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AERODYNAMIC PARAMETERS OF THE NAVION AIRPLANE EXTRACTED FROM FLIGHT DATA

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16. Abstract

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Results from the study showed that the parameter-extraction program can produce aerodynamic parameters which will permit close estimation of the aircraft time histories used in the extraction process. The program determined an estimate of the standard deviations of the states and parameters. These estimates were used to indicate how well the calculated states fit the flight data and the confidence in the values of the estimated parameters. The study also showed that the values of the parameters were affected by the data and mathematical model used during the extraction process. Because of the lack of confidence in the parameters extracted by use of some of the sets of data, several parameters were estimated by other methods. By using a combination of methods, a set of parameters which gave a fit to the data was obtained.

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SUMMARY

An iterative method, which is characterized as a maximum-likelihood minimum-variance technique, was used to extract the aerodynamic parameters of a Navion airplane from flight data. The purposes were to compare the results with parameters obtained from wind-tunnel tests and with results obtained by analog matching of the same data, and to develop techniques for application of the parameter-extraction program.

Results from the study showed that the parameter-extraction program can produce aerodynamic parameters which will permit close estimation of the aircraft time histories used in the extraction process. The program determined an estimate of the standard deviations of the states and parameters. These estimates were used to indicate how well the calculated states fit the flight data and the confidence in the values of the estimated parameters. The study also showed that the values of the parameters were affected by the data and mathematical model used during the extraction process. Because of the lack of confidence in the parameters extracted by using some of the sets of data, several parameters were estimated by other methods. By using a combination of methods, a set of parameters which gave a fit to the data was obtained.

The extracted parameters agreed reasonably well with the values obtained by analog matching the same data, with the exception of the change in normal-force coefficient with angle of attack $\left(C_{Z_\alpha}\right)$. The agreement with wind-tunnel parameters was not as good for the variations of pitching-moment coefficient with angle of attack $\left(C_{m_\alpha}\right)$, side-force coefficient with sideslip angle $\left(C_{Y_\beta}\right)$, rolling-moment coefficient with sideslip angle $\left(C_{l_\beta}\right)$, and yawing-moment coefficient with sideslip angle $\left(C_{n_\beta}\right)$. However, of the parameters determined by the program, only one had a standard deviation greater than 15 percent of the value of the parameter and the parameters determined gave a reasonable fit to the flight data.

INTRODUCTION

Mathematical analyses of flight dynamics and handling qualities of an aircraft are required for determining the suitability of the aircraft for its mission. In order to make such analyses, it is necessary to have available the aerodynamic parameters of the aircraft. There are several methods of obtaining the parameters. These methods include those presented in various books, wind-tunnel tests, and extraction of derivatives from flight-test data. Of these methods, derivatives from flight tests should be the most accurate since such results are obtained with the actual aircraft in its proper environment. There is, therefore, a continuing interest in developing and evaluating improved methods of extracting derivatives from flight data.

In a recent study (ref. 1), a comparison was made between various analytical methods, wind-tunnel measurements with a full-scale airplane, and results obtained from flight-test data for a Navion airplane. In that study, an analog-matching technique (ref. 2) was used in extracting parameters from the flight data. Some rather large differences were found between the various methods. In particular, some large differences were obtained between the wind-tunnel and the flight-test results. Analog matching requires a highly experienced operator to match flight data properly. It appeared desirable to use an alternate method of extracting the derivatives from the flight data. The method used in this study is a mathematical formulation of the logic required to select derivatives to best match a set of flight data. The method selected is an iterative procedure which selects the aerodynamic parameters to maximize a conditional maximum likelihood function and is equivalent to determining the set of aerodynamic parameters which will maximize the probability that the calculated state of an airplane will match the measured state for the same control inputs (ref. 3). The maximization process used minimizes the measurement error covariance matrix. The resulting parameter adjustment equations are of the same form as those obtained by use of a modified Newton-Raphson or weighted least-squares technique (ref. 4). The main difference is that with the maximum likelihood formulation, the weights are updated at each iteration. The program will speed up derivative determination, give a fit to the flight data based on mathematically minimizing a cost criterion, and determine a matrix which indicates the variances of dependency between the estimated derivatives.

The primary purpose of the present paper is to use the flight data employed in the analysis reported in reference 1 and extract the aerodynamic parameters for comparison with the results presented in reference 1. A second purpose of this paper is to indicate the procedure used in applying the parameter estimation program to the data herein. A third purpose is to relate the experience gained from this investigation and to point out the advantages of the program used. A fourth purpose is to indicate the confidence in the parameters obtained.

SYMBOLS

Values are given in both SI and U.S. Customary Units. The measurements and calculations were made in U.S. Customary Units. The aerodynamic parameters are referenced to a system of body axes with the origin at the aircraft center of gravity, and with body axes orientation as shown in figure 1.

a	acceleration, m/sec^2 (ft/sec^2)
b	wing span, m (ft)
$\bar{c}$	wing mean geometric chord, m (ft)
F	force, N (lb)
g	acceleration due to gravity, m/sec^2 (ft/sec^2)
I	moment of inertia, kg-m^2 (slug-ft^2)
i	index
i_t	tail incidence angle, radians or degrees
K	weighting factor
L	likelihood function
l_t	distance from aircraft center of gravity to center of pressure of horizontal tail, m (ft)
M	moment, N-m (ft-lb)
m	mass, kg (slugs)
N	number of data points
ΔP	change in parameter from iteration to iteration
p	rate of roll, radians/sec
q	rate of pitch, radians/sec
$\bar{q}$	dynamic pressure, $\frac{1}{2}\rho V^2$, N/m^2 (lb/ft^2)
R	estimate of error covariance matrix

r	rate of yaw, radians/sec
S	wing area, m^2 (ft^2)
T'_c	$\frac{\text{Thrust}}{\text{(Dynamic pressure)(Wing area)}}$, nondimensional
u	velocity along X body axis, m/sec (ft/sec)
V	aircraft total velocity, m/sec (ft/sec)
v	velocity along Y body axis, m/sec (ft/sec)
w	velocity along Z body axis, m/sec (ft/sec)
α	angle of attack, radians
β	sideslip angle, radians
γ	flight-path angle, radians
δ	control deflection, radians or degrees
θ	pitch angle, radians
ρ	air density, kg/m^3 ($slugs/ft^3$)
ϕ	roll angle, radians
C_l	rolling-moment coefficient, $M_X/\bar{q}Sb$
C_m	pitching-moment coefficient, $M_Y/\bar{q}S\bar{c}$
C_n	yawing-moment coefficient, $M_Z/\bar{q}Sb$
C_X	axial-force coefficient, $F_X/\bar{q}S$
C_Y	side-force coefficient, $F_Y/\bar{q}S$
C_Z	normal-force coefficient, $F_Z/\bar{q}S$

$$C_{Z_\alpha} = \frac{\partial C_Z}{\partial \alpha}$$

$$C_{m_\alpha} = \frac{\partial C_m}{\partial \alpha}$$

$$C_{X_\alpha} = \frac{\partial C_X}{\partial \alpha}$$

$$C_{m_q} = \frac{\partial C_m}{\partial \frac{q\bar{c}}{2V}}$$

$$C_{m_{\delta_e}} = \frac{\partial C_m}{\partial \delta_e}$$

$$C_{l_\beta} = \frac{\partial C_l}{\partial \beta}$$

$$C_{n_\beta} = \frac{\partial C_n}{\partial \beta}$$

$$C_{Y_p} = \frac{\partial C_Y}{\partial \frac{pb}{2V}}$$

$$C_{Y_\beta} = \frac{\partial C_Y}{\partial \beta}$$

$$C_{l_p} = \frac{\partial C_l}{\partial \frac{pb}{2V}}$$

$$C_{n_p} = \frac{\partial C_n}{\partial \frac{pb}{2V}}$$

$$C_{Z_q} = \frac{\partial C_Z}{\partial \frac{q\bar{c}}{2V}}$$

$$C_{Z_{\delta_e}} = \frac{\partial C_Z}{\partial \delta_e}$$

$$C_{m_{\dot\alpha}} = \frac{\partial C_m}{\partial \frac{\dot\alpha\bar{c}}{2V}}$$

$$C_{Y_r} = \frac{\partial C_Y}{\partial \frac{rb}{2V}}$$

$$C_{l_r} = \frac{\partial C_l}{\partial \frac{rb}{2V}}$$

$$C_{n_r} = \frac{\partial C_n}{\partial \frac{rb}{2V}}$$

$$C_{l_{\delta_a}} = \frac{\partial C_l}{\partial \delta_a}$$

$$C_{n_{\delta_a}} = \frac{\partial C_n}{\partial \delta_a}$$

$$C_{Y_{\delta_r}} = \frac{\partial C_Y}{\partial \delta_r}$$

$$C_{l_{\delta_r}} = \frac{\partial C_l}{\partial \delta_r}$$

$$C_{n_{\delta_r}} = \frac{\partial C_n}{\partial \delta_r}$$

Subscripts:

a	aileron
b	body
c	computed

e	elevator
f	flap
m	measured
o	indicates coefficient at trim conditions
r	rudder
t	indicates state at trim conditions
X	X-axis
Y	Y-axis
Z	Z-axis

Superscript:

T	transpose

A dot over a symbol signifies a derivative with respect to time.

DESCRIPTION OF AIRPLANE AND FLIGHT TESTS

Instrumentation

The flight data used for extracting derivatives was obtained from flight tests of the Princeton University variable-stability Navion airplane, N91566. The physical characteristics of the Navion are presented in figure 2 and table I. The data recorded for this study included:

	Accuracy	Response frequency	
		Hz	cps
Normal acceleration	±0.01 g	2	2
Roll rate	±0.044 radians/sec		
Pitch rate	±0.024 radians/sec		
Yaw rate	±0.010 radians/sec		
Angle of attack	±0.8°	4	4
Altitude	±30.48 m (±100 ft)	2	2
Indicated airspeed	1.03 m/sec (±2.3 mph)	2	2
Control surface position	±1 percent	2	2

The data were sequenced by a commutator at a rate of 10 points per second and telemetered to a ground station as a frequency-modulated signal where it was recorded on magnetic tape. The angle of attack was corrected for upwash effects but required no correction for angular rates. The accelerations and angular rates needed no correction.

Flight Data

The flight-test data used in the present study were obtained from tests made by personnel of the Princeton University Aeronautical Laboratory. The data were recorded on magnetic tape and were processed at the Langley Research Center for parameter extraction. The calibrations used during processing were furnished by Princeton University. The data processing included digitizing the data, converting the recorded signal to engineering units, and interpolating to give all the data at the same times on the tape. The interpolation was required because the commutated data gave each state at a different time. Flight-test conditions are listed in table II. The applied control disturbances included an elevator doublet, an aileron doublet, and a rudder pulse.

PARAMETER-ESTIMATION PROCEDURE

The parameter-estimation procedure used in this study is an iterative procedure which maximizes the conditional likelihood function L(aerodynamic parameters, weights, initial conditions):

$$L = \frac{1}{(2\pi)^{1/2}|R|^{1/2}} \exp\left[-\frac{1}{2}\sum_{i=1}^{N}\left(X_{im} - X_{ic}\right)^T R^{-1}\left(X_{im} - X_{ic}\right)\right]$$

where R is the estimate of the error covariance matrix and X is the vector describing the state of the aircraft. Maximizing the likelihood function minimizes the difference between the measured and calculated aircraft motions.

The weighting matrix R^{-1} can be the complete error covariance matrix, the diagonal terms of the error covariance matrix, or a diagonal matrix with fixed weights on the diagonal, at the discretion of the investigator. If the diagonal form of the weighting matrix is used, the weights represent the estimated lower bound of the noise on the measured states. The use of the likelihood function in parameter identification is discussed in reference 3. Maximizing the likelihood function results in a parameter updated equation which is given by

$$\Delta P = \left(M^T R^{-1} M\right)^{-1} M^T X_m$$

where M is the matrix of sensitivities of the calculated states with respect to the unknown parameters (ref. 5). The matrix will be called in this paper the estimated parameter covariance matrix. The updated equation is determined by forming a set of differential equations with the changes in the unknown parameters as the variables. This set of simultaneous equations is then solved by least squares to give the updated equation. (See ref. 5.)

The steps in the iteration procedure are outlined in figure 3. The procedure is to write a set of equations of motion for the aircraft under consideration. These equations will include a number of aerodynamic parameters. The parameters must be initially estimated so that the motions of the aircraft can be calculated. These calculated motions are then compared with the motions of the actual aircraft for identical control inputs. The parameter estimation program uses the differences between the measured and calculated data to calculate the updated values of the parameters. The parameters are then corrected and new aircraft motions are calculated. The process is repeated with updated parameters until the difference between calculated and measured motions are within some acceptable range. The complete details are given in reference 3.

During this investigation the mean-squared error between the measured and calculated states was displayed at each iteration. When the mean-squared error became constant for several iterations, the problem was terminated. A printout of an estimate of the variance of the states, the changes in unknown parameters at each iteration, the estimated lower bound on the standard deviations of the unknown parameters, and the determinant of the R matrix were obtained. Data from the printout were examined to determine the fit to the flight data and the confidence in the extracted parameters. If all these criteria indicated a fit in the order of the instrument uncertainty and if the unknown parameters had changes less than 5 percent from iteration to iteration, the parameters obtained were considered to be as good as could be determined.

The program is set up so that the fit to the flight data can be monitored on a cathode ray tube (CRT) as the parameters are updated. The mean-squared error was displayed on a digital voltmeter. The program can be stopped at any iteration and the states used in the likelihood function changed or unknown parameters added to or taken from the mathematical model. Also, any of the parameters in the mathematical model can be considered to be known and fixed at specified values. These characteristics of the program allow the operator to have a close interaction with the program and give the operator a very flexible tool to use. Because the operator can see any large effect of changes in the parameters, there is a considerable saving in time over letting the computer run unobserved for a fixed number of iterations and then examining a printout of the results. Also, the turnaround between runs is seconds rather than hours if batch processing had been used. The operator's console and cathode ray tube (CRT) are shown in figure 4.

PRELIMINARY CONSIDERATIONS

Several decisions must be made prior to application of the parameter-extraction program. These decisions are:

(1) What parameters should be extracted?

(2) How good an initial estimate is required to start the procedure?

(3) What amount of data and what sections of the flight records should be used?

In this study, all the parameters used in the equations of motion along with the initial conditions on the states were extracted initially. If it was found that some of the parameters could not be well determined, then these were fixed at an assumed value, dropped from the mathematical model, or the mathematical model was changed to determine the parameter better. The experience of the investigator in this study indicates that initial estimates of the aircraft parameters obtained from reference 6 gave a fit good enough to allow the problem to start.

An important aspect of selecting data for analysis is that of using a section of the flight record which contains motion of reasonable amplitude (well above the measuring instrument noise level). It also is advisable to include several seconds of flight record at trim conditions. The flight data used in the present analysis were recorded on magnetic tapes. The data for the analyses were taken from the tapes at the rate of 20 data points per second for each state used.

EQUATIONS OF MOTION

The equations of motion used herein are written relative to the body axes and are essentially the same as the equations given in reference 7. The assumptions used when writing the equation are:

(1) The moment of inertia $I_{XZ} = 0$. (See ref. 8.)

(2) The quantities q, β, r, and p are initially zero.

(3) The equations are basically uncoupled so that α and q variations are negligible during lateral maneuvers.

(4) The quantities θ and ϕ were initially assumed to be zero.

(5) When θ and ϕ were used in the equations, they were calculated by using $\dot{\theta} = q \cos \phi - r \sin \phi$ and $\dot{\phi} = p + (q \sin \phi + r \cos \phi)\tan \theta$.

(6) The velocities u, v, and w were calculated from the α, β, and V flight data by $u = V \cos \alpha \cos \beta$, $v = V \sin \beta$, and $w = V \sin \alpha \cos \beta$.

(7) In the $\dot{v}$ equation, u and w were assumed to be constant and fixed values were put in.

By using these assumptions, the equations of motion used were for the longitudinal equations:

$$\dot{u} = -qw - g \sin \theta + \frac{1}{2}\rho \frac{V^2S}{m}\left[C_{X,o} + C_{X_\alpha}\left(\alpha - \alpha_t\right)\right] \tag{1}$$

$$\dot{w} = g \cos \theta + qu + \frac{1}{2}\rho \frac{V^2S}{m}\left[C_{Z,o} + C_{Z_\alpha}\left(\alpha - \alpha_t\right) + C_{Z_{\delta_e}}\left(\delta_e - \delta_{e_t}\right)\right] \tag{2}$$

$$\dot{q} = \rho \frac{V^2S\bar{c}}{2I_Y}\left[C_{m,o} + C_{m_\alpha}\left(\alpha - \alpha_t\right) + C_{m_{\dot{\alpha}}} \frac{\dot{\alpha}\bar{c}}{2V} + C_{m_q} \frac{q\bar{c}}{2V} + C_{m_{\delta_e}}\left(\delta_e \delta_{e_t}\right)\right] \tag{3}$$

for the lateral equations:

$$\dot{v} = -ru + pw + g \cos \theta \sin \phi + \rho \frac{V^2S}{2m}\left(C_{Y,o} + C_{Y_\beta}\beta + C_{Y_r} \frac{rb}{2V} + C_{Y_p} \frac{pb}{2V} + C_{Y_{\delta_r}}\delta_r\right) \tag{4}$$

$$\dot{p} = \rho \frac{V^2Sb}{2I_X}\left(C_{l,o} + C_{l_\beta}\beta + C_{l_r} \frac{rb}{2V} + C_{l_p} \frac{pb}{2V} + C_{l_{\delta_a}}\delta_a + C_{l_{\delta_r}}\delta_r\right) \tag{5}$$

$$\dot{r} = \rho \frac{V^2Sb}{2I_Z}\left(C_{n,o} + C_{n_\beta}\beta + C_{n_p} \frac{pb}{2V} + C_{n_r} \frac{rb}{2V} + C_{n_{\delta_a}}\delta_a + C_{n_{\delta_r}}\delta_r\right) \tag{6}$$

Because of the nature of the control inputs, it was possible to separate the longitudinal and lateral modes of motion; therefore, the parameter-extraction program solved for the longitudinal and lateral aerodynamic parameters separately.

APPLICATION OF EXTRACTION PROGRAM

Longitudinal Aerodynamic Parameters

High-speed configuration.- Initial values of the various aerodynamic parameters which were required for the equations of motion were estimated by use of reference 6 and are listed in table III. The likelihood function initially included the states u, w, and q. An attempt was made to extract all the parameters listed in the longitudinal equations of motion and also the initial conditions for a best fit. (See run 1, table IV.) For this case, the mean-squared error decreased for several iterations and then began to increase. After about 15 iterations, the extracted values and the covariance matrix for the unknown parameters were read out.

Examination of the printout showed that at termination, changes in some of the unknown parameters were still large. (See run 1, table V.) The parameters with large variations from iteration to iteration changed in pairs or groups of three. Also, most of the parameters which were still varying had values significantly different from those predicted by aerodynamic theory. Some groups of parameters were varying together and had high estimated correlation coefficients in the covariance matrix. (See table VI.) However, since the problem had not converged, the numbers in the covariance matrix could only be considered estimates of the actual covariances. In this problem, it was found that when the estimated correlation coefficients were near 1, the coefficients involved tended to deviate toward questionable values. The parameters which appeared to be related were C_{m_q} and $C_{m_{\dot{\alpha}}}$; C_{m_q} and $C_{m_{\delta_e}}$; C_{m_α} and $C_{m_{\dot{\alpha}}}$. Examination of the flight data (fig. 5) showed that q, w, a_Z, and δ_e were all damped sine waves and approximately in phase so that a relation between these parameters might be expected.

Since the pairs of coefficients which appeared to be related were coefficients which varied together from iteration to iteration, fixing one of each pair at a specified value could possibly force the other parameter in the pair to a specific value. Therefore, $C_{m_{\dot{\alpha}}}$ was set equal to zero so that C_{m_q} assumed the whole effect of the C_{m_q} and $C_{m_{\dot{\alpha}}}$ combination and $C_{m_{\delta_e}}$ was set equal to the value used in reference 1 which was taken from reference 9. Fixing the two coefficients reduced the number of unknowns being extracted. It was felt that if the number of unknowns were reduced further, there would be a better chance for convergence, and then more unknowns could be added. Therefore, the forward velocity equation was deactivated, and the initial conditions of w and q were considered to be those of the flight data before any controls were activated, and $C_{Z,o}$ and $C_{m,o}$ were calculated to balance the equations initially. Therefore, u_o, $C_{X,o}$ C_{X_α}, w_o, $C_{Z,o}$, q_o, and $C_{m,o}$ also were eliminated as unknowns in the mathematical model. (See run 2, table IV.) The program was rerun. The mean-squared error stabilized and became constant after about 10 iterations.

By using these newly selected parameters as starting values, the initial conditions on u, w, and q, and the parameters $C_{X,o}$, $C_{Z,o}$, $C_{m,o}$, and C_{X_α} were added as additional unknowns. (See run 3, table IV.) Again, this program stabilized to a lower performance index and a set of parameters was obtained. At this point $C_{X,o}$, $C_{Z,o}$, and $C_{m,o}$ seemed to be well determined. A set of parameters had been obtained but $C_{m_{\delta_e}}$ and $C_{m_{\dot{\alpha}}}$ had been set to specific numbers. Also, the value obtained for $C_{Z_{\delta_e}}$ had a standard deviation of about 100 percent; the value obtained for C_{Z_α} was about 85 percent higher than expected (table III); and C_{m_α} was about one-third the value expected.

These results were not felt to be satisfactory. The large standard deviation in $C_{Z_{\delta_e}}$ indicated that this parameter could not be determined accurately from the available data. It was, therefore, decided to use the geometric relationship

$$C_{Z_{\delta_e}} = \frac{\bar{c}}{l_t} C_{m_{\delta_e}} \tag{7}$$

to estimate $C_{Z_{\delta_e}}$ and to keep that value constant during further parameter extractions. In addition, previous unpublished results of a derivative-extraction study had indicated that inclusion of a parameter C_{Z_q} in the mathematical model might have an appreciable effect on C_{Z_α}. The parameter C_{Z_q} therefore was added to the model, and equation (2) became (see also run 4, table IV)

$$\dot{w} = g \cos \theta + qu + \frac{1}{2}\rho \frac{V^2 S}{m}\left[C_{Z,o} + C_{Z_\alpha}\left(\alpha - \alpha_t\right) + C_{Z_q} \frac{q\bar{c}}{2V} + C_{Z_{\delta_e}}\left(\delta_e - \delta_{e_t}\right)\right] \tag{8}$$

Adding C_{Z_q} to the model changed C_{Z_α} from -6.89 to -5.5, which appeared to be more reasonable, but probably is still too negative. The parameter C_{Z_q} was found to be -26.9; however, it had a large estimated standard deviation. (See run 2, table V.) In addition, the estimated standard deviation of w was high, 1.25 m/sec (4.1 ft/sec) (run 2, table V) compared with the flight range of from about 3.048 to -6.096 m/sec (10 to -20 ft/sec) (fig. 5). Most of this difference is attributable to the uncertainty in measured angle of attack. The uncertainty was $\pm 0.8^{\circ}$, which resulted in an uncertainty of 0.975 m/sec (±3.2 ft/sec) in w. It appeared advisable, therefore, to expand the likelihood function to include additional flight data. In this case, the measured normal accelerations appeared to be pertinent and reasonably accurate and were therefore added to the likelihood function.

After a_Z was put into the likelihood function (in place of w) along with u and q, the value of C_{Z_α} changed from -5.5 to -4.33, the values of the other unknown parameters changing less than 5 percent. At this point u, w, q, and a_Z were all used in the likelihood function, and the program was run again. (See run 3, table V.) The values previously obtained changed by less than 2 percent. The program weighted the a_Z data so that the u, q, a_Z likelihood function gave about the same results as the u, w, q, a_Z likelihood function. At this point the estimated derivatives appeared to be reasonable, except possibly for C_{m_α} which was somewhat less negative than had been expected. (Compare table III with run 3 of table V.)

It should be noted that for runs 2 and 3 of table V, the parameter $C_{m_{\dot{\alpha}}}$ had been set equal to zero, with the expectation that the extracted C_{m_q} would reflect any $C_{m_{\dot{\alpha}}}$ contribution to the aircraft motion. As a matter of interest, it was decided to now insert and hold constant a value of $C_{m_{\dot{\alpha}}}$ equal to the value given in table III. The results from the extraction program are given in table VII. A comparison of run 3 of table V and the values of table VII show that

(1) The value of $C_{m_{\alpha}}$ changed from -0.384 to -0.63, which is close to the value extracted in the study of reference 1.

(2) The value of C_{m_q} changed from -24.7 to -18.1. However, note that the sum $C_{m_q} + C_{m_{\dot{\alpha}}}$ is about the same for runs 2 and 3 of table V and the values given in table VII. The other parameters were not affected. The standard deviations of the parameters determined, except $C_{X_{\alpha}}$, were less than 15 percent of the value of the parameter and indicated that they were well determined for the model being used. The fit obtained to the flight data is shown in figure 5 and the derivatives are given in table VII.

As a matter of interest, it was decided to make additional runs to examine the significance of the high estimated correlations indicated in table VI. These additional runs held fixed values of one of the parameters involved and allowed the program to extract the remaining parameters. The results were then examined to note how the remaining parameters changed as a function of the fixed parameter. In addition, the changes in the determinant of R were also observed. Some of the results are shown in figures 6 and 7. The parameters shown in the figures are those that varied by more than about 5 percent as the "fixed" parameter was changed about ±100 percent. Figure 6 shows that since the determinant of R remained very nearly constant over the range of $C_{m_{\dot{\alpha}}}$ values, and since the extracted $C_{m_{\alpha}}$ and C_{m_q} values varied linearly with $C_{m_{\dot{\alpha}}}$, there exists an approximately true linear correlation between C_{m_q} and $C_{m_{\dot{\alpha}}}$ and between $C_{m_{\alpha}}$ and $C_{m_{\dot{\alpha}}}$. However, as shown in figure 7, the determinant of R varied somewhat as $C_{m_{\delta_e}}$ was changed over the range shown. Therefore, a linear correlation between C_{m_q} and $C_{m_{\delta_e}}$ or between C_{Z_q} and $C_{m_{\delta_e}}$ cannot be definitely established.

The preceding discussion illustrates the considerations necessary to obtain coefficients for the set of data used herein. These considerations are

(1) Obtain as good a set of starting values as feasible

(2) Even if the program has not truly converged, the estimated covariance matrix for the unknown parameters will show potentially related parameters.

(3) If the program is slowly diverging because of pairs of parameters varying, fixing one of the parameters based on other factors such as wind-tunnel data or reference 6 will generally cause the program to converge to a set of derivatives.

(4) Once the performance index has stabilized for a particular set of parameters, the states in the performance index and the parameters in the mathematical model can be changed to determine whether any improvements in convergence occurs.

(5) After investigating the importance of coefficients which could affect the calculated response, and determining a minimum performance index, curves to show possible relations between coefficients can be established.

(6) Based on all this information, a set of parameters for the airframe investigated can be determined.

Low-speed configuration.- The general procedures given for the high-speed configuration were followed for the low-speed configurations. The final values of the extracted parameters are shown in table VII. Time histories of the measured and calculated motions are shown in figure 8.

Lateral Aerodynamic Parameters

High-speed configuration.- There were two sets of flight records from which the lateral aerodynamic parameters could be obtained: the response to an aileron doublet, and the response to a rudder pulse. As will be indicated, both sets of flight records were used in determining the parameters.

The initially estimated lateral parameters required to start the computing and extraction process were computed by using reference 6 and are shown in table III. The program gave a fair match to the aircraft motions in about six iterations; however, the mean-squared error, after decreasing for three iterations, began to slowly increase. The derivatives extracted for this case (high-speed, aileron doublet input) are indicated in table VIII (run 1). Since the standard deviations were small for all parameters, with the exception of C_{Y_p} and C_{Y_r}, the parameters appeared to be well defined. (See table IX.) Varying C_{Y_p} and C_{Y_r} by ±100 percent did not affect the values of the other parameters or the fit, and since they were not important, they were subsequently dropped from the mathematical model.

The off-diagonal terms of the estimated covariance matrix are shown as table X. A possible relation appeared to exist between C_{l_p} and $C_{l_{\delta_a}}$ and between C_{n_p} and $C_{n_{\delta_a}}$. In a similar situation for the longitudinal case, one of the apparently related variables assumed to be known, resulted in a nonincreasing mean-squared error after several iterations. For the lateral runs, it was thought that the rudder pulse input data could be used to determine C_{n_p} and C_{l_p}. However, when rudder data were run (run 2, table VIII), the mean-squared error began to increase rapidly after one or two iterations. The printout showed that the roll and sideslip parameters had large variations and the estimated covariance matrix implied a possible relation between C_{n_p} and C_{n_β}, and between C_{l_p} and C_{l_β}. Because these possible relations prevented a nonincreasing mean-squared error, a procedure for using both sets of lateral data was developed. The procedure is as follows:

(1) Obtain the control parameters $C_{l_{\delta_a}}$ and $C_{n_{\delta_a}}$ from wind-tunnel tests (ref. 9), and hold these values constant during the following parameter extraction process. Fix the initial conditions at the flight values and calculate the trim coefficients to initially zero the equations, and fix the trim coefficients at these values. (See run 3, table VIII.) Extract the remaining parameters. By using the initial conditions and these parameters as initial estimates, rerun the problem with the initial conditions and trim coefficients as additional unknowns. (See run 4, table VIII.)

(2) A similar procedure is used with the rudder input flight data. The initial conditions and trim were first assumed to be known, and C_{l_p} and C_{n_p} were fixed at the values obtained from run 4. With these conditions, the problem was run until a nonincreasing mean-squared error was obtained. (See run 5, table VIII.) The extracted parameters were then used as initial estimates, and the initial conditions and the trim coefficients were assumed to be unknown and the run repeated. (See run 6, table VIII.)

(3) The results of runs 4 and 6 were compared and it was found that the differences in the parameters obtained varied less than 10 percent with the exception of C_{n_r} which had a variation slightly greater than 10 percent.

(4) The aileron input case was run with the yawing and sideslip derivatives obtained from the rudder input runs. (See run 7, table VIII.) The rudder input case was then run with the roll, sideslip, and yaw derivatives obtained from the aileron input runs. (See run 8, table VIII.) Runs 7 and 8 were repeated several times and a set of derivatives which best fit both sets of data was determined. These values are given in table XI and the resulting motion time histories are given in figures 9 to 12.

As was done in the longitudinal runs, curves showing any significant percent changes in the parameters or in the determinant of R due to changes in the parameter assumed to be known are shown as figures 13 to 15. As before, in addition to the parameters primarily affected, there was some effect on other parameters. Therefore, the determinant of R did not remain constant and correlations could not be determined.

DISCUSSION OF RESULTS

The results shown in figures 5, 8, 9, 10, 11, and 12 show that the parameter-extraction program was able to obtain a set of aerodynamic parameters which permitted estimates of the aircraft motion compatible with the accuracies of the measurements of the flight data. The results of this study are compared with results of references 1, 8, and 9 in tables XII and XIII for the longitudinal and lateral parameters, respectively. Major differences in values of the parameters are discussed. In this discussion the T'_c for the wind tunnel was approximately the same as the T'_c for flight.

Longitudinal Parameters

The longitudinal parameters obtained from the present study and those of references 1, 8, and 9 are in reasonable agreement in most instances. However, there are several important differences which require discussion.

$\underline{C_{X_\alpha}}$.- There was not enough variation in forward velocity to permit accurate determination of C_{X_α} from the flight data.

$\underline{C_{Z_\alpha}}$.- All references had about the same value of C_{Z_α} except that for reference 1 which is somewhat higher than appears to be reasonable. In searching for reasons for this difference, it was noted that the mathematical model used in reference 1 did not include the parameter C_{Z_q}, whereas it was included in the present study. The parameter C_{Z_q}, therefore, was set equal to zero in the present study, and the program was used to extract the remaining parameters. It was found that most of the parameters remained at the values of table XII; however, C_{Z_α} increased (negatively) from -4.33 to about -6.0. It appears that, at least for the Navion, it was important to include C_{Z_q} among the parameters in the mathematical model.

$\underline{C_{m_\alpha}}$.- The present study yielded a value of C_{m_α} which was less than one-half the wind-tunnel value (ref. 9) if $C_{m_{\dot{\alpha}}}$ were set to zero in the mathematical model. However, if $C_{m_{\dot{\alpha}}}$ were set equal to -6.5 for the high-speed case and to -6.0 for the low-speed case, C_{m_α} was about 25 percent less than the value obtained from wind-tunnel tests (ref. 9). The difference between the wind-tunnel value of C_{m_α} and the value extracted in the present study raised a question of uniqueness of derivatives. It was, therefore, decided to insert the wind-tunnel values of the parameters as constants in the parameter-extraction program, and to let the program extract the remaining parameters. The results of the time-history calculation are shown in figures 16 and 17, and the fixed wind-tunnel parameters and the remaining (extracted) parameters are listed in table XIV for the high-speed and low-speed configurations, respectively. Although the computed time histories are in fair agreement with the flight data, the agreement is not as good as that shown in figures 5 and 8. Also note that the period of the calculated time histories is somewhat shorter than that of the flight data (figs. 16 and 17), which is a consequence of increasing the value of C_{m_α} to the wind-tunnel value.

Lateral Parameters

For the high-speed lateral runs, the agreement between the results of reference 1 and the present study was generally good. (See table XIII.) The major exception was a difference in C_{n_β} of about 15 percent. Also C_{l_r} showed a 40-percent difference and $C_{n_{\delta_r}}$ a 20-percent difference. However, the value obtained for C_{l_r} in reference 1 was based on flight data different from those used in this study. More serious differences arose between the present study and reference 9 where C_{Y_β}, C_{l_β}, and C_{n_β} were all different by about 40 percent. Significant differences exist between the values of $C_{Y_{\delta_r}}$, $C_{l_{\delta_a}}$, and $C_{n_{\delta_r}}$ obtained in reference 9 and those of the present study.

By examining the low-speed lateral runs, differences between the results of reference 1 and the present study were found in C_{l_r} (100 percent), C_{n_r} (35 percent), and $C_{n_{\delta_r}}$ (30 percent). Differences between reference 9 and the present study occurred in C_{l_β} (90 percent), C_{n_β} (50 percent), and $C_{l_{\delta_r}}$.

CONCLUDING REMARKS

An iterative procedure, which is characterized as a maximum-likelihood minimum-variance technique, was used to extract the aerodynamic parameters of a Navion airplane from flight data. Results from the study showed that the parameter-extraction program can produce aerodynamic parameters which will permit close estimation of the aircraft time histories used in the extraction process. The study also showed that good judgment and ingenuity is required to circumvent problems occurring because of apparent relations between some of the parameters. It also became evident that the mathematical model selected to represent the aircraft is very important.

The extracted parameters agreed reasonably well with the values obtained by analog matching the same data, with the exception of the change in normal-force coefficient with angle of attack $\left(C_{Z_\alpha}\right)$. The agreement with wind-tunnel parameters was not as good for the variations of pitching-moment coefficient with angle of attack $\left(C_{m_\alpha}\right)$, side-force coefficient with sideslip angle $\left(C_{Y_\beta}\right)$, rolling-moment coefficient with sideslip angle $\left(C_{l_\beta}\right)$, and yawing-moment coefficient with sideslip angle $\left(C_{n_\beta}\right)$. However, of the parameters determined by the program, only one had a standard deviation greater than 15 percent of the value of the parameter and the parameters gave a reasonable fit to the flight data.

The parameter estimation process was greatly aided by the capability of the operator to interact with the program. The hands-on feature was especially useful during the initial stages of the estimation procedure when a number of different parameters were being considered as known or unknown in the mathematical model. Some of the other advantages of this parameter estimation program were found to be:

(1) The mathematical logic for updating unknown parameters is mechanized.

(2) A "best fit," within the limits of the mathematical model and the data used, is obtained.

(3) The estimated standard deviation of each unknown parameter is obtained.

The control inputs used to obtain the aircraft responses used herein caused these responses to be approximately sine waves which were close to being in phase. Uniquely determining the parameters using this flight data was difficult and the values of several parameters had to be assumed to be known from theoretical calculations or wind-tunnel data.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., January 20, 1972.

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TABLE I.- NAVION AIRPLANE DIMENSIONS

Wing:	
Area, S, m^2 (ft^2)	17.112 (184)
Sweep, leading edge, deg	2.996
Aspect ratio, A	6.04
Taper ratio, λ	0.54
Mean aerodynamic chord, m (ft)	1.74 (5.7)
Dihedral, deg	7.5
Incidence at root, deg	+2
Incidence at tip, deg	-1
Airfoil:	
Tip	NACA 6410 R
Root	NACA 4415 R
Horizontal tail:	
Area, m^2 (ft^2)	4.0 (43)
Sweep, leading edge, deg	6
Aspect ratio	4.0
Taper ratio	0.67
Airfoil	NACA 0012
Incidence, deg	-3
Vertical tail:	
Area (above horizontal stabilizer), m^2 (ft^2)	1.163 (12.5)
Airfoil:	
Root	Modified NACA 0013.2
Tip	Modified NACA 0012.04
Fin offset, deg	2
Propeller characteristics:	
Diameter, m (in.)	2.14 (84)
Number of blades	2
Side-force factor	100
Power plant:	
Continental engine	Model no. 10520B
Horsepower rating at take-off at 2700 rpm	285
Mass and inertia characteristics for data of this report:	
Gross mass, kg (lb)	1335.76 (2948)
Center of gravity, percent $\bar{c}$	25
I_X, kg-m^2 (slug-ft^2)	1742.33 (1284.08)
I_Y, kg-m^2 (slug-ft^2)	3762.4 (2772.86)
I_Z, kg-m^2 (slug-ft^2)	4389.10 (3234.72)

Control surfaces:

Control surface	Area		Deflection, deg
	m^2	ft^2	
Flaps (plain)	7.775	83.6	40
Stabilizer	2.79	30.0	-------------
Elevator	1.31	14.1	Up 30, down 20
Aileron	.502	5.4	20
Rudder	.558	6.0	15

TABLE II.- FLIGHT-TEST CONDITIONS FOR THE VARIABLE-STABILITY NAVION

	Condition I	Condition II
Altitude, km (ft)	1.525 (5000)	1.525 (5000)
Velocity (true airspeed), m/sec (ft/sec)	73.2 (240)	43.9 (144.1)
Flaps, deg	0	20
Lift coefficient	0.271	0.753
Thrust coefficient (estimated), T_C'	0.02255	0.05716

TABLE III.- INITIAL ESTIMATES OF THE AERODYNAMIC COEFFICIENTS*

Coefficient	Value
$C_{X,o}$	0.0015
C_{X_α}	0
$C_{Z,o}$	0.02
C_{Z_α}	-5.23
$C_{Z_{\delta_e}}$	-0.40
$C_{m,o}$	0
C_{m_α}	-1.26
$C_{m_{\dot{\alpha}}}$	-6.50
C_{m_q}	-13.00
$C_{m_{\delta_e}}$	-1.4
C_{Y_β}	-0.35
C_{Y_r}	0.50
C_{Y_p}	-0.30
$C_{Y_{\delta_r}}$	0.08
C_{l_β}	-0.06
C_{l_r}	0.04
C_{l_p}	-0.45
$C_{l_{\delta_a}}$	0.15
$C_{l_{\delta_r}}$	0.30
C_{n_β}	0.049
C_{n_p}	-0.024
C_{n_r}	-0.082
$C_{n_{\delta_a}}$	-0.005
$C_{n_{\delta_r}}$	-0.08

*Estimated by using methods presented in reference 6.

TABLE IV.- PARAMETERS USED IN MATHEMATICAL MODEL FOR LONGITUDINAL RUNS

[Key: - denotes parameter assumed known; ✓ parameter assumed unknown and to be determined; * changed fixed value of $C_{m_{\dot{\alpha}}}$]

Parameter	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6
u_o	✓	-	✓	-	-	-
$C_{X,o}$	✓	-	✓	-	-	-
C_{X_α}	✓	-	✓	✓	✓	✓
w_o	✓	-	✓	-	-	-
$C_{Z,o}$	✓	-	✓	-	-	-
C_{Z_α}	✓	✓	✓	✓	✓	✓
C_{Z_q}	Not in model			✓	✓	✓
$C_{Z_{\delta_e}}$	✓	✓	✓	-	-	-
q_o	✓	-	✓	-	-	-
$C_{m,o}$	✓	-	✓	-	-	-
C_{m_α}	✓	✓	✓	✓	✓	✓
$C_{m_{\dot{\alpha}}}$	✓	-	-	-	-	*
C_{m_q}	✓	✓	✓	✓	✓	✓
$C_{m_{\delta_e}}$	✓	-	-	-	-	-

TABLE V.- PARAMETERS DETERMINED AT VARIOUS STAGES OF FITTING THE LONGITUDINAL FLIGHT DATA

Parameter	Run 1			Run after establishing better starting values and adding C_{Z_q} to the model (Run 2)			Run after adding a_Z to the performance index to the model and fixing $C_{m_{\delta_e}}$ and $C_{Z_{\delta_e}}$ (Run 3)		
	Value	ΔP	Standard deviation	Value	ΔP	Standard deviation	Value	ΔP	Standard deviation
C_{X_α}	-0.69	-0.15	0.04	-0.09	6.6×10^{-6}	0.04	-0.265	5.3×10^{-6}	0.05
C_{Z_α}	-6.87	0.16	0.12	-5.47	6.0×10^{-4}	0.15	-4.33	7×10^{-5}	0.007
C_{Z_q}	0	0	----	-26.9	0.1	19.6	-15.9	-4.23×10^{-4}	2.28
$C_{Z_{\delta_e}}$	-4.11	0.33	0.15	-0.511	0	----	-0.511	-----------	-----
C_{m_α}	3.00	-0.05	0.43	-0.584	2.3×10^{-7}	0.02	-0.39	2×10^{-7}	0.011
$C_{m_{\dot{\alpha}}}$	124.9	-1.2	15.5	0	0	----	0	-----------	-----
C_{m_q}	-144.2	1.18	16.3	-24.7	9×10^{-5}	0.43	-24.5	8×10^{-5}	0.41
$C_{m_{\delta_e}}$	-0.91	-0.23	0.14	1.42	0	----	-1.42	-----------	-----
Standard deviation of u	28				4.4		1.7		
Standard deviation of w	8				4.1		2.5		
Standard deviation of q	0.003				0.0004		3.7×10^{-5}		
Standard deviation of a_Z							1.6×10^{-3}		

TABLE VI.- ESTIMATED CORRELATION MATRIX FOR ELEVATOR DOUBLET INPUT FOR RUN 1 OF TABLE V

[V = 73.2 m/sec (240 ft/sec)]

	C_{X_α}	C_{Z_α}	$C_{Z_{\delta_e}}$	C_{m_α}	$C_{m_{\dot{\alpha}}}$	C_{m_q}	$C_{m_{\delta_e}}$
C_{X_α}	1	0.08	0.15	-0.04	0.03	0.015	-0.13
C_{Z_α}	0.08	1	0.69	-0.11	0.09	-0.06	-0.10
$C_{Z_{\delta_e}}$	0.15	0.69	1	0.07	0.10	-0.11	0.034
C_{m_α}	0.04	-0.11	0.07	1	0.978	0.762	0.15
$C_{m_{\dot{\alpha}}}$	0.03	0.09	0.10	0.978	1	0.997	-0.014
C_{m_q}	0.015	-0.06	-0.11	0.762	0.997	1	0.968
$C_{m_{\delta_e}}$	-0.13	-0.10	0.034	0.15	-0.014	0.968	1

TABLE VII.- LONGITUDINAL AERODYNAMIC PARAMETERS OF THE NAVION AIRPLANE EXTRACTED FROM FLIGHT DATA

(a) $V_o = 73.2$ m/sec (240 ft/sec); $\delta_f = 0^o$

Parameter	Value	Standard deviation
C_{X_α}	0.262	0.046
C_{Z_α}	-4.33	0.007
C_{Z_q}	-15.9	2.28
$C_{Z_{\delta_e}}$	-0.511	
C_{m_α}	-0.63 (-0.77 corrected to 25% c.g.)	0.006
$C_{m_{\dot{\alpha}}}$	-6.5	0.52
C_{m_q}	-18.1	
$C_{m_{\delta_e}}$	-1.42	

(b) $V_o = 43.9$ m/sec (144.1 ft/sec); $\delta_f = 20^o$

Parameter	Value	Standard deviation
C_{X_α}	1.37	0.09
C_{Z_α}	-4.86	0.09
C_{Z_q}	-27.13	1.75
$C_{Z_{\delta_e}}$	-0.52	
C_{m_α}	-0.70 (-0.84 corrected to 25% c.g.)	0.011
$C_{m_{\dot{\alpha}}}$	-6.0	0.25
C_{m_q}	-16.4	0.035
$C_{m_{\delta_e}}$	-1.55	

TABLE VIII.- PARAMETERS USED IN MATHEMATICAL MODEL FOR LATERAL RUNS

[Key: - denotes parameter assumed known; ✓ parameter assumed unknown; * not in mathematical model. Subscript 1 indicates aileron-input data; subscript 2 indicates rudder-input data]

Parameter	Run 1_1	Run 2_2	Run 3_1	Run 4_1	Run 5_2	Run 6_2	Run 7_1	Run 8_2
v_o	✓	✓	-	✓	-	✓	-	-
$C_{Y,o}$	✓	✓	-	✓	-	✓	-	-
C_{Y_β}	✓	✓	✓	✓	✓	✓	✓	✓
C_{Y_p}	✓	✓	-	-	-	-	-	-
C_{Y_r}	✓	✓	-	-	-	-	-	-
$C_{Y_{\delta r}}$	*	✓	*	*	✓	✓	*	✓
p_o	✓	✓	-	✓	-	✓	-	-
$C_{l,o}$	✓	✓	-	✓	✓	✓	-	-
C_{l_β}	✓	✓	✓	✓	✓	✓	✓	✓
C_{l_p}	✓	✓	✓	✓	-	✓	✓	-
C_{l_r}	✓	✓	✓	✓	✓	✓	✓	✓
$C_{l_{\delta r}}$	*	✓	*	*	✓	✓	*	✓
$C_{l_{\delta a}}$	✓	*	-	-	*	*	-	*
r_o	✓	✓	-	✓	-	✓	-	-
$C_{n,o}$	✓	✓	-	✓	-	✓	-	-
C_{n_β}	✓	✓	✓	✓	✓	✓	✓	✓
C_{n_p}	✓	✓	✓	✓	-	-	✓	-
C_{n_r}	✓	✓	✓	✓	✓	✓	✓	✓
$C_{n_{\delta r}}$	*	✓	*	*	✓	✓	*	✓
$C_{n_{\delta a}}$	✓	*	-	-	*	*	-	*

TABLE IX.- AERODYNAMIC PARAMETERS FOR NAVION AIRPLANE EXTRACTED FROM AILERON DOUBLET RUN

[V = 73.2 m/sec (240 ft/sec); $\delta_f = 0^{\circ}$]

Parameter	Value	Standard deviation
C_{Y_β}	-4.87	0.12
C_{Y_p}	4.70	1.10
C_{Y_r}	5.24	0.98
C_{l_β}	-0.084	0.003
C_{l_p}	-0.596	0.012
C_{l_r}	0.027	0.013
$C_{l_{\delta_a}}$	0.089	0.002
C_{n_β}	0.085	0.002
C_{n_p}	-0.16	0.006
C_{n_r}	-0.11	0.008
$C_{n_{\delta_a}}$	-0.017	0.009

TABLE X.- ESTIMATED CORRELATION MATRIX FOR AILERON DOUBLET FOR

V = 73.2 m/sec (240 ft/sec) AND $\delta_f = 0^{\circ}$

$$\left[\left(M^{T}R^{-1}M\right)^{-1} \text{ with the diagonal terms set to } 1\right]$$

	C_{Y_β}	C_{Y_p}	C_{Y_r}	C_{l_β}	C_{l_p}	C_{l_r}	$C_{l_{\delta_a}}$	C_{n_β}	C_{n_p}	C_{n_r}	$C_{n_{\delta_a}}$
C_{Y_β}	1	-0.17	-0.14	0.02	-0.003	0.31	-0.03	0.03	-0.04	-0.84	-0.10
C_{Y_p}	-0.17	1	0.23	0.10	0.33	0.004	-0.30	-0.28	-0.61	0.13	-0.65
C_{Y_r}	-0.14	0.23	1	-0.34	0.13	0.001	-0.11	0.76	-0.22	0.11	-0.22
C_{l_β}	0.02	0.10	-0.34	1	0.18	0.04	-0.18	-0.07	-0.02	0.04	0.04
C_{l_p}	-0.003	0.33	0.13	0.18	1	0.47	-0.94	-0.02	0.12	0.12	0.08
C_{l_r}	0.31	0.004	0.001	0.04	0.47	1	-0.35	-0.07	0.19	0.11	0.10
$C_{l_{\delta_a}}$	-0.03	-0.30	-0.11	-0.18	-0.94	-0.35	1	0.02	-0.02	-0.03	0.03
C_{n_β}	0.03	-0.28	0.76	-0.07	-0.02	-0.07	0.02	1	0.14	-0.01	0.15
C_{n_p}	-0.04	-0.61	-0.22	-0.02	0.12	0.19	-0.02	0.14	1	0.26	0.97
C_{n_r}	-0.84	0.13	0.11	-0.04	0.12	0.11	-0.03	-0.01	0.26	1	0.25
$C_{n_{\delta_a}}$	-0.10	-0.65	-0.22	0.04	0.08	0.10	-0.03	0.15	0.97	0.25	1

TABLE XI.- LATERAL PARAMETERS

Parameter	V = 73.2 m/sec (240 ft/sec); $\delta_f = 0^\circ$		V = 43.9 m/sec (144.1 ft/sec); $\delta_f = 20^\circ$	
	Value	Standard deviation	Value	Standard deviation
C_{Y_β}	-0.6	0.078	-0.74	0.059
$C_{Y_{\delta_r}}$	0.33	0.05	0.68	0.03
C_{l_β}	-0.07	0.0016	-0.053	0.0012
C_{l_p}	-0.49	0.003	-0.53	0.0037
C_{l_r}	0.11	0.008	0.114	0.0068
$C_{l_{\delta_a}}$	0.154	0.0004	0.16	0.0004
$C_{l_{\delta_r}}$	0.026	0.0018	0.0007	0.0013
C_{n_β}	0.073	0.0004	0.083	0.0005
C_{n_p}	-0.04	0.0005	-0.147	0.0016
C_{n_r}	-0.09	0.005	-0.108	0.0044
$C_{n_{\delta_a}}$	-0.004	8×10^{-6}	-0.0015	0.0001
$C_{n_{\delta_r}}$	-0.063	0.0007	-0.067	0.0005

TABLE XII.- COMPARISON OF LONGITUDINAL PARAMETERS WITH THOSE OF REFERENCES

Parameter	Present study	Reference 1	Reference 9	Reference 8
		$\delta_f = 0^\circ$		
C_{X_α}	0.262	------	0.12	0.33
C_{Z_α}	-4.33	-6.04	-4.52	-4.44
C_{Z_q}	-15.9	------	-----	-------
$C_{Z_{\delta_e}}$	-0.511	------	-0.43	-0.355
C_{m_α}	-0.77	------	-0.95	-------
$C_{m_q} + C_{m_{\dot{\alpha}}}$	-24.6	-18.3	-----	-14.32
$C_{m_{\delta_e}}$	-1.42	-1.42	-1.42	-------
		$\delta_f = 20^\circ$		
C_{X_α}	1.37	------	0.30	
C_{Z_α}	-4.86	-6.40	-4.52	
C_{Z_q}	-27.13	------	-----	
$C_{Z_{\delta_e}}$	-0.52	------	-0.53	
C_{m_α}	-0.84	------	-----	
$C_{m_q} + C_{m_{\dot{\alpha}}}$	-22.4	-15.5	-----	
$C_{m_{\delta_e}}$	-1.55	-1.55	-1.55	

TABLE XIII.- COMPARISON OF LATERAL PARAMETERS WITH THOSE OF REFERENCES

Parameter	Present study	Reference 1	Reference 9	Reference 8
$\delta_f = 0^\circ$				
C_{Y_β}	-0.6	-0.61	-0.8	-0.56
$C_{Y_{\delta_r}}$	0.33	-------	0.143	0.157
C_{l_β}	-0.07	-0.067	-0.109	-0.074
C_{l_p}	-0.49	-0.46	-------	-0.41
C_{l_r}	0.11	0.07	-------	0.107
$C_{l_{\delta_a}}$	0.154	0.152	0.152	0.134
$C_{l_{\delta_r}}$	0.026	-------	0.029	0.012
C_{n_β}	0.073	0.086	0.109	0.070
C_{n_p}	-0.04	-0.038	-------	-0.058
C_{n_r}	-0.09	-0.088	-------	-0.125
$C_{n_{\delta_a}}$	-0.004	-0.0047	-0.0047	-0.0035
$C_{n_{\delta_r}}$	-0.063	-0.075	-0.077	-0.07
$\delta_f = 20^\circ$				
C_{Y_β}	-0.74	-------	-0.77	
$C_{Y_{\delta_r}}$	0.68	-------	0.143	
C_{l_β}	-0.053	-0.051	-0.096	
C_{l_p}	-0.53	-0.48	-------	
C_{l_r}	0.114	0.27	-------	
$C_{l_{\delta_a}}$	0.16	0.150	0.09	
$C_{l_{\delta_r}}$	0.007	-------	0.015	
C_{n_β}	0.080	0.084	0.126	
C_{n_p}	-0.147	-0.141	-------	
C_{n_r}	-0.12	-0.163	-------	
$C_{n_{\delta_a}}$	-0.0015	-0.0013	0	
$C_{n_{\delta_r}}$	-0.075	-0.093	-0.08	

TABLE XIV.- VALUES OBTAINED BY FIXING PARAMETERS OBTAINED FROM REFERENCE 9 AND DETERMINING $C_{m_q} + C_{m_{\dot{\alpha}}}$

(a) $V_o = 73.2$ m/sec (240 ft/sec); $\delta_f = 0^o$

Parameter	Value	Standard deviation
C_{X_α}	0.12	
C_{Z_α}	-4.52	
C_{Z_q}	-5.25	
$C_{Z_{\delta_e}}$	-0.403	
C_{m_α}	-0.95	
$C_{m_q} + C_{m_{\dot{\alpha}}}$	-15.6	1.63
$C_{m_{\delta_e}}$	-1.42	

(b) $V_o = 43.9$ m/sec (144.1 ft/sec); $\delta_f = 20^o$

Parameter	Value	Standard deviation
C_{X_α}	0.3	
C_{Z_α}	-4.52	
C_{Z_q}	-6.06	
$C_{Z_{\delta_e}}$	-0.53	
C_{m_α}	-0.95	
$C_{m_q} + C_{m_{\dot{\alpha}}}$	-13.12	1.31
$C_{m_{\delta_e}}$	-1.55	

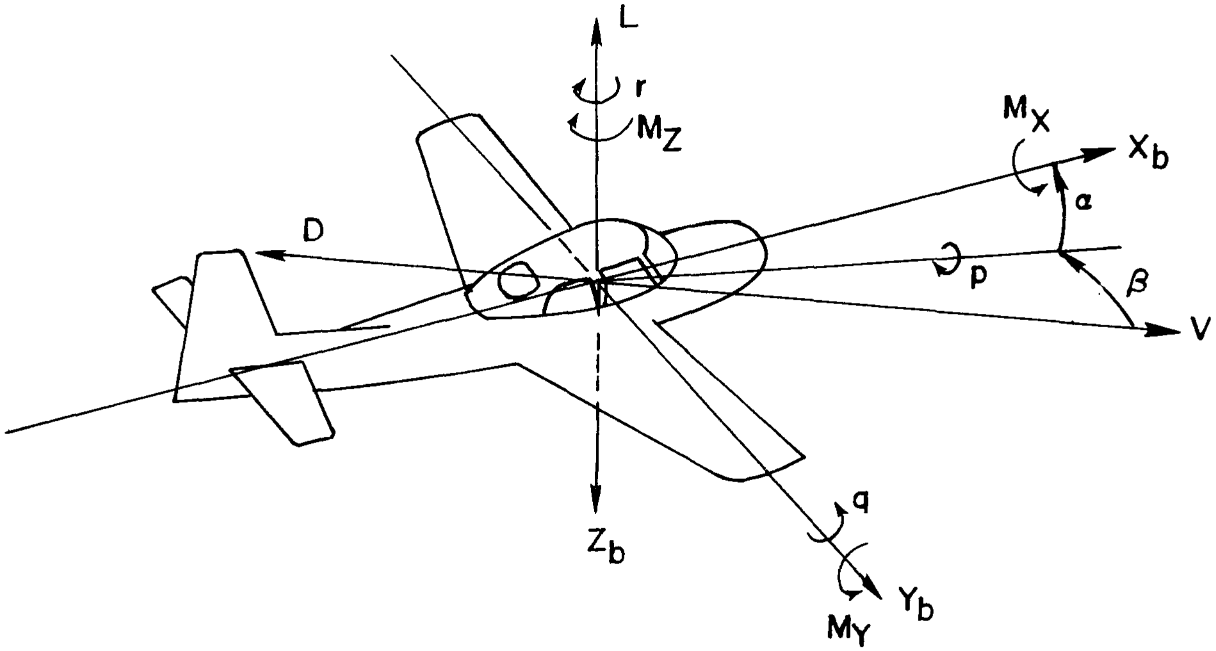

Figure 1.- System of body axes showing positive sense of angles, forces, and moments.

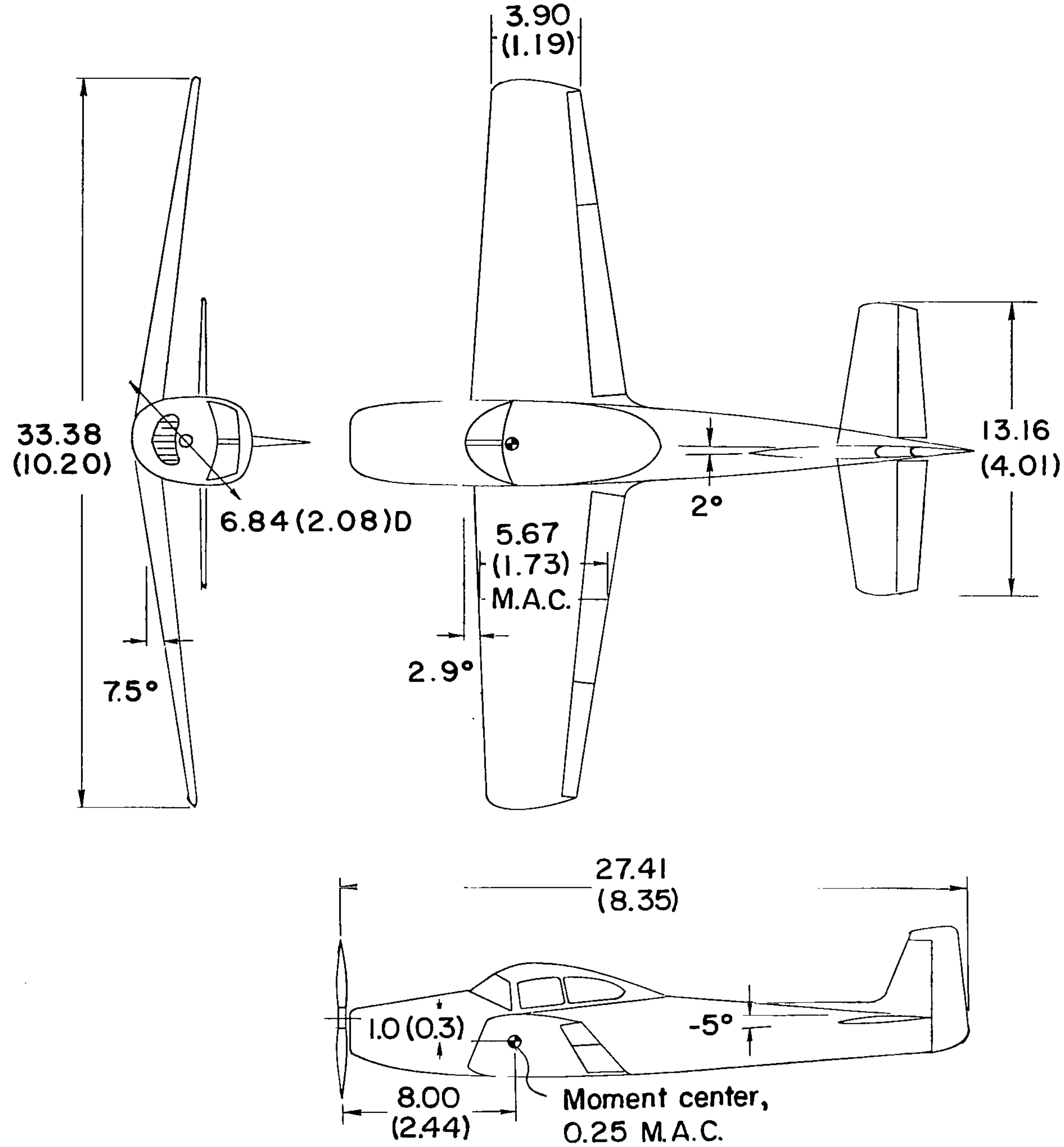

Figure 2.- Three-view drawing and principal dimensions. All dimensions are in feet (meters). Tail incidence angle i_t is normally set at 0°; it was set at -5° for these tests.

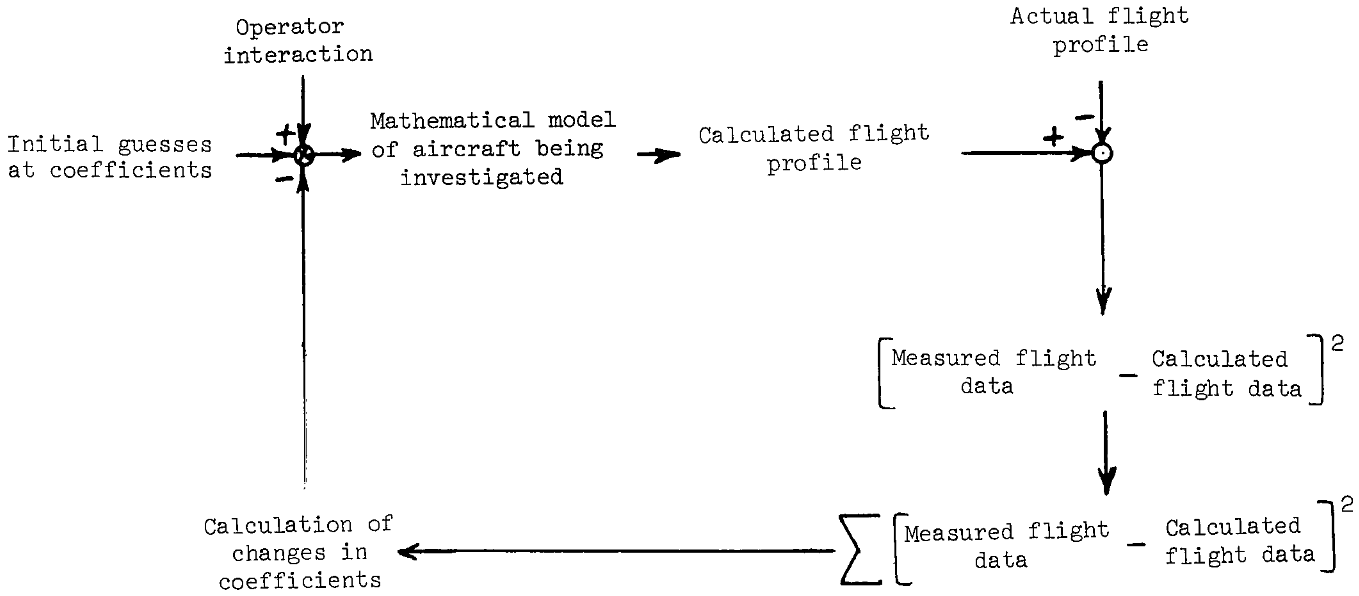

Figure 3.- Flow diagram for parameter-extraction program.

L-69-8763

Figure 4.- Photograph of control area showing console operator, control console, and cathode ray tube (CRT).

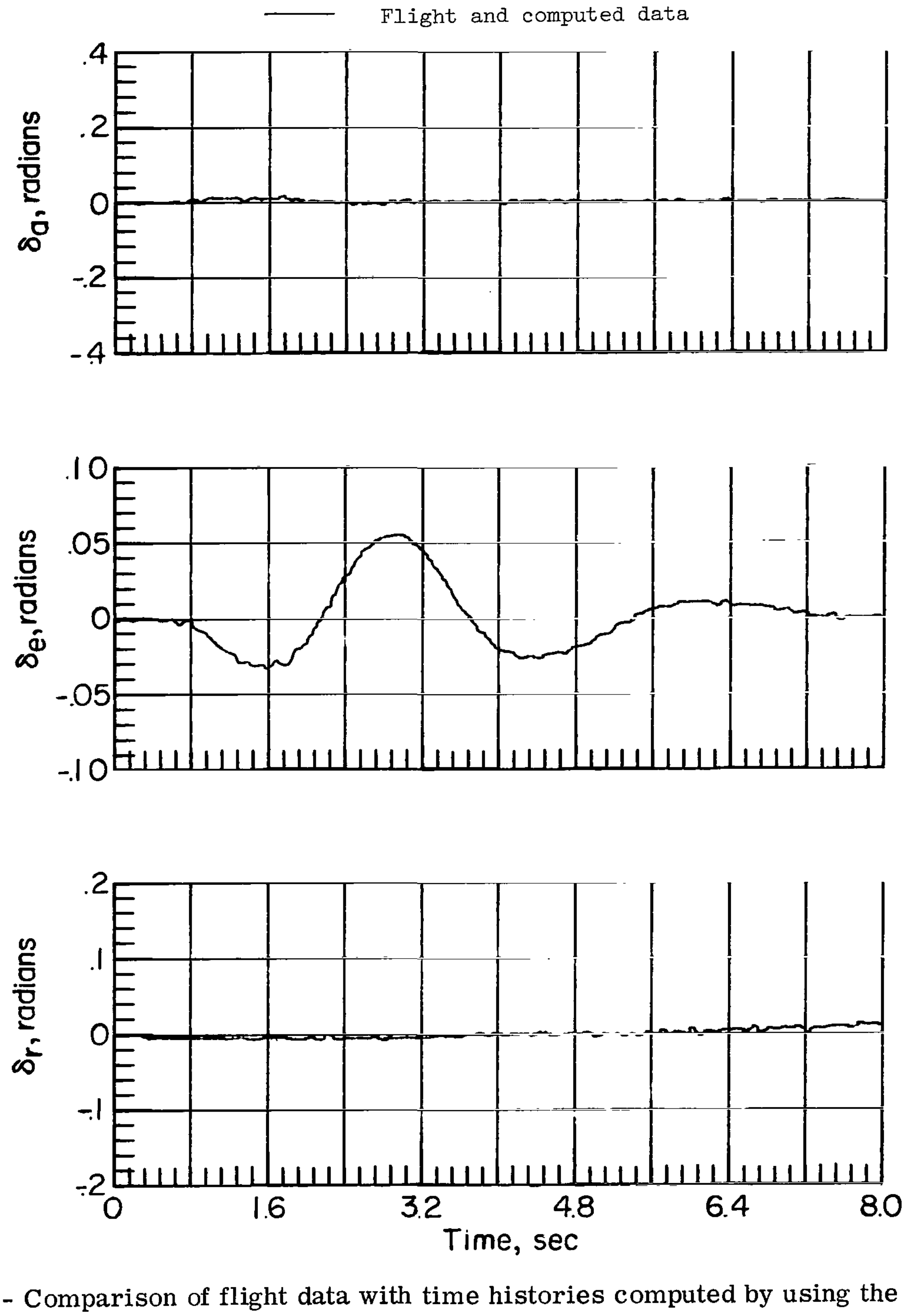

Figure 5.- Comparison of flight data with time histories computed by using the aerodynamic parameters of table VII for elevator doublet input. V = 73.2 m/sec (240 ft/sec); $\delta_f = 0^\circ$. Flight and computed data are the same for control inputs.

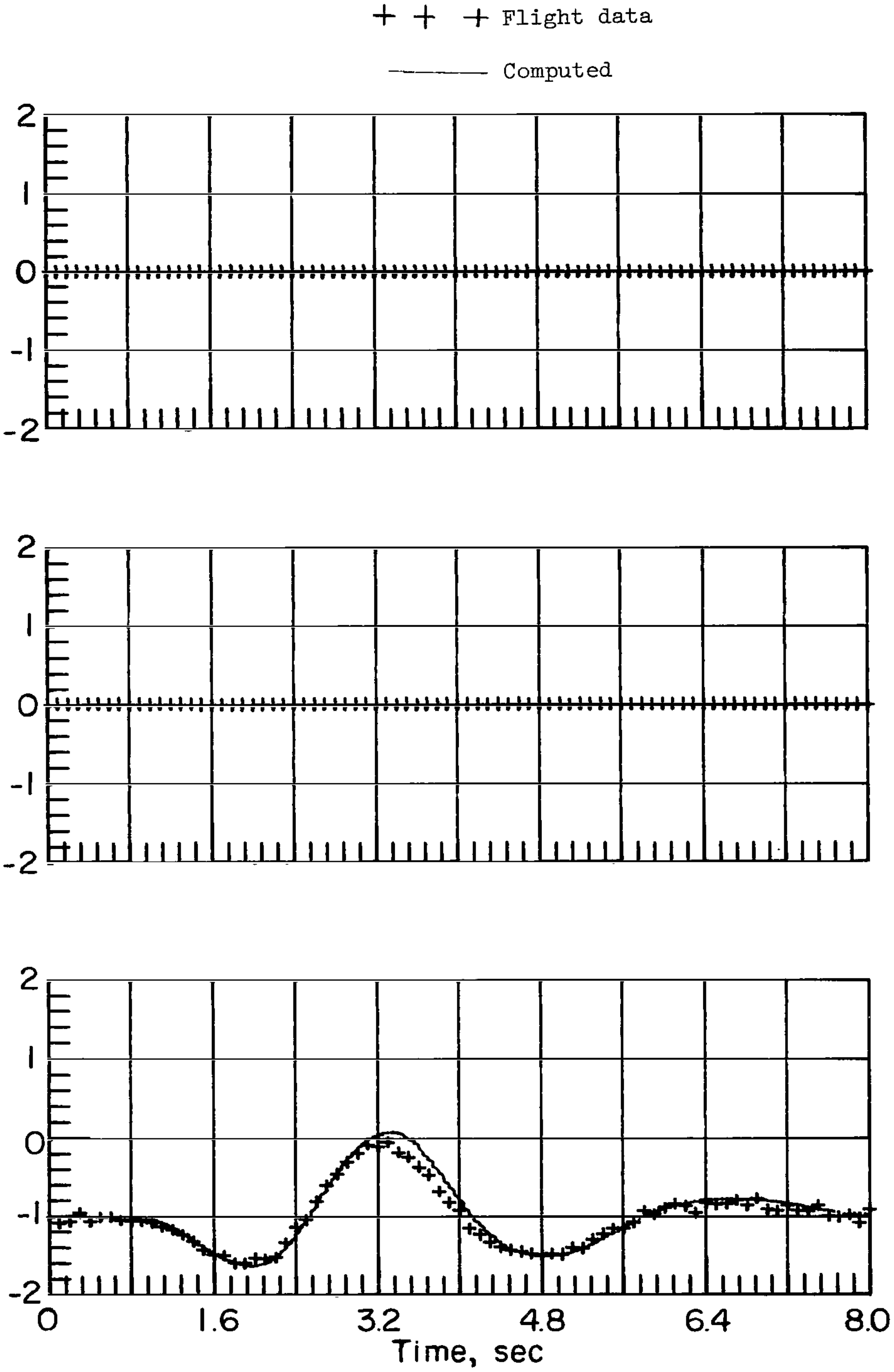

Figure 5.- Continued.

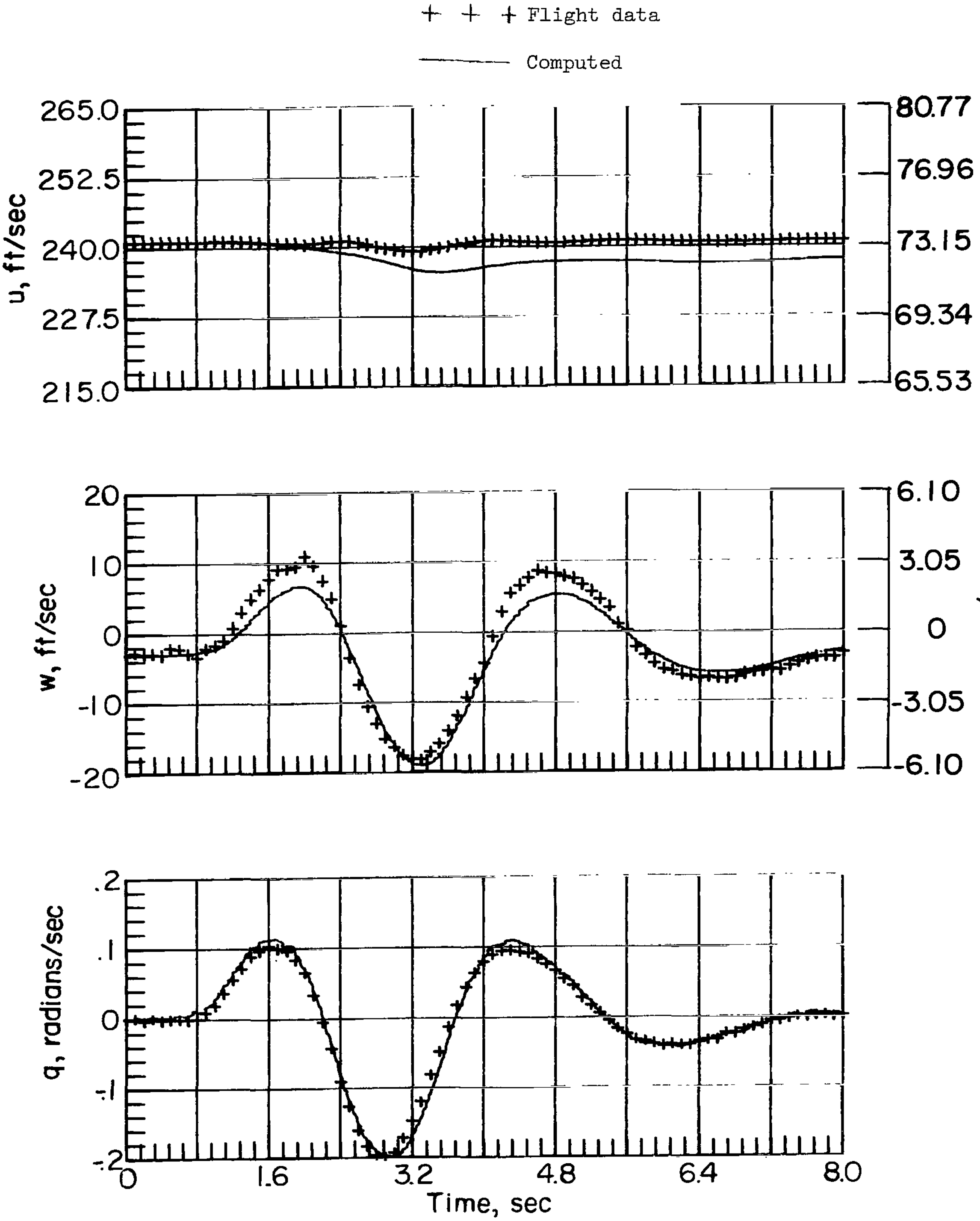

Figure 5.- Concluded.

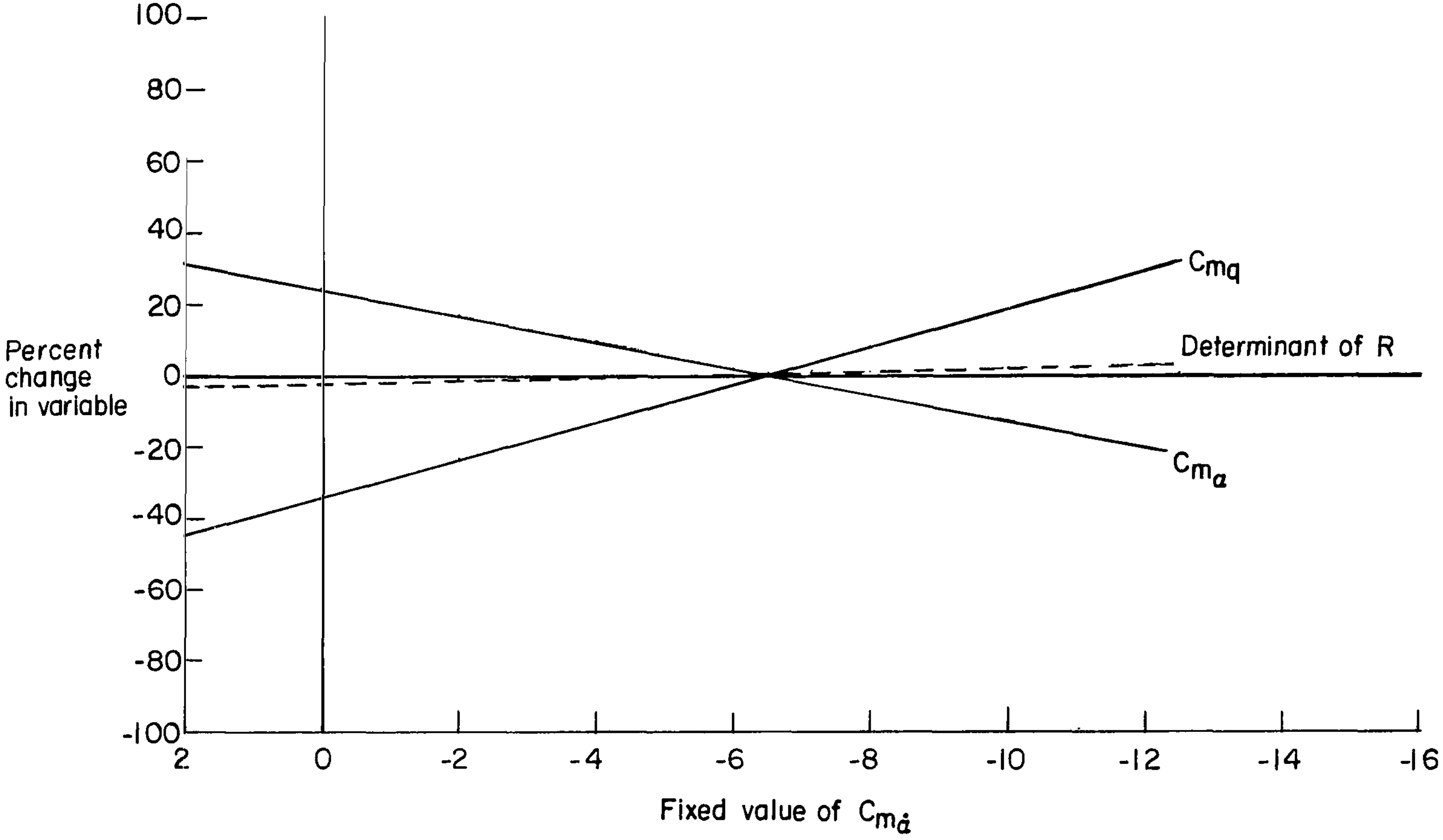

Figure 6.- Percent changes in the values of C_{m_q}, C_{m_α}, and the determinant of R for changes in the value of $C_{m_{\dot{\alpha}}}$.

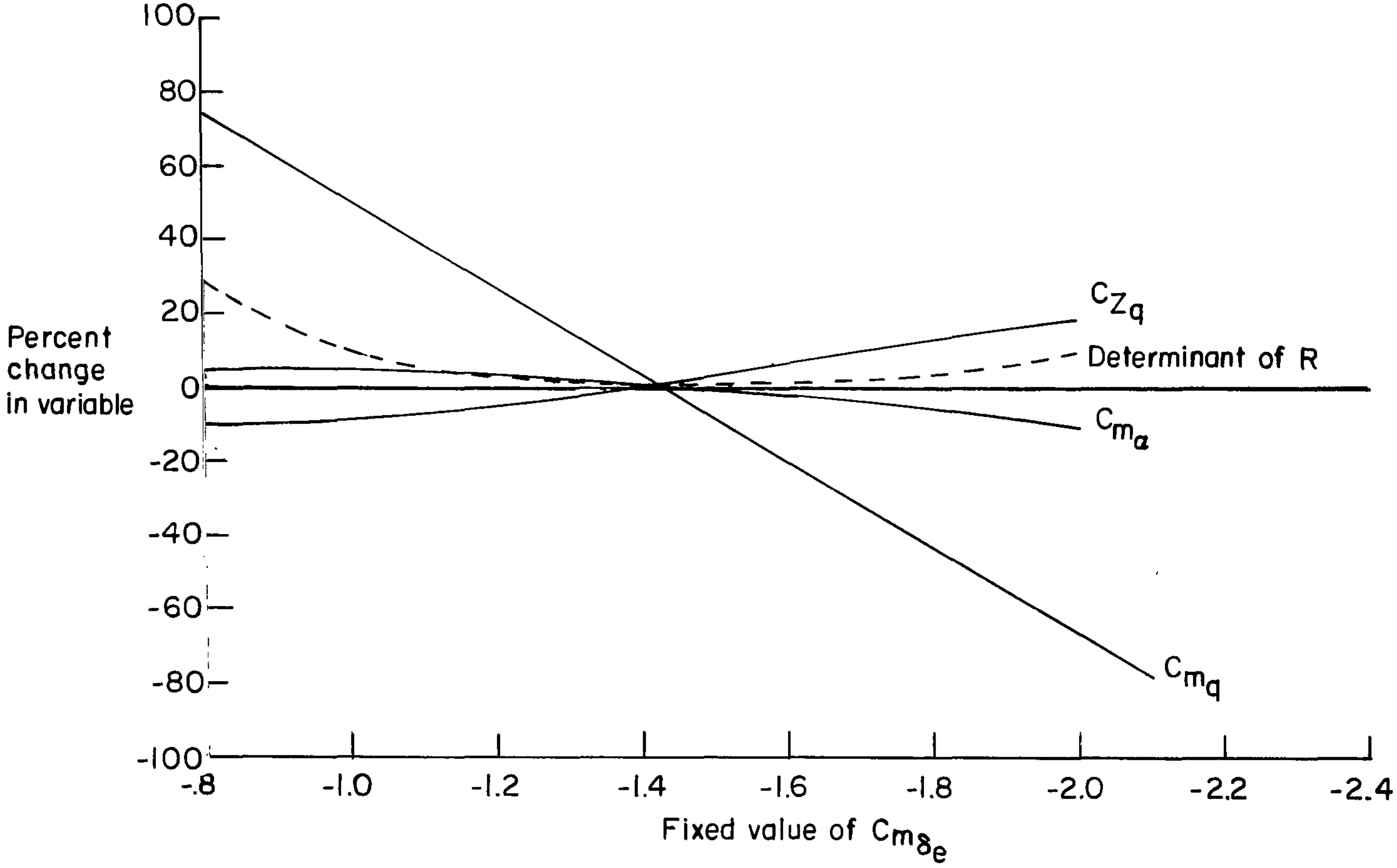

Figure 7.- Percent changes in the values of C_{m_q}, C_{m_α}, C_{Z_q}, and the determinant of R for changes in the value of $C_{m_{\delta_e}}$.

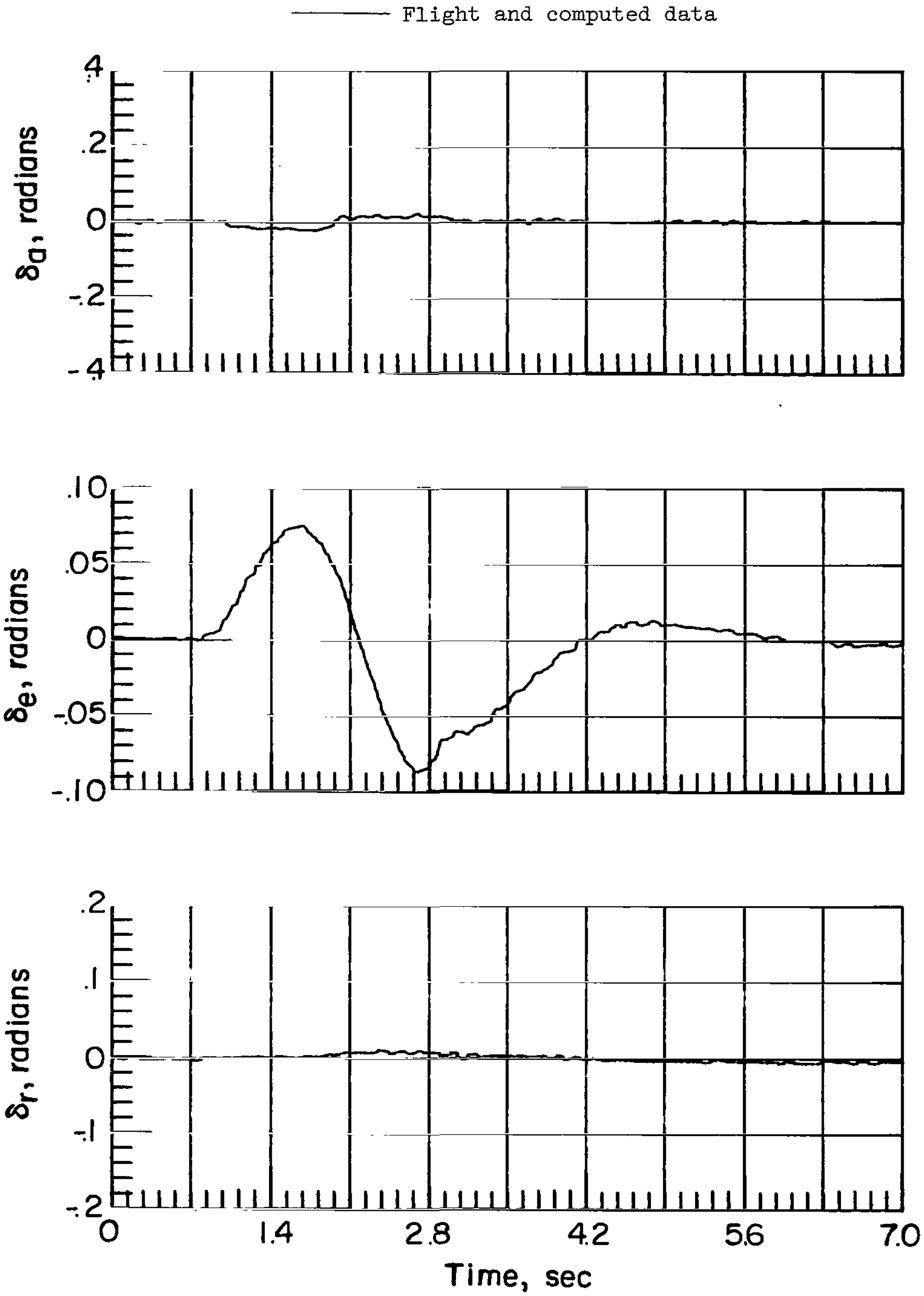

Figure 8.- Comparison of flight data with time histories computed by using aerodynamic parameters of table VII for elevator doublet input. V = 43.9 m/sec (144.1 ft/sec); $\delta_f = 20^\circ$. Flight and computed data are the same for control inputs.

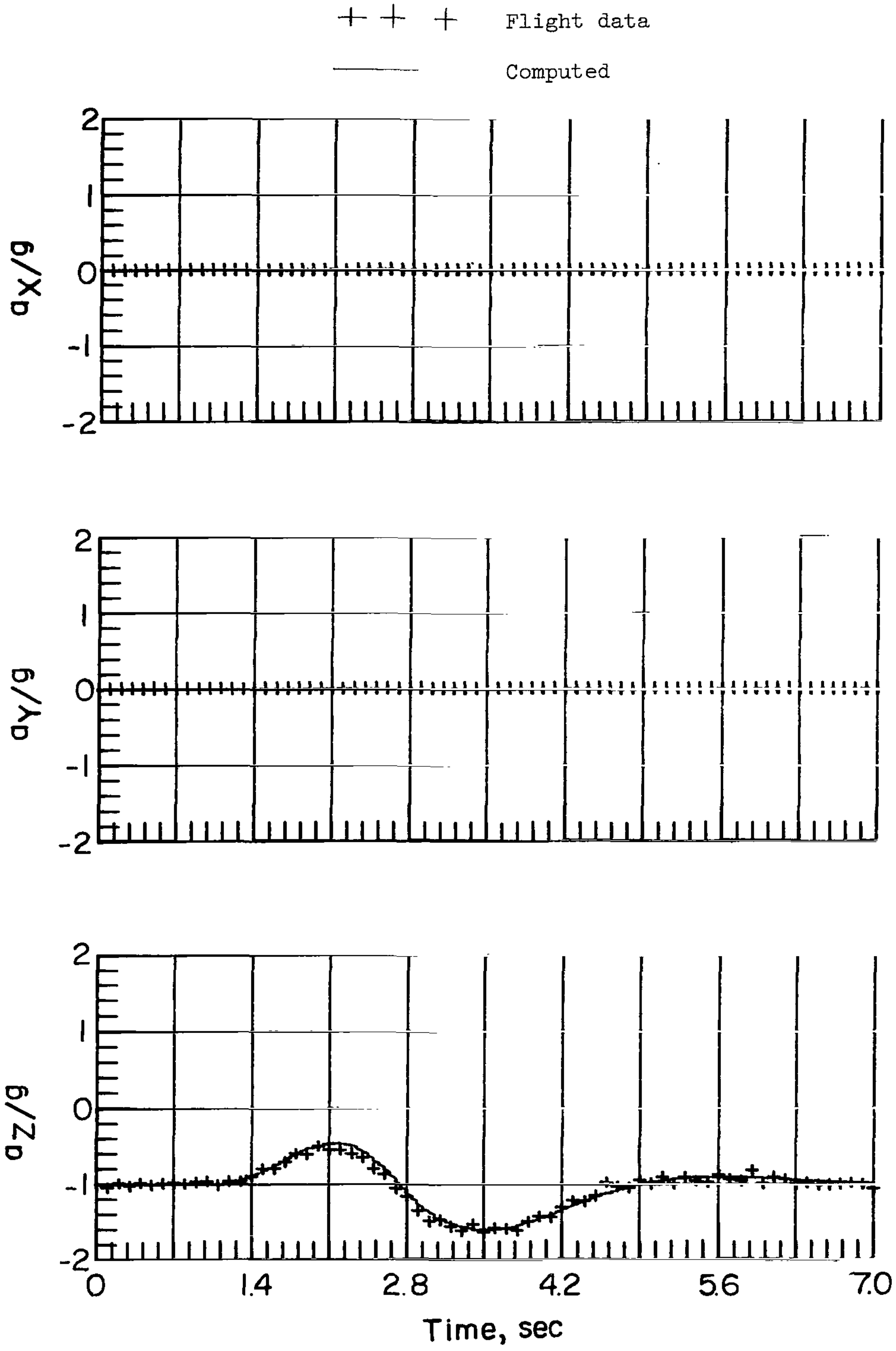

Figure 8.- Continued.

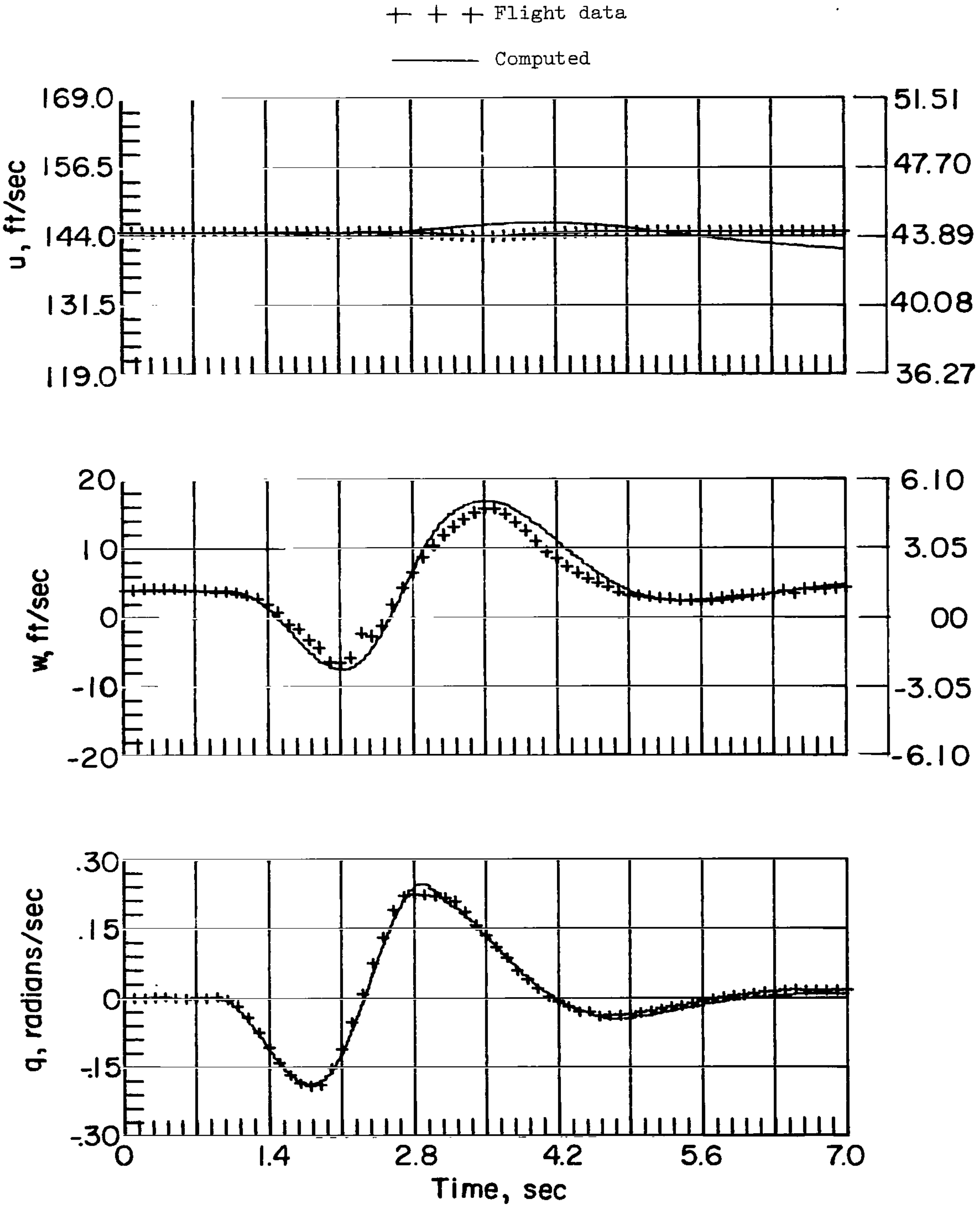

Figure 8.- Concluded.

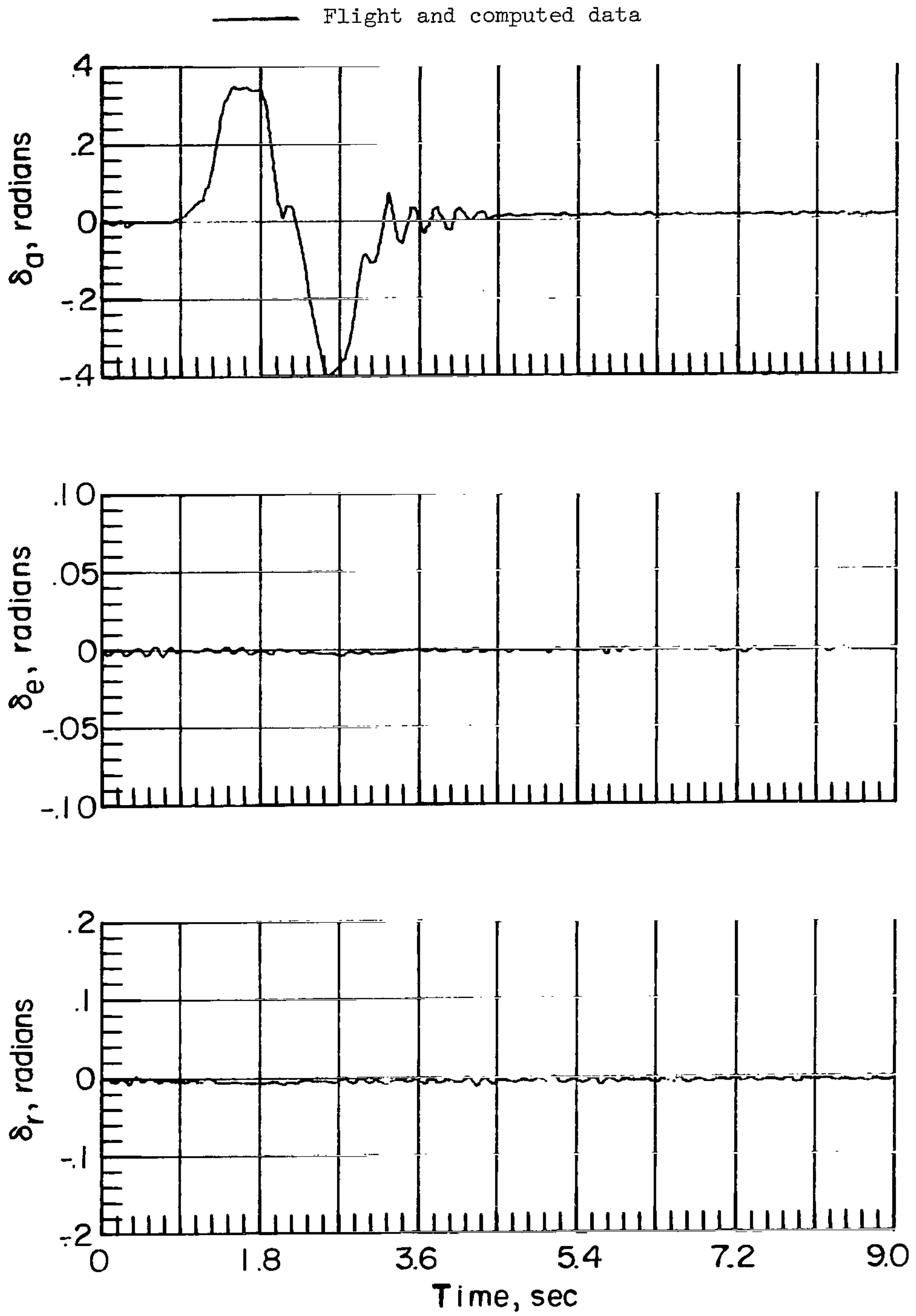

Figure 9.- Comparison of flight data with time histories computed by using aerodynamic parameters of table XI for aileron doublet input. V = 73.2 m/sec (240 ft/sec); $\delta_f = 0^{\circ}$. Flight and computed data are the same for control inputs.

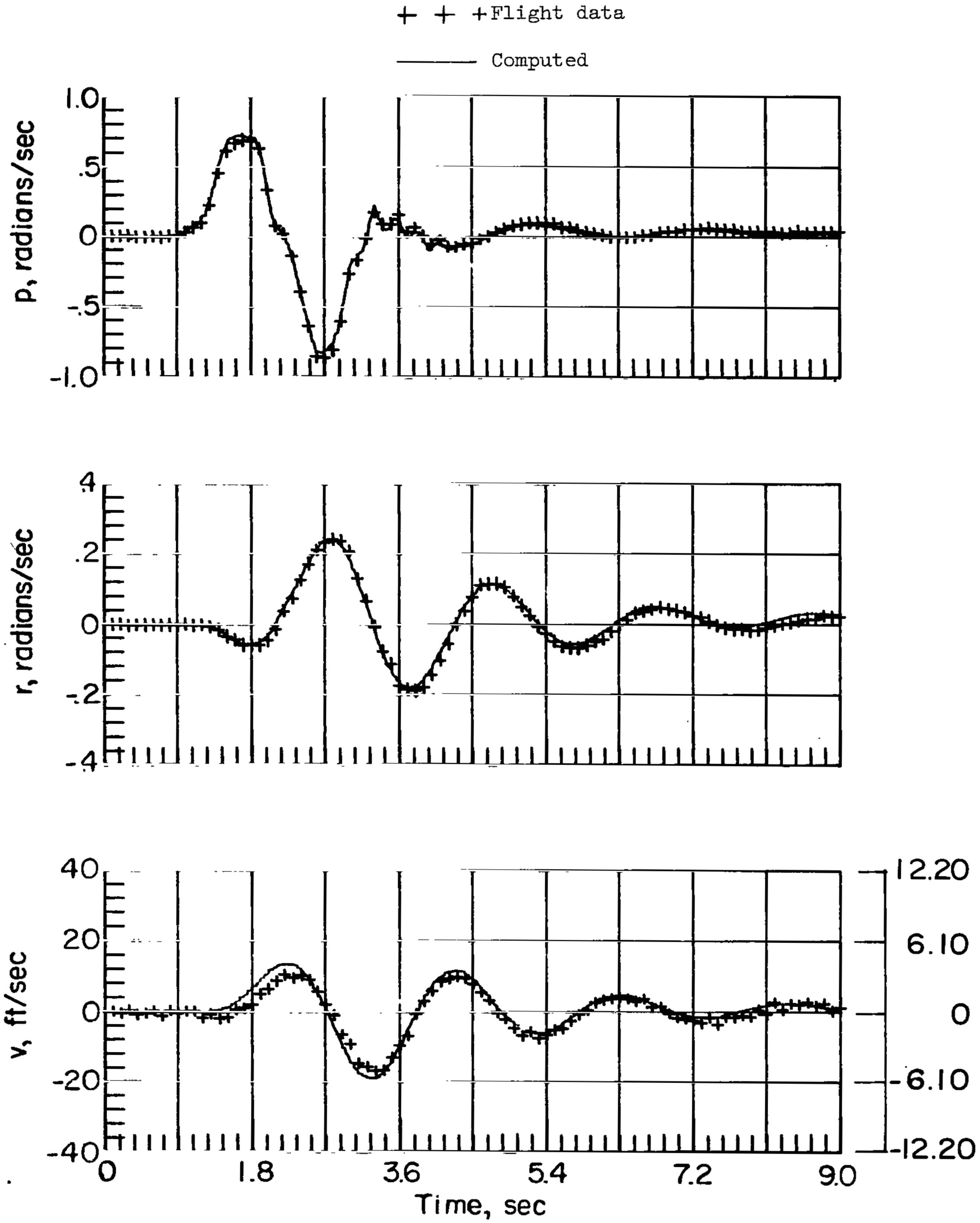

Figure 9.- Concluded.

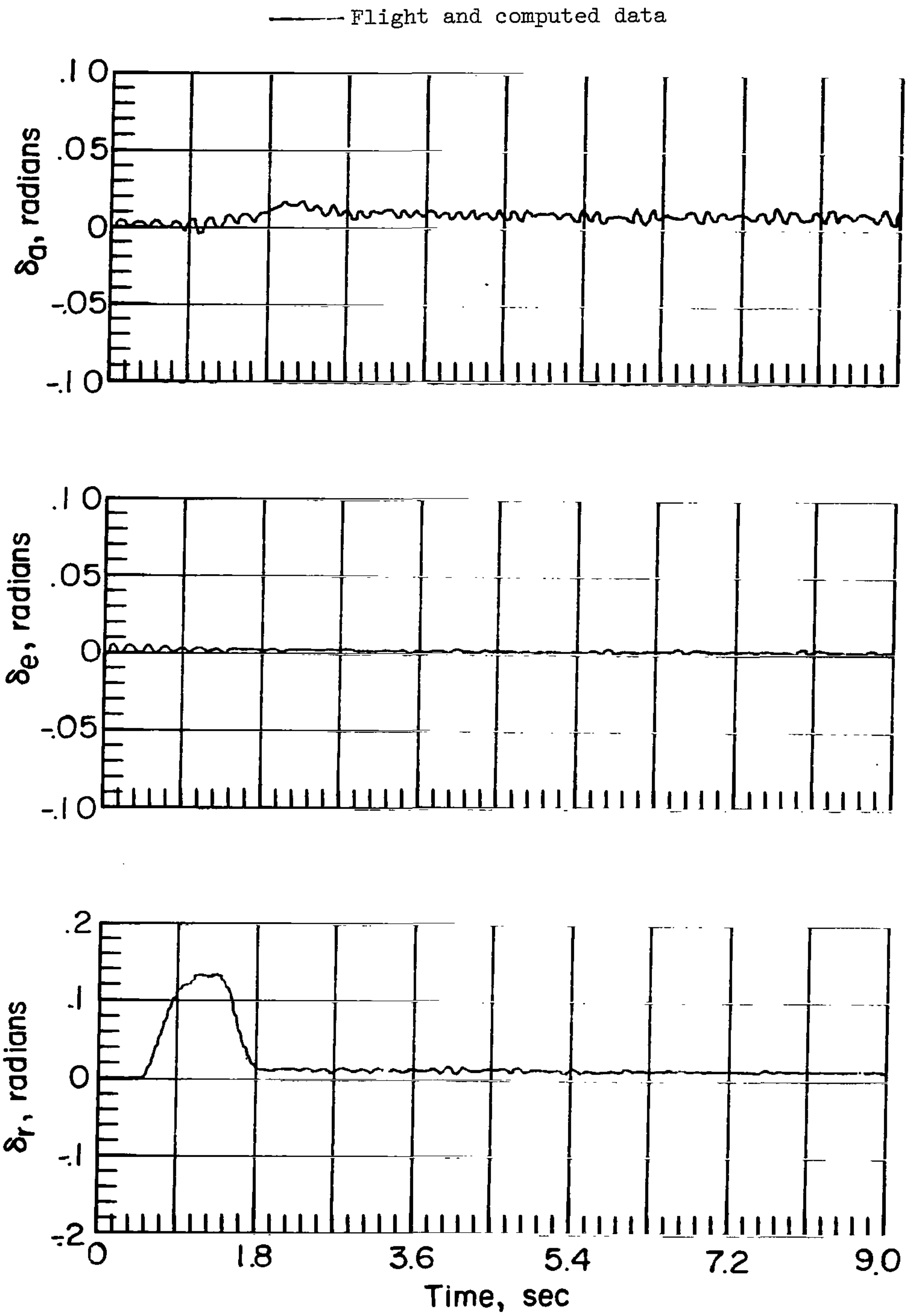

Figure 10.- Comparison of flight data with time histories computed by using aerodynamic parameters of table XI for rudder pulse input. V = 73.2 m/sec (240 ft/sec); $\delta_f = 0^{\circ}$. Flight and computed data are the same for control inputs.

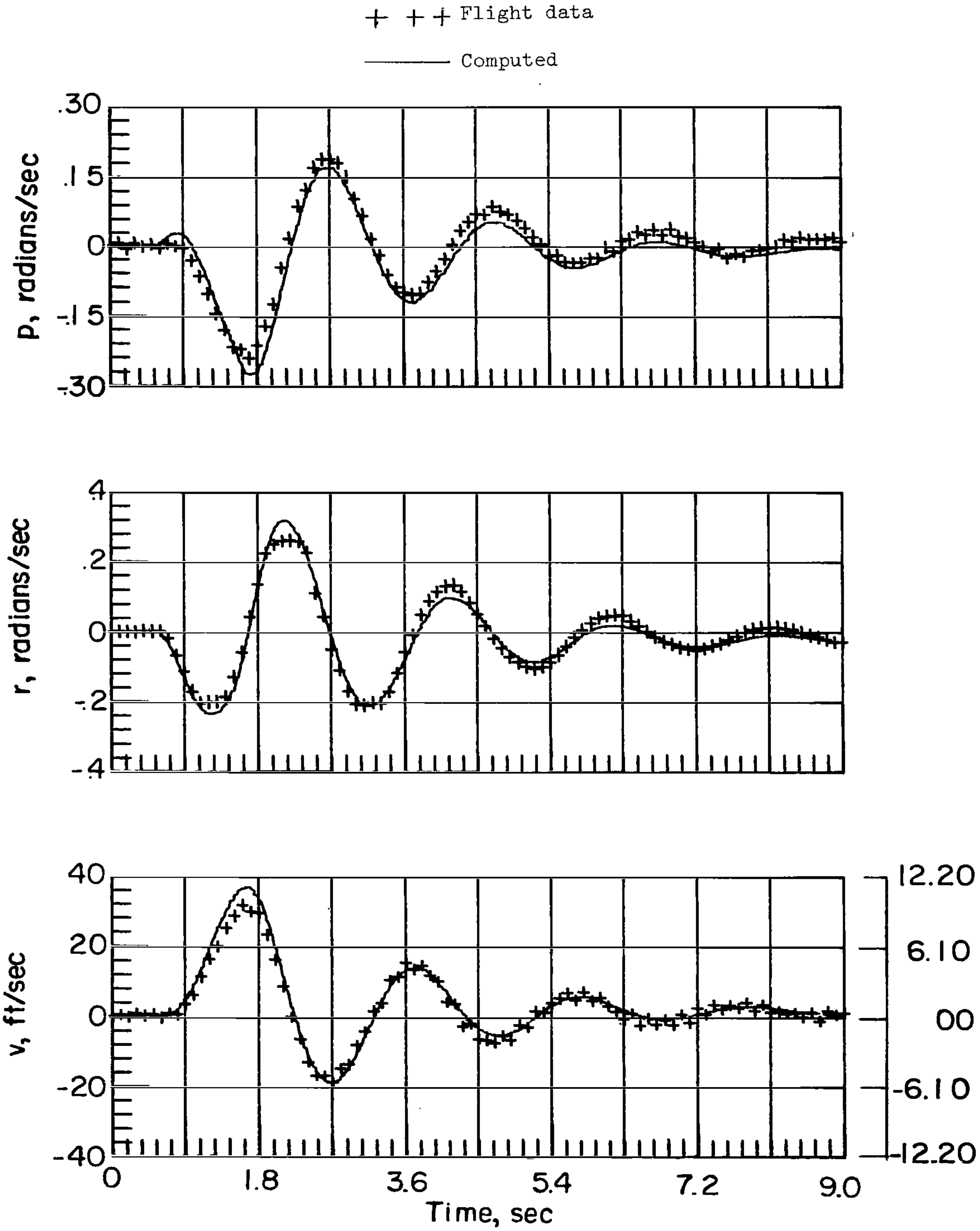

Figure 10.- Concluded.

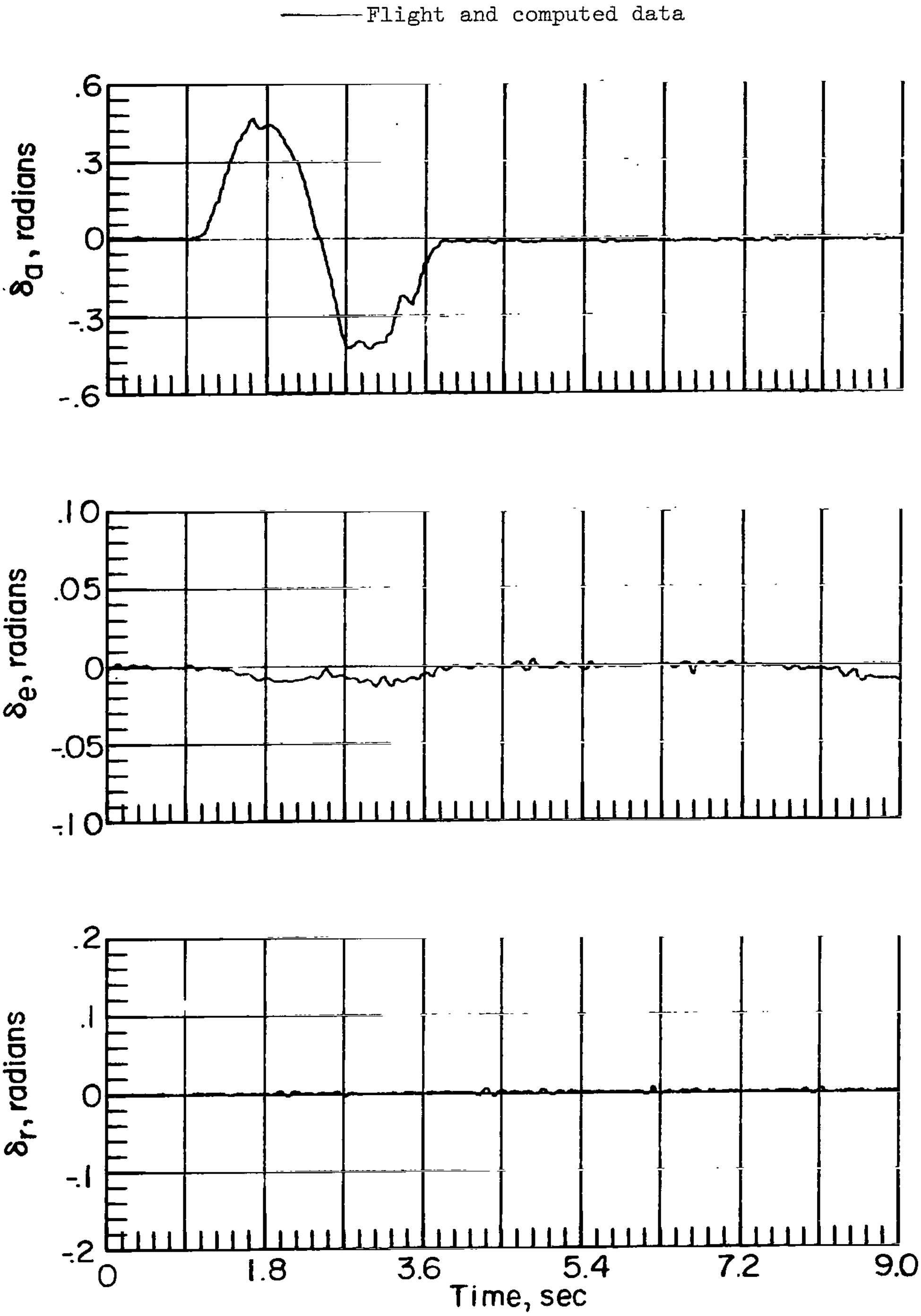

Figure 11.- Comparison of flight data with time histories computed by using aerodynamic parameters of table XI for an aileron doublet input. V = 43.9 m/sec (144.1 ft/sec); $\delta_f = 20^{\circ}$. Flight and computed data are the same for control inputs.

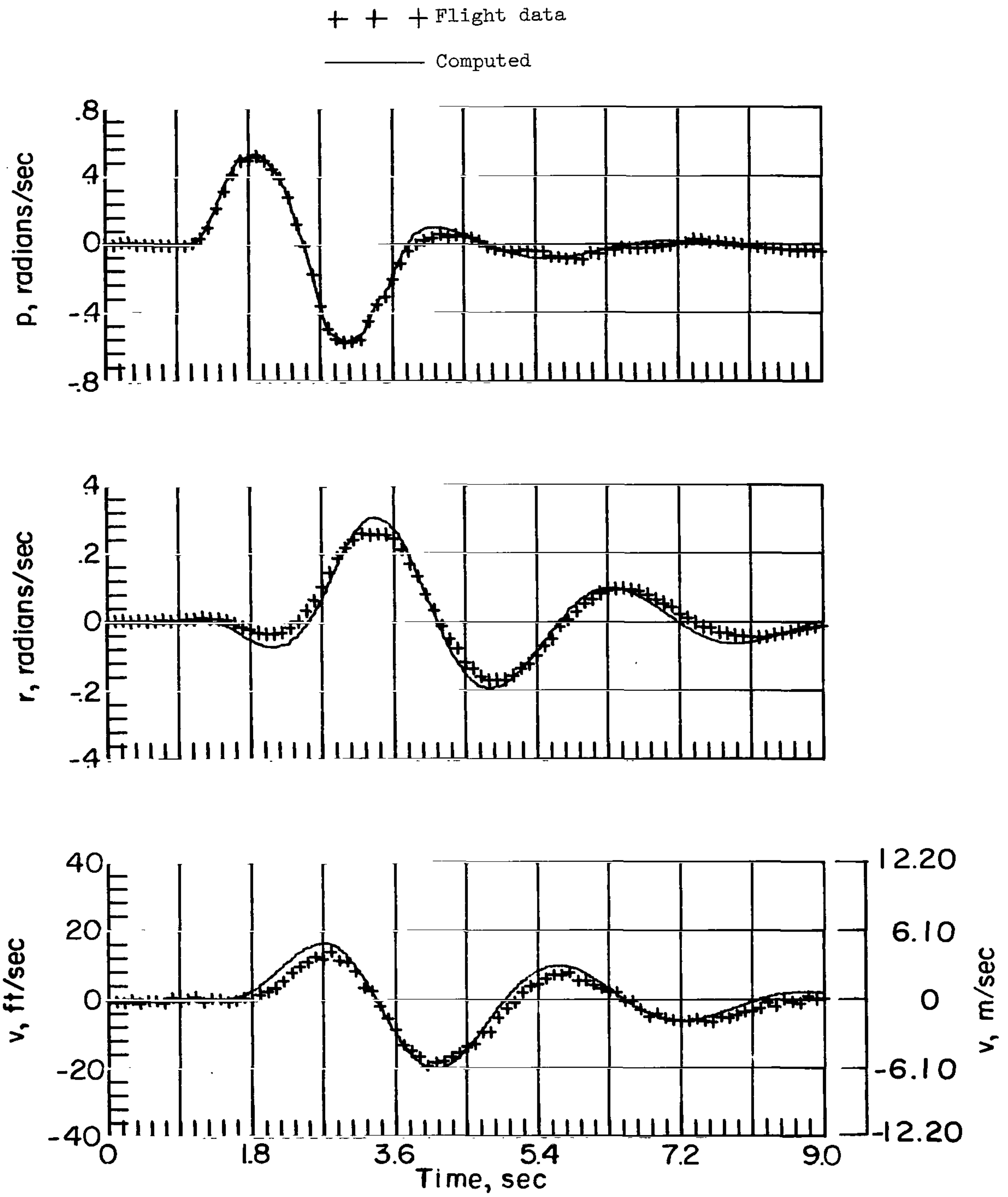

Figure 11.- Concluded.

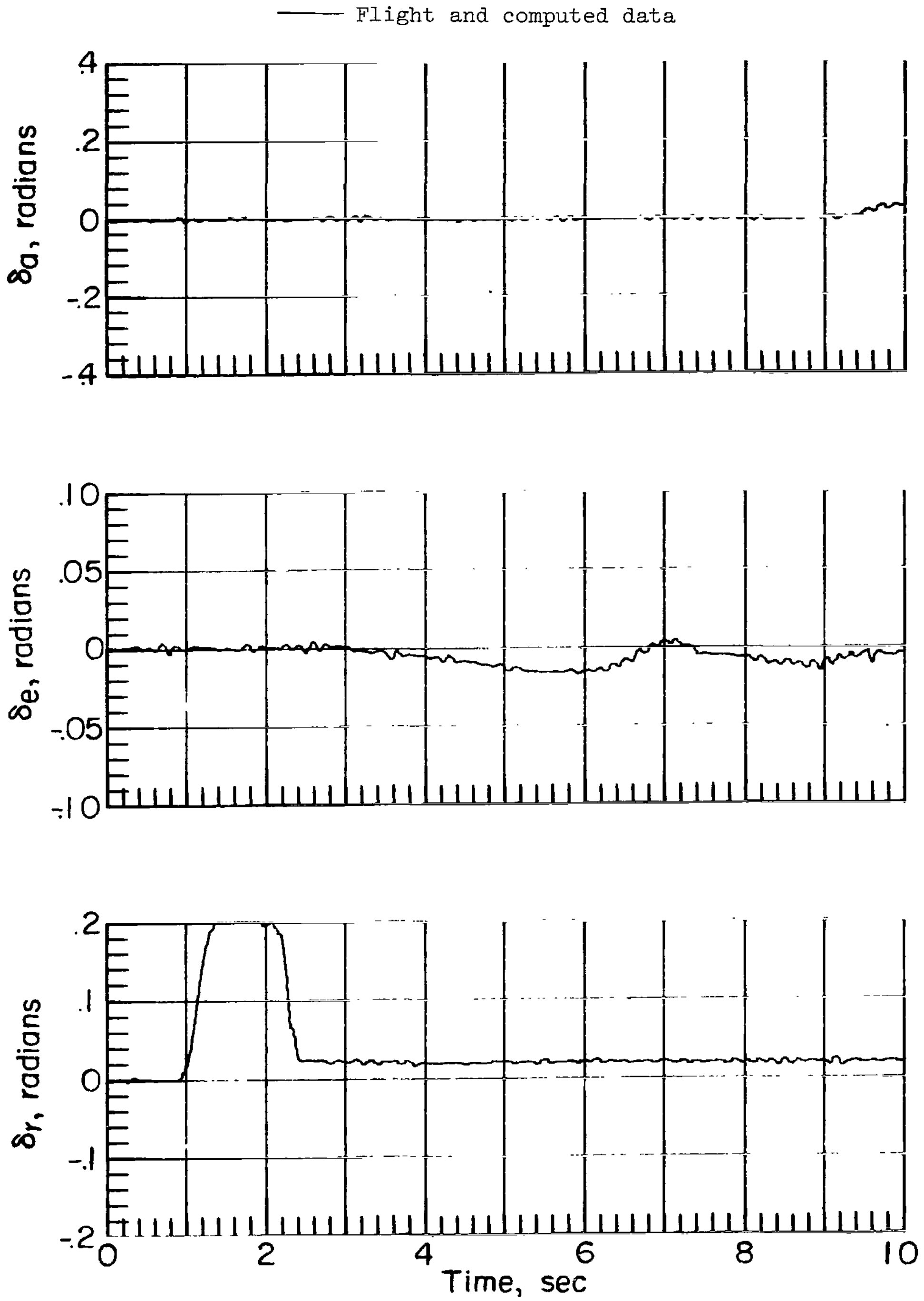

Figure 12.- Comparison of flight data with time histories computed by using aerodynamic parameters of table XI for a rudder pulse input. V = 43.9 m/sec (144.1 ft/sec); $\delta_f = 20^{\circ}$. Flight and computed data are the same for control inputs.

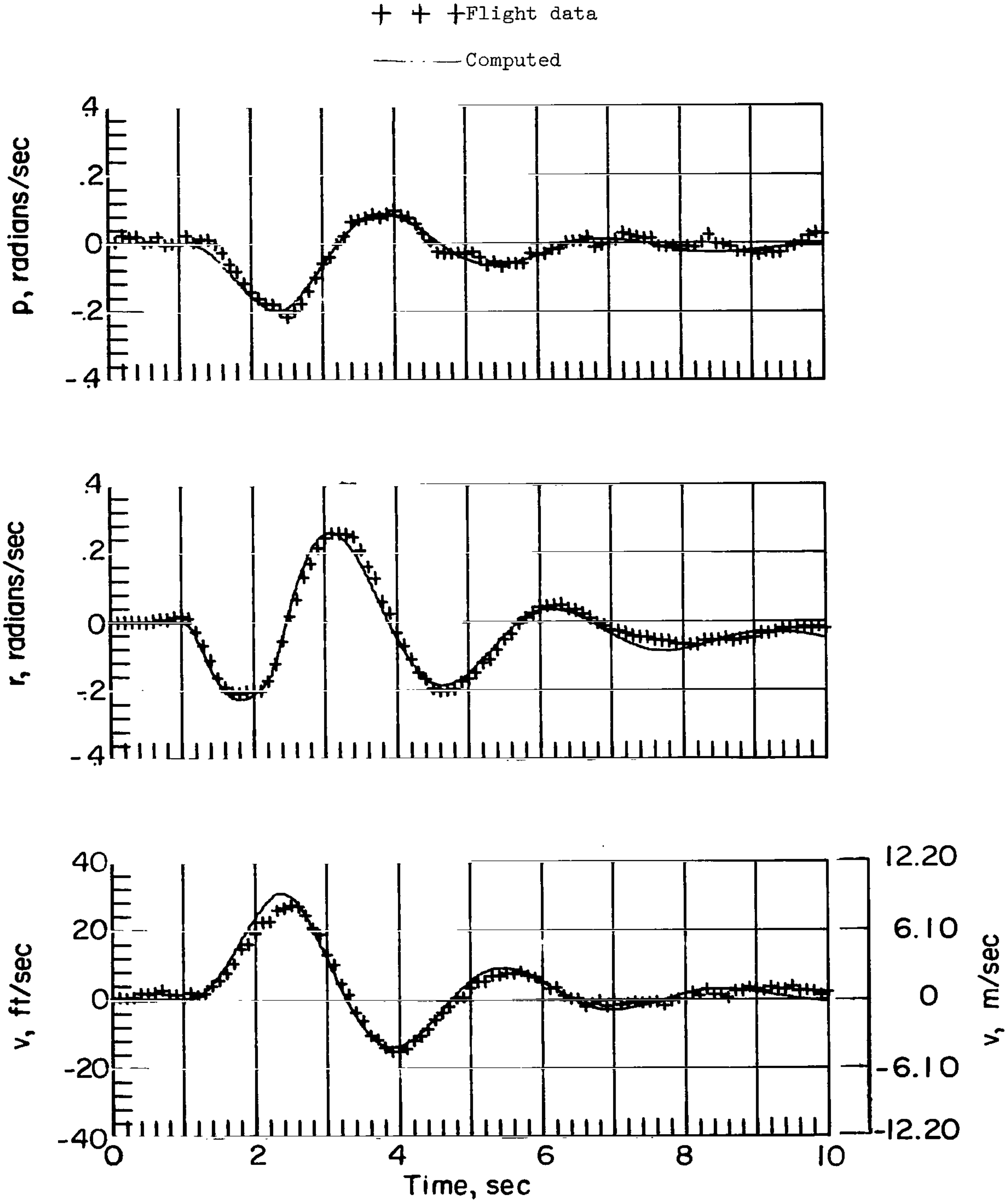

Figure 12.- Concluded.

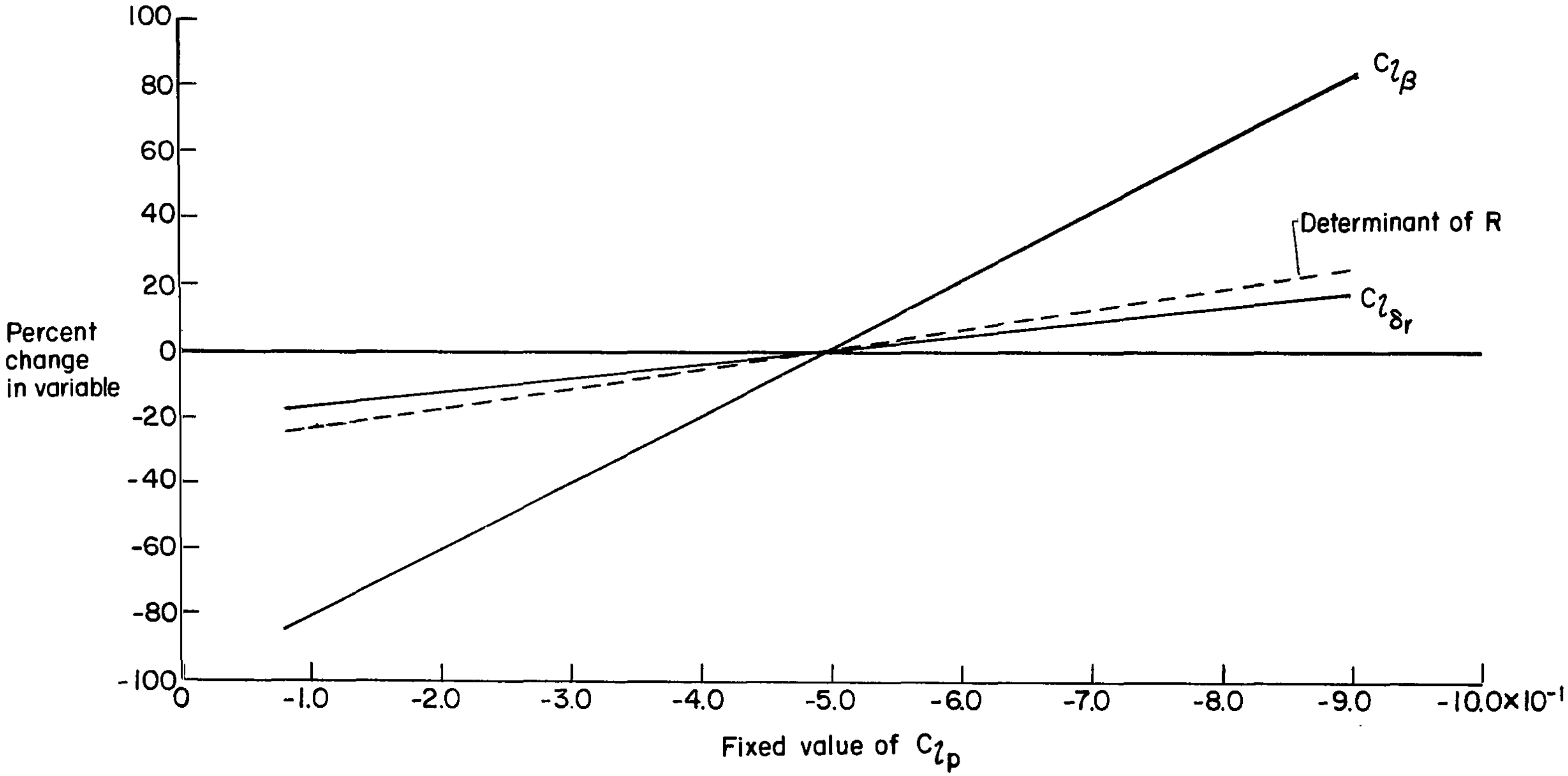

Figure 13.- Percent changes in the values of C_{l_β}, $C_{l_{\delta_r}}$, and the determinant of R for changes in the value of C_{l_p}.

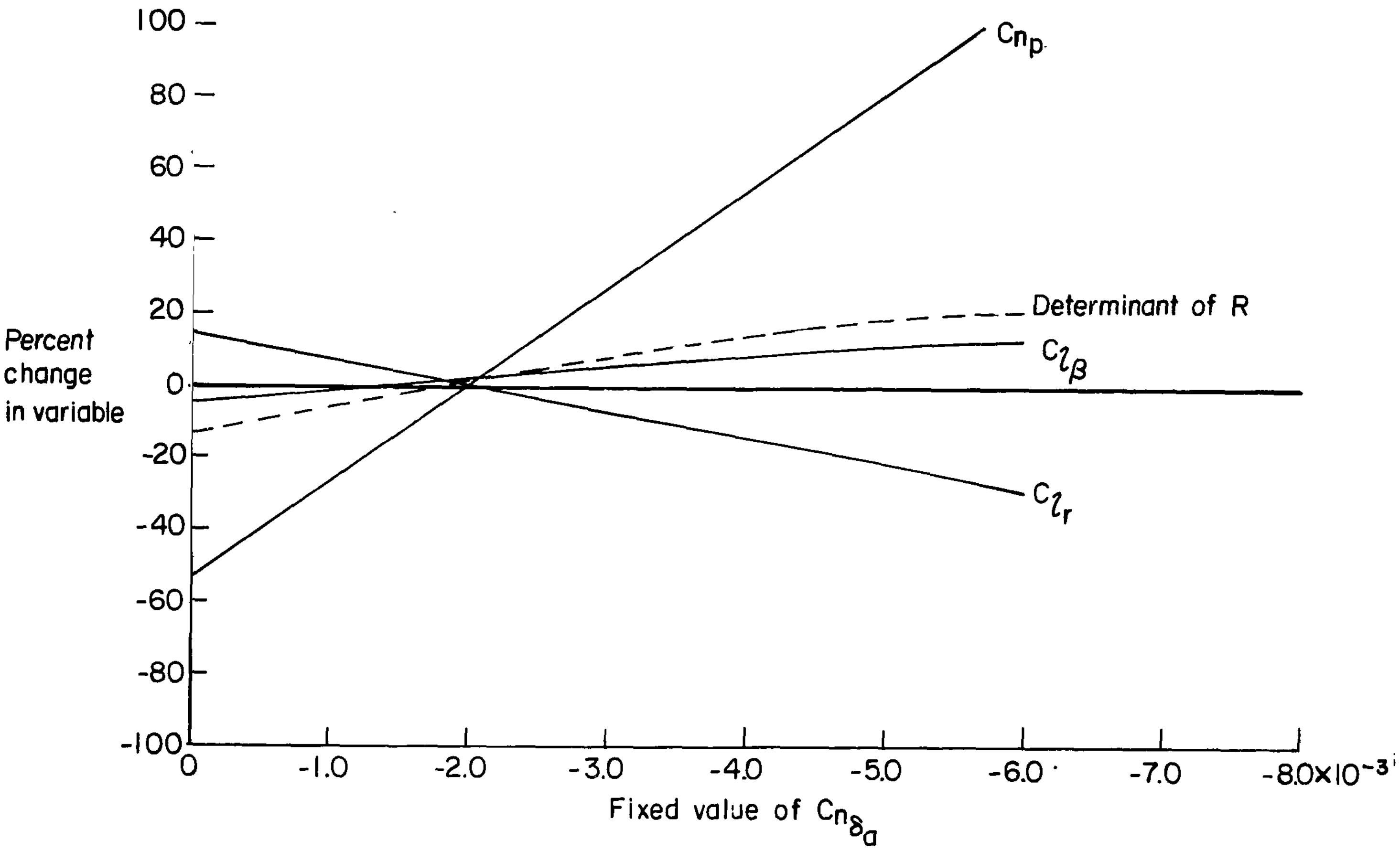

Figure 14.- Percent changes in the values of C_{n_p}, C_{l_r}, C_{l_β}, and the determinant of R for changes in the value of $C_{n_{\delta_a}}$.

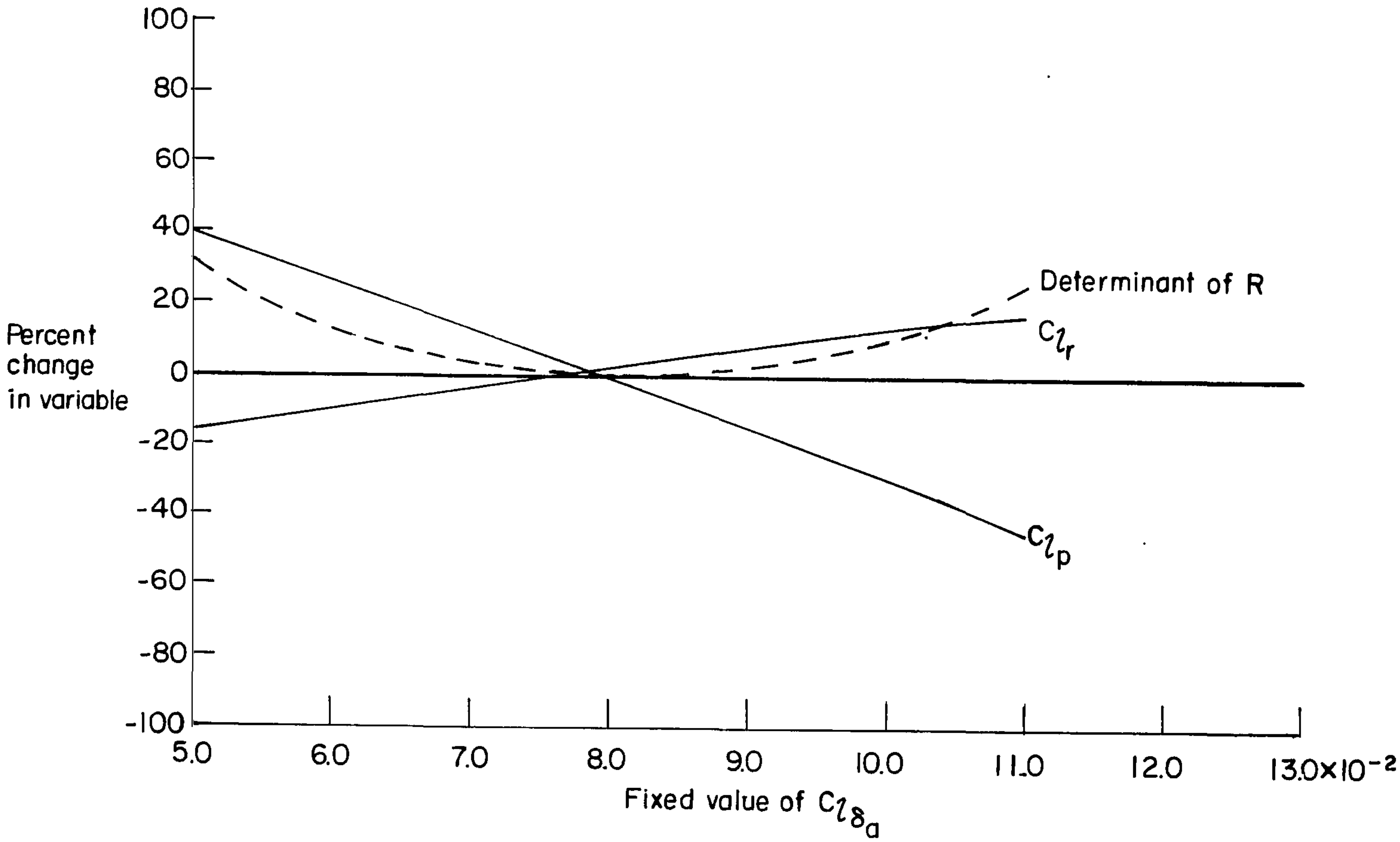

Figure 15.- Percent changes in the values of C_{l_p}, C_{l_r}, and the determinant of R for changes in the value of $C_{l_{\delta_a}}$.

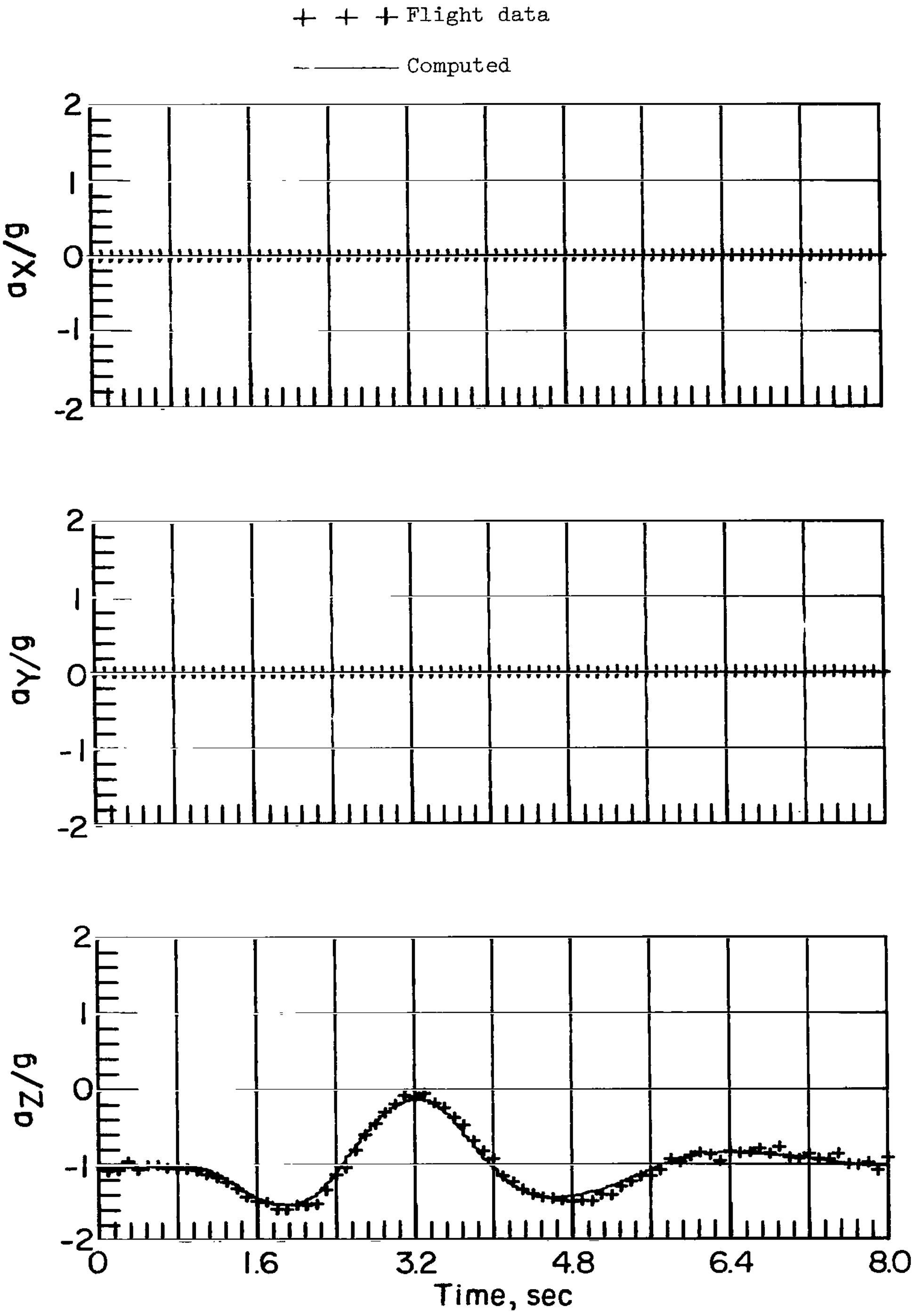

Figure 16.- Response of the mathematical model using derivatives from reference 9 and from the extraction program as compared with flight data for the 0° flap, V = 73.2 m/sec (240 ft/sec) case when the control input was an elevator doublet.

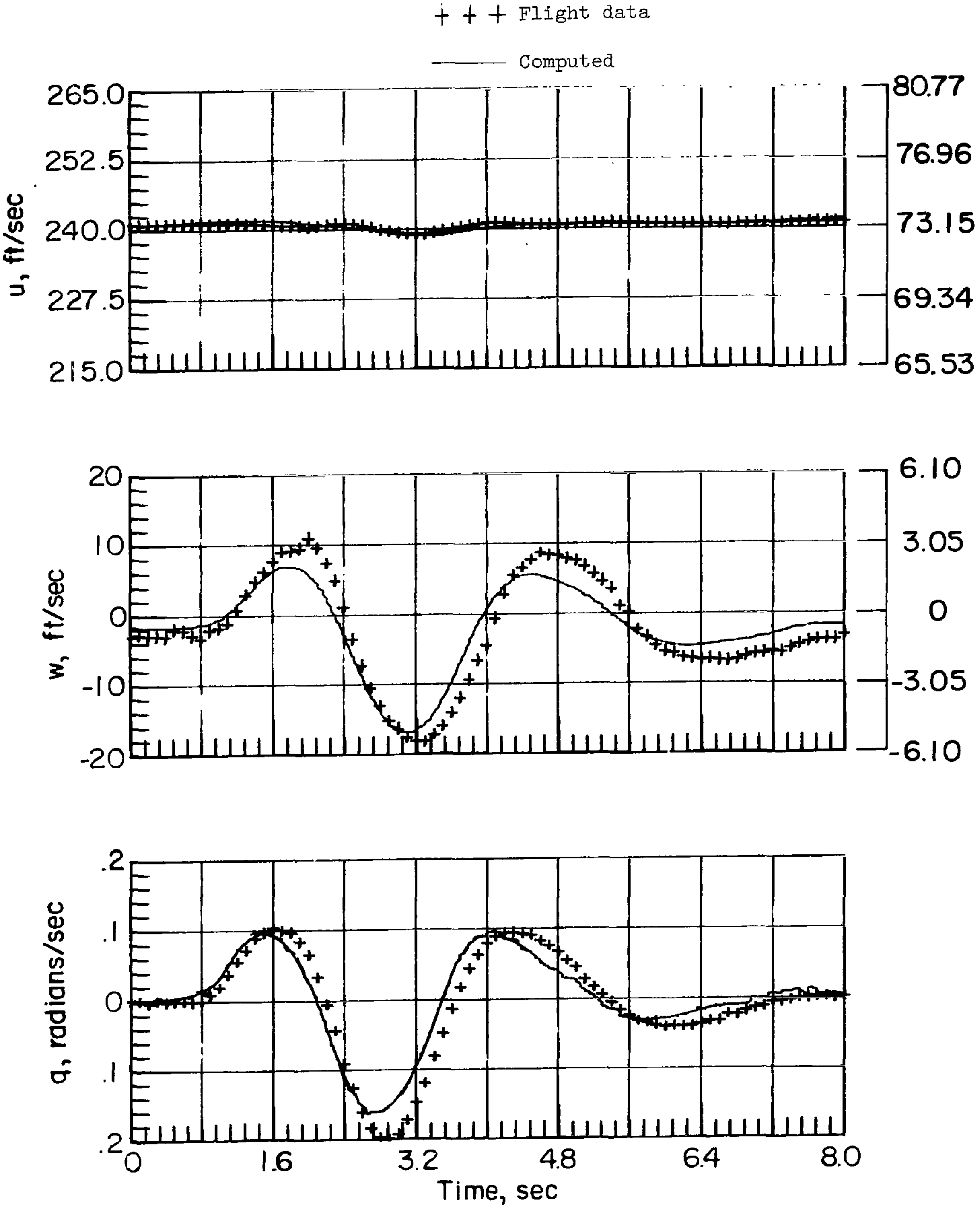

Figure 16.- Concluded.

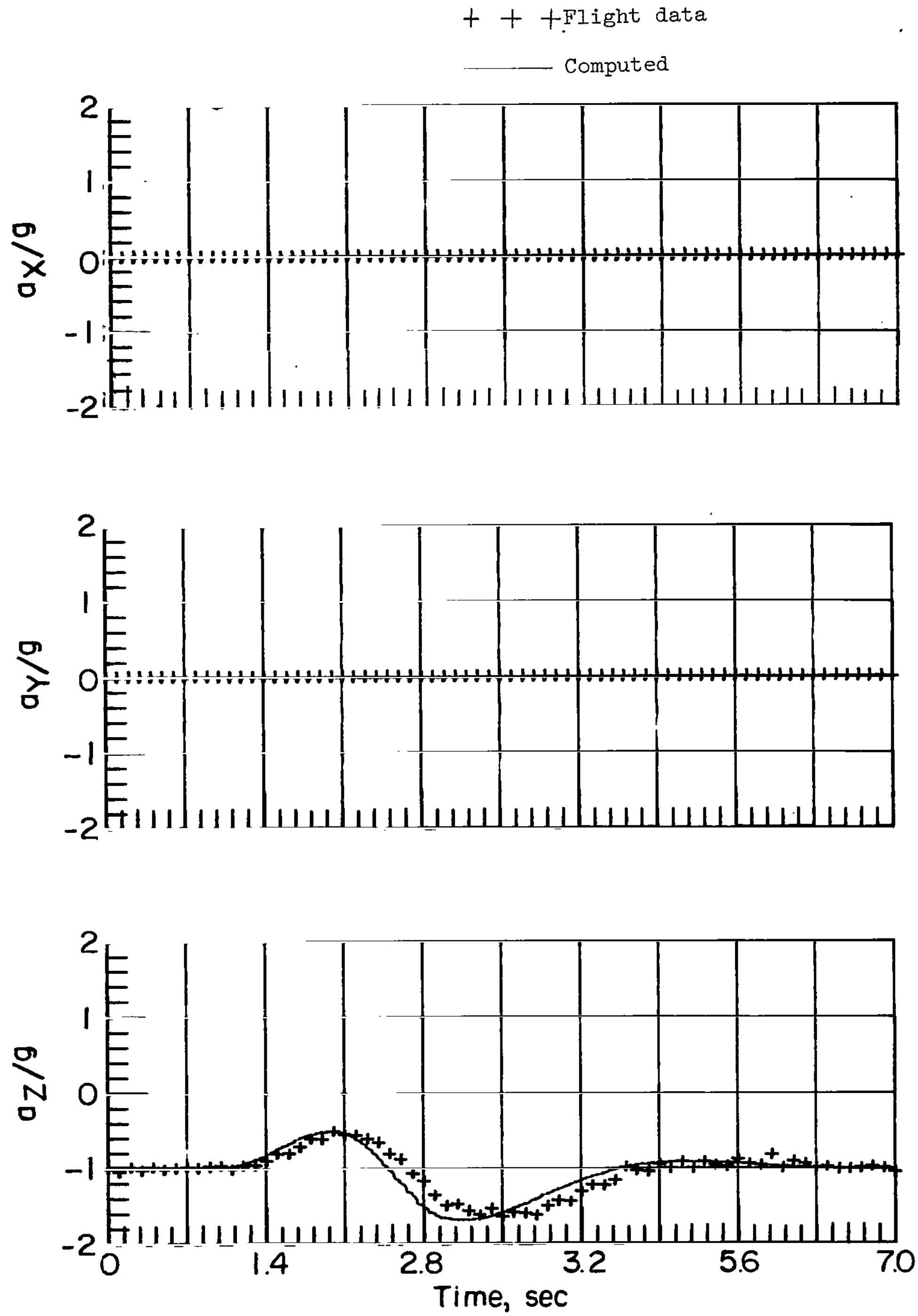

Figure 17.- Response of the mathematical model using derivatives from reference 9 and from the extraction program as compared with flight data for the 20° flap, V = 43.9 m/sec (144.1 ft/sec) case where the control input was an elevator doublet.

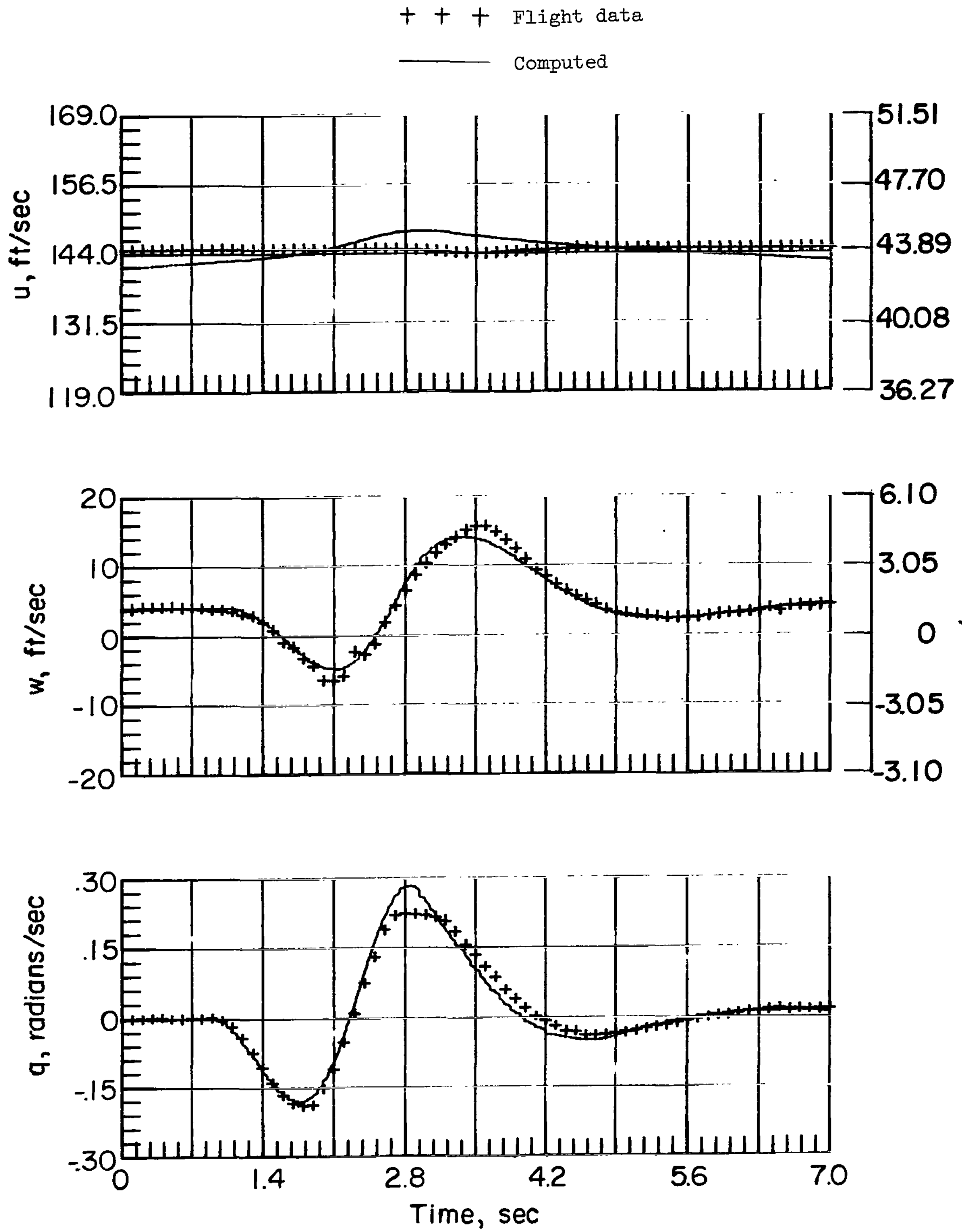

Figure 17.- Concluded.

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