AN ANALYSIS OF THE EXPECTED ECCENTRICITY PERTURBATIONS FOR THE SECOND RADIO ASTRONOMY EXPLORER (RAE B)

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The second spacecraft in the Radio Astronomy Explorer series, RAE B, is to be placed in a nearly circular lunar orbit. The orbit is to remain circular, with an eccentricity constraint of less than 0.005 for a period of a year or more, if possible. This constraint is made because of factors concerning the dynamical stability of this gravity gradient spacecraft and the economical reduction of the experimental data. Other constraints upon the orbit are the result of sunlight conditions and the precession of the orbital plane. Thus, original plans called for a circular orbit about 1100 m above the lunar surface with a selenographic inclination between 120 and 130 deg and an initial longitude for the ascending node of 282 deg. The initial values of the argument of perilune and the eccentricity were arbitrary so long as the latter was less than 0.005.

Past experience with the Lunar Orbiter and Apollo missions has shown that eccentricity perturbations have been quite severe. The Apollo orbits were close, circular, and nearly equatorial; the Lunar Orbiter spacecraft flew more distant elliptical orbits with either low or nearly polar inclinations. The RAE B orbit is to be circular, as in the Apollo missions, but more distant, as in the Lunar Orbiter missions, and with a moderate inclination unlike either of these two previous missions. Nevertheless, eccentricity histories for these missions can be useful in planning for RAE B. One such history is presented in Figure 1. The dots in this figure are the actual elements determined from the tracking data. The solid line represents a numerical integration that used the current operational Apollo field, the L1 model. Two points can be made: First, the eccentricity perturbations are large relative to the RAE B eccentricity constraint, and second, the L1 model predicts this eccentricity variation quite well. It should be mentioned at this point that the results presented in this study were obtained by the use of eight different lunar gravity models developed at the Jet Propulsion Laboratory, the Langley Research Center, and GSFC. The main results of this study are essentially independent of the gravity field chosen. All the results are documented in a GSFC report, but because of time limitations we will concentrate on the results obtained from the L1 model.

Eccentricity perturbations for RAE B are of two types: The first type has a monthly period, while the second type has a period of the order of many years and hence appears to be linear during the course of a year. An analytical development of these perturbations follows.

The monthly eccentricity variation is

$$\delta e = -\frac{3}{16}\frac{n}{n}\left(\frac{b}{a}\right)^3 C_{3,1}\left\{\left[5\sin^2 i(1+3\cos i)-4(1+\cos i)\right]\cos(\omega+\Omega) - \left[5\sin^2 i(1-3\cos i)-4(1-\cos i)\right]\cos(\omega-\Omega)\right\}$$

It can be seen that the amplitude is a function only of the semimajor axis and inclination of the orbit and of certain physical parameters such as the rotation rate, mean radius, and gravity coefficients of the Moon. If one evaluates this amplitude for various orbits within the RAE B nominal range and for various lunar models, it will be observed that this amplitude hardly changes. For instance, if $a = 2838$ km, and $i \approx 120$ deg then from the L1 model,

$$\delta e = .00185 \cos\omega \cos\Omega \approx .00185 \cos\Omega.$$

It should be noted that this would result in a peak-to-trough variation of 0.0037. In other words, 74 percent of the eccentricity constraint would be reached in a month or less. Therefore, any appreciable long term perturbation would be most unwelcome. Unfortunately, for any value of the inclination in the originally proposed range and for any of eight lunar gravity models tested, the envelope of the eccentricity variation had a very substantial positive slope. After tabulating the results of some parametric inclinations studies and after performing an analysis of the perturbation equations in analytical form, we discovered that the positive slope would vanish for an inclination of 116.565 deg, just a few degrees under the nominal inclination range. This value is that of the so-called retrograde "critical inclination", which was given much attention in the literature during the early stages of the development of artificial satellite theories. As an approximation, one may write for the long-term perturbations:

