THE GEOPOTENTIAL AT SYNCHRONOUS-ORBIT ALTITUDES

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I would like to discuss briefly the latest solution I have obtained for the Earth's gravity potential at synchronous-orbit altitudes.

As you may know, a distant satellite in a synchronous orbit is affected significantly by only a few low degree terms in the conventional spherical harmonic representation of the field. These terms include the oblateness of the Earth and, because of the commensurability of the synchronous orbit, all the longitude-dependent terms as well.

It has been 7 years since the first such satellite was put into orbit. Since that time, about 50 others have followed, making synchronous orbits probably the most popular variety of orbit. However, only about 15 of these objects are close enough to exact commensurability to be really useful in studies of the small but persistent effects of the so-called resonant harmonics of gravity.

I have obtained the present solution for these harmonics from the most representative sample of tracking data from eight of these satellites in 21 distinct free-drift arcs. The length of time for these arcs ranges from 6 weeks to 3½ years (Table 1).

The observations I use in my solution are the mean Kepler elements for the satellites reported by the various responsible tracking agencies, including GSFC. The element most sensitive to the resonant effect is the mean geographic longitude or equator crossing of the satellite. This is a simple combination of the Kepler orbit arguments and the position of the Earth and is the principal observable recovered in the solutions. The quality of this observable varies from arc to arc, but in no case does it approach the fine precision of the tracking required for geodetic recovery from nonresonant satellites. The geodetic solutions with these data demonstrate conclusively that we are at last getting absolute determinations of the low-degree field.

This can be seen from the answers to three questions concerning these sensitive resonant data. First, if there were no resonant gravity harmonics, what kind of recovery of these data would we get? Figure 1 shows the evolution of the longitude of Syncom 2 between 1965 and 1969. Without resonant harmonics, this evolution would be almost linear, and, since the data in this arc have an accuracy of about 0.05 deg, the mean weighted residual in a best-fit trajectory would be over 100. Second, how well can the data be recovered with the use of recent gravity models that include resonant harmonics but not the data itself? Table 2 gives results of orbits for these satellites calculated with recent fields through fourth degree. The first two rows show the overall recovery with two recent Smithsonian geopotentials. From about 100 without low degree coefficients, the mean weighted residual (rms) comes down to below 3 with the latest SAO field, which includes ground gravity information. An improvement in the fit to less than 2 is found with a 1969 all-satellite SAO field, very little different in these critical harmonics from the 1970 field.

Third, what field adjustment is necessary for complete recovery of the data? This question is answered in the last two rows of Table 2. In the unconstrained solution, the dominant harmonics of second and third degree were freely derived along with the orbits. The residuals were lowest, but the (3,1) harmonic was not realistic and many correlation coefficients were large. The last row shows the most satisfactory solution obtained from the 24-hour data. Here, all five significant harmonics were derived with reasonable *a priori* constraints from the SAO COSPAR coefficients. This constrained field shows adequate data recovery with realistic coefficients that are only moderately correlated.

There should be little doubt that we can now predict the long term drift of these satellites to considerable accuracy even over periods of many months. In fact, a dramatic test of this proposition came in September 1970 when Syncom 3 was reacquired by GSFC within 0.4 deg of its predicted position after being silent for almost 2 years. This prediction was made with the constrained field.

The economics of computing these fields from mean element data is also interesting. Only 279 observations were used, compared with over 50,000 for the SAO solutions. The solutions themselves involved about five differential corrections to both orbits and field parameters or a total of 60 orbit years of calculation, and took less than 4 min of computing time on an IBM 360/95 machine.

The overall results of this study of the geopotential for synchronous satellites are as follows:

1. Absolute accuracy of second-degree coefficients has been established to better than 3 percent. Coefficients through fourth degree have been established to better than 15 percent.

2. The positions of equilibrium points for geostationary satellites have been established to better than ½ deg.

3. Accuracy in prediction of orbits for 24-hour satellites has been established to better than 1 deg for periods greater than 2 years.

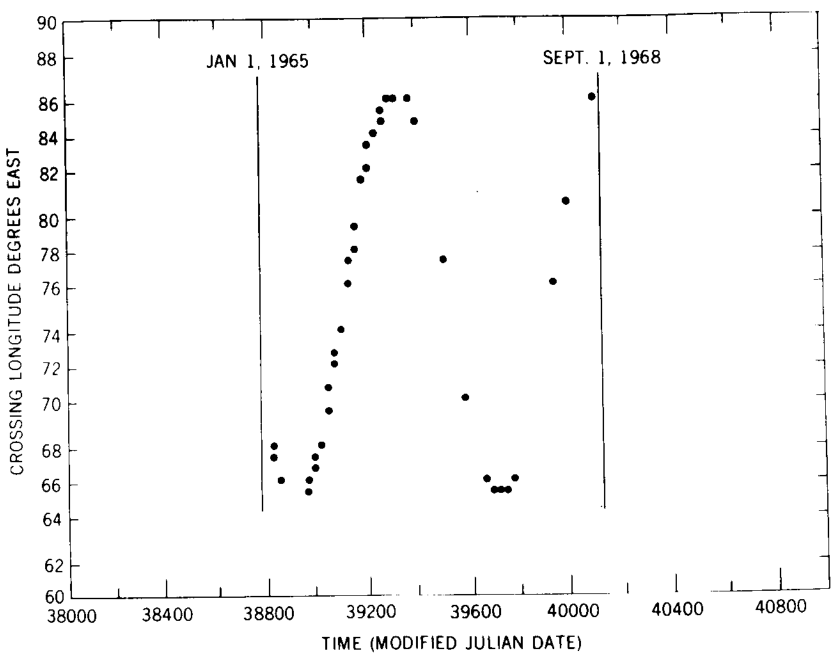

Figure 1—Equator crossings for SYNCOM 2, 1965-1968.

