICAL SCANNING SYSTEM

A schematic diagram of the optical system is shown in Figure 2. Light from a Perkin-Elmer helium-neon continuous gas laser passes through a converging lens into a rotating octagonal prism. The rotating prism imparts the scanning motion to the laser beam. The beam is split by the action of a beam splitting prism cluster placed immediately beyond the exit fact of the rotating prism. The three beams obtained from the prism cluster are shown in the figure; one beam with 50% of the beam energy goes upward through the relatively dense original plates; the second beam with 25% of the energy passes downward through the relatively clear second epoch plates. The third beam with 25% of the energy passes into the reference reticle which is essentially a 64 micron scale from which the instantaneous position of the spot is obtained. Light passing through each of these barriers is collected by a special optical system and is sensed by three separate photomultipliers. Light sources and pivoted mirrors for visual observation of the star plate pairs for alignment purposes are shown adjacent to the photomultipliers for the two star plates.

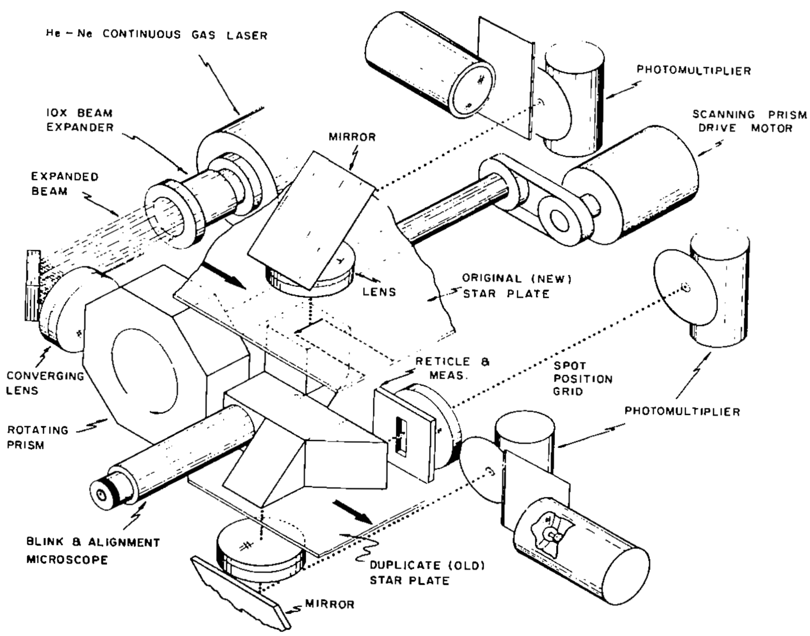

FIGURE 2

An optical Schematic of the Star Plate Scanning Microdensitometer. Monochromatic light at .6328 microns wavelength converges to a spot after passage through a positive lens. The converging light beam is given a scanning motion by its passage through a rotating octagonal glass prism. A beam splitting prism cluster divides the converging beam into three parts - two portions of the beam scan the star plates and one scans the reticle. Individual light collection systems monitor the intensities of the three beams.

Scanning action on the star plates is produced by a combination of the motion of the spot in the Y direction over a strip 16mm wide and the motion of the star plates in the X direction over a path 356mm long. Both the rotating prism and the X axis drive motors are synchronous and their speeds so related that individual adjacent scans of the moving spot are five microns apart. The size of the star images of interest ranges from 900 microns to about 25 microns.

The average size is about 80 microns. Thus, an average stellar image would be scanned 16 times. After a strip has been scanned, the plate holders are automatically moved to the next strip position before the next scan begins. At the maximum scan rate, the plates move at a velocity of 6mm per second, and the prism rotates at 9000 rpm. Velocities at one-half and one-fourth of this value are available for plates of higher star densities.

A photograph of the optical scanning head is shown in Figure 3. The beam splitting prism cluster is shown mounted to the right of the rotating prism. The mirror and converging lens collecting light from the laser beam are in the housing to the left of the rotating prism. This housing contains the motor and advance system for focusing the converging lens. Directly above the prism cluster is the housing for the light collection system for the upper (original) star plate.

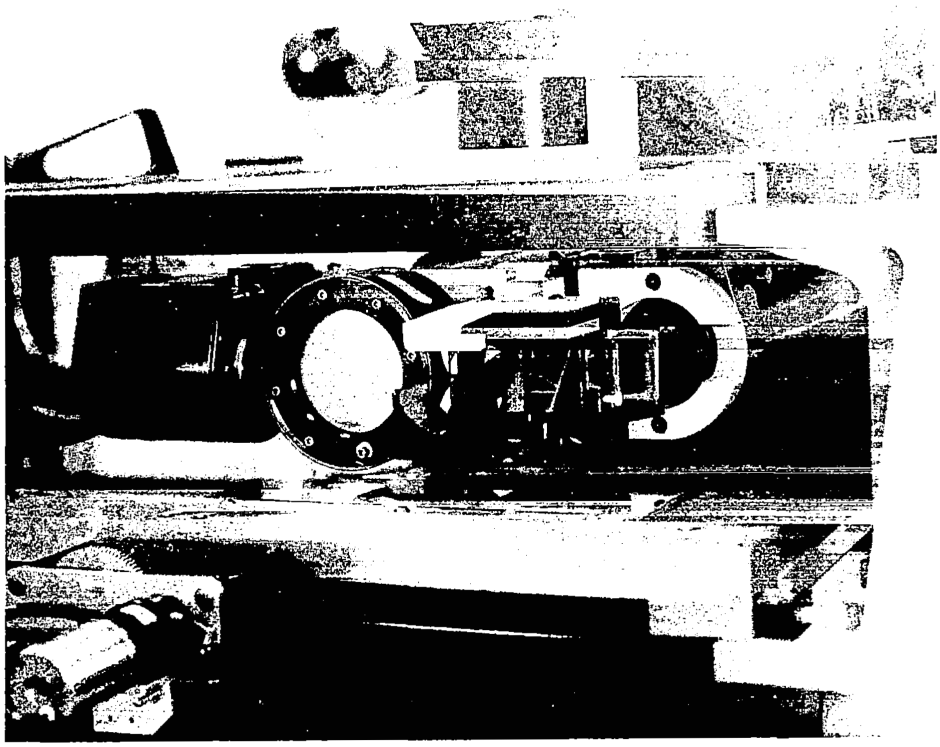

FIGURE 3

The Scanning Head of the Microdensitometer. The rotating octagonal prism is in the center of the photograph; the prism cluster is to the right of the rotating prism and the converging lens is in a focusing mount on the left.

MECHANICAL CONFIGURATION

The measuring system on which the scanning is based is the orthogonal lead screw measuring system of the Moore Measuring Machine #2. Star plates clamped in holders mounted on the ways of the measuring machine move by the optical scanning head to provide part of the scanning action. A general view of the scanner as placed in our laboratory for testing and checkout is shown in Figure 4. The blink microscope used for visual observation of the star plates projects out of the prism cluster assembly to the front of the machine. The Y axis lead screw and ways are parallel to the microscope tube; the X axis ways and lead screw are at right angles to the tube. Directly below the eyepiece of the microscope and parallel to it is the incremental angle encoder directly connected to the Y axis lead screw. At right angles to the encoder are the lead screw drive motor and electric brake. A similar assembly is connected to the X axis lead screw. On either side of the microscope support stand are located the two control boxes used to provide control during visual alignment, blinking, plate change, and testing activities. The three light collection systems with a photomultiplier and preamplifier in a housing on each are shown in Figure 5. The upper light collection tube is for the second epoch star plate; the center tube collects light from the reticle; the lower tube collects light from the first epoch star plates.

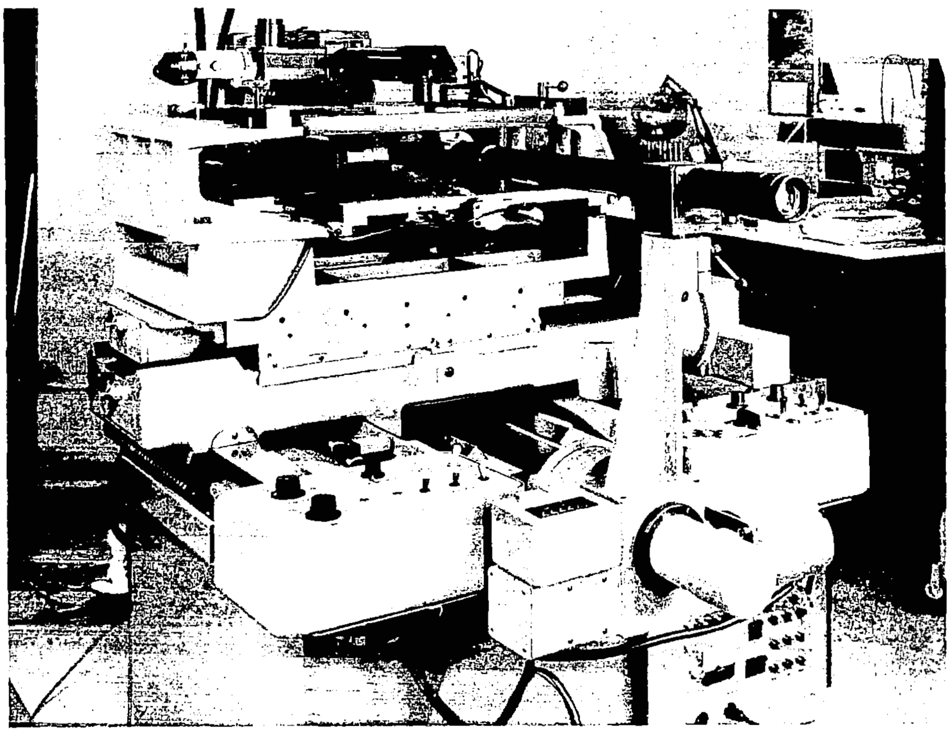

FIGURE 4
The Scanning Microdensitometer. The star plates being scanned are in holders immediately above and below the blink microscope which projects out of the front of the scanning head. Parallel to the microscope is Y axis motion of the star plates; at right angles to it is X axis. Controls for local operation of the machine are located on either side of the Y axis ways.

FIGURE 5

Side View of the Microdensitometer Showing the Three Light Collection Systems. The upper horizontal tube collects light from the second epoch star plate and reflects it to the photomultiplier housed at the near end of the tube. The lower tube collects light from the first epoch star plate; the center tube is the light collection system for light passing through the reference reticle.

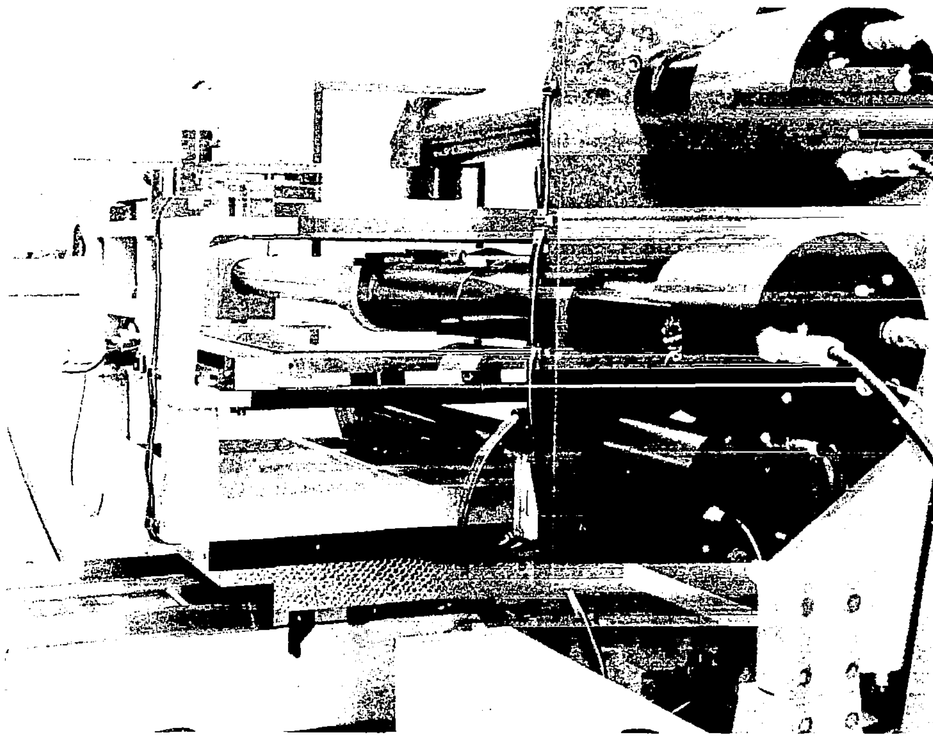

IMAGE WIDTH AND POSITION IDENTIFICATION

Signals from the two star plate scannings pass from the photomultiplier into the detection and data display system. An average star plate background signal level is computed for each plate and the average maintained at a constant level by automatic gain control of the photomultipliers. An image detection threshold level is set by the operator for each star plate. When the signal from a star plate drops below this threshold level (because the moving spot has encountered a dense image) the X and Y positions of the scanning spot are entered into the buffer memory through an electronic gate. The X position is obtained from a bidirectional counter which keeps track of the pulses from the X axis incremental encoder. The Y position is obtained from two sources - the counter accumulating the pulses from the Y axis angle encoder, and the counter system that measures the instantaneous position of the moving spot at the time the image was encountered. As the spot traverses the image, a separate counter records the distance travelled. When the spot emerges from the image and the signal level rises beyond the threshold level, the contents of the image width counter are entered into the buffer.

THE CONTROL CONSOLE

Since the plate scanning is controlled in a room separate from the scanner room, adequate controls and displays must be supplied to the operator. These controls and displays as well as the special purpose computer needed for data acquisition and transfer are housed in a separate console. A photograph of the console is shown in Figure 6. The left section of the console provides continuous monitoring of the output of the three photomultipliers plus automatic gain controls and focusing switches.

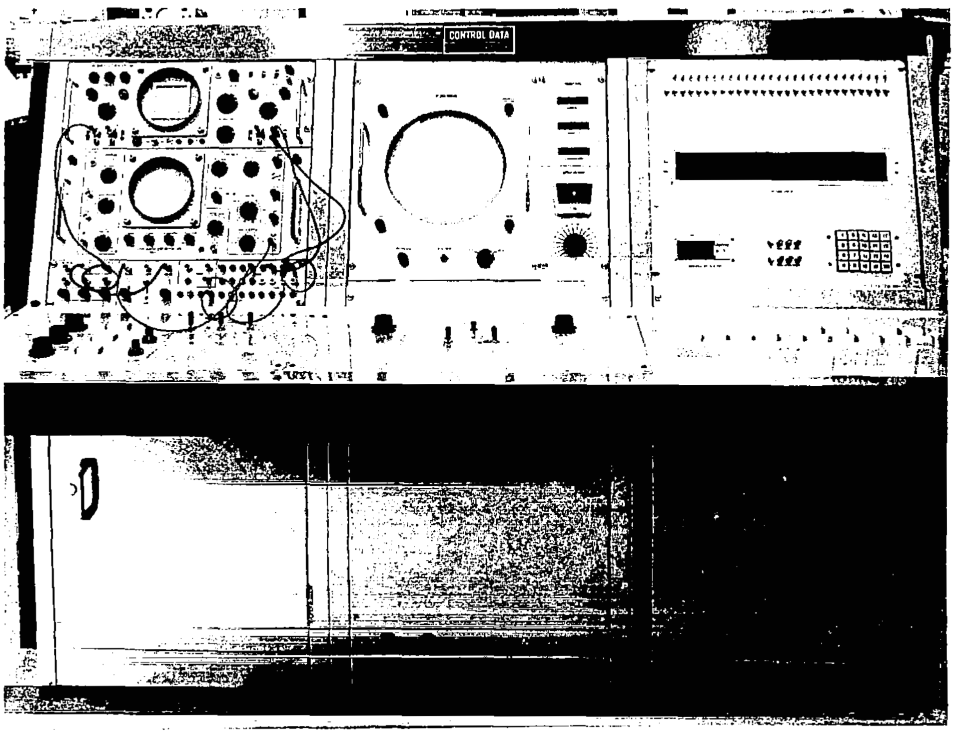

FIGURE 6

The Control Console. The actual scanning of star plates and the writing of data on digital magnetic tape is monitored and controlled from this console. The photomultiplier outputs are observed and the threshold levels controlled from the left hand section of the console. Star plate motion controls and a special bright display of the detected images are available from the central section of the console. Control, monitoring, and testing of the digital data system occur in the right section of the console.

The center panel provides a continuous bright display of the images detected by the system as well as the scanner motion controls. A sample of the image display of a star plate is shown in Figure 7, as photographed from the screen of the circular tube in the central panel. Note the diffraction spikes of the bright star, the scratch on the plate and some dust particles and/or plate flaws. The right end of the console contains the data acquisition logic, the core memory, system status displays, logic system test equipment, and the basic scan and stop controls for standard scanning. Cables connect the scanner to the console, and the console to the magnetic tape unit. Thus, contained in the console is the data "filtering" of the threshold level system, the data storage in the memory system, and the data transfer to the magnetic tape. Data is read into the tape in complete records from the memory so that the tape contains only completely filled records.

FIGURE 7

Star Plate Image Display. This is a photograph of the face of the circular tube in the center of the console as the microdensitometer scanned an actual star plate. The display covers a 16 x 16mm area of the star plate.

DATA REDUCTION IN THE COMPUTER

The tapes are read into the computer for data reduction as fast as the tape can be read. This means that although the data flow from the scanner is completely random, the data flow into the computer is now continuous.

Computer processing of the star plate measurements begins with the reconstruction of the images from the serial measurements obtained from the tape. The assembly of the individual transits into an image is accompanied by simple tests of image shapes which decide whether an image is sufficiently star-like to be retained. Borderline cases would be rejected but marked as available for review by the astronomer.

The centers of nearly circular star-like images are calculated by determining the centroids of the isophotal area describing the images. The location of this center in plate coordinates is used throughout the rest of the calculation as the star image location. The area of the star is calculated by the trapezoidal approximation. Star images of a more complex shape, such as those produced by binaries, will be described by a least squares fit with the use of residuals as a criterion of acceptability.

The second step in the data processing is the alignment and analysis of a 16mm square area on the two star plates. The centroids of star images on the two plates are compared, and the best fit of alignment between the two areas is obtained, based on iterative coordinate transformations of rotation, translation, and stretch of emulsion. Reference will be made to the resultant transformations of previous nearby areas to guide the iterative process on successive 16mm square areas. Provision has been made on the scanner itself for visual alignment of the plates before the plate scanning begins, in order to reduce the number of computer iterations necessary to produce optimum coincidence. After optimum coincidence has been obtained for a given 16mm square region, star image pairs not fitting the coincidence pattern are measured and documented with respect to nearby stars of comparable area. Right ascension and declination coordinates for the star images are obtained from reference stars located visually on the star plates before the scanning begins.

DATA FLOW RATES

The expected data flow rates from the star plates have been a pivotal factor in the system design. Measurements of samples of data flow rates made with an engineering test model built before the actual scanner showed that star image measurements could be expected to come in random bursts with wide variations in the numbers of measurements made in each moving spot scan along a strip being being scanned, along different strips in the same plate, and in different plate pairs. Estimates of the average data flow for various regions of the sky were used to evaluate the probabilities of losing significant quantities of data at various scan speeds because the data flow was too great against the estimated scan time for a complete plate pair as a function of scanning velocity. The basic design conclusion reached from these measurements was that any computer fast enough to accept data from closely spaced images would be too expensive to sit idle during the time no data was being delivered to the machine.

The system design philosophy has been to detect and record all images whose density exceeds the detection threshold level. The astronomical information obtained from these measurements is determined by the computer program reducing the data; documenting stellar proper motions will be the first but need not be the only information obtainable from these plates - the change in information output is the result of a change in computer program, not a change in the hardware of the scanning system.

Thus, we hope to produce by this system a statistically significant number of documented proper motions of stars from the tenth through the twentieth red magnitude in forms readily usable by the scientific community.

Acknowledgement

This work was supported by the National Aeronautics and Space Administration through the University of Minnesota, prime contract NSR 24-005-062.

DISCUSSION

Gatewood: *How long does it take to measure a pair of plates on this machine?*

Newcomb: *About 45 minutes.*

Gatewood: *What is the accuracy of a measured position?*

Luyten: *We are aiming at an accuracy of 5 microns on the plate which corresponds to about 0."3, as in the Astrographic Catalogue. This is a relative accuracy in a small field. At this time, without using the computer, we get absolute positions with an accuracy of 1 minute of arc. With the aid of the computer, we hope to eventually get the positions accurately to one second of arc, corresponding to 15μ.*

Mueller: *What precisely does that figure mean?*

Luyten: *This refers to the accuracy with which the location of the image in the emulsion is measured. With the overlaid errors of emulsion shift, we may well get ultimately mean errors near 5".*

Strand: *Will you be able to obtain this with the system of your reference stars?*

Luyten: *I believe so. We have an average of 58 AGK2 stars on each plate.*

Herget: *My own experience is this: I reduced forty-eight Palomar Schmidt plates and used the Yale Zone Catalogues as a source of reference stars. The plates had been measured on a manually operated measuring machine by van Houten. The rms residuals on some plates were as low as 0$''$.3, others were 0$''$.7 . These figures describe the agreement between the measurements and the positions printed in the Yale Catalogues. So maybe Luyten's figures are too pessimistic.*

Luyten: *Were these "good" plates?*

Herget: *I don't know. They were taken in 1960 by Gehrels.*

Luyten: *It is important to consider that all our plates were taken through a plexiglass filter, while those that Herget talks about were taken without a filter. Since plexiglass filters do not last for 15 years, I am sure different filters were used at the first and second epoch. The regions close to the edges of the 4 in. by 4 in. plates are particularly sensitive to this, and we have noticed in fact the effects of different filters at the different epochs in the proper motions. These are our reasons for being somewhat pessimistic.*

Newcomb: *The effects of granularity of the emulsion seems to set the limits for the achievable accuracy in connection with the fact that every image is covered by several scans. We notice that the least accurately measured stars are those of 30μ diameter, the smallest we measure, and that the accuracy rises as the image diameters rise. Also, we notice as the results of experiments which we have conducted, that the spacing between the scan lines is insignificant for the ultimately achieved accuracy.*

COMPUTER CONTROLLED PRECISION DIGITIZERS OF OPTICAL IMAGE DATA EXTRACTION

Richard C. Strand

Brookhaven National Laboratory

ABSTRACT

The Flying Spot Digitizer at the Brookhaven National Laboratory and its applications are described. Suggestions are made as to how this apparatus might be used for the precise and fast measurement of star images on photographic plates.

AUTOMATIC EXTRACTION OF DATA FROM BUBBLE CHAMBER FILM

Among the many applications for information storage and extraction by use of photographic film are the several million bubble chamber stereo triads exposed each year for elementary particle experiments (Brookhaven National Laboratory, 1966). One third of a representative stereo triad, or one "view", is reproduced in Figure 1 . This exposure of the brookhaven National Laboratory (BNL) twenty-inch hydrogen chamber was made to a beam of secondary π^+ mesons which have a momentum of 1.0 GeV/c from the BNL Alternating Gradient Synchrotron, a 30 GeV proton accelerator. The beam tracks start at the right and are deflected down by the chamber magnetic field of 17 kilogauss. The diameters of the images of the individual bubbles on the 35mm film strip are about 30 microns. The linear bubble density of a track is proportional to the square of the inverse of the particle velocity. Consequently, the relativistic π^+ beam tracks and the low momentum electron spirals produce "minimum" bubble density while some nonrelativistic particles leave more continuous tracks. A pair of "two-prong" interactions occur but most of the beam tracks pass through and thereby form a background of noise which is rejected by the data extraction system. For stereo-reconstruction purposes, "×" - shaped and "+" - shaped fiducial marks are etched into the bubble chamber window. The locations of these marks with respect to the camera optics are known to within a few microns. A binary coded picture number for automatic reading also appears in Figure 1.

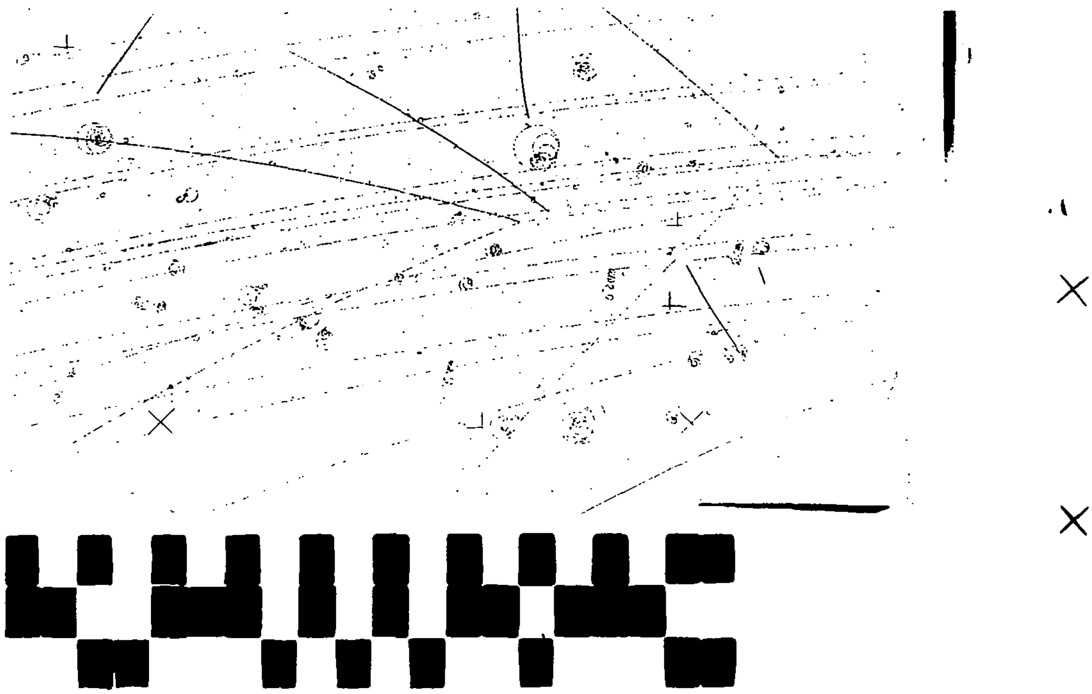

FIGURE 1

Reproduction of film-strip for one exposure of the BNL twenty-inch hydrogen bubble chamber to π^+ mesons at 1.0 GeV/c .

For faster measurement a flying spot digitizer (FSD) (Hough and Powell, 1960; Schutt, 1967; Strand, 1967) has been developed that automatically measures and records about 50 000 coordinates of bubble-like images from one frame. Digitizings from the film-strip reproduced in Figure 1 were plotted on a CRT, a photograph of which appears in Figure 2. The quality of the result is limited by the plotting CRT, but a close resemblance to the original image is preserved. Notice that only the × - shaped fiducial marks are detected by this FSD. Since extended images that are parallel to the direction of the flying spot are not digitized, an orthogonal scan direction is employed for the wide angle tracks of interest.

Hough and Powell (1960) proposed the mechanically deflected flying spot digitizer (HPD), Hough-Powell Digitizer, for the measurement of bubble chamber film, that was developed cooperatively at BNL, the Lawrence Radiation Laboratory at the University of California at Berkley (LRL), and the European Organization for Nuclear Research, Geneva, Switzerland (CERN). The flying spot traces a serial path over the film in much the same manner as the electron beam of a television tube. Each scan line of the raster is formed as the spot moves orthogonally to the uniform motion of the film on a precision stage. The spot generation scheme is shown in Figure 3. The flying source is the aperture defined by the intersection of a fixed straight slit with a curved slit which is located along the radius of a rotating disk. The spot is focused onto the film and a precision grating.

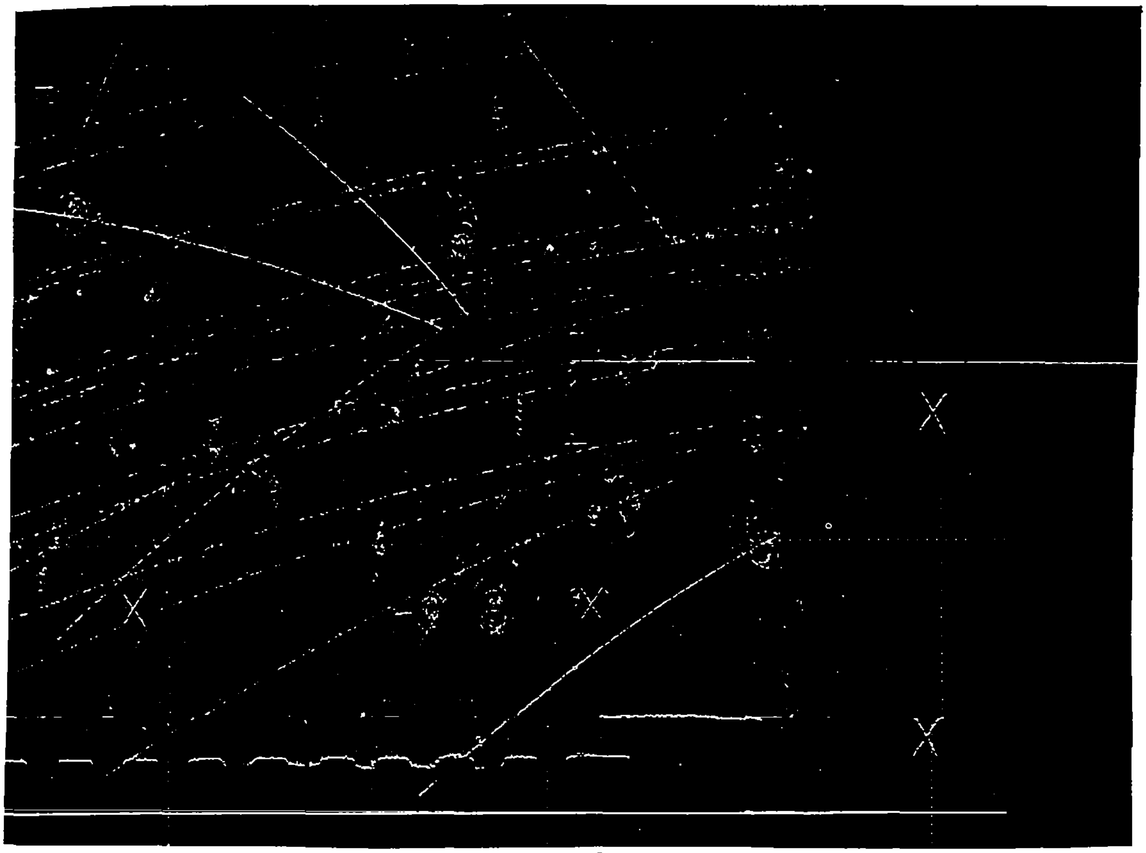

FIGURE 2

Composite photograph of plotting CRT for the HPD digitizings from film-strip of Figure 1.

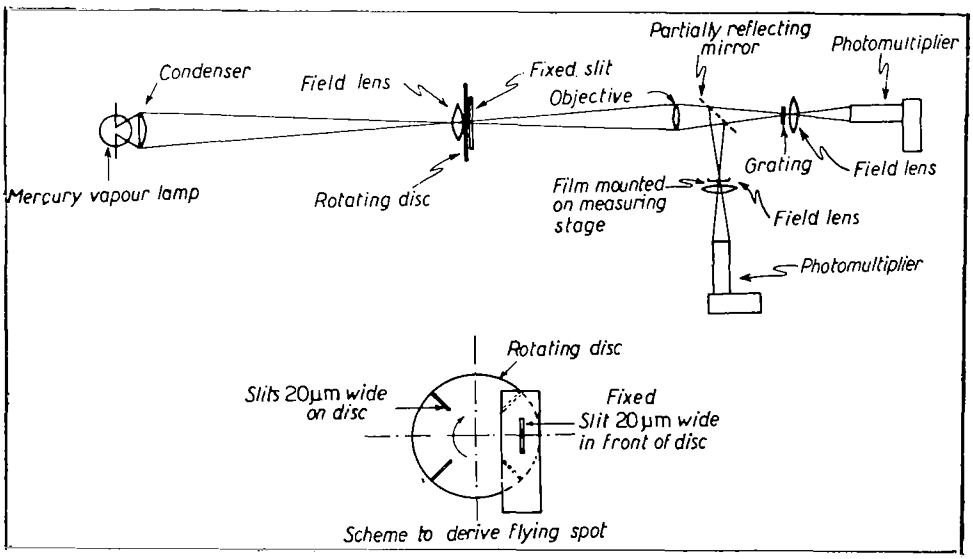

FIGURE 3

Scheme to derive flying spot.

When the spot intensity is weakened due to the opacity of an image on the film, the precision grating line counter is interrogated for the distance of the spot from an origin. The other coordinate is taken from the position of the moving precision stage at the end of each flying spot scan line. The coordinates are sent to the computer during the scan from a local buffer used to accomodate the peak rates.

The precise coordinates of all images within an area of $100 \times 50\text{mm}^2$ are measured in four seconds. Their characteristics are matched to the requirements of bubble chamber experiments. Kinematic least squares analysis (Crennell et al., 1965) of well measured bubble chamber interactions show that individual track point measurements have effective setting errors that vary between four and nine microns on the film. Included here are both the systematic and random uncertainties of the whole bubble chamber instrument. Manual track point measurement precisions from two to six microns are regularly used for experiments. The HPD coordinate precision can be between three and six microns. The spot diameter is about 15 microns and electronic speeds enable a 35 micron bubble

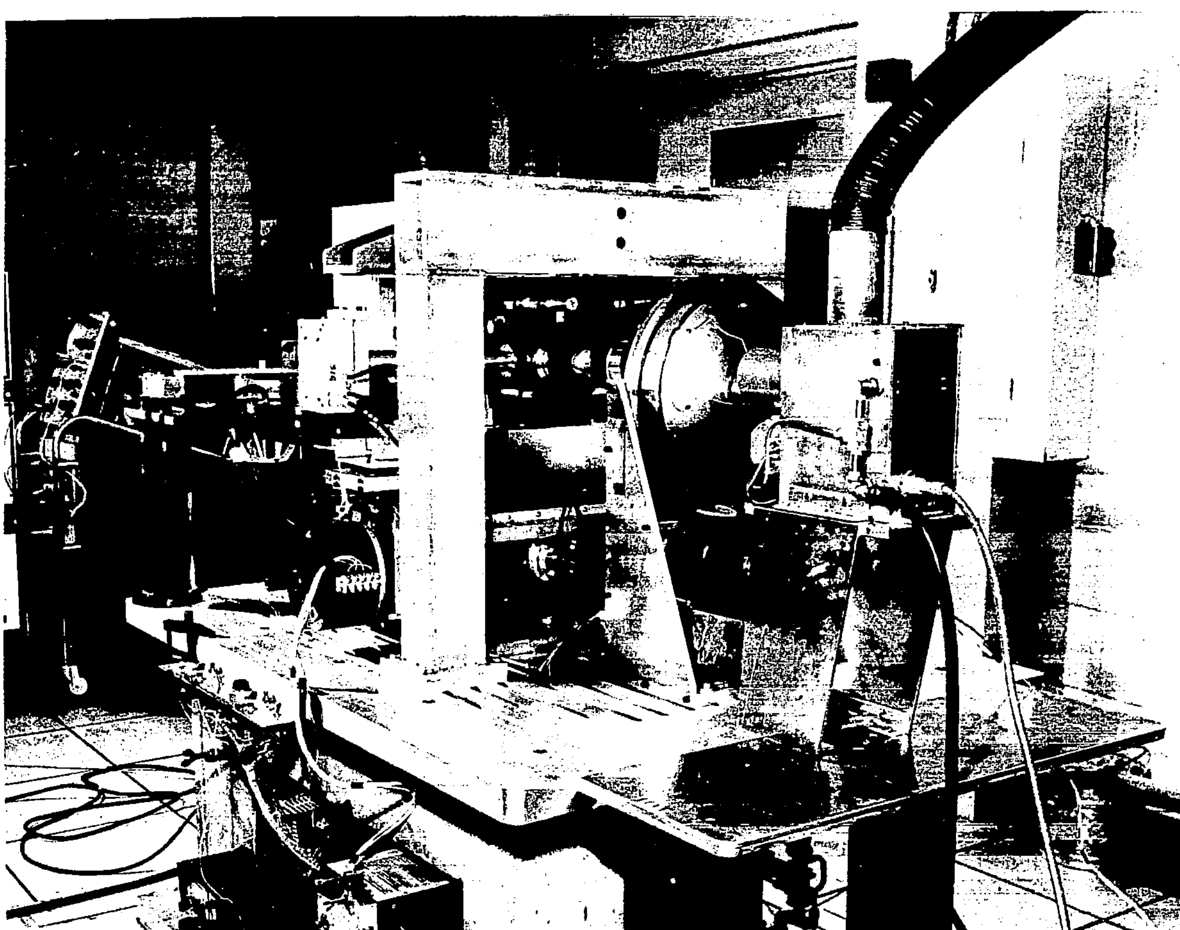

FIGURE 4

Early appearance of the BNL 35mm HPD.

center to center resolution. The coordinate system is stable and accurate to a few microns as determined by precision grid measurements.

The HPD apparatus was developed by 1962. The BNL HPD for 35mm film is shown in Figure 4 and the HPD for 70mm film is shown in Figure 5. At the three laboratories, adequate computer track recognition programs were developed in another one to three years. Existing computer speeds were too slow for automatic scanning, so manual guidance for the HPD measurement is provided by a scan table operator who finds the event of interest and makes rough settings on three points per view on each track. These three points define a circular 400-micron-wide "road" on the film. At BNL during the HPD scan, points within the road are saved by the computer and after the scan they are recorded on magnetic tape for later processing by the track finding program, FILTER (Hough, 1965). Development of accurate track finding programs to find the points that belong to the track was more difficult than expected (Hough and Powell, 1965). It was necessary to use the redundancy of the third stereo view (Crennell, 1965) to check the quality of each track measurement.

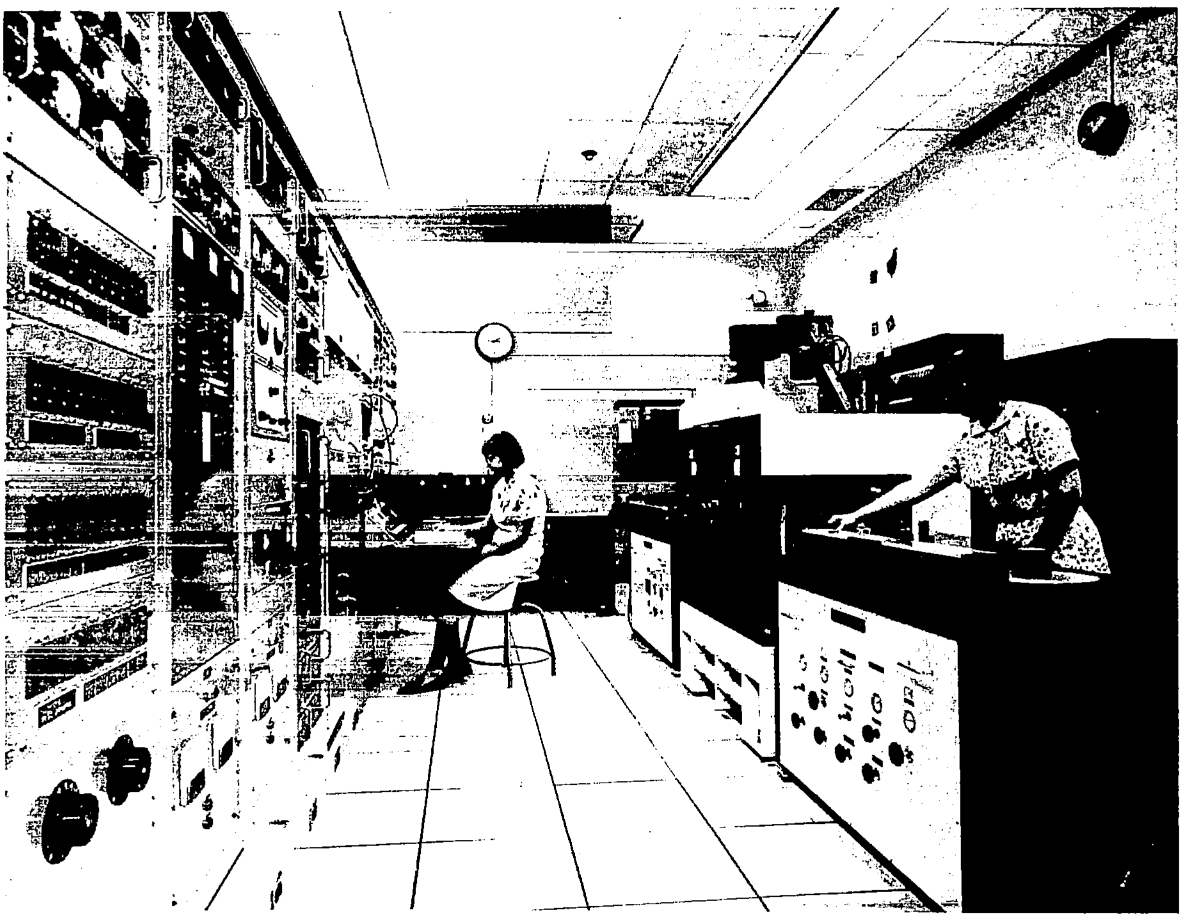

FIGURE 5

BNL 70mm HPD.

At BNL, CERN, and LRL the HPD systems are being used for experiments between one-quarter and three-quarters of a million events per year. These events are limited by the manual scanning and road making. Several second generation HPD's are coming into operation. The Sogenique company of England manufactures most of these machines. Two groups, at LRL (White et al., 1968) and Columbia University (Newman et al., 1968) are working on the automatic scanning of bubble chamber pictures. The HPD group at CERN (Altaber et al., 1968) is developing the programs for the vertex guidance where the scan table operator finds the event and measure the approximate coordinates of only the interaction point in each view.

Bubble chamber HPD's have been applied to the automatic scan-measuring of spark chamber film (Lassalle and Zanella, 1968) from elementary particle experiments where the number of information elements per photograph is about three orders of magnitude lower. Recently, the BNL 70mm HPD was used to digitize some astronomical plates for H. Eichhorn and G. Gatewood (1970) of the University of South Florida.

ASTRONOMICAL MEASUREMENTS: HARDWARE POSSIBILITIES

A flying spot can be generated mechanically as in the HPD or electronically by a precision CRT, called PEPR (Shutt, 1967) for Precision Encoding and Pattern Recognition in its bubble chamber form. Nine-inch tubes have been manufactured that have a 13 micron spot at the center and about a 20 micron spot at the edge. Phosphor decay times are available that allow speeds comparable to the present HPRD's and the spot location is directed by a computer. Here the film appears to be a random access memory.

Recently a new Videcon tube was developed by RCA for space applications with a 5000 line resolution on a $4^1/_2$ - inch tube. This device could have potential use for data extraction from film.

The one micron precision needed for star image coordinates could be obtained from bubble chamber FSD's. If each star image were "digitized" several times, the average coordinate precision would be improved. If the "rough" location of each star image were known, then a precision stage could position the plate for a very fine scan over a small area.

Dimension and shape of stellar images (star pulses) are less uniform than those of the bubble pulses. The digitzation of pulse width and height as well as position could be obtained. Star recognition programs could make good use of width and height information. For instance, if a star is recognized in two mutually orthogonal scans, then it should have consistent widths and heights. Such redundancies may be used to guarantee the quality of the result.

The value of manual assistance for an automatic data extraction system is well established. For example, there must be some star images within the fringe of other images that a human eye can measure which a flying spot device could not be expected to resolve. These "sub-spot data" can be conveniently extracted by proper provision for an operator on-line. Another example inevitably occurs when an important plate is so bad that some images will be classified as "sub-spots". The operator should be able to view, for comparisons, a projection of the plate and a digital display of the raw coordinates or recognized stars on a display scope. The computer program can provide a suitable range of magnifications. A "light pen" or a "track ball" device can be used by the operator to indicate the exact location of stars to the computer. A display also is very helpful for developing and operating the digitizer.

Three general computer configurations are possible. A separate medium size computer, big enough to control the apparatus and also recognize the star images, is the least complicated. To hold down development costs, the initial configuration can have the minimum amount of central memory and peripherals necessary while the hardware is developed. However, compilation of FORTRAN programs should be possible with this configuration or else the control program development costs might get out of hand.

One could also use a small computer to drive the hardware and collect the raw digitizings and send them to a large central computer for star recognition. This, however, does introduce a problem. It is desirable to be able to complete the scanning and measurement of one plate before going onto the next. The large computer must therefore provide time-shared service. The time sharing system becomes one extra complication.

The least desirable configuration has the digitizer directly on-line to a large time-sharing computer. Here the digitizer developers are often constrained to working at odd hours to check out their programs without temporarily destroying the larger host system. Undoubtedly, time-sharing systems will be improved because the larger computers seem to provide the least expensive computation.

ASTROMETRIC MEASUREMENTS: SOFTWARE CONSIDERATIONS

Computer programs for scientific research tend to require frequent revision to meet the needs of new experimental situations. The development of scanning and measuring programs leads to prolonged periods of trial and error. For these reasons, the FORTRAN language is preferable to the basic machine assembly languages because it is intrinsically easier to document and modify. For many applications, the documentation is inserted with the code in the form of comment cards. Where required, machine language subroutines can be linked to the FORTRAN structures. The major programs should be controlled by one large routine, called a dispatcher, that contains only logical branches to call the appropriate subroutines for particular sub-tasks. Each routine should begin with a short paragraph on its purpose, the definition of variables and a warning about any complicated logic or approximations used. At appropriate places in the code, descriptive comments are inserted.

First the hardware drivers must be prepared for testing, maintenance, and measurement operations. Digital display drivers for graphic communication could come next. Then these two drivers should be integrated into an overall measurement-system-monitor that will control the sequence of operations to be performed for testing, calibration, and measurement.

Pattern recognition programs must have the most human-like decision making ability. There is a tendency to solve part or all of the pattern recognition problem with special equipment. This tendency would be resisted intitially because FORTRAN programs are much more quickly modified than electronic circuits. The redundancy in the data can be exploited to insure the quality of the results. Comparison of previous manual measurements of the same plate with automatic measurements will "teach" the pattern recognition program many new tricks.

Finally there will be experiment dependent subroutines, and data display routines for processing and publication of results. Where many laboratories collaborate, common magnetic tape formats for data interchange must be specified.

CONCLUSIONS

The bubble chamber experiments in elementary particle physics are now benefiting from automatic film data extraction provided by flying spot digitizers. The methods are general enough, so that their extension to other data extraction bottlenecks seems practical. The FSD's have been used for spark chamber film and their application to photographic astrometric technique has been tested.

Acknowledgement

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DISCUSSION

Rosenberg: *Which size plates did you scan, and how long did it take to scan them?*

R. Strand: *Plates of the format 100mm × 150mm were measured in six seconds on the 70mm machine with a 50 micron distance between the scan lines. Also, the machine can scan backward and forward, but in our work, we have not scanned any frame twice since we did not consider this necessary.*

THE USEFULNESS OF A FLYING SPOT DIGITIZER FOR THE MEASUREMENT OF THE COORDINATES OF STAR IMAGES ON PHOTOGRAPHIC PLATES*

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University of South Florida

ABSTRACT

The Flying Spot Digitizer (FSD) of the Brookhaven National Laboratory (BNL) can measure the coordinates of star images on photographic plates in one pass with an accuracy of 4.8 microns r.m.s. error. A field of the size 50 × 100 square millimeters is measured in a few seconds. Suggestions are made how a better accuracy can be achieved through the way in which the measurements are arranged and by repetition of the measurements. It is suggested that the speed of photographic astrometric measurements can be improved by about two orders of magnitude, at no loss of accuracy, by proper use of the FSD.

1.

The principle of measuring coordinates of images of stars on photographic plates has in the past been, to accurately align with them a measuring device (center of a rotating sector, cross hairs, etc.) by a measurable motion. From this, the coordinates of the image are computed. In recent times, several now well-known types of measuring machines have been put into operation in which human judgment as a measuring principle is eliminated and in which the human operator only initiates the measuring process and overrides the measuring machine when it is about to commit a serious blunder, such as measuring an obviously deformed image.

*Final report, NASA grant # NGR 10-008-011.

Even under the most favorable circumstances, this mode of measuring the coordinates by going from one image to the next will be time consuming. Also, unless some kind of an image sensor is employed this will require either a human operator to select the images, or a program which brings the measuring device sufficiently near to the images to be measured in a predetermined sequence.

The alternative to this "star-by-star" mode of automatic (and semi-automatic) measuring is a scanning device, with which the plate is systematically searched for images. As soon as these are recognized, their coordinates are measured and recorded. Since the entire plate is covered by the scanning process, the mechanical movements which produce it may follow a regular pattern which need not involve much change of momentum, so that it can be expected to proceed much faster than any "star-by-star" measuring process.

2.

The scanning mode of measuring images on photographic material is being employed in an apparatus, now receiving its final checkout, for the determination of relative proper motions (Luyten, 1970 and Newcomb, 1970). At the Brookhaven National Laboratory, Hough and Powell (1960) first designed and constructed a scanning device for the measurement of the coordinates of the trails of subatomic particles on films which were exposed in bubble chamber experiments. This apparatus, called "Flying Spot Digitizer", will record measurements of the coordinates of images in an area of about 5 × 7 centimeters in a few seconds. This is several orders of magnitude faster than any, even automatic, image-by-image machine would perform the same task. A light beam is sent through the film (or plates) and a receiver reacts when the intensity of the passing light decreases. While the light beam is kept in a fast up and down motion, the field to be measured is transported at right angles to the direction of the up and down motion of the scanning beam. The distance between two vertical scan lines (scan width) can be controlled and is about ten microns at minimum. The height of the scanned area (i.e., the length of a scan stroke) can also be controlled and is divided into 40 000 discrete fields, so that each of these would be one micron long for a scanning stroke length of four centimeters. Accurate coordinate measurement is therefore possible only in the direction of the scan stroke.

It follows from the mechanics of the FSD that the scanning beam will encounter every image whose diameter is at least equal to the scan width at least once, and larger images several times. For the purpose of measuring the coordinates of points on bubble chamber trails, the FSD is provided with a device which will record that position of the measuring beam where the intensity of the blackening reaches its maximum. This is, of course, not the ideal digitization position for star images, since on a typical good plate the blackening intensity profile along a scan through a stellar image rises rather steeply to a plateau, at which it remains in the inner regions of the star image. The digitization would then occur whenever this plateau is reached, so that a large stellar image would be recorded as a series of points (each obtained by a different scanning stroke) that lie on a somewhat semi-circle shaped curve.

For photographic-astrometric applications, a digitizing device for the FSD would be desirable which records the center line of the intensity profile or the equivalent thereof, instead. The authors understand that the construction of such a device is entirely feasible (Strand, 1968).

3.

The purpose of the investigations described in this paper was primarily to find out whether the FSD could usefully measure the coordinates of stellar images on photographic plates at all, and if so, how accurately. Since without mechanical changes, the FSD cannot accommodate films or plates of more than 50mm width, it was somewhat difficult to find appropriate material for measuring because good plates, once taken, are rarely disposable, or available for trimming to size. Dr. A. Upgren of the Van Vleck Observatory put at the authors' disposal a plate from the Van Vleck parallax program on which several misidentified fields were photographed and where the exposures had been taken with a coarse objective grating. Several scanning runs were made on a piece from this plate that had been cut to size so that it would fit into the FSD film stage. Measurement runs made on a piece of a negative copy of a plate which had originally been taken at the 26-inch refractor of the Leander McCormick Observatory proved to be unusable, apparently because the two fold copying process had rendered the images too hard (i.e., contrasty).

The measurements were recorded on magnetic tape. Many of the digitization records do not indicate stars. A particularly strong fluctuation in background blackening can trigger the digitization signal, and so will any sufficiently intense blackening (or object), whether associated with a star or not.

4.

Two of the main probelms in processing the output from the FSD are therefore the decision which digitizations are due to star images and which are not, and the computation of the effective measured coordinate (emc) from the various digitizations on one image.

The first problem could be solved by regarding as legitimate only those marks which produced records of at least two encounters with the digitizing beam, which would be indicated by the fact that the accurate coordinates (measured by the scan) measured at successive scan strokes differ by less than a fixed amount (no more than a few microns). Whether a legitimate mark originates from a star or plate imperfection, etc., can usually be decided by the pattern of the coordinates of its digitized points and proven either by correlating the measured image coordinates with the preciously known coordinates of the stars in the field, or by comparing the measurements on two plates of the same field, exposed under as identical conditions as possible. We used the computer only to eliminate isolated digitizations, i.e., those for which there was not at least one at a sufficiently similar position on a neighboring scan stroke. This simple procedure got rid of most of the spurious digitizations and rendered the remaining material usable for further processing.

This was done by plotting (on an appropriate scale) the measured coordinates of the digitized points. Those belonging to the same image lied typically on curved and sloped lines. Figure 1 shows the points originating from a fairly strong central image. The unit on the vertical scale is in ten thousandths of an inch, and the separation of the points on the abscissa is 15 microns. Figure 2 shows the plots of the points which originated from the diffraction images of the same star whose central image produced the points plotted in Figure 1.

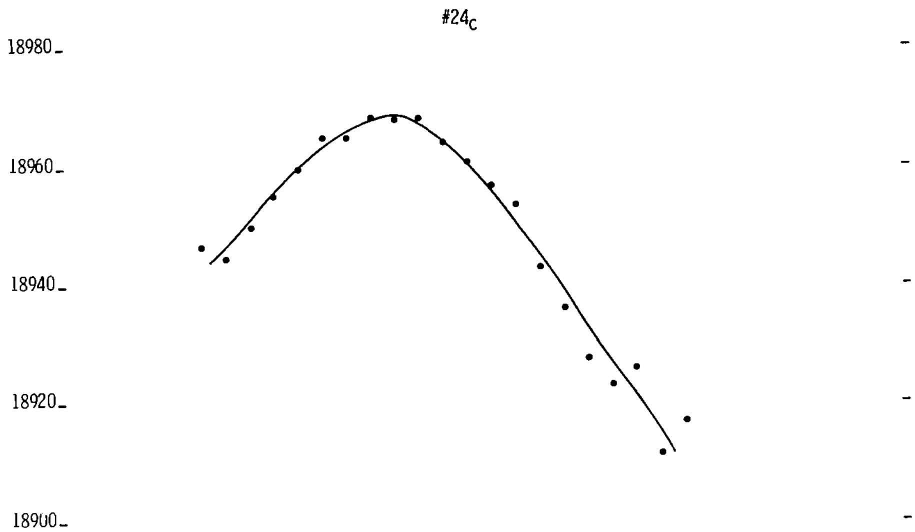


FIGURE 1

The way the machine sees a strong central image. The scan lines are 15 microns apart.

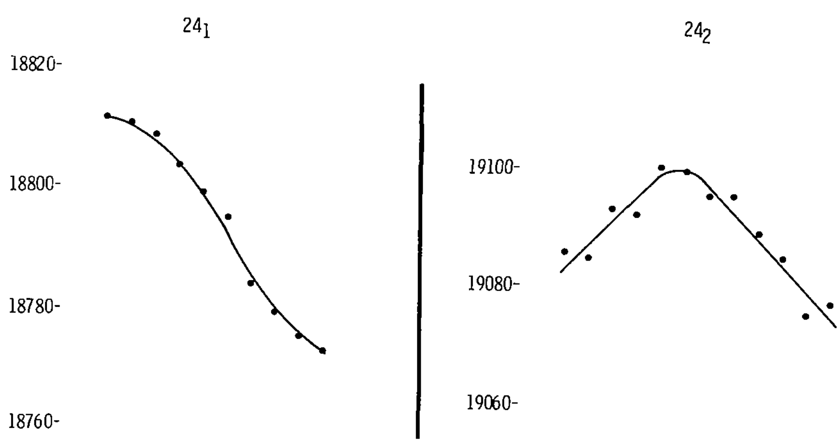


FIGURE 2

The way the machine sees diffraction images (of $\#24_c$).

Altogether, 31 images were thus identified. As the effective measured (y) coordinate of an image we regarded the straight mean of the individual y values given by all scan strokes that encountered this image. Undoubtedly, a more sophisticated definition would have resulted in emc's of higher ultimate accuracy. In order to obtain an estimate of the accuracy of these emc's which is a lower limit for the accuracy the coordinates of the same images were measured on the two-screw Mann measuring machine of the University of South Florida Observatory. During this manual measurement, the quality of the images was judged on a scale from zero (very bad image) to five (perfectly round image). Next, the y coordinates measured by the screw (y_s) and those measured by the FSD (y_f) were compared by a least squares solution of the form

$$y_s = Ax_f + By_f + C + Dd + Eq$$

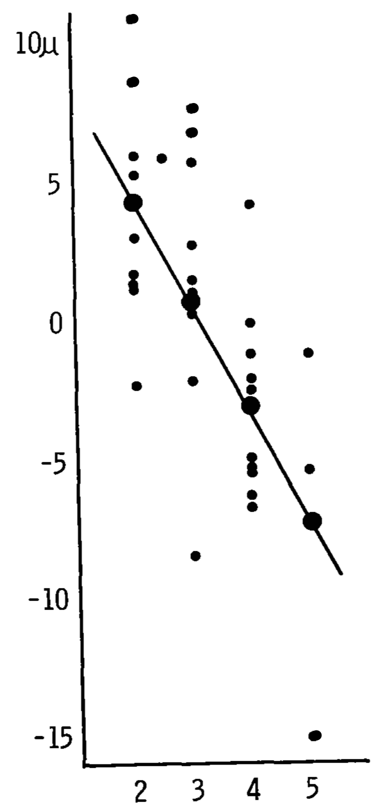
where d is the effective diameter of the image, expressed in terms of the number of scan strokes during which it was digitized, and q is the quality of the image, expressed in terms of the discussed scale above. This solution showed the following features: the standard deviation of D is one tenth of its value, thus the term with D is essential in the reduction model. The term Eq was added after a preliminary reduction had suggested a correlation of the residual to the image quality, and E turned out to be nearly five times its standard deviation, thus highly significant. The standard deviation of the residuals, i.e., the differences $y_s - (Ax_f + \ldots + Eq)$ results as 0.0048mm. If one assumes a standard error of 0.0015mm for the measurements on the Mann measuring machine, the standard error of an emc measured on the FSD turns out to be 0.0045mm. The establishment of this figure was the primary purpose of the investigation described in the current paper.

FIGURE 3

Preliminary residual vs. image quality.

5 = perfectly round

2 = quite oblong



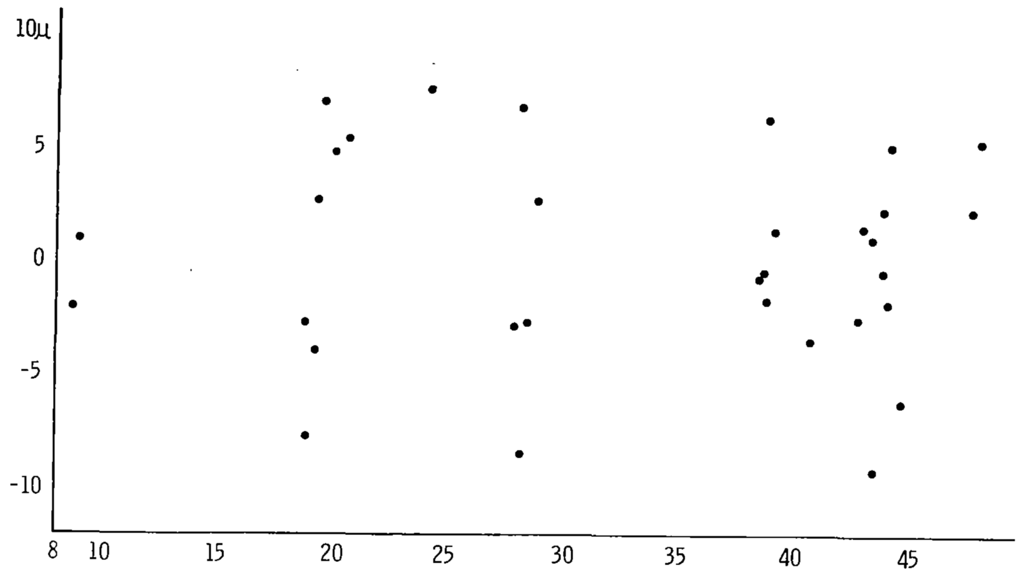


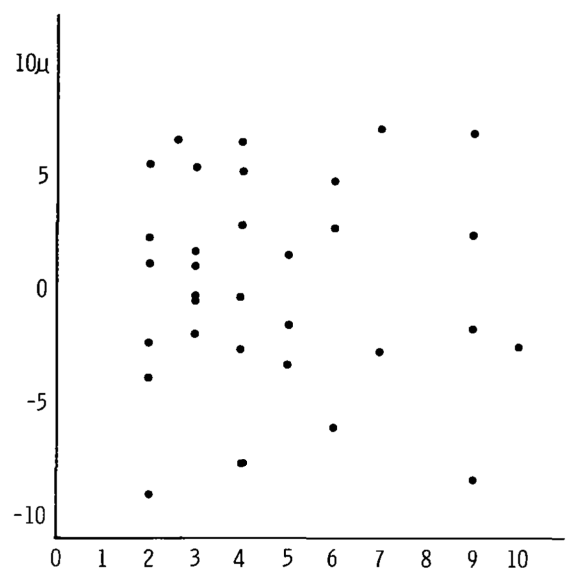
FIGURE 4

Final residuals vs. y

FIGURE 5

Final residual vs. image diameter

(in arbitrary scale)



5.

This number is of course quite large and would at first sight suggest that the FSD is useful for only very crude astrometric purposes. However, the accuracy available through more sophisticated use of this instrument can, in our opinion, be improved to where the emc's would be at least as accuracte as the coordinates obtained on a manually operated screw measuring machine. The measuring and reduction process described in this paper would, for this purpose have to be modified as follows:

a. It would probably be unwise to increase the scan width, since the least count of the digitization is 1/40 000 of the scan width. Since the scan width is thus limited, the plate will have to be shifted in y between various scanning sweeps. If it is shifted by exactly half the scan width or somewhat less, every image will be covered by at least two scanning sweeps. The multiple coverage of the images will thus aid in increasing the accuracy. The measurements obtained in different scanning sweeps can be reduced to the same frame of reference by a procedure analogous to the plate overlap method. This can be made even more accurate by shifting the plate by an accurately measured distance. It is clear that, even considering the limited field that can be scanned on the FSD, a mechanical device can be constructed which will allow plates of any reasonable size to be measured on the FSD.

b. Since only one coordinate is measured accurately, the plate (or the direction of scanning) must be rotated by 90° in order to measure both coordinates. It would then be natural to measure the plate in at least four different positions, between each of which the plate and (or) the direction of the scanning would be turned on an angle of 90°.

c. The emc's will have to be computed by a considerably more sophisticated procedure than taking the straight mean of the digitized coordinates on adjoining scanning strips.

d. The FSD can be made to digitize equal positive and negative values of a certain absolute calue of the slope of the blackening intensity curve along the chord instead of the zero slope of this curve. Every stroke would then yield two digitizations on every image, one when the beam enters and one when it leaves the image. The mean of these two would probably be better suited for the astrometric purposes than the present mode of digitizing the first occurance of zero slope of the curve of the blackening intensity along the chord.

Both a. and c. above will mean that every image is measured at least about eight times in both coordinates. This redundancy, in connection with the modification of the criterion for digitizing, could result in a standard error of the emc's of the various images in the order of between one and two microns. In view of the rapidity of the scanning sweeps, even the multiple measurement of individual plates necessary for obtaining the coordinates accurately will be about 100 times faster than measuring the same plates on a modern semi-automatic machine.

Acknowledgement

The authors are grateful for the National Aeronautics and Space Administration grant, #NGR 10-008-011, to the senior author. Thanks are due to Dr. Richard Strand of the BNL who was instrumental in our being able to use the FSD, and to Mr. Richard Powers of the Tampa, Florida IBM office who helped with the interpretation of the digitizer magnetic tape. Miss Marie de Luccia and Mr. William J. Veigel contributed a considerable amount of data handling.

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DISCUSSION

Jefferys: *Did you scan your plate in direct or reverse? That would have eliminated some of the magnitude dependent and quality dependent effects from the means of the direct and reverse measurement.*

Eichhorn: *Unfortunately we were pressed for time. The machine is designed to be operated with film frames, and to get a piece of glass to fit it proved to be quite cumbersome and time consuming.*

Murray: *How was the image quality judged?*

Gatewood: *At the measuring microscope in terms of roundness. Perfectly round images were classified as "five", the worst as "zero".*

Newcomb: *In our system, a counter is started when the scanning spot strikes a dense area, and stopped when it leaves it. This way we get the center of the dark area as well as its size, which is a measure for the magnitude of the star.*

Eichhorn: *I remember Dr. R. Strand as having told me that Dr. Hough, one of the designers of the machine, has said that the digitizing circuit of the Brookhaven Flying Spot Digitizer could be modified to do essentially the same thing.*

R.Strand: *(to Newcomb) Will your machine be commercially available? And what will it cost?*

Newcomb: *An exact duplicate would cost about $245 000.00*

THE LICK-GAERTNER AUTOMATIC MEASURING SYSTEM

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Santa Cruz, California

and

W. A. Popov

Gaertner Scientific Corporation

ABSTRACT

The Lick-Gaertner automatic equipment has been designed mainly for the measurement of stellar proper motions with reference to galaxies, and consists of two main components: the survey machine and the automatic measuring engine. The survey machine is used for initial inspection and selection of objects for subsequent measurement. Two plates, up to 17 × 17 inches each, are surveyed simultaneously by means of projection on a screen. The approximate positions of objects selected are measured by two optical "screws": helical lines cut through an aluminum coating on glass cylinders. These approximate coordinates to a precision of the order of 0.03mm are transmitted to a card punch by encoders connected with the cylinders.

This card deck serves as input for the automatic measurement process. A card is read, the automatic engine goes to that position, finds and centers the object whose precise coordinates and a photometric parameter are then punched. The next card is then read, the measurement and recording process repeated, and so on until the last selected image is measured. The measuring engine is in a separate, specially air and humidity conditioned room, and normally needs no supervision during the measurement. Remote control and supervision are possible, however, from another room.

The precise positioning of star images is accomplished by a spinning-sector scanner, and two crystal-quartz scales ruled in millimeters serve for coordinate measurement. An iris system is used for photometry. Although all the electronics are transistorized, the automatic engine needs a few hours of warm-up for dimensional stability. When the latter is achieved, the coordinate measurements repeat within 0.5 microns for practically indefinite periods; runs separated by 12 hours have not revealed any significant systematic changes in coordinates. The absolute accuracy of a measurement over the whole 17-inch range is within one micron for stellar images from approximately one magnitude above the plate limit to those reaching 0.25mm in diameter. Outside these limits the accuracy is lower. On the average, galaxies are measured with an approximately 50% larger error than stars. Slightly less than three hours are required for automatic measurement of 300 objects on a 17 × 17 inch plate, i.e., approximately 35 seconds per image.

INTRODUCTION

The Lick-Gaertner equipment for automatic measurement of astronomical plates was specially designed for determination of stellar proper motions with reference to galaxies, as part of the Lick program intiated by Wright (1950). The first-epoch plates of 1246 fields were taken with the blue-corrected lens of the Carnegie 20-inch Astrograph from 1947 to 1954. A 6° × 6° field was photographed on each 17 × 17 inch plate; the sky from -23° to +90° is covered, with a minimum overlap of one degree between adjacent fields. A 20-year interval between photographs was chosen for measurement of proper motions (Vasilevskis, 1954); consequently the second-epoch photography was started in 1967.

In planning the second-epoch series, it was decided to enhance the value of the program by determination of magnitudes and colors of those stars measured for proper motion. Accordingly, the Carnegie Astrograph was equipped with a second lens, corrected for yellow light (Vasilevskis, 1964), and the second-epoch photography is being carried out in two colors. This means that at least three plates will be available for each field, and a total number of approximately 4000 plates will have to be measured. Since approximately 300 objects will be selected on each plate, it would take several decades to carry out all the position and photometric measurements by conventional means. The proper selection of those objects to be measured is also a major task and must be considered carefully. It was decided to develop and build two separate machines, one for the efficient surveying of the plates and the selection of objects, and the other for carrying out a completely automatic process of measurement.

One of us (S.V.) did the original planning and prepared the instrumental specifications, while the other (W.A.P.) has been responsible for every phase of the technological process. Both have worked together closely during the full duration of the project, which was sponsored by the National Science Foundation.

The present report will be concerned primarily with the operation and performance of the equipment; a detailed technical account will be published at a later date. An outline of the specifications and the means chosen to satisfy them has been published elsewhere (Vasilevskis, 1960) and need not be repeated in full here.

THE SURVEY MACHINE

FIGURE 1

Survey Machine. To the right of the operator is the control rack of the automatic measuring engine, located in another room.

Objects for measurement are selected and their approximate coordinates recorded on punched cards with the aid of the survey machine shown in Figure 1. Two plates, up to 17 × 17 inches each, can be placed in adapters on a horizontal carriage at the top of the main structure. A high-pressure mercury lamp is located above the carriage, to illuminate areas of the plates that are projected on a 12 × 12 inch screen in front of the operator; the optical layout is shown in Figure 2. A 3 × 3 inch area of the right-hand plate can be viewed at a magnification of 4x for a general inspection of a large field; this corresponds to 1°1 × 1°1 on Carnegie Astrograph plates. For more detailed inspection and final selection of objects, both plates can be projected at magnifications of either 10x or 20x. For changes of magnification, optical components are placed in or removed from the light paths by remotely operated rotary solenoids. Either field can be blocked or both opened by shutters. A variable-speed blink device is also incorporated into the machine.

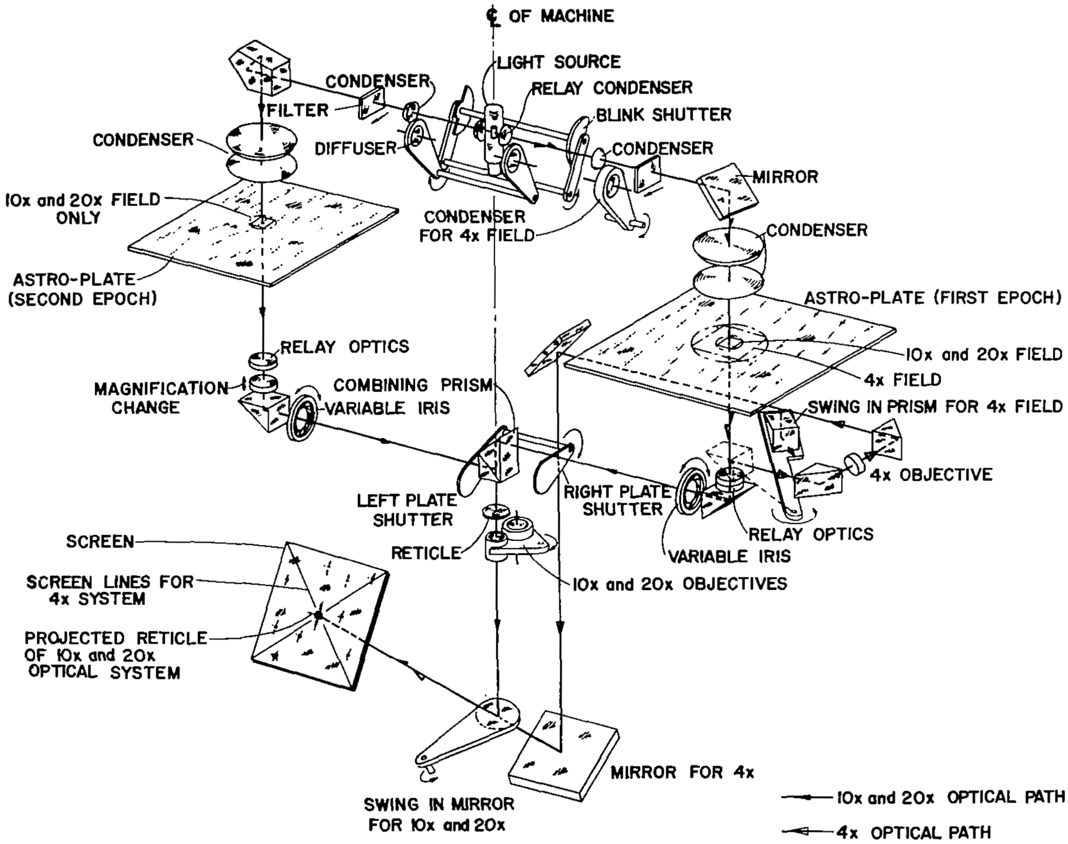


FIGURE 2

Optical layout of the survey machine.

Controls for these operations and also for those to be described later are located at the screen, as shown in Figure 3. In the center, under the screen is a joy-stick for controlling the rapid motion of the plate carriage, with a variable speed up to 2.5 inches per second. The final setting is done by four push-buttons located to the right of the screen. While these controls cause a motion of the whole carriage in two coordinates, the left plate adapter can be moved relatively to the right in two-coordinate linear and angular directions. Toggle and selector switches at the extreme left of the control panel actuate motors for these relative motions, thus permitting efficient and precise superimposing of images on both plates.

Other controls on the panel are: a switch for the mercury lamp, a knob for a slight change of magnification of one field relative to the other, two knobs for brightness adjustment of individual fields in order to match plates with different background fog, and finally, two controls for coordinate punching.

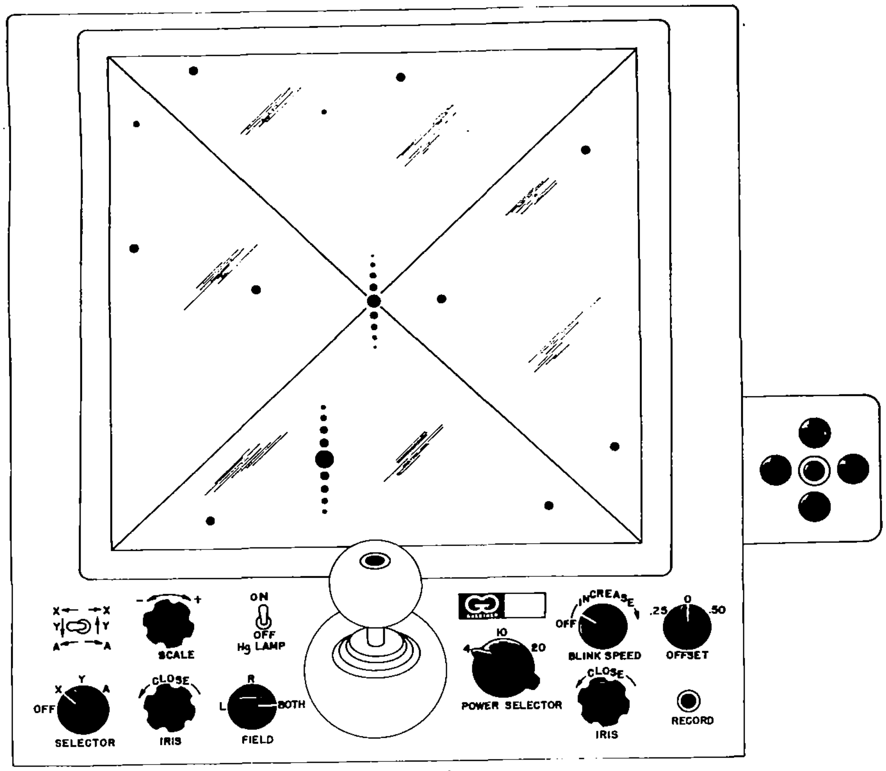


FIGURE 3

Projection screen and controls of the survey machine.

The coordinates obtained are to serve as input for subsequent automatic measurement, and this fact had to be considered in the development of the survey machine. On one hand, the process of surveying must be efficient enough to keep up with the automatic measurement, and so no time should be wasted in pursuit of excessive precision. On the other hand, the coordinates must be accurate enough for positioning the image within the scanner of the automatic machine. A precision of 0.1mm was specified in order to satisfy both these requirements. The actual precision of the measuring device is higher, as will be mentioned later.

To select an object for automatic measurement, its image is centered on a cross-wire reticle projected on the screen (Figure 2). Two rotating glass cylinders, with a helical line cut through an aluminum band deposited on each, serve to provide position reference, as shown in Figure 4. The pitch of these optical screws is 50mm with a precision higher than 0.01mm. Each cylinder drives a digital encoder and is also geared to the lead screw that moves the corresponding stage. The cylinders and the lead screws rotate in synchronization, to within their relative errors. When the carriage stops, its position with respect to the helical lines is sensed by photoelectric error detectors on the carriage. An error signal causes a servomotor to rotate the cylinder and encoder for correction of the error. A differential in the gear-train connecting the cylinder with the lead screw prevents motion of the carriage during the

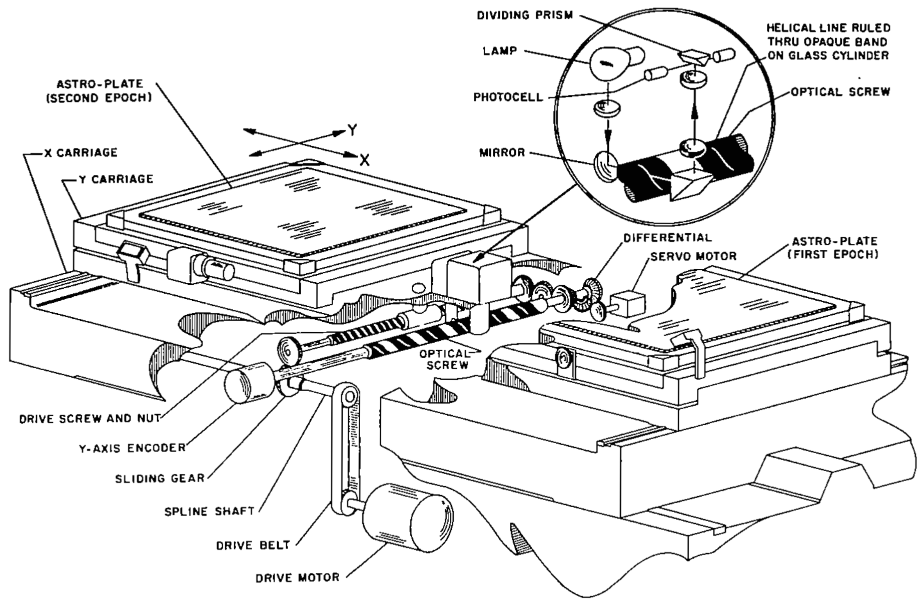


FIGURE 4

Carriage drive and measurement device for Y coordinate, survey machine.

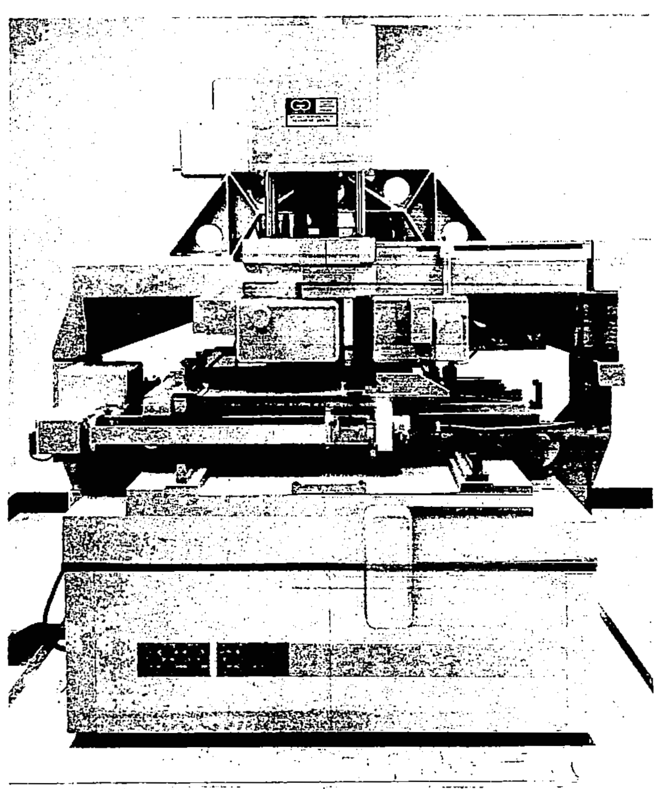


FIGURE 5

Automatic Measuring Machine

the correction process. Thus the measurement does not depend on errors, backlash, or wear of the lead screws. A push-button at the right lower corner of the control panel, Figure 3, causes the transfer of coordinate information from the encoders to an IBM 526 punch, shown in Figure 1; the read-out is to the nearest 0.05mm.

A selector switch above the recording push-button permits a considerable saving in time during measurement of either the first or second order spectra that are produced by coarse gratings placed over the Astrograph lenses. The grating images are displaced in the north-south direction at distances of 0.25mm and 0.50mm from the central image for the first and second order, respectively. During the survey operation the central image is always put on the cross-wire, and if inspection shows that this image is to be automatically measured, then the selector switch is left in its zero position. If, however, it is decided to measure the grating images instead, the switch is turned to one of the other two positions depending upon whether the first or second order spectra are to be measured. A constant is thus added to and subtracted from the $\underline{Y}$ coordinate, and two values of these modified coordinates are punched, corresponding to positions of the selected grating images.

Tests and extensive use of the survey machine since its installation in 1963 have demonstrated that its performance surpasses the original specifications and expectations. Since two plates are surveyed simultaneously, it is possible to keep pace with the subsequent automatic measurement of individual plates. If particular care is exercised in centering images on the cross-wire, coordinate errors do not exceed 0.03mm, the rounding error of the read-out to 0.05mm. In a normal survey when efficiency is of primary importance, the operator cannot spend too much time on exceptionally careful centering. Then the coordinate errors become larger, but normally they still do not exceed 0.1mm, and consequently do not cause any difficulties in the automatic measurement. Although the machine was primarily designed for the Lick Proper Motion program, it is being successfully used in other work, such as the blinking of plates for large proper motion stars or the discovery of variable stars, coordinate measurement for identification purposes, estimation of stellar colors, etc.

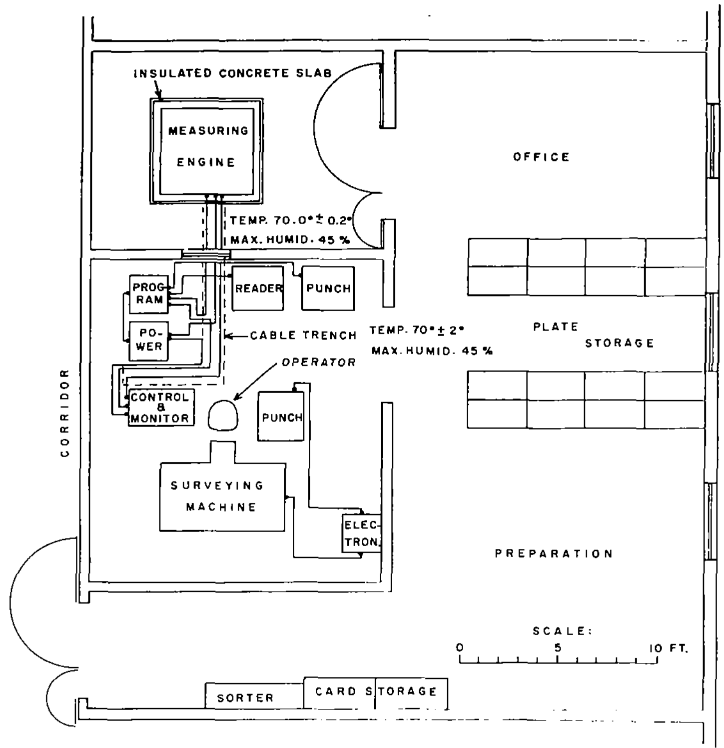
THE MEASURING ENGINE

The automatic measuring engine, depicted in Figure 5, is mounted on an isolated concrete slab in a separate air-conditioned and humidity-controlled room, as shown in Figure 6. Auxiliary equipment consisting of two IBM 526 punches and three electronic racks, is located in the survey room. One of the punches reads the survey cards, while the other records the results of the automatic measurement.

The usual measuring procedure is as follows: One of the two simultaneously surveyed plates is put into the measuring engine, where its position is defined by three studs against which two edges of the plate rest. During the survey process, the positions of both plates were also defined by exactly the same type and arrangement of studs, and the coordinate origin in the measuring engine is the same as in the right-hand adapter of the survey machine. This means that

FIGURE 6

Plan of rooms with the automatic measuring equipment, on the ground floor of the Natural Science Unit I building of the University of California, Santa Cruz.



a plate surveyed in this adapter needs no positional adjustment prior to automatic measurement. As a rule, the center and orientation of the other plate differ slightly from those of the first, so the position of the left-hand survey machine adapter must be adjusted so as to superimpose both fields on the survey screen. For automatic measuring of this plate, an adapter with adjustable studs is available for the measuring engine. It has been found, however, that it is more convenient to produce an adjusted survey deck by use of a computer. A few stars are measured semi-automatically and a comparison of these measurements with the survey coordinates gives the necessary reference information for adjusting and punching the original input data for subsequent automatic operation.

Normally the operator of the survey machine also supervises the automatic measuring activity. After placing one plate in the engine he returns to the survey room, loads the reader with the input cards, starts the engine by a switch on the control rack shown in Figure 1 and Figure 6, and resumes his survey of another plate-pair. Measurement proceeds automatically there after until the last object selected is measured and recorded. If a malfunction occurs during the process, a buzzer calls the attention of the operator, who usually identifies the reason for the failure by the indicator lights, and corrects it by the remote controls.

The principle of the automatic operation is shown in Figures 7 and 8. When the survey coordinates are read by the measuring machine, they are stored in a comparator unit and also transmitted to servo systems which drive the carriage to the position called by the survey data. As at the survey machine, the car-

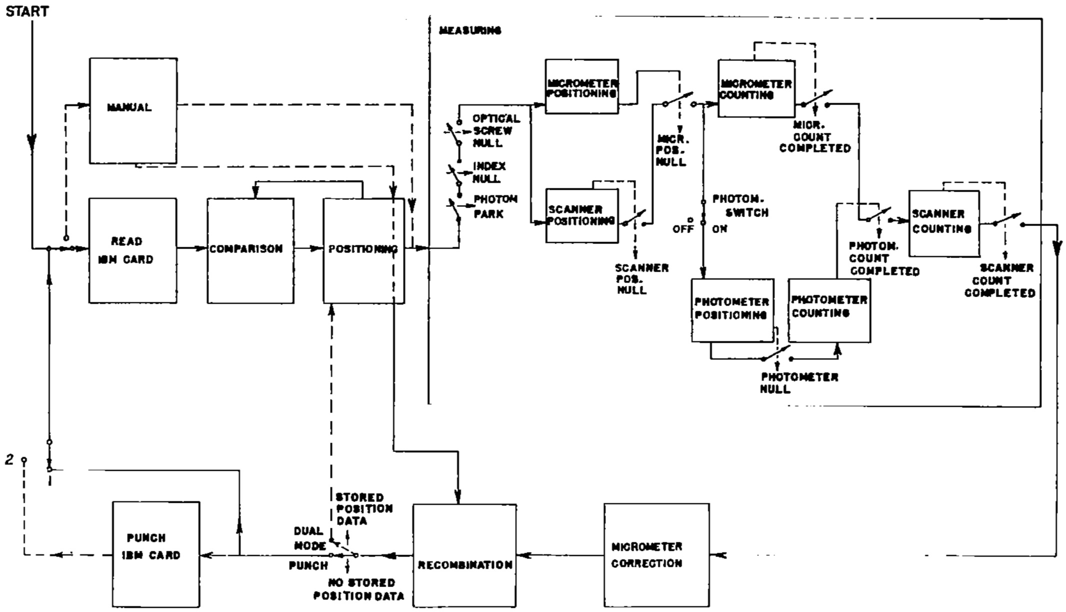


FIGURE 7
Block diagram showing the operation of the automatic measuring engine.

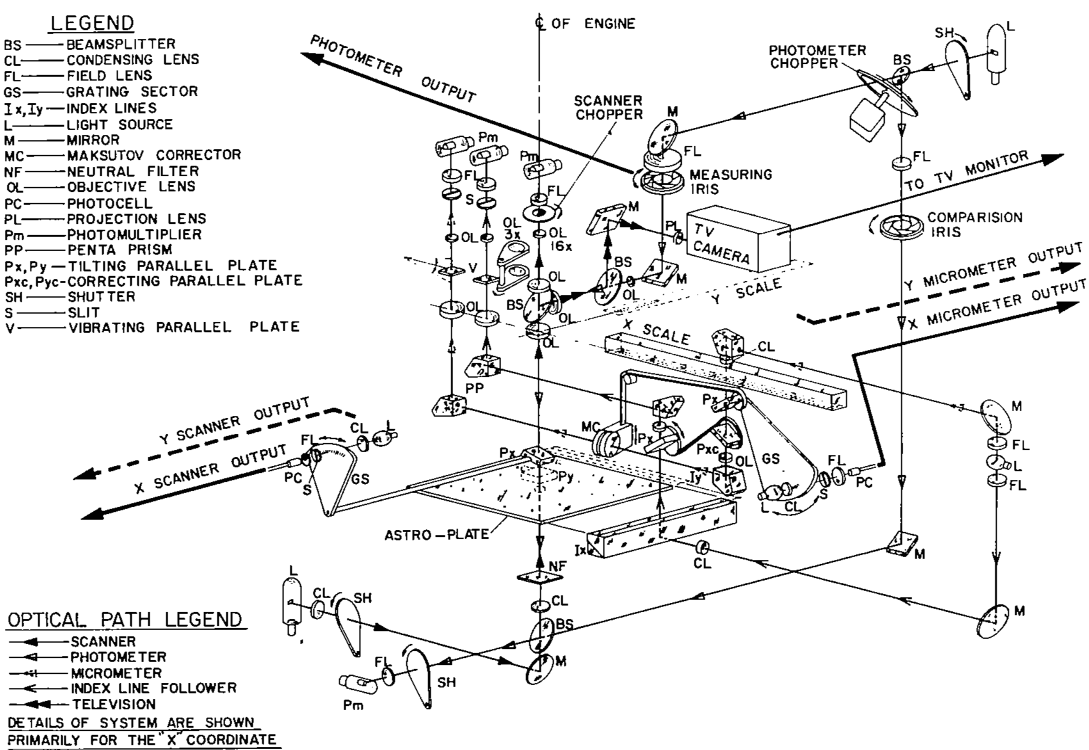


FIGURE 8
Optical layout of the automatic measuring engine.

riage position is referred to two glass cylinders with helical lines whose encoders continuously transmit the position of the carriage to the comparison unit. When this position information agrees with the stored survey data, the carriage stops and measurement starts. These steps can be bypassed if manual positioning is desired. In that case the carriage is positioned by push-buttons, and measurement initiated by pressing a null-detect control. In any case all the subsequent precise-setting operation proceeds with the carriage stationary, thus insuring that no possible vibration caused by motion of the heavy stages can enter.

Setting on the image is performed by a spinning-sector type scanner first developed and employed by the Watson Computing Laboratory of IBM (Lentz and Bennett, 1954). The image is centered on the scanner axis by two plane-parallel plates placed in the light beam, as Figure 8 shows. The amount of displacement is measured by counting divisions on sectors attached to the plates; each count is equivalent to 0.1 micron on the plate.

Two scales divided into millimeters are used for coordinate measurement. The scales are made of crystal quartz for dimensional stability, and the divisions are engraved through an aluminum coating. The position of the carriage is referred to the scales by two index lines ruled on glass bars that are attached to the carriage stages at right angles to the corresponding scales. Optical systems moving with the stages compare the index lines with the scales by means of tilting plane-parallel plates that act as micrometers for interpolation between the scale lines, as shown in Figure 8. A Maksutov type optical system is used for correction of the optical aberrations of these plates which appear at large plate tilts, for example when displacements of the order of 1mm are required. Again, the unit count of measuring the tilt of the plates is equivalent to 0.1 micron on the scales. Errors of these scales were determined in the process of calibration, and are permanently stored and automatically combined with the scanner and scale read-outs in every measurement. In this way the four decimals of millimeters are obtained; the full millimeters are taken from the encoders of the helically-engraved glass cylinders. When the decimals of the survery coordinate are near zero, one full millimeter may need to be added to or subtracted from the final combination; a computer unit for this operation is incorporated in the program rack. If the central image is measured, the coordinate information is transmitted directly to the punch for recording; if a grating image is measured, the information is stored until the second image is also measured, and then coordinates of both the images are recorded on the same card.

Errors of the carriage ways are eliminated by mounting the scales as suggested by Abbe (1890), namely in such a way that the axes of both the scales and of the scanner intersect. There is, however, a departure from the way chosen by Zeiss (König, 1932) to carry out the idea of Abbe. The mechanical contact between the edges of the carriage and the sliding scales has been replaced by an optical "contact" between the index-lines and the fixed scales. Since, if the scales were placed in the plane of the plate emulsion, as suggested by Abbe, a machine for measurement of 17 × 17 inch plates would be prohibitively large, the scales are placed above the carriage. The scale-reading system is so designed that any small tilt of this system is eliminated from the measurement.

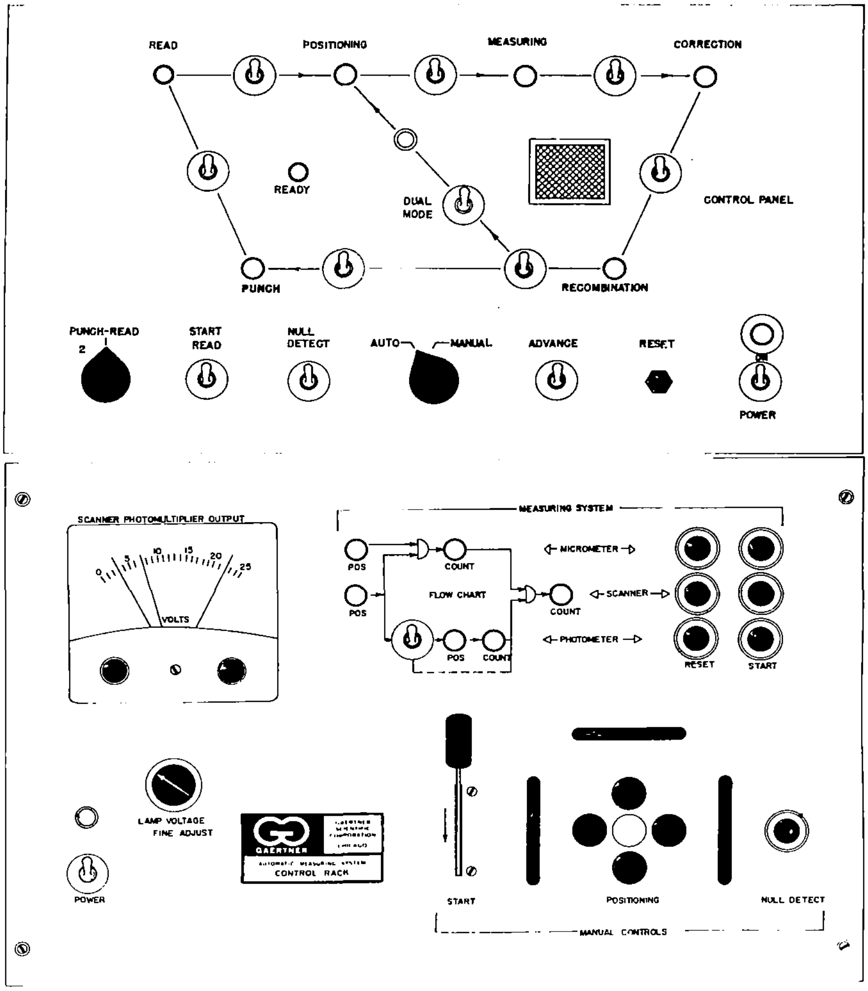


FIGURE 9

Indicator lights and controls of the automatic measuring machine.

As soon as an image is centered, its photometric measurement is initiated. The photometer contains an iris diaphragm as proposed by Eichner et al. (1947). The light from the photometer lamp is split into two beams, one passing through the iris with the star image centered, while the other serves as a comparison beam. The imbalance between the beams actuates a servosystem which rotates the iris ring until a balance is achieved. A sector with 5000 divisions is attached to the ring, and the divisions count provides a photometric read-out which is finally recorded together with the coordinate measurements.

The process of measurement is monitored by the control rack shown in Figure 1. At the top is a numerical display of coordinates transmitted from the positioning encoders connected with the "optical screws"; these coordinates agree with the survey input when the plate carriage stops. Under this display is a television screen showing a plate area of 5mm across enlarged 40x. A circle projected in the center of the screen shows the 250 micron opening of the scanner, and at least a part of the image selected must be within the circle for automatic measurement. In manual operation, images are brought within this circle by push-buttons. Under the screen there are indicator lights and controls located on two panels, one vertical and the other, below it, inclined; both these panels are shown in Figure 9.

In the upper part of Figure 9 appear indicator lamps which light up in sequence as operation of the automatic engine progresses. In case of a failure the last light shows the step whose completion has been interrupted. Switches between the lights permit stopping the automatic operation at any step desired; these are used mainly for testing and adjustment purposes. Under the bank of lights there is a row of controls, including a selector switch for automatic or manual mode.

In the middle, at the top of the lower panel, there are six additional lamps representing the sequence of measurement; they light when the measuring light on the upper panel goes on. These additional lights show the progress of centering the image, reading the scales, and operation of the photometer; a switch permits bypassing the photometer when it is not needed. Eleven more lights on the measuring engine, which can be seen through a window from the survey room, indicate the position of the plane-parallel plates and the iris, and provide additional diagnoses if a failure occurs. Controls for manual measurement are mounted in the lower right part of the inclined panel.

The 17 × 17 inch proper motion plates, and other plates with many images, are measured automatically. Completely manual operation is used only on rare occasions. More frequent use is made of the semi-automatic mode. In this case the input coordinates do not need to bring images within the scanner; this is performed by manual controls as soon as the image appears on the screen and the carriage stops. This mode is used for producing adjusted input data for plates surveyed in the left adapter, as mentioned before, and also for measurement of a small number of images per plate, as in parallax programs.

The 17 × 17 inch plate with 300 nearly uniformly distributed images can be measured automatically in nearly three hours; the average speed for such plates is 35 seconds per image. The efficiency is of course slightly higher for clustered images, and lower if a few widely scattered objects are measured. Needless to say, the photometric measurements alone would require a considerably longer time with a conventional iris photometer.

Accuracy is the main concern in a measuring engine, and tests of various components were made as the construction of the machine progressed. After the engine was completed, its performance was tested at the Gaertner plant in Chicago, and again after it was installed at Lick Observatory in Santa Cruz. Two types of tests were made: those of repeatability, and of the total error of measurement.

The repeatability is determined by comparison of successive measurements of the same image with minimum time intervals between. From many tests it was found that for stellar images of good quality the average deviation is 0.35 and 0.25 microns in _X_ and _Y_ respectively. The systematically larger errors in _X_ are probably due to the fact that the corresponding stage is carried by the mobile _Y_ carriage, with the added complication of a double motion and more involved arrangement of the measuring components. Quite recently Dr. A.R. Klemola made tests of repeatability over a long period of time; his results are presented in Figures 10 and 11. The abscissae in both graphs are the time intervals

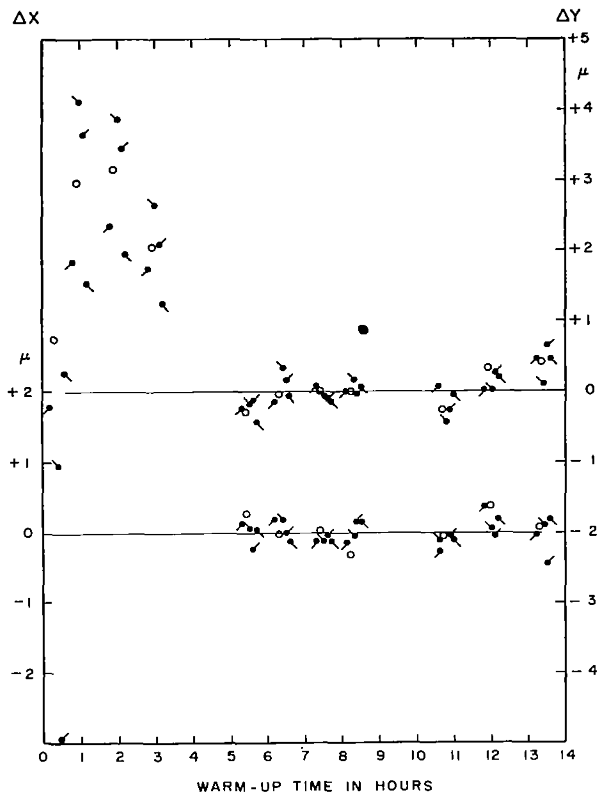


FIGURE 10

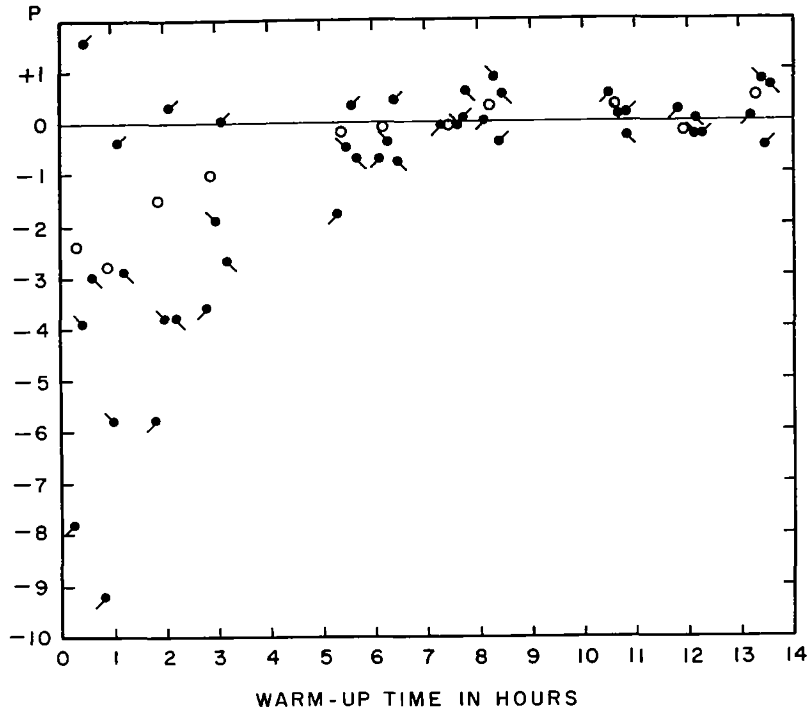
Repeatability of coordinate measurement as a function of the warm-up period after a two-day shut-down. Ordinates in microns.

after the engine was turned on following a two-day shut-down. Five stars were selected, one in the center and others in the corners of a 17 × 17 inch plate, and the cycle of measuring these stars was automatically repeated for almost 14 hours. Open circles represent measurements of the central star, bars of filled circles point towards the corresponding corners, with north up and east to the left. Each symbol represents a mean of approximately 10 measurements. The agreement between successive individual measurements confirmed the short-term repeatability quoted above. The graphs, however, exhibit the long-term stability of the measuring engine.

In Figure 10, only measurements in Y are represented for the initial five hours; the ordinate in microns is shown on the right side of the frame. The systematic departures and large scatter clearly show that this interval is the minimum warm-up time. The differences in X are several times larger, and they are not represented in Figure 10. Except for the television system, all the control and power circuits are built with solid-state components and should not require any significant warm-up time. The warm-up time presumably is necessary for the machine to reach thermal equilibrium and dimensional stability after the many lamps are turned on. The repeatability in both coordinates is shown for measurements after the first five hours; the ordinate for X is marked on the left side. For each star the mean of all the measurements during this stable

FIGURE 11

Repeatability of photometric measurements. Ordinate unit corresponds to 0.004 magnitudes on plates taken with the blue lens of the Carnegie 20 inch Astrograph of Lick Observatory.



period was compared with the individual means, and the differences plotted. As can be seen, the long-time repeatability is within half a micron.

The repeatability of photometric measurements is shown in Figure 11; the ordinate is in units of the last digit of the photometer output. Approximately 1200 units correspond to a range of five stellar magnitudes on blue plates taken with the Carnegie 20 inch Astrograph, or 0.004 magnitudes per unit. Figure 11 shows that the photometer also needs a five-hour warm-up period, after which the stability is within one unit. The plate errors are much larger, of course, and the precision quoted means only that the photometer errors can be neglected in discussions of photometric measurements.

For determination of total errors, not just repeatability, several 17 × 17 inch plates were chosen and up to 300 objects, both galaxies and stars, were selected for measurement on each plate. Objects were divided into several groups according to certain properties of their images; codes for use in computing were assigned to each group in the process of surveying the plates. Each plate was measured in four positions, at 90° intervals in orientation. The measured coordinates were then transformed to the same position angle by a computer, and the measurements of the same objects obtained in all four positions were intercompared. Since each rectangular coordinate of every object was measured with both scales at two different positions of each scale, it was possible to make an analysis of various errors. The corresponding least-squares solutions revealed immediately a systematic error of up to three microns in measurements made against the *X* scale, and further investigation showed that a curvature of the corresponding index-line was responsible for the error. The departure from a straight line was found to be rigorously proportional to the square of the distance from the center, and amounts to 6.49 microns at 200mm from the center, i.e., at the edges of 17 × 17 inch plates.

The index-lines ruled on glass bars were tested for straightness after their manufacture, and the curvature must have been produced during the process of attaching the _X_ -bar to the carriage. Repeated determinations at intervals of several months showed that the curvature is constant, and therefore of no concern. In proper motion and other differential measurements the error is eliminated; correction of absolute positions requires a single arithmetic statement in the computer program. Still, we are tempted to store the errors in the program rack of the measuring engine for correction in the measurement process.

No other systematic effects could be detected, and after the curvature mentioned was taken into account, the total accidental error was found from residuals of least squares solutions. Stars were divided into four groups: 1) very faint, barely above the plate limit, 2) faint, approximately one magnitude above the plate limit, 3) medium, with images of the order of 100 microns in diameter, 4) bright, with images almost filling the scanner area. The probable errors of a measurement of an image in these groups are: in _X_, 1.15, 0.79, 0.69, 0.73 microns, and in _Y_, 0.74, 0.56, 0.54, 0.55 microns, respectively. As before, the errors in _X_ are slightly larger than in _Y_ . The images of very faint stars produce a small signal with the 250 micron scanner, and consequently those errors are the largest. The errors of measurement of galaxies depend even more on the quality of images. For galaxies normally selected for standards on the Lick Proper Motion program, the errors are about 50% higher than those for stars.

CONCLUSION

The basic technological ideas regarding the described equipment were formulated in 1959. The project, however, was not constrained in its progress by the original ideas, and appropriate new technological developments were incorporated as they became available. This fact has permitted the system to meet the severe requirements of versatility and precision originally specified for the equipment.

The versatility was dictated by the special needs of the Lick Proper Motion program, and it is difficult to visualize a program demanding a greater flexibility. Indeed, individual components of the present equipment might well serve as independent units for many astronomical efforts; some of these possibilities have been outlined elsewhere (Vasilevskis, 1960).

Precision cannot be discussed separately from the plate size and speed of measurement. The limits of dimensional stability may have nearly been reached in this large engine for 17 × 17 inch plates. As long as mechanical motions of heavy stages are required, the carriage speed that is compatible with positional stability is limited, regardless of the means used for the actual coordinate measurement. Finally, the appreciable intrinsic dimensional errors that are inherent in astronomical photography do not justify excessive effort and expense in achieving precision which cannot be utilized in practice.

Acknowledgement

We are greatly indebted to the many persons at both Gaertner Scientific Corporation and the University of California who took interest and participated in the project; the late Dr. Samuel Jacobsohn of Gaertner contributed many important ideas during the initial stage. The project could not have been undertaken without the support of the National Science Foundation.

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DISCUSSION

R. Strand : *Do you use a digital computer anywhere in the measuring process directly?*

Vasilevskis: *No. Our output is on punched cards, which are then computer processed.*

Murray : *The combined action of grating dispersion and of atmospheric dispersion will make the grating images on top look different from the grating images at the bottom. Do you anticipate that this will cause any difficulties?*

Vasilevskis: *We now have in operation a program, consisting of over 100 fields which have as the main purpose the detection of error sources. I agree that the line connecting the grating and central images ought to be horizontal, but I would be against changing the already established practice for our second epoch plates. Since our measurements are essentially differential, the effects, if any, may be eliminated.*

Wesselink : *The plates being taken now in Argentina for the Yale-Columbia program, the line connecting the images lies horizontally for the very reason mentioned by Murray.*

A SEMI-AUTOMATIC MEASURING MACHINE

K. Aa. Strand

U. S. Naval Observatory

ABSTRACT

The U. S. Naval Observatory acquired, in 1966, a machine which is designed to locate automatically a star image on a photographic plate from approximate coordinates on a punched card and to measure and record its position to a micron. The machine was manufactured by Nuclear Research Instruments, Inc., Berkeley, California, according to the Observatory's design specifications.

The main frame of the machine is of granite, as are the x and y coordinate carriages which move on air bearings against granite ways.

The system is capable of making measurements continuously over a 10 × 10 inch range by means of a Ferranti moiré fringe system, with a least count of one micron.

A SEMI-AUTOMATIC MEASURING MACHINE

A semi-automatic measuring machine was acquired by the U. S. Naval Observatory in 1966 for the purpose of measuring the extensive plate material expected to accumulate from the trigonometric stellar parallax program being carried out with the 61 inch Astrometric·Reflector at the Observatory's Flagstaff Station, Flagstaff, Arizona. It was realized that a considerable savings in manpower would be possible if; 1) Automatic centering of images replaced visual bisection, 2) Measurements could be recorded directly on punched cards or tape, 3) Automatic prepositioning on selected images was made available, 4) Easy access to the machine was made available for rapid change-over from plate to plate.

The last two features are of particular importance in view of the relatively few star images which are being selected for measurement on each plate and because a parallax series consists of some 50 plates with identical reference stars. These features, as well as a series of others, were incorporated in the design specifications drawn up by the Naval Observatory.

The machine was built by Nuclear Research Instruments (NRI) in Berkeley, California -- the same firm that built the Franckenstein machine which was first developed at the Lawrence Radiation Laboratory for measuring nuclear tracks in bubble chamber photographs.

The measuring system consists of: 1) The opto-mechanical assembly called "the machine", 2) A three rack electronic cabinet, 3) Two IBM-526 Card Punches.

The main structural members of the machine are heavy granite components, as are the two carriages which carry the photographic plate over a 10 × 10 inch area. The lower carriage (the saddle) rides over the base plate on eight lift air bearings (pressure 60 lbs./sq. inch), and its motion is controlled by granite guideways mounted on the base-plate. Guide air bearings maintain the linearity of the motion.

As shown in Figure 1, the upper carriage (the stage) rests above the base on four lift air bearings (pressure 45 lbs./sq. inch) and is held perpendicular to the saddle by two guideways mounted on it, again using guide air bearings moving against the granite guideways.

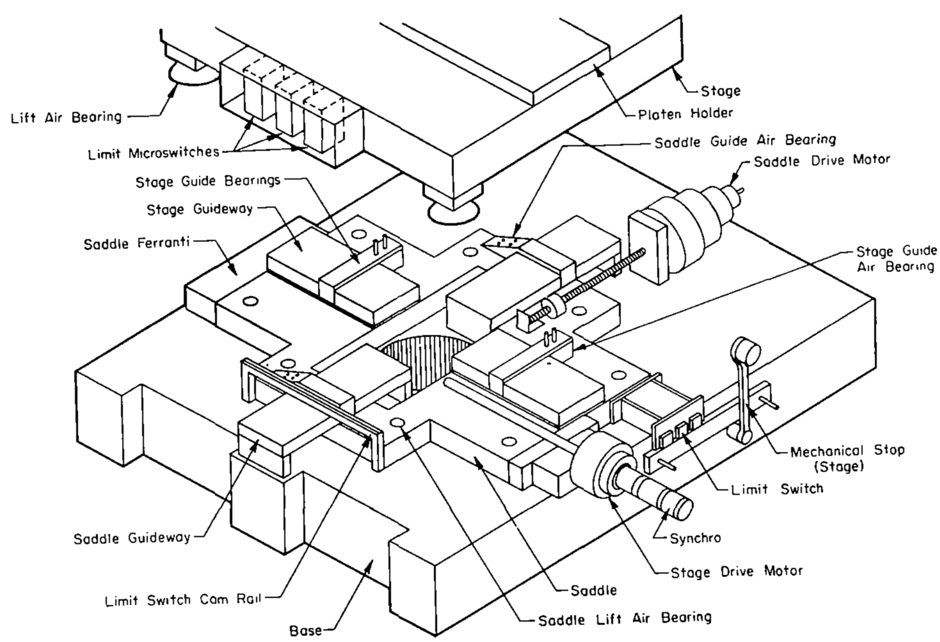

FIGURE 1

Schematic drawing of the U. S. Naval Observatory's semi-automatic measuring machine, showing the base plate with the measuring stages, their guideways and synchro-motor drives.

Air bearings separate carriages and ways by approximately 2.5 microns (noise 1/4 micron).

Both carriages are moved by means of ball screw assemblies driven by synchro motors. However, two opto-electrical transducers, employing Ferranti gratings, monitor carriage movement along the two axes for measurements to a least count of ±1 micron.

The photographic plate mounted on the platen is illuminated by a 450 watt Xenon arc lamp, the light of which passes through the platen and the photographic plate to a 40x or 3x projection lens.

The light is then directed to a beamsplitter, which divides it into two paths. One beam forms an image on the viewing screen at the front of the machine; the other beam is reflected off a 45° mirror to an opto-mechanical detecting head which generates signals to control the auto-centering.

The image beam enters the port at the front of the detecting head and is divided into two light paths by a small beamsplitter for two orthogonal scans. Each beam then reaches a photomultiplier tube mounted behind the scanning disk.

The auto-centering is performed by means of a 5 inch scanning disk with 24 slits rotating at 3600 rpm, scanning the image at a rate of one micron in one microsecond.

As each slit of the disk "scans" over the image beam, the photomultiplier receiving this beam generates a "track" pulse. A second pulse (the marker pulse) is generated for each track pulse via the marker light, mirrors, scanning disk, and marker light phototubes. The marker assemblies are orientated so that the marker pulses fall at the optical reference plane and thereby establish a time reference for the electronic circuits.

The detecting head scans an area of 350μ in diameter at the plane of the photographic plate. The maximum size image which can be centered automatically is 200μ. The smallest size is of the order of 50μ.

Two oscilloscopes, located below the viewing screen, monitor detecting head operation to enable the operator to view the interpretation of the image by the scanner.

The centering of the image, once within the scanning area, is practically instantaneous -- while the carriage traveling speed, using the joy stick, is 1 in./2 sec.. In the prepositioning mode the speed is one-third this value.

A special feature of the machine is the easy attachment and detachment of the photographic plate to the machine. The plate is held to a glass plate-holder by means of a vacuum. The plate-holder, in turn, is held in place by a vacuum against a rotatable glass platen, which is also held in position with a vacuum on top of the stage. When the platen is being rotated, a pressure is applied for lift-off for easy movement. Similar lift-offs are used for the plate and the plate holder.

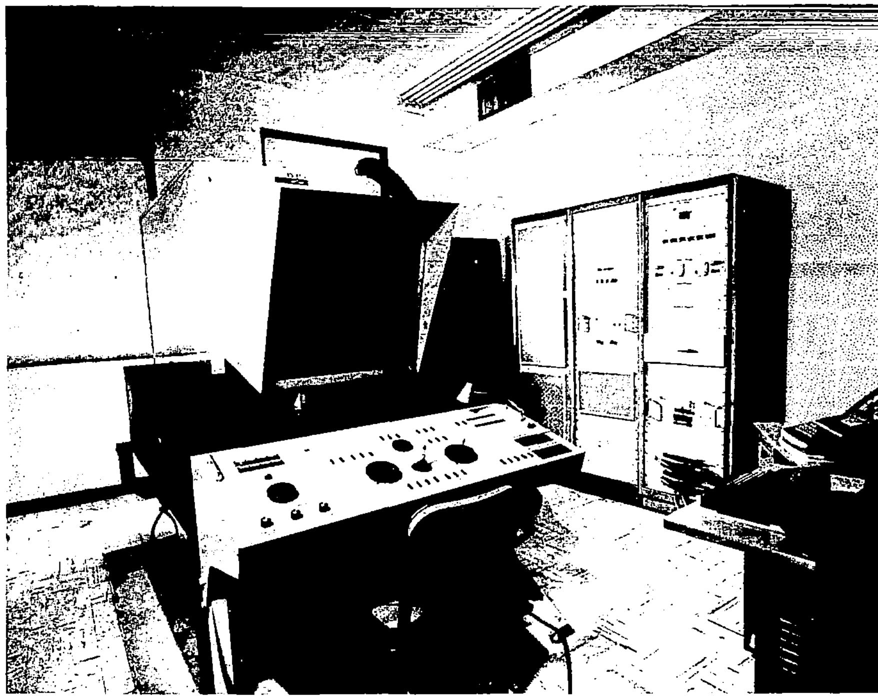

FIGURE 2

Front view of the semi-automatic measuring machine showing the viewing screen and control console. Shown at the right center is the three rack electronic cabinet.

Figure 2 shows a front view of the finished machine with the control console from which the machine is operated.

The first plate in a parallax series is measured in the manual mode. This provides the deck of punched cards which are used as a program deck for the remaining plates in the series to be measured in the semi-automatic mode. It is intended that the measurer be required to examine the image to be measured, either on the screen or by means of the oscilloscope tracing, for acceptance or rejection prior to recording the position.

The machine has been found to have a slight bias depending upon magnitude. This can be compensated for, however, by measuring each plate in two positions, turning the plate 180° between them. With this method 6 to 8 plates can be completely measured in one hour, as compared to two plates on a manually operated machine.

The measurements obtained with the semi-automatic machine compared to those with a machine using conventional bisection of images, show a decrease in their mean error of 25%. The machine has a repeatability of 0.8μ.

The significant increase in accuracy and measuring speed has not only had a significant impact on the Naval Observatory's astrometric programs, but has also assisted several other observatories in their programs.

DISCUSSION

Murray: *Are you talking about the accuracy of a position of a star or that of its image on the plate?*

Strand: *The error I was referring to is what we call "mean error of unit weight" in a parallax solution.*

Murray: *This then depends on the position of the image on the plate, and its structure?*

Strand: *That is correct.*

Vasilevskis: *Have you investigated the ways and their orthogonality?*

Strand: *Yes. This was done by Dr. Wesselink during one of his recent visits to the Naval Observatory when he found a deviation of 13″7 ± 0″7 (m.e.)*

Schmid: *What is the sensitivity to vibrations, and the long term stability of the apparatus, especially of the zero points?*

Strand: *The original Ferranti counting system was modified by Nuclear Research, Inc., and is very stable.*

Fredrick: *According to some of my own experience with airborne equipment, I find that laser interferometer measuring machines are sensitive to even the clapping of hands, which causes them to run up counts due to the change in the air density.*

van de Kamp: *Dr. Strand's measurement procedures appear to have increased the weight of parallax plates by a factor of 2. It would be interesting to find out exactly, which feature of the automatic measuring process is responsible for this. Perhaps the personal equation in bisecting by different observers is the reason for this.*

Strand: *It may be due to the automatic machine centering as compared to the bisecting by human measurers. The faintest images measured by our machine are two magnitudes above the plate limit, therefore quite black, and the blackening density profile goes up rapidly as one enters an image. The machine, in effect, then measures the center of the image's geometric boundary.*

Fredrick: *The fatigue of the measurer which sets in after about one hour is probably responsible for a good share of the larger error on manual and visual machines as compared to that on the USNO automatic machine.*

Vasilevskis: *I know from my own experience that measuring visually for many hours is extremely difficult and tedious, even for ideal, round images. Since the setting accuracy of a bisection is about a thirtieth of a diameter, large images are measured visually less accurately than small ones. This is not true for the automatic machine.*

Strand: *We have found that we can measure very faint images visually more accurately than automatically. This is obviously because in their case the blackening density profile is very shallow.*

van de Kamp: *This investigation shows that the process of visually bisecting an image in the measuring machine has indeed been a principal contributor to the total error, along with emulsion shifts, etc.*

Strand: *The automatic measuring machine probably approaches the best measuring accuracy we can expect, since out of the 1.7 microns (m.e. 1) achieved with it, 0.8 or so might be due to emulsion shifts.*

Eichhorn: *What is the dispersion of the difference between measurements of the same stars when a plate measurement is repeated?*

Strand: *Very small.*

Eichhorn: *Does this mean that the measurements of the same plates but in different runs give practically identical results within one micron, which is the least count of the machine?*

Strand: *Yes.*

Vasilevskis: *One of the error sources of manual measurements are periodic and progressive errors of the screws which are improperly accounted for. Unfortunately, these have to be constantly kept under surveillance, since they change in time due to the wear of the screws. In the Gaertner machine at Lick Observatory, this problem was avoided by not having mechanical measuring screws at all.*

THE EXTRACTION OF ACCURATE COORDINATES OF IMAGES ON PHOTOGRAPHIC PLATES BY MEANS OF A SCANNING TYPE MEASURING MACHINE

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INTRODUCTION

A problem of the Moiré method experimental stress analysis is similar to a problem encountered in astrometry. It is necessary to extract accurate coordinates from images on photographic plates. This paper will discuss the solution to the mutual problem found applicable to the field of experimental stress analysis. It is hoped that this discussion will stimulate further thought rather than suggest an ultimate solution to the problem of photographic measurement.

The photographic measurement problems are similar in that in both disciplines comparative measurements must be made from two large format photographic plates. Several significant differences in the problem exist. One difference is the number of plates to be measured is much less in experimental stress analysis. Therefore, there is less pressure to measure rapidly. Another difference is the fact that in experimental stress analysis continuous tone images are being measured whereas in astrometry the existence and position of singularities in the optical image are being measured.

A brief discussion of the Moiré method of experimental stress analysis is presented to outline the measurement problem. A discussion of the photo-reading device developed to make the measurements follows.

THE MOIRE METHOD

The object of the Moiré method of experimental stress analysis is to produce a set of fringes in a photographically recorded image which indicate the local displacements on a deformed body.

If the displacements are known, strains and stresses can be determined. Fringes are formed by embedding, imprinting or etching a set of crossed lines on the object in an undeformed state. These are photographed before and after deformation of the surface to which the lines have been applied. A reference line set at the original spacing is superimposed on the resulting negatives and a new negative is obtained containing Moiré fringes.

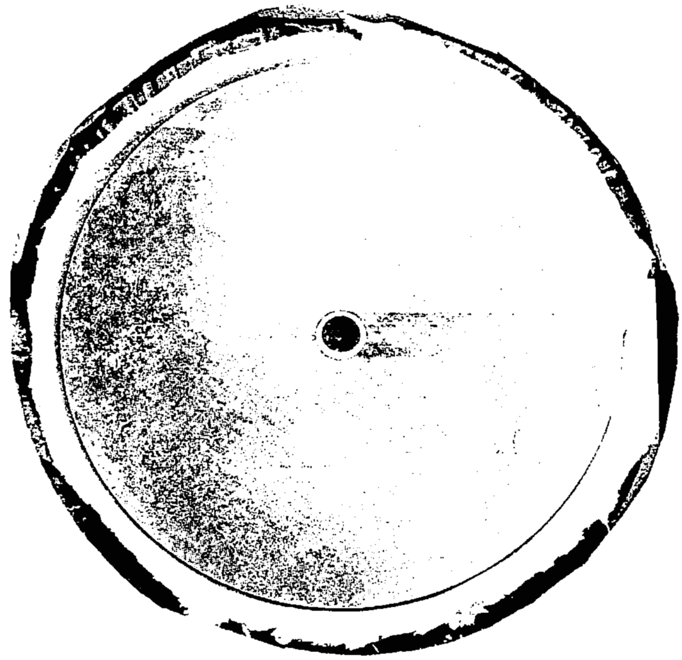


FIGURE 1

Stainless steel ring number two at room temperature.

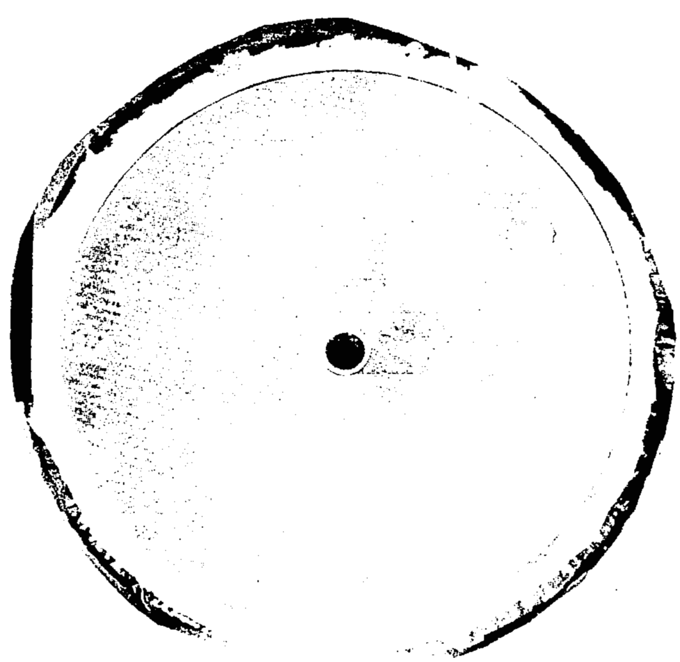
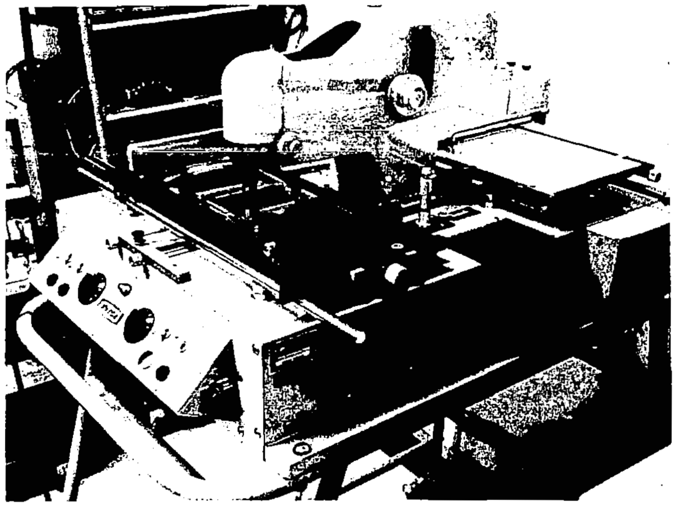


FIGURE 2

Stainless steel ring number two heated to 1580°F O.D. and 900°F I.D. .

The fringes represent lines of equal displacements of the deformed body. An example of the data expected is shown in Figures 1 and 2 which show a ring at room temperature and at an elevated temperature respectively. Temperature gradients have introduced thermal stresses in the ring. Small differences in positions of the maximum densities of the fringes must be measured.

FIGURE 3
Photo-reader used to Interpret fringe patterns.



MEASUREMENT OF MOIRE FRINGES

The machine that is most commonly used to extract coordinate intensity information in experimental stress analysis work is the Joyce-Loeble recording microdensitometer, shown in Figure 3.

The machine uses a dual beam density measuring system. "Dual Beam" means that one source of light is split into two beams. One beam travels through the photographic plate and by a microscope arrangement is focused alternately upon a single photo cell. The photo cell also "sees" the second beam after it has passed through a calibrated variable density plate. A servo motor driven by the amplified differences in the two beams establishes the position of calibrated plates. A pen attached to the moving calibrated plate records intensity along one axis on a sheet of paper. Along the other axis of the paper is a multiplied coordinate dimension accomplished by the use of levers. Mechanical multiplications of 2000 result in increased accuracy of the final measurement performed on the paper record. An alternate arrangement provides either analog on digital electric output signals.

This machine provides very accurate intensity information. It also provides accurate coordinate positions over an extremely small range (on the order of 1/4mm). The requirement to scan large plates involves starting and stopping thereby reducing coordinate position accuracy. A machine was designed to overcome this difficulty.

THE NEW SCANNING MACHINE

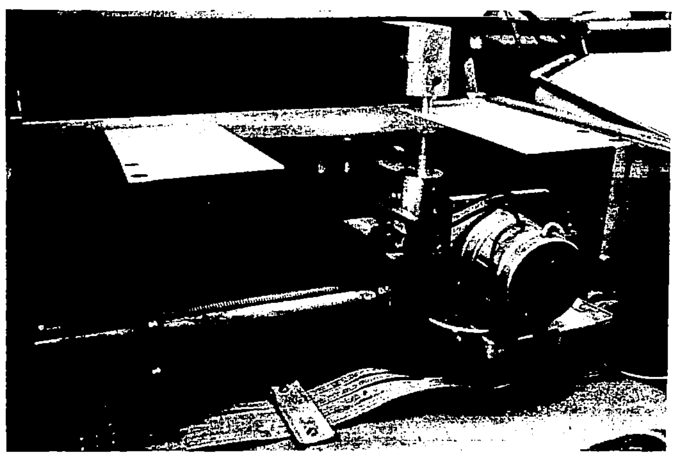
The new device consisted of a light source, optical system, and photo cell mounted in a relatively massive yoke, which traveled in carefully machined ways on roller bearings. The yoke had a travel slightly longer than the longest plate to be measured. The yoke was driven by a machined lead screw by a constant velocity motor. A fundamental concept in the design was that time can be divided into smaller increments more accurately than can linear motion. The photographs were, therefore, scanned at constant speed and a very accurate oscillator controlled the time of intensity sampling. Time increments could be directly related to positions on the photograph. One problem in achieving constant velocity was the existence of small progressive errors in the machined lead screw that were periodic with a period of several revolutions of the screw. These were removed by using a soft elastic nut which followed the average pitch over a considerable length of the screw. A set of adjustable opaque plates beneath the photographic plates actuated auxiliary switches which started and stopped the sampling electronic circuits. The construction is illustrated in Figure 4.

The combination of slow speeds and rapid sampling yielded accuracies of position coordinates of one micron. Plates were read in succession and data were compared in a digital computer.

FIGURE 4

Photo-reader

components



GENERAL DISCUSSION

This machine operated accurately enough to obtain the desired results. However, it suffered one defect which has been in evidence in other machines described in this conference. That is, it operated "open ended" in the photo cell circuit and in the scanning mechanism. The term "open ended" implies the lack of error detection and correction inherent in the system. The dual beam feature of the Joyce-Loeble microdensitometer is an example of a null balance principle. This is called a "closed system" and is desirable for any system in general. The problem in photo cell and motor driven circuits is that power supply voltages, values of passive elements and mechanical motions are subject to changing conditions. Only in a true null balance system can the effect of component elements be minimized.

A new system is under design which uses a null balance control on both the intensity measurement (photo cell) and on the constant speed carriage travel. The photo cell will operate in a dual beam circuit which is inherent in any light intensity measuring device. The table travel will yield a frequency signal generated by fine moiré grids or alternately monochromatic light interference fringes. The reference frequency is contained in an environment controlled station. The error signal between the generated frequency and the reference frequency will be used to control the constant speed servo system. Thus the machine can become independent of temperature changes, local machine difficulties, and electronic system components.

Unfortunately the system is designed for low speed scans.

DISCUSSION

Rosenberg: *How well does your constant velocity principle work?*

Ross: *We achieve a reproducibility of about a tenth of a micron.*

Rosenberg: *Can you measure photographic plates with your machine?*

Ross: *Yes, but in the present form not for making accurate measurements over the whole field.*

Eichhorn: *Could the combination of screw motion and time be used as a digitization principle to be used for measurements on astrometric plates?*

Ross: *This is essentially what is happening with the flying spot digitizers.*

IMPROVEMENTS IN ROSS TYPE ASTROMETRIC OBJECTIVES

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ABSTRACT

It is shown that aspheric deformations of the first and fourth elements of the four element Ross objective can be introduced to permit one to obtain improved color correction for astrometric purposes. The usual monochromatic aberrations are as well corrected as for the standard Ross lens. In addition, one can eliminate or reduce additional aberrations, such as secondary spectrum, chromatic spherical aberration, chromatic coma and chromatic distortion. The resulting objectives are suitable for use as intermediate and long focus astrometric objectives covering large angle fields.

INTRODUCTION

The availability of high speed electronic computers has brought a completely new perspective into the procedures of optical design. Problems can now be tackled and in many cases adequately solved in a few minutes running time on an electronic computer such as the IBM 7094 or the CDC 6400 which previously would either have taken years to complete on a desk calculator or have been too complicated to solve.

One of the most widely used optical systems for the photography of star fields has been the four element refractive system introduced into professional astronomy by F. Ross in the nineteen-twenties. This type of optical system comprises an axially aligned array of lens elements in the order positive, negative, negative and positive. Such an array allows the designer to vary a large number or parameters related functionally to the aberrations affecting the quality of star images over the field, aperture and spectrum. The range of solutions is particularly large when one considers that the four elements can be chosen from as many as three hundred currently available types of optical glass manufactured in many countries.

The original design by Ross is nearly symmetrical and uses the normal crown and flint glasses which are still the most practicable glass types for instruments intended for a long useful service life in unheated observatory domes. The choice of near symmetry has continued to be very suitable for astrometric objectives that must be as insensitive as possible to thermal variations and to small errors in alignment that may change with time.

The one design feature that has led to the present investigation is the use of aspheric deformations on the first and fourth elements to achieve the elimination of chromatic spherical aberration even though for astrometric reasons one may have designed a Ross system with a very compact barrel length. For example, the 20-inch blue astrographic objective at Lick Observatory, designed by Ross in the thirties, retains a large residual chromatic spherical aberration that affects the limiting magnitude unduly and that leads to overly enlarged diameters for the more heavily exposed star images on a given plate. One must also hasten to state, however, that the aspheric deformations required for the elimination of the chromatic spherical aberration for so large a lens system are of substantial depth and their fabrication would have been beyond the technology of the time. Increased cost and delivery time occasioned by strong aspherics are also very significant factors.

In proceeding with the investigation the author has also become interested in carrying out a few comparative studies with respect to combinations of glass types, some suitable for reducing the secondary spectrum while at the same time possessing matched physical properties about which more will be said below. The selection of matched glass types has become feasible from the availability of precise data in the new comprehensive Schott Optical Glass Catalogue. (One should note that substantial progress has been made recently on the applications of hard protective coatings for sensitive glasses. When such coatings are covered in turn with hard non-reflection coatings for high efficiency in desired wavelength intervals, the optical systems used in astrometry may soon provide improved performance and a lengthy service life together.)

It is well known that the most longstanding problem connected with refractive optical systems is that of the color curve of the system. That is to say, a wide range of spectral wavelengths cannot readily be brought to a sufficiently good common focus. The problem lies more in the physical limitations of optical materials than in the underlying mathematical principles. For astrometric purposes the existence of an inadequate color curve affects mainly the efficiency of the system, inasmuch as a reduction in the wavelength interval used will cause a corresponding lessening of the defocusing errors but at the expense of total light energy.

The mathematical theory is customarily formulated in terms of a power series that for rotationally symmetrical systems is expanded as a function of three rotational variables and one sub-function of the wavelength. The individual term, if of higher order than the first, may be considered an aberration and the coefficients of the respective terms become functions of the parameters of the system, including radii of curvature, thicknesses and spacings, glass indices and dispersions, aspheric deformations and the like.

The linear terms are, of course, related to the focal length and to the focal position, and are said to be gaussian. The description of a perfectly designed system would be complete in terms of these two parameters. However, the higher order aberrational terms in practice are rarely precisely zero and must instead be brought into a state of balance with one another at a magnitude so small that the purposes of the optical system are achieved.

If instead of making use of rational variables one carries out the power series expansion for a rotationally symmetrical optical system in terms of the y and z intercepts in the adopted entrance pupil and of the two independent direction cosines of any given ray in object space, he will then be setting up an expansion in four independent variables, or in five, if the wavelength is included. Because of the underlying rotational symmetry there will exist certain relationships among the various coefficients. One speaks of first order, third order, fifth order, seventh and higher order terms. The first order is properly called the gaussian, the third order is called the Seidel, and the fifth is not customarily given any similar designation, although K. Schwarzschild (1905-1906) was the first to study the geometrical nature of the fifth order aberrations.

There are five monochromatic coefficients of the third order which comprise spherical aberration, coma, astigmatism, curvature of field and distortion. In addition, there are two chromatic aberrations which are the two chromatic variations of the first order gaussian terms and which may be considered roughly of third order magnitude among the hierarchy of terms. These seven conditions through the process of design must be reduced to more or less negligible magnitude. These conditions represent the aberrations. One, of considerable importance to astrometrists, is the wavelength dependent difference in magnification over the field, which is essentially a dependence of the scale and therefore of the focal length on wavelength.

The longitudinal color aberration or primary spectrum can be eliminated in all cases insofar as the Ross type system is concerned. Similarly, the lateral color aberration, which is the above mentioned chromatic difference of magnification, can readily be eliminated in a Ross type system, but here we are speaking so far only of the chromatic variation of the gaussian term. The monochromatic aberrations can also be eliminated or rendered harmless through the third order in any practicable Ross system design, but even so, the state of correction is only a very good first approximation to a final optimized system.

The asymmetric aberrations of coma, distortion and lateral color all are of importance in astrometric considerations and must be held to very small residuals in any final design. On the other hand, the symmetrical aberrations of spherical aberration, astigmatism and curvature of field affect mainly the efficiency of the system but are not particularly dangerous because they do not displace the images laterally, unless by some overlaid secondary effect, for example, by unsymmetrical vignetting of the pupil in the outer field. Then, too, any significant deterioration of the diffraction structure of the image, whether monochromatically or in outlying wavelengths in the spectral interval employed, may cause some slight increase in measuring errors because of a partial enlargement of the star image and because the edge gradients in the image may become reduced in combination with the usually much more serious seeing errors.

THEORY AND PROGRAMS

The next higher order of approximation is the fifth and here the situation becomes considerably more complicated. Because of the cross-multiplication of terms in matrix operations, the first and third order coefficients reappear in various ways among the more elaborate coefficients of the fifth order, and in addition, intrinsically new terms of the fifth order appear. Schwarzchild found that the total effect is to have nine independent fifth order conditions.

In addition, one notes the existence of the chromatic variations of the five Seidel third order coefficients, which may be thought of as fifth order in magnitude among the hierarchy of terms in the expansion, and the secondary chromatic variations of the gaussian terms, which then become the secondary spectrum in the longitudinal aberration and the secondary spectrum in the lateral color. Thus, one now has a total of sixteen aberrations of fifth order nature. If one adds in the previously discussed lower order aberrations, he will then have a total of 25 conditions to consider.

The secondary spectrum in the longitudinal focal error is more simply known as the secondary spectrum, and the plot of the differential focal position against wavelength is called the color curve of the lens system. The secondary spectrum in the lateral color is less well known as a direct error in astrometry and is one of several aberrations sqallowed up in the astrometrically well known and troublesome residual dependence of scale or magnification on wavelength. Contributing to the astrometric chromatic shifts is the chromatic variation of distortion, which varies over the field as the cube of the off-axis field angle, or if expressed in relative terms, as the square. Contributing also is the chromatic coma.

In general, at any given point off-axis can make a plot of the differential lateral shift of the centroid of the image with respect to some mean centroid in the image against the wavelength. The nature of the resulting curve will be directly related to the contributing aberrational terms. If the plot is only of a slightly tilted straight line, the primary spectrum in the lateral color would be the cause, but only if the slope is proportional to the off-axis angle. Otherwise, chromatic distortion would also be involved in whole or together with some residual in the primary spectrum. If at a given off-axis angle the slope varies with aperture, then chromatic coma is responsible.

In the absence of other chromatic aberrations one would note that for the secondary spectrum in the lateral color (which is purely the lateral chromatic shift of the chief rays) the plot will be approximately parabolic in shape, turned either outwardly or inwardly, according to the sign of the aberration. One might find, for example, that in the outer field of a lens system afflicted only with secondary spectrum in the lateral color, B and M stars might have the same effective centroid, but that G stars might be shifted. In the presence of all the contributing terms it is easy to see that the spectral dependence becomes a troublesome affair, particularly if the lens system is further afflicted with slight decentering of the elements.

The algebraic form of the coefficients of the fifth order can be expressed fairly well in terms of single and double summations of the lower order surface by surface coefficients, and in terms of single summations over the surfaces of the intrinsic fifth order errors. However, if one were to write out such coefficients in explicit form prior to an attempted algebraic solution, he would find an almost totally unwieldy array of algebraic quantities with cross-product terms extending into very large algebraic degrees. Even the terms of lower degree, if isolated, afford multiple solutions and hence one must seek other and more tractable means for useful procedures in design. The combined use of the least squares method and successive approximations to overcome the strong non-linearity become the most useful devices.

The number of independent parameters derived from the Ross system array is also fairly great, particularly if aspheric coefficients for two of the eight surfaces are drawn upon. One has 8 radii or curvature, 4 lens element thicknesses, 3 air spaces, 4 mean indices of refraction, 4 mean dispersion coefficients, 4 secondary dispersion coefficients, and effectively 4 aspheric coefficients or a total of 31 parameters available for coping with the 25 conditions previously described. However, just as the conditions vary widely among themselves with respect to aberrational magnitude and geometrical complexity, the parameters also vary among themselves with respect to effectiveness and range of values. In addition, there are underlying conflicts between conditions that cause any simultaneous exact solutions, if real, to take on totally unacceptable values for the parameters. One must also remember that even though in principle multiple exact solutions may exist in real space for some selected sub-set of conditions and parameters, the large values for the parameters usually so obtained lead to hopelessly large aberrational values when inserted into the remaining neglected conditions. Even if one were so fortunate as to find a real simultaneous solution to the 25 conditions, there are many other hybrid conditions of seventh and higher order in the appplicable power series which would collectively lead to large aberrations.

There are also other conditions that should be kept in mind, and in fact in some recent computer programs these additional conditions have been incorporated. These have to do with the sensitivity of the system to misalignments, to thermal changes, and even to cost. Sometimes it is necessary also to put in conditions for total transmission, vignetting control, weight, barrel length, position of the cardinal points of the system and the like, particularly where the lens system must have certain physical characteristics in addition to optical performance. Athermal conditions have often been added, and then again, one might add in one or more conditions having to do with the sensitivity of the system to small thermal changes. Finally, one must make use of optical materials that are physically compatible with the circumstances of usage.

In considering lens systems for astrometric applications one is usually not too concerned with limitations of weight and bulk except for the very largest systems. On the other hand, one is necessarily concerned with considerations of transmission, durability, insensitivity to misalignment, and to thermal changes, and very often, of costs, particularly where aspheric surfaces and exotic glass types are to be employed in any effort on the designer's part to gain some small increment in performance over previous practice. In such matters the law of diminshing returns applies with full force and one finds, for example, that the total cost of a large astrometric lens system may easily triple as the result of an effort to obtain perhaps a 10% improvement in performance.

There are also pitfalls along the way. Many of the newer glass types are not manufactured in large quantities or in large diameters and hence the glass companies may find it very difficult to provide the desired optical quality from a limited production run. As a consequence, the total glass cost and total delivery time may both be very much increased.

Similarly, the use of aspheric surfaces is quite likely to cause greater difficulties than the optical aberrations the aspherics were introduced to allay, except in the hands of well equipped, very experienced opticians who in turn must be given all the time they need and the corresponding financial support en route. That is to say, an improved optical design imperfectly made may in practice be inferior in performance to that provided by a more conservatively designed system with all spherical surfaces made with high precision by any one of a large number of capable opticians.

To put the matter briefly, one can say that the concept of optimization must be applied within the constraints of the entire problem and not just to portions of the problem. Conversely, the initiator of a lens project should make himself thoroughly familiar with all the above considerations whereafter he should set forth the constraints with great care. The designer's task concurrently may often be to bring a range of solutions to the attention of the initiator on the basis of which reasoned decisions can be made. In many cases further design work will be required before a final specification can be prepared and the work released to a manufacturer.

With respect to the optical part of the problem, in the generic Ross design there are not enough effective parameters available substantially to satisfy the various conditions through the fifth order. For example, the thicknesses of the elements play no design role and usually must be assigned minimum values. One must then set forth the array of conditions and assign weighting factors or tolerances that are tailored to the desired optical performance. Astrometrists are typically not as much concerned with resolving power (which is the principal concern of most other users) as with the elimination of asymmetric and lateral aberrations. Even so, not all of these can be eliminated.

One of the several programs developed can handle twenty-four different conditions. Thirteen of these are optical conditions of the sort already described, having to do with gaussian and Seidel optics and the chromatic variations of these, and including the secondary spectrum. Five more have to do with the least squares minimization of the surface by surface contributions in the Seidel region, a sub-set that can be called upon in whole or in part, which with suitable weighting factors can often be used to isolate solutions of minimum sensitivity to errors of alignment. The remaining conditions have to do with the physical requirements on the lens system, including focal position and focal length, stop positions in object and image space, relative heights of the refracted paraxial rays and the like.

As many as 150 parameters can be inserted into the program to go with the 24 conditions and the solutions can be obtained either by assigned exact conditions, or by assigned least squares conditions, or more generally by some assortment of exact and least squares conditions with assigned weighting factors or tolerances. The program internally goes through many successive approximations in which the individual increment of any given parameter is sytematically reduced as the number of internal cycles increases, and then, of course, there are safeguards to prevent run-away calculations. For very complicated systems the solutions may fail to converge or may oscillate until brought ot a conclusion by time or a limit on the number of allowed cycles. However, for a system as simple as the four element Ross design it is the usual experience that the solution locks on rather quickly and thereafter converges satisfactorily to the required minimum. The program permits one to run a number of simultaneous optical problems, where necessary, and easily permits one to change or add conditions, weighting factors, or new parameters, or to freeze one or more parameters already in use.

A second program can make use of the results of the first for making a complete analysis of the optical performance either by geometrical rays or by optical path errors or by a mixture of both. This program is simply an analytical one that establishes the state of correction of the design for the assigned purposes. The results so obtained can readily be used to assign improved target values for the non-zero conditions in the first program and improved tolerances. The two programs used in sequence will often lead to a sufficiently good final result and the final analysis will inform one as to the predicted performance.

A third program can be drawn upon to squeeze the utmost out of the design parameters whose values are already quite well obtained from the application of the first two programs. This third program requires a large amount of setup time in its present form but basically its input is from the output of the first two programs. As many as 1000 rays can be inserted into this program as conditions and as many as 50 parameters can be optimized by least squares in accordance with either calculated or assigned tolerances for the individual rays. A novel feature of the program is that the weighting factors or tolerances can be readjusted internally from cycle to cycle automatically to favor the use of the parametric array to solce the more difficult conditions at the expense of the more easily met conditions, as determined during the calculations.

Thus, if there are say 300 conditions in all in a given problem, it may be that in the first internal cycle 250 of these may have been satisfied to some fraction of a single tolerance unit as previously assigned, and that the other 50 conditions have not been satisfactorily met, and may in the worst instance be off by, say, 30 tolerance units. If the program has received the proper initial instructions, it will then relax somewhat on the tolerances assigned to the 250 easily met conditions and tighten up on the tolerances for the remaining unmet conditions. In the subsequent least squares solution more emphasis will then have been given to the harder to satisfy conditions and the array of parameters will have been more properly used.

This third program has been used on many past problems but is in need of further streamlining and amplification to include diffraction calculations of image quality. Experience has shown too that if a very large number of conditions has been assigned, with many inter-relationships as must be the case from the fact that the underlying power series already described has a far smaller number of significant conditions, the recalculation of the tolerances and reapplications of the least squares colution does only partial good. Quite often a given condition simply cannot be met satisfactorily no matter how the tolerances are calculated or assigned. However, one does benefit from improvements among the more tractable conditions, and that is all that the mathematical situation permits. The question thereafter is whether the resulting system can be used or whether further parameters or new concepts must be introduced in order for one to obtain the desired results.

Most of the calculations supporting the conslusions in this investigation have been performed with the first of the above mentioned programs. Any one optical system used in the comparative study could be brought to full optimization by the application of the second and third programs for any assigned aperture-ratio, focal length and spectral range. Indeed, the first program is one that is very useful for exploratory studies of various optical systems for sundry purposes.

The actual calculations discussed here involve four species of four element systems which for astrometric purposes can be thought of generically as Ross systems. The first of the four species is that of the standard Ross lens, by which is meant a nearly symmetrical array of elements having roughly equal absolute dioptric powers. The first and fourth elements in general will be weakly bi-convex with the stronger curvatures on the outward surfaces. The second and third elements in general will be bi-concave, though not necessarily equi-concave.

The second of the four species is derived from an alternate solution of the same equations of condition. In general the first and second elements may be of meniscus form curved around a central stop, and the third and fourth elements may often be menisci also curved in the opposed sense about the same central stop, thought there are exceptions in the meniscus forms of all four elements.

The third and fourth species represent systems that are merely Cooke triplets in disguise. Here one finds that some of the negative power of the normal central element of a Cooke triplet has been split off and formed into an additional negative element of no great power of itself. Such systems in principle represent improvements on the Cooke triplet and are by no means to be neglected.

Designwise one may characterize the four species in a different way. Usually, only two of the four surfaces of the two negative elements in the Ross system contribute significantly to the correction of the spherical aberration of the system and the other two surfaces are of much lesser importance in this respect. Thus, if the third and sixth surfaces are the strong ones, a standard Ross system is indicated, which is the first of the four species. If the fourth and fifth surfaces are the strong ones, then the second of the four species is indicated. If the third and fourth surfaces are the strong ones, one has the third of the four species, and finally if the fifth and sixth surfaces are the strong ones, one has the fourth of the four species.

It is no longer advisable for a designer to outguess the computer, but in the absence of a very prolonged study one may say that the superior astrometric objectives are more likely to be found among the first two species which are at least more or less symmetrical. In addition, it is probable that systems of the first species, which are the standard Ross objectives, are well suited to medium to small angular fields at intermediate aperture-ratios, such as f/7 to f/15 and beyond, whereas systems of the second species are better suited to medium to large angular fields at the slower aperture-ratios, such as f/15 to f/30, though there can be a large overlap of well-corrected systems.

As an example of a system of the second species derived from the investigation, there are given below the data for an astrometric objective of 100 inches focal length with an aperture-ratio of f/ 15, corresponding to a clear aperture for the entrance pupil of 6.667 inches. In metric terms the focal length is 2540 mm and the clear aperture is 169.3 mm. The system is a semi-apochromat constructed of two glass types, namely, Schott types SK-11 and KzF-2. The glass types have been chosen also on the basis of moderate cost, acceptable stability if coated and not too exposed to dampness, for excellent transmission in the visual part of the spectrum and for closely matched thermal expansion. The intended field is that given by a 17 × 17-inch photographic plate in combination with the 100-inch focal length, whereby the total diagonal field is 13.75 degrees.

Table I. - Data for Astrometric Objective.

Surface	Curvatures	Separations		Glass Types
		Glass	Air	
1)	7.6611	0.0080		SK-11
2)	-0.8415*		0.00908	
3)	-0.0243	0.0050		KzF-2
4)	7.9832		0.00800	
5)	-8.6765	0.0050		KzF-2
6)	2.7512		0.00796	
7)	2.0581*	0.0060		SK-11
8)	-8.2239		0.9776	

*aspheric

The above data are in units of the focal length at the 5461 mean wavelength. For the intended system at hand all dimensional quantities are to be multiplied by the assigned focal length, either 100 inches or 2540mm. It will be seen that the total distance between the first and last vertices is only a little more than 5 inches and therefore that the objective is very compact. By the same token, only a slight enlargement of the outlying apertures will be needed to hold vignetting to a minimum.

The data above show that the first and fourth elements are bi-convex and that the second and third elements are more or less bi-concave, and yet the implicit meniscus character of the solution is preserved in the strong curvatures of the first, fourth, fifth, and eighth surfaces. The main reason for the large departure from the meniscus form for the individual elements is that the semi-apochromatic nature of the solution derives from glass types with fairly small differences in the dispersions, leading to strong individual dioptric powers.

The second and seventh surfaces are aspheric. The curvatures given in Table I above are the vertex curvatures. The usual formula describing the meridional section of the particular aspheric surface is:

$$\xi = \frac{c\eta^2}{1 + \sqrt{1 - c^2\eta^2}} + \beta\eta^4 + \gamma\eta^6 + \delta\eta^8$$

For surface 2) the constants have the following values;

$c = -0.84154$
$\beta = 14.362$
$\gamma = -948.4$
$\delta = -44663.$

For surface 7) the constants have the following values:

$c = 2.05814$
$\beta = -12.454$
$\gamma = -364.4$
$\delta = 0.0$

The Seidel results in three wavelengths are:

	5461	4800	6438
Spherical Aberr.	-0.000042	-0.014621	-0.014602
Coma	-0.000027	0.000058	0.000057
Astigmatism	0.000000	0.001999	-0.002119
Petzval	0.020000	0.016338	0.023357
Distortion	0.000000	-0.000010	0.000009

The above tabulation is for a focal length of unity. The values entered for the Petzval sum (related to curvature of field in the absence of astigmatism) must be doubled to bring them into agreement with the accepted definition for the textbook Petzval sum. The values are the B, F, C, (P), and E of the Schwarzschild-Kohlschutter (A. Kohlschütter, 1908) form. Six decimal accuracy is given above for the purpose of showing the chromatic variations, whereas actually for the f/15 system at hand, three decimals would suffice. The reader is referred to Schwarzschild or to Kohlschutter for the quantitative meaning of the above values in terms of seconds of arc in object space.

One can see that the spherical aberration has been eliminated and that the chromatic spherical aberration has also been eliminated (4800 and 6438 have the same values, differing from 5461 by a small secondary spectrum in the chromatic spherical aberration.)

Similarly, coma and distortion have been eliminated, and chromatic coma and chromatic distortion have also been nearly eliminated. The astigmatism has been eliminated, and the Petzval sum as redefined above has the value one needs if the field is to be approximately flat when higher orders are taken into account in a final optimization.

The residual aberrations of the system arise from the color curve and from the reappearance of spherical aberration off-axis, a normal characteristic of Ross systems. While the semi-apochromatic character of the solution has led to a reduction in the secondary spectrum, the residual color curve is by no means negligible. In practice one would find that a significantly larger range of wavelengths can be used with this design as compared to that for the normal combinations of standard crown and flint glass types.

The elimination of the chromatic spherical aberration is a unique feature of this type of Ross system having appropriate aspheric shapes for the second and seventh surfaces. An all-spherical design with so short a barrel length would inevitably have a substantial amount of chromatic spherical aberration. Physically, the elimination of chromatic spherical aberration arises in the following way. In the normal lens of the second species and for that matter in the normal Ross lens of the first species the negative spherical aberration is conributed by only two surfaces, such as the fourth and fifth, or the third and sixth, to compensate the positive spherical aberration introduced by the first and eighth surfaces and substantially also by the second surface. For compact systems it is invariably the case that the shorter wavelengths are overcorrected and the longer wavelengths undercorrected.

The use of negative aspherics applied to the second and seventh surfaces, or to the first and eighth, in effect spreads out the negative spherical aberration among four surfaces instead of on only two surfaces, which in turn suppresses the over-correction of the shorter wavelengths. By appropriate solution one can find an exact elimination of the chromatic spherical aberration, preferably for some mean value in the intermediate field angles and for some compromise between meridional and skew rays of the aperture. In any final optimization the elimination of chromatic spherical aberration must be performed in terms of the path errors for the respective outlying wavelengths, rather than in geometrical terms. The example above demonstrates how effective the design technique can be.

With respect to astrometry it will be of interest to record the calculated transverse ray errors in the image plane in terms of displacements from the gaussian point in millimeters for the prescribed lens and focal length. Table IIa gives a tabulation for 0.085 radians off-axis and Table IIb for 0.120 radians off-axis. The displacements are given for the rays in a skew fan which best represent the effect of distortion, coma and their chromatic variations as well as the primary and secondary spectra in the lateral color.

It should be noted that the system of the example has not yet been fully optimized. When optimization has been completed, one would expect that the transverse residuals over the 17 × 17-inch format would shrink to less than one micron and perhaps to less than 0.5 microns.

Table IIa. - Transverse Ray Errors: 0.085 radians off-axis.

Skew Zone	Wavelengths 5461	4800	6438	
1.0	-0.0003	-0.0004	0.0000	rim ray
0.8	-0.0006	-0.0006	-0.0003	
0.6	-0.0005	-0.0004	-0.0002	
0.3	-0.0002	-0.0001	0.0001	
0.0	0.0000	0.0000	0.0002	chief ray
	(all dimensions above given in millimeters)			

Table IIb. - Transverse Ray Errors: 0.120 radians off-axis.

Skew Zone	Wavelengths 5461	4800	6438	
1.0	-0.0030	-0.0031	-0.0026	rim ray
0.8	-0.0024	-0.0024	-0.0020	
0.6	-0.0015	-0.0015	-0.0011	
0.3	-0.0005	-0.0004	0.0000	
0.0	-0.0001	0.0000	0.0002	chief ray
	(all dimensions above given in millimeters)			

It is clear also in the above tables that the mean distortion has been brought to a very small residual, particularly if one replaces the gaussian focal length used in the calculations of the tabulated residuals by a mean focal length adjusted by least squares to all the residuals.

The aspheric depths in the example above turn out to be 46 fringes of mercury green light from the nearest sphere for the second surface and 43 fringers from the nearest sphere for the seventh surface where the respective clear apertures are adopted for zero vignetting in the corner of the format. Such aspheric depths for this f/15 case are not unreasonable, though costly, and can be produced with adequate precision either from careful fringe counting and testplates, or better, from null test methods and interferometry.

For purposes of comparison it will be of interest here to tabulate some of the data for a species one standard Ross lens of crown and flint, taken to be SK-2 and F-2. These data are not optimized for any particular lens, though the data can be used as a very good first approximation to Ross systems from f/7 to any slower aperture-ratio and for apertures up to 20 inches.

Table III. - Data for Standard Ross Objective (All Spherical Case).

Surface	Curvatures	Separations		Glass Types
		Glass	Air	
1)	4.0662	0.0125		SK-2
2)	-0.5196		0.07803	
3)	-2.3803	0.0075		F-2
4)	1.2298		0.01250	
5)	-0.9454	0.0075		F-2
6)	3.8772		0.10595	
7)	0.9069	0.0125		SK-2
8)	-3.1659		0.8657	

The Seidel results in three wavelengths are:

	5461	4800	6438
Spherical Aberr.	0.000001	-0.102081	0.080168
Coma	0.000000	-0.008157	0.007145
Astigmatism	0.000000	0.011712	-0.010821
Petzval (@ 1/2)	0.020000	0.014866	0.024406
Distortion	0.000000	-0.000435	0.000650

The aspheric case for the species one Ross system in this instance is derived from exact conditions for the elimination of chromatic coma and chromatic distortion, but from least squares minimization of the chromatic spherical aberration. The constraints depart somewhat from the previous discussion but were adopted for astrometric reasons in the solution at hand.

Table IV. - Special Aspherica Ross Type Objective (Special Aspheric Case).				
Surface	Curvatures	Separations		Glass Types
		Glass	Air	
1) 2)	3.4733 -0.5513*	0.0125	 0.0909	SK-2
3) 4)	-2.2174 1.2697	0.0075	 0.0125	F-2
5) 6)	-1.0610 3.8656	0.0075	 0.0902	F-2
7) 8)	0.8581* -3.7567	0.0125	 0.8853	SK-2
*Aspheric Surfaces		β_2 = 2.1226	β_7 = 2.2335	

The Seidel results in three wavelengths are:

	5461	4800	6438
Spherical Aberr.	-0.002756	-0.112297	0.084442
Coma	-0.001068	-0.001412	-0.001403
Astigmatism	0.000018	0.011321	-0.010425
Petzval (@ 1/2)	0.020000	0.014878	0.024396
Distortion	0.000005	0.000111	0.000111

The non-zero values in 5461 for spherical aberration, coma, astigmatism and distortion mean only that a further run or two of the program is needed. However, in an optimized design the values are non-zero anyway to preferred target values and hence the tabulation above suffices for the purpose at hand. It is clear that the chromatic coma and chromatic distortion have indeed been eliminated inasmuch as the values for 4800 and 6438 agree.

It will be recalled that the overall axial barrel length of the example in Table I, a species two system, is 5.104 inches for a focal length of 100 inches. From Table III the barrel length of the all spherical, species one standard Ross lens of the example 23.648 inches for a focal length of 100 inches, and for the aspheric version in Table IV, 23.360 inches.

The difference in barrel length is derived mostly from the choice of glass types, however, and not much at all from the species one or species two form of solution. Semi-apochromats will rend toward compactness for either case because of the smaller difference in dispersion between crown and flint. On the other hand, long barrel species one systems are quite usable for moderately fast Ross systems to f/7 in large diameters or to f/5 in small diameters for medium to small angular fields, whereas species two systems will usually have an excessive amount of oblique spherical aberration. A comparison of the data in Tables I, III, and IV will show that the curvatures of the semiapochromat are large numerically, and yet the short barrel offers a distinct advantage. Thus, a species two semiapochromat in a short barrel probably should not be fabricated at aperture-ratios any faster than f/15, whereas the species one systems above are better suited to faster systems with longer barrels but with restricted wavelength range.

It is obvious that there will be many intermediate solutions. Quite possibly a species one system with choice of glass types tending toward semi-apochromatism will be favorable for system around f/10 for plate sizes up to 14 × 14 inches for a 100-inch focal length. For larger field angles and for slower aperture-ratios, the choice tends toward the species two system.

Another example of the species one standard Ross lens has been included in the investigation, this time intended for an aperture-ratio in the vicinity of f/12 to f/15. In this instance aspherics have been used on surfaces two and seven for the elimination of chromatic spherical aberration and chromatic coma, but chromatic distortion has only been minimized by least squares. Such a system in a 100-inch focal length might be favorable for use at f/12 on an 8 × 10-inch format. The color curve of the system is normal, rather than reduced, and the glasses chosen, SK-2 and F-2, are known to be very stable against atmospheric attack if kept dry.

Table V. – Data for Alternate Ross Type Objective System with Two Aspheric Surfaces.

Surface	Curvatures	Separations		Glass Types
		Glass	Air	
1)	3.2520	0.0080		SK-2
2)	-0.5738*		0.09786	
3)	-0.8926	0.0050		F-2
4)	3.3168		0.0080	
5)	-2.9425	0.0050		F-2
6)	1.2522		0.08464	
7)	1.0338*	0.0080		SK-2
8)	-3.7700		0.9004	

*β_2 = 3.8995 β_7 = -3.8999

The Seidel results in three wavelengths are:

	5461	4800	6438
Spherical Aberr.	0.001082	-0.003943	-0.003974
Coma	0.000254	0.000119	0.000111
Astigmatism	-0.000082	0.009839	-0.009226
Petzval (@ 1/2)	0.020000	0.014884	0.024391
Distortion	0.000012	0.000061	0.000137

It is clear from these results that the chromatic spherical and chromatic coma have indeed been eliminated. The distortion chromatically is very small and amounts to about 0.0001mm in the corner of an 8 × 10-inch plate for a focal length of 100 inches. The barrel length from the above data comes out to be 21.650 inches for a focal length of 100 inches. The curvatures are quite moderate and the surface by surface contriubtions to the aberrations are at a minimum also.

Similar solutions are at hand for the mean wavelengths of 4358 and 6438 respectively, with approximately the same suppression of aberrations. The barrel length comes out to be 27.434 inches for the blue lens, and 19.335 inches for the red lens, the glass types being unchanged. It is obvious that such changes in the wavelength of the minimum point on the color curve profoundly affect the lens data.

The next system studied in the investigation is a semiapochromat of the species one, standard Ross form, first with all spherical surfaces and then with aspherics. It is of interest that in both cases the chromatic coma and chromatic distortion have been eliminated. The system is intended for use at about f/12 to f/15, but only in small diameters.

Table VI. - Data for Objective (All Spherical Case).

Surface	Curvatures	Separations		Glass Types
		Glass	Air	
1)	4.8700	0.0125		SK-2
2)	-1.0596		0.03154	
3)	-3.0239	0.0075		KzFS-N4
4)	2.5635		0.01250	
5)	-4.0511	0.0075		KzFS-N4
6)	3.8031		0.02418	
7)	1.8501	0.0125		SK-2
8)	-5.8595		0.9556	

The barrel length for a system scaled to a 100-inch focal length turns out to be 10.822 inches. The system is therefore a rather compact one with fairly strong curvatures arising from its semiapochromatic form.

The Seidel results in three wavelengths are:

	5461	4800	6438
Spherical Aberr.	0.000034	-0.436772	0.391306
Coma	-0.000012	0.000047	0.000047
Astigmatism	0.000000	0.007230	-0.006919
Petzval (@ 1/2)	0.020000	0.016043	0.023569
Distortion	0.000000	0.000006	0.000006

The chromatic spherical aberration is quite large and is characteristic of the compact, species one, systems having all spherical surfaces. It is for this reason that either the f/number must be large, or the diameter small, or both together.

Table VII. - Data for Objective (Aspheric Case).				
Surface	Curvatures	Separations		Glass Types
		Glass	Air	
1) 2)	5.6795 -1.1540*	0.0215	0.02538	SK-2
3) 4)	-1.5741 5.6488	0.0075	0.01250	KzFS-N4
5) 6)	-3.7261 2.5905	0.0075	0.02548	KzFS-N4
7) 8)	1.4508* -5.4535	0.0125	0.9485	SK-2
*aspherics	[illegible] = 23.441	[illegible] = 9.837		

The barrel length for a system scaled to a 100-inch focal length turns out to be 10.336 inches.

The Seidel results in three wavelengths are:

	5461	4800	6438
Spherical Aberr.	-0.000066	-0.174915	0.148823
Coma	0.001219	0.001053	0.001015
Astigmatism	-0.000074	0.006260	-0.006162
Petzval (@ 1/2)	0.020000	0.016013	0.023596
Distortion	-0.000003	0.000005	0.000006

The use of the aspherics has resulted in a substantial reduction of the chromatic spherical aberration, which could be further eliminated in a recalculation if the chromatic distortion were to be allowed to vary from zero. The aspheric system can thus be built in apertures somewhat larger than for the all spherical case and to somewhat faster aperture-ratios.

Solutions have been carried through for a combination of the glass types SK-12 and KzF-2 for blue, yellow, and red correction. No new principles are demonstrated but as would be expected, there is a further slight reduction in the color curve for the yellow system and almost complete three wavelength correction for the red solution. The improvement is obtained at the expense of still stronger curves and a still shorter barrel, which for the yellow solution is only 4.201-inches for a focal length of 100-inches. The systems would be suitable for long focus astrometry at aperture-ratios of f/20 and slower. The two types of optical glass used are readily available at moderate cost up to quite large diameters. The solutions at hand are for the second species.

Still another solution has been carried through for a system of the second species in which the glass types SSK-3 and KzFS-N4 have been used. The combination offers very little improvement in the blue over that already obtained for the combination of SK-11 and KzF-2. Yellow and red solutions have not been run.

Calculations have been carried through for a combination of PK-50 and KzF-2 for blue, yellow and red systems. The glass type PK-50 appears to be one of considerable value in several types of optical designs intended for use in blue and violet light, but the cost of glass discs of objective quality for the larger diameters is manyfold that for standard crown types. It was intended that the solution would be of the second species but the computer came up with a solution that borders on the fourth species. The second element is substantially weaker numerically than the third, which is to say, that the solution physically is somewhat unsymmetrical. More study is certainly warranted but it will be of interest here to reproduce the results already obtained.

Table VIII. - Data for Objective.

Surface	Curvatures	Separations		Glass Types
		Glass	Air	
1) 2)	5.3989 -1.2481*	0.0080	0.04005	PK-50
3) 4)	-0.0382 5.4775	0.0050	0.00800	KzF-2
5) 6)	-4.8593 3.1643	0.0050	0.04070	KzF-2
7) 8)	1.4976* -5.6732	0.0080	0.9430	PK-50
*aspherics		β_2 = 21.056		β_7 = 2.486

The barrel length for a system scaled to a 100-inch focal length is 11.475-inches.

The Seidel results in three wavelengths are:

	5461	4800	6438
Spherical Aberr.	-0.008246	-0.015819	-0.015682
Coma	0.000315	0.000282	0.000278
Astigmatism	-0.000005	0.005866	-0.005753
Petzval (@ 1/2)	0.020000	0.016092	0.023529
Distortion	-0.000001	-0.000161	0.000189

The system is well corrected for chromatic spherical aberration and for chromatic coma. The chromatic distortion is very small, and amounts to a transverse displacement of about 0.0002mm in the corner of an 8 × 10-inch plate for a system scaled to the 100-inch focal length. This type of system would be especially good for a blue lens in apertures up to 10 inches and for aperture-ratios of f/15 or slower.

GLASS TYPES AND OTHER CONSIDERATIONS

The examples above have been selected from what could be a very large number of good to excellent designs, gleaned from various combinations of glass types. All of the examples employ only two types of glass but it is obvious that a still larger variety of systems will result if as many as three or even four types of optical glass are employed.

During the last twenty years a considerable number of new glass types, representing previously unobtainable combinations of index and dispersive power have become available to the optical designer. For the time being, only a relatively small number of these are useful for astronomical purposes, i.e., will tolerate the atmospheric, thermal, and other conditions to which the optical surfaces of astronomical instruments are typically subjected.

It has been mentioned that recent progress has been made in the hard coating of sensitive glasses for the purpose of protecting the optically polished surfaces from tarnish and harsh treatment. The Schott Company, for example, are now marketing an evaporative glass that can be used for such coatings preceding the non-reflection coating in the same vacuum set-up. Similarly, much progress has been made in the electrical sputtering of protective coatings of quite appreciable thickness which in turn will be usable for protecting sensitive glass types. For example, the Itek Corporation has succeeeded in coating the very sensitive but desirable glass, KzFS-2. The sample, placed outdoors for several weeks in rainy and sunny weather, shows no deterioration on the coated portion of the two polished faces, whereas the uncoated surfaces degraded into only a whitish, unpolished smear. Further experimental work will be needed before finished elements can be adequately coated since uniformity of coating is also needed. Conversely, ultimately the added layers might be made of variable thickness for the purpose of aspherizing the surface to prescribed form, where needed.

With respect to the Ross design, only the first element is seriously exposed, and experience would indicate that the rate of deterioration of the first surface will be manyfold that of the internal surfaces. The Ross designs benefit from having fairly high indices for the crown outlying elements which in current practice are limited to a maximum index of about 1.60 . Most of the rare earth glasses in the region of the lanthanum crowns are unsuitable for use as the first and last elements, even though protectively coated, because of a common characteristic of having weakened dispersive power in the blue-violet. A few of these, however, have normal trends of dispersion and might be incorporated into a more detailed study. The lanthanum glasses, however, are heavy and therefore would not be suitable for use in large lenses where flexure becomes a problem.

The restrictions on the glass types used for the inner elements are not so stringent and there are a number of more or less equally suitable ones available. The so-called short flints and a few others have unusual properties concerning the compensation or reduction of color aberration and therefore can be used with caution, as in some of the examples above, for reducing the focal error with wavelength, increasing the efficiency and generally improving the astrometric performance. The standard flint glasses normally used for the inner elements occupy a nearly continuous region in the optical glass diagram where the V-value is plotted against index of refraction. In several of the examples above, F-2 has been used, since this type has long been available at low cost and in large diameters in objective quality. Ross systems making use of F-2 would have the usual secondary spectrum, however, but monochromatically, at least, can be designed and fabricated with high precision.

The short flints, such as KzF-2, KzF-3, KzF-4, and KzF-5, have long been available but only KzF-2 has the combined qualities of moderate cost, good transmission and fairly good resistance to atmostpheric attack. It is for such reasons that KzF-2 has been used in the examples above for those having some degree of reduction in color curve. Similarly, such types as KzFS-1, KzFS-2, and KzFS-4 have been available for a very long time but have been known from the very beginning to be extremely sensitive to weathering and to reagents. Recently, Schott has introduced a greatly improved KzFS-N4, which has been used in several of the solutions in this investigation. Schott has also developed three additional types, KzFS-5, KzFS-6, and KzFS-7 and soon promises a fourth. Glasses of the KzFS series have an enhanced ability to reduce the secondary spectrum of optical systems when combined with the proper crown glass types, but are not very useful in the blue-violet for large lenses because of reduced transmission. Only KzFS-N4 has sufficient resistance to weathering when used in an astronomical system as an internal element. However, when the hard protective coatings become standard practice, then the entire subject is opened up once more and improved designs will undoubtedly become available.

One of the important considerations for astrometric systems is a matching of the glass elements for coefficient of thermal expansion. In the examples SK-2 and F-2 are not well matched but represent more or less past practice. On the other hand, SK-11 and KzF-2 are well matched, as are also SK-12 and KzF-2, SSK-3 and KzFS-N4, and SK-2 and KzFS-N4. The matching will be of increased importance for the compact systems with strong internal curvatures. Similarly, the optical engineer planning the lens cells can select alloys that have the same mean coefficient of expansion as the lens elements. In this way it can be expected that the astrometric quality will be preserved over a wide range of summer and winter temperatures, as long as the lens system is kept in a reasonable thermal equilibrium.

Some caution must be exercised in the coating of lens elements intended for precision use in astrometry. The hard, non-abrading coatings of magnesium flouride normally used are generally applied to lens elements heated to as much as 450 degrees F. Some glass types have low transformation temperatures and can be damaged with respect to annealing and homogeneity if so heated. Such deterioration has been noted with KzFS-4, for example, in 8-inch diameters where not only was the annealing degraded but the lens element slumped slightly out of shape. Fortunately, the crowns such as SK-11, SK-12, and SK-2 have quite high transformation temperatures and are probably immune to damage. Glasses such as F-2, KzF-2 and the KzFS-series should all be cleaned and coated with caution at reduced temperatures.

Similar caution must be exercised with respect to the quality of polish of the soft flints. The work-function of such glasses is low so that ion bombardment can result in a pitting of the optical polish that can reach disastrous proportions. Experience on KzFS-4 (not N-4) is a case in point, where a number of finished elements were too damaged for use and had to be repolished and refigured, owing to extensive pitting vaused by surface cleaning with a high voltage discharge. Several of the surfaces seemed almost to be fine-ground.

Optical glasses of the types discussed in this investigation are almost all available in large diameters in objective grade but some are far more expensive than others. The crowns, such as SK-2, SK-11, and SK-12, can be considered to be of low to moderate cost, whereas PK-50 would be very expensive. A flint glass like F-2 would be of low cost. The more exotic short flint, KzF-2, can still be considered to be of moderate coast, whereas KzFS-N4 would be much more expensive. Glass types such as SK-2, SK-11, SK-12, F-2 and KzF-2 can be made in diameters much greater than would be of interest in astrometry, perhaps up to 40 and 50-inch diameters, or greater. To date, at least, KzFS-N4 has been made in diameters in objective grade to perhaps 24 inches, but not reliably and in addition, this type of glass absorbs light too much in the violet to be used for very large elements in blue-corrected systems.

IMPROVEMENT OF COLOR CORRECTION

There are various good ways to present the state of color correction of a lens system but for comparative purposes here it will suffice simply to give the calculated longitudinal displacement of the mean focal position of the outlying wavelengths from the focal position of the central wavelength. Thus, if we have wavelengths 4800, 5461, and 6438, we average the focal positions of 4800 and 6438, and subtract from this mean the focal position of 5461. Similarly, for a system corrected for a central or mean wavelength of 4358, called here a blue correction (although actually blue-violet), the outlying wavelengths will be 4047 and 4800. For a system corrected for a central or mean wavelength of 6438, called a red correction, the outlying wavelengths will be 5461 and 8521. The state of color correction of the lens systems of the examples can be summarized in Table IX.

Table IX. - Color Correction for Lens System.

Glass Combinations	Blue Corr.	Yellow Corr.	Red Corr.
SK-11 & KzF-2	0.000344	0.000381	0.000057
SK-2 & F-2, Case 1		0.000779	
" Case 2		0.000779	
SK-2 & F-2 (f/15) Case 1	0.000530	0.000791	0.000378
" Case 2	0.000529	0.000795	0.000395
SK-2 & KzFS-N4 Case 1		0.000440	
" Case 2		0.000455	
PK-50 & KzF-2	0.000332	0.000482	0.001179
SSK-3 & KzFS-N4	0.000500		
SK-12 & KzF-2	0.000465	0.000330	-0.000019

In Table IX the standard color curves for the blue, yellow and red cases are those associated with the combinations, SK-2 and F-2. The semi-apochromatic character of the other combinations can then be observed in the table. The combination of PK-50 and KzF-2 is a special case, where the highly unusual nature of PK-50 as an optical material shows an improvement in the blue and a marked worsening in the red. Presumably, PK-50 and KzF-2 as a combination would excel in the near ultraviolet where both glass types remain quite transparent. Table IX shows also the excellent state of correction in the red for the SK-11 and SK-12 pairings with KzF-2.

No calculations have as yet been made for Ross systems using KzFS-2 for the negative elements. At the time the examples were calculated, there seemed to be little hope that KzFS-2 could be adequately protected for many years of exposed usage. In view of the likely future availability of hard protective coatings, one should incorporate KzFS-2 in any further study, and possibly also KzFS-1, at least for yellow and red systems.

There are no general recommendations to be made on the basis of the examples. Each astrometric requirement should be studied carefully in its own right in the light of all the information at hand. The full correction for chromatic coma and in some cases for chromatic distortion should be of interest to astrometrists. Optical designers may find it of interest also that the cautious use of aspherics for the first and fourth elements can be very helpful for the elimination of chromatic spherical aberration in short barrel semi-apochromatic systems.

One can always achromatize the spherical aberration in compound systems in about the same fashion that ordinary achromatism is achieved, but to do so requires the use of aspherics on both crowns and flints. To preserve the symmetry for astrometric needs in a Ross lens, such an approach might well involve the introduction of four aspheric surfaces. Instead, it has been shown above that the chromatic spherical aberration can be eliminated by adding only two, preserving symmetry, and indeed, because of the distribution of the negative corrections among more surfaces, one can expect some improvement also in the oblique spherical aberration.

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DISCUSSION

Newcomb : *What was your computer program run on?*

Baker : *Since it is written in Fortran II, it could easily be converted to Fortran IV and thus be run on all machines which accept this program. This more elaborate program also includes the calculation of the photo-optical path, so that, as the design nears completion, one can drop the ray tracing per se and switch to wave front calculations.*

Eichhorn : *From the standpoint of achieving a certain task, would an objective of this type be the most economical, or might it be that a catadioptric system which achieves the same optical effect might be cheaper to make?*

Baker: : *There are two problems involved. The objective I described would be most useful for wide angle (astronomically speaking) coverage, for plates of about 17 inches square. Catadioptric systems would be easier to construct, would be more temperature stable, and have longer effective focal lengths, but they are better suited for narrow angle fields. As an example, I designed a system (with the view of space application at some later time) of two mirrors where every component is used twice. For astrometric purposes in space, one may achieve a focal length of 8000 inches with two mirrors only. This instrument itself would be only 400 inches long, and well corrected.*

Gatewood : *What is the practical size of a corrector plate?*

Discussion continued on the following page.

Baker : *Different optical scientists will give different answers to this question. From the standpoint of glass technology, the limit is a diameter of well over 100 inches. Schott has made a disc of 93 inches and is able to make almost objective grade discs of 150 inches diameter, if required. I think these could be used in a telescope without much danger in spite of their weight and other technical problems.*

K. Strand : *What would the cost be for the objective which you described?*

Baker : *That depends, of course, on the rigidity of the specifications. An eight inch aperture system for astrometric applications would come to about $30 000.00, I guess.*

Dieckvoss : *At Hamburg we paid $6000.00 for a standard 20cm Ross objective of 200cm focal length, made by Zeiss in Jena.*

Baker : *The cost also depends, of course, on how much testing the manufacturer will have to do. The Lick Carnegie yellow objective has one aspheric surface which was used to eliminate some residual coma, which is a monochromatic error. The aspheric surfaces in the system on which I reported have the purpose of allowing the other component parameters a greater latitude so that we get a very short system which is still fully corrected for chromatic spherical aberration. Even over the field with the longitudinal color, the secondary spectrum accounts for practically everything that is calculated by the ray tracing. The image radius in red light, for example, is 30 microns, and it is 38 microns for the blue color. There is hardly any dependence of the spherical correction on color over that full range. The cost of such a system will be high because of the aspheric work and mandatory precision.*

COMPARISON OF PHOTOGRAMMETRIC AND ASTROMETRIC DATA REDUCTION RESULTS FOR THE WILD BC-4 CAMERA

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ABSTRACT

This paper compares the results of astrometric and photogrammetric plate reduction techniques for a short focal length camera. Several astrometric models are tested on entire and limited plate areas to analyze their ability to remove systematic errors from interpolated satellite directions using a rigorous photogrammetric reduction as a standard. Residual plots are employed to graphically illustrate the analysis. Conclusions are made as to what conditions will permit the astrometric reduction to achieve comparable accuracies to those of photogrammetric reduction when applied for short focal length ballistic cameras.

INTRODUCTION

We are concerned here with the optical determination of a satellite's position using wide angle ballistic type cameras in which the satellite is photographed against a background of stars. Basically, there are two methods of deriving the necessary information from these photographs, one by the "photogrammetric" reduction method and the other by the "astrometric" method. In the photogrammetric approach, the attempt is to describe term by term the physical nature of the various phenomena which displace the image on the photographic plate with respect to its true geometric position and to determine the parameters defining these phenomena in a single least squares solution together with the elements of exterior and interior orientations. The astrometric technique is based on the projective equations in which certain effects causing systematic image displacements are not modeled sufficiently. For this reason in the case of the large field ballistic cameras, the astrometric technique can be successfully used only after the a priori removal of certain systematic distortions or by limiting the useful area of the plate.

The purpose of this investigation was to find out whether the astrometric technique could be used for the plates taken by the Wild BC-4 camera (focal length, 303mm; field, 33° × 33°): what kind of systematic effects need to be removed a priori or how large a portion of the plate can be reduced without a priori corrections.

Dr. Hellmut Schmid, Director of the Geodetic Laboratory of ESSA, kindly provided measured star and satellite plate coordinate data from three photographic plates taken for the USC&GS satellite triangulation project. The plates had been exposed simultaneously at three stations, the satellite being Echo II. With these data, the possibility of reducing them with various astrometric models was investigated. The results were compared to those of the USC&GS highly sophisticated photogrammetric reduction. Herein, the results of the exercise for one plate only, representative of all three, will be discussed.

The results support previous knowledge and as such are not startling. However, they do demonstrate that in the case of such a large field of view, the photogrammetric method of plate reduction in fact models the significant systematic displacement effects successfully while the various astrometric methods, without the a priori removal of certain systematic errors, are incapable of doing so.

THEORETICAL BACKGROUND

The Photogrammetric Reduction

As mentioned above, the photogrammetric approach attempts to identify the main sources of systematic errors and model them mathematically for the least squares solution. Without regard to any physical influences on the emulsion, the systematic errors usually modeled are those contributed by the comparator which do not cancel after repeating the measurements in different positions, and the various errors contributed to the distortion of the camera objective. Since stars provide the reference frame, the apparent stars' positions are corrected for astronomical refraction and diurnal aberration, before beginning the adjustment.

The formulas used by the USC&GS in the photogrammetric reduction are shown in Table I. This method of reduction is considered representative of the photogrammetric approach in general.

TABLE I
PHOTOGRAMMETRIC FORMULAS

A. Corrections for Systematic Effects on Image and Object Space

(1) The apparent reference star positions for the epoch and equator and equinox of observation are corrected for diurnal aberration.

(2) Astronomical refraction (Garfinkel model) is applied to the updated star positions from (1):

$$Z_O - Z_R = T^{\frac{1}{2}} W \left(\eta_1 \tan \frac{B}{2} + \eta_2 \tan^3 \frac{B}{2} + \eta_3 \tan^5 \frac{B}{2} + \eta_4 \tan^7 \frac{B}{2} \right) \text{ , where}$$

$$W = \frac{P}{T^2}, \quad P = \frac{P_S}{P_O}, \quad T \doteq \frac{T_S}{T_O} \quad , \quad \tan B = \frac{T^{\frac{1}{2}}}{8.7137} \tan Z_R \quad ,$$

$\eta_1 = 1050.61030, \quad \eta_2 = 706.11502, \quad \eta_3 = 262.06086, \quad \eta_4 = 142.67293.$

(3) Nonperpendicularity of the comparator axes is considered by

$$\bar{x} = x_B + \varepsilon y_B \ , \qquad \bar{y} = y_B$$

(4) Radial lens distortion is modeled as follows:

$$x^* = \bar{x} - \frac{\Delta R}{d}(x - x_s) \text{ , and } y^* = \bar{y} - \frac{\Delta R}{d}(y - y_s) \text{ , where}$$

$$\frac{\Delta R}{d} = K_1 d^2 + K_2 d^4 + K_3 d^6, \text{ and}$$

$$d^2 = (x - x_s)^2 + (y - y_s)^2$$

(5) Lens decentering distortion requires the following formulas:

$$x^* = \bar{x} - \Delta T[DC_3 \sin\phi_T + DC_1 \cos\phi_T] \ ,$$

$$y^* = \bar{y} - \Delta T[DC_3 \cos\phi_T + DC_2 \sin\phi_T] \text{ , where}$$

$$DC_1 = 2(x - x_s)^2/d^2 + 1$$

$$DC_2 = 2(y - y_s)^2/d^2 + 1$$

$$DC_3 = 2(x - x_s)\ (y - y_s)/d^2$$

$$\Delta T = K_4 d^2 + K_5 d^4$$

B. The Central Projection

$$x = c_x \frac{(A_1 X + B_1 Y + C_1 Z)}{q} + x_p \ ,$$

$$y = c_y \frac{(A_2 X + B_2 Y + C_2 Z)}{q} + y_p \text{ , with}$$

$$q = DX + EY + FZ \text{ , where}$$

$$A_1 = -\cos\alpha\cos\kappa + \sin\alpha\sin\omega\sin\kappa$$

$$B_1 = -\cos\omega\sin\kappa$$

$$C_1 = \sin\alpha\cos\kappa + \cos\alpha\sin\omega\sin\kappa$$

$$A_2 = -\cos\alpha\sin\kappa - \sin\alpha\sin\omega\cos\kappa$$

$$B_2 = \cos\omega\cos\kappa$$

$$C_2 = \sin\alpha\sin\kappa - \cos\alpha\sin\omega\cos\kappa$$

$$D = \sin\alpha\cos\omega$$

$$E = \sin\omega$$

$$F = \cos\alpha\cos\omega, \text{ and}$$

$$X = \cos a \cos A$$

$$Y = \cos a \sin A$$

$$Z = \sin A$$

The various symbols in the table denote the following:

(A)	(2)	z_0	unrefracted zenith distance
		z_R	refracted zenith distance
		T_s	temperature in degrees Kelvin
		P_s	barometric pressure in millimeters
		P_0, T_0	standard pressure and temperature
	(3)	x_B, y_B	measured plate coordinates
		ε	angle of nonperpendicularity (unknown)
	(4)	x, y	undistorted plate coordinates
		x_s, y_s	coordinates of the origin of distortion (unknown)
		K_1,K_2,K_3	coefficients of radial lens distortion (unknown)
	(5)	ϕ_T	angle between the $\bar{y}$ axis and the axis of maximum tangential lens distortion (unknown)
		K_4, K_5	coefficients of tangential lens distortion (unknown)
(B)		x_p, y_p	coordinates of the principal point (unknown)
		c_x, c_y	principal distances from the projection center to the $\bar{x}$ and $\bar{y}$ axes (unknown)
		X,Y,Z	coordinates of the point in object space
		α,ω,κ	Eulerian rotation angles to transform from object space to the plate coordinate system x, y,c (unknown)
		a,A	altitude and azimuth

The parameters to be determined from the least squares adjustment are designated "(unknown)" above. There are 16 of them.

The Astrometric Reduction

In contrast to the photogrammetric reduction in the astrometric technique generally no physical interpretation is attempted except the implicit relationship between the plane of the photograph and a plane tangent to the celestial sphere. The models tested contain six or more plate constants that are coefficients in linear or higher order equations relating object and image space. These constants are not arranged to correct for specific systematic errors (except in the case of a translation term), but they are expected to absorb certain portions of the combination of various systematic errors, such as astronomical refraction, annual and diurnal aberration (since astronomers usually work with mean rather than apparent positions), errors resulting from unknown orientation of the tangent plane, and even lens distortions in some cases. The astrometric technique is simple in concept and is easy to apply. If accuracies comparable to those from the photogrammetric technique could be obtained, i.e., if the same systematic errors could be removed, its economical aspects would make it extremely appealing.

The following astrometric models were tested:

Model 1: Projective Equations

$$\left.\begin{aligned} x &= \frac{A\xi + B\eta + C}{a\xi + b\eta + 1} \\ y &= \frac{D\xi + E\eta + F}{a\xi + b\eta + 1} \end{aligned}\right\} \quad \text{8 (6 independent) plate constants}$$

Model 2:

$$\left.\begin{aligned} x &= A + B\xi + C\eta + D\xi^2 + E\xi\eta + F\eta^2 + G\xi^3 \\ &\quad + H\xi^2\eta + P\xi\eta^2 + Q\eta^3 \\ y &= A' + B'\xi + C'\eta + D'\xi^2 + E'\xi\eta + F'\eta^2 \\ &\quad + G'\xi^3 + H'\xi^2\eta + P'\xi\eta^2 + Q'\eta^3 \end{aligned}\right\} \quad \text{20 plate constants}$$

Model 3:

$$\left.\begin{aligned} x &= A\xi + B\eta + C \\ y &= D\xi + E\eta + F \end{aligned}\right\} \quad \text{6 plate constants}$$

In this exercise the quantities x and y represent the *measured* (uncorrected) plate coordinates; ξ and η are the standard coordinates computed from the apparent positions of the reference stars corrected for diurnal aberration and astronomical (Garfinkel) refraction.

In all the astrometric reductions the measured plate coordinates were considered observed quatities and the standard coordinates were regarded as known. All observed coordinates had equal weights. The same image is used to be the origin of the plate coordinate system as the origin of the standard coordinate system.

RESULTS OF THE EXPERIMENT

Photogrammetric Residuals

Figure 1 shows the star (and satellite) image locations on one of the plates that were used in the reductions. The values of the sixteen photogrammetric parameters obtained from the USC&GS were applied with the updated star positions to compute the "adjusted" plate coordinates.

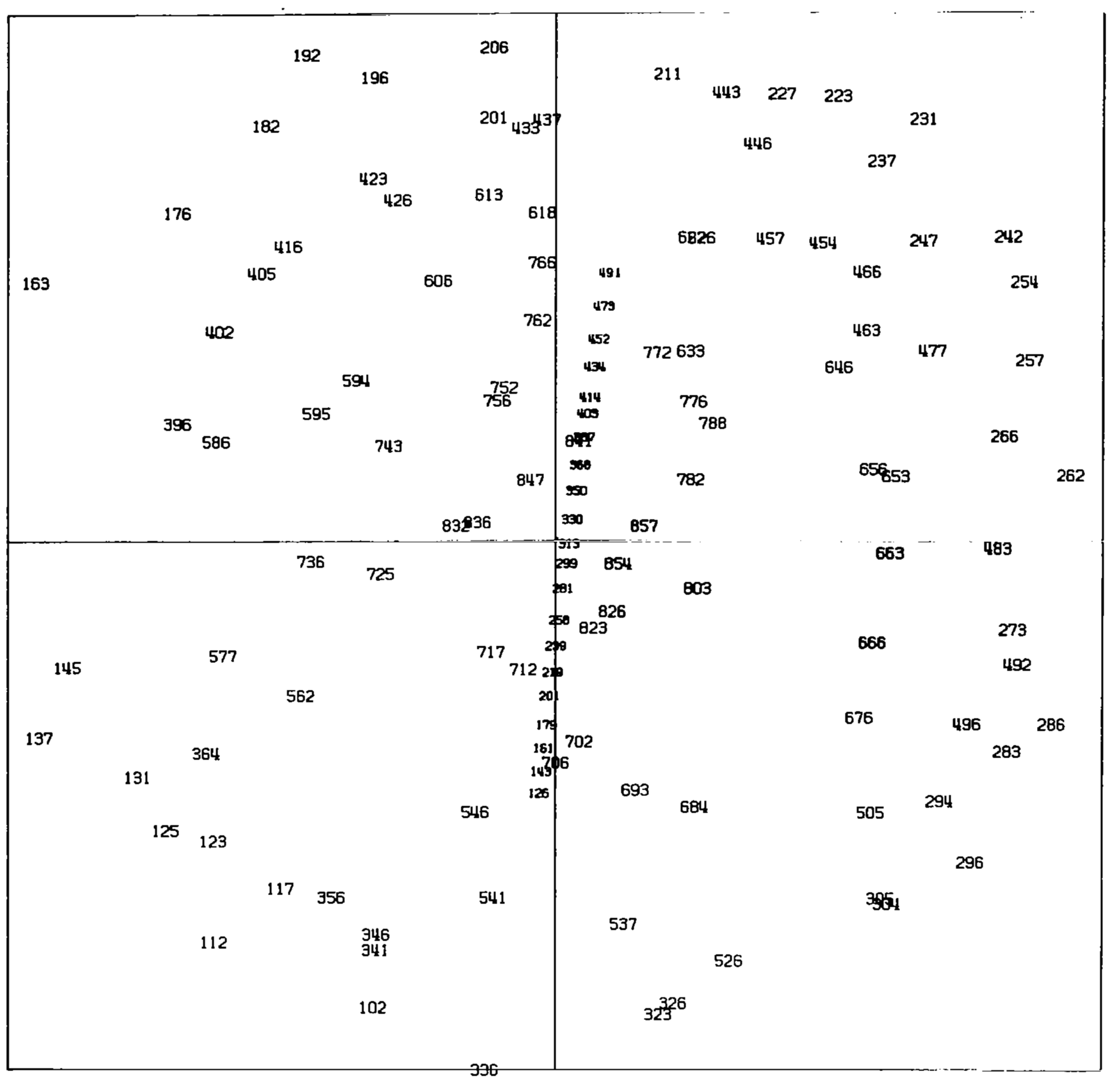


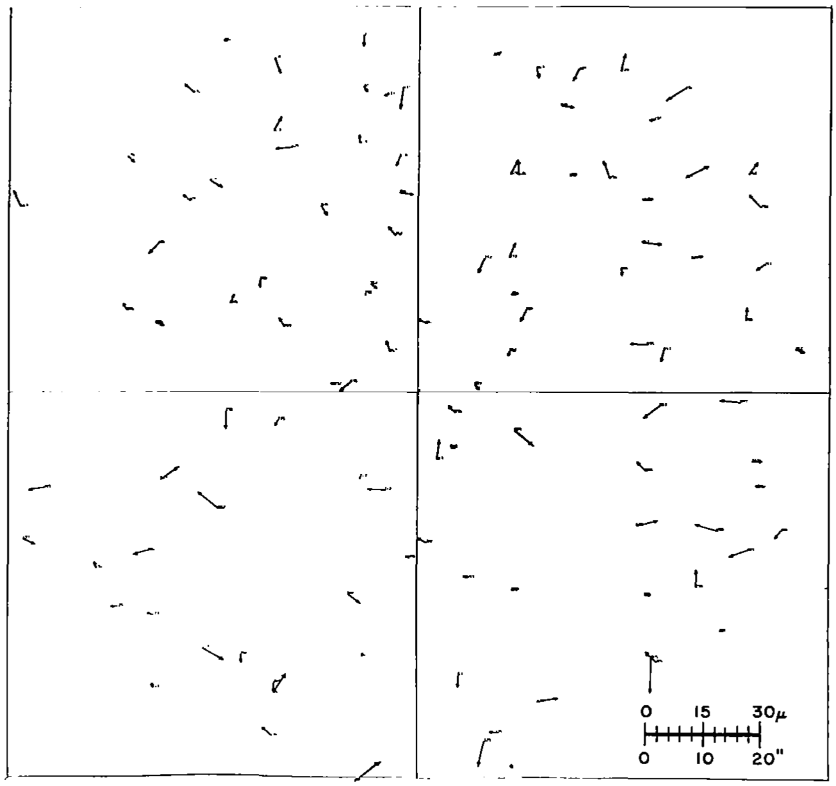
FIGURE 1

Observed Star and Satellite Locations
Satellite images are numbered 126-494

FIGURE 2

Photogrammetric Residuals

$m_o = 2.80\ \mu$



The residuals between the adjusted and measured coordinates of the stars were plotted and are shown in Figure 2 . The standard error of unit weight (m_o) as quoted from the original USC&GS reduction was 2.80μ. Notice the apparent randomness of the residual plot and the small magnitude of the residuals.

Residuals from Astrometric Model 1

Test 1. In this test Model 1 was applied to the entire plate area. This involved 106 reference star images. The standard error of unit weight after reduction was 7.81μ. Figure 3 shows the residual plot, systematic errors obviously remaining and sizeable after reduction.

Test 2. It was desired to find out how great the effects of decentering distortion were in regard to Model 1. Therefore, decentering distortion was removed from the measured coordinates. Still remaining, then, were radial distortion effects and the nonperpendicularity of the comparator axes. The reductions were performed as in Test 1. The change in the plotted residuals compared to those in Figure 3 appeared to be minimal. The standard error of unit weight was 7.77μ.

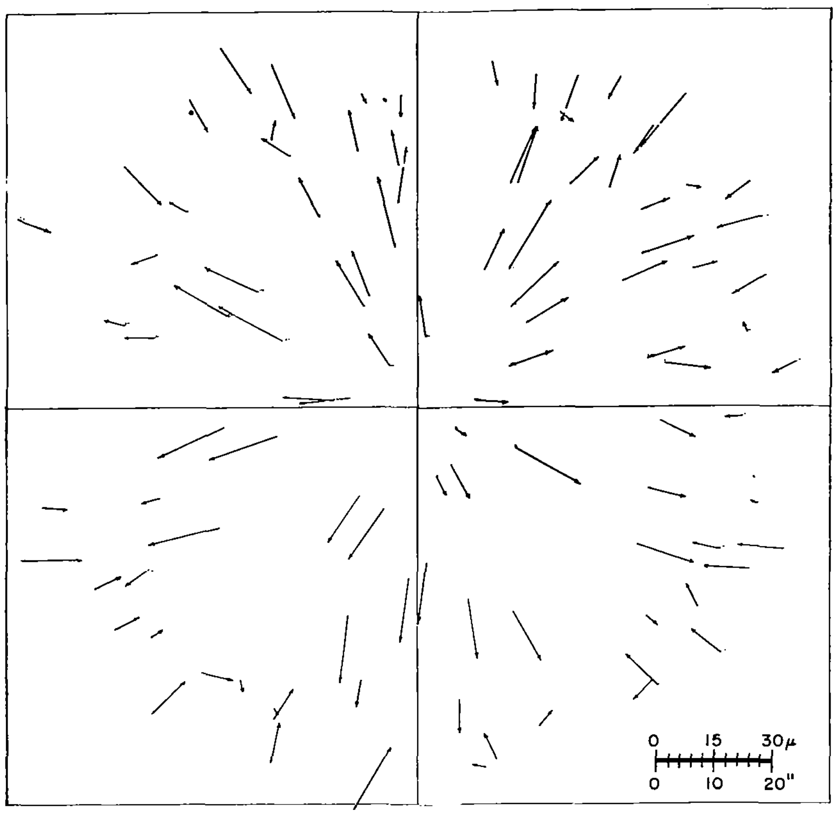


FIGURE 3

Astrometric Model 1 Residuals: Test 1

Projective equations applied to measured coordinates

$m_o = 7.81\mu$

Test 3. This time all distortions (including nonperpendicularity of the comparator axes, although this effect is also almost negligible) were removed a priori from the measured coordinates. The reductions were performed as in Test 1. The residuals are shown in Figure 4 . These, as expected, are almost identical to the residuals of the photogrammetric reduction (Figure 2). The standard error of unit weight was 2.62μ.

Test 4. The question to be answered in this test was how far out from the origin can the reduction be performed without a priori removing the distortions and still get satisfactory results.

The photogrammetric model was expanded into polynomials in x and y . The results are as follows:

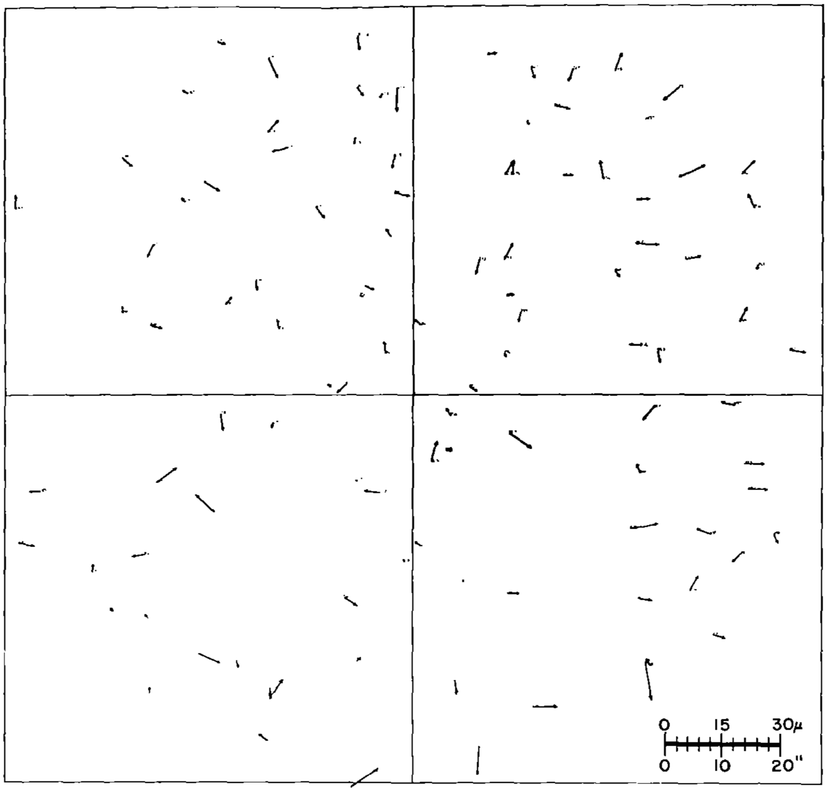


FIGURE 4

Astrometric Model 1 Residuals: Test 3

Projective equations applied after all distortions were removed from measured coordinates

$m_o = 2.62\mu$

$$
\begin{aligned}
x_{measured} = {} & x - y(\varepsilon) + x^2(3K_4\cos\phi_T) + x^3(K_1) + x^4(3K_5\cos\phi_T) + x^5(K_2) \\
& + x^7(K_3) + xy(2K_4\sin\phi_T) + xy^2(K_1) + xy^3(2K_5\sin\phi_T) + xy^4(K_2) \\
& + xy^6(K_3) + x^2y^2(4K_5\cos\phi_T) + x^3y(2K_5\sin\phi_T) + x^3y^2(2K_2) \\
& + x^3y^4(3K_3) + x^5y^2(3K_3) + y^2(K_4\cos\phi_T) + y^4(K5\cos\phi_T)
\end{aligned}
$$

$$
\begin{aligned}
y_{measured} = {} & y + y^2(3K_4\sin\phi_T) + y^3(K_1) + y^4(3K_5\sin\phi_T) + y^5(K_2) + y^7(K_3) \\
& + yx(2K_4\cos\phi_T) + yx^2(K_1) + yx^3(2K_5\cos\phi_T) + yx^4(K_2) + yx^6(K_3) \\
& + y^2x^2(4K_5\sin\phi_T) + y^3x(2K_5\cos\phi_T) + y^3x^2(2K_2) + y^3x^4(3K_3) \\
& + y^5x^2(3K_3) + x^2(K_4\sin\phi_T) + x^4(K_5\sin\phi_T)
\end{aligned}
$$

where x and y are the undistorted coordinates as defined by the projection equations.

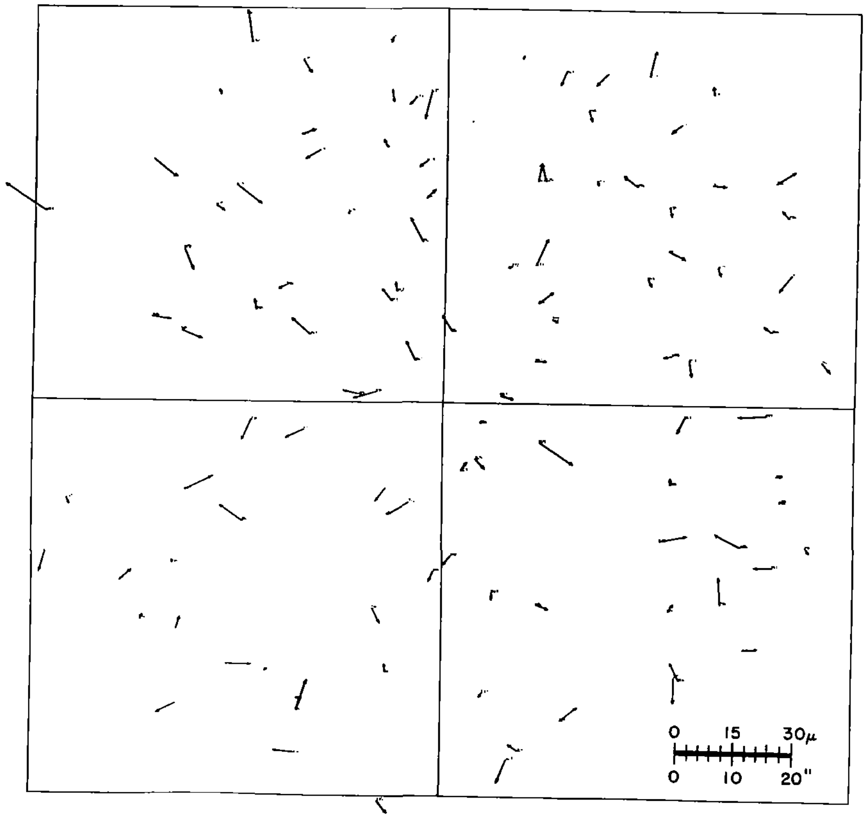


FIGURE 5
Astrometric Model 2 Residuals
$m_o = 3.39\mu$

For ease of analysis, assume now that an image is projected, without distortions, onto the x axis. In this case the first equation above becomes

$$x_B = x + x^2(3K_4\cos\phi_T) + x^3(K_1) + x^4(3K_5\cos\phi_T) + x^5(K_2) + x^7(K_3) \ .$$

The error committed by using the projective equations is therefore given by

$$x_B - x.$$

After some preliminary investigation, the maximum allowable distortion arising from the above equations was set to 16μ. This limit was found to be reached at at radius of about 4cm. After applying Model 1 for such a restricted area around the center of the plate, the residuals were found to be apparently random and the standard error of unit weight $m_o = 3.24\mu$, nearly the same as in Tests 1 and 3. When the area was decreased to a circle of 3.5cm radius, m_o decreased to 2.40μ.

Residuals from Astrometric Model 2

Figure 5 shows the residual plot when Model 2 is applied to the entire plate area without a priori corrections for systematic camera or comparator errors. These plots are approaching in randomness and magnitude the residuals of the photogrammetric reduction. The standard error of unit weight was found to be 3.40μ, slightly higher than that of the photogrammetric reduction.

When the polynomials are carried out to fifth-order terms, it is conjectured that the error would further decrease by about 1μ. In this case, however, the large number of unknown parameters would render the method even less economical than Model 2 or the photogrammetric method.

Residuals from Astrometric Model 3

To find the area where Model 3 may be applied with acceptable results, the following empirical approach was taken: Assume that Model 2 is rigorous. If the significant terms are kept in the polynomials after the adjustment, the following equations define the transformations on the plate:

$$x_B = 0.046\xi - 0.30\eta - 0.00019\xi^2 + 0.00089\xi\eta + 0.0010\eta^2$$

$$y_B = 0.30\xi + 0.046\eta - 0.0010\xi^2 - 0.0011\xi\eta - 0.00017\eta^2$$

In a preliminary adjustment in which Model 3 and stars in a circular area of about 2.5cm radius were employed, the following empirical relationships were found (disregarding the translation term):

$$x_B = 0.046\xi - 0.30\eta \ ,$$

$$y_B = 0.30\xi + 0.046\eta \ .$$

It is evident that the maximum magnitude of the neglected terms is in the order of $0.0011\xi\eta$. If the error committed by using this model is to be less than 3μ, the usable area appears to have a radius of about 2.2cm. Applying Model 3 for such an area after iterating for the plate center, it was found that the standard error of unit weight was 2.59μ .

CONCLUSION

The projective equations cannot be used to reduce the entire area of a Wild BC-4 stellar plate. Unmodeled distortion effects are too great toward the outer edges of the plate. However, if the lens distortion parameters are known and fairly constant for a given camera, then the projective equations (with two conditions) could be used after the measured coordinates have been corrected for lens distortion effects and the apparent stellar coordinates are corrected for diurnal aberration and refraction. The behavior of the distortion characteristics of the lens used over a period of time is not known to the authors. However, if it is not necessary to recalibrate the camera after each exposure, it appears that the projective equations could be applied by means of the procedure just described to obtain results practically equal to those which a complete photogrammetric reduction would provide. If no a priori corrections are made for lens distortions but still the apparent stellar coordinates are corrected for diurnal aberration and refraction, then, as the experiment shows at least in the case of the three plates discussed here, a confined area not greater than 4cm (about 8°) in radius from the plate center can still be reduced with good results. An image as close as possible to the gemometrical plate center should be selected as the origin, but no iteration technique need be applied to obtain a better approximation.

It is also obvious from the results of the experiment with Model 2 (polynomial) that if enough higher order terms are included in the equations, the reduction results can be made comparable to those obtained photogrammetrically without applying a priori corrections for systematic camera and comparator errors. However, the 20 unknowns (up to third-order terms) of Model 2 already justify the use of the complete photogrammetric reduction.

Even Model 3 can be applied without a prior corrections with caution under the following conditions:

(a) The satellite image(s) should be located close to the geometrical plate center and completely contained in the field of reference stars used in the reduction.

(b) The area of reduction should be no greater than a circle of 2cm (4°) radius.

(c) The choice of the origin should be made with extreme care. After approximating the geometrical center as closely as possible, an iteration technique should be used to obtain the best origin for both the plate and standard coordinate systems.

Acknowledgement

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DISCUSSION

Vasilevskis: *I would like to see another term used for what you call the Astrometric Method. Astrometrists have known about the rigorous reduction for a long time. The use of the necessary terms in the reduction method requires a computation effort that used to be very time consuming before the advent of computers. Also, every sophistication requires new constants. These cannot be reliably determined unless there is a sufficient number of reference stars available. The large fields of satellite photographs present, of course, different problems than the smaller, narrow ones in traditional photographic astrometry.*

Schmid: *This appears to be a difference in nomenclature only. From a mathematical standpoint, the two are in effect identical. The variety in the modeling approaches occurs mostly because different lens systems require different terms in the reduction model for the consideration of the imperfections of the lens system's imaging quality. The wider the field, the more sophisticated the reduction model must be.*

Eichhorn: *Mr. Hornbarger's comparison is not quite valid. The superiority of what he calls the "photogrammetric method" cannot be demonstrated by reducing certain measurements first rigorously and then by means of an inadequate reduction model. The radial distortion terms would have to be included in the so-called astrometric model as well as in the rigorous so-called photogrammetric model, otherwise there are obviously going to be significant systematic residuals in the former.*

Mueller: *The purpose of Mr. Hornbarger's investigation was not to prove the superiority of the photogrammetric method. Its point was rather to demonstrate the usefulness of cameras with focal lengths between 300 and 1000 millimeters for obtaining the coordinates of satellites, which possibly had been doubted by some of the participants in the Conference on Star Catalogues at the University of Maryland in 1967, who claimed that high accuracy could be obtained only with long focus cameras.*

Veis: *What really determines the approach to use for the proper reduction method is not the focal length but rather the size of the field. Large fields require sophisticated reduction models, but for small fields the application of the same complicated reduction models with so many parameters would be a waste of effort.*

Brown: *Everyone speaking before me in this discussion has missed the fundamental difference between photogrammetric and astrometric methods: while the formulas used in the latter may be regarded as series expansions of the former, the two sets of formulas would be equivalent only if the two existing, but always unenforced relationships between the eight parameters in the astrometric model (Hornbarger 1970) were enforced. Thus, the photogrammetric method preserves properties of the central projection while the astrometric method typically fails to do this.*

Strand: *In astronomical practice it is usually not necessary to locate the tangential point by calculation, and this removes two unknowns from the adjustment. At the 61-inch telescope of the USNO, for instance, the tangential point is found by means of a special collimation system.*

Rosenfield: *The essential point is not how many parameters are in the reduction model, but rather how many of these are determined by adjustment rather than previous calibration. Only parameters determined in the reduction carry a danger of lowering the all-over accuracy of the results.*

Vasilevskis: *In astronomical work, an affine transformation model is usually adequate for long focus work. This has been very fully documented by van de Kamp. Zurhellen in his dissertation and A. Koenig in the "Handbuch der Astrophysik" have treated the case of wider angles and used what goes now by the name of "Photogrammetric model". Beyond these classical wide angle applications, there is now a third generation "super wide angle" astrometry, which is in principle not in any way different from the others, even if it requires some modification in the technique.*

ASTROMETRIC INVESTIGATIONS AT THE VIENNA OBSERVATORY AND ASTROMETRIC PLANS FOR THE 60" REFLECTOR OF THE L. FIGL OBSERVATORY

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ABSTRACT

A short discussion of the astrometric instruments at the Vienna Observatory is given. The essential features of design and organization of the planned 60 inch telescope are described. Furthermore, several points of view concerning modern astrometry are discussed, especially the usefulness of comparing astrometric results based upon different instruments, for instance the comparison between mirrors and refractor based data. Some examples are given together with new immediate results. Lastly, as far as possible at this moment astrometric plans for the new telescope are formulated.

THE SITUATION OF THE VIENNA OBSERVATORY

The situation of the Vienna Observatory is typically that of an observatory founded in the 19th century in connection with a university: immediate neighborhood of large settlements and towns, and thereby continuously deteriorating observing conditions. Most instruments were built in the last century.

The Observatory possesses thus the 27 inch, 10 meter focal length, "Large Refractor", two refractors of 12 inches and 8 inches each, and the 13 inch Normal Astrograph. Beside these, there is a 16 inch mirror, mostly used for photoelectric measurements.

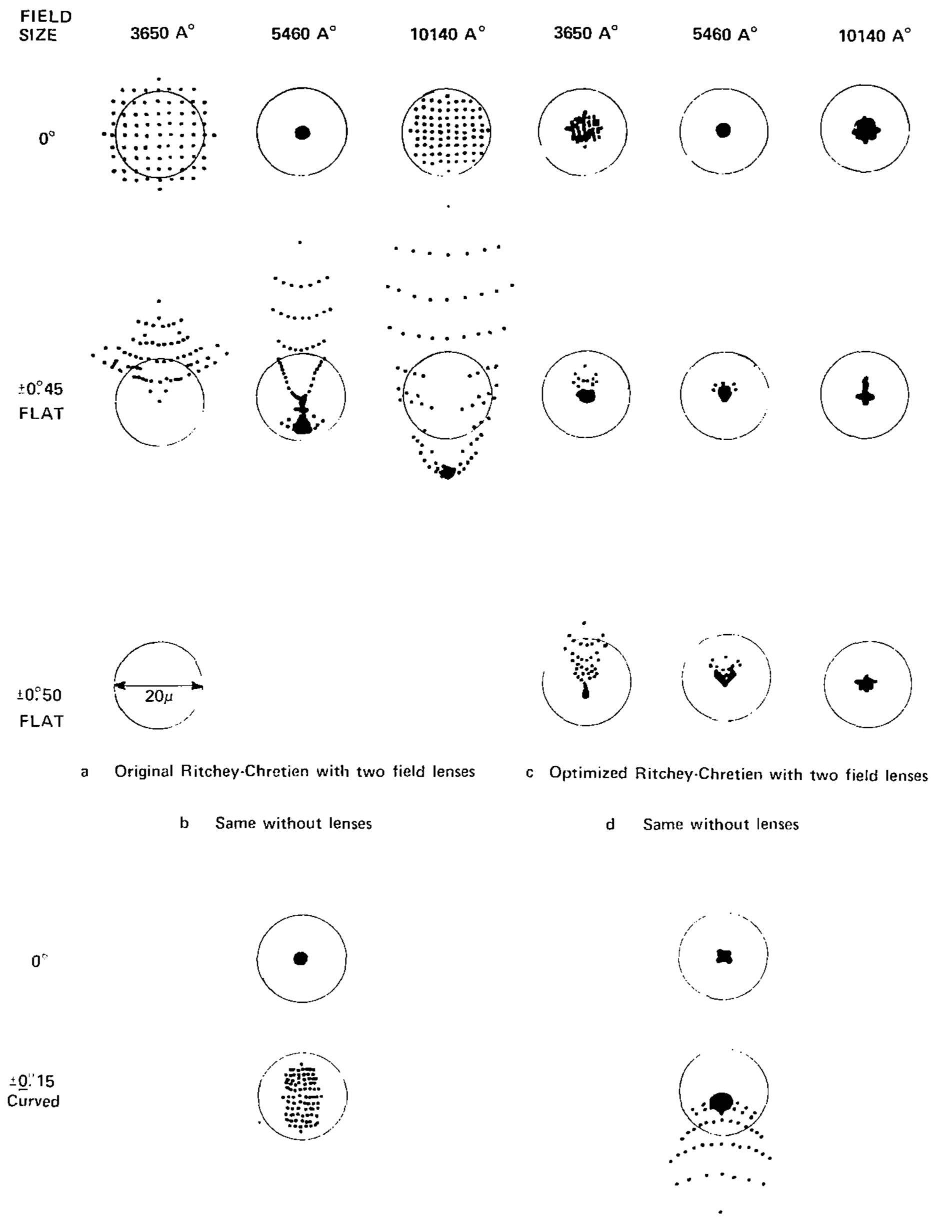

FIGURE 1

Ray tracing diagrams of two possible optical solutions for the Vienna Ritchey-Chrétien from the astrometric point of view.

Equipment of this type may still be considered "valuable for science" as long as there are problems to be solved with any of it which cannot be treated better and much more efficiently by more modern instruments. Such a definition of "value" is useful for preventing the evaluation of equipment only on the basis of emotion. Refractors, in our opinion, are generally not obsolete in the sense of the definition above for the following reason: although practically all astrophysical problems and questions are the legitimate domain of optical reflectors and radio telescopes, refractors still have their place for the exact imaging of the sphere onto the photographic plate. Also, the quality of photographic images intended for measurement of their positions which one can achieve with refraction optics illustrates very well the continued usefulness of large and medium sized refractors. This is demonstrated, for instance, by the important, and as may be hoped, successful enterprise of the photographic AGK3 at the Hamburg-Bergedorf Observatory. The most serious limitation of refractors with respect to astrometric problems is probably the faintness of the stars; one can reach about the 13th photographic magnitude, at least in northern climates, with normal exposure times. However, modern Schmidt telescopes and Ritchey-Chrétien mirrors provide excellent measurable images covering larger fields down to magnitudes, normally out of reach of most refractors. One thus wonders if in view of this, refractors are not outdated and definitely obsolete after all. This is the principal problem facing Vienna Observatory and its one-half to nearly one full century old refractors.

THE L. FIGL OBSERVATORY AND ITS TELESCOPE

In spite of the development of modern astrometric reflectors, the refractors are still useful instruments in astrometry. This is shown, for instance, by the work of Jackson (1968), who used the Vienna Large Refractor. van de Kamp's (1962) ideas must also be noted. But the main problem now is the coordination of refractors and modern mirrors for astrometric work. Since, however, reflector plates contain always more information than refractor plates, refractors, in comparison with mirrors, are useful mainly for those special purposes that require large focal lengths. Examples for this are astrometric relative positions of narrow binaries, parallaxes and proper motions of objects of small angular size, such as individual stars and star clusters.

The new telescope of the L. Figl Observatory in the Vienna Woods has been designed with an eye on these considerations. It will be housed 880m above sea level and 50 kilometers southwest of the city. Of course, it may turn out that this 60 inch telescope will be used chiefly for astrophysical purposes, but it will also be useful for astrometric work, since it was designed as a multi-purpose telescope. It may be used as an f:8.3 Ritchey-Chrétien, with an f:15 Cassegrain and an f:30 Coudé. The astrometric work will be carried out on the Ritchey-Chrétien mode. Since it was impossible to have even a Ritchey-Chrétien configuration that would have been ideal for astrophysical as well as astrometric purposes, the adopted optimum design had to be a compromise, shown on the right hand side of Figure 1 (after B.G.Hooghoudt, Leiden). This so-called Optimized Ritchey-Chrétien System employs two field lenses. This implies that the mirrors will not be figured exactly as in the classical Ritchey-Chrétien design. The upper part of Figure 1 shows images at several wavelengths for different distances from the center of a flat field. The left hand side shows the images for the classical, the rights hand side for the optimized Ritchey-Chrétien system, which was chosen as the apparent best of several calculated by Dr. Wilson of the Zeiss Works at Oberkochen in West Germany. The advantage of the optimized design for astrometric puposes is obvious at first sight, especially the marked compactness of the images up to a field of one degree diameter. Note the insignificant dependence of the image positions on wavelength. The disadvantages of the optimized design are illustrated in the lower part of Figure 1. For working in the center of the field and without corrector lenses, the classical design is clearly superior to the optimized one. The optimized design for the telescope was chosen, however, since the telescope will also have a Cassegrain mode. Besides, for extended fields the Schmidt telescopes are the most suitable instruments anyway, although they are not very suitable as multipurpose telescope.

It is hoped to preserve thus the connection with the astrometric tradition of the Vienna Observatory and its refractors, without preventing or restricting the astrophysical work at the new telescope.

POINTS OF VIEW CONCERNING MODERN ASTROMETRIC MEASUREMENTS

The question in which way the telescope of the L. Figl Observatory could be used for astrometric purposes will have a definitive answer only after an extensive investigation of the optical system, which cannot be carried out before the telescope is mounted.

The suitability of reflectors for astrometry has been extensively discussed and methods towards this purpose have been developed, witness the papers by van de Kamp (1963), Eichhorn (1963), Vasilevskis (1962), and Strand (1963), to mention only a few. The results show clearly that the field of each mirror system has special and individual properties. The aberrations are apparently generally much larger than those of the refractors. Before specialized plans may be formulated, it will therefore be necessary to first test the properties of the field of the new Vienna telescope; and all future astrometric work on it will have to be suitably connected to former refractor based astrometric work, if it is to be useful. This is so because all astrometry (including meridian work) up to the middle of the current century was carried out on refractors. Perhaps no field in astronomy depends as much on previous work as astrometry. In the opinion of this author it is indispensable in establishing a connection between the classical refractor based, and the modern mirror based astrometric work. Noteworthy in this connection are the investigations by Dieckvoss (1955, 1960), in which he juxtaposed astrometric work on the Hamburg big Schmidt and on the AG astrograph. There, a direct connection between mirror and refractor was established. A similar comparison of performance of the new Vienna system and that of not only the Vienna, but also other refractors, is planned, to collect experiences with the new Vienna telescope and establish its individual properties as they are relevant to astrometric work. The results of Dieckvoss' paper demonstrate how very individual the properties of an astrometric mirror system may be. The importance of comparisons of this type for modern astrometric work is not only restricted to the testing of optical systems; many modern astrometric problems, for instance expansions of stellar associations, concern the proper motions and positions of single, relatively isolated objects. Most suitably, such problems ought to be attacked on several telescopes and measuring machines to establish their relations to one another, and to get the final, definitive values for the objects' positions and proper motions from a combination of the results obtained on the individual instruments. This practice will guarantee the highest obtainable astrometric accuracy and will undoubtedly be effective. Considerations of this type, however, may be valid for astrophysics as well, and therefore for astronomy as a whole. This may be the deeper hidden meaning of a comparison between mirrors and refractors for astrometric purposes.

A COMPARISON OF ASTROMETRIC MEASUREMENTS WITH SEVERAL TELESCOPES

A few examples may illustrate our point. Dieckvoss (1955) was the first to find a relative motion of the clusters h and χ Persei. In order to check this result, Meurers and Aksoy (1960) investigated the relative proper motions with the 5m f.l. Large Refractor of the Bonn Observatory. The results compare as follows with Dieckvoss', whose results are based on a combination of several different instruments:

	$\Delta\mu_\alpha$	$\Delta\mu_\delta$	
Bonn I	$-0''.34$	$+0''.04$	$\pm0''.05$
Hamburg	$-0''.19$	$+0''.10$	$\pm0''.04$

($\Delta\mu_\alpha$ and $\Delta\mu_\delta$ in this and all following tables, are in units of seconds of arc on the great circle per century.)

The agreement is reasonably good and confirms the relative motion of the two clusters to one another by two completely independent sets of data. Another example of the idea to confirm astrometric results concerning isolated objects and to establish connections is this: there is a star in the neighborhood of h and χ Persei with a very large proper motion, whose exact magnitude is of special interest, for reasons conerning galactic kinematics, etc. In this case, there were two independent investigations. The proper motion of the star has been determined by Meurers and Askoy (1960) and then, in another investigation, by Meurers and Voosholz (1968). In each case the plate material is different, but the telescope was the same. The results are:

	μ_α	μ_δ
M & A	$+17''.09$	$-2''.19$
M & V	$+17''.60$	$-2''.44$

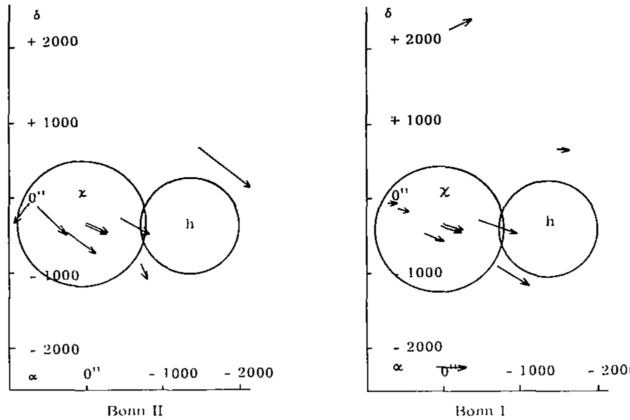

FIGURE 2

Proper motions of bright M stars around h and χ Persei according to two independent series of measurements (different plates and observers). The double arrow is the motion of χ Persei.

On the basis of these two independent measurements, the motion of this star seems to be very well established. The results from these two independent investigations concerning the whole field around the double cluster and the individual motions of the clusters also agree rather well.

	M & A		M & V	
	μ_α	μ_δ	μ_α	μ_δ
Field	$-0\overset{''}{.}05$	$-0\overset{''}{.}03$	$-0\overset{''}{.}01$	$-0\overset{''}{.}03$
h	$-0\overset{''}{.}08$	$-0\overset{''}{.}03$	$+0\overset{''}{.}07$	$-0\overset{''}{.}08$
χ	$-0\overset{''}{.}42$	$+0\overset{''}{.}01$	$-0\overset{''}{.}42$	$-0\overset{''}{.}19$

This is another example in which the results from independent investigations are in good agreement and thus confirm their reliability. Some of the M stars around h and χ Persei were also measured in both of these investigations, and the results agree well, as shown in Figure 2. The arrows represent relative proper motions of the M stars which apparently is the same as that of Persei. These independent measurements demonstrate thus that these M stars belong to χ Persei, confirming thereby (through the distance of χ Persei) the luminosity, which had been predicted for them from astrophysical investigations. The agreement between the absolute proper motions of h and χ Persei, based on the investigations by Dieckvoss (1955) on the one hand, and Meurers and Voosholz on the other, is equally impressive since in those investigations observers, plates, telescope , and measuring machines were all different.

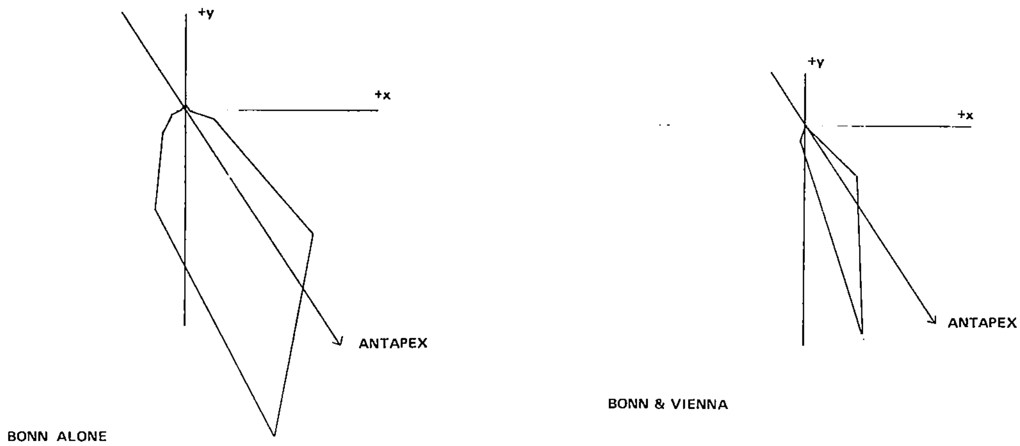

FIGURE 3

Proper motion (velocity diagrams) of NGC 6838 according to two independent series of measurements (different refractors and plates).

h - Persei

	μ_α		μ_δ	
D	$-0''.02$	$\pm0''.09$	$+0''.51$	$\pm0''.07$
M & V	$-0''.04$	$\pm0''.16$	$+0''.20$	$\pm0''.08$

χ - Persei

	μ_α		μ_δ	
D	$-0''.21$	$\pm0''.09$	$+0''.61$	$\pm0''.08$
M & V	$-0''.22$	$\pm0''.01$	$+0''.58$	$\pm0''.05$

One final example: NGC 6838 may be either a globular or an open cluster. Its absolute proper motions were investigated on the basis of two independent sets of measurements by Meurers and Prochazka (1969), made at the Large Refractors at Bonn and Vienna respectively. Absolute proper motions of the cluster were determined first with the Bonn refractor alone from two old and two new plates, then with the Bonn and Vienna refractor from two old Bonn and two new Vienna plates. The results agree well within their errors:

	Bonn alone		Bonn and Vienna	
μ_α	$+0''.28$	$\pm0''.19$	$+0''.34$	$\pm0''.19$
μ_δ	$-0''.78$	$\pm0''.26$	$-0''.91$	$\pm0''.26$

They also show that the Vienna Large Refractor, previously not used for photographic astrometric work, yields the same results as Bonn and was in this way (and also on the basis of other comparisons) shown to be suitable for photographic astrometric work. These two independent measurements of the absolute proper motions of NGC 6838 demonstrate also that the cluster's motion is directed toward the antapex, and therefore it is extremely probable that the cluster is a galactic one. (See Figure 3)

CONCLUSION

The reliability of, and confidence in astrometric results is, in the author's opinion, greatly strengthened by the comparison of measurements which are based on independent instruments and observers which should become more prevalent in the future. It is moreover necessary to test each new mirror system for astrometric work, whenever possible, by comparison of results obtained with it, with those obtained from preferably several independent measurements of single objects. This is planned for the new Ritchey-Chrêtien telescope of the L. Figl Observatory, whose performance will be tested at first by comparing it with other instruments, mirrors and refractors, especially with the Large Refractors at Vienna and Bonn, as well as with the excellent astrometric mirrors in the USA. Results obtained on new systems must be compared with those obtained on other already existing modern astrometric telescopes, in order to minimize systematic errors in future work. The author would thus propose that the modern astrometric mirror systems should be "harmonized" one to another, i.e., that the relationships of their field properties should be established very carefully. We suggest the concept of a world wide net of large astrometric mirrors. The relationships between the properties of their fields should be known as accurately as possible so that on any one of these instruments the same problems may be attacked on the basis of highly reliable, independent measurements.

This world wide net of astrometric mirrors, "harmonized" on to another requires, and this idea is not a new one, a system of calibrating fields on the sphere, something of the type of, but not as numerous as the Selected Areas. In these fields the positions of stars as faint as possible should be determined as exactly as possible for calibrating the mirrors of the network. This would also be the fastest and most efficient method to investigate the field properties of the individual mirror systems.

The author also suggest that today, astrometric work on the basis of modern mirrors, in order to achieve optimum results, should be organized on a world wide basis, including the observations of artificial satellites. The establishment of the new Vienna telescope will be attempted with these considerations in mind. However, not everybody will agree with the author's propositions and suggestions.

The special long range scientific plans for the Ritchey-Chrêtien reflector of the L. Figl Observatory essentially are geared to the above mentioned concepts Other things can be done even now: the small field of the Ritchey-Chrêtien requires the determinations of the positions of faint reference stars from, for instance, the AGK3 or the Yale Catalogues. An example for this is the investigation of M56 by Küstner (1920). Measurements of proper motions, especially of faint objects, are the "natural" domain for such systems.

The establishment of plate archives for the future, especially of the Milky Way clouds, is also worth mentioning. Note, however, that the establishment of plate archives for astrometric work is always overshadowed by the uncertainty of which objects' proper motions will eventually be needed. One telescope alone, after all, cannot cover the whole sky, or even the Milky Way. However, this could perhaps be realized. Only in this way will astrometry maintain its position within astronomy at large, especially within astrophysics which, after all, is traditionally the "proper domain" of the mirrors.

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DISCUSSION

Murray : *I agree with the speaker that the results from refractors and reflectors should be compared to obtain information about the color and magnitude equations of the two systems. Also, how do the proper motions of h and χ Persei differ with regard to different reference frames?*

Meurers : *Two different, but numerous sets of reference stars were used for the two sets of computations, but there are so many reference stars that there should not be any significant differences. For the absolute motions we took Dieckvoss' reference stars.*

Dieckvoss : *The reference stars used at Hamburg are on the FK3 system.*

Strand : *About six or seven years ago I compared the parallaxes obtained from plates taken at the Cassegrain focus by van Maanen with the 60 inch Mt. Wilson reflector (whose configuration is well known for its stability) with the parallaxes obtained at various refractors. The agreement between the values from the two sets was excellent, and the errors also were about the same.*

Murray : *(Asking Strand): Several years ago, some cluster plates were taken on the 40 inch Ritchey-Chrêtien of the USNO. Are these going to be used to proper motion studies?*

Strand : *Probably not. Tests have shown that the plates taken at the Ritchey-Chrêtien are not very well suited for astrometry of this kind. Moreover, the present glass mirrors will be replaced with new optics made of ultra-low expansion silica by Corning.*

Fredrick : *Would the Vienna Observatory be willing to furnish a set of overlapping plates of the Praesepe, both with and without grating, for the purpose of establishing a calibration field?*

Meurers : *Yes.*

AUTOMATIC PLATE MENSURATION AND REDUCTION AT AERONAUTICAL CHART AND INFORMATION CENTER

Thomas O. Seppelin

Aeronautical Chart and Information Center

ABSTRACT

The use of optical satellite observation data for geodetic purposes depends on the ability to extract pertinent information from photographic plates by identifying points on the plates and correlating them with known star positions for use in geodetic positioning. At the Aeronautical Chart and Information Center (ACIC), this procedure is accomplished with a new semi-automatic measuring system, an advanced analytical plate reduction program for the IBM 7094 computer, and a triangulation adjustment. The advantages gained with the measuring system, a brief discussion of the system itself, and the plate reduction and triangulation adjustment are described.

The successful launching of the ANNA geodetic satellite into orbit in 1962 created a need for ballistic camera plate measurement, and for the reduction of satellite observation data for geodetic purposes. The ACIC is responsible for a major portion of the geodetic plate reduction supporting U.S. Air Force (USAF) programs.

Ballistic camera plates are developed and forwarded with reduced timing information to the Center where they undergo a series of evaluation and reduction processes. During the *preliminary phase*, the plate is plotted on a star chart; images (star and satellite) are selected for reduction and coded to correlate image measurements with exposure times; and four "index stars" are selected for general plate orientation. The coordinates of the images are then measured.

From the very beginning of the operation, the ACIC staff was aware of the several well known significant error sources or excessive cost, especially in terms of time: the setting error, operator fatigue, environmental control, and the requirement for always approaching an image from the same direction to avoid "backlash". Therefore, they were constantly looking for ways and means of minimizing or eliminating these factors.

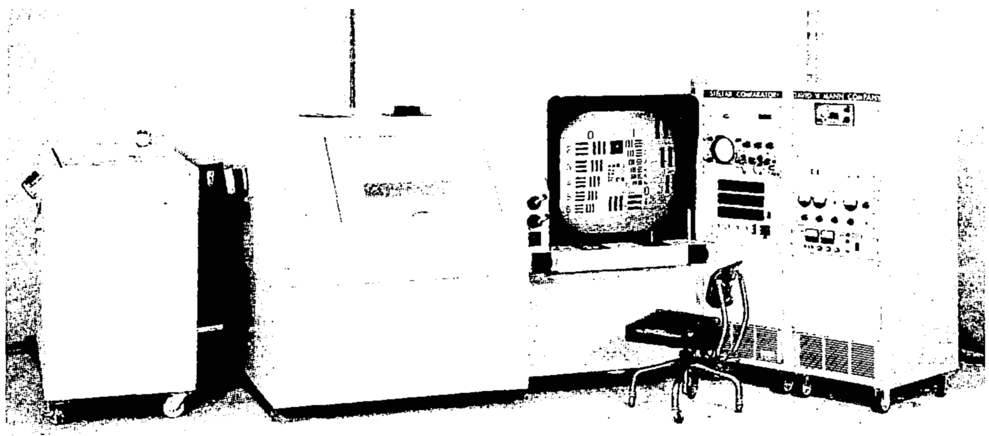


FIGURE 1

Type 1205 Semi-automatic Stellar Comparator

Eventually, a set of specifications was developed for the "Stellar Comparator" - a semi-automatic measuring instrument designed to satisfy the requirements of ACIC. The specifications were submitted to manufacturers, and the David Mann Company was finally selected to fabricate the equipment which was delivered in December, 1965.

Basically, the Stellar Comparator consists of an air-bearing measuring engine, environmental control, an optical system, and an electronic image sensing system with servomechanisms to control motions. The 10 × 10 inch measuring area accepts both positive or negative glass plates and film as large as 10 1/2 × 10 1/2 inches ranging from .0025 to 1/2 inches in thickness.

The output consists of x and y Cartesian coordinates in microns for the images measured, 18 additional digits for identification and recovery, and a 6 digit frame count. Output is automatically recorded on standard 80 column cards with an IBM 526 keypunch. Although the comparator is designed for semi-automatic operation, it may be manually operated at the measurer's discretion.

The measuring engine is similar to a standard two-coordinate screw comparator. However, air bearings support both the x and y plate stages for maximum stability without detrimental load shifting. Since the location and configuration of the machine make direct dial reading impractical, the dial presentations are relayed optically to the operator's position at the console.

The screws, whose pitches are one millimeter, are mounted in high precision bearings and equipped at one end with the optical dial reading system and a servo drive mechanism.

The illumination system provides a field of light of uniform intensity for the projection systems. A built-in spherical reflecting mirror directs the light from an air-cooled 500 watt projection lamp through a set of condenser lenses. The infrared portions of the spectrum are removed by a filter and deflected to a heat sink to minimize the heat at the film gate. Only the blue and green portions of the light beam are used for the photoelectric and projection systems. The remainder of visible light is removed by a system of filters to further reduce the heat generated within the system.

The photoelectric setting system which consists of an optical and an electric sub-system automatically sets the film stage so that the center of an image - 20 to 450 microns large - is located in the center of the optical system. This is done as follows: the optical system automatically scans an area, measures the displacement of the image from the optical axes, and generates an electric signal proportional to the size and displacement of the image. The electronic system converts this signal into a display on the scope and another signal to drive the servomotors which center the image.

In the semi-automatic mode, the stage movement is controlled by a joy stick. The selected image is brought into any of the three scan areas - concentric circles 500, 250 and 125 microns in diameter - the operator presses a button on the console to automatically lock on the image center and activate the digitizing device to read out the coordinates and any related special purpose data indicated by the operator. Coordinated axis curves, displayed on an oscilloscope, permit the operator to verify that the image center has been recorded. Unless the equipment is properly centered, the curves will not coincide.

In the manual mode, the image is centered by manipulating x and y axis hand dials to set the image center on a projected reticle. The oscilloscope display is then used to accomplish the final image centering.

The viewing screen is approximately 25 × 25 inches in size. A clear line reticule, adjusted to coincide with the photoelectric center position, is projected on the viewing screen. The image on the film plane is projected on the screen through a system of mirrors reflecting the green portion of the illumination.

A climate controlling system is linked to the instrument housing to provide a steady flow of dust and lint free air of constant temperature and humidity for the measuring engine area, the photoelectric system, and the plate storage area. This also furnishes an "air wash" which insures against any foreign particles settling on the plate being measured. A variety of safety devices, built into the equipment, guard it against damage from extreme external heat or humidity.

The Stellar Comparator described above has been in operation at ACIC for more than four years. The experiences with it exceeded by far the expectations. The automatic centering device measures coordinates with an accuracy of one micron and cuts reading time from the five to six hours required with a manual system to less than one hour, and fatigue is no longer a problem. Even when operated in the manual mode, the Stellar Comparator is free of "backlash" to the extent that it is no longer necessary to always approach the image from the same direction. The control and viewing system provide convenient and easy operation. And the internal climate control permits a flexibility in machine location not possible before and makes the machine more easily accessible.

The measured x and y plate coordinates then become major input for the Plate Reduction Program, whose features are described below:

Camera calibration.- Periodically each camera is calibrated to determine distortion coefficients and focal length. For this purpose, the measured coordinates of about 200 star images from a plate exposed in the camera to be calibrated are used to determine the coefficients of radial lens distortion, decentering distortion, and the focal length of the camera.

Computing the Astronomical Refraction According to Garfinkel.- It has been known for a long time that the elements of orientation resulting from a stellar calibration can readily compensate for moderate errors in the refraction corrections applied to the stellar directions. In view of these compensatory capabilities, the computed refraction coefficients are adequate for zenith distances as great as 70°.

During this phase, the measurements are also freed from the effects of radial and decentering distortion.

Automatic star identification (ID). - Essentially the star ID program accomplishes the following:

(1) Calculates preliminary values of azimuth (AZ), altitude (EL), and roll, based on four index stars identified manually before plate measurement.

(2) Computes from these the direction cosines of the camera axis.

(3) Converts the measured plate coordinates of the remaining reference stars to the right ascensions (RA) and declinations (DEC), using the direction cosines for the camera axis calculated in (2)

(4) Determines the maximum and minimum limits for RA and DEC to define the search area.

(5) Performs the star identification.

(6) Calculates apparent star coordinates for the epoch of observation.

(7) Computes weights for the catalogued reference star positions from information in the star catalogue which is on tape and accessible to the operation.

(8) When the catalogue based apparent coordinates of the reference stars and those computed in (3) from the measurements exceed a pre-determined limit, the star concerned is rejected.

Data preparation. - A routine which prepares data for the orientation adjustment to follow. Previously determined Garfinkel refraction terms are used to convert the apparent star positions to the observed positions.

Preliminary orientation adjustment. - In this phase, very good approximations for AZ, EL, roll and principal point coordinates, and focal length are computed. All of the corrected star coordinates which were not lost in star ID (8) are used in the adjustment. Editing of the measurement data (X and Y coordinates) is accomplished by eliminating the X and Y coordinates having the greatest residual errors at the end of each iteration.

General analytical plate reduction. - Final orientation parameters are next determined, together iwth corrections to the refraction coefficients and to the catalogued positions of the stars. In this adjustment, the catalogue values of the RA and DEC are not regarded as known input parameters, as they are in the preliminary orientation. The adjustment rather treats both plate measurements and catalogue positions as observations and thus affected by random errors.

For each of the above described iterations, unweighted errors in each X and Y measurement, their mean value, unweighted errors in each star RA and DEC, and their mean value are computed. Finally, the weighted mean error of measurements and star coordinates are obtained.

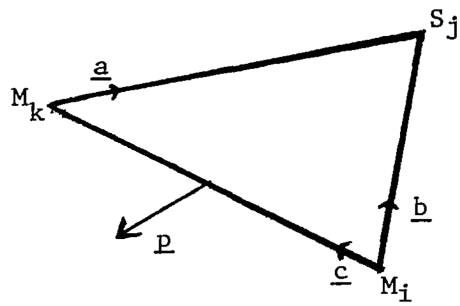
Computation of the azimuth, altitude, right ascension and declination of the satellite. - With the direction cosines of the camera axis obtained by the final adjusted orientation parameters, azimuth and altitude of each satellite image are calculated from the measured plate coordinates of the satellite images and converted to right ascension and declination.

Also performed is the error propagation for azimuth, altitude, right ascension and declination. The celestial coordinates determined for the satellite points are topocentric right ascensions and declinations referred to the epoch reckoned in UT1 of the satellite observation. The correction for parallactic refraction is applied before the final horizon system coordinates are computed.

Phase angle. - Images of a passive satellite must be corrected for phase angle effect, to refer the observation to the satellite's center, and the epoch must be corrected for the light time from satellite to station.

The phase angle program furthermore fits an overlapping interval of observations from two or more stations to polynomials and produces simultaneous data at a specified number of equal intervals of time. Two polynomials are formed for the reduced data from each plate - one for the RA and one for the DEC.

The adjusted information from the Plate Reduction Program is then used in the *Method of Intersecting Planes* to derive station positions.



In the diagram, M_i and M_k are camera stations which have observed the satellite flash S_j simultaneously. The unit vectors a and b represent the observed directions from M_k to S_j and M_i to S_j, respectively. Vector c points in the direction of the chord from station M_i to M_k and is a function of the station coordinates.

The apparent topocentric right ascension (α) and declination (δ) are the observed directions used by ACIC. The observation time (UT1) is converted to apparent Greenwich sidereal time (γ) and all observations are referred to the Greenwich meridian through the angle $G = \alpha - \gamma$, which is the negative Greenwich hour angle of the object.

<u>Basic condition and Equation</u>. - Development of the mathematical formulation for the method of intersecting planes starts with the basic condition that a, b, and c, are coplanar; we have

$$\underline{a} \times \underline{b} \cdot \underline{c} = 0 \qquad (1)$$

The components of the $\underline{c}$ are:

$$\underline{c} = \begin{pmatrix} X_k - X_i \\ Y_k - Y_i \\ Z_k - Z_i \end{pmatrix}$$

The $\underline{a}$ and $\underline{b}$ are transformed from the equator system to which they are originally referred, to the geocentric system that refers to the mean geocentric north pole of 1903.0 . After that, $\underline{a} \times \underline{b} \cdot \underline{c} = 0$ becomes equivalent to

$$A(X_i - X_k) + B(Y_i - Y_k) + C(Z_i - Z_k) = 0. \qquad (2)$$

where:

$$A = tr - un$$

$$B = um - sr$$

$$C = sn - tm$$

with

$$m = \cos\delta_i \cos G_i + R_i$$

$$n = \cos\delta_i \sin G_i + S_i$$

$$r = \sin\delta_i + T_i$$

$$s = \cos\delta_k \cos G_k + R_k$$

$$t = \cos\delta_k \sin G_k + S_k$$

$$u = \sin\delta_k + T_k$$

where i refers to unknown and k to either known or unknown quantities.

Furthermore

$$R = x_p \sin\delta$$

$$S = -y_p \sin\delta$$

$$T = -x_p \cos\delta \cos G + y_p \cos\delta \sin G$$

where x_p and y_p represent the polar motion (in radians).

Observation equations. - For the actual least squares adjustment, the basic equation (2) is expanded into two first order series expressions so that two different observational situations or cases can be utilized.

Case 1: Simultaneous observation of a flash from two unknown stations, i and k .

$$A\Delta X_i + B\Delta Y_i + C\Delta Z_i - A\Delta X_k - B\Delta Y_k - C\Delta Z_k + 1' = 0 \quad (3)$$

where:

$$1' = A(X_i - X_k) + B(Y_i - Y_k) + C(Z_i - Z_k)$$

The approximate latitude, longtitude, and height of the unknown stations are used to compute X_i , Y_i , Z_i and X_k , Y_k , Z_k . Approximate coordinates should be known to the nearest minute of arc or better with respect to the datum of the known stations.

Case 2: Simultaneous observations from one unknown (i) and one known (k) station.

$$A\Delta X_i + B\Delta Y_i + C\Delta Z_i + 1^2 = 0 \quad (4)$$

where:

$$1^2 = A(X_i-X_k) + B(Y_i-Y_k) + C(Z_i-Z_k)$$

The adjustment, as presently programmed at ACIC for the IBM 7094, will handle a maximum of twenty unknowns and ten knowns. However, there is no practical limit to the number of observations which can be included in a solution because the satellite positions are not computed and, therefore, the matrix to be inverted remains at the sixty by sixty size for the maximum of twenty unknown stations.

Results from the ACIC reduction have proven to be geodetically sound. The entire procedure has been used successfully not only with data from the USAF PC-1000 ballistic camera but also with combined data from the PC-1000 and BC-4 camera used by the U.S. Coast and Geodetic Survey.

DISCUSSION

Strand : *Are the images ever so densely distributed that they interfere with the image on which you are centering?*

Seppelin : *Yes, in the sense that sometimes double stars are not sufficiently widely separated and thus are not measurable. The satellite images are, however, more widely spaced and present no problem.*

Strand : *What is the increase in accuracy as compared to that of manual measurement?*

Seppelin : *The r.m.s. error of one measurement is about 1.1μ or less. This represents an improvement of about 30% to 40% as compared to the manual mode in a normal production operation. The automatic comparator is operative 95% of the time, the balance goes mostly to preventative maintenance.*

Brown : *I would recommend the direct computation of the coordinates of the flash points in the reduction above the intersecting plane method. The computing labor in connection with it grows only linearly with the number of flash points, while the reduction by means of the intersecting plane method becomes involved, cumbersome and mistake prone whenever more than two observing stations are involved.*

Rosenfield : *Who made the grid used with the ACIC machine, and who calibrated it?*

Seppelin : *David Mann Co. made it, and it was calibrated by them against a National Bureau of Standards bar.*

Dieckvoss : *You chop the satellite trail into several images in order to guard against the influence of scintillation. Would the influence of the chopping shutter delay produce an apparent shift of the chopped satellite images in the direction of motion?*

Seppelin : *Scintillation affects both azimuth and altitude randomly. The normal exposures used for recording stellar control point images are exposed sufficiently long to (.3^s to 2^s) average out scintillation effects which produce only somewhat enlarged images of the stellar points. On the other hand, flashes or chops of extremely short duration (a few milliseconds), are frozen in their displaced "scintillated" position and hence do not average out in time. This may result in position errors of about 1.5 microns.*

Eichhorn : *Are the measuring machines used at the Army Map Service and the ACIC identical?*

Seppelin : *Yes, except for minor differences.*

Berbert : *How do you achieve the reduction of the measuring time from one plate from about five hours to a half hour? Furthermore, does the machine measure the interruptions in a star trail, or only round images?*

Seppelin : *The machine measures round images, and we select, for the sake of consistency, images whose size is the same as that of the satellite trail. The saving in measuring time is achieved by measuring the plate in one position only, by using only one measurer, and due to the fact that on our machine the travel time from one star to the next is greatly reduced. On one plate we measure between 200 and 300 images. Even considering the preparation of the plate for measuring, the automatic machine mode is altogether 10 times as fast as the manual mode.*

Vasilevskis : *Do you control the temperature in the measuring room?*

Seppelin : *Yes, the measuring engine enclosure is controlled to within about one degree around 70° F.*

Strand : *Do you get any noticeable effect from the cooling of the plate when it is inserted into the machine directly?*

Seppelin : *No, because we store the plates inside the measuring engine enclosure for one hour before starting to measure them.*

Strand : *Have you compared the measurements of the same stars after the plate was rotated in its plane by 180° ?*

Seppelin : *Yes, exhaustively. We found no significant differences.*

Mueller : *Do you express the coordinates of the optical center in terms of altitude and azimuth or in terms of right ascension and declination? If you use the former, you need approximate station coordinates to compute them.*

Seppelin : *We use approximate station coordinates for the computation of the three so-called exterior elements, namely azimuth, altitude and parallactic angle.*

Mueller : *I wonder if the thus necessitated use of inaccurate station coordinates is significantly propagated into the results.*

Veis : *Not as far as I can judge.*

THE REDUCTION OF THE NATIONAL AERONAUTICS AND SPACE ADMINISTRATION PLATES AT THE NEW MEXICO STATE UNIVERSITY

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ABSTRACT

The method of reducing GEOS satellite plates at New Mexico State University is described. The catalogue positions of stars, whose images were measured, are updated to the time of the photographic observation. This process requires the customary corrections for precession, proper motion, nutation, annual aberration and diurnal aberration. The apparent star places are corrected for astronomical refraction. Plate constants are developed from a procedure described by Duane C. Brown. The plate measures are corrected for symmetric radial lens distortion and for the decentering distortion. The projective equations are solved by an iterative procedure for eight parameters defining the interior and exterior orientations.

The plate constants are then used to compute the positions of the seven (or fewer) flashing light images which were measured. These are corrected for refraction, parallax, and radial and decentering lens distortions. With sidereally tracking cameras, the exterior orientation is further corrected to the time of the particular flash, and the change of refraction with time is allowed for. First and second differences and standard deviations and covariances for the flashing light images are developed as a measure of accuracy.

The procedures adopted for the reduction of the GEOS satellite plates follow closely those developed over the last years by D. Brown. We had, previously to the adoption of the present methods, used an "astrometric" reduction, which originally had been developed for the reduction of the plates taken in connection with the minitrack program. When the GEOS program started, we made several tests on these sidereally driven plates, comparing the results obtained by reducing them with "astrometric" and "photogrammetric" procedures. These test plates used were taken over a considerable range in angular elevation, and also in high declinations. The "photogrammetric" reductions produced results of about 20% better accuracy than the "astrometric" ones, and were therefore adopted as standard procedure. The computer presently used is an IBM 360/30.

Our source of reference star positions is the SAO Star Catalog stored on a disc. This proved to be a very effective saver of search time.

The reduction procedure works essentially as follows:

First, the reference stars whose images were measured on a particular plate are identified. The catalogue is searched for their positions. From the catalogued positions, the observed positions for the epoch of the observations must be calculated. This is done in several steps that will be described below. These catalogue derived reference star data are merged with the stars' measured positions (about 40 to 50 on each plate), and from all this, the reduction parameters for the particular plate are calculated by a least squares adjustment with a number of iterations. The final reduction parameters (plate constants) are then used to derive the positions of the satellite at the time of the flashes as final results of this operation.

In detail, then, the stars are identified by placing the plates over BD charts. For the convenience of the measurers, the star images are circled, while those of the satellite images which were selected for measurement are enclosed in triangles. (The location of the satellite images on these sidereally driven plates is often difficult.) The plates are measured manually on a Mann two-screw measuring machine (pitch 1mm., screw length 265 and 250mm., respectively) with a binocular eyepiece. A Telecordex encoder automatically punches the measurements on cards. Five settings are customarily made on both star and satellite images.

The catalogued star positions are "updated" in the following steps:

Proper motion is applied by a rigorous spherical trigonometric procedure. Secular acceleration is neglected as well as the change of proper motion by precession. Every star position is precessed by rigorous trigonometric formulas to the mean coordinate system at the plate epoch.

Then, the true positions at epoch are computed by applying the nutation effects to each star individually by rigorous equations. Some of the parameters in these equations are taken from the ephemeris; others are computed directly.

The apparent positions of the stars are next computed for each star on the basis of Keplerian motion of the Earth around the Sun. To these apparent positions, the effects of diurnal aberration are applied.

The preliminary values of azimuth and altitude of the camera's optical axis, and of the parallactic angle at the principal (tangential) point which are needed for setting up the adjustment, are calculated from considering the astronomical triangle at the mean plate epoch. The model for the refraction corrections required for reduction of the star positions to "observed" positions is based on Garfinkel's theory. The measured coordinates are corrected also for radial and decentering distortion of the lens, which are assumed to be known. Since these are reckoned with respect to the principal point, they are slightly changed after each iteration as successively better estimates of the location of the principal point become available. The adjustment involves a model with eight parameters, namely, the three angles of the exterior camera orientation, the coordinates of the principal (tangential) point, the focal length of the camera and two refraction coefficients.

After the iterations have converged and we have final estimates of the parameters, these are used to derive the positions of the flashes. For those cameras which are driven sidereally, the orientation (with respect to the horizon system, i.e., azimuth and altitude of optical axis, and parallactic angle at principal point) are different for every flash, since they occurred at different epochs. The reduction from the average external camera orientation angles, which are the first three of the above mentioned adjusted parameters, to the values at the epoch of the individual flash was originally calculated by differential formulas. An exact updating procedure for the three external parameters was later adopted since the exact method gave improved accuracies at certain critical geometries. The refraction correction applied to the flashes takes into account the finite distance of the satellite.

Also computed are the standard deviations of the results, and likewise, those of the measured plate coordinates. In our experience, on "good" plates, the standard deviations of the measured coordinates are about three microns. Plates on which deviations are seven or eight microns are considered bad.

A listing of the rectangular coordinates of the flash points, together with their first and second differences serve to spot misidentifications, etc. The final results are the measured plate coordinates, and right ascension and declinations as well as azimuth and altitude for every measured flash point image, together with their epochs.

DISCUSSION

Eichhorn : *I dislike seeing obviously inadequate reduction procedures labeled as "astrometric". No competent astrometrist would apply this so-called astrometric reduction process to the problem at hand. Do you apply the effect of nutation to the entire plate or to each star individually?*

Haas : *To each star individually.*

Eichhorn : *Nutation enters the problem only as a rotation of the coordinate system, and would therefore be worked summarily into the reduction constants. This would save you some computer time. Do you consider the influence of radial velocity when you calculate the effect of proper motion?*

Haas : *No.*

Strand : *Some people involved in satellite plate reduction might read with profit A. König's article on plate reduction in the first volume of the "Handbuch der Astrophysik", where efficient methods for applying many kinds of corrections, such as refraction, are fully described.*

Veis : *I understand that you select the stars to be measured from a B.D. Chart. The Smithsonian Astrophysical Observatory is preparing a chart with all the stars in the SAO Star Catalogue at the scale of the Baker-Nunn photographs.*

Haas : *We intend to use this when it comes out.*

Eichhorn : *Have you tried to find if your effective accuracy is significantly reduced it you make two or three settings on every image instead of five?*

Chavez : *Originally we made five settings on every image and excluded the most discordant setting from the mean. But I think the accuracy would not be significantly decreased by making only three settings per image.*

Eichhorn : *If you changed your procedure to making three readings only you would save a considerable amount of time.*

Vasilevskis : *Do you measure in direct and reverse?*

Haas : *In direct only. We made some experiments with direct and reverse measurements and found no significant differences between them.*

Vasilevskis : *Then your measurers must be exceptional, because such differences occur even with very experienced measurers. I would suggest that you cut the number of settings from five to three, but that you measure in direct and reverse. This should increase your accuracy.*

Brown : *The situation at hand is somewhat different from general astrometric practice, since in measuring satellite plates we may restrict the measurements to images whose diameters are more or less the same. Therefore, there will be no personal magnitude equation and one need not make the measurements in both direct and reverse.*

Mueller : *Even though modifications of measuring practice may result only in a seemingly insignificant time saving as far as the measurement of a single image is concerned, they should be carefully considered. In the application of photographic astrometry to satellite geodesy, dozens to thousands of coordinate pairs of star images are measured, and the savings in measuring time on a single image may add up to significant savings of time - and money - as related to the entire plate.*

COMMENTS ON THE ACCURACY OF BAKER-NUNN OBSERVATIONS

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INTRODUCTION

The Smithsonian Astrophysical Observatory's (SAO) Baker-Nunn cameras have now been in operation for some ten years, and have furnished thousands of precise optical direction observations for a large number of satellites. From these observations, considerable insight has been gained into the nature of the earth's gravitational field, the structure of the atmosphere, and the precise location of the camera stations themselves. The upper limit to the accuracy of the observations has been nominally set to 4 seconds of arc in direction and 2msec in time, but it has been known for some time, on the basis of *a posteriori* evidence, that the real accuracy is nearer 2 seconds of arc than 4 seconds of arc.

Many factors influence the accuracy with which the satellite positions may be derived from Baker-Nunn observations. Some of these factors are more important than others. Some vary randomly from film to film, and others systematically influence the precision of the observations for the duration of an observation sequence, or for a longer time period. But all will contribute in some degree to the total positional uncertainty.

In an accuracy study of satellite tracking methods there are invariably two distinct aspects to be investigated: the accuracy with which the satellite position can be referred to the reference system used - the stellar framework in the case of the optical observations, and the accuracy of the instant of observation. It is generally desirable to keep time as an independent variable so that any uncertainty in measuring the time of observation will be reflected in the positional accuracy.

ACCURACY OF THE SATELLITE POSITION IN THE STELLAR FRAMEWORK

The more obvious phenomena affecting the accuracy of the film reduction procedures are:

a) measuring uncertainties in both the star and the satellite positions;
b) limitations in the comparators used;
c) film distortions;
d) limitations imposed by the atmosphere;
e) deficiencies in the mathematical models employed; and
f) uncertainties in the definitions of the stellar framework itself.

Table I summarizes the precision estimates of the star positions, and Table II gives a similar summary of the factors determining the satellite's positional accuracy.

When the camera is used in the stationary mode, the *a priori* standard deviation estimate of a single satellite position is of the order of 1$''$.8 in the along track direction, and 1$''$.5 in the across track direction, while for observations made with the camera in the tracking mode the positional accuracy is of the order 1$''$.6 in both directions. The estimates are averages for satellite positions distributed uniformly against the sky background and above an altitude of about 15° . They are valid for any of the exposure times used in the normal Baker-Nunn operating routines.

The tracking mode appears to give slightly more precise results than the stationary mode because the atmospherically induced image motion is now of lesser importance, and because the film measurements of the satellite position are of similar accuracy in either mode.

The image motion or shimmer is one of the major factors affecting the positional accuracy, and it cannot be reduced for the stationary camera mode of operation.

Other major contributors affecting the overall accuracy are measuring errors, film distortions and emulsion shifts, and the combined influence of the standard deviations in the star positions.

Measuring errors can theoretically be removed by increasing the number of settings made on the satellite image. In many cases, however, this may be misleading: increasing the number of settings may only improve the consistency of the measurements without increasing the reliability of their average. This would not appear to be the case, however, for GEOS flashes and similarly defined images.

TABLE I. - Summary of *a priori* precision estimates of phenomena contributing to uncertainties in star positions.

Measuring errors	$(\sigma)_{mean} = 3\mu (\equiv 1''.2)$
Calibration of comparator	0.5μ ($\equiv 0''.2$)
Film distortion and emulsion shift	2μ ($\equiv 0''.8$)
Atmospheric refraction	1''.1 (image motion for tracking camera) 0''.8 (differential refraction) 0''.3 (wandering)
Approximations in reduction method	0''.5μ ($\equiv 0''.2$)
Star positions from catalog	0''.5 random 0''.2 systematic
Total standard deviation of each star position	1''.8 (+0''.2) stationary 2''.1 (+0''.2) tracking

TABLE II. - Summary of *a priori* precision estimates of phenomena contributing to uncertainties in satellite positions. (Excluding Measuring errors, calibration of comparator, film distortion and emulsion shift, as shown in Table I).

Atmospheric refraction	1''.1 (image motion along track, or flash images) 0''.5 (image motion across track) 0''.3 (wandering) 0''.1 (parallactic refraction)
Contribution of standard deviation of n stars	$\left\| \frac{(1.8)^2}{n-3} + (0.2)^2 \right\|^{1/2} = 0''.8$ stationary
Average of 8 stars	$\left\| \frac{(2.1)^2}{n-3} + (0.2)^2 \right\|^{1/2} = 0''.9$ tracking
Total standard deviation of satellite position	1''.8 stationary - along track 1''.5 stationary - across track 1''.6 tracking

Now, if the number of measurements were increased from the present six settings to nine settings in X and Y in both the direct and reverse directions, which requires little additional measuring time, the standard deviation of the mean position would be reduced to

$$[\sigma_{mean}]_{sat} = \frac{0.0055}{\sqrt{2}\ \sqrt{8}}\ \mu = 0''.6$$

from the present $0''.8$.

The effect of emulsion shift will require considerable study to determine its magnitude and whether it can be reduced by changing the processing techniques or by improving the interface condition between the emulsion and the film base.

The most readily available means of reducing the effect of star position uncertainties on the satellite position is to increase the number of reference stars used in the reduction, but without increasing the average distance between the star positions and the satellite. An increase from 8 to 12 stars will result in an improvement in the combined effect of the star positions on the satellite position to $0''.6$ for the stationary mode.

With these two modifications the total *a priori* standard deviation of the satellite position becomes:

$1''.6$ stationary - along track
$1''.3$ stationary - across track
$1''.3$ tracking

MEASUREMENT OF THE TIME AT THE INSTANT OF EXPOSURE

The measurement of the time of an event on the film involves two steps: relating the event on the film to the station clock, and relating the station clock to the adopted time scale. Before 1965-1966, when the Norrman crystal clocks were in operation at the Baker-Nunn stations, the second step was probably the source of greatest contention as it relied on VHF radio signals for both the time base and the clock rate at the stations. Uncertainties of several milliseconds could be expected, particularly at the more distant stations.

With the introduction of the Eeco clocks and the use of portable clocks, the accuracy of relating the station clock to the time standard has been considerably improved. Monthly timing-accuracy records kept by Station Operation Engineering indicate that during 1967 the maximum possible errors in relating the station clock with the National Bureau of Standards UA time scale, and later with the National Observatory UTC time scale, only very rarely exceeded 0.5msec, and more generally seldom exceeded 0.3msec. Now the problem of maintaining high-accuracy timing standards revolves around the relation of the time of an event on the film to the station clock. This is only the case for the Baker-Nunn stations. The K-50 stations still rely on VHF for time base and clock rate.

The essentially different nature of the error sources inherent in the two steps should be noted. Any uncertainties in relating the station clock to the time scale will remain constant for at least one night's observing, and possibly even longer. The greatest uncertainty in this step is the determination of the the propogation time of the radio signals, which is based on an "average" propagation velocity. The error sources in the first step relating the event on the film to the station clock will be largely random from event to event.

Table III presents a summary of the accuracy of measuring the instant of an event on the film for different cycle rates.

TABLE III. - Summary of the accuracy of measuring the instant of an event on the film for different cycle rates.

Cycle rate	Accuracy of relating event on film to station clock	Accuracy of relating station clock to absolute time		Total time accuracy of event	
		w/VHF	w/port. clock	w/VHF	w/port. clock
	Seconds	Seconds	Seconds	Seconds	Seconds
32 sec	0.5×10^{-3}	1.5×10^{-3}	0.3×10^{-3}	1.6×10^{-3}	0.6×10^{-3}
16	0.25×10^{-3}	1.5×10^{-3}	0.3×10^{-3}	1.5×10	0.4×10^{-3}
8	0.17×10^{-3}	1.5×10^{-3}	0.3×10^{-3}	1.5×10	0.4×10^{-3}
4	0.13×10^{-3}	1.5×10^{-3}	0.3×10^{-3}	1.5×10	0.3×10^{-3}
2	0.11×10^{-3}	1.5×10^{-3}	0.3×10^{-3}	1.5×10	0.3×10^{-3}

The uncertainty involved in relating the time at the station to the time scale will always dominate the other uncertainties when this relation is established from VHF radio signals, whereas the reverse is true when portable clocks are used to make this comparison. The exception is for the very short cycle rates where this comparison again becomes the dominant contributor to the overall timing uncertainty.

If the timing accuracy is incorporated into the positional accuracy, the overall accuracy of a Baker-Nunn observation as a function of the camera cycle rate - or the satellite's topocentric velocity - can be established, as is shown in Table IV .

TABLE IV. - Overall accuracy of Baker-Nunn observations as a function of camera cycle rate.				
Cycle rate	(Angular velocity) of object	Stationary mode		Tracking mode
		VHF	Port. clock	
32 sec	0-250'/sec	1".8	1".8	1".6
16	250-500	2".1	1".8	1".6
8	500-1000	2".3	1".9	1".6
4	1000-2000	2".7	1".9	1".6
2	2000	3".7	2".0	1".6

STATISTICAL TESTING OF BAKER-NUNN OBSERVATIONS

There exist several possibilities for estimating the accuracy of Baker-Nunn observations from *a posteriori* evidence. Examples are the examination of orbit observation residuals or of simultaneous observations, and a careful interpretation of these results will give a useful insight into the nature of the observational accuracy and lead to either a rejection or a verification of the theoretical - *a priori* - estimates.

The reduction procedure provides the first means of estimating the accuracy of the observations, as it should provide a covariance matrix of the satellite position that reflects a number of the phenomena discussed earlier. In the case of Baker-Nunn reduction procedures, these phenomena include the indirect factors that result in discrepancies between the observed and the computed star coordinates - with the exception of star catalog errors - and the measuring accuracy of the satellite position. The other direct factors affecting the satellite's positional accuracy are not considered, and neither are timing errors.

The summary of the error sources listed in Table II enables the precision (σ_I) corresponding only to those factors considered explicitly or implicitly in the reduction to be estimated; that is,

σ_I = 1".13 camera in stationary mode,
σ_I = 1".21 camera in tracking mode.

A sample of about 250 reductions of Baker-Nunn films, taken in both operating modes, gives an average value of 1".15 for the positional standard deviation and is in good agreement with the theoretical values. The comparison indicates that the theoretical results are overestimated by an amount of about 0".2 or 0".3, but this is hardly significant.

In general, a satellite is tracked along a small part of its orbit, resulting in a number of consecutive frames in which the satellite image appears. A usual procedure is to fit a polynomial through the sequence of consecutive satellite images and to interpolate at a desired instant for a fictitious position that will have a higher precision than a single observation. This computational procedure provides a further estimate of the accuracy of a single observations.

The polynomial curve-fitting procedure currently used at SAO is based on the following assumptions:

1) The along-track and across-track accuracy components of each satellite position are equal, and there is no correlation between them; and

2) All observations in the sequence are of equal accuracy, and no correlation exists between the various components in the sequence.

Neither of these conditions is satisfied for individual observations, because the same reference stars may be used in two or three consecutive frames, and timing errors may behace in a systematic manner during the sequence. A check on the above assumptions is possible if the covariance matrix of the adjusted individual satellite positions is computed from the polynomial fit, but the current program does not provide for this computation. The covariance matrix of the synthetic position, however, is computed.

Included in these estimates now are all those factors that affect the satellite position randomly from frame to frame: those included in the photoreduction stage, and those affecting the satellite positions directly on each frame - emulsion creep, random refraction, random timing errors, and other minor factors. From the results in Tables I and II the theoretically expected precision (σ_{II}), for a satellite of the Midas 4 type, with the camera operated in the stationary mode, is

$$\sigma_{II} = 1''.8 \text{ along track,}$$
$$\sigma_{II} = 1''.5 \text{ across track.}$$

The standard deviations in the along-track and across-track directions for about 400 synthetic observations, derived from sequences of seven Midas 4 frames with the camera operating in the stationary mode, yielded average values of 0''.85 along track and 0''.68 across track, corresponding to about 1''.70 and 1''.36 for the along- and across-track components of a single observation. These estimates are somewhat smaller than the theoretically expected results, and indicate that one phenomenon or a combination of the phenomena listed in Table II is overestimated by

$$[\ (1''.8)^2 - (1''.7)^2\ ]^{1/2} = 0''.6 \text{ along track,}$$
$$[\ (1''.5)^2 - (1''.4)^2\ ]^{1/2} = 0''.5 \text{ across track.}$$

It is not possible to determine which phenomena have been overestimated. That the overestimation is only marginally due to the measuring process is indicated by the good comparison between the theoretical and the practical results obtained from the photoreduction step. More likely it is caused by overestimation of the emulsion creep and/or atmostpheric refraction. In any case, because of the approximations made in the curve-fitting process, this difference is hardly significant.

The remaining factors affecting the accuracy of the observations that could have avoided detection are those that are systematic over the entire sequence of observations; systematic timing errors resulting from the use of VHF signals to maintain the station clock are the most obvious. A check on these errors is, however, provided from an analysis of simultaneous observations of the same object from two or more stations.

The mathematical model presently used by SAO for adjusting the simultaneous observations and computing the directions between the stations from which the observations are made is based on a number of approximations, and the resultant accuracy estimates are not entirely realistic. But the standrad deviations of unit weight obtained for the solution of each station-station vector will nevertheless indicate the accuracy of a single synthetic observation. These estimates indicate a directional accuracy of the order of $1''.1$ for a synthetic observation, and on the assumption that the average sequence length is seven frames, the new estimate of the standard deviation of a single observation is $2''.2$. This is greater than the earlier estimates, and it means that systematic errors totaling about $[\ (2''.2)^2 - (1''.8)^2\ ]^{1/2} = 1''.3$ must be postulated.

Midas 4 has been used for the majority of these observations, and if the timing errors are assumed to be constant over an arc, but variable from arc to arc, the magnitude of the timing error must be about 2.5msec at each of the two stations from which the simultanrous observations are made. It is more likely that the timing error is smaller but systematic over a longer time interval.

Some attempt has been made to isolate these discrepancies by dividing the solutions for the various station-station vectors into subgroups. One such grouping was made according to the satellite observed and another according to chronological order. However, neither case presented any evidence suggesting the true nature of the timing discrepancies, essentially because of poor time distribution of the observations.

The above analysis applies only to observations collected before the use of the portable clock to provide the time base. At present, there are insufficient date available for a similar analysis from observations taken since the improvement in time keeping. But more direct measurement of the uncertainties in determining time by means of VHF reception is now possible by comparison of such signals received at the station with the time carried by the portable clock.

Acknowledgement

This work was supported in part by grant NsG 87-60 from the National Aeronautics and Space Administration.

DISCUSSION

Eichhorn : *What exactly do you consider a "frame" ?*

Lambeck : *In essence, one satellite image.*

Eichhorn : *How many reference stars do you use for the determination of its position?*

Lambeck : *Between eight and ten.*

Eichhorn : *What is the cost of making the measurements to get a position with a mean error of one and a half second of arc ?*

Lambeck : *About thirteen to fifteen dollars.*

Rosenfield : *At the Eastman Kodak Company extensive work has been done with regard to the stability of emulsions and of emulsion carriers. This information is available from Kodak. Also, I should like to ask which projection is used for the sky to the curved focal surface of the Baker-Nunn film ?*

Lambeck : *Azimuthal equidistant projection.*

Veis : *The reason is that although this is not a rigorous mathematical model for the projection, it represents very well the distortions introduced by the elastic deformation during exposure.*

Rosenfield : *Under the circumstances, how do you find the tangential point ?*

Lambeck : *Since we use only an area of about 2 degrees diameter in every frame, the choice of the tangential point has no noticeable influence on the final result.*

Luyten : *If it costs fourteen dollars to reach an accuracy of a little better than two seconds of arc, I can't quite understand why it is necessary to go to all the extremely complicated procedures and all this computerization. It seems to me that at my institution we could do the same thing for about half the cost.*

Veis : *But then we would have to pay for a lot of peripheral items. It should also be kept in mind that we need to produce a great many positions.*

Mueller : *How many frames were reduced so far at SAO ?*

Lambeck : *Nearly one quarter of a million.*

Mueller : *And at ACIC ?*

Seppelin : *About fifteen hundred.*

Mueller : *How many plates were reduced for NASA ?*

Berbert : *About three thousand, at a cost of roughly forty-five to fifty dollars per plate.*

Lacroute : *At Strasbourg we proceed in the same way as at the SAO.*

OBSERVATIONS AND REDUCTION TECHNIQUES OF THE U.S. COAST AND GEODETIC SURVEY

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ABSTRACT

The plate reduction method and other aspects of the world geodetic net are described. About one hundred stars from the SAOC are used as reference stars on the PAGEOS satellite plates obtained on 300mm f.1. and later 450mm f.1. Wild BC-4 cameras. The measurements are made manually now on diapositives since these can, according to experience, be measured with higher accuracy (about 1.6μ r.m.s.), and are adjusted following a rigorous projective geometric model. The satellite images are analytically represented by a polynomial curve, of which seven fictitious points (positional accuracy about 0ʺ.3 within the system) are regarded as the result from this plate. Some other aspects of the program are also discussed.

We are at this time engaged in a very extensive process of documentation, which will describe our method of data processing in every detail. We consider such a documentation necessary simply because in the operation of a worldwide geodetic program one arrives at a point where the amount of data is so overwhelming that one can no longer afford to surrender to the temptation of further refining the method. One is forced to freeze the operational procedure in order to deliver the end results on schedule.

At the moment we are still in a position to recompute all our present data, amounting to the reduction of about 1000 plates. We enter this final phase of data processing with an entirely new generation of data reduction programs and with the feeling of having a system which will, for all practical purposes, produce an optimum end result.

Our star-updating procedure, aside from some rearrangements in the computational steps, is identical to that used at the U.S. Naval Observatory (Scott and Hughes, 1964). After updating the catalog information by considering the effects of proper motion, radial velocity, precession, nutation, diurnal aberration and parallax, we introduce local sidereal time, thus referring our system to a local station meridian. We would definitely consider it inadequate to apply to the total field of view of 20° a single refraction correction only, and therefore we apply individual corrections to each ray (i.e., image). In some places in the world, islands in particular, we have found that the available coordinates for the stations are rather inadequate, sometimes as much as a mile in error. When such discrepancies are discovered in the preliminary triangulation, we use these results as preliminary approximations in order not to impair the refraction correction which obviously assumes a normal stratification of the atmosphere.

For star reference information we use the SAO Catalog. From this catalog we have selected those stars which we feel are suitable for our purpose. The selection was made on the basis of magnitude and listed mean error of the proper motion. Because we need one hundred individual stars on each photograph, we found it necessary to incorporate stars to the eighth magnitude whose positions at the epoch of the catalog had an accuracy of at least 0".4 . Thus we have about 20 000 stars available. Somewhat handicapped by the lack of an entirely even distribution, we can expect a minimum number of 45 stars on any one individual plate, while the maximum is about 425 stars. The average is somewhat higher than 150 stars per photograph.

Our data processing procedure consists of four distinct steps. First, we have programs concerned with the reduction of the comparator measurements. Second, we have programs that deal with the reconstruction of the photogrammetric bundle, that is to say, to provide interpolation parameters necessary for reconstructing the direction toward the satellite relative to the surrounding star background. The third step, which deals simultaneously with all photographs belonging to an individual event, determines these directions incorporating a curve fit through the multiple imagery of the chopped satellite trail. The fourth and final step in the data reduction procedure deals with the spatial triangulation of the positions of the observation stations.

Associated variance - covariance matrices are carried through all the individual steps in such a way as to guarantee a result which is rigorous in the sense of a least squares adjustment.

Analysis of the precision of the plate measurement from the reduction of about 1000 plates gives the following quantitative data. On the average, 103 different stars are measured on each plate and each star is photographed seven times. Consequently there are, on the average, about 700 star images measured on each plate that are obtained from the star photography taken before, during and after the recording of (on the average) 308 satellite images. Therefore, typically more than 1000 images are measured on each plate in direct and reverse position. After a least squares fit of the direct and the reverse measurements to each other (which allows for two translations, one rotation and a scale correction), the precision of the measuring procedure can be judged from the dispersion of the difference between the double measurements. The mean error of the average of a double setting as obtained from the differences between about a million pairs of double measurements is ±1.6 microns.

In the earlier stages of our work we measured on negatives. Then we were faced with the problem that a black point-shaped measuring mark had to be set into the not quite black star or satellite image. This procedure caused systematic bias errors, differing from operator to operator. By turning the plate through 180° in its own planes, i.e., measuring indirect and reverse, such errors cancel. However, they were uncomfortably large. In order to reduce these systematic setting errors, we now produce from each original negative a diapositive by copying the original plates under vacuum pressure. The effect is that we can now set the black measuring mark into a white star or satellite point image. This reduces the systematic setting error of the operators. We tested this method on a considerable number of plates, assuring ourselves that no additional bias errors had been introduced. The overall mean error of the averaged, measured image coordinates fell from ±1.8 to the present ±1.6 microns, after we changed from measuring negatives to diapositives. Statistical analysis of all our data showed, furthermore, a slight improvement after the 300mm were replaced by the new 450mm focal length lenses. This may be explained by the better imaging quality of the new lens, especially in the peripheral portion of the now narrower field.

A first measure of the accuracy is obtained when the measured plate coordinates are matched with the star catalog data. The mean error of the image coordinate obtained from the individual plate reductions is between ±1.6 and ±2.5 microns. The average is ±2.3 microns. This increase from ±1.6 to ±2.3 microns is caused by additional error sources such as random emulsion shifts and scintillation effects, which are due to the turbulence of the atmosphere. Emulsion shifts are of the order of one micron while scintillation differs from station to station depending on atmospheric conditions. In order to determined the magnitude of scintillation, a preliminary curve fit through the satellite images is executed providing a noise trace which is used to establish the appropriate weights in the individual single camera reduction process. In this way we make sure that the weights are assigned to the star positions in the SAO Catalog.

A statistical analysis of the results of the single camera orientation reductions occasionally showed a mean error of unit weight that was larger than explainable by the magnitude of the aforementioned noise sources and thus indicated some kind of systematic error in our data acquisition procedure. This effect disappeared when two independent orientation matrices were introduced, the first one referring to the star imagery before the event, and the second one referring to the star imagery taken after the event. Thus it became evident that, occasionally, the camera was not stably mounted. For this reason, we changed the plate taking procedure in the middle of 1966. We are now obtaining additional star images during the satellite mission and through the rotating disk shutters. The plates record, on the average, the images of 20 different stars during the time the satellite mission is photographed. These are typically FK4 stars, the most desirable ones (because of the accuracy of their catalogued positions) for determining the orientation of the camera at the event. Consequently we revised our program of reduction so that we now have three independent rotation matrices referring to the orientation: before, during and after the event respectively. The remaining parameters describing the geometry of the bundle of rays can be assumed as stable during the 20 minutes, which is the typical duration of an event. Thus the information from all stars is used to determine the stable parameters while the images of the stars taken before, during and after the event are used to determine the individual camera orientation parameters valid during the corresponding time intervals.

While we prepared our data for re-reduction, we studied the problem of the use of Universal Time in detail and discovered that UT1, especially during the period from 1956 to 1968, was not strictly a measure of earth rotation. During this period the Bureau International de l'Heure entertained the concept of a moving pole, in other words, periodically changed the position of the epoch pole. In addition, during this period the zero meridian of the UT1 system was defined as a meridian originating from the epoch pole and passing through Greenwich. As a consequence, each time the pole was shifted to a new position, this meridian would intersect the conventional equator at a different point. Therefore the orientation of the earth, relative to the same inertial frame, was different at 0^hUT for each of the pole positions chosen for the various epochs.

At the Lucerne meeting of the International Association of Geodesy and Geophysics in 1967, a recommendation was adopted to use the Conventional International Origin, that is, the pole of 1902-03 as the origin for both polar motion and the reckoning of time. Because of the aforementioned situation, a discontinuity of 18 milliseconds arose on January 1, 1968, either as a jump in time or as a displacement of the conventional geodetic zero meridian. Whatever their reason, these facts and their consequences must be taken into account when precision geometric satellite triangulations are carried out.

Another subtle point came to our attention during the preparations for the data re-reduction, namely an effect in the nature of an additional aberration. This is not typical for astronomical work, but occurs here because of our choice of an earth-fixed coordinate frame for our final triangulation reductions. Consider the photography of one flash from two stations. According to conventional aberration theory, the observed direction is space parallel to the required actual direction, at least sufficiently close to the order of uniform linear motion. This statement, however, is only correct if this direction is referred to a nearly inertial system. When these observations are referred to an earth-fixed system, the direction is affected by the rotation of the earth during the light travel time interval. Because the distances between the flash and the various observing stations are different, the appropriate corrections will be different for the different stations, and therefore, will cause a slight systematic error in the spatial triangulation when neglected. In the case of a passive satellite, the problem is somewhat more complex but readily handled because the curve fitted to the measured satellite images will then represent a known function of time.

I am afraid I will make a bad public impression by stating we are spending about $1000 per plate. The accuracy which we obtain for a single reduced direction corresponds to a mean error of about 0.″4 in each coordinate. However, it is evident that a reduction method producing a direction with a mean error of 2″, which is five times our figure would have to be carried out twenty-five times to produce the same accuracy as our final results, disregarding the consequences of the use or abuse of the square-root error law. When we thus compare the cost of our work with that described by another speaker, who stated $20 for an individual reduction, yielding an accuracy of two seconds, we are still by a factor of two on the expensive side with our reductions. On the other hand, the cost of the data reduction effort constitutes only about 25% to 30% of the total program expenditures which amount to about $12 000 000 or, in addition to the cost of the PAGEOS satellite launch, estimated to be $8 000 000. Thus the Geodetic World Net Program constitutes a budgetary expense of about $20 000 000, and therefore I think it is quite justifiable to obtain the best results possible from the raw data. The value of the final result cannot, in such programs, be readily expressed by cost-benefit considerations. It could be successfully argued that our reduction operation incorporates redundant information beyond the point of return in terms of accuracy improvements of the final result. Our approach, however, contributes to a high level of reliability of the solution which is based on the concept that information, even if it is basically not of optimum value for geometric or physical reasons, must nevertheless fit into the frame of a least squares adjustment without unduly increasing the mean error of unit weight after adjustment.

At this time, we have adjusted about one third of the world net by our preliminary methods. The final result, which corresponds to some 3000 plates will provide a three-dimensional triangulation system approaching an accuracy of one part in one million (Schmid, 1969).

In closing, I want to express my appreciation for the quality of the astronomical system of star positions which, at least in the northern hemisphere where we have used it in our triangulation, constitutes an exceptionally consistent reference frame.

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DISCUSSION

Brown : *The error calculated by means of Gaussian error propagation of a position from one of your plates comes out to 0".3 to 0".4 . The actually observed closure error, however, usually comes out twice that large. This seems to indicate that unaccounted for systematic errors enter. Could this have anything to do with your calibrating all your comparators against the same grid plate?*

Schmid : *Probably not. We determine the x and y scale values independently from the star positions and do not assume that the comparator axes are perpendicular to each other. Our measurements should be influenced only by the periodic errors of the screws, since we cannot detect any systematic differences between direct and the reverse measurements.*

Brown : *So this still leaves the question open why you don't get closures of 0".4 ?*

Schmid : *This number applies to the world net, and I believe it is really wishful thinking to expect the closure to come to 0".3 or 0".4 after all the elaborate processing. It may be that the systematic errors of the system of star positions which we use, has something to do with this, good as it is. I should also like to point out that we shall have made, after completion of the world net, a total of about 240 000 star image measurements on altogether 13 000 stars, so that the position of every star will have been used about twenty times. Could it be that the residuals from these measurements are valuable to fundamental astronomy for the improvement of the fundamental system ? It would also be interesting to eventually compare our results with those found from a dynamical Doppler solution. Whatever comparisons of this kind have been made up to this time show excellent agreement between the results from the different groups of data.*

Eichhorn : *The care and thoughtfulness incorporated into these reductions makes them one of the most impressive astrometric procedures in existence. If you had applied the same care to reducing photographs from a longer focal length camera, (and focal length is regarded by astrometrists as the principal single determinator of relative accuracy) might you not have improved the accuracy, with no extra expenditure of work, to, say, 0".1 ? Has this maybe something to do with the systematic errors that Mr. Brown alluded to earlier? I can imagine that the 240 000 adjustment residuals of the 13 000 star positions which you have would be even more valuable for the determination of individually improved star positions as well as the systematic errors of the underlying fundamental system if they had that higher accuracy.*

Schmid : *Our working with the presently used focal length has been largely necessitated by the emergence of this project from what was originally missile tracking. Since time was important, the 300mm objective was designed to fit the then existing missile tracking requirements. When geodetic considerations became important, it was necessary to adapt the focal length to other already well functioning components, that would have had to be completely redesigned, for instance, the shutter mechanism. Considerations of this type also restricted our possibilities of enlarging the aperture. We also had to have an instrument that would work in the climatic conditions of Greenland as well as of those at the equator. Finally we felt that the limited quantity of available positions of reference stars made it necessary to have a wide field so that more stars per frame could be available for the establishment of a systematically more accurate reference frame, in so far as an overall systematic error of the fundamental system is considered. If we had more and better reference star positions, we might well have utilized with profit cameras of larger focal length.*

Eichhorn : *The work to get these star positions is already under way, partly as the AGK3 and partly as an effort in the Southern Hemisphere. Do you feel that after these positions have become available, it would be meaningful to repeat your present effort with a one- or two meter focal length camera ?*

Schmid : *Probably not for a world wide effort, but perhaps for special investigations involving smaller areas, such as studies in seismology and involving the investigation of continental drift. However, it is rather to be expected that radically new and different systems will be developed for an attack on these problems, especially when they involve continental drift at an assumed rate of about two centimeters per year.*

Vasilevskis : *Scintillation is also a factor which limits the obtaining of higher accuracy through increased focal length. How do you handle this problem ?*

Schmid : *We expose about four minutes and during this period produce two satellite images per second, thus we integrate the effect of scintillation over about 500 images.*

Vasilevskis : *Astrometric experience indicates that integration over a minimum period of two minutes is necessary to eliminate harmful effects of scintillation on the positions.*

Schmid : *Conditions vary from station to station and must be particularly carefully considered in the field. At Troms, Norway, for instance, the seeing is particularly bad. An attempt to explain this may be to realize that the presence of the Gulf Stream, and typically cold continental high pressure weather systems may cause unusual atmospheric turbulence. This is why we need to integrate over as long a time span as is feasible.*

Murray : *Why don't you use the apparent places at the mean 1950.0 system instead of the apparent places in the true system at the epoch ?*

Veis : *At some stage, the apparent positions in the true system of epoch must be used because after all, the directions must be related to a geocentric terrestrial system. At the SAO, the bulk of the reductions is carried out in the mean 1950 system and the reduction to the terrestrial system is done at a later step which involves, of course, polar motion and a rotation through UT1. The only reason for using the apparent positions in the true system at the epoch, would be that these, and not the mean 1950 positions, are strictly speaking, the arguments for the computation of refraction.*

Murray : *But the refractions could also be properly computed from the 1950 mean positions.*

Veis : *This is true since the difference will be small, but the method by which the reductions are carried out at various places can be explained better by the history of the project. Another point: Polar motion and the UT1 are not independent of each other since they should use the same mean pole, and it would be inconsistent to pair the polar motion as determined at one place with the UT1 determined at another. I understand you use UT1 as defined by the Bureau de l'Heure. However, at the SAO we have used the Naval Observatory's UT1 for the Standard Earth solution because during this period the mean pole of the earth was in the direction of the longtitude of the Naval Observatory with respect to the 1900.0 - 1905.0 mean pole. This can lead to a discrepancy of a hundredth of a second to a millisecond.*

Schmid : *The only important thing is that consistent data are used.*

Herget : *Do you consider the fact that the satellite is at a finite distance when you calculated the refraction ?*

Schmid : *Yes, we do. We call this "parallactic refraction" and it is a purely geometric effect and its proper consideration requires knowledge of the distance to the satellite.*

AN F1 SCHMIDT SATELLITE CAMERA AND THE METHODS OF PLATE MEASUREMENT AND REDUCTION

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ABSTRACT

The f1 Hewitt Camera is a field flattened Schmidt system of 60cm aperture. The salient features of this equipment are briefly described. Details of the methods of plate measurement are then given. The plate reduction is carried out in two stages. The plate is first calibrated using the photogrammetric method. The formulae usually quoted have been extended to take account of the large distortion introduced by the field flattening lens. In the second stage, the satellite measurements are reduced to satellite positions corrected for refraction, aberration, and when necessary, phase.

Precise measurements of satellite positions are made in the United Kingdom using two flat field f1 Schmidt Cameras (Hewitt, 1965) which were designed and built at the Royal Radar Establishment at Malvern, Worcester. The cameras (as shown in Figure 1) cannot follow the satellite and thus are used in the fixed mode recording the satellites as a trail against the star background. The optical system has an aperture of 610mm and consists of a spherical mirror and aspheric plate of the usual Schmidt type with the addition of a three element field flattening lens mounted approximately one centimeter in front of the photographic plate. The mirror diameter is 800mm and the field of 10° diameter is entirely free of vignetting. The optical system was designed to give an image diameter of 30μ at the center of the field with 90% of the light between wavelengths 480mμ and 650mμ falling within this image.

The camera has two shutters. One is a five bladed iris shutter, mounted immediately in front of the aspheric plate of the Schmidt system and carried on its own turntable, concentric with and independent of that of the optical system. This shutter opens (or closes) in approximately 100 milliseconds and is used for

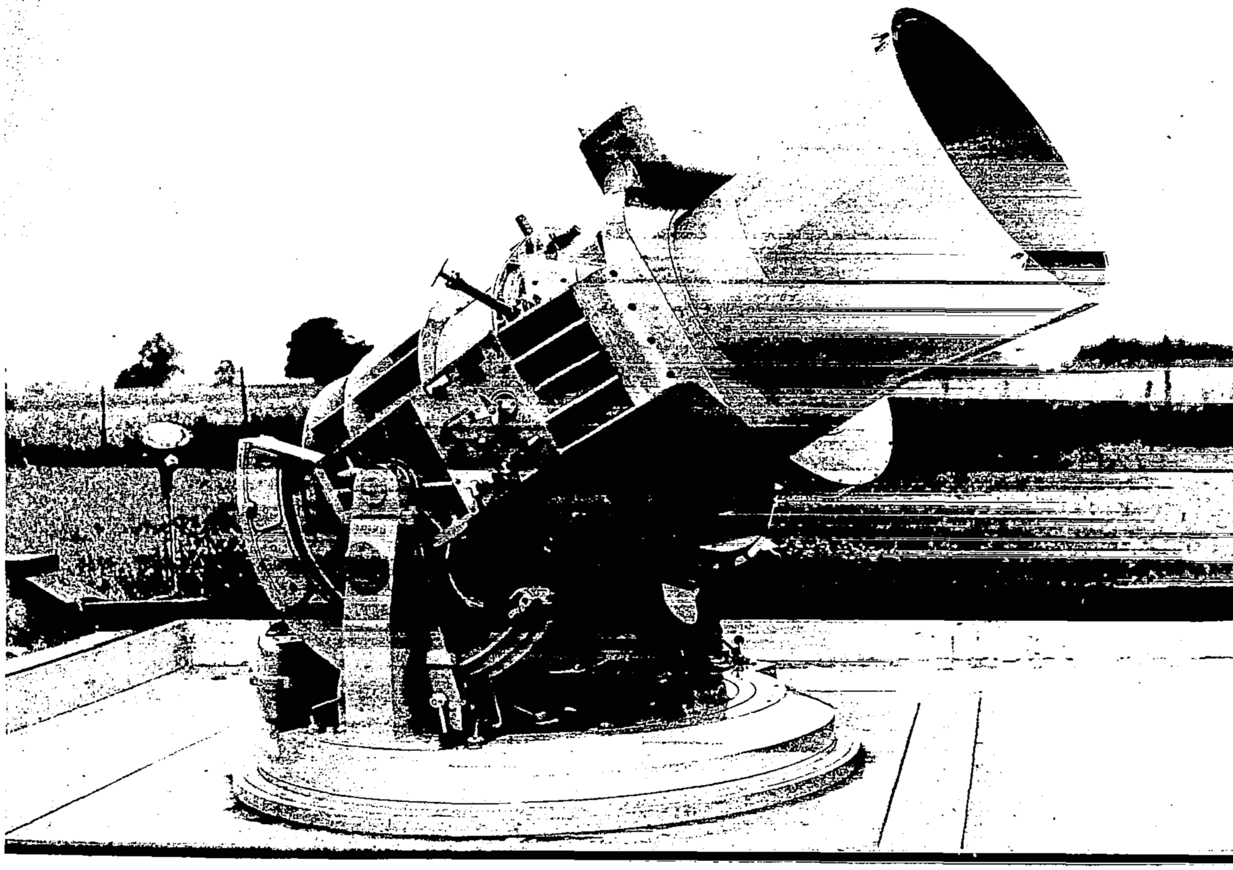

FIGURE 1

An f1 Field Flattened Schmidt System for Precision Measurements of Satellite Positions.

star exposures and for coding the breaks in the trail of the satellite on the photographic plate. The duration of the star exposure can be set to 0.3 , 0.6 , or 1.2 seconds depending upon the speed of the photographic emulsion being used, but all the star images on one plate have the same exposure. The time at which a star exposure is made is obtained by recording both the time when the shutter is near the middle of the opening cycle and the time when it is in the corresponding position in the closing cycle, so that the mean of these gives the time of the exposure of the star image.

The second shutter is a rotating sector mounted so that the blade of the shutter crosses the field of the camera in the gap between the photographic plate and the field flattening lens. When recording passive satellites, the shutter occults the image of the satellite in every rotation, producing a series of breaks in the satellite trail. The speed of the shutter can be varied in discrete steps and depending upon the velocity of the satellite, is set to produce breaks in the trail approximately 0.1mm long. In every revolution of the shutter the time is recorded at the point where the blade is about to enter the field of the camera. This point is defined by a phototransistor unit with a very narrow slit. The photographic plate carries four fiducial marks, two of which define the position of the leading edge of the sector shutter when the time is recorded. From the measurements of these two fiducial marks, and those of a break in the satellite trail, an increment of time can be calculated and added to the corresponding recorded time. This calculation assumes a uniform velocity during one rotation of the shutter. Tests have shown in fact that the r.m.s. error in the recorded time is better than ±0.05% of the period of rotation of the shutter.

The time is displayed in digital form to 0.0001 sec. using a quartz clock driven by a 1MHz oscillator accurate to 2 parts in 10^9. The decimal seconds change only when a pulse is received from the camera, this pulse transferring the count to storage tubes and hence to the display which is recorded photographically. The timing pulses are derived directly from the shutter blades and the delays in the pulses between the camera and the timing equipment are less than 25 micro-seconds.

The recorded times are corrected for the clock error, determined from radio time signals transmitted on 60 KHz. The times of the star exposures are converted to UT1 using the provisional values for this correction issued by the Royal Greenwich Observatory. The recorded times of the breaks in the trail of a passive satellite are corrected for the increment of time introduced by the rotating shutter. The finally corrected time is in UTC. No corrections are applied to the nominal flash times of the GEOS 1 satellite.

PLATE MEASUREMENT

The photographic plates are measured by two independent observers. Each observer makes two readings, one forward and one reverse. The difference between the means of the two observers is used to assess the quality of the image and to derive a weighting factor for the corresponding observation equation in the calibration routine. The measured position of the image is the mean of all four readings.

The means of the measurements of the images are corrected for;
1) The difference between the x and y scale factors.
2) The non-rectangularity of the comparator axes.
3) The weave of the guides.

The parameters used for calculating these corrections are pre-determined from tests made on the comparator. Periodic errors in the comparator micrometers have been found to be negligible.

PLATE REDUCTION PROCEDURE

Camera Calibration

The procedure for calibrating the camera is based on the projective relationship between the object space and the image space and the methods have already been described (von Gruber, 1932; Schmid, 1953; Brown, 1957; Currie, 1964). The following description therefore summarizes the method and discusses the modifications necessary with the use of a camera such as the f1 Schmidt Camera which has a field of narrow angle and considerable distortion.

The projective relationship is given by

$$\begin{pmatrix} x - x_p \\ y - y_p \\ -c \end{pmatrix} = A \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

where x and y are the measured coordinates of the photographic image; x_p, y_p are the plate coordinates of the principal point; c is the principal distance; u, v, w, are the coordinates of the object point referred to the front nodal point of the camera lens and corresponding to the measured image point; and A is a well known 3 × 3 matrix, the elements of which are functions of the orientation angles of the optical axis of the camera.

The x and y coordinates of the measured image are affected by errors due to the comparator, optical distortion and random errors from emulsion shift and other sources. Optical distortion gives errors in both the radial and tangential directions. The radial distortion is expressed as:

$$\delta x = (x - x_p) \sum_{i=1}^{n} h_i r^{2i}$$

$$\delta y = (y - y_p) \sum_{i=1}^{n} h_i r^{2i}$$

where $r^2 = (x - x_p)^2 + (y - y_p)^2$ and h_i are the radial distortion coefficients.

The tangential distortion, which is often referred to as decentering distortion or off center correction is given by (Brown, 1964)

$$\Delta_x = -\sin\beta \sum_{i=1}^{n} j_i r^{2i}$$

$$\Delta_y = \cos\beta \sum_{i=1}^{n} j_i r^{2i}$$

where j_i are the tangential distortion coefficients and β is the angle between the positive x axis and the axis of maximum tangential distortion.

The measured coordinates of the images are corrected for the errors in the comparator using the predetermined values. The effects of optical distortion are combined with the expression for the projective relationships which now become

$$\begin{pmatrix} x + \delta_x + \Delta_x - x_p \\ y + \delta_y + \Delta_y - y_p \\ -c \end{pmatrix} = A \begin{pmatrix} u \\ v \\ w \end{pmatrix}$$

The coordinates of the reference stars used in the plate reduction are the standard coordinates, ξ and η, in a plane perpendicular to the optical axis of the camera and at unit distance from the projection center. The coordinates u, v, w, in the above expression are therefore replaced by ξ, η and 1 respectively.

Star positions are taken from the SAO Star Catalog and reduced to the apparent position of date. Precession from the equinox of the catalogue to the beginning of the nearest Besselian year is first applied by means of the rigorous trigonometric formulae. The apparent place at the epoch of observation is then computed using the Independent Day Numbers taken from the Astronomical Ephemeris. Correction for the proper motion is also applied at this stage. Finally the star positions are corrected for diurnal aberration.

A correction for atmospheric refraction is applied to each star. The astronomical refraction is computed after the method of Garfinkel (1944) and is given by

$$R = K_0 \sum_{i=0}^{4} C_i t^{2i+1}$$

where R is the astronomical refraction (observed minus true zenith distance), and

$$t = \sin z \left[(\sin^2 z + y_0^2 (\mu_0^2 - \sin^2 z))^{1/2} + y_0 (\mu_0^2 - \sin^2 z)^{1/2} \right]^{-1}$$

z being the true zenith distance of the star.

If P is the atmospheric pressure in units of 760 millimeters, and T the temperature in units of 273° Kelvin, then

$$\mu_0 = 1 + 0.0002924\ p/T$$

$$y_0 = 8.2223\ T^{-1/2}$$

$$K_0 = 4''.952\ pT^{-3/2}$$

$$B_0 = 0.03916\ pT^{-1/2}$$

$$C_0 = 0.2$$

$$C_1 = \frac{2}{15} + \frac{1}{5}B_0$$

$$C_2 = \frac{2}{35} + \frac{17}{55}B_0 + \frac{2}{5}B_0{}^2$$

$$C_3 = \frac{1}{70} + \frac{87}{385}B_0 + 0.88091B_0{}^2$$

$$C_4 = \frac{1}{630} + \frac{188}{1430}B_0 + 0.975B_0{}^2$$

The apparent positions of the stars, corrected to include the effects of diurnal aberration, reduced to the epoch of the observation and corrected for refraction, are used to compute their standard coordinates with the optical axis of the camera as origin. The approximate direction of this axis in right ascension and declination is calculated from the azimuth and altitude to which the camera was set during the observation.

Approximately 120 star images distributed fairly uniformly over the plate are used in the camera calibration. About 60 of these are taken from star exposures made prior to the passage of the satellite across the plate and the remainder from star exposures taken after the passage of the satellite. In selecting the images for measurement, those from stars having magnitudes between 6 and 8.5 are chosen where possible.

The projective equations are solved by an iterative method. The equations are converted to linear functions of x and y by means of a Taylor expansion about approximate values of the parameters the linearized equations being:

$$v = \varepsilon_i^0 + \frac{\partial F_i^0}{\partial x_p}\Delta x_p + \frac{\partial F_i^0}{\partial y_p}\Delta y_p + \frac{\partial F_i^0}{\partial c}\Delta c + \ldots + \frac{\partial F_i^0}{\partial h_n}\Delta h_n + \ldots + \frac{\partial F_i^0}{\partial j_n}\Delta j_n$$

$$v' = \varepsilon_i'^0 + \frac{\partial G_i^0}{\partial x_p}\Delta x_p + \frac{\partial G_i^0}{\partial y_p}\Delta y_p + \frac{\partial G_i^0}{\partial c}\Delta c + \ldots + \frac{\partial G_i^0}{\partial h_n}\Delta h_n + \ldots + \frac{\partial G_i^0}{\partial j_n}\Delta j_n$$

where

$$\varepsilon_i^0 = (x_p^0 - x_i)\ (1 + \Sigma_s h_s r_i^{\ 2s}) + \sin\beta(\Sigma_t j_t r_i^{\ 2t}) - \frac{c^0 m_i^0}{q_i^0} \equiv F_i^0$$

$$\varepsilon_i'^{0} = (y_p^0 - x_i)\ (1 + \Sigma_s h_s r_i^{\ 2s}) - \cos\beta(\Sigma_t j_t r_i^{\ 2t}) - \frac{c^0 n_i^0}{q_i^0} \equiv G_i^0$$

and

$$\begin{pmatrix} m_i \\ n_i \\ q_i \end{pmatrix} = A \begin{pmatrix} \xi_i \\ \eta_i \\ 1 \end{pmatrix}$$

The formulae for the partial derivatives in the linear equations have been given by Schmid (1953), Brown (1957), and others, the angles of orientation of the camera being the azimuth, altitude and the parallactic angle, sometimes called swing angle or roll angle, of the photographic plate. In the procedure used at Malvern, the orientation angles are the Eulerian angles θ , ϕ , and ψ . θ is a rotation about OY of a set of axes X^1, Y^1, Z^1, with respect to X , Y , Z , and ϕ is the rotation about OX^1 of a set of axes, X^{11} , Y^{11} , Z^{11} , with respect to X^1, Y^1, Z^1 The angle ψ is the parallactic angle of the plate and is the rotation of the plate axes about the z axis which is coincident with the Z^{11} axis. The elements of the matrix A in terms of these Eulerian angles are:

$$a_{11} = \cos\theta\cos\psi - \sin\theta\sin\phi\sin\psi$$

$$a_{12} = -\cos\phi\sin\psi$$

$$a_{13} = -\sin\theta\cos\psi - \cos\theta\sin\phi\sin\psi$$

$$a_{21} = \cos\theta\sin\psi + \sin\theta\sin\phi\cos\psi$$

$$a_{22} = \cos\phi\cos\psi$$

$$a_{23} = -\sin\theta\sin\psi + \cos\theta\sin\phi\cos\psi$$

$$a_{31} = \sin\theta\cos\phi$$

$$a_{32} = -\sin\phi$$

$$a_{33} = \cos\theta\cos\phi$$

The partial derivatives with respect to x_p and y_p usually neglect the terms containing the distortion coefficients. However, under these conditions the process will only converge if $|2hc^2| < 1$. The introduction of the field flattener into the f1 Schmidt Satellite Camera gives a photographic image with considerable distortion, the value of $|2h_1c^2|$ being approximately equal to 1.3. The iterative process, therefore, does not converge unless terms in h , the distortion coefficients, are included in the partial derivatives with respect to x_p and y_p . It has been necessary therefore, to include terms in h_1 in these partial derivatives which now take the form

$$\frac{\partial F_i}{\partial x_p} = 1 + h_1(r_i^2 + 2(x_p - x_i)^2); \quad \frac{\partial F_i}{\partial y_p} = 2h_1(x_p - x_i)\ (y_p - y_i);$$

$$\frac{\partial G_i}{\partial x_p} = 2h_1(x_p - x_i)\ (y_p - y_i); \quad \frac{\partial G_i}{\partial y_p} = 1 + h_1(r_i^2 + 2(y_p - y_i)^2)$$

A change in the coordinates of the plate center, x_p , y_p , can be almost exactly compensated by a change in the angles of orientation, θ and ϕ . This correlation between the coordinates and the camera orientation angles leads to ill conditioning in the normal equations and consequently the convergence of the iteration proceeds slowly. The ill conditioning has been minimized by using auxiliary variables, $\Delta x_p'$, $\Delta y_p'$, in the normal equations. These auxiliary variables are expressed as:

$$\Delta x_p' = x_p + c\ a_{22}\Delta\theta + c\ \sin\psi\Delta\phi$$

$$\Delta y_p' = y_p - c\ a_{12}\Delta\theta - c\ \cos\psi\Delta\phi$$

where a_{22} and a_{12} are elements of the matrix A . When these expressions are inserted into the linearized observation equations, the partial derivatives with respect to θ and ϕ become

$$\frac{\partial F_i}{\partial \theta} = c\left[-a_{22}\left(\frac{\partial F_i}{\partial x_p} - 1\right) + a_{12}\frac{\partial F_i}{\partial y_p} - \frac{n_i}{q_i}a_{32} + \frac{m_i}{q_i}\left(a_{22}\frac{m_i}{q_i} - a_{12}\frac{n_i}{q_i}\right)\right]$$

$$\frac{\partial F_i}{\partial \phi} = c\left[-\sin\psi\left(\frac{\partial F_i}{\partial x_p} - 1\right) + \cos\psi\frac{\partial F_i}{\partial y_p} + \frac{m_i}{q_i}\left(\frac{m_i}{q_i}\sin\psi - \frac{n_i}{q_i}\cos\psi\right)\right]$$

$$\frac{\partial G_i}{\partial \theta} = c\left[\frac{m_i}{q_i} a_{32} + a_{12}\left(\frac{\partial G_i}{\partial y_p} - 1\right) - a_{22}\frac{\partial G_i}{\partial x_p} + \frac{n_i}{q_i}\left(\frac{m_i}{q_i} a_{22} - \frac{n_i}{q_i} a_{12}\right)\right]$$

$$\frac{\partial G_i}{\partial \phi} = c\left[\cos\psi\left(\frac{\partial G_i}{\partial y_p} - 1\right) - \sin\psi\frac{\partial G_i}{\partial x_p} + \frac{n_i}{q_i}\left(\frac{m_i}{q_i}\sin\psi - \frac{n_i}{q_i}\cos\psi\right)\right]$$

These modifications have been discussed by Currie (1964). In the first stage of the solution of the normal equations only the three orientation parameters are treated as unknown. A gross error in an individual star position or its plate measurements can then be detected and the corresponding observation rejected. This stage of the iteration is completed when the weighted sum of the squares of the residuals is reduced by less than one half of the sum of the squares of the residuals in the previous iteration.

The second stage of the solution is carried out regarding all the parameters which are treated as unknown. The Malvern model uses eleven parameters: the three orientation angles, the two coordinates of the principal point, the principal distance, two radial distortion coefficients and three parameters for the tangential distortion. One or more of the parameters can be taken as being predetermined. Star images are rejected if the residuals are greater than three times the root mean square error of unit weight. Iteration is continued until the weighted sum of the squares of the residuals is reduced by less than one hundredth of the weighted sum of those of the previous iteration. On completion of the iteration, the direction matrix of the angles of orientation is converted to the topocentric axes defined by east, north, and zenith, i.e., the astronomical horizon system.

Reduction to the Satellite Position

The measured plate coordinates of the satellite images are corrected for the comparator errors, radial distortion and tangential distortion. The corrected measurements are then fitted to polynomials as a check for errors. Fitting the measurements to a second order polynomial in x with respect to y checks for large errors in the plate readings. The measurements are then fitted to a third order polynomial in time with respect to x, checking for errors in the x coordinate or in the time of a satellite position. For a passive satellite the times used in the polynomial are the recorded times corrected for the sweep of the radial sector shutter. No correction has so far been made to the nominal times of the flashes from the GEOS satellites.

For passive satellites the time of the observation at the camera is corrected for satellite (planetary) aberration.

In computing a satellite position the refracted direction in the local horizon system is first determined using the projective relationship

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = B \begin{pmatrix} \dfrac{x_p - x}{c} \\ \dfrac{y_p - y}{c} \\ 1 \end{pmatrix}$$

where B is the inverse (or, since this is an orthogonal matrix, the transpose) of the direction matrix with respect to the local topocentric axes obtained from the plate calibration. A correction for refraction is then applied to the plate measurements, using the expressions

$$x' = x - \left(b_{31} - \frac{xw}{u^2 + v^2 + w^2}\right) mr$$

$$y' = y - \left(b_{32} - \frac{yw}{u^2 + v^2 + w^2}\right) mr$$

where b_{31}, b_{32} are elements of the matrix B and

$$m = (u^2 + v^2 + w^2)\ (u^2 + v^2)^{-1/2}$$

r is the astronomical refraction, calculated from the Garfinkel formula as for star positions and corrected for parallactic refraction.

Large passive spherical satellites, having a perfectly reflecting surface are corrected for phase in the following manner:

Let L , M , N be the sun's direction cosines derived from

$$\begin{pmatrix} L \\ M \\ N \end{pmatrix} = \begin{pmatrix} \cos a & \sin a & 0 \\ -\sin a & \cos a & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

where $(X, Y, Z)^T$ is the unit vector of the sun's rectangular coordinates which may be computed from the Astronomical Ephemeris and a is the Local Apparent Sidereal Time at the epoch of observation. Also let l , m, n be the topocentric direction cosines of the satellite image referred to a set of right-handed orthogonal coordinates in which the X and Z axes are in the plane of the local meridian, the Z axis being parallel to the Earth's axis and positive northwardly, and the positive X axis pointing in the direction of zero hour angle and zero declination polar axis. Then the directed cosines of the bisector of the angle at the satellite between the sun's direction and that of the observer are

$$\frac{l - L}{s}, \frac{m - M}{s}, \frac{n - N}{s}$$

where $s = [2 - 2(lL + mM + nN)]^{1/2}$. The corrected coordinates of the satellite are given by

$$x = Dl + d\lambda$$

$$y = Dm + d\mu$$

$$z = Dn + d\nu$$

where D is the distance of the satellite and d its radius.

The directions of the satellite are finally expressed in terms of right ascension and declination, or Greenwich Hour Angle and declination.

APPLICATION OF THE METHOD OF CAMERA CALIBRATION

A number of plates from the f1 Schmidt Camera have now been reduced by the method described. In a sample of 23 plates the calibration procedure with eleven parameters gave an average weighted root mean square error of a single star position of 1.″43 ± 0.″21 . The root mean square error in the direction of the camera axis, due only to the parameters, is given by

$$\sigma_p = \sigma\sqrt{\frac{p}{2n}}$$

where σ is the r.m.s. error of a single star position, p the number of parameters and n is the number of stars in the calibration. The number of stars used in the camera calibration is approximately 120, and therefore the direction of the camera axis is determined to ±0.″32 ±0.″06 .

It takes considerable time to measure a plate with 120 reference stars and up to 100 satellite images with a manual measuring machine. The possibility of reducing the number of reference stars has therefore been examined. To maintain the accuracy in the direction of the camera axis with fewer reference stars, either the r.m.s. error of a single star position or the number of parameters carried in the solution must be reduced. As it is doubtful whether the r.m.s. error of a star position can be significantly reduced when a smaller number of reference stars is employed, some of the parameters must be predetermined. The effect of using fixed values for the radial distortion coefficients and the effect of the tangential distortion terms, on the accuracy of the plate reductions has therefore been investigated, the position of the plate center being fixed relative to fiducial marks.

Before the results from the plates reduced with fewer stars and fixed distortion parameters are discussed, it is interesting to consider the distortion parameters. Over the sample of 23 plates, the radial distortion parameters and the tangential distortion parameters showed a wide variation. The parameters of tangential distortion had standard deviations nearly equal to the mean values of the parameters themselves. Plate calibrations with eleven parameters which included two parameters of tangential distortion had an average weighted r.m.s. error for a single star position of $1''.43$. Calibrating the same plates including one parameter of tangential distortion gave an average weighted r.m.s. error of $1''.44$. When the tangential distortion terms were omitted the r.m.s. error was also $1''.44$. With 120 stars in the reduction there is no significant difference between these three estimates of the r.m.s. error and therefore it should be possible to omit the tangential distortion terms from the plate reductions.

The first and second radial distortion terms have values of -1.7835 ± 0.190 and -266.10 ± 83.92 respectively. The two coefficients are negatively correlated and their standard deviations are approximately 1.5 times those derived from the covariance matrix of normal equations. However, the errors introduced by using predetermined values for the radial distortion parameters are offset, to a large extent, by a change in the principal distance, equivalent to a change in scale. With approximately 120 stars, and using fixed radial distortion parameters and a predetermined plate center, the weighted r.m.s. error of a single star position from the plate calibration is $1''.55$. This figure is not significantly different from that for the average weighted r.m.s. error of a single star position from reductions of plates using the eleven variable parameters.

To investigate the effect of reducing the number of reference stars, groups of 50 stars have been taken. With this number of reference stars and predetermined values for the distortion parameters the r.m.s. error in the direction of the camera axis σ_p , is maintained at approximately $0''.3$. Taken over different groupings of 50 stars, the average weighted r.m.s. error of a single star position in the plate calibration is $1''.33 \pm 0''.20$. Compared with the r.m.s. error from the plate reductions with eleven variable parameters, the value of $1''.33$ is not significant. The range of values of the r.m.s. error over the different groupings of 50 reference stars is most probably due to the quality of the star images and the accuracy of the star positions and of the plate measurements.

The influence of predetermining the values of some of the parameters on the positions of the satellites has been investigated with the use of the recordings of GEOS 1 flashes. The positions in right ascension and declination have been computed with the use of different groups of predetermined parameters and compared with the positions computed from those plate reductions that used approximately 120 stars and the total of eleven variable parameters. The differences in right ascensions were converted to arc and combined with the differences in declination to give the angular displacement of the satellite positions. The mean angular displacement was obtained from the seven GEOS flashes recorded on the photographic plate. There was no correlation between the groupings of predetermined parameters and the mean angular displacement, the displacement ranging from $0\overset{''}{.}4$ to $1\overset{''}{.}0$.

Two plates, measured and reduced with 120 reference stars and eleven unconstrained parameters, have also been measured and reduced by the SAO with their procedures. The differences in the right ascension and in the declination between the first method and the SAO method of reduction are given in Column A of Table I, for the seven flashes on one of the plates. The mean angular displacement in the positions of the satellite images from the two plates is $1\overset{''}{.}7 \pm 1\overset{''}{.}0$.

TABLE I

A - Measured and reduced by Smithsonian Astrophysical Observatory
B - Measured by two independent pairs of observers at Malvern
C - Reduced using 50 reference stars and fixed distortion parameters

Satellite Position	A		B		C	
	ΔRA cosδ	$\Delta\delta$	ΔRA cosδ	$\Delta\delta$	ΔRA cosδ	$\Delta\delta$
1	$-0\overset{s}{.}048$	$-0\overset{''}{.}20$	$+0\overset{s}{.}001$	$+0\overset{''}{.}57$	$-0\overset{s}{.}010$	$+0\overset{''}{.}06$
2	-0.003	-0.56	+0.018	-0.40	-0.021	-0.36
3	+0.014	-0.81	+0.031	-1.00	-0.041	-0.49
4	-0.028	+0.01			-0.010	-0.49
5	-0.002	-0.02	-0.005	+0.50	+0.005	+0.07
6	+0.041	+0.97	-0.049	+0.07	+0.023	+0.50
7	+0.210	+1.04	+0.029	+1.12	+0.058	+0.61

Also, another plate has been measured twice at Malvern using a different pair of observers for each set of measurements and the measurements have been reduced with eleven unconstrained parameters. The displacements in right ascension and declination are given in Column B of Table I . The mean difference in the positions of the satellite images is $0\overset{''}{.}8 \pm 0\overset{''}{.}3$. The displacements of the satellite image positions when only 50 reference stars are used and the distortion parameters are fixed, are shown in Column C.

These results indicate that there is an uncertainty of approximately 1" in the derived positions of the satellite. The discrepancies in the positions calculated by the use of different groups of predetermined parameters lie within this limit of uncertainty and therefore predetermined values of the distortion coefficients with a reduction in the number of reference stars do not significantly reduce the accuracy of the plate reductions. However, these results have been obtained from the reduction of plates taken with a narrow (10° diameter) field camera and it is very probable that they may not be entirely valid for plates covering a much wider field.

Acknowledgement

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DISCUSSION

Baker : *Do focus and scale of your camera depend on the temperature?*

Hewitt : *No. During the daytime we keep the camera in a refrigerated hut so that its temperature is only a few degrees above the outside temperature when we start observing. The camera has a temperature compensation between the mirror and the spider which carries the plate holder. Thus, in the four years during which the camera has been in operation no changes in focus or scale due to temperature have been detected.*

PLATE MEASUREMENT TECHNIQUES AND REDUCTION METHODS USED BY THE WEST GERMAN SATELLITE OBSERVERS, AND RESULTING CONSEQUENCES FOR THE OBSERVATION

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ABSTRACT

The West German tracking stations are equipped with ballistic cameras. Plate measurement and plate reduction must therefore follow photogrammetric methods. Approximately 100 star positions and 200 satellite positions are measured on each plate. The mathematical model for spatial rotation of the bundle of rays is extended by including terms for distortion and internal orientation of the camera as well as by providing terms for refraction which are computed from the measured coordinates of the star positions on the plate. From the measuring accuracy of the plate coordinates it follows that the timing accuracy for the exposures has to be about one millisecond, in order to obtain a homogeneous system. By use of one fictitious point for each plate there is no need for exact synchronization between the observation stations.

Satellite tracking systems are complete units within which various steps are highly dependent upon one another. The following explanations concerning the methods of plate measurement and reduction, therefore, need some general remarks concerning the basic system which is used.

Satellite tracking in Western Germany was developed at the German Geodetic Research Institute at Munich. The other six German observation stations adopted the methods unchanged or with small modifications. About 90% of the observations are made as a contribution to the West European Satellite Triangulation.

At the beginning we decided to employ rigorous photogrammetric methods. This decision was made primarily on the basis of the necessity of equipment mobility. Therefore, ballistic cameras such as the BC-4, IGN-equipment and similar ones are used.

The observations yield as their final product photographic plates 7 1/2" by 7 1/2" on which the star trails are interrupted by means of the shutter, by three different groups of four identical intervals during pre-calibration and post-calibration. The satellite trail is a coded sequence of points. For each interruption (gap) or satellite position the shutter time is known with an accuracy of 1 to 0.1 milliseconds, respectively.

PLATE MEASUREMENT

The stars to be used in the computation are selected during plate measurement. The circular measuring field of 7" diameter is divided into 112 squares by a superimposed grid. In each square, the two to four most suitable trail gaps of one star are selected for pre and post-calibration. In addition, 4 or 5 very bright stars are measured for primary identification and orientation. The identification of the 100 reference stars is done by computer during the reduction. Each satellite position is also measured within the above mentioned field.

The plates are measured on Wild or Zeiss stereo comparators such as are normally used at the various institutions for photogrammetric purposes. Both types have binocular eyepieces. The instruments are calibrated every 3 to 6 months by measuring grid-plates by use of a specially developed program. The non-perpendicularity of, and scale difference between the x and y axes are of main interest since the absolute scale factor is determined later during the computation of the camera orientation and therefore is not important. These effects are not removed by adjusting the instrument; instead of this, corrections based on the calibration values are introduced in the computations.

The satellite images are not such well defined points as one may assume from looking at paper prints. Observing the original plates with a magnification up to 30 x one finds that for a number of points one is not sure where to set the measuring mark. In order to eliminate a subjective decision whether a point can or cannot be accurately measured, we measure the plate again after rotating it by 180°. In this way, the point to be measured presents a completely different subjective impression.

The measurement of one plate lasts about 3 days. It cannot be assumed that the position of the plate in the instrument, as well as the instrument itself, are stable enough during this interval. Each measuring period of some hours is therefore considered as a unit, all of which are in the end "homogenized" by a Helmert transformation. The transformation parameters should be determined with a higher accuracy than the star positions. For this purpose, eight very fine holes are punched into the emulsion in the plate corners and centers of the edges. These points can be measured four to five times more accurately than the photographic images, including the fiducial marks on the plate. Each plate measurement period starts and ends by measuring these punched marks in order to check on possible plate shifts during the measuring session. The alignment marks are afterwards used to compute parameters for reducing the individual units to a common system for the direct plate position (Position 1). The reverse measurement after the plate has been rotated 180° (Position 2), is handled in the same way.

After the plate coordinates measured in Positions 1 and 2 have been corrected for the systematic errors of the comparator they are combined by another transformation. During this procedure, measuring points with a difference between the two positions of more than 10 microns are eliminated. In these cases one may assume that these points are difficult to identify or that there was a mistake. The measurement of these eliminated points is not repeated since there is enough redundancy in the procedure.

After the transformation, the coordinates of the star images and satellite images, and those of the fiducial marks on the plate are printed. The number of eliminated measured points indicates the quality of the plate and the measurement. Following this, we obtain a mean square error, after transformation, computed from double measurement, of 1.5 to 2 microns for star points and satellite points. Considering the great number of stars and satellite positions used in one reduction we accept this as satisfactory.

DATA REDUCTION

To explore the requirements for the mathematical model, we carried out a critical investigation of the feasible systematic errors. Assuming that the residuals of these should be less than 0.1 micron, we came to the following conclusions:

1. The elements of the interior orientation of the camera can be considered as constant, with the exception of the focal distance and the position of the principal point.
2. The distortion values cannot be considered as constant, but the radial symmetry of the distortion is maintained.
3. The meteorological data at the station are not sufficient to calculate the refraction corrections with sufficient accuracy.

We thus carry in our data reduction model correction parameters for these effects and for these effects only. The large number of stars used as control points allows us to evaluate these parameters with high accuracy. To the matrix of spatial rotation, terms are added to account for change of focal distance, location of the principal point, distortion and refraction. The standard distortion and standard refraction are used as first approximate values. At least for a precision camera with small distortion it is sufficient to model them by a polynomial with constant terms up to the 4th order, and one to two variables for change of distortion. Concerning the refraction, we developed a direct solution to compute a correction for the refraction from the zenith distance of the camera axis and plate coordinates:

$$\Delta x'_R = \frac{x'_0}{c_0} \cdot \frac{x'^2_0 + y'^2_0 + c^2_0}{c_0 + y'_0 \tan z} \cdot (A_{1_{korr}} - A_2 \tan^2 z) \cdot \frac{1}{\rho''}$$

$$\Delta y'_R = (\tan z - \frac{y'_0}{c_0}) \cdot \frac{x'^2_0 + y'^2_0 + c^2_0}{c_0 + y'_0 \tan z} \cdot (A_{1_{korr}} - A_2 \tan^2 z) \cdot \frac{1}{\rho''}$$

where x', y', c are the image coordinates, z the zenith distance of the camera axis and A_1, A_2 the coefficients for astronomical refraction.

This also yields the value for the differential refraction, which we shall need later on, with a higher degree of accuracy than available from meteorological data. The residuals after computation of the camera orientation show how well each plate orientation fits the mathematical model.

Corrections for satellite phase, propagation of electromagnetic waves, timing, epoch and so on are made in the usual way. Strictly following the rigorous procedures practiced in photogrammetry, we consider the complete bundle of rays as one unit. The satellite positions have to be "condensed" to one ficticious point near the plate center as a final step of the reduction. This is not done for observations within the net of the Western European Triangulation, due to a Commission resolution. Up to now, the location of the ficticious point is computed by normal polynomials. This is certainly not satisfying because this does not consider the actual structure of the relationships involved, and only serves as an interpolation formula. The investigations toward a better process have not yet been finished.

TIMING

As mentioned above, the measuring accuracy of a single star position or satellite position corresponds to an r.m.s. error of 1.5 to 2 microns. The timing accuracy corresponding to this is two to four milliseconds. It is therefore not necessary to have the timing accuracy for star positions and satellite positions better than 1msec, except for the comparison of the time signal with the time system at the station. The latter is very sensitive to systematic errors.

There is a second process which can produce systematic timing errors: the formation of the photographic image. It has not been proven that the center point of the image corresponds to the middle position of the shutter opening. It is very difficult to research this effect, but it is possible to make the time calibration in such a way that this effect is eliminated. For this purpose we used the following set-up (see Figure 1). In front of the camera we placed a two beam oscilloscope. One beam is continuously fed with a frequency of 1000Hz.

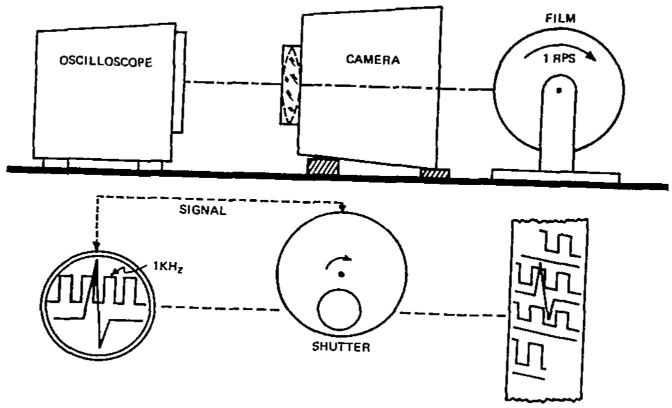

FIGURE 1
Setup for camera calibration.

The other beam shows the signal given by the shutter. On the other side of the camera we put a film, mounted on a rotating drum at such a distance that the image of the oscilloscope is in focus at the film. The rotating shutter is then activated. The film will be exposed during one shutter opening. The position of the shutter signal can now be counted directly at the image of the 1000Hz frequency, which stars and ends with the opening and closing of the shutter. By the rotation of the drun, the overlapping of pictures is avoided.

The third effect to consider is this: if one uses a fictitious point there is no need to have the synchronization between the cameras on different stations better than a few seconds. Also the sequence of exposures may be different. Computing the fictitious point from more than 100 single positions will eliminate all accidental errors along the trail.

These consequences are important for the observation equipment. It can become less complicated and less expensive.

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DISCUSSION

Rosenfield: *How do you put your fiducial marks on the emulsion?*

Deker : *With what I think is called a "point transfer device" which is essentially a very exactly dropped needle. This produces images that resemble shooting targets due to the displacement of the emulsion which can be very accurately set on.*

Mueller : *What, so far, have been your experiences with the I.G.N. camera?*

Deker : *They are very ingeniously designed, but have their drawbacks. We had to replace the timing equipment which produced irregular errors up to an amount of ten milliseconds.*

THE AGK3, A BASIS FOR A GENERAL (NORTHERN) REFERENCE CATALOGUE OF POSITIONS AND PROPER MOTIONS

Wilhelm Dieckvoss

Hamburger Sternwarte

ABSTRACT

The original concept of the international AGK3 program is presented and a report on the methods applied and on the progress of the work is given. The results in the form of revised positions of the AGK2 reduced to the FK4 system, and of AGK3, can be extrapolated easily to earlier epochs, as far back as those of AGK1, and the mean error for these epochs will permit one the deriving of reliable values for systematic corrections of the old catalogues. New experience with the Astrographic Catalogue confirms former conclusions as to furnishing excellent positions for the reference stars (from AGK2 / 3) by the automatic process of smoothing the positions inserted into the plate solutions.

By using the AGK2 / 3 positions a general catalogue of roughly to the ninth magnitude can be constructed by including large catalogues such as AGK1, AC, Yale, while the systematic corrections to the individual catalogues need no longer be taken from the GC but can be derived uniquely and independently from the AGK2 / 3 data. Thus a new edition of the type of the SAO Catalogue for the northern hemisphere can be produced.

In 1953 O. Heckmann (1954) at the Evanston conference gave an account of the value of a third AG Catalogue. In a way, he gave apologies for the AGK2 (Schorr and Kohlschütter, 1951), the makers of which had decided not to publish proper motions (p.m.) with the positions. While Heckmann's arguments are all right as they stand, the lack of proper motions was frequently felt to be rather unfortunate, the more so as the work of Schlesinger and his successsors in the Yale Zone

Catalogues contained proper motions, and as the AGK2 can be viewed as a concerted effort in an application of Schlesinger's methods to half the celestial sphere during a small time period.

Heckmann's appeal of 1953 to furnish the proper motions for all stars in the AGK2 by a combined activity of many astronomers and institutes gained wholehearted support, and his demonstration of the usefulness of such an addition of proper motions in a homogeneous and unified system was at once highly appreciated in connection with the coordination of galactic research, e.g., at the Groningen Conference in 1953 (Blaauw, 1955).

At Bergedorf and Bonn, the two observatories that published the AGK2 (beside the Poulkovo volume for the polar cap) in accordance with the original AGK2 plans, numerous ledgers exist with lists of differences AGK2 minus AGK1. These data do not lend themselves easily to the derivation of proper motions, and, although the General Catalogue (GC, Boss, 1937) gives tables of systematic corrections to the AGK1 positions the final incorporation into the AGK2 of the differences against AGK1 was not carried out. Any worker in single fields or with individual stars has to compute these differences anew and has to apply corrections from well known tables to put them on a definite system, preferably that of the FK3 or the FK4.

In the AGK3 program, selection and observation of reference stars was begun together with the re-exposure of the plate fields in 1956, and the originally established procedures went according to plan. On the original measuring sheets of the AGK2 (used at Bergedorf and Bonn after 1930) the coordinates of the stars measured on the new plates, whose centers were identical to those taken in 1930, were entered and the differences in the sense new coordinate ("AGK3") minus old coordinate (AGK2) were transformed into relative proper motions by means of a few suitably selected stars. All reductions were made such that after the publication of the proper motions of the reference stars, (the AGK3 R; Scott, 1967) only a short time would be needed in order to transform the relative proper motions to absolute ones and to take the arithmetic means for publication in the catalogue that was to give proper motions only, for the stars of the AGK2, identified by their reference number.

While the meridian circles were busy on the program, and while at Bergedorf the first steps with exposing and measuring plates were under way, the development of new ideas and technological equipment showed better possibilities for achieving results. At Hamburg, the extensive use of automatic data processing machines began in 1958, and we entered the computer age slowly, first of all by having the IBM 650 perform much of the work formerly done by hand on desk calculators; the punching of the star coordinates, however, had to be done manually on the key punch, and with this transforming of the data from human readable form to machine readable form, the basis was laid for more and more sophisticated data processing.

The systematic errors introduced into the coordinates by the faulty objectives depend on image position on the plate, and on magnitude and color. These errors were not removed from the coordinates published in the volumes of the AGK2.

and in volumes 10 and 11, preliminary tables were printed that were intended to serve as a substitute for publishing corrected positions. In principle, however, it is far better to apply systematic corrections to the measured coordinates directly before the plate solutions are performed instead of relying on the conviction that the plate constants will be affected only slightly. Regardless of how deeply this confidence might be rooted in the minds of the people responsible for the work, I nevertheless want to state that it is a dangerous policy to rely on applying systematic corrections after the catalogue has been formed. As to the AGK3 work in Bergedorf, reports on systematic errors and on reduction methods have been published in the Astronomical Journal (Dieckvoss, 1960, 1962). About 1964 it was decided to re-reduce the AGK2 by means of the appropriate reference catalogue (AGK2 A, Kopff, 1943) that was put on the system of the FK4. The original coordinates of the AGK2 were available on punched cards anyway, and on the basis of experience gathered in the meantime, these coordinates were corrected for systematic errors before the new plate reductions were performed. The reduction of the coordinates of the new plates taken for the AGK3 was a natural sequel. In a way, the lack of proper motions for the AGK3 R positions aided in reaching these decisions.

The task was a big one as to computer time consumed. Fortunately we at Bergedorf could use the GIER computer installed there, from 1964 to 1967. For almost two years we never had any difficulty in getting as much time as we wanted, altogether more than 1000 hours. Meanwhile, we shifted to a big computer at Hamburg with extensive use of magnetic tapes, but even there we used almost 100 hours. In 1965 we used nearly 500 hours to apply the primitive method of artificially enlarging the field of a plate by attaching up to four neighboring plates to a central plate by means of all stars common to the common regions (Dieckvoss, 1962) for old and new epochs separately. It turned out, however, that when taking the final means for each star the mean error was 10 to 15% larger than the mean errors found by the classical method of solution for single plates. The obvious reason was the frequently too small number of common stars with the consequence of excessively high mean errors of the constants that transform one plate to the other.

Table I. - Mean Errors of Positions for Different Epochs

AGK2 $\pm0''.16$	+ p.m. $\pm''.008$/yr.	AGK2 and AGK3
1870 (AGK1)	$\pm''.51$:	$\pm0''.80$
1900 (AC)	.29:	.36
1930 (AGK2)	.16	.16
1945		.12
1960 (AGK3)	.29:	.16
1990	.517:	.36

: uncertain positions and proper motions interdependent.

Table I gives an indication of the increase in accuracy if one changes from adding 1930 positions and proper motion influence, to the use of two independent positions for two separate epochs. It should be remarked that the left hand side of the table gives incorrect values as in this case the proper motions are not independently known.

While I need not go into detail here, some remarks will be in order to indicate how we proceeded at Bergedorf. F.P. Scott of the U.S. Naval Observatory sent in 1965 a preliminary copy of the AGK3 R on punched cards, and this was used for deriving a preliminary version of the AGK3 that contains the results of plate solutions for both epochs. As the AGK3 R had no proper motions, and the proper motions of the AGK2 A were of very inferior quality, some manipulating was necessary for the identification of the stars because neither catalogue contained our AGK2 numbers. At Bergedorf we have available an AGK2 Revised and an AGK3 Preliminary, and we have lists of stars that showed large discrepancies when the means were taken from neighboring fields. The final plate solution with the definitive AGK3 R have been performed, and the zones from the pole down to about +35° declination have been inspected for punching errors or faulty measuring and a final catalogue is ready from the pole down to +49°, but once more the data for some stars turned out to be faulty, but in due course they will be remeasured with our Mann comparator and the necessary corrections will be made. The catalogue has been punched on cards that have to serve for manufacturing the manuscript.

Table II. - Sample Pages from the AGK3 Manuscript.

+83	MAGN	SP	RA(1950.0)	DKL(1950.0)	EP.	N	P.M.		DT	BD
1	10.7	F0	0 02 01.108	+83 22 44.36	58.7	2	- 24	+ 30	28.7	+82 0750
2	9.1	F5	0 07 06.571	+83 52 25.90	58.7	3	+ 38	+ 18	28.7	+83 0001
3	9.7	F5	0 10 00.277	+83 45 59.03	58.7	2	+ 116	- 15	28.8	+83 0003
4	10.7	K2	0 12 39.700	+83 06 47.95	58.7	2	- 7	+ 5	28.8	+82 0003
5	9.4	F0	0 12 57.159	+83 05 33.42	58.7	2	- 42	- 1	28.8	+82 0004
6	11.0	F5	0 14 42.949	+83 39 34.21	58.7	2	+ 20	+ 17	28.8	+83 0004
7	10.1	K5	0 15 08.999	+83 05 23.97	58.7	2	+ 31	+ 23	28.8	+82 0005
8	11.5	G5	0 21 17.743	+83 20 27.87	58.7	2	+ 41	- 32	28.8	+82 0009
9	10.9	G5	0 24 23.086	+83 59 49.18	58.7	2	- 18	- 3	28.8	+83 0006
10	11.0	A5	0 26 12.261	+83 51 51.24	58.7	2	- 7	+ 12	28.8	+83 0007
11	11.5	F2	0 30 36.139	+83 15 03.24	58.7	2	- 35	+ 34	28.8	+82 0013
12	8.3	A2	0 31 35.578	+83 21 22.81	58.7	3	+ 33	+ 10	28.7	+82 0014
13	11.2	K0	0 32 55.071	+83 57 04.92	58.7	2	- 6	+ 2	28.8	+83 0008
14	11.3	F0	0 39 18.299	+83 58 17.19	58.7	2	- 19	+ 15	28.8	+83 0013
15	12.2	K0	0 39 25.583	+83 01 30.43	58.7	1	- 271	- 49	28.7	+82 0016
16	11.3	A5	0 44 57.006	+83 36 17.09	58.7	2	+ 3	+ 6	28.8	+83 0014
17	9.6	A2	0 50 03.217	+83 28 11.64	58.7	2	+ 66	- 31	28.8	+82 0020
18	9.7	A0	0 52 27.609	+83 26 16.39	58.7	2	+ 4	+ 16	28.8	+82 0021
19	10.1	A2	0 54 28.451	+83 24 48.41	58.7	2	+ 18	+ 4	28.8	+82 0022
20	11.3	F0	0 58 01.444	+83 48 34.44	58.7	2	- 7	+ 0	28.8	+83 0021
21	12.1	K0	1 03 17.612	+83 08 42.46	58.7	2	- 5	- 1	28.8	+82 0027
22	11.8	K2	1 03 24.019	+83 13 37.63	58.7	2	+ 19	- 7	28.8	+82 0028
23	11.9	K5	1 05 42.357	+83 53 06.36	58.7	2	+ 20	+ 10	28.8	+83 0022
24	11.8	G5	1 06 19.737	+83 29 38.81	58.7	2	- 12	- 5	28.8	+82 0029
25	9.4	A2	1 06 20.151	+83 11 28.47	58.7	2	- 9	- 3	28.8	+82 0030
26	11.0	G0	1 08 12.519	+83 38 12.85	58.7	2	+ 23	+ 7	28.6	+82 0031
27	11.5	K0	1 11 01.261	+83 59 34.37	58.7	2	+ 15	+ 16	28.6	+83 0024
28	12.1	K5	1 13 21.243	+83 46 09.74	58.7	1	+ 8	+ 1	28.6	+83 0026
29	9.9	F2	1 14 45.995	+83 57 53.18	58.7	2	+ 33	+ 8	28.6	+83 0027
30	10.5	G5	1 16 45.517	+83 32 50.18	58.7	2	+ 209	+ 86	28.6	+83 0028
31	12.0	G5	1 24 19.501	+83 29 37.13	58.7	2	+ 28	- 1	28.6	+82 0035
32	10.8	F2	1 27 23.506	+83 29 11.08	58.7	2	+ 9	+ 14	28.6	+82 0037
33	9.7	F5	1 28 55.247	+83 05 31.20	58.7	2	+ 26	+ 9	28.6	+82 0039
34	11.1	G0	1 29 03.165	+83 05 52.34	58.7	2	+ 80	- 12	28.6	+82 0039
35	11.8	K0	1 36 31.875	+83 36 20.92	58.7	2	+ 7	+ 26	28.6	+83 0034
36	10.8	F5	1 38 45.355	+83 50 51.65	58.7	2	+ 4	+ 8	28.6	+83 0035
37	11.3	K2	1 45 01.466	+83 21 20.08	58.7	2	+ 11	+ 6	28.6	+82 0042
38	10.2	F2	1 48 01.633	+83 49 08.02	58.7	2	+ 12	+ 4	28.6	+83 0039

+83	MAGN	SP	RA(1950.0)	DKL(1950.0)	EP.	N	P.M.		DT	BD
101	11.0	F5	4 16 30.288	+83 36 21.95	58.1	3	+ 22	- 28	27.9	+83 0105
102	11.2	K0	4 17 07.113	+83 34 40.91	58.1	2	- 3	+ 24	27.9	+83 0106
103	10.0	G0	4 17 42.059	+83 15 39.50	58.1	2	+ 38	- 78	27.9	+83 0109
104	11.8	G5	4 17 45.219	+83 35 40.78	58.1	1	+ 24	+ 22	27.9	+83 0106
105	9.6	F5	4 18 03.617	+83 41 58.37	58.1	2	+ 39	- 16	27.9	+83 0107
106	11.2	G5	4 18 42.985	+83 43 40.98	58.1	2	+ 27	+ 30	27.9	+83 0108
107	6.5	G5	4 18 49.253	+83 13 35.55	58.1	2	- 45	+ 118	27.9	+82 0113
108	10.9	K0	4 19 51.607	+83 06 20.28	58.1	2	+ 12	- 8	27.9	+82 0114
109	11.2	F8	4 20 00.351	+83 43 15.99	58.1	2	- 2	+ 32	27.9	+83 0110
110	10.6	K2	4 26 11.073	+83 13 38.53	58.1	2	+ 33	+ 33	27.9	+82 0116
111	8.3	K0	4 33 38.347	+83 56 05.84	58.1	2	- 7	- 10	27.9	+83 0114
112	11.1	G5	4 35 34.707	+83 50 06.97	58.1	2	+ 9	+ 11	27.9	+83 0115
113	10.7	G5	4 37 28.757	+83 50 16.63	58.1	2	+ 19	- 4	27.9	+83 0116
114	10.9	F0	4 39 11.938	+83 47 25.49	58.1	2	- 1	+ 11	27.9	+83 0117
115	9.3	K0	4 39 51.063	+83 39 28.11	58.1	2	+ 4	- 20	27.9	+83 0118
116	10.8	F8	4 44 02.791	+83 19 12.72	58.1	2	+ 25	+ 13	27.9	+83 0119
117	8.8	F5	4 45 24.791	+83 13 03.06	58.1	2	+ 15	- 16	27.9	+83 0121
118	10.0	K0	4 45 29.387	+83 35 28.00	58.1	2	+ 13	+ 34	27.9	+83 0120
119	10.3	G0	4 48 00.293	+83 24 50.91	58.1	2	- 13	+ 39	27.9	+83 0122
120	7.7	B9	4 48 20.289	+83 07 00.37	58.2	2	- 3	+ 21	27.9	+82 0125
121	10.0	G5	4 50 22.284	+83 38 23.78	58.1	2	+ 7	+ 18	27.9	+83 0123
122	9.3	K2	4 54 06.331	+83 24 20.82	58.1	2	+ 6	+ 6	27.9	+83 0126
123	10.5	K0	4 55 01.927	+83 57 13.57	58.1	2	+ 14	+ 10	27.9	+83 0125
124	10.0	F8	4 57 13.477	+83 18 47.70	58.1	2	+ 29	- 15	27.9	+83 0127
125	11.2	K0	4 59 08.883	+83 03 14.30	58.1	2	+ 24	- 11	27.9	+82 0131
126	9.7	K0	5 01 40.969	+83 28 37.47	58.1	3	+ 16	+ 10	27.9	+83 0129
127	11.3	G5	5 02 12.775	+83 29 22.64	58.1	2	+ 27	+ 16	27.9	+83 0131
128	11.0	G5	5 08 10.010	+83 31 27.19	58.1	2	+ 15	- 12	27.9	+83 0132
129	10.7	F5	5 12 05.465	+83 28 32.11	58.1	2	+ 31	- 24	27.9	+83 0134
130	11.3	G0	5 13 34.983	+83 49 13.47	58.1	2	+ 16	+ 7	27.9	+83 0135
131	11.2	K0	5 13 53.241	+83 30 19.85	58.1	2	+ 20	- 13	27.9	+83 0136
132	9.2	F5	5 17 23.419	+83 46 46.72	58.1	2	+ 6	- 18	27.9	+83 0137
133	9.8	G0	5 18 14.127	+83 55 58.43	58.1	2	+ 18	+ 10	27.9	+83 0138
134	9.8	G0	5 20 44.229	+83 21 52.19	58.1	2	- 56	- 29	27.9	+83 0139
135	10.7	G0	5 21 49.727	+83 22 49.45	58.1	2	- 4	- 30	27.9	+83 0140
136	7.2	A0	5 24 27.020	+83 49 34.77	58.1	2	+ 11	- 16	27.9	+83 0141
137	10.1	K5	5 26 49.547	+83 19 24.25	58.1	2	+ 4	- 3	27.9	+83 0142
138	10.0	K0	5 27 52.976	+83 06 30.79	58.1	2	+ 22	- 36	27.9	+83 0144

Table II shows what the AGK3 might look like when published. In the next few weeks the Hamburg people will finally finish the program necessary for a special tape unit that accepts and writes IBM tapes which will be lent to interested institutions at their request.

The printed proper motions were computed from differences AGK3 minus AGK2 and the last column next to the last epoch gives the differences. Thus, the printed catalogue will actually contain two individual catalogues, while the new magnetic tapes contain two sets of positions.

The Astrographic Catalogue (AC, Carte du Ciel) was conceived in the last century, and the numerous volumes printed represent a vast amount of information relating to the relative positions of stars at approximately the turn of the century. In order to become usable, however, accurate constants for the individual plates have to be derived. The latest contribution in this context can be found in a paper by Eichhorn and Gatewood (1967). The possibility to derive good plate constants by the use of AGK3 data was stressed by Heckmann in 1953. The task is not an easy one because the coordinates are affected by systematic errors depending on measured coordinates and magnitude.

Over the years, experience has shown that by deriving plate constants, the automatic smoothing process gives excellent positions of the reference stars for the epoch of the plate, and the following simple example may serve as an illustration: Position (AC) = a + bx + cy to be found from the right hand member of the equation of condition

$$= \text{pos (AGK3)} - \mu\Delta t + v$$

The new proper motion based on a normally larger time interval (from 1900 to 1960 instead of the 30 years in the AGK2/3) will become:

$$\mu' = (\text{pos (AGK3)} - \text{pos(AC)}) /\Delta t$$

$$= \mu - v/\Delta t$$

The mean errors of the AGK2 and AGK3 positions have values of 0".16; for the AC and referring to the mean value from two plates, the mean errors lie between 0".16 and 0".22 . In the weighting system introduced in the GC with the unit weight belonging to a probable error of 0".300 the weights of AGK2/3 and of the AC measured with high precision will be 7.7 and for the AC with lesser accuracy, 4.1 .

If and when the work on definitive plate constants for the AC will be finished we can assume that a system of positions and proper motions will have to be established. The simplest method has been described by S. Newcomb (1906). In the best zones of the AC with reasonably early epochs the central epoch will lie around 1930, and the proper motions will have mean errors of the order of 0".004 per year. In zones less favorable, the AGK2/3 with 1945 as mean epoch and proper motions with mean errors of 0".008 per year will give a homogeneous system of absolute proper motions. In any case, we have a basis for extrapolating back to the epoch of older catalogues such as the AGK1, as well as to other, more modern catalogues.

Table III. - Actual Mean Errors for Different Epochs.

Epoch	Type of Catalogue	Mean Error	
1870 ±10	AGK1	±″.3 to > 1″.0	
1900...	AC	.16	.22
1930 ±1	AGK2	.16	
1960 ±2	AGK3	.16	

Table IV. - Examples for Different Zones of AGK1 and AC combined with AGK2/3 (weight 7.7).

AGK1	Helsingfors-Gotha	Bonn	Berlin B.
Decl.	+55° to +65°	+40° to +50°	+20° to +25°
Epoch	1875	1875	1880
weight ("GC")	.3	.2	1.5
m.e.	″.8	1″.0	″.36
AC (1900)	Vatican	Helsingfors	Paris
weight	4.1	7.7	7.7
centr. Ep.	1934	1929	1923
m.e. of pos.	″.10	″.09	″.09
m.e. of p.m.	″.004/yr.	″.004/yr.	″.003/yr.
m.e. 1980	″.22	″.21	″.21

Table III indicates the mean errors of the principal catalogues discussed in this paper.

The last table (Table IV) presents three examples for how a general catalogue envisaged at the beginning of this paper might yield different mean errors for the results proper, and as an example, an extrapolation to the year 1980.

Such a general catalogue for the northern hemisphere containing the stars to the ninth magnitude originally selected for the AG zone work will be far from homogeneous owing to differences in epoch and precision for the various zones of the AC, and the other older catalogues. It will, however, in all probability exhaust the data presently available and will present the information concerning the motions of the stars in a form in which they can be used profitably for many purposes of astrometry.

I hope that some central institute will tackle this task, and will use its resources with the incorporation of modern results from different sources, viz. from our AGK3, from the AC people and last but not least, from those who patiently go ahead with keypunching and proofreading the contents of old catalogues with pages yellow from old age. Herget's (1967) endeavors in the Bordeaux zone of the AC will probably set the pace and will allow an estimate of the amount of work involved. Only by such honest work, the magnificent work of the Smithsonian Astrophysical Observatory with its Star Catalogue so widely used today can be superseded by a new catalogue with a considerable improvement of the weights of the relevant numbers.

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DISCUSSION

Gatewood : *How did you get the mean error of the AC positions? Some of my own experience indicates somewhat larger errors.*

Dieckvoss : *From a comparison of the discrepancies between neighboring fields. Work is now going on at Hamburg and at Paris to find the systematic errors common to whole AC zones. When these are applied, our mean errors should be comparable to those of the typical catalogue zone astrographs. In a few weeks, our results will be available on magnetic tapes.*

AN ANALYSIS OF THE AGK3 COMPARISON STAR POSITIONS

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ABSTRACT

Proper motions formed with the use of positions in the AGK3 and other, older catalogues are investigated for systematic trends.

For everything in the present paper I am indebted to Dr. Dieckvoss, because he very kindly sent me, at the earliest possible time that he could, the information of the AGK3-AGK2 on packed punch cards. I converted these onto regular star catalogue magnetic tapes and began to do a number of things with them, the first of which is a kind of fiendish trick: I analyzed all the information which he gave me, in order to see whether or not the proper motions in right ascension would average zero around the whole sky. The material was divided into two zones (+10° to +15° to correspond with AGK1-Leipzig, and +15° to +20° to correspond with AGK1-Berlin), each zone was divided into five magnitude groups brighter than 7.0, 7.01 to 8.0, 8.01 to 9.0, 9.01 to 10.0 and fainter than 10.0). Then all the proper motions in right ascension and declination, separately within each group, were represented by a least squares solution of the form

$$\mu = C_0 + C_1 \cos\alpha + S_1 \sin\alpha\,.$$

It is apparent that for the right ascension the constant term is nearly zero, as it should be, the phase corresponds to the direction of the Sun's way toward Hercules, and the amplitude diminishes as it should for the successively fainter magnitude groups. The declination solutions all have a negative bias, as they should. These results gave me full confidence to go ahead and use the material.

Next we undertook a comparison of (AGK3-AGK2) with the Yale Zone Catalogue. The position from (AGK3-AGK2) was extracted for the epoch of the Yale position and the two were then differenced. Using the same ten groups as before, the differences within each half hour strip of right ascension were represented by a mean value and its probable error. The results in right ascension are fairly satisfactory, in the sense that the range shown by the probable error straddles the zero line fairly well. There is a slight positive bias between 20^h to 24^h, more pronounced in the lower zone than in the upper one. But the declinations show a definite positive bias. Since we already had the computer program, we also analyzed these differences in the same way as we had done before for the proper motions. The results in the five magnitude groups are fairly similar, and so we show only a weighted mean (based on the number of stars in each group) of the resulting coefficients.

$$+10° \text{ to } +15° : (\Delta\alpha)^s = +0.0062 - 0.0053 \cos\alpha + 0.0061 \sin\alpha$$
$$(\Delta\delta)'' = + 0.178 - 0.017 \cos\alpha + 0.030 \sin\alpha$$

$$+15° \text{ to } +20° : (\Delta\alpha)^s = - 0.0012 - 0.0021 \cos\alpha + 0.0059 \sin\alpha$$
$$(\Delta\delta)'' = + 0.333 - 0.030 \cos\alpha + 0.035 \sin\alpha$$

None of these coefficients is of much significance, except the constant term in declination.

I do not know how to explain this. The plate of the Yale Zones covered slightly more than ten degrees in declination and fourteen degrees in right ascension. All of the stars in both of my five degree zones of declination were on the same Yale plates. Yet when I split them into these two zones I get this decided difference of +0"18 and +0"33 . I cannot explain it; I can just point it out.

Finally we undertook a comparison of (AGK3-AGK2) with the Leipzig and Berlin zones of AGK1, in the same way as we had for Yale. The AGK1 positions were precessed to 1950.0 with Newcomb's precession constants. These plots show that the differences in declination are inconsequential: the range of the probable errors straddles the zero line, and the probable errrors are undoubtedly larger because of the greater individual errors in the AGK1 observations. But the right ascensions have a definite negative bias amounting to 1"8 . There is a slight magnitude effect which seems to be smaller for the brighter magnitudes, and again I do not know how to explain this. However, this is a visual demonstration of the systematic differences which will exist if one is to adopt as the fundamental baseline the system of star positions and proper motions defined by (AGK3-AGK2).

If one attempts to construct some "general catalogue" as we propose to do for the zone of the Bordeaux Astrographic catalogue, then these are the kinds of problems that one will face in determining the systematic corrections. If anyone has any comments or thoughts on these problems, I would be glad to hear them.

DISCUSSION

Murray : *The fundamental system to which those catalogues are referred is of critical importance. Were the reference catalogues used for the reduction of the two half-zones of the Yale catalogues always the same?*

Herget : *The reference catalogues were the same; the reference stars must have come from a 10° wide zone at the same meridian circle. There is, however, a large error expressed by a quadratic term in declination; this is probably the reason why Yale eventually repeated this catalogue.*

Murray : *Is the difference here, then, the same as that between the NFK and the FK4 equinoxes? Maybe Dr. Dieckvoss could say something about the systematic right ascension differences between the AGK1 and the AGK3?*

Dieckvoss : *I don't have these data at my fingertips, and I don't remember exactly what has been done with the Yale Catalogues. They were compared with the AGK1 to get the proper motions, and there are tables to correct the positions in them to the GC system.*

Herget : *I'll have to see how my values compare with that material.*

Dieckvoss : *Your correction of 1".8 seems large. As I remember the values in Volume One of the GC for the corrections from one system to another are usually in the order of a few tenths of a second of arc.*

Hoffleit : *These northern zones were computed long before I was involved in the Yale project. The reason (if I remember correctly) why the Yale Zones were published in zones narrower than the ten degrees covered by the plates, was, that the older catalogues, which furnished the material for the determination of the proper motions, had been published in narrower zones. And the proper motion systems in different "subzones" depending on the same set of plates may therefore be different, although any set of plates was reduced with a homogeneous set of reference positions. One notes, when reading Schlesinger's introductions to his earlier catalogues, in how much detail he describes the corrections to the proper motions in right ascension which were based on a hypothesis concerning reflected solar motion; yet, this was in no way used to correct the proper motions in declination, which in some cases were simply reduced to the proper motion system of the GC and not to whatever the system of the right ascension proper motions was.*

Herget : *When I first saw this 1".8, I believed I had made some slip. However, this is exactly the amount by which Schlesinger corrected the crude proper motions to make their average come out to zero.*

Scott : *The Yale Zone +10° to +20°, which was also published in two subzones, was reduced by means of the First Greenwich Catalogue for 1950 as a source of reference stars. We compared it at the USNO with the Katalog con 3356 Schwachen Sternen by Larink. The northern as well as southern half of this Yale Zone show constant right ascension and declination differences as compared to Larink's catalogue, the differences being $(Y_N - L) = +0^s.004$ in α and $- 0''.33$ in δ; $(Y_S - L) = - 0^s.005$ in α and $- 0''.14$ in δ. Furthermore, a comparison of the upper and lower halves of this Yale Zone with the "First Greenwich Catalogue for 1950" indicates that the two halves of the Yale Zone are not on the same system; Yale North minus Yale South being $+0^s.006$ in α and $- 0''.13$ in δ. Maybe a close study of the Greenwich positions would reveal the reason for this. I suspect that there is, somewhere in them, a jump of nearly 0".2 in declination.*

Eichhorn : *I am somewhat disappointed that the analysis of your right ascension proper motions showed a zero constant term. This should have been the case only if the proper motions were on a system which is inertial. But since there seems to be agreement that the constant of precession requires a correction of about 1" per century to render this system inertial, a corresponding constant term should have appeared in the analysis of your proper motions.*

Herget : *The bases are Dieckvoss' 1930 and 1960 photographs, which depend on Scott's reference catalogue.*

Scott : *Our reference stars were for use with the AGK3 only, not the AGK2.*

Dieckvoss : *For the re-reduction of the AGK2A we used the AGK2 reduced to the FK4 system. Our proper motions are therefore based on Newcomb's constant of precession, which is being retained at this time by international agreement.*

Murray : *I agree with Eichhorn that the proper motions should have shown the correction to the precession.*

Vasilevskis : *What matters is only the total change in a star's positions, and when the constant of precession is not inertially true, the proper motion of the star will absorb the correction to precession.*

RECENT PROGRESS ON THE YALE ZONE CATALOGUES

Dorrit Hoffleit

Yale University Observatory

ABSTRACT

The Yale University Observatory has been engaged in the measurement or reduction of stars in southern zones.

We have been handicapped by frequent turn-over and shortage of scientific personnel, and by a vast assortment of machine failures. Most of these conditions have recently been greatly ameliorated by the installation of a more efficient air conditioner in the measuring room, numerous instrumental improvements, and the appointment of Mr. Phillip Lu who has had previous experience and training in astrometry.

The catalogue for the -35° to -40° zone is rapidly nearing completion. All the reductions and comparisons with other catalogues have been completed. The tables and star charts are being prepared in final form for publication by Myles Standish. Although the introduction to Volume 28 of the Transactions of the Yale Observatory basically also applies to this zone, since both were measured on the same Yale 11° × 11° plates, new analyses of the relative errors of the proper motions determined here and at the Cape were necessary in view of the fact that the Cape values depended upon different older catalogues. Two items of particular interest or concern are the problem of magnitude corrections, especially in declination, and comparisons with Luyten's Bruce Proper Motion Survey.

Under a contract with the U.S. Army Map Service (now named U.S. Army Topographic Command), the Yale University Observatory has been engaged in the measurement and reduction of positions and proper motions of stars in the southern hemisphere. The present status of the work is briefly summarized as follows:

-30° to -35° 12 876 stars. Published, Vol. 28, 1967, *Transactions* of the Astronomical Observatory of Yale University.

-35° to -40° 12 120 stars. Vol. 29, *ibid*, in press.

-40° to -50° Measured by D. Eckert and assistants at the IBM Watson Scientific Laboratory. Reductions resumed at Yale.

-60° to -70° Measured at Yale under the supervision of Dr. A. Klemola. Reductions assumed by the Army Map Service.

-70° to -90° Measurements at Yale over 50% completed. In this zone, with highly overlapping plates in both right ascension and declination, an average of nearly 4000 stars per plate has been selected for measurement, including many faint stars.

The programs of stars to be measured have been mainly the stars in the corresponding catalogues published or in preparation at the Royal Observatory at the Cape. These catalogues are the basis for the Yale determinations of proper motion. Only in the "South Polar Cap" zones, -70° to -90°, have appreciable numbers of stars been added from other sources, particularly faint stars from Melbourne Astrographic Catalogues, for a study of magnitude and coma corrections.

The -35° to -40° zone just completed was measured on the same 11°×11° plates as the stars in the -30 to -35° zone, published as Volume 28 of the *Transactions* of this observatory. A large part of the introduction to that catalogue hence also applies to the forthcoming Volume 29 and will not be repeated there. However, new analyses of the relative errors of the proper motions determined here and at the Cape were necessary in view of the fact that the Cape values (Jackson and Stoy, 1955) depended upon different older catalogues. Of particular concern or interest have been the problem of magnitude corrections, especially in declination, and comparisons with the Bruce Proper Motion Survey (Luyten, 1960) of the stars with comparatively large proper motions.

The observed proper motions are the resultant of reflex solar motion, galactic rotation, and peculiar motions. Schlesinger had long ago (1925) pointed out that if the peculiar motions have effectively random distribution, then the hourly means of the proper motions in right ascension should closely follow a sine curve and the average of the 24 hourly means in any zone should be zero. If, then, the stars are grouped by apparent magnitude, the amplitudes of the sine waves should be greatest for the brighter stars (if they are on the average the more nearby) and smallest for the faintest stars, provided that the percentages of high and low intrinsic luminosities are about the same in all magnitude groups. But in each of the groups the average of the 24 hourly means should be close to zero. On this assumption, the proper motions in right ascension provisionally derived from the difference between the Yale and the Cape positions (for a twenty-year interval) were corrected by $+0^{s}.0001(m - 6.5)$.

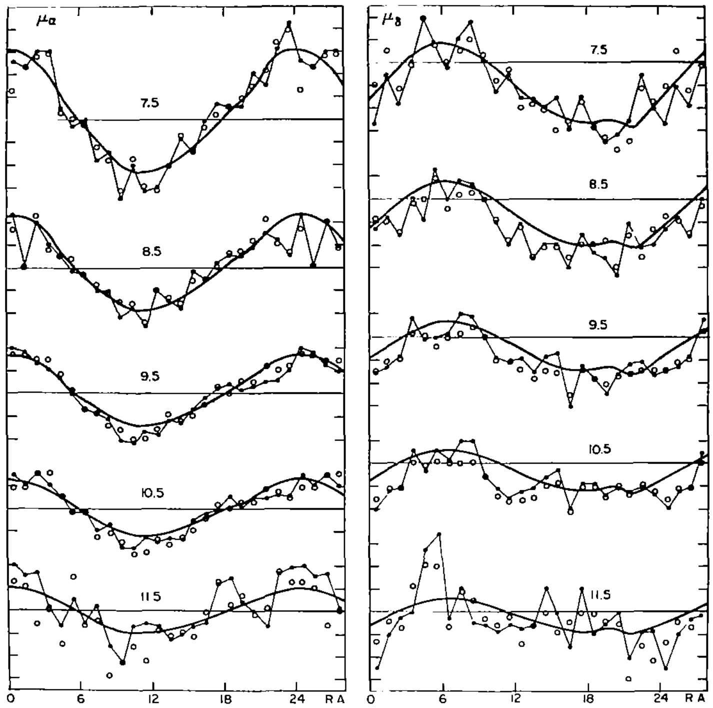

FIGURE 1
Expected trends of mean proper motions (curves) with recently derived values in the -35° to -40° zone.

In declination the pattern of the expected systematic motions is less simple and Schlesinger by-passed the problem by simply correcting the Yale proper motions for northern stars to the system of the Boss Preliminary General Catalogue available at that time. In the southern hemisphere, the proper motions given in the General Catalogue are less accurate than for the more widely observed northern stars and may not provide a reliable basis for magnitude corrections, nor is there a sufficient number of the fainter stars. Instead, I have compared the average Yale proper motions in both right ascension and declination with systematic motions derived from tables by van Rhijn and Bok (1931) based upon their analysis of statistical parallaxes. The comparisons are shown in Figure 1, where the curves represent the expected trends, the dots the Yale proper motions, and the open circles the weighted means of the Cape and Yale proper motions all ostensibly reduced to the FK4 system (Fricke et al, 1963) - a reduction depending on only 220 stars between -30° and -40° common to the FK4 and its Supplement. The reliability of this procedure depends upon the accuracy of the statistical parallaxes and on whether or not the percentages of stars with different spectral and luminosity classes in the various apparent magnitude groups are sufficiently the same in the van Rhijn-Bok analysis as in the Cape and Yale zone catalogues. It would seem highly important that the van Rhijn-Bok analysis be repeated with more modern data.

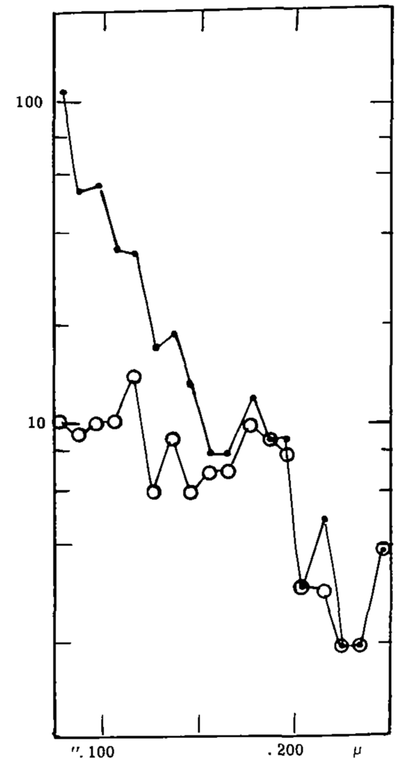

FIGURE 2
Frequency distribution of moderately large Yale proper motions, compared with frequency of those included in the BPM.

Table I. - Comparisons with Bruce Proper Motion Survey.

RA	n	m(pg) Cape	Δm C-BPM	Differences in Proper Motion: Yale - BPM α	δ	Mean - BPM n	α	p.e.	δ	p.e.	Solar Reflex α	δ
0 - 2	30	9.6	+0.8	+″.024	-″.014	25	+″.019	±″.019	-″.006	±″.020	+″.016	-″.005
6 - 8	24	9.8	-1.3	-.012	+.028	20	-.021	±.020	+.028	±.022	-.004	+.005
12 - 14	67	9.4	-0.9	-.010	+.001	49	-.011	±.027	-.003	±.024	-.013	-.007
18 - 20	32	9.2	-0.9	+.009	-.001	25	+.009	±.021	-.011	±.034	+.003	-.015

For a large number of stars proper motions exceeding $0''.100$ annually were found either at Yale or at the Cape, but not confirmed by the other. In many of these instances no Cape values were available because the stars did not appear in older catalogues. All such stars were searched in the Bruce Proper Motion Survey (Luyten, 1960) for confirmation. Unfortunately the absence of a star with ostensible proper motion less than $0''.200$ annually could not necessarily be considered proof that the star in fact has only a small proper motion. The completeness of the BPM is not well established either as to smallness of the proper motion or span of apparent magnitude included. A sampling in eight hours of right ascension, namely $0 - 2^h$, $6 - 8^h$, $12 - 14^h$ and $18 - 20^h$, indicated 111 stars for which both the Cape and Yale found motions exceeding $0''.100$ in either coordinate. Of these 76 stars or 69% are in the BPM; 18 others are in the GC; while 17 stars or 15% are neither in the BPM nor the GC. The total proper motions of the 17 stars range from $0''.100$ to $0''.182$. Their average Cape photographic magnitude is $9^m.48$, the individual values ranging from $8^m.56$ to $10^m.6$. These stars are in general too faint for inclusion in the GC, while their images might well be overexposed on the Harvard Bruce plates used in Luyten's survey.

Figure 2 shows the frequency distribution (on a logarithmic scale) of Yale total proper motions between $0''.070$ and $0''.250$ in the same intervals of right ascension. The numbers of these stars that also occur in the Bruce Proper Motion catalogue are represented by the open circles. The comparison indicates that practically all stars with proper motions exceeding $0''.150$ are included in the BPM but that the percentage drops to about 30% for total motions of $0''.100$. Further comparisons are given in Table I. It is noted that there are large discordances between the photographic magnitudes given by Luyten and those determined at the Cape, the average discordance in each of the four regions amounting to nearly a magnitude. Curiously the sign of the discrepancy in the first region is opposite from the other three; this region is near the galactic pole while the other three are all within 30° of the Milky Way. The table also suggests a systematic trend with right ascension of the difference between the Yale system and the BPM, trends apparently somewhat correlated with the reflex solar motion. This would be expected if the BPM values are relative while the Yale are absolute values of the proper motion.

The probable errors of the differences in proper motion in the various groups yield an average value, BPM minus weighted mean of Cape and Yale, amounting to $\pm 0''.0234$ in each coordinate. The average probable error of the weighted mean of the Cape and Yale values is approximately $0''.010$. This would indicate a probable error of $0''.021$ in the BPM values. As the stars under consideration here are mainly the brighter stars of the BPM survey, it may be expected that the accuracy of the BPM as a whole is somewhat higher. Luyten himself published a value of $0''.021$ as the average mean error estimated from overlapping plates.

In the search of the BPM it was noted that the $12^m.5$ star, BPM 46575, has approximately the same proper motion as the $8^m.4$ catalogue star no. 196. Four Harvard Bruce plates were borrowed to check on the common proper motion. The plates have been measured by Phillip Lü. His results and previous determinations (not reduced to a common system) are shown in Table II.

Table II. - A New Wide Double Star, sep. $50''.7$, P.A. 350°.

Star	m_{pg}	Proper Motion				Source
		RA	Dec	Total	P.A.	
Cape 196*	8.4	$+^{s}.0050$	$-''.114$	$''.128$	153°	Cape
		+.0048	-.111	.125	153	Yale
		+.0061	-.105	.127	146	GC
		-.0009	-.141	.141±12	183±8	Lü
BPM 46575	12.5	--	--	.127	159	BPM
		+.0017	-.107	.109±10	170±6	Lü

*CoD - 38°223 = CPD - 38°54

Attention is called to CoD - 35°4257, listed by the Cape (No. 3488) as $8^{m}.6$ vis, F5. This star could not be found on either of the two overlapping Yale plates which reach considerably fainter than 11^{m} pg. It was then noticed that the star had been suspected of variability in the Cordoba Durchmusterung (1894) where four visual magnitudes appeared discordant by $1^{m}.3$. I then checked the star on Harvard objective prism plates. The F5 spectral class evidently refers to a nearby star whereas the best available spectrum plate revealed in the position of CoD - 35°4257 a barely visible feature resembling an N-type spectrum. Two Yale students taking my course on variable stars, Susan Hess and Wayne Osborn, and I therefore went to Harvard to examine the star on Harvard patrol plates. On a red sensitive plate the star is extremely bright, confirming its assumed late-type spectrum. On the ordinary blue sensitive plates the star varies from brighter than 12^{m} to about $15^{m}.3$, with evidence of multiple periodicity char-racteristic of long period variables with carbon spectra. The primary period is about 500 days and a beat period is some 2500 to 2700 days.

One by-product of the zone catalogue work is the detection of widely separ-ated common proper motion stars, or group motions, from the proper motion star charts accompanying the newly-published catalogues. Among 29 such groups or pairs between -30° and -40° many warrant further investigation, particularly radial velocity determinations.

Acknowledgement

I am deeply indebted to a large number of persons both at the Yale Observatory and the Computer Center who have participated in the vast tasks of key punching input catalogue data, performing plate measurements, and computer programming necessary for the production of new zone catalogues, as well as to the Army Map Service for financial support.

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DISCUSSION

Stoy : *I am very pleased to see that through this work the positions in the -30° to -40° zone, which were formerly among the most weakly determined positions in the sky, have now become very well determined.*

COMPARISON OF THE SAO AND AGK3R STAR CATALOGUES

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ABSTRACT

The SAO Star Catalog positions from the GC and the AGK2 and Yale photographic catalogs are compared with the AGK3R catalog. Both catalogs are alleged to be on the system of the FK4. When the differences are considered as accidental errors, the mean errors deduced for positions from the SAO at the epoch 1958.5 range from ±0ʺ61 to ±0ʺ25. The systematic differences in right ascension between SAO and AGK3R positions to the north of 60° declination indicate that each SAO source represents the FK4 somewhat differently than the AGK3R.

THE SAO CATALOG

The Star Catalog, SAO Catalog, prepared by the Staff of the Smithsonian Astrophysical Observatory (1966) contains the positions and proper motions of 258 977 stars for the equinox and epoch 1950.0. This catalog was compiled by extracting positions, and proper motions when possible, from available sources and reducing them to the system of the FK4 by use of existing tables of systematic differences. This catalog is now widely used in the reduction of photographic plates of artificial satellites.

The principal sources for star positions used in preparing the SAO Catalog for the northern hemisphere were the Albany General Catalogue, (GC) and the AGK2 and Yale photographic catalogs. The epochs of observation of the source positions range from about 1890, for stars taken from the GC, to 1951, for stars taken from the most recent Yale catalog; thus, a wide range of uncertainty should be expected in star positions updated from the SAO Catalog. Considering all stars together, Haramundanis (1967) estimated that the mean standard deviations of SAO stars updated to 1964.5 would range from a minimum of about 0".25 in the zone +50° to +60° declination, to a maximum of about 0".6 in the zone +30° to +50° declination.

THE AGK3R CATALOG

The AGK3R Catalog contains the positions at the observed mean epoch 1958.5 of 21 499 northern reference stars evenly spaced on the surface of the celestial sphere from -5° to +90° declination. This catalog was compiled from upwards of 300 000 observations made with 11 northern meridian circles during the period 1956 to 1963. All observations were reduced on a nightly basis by the observers to the system of the FK3R and, later, to the FK4 by the observer himself or at the U.S. Naval Observatory. The final catalog was compiled by three successive approximations during which weights were developed for the results produced at each observatory. The mean errors of the final right ascensions and declinations are shown in Tables I and II.

Table I. - AGK3R: Mean Errors In Right Ascension

Unit: m.e. cosδ = 0^{s}0001

Declination	(C + S) All Stars				Z Stars, M≤8.8				Z Stars, M≥8.9			
	No. Stars	Avg.	50%	95%	No. Stars	Avg.	50%	95%	No. Stars	Avg.	50%	95%
- 5° to + 5°	2154	54	52	93	734	61	56	124	475	71	62	144
+ 5 +15	2168	50	48	88	703	56	51	110	501	63	58	128
+15 +20	1089	49	46	90	349	47	44	88	210	58	54	119
+20 +25	1010	48	46	89	342	51	49	95	258	56	50	113
+25 +40	2763	44	42	80	1187	47	44	91	507	48	45	99
+40 +50	1573	49	46	87	629	53	50	105	235	54	51	103
+50 +70	2252	45	43	78	852	47	44	94	317	43	41	87
+70 +90	802	49	47	85	261	51	48	99	132	48	43	96

All Stars: Avg. = 50, 50% = 47, 95% = 95.

In examining Tables I and II, it should be recalled that the AGK3R star list is composed of two lists; one selected at the U.S. Naval Observatory and the other, the KSZ list, selected at the Pulkovo and Sternberg Observatories. The C stars are those that were found to be in both star lists. The S and Z stars are those that remained in the U.S. Naval Observatory and KSZ lists, respectively, after removing the C stars. All C and S stars, and the bright Z stars in the zone +5° to +40° declination, were observed at least 10 times each during the program. The remaining Z stars were observed only eight times each due to the failure of one observatory to complete its commitment.

Another reason for listing the C, S, and Z stars separately is due to the manner in which commitments were made for their observations. In general, the C and S stars were observed twice on each of five different meridian circles.

Table II. - AGK3R: Mean Errors In Declination.
Unit: m.e. = 0".001

Declination	(C + S) All Stars				Z Stars, M≤8.8				Z Stars, M≥8.9			
	No. Stars	Avg.	50%	95%	No. Stars	Avg.	50%	95%	No. Stars	Avg.	50%	95%
- 5° to + 5°	2154	115	112	194	734	121	108	252	475	139	126	290
+ 5 +15	2168	108	103	186	703	108	102	204	501	127	116	264
+15 +20	1089	112	109	194	349	113	107	217	210	137	126	264
+20 +25	1010	113	107	204	342	102	94	201	258	125	117	250
+25 +40	2763	117	111	210	1187	103	95	196	507	121	114	237
+40 +50	1573	121	116	204	629	118	108	240	235	142	130	282
+50 +70	2252	116	111	201	852	105	96	204	317	116	105	240
+70 +90	802	134	129	225	261	124	112	259	132	133	124	272

All Stars: Avg. = 116, 50% = 109, 95% = 214.

The commitments to the Z stars required that they be observed four times each at either Bordeaux, Paris or Strasbourg, and twice each at two other observatories. Such division of effort could, possibly, cause small systematic differences between the final positions of the Z stars and other stars in the catalog. The division of Tables I and II into declination zones was made where some change took place in the combination of observatories involved in the observation of the AGK3R stars.

DATA FOR THE COMPARISON OF THE SAO AND AGK3R CATALOGS

An examination of these catalogs revealed that they had 21 377 stars in common. The common stars ranged in magnitude from 6.9 to 9.2, the average magnitude being close to 8.3 . It should be remarked that no FK4 stars are included in the comparison.

The SAO positions of the common stars were advanced to the epochs corresponding to their positions in the AGK3R by use of the proper motions given in the SAO. Differences were then computed in the sense, SAO-AGK3R and those in right ascension reduced to the equator through multiplication by $\cos\delta$. The symbols $\Delta\alpha\cos\delta$ and $\Delta\delta$, used hereafter, denote the differences in right ascension and declination, respectively.

ACCIDENTAL ERRORS

For this study the differences, $15\Delta\alpha\cos\delta$ and $\Delta\delta$, were assumed to have been caused by accidental errors in the positions in both catalogs and in the SAO proper motions. Following this assumption, all differences were collected into class intervals and the percentage of the total number of differences in each

Table III. - Accidental Errors.
Distribution of SAO-AGK3R Differences.

Zone	No. Stars	SAO Source Cat.	Epoch	50% $15\Delta\alpha \cos\delta$	50% $\Delta\delta$	68.26% $15\Delta\alpha \cos\delta$	68.26% $\Delta\delta$	95% $15\Delta\alpha \cos\delta$	95% $\Delta\delta$
$+90^\circ$ to -5°	21377	all	———	±″28	±″29	±″43	±″44	±0″97	±0″96
+90 to − 5	4681	GC	———	.40	.38	.61	.58	1.40	1.31
+90 tp +85	55	Y26-1	1951	.24	.23	.35	.32	0.58	0.63
+85 to +70	844	AGK2	1930	.28	.40	.43	.59	0.83	1.16
+70 to +60	988	AGK2	1930	.32	.36	.47	.53	0.97	1.13
+60 to +55	722	Y27	1947	.19	.17	.29	.25	0.57	0.53
+55 to +50	817	Y26-2	1947	.20	.17	.29	.25	0.52	0.55
+50 to +45	857	AGK2	1930	.34	.34	.50	.51	0.99	1.05
+45 to +40	1005	AGK2	1930	.34	.36	.51	.55	1.07	1.07
+40 to +35	1118	AGK2	1930	.30	.30	.44	.48	0.90	0.97
+35 to +30	1080	AGK2	1930	.26	.27	.39	.41	0.78	0.83
+30 to +25	1349	Y24	1929	.32	.26	.48	.39	0.93	0.76
+25 to +20	1360	Y25	1929	.20	.24	.30	.37	0.64	0.75
+20 to +15	1182	Y18	1940	.21	.24	.32	.36	0.67	0.71
+15 to +10	1386	Y19	1940	.24	.21	.36	.31	0.69	0.64
+10 to + 9	268	Y22-2	1940	.26	.30	.40	.44	0.76	0.92
+ 9 to + 5	1145	Y22-1	1936	.26	.28	.39	.41	0.76	0.82
+ 5 to + 1	904	Y20	1936	.24	.26	.35	.37	0.70	0.73
+ 1 to − 2	863	Y21	1936	.28	.32	.42	.47	0.85	0.91
− 2 to − 5	753	Y17	1933	.33	.29	.48	.43	0.96	0.88

class interval computed. These percentages were used to plot cumulative frequency curves from which the data in Table III were read. The first line of the table gives the data obtained by considering the entire 21 377 differences as one lot. The remainder of the table gives the data according to SAO source catalogs arranged according to declination. The interpretation to be given the data in the percentage columns may be illustrated by stating that 50%, 68.26% and 95% of the $15\Delta\alpha\cos\delta$ differences for which the GC was the source catalog, were less than or at most equal to ±0″40, ±0″61, and ±1″40, respectively.

It will be noted that the size of the numbers in the percentage columns is closely related to the interval from the epoch of the source catalog to 1958.5. This is in accordance with the well known fact that uncertainties in the proper motions are the principal cause of errors in the updated positions of the stars.

The ratios of the numbers in the percentage columns are approximately 2:3:6, as they should be for normal distributions.

Considering that the FK4 stars and all stars in Table III are mixed together in the SAO Catalog, and making some allowance for the epoch difference 1958.5 to 1964.5, one could conclude, without further computation, that Mrs. Haramundanis' estimates of the mean standard deviations of northern SAO stars are fairly well supported by modern observations.

SYSTEMATIC ERRORS

In this study the differences, $\Delta\alpha\cos\delta$ and $\Delta\delta$, were assumed to have been caused by failures of tables of systematic differences to reduce the various SAO source catalogs to the system of the FK4 in the same manner, or at least, by failures to reduce them in the same manner to the system represented by the AGK3R. To examine this assumption, the individual star differences for each SAO source catalog were averaged over three-hour groups of right ascension in each 5° zone of declination and their standard deviations computed. Tables IV and V show the collected results for all stars for which the GC was the source catalog. Tables VI and VII show corresponding results for all stars for which photographic star catalogues were the source catalogs.

The precision of the numbers in Tables IV and V is only about half that in Tables VI and VII for two reasons: (1) the ratio of the number of stars from the GC to the number from photographic catalog sources is approximately 1/3.57, and (2) proper motions had to be applied to the GC stars for a much longer interval to reduce them to 1958.5 than for the photographic stars.

If the AGK3R is assumed to truly represent the system of the FK4 in all parts of the sky then the following deductions may be made.

1. Table IV shows that the right ascensions north of 60° declination, obtained by reducing the GC to the FK4, are largely positive with respect to the AGK3R, often exceeding $\pm 0^{s}.02 \sec\delta$ in time. The weighted average column indicates a rather well defined term of the form $(\Delta\alpha\cos\delta)_{\delta}$.

2. Table V indicates that the reduction of the GC declinations to the FK4 is about as good as could be expected to the north of +30° declination. There are many residuals in the remainder of the sky which exceed twice their standard deviations but not enough to be of serious concern. The last column of this table suggest that there may be a small term of the form $\Delta\delta_{\delta}$ in comparison with the AGK3R. This term appears to be well defined to the south of +35° declination.

3. Tables VI and VII. The great number of underlined quantities in these tables strongly support the contention that they do, indeed, represent real systematic differences with respect to the AGK3R.

The most striking aspect of the comparison of the SAO positions from the GC and from photographic star catalogs with the AGK3R is the contrariness of algebraic signs, especially in the northern part of the sky. Since all quantities under comparison were reduced to the FK4 system by one route or another, one would expect that the results should be the same in a statistical sense.

The equivalency of the routes used for reducing the GC and photographic catalogs to the FK4 system may be tested by eliminating the AGK3R through a comparison of Tables IV and V with Tables VI and VII. The results of such a comparison are shown in Tables VIII and IX. In making the comparison, the standard deviation of each difference was computed and used as a basis for inserting the dotted and solid underlines in Tables VIII and IX. The significance of each type of underline is explained at the foot of each table.

Table VIII clearly indicates that the end results of reducing right ascensions of GC and photographic star catalogs to the system of the FK4 are not the same for stars north of 60° declination. The presence of a few dotted underlined quantities, as well as other quantities nearly as large, suggests a dissimilarity of end results in a few other declination zones, especially the AGK2 zone at 37.5° declination.

Although a number of differences in Table IX do exceed twice their standard deviations, there is not strong indication that different end results were obtained in reducing the GC and photographic catalog declinations to the FK4.

Table IV. - Systematic Differences, SAO-AGK3R, $\Delta\alpha\cos\delta$
SAO source: GC.

Decl.	Right Ascension $1^h.5$	$4^h.5$	$7^h.5$	$10^h.5$	$13^h.5$	$16^h.5$	$19^h.5$	Unit: $0^s.0001$ $22^h.5$	Wt'd Avg.
+85°	-008		-094		-026		+102		$-^s.0003$
+77.5	+251	+178	+109	+174	+253	+004	-154	+168	+.0121
+72.5	+238	+240	+184	-219	+232	-089	-039	+119	+.0093
+67.5	+261	+214	+221	-048	+122	+015	+105	+113	+.0136
+62.5	-132	-024	+150	+165	-041	-080	-030	+070	+.0007
+57.5	-122	-141	+061	-084	+016	+008	-004	-148	-.0054
+52.5	+035	-032	+023	-124	-139	+003	+040	+020	-.0011
+47.5	+052	-048	+080	-115	+055	+066	-035	+070	+.0017
+42.5	+076	-039	+041	-049	-125	-082	-030	-028	-.0033
+37.5	-039	-025	+009	-104	-029	+037	+129	+038	-.0001
+32.5	-210	-148	+043	-113	-054	+044	-031	-090	-.0070
+27.5	-006	-231	-008	-036	-050	-084	-045	-082	-.0071
+22.5	-067	+007	+013	-010	-005	-025	-127	-188	-.0052
+17.5	+036	+014	+038	-005	+088	+010	+066	+145	+.0045
+12.5	+026	-023	+080	-002	+023	+123	+016	+153	+.0048
+ 7.5	+049	+021	-018	-117	+044	+107	-059	+120	+.0009
+ 2.5	-009	-109	-109	-004	+027	-087	-040	+005	-.0037
- 2.5	-042	-098	-167	+044	-049	+077	-084	+024	-.0040

Underlined quantities: --- 2 s.d.<$|\Delta\alpha\cos\delta|\leq 3$ s.d., ___ $|\Delta\alpha\cos\delta|>3$ s.d.

Table V. – Systematic Differences, SAO-AGK3R, $\Delta\delta$
SAO source: GC.

Decl.	$1^h.5$	$4^h.5$	$7^h.5$	$10^h.5$	$13^h.5$	$16^h.5$	$19^h.5$	$22^h.5$	Wt'd Avg.
	Right Ascension								
+85°	–".03		–".03		+".06		–".03		".00
+77.5	–.18	.00	.00	+.13	+.12	–.03	+.12	–.13	.00
+72.5	+.01	.00	–.02	+.16	+.15	+.04	–.24	+.10	+.04
+67.5	+.17	+.27	–.04	–.08	–.05	–.21	+.04	–.04	+.02
+62.5	+.07	+.21	+.03	+.07	+.04	+.05	+.08	–.01	+.06
+57.5	–.07	–.09	–.13	–.05	.00	+.03	–.02	+.14	–.02
+52.5	.00	–.17	+.04	–.18	+.21	–.05	+.10	–.08	–.02
+47.5	+.06	+.03	–.08	–.11	–.02	–.03	+.01	–.01	–.02
+42.5	+.08	+.08	–.03	+.05	–.05	–.14	–.16	–.02	–.03
+37.5	–.14	+.03	–.02	–.06	+.01	–.24	–.04	+.04	–.05
+32.5	.00	+.05	+.09	+.25	+.03	+.05	+.14	+.16	+.09
+27.5	+.01	+.03	+.24	+.04	+.07	+.08	+.13	+.36	+.13
+22.5	+.10	–.01	–.01	+.01	–.03	–.28	–.07	+.07	–.02
+17.5	.00	–.16	+.02	–.05	–.10	–.23	+.01	+.04	–.06
+12.5	–.05	+.10	–.05	–.03	+.04	–.01	+.03	–.09	.00
+ 7.5	+.06	+.16	+.01	+.06	+.01	+.03	.00	+.24	+.06
+ 2.5	–.13	–.06	–.13	–.04	–.03	–.23	.00	–.10	–.09
– 2.5	–.19	–.18	–.15	+.04	–.04	+.01	+.09	–.12	–.07

Underlined quantities: ___ 2 s.d.<|$\Delta\delta$|≤3 s.d., ____|$\Delta\delta$|>3 s.d.

Table VI. – Systematic Differences, SAO-AGK3R, $\Delta\alpha\cos\delta$.
SAO Sources: Yale and AGK2.

Decl.	SAO Source Cat.	Epoch	$1^h.5$	$4^h.5$	$7^h.5$	$10^h.5$	$13^h.5$	$16^h.5$	$19^h.5$	$22^h.5$	Wt'd Avg.
			Right Ascension Unit: $0^s.0001$								
+87°.5	Y26-1	1951	–139		+150		+240		+070		+^{s}0075
+82.5	AGK2	1930	–137	–046	–232	–187	–397	–140	–233	–035	–.0168
+77.5	AGK2	1930	–064	–054	–160	–144	–124	–094	–102	–035	–.0098
+72.5	AGK2	1930	+065	–005	+026	+067	–122	–108	+072	–057	–.0006
+67.5	AGK2	1930	–074	–044	–031	–102	–083	–099	–002	–074	–.0064
+62.5	AGK2	1930	–183	–230	–191	–262	–123	–133	–193	–199	–.0188
+57.5	Y27	1947	–192	–046	+183	+027	–060	+022	+060	–053	–.0006
+52.5	Y26-2	1947	+122	+069	+068	–106	–084	+113	+071	+017	+.0032
+47.5	AGK2	1930	–022	+043	+140	–043	–047	–023	–069	–032	–.0006
+42.5	AGK2	1930	+015	+069	+080	–050	+042	–017	–102	–060	–.0002
+37.5	AGK2	1930	–083	+142	+100	+062	+040	+029	–067	–072	+.0020
+32.5	AGK2	1930	–086	–062	–043	–039	+038	–022	+033	–152	–.0039
+27.5	Y24	1929	–047	–077	+073	+054	+078	+181	+084	–060	+.0034
+22.5	Y25	1929	–056	–027	+028	–005	–016	+084	–018	–143	–.0018
+17.5	Y18	1940	+096	+041	+062	+030	+084	+174	+072	+004	+.0069
+12.5	Y19	1940	+067	–034	+050	–013	+011	+025	+066	+105	+.0035
+10.0	Y22-2	1940	+093	+052	–121	–062	+155	+054	–188	–043	–.0009
+ 7.0	Y22-1	1936	+029	–014	–033	–119	+008	+035	–048	–035	–.0021
+ 3.0	Y20	1936	–027	–057	–133	–019	–012	–026	–022	–023	–.0041
– 0.5	Y21	1936	+034	–116	–241	–081	–041	–067	–150	–112	–.0097
– 3.0	Y17	1933	–164	–103	–127	–102	+004	+018	–037	–002	–.0066

Underlined quantities: ____|$\Delta\alpha\cos\delta$|≥3 s.d.

Table VII. - Systematic Differences, SAO-AGK3R, Δδ.
SAO Sources: Yale and AGK2.

Decl.	SAO Source Cat.	Epoch	$1^h.5$	$4^h.5$	$7^h.5$	$10^h.5$	$13^h.5$	$16^h.5$	$19^h.5$	$22^h.5$	Wt'd Avg.
+87°.5	Y26-1	1951	+".16		-".04		-".17		-".02		-".01
+82.5	AGK2	1930	-.14	-.39	-.17	-.32	-.56	-.09	-.34	-.18	-.26
+77.5	AGK2	1930	-.11	-.18	-.10	+.06	-.20	+.04	+.12	-.17	-.07
+72.5	AGK2	1930	-.06	-.11	-.29	-.39	-.29	+.14	+.20	-.06	-.11
+67.5	AGK2	1930	+.10	+.01	+.23	+.02	-.14	-.13	+.04	-.06	+.01
+62.5	AGK2	1930	+.08	-.25	-.03	-.07	-.14	-.14	+.08	-.02	-.06
+57.5	Y27	1947	.00	-.07	+.09	+.06	+.05	.00	+.16	+.03	+.04
+52.5	Y26-2	1947	-.02	-.05	-.01	+.08	+.04	-.03	+.04	-.04	.00
+47.5	AGK2	1930	-.18	-.11	+.21	+.11	+.17	+.16	+.12	+.09	+.07
+42.5	AGK2	1930	-.28	-.06	-.03	+.04	-.01	-.04	-.02	-.04	-.06
+37.5	AGK2	1930	.00	-.03	-.06	-.13	-.05	-.13	-.09	+.21	-.03
+32.5	AGK2	1930	-.13	-.06	+.05	+.14	+.12	-.19	-.11	+.02	-.02
+27.5	Y24	1929	-.08	+.20	+.12	+.19	+.09	-.13	-.01	+.13	+.06
+22.5	Y25	1929	-.04	+.15	+.05	+.19	.00	-.14	-.09	+.05	+.02
+17.5	Y18	1940	+.03	+.01	-.08	-.06	-.19	-.07	-.06	+.01	-.05
+12.5	Y19	1940	.00	+.04	+.01	-.02	+.08	-.04	-.04	-.05	.00
+10.0	Y22-2	1940	+.24	+.21	+.01	-.18	+.33	.00	+.05	+.22	+.11
+ 7.0	Y22-1	1936	.00	-.04	+.15	+.05	+.06	+.11	+.14	-.06	+.05
+ 3.0	Y20	1936	-.17	+.01	-.22	-.05	-.25	-.10	-.05	-.08	-.11
- 0.5	Y21	1936	-.11	-.18	-.26	-.14	-.29	-.26	-.13	-.03	-.18
- 3.0	Y17	1933	-.05	+.01	+.10	-.02	.00	+.26	+.20	+.04	+.07

Underlined quantities: ____ $|\Delta\delta| \geq 3$ s.d.

Table VIII. - (GC-Photographic), Δαcosδ
SAO Sources

Decl.	GC Minus	$1^h.5$	$4^h.5$	$7^h.5$	$10^h.5$	$13^h.5$	$16^h.5$	$19^h.5$	$22^h.5$	Wt'd Avg.
		Right Ascension				Unit: $0^s.0001$				
+77°.5	AGK2	+315	+232	+269	+318	+377	+098	-052	+203	$+^s.0219$
+72.5	AGK2	+173	+245	+158	-286	+354	+019	-111	+176	+.0099
+67.5	AGK2	+335	+258	+252	+054	+205	+114	+107	+187	+.0200
+62.5	AGK2	+051	+206	+341	+427	+082	+053	+163	+269	+.0195
+57.5	Y27	+070	-095	-122	-111	+076	-014	-064	-095	-.0048
+52.5	Y26-2	-087	-101	-045	-018	-055	-110	-031	+003	-.0043
+47.5	AGK2	+074	-091	-060	-072	+102	+089	+034	+102	+.0023
+42.5	AGK2	+061	-108	-039	+001	-167	-065	+072	+032	-.0031
+37.5	AGK2	+044	-167	-091	-166	-069	+008	+196	+110	-.0021
+32.5	AGK2	-124	-086	+086	-074	-092	+066	-064	+062	-.0031
+27.5	Y24	+041	-154	-081	-090	-128	-265	-129	-022	-.0105
+22.5	Y25	-011	+034	-015	-005	+011	-109	-109	-045	-.0034
+17.5	Y18	-060	-027	-024	-035	+004	-164	-006	+141	-.0024
+12.5	Y19	-041	+011	+030	+011	+012	+098	-050	+048	+.0013
+ 7.0	Y22-1	+020	+035	+015	+002	+036	+072	-011	+155	+.0030
+ 3.0	Y20	+018	-052	+024	+015	+039	-061	-018	+028	+.0004
- 3.0	Y17	+122	+005	-140	+146	-053	+059	-047	+026	+.0026

Underlined quantities: _ _ _ 2 s.d.$<|\Delta\alpha\cos\delta| \leq 3$ s.d., ____ $|\Delta\alpha\cos\delta| > 3$ s.d.

Table IX. - (GC-Photographic), Δδ.
SAO Sources.

Decl.	GC Minus	Right Ascension $1^{h}.5$	$4^{h}.5$	$7^{h}.5$	$10^{h}.5$	$13^{h}.5$	$16^{h}.5$	$19^{h}.5$	$22^{h}.5$	Wt'd Avg.
+77°.5	AGK2	-".07	+".18	+".10	+".07	+".32	-".07	".00	+".04	+".07
+72.5	AGK2	+.07	+.11	+.27	+.55	+.44	-.10	-.44	+.16	+.15
+67.5	AGK2	+.07	+.26	-.27	-.10	+.09	-.08	.00	+.02	+.01
+62.5	AGK2	-.01	+.46	+.06	+.14	+.18	+.19	.00	+.01	+.12
+57.5	Y27	-.07	-.02	-.22	-.11	-.05	+.03	-.18	+.11	-.06
+52.5	Y26-2	+.02	-.12	+.05	-.26	+.17	-.02	+.06	-.04	-.02
+47.5	AGK2	+.14	+.14	-.29	-.22	-.19	-.19	-.11	-.10	-.09
+42.5	AGK2	+.36	+.14	.00	+.01	-.04	-.10	-.14	+.02	+.03
+37.5	AGK2	-.14	+.06	+.04	+.07	+.06	-.11	+.05	-.17	-.02
+32.5	AGK2	+.13	+.11	+.04	+.11	-.09	+.24	+.25	+.14	+.11
+27.5	Y24	+.09	-.17	+.12	-.15	-.02	+.21	+.14	+.23	+.07
+22.5	Y25	+.14	-.16	-.06	-.18	-.03	-.14	+.02	+.02	-.04
+17.5	Y18	-.03	-.17	+.10	+.01	+.09	-.16	+.07	+.03	-.01
+12.5	Y19	-.05	+.06	-.06	-.01	-.04	+.03	+.07	-.04	.00
+ 7.0	Y22-1	+.06	+.20	-.14	+.01	-.05	-.08	-.14	+.30	+.01
+ 3.0	Y20	+.14	-.07	+.09	+.01	+.22	-.13	+.05	-.02	+.02
- 3.0	Y17	-.14	-.19	-.25	+.06	-.04	-.25	-.11	-.16	-.14

Underlined Quantities: ___ 2 s.d. < |Δδ| ≤ 3 s.d., ___ |Δδ| > 3 s.d.

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DISCUSSION

Dieckvoss : *I think the reductions of the SAO Catalogue to the FK4 system were obtained through GC and FK3.*

Eichhorn : *This was the only possible way since at the beginning of the work on the SAO Star Catalogue, the FK4 had not yet been available. Thus, according to the availability of reduction tables, the material was first reduced to the FK3 system, and after publication of the FK4, to the FK4 system.*

Mueller : *Which criterion did you use to decide whether a systematic difference was significant or not?*

Smith : *Differences were regarded as significant if they exceeded between two and three times the dispersion of the differences. This also takes into account the uncertainty of the AGK3R positions.*

Scott : *At present, this is quite a rough procedure, but Dr. Smith is working on a representation of the observed differences between SAOC and AGK3R in spherical harmonics. He is then going to apply rigorous statistical procedures to decide whether a term should be retained or not. This requires a large amount of computer time, which is not always available.*

Veis : *The GC positions, which were less accurate, were retained (after reduction to the FK4 system) and not replaced by positions from other sources, at the request of several investigators, who use GC positions in their work.*

PLANS FOR A STANDARD REGION FOR LONG FOCUS ASTROGRAPHS

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ABSTRACT

The construction of a catalog of precise star positions in the region of Praesepe is proposed. A list of the 408 stars to be included is given. The program stars were chosen on the basis of their position, magnitude and color, and the availability of accurate early epoch positions. The present status of the project is noted.

A very accurate catalog, of a small densely populated region of stars, free from sensible systematic effects would be of use in determining the geometry of projection of long focus photographic telescopes, and in determining whether the imaging characteristics, i.e. scale, "magnitude equation", coma, "color equation", color magnification, tilt, distortion, etc. are functions of time or ambient conditions. A catalog of this type will enable one to determine for instance what effects if any are introduced by removing, cleaning, and replacing the objective (Lippincott, 1957) of a telescope.

The region of Praesepe was chosen because of the large number of early epoch positions available and because of the wide range of color indices of its brighter stars. It is observable from most southern hemisphere observatories as well. Selection of catalog members was based on four criteria;

1.) the star's inclusion in previous reference catalogues
2.) the availability of highly accurate early epoch relative positions
3.) freedom from close companions
4.) color and magnitude

About one half of the program stars are cluster members (Klein-Wassink, 1927), the rest were included to weaken the natural correlation between magnitude

Table I. - Transit Stars.

No.	BD#	m_V	No.	BD#	m_V
57	20 2132	7.79	199	20 2166	6.44
65	19 2053	6.75	209	20 2171	6.30
80*	20 2136	7.83	223*	19 2073	7.80
110	20 2143	7.45	230	20 2172	6.85
113	20 2144	8.50	246	20 2175	6.78
142	20 2149	6.67	286	20 2182	8.50
145*	19 2064	7.67	297*	20 2185	6.90
180	20 2158	6.39	301*	20 2186	8.53
186	20 2159	6.61	311*	19 2083	7.96

and color, or because of individual interest. UBV magnitudes exist for many of the catalog stars (Johnson, 1952). New BV magnitudes for selected catalog members are being obtained by one of the authors (L.W.F.).

Eighteen stars have been selected for reobservation on transit circles (Table I). Nine of these stars appear in the Preliminary General Catalogue, two in the AGK3R, and the rest have been selected by their magnitude and position within the region. The Six Inch Transit Division of the U.S. Naval Observatory has agreed to observe eleven of these stars. It is hoped that other observatories will observe a similar number, especially those six stars, whose numbers are followed by an asterisk.

This study, it is hoped, will receive the backing of many observatories with various size astrographs. Measuring will be a cooperative effort of the Leander McCormick Observatory and the University of South Florida. The reductions will be carried out using techniques proven to eliminate systematic errors as much as possible (Eichhorn and Gatewood, 1967; Eichhorn, Googe, Lukac and Murphy,1970).

Overlapping plates have been or are being taken with the U.S. Navy 26-inch and 61-inch telescopes, the Yerkes 40-inch refractor, the Sproul refractor, the Leiden astrograph, the Lick astrograph, the University of South FLorida 26-inch astrograph, as well as at Leander McCormick Observatory where an almost continuous study of that telescope has been under way since 1955 (Eichhorn, 1956, 1968; Fredrick, 1969; Hershey, 1967; Knappenberger, 1967; Gatewood, 1967). Measurement of these plates has begun at both observatories.

In the following catalogue column one lists the catalog number, column two is the visual magnitude, and column three the color index. Columns four and five are the standard coordinates in units of 1" with respect to the tangential point $\alpha_0 = 8^h37^m30^s.0$, and $\delta_0 = 19°50'52"$ (equinox 1950). The root mean square errors are, for visual magnitude: $0^m.13$ and for B-V: $0^m.16$ and were found from a comparison of the values listed here and by H. Johnson(1952).

1950

CATALOG

$8^h37^m30^s.0$ 19°50'52"

#	Mv	C	ξ	η
1	8.1	.3	-6516	1758
2	14.4	1.4	-6498	1756
3	5.7	1.0	-6461	2817
4	8.0	.7	-6224	-2371
5	15.7	.2	-6041	1365
6	14.5	1.4	-5650	-2035
7	14.6	1.4	-5646	-2037
8	12.1	.4	-5413	3806
9	9.5	.6	-5403	-1434
10	15.9		-5393	-1435
11	7.3	1.2	-5107	-4558
12	12.9	.7	-4870	5177
13	12.2	.8	-4862	5170
14	8.5	.8	-4751	3232
15	13.7	.7	-4636	-2140
16	13.2	1.0	-4533	-3857
17	9.1	.7	-4506	-528
18	10.1	1.3	-4463	3579
19	10.0	1.0	-4426	-1854
20	13.1	.7	-4323	3621
21	8.5	.3	-4274	3235
22	6.5	.7	-4275	-270
23	14.6	-.1	-4238	454
24	11.2	.2	-4230	453
25	8.2	.1	-3921	2353
26	9.0	.0	-3901	4254
27	7.1	.4	-3909	379
28	14.2	.7	-3879	2649
29	12.0	1.1	-3514	-2773
30	12.5	.5	-3788	5041
31	14.0	.7	-3667	-1986
32	13.1	1.5	-3661	-1986
33	7.5	.0	-3440	2430
34	11.1	.4	-3470	-3565
35	13.5	.9	-3460	-3575
36	15.3	.4	-3291	-853
37	9.3	.7	-3293	-2524
38	14.3	1.3	-3257	-2373
39	9.8	1.1	-3229	-2928
40	8.4	.4	-3036	-2824
41	13.9	1.3	-3023	-1475
42	11.4	.9	-3016	-1476
43	12.7	.2	-2815	4254
44	15.3	.6	-2807	4295
45	9.2	.5	-2825	-226
46	15.3	.9	-2631	-2871
47	8.4	.4	-2608	-2873
48	13.9	1.1	-2587	-3901
49	14.7	1.3	-2586	-3905
50	11.4	.7	-2531	1833
51	15.3	-.1	-2533	-4616
52	8.1	.0	-2469	2948
53	5.7	.6	-2460	-379
54	9.5	.5	-2462	-1821
55	15.1	1.1	-2445	-658
56	8.7	.3	-2370	1244
57	7.8	.2	-2329	232
58	12.9	.9	-2305	1126
59	14.6	1.4	-2302	1102
60	10.7	.4	-2289	313
61	8.3	.2	-2280	-539
62	10.4	.6	-2273	232
63	9.8	.5	-2262	-1924
64	10.7	.6	-2158	-828
65	6.8	.2	-2197	-1444
66	10.9	.5	-2120	2298
67	13.0	.3	-1956	850
68	14.5	.8	-1975	-5217
69	14.3	1.0	-1968	-3501
70	12.0	1.6	-1970	-5212
71	12.8	.0	-1940	1661
72	9.4	1.6	-1933	1661
73	12.3	.7	-1943	-1746
74	12.2	.5	-1879	4170
75	15.4	.7	-1870	204
76	14.6	.3	-1868	-3518
77	11.7	.4	-1818	384
78	11.0	.7	-1805	-422
79	10.9	.7	-1807	-1092
80	7.8	.5	-1675	1938
81	12.3	.7	-1674	-107
82	10.6	.6	-1658	1574
83	11.2	1.1	-1654	1409
84	11.3	.6	-1557	-2020
85	14.0	1.3	-1585	657
86	11.6	.5	-1566	-551
87	10.0	1.0	-1554	-733
88	5.0	1.4	-1520	688
89	12.5	.9	-1503	188
90	12.0	.6	-1500	152
91	10.5	.7	-1473	-79
92	13.0	.9	-1477	-1930
93	8.1	.2	-1468	1154
94	13.4	.5	-1471	-1299
95	10.6	.7	-1429	4083
96	9.0	.3	-1343	-605
97	11.8	.6	-1294	-319
98	11.5	.4	-1260	1266
99	10.1	.9	-1231	-1695
100	14.0	1.3	-1193	1844
101	12.3	.8	-1185	-1486
102	11.2	.3	-1170	-881
103	9.4	.7	-1127	4028
104	11.9	1.2	-1131	1211
105	12.8	.8	-1121	448
106	9.3	.5	-1117	199
107	14.4	1.4	-1110	1348
108	8.3	.2	-1105	1189
109	9.4	.4	-1081	1612
110	7.5	.3	-1071	27
111	13.9	.6	-1060	-1183
112	8.6	.8	-1041	4062
113	8.5	.3	-1030	-276
114	8.7	.6	-1031	-2583
115	9.4	.4	-1014	32
116	11.4	1.0	-999	-2643
117	14.6	1.0	-987	-446
118	10.6	.6	-989	-1993
119	13.7	.4	-981	-561
120	14.8	.9	-967	191
121	14.9	1.5	-933	1444
122	14.9	.9	-920	-810
123	13.0	1.3	-891	472
124	9.2	1.1	-879	1381
125	14.8	.6	-879	480
126	12.5	.9	-848	690
127	10.5	.6	-806	-756
128	12.5	.2	-787	1495
129	14.1	.8	-779	-490
130	12.6	1.0	-742	422
131	10.3	.7	-727	1438
132	12.5	.3	-707	-73
133	11.0	.3	-657	-1246
134	9.1	1.3	-684	1800
135	13.4	.2	-646	1223
136	13.7	-.1	-642	1219
137	13.1	.5	-638	-259
138	15.4	.8	-619	2842
139	13.8		-617	-821
140	13.2	.2	-571	-72
141	14.0	.2	-568	-70
142	6.7	.2	-555	392
143	7.7	.2	-552	1500
144	11.7	1.2	-546	-893
145	7.7	.2	-528	-1419
146	10.7	.6	-512	-1089
147	12.7	1.0	-493	569
148	13.6	.5	-453	730
149	8.6	1.0	-442	-462
150	11.8	.8	-440	-427
151	14.0	.3	-433	-383
152	10.2	.5	-419	-1284
153	11.4	.4	-413	1001
154	13.9	.5	-414	-1841
155	10.5	.8	-404	3131
156	9.4	.1	-389	3207
157	13.1	.2	-387	548
158	10.1	.5	-379	1423
159	12.9	.6	-359	4279
160	16.4	.5	-361	-87
161	7.3	.2	-360	-419
162	15.1	.9	-357	167
163	13.6	.9	-358	-3025
164	7.5	.3	-342	-461
165	9.5	.4	-338	-1684
166	8.9	.3	-333	1759
167	9.2	.4	-323	1302
168	12.8	1.0	-311	-329
169	10.3	.5	-301	493
170	9.7	.4	-291	1678
171	10.2	.5	-287	-2411
172	11.1	.9	-249	1881
173	14.6	1.0	-244	315
174	9.8	.5	-241	215
175	12.8	.5	-235	1855
176	15.2	.8	-229	222
177	12.8	.8	-228	-49
178	14.1	.3	-227	-308
179	10.2	.6	-222	-775
180	6.4	1.0	-219	1217
181	15.7	.9	-213	45
182	9.6	.0	-200	5016
183	13.0	1.3	-192	2534
184	14.6	.6	-188	2540
185	12.0	.8	-174	-173
186	6.6	.0	-148	1085
187	9.9	.5	-136	-108
188	13.1	1.1	-120	393
189	14.7	1.1	-106	246
190	8.8	.3	-93	1169
191	12.7	1.0	-88	883
192	12.4	.1	-67	1469
193	10.0	.6	-61	424
194	7.5	.2	-54	-494
195	14.7	.6	-46	306
196	6.8	.3	-26	-1154
197	15.1	.5	-23	-372
198	7.7	.2	-17	61
199	6.4	1.0	1	1
200	10.1	.5	4	1574
201	14.5	.4	5	-12
202	15.2	.6	9	229
203	10.7	.6	10	-737
204	10.4	.6	17	13
205	8.0	.2	20	595
206	9.9	.5	50	-697
207	9.4	.4	58	60
208	8.2	.3	66	1844
209	6.3	.2	70	-449
210	13.2	1.1	77	-50
211	14.0	1.2	110	1724
212	14.0	.3	120	-941
213	15.3	1.0	134	92
214	8.7	.3	155	-1710
215	12.2	.9	161	-129
216	14.4	1.0	165	-2570
217	15.5	.8	166	-2574
218	13.0	.6	184	-488
219	11.6	1.3	189	-1738
220	13.7	.2	189	373
221	14.8	.5	214	290
222	11.7	.3	231	843
223	7.8	.2	244	-1588
224	10.9	.7	252	-1
225	15.2	.7	273	11
226	14.7	.3	274	-263
227	10.6	.6	282	-1605
228	11.3	.7	289	-373
229	11.8	.5	290	1242
230	6.9	.2	259	178
231	13.9	.3	334	395
232	9.6	.4	341	-1295
233	15.7	.4	350	-436
234	10.5	.8	355	-453
235	11.0	.7	366	-38
236	11.0	.7	371	908
237	11.5	.5	384	-1946
238	13.3	.9	401	574
239	15.3	.7	415	-248
240	8.5	.3	428	2149
241	10.3	.5	431	-670
242	15.6		437	2136
243	16.0	.2	448	-1132
244	14.3	.4	452	-1166
245	13.4	.5	453	-1752
246	6.8	.2	484	-321
247	15.0		486	564
248	12.2	.9	489	235
249	8.7	.3	492	955
250	11.2	.7	603	-528
251	13.2	.9	609	277
252	15.6	.3	619	3933
253	11.8	.4	628	-1077
254	12.4	.9	640	-801
255	10.2	.6	643	-2154
256	13.8	.4	643	-227
257	10.7	.7	671	668
258	13.6	1.2	673	957
259	13.7	.2	676	1462
260	10.1	.5	679	-578
261	11.7	.5	681	536
262	9.1	.3	686	576
263	13.3	.2	713	1616
264	13.6	-.3	717	-694
265	14.7	1.5	717	1612
266	8.3	.2	729	909
267	6.7		747	2906
268	14.3	1.4	752	1326
269	13.7	.8	762	281
270	13.4	.7	774	-263
271	7.9	.2	798	-1470
272	12.0	.4	843	1120
273	12.8	.7	863	-3505
274	14.7	1.5	870	2087
275	11.6	.2	871	4161
276	10.7	.6	900	987
277	15.9	.0	909	-458
278	9.8	.5	918	-456
279	15.5	.8	924	-655
280	13.2	1.0	942	-3349
281	10.9	.6	941	278
282	13.6	.9	1005	767
283	15.6	.4	1012	33
284	11.7	.8	1012	1079
285	12.5	.6	1025	223
286	8.5	1.1	1031	-24
287	9.3	.4	1051	-1896
288	15.4	.3	1061	793
289	9.8	.4	1094	3611
290	11.3	.4	1100	4866
291	9.6	.4	1134	-31
292	13.5	.8	1178	-347
293	10.2	.5	1182	-1447
294	11.5	.2	1201	-1934
295	12.2	.3	1211	-1158
296	11.4	.5	1213	-925
297	6.9	1.0	1242	737
298	13.3	.2	1235	3691
299	15.5	.1	1265	119
300	12.0	.8	1266	1792
301	8.5	.3	1283	1765
302	11.4	.7	1308	-1482
303	11.0	.4	1313	-124
304	11.1	.6	1325	73
305	9.3	.6	1360	-2727
306	12.7	.5	1372	890
307	12.3	.7	1380	3478
308	14.4	.1	1416	465
309	14.2	.4	1429	-616
310	10.6	.9	1461	-249
311	8.0	.2	1478	-928
312	7.8	.5	1544	-2645
313	13.2	1.1	1540	1713
314	9.9	.5	1603	67
315	11.5	.2	1632	194
316	10.8	.9	1677	-3610
317	9.7	.6	1664	1323
318	9.2	.4	1683	1846
319	12.1	.9	1699	1618
320	11.0	.5	1733	-289
321	11.0	.6	1843	-981
322	13.4	.1	1854	910
323	12.7	.2	1920	-1776
324	9.8	.4	1961	-451
325	11.6	.8	1591	-2048
326	15.2	.5	1592	-65
327	9.7	.4	2012	-319
328	13.5	.7	2024	-672
329	15.6	.9	2026	-663
330	12.2	.5	2034	1194
331	15.4	1.1	2041	1198
332	11.7	.7	2067	-724
333	12.3	.4	2093	-1186
334	9.0	.0	2117	4143
335	13.5	.5	2149	659
336	12.5	-.1	2252	-19
337	12.0	.5	2297	-1353
338	11.2	.2	2299	-264
339	9.9	.7	2319	-830
340	9.6	.5	2340	-2160
341	12.3	.3	2364	-3993
342	11.3	.2	2397	2041
343	12.3	.3	2413	2026
344	8.5	1.5	2458	3356
345	14.2	.8	2478	-1090
346	13.5	.9	2483	-1082
347	12.5	1.0	2486	581
348	10.8	.6	2515	363
349	14.1	1.0	2594	-2695
350	14.7	.9	2620	-3889
351	13.7	.5	2626	-3889
352	10.1	1.5	2738	-5106
353	10.1	.5	2724	1878
354	11.1	.2	2833	1830
355	8.3	.1	3036	-2280
356	8.3	.8	3324	4357
357	14.4	.8	3371	2047
358	12.6	.3	3378	2040
359	9.6	1.0	3458	1070
360	16.3	.0	3505	-5045
361	15.4	-.1	3544	-4973
362	14.0	.5	3552	2245
363	15.4	.7	3597	-4853
364	14.5	.7	3557	2241
365	12.4	.8	3728	113
366	9.4	1.3	3725	821
367	11.2	.7	3737	111
368	13.5	.0	3749	4330
369	13.1	1.0	4085	-3672
370	9.7	.0	4099	4797
371	8.4	.3	4238	-2844
372	9.7	.7	4289	558
373	12.0	.3	4286	1073
374	13.7	.9	4291	1754
375	8.6	.1	4306	2629
376	6.8		4370	-3052
377	9.8	.6	4332	3331
378	15.6	-.1	4527	219
379	8.0		4602	-2215
380	7.8	.3	4621	1289
381	15.4	.8	4637	1269
382	9.9	1.1	4867	-1355
383	15.1	.2	4874	-1361
384	9.9		4973	-1481
385	14.7	1.1	4966	162
386	8.4	.4	4992	163
387	7.1	.2	5029	3432
388	7.6	.5	5051	2701
389	14.5	.6	5154	4287
390	15.8	.1	5184	4263
391	11.0	.8	5353	-1538
392	10.7	.7	5362	-1545
393	11.0	.1	5352	3271
394	14.6	.4	5362	3267
395	10.0	1.2	5410	553
396	9.1	1.2	5454	621
397	8.9	.2	5464	2332
398	15.6	.3	5711	-15
399	15.8	.4	5746	-111
400	10.4	.4	5804	89
401	11.6	.8	5861	1307
402	12.6	.4	5847	4217
403	13.7	.5	5850	4211
404	15.5	.0	6007	-448
405	11.0	.4	6155	-732
406	11.7	.8	6200	-2150
407	13.6	1.0	6592	-3837
408	9.7	.1	6613	-3800

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DISCUSSION

Jefferys : *I take it that you will want plates from many different places for the construction of the test field position catalogue. Do you want these plates taken in some kind of an overlapping pattern?*

Gatewood : *It might be better if the pattern was rather random. A short focus instrument, for instance, might be centered on the cluster, and thus provide a homogeneous frame of reference with which an overlap solution from the other material could be compared.*

Luyten : *Sandage and I have a plate covering the whole field of 40 square degrees taken at the Palomar 48" Schmidt which contains Praesepe and goes to about 19^m.*

Gatewood : *This would be useful.*

Strand : *It seems to me that the stars you are interested in are all too bright for the Astrometric Reflector.*

Gatewood : *Dr. Riddle of the Naval Observatory pointed this out to me, and immediately thereafter we added a list of 15^m and 16^m stars.*

Murray : *What baseline in time is available for plates that have galaxies recorded?*

Gatewood : *I believe Dr. Strand's plates were taken sometime in the fifties, and they are the earliest. Therefore, galaxies will not be available for proper motion reductions for quite a while yet.*

Vasilevskis : *I made attempts to use galaxies in proper motion reductions from plates taken with the Crossley reflector by Keeler and others. Unfortunately, the old plates were taken on the telescope that was almost never properly collimated and was poorly guided. The best images were usually found, not in the center, but somewhere on one side due to the combined effect of coma and guiding. The determination of proper motions by means of referring to the galaxies requires that the plate be repeated under exactly the same conditions. However, while it is difficult to repeat a good plate, it is impossible to exactly repeat a bad one.*

CAPE PHOTOGRAPHIC CATALOGUES

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ABSTRACT

A history and discussion of the photographic star catalogue work at the Cape Observatory is given.

The first general photographic catalogue of star positions was the Cape Photographic Durchmusterung (CPD) which was begun 1885, quite soon after the introduction of the dry photographic plate. A Dallmeyer Rapid Rectilinear f/9 lens of 137cm focus was used and each plate covered an area of $5^\circ \times 5^\circ$. The star positions were measured only to Durchmusterung accuracy but Gill and Kapteyn fully realized that the plates were capable of yielding positions with much higher precision. It does not seem to be generally known that charts to match the CPD are available. The Union (now the Republic) Observatory, Johannesburg has published a complete set of specially ruled charts based on photographs taken with the Franklin Adams lens, while the National Observatory of Chile has issued a rather more conventional series prepared by Dr. Ristenpart. Unfortunately, owing to Dr. Ristenpart's untimely death, these charts cover only the southern section.

The second big photographic catalogue undertaken at the Cape was that of the Carte du Ciel zone between -40° and -52°. The rectangular coordinates of stars in this zone, as carefully measured with specially constructed Repsold machines employing screw micrometers, are given in eleven large volumes. Mr. Hough used this material to derive accurate equatorial coordinates for 20 843 stars in the zone, including all those with magnitude 9.0 or brighter in the CPD (1). Photographic magnitudes were derived for these same 20 843 stars (2) and later proper motions based on a repetition of the Carte du Ciel plates after an average interval of about 30 years (3). This work was extended to another 20 554 stars consisting mainly of those with CPD magnitudes between 9.1 and 9.5 (4). These two catalogues (3,4) give accurate positions, proper motions, spectral types and photographic magnitudes for over 40 000 stars in a zone of the sky which includes a wide variety of galactic latitudes and should be very useful for statistical investigations.

The Carte du Ciel work was closely followed by that for the Cape Photographic Catalogue for 1950.0 (CPC50) which was intended to be the southern extension of the Yale and AGK2 work which was then being carried out in the north. This catalogue, which was only recently completed (5), grew into something more than an extension of the AGK2. It gives for each star an accurate position for 1950.0 and where possible a proper motion, spectral type, photographic magnitude and color. The CPC50 has an approximately even distribution of ten stars per square degree and covers the sky south of -30° except for the Cape Carte du Ciel zone, which was already well observed. The spectral types are from the Henry Draper Catalogue or specially determined on the HD system from Harvard plates by Professor Cillié, Dr. Hoffleit and Mrs. Mayall. The photographic and photovisual magnitudes are completely new determinations and are on a rigid photometric system based on photoelectrically determined standards. For the part of the sky between -30° and -37 1/2°, only one early place taken from the Cordoba B or C catalogues was used for the determination of the proper motion. South of -37 1/2° a wide variety of sources had to be used and, whereever possible, more than one source for each star. Considerable use was made of the places given in the La Plata Catalogues A, B, C, D, and E; the Argentine General and Zone Catalogues; Gillis' Catalogue for 1850.0; and the Melbourne, Perth and Sydney Carte du Ciel catalogues. The weakest determinations of proper motion are those for the stars between -30° and -40° and, fortunately, these can now be replaced by those determined by Dr. Hoffleit in the two new Yale catalogues covering this zone (6).

The lens used for the CPC50 positions was a specially constructed Taylor Hobson three component f/16, two meter focus, giving a scale of approximately 100" per mm. For the first two zones (those between -30° and -40°) an area of 5° × 5° was measured on each plate, but for the more southerly zones this was reduced to 4° × 4°. This was done partly to avoid working close to the edge of the plate and partly to reduce the effect of the rather complicated magnitude-color-distance corrections which increase rapidly in size with increasing distance from the plate center. Although these corrections were evaluated as carefully as possible it is very doubtful if their effect was entirely eliminated from the CPC50.

To reduce this source of uncertainty in any further Cape astrometric work a Taylor Hobson four component f/10, two meter focus, lens was ordered. This lens was specially computed to work with a plate-filter combination which gives only a narrow pass band close to the D lines of sodium.

Photography of the southern sky with this new lens began some time ago and the zones between -30° and -52° and have already been completed. The plates have been so spaced that there are generous overlaps and every star appears on four plates, once in each quadrant. How we hope to treat these plates and others still to be obtained is the subject of Dr. Clube's paper.

(1) Zone Catalogue of 20 843 Stars included between 40° and 52° of South Declination, referred to the Equinox 1900 derived from meridian observations and photographs made at the Royal Observatory, Cape of Good Hope. London, 1923.
(2) Magnitudes of Stars contained in the Cape Zone Catalogue of 20 843 Stars for the Equinox 1900, Zones -40° and -52°. London, 1927. (All magnitudes for the zone stars are repeated in (3)).
(3) Proper Motions of Stars in the Zone Catalogue of 20 843 Stars. London, 1936.
(4) Catalogue of 20 544 Faint Stars in the Cape Astrographic Zone -40° to -52°. London, 1939.
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THE IMPROVEMENT OF STAR POSITIONS BY PHOTOGRAPHIC METHODS

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ABSTRACT

After a short account of earlier Cape programs for the determination of star positions by photographic methods, a brief description is given of the new astrometric camera and of the program on which it is being employed. This is the photography of the southern sky ($0 > \delta > -90°$, $m_v < 11.5$, $\overline{Ep} = 1968$) *so that each star appears on at least four overlapping plates each* $4° \times 4°$. *It is intended that the plates should be measured with some form of automatic machine. Sets of star coordinates* $\{x, y\}_p$ *measured on each plate* p *will be combined with a limited set of provisional meridian places* $\{\alpha, \delta\}$ *to give an accurate set of relative star positions* $\{\alpha'', \delta''\}$ *which should be independent of the meridian system together with a set of residuals for each plate-star* $\{ \{\Delta\alpha'', \Delta\delta''\}_p\}$. *The standard errors associated with these residuals for the Cape camera are* $\sim 1\mu \equiv 0''.1$,

so that with an average of 150 measurable stars per plate, it is anticipated that errors in the <u>*system*</u> *of places derived may be as low as 0"01 over wide areas of the sky. It is further planned to tie this system with appropriate deep plates to extragalactic nebulae at suitable points over the sky, thereby making it possible for a repetition of this program in 10 years, say, to establish an absolute frame of reference for proper motions to* $\sim 0''002/y$. *The technique by which* $\{\{x, y\}_p\}$ *are transformed to* $\{\alpha'', \delta''\}$ *is based on a simple geometrical model of image formation on photographic plates. Current investigations at the Cape are concerned with the elimination of sources of systematic deviation from this model (e.g., due to telescope objective and filter, refraction anomalies, etc.).*

INTRODUCTION

The Cape Observatory has recently embarked on a program to produce relative star positions from overlapping photographic plates over the whole of the southern hemisphere (Clube, 1967). It is at present intended that the internal system of these positions will be completely photographic, but there is some fear that an inexact formulation of the factors which determine where a stellar image is formed on a photographic plate may give rise to intolerable systematic errors. In these circumstances, it is suggested that there may be some merit in computational procedures which permit a step-by-step comparison between the transit circle system (usually available initially) and the derived photographic system. The local differences between the systems may therefore be examined with a view to specifying the image forming process to a sufficiently high accuracy. The likely effectiveness of such a technique is discussed in the light of experience gained in forming past Cape catalogues.

THE CAPE CATALOGUE FOR 1900

It is not widely known that one of the first clear statements and successful applications of a plate-overlap technique was made by Hough as long ago as 1920. His work was devoted to the improvement of the 1900 transit circle places with the Cape astrographic zone photographic material of that epoch, and occupied him for some eight years before the publication of the Cape Zone Catalogue for 1900 (Gill and Hough, 1923).

A brief review of this work follows. He used meridian star positions (α_M, δ_M) and photographic plate coordinates (x_M, y_M) in the usual way with linear plate constants to get improved estimates of the positions (α'_M, δ'_M) together with positions of faint stars having better shaped images, on each plate (α'_F, δ'_F). In order to produce a system of exact overlaps over the whole Cape zone, he then assumed that faint star positions derived from each plate required corrections ($\alpha''_F - \alpha'_F$, $\delta''_F - \delta'_F$) to their true positions (α''_F, δ''_F), resulting from errors in four plate constants (p, q, a, b). Thus:

$$\alpha''_F - \alpha'_F = \frac{x}{1/2\ell}\,\Delta p + \frac{y}{1/2\ell}\,\Delta q + \Delta a$$

$$\delta''_F - \delta'_F = \frac{x}{1/2\ell}\,\Delta q + \frac{y}{1/2\ell}\,\Delta p + \Delta b$$

where the plate side = 2ℓ. When the average over each plate quadrant is taken, it follows that

$$\overline{\alpha''_F - \alpha'_F} = \pm\,\Delta p \pm \Delta q + \Delta a$$

$$\overline{\delta''_F - \delta'_F} = \pm\,\Delta q \pm \Delta p + \Delta b$$

Differencing such equations over all overlapping quadrants (e.g., plate i with plate j), equations of the following kind are formed:-

$$\begin{aligned} \Delta p_i - \Delta p_j + \Delta q_i - \Delta q_j + \Delta a_i - \Delta a_j &= m_{ij} \\ \Delta q_i - \Delta q_j + \Delta p_i - \Delta p_j + \Delta b_i - \Delta b_j &= n_{ij} \end{aligned} \qquad (1)$$

where

$$m_{ij} = \overline{\alpha'_{Fi} - \alpha'_{Fj}}$$

$$n_{ij} = \overline{\delta'_{Fi} - \delta'_{Fj}}$$

involving quantities derived from the initial solutions.

Hough solved equations (1) for the whole Cape zone subject to the condition that

$$\Sigma\Delta p^2 + \Sigma\Delta q^2 + \Sigma\Delta a^2 + \Sigma\Delta b^2 = \text{Min}$$

Although this formulation neglects many of the factors affecting photographic star positions, its intention corresponds closely to that of more sophisticated developments in recent times. It is therefore of some interest to examine the solutions obtained from this analysis. It was found that Δp, Δq, Δa, Δb varied from plate to plate with some mean characteristics prevailing. Thus

$$\overline{\Delta p} \approx 0''.35 + 0''.02 \sin(\alpha - 3^h)$$

$$\Delta(\overline{\Delta a}) \approx 0''.05 \ (\delta - 44.5)$$

These represent differences between the meridian system and the photographic overlap system. It is difficult to see how the meridian system could have induced the first term of $\overline{\Delta p}$ locally over every plate, and it is more likely that this term arises from a differential photographic effect as between M (meridian) and F (faint) stars, e.g., coma. Similarly, the second term could be due to a seasonal variation in coma. These conclusions could in principle have been tested by overlapping on the meridian stars themselves. Since there were fewer of these this was not done, but Hough was probably perfectly correct in attributing $\overline{\Delta p}$ to a differential photographic effect rather than an error in the meridian system. Likewise, the run in $\overline{\Delta a}$ could have been due to a large declination "shear" in the meridian system, but it was more readily attributable to a small deviation from orthogonality of the reseau lines on the plates to which the measurements of star positions were referred. Expressed another way, the run in $\overline{\Delta a}$ exhibited a defect in the analytical model (having regard to the circumstances of measuring) which could have been avoided.

Hough therefore allowed for the differential photographic effects and restored the overall meridian system in declination by appropriate modifications of Δp for brighter stars, and Δa.

The resultant r.m.s. values of Δa and Δb represent average local deviations between the photographic and meridian systems at the center of a plate, and were found to be 0''.20. Typically, such differences remained steady over arcs of 10° - 20°. There are three sources of error contributing to this discrepancy such that

$$x^2 + y^2 + z^2 = (0''.20)^2$$

where the accuracy of tie-in of a single photographic plate to the meridian system $x = \{(\varepsilon_M^2 + \varepsilon_{PH}^2)/n_M\}^{1/2}$, the accuracy of a single photographic overlap $y = \{(\varepsilon_{PH}^2 + \varepsilon_{PH}^2)/n_F\}^{1/2}$, and z is the r.m.s. difference between photographic and meridian systems (ε_M is the standard error of meridian circle observations, ε_{PH} the standard error of a photographic position, n_M the average number of meridian reference stars per plate, and n_F the average number of faint overlapped stars on each half-plate).

Extracting the following values from the Cape Catalogue

$$\varepsilon_M = 0''.30 \qquad \varepsilon_{PH} = 0''.35 \qquad m_M = 10 \qquad n_F = 100$$

we derive

$$z \sim 0''.13$$

Thus, unless there is some unsuspected correlation between the photographic and meridian systems, this figure represents an upper limit to the absolute accuracy of the photographic system. Since there is every reason to believe that a substantial part of this figure may be attributed to flexure errors and the like in the absolute meridian system (say 0''.10), and since also, the photographic overlap has, by modern standards, been performed with crude measuring facilities, unsatisfactory star images, and an overly simple model, Hough's work suggests that a good quality photographic system is quite feasible.

THE CAPE PHOTOGRAPHIC CATALOGE FOR 1950

The more recent CPC50 has been formed by using linear plate constant theory and straightforward averages from partially overlapping plates, no attempt being made to improve the transit circle system with the photographic positions. Essentially therefore, each star position is tied to the system only through the meridian stars on those plates on which it appears - that is through about 30 stars with a typical T.C. standard error of 0''.24 and photographic error of 0''.15. It is to be expected then, that the system of star positions in this catalogue has localized deviations of $\sim$ 0''.05 from the T.C. system over areas on the sky of a plate width or so across. Had the improved photographic positions been used in determining revised plate constants, successive iterations would have smoothed out the transit circle system over wider areas of the sky. The prohibitive amount of computing involved however made such a step impossible. Nevertheless, a study was made of the residuals between the T.C. and photographic star positions on each plate, and these showed a reassuring consistency over a long period of time.

That part of each residual Δ which depended on position alone was represented by the formula

$$\Delta = Ax + By + C + Dx^2 + Exy + Fy^2 + Gx^2y + Hxy^2 + Lx^3 + My^3$$

and the values of the coefficients found for each declination zone. These are given in Table I. Since the plate-average of Δ is zero, the zonal values of A, B, C represent compensating corrections to the linear plate constants arising from the introduction of the higher order terms.

TABLE I

Zone	Approx area of plate examined	A_x	A_y	B_x	B_y	C_x	C_y	D_x	D_y	E_x	E_y	F_x	F_y	G_x	G_y	H_x	H_y	L_x	L_y	M_x	M_y
30°- 35°	$5^\circ \times 5^\circ$	-3.7	-1.0	-1.2	-2.8	0.0	-12.0	.00	.00	+.50	+.20	.00	+.36	+.020	+.125	+.070	+.030	+.040	.000	+.020	.000
35°- 40°	$5^\circ \times 5^\circ$	-4.1	-0.4	-0.8	-4.3	-3.5	-15.7	-.01	+.09	+.35	-.06	+.14	+.52	+.039	+.132	+.087	+.028	+.058	-.018	-.002	+.042
52°- 56°	$4^\circ \times 4^\circ$	-6.5	0.0	-0.4	-4.4	-4.6	- 5.8	+.10	+.11	+.47	+.03	+.16	+.20	+.010	+.167	+.097	+.022	+.163	-.013	+.006	+.048
60°- 64°	$4^\circ \times 4^\circ$	-6.7	-1.1	-0.7	-6.3	-3.8	- 2.5	+.04	+.06	+.23	+.02	+.14	+.06	+.018	+.132	+.095	+.038	+.170	+.008	+.013	+.137
64°- 68°	$4^\circ \times 4^\circ$	-6.1	-1.2	-1.9	-6.4	-3.9	- 9.0	-.06	+.16	+.27	-.08	+.27	+.29	+.046	+.147	+.115	+.028	+.156	+.024	+.046	+.147
68°- 72°	$4^\circ \times 4^\circ$	-8.5	0.0	-1.4	-8.5	-4.4	- 9.0	+.06	+.13	+.34	-.03	+.14	+.33	+.039	+.134	+.111	+.021	+.245	-.020	+.026	+.222

(Δ measured in 10^{-4} mm, and (x, y) in cms).
Scale = 100" per mm

The general equality $(D_x, E_x) \equiv (E_y, F_y)$ indicates a persistent but steady displacement of the tangent point from the measuring center by 1.2mm = 2' while the values of F_x and D_y are suggestive of a constant periodic screw error. This suggestion is confirmed by the different values for F_x and D_y (approximately zero) in the first two zones where an attempt was made to eliminate the progressive screw error empirically. Although at first sight, the variation of the third order terms with declination might be attributed to refraction, the effect is too large. It probably results from the fact that the effective area of plate in which the residuals were examined diminshes as the south pole is approached. However, in Table II are given the differences between the scale plate constants in y and x together with the zenith distance.

TABLE II	
ξ	$b_y - a_x$
28°	+0.57 $\times 10^{-4}$
32°	+0.87
36°	+1.25
40°	+1.74
44°	+2.38
48°	+3.21
51°	+3.99
54°	+4.95
56°	+5.74

These correspond to

$$\frac{\Delta(b_y - a_x)}{\Delta(\tan^2\xi)} \sim 53'' \pm 2$$

in tolerably close agreement with the value of 59" predicted by simple refraction theory.

Neither this survey nor the earlier one give a suggestion of any obvious inadequacy in the simple geometrical theory of plate constants coupled with constant higher order terms (including color-magnitude dependencies) characteristic of the telescope. It is this fact which gives us some confidence in embarking on a more ambitious program to produce an entirely photographic system with the new astrometric camera.

THE NEW CAPE PHOTOGRAPHIC SURVEY

The present Cape photographic survey is being carried out concurrently with the S.R.S. transit circle program. The analytical technique which is applied to the photographic survey may be summarized briefly. We denote star direction cosines by

$$\underline{\tau} \equiv (\cos\alpha\cos\delta,\ \sin\alpha\cos\delta,\ \sin\delta) \equiv (\alpha,\delta)$$

and plate-star coordinates corrected for aberration and refraction by

$$\underline{r} \equiv (x,\ y,\ f_0)$$

Then, in general

$$\{\underline{r}\} = \{M\underline{\tau}\}$$

where { } denotes a set pertaining to stars on one plate, and M is a 3 × 3 geometrical transformation matrix peculiar to this plate. At first, we adopt a limited number of provisional meridian star coordinates $[\underline{\tau}_0]$ where [ ] denotes a set taken over the area of sky examined. The first estimates of M are then rigorously derived from $\{M^{-1}r\} = \{\underline{\tau}_0\}$ and followed by determinations of coordinates $\underline{\tau}'$ for each plate-star from $[\{\underline{\tau}'\}] = [\{M^{-1}\underline{r}\}]$. These values of $\underline{\tau}'$ are then averaged over all the overlapping plates to form a weighted means $\underline{\tau}''$ for each star: thus $[\underline{\tau}''] = [\{\underline{\tau}'\}]$. These values of $\underline{\tau}''$ are then adopted as a new set of provisional values of $\underline{\tau}$ and the whole process repeated iteratively until the changes in $\underline{\tau} - \underline{\tau}''$ through each iteration are smaller than some chosen limit.

Although it is analytically rather cumbersome, this procedure has an operational advantage in that the plate-star residuals $\{\underline{\tau}' - \underline{\tau}''\}$ can be examined at each iteration, and some view formed as to the suitability of the geometrical model adopted. Smoothed formulations of any general systematic deviations (attributable to the telescope-filter combination for example) may be negatively applied to the measured $\{\underline{r}\}$ at each iteration and the convergence process judged by the existence or not of any significant non-random deviations amongst the residuals.

The minimum number of necessary iterations to secure photographic smoothing over the whole sky may be estimated as follows: the first iteration links 4 overlapping plates, the second 4 × 9, the third 4 × 9 × 9 and so on. Each center has one degree square of sky associated with it, and therefore, the number of iterations N required to link the whole sky is such that

$$4 \times 9^{N-1} \sim 4 \times 10^4 \text{ or } N \sim 5$$

It is important to realize that a basic weakness of the overlap technique which Eichhorn et al. have discussed, namely the propagation of photographic errors (1967), to some extent disappears when a whole sky coverage is considered. Thus, any closing error resulting from a *constant* source of systematic deviation, otherwise undetectable, over each plate area (equal weights assumed for each plate) will be uniformly redistributed amongst each plate, thereby rendering the error non-cumulative. Sources of error which are likely to be troublesome are therefore the periodic ones such as seasonal variations in refraction and the optical parameters of the telescope. Nevertheless, the method of solution is designed to reveal any periodicity in the photographic system not present in the meridian system. By itself, of course, the Cape cannot hope to obtain an all-sky coverage, and the final photographic positions will suffer to some extent from errors of the kind discussed here. It may be hoped, however, that some link will be established in the equatorial zone with similar surveys in the n northern hemisphere.

Preliminary investigations of overlap solutions with the new Cape camera indicate that star positions on overlapping plates can be estimated to within $1.5\mu \equiv 0''\!.15$ (standard error) though no systematic deviations from this simple geometrical model have as yet been studied. This means that with an average of 150 measurable stars per plate, the standard error in the relative system of two overlapping plates may be as low as 0".015 . An argument has been presented elsewhere (Clube, 1967) for supposing that this figure is an upper limit to the all sky accuracy of the system of photographic places. Even if it proves in the long run, impossible to evaluate all the sources of systematic error in the photographic star positions, it is very likely that such an accuracy can be maintained over wide areas of the sky. In this case, the overlap method will be extremely useful in establishing an absolute framework for proper motions for it is necessary only to anchor star positions with respect to extragalactic nebulae at convenient separations across the sky (e.g., across the zone of avoidance) at two different epochs. It is therefore planned to tie the system of places obtained from the current Cape program with appropriate deep plates to extragalactic nebulae at suitable points over the sky thereby making it possible for a repetition of this program in 10 years, say, to establish an absolute frame of reference to ∿ .002"/y.

Acknowledgement

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DISCUSSION

See page 239.

PHOTOGRAPHIC ASTROMETRY AND OVERLAP REDUCTION TECHNIQUES

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ABSTRACT

Rigorous adjustment algorithms are discussed which extract hidden information contained in overlapping photographs of star fields.

In particular, the development of adequate physical and statistical models is emphasized.

INTRODUCTION

The design of the process for the reduction of any series of stellar photographs can be separated into several distinct but related phases. The first (the development of the physical model of the imagery) probably requires the greatest knowledge and insight on the part of the astronomer. Next, the reference system (or systems) requires consideration, for only through the observations of the reference stars does absolute information enter the system. Then a statistical model is chosen. This amounts to a series of assumptions about the nature of the errors in the measurements and in other parameters that are introduced into the reduction. Finally there is the choice of the reduction methods themselves. Before large electronic computers were available, the physical labor of accomplishing the adjustment imposed severe limitations upon the choices at each of these steps. These limitations are now disappearing.

The overlap algorithm for plate reduction is a direct result; it is now feasible to better characterize the relationships of the measurements to the celestial coordinates of the stars by making full use of the redundancy implicit in the appearance of one star on more than one photograph. This is merely one

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of the advances that modern computers have allowed in the reduction process. In addition, better models can now be developed by systematic experimentation. Experiences with the overlap method show that adequate models are not only desirable but mandatory.

This paper will discuss the various steps in the construction of a plate reduction method. It also relates some of the results and difficulties encountered at the Army Map Service (AMS; now the U.S. Army Topographic Command) while implementing the overlapping plate adjustment technique.

THE PHYSICAL MODEL

Extensive investigations have produced a great deal of knowledge about the various phenomena that effect the formation of a stellar image on a photographic plate. With the use of this information, corrections can be applied to the measurements in order to simplify their relationship with the celestial coordinates. Unfortunately, some of the parameters which specify these corrections vary randomly with time, for example, the coefficients in the expansion of refraction as a function of zenith distance. Such parameters must be treated as unknown in the adjustment.

With the use of all the knowledge one can gather about the significant imaging effects present, the physical model is formulated. At AMS the so-called "astrometric" method has been used. The measured coordinates are assumed to be polynomial functions of the standard coordinates (ξ,η), magnitude (m) and color index (c); that is

$$x = \Sigma a_{ijkl}\xi^i\eta^j m^k c^l$$

$$y = \Sigma a'_{ijkl}\xi^i\eta^j m^k c^l \qquad (1)$$

These equations, together with the well known expressions for ξ and η as functions of the right ascension (α) and declination (δ) form the basis of the measurement observation equations - they relate the observations (x,y) to the unknowns (α, δ, a_{ijkl}, a'_{ijkl}).

In addition to the basic observation equations, the model may include additional conditions relating the plate constants. For example, for plates exposed with the same telescope it may be felt that lens distortion will not vary from plate to plate; that is, the parameters for radial distortion are not "plate" but "telescope" constants. Thus, there my be condition equations

$$h(q_1, q_2, \ldots) = 0 . \qquad (2)$$

These can be handled in two ways. Equation (2) could be solved for one of the q's and then a substitution made in the basic measurement observation equation to eliminate one unknown, or (2) could be carried explicitly as a condition equation using the method of Lagrange multipliers.

Another situation that can occur is that certain conditions hold but only to a specified accuracy. Thus, there is a condition

$$g(q_1, q_2, \ldots) = 0 \tag{3}$$

which is valid only with a certain weight. This is enforced merely by treating (3) as an extra observation equation which is weighted and combined with the others. This method was used by Eichhorn and Gatewood (1967) to constrain the coma coefficient on individual plates to lie near the mean value over all plates.

By giving such a pseudo observation equation a large weight it can be made to serve as a condition equation. This trick has been used at AMS in the reduction of zone catalogues. In particular, x coma has been forced to equal y coma on a given plate and distortion coefficients have been constrained to the same value on all plates by writing simple equations relating these terms and enforcing them with large weight.

THE REFERENCE STAR MODELS

The basic aim of any plate reduction is to relate the measurement coordinate system to a celestial system. To achieve this requires some information about the celestial system, information which the reference star observations supply.

One method of utilizing this information is to allow the overlap configuration to form a single system, that is, tie the plates into what is effectively one large plate. The reference star observations are then used to relate this single system to the system of the reference stars in what amounts to a super plate reduction. This is the method employed by Lacroute (1964, 1967). The major difficulty with this approach arises from the ambiguous nature of the unified plate system. Undoubtedly, some of the distortions in the systems of the individual plates will cancel out but others will perhaps magnify in very surprising ways. The latter effect is more apt to occur if care is not taken to insure the nonsingularity of the normal equations of the adjustment.

Inadvertently such an adjustment was performed using the Pleiades material (Eichhorn, Googe, Lukac and Murphy, 1970) by introducing the reference observations with negligible weights. The residuals were naturally very small. But the coefficients of the η^2 term in the x equations turned out twice as large as those of ξ^2 and $\xi\eta$. Thus, the adjustment unified the plates into a single system but it was a system that was connected to that of the reference stars in a completely unexpected and unnatural manner.

The approach used at AMS is to adjust the plates to one another while simultaneously reducing the complex of plates to the reference stars. A model is then required for the reference star observations. Normally, this will be given by the condition that the observed position of the reference star is its actual position except for the error of the observation. If several catalogues of reference stars are used, more complicated models may be needed to account for systematic differences.

Other situations could call for more elaborate models. For example, suppose the plates to be reduced had been exposed with a diffraction grating. Then in a preprocessing step, magnitude effects could be removed by a comparison of central images to averaged spectral images. Thus, the system of coordinates measured on the plate might well be more free of magnitude dependent distortions than that of the reference stars. If this were true one could use a model such as

$$\alpha_o - (\alpha + am) = 0$$

where

α_o is the observed right ascension of the star

α is the unknown actual value

a is an unknown magnitude error rate, and

m is the stellar magnitude.

The effect would be to correct the system of the reference stars.

STATISTICAL MODELS

After selecting physical models for the relationships between the observed variables and the unknowns, assumptions must be made about the distribution of errors. This is usually a straightforward matter unless many reference star sources are to be used, or unless, for some reason, photographic images of a wide range of quality were measured.

The latter condition obtains with respect to the Pleiades data now being reduced at the Army Map Service. Rather elaborate procedures have been employed to express the variance of an individual measurement as a function of the magnitude of the star, the spectral type of the image, etc.

Normally, the specifications of the observation and mensuration programs will insure that the quality of the measurements is uniform.

If constraints are to be employed, their weights must be chosen but needn't be statistically valid. They are essentially parameters of the model and, as such, may be empirically or arbitrarily established.

ADJUSTMENT METHODS

Once physical and statistical models have been chosen, the adjustment of the data follows the standard least squares algorithm. If the formulation is in terms of observation equations, then a system of weighted and linearized observation equations are formed,

$$AX = K,$$

normal equations are generated,

$$A^TAX = A^TK,$$

and solved. (If condition equations are used, then minor modifications are necessary.) Theoretically, this is all very simple.

Unfortunately, the large number of equations and unknowns usually forces the use of some ingenuity to actually carry out these steps.

For the Pleiades adjustment each of the steps was done explicitly because working computer programs were available. But the size (forty thousand equations in sixteen hundred unknowns) was just about the limit for this approach without incurring great expense in computer time.

It is much more efficient to take advantage of the special nature of the matrices to optimize the data flow. The normal equations can be formed as the observation equations are generated, and at the same time the star unknowns eliminated.

When this is done a patterned matrix emerges. Each row in the partitioned normal equation matrix represents a plate as does each column. The rule is that at the intersection of the i^{th} row and j^{th} column, a nonzero element occurs whenever a star appears on the i^{th} and j^{th} plate simultaneously. Thus, for a zone with the plates numbered consecutively, we have the following pattern:

$$\begin{bmatrix} X & X & & & & & & X \\ X & X & X & & & & & \\ & X & X & X & & & & \\ & & X & \cdot & & & & \\ & & & & \cdot & & & \\ & & & & & \cdot & & \\ & & & & & & X & X \\ X & & & & & & X & X \end{bmatrix}$$

In the case of three zones the picture is

$$\begin{bmatrix} A & B & & & & & B \\ B^T & A & B & & & & \\ & B^T & A & B & & & \\ & & B^T & \cdot & & & \\ & & & & \cdot & & \\ & & & & & \cdot & \\ & & & & & A & B \\ B^T & & & & & B^T & A \end{bmatrix}$$

where

$$A = \begin{bmatrix} X & X & X \\ X & X & X \\ X & X & X \end{bmatrix} \qquad \text{and} \qquad B = \begin{bmatrix} X & & X \\ X & X & X \\ X & & X \end{bmatrix}$$

This pattern is predicated upon having corner to center overlap between zones, some overlap in right ascension within each zone, and the following numbering system

$$\begin{array}{cccccccccccc} & n-2 & & 1 & & 4 & & 7 & & 10 & & \\ \cdots & & n & & 2 & & 5 & & 8 & & 11 & \cdots \\ & n-1 & & 3 & & 6 & & 9 & & 12 & & \end{array}$$

If, on the other hand, the plates are ordered

$$\begin{array}{cccccccccc} & m & & 1 & & 2 & & 3 & & \\ \cdots & & 2m & & m+1 & & m+2 & & m+3 & \cdots \\ & 3m & & 2m+1 & & 2m+2 & & 2m+3 & & \end{array}$$

the matrix takes the form

$$\begin{bmatrix} A & B & B \\ B^T & A & B \\ B^T & B^T & A \end{bmatrix}$$

with

$$A = \begin{bmatrix} X & X & & & & & & X \\ X & X & X & & & & & \\ & X & X & X & & & & \\ & & X & \cdot & & & & \\ & & & & \cdot & & & \\ & & & & & \cdot & & \\ & & & & & & X & X \\ X & & & & & & X & X \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} X & & & & & & & X \\ X & X & & & & & & \\ & X & X & & & & & \\ & & X & X & & & & \\ & & & & \cdot & & & \\ & & & & & \cdot & & \\ & & & & & & \cdot & \\ & & & & & & X & X \end{bmatrix}$$

If "telescope" constants are present, or if there are parameters for systematic reference star differences, these can be inserted after the plate unknowns. For the zonal adjustment, the matrix appears as

$$\begin{bmatrix} X & X & & & & & & X & X \\ X & X & X & & & & & & X \\ & X & X & X & & & & & X \\ & & X & \cdot & & & & & X \\ & & \cdot & & \cdot & & & & \cdot \\ & & & & & \cdot & X & X & X \\ & & & & & & X & X & X \\ X & X & X & X & \cdot & \cdot & X & X & X \end{bmatrix}$$

The extreme sparseness of these matrices naturally facilitates the solution, and makes the rigorous overlap adjustment possible.

But as these examples show, considerable change occurs in the mosaic of the normal equation matrix with even a simple renumbering of the plate. Advantage should be taken of this to achieve the optimum form for computer efficiency.

EXPERIMENTS WITH ZONAL REDUCTIONS

Work is now in progress at AMS on several photographic star catalogues. In particular, for the region - 48° to - 54°, a series of plates were exposed for AMS by H. Wood at the Sydney Observatory. These plates were measured at the University of South Florida and are now being reduced at AMS. Similarly, the measurements for the Yale Zone (- 60° to - 70°) are being prepared for reduction by AMS.

The overlap method will be used for these zones and computer programs have been prepared in anticipation of these adjustments. In order to test the coding, as well as the method itself, real data has been obtained. Dr. Stoy of the Cape Observatory kindly allowed access to the original measurements used to produce the Cape Photographic Catalogue for the zone - 52° to - 56°.

Several overlap reductions of this data have been carried out. The statistical model in each of these has been based on the assumption that all plate mensuration errors have equal variance, that all reference star observations have variances four times as large, and that there is no correlation between any of the observations. What has varied is the imaging model.

One model which gave interesting results used the terms

$$1,\ \xi,\ \eta,\ \xi^2,\ \xi\eta,\ \xi(\xi^2+\eta^2),$$

$$m,\ m\xi,\ c,\ c\xi,\ mc$$

in the x-equation and symmetric terms in the y-equation. This choice was rather unfortunate. For example, the variance of the constant term on a typical plate was a tenth of a second of arc - implying that the systematic error due to the error in the plate constant is almost as large as the random error. This large error is due in part to the rather narrow range of magnitude of the reference stars. The total range is perhaps seven to eleven but for the reference stars it is only about eight to ten. This result again demonstrates that the overlap algorithm cannot find absolute information that is not actually present.

Another interesting result from this adjustment was that the color magnification term $c\xi$ remained remarkably constant from plate to plate despite the high correlation of magnitude with color and despite the overall weakness of the solution.

CONCLUSION

The reduction of overlapping stellar photographs is a standard least squares adjustment problem - a large problem with many unknowns and many equations, but in no way fundamentally different from other least squares adjustments. Thus, there are well known and much used algorithms for accomplishing the reduction once the statistical and physical models have been defined. The definition of these models is the major difficulty in successfully implementing an overlap plate solution.

The size of the matrices, even a few years ago, appeared to be an insurmountable obstacle. However, advances in techniques for manipulating large structured matrices have made size no longer the primary consideration. In fact, large adjustments are already routine for problems in photogrammetry and satellite geodesy. An overlap adjustment of twenty thousand star images on ninety plates takes only a half hour on a modern computer. In a few years, the simultaneous adjustment of a collection of plates covering the whole sky seems entirely feasible. Of course, it is extremely doubtful whether sufficiently simultaneous data will be available.

A much more serious problem than mere size of the matrices is that of establishing adequate models. Because of the sheer number of stars considered in catalogue work, the overlap conditions are so strong that the reference stars can easily be overwhelmed. This occurs if the measurement model is not flexible enough, that is, if it does not account for all distortions present.

In a classical solution the same cause will only produce local effects and local errors since there is no correlation between results from two plates that do not overlap. There is, of course, no general algorithm for obtaining a complete model. The model must uniquely reflect the whole spectrum of observational instrumentation and of mensuration procedures, and must be solvable in terms of the amount of reference material available.

With an adequate model the overlap algorithm yields results which are systematically more accurate than those obtained using classical techniques. Moreover, it can do this with fewer reference star observations than the classical method requires and without inordinate computational demands.

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DISCUSSION

See page 239.

THE OVERLAP METHODS

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ABSTRACT

In the first part the different methods of reducing measurements on photographic plate by the overlap methods are compared. Our principal objective is the practical applications. We attempt to distinguish the methods accepted by most astronomers from those not generally agreed upon and from those that are still being studied.

In the second part some questions are examined: organization of future works, improvement of AGK2 and AGK3, and solutions for the Astrographic Catalogue, as well as considerations of some empirical methods.

INTRODUCTION

For many years, some astronomers thought that it would be helpful to use the information given by the overlapping of plates to study the field coorections and sometimes to improve the plate constants. Although some experiments were made in the past, it is only since the introduction of electronic computers that Eichhorn (1960) proposed to utilize this information in a broad sense.

Since then, a large number of research workers studied the overlap methods, theoretically and practically, namely, in chronological order: Eichhorn (1960), Dieckvoss (1962), Eichhorn and Williams (1963), Jefferys (1963), Lacroute (1964, 1964a), Lacroute and Bacchus (1964, 1964a, 1964b), Henriksen (1967), Lukac and Haligowski (1967), Lukac(1967), Googe (1967), Eichhorn, Googe and Gatewood (1967), Herget (1967), de Vegt (1967), Lacroute (1967), Clube (1967), de Vegt (1967a), Dieckvoss and de Vegt (1967) and finally, Eichhorn and Gatewood (1967).

The great number of papers shows the general interest in this problem. Some points seem to be well established, others are not yet clear, or questionable.

We will try to make a distinction between the problems resolved and the problems in doubt, and draw attention to the problems not yet considered.

SUPERIORITY OF OVERLAP METHODS

Improvements of the Constants

All the authors agree that it is possible to obtain much better plate constants, for example, Lacroute and Bacchus (1964a), Lukac (1967), Lukac and Haligowski (1967), Eichhorn et al. (1967), Lacroute (1967). The results of practical computing prove this, cf. Lacroute and Bacchus (1964a), Lukac (1967), Googe (1967), Eichhorn et al. (1967), de Vegt (1967), and Lacroute (1967).

However systematic studies of the increase of the weight of the constants are more rare. Ecihhorn (1960) gave some principles of theoretical formulation. Eichhorn and Gatewood (1967) showed in a practical case the benefit on the results, and especially on the constants. De Vegt (1967a) gave. on an artificial field, some attractive results, but maintains that the calculations only on an artificial field allow one to study the benefit of the overlap methods, because only in this case do we know the exact values which we want to approach.

I have systematically studied (Lacroute, 1967) the possible increase of the weights. Evaluations have been made assuming that corrections to the constants obtained in the first approximation are determined.If the type of relation between standard coordinates and measured coordinates were correct, if we had no errors in the coordinates of the reference stars, no errors in the tangential point, and no errors in the measurements of images on the plates, the results of the first approximation would be rigorously correct for all stars. Then the system of least squares normal equations for the determination of corrections to the constants would have a zero right hand side. The corrections calculated would be zero also. From the above mentioned least squares solution, the effect of introducing errors in the reference star coordinates and in the measurements is calculated. This method is as correct as that of using an artificial field, more convenient for the systematic study of influencing factors, but at the cost of some schematic hypothesis as to which effects should be studied.

For example, with the tables in (Lacroute, 1967) it is possible to evaluate the decrease of the differences calculated by de Vegt (1967a). With the mean values for the star numbers, a factor 0.91 is computed; de Vegt found 0.87. The difference comes from my schematic treatment; with a uniform density in stars the first approximation is too good and the computed benefit is smaller than with a random star density.

In my theory (Lacroute, 1967) errors in the calculated coordinates are put in linear form as function of errors in the reference catalogue, and or errors in the measurements on the plates. Until now I have omitted the effect of errors in the measurements of the catalogue stars, which are not reference stars, except in the calculated stars. The effects are certainly small, but it is necessary to evaluate them if we want to use the overlap method at the limit. (see Appendix with A. Valbousquet).

DECREASE OF THE REFERENCE STAR NUMBER

All the workers think that it is possible to obtain such good constants with less reference stars; namely, Lacroute and Bacchus (1964a), Lukac and Haligowski (1967), de Vegt (1967), Eichhorn et al. (1967), and Lacroute (1967). However, the reduction number of the reference stars can decrease the quality of the final system; that is said in Lacroute and Bacchus (1964a), de Vegt (1967), and Lacroute (1967). Henriksen (1967, p. 607) rightly shows that by reducing the plates separately, we cannot say that the results are on the system of the reference catalogue. De Vegt (1967) points out that the number of reference stars should be sufficient to accomplish a meaningful relation to an absolute system. I studied, in detail, the problem of the systems; first for meridian observations in (Lacroute, 1964), then for photographic catalogues in Lacroute and Bacchus (1964b) and Lacroute (1967).

With Bacchus, I studied the problem of establishing a relation, by means of a photographic catalogue, between a fundamental catalogue of bright stars, and the faint stars or nebulae. This study shows what is to be done and the benefit of the overlap method.

Moreover, we show (Lacroute, 1967) the relations between the calculated photographic system and the reference system which has been used. The study on the systems is not finished. More computing is required to find the relations between the systems when the star densities are not uniform.

EFFECTS OF MAGNITUDE

Those systematic errors in photographic catalogues, which are functions of magnitude, posed a problem that seemed insurmountable and induced the astronomers of the USSR to establish a fundamental catalogue of faint stars by meridian observations.

The overlap methods and the use of grating are particularly beneficial to reveal and to correct these systematic errors. For example, the work on the Pleiades with many plates proves this (Lukac, 1967; Lukac and Haligowski, 1967). My computations on the AGK2 (Lacroute and Bacchus, 1964a) and then especially in (Lacroute and Bacchus, 1964) indicate the efficacy of these corrections based on a large number of plates. Lukac (1967), Lukac and Haligowski (1967), and Lacroute (1967) agreed that the magnitude correction functions are better determined by the overlap method for two reasons. First because as for the other constants, the number of stars used is larger; second, because often the range of the magnitude values is more important.

However, the use of overlapping is not always sufficient to resolve this problem satisfactorily, cf. Eichhorn and Gatewood (1967). The effects of magnitude are modeled by a linear or very simple function because it is not possible to determine many parameters. But Lacroute and Bacchus, (1964a), and Lacroute (1967) note that the effects of magnitude may be non linear and, for example, may appear only from a certain magnitude on down. The observed effects suggest that more caution be used in guiding the plates more carefully.

I think a richer documentation could be obtained by a grating and several exposures. But it would be necessary that the magnitude differences by the grating as well as by the exposures be lower than 2.5 magnitudes.

COMPUTATIONAL METHODS

Linear Relations

Almost all the authors favor putting the problem in a linear form after a first approximate solution. However, not Clube (1967); he intends to establish a sphere by using the best values of angles between plate pairs based on the overlapping. In other respects, de Vegt (1967, 1967a) gives exact relations between the plate constants of neighboring plates.

However, though his form is not linear, he draws attention to the fact that other methods for the solution of the system might exist.

It seems to me that for the work on an important catalogue, the easiest way would be to linearize the problem. We can always obtain a first solution which is accurate enough for the purposes of linearization (Googe, 1967).

CHOICE OF THE UNKNOWNS

Certainly, all star coordinates α, δ and plate constants are to be unknowns; that was demonstrated by Eichhorn (1960), Lukac and Haligowski (1967), and Googe (1967). However, for the sake of simplicity, I had (Lacroute and Bacchus, 1964a) wrongly considered only the constants as unknowns. The problem therefore was formulated more simply, but not absolutely correctly. This simplification is now useless because Googe (1967) showed that it was easy to reduce the entire matrix to a matrix pertaining to the plate constants only. At the Prague IAU meeting, (Lacroute, 1967) I indicated that it was possible to obtain the reduced matrix directly by comparing simultaneously the first approximation coordinates of each star. We could even obtain the reduced matrix directly by considering proper motion unknowns for all the stars, also.

EQUATIONS

In order to satisfy the principle of least squares, we must find that set of unknowns which produces the lowest value of the sum of the squares of differences between calculated values and measured data. The measurements are measures of x and y on the plates, as was amply emphasized by Lukac (1967), and by Googe (1967). The x and y were used in the computations by Eichhorn and Gatewood (1967). Henriksen (1967) also shows that to be absolutely correct, we should not consider the coordinates α,δ in the reference star catalogue as measured data, but we should use them with the appropriate weight matrix in the equations that utilize the positions in the reference star catalogue.

All these considerations are perfectly correct. But it is more difficult to establish the equations solved with respect to x and y rather than with respect to α and δ, and the problem is to know when the extra work incurred by regarding the x and y as the measurements to be adjusted is really justified in terms of the returns.

When we check the calculated values of x and y against the measured values, we must express x and y as functions of the standard coordinates and not the other way around as usually. This is of no importance. After the first approximation with the reference stars, we obtain values for constants for each plate and for each star, first approximation values of its equatorial coordinates, for example α_1 and δ_1. The, the values x and y corresponding to this α_1 and δ_1 are computed and also the partial derivatives of x, y with respect to the constants and the coordinates α_1 and δ_1. Thereafter we establish a system whose solution gives corrections to the initial values of the constants and the equatorial coordinates α_1 and δ_1; these corrections produce the smallest value of the sum of the squares of the differences between the calculated and measured x and y.

If we want to compare α and δ, it is better to express the standard coordinates as function of the x and y, as astrometrists usually do. After the first approximation, it is sufficient to compute the partial derivatives of α and δ with respect to the constants. Then we establish a system giving the corrections to the intial values of the constants and the equatorial coordinates; these corrections give the smallest value for the sum of squares of differences between the final α and δ and the values obtained from each plate.

The computation of α and δ is easier than for x and y ; we do not have to go back from the α and δ to the x and y, and the number of partial derivatives to compute is smaller. Most important is probably that for extended work, it is easier to detect the clerical errors by the comparisons in α and δ.

I agree that this principle is not absolutely correct, but considering that

$$\frac{\partial(\alpha\cos\delta)}{\partial x} \text{ and } \frac{\partial\delta}{\partial y}$$

are near to 1 (in appropriate units) and that the other partial derivatives are small, it is quite all right to regard the errors in $\alpha\cos\delta$ and δ and the errors in x and y as equivalent.

In order to rigorously adhere to the principles and to preserve the comparison in α and δ, we could give to equations with the coordinates α and δ weights in inverse ratio to:

$$\left[\frac{\partial(\alpha\cos\delta)}{\partial x}\right]^2 + \left[\frac{\partial(\alpha\cos\delta)}{\partial y}\right]^2 \quad \text{and:} \quad \left(\frac{\partial\alpha}{\partial x}\right)^2 + \left(\frac{\partial\delta}{\partial y}\right)^2$$

In the case of AGK2, AGK3, or in the case of the Astrographic Catalogue it seems to me that this would be useless since all the weights would be very close to 1. Surely it would be more important to take into account the differences in quality of the images depending on the positions on the plates and specially the differences in quality of the plates (see further calculations on the AGK2).

However, next to practical considerations, it is evidently necessary to pay close attention to the correct applications of the fundamental principles and to investigate each case and find out if simplifications are valid. For example, in the case of the big field plates taken with a Schmidt telescope, where the images are good even at the edges, it will probably be necessary to work in x and y or assign weights in α and δ.

MATHEMATICAL MODELS

The relationships between the equatorial corrdinates and the standard coordinates are geometrically rigorously defined. The choice of relationships between measured coordinates (x,y) and standard coordinates (X,Y) regardless of whether we consider X(x,y) and Y(x,y) or x(X,Y) and y(X,Y) is difficult.

The introduction of superfluous constants must be avoided. That was well established and is demonstrated by Eichhorn, Googe, and Gatewood (1967, p.626). The problem has been studied in detail by Eichhorn and Williams (1963) for solutions on separated plates and by Lacroute (1967) for overlap solutions. Eichhorn and Williams (1963) on p. 230 give a criterion for a choice between different forms of relations. Eichhorn, Googe, and Gatewood (1967) on p. 629, recommend the use of reference star residuals to investigate which modifications would produce more adequate relationships. These problems are studied again in detail by Eichhorn and Gatewood (1967).

Lacroute (1967) has made many calculations to evaluate the advantages of a decrease in the number of constants to increase the efficienty of the overlap methods.

In order to simplify the relationships it is helpful to correct x, y first for the effects of those phenomena which can be exactly predicted; for example, refraction, field corrections and aberration.

However, it is necessary to remember that the application of these corrections improves the results only if they permit a reduction in the number of constants to be determined by the least squares method, otherwise they are useless.

In order to reduce the number of constants in the least square method in future work, careful precautions should be taken in designing astrographs, and during the exposures in order that the field defects are well defined and known and can be corrected.

Dieckvoss has carefully investigated the systematic corrections for the AGK2 and AGK3 plates. Therefore this catalogue is very appropriate for the application of overlap methods.

On the Astrographic Catalogue, Gunther and Cox have thoroughly studied the systematic errors in the Catania zone, Eichhorn and Gatewood (1967) the errors in the Hyderbad zone. At the French observatories, an investigation of the zones with $\delta \leq +32°$ is being started.

SOLUTIONS

We have to invert a large least squares matrix. Jefferys (1963) proved the eventual convergence of iterative solutions. However, Googe (1967, p. 624) would prefer at least temporarily, direct solutions fearing that the convergence is too slow. Henriksen (1967) believes that a direct solution would be too difficult to carry out and estimates the computing labor in the case of iterative solutions to be much smaller. In fact, direct solution is convenient for simple cases,(see for example Eichhorn and Gatewood (1967)), but for a large number of plates, only the iterative solutions are possible. In the case of AGK2-AGK3 the convergence is excellent (Lacroute, 1967). I have also systematically studied the variations in the speed of convergence as it depends on the relations used and the relative weights assigned to the field stars and the reference star equations. In the iterative process adopted, the constants for each plate are arranged in groups for the estimation of corrections at each step. The convergence is slow when we "load" the overlap method "heavily", namely when the number of reference stars becomes very low and the number of field stars increases. The convergence is then slow because it is necessary to take into consideration the indirect relations between the distant plates and because at each step, the method extends the relations only by the length of half a plate diameter.

It would be necessary to improve the convergence by adding small groups of (4 or 5, or more) plates at a time to the large matrix of the constants. The partial matrix of this group would again be easy to invert.

CONCLUSION

All that was said above proves the feasibility, and indeed the efficiency of the overlap methods. We now know how to use them and although some points must still be studied, we know quite well what we may expect from these methods. We must then use them to reduce the plates in our possession, but we must also consider them for the organization of our future work. I have given (Lacroute and Bacchus, 1964a) the elements of this problem and later I went into more detail (Lacroute, 1967).

PLANS FOR FUTURE WORK

Lacroute (1967) gave a formula that describes the random error E' of the photographic system as a function of ε_1; the random error of the reference catalogue coordinates, ε_2; the random error of the measurements on the plates, S; the area of the plates, m; the number of the constants per plate, σ; and the densities of reference stars and total number of measured stars, ε; respectively.

$$\frac{E_2'^2}{\varepsilon_2^2} = 0.35 \frac{m}{S} \cdot \frac{1/3 + \left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2}{\left\{\sigma^2\varepsilon\left[1+2\left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2\right]\right\}^{1/3}}$$

This relation was used to discuss the choice of astrographs and the number of reference stars necessary to obtain a desired result.

With the introduction of K, the ratio of the total number of the stars to the number of reference stars, this becomes:

$$\frac{E_2'^2}{\varepsilon_2^2} = 0.35 \frac{m}{\sigma S} \cdot \frac{1/3 + \left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2}{\left\{K\left[1+2\left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2\right]\right\}^{1/3}}$$

In this form we see that for any given σ and ε_1, which characterize the reference catalogue, E' decreases only slowly with a corresponding increase in K due to an increase in the number of measured stars. This consideration is not important in the cases of the AGK2 and the AGK3 or of the Astrographic Catalogue because we are interested in the coordinates of the stars themselves.

But this relationship will be useful for establishing a photographic catalogue to serve as a connection between the bright stars, and the nebulae and quasars. In this case it will be judicious to measure only the minimum number of weak stars necessary to obtain the desired quality of the system. If the measuring labor is not negligible compared to the observation work of the reference catalogue, perhaps we will be encouraged to spend a little more time on the reference catalogue to avoid too big a measuring job. Indeed, E'^2 grows in inverse proportion to the work on the reference catalogue, through σ, but the actual measuring work is proportional to σK, and E'^2 varies only in inverse proportion to $\sigma K^{1/3}$.

COMPUTATIONS OF THE AGK2 AND THE AGK3

Dr. Dieckvoss has already reported on the AGK2 and the AGK3. However, I can also report on some results which I have recently obtained during the studies on the possibilities for refining these catalogues.

TEST OF THE WEIGHT INCREASE

Some tests were given (Lacroute, 1967), particularly on the effects of constants describing the magnitude correction function. I have made a close study of the polar zone.

I calculated the mean quadratic values $\overline{\Delta^2}$ of the differences between the equatorial coordinates deduced from different plates in the case of classical separated solutions (AGK2, given by Dieckvoss) and in the case of overlap method. These values Δ^2 are calculated in units of $(0''.01)^2$ after elimination of the $\Delta > 1''.00$.

With the overlap method, the number n of the available differences is larger than in the classical method (by separated plates), and the values of are smaller.

In this case of the AGK3, the same calculation gives results of inferior quality. The first solutions were only rough estimates. However, this is not the reason, because the calculated corrections were sufficiently small to be sure that the use of a linear scheme was valid. The inferior results certainly come from inferior precision of the initial measurements. The results are contained in Table I.

Fortunately, the new measurements executed by Dieckvoss will make it possible now to have better measures for the definitive work.

Table I. - The Imrpovement of Positional Agreement Due to Plate Overlapping.						
	AGK2		AGK2		AGK3	
Plate Zone	n	$\overline{\Delta^2}(0\overset{''}{.}01)^2$		improved $\overline{\Delta^2}$	n	improved $\overline{\Delta^2}$
90° and 87°	14460	1670	14937	1077	14103	1425
85°	7755	1286	7790	945	7633	1325
82°	7854	1205	7929	947	7712	1410
80° (partial)	5416	1018	5427	889	5344	1341

The benefit is especially great very near the pole. This is, perhaps so because the coordinates of the reference stars near the pole are affected by rapidly variable systematic errors. The "plate systems" obtained by the separated solutions are thus not very consistent. With overlapping, especially generous near the pole, we obtain a consistent photographic system. Thus it is possible to obtain information on the relative systematic errors of the reference catalogue, near the pole.

Table II below gives the average difference between the calculated coordinates and the introduced coordinates for the reference stars for the epoch 1930.0. i.e., that of the AGK2 , in units of 0".01. The averages of the absolute values are 0".12 for $\Delta\alpha\cos\delta$ and 0".17 for $\Delta\delta$.

Table II. - Systematic Errors of Reference Star System Near the CNP.			
Stars B.D.	Number	$\overline{\Delta}(\alpha\cos\delta)$	$\overline{\Delta}(\delta)$
84°	24	-0".05 ±0.025	+0".05 ±0.04
85°	22	+0".06 ±0.025	-0".03 ±0.04
86° and more	22	+0".04 ±0.025	-0".07 ±0.04

It seems that the reference catalogue has some systematic errors in $\alpha\cos\delta$ and δ, and that the photographic system using the overlapping method corrects a part of these errors.

POSITION ACCURACY AS A FUNCTION OF THE POSITIONS ON THE PLATES

In the polar zone ($\delta \geq 85°$), with a star occurring on n plates ($n \geq 3$) we calculate $(\delta_i - \overline{\delta})^2$ and the same expressions for $\alpha\cos\delta$. These give estimates of the errors of the calculated coordinates. In these estimates, the weight of the error ε_i of the determination on the plate i is overwhelming; it is nearly an estimation of ε_i^2.

Table III. - Position Errors in Dependence on the Distance from the Plate Center.		
Distances from plate center	Number of Estimates	$\varepsilon^2 (0''\!.01)^2$
$0° < \rho < 1^\circ\!.28$	614	398
$1^\circ\!.29 \leq \rho \leq 1^\circ\!.81$	602	507
$1^\circ\!.82 \leq \rho \leq 2^\circ\!.21$	596	467
$2^\circ\!.22 \leq \rho \leq 2^\circ\!.55$	650	569
$2^\circ\!.56 \leq \rho \leq 2^\circ\!.86$	545	591
$2^\circ\!.86 \leq \rho$	450	700

Table III gives the statistics of these estimations as a function of the distance to the center of the plate.

It will be noted that these results were obtained with the hypothesis that the measurements are of the same quality on all parts of the plates; however we observe that the quality decreases as we leave the center.

In the case of the AGK2 and the AGK3 in which we are working in α and δ, it will be more useful to account for the weights as a function of the distance from the plate center than to use the calculated partial derivatives

$$\frac{\partial(\alpha\cos\delta)}{\partial x}, \quad \frac{\partial(\alpha\cos\delta)}{\partial y}, \quad \frac{\partial\delta}{\partial x} \quad \text{and} \quad \frac{\partial\delta}{\partial y}$$

as weight factors.

DIFFERENT QUALITIES OF THE PLATES

After application of the calculated corrections, we calculated in the polar zone the mean quadratic value $\overline{\Delta_i^2}$, of the calculated coordinate differences between plate i and the neighboring plates.

If ε_i^2 is the mean quadratic value of the errors of the calculated coordinates on the plate i, and $\overline{\varepsilon^2}$ is the mean quadratic value of the errors of the calculated coordinates on the neighboring plates, we have:

$$\overline{\Delta_i^2} = \varepsilon_i^2 + \overline{\varepsilon^2}$$

It is possible to correct $\overline{\Delta_i^2}$ by $\overline{\varepsilon^2}$ to obtain ε_i^2, since the number of comparisons in the polar zone (between 600 and 1000) is big enough to have valid estimations.

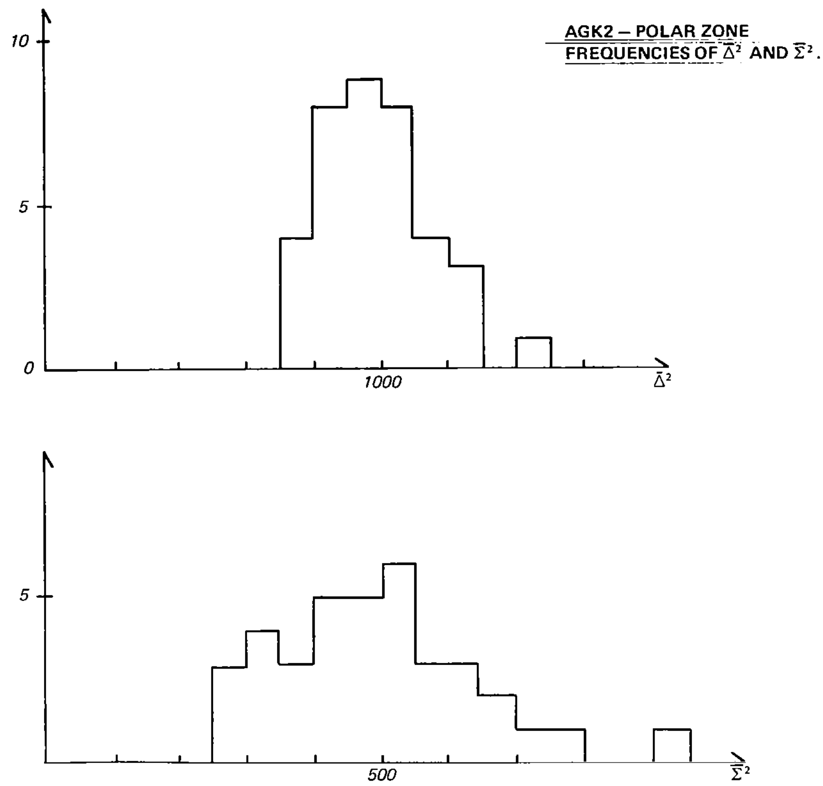

Fig. 1

Figure 1 shows the frequencies of the different values of Δ_i^2 and ε_i^2 We have to remember that these results are obtained by supposing that the plates are of equal quality, decreasing then the dispersion of Δ_i^2 and ε_i^2 .

Then, we come to the conclusion that the quality of the plates is very different. Perhaps it would be advisable to calculate a new solution by using the estimated weights obtained after the first unweighted calculation. It would be easy and useful to give weights to the plates at least for the calculation of the published mean values.

WORK TO REDUCE THE AGK2 AND AGK3

There is no point in using the results I have obtained so far, because they have been calculated with relations that are not perfectly rigorous and from a documentation which has since been improved by some identification corrections and some new measurements by Dieckvoss.

I could refine the overlap method again by using the data in their latest available form. If they were available on magnetic tape, the whole work could be finished within approximately two months. The results would be better than

than the ones to be published, but not very much better. The whole difficulty lies in the fact that the data are available only on German magnetic tapes, which is incompatible with IBM tape.

For this work, I believe it is useless to consider the proper motions also as unknowns. Indeed, the epochs of the plates scatter very little about the mean of the group and are widely different for the two groups, AGK2 and AGK3. If we consider the proper motions as unknowns, we would obtain after their elimination, a big matrix, four times bigger than without them, and the coupling between the two epochs would not be important enough to modify the results very much. However, if that does not require too much work, we will use also the proper motions as unknowns.

The difference in plate quality and of image quality in dependence on the distance from the plate center are larger than the ones disregarded by substituting the equations in $\alpha\cos\delta$ and δ, for the equations in x and y. I therefore believe we could work in $\alpha\cos\delta$ and δ. I also believe that we whould give different weights to the different plates and compute weighted averages of the coordinates for publication.

REFLECTIONS ON SOME EMPIRICAL METHODS

In the past, some empirical methods have been used to improve the results. For example (Lacroute, 1967) in the case of meridian observations, I suggested the "methode de synthèse". After a preliminary reduction on each series, the coordinates are calculated for all the stars. Then, we calculated the constants again, but by using all the information given by the results of the first calculation, on all the stars. We may repeat this process several times. This method is efficient and gives interesting results (Lacroute and Bacchus, 1964).

Similar methods can be used in the case of overlapping plates; Clube (1967) proposes such a method. They are not mathematically correct because the problem is not correctly presented. But these methods improve the results and they do not require a powerful computer. The calculations are simply the same as in the initial reduction. Then, it is interesting to compare the results obtained by these empirical methods with the results of a rigorous solution.

I examined (Lacroute and Bacchus, 1964a, p. 98) an application of this type of method in the case of overlapping plates. It is possible to establish the linear relationship between the errors of the results and the errors of the reference coordinates, and of the x and y measure on the plates. It is possible to follow the changes of the linear form at each step. Then we obtain the relationships between the system of the reference stars and the resultant photographic system. We also obtain the random errors on the system at one point. The above method is exactly the same as I used afterwards (Lacroute, 1967) on the correct solution of the problem. Some calculations have already been made and others will be carried out before long.

In the first results, the "dependences", after each step, spread over the area covered by the plates, decreasing approximately linearly in relation to the distance from the studied point, as in the case of the solution by iteration on a rigorous matrix (Lacroute, 1967). In the first iterations, the spread is smaller in the empirical method than in the rigorous method, but thereafter, it becomes larger, so that the spread extends farther than the limit obtained in the rigorous method. Therefore, it seems possible to obtain, with the empirical method, almost exactly the same results as with the rigorous method, if the iterations are judiciously stopped; but that remains to be confirmed and the purpose of the calculations in progress is to evaluate the influence of some factors on the number of iterations.

If the "dependences" are the same, the random errors on the system will be the same too.

If it can be confirmed that the empirical methods yield valid results by choosing the number of iterations, the reductions would become thereby less laborious because neither the inversion of a large matrix nor the use of a powerful computer would be necessary, but for each iteration the calculations are more difficult than for the iterations toward the solution of the large matrix.

These empirical methods are not specially suitable in the case of a large amount of plates because the establishment and the solution of the matrix could be reduced by considering that only the neighboring plates are related. But, in the case of resolution of a large number of meridian series, with well mixed stars on the sphere, the proportion of null terms in the large matrix would be smaller. The large matrix is more difficult to handle and that is a point in favor of the empiric methods.

In conclusion, the empirical methods, after justification by comparison with the theory, would perhaps be useful to the researchers who must work without large computers, or who do not want to use these for some of their experiments.

COMPUTATION OF CONSTANTS FOR THE ASTROGRAPHIC CATALOGUE

In January, 1966, the astronomers of Hamburg Observatory and the astronomers of French Observatories agreed to determine the constants of the Astrographic Catalogue in the northern hemisphere with the use of the AGK2 and the AGK3.

Messieurs Kox and Günther of the Hamburg Observatory will calculate the constants for the plates for $\delta>+32°$. Already they have made a very interesting study on the field corrections in the Catania zone.

At the beginning of 1968, the French "Centre National de la Recherche Scientifique" charged me with the task, with the collaboration of the Observatories at Paris, Bordeaux and Toulouse, to determine the constants for the zones $+32°>\delta>-2°$. Now the data from the Paris zone and part of the data from the Oxford and Toulouse zone are on computer cards.

The overlap method will be used to obtain superior results. This is especially useful here because by the extrapolation from the AGK2-AGK3 catalogues the reference coordinates will be on a good system but have large random errors. We are planning to calculate all the coordinates α, δ of all the stars because these old epoch coordinates (near 1900) on a well defined system, will be very useful for future work on proper motion. Moreover, the comparison between the α, δ calculated from different plates will be useful for judging the value of a coordinate from the agreement on the two plates.

The results obtained at the Hamburg Observatory and the French Observatories should increase the weights of the proper motions of AGK2-AGK3 by a factor of between 2 and 3, and furthermore, they will be on the same system.

PUBLICATION OF THE RESULTS

The IAU Commission 23 examined at the Prague meeting in 1967, the question concerning the form in which the results will be released. No decision was made, because (the most urgent need being the calculations themselves) with modern equipment it will be easy to transform the results into any desired form. However, it will be worth while to discuss this problem wherever interested astronomers are assembled.

My personal opinion is as follows: We have two cases.

A. Only the plate constants and the field corrections are to be determined.

B. The coordinates for all stars are also to be determined.

On the other hand, there are two points of view regarding the use of the Astrographic Catalogue:

I. To provide reference coordinates for photographic astrometric work; this was the initial purpose of the Astrographic Catalogue.

II. To obtain precise old positions for the purpose of proper motions.

The use of the Astrographic Catalogue as a source of reference coordinates for the purpose of obtaining accurate positions at recent and contemporary epochs loses its interest progressively due to proper motions.

For instance, on the basis of statistical data taken from the proper motions of the Smithsonian Star Catalogue we verify that with the Astrographic Catalogue, $m \leq 11.5$ - the average dispersion of the reference positions due to proper motions for a 70 year interval would still be $0''.5$ if we take care of eliminating half the reference stars according to their disagreement with positions from contemporary plates.

By adding to this the random errors due to the measurements on the old and the new plates, we see that it is impossible to obtain very accurate positions for contemporary epochs.

On the other hand, the use of old positions for the computation of proper motions becomes more and more useful.

In case A, the only really practical use of results will be to obtain reference positions of individual, isolated stars.

Evidently, in this case, we would have to publish the constants and the field corrections of all the plates. The use of these solutions would be practical enough for individual, isolated stars; but extremely badly suited for statistical purposes.

In case B it would be easy enough to also publish the constants and field corrections which would be useful for the employment of the Astrographic Catalogue as a source of reference positions. I do not think it would be useful however, to publish all the α, δ in print. We should store them on magnetic tapes in several formats: each format would be geared to a precise task. For instance one tape could be used for deriving the proper motion by combination computation with newly available measurements. We would then store on this tape the matrices for the determinations of the proper motion.

About ten stations scattered around the globe would be necessary (at observatories equipped with computers) and the results should be communicated in the form of copies of magnetic tapes, partial listings, or even in converted forms, for instance in the form of standard coordinates with specified centers. Communications should be rapid and billed at cost.

APPENDIX

Complements to Theoretical Studies on Overlap Methods

P. Lacroute and A. Valbousquet

Improvement of Constants

Using the method described in (Lacroute, 1967) we calculate the "dependences" between all the constants from the coordinates of the reference stars, and then, we calculate $\Sigma(\beta^2 + \gamma^2)$ for each constant.

The computation was made for 16 reference stars, K=5, that is a total of 80 stars, in the case of the relations of type II, linear with correction for tangential point:

$$X = ax + by + c + py^2 + qxy$$
$$Y = a'x + b'y + c' + pxy + qy^2$$

The values of $\Sigma(\beta^2 + \gamma^2)$ are the same for a and b', b and a', c and c', and p and q. We include then in the Table IV only the results for a, b, c, and p.

Included also are the values of $\Sigma(\beta^2 + \gamma^2)$ for the average X and the differences D of the calculated values of X on two plates.

Table IV. - Relation linear with tangential point, $\Sigma(\beta^2 + \gamma^2)$ in units of 0.001.

16 reference stars (K=5)

	a	b	c	p	$\overline{X}$	D
1st solution	200	200	100	390	187	249
1st iteration	104	103	69	103	112	62
2nd iteration	79	72	61	68	90	39
3rd iteration	64	54	55	51	79	21
4th iteration	56	47	52	45	72	17
5th iteration	51	42	49	39	68	15

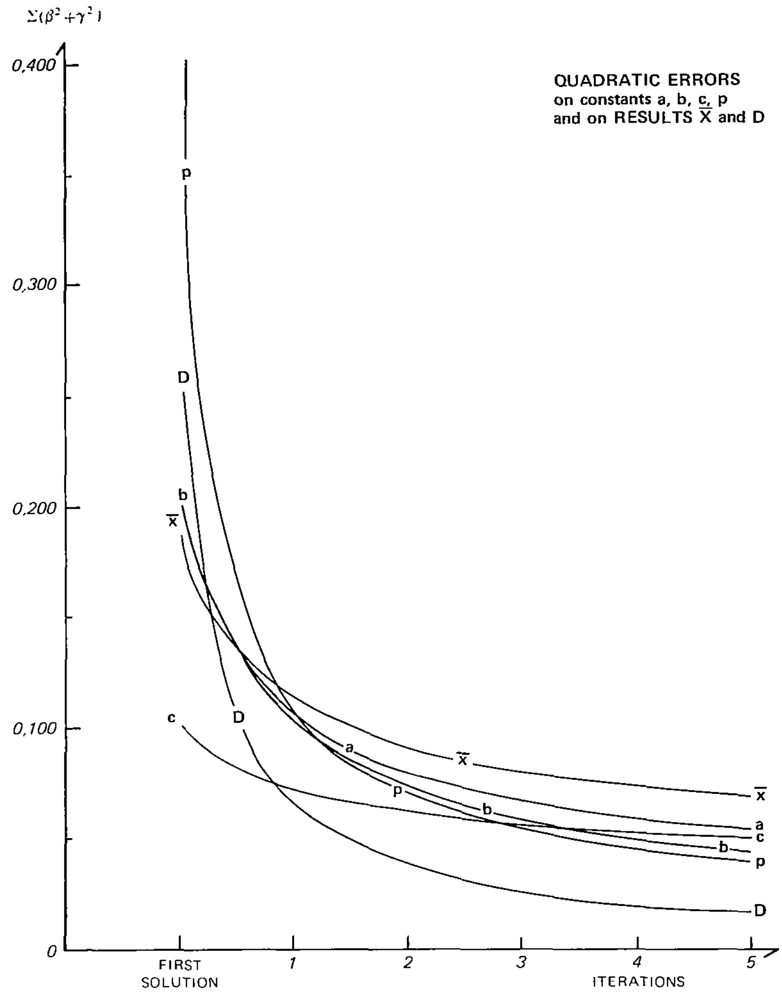

FIGURE 2

The results of Table IV in graph form.

As should have been expected, we see that the improvement is apparent primarily in the quadratic constants p and q with less improvement in the constants of magnification and orientation, a and b, and much less for the additive constants, c.

These results illustrate well the working of overlap method: the solutions for each plate are much improved, the plates are reduced precisely to a unique system. The improvement for the average random positions is not spectacular.

In a practical case, Eichhorn and Gatewood (1967) in their Table III give results in agreement with those shown above.

Validity of Tests on Coordinate Differences

Many authors use the differences between the determinations of the same equatorial coordinates on two different plates to test the quality of the results. This is especially appropriate for the overlap method.

With the notations used by Lacroute (1967), ε_1 is the random error of the reference star coordinates, ε_2 is the average random error of the measurements on the plates. We have for the average value of the difference D between two determinations of coordinates:

$$\overline{D^2} = 2\varepsilon_2^2 + \Sigma(\beta^2 + \gamma^2)\left(\varepsilon_1^2 + \frac{\varepsilon_2^2}{2}\right)$$

With the values on the table we now have:

$$\overline{D^2} = 2\varepsilon_2^2 + 0.015\left(\varepsilon_1^2 + \frac{\varepsilon_2^2}{2}\right)$$

The test on the differences gives almost exactly the random errors of the measurements on the plates. The errors by the solutions of the plates are very small.

This shows well to what extent we can be sure that the calculated results are on the same system, no matter which plate is used to deduce the coordinates.

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DISCUSSION

The following is a joint discussion of the Stoy and Clube papers, the Googe and Lukac paper, and the Lacroute paper.

Vasilevskis : *Googe's results are interesting. It is mathematically elegant and modern to take a large number of unknowns, subject them to special conditions and work everything in a large computer. But would it not have been better in the beginning to make a simple conventional reference star solution on two overlapping plates and then check for systematic trends in the residuals that require the introduction of additional reduction parameters?*

Eichhorn : *Decisions as to the most appropriate procedure are extremely difficult. If the accuracy of the position measurements on overlapping plates is intrinsically high, the positions may "interlock" and distort the system with them. If therefore, one uses a reduction model with not enough terms, true systematic trends may go unaccounted for; for example, the brightest and the faint stars may represent different systems. On the other hand, a reduction model which contains superfluous terms may introduce rather than remove systematic errors because of the correspondingly high parameter variance. Furthermore, a reduction model which is perfectly adequate for a conventional reduction may be totally unsuited for an overlap solution. I have a good example for this. Pointers as to the most appropriate procedure would certainly be valuable.*

Baker : *The various coefficients in the reduction model are also dependent on temperature, and, in particular, temperature gradients. The reason why you found the radial distortion to be a rather constant characteristic of the optical system is that the central rays which are mainly responsible for it are very little influenced by temperature gradients, which is not true for the rays from the periphery which produce the coma, so that coma terms will vary, to the extent of even reversing the sign, depending on whether the optics was used under certain cooling conditions or at a stable temperature.*

Schmid : *I believe that one ought to disregard the historical developement of astrometry in this respect and introduce, in the reduction model, physically meaningful parameters rather than such ones which produce only an interpolation formula. Only in this way is it possible to realistically utilize some a priori knowledge about the behavior of the optical system, as for instance, boundary values for the variability of certain reduction parameters which we just heard being described.*

Dieckvoss : *In preparation to reducing the plates of the AGK3 at Hamburg, Kox and Güenther performed a new reduction of the Catania Zone of the Astrographic Catalogue, which incorporated about 30 000 residuals on 1010 plates. These were used to find the non-linear terms such as coma. As an experiment, it was investigated whether the non-linear terms show a variation with temperature, by determining them separately for three different temperature groups. The result of this was negative, none of the non-linear effects depend in any way on the atmospheric temperature while the plate was taken.*

Baker : *I believe it would be more significant to look at the temperature gradient during the exposure. To do this, one probably ought to record the temperature at the beginning and the end of the exposure.*

Vasilevskis : *It is important to have the dome open and the lens exposed to the sky for as long as possible before starting the exposures.*

Baker : *In the laboratory, we found that some objective systems required a period of as long as four hours before they were usable after they had been handled.*

van de Kamp : *At our observatory we have found that it takes the lens occasionally six to seven hours to reach equilibrium. This shows again that instruments are unavoidably imperfect, and we have to learn to work with them. This introduces into science an element of art and sensual enjoyment.*

THE BEHAVIOR OF MAGNITUDE DEPENDENT SYSTEMATIC ERRORS

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ABSTRACT

Coma and other magnitude dependent errors cause a dependence of the relationship between measured and standard coordinates on magnitude. In order to establish this relationship, the reference stars must either adequately cover the magnitude range of the field stars, or faint and strong images of the same star must be directly related, for instance by means of a coarse objective grating.

The objective grating technique is discussed.

From the investigation of actual material, it is established that the effect of coma varies significantly from plate to plate, also that a model linear in diameter and/or coordinate may be inadequate for its removal, and even introduce systematic errors.

From this standpoint, the requirements of reference star systems are discussed.

INTRODUCTION

Optical systems are of necessity imperfect, that is the images of the sky which they produce on photographic plates or films follow only almost, but not quite the intended geometry, which is usally that of gnomonic projection. If the deviations from ideal geometry are generally smaller than the random error with which the position of an image can be measured, they are not particularly dangerous; however, when they behave sufficiently systematically, they can be very detrimental to the accuracy of the finally derived positions.

Since the essential characteristics of a star are its position, its brightness and its spectrum (for which color is a good substitute), the deviations Δx, Δy of the measured positions from what they would be if imaging followed strictly the ideal geometry are functions of the location of the image on the plate, its intensity and the color of the producing star. In practice, corrections which depended on all these arguments in a non-separable form such as $\Delta x = \Delta x(x,y) + \Delta x(m) + \Delta x(c)$, etc. were actually found in the reductions of some of the Yale photographic zones. These corrections must be established and applied in order to reduce the measurements properly. This is done in two steps: First, by establishing an adequate model to represent them; and Second, by the estimation of the parameters in this model. We shall first discuss the second step, and make some remarks concerning the first one later on.

One of the possibilities for finding the corrections Δx and Δy is to use a set of reference positions. It is essential that these not only cover the area of the entire plate, but also that they cover the entire magnitude and color range of the field stars. While often the set of reference stars will adequately cover the color range of the field stars, the faintest reference stars are almost universally, by necessity, several magnitudes brighter than the faintest field stars.

Fortunately, color dependence of the corrections Δx, Δy, seems to be much rarer than their magnitude dependence, and when it occurs, it is generally smaller than that of a magnitude (dependent) correction. We shall therefore discuss only the latter.

THE COARSE OBJECTIVE GRATING TECHNIQUE

We assume the systematic corrections Δx, Δy (the systematic errors would be their algebraic opposites) to be of the form $\Delta x = \Delta x(x,y,m,a_1,\ldots,a_n)$ where m is the magnitude or a magnitude equivalent such as image diameter, etc. (When not expressly stated otherwise, we shall throughout what follows, assume that completely analogous formulas and expressions hold for Δy). x, y are, of course the image's measured coordinates and $a_1,\ldots,a_n$ are the parameters in the analytical model of Δx. As an alternative to the straightforward determination of $a_1,\ldots,a_n$ through comparison with reference positions, the $\{a\}$ could be determined if, for at least a sufficiently large sample of stars, another set of images were available which require identical Δ whose m are different. Since m measures the blackening of the image on the plate and not its brightness in the sky, such a set might be produced by another exposure of different duration (on the same plate with the same camera). Note that in principle the second image need not be close to the first, although this would make things simpler in general. Properly done, the comparison of the measured positions of the two images will yield equations that can be used as condition equations for the establishment of $\{a\}$. For example, a shift of the exposure, (i.e., a change of the direction of the camera's optical axis) helps to establish parameters pertaining to the location of the image only, for instance the coefficient of radial distortion, while a variation of intensity will, when appropriate precautions are taken, lead to an establishment of those parameters that characterize the magnitude dependence of the Δ's. Note that this principle would also work for the establishment of color parameters. There is, however, at present, no satisfactory technique known for generating a set of images that require identical corrections, but with different color characteristics of the second set of images.

An ingenious device for creating a favorable situation for the determination of magnitude parameters is the well known coarse objective grating. Frequently, sufficiently bright stars produce up to three orders of measurable diffraction spectra, provided the wave length interval is kept sufficiently narrow. Theoretically, the diffraction spectra of a certain order show a blackening identical to that of the central image produced by a star Δm magnitudes fainter. Δm is called the grating constant. Since the whole complex of grating images is produced during the same exposure, one may assume that those magnitude dependent effects which depend on the peculiarities of a single exposure (such as a magnitude equation caused by faulty guiding) will be identical on all images produced by the same exposure. Furthermore, the average x_D of the x-coordinates of a pair of corresponding diffraction images is ideally equal to x_C, the x-coordinate of the central image. Hence we may write

$$\Delta(x,y,m) - \Delta(x,y,m + \Delta m) = x_C - x_D \tag{1}$$

The differences $x_C - x_D$ are immediately available from the observations before any other reductions are performed on the plates, and may themselves be regarded as observations that are subjected to random errors. Therefore, every comparison of a grating image pair with a central image will give an equation of type (1). The system thus generated can be used as condition equations in a least squares system for the determination of the parameters $\{a\}$, provided that an analytical model for Δ has been chosen.

In this way, Δ can be determined independently of reference stars and from images whose blackening covers the entire range on the plate. For this and other reasons, a grating ought to be de rigeur in photographic astrometry whenever images of different blackening are simultaneously considered.

THE VARIATION OF THE COMA EFFECT FROM PLATE TO PLATE

A minimum model for Δ is:

$$\Delta x = a_1 m + a_2 mx$$
$$\Delta y = b_1 m + a_2 my \qquad (2)$$

whereby one assumes that the coma effect is strictly radial.

This requirement can be relaxed to:

$$\Delta x = a_1 m + a_2 mx$$
$$\Delta y = b_1 m + b_2 mx \qquad (3)$$

where the coma is still linear. In essence this was done in the Yale Photographic Zones -30° to -35° and -35° to -40° (Hoffleit, 1967) and, somewhat modified in a determination of new plate constants for the northern Hyderabad Zone of the Astrographic Catalogue (Eichhorn and Gatewood, 1967).

At Yale, the coma coefficients a_2 and b_2 were determined independently for each direction on each plate from comparisons with reference stars only. They are there called G and J, respectively. In order to test whether the assumption that the coma coefficients are completely independent in each direction and on each plate, the correlation between the coma coefficients in the two directions on each plate can be used. This is so, because, if there are significant variations of the coma effect from plate to plate, the coma coefficients on each plate would deviate from the mean in both coordinates in the same sense, so that there would be a correlation between the a_2 and b_2. The correlation between the published G and J is 34% in a sample of 60 plates which shows that in this zone the coma does indeed vary significantly from plate to plate. Likewise, this shows that a model in which the x and y coma coefficients were not allowed to change completely independently of each other, as for instance in Eichhorn and Gatewood's (1967) paper, would have been more realistic.

The following data in Table I kindly supplied by Dr. Hoffleit, compare in three random sample regions the published coma coefficients G and J with those newly calculated from grating images. Every number is followed by its r.m.s.; the unit is 10^{-5}"/mm mag.

Table I. - Coma Coefficients of three plates in the Yale Photographic Catalogue.

Reg	G	δ_G	a_2	δ_{a_2}		J	δ_J	b_2	δ_{b_2}
4	+182	275	+ 258	242		+ 293	264	+ 38	289
24	+681	234	+ 506	222		+ 532	250	+ 767	192
52	+916	245	+1376	160		+1438	337	+1621	200

Although the sample in this table is too small, one can still see that the coma coefficient can, generally, be at least as accurately determined by the grating method as by comparison with reference stars. If this fact is utilized the resulting positions will be systematically more accurate because there will be fewer parameters which will depend on the reference stars alone. Table I also shows perfect agreement (within their errors) between the coma coefficients determined by the two different and completely independent methods. Inspection of the values also shows that one cannot doubt the reality of the variations of the coma effect from plate to plate.

THE ANALYTICAL FORM OF THE COMA EFFECT

During the work by Eichhorn and Gatewood, (1967) on new reduction parameters for the northern Hyderabad zone of the Astrographic Catalogue, expressions (3) were used for the modeling of the magnitude equation and coma, after extensive experiments with single plates had shown that carrying additional terms would only be an additional load on the reference stars and decrease the systematic accuracy of the final positions. Since a very powerful computer was not available, the plate overlapping was done in 16 regular complexes of six plates each (and an additional complex covering the galactic cluster NGC752). Table II gives the averages of the values of the coefficients a_2 and b_2 over the 16 plates in analogous positions in the plate complex. The individual values were taken from Eichhorn and Gatewood (1968) and are there called α_5 and β_6, respectively.

Table II. - Linear Coma Coefficients for Varying Positions in Overlapping Complex.

	$x(a_2)$				$y(b_2)$			
Zone	preceding	center	following	average	preceding	center	following	average
+39°	-1162		-2187	-1674	-1238		-1380	-1309
+38°		-1490		-1490		-1007		-1007
+37°	-1205		-2252	-1728	-2136		-2110	-2123
+36°		-1983		-1983		-3196		-3196
Avg.	-1184	-1736	-2219		-1687	-2102	-1745	

There is a pronounced trend of the a_2 to increase with the right ascension of the plate center, and a curved trend with the declination is likewise indicated. The b_2 values curve strongly with declination and with right ascension. Inspection of the values in Eichhorn and Gatewood (1968) also shows that these dependences are more pronounced in the Milky Way regions where the large number of field stars creates a stronger overlap tie than off the Milky Way, where the fit to the reference stars is stronger. The strongest of these, although different in character, are $a_2(\alpha)$ and $b_2(\delta)$. $a_2(\alpha)$ is practically linear and could probably be made constant by allowing for a term mx^2 in the x reductions. $b_2(\delta)$ is smallest for the center plates at +38°, the only one completely covered by overlapping plates. If the y coma had a term m^2y which was neglected, the effect would be similar to what is observed in Table II.

The adjustment of the plates to each other is mainly determined by the faint stars which can actually be on a system that deviates from that of the reference stars since none of the reference stars are directly compared with the faint field stars.

The other trends, namely $a_2(\delta)$ and $b_2(\alpha)$ are perhaps not pronounced enough to be of serious concern. They might be produced by terms mxy^2 or m^2xy, and myx^2 or m^2xy respectively. Note that terms m^2 and my^3, respectively, might also be responsible for the observed $a_2(\alpha)$ and $b_2(\delta)$. From the material at hand, one cannot decide whether higher order terms in m or the coordinates are indicated. Special ad hoc investigations would in each case have to be conducted. If for instance, terms containing m^2 are responsible for the trends, they would show up if coma values determined from investigations in which stars within different magnitude ranges were compared. If, for instance, terms that contain m^2 were significant in the Yale Zone discussed above, the values of G and J in Table I would be significantly different from a_2 and b_2, since the former were determined from material in the magnitude range of the reference stars only, while the latter were determined from images that cover the entire magnitude range of the plate.

However, even knowing which terms to include in the model solves only part of the problem. How variable will the coefficients of these terms be from plate to plate? What are the zero points of magnitude and coordinates to be used when higher order powers of these quantities are taken? (These will furthermore have to be carried as parameters and thus contribute to the systematic uncertainty.) Will their inclusion in the system increase the parameter variance to the point where systematic errors are increased rather than decreased? All these are questions that only further experimentation can answer.

CONCLUSION

Astrometric work almost always covers stars in a considerable magnitude range. Since particularly non-linear magnitude effects which are independent of the images' positions on the plate can be established only by a grating or if the reference material covers the same magnitude interval as the field stars, no critical work ought to be undertaken without a coarse objective grating (or an equivalent device) or faint reference stars, preferably both.

A warning must also be sounded against over confident predictions of systematic accuracy obtainable from overlapping without or with only a minimum of reference stars. These considerations usually are based on the (entirely wrong) assumption that the optical field properties of the cameras used are either known or can be established with any desired accuracy. In this author's opinion, much more research will be necessary to find out for sure how the application of the plate overlap method influences the need for reference stars, especially faint ones.

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DISCUSSION

Gatewood : *In spite of the imperfections mentioned, our results on the Hyderabad Zone are still more accurate than previous results.*

Baker : *There are actually three kinds of coma which depend in different ways on the aperture ratio and the angular width of the field. The ordinary linear coma is the most prevalent for relatively narrow fields. For wider angle fields, coma terms that involve the square of the field angle do become important, likewise terms that go as* mx^3. *The* mx^2 *term, I believe, originates from a decentering of the elements.*

Lacroute : *When one reduces a large field, preferably in a ring around the sky, the coma obtained will have to adjust itself to the average value.*

Eichhorn : *This is quite true, but we had only small fields.*

Baker : *When wide angle fields are observed, another cubic magnitude effect is introduced. The principal ray is actually inclined to the emulsion, which has a finite thickness of, say 10 microns. As the light penetrates the emulsion at a skew angle, the images of faint stars are formed close to the emulsion surface, while those of bright stars are formed throughout the emulsion. Since the images are not measured from the direction in which the principal ray came, but vertically, a magnitude dependent displacement from the center will thereby be introduced.*

van de Kamp : *Originally, when long focus astrometry got started, it was assumed that the lenses were perfect and that there was no coma. We now realize that long focus telescopes produce a coma effect and we have learned to live with it.*

INVERSION OF VERY LARGE MATRICES ENCOUNTERED IN LARGE SCALE PROBLEMS OF PHOTOGRAMMETRY AND PHOTOGRAPHIC ASTROMETRY

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ABSTRACT

The simultaneous adjustment of very large nets of overlapping plates covering the celestial sphere becomes computationally feasible by virtue of a twofold process that (a) generates a system of normal equations having a "bordered-banded" coefficient matrix, and (b) solves such a system in a highly efficient manner. Numerical results suggest that when a well constructed spherical net is subjected to a rigorous, simultaneous adjustment, the exercise of independently established control points is neither required for determinancy nor for production of accurate results.

INTRODUCTION

During the last decade the disciplines of analytical photogrammetry and photographic astrometry have evolved, quite independently of one another, along closely parallel lines. In astrometry, the initial impetus was provided by Eichhorn's (1960) general development of the "plate overlapping" method. This method is now well known to specialists in astrometry and is being adopted in more and more investigations (a fact well attested to by several papers at this conference). In the field of photogrammetry the theory of analytical aerotriangulation developed by Brown (1958) constitutes an independent development akin to the plate overlapping method. The astrometric and photogrammetric theories are concerned with fundamentally the same problem, namely, the reconstruction of positions in object space from photographic measurements. In the astrometric problem relating to stellar positions, the points in object space are located on the celestial sphere at an essentially infinite distance. Moreover, if stellar parallaxes are ignored, the location of all exposure stations may be considered to coincide with the center of the celestial sphere. Thus in stellar astrometry one is concerned primarily with the geometrical reconstruction of a *two dimensional* object space from photographs made by cameras of known and coincident location. By contrast, in aerial photogrammetry the points in object space are at a finite distance and are distributed in three dimensions. Moreover, the exposure stations are not necessarily coincident nor are they necessarily of known location. Thus in aerial photogrammetry one is concerned primarily with the geometrical reconstruction of three dimensional object space from photographs made by cameras of unknown and noncoincident location. It follows, then, that basic developments in analytical aerotriangulation can usually be specialized in order to apply to astrometry, whereas developments in astrometry must usually be generalized in order to apply to analytical aerotriangulation. In this paper, results in analytical aerotriangulation originally developed to establish a Lunar Control Network (Brown, 1968) will be specialized to apply to the analogous astrometric problem of determining stellar positions from a possibly large net of overlapping photographs covering the entire celestial sphere. The rigorous simultaneous adjustment of such nets can lead to the generation of systems of normal equations involving tens of thousands of unknowns. The problem of obtaining a practical, direct solution to such a system (as opposed to an indirect or iterative process such as that of a Gauss-Seidel) is one that has been successfully solved in analytical photogrammetry. As will be shown, an analogous solution applies to the astrometric problem of simultaneous adjustment of a large block of photographs covering the celestial sphere.

STRUCTURE OF THE NORMAL EQUATIONS

In considering the problem of adjusting a large block of overlapping photographs providing complete coverage of the Moon, Brown (1968) showed that by following a process of "pole-to-pole spiral ordering" of photographs one generates a banded system of normal equations amenable to an efficient direct solution. We shall review these results briefly before considering their application to the corresponding astrometric problem.

Photographic coverage of a sphere from external points can be systematized in a convenient manner by means of a scheme based on successive bisections of the sides of an icosahedron inscribed in a sphere (Figure 1). A total of k successive bisections generates a set of approximately equal spherical trinagles having a total of $10 \times 4^k + 2$ vertices. Each vertex generated by a given level of bisection may be regarded as the approximate nadir of a vertical photograph. For a specified cone angle of the camera the heights of exposure stations may be taken to be comfortably sufficient to encompass all immediately adjacent nadirs (generally six in number, except when the exposure station is over a vertex of the original icosahedron, whereupon the number of adjacent nadirs is five). Thus points in the vicinity of photo nadirs generally appear on seven different photos and most other points appear in four or more photos.

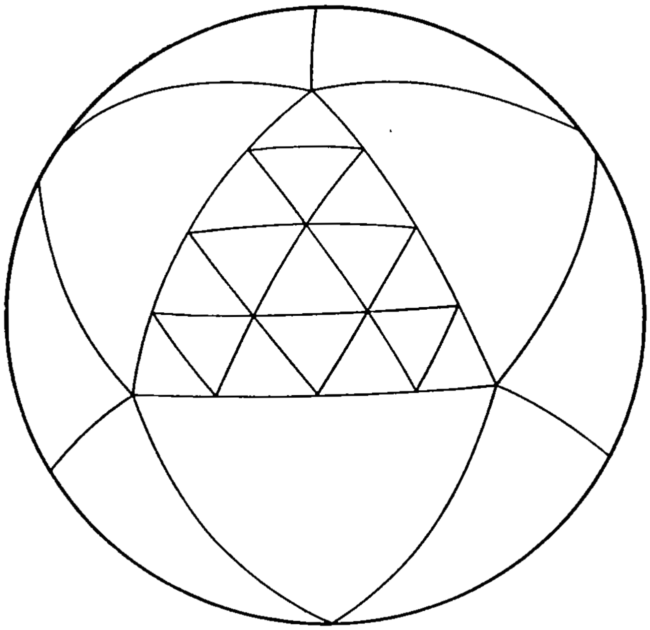


FIGURE 1
Systematic method of subdivision of a spherical surface by repeated bisections of the sides of an initial set of spherical triangles connecting adjacent vertices of an inscribed icosahedron.

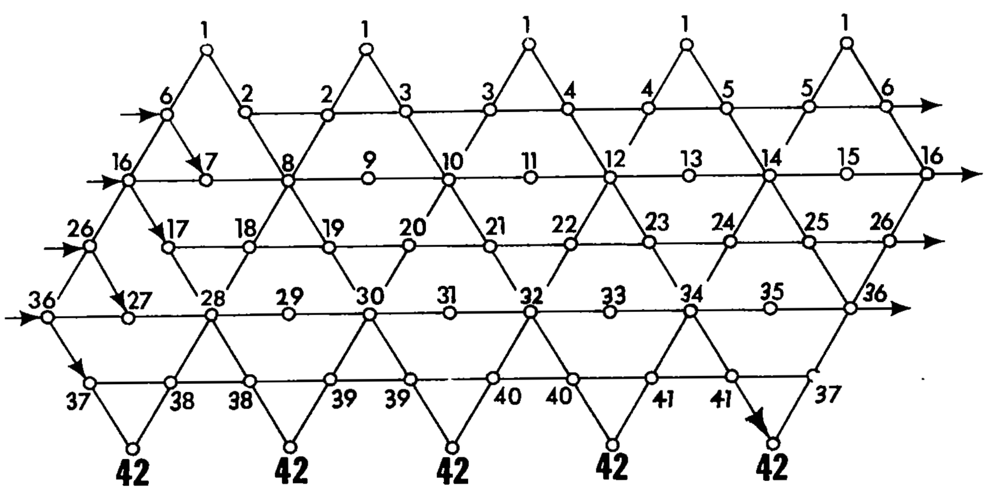


FIGURE 2
Illustrating pole-to-pole spiral ordering for 42-photo net covering a sphere.

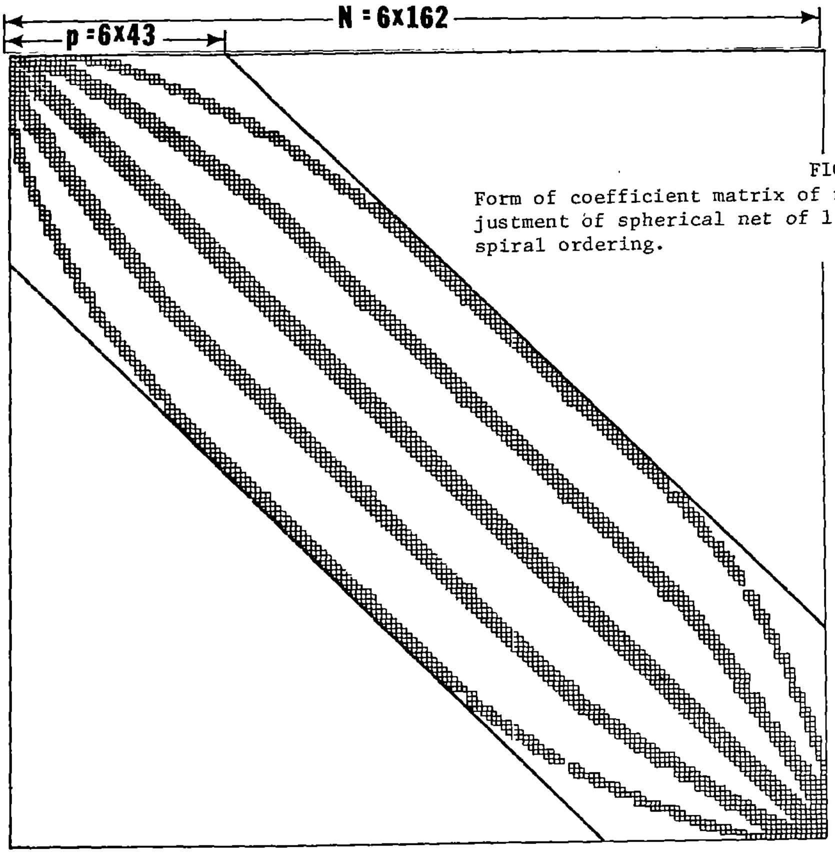
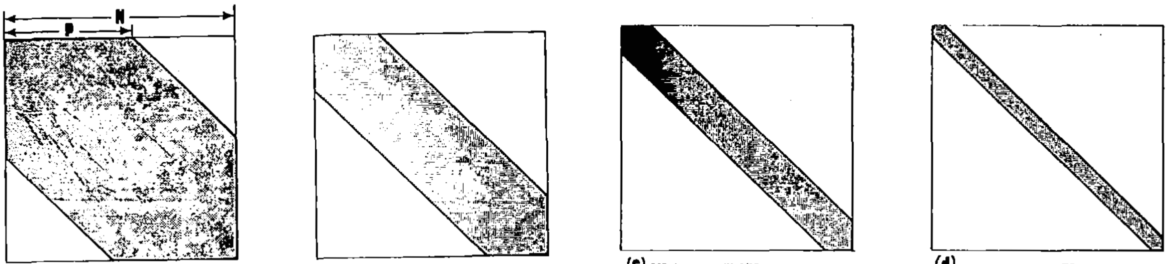


FIGURE 3
Form of coefficient matrix of normal equations generated by adjustment of spherical net of 169 photos subjected to pole-to-pole spiral ordering.

The detailed structure of the normal equations for the photogrammetric adjustment depends directly on the photo-ordering scheme adopted. The most desirable scheme appears to be what may be called "pole-to-pole spiral ordering". The application of such a scheme to the 42 photo net generated by the first level of bisection of an icosahedron is illustrated in Figure 2. Here, the icosahedron providing the framework for the division of the sphere is presented in its well known development as a surface in a plane. The flow of the numbering scheme characterizing pole-to-pole spiral ordering is indicated with the aid of arrows and is largely self explanatory. The only subtlety of the scheme is in the manner in which the transition is made from one zone to the next. If spiraling is to the right, as in the figure, the step down from one zone to the next must also bear to the right, as shown, for otherwise the form of the normal equations will be adversely affected.

When pole-to-pole spiral ordering is adopted, the coefficient matrix of normal equations for simultaneous block adjustment assumes the form typified in Figure 3, which applies specifically to the 162 photo net resulting from the second level of icosahedronal bisection. Each small square in the figure represents a 6 × 6 block of nonzero elements. The unknowns consist of six projective parameters for each photo, namely, corrections (δX^C, δY^C, δZ^C) to the



(a) 42 photo net, p/N≃1/2 (b) 162 photo net, p/N≃1/4 (c) 642 photo net, p/N≃1/16 (d) 2562 photo net, p/N≃1/32

FIGURE 4
Illustrations showing the increase of diagonal dominance of the normal equations for a spherical net with an increase in the number of photos in the net.

adopted position of each exposure station and corrections ($\delta\alpha$, $\delta\omega$, $\delta\kappa$) to the adopted angular elements of orientation. The coordinates X, Y, Z of photographed points do not appear explicitly in the normal equations for they have been eliminated according to the method developed by Brown (1958). The nonzero elements of the coefficient matrix are seen to be confined to five diagonal bands that are parallel except where they draw together near the upper left and lower right hand extremeties of the matrix. The matrix falls within the general classification of banded matrix. An $N \times N$ matrix $\{a_{ij}\}$ is said to be banded with a minimum bandwidth of p if $a_{ij} = 0$ for all $|i-j| \geq p$, and if $a_{i,\,i+p-1}$ $a_{i,\,i-p+1} \neq 0$ for at least one value of i. The bandwidth ratio is defined as p/N. In general, with pole-to-pole spiral ordering one finds that the coefficient matrix of the normal equations for the adjustment of the spherical net defined by the k th bisection of an icosahedron constitutes a banded system having the following properties.

Number of photos, m: $10 \times 4^k + 2$

Order of normal equations, N: ℓm (ℓ = number of unknowns per photo)

Bandwidth, p: $\ell[10 \times 2^k + 3] \simeq \ell\sqrt{10m}$

Bandwidth ratio, p/N: $(10 \times 2^k + 3)/(10 \times 4^k + 2)$

$$\simeq 1/2^k$$

$$\simeq 1/\sqrt{m/10}$$

The fact that the bandwidth ratio is inversely proportional to the square root of the number of photos means that the normal equations become increasingly diagonally dominant as the number of photos in the net is increased (see Figure 4). By exploiting this fact, one can (as will be shown shortly) effect the solution of the normal equations with an efficiency relative to a conventional solution that increases with the number of photos.

The development, as reviewed thus far, pertains to a photogrammetric block with unknown exposure stations external to the sphere. The specialization of the development to apply to the astrometric problem involves the following steps:

a) the radius of the sphere is assigned the value of unity;

b) all exposure stations are assigned a height of minus unity (this places all stations in coincidence at the center of the unit sphere; also, nadir points become tangential points);

c) the coordinates of exposure stations are not allowed to adjust;

d) the constraint $X^2 + Y^2 + Z^2 = 1$ is introduced for all photographed points, or, equivalently, the height of each point is constrained to zero (in either case, the X, Y, Z's become direction cosines);

e) the projective model is revised, if desired, to include additional parameters beyond the three rotational parameters remaining after step c);

f) a camera cone angle is assigned that is sufficiently large to encompass the tangential points of the five or six plates immediately adjacent to that of any given plate.

With regard to f), one finds that for a plate format 23cm in diameter, the cone angles and focal lengths appropriate to the various levels of bisection of the icosahedron are as indicated in Table 1. It is to be noted that the degree of overlap provided by the recommended scheme (i.e., points common to as many as seven plates) is greater than is generally practiced in astrometry. Customary astrometric overlap of fifty percent (and often less) would be considered by photogrammetric standards to be too weak for an acceptable analytical reconstruction.

TABLE 1. Characteristics of camera coverage of celestial sphere corresponding to various levels of bisection of icosahedron, (for adopted plate format 23 cm in diameter).

No. Bisections k	No. Plates m	Cone Angle 2θ	Focal Length (m) $f = .115 \cot \theta$
1	42	72°	.16
2	162	36°	.35
3	642	18°	.73
4	2562	9°	1.46
5	10242	4°.5	2.92

When the steps outlined above are taken, the normal equations for the astrometric problem assume precisely the form considered above for the corresponding photogrammetric problem. The only difference is that the 6 × 6 nonzero submatrices, such as those in Figure 3, become $\ell \times \ell$ submatrices, where ℓ denotes the number of projective parameters considered to be unknown for each plate.

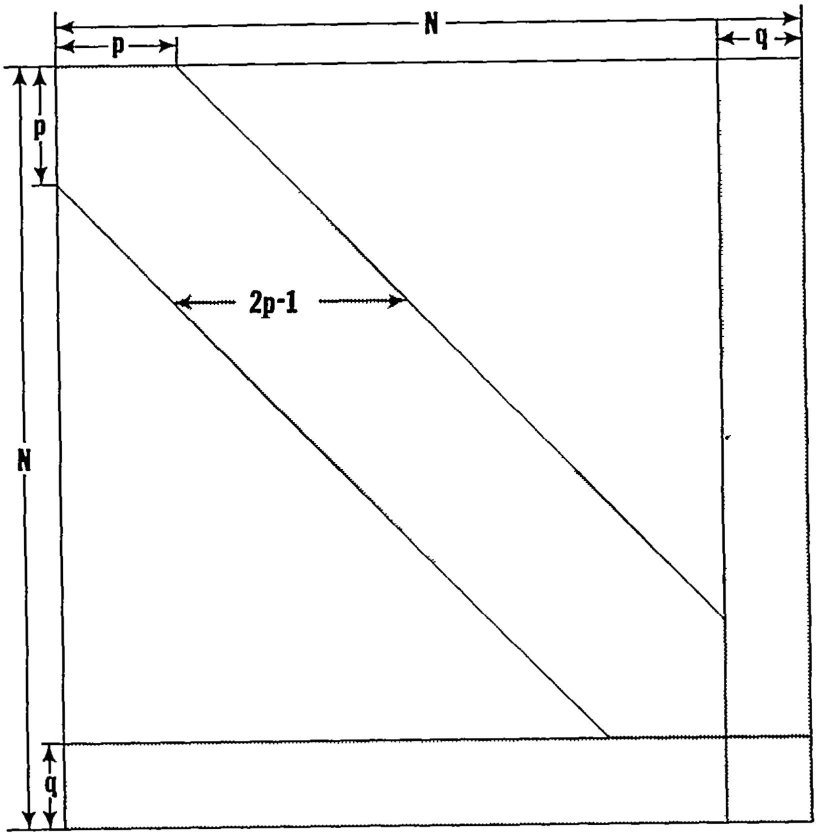


FIGURE 5
General form of bordered-banded coefficient matrix.

SOLUTION AND INVERSION OF BANDED SYSTEMS

The banded system of Figure 3 may be regarded as a special case of the general matrix depicted in Figure 5. This is a bordered-banded matrix having bandwidth of p and border width of q. In early 1966 we developed a special algorithm called Recurrent Partitioning to effect the direct solution and (on option) inversion of the matrices of normal equations having bordered-banded coefficient matrices. The algorithm was incorporated into our existing analytical aerotriangulation program, which in June 1967 was successfully used to effect the adjustment of a simulated 1000 photoblock of aerial photographs (5 strips of 200 photos). Total computing time for the formation of the normal equations and the computation of their solution by Recurrent Partitioning amounted to 40 minutes on a computer in the class of an IBM 7094. A conventional direct solution, by contrast, would have taken well over 500 times longer. In general, the computing time required to solve a bordered-banded system of normal equations by Recurrent Partitioning is on the order $T \simeq K(p + q)^2N$ where K is a constant depending on the computer used. A completely filled coefficient matrix corresponds to the special case where $p + q = N$, whereupon Recurrent Partitioning becomes equivalent to Gauss Elimination, and the computing time

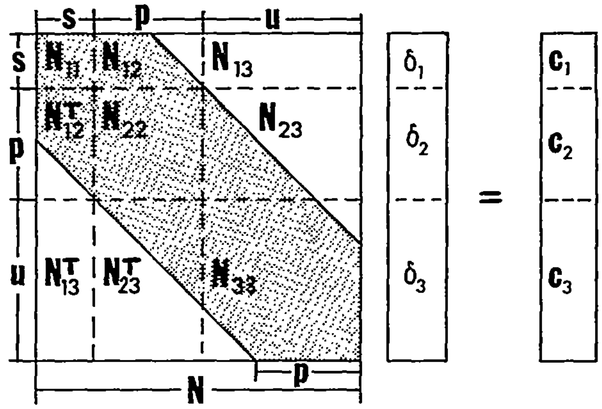


FIGURE 6
Partitioning of banded system of normal equations.

becomes $T_0 \approx KN^3$. It follows that a solution by Recurrent Partitioning requires only $T/T_0 = (p + q)\,/N$ as much time as a solution by direct method that does not specifically recognize and exploit the bordered-banded form of the normal equations.

Because the detailed development of Recurrent Partitioning is provided by Gyer (1967), we shall limit consideration here to the bare essentials of the method. Furthermore, we shall limit consideration to a banded system for this suffices to illustrate the principles involved. Illustrated in Figure 6 is a banded system of normal equations which has been subjected to triple partitioning. The number of elements in the first partition is arbitrary except that $s \leq p$, the bandwidth. The number of elements in the second partition is p, which automatically leaves $u=N-p-s$ elements in the third partition. If we now apply the method of partitioning to eliminate the vector δ_1 from the above system, we shall obtain the reduced system:

$$\left\{\begin{bmatrix} N_{22} & N_{23} \\ N_{23}^T & N_{33} \end{bmatrix} - \begin{bmatrix} N_{12}^T \\ N_{13}^T \end{bmatrix} N_{11}^{-1} [N_{12} N_{13}] \right\} \begin{bmatrix} \delta_2 \\ \delta_3 \end{bmatrix} = \begin{bmatrix} c_2 \\ c_3 \end{bmatrix} - \begin{bmatrix} N_{12}^T \\ N_{13}^T \end{bmatrix} N_{11}^{-1} c_1 .$$

Because N_{13}, N_{13}^T are zero matrices by construction, this reduces to:

$$\begin{bmatrix} N_{22} - N_{12}^T N_{11}^{-1} N_{12} & N_{23} \\ N_{23}^T & N_{33} \end{bmatrix} \begin{bmatrix} \delta_2 \\ \delta_3 \end{bmatrix} = \begin{bmatrix} c_2 - N_{12} N_{11}^{-1} c_1 \\ c_3 \end{bmatrix} .$$

Comparing this system with its predecessor in Figure 6, we see that the banded form and the bandwidth of the original coefficient matrix are preserved;

only the N_{22} portion of the matrix is altered by the elimination of δ_1. One could therefore partition the reduced system in the same manner as the original system, and then could repeat the process of elimination. It is clear that at no stage of the repeated application of this process would one have to operate outside the original band. Moreover, at any step of the process, only the $p \times p$ portion of the matrix corresponding to the current N_{22} is subject to alteration.

The overall process thus constitutes a simple repetitive reduction that can be formulated as a recurrent process (hence the name Recurrent Partitioning). Repeated application of the process will ultimately lead to a system sufficiently small to be solved directly for all of the remaining unknowns. This partial solution can then be used in backward substitution, whereby the unknowns eliminated at each step of the forward course are recovered in reverse order. As in the forward reduction, the bandwidth is preserved throughout the backward reduction. Specific exploitation of the fact that zeroes outside the original band are never annihilated provides the key to the computational efficiency of the algorithm. A similar statement holds for the extended version of Recurrent Partitioning applying to a bordered-banded matrix.

Not only can Recurrent Partitioning be used to solve a bordered-banded system of normal equations but it can also be adapted to invert the coefficient matrix. In the application to inversion, the algorithm can, on option, be exercised to reconstruct only that portion of the inverse corresponding to the original band and border. Because it requires only as much additional time as is needed for the solution alone, this mode of inversion is especially efficient. Yet, it provides all the elements of the inverse that are needed for subsequent error propagation through the determination of positions in object space.

The border in a bordered-banded system allows one to accommodate unknowns that may be common to many plates - parameters for distortion, for example (perhaps with temperature dependent coefficients). Another possibility is to employ the border for the coefficients of a tentative, empirical error model intended to account for unknown systematic errors that are regarded as common to all plates or to large subsets of plates. For example, one might postulate that systematic errors common to all plates can be described empirically by selected terms of general polynomials such as:

$$\Delta x = \sum_{i=2}^{r} \sum_{j=0}^{i} \sum_{k=0}^{j} a_{ijk} x^{i-j} y^{j-k} z^{k}$$

$$\Delta y = \sum_{i=2}^{r} \sum_{j=0}^{i} \sum_{k=0}^{j} b_{ijk} x^{i-j} y^{j-k} z^{k},$$

where x, y denote plate coordinates, and z denotes either image diameter or stellar magnitude. The unknown coefficients a_{ijk}, b_{ijk} would be determined within the adjustment (note: zero and first order terms are not included in the above expressions because they would ordinarily be equivalent to the parameters in the banded portion of the matrix). When all plates contribute to a common empirical model, the result is far more deterministic than when independent empirical models are postulated for each plate. For this reason, one can accommodate rather extensive models in the border without serious risk of inducing ill-conditioning. The border can also serve as a convenient place to assign occasional parameters that would otherwise increase the bandwidth of the banded portion of the matrix. Thus one sometimes finds that a logical ordering scheme leads to a very narrow bandwidth except for a few outlying blocks requiring a significantly wider bandwidth for their accomodation. One can easily overcome this problem by reordering the offending parameters so that they become relegated to the border.

From the foregoing it is clear that a bordered-banded system allows the investigator considerably greater flexibility than does a banded system alone. By achieving an ordering that generates a banded bordered system of normal equations having minimum p + q, one maximizes the efficiency of the solution.

THEORETICAL COMPUTING TIMES

The significance of pole-to-pole spiral ordering in combination with Recurrent Partitioning is best appreciated through a comparison of the theoretical computing times required for this approach versus a conventional unstructured approach. Such a comparison is provided in Table 2.

TABLE 2. *Theoretical computing times for solution of normal equations for simultaneous adjustment of astrometric net covering entire celestial sphere.*

Number Bisections k	Cone Angle 29	Focal Length (meters) f	Degree of Normal Equations N	Borderwidth Plus Bandwidth p+q	Border-Bandwidth Ratio (p+q)/N	*Comp. Time, Conventional Solution (hr) $T_o \approx KN^3$	*Comp. Time, Solution By Rec. Part. (hr) $T \approx [(p+q)/N]^2 T_o$
1	72°	.16	36+6x42	36+6x23	1/ 1.7	.0029	.0011
2	36°	.35	36+6x162	36+6x43	1/ 3.4	.124	.009
3	18°	.73	36+6x642	36+6x83	1/ 7.3	7.11	.13
4	9°	1.46	36+6x2562	36+6x163	1/15.2	442.6	1.9
5	4°5	2.92	36+6x10242	36+6x323	1/31.2	28,129.	29.0

*Based on CDC-6600 ($k \approx 1.21 \times 10^{-10}$)

Here, we have made the following assumptions;

a) 6 independent projective parameters are exercised for each plate (thus, $\ell = 6$);
b) 36 parameters are considered to be common to all plates (thus, $q = 36$);
c) a CDC-6600 computer is used for the reduction (thus, $K \simeq 1.2 \times 10^{-10}$hr in the expression $T_0 \simeq KN^3$).

Because some of the matrices considered become too large to be stored in any realistically postulated core, the further assumption is made that external storage is used for the coefficient matrix and that each reduction is perfectly buffered so that no time is lost in transfer of data in and out of core. This assumption is one that can be realized in practice.

Table 2 indicates that while a conventional direct solution of the normal equations becomes prohibitively time consuming for nets embracing thousands of photos, this is not the case when Recurrent Partitioning is employed. We see, for example, that a conventional reduction of a 2562 photo net would require well over two weeks of steady computing on a CDC-6600, whereas a reduction by Recurrent Partitioning would require under two hours. Table 2 suggests that even the adjustment of net containing as many as 10 000 photos remains a practical possibility with Recurrent Partitioning.

RESULTS OF NUMERICAL SIMULATIONS

Simulations reported on by Brown (1968) indicate that in establishing a Lunar control network of uniform accuracy by simultaneous adjustment of all photos covering the surface, one can do totally without the exercise of pre-established control points. It suffices instead simply to exercise a set of arbitrary constraints that serves to define uniquely the origin, orientation and scale of the coordinate system to be adopted. One would expect a similar result to hold for the corresponding astrometric reduction. In this case, the specification of the coordinate system could be fully accomplished by assigning direction cosines (0,0,1) to one particular star and by assigning a value of zero to either the first or second direction cosines of another star. The first specification would establish an arbitrary pole and the second an arbitrary zero meridian.

In order to gain insight into the fundamental strength of the adjustment, we performed a series of numerical simulations of the 42 photo astrometric net resulting from the first bisection of an icosahedron. Camera focal length was taken as 150mm, which is close to the 160mm of Table 1 and corresponds to a typical aerial mapping camera. Plate measuring accuracy was taken as three microns (one sigma) which corresponds to a basic angular accuracy of about four seconds of arc. Only the minimal case was considered in which the measured stars on each plate are limited to stars near the tangential points of overlapping plates (thus only 6 or 7 stars are measured on each plate). In order

TABLE 3. Expected accuracies of stellar positions from simultaneous adjustment of 42 plate net covering celestial sphere (camera cone angle 72°, focal length 150mm).

		σ_φ	$\sigma_\lambda \cos\varphi$
Case 1. Limiting case of perfectly known projective parameters for all plates.	Min.	1".27	1".23
	Max.	1.32	1.32
	Av.	1.29	1.25
Case 2. All projective parameters unknown; twelve well distributed control points exercised in adjustment.	Min.	1.34	1.33
	Max.	1.51	1.49
	Av.	1.42	1.41
Case 3. All projective parameters unknown; no control points exercised in adjustment.	Min.	1.58	1.54
	Max.	1.96	1.94
	Av.	1.74	1.69

to use, without modification, the computer program developed for the simulation of the Lunar control net, we limited the unknown projective parameters to the three angular elements of orientation for each plate (thus $\ell = 3$). Inversion of the coefficient matrix of the normal equations was accomplished by Recurrent Partitioning. The resulting covariance matrix of the projective parameters was then used to perform the error propagation associated with the determination of stellar coordinates in accordance with results developed by Brown (1958).

Key results of the simulations are summarized in Table 3. In Case 1 we assumed that all projective parameters were flawlessly known. This limiting case provides a standard against which other less restrictive cases can be evaluated. The angles λ, ϕ referred to in the table are spherical coordinates analogous to right ascension and declination. The table lists the minimum, maximum and average values of σ_ϕ and $\sigma_\lambda \cos\phi$ resulting from the propagation of plate measuring error through the simultaneous adjustment of the entire block of plates. For Case 1 we see that, given perfect projective parameters, accuracies in stellar positions on the order of 1".3 are to be expected from the postulated 42 photo net.

In Case 2 none of the projective parameters is considered to be known. However, a set of twelve, perfectly known stellar control points is exercised in the reduction. These control points are located at the vertices of the inscribed icosahedron from which the net was generated. In this case, the propagated effect of errors in the projective parameters reconstructed from the adjustment results in an increase in typical standard deviations to slightly over 1".4, or about seven percent higher than the limiting case.

The most interesting and most significant results are provided by Case 3. Here, all projective parameters are regarded as unknown, and not a single control point is exercised. The coordinate system is established by assigning direction cosines (0,0,1) to Point 1, the adopted pole, and by defining the zero meridian to pass through Point 17 (see Figure 2). The consideration of overriding importance from Case 3 is that the adjustment remains determinate even though no control points are exercised. Moreover, standard deviations are fairly uniform throughout the celestial sphere and are only about one third higher than those for the limiting case (about 1''.7 versus 1''.3). This result, it is to be emphasized, depends on the reduction of only six or seven stellar images per plate. If the number were increased tenfold to sixty or seventy per plate, one could expect the resulting average standard deviations to approach to within five percent of the limiting value of Case 1. This statement is based on the following considerations. Let us set:

n = typical number of stars carried on each plate;
$\sigma_p^2(n)$ = variance in stellar coordinates attributable solely to errors in projective parameters determined from average n stars per plate;
σ_0^2 = variance in stellar coordinates for limiting case of perfectly known projective parameters.

Then the following relation developed by Brown (1958) holds:

$$\sigma^2(n) = \sigma_0^2 + \sigma_p^2(n)$$

in which $\sigma^2(n)$ is the total variance in stellar coordinates. Now, if the number of stars per plate is changed from n well distributed stars to kn well distributed stars, one has

$$\sigma^2(kn) \simeq \frac{1}{k}\sigma^2(n).$$

We know from Table 3 that $\sigma_0^2 \simeq (1.3)^2$. Also, $\sigma^2(n) \simeq (1.7)$ for the case in which $n = 7$. It follows that for $n = 7$, $\sigma_p^2(n) \simeq (1.7)^2 - (1.3)^2 = 1.20$. Accordingly, for $n = 10 \times 7$, $\sigma_p^2(n) \simeq .12$. Thus when approximately 70 stars are reduced per plate, one has $\sigma^2(70) \simeq (1.3)^2 + (.12)$ or $\sigma(70) \simeq 1.36$. This is only about five percent greater than $\sigma_0 \simeq 1.30$, and, in turn, is superior to the results to be expected from Case 2 in which 12 control points are exercised.

Our results indicate that if a moderately great number of stars are exercised on each plate, a simultaneous adjustment of a net of 42 plates covering the celestial sphere can establish stellar coordinates to accuracies closely approaching theoretical limits even when no stellar control points are used. More extensive numerical simulations should be undertaken to establish whether or not this statement has general validity. Such simulations should consider not only very large nets, but also should exercise more extensive projective models and varying degrees of plate overlap. Of particular interest would be an investigation of the effects of empirical modeling of systematic errors common to large numbers of plates. Experience in other fields leads us to believe that in large scale adjustments, rather extensive empirical modeling of this nature can be incorporated without danger of significant dilution of attainable accuracies. If this be so, final results can conceivably be rendered almost immune to effects of unknown systematic errors that persist from plate to plate. We believe a practical test of this thesis would constitute a most worthwhile undertaking. As an initial venture we would suggest the execution of actual photography to generate a net of 162 plates covering the celestial sphere. A suitable camera for this purpose would be one employing a 360mm Schneider Symmar lens and Kodak 24 × 24cm microflat plates. Accuracies in stellar positions theoretically to be expected from such an exercise would approach 0".6. By comparing suitably transformed positions from actual reductions with corresponding catalogued positions, one could determine the extent to which theoretical expectations can actually be realized, and, more importantly, one could ascertain to what extent systematic error can actually be overcome (both with and without special modeling). A limited experimental investigation along such lines should, we feel, be executed and thoroughly evaluated before the undertaking of more extensive experimental investigations is seriously considered.

When plate measuring accuracies of 2 to 3 microns (one sigma) are attained, our extrapolations suggest that one can expect to produce stellar positions accurate to between 0".1 and 0".2 from the simultaneous adjustment of an uncontrolled 2562 plate net produced by a camera having a focal length of about 1.5m. Let us assume momentarily that this expectation is indeed correct. The consequences would be far reaching, for photographic astrometry would then be freed from all dependence on control stars established by meridian observations (except to the minor extent required to effect a rigid body rotation of arbitrarily established λ, ϕ system into the right ascension-declination system). This raises the intriguing possibility that photographic astrometry might even one day serve to uncover systematic errors in the system of fundamental stars. This would, of course, constitute a complete reversal of the traditional roles of photographic and visual astrometry. Until now, the plate overlapping method has been universally regarded as primarily an interpolative process, of greater effectiveness than earlier methods, but nonetheless fundamentally dependent on a framework of pre-established control. We now appreciate that such a framework is not inherently essential to the analytical reconstruction of a well constructed spherical net.

CONCLUSIONS

In the simultaneous adjustment of a spherical net, we find that the processes of pole-to-pole spiral ordering and modeling of effects common to large numbers of plates combine to generate normal equations having a bordered-banded coefficient matrix. The direct solution and inversion of such a system of normal equations becomes computationally feasible by means of Recurrent Partitioning, even when tens of thousands of unknowns must be recovered simultaneously. Results on a limited scale indicate that spherical nets can be successfully adjusted with but slight dilution of theoretically attainable accuracies, even in the complete absence of independently established control. Should these findings hold for very large nets, the implications to astrometry could be profound. In any event, practical means are now available to permit the undertaking of more extensive investigations into this matter.

As a final note we would suggest that a greater degree of interdisciplinary awareness between astrometry and photogrammetry could well be of considerable benefit to both fields.

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DISCUSSION

Clube : *What happens if you impose a direction on your solution?*

Brown : *In Case 2 I assumed that twelve reference stars were available. However, Case 3 showed that these were not really essential to determinacy. In this sense I agree with your comment that in principle, the photographic method can produce star positions entirely without reference stars. For that reason, it could be used as a means of investigating systematic errors in a fundamental star catalogue.*

APPENDIX

Comparison of measuring and reducing techniques of satellite observations and their accuracy.

There were six agencies represented in the meetings of the Conference, which were actively engaged in obtaining, measuring, and reducing satellite data. The techniques used toward this purpose at the various institutions varied considerably and this appendix has been compiled to give a comparison of the data acquisition and handling methods. The place where the original data are stored is also indicated so that, if some time in the future improved star positions should become available, some of the reductions might be repeated with the improved star positions and lead to more accurate results.

The agencies from which information was available and the instruments they used for measuring the photographic material were as follows:

The Smithsonian Astrophysical Observatory (SAO) at Cambridge, Massachusetts used a two lead screw David Mann model 829A comparator.

The Deutsches Geodätisches Forschungsinstitut (DGFI) der Deutschen Geodätischen Komission at Munich, Germany used a Wild Stereo comparator model STK1.

The Aeronautical Chart and Information Center (ACIC) at St. Louis, Missouri used a David Mann type 1204 semiautomatic stellar comparator.

At the PSC of New Mexico State University (NMSU) at University Park, a two screw David Mann type 422F measuring machine was used.

The workers at the Royal Radar Establishment (RRE) at Great Malvern, Worcs., England used a 30cm × 30cm Zeiss coordinatograph.

At the Coast and Geodetic Service (CGS) of the Environmental Service Administration of the U.S. Dept. of Commerce in Rockville, Maryland, six different Mann monocular two screw comparators were in use.

The rows of the Table below give the following information (n.i. means that no information was available).

1. the measuring machines were operated either manually (m) or semiautomatically (s.a.).
2. mode of digitization (T = Telecordex, n = none, M = David Mann).
3. the output of the measurements appeared on punched cards (c) or punched tape (t).
4. type of emulsion carrier (f = film, p = glass plates).
5. size of the field in linear units.
6. effective focal length of camera (in units of millimeters).
7. mode of camera operation (f = earth fixed, s = following the stars).
8. a satellite image is understood to be a point shaped trail section (s), an interruption in the trail (i) for passive satellites, or a flash (f) for active satellites.

9. standard deviation of the measured coordinates of one stellar image, as it is used in the reduction (in units of microns).

10. number of settings made on each stellar image.

11. were the measurements made in direct or reverse positions of the frame?

12. number of images of the same (reference) star measured on any one frame (average).

13. (average) number of satellite images measured on any one frame.

14. standard deviation of the measured coordinates of an individually recorded satellite image (in units of microns).

15. average number of reference stars used.

16. standard deviation of the rectangular coordinates of the reference star positions as computed from the measured coordinates of one of its images on the frame.

17. number of satellite images on one frame that are combined into a fictitious point.

18. standard deviation of the coordinates of a fictitious (combined) satellite image.

19. standard deviation of the timing associated with the satellite images (in units of millisecdons).

20. form in which the data at the quoted institutions are preserved (c = punched cards, s = typed sheets, p = computer printout, m = magnetic tape, t = punched tape, f = microfilm).

Table entries which are numbers in parentheses refer to remarks at the bottom of the table.

APPENDIX TABLE.- Information on procedures used for the establishment of spherical positions of artificial satellites.

	SAO	DGFI	ACIC	NMSU	RRE	C&GS
1	m	m	sa	m	m	m
2	T	n	M	T	n.i.	T
3	c	c	c	c	t	t
4	f	g	g	g	g	g
5	n.i.	18 × 18 cm^2	10 × 10 in^2	8 × 10 in^2	20 × 15 cm^2	18 × 21 cm^2
6	500	450	(1)	1000	611	450
7	(2)	f	f	s	f	f
8	(2)	s	(3)	f	(3)	s
9	2 to 3	2	1	4 to 5	1.5	1.6
10	6	2	1	5	4 (4)	2
11	yes	yes	no	no	yes	yes
12	1	8	6	1	2	5
13	1	200	(5)	7	60	300
14	2 to 3	2.5	1	4 to 5	2.2	1.4
15	8	50	25	50	50	120
16	$0''.5$	3.5μ	$0''.7$	4μ	4.2μ = $1''.4$	n.i.
17	(6)	all	∿100 (7)	n.a.	n.a.	(8)
18	1" to 2"	0.2 to 0.3μ	$0''.7$	n.a.	n.a.	0.65μ
19	1	1	0.5	n.i.	0.5 to 1	(9)
20	c	s,c	c,p	c,m	t,p	m

Remarks:

(1) Three camera types were used, PC 1000 with focal length 100mm, and two types of BC-4 cameras with 450 and 300mm focal length, respectively.

(2) The Baker-Nunn cameras could be used in the fixed mode, or tracking a satellite, or tracking the stars. All of these were employed on different occasions, and accordingly, yielded different accuracies.

(3) Flashes or active satellites and shutter produced trail interruptions for passive satellites.

(4) Two settings each by two different observers.

(5) 7 for active, 100 for passive satellites.

(6) Fictitious points were not normally produced - this was done only for special projects.

(7) Number valid for photographs with the PC-1000 camera.

(8) 7 Fictitious points were created from the 300 satellite images on any one frame.

(9) The timing accuracy of the satellite images depends on the speed of the rotation section that produces the interruptions in the trail. At 60 rpm the timing s.d. is 0.5ms, at 30 rpm, 1 ms. Most satellites were recorded with the shutter speed between these extremes.