$$\dot{e} = -A\cos\omega$$

$$\dot{\omega} = (A/e)\sin\omega,$$

where

$$A = \frac{3}{8}\left(\frac{b}{a}\right)^3 nC_{3,0} \sin i(1 - 5\cos^2 i).$$

When these equations are solved, the results are

$$e \sin \omega = e_0 \sin \omega_0$$

$$e \cos \omega = e_0 \cos \omega_0 - At.$$

It can be deduced from these equations that, regardless of the sign of $C_{3,0}$, the eccentricity will grow linearly in time so long as A is nonzero. However, A vanishes at the recommended inclination of 116.565 deg. The results of this analytical treatment were confirmed by numerical integration of an RAE B orbit. One set of integrations is shown in Figure 2. After considering all the results of a lowering of the value of the nominal inclination, the RAE project accepted the recommendation for a new nominal inclination.

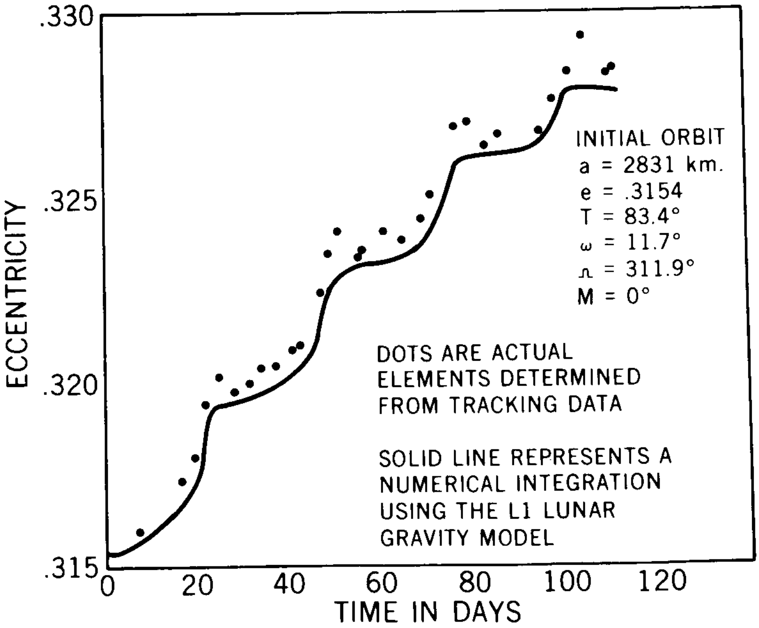

Figure 1—Eccentricity versus time for Lunar Orbiter 5 (arc 4).

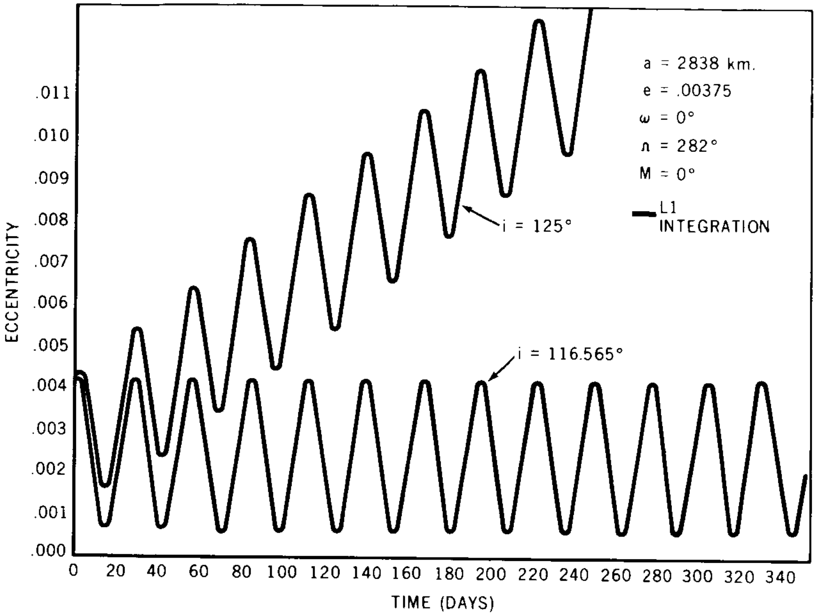

Figure 2—Eccentricity versus time for RAE B.