Table 1–Twenty-four hour satellite arcs in 1970 resonant geopotential solutions.

SATELLITE	ARC	NUMBER OF KEPLER ELEMENT SETS USED	SPAN OF DATA (modified Julian Days)	GEOGRAPHIC LONGITUDE SPAN IN ARC (degrees)	ORBIT INCLINATION (degrees)	ESTIMATE OF QUALITY OF DETERMINATION OF THE GEOGRAPHIC LONGITUDE, σ (λ), IN ARC, FROM INDIVIDUAL ARC ANALYSES (degrees)	RMS RESIDUALS IN LONGITUDE FROM COMBINED ARC SOLUTIONS (degrees)		
							WITH 24 HOUR SATELLITES ALONE, SOLVING FREELY FOR 6 RESONANT COEFFICIENTS	WITH 24 HOUR SATELLITES COMBINED WITH SAO 1969 COSPAR DATA, IN CONSTRAINED SOLUTION FOR 10 RESONANT COEFFICIENTS	WITH SAO 1969 COSPAR FIELD ALONE
SYNCOM 2	1	16	38263-38351	302-305	33.0	0.025	0.013	0.013	0.013
SYNCOM 2	2	11	38361-38443	296-301	32.8	0.025	0.034	0.034	0.035
SYNCOM 2	4	9	38510-38570	196-243	32.5	0.020	0.015	0.014	0.014
SYNCOM 2	5	24	38600-38807	72-189	32.3	0.036	0.039	0.038	0.035
SYNCOM 2	8	15	38816-38918	65-68	31.8	0.020	0.018	0.018	0.018
SYNCOM 2	DoD	42	38816-40104	65-86	29-32.0	0.040	0.037	0.038	0.103
SYNCOM 3	6	9	38699-38750	178-180	0.0	0.015	0.014	0.015	0.013
SYNCOM 3	7	10	38775-38835	174-181	0.0	0.055	0.054	0.055	0.052
SYNCOM 3	11	16	39075-39262	165-172	0.5	0.020	0.012	0.014	0.013
SYNCOM 3	13	4	39385-39491	160-161	1.3	0.015	0.006	0.006	0.010
SYNCOM 3	14	15	39663-40175	146-160	2-3.0	0.045	0.061	0.061	0.110
ATS 1	1	12	40204-40248	210-211	1.2	0.005	0.001	0.001	0.001
ATS 3	1	12	40197-40241	288-312	0.6	0.005	0.002	0.003	0.003
ATS 3	2	13	40267-40337	287-288	0.3	0.006	0.004	0.003	0.002
ATS 3	3	10	40524-40577	313-315	0.3	0.005	0.002	0.003	0.003
INTELSAT 2-F3	1	12	39607-39905	349-352	1.0	0.025	0.028	0.028	0.030
INTELSAT 2-F3	2	10	40406-40642	346-349	1.0	0.020	0.011	0.012	0.015
INTELSAT 2-F4	1	8	40323-40600	179-194	1.0	0.040	0.043	0.046	0.062
EARLY BIRD	1	11	38897-39080	324-332	0.5	0.025	0.025	0.026	0.026
EARLY BIRD	2	6	39096-39219	321-331	0.8	0.010	0.007	0.009	0.009
ATS 5	1	14	40476-40556	253-256	2.6	0.005	0.004	0.006	0.009

Table 2–Resonant geopotential coefficients for 24 hour satellites and rms fits to 21 arcs.

FIELD	OVERALL WEIGHTED RMS IN 21 ARC SOLUTION	2,2		3,1		3,3		4,2		4,4	
		C	S	C	S	C	S	C	S	C	S
STANDARD SAO EARTH II (1970)	2.46	1.558	-0.881	2.128	0.281	0.096	0.199	0.074	0.158	-0.0017	0.0072
SAO COSPAR (1969)	1.38	1.566	-0.896	2.040	0.262	0.096	0.198	0.073	0.148	-0.0028	0.0078
WAGNER (UNCONSTRAINED) 2,2-3,1-3,3 (1970)	0.849	1.568	-0.907	1.687	0.483	0.103	0.204	0.074	0.158	-0.0017	0.0072
WAGNER-SAO COSPAR (CONSTRAINED) (1970)	0.896	1.570	-0.908	2.029	0.289	0.098	0.205	0.075	0.150	-0.0028	0.0078

UNNORMALