FREQUENCY OF LIGHT-FLASHES INDUCED
BY CERENKOV RADIATION FROM HEAVY COSMIC-RAY NUCLEI

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Astronauts on Apollo missions 11 through 14 have reported seeing light-flashes at a typical rate of about one per minute. We have calculated the expected frequency of light-flashes induced in the dark-adapted eye by Cerenkov radiation from the flux of heavy nuclei that exists in the region of space beyond the geomagnetic field. In order to induce a visual sensation, at least two Cerenkov photons must be absorbed in coincidence by the rhodopsin molecules in a small area of the retina served by a single optic nerve fiber. The fact that the photons must be coincident on a small cluster of rods restricts the path length of the heavy nuclei for the production of relevant Cerenkov photons to a thin layer of the nervous tissue of the retina just ahead of the rods. The expected frequency of light-flashes depends on the threshold number of photons that must be absorbed in a rod cluster. It is known that individual threshold values in the dark-adapted eye can vary by a factor of five even among young normal human subjects. The result of our calculation can be presented as a curve of the mean frequency of light-flashes versus the threshold number of absorbed photons. The results are not sensitive to variations in the path length from 5 to 15 grams per square centimeter of water-equivalent before the nucleus reaches the retina. Our calculations are based on the fluxes and energy spectra of galactic cosmic-ray nuclei helium to iron measured at a time of minimum solar modulation. The expected light-flash frequencies induced by Cerenkov radiation are consistent with the frequencies reported by the astronauts on Apollo missions 11 through 14.

Astronauts on Apollo missions 11 through 14 have reported seeing light-flashes in a region beyond the geomagnetic field when their eyes were dark-adapted (1). The mean interval between the light flashes is typically about one minute. According to physiological experience, every stimulus that is capable of exciting the visual sensory nerve fiber can produce the sensation of light (2). That part of the nervous system where light sensations can be excited comprises the retina, the optic nerve, and a part of the brain containing the fibers of the optic nerve. In addition to visible photons, types of stimuli that are known to induce light sensations include mechanical stimulation, electric currents, and X-rays (2)-(4). Pressure on the eyeball produces a pressure-image or phosphene (5). Light flashes can be induced by current discharges through the optic nerve. Collimated X-rays (3)(4)(6)-(11) have been shown to stimulate spots on the retina of the dark-adapted eye. Previously, we have given a brief report (12) of a calculation of the expected frequency of light-flashes induced in the dark-adapted eye by Cerenkov radiation from the flux of heavy nuclei that exists in regions of space beyond the geomagnetic field. In this paper, we present some of the details of this work. Fazio et al (13) have proposed that the Cerenkov mechanism may be the dominant one responsible for the light flashes observed by the astronauts. Tobias et al (14) indicate that the dominant mechanism is probably ionization.

VISUAL SENSATIONS INDUCED BY CERENKOV RADIATION

Cerenkov radiation occurs when a charged particle traverses a dielectric medium at a speed greater than the speed of light in the medium. The Cerenkov wave is conical. The apex of the cone is at the position of the charged particle. The radiation is emitted in the forward direction at an angle θ with the direction of the particle velocity. The angle θ is the half-angle of the cone. It is given by the relation

$$\cos \theta = 1/n \beta \qquad (1)$$

where n is the index of refraction of the dielectric medium at the frequency of interest, and β is the speed of the particle in units of the speed of light.

Since the Cerenkov radiation is an electromagnetic shock wave, it contains all frequency components that can satisfy Eq. (1). Per unit path length traversed by a particle of charge Ze and speed β, the number of photons with frequency ω in the interval $\Delta \omega$ is given by

$$\frac{dN}{d\ell} = Z^2 \frac{\alpha}{c} \sin^2 \theta \, \Delta \omega \qquad (2)$$

Here the fine structure constant $\alpha = e^2/\hbar c \simeq 1/137$. This expression assumes that the medium is non-dispersive; that is, the index of refraction is assumed to be constant over the frequency interval $\Delta \omega$ of the emitted light. Wald and Brown (15) have measured the relative absorption spectrum of human rhodopsin. The wave-

lengths at the half-amplitude points of the scotopic (rod) spectral sensitivity curve at the retinal surface are $\lambda_1 = 4.4 \times 10^{-5}$ cm and $\lambda_2 = 5.4 \times 10^{-5}$ cm. Thus, in the wavelength interval corresponding to the sensitive region of the eye, Eq. (2) becomes

$$\frac{dN}{d\ell} \text{ (visible photons/cm)} = 193\, Z^2 \sin^2 \theta \qquad (3)$$

Hecht et al (16) analyzed data on threshold sensitivity and showed that a fully dark-adapted human rod can be excited by the absorption of a single (5100 angstrom) visible photon. Although a rod responds to a single absorbed photon, a single rod response will not activate the optic nerve to produce a visual sensation. For vision, it is necessary that there be an N-fold coincidence of photons absorbed in a cluster of rods served by a single optic nerve fiber (17). Threshold sensation under optimal physiological conditions requires at least a two-fold coincidence and perhaps up to a 14-fold coincidence. In the experiments of Hecht et al (16), the probability is negligible that two photons would be absorbed in a single rod because of the small number of photons incident on a much larger number of rods in a cluster served by a single optic nerve. For the Cerenkov radiation from a charged particle to induce a visual sensation, there must be a coincidence of at least two photons absorbed in a small area of the retina that contains a cluster of rods served by a single optic nerve fiber. It turns out that this requirement restricts the path length to a small region in the retina for the production of Cerenkov photons that can induce a visual sensation. The fact that the Cerenkov photons must be coincident on a small area of the retina restricts the relevant path length to a thin layer of the nervous tissue of the retina just ahead of the rods. The equations that govern the production of a visual sensation by Cerenkov light can be obtained with reference to Fig. 1. Let r_o denote the radius of a small retinal area that contains a cluster of rods served by a single optic nerve fiber. The Cerenkov photons that are incident on a rod cluster are produced by a particle that traverses a path length ℓ_o just ahead of the rods. The path length ℓ_o is given by

$$\ell_o = r_o / \tan \theta \qquad (4)$$

Thus, the number M of Cerenkov photons (in the wavelength interval between 440 and 540 nanometers) incident on a rod cluster is

$$M = \int_0^{\ell_o} \left(\frac{dN}{d\ell}\right) d\ell = 193\, Z^2 r_o \sin\theta \cos\theta \qquad (5)$$

Eq. (5) may be rewritten

$$M = 193\, Z^2 \frac{(n^2 \beta^2 - 1)^{1/2}}{n^2 \beta^2}\, r_o \qquad (6)$$

The number of Cerenkov photons incident on a rod cluster is directly proportional to the radius of the cluster. A single ganglion cell serves a circular area on the retina with a diameter that corresponds to visual angle of about 20 minutes of arc (18). For a posterior
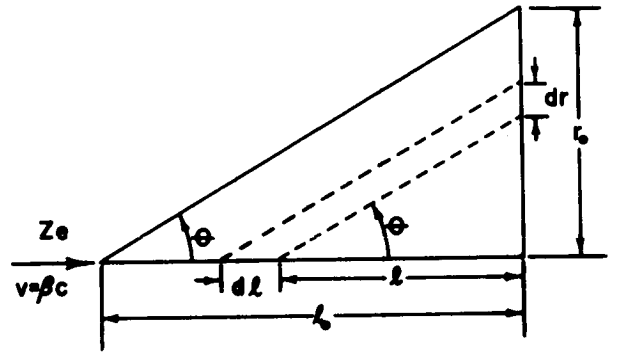
CERENKOV PHOTON PRODUCTION

FIGURE 1. Geometry for the Production of a Visual Sensation by Cerenkov Radiation from a Heavy Nucleus Traversing the Retina

nodal distance of 16.68 mm (19), a visual angle of 1 minute of arc corresponds to an image size on the retina of 4.85 microns. Thus, the radius of a circular area on the retina served by a single ganglion cell is about 48.5 microns (= 4.85×10^{-3} cm). For this value of r_o, Eq. (6) becomes

$$\frac{M}{Z^2} = 0.935\, \frac{(n^2 \beta^2 - 1)^{1/2}}{n^2 \beta^2} \qquad (7)$$

In Fig. 2, we plot the normalized number M/Z^2 of visible Cerenkov photons incident on a rod cluster as a function of the speed β of a stripped nucleus with charge Z moving in the nervous tissue of the retina just ahead of the rods. For the index of refraction n of this nervous tissue, we use 1.338. For a given nucleus of charge Z traversing the retina with a speed β, the number M of visible Cerenkov photons incident on a rod cluster is equal to the product of Z^2 and the value of the ordinate of the visible-photon production function in Fig. 2 that corresponds to the speed β. The minimum speed β_{min} at the retina of a nucleus of charge Z that can produce a specified number M of relevant Cerenkov photons can be found from this charge-normalized production function in Fig. 2. In Fig. 3, we plot β_{min} versus Z for M = 2, 5, 10, 20, 50, 100, 200, and 500. The absolute threshold speed β_{th} for the production of Cerenkov photons can be found from Eq. (1) for the limiting case $\cos\theta \to 1$; thus,

$$\beta_{th} = \frac{1}{n} \qquad (8)$$

For n = 1.338, $\beta_{th} = 0.7474$. This value for the threshold speed corresponds to a threshold kinetic energy per nucleon of 470 MeV per nucleon.
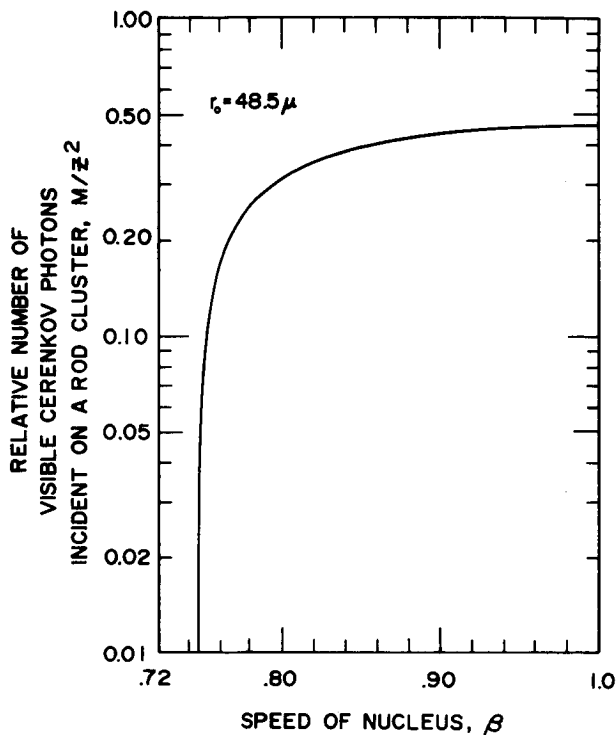
FIGURE 2. The Relative Number of Visible
Cerenkov Photons Incident on a Rod
Cluster versus the Speed of a Nucleus

FIGURE 3. The Minimum Speed of a Nucleus for the
Production of a Specified Number M of
Visible Cerenkov Photons Incident on
a Rod Cluster versus the Charge Number
Z of the Nucleus

At least two photons must be absorbed by rhodopsin
in order to induce a visual sensation. Hecht et al (16)
estimated that from 5 to 20 percent of the visible (510mμ)
photons incident on the retina are actually absorbed
by the rhodopsin. Later estimates indicate that the
probability of absorption of a visible (505 mμ) photon that
interacts with a rod is about 30 percent (20)(21). Some
of the photons incident on the retina interact with the
cones or with material between the rods. The rod density
is greatest at an angle of about 20 degrees to the right
of the observer's fovea (22); in this region, about 70
percent of the area is occupied by rods. Thus, the pro-
bability of absorption by rhodopsin of visible (505 mμ)
photons incident on the retina near 20 degrees is about 20
percent. If M photons are incident on the retina, the pro-
bability that exactly (M-T) photons will be absorbed is given
by the binomial probability distribution:

$$P(M-T) \equiv \binom{M}{T} p^{M-T} q^{T} \tag{9}$$

where p = the probability that a photon will be absorbed,
and q = $1-p$ = the probability that a photon will not
be absorbed.

If M photons are incident, the probability that at least
R photons will be absorbed is

$$Q_{M}(\geq R) \equiv \sum_{T=0}^{T=M-R} P(M-T) = \sum_{T=0}^{T=M-R} \binom{M}{T} p^{(M-T)} q^{T} \tag{10}$$

In Eqs. (9) and (10), the symbol $\binom{M}{T}$ denotes the bi-
nomial coefficients:

$$\binom{M}{T} \equiv \frac{M!}{T! \ (M-T)!} \tag{11}$$

These coefficients are tabulated in standard handbooks.
The probability $Q_{M}(\geq R)$ that at least R photons will
be absorbed in rhodopsin is listed in Table 1 for R = 2,
5, 10, and 20 as a function of the number M of incident
photons for a single photon absorption probability $p = 1/5$.
These data are plotted in Fig. 4.

FLUXES AND ENERGY SPECTRA OF GALACTIC
COSMIC-RAY NUCLEI

Comstock, Fan and Simpson (23, 24) have
measured the fluxes and energy spectra of the galactic
cosmic-ray nuclei helium to iron at the time of min-
imum solar modulation (viz, October 1964 - November
1965). Based on the reported differential energy spectra
of the various nuclei, we have represented the unidirectional
differential flux spectra of nuclei by an inverse power
law in the total energy per nucleon E above E_1, and by a
constant value in the kinetic energy per nucleon in the
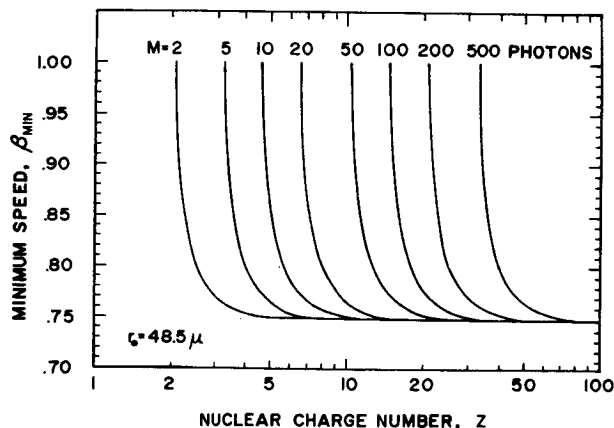interval between T_o and T_1:

Number of Incident Photons M	Probability of Absorbing at Least R Photons, $Q_m(\geq R)$			
	R = 2	R = 5	R = 10	R = 20
2	0.0400			
3	0.1040			
4	0.1808			
5	0.2627	0.0003		
6	0.3446	0.0016		
7	0.4233	0.0047		
8	0.4967	0.0104		
9	0.5638	0.0196		
10	0.6242	0.0328		
11	0.6779	0.0504		
12	0.7251	0.0726		
13	0.7664	0.0991		
14	0.8021	0.1298		
15	0.8329	0.1642	0.0001	
16	0.8593	0.2018	0.0002	
17	0.8818	0.2418	0.0005	
18	0.9009	0.2836	0.0009	
19	0.9171	0.3267	0.0016	
20	0.9308	0.3704	0.0026	
30	0.9895	0.7448	0.0611	
40	0.9985	0.9241	0.2882	
50	0.9998	0.9815	0.5563	0.0009
60	1.0	0.9961	0.7868	0.0107
70	1.0	0.9992	0.9155	0.0545
80	1.0	0.9998	0.9713	0.1634
90	1.0	1.0	0.9914	0.3382
100	1.0	1.0	0.9976	0.5398
110	1.0	1.0	0.9994	0.7190
120	1.0	1.0	0.9998	0.8483
130	1.0	1.0	1.0	0.9267
150	1.0	1.0	1.0	0.9680
160	1.0	1.0	1.0	0.9872
170	1.0	1.0	1.0	0.9953
180	1.0	1.0	1.0	0.9984
190	1.0	1.0	1.0	0.9994
200	1.0	1.0	1.0	1.0

$$n(E) = CE^{-\gamma} \qquad (E > E_1) \qquad (12)$$

$$n(T) = C_1 \qquad (T_0 < T < T_1) \qquad (13)$$

where

$$E(BeV/nucleon) = T(BeV/nucleon) + 0.9315 \quad (13)$$

The spectral exponent γ has an observed value of about 2.5. The representation in Eq. (13) is valid for a kinetic energy T_0 in the neighborhood of 100 MeV per nucleon. The value of the constant C_1 for oxygen, as reported by Comstock et al (23), is 5.4 nuclei/m^2-sec-sr-(BeV/nucleon) in the time interval from October to November, 1964. The values of C_1 for other nuclei come from measurements on the relative abundance of the elements in the cosmic radiation. There seems to be no significant change in the relative abundances with time even though the flux level changes with the solar cycle. Also, within the present experimental errors, the relative abundances appear to be independent of energy. The relative abundance of the nuclei in the cosmic radiation from the (IMP IV and OGO I) satellite measurements by the University of Chicago group are listed in Table 2 and plotted in Fig. 5. The coefficient C in the high-energy portion of the differential flux Eq. (12) is given by Comstock et al (23) for some of the nuclei (namely, C, O, B, Ne, Mg, Si and the Fe-Co-Ni group). These values for the coefficient C are tabulated in Table 2. For these nuclei, values for the total energy per nucleon E_1 can be found by equating Eqs. (12) and (13):
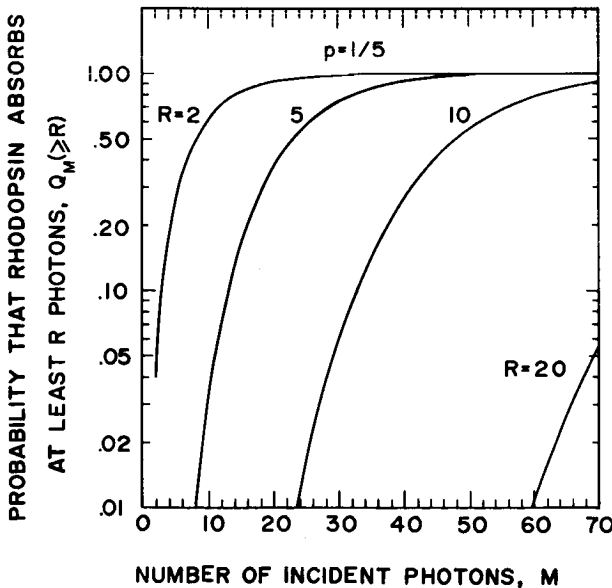
$$E_1 = (C/C_1)^{1/\gamma} \qquad (15)$$

FIGURE 4. The Probability that Rhodopsin Absorbs at Least R (= 2, 5, 10 and 20) Photons versus the Number of Photons Incident on the Retina for a Single Photon Absorption Probability of Twenty Percent

Table 2

RELATIVE ABUNDANCE OF NUCLEI IN THE COSMIC RADIATION AND PARAMETERS
FOR REPRESENTING THE DIFFERENTIAL FLUX SPECTRA AT A TIME OF MINIMUM
SOLAR MODULATION

Nucleus	Relative Abundance	Coefficient C_1 [p/m^2-s-sr-(BeV-nuc^{-1})]	Coefficient C [p/m^2-s-sr-(BeV-nuc^{-1})]	Total Energy E_1 (BeV/nucleon)	Kinetic Energy T_1 (BeV/nucleon)
$_2$He	(4.2 ± 0.6) x 10^3	228 ± 32			
$_3$Li	19.5 ± 5	1.05 ± 0.03	2.8	1.485	.554
$_4$Be	6.5 ± 2.2	0.35 ± 0.12	.94	1.485	.554
$_5$B	29 ± 6	1.6 ± 0.3	(4.3)	1.485	.554
$_6$C	100 ± 6	5.4 ± 0.3	(12)	1.376	.445
$_7$N	25.5 ± 2	1.4 ± 0.1	3.1	1.376	.445
$_8$O	100 ± 6	5.4 ± 0.3	(12)	1.376	.445
$_9$F	1.7 ± 0.2	0.092 ± 0.011	.14	1.168	.236
$_{10}$Ne	17.5 ± 3	0.95 ± 0.16	(1.4)	1.168	.236
$_{11}$Na	4.1 ± 1.1	0.22 ± 0.06	.25	1.064	.132
$_{12}$Mg	23 ± 1	1.2 ± 0.1	(1.4)	1.064	.132
$_{13}$Al	3.4 ± 0.4	0.18 ± 0.02	.27	1.183	.251
$_{14}$Si	17 ± 3	0.92 ± 0.16	(1.4)	1.183	.251
$_{15}$P	1.95 ± 1.0	0.11 ± 0.06	.18	1.170	.240
$_{16}$S	5.0 ± 1.3	0.27 ± 0.07	.40	1.170	.240
$_{17}$Cl	1.01 ± 0.29	0.055 ± 0.016	.081	1.170	.240
$_{18}$A	2.08	0.112	.17	1.170	.240
$_{19}$K	1.37	0.074	.11	1.170	.240
$_{20}$Ca	3.24 ± 0.65	0.17 ± 0.04	.25	1.170	.240
$_{21}$Sc	1.25	0.068	.10	1.170	.240
$_{22}$Ti	2.62	0.142	.21	1.170	.240
$_{23}$V	2.38	0.129	.19	1.170	.240
$_{24}$Cr	3.99 ± 1.25	0.215 ± 0.066	.32	1.170	.240
$_{25}$Mn	2.65	0.143	.21	1.170	.240
$_{26}$Fe-$_{27}$Co-$_{28}$Ni	8.15 ± 1.73	0.44 ± 0.09	(1.4)	1.589	.857
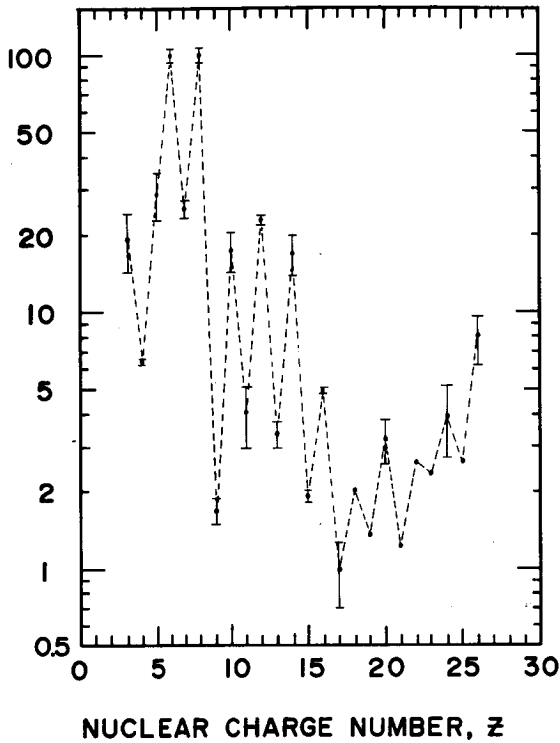
FIGURE 5. The Relative Abundance of Nuclei in the Galactic Cosmic Radiation

Table 3

INTEGRAL FLUX SPECTRA OF GALACTIC COSMIC-RAY NUCLEI AT A TIME OF MINIMUM SOLAR MODULATION

Nucleus	Coefficient $C/(\gamma-1)$ $[p/m^2\text{-s-sr-}(BeV/nuc)^{-(\gamma-1)}]$	Flux Above Cerenkov Threshold $N(E > 1.40\,BeV/nucleon)$ $(p/m^2\text{-s-sr})$
$_3Li$	1.88	1.04
$_4Be$	0.627	.60
$_5B$	2.87	1.58
$_6C$	8.00	4.94
$_7N$	2.07	1.28
$_8O$	8.00	4.94
$_9F$	0.091	.076
$_{10}Ne$	0.933	.78
$_{11}Na$	0.171	.43
$_{12}Mg$	0.933	.97
$_{13}Al$	0.183	.15
$_{14}Si$	0.933	.77
$_{15}P$	0.109	.091
$_{16}S$	0.267	.22
$_{17}Cl$	0.054	.045
$_{18}A$	0.107	.090
$_{19}K$	0.073	.061
$_{20}Ca$	0.168	.14
$_{21}Sc$	0.067	.056
$_{22}Ti$	0.140	.17
$_{23}V$	0.127	.11
$_{24}Cr$	0.212	.18
$_{25}Mn$	0.141	.18
$_{26}Fe\,-_{27}Co\,-_{28}Ni$	0.933	.47
		19.37

The values of E_1 are listed in Table 2. We have used Eq. (15) with $\gamma = 2.5$ to compute values for the coefficient C for the other nuclei for assumed (tabulated) values of E_1 and the measured values of C_1.

For $E > E_1$ (or $T > T_1$), the unidirectional integral flux spectra of the galactic cosmic-ray nuclei can be represented by the following power-law:

$$N(>E) = \int_E^\infty n(E)\,dE = \frac{CE^{-(\gamma-1)}}{\gamma-1} \quad \text{(for } E > E_1) \quad (16)$$

For $E_0 < E < E_1$ (or $T_0 < T < T_1$), the unidirectional integral flux spectra of galactic cosmic-ray nuclei can be represented as follows:

$$N(>E) = \int_E^{E_1} n(E)\,dE + N(>E_1) = C_1(E_1 - E) + \frac{C}{\gamma-1}E_1^{-(\gamma-1)}$$

$$(17)$$

Values of the coefficient $C/(\gamma-1)$ for integral flux spectra of galactic cosmic-ray nuclei for an integral spectral exponent $\gamma -1 = 1.5$ at a time of minimum solar modulation are listed in Table 3. Also listed in Table 3 are unidirectional integral fluxes above a total energy of 1.4 BeV per nucleon, which is the threshold for the production of Cerenkov radiation in the retina.

EXPECTED FREQUENCY OF LIGHT-FLASHES

To calculate the mean frequency of light-flashes expected from the Cerenkov photons associated with heavy nuclei that reach the retina, we have prepared a computer program that calculates the contribution to the light-flash frequency from each nucleus of charge number Z. For convenience, we introduce primed symbols to denote quantities at the retina. Unprimed symbols will refer to corresponding quantities in the incident spectrum. We sketch the steps involved in the calculation:

a. Find the minimum speed β'_{min} at the retina of a nucleus of charge Z as a function of the number M of visible Cerenkov photons incident on a rod cluster. This limiting speed is the solution of Eq. (7). In Fig. 6, we plot β'_{min} vs M for Z = 6, 10, 14, 20 and 28.

b. Calculate the minimum kinetic energy per nucleon T' at the retina for the production of M relevant photons:

$$T'_M = Mc^2(\gamma'_M - 1) \quad (18)$$

$$\gamma'_M = (1 - \beta'^2_M)^{-1/2} \quad (19)$$

c. Find the range (in tissue) of a proton R'_p with this kinetic energy T'_M . For this purpose, we have represented the range–energy relation by a power–law of the form:

$$R'_p = kT'^{\alpha}_M \tag{20}$$

To obtain the coefficient k and the exponent α, we use the range–energy curve for water (28). For protons in the energy interval from 100 to 1000 MeV, $k = 4.25 \times 10^{-3}$ gm/cm^2 H$_2$O=MeV$^{\alpha}$ and $\alpha = 1.627$; thus, for $100 < T'$ (MeV) < 1000,

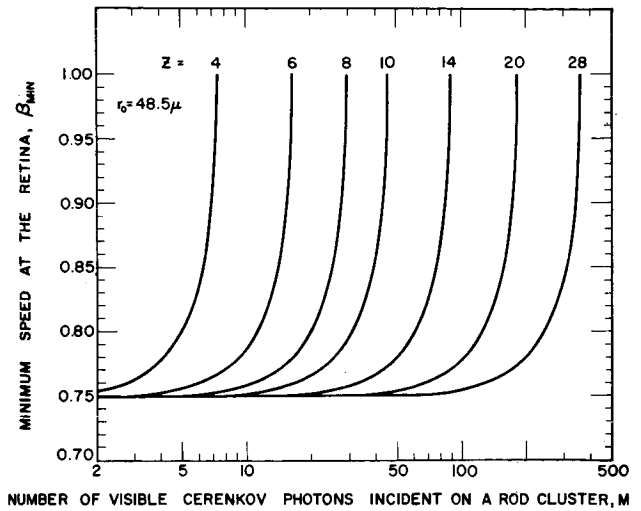$$R'_p (\text{gm/cm}^2 \text{H}_2\text{O}) = 4.25 \times 10^{-3} T'^{1.627}_M \tag{21}$$

FIGURE 6. The Medium Speed of a Nuclear of Charge Number Z for the Production of M Visible Cerenkov Photons Incident on a Rod Cluster versus the Number M

d. Find the minimum kinetic energy per nucleon T_M in the incident spectrum for the production of M photons incident on a rod cluster. The nucleus of charge number Z and mass number A must traverse a path of length ℓ before reaching the retina. The path length in the vitreous body is typically 2.4 grams per cm^2. The nucleus must also penetrate the skin of the vehicle and possibly additional shielding material. We have carried out the calculations for typical path–lengths ℓ of 5 and 15 grams per cm^2 of water–equivalent. The range of a proton ℓ_p that is equivalent to the path–length ℓ traversed by a nucleus of charge number Z and mass number A is given by

$$\ell_p = \frac{Z^2}{A} \ell \tag{22}$$

For convenience, we calculate

$$R_p = R'_p + \ell_p \tag{23}$$

From the proton range R_p, we find the minimum kinetic energy per nucleon T_M in the incident spectrum:

$$T_M = (R_p/k)^{1/\alpha} = [(R'_p + \ell_p)/k]^{1/\alpha} \tag{24}$$

e. Calculate the minimum total energy per nucleon in the incident spectrum for the production of M relevant photons at the retina:

$$E_M (\text{BeV/nucleon}) = T_M \ (\text{BeV/nucleon}) + 0.9315 \tag{25}$$

f. Find the unidirectional integral flux density N_M ($> E_M$). The symbol N_M ($> E_M$) denotes the unidirectional integral flux density of a nucleus of charge Z in the incident spectrum that can produce at least M relevant photons at the retina. From Table 2, we see that the threshold value of E_M for the production of M relevant Cerenkov photons exceeds E_1 for $5 \leq Z \leq 25$.

g. Find the incident unidirectional flux density $\Delta N_{Z,M}$ of a nucleus of charge Z that produces M relevant photons at the retina:

$$\Delta N_{Z,M} = N_{Z,M-1/2} (\geq E_{M-1/2}) - N_{Z,M+1/2}$$

$$- N_{Z,M+1/2} (\geq E_{M+1/2}) \tag{26}$$

h. Calculate the omnidirectional flux $\Delta F_{Z,M}$ of a nucleus of charge Z that can produce M relevant photons at the retina:

$$\Delta F_{Z,M} (\text{nuclei/minute}) = 120 \, G \, \Delta N_{Z,M} (\text{nuclei/m}^2\text{-sec-sr}) \tag{27}$$

Here the factor 120 consists of a factor 2 to take into account both eyes and a conversion factor of 60 seconds per minute. The factor G (m^2-sr) is the geometric factor for the sensitive region of the retina. From a curve (29) of the relative sensitivity of the eye as a function of the angular position of the test flash with respect to the fovea, we estimate that the sensitive

region of the retina is an annulus on the surface of a sphere. For visual angles α_1 and α_2 between about 10 and 50 degrees, this sensitive annulus on the retina subtends angles β_1 and β_2 at the center of a spherical eyeball of radius R. The geometric factor G of the sensitive region of the retina for omnidirectionally incident radiation is

$$G = (4\pi)(\pi R^2)(\cos\beta_1 - \cos\beta_2) \qquad (28)$$

For R = 1.2 cm, $\beta_1 = 14^\circ$ and $\beta_2 = 70^\circ$, we obtain $G = 35.7$ cm^2-sr = 35.7×10^{-4} m^2-sr.

i. Use Eq. (10) to calculate the probability $Q_M(\geq R)$ that at least R photons will be absorbed if M photons are incident on a rod cluster. The probability p that the rhodopsin in a rod cluster absorbs a visible ($\lambda = 505$mμ) is about 20 percent.

j. Find the contribution $c_{Z,M}(\geq R)$ to the mean frequency of light flashes induced by the absorption of at least R photons from the M relevant photons produced at the retina by the flux $\Delta F_{Z,M}$ of a nucleus of charge number Z.

$$c_{Z,M}(\geq R) = Q_M(\geq R)\,\Delta F_{Z,M} \qquad (29)$$

k. Find the total contribution $C_Z(\geq R)$ from nuclei of charge number Z to the mean frequency of light flashes induced by the absorption of at least R photons by summing over all allowed values of M:

$$C_Z(\geq R) = \sum_{M=R} c_{Z,M}(\geq R) \qquad (30)$$

The numbers in Table 4 represent the contribution from each nucleus to the number of light flashes per minute, $C_Z(\geq R)$, expected at a time of minimum solar modulation for an observer with a threshold that corresponds to the absorption of at least R (= 5, 10, and 20) visible Cerenkov photons. The light-flash frequencies in Table 4 are for a path-length of 15 grams per cm^2 of water-equivalent ahead of the retina. Since the frequencies increase slightly for a smaller path-length of 5 grams per cm^2 of water-equivalent, we note that they are not sensitive to small variations in the path-length.

l. Finally, find the mean frequency of light-flashes induced by the absorption of at least R photons by summing the contributions from each nucleus of charge number Z.

$$C(\geq R) = \sum_Z C_Z = \sum_Z \sum_{M=R} Q_M(\geq R)\,\Delta F_{Z,M} \qquad (31)$$

Table 4

THE CONTRIBUTION FROM EACH NUCLEUS TO THE LIGHT-FLASH FREQUENCY, $C_Z(\geq R)$, EXPECTED AT A TIME OF MINIMUM SOLAR MODULATION FOR AN OBSERVER WITH A THRESHOLD THAT CORRESPONDS TO THE ABSORPTION OF AT LEAST R (= 5, 10, and 20) VISIBLE CERENKOV PHOTONS. (This Calculation Assumes That Each Nucleus Traverses a Path-Length of 15 Grams/cm² of Water-Equivalent Ahead of the Retina)

Nuclear Charge Number Z	Frequency [(Flashes per Minute) x 10^4]		
	R = 5	R = 10	R = 20
3	---	---	---
4	---	---	---
5	68	---	---
6	1227	---	---
7	998	6	---
8	7695	247	---
9	130	14	---
10	1611	429	---
11	324	156	---
12	1805	1245	19
13	358	299	19
14	1805	1648	315
15	210	200	78
16	506	490	301
17	102	100	78
18	203	200	175
19	134	133	123
20	303	300	286
21	73	72	68
22	67	65	59
23	38	37	32
24	44	43	36
25	22	21	18
26-27-28	110	105	87
	17,833	5810	1694

In Fig. 7, we plot the number of light-flashes per minute, C (≥R), versus the threshold number R of absorbed Cerenkov photons. The upper and lower limits of each band in this figure correspond to typical path lengths of 5 and 15 grams per cm^2 of water-equivalent. It should be noted also that the curves in Fig. 7 are based upon fluxes of heavy nuclei observed at a time of solar minimum.
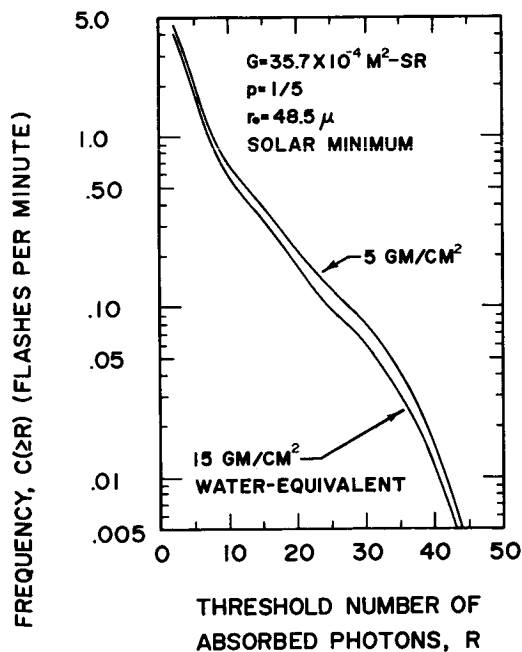

FIGURE 7. The Expected Mean Frequency of Visual Sensations Induced by the Absorption of at Least R Visible Cerenkov Photons in a Rod Cluster versus the Threshold Number R

CONCLUSION

From the results of our calculations, we can draw the following conclusions:

1. The expected frequency of light-flashes depends on the threshold number of photons that must be absorbed in a rod cluster. It is known that individual threshold values can vary by a factor of five even among young normal human subjects (30, 31). Thus, the observed frequency will depend on the physiological and psychological conditions of the observer. The calculated frequency decreases rapidly at first as the threshold number of photons increases from two to about eight; then it decreases relatively slowly in the region of eight to thirty absorbed photons; and finally it decreases more rapidly at higher values of the threshold number of absorbed photons.

2. The expected frequencies of visual sensations induced in the dark-adapted eye by Cerenkov radiation from heavy nuclei in the galactic cosmic radiation beyond the geomagnetic field are consistent with the frequencies reported by astronauts on Apollo missions. Apollo 13 astronaut Fred W. Haise reported ten flashes in five minutes; astronaut James A. Lovell reported one every two minutes; and astronaut John L. Swigert saw two flashes in about 30 minutes. The Apollo 13 observations were made at a period of maximum solar modulation.

3. The curve of the expected frequency of visual sensations induced by Cerenkov radiation is not sensitive to variations in the path length from 5 to 15 grams per cm^2 of water-equivalent before the nucleus reaches the retina.

4. The expected frequency of visual sensations induced by Cerenkov radiation decreases rapidly as the threshold number of absorbed photons increases. The observation of light-flash frequency of the order of two per minute should correspond to the absorption of a few photons and to peak efficiencies for the detection of visual sensations (16) (31-34). The steepness of the curve of frequency versus number of absorbed photons in the region of a few absorbed photons has interesting implications for studying the threshold sensitivity of the eye. A measurement of the light-flash frequency with a precision of the order of ten percent may permit a determination of the threshold number of absorbed photons. The electromagnetic radiation fields of heavy nuclei offer an alternative to weak light sources used in laboratory experiments.

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