COMPARISON OF SATELLITE THEORIES

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Goddard Space Flight Center is engaged in computing orbits of artificial satellites on a routine basis. For that reason it is appropriate to examine the mathematical models used with respect to accuracy and efficiency. Here, results of an investigation concerning the accuracy only will be discussed.

To determine the motion of an artificial satellite mathematically, one must start out with a position-velocity vector at a specified time, called the epoch, or use six equivalent quantities and specify a force model. This model may have various degrees of complexity. In the simplest case, one may consider merely the attraction of a spherical Earth. More complex models would imply the attraction of an Earth with a potential due to zonal harmonics only or due to zonal and tesseral harmonics. The complexity would be increased by adding drag or solar radiation pressure or both.

Various mathematical methods used in determining the motion (using one of the possible force models) lead to various mathematical models. There are two types of such models: numerical integration theories and analytical theories.

In numerical integrations the solution is constructed stepwise, while analytical theories permit the solutions to be constructed for individual points. Numerical integrations therefore tend to be slow; furthermore, they offer no physical insight. Analytical theories allow quick determinations for the points required and also offer physical insight into the structure of the solution. On the other hand, it is fairly easy to add forces to the force model employed in the numerical integration. It is also easy to attain a specified accuracy in a numerical integration. The addition of additional forces and the attainment of a specified accuracy in case of analytical theories, on the contrary, may be a very involved process.

Since the accurate solution for the motion is not known, only indirect tests of the accuracy are possible. The solution for one particular model may be used as a standard, and solutions based on other models may be compared with the standard solution. If the force model is restricted to the forces due to a potential consisting of zonal harmonics only, two integrals exist. One implies that the total energy is constant, and the other that the z-component of the angular momentum is constant. Because of numerical inaccuracies, strict constancy cannot be achieved. The amounts of spread in the values of the total energy and the z-component of the angular momentum for a set of times may be considered as measures of the accuracy of the mathematical models with which these values are computed.

Five mathematical models have been constructed using a position-velocity vector which is associated with a nominal orbit for the Vanguard 2 satellite. In all models the same force model was used, namely the one which corresponds to a potential with the zonal harmonics to order four. Models 1 and 2 are numerical integrations with step sizes of 1 min and 0.5 min, respectively. The integrations are based on programs created by Juergensmeyer.* Models 3, 4, and 5 are analytical theories. Model 3 corresponds to the Brouwer theory as originally published (Reference 1) and is a first-order theory. A modification of this theory, which is still of the first-order, is used in routine GSFC operations. Model 4 incorporates a modification introduced by the author (Reference 2) to the original Brouwer theory. Model 5 is the second-order theory by Aksnes (Reference 3).

The orbit based on model 1 (i.e., the 1-min numerical integration) was taken as a standard orbit.** The rectangular coordinates in the orbits based on models 2 through 5 were compared with the standard orbit every 40 min in the case of model 2 and every 4 hr for models 3 through 5 for an arc of 60 days. The largest maximum absolute values of the corresponding differences are shown in Table 1. The two integrations differ thus only by a few meters. The original Brouwer differs from the 1-min integration by as much as nearly 27 km. The modification inherent in model 4 reduces the maximum difference to less than 200 m. The maximum differences between Aksnes and the 1-min numerical integration are even smaller.

*Juergensmeyer, C. H., "A Double Precision Cowell Integration Program", Goddard Space Flight Center Document X-542-63-139, July 1962, Goddard Space Flight Center, Greenbelt, Maryland.

**This is reasonable, since numerical integrations tend to be more accurate.

Table 2 shows that the spreads in the values of the total energy and the z-component of the angular momentum are much less for model 2 than for model 1. This suggests that it might have been better to have taken the 0.5-min numerical integration as the standard orbit.

The modification introduced in model 4 reduced the spread in the values of the energy but increased the spread in the values of the z-component of the angular momentum. The spread of the corresponding values for the Aksnes theory is less than that for the Brouwer theory. It is remarkable that the spread in the values of the z-component of the angular momentum is even less than it is in the case of the numerical integrations. Part of the results presented here were published previously (Reference 4).

This investigation shows that a considerable increase in the accuracy for the Vanguard 2 orbit may be obtained by the use of models different from the ones used in the Goddard Space Flight Center routine operations. It is planned to continue these investigations with other orbits and other theories.

REFERENCES

1. Brouwer, Dirk, "Solution of the Problem of Artificial Satellite Theory without Drag", Astronomical Journal, 64: 378,1959.

2. Hertz, Hans G., "A Computer Program Version of the Brouwer Orbital Theory with Optional Modifications", NASA Technical Memorandum X-63749, August 1969.

3. Aksnes, Kaare, "A Second Order Artificial Satellite Theory Based on An Intermediate Orbit", Astronomical Journal, 75: 1066, 1970.

4. Hertz, Hans G., "Numerical Integration Orbits and Brouwer and Modified Brouwer Orbits", NASA Technical Memorandum X-63809, August 1969.

Table 1—Comparison of mathematical models with Cowell 1-min integration (60-day arc) (nominal Vanguard 2 orbit).

POINTS	0.5 MINUTE INTEGRATION 40 MIN (m)	ORIGINAL BROUWER 4 HR (m)	MODIFIED BROUWER 4 HR (m)	AKSNES 4 HR (m)
ΔX	3	26840	121	94
ΔY	2	23378	76	82
ΔZ	1	13870	161	48

Table 2—Maximum relative changes in the constancy of integrals.

	INTEGRATION		BROUWER		AKSNES
	1-MIN	0.5-MIN	ORIGINAL	MODIFIED	
ENERGY	8×10^{-9}	8×10^{-12}	5×10^{-6}	1×10^{-8}	1×10^{-8}
Z-COMP. ANG. MOM.	3×10^{-9}	3×10^{-12}	9×10^{-7}	2×10^{-6}	$< 1\times10^{-12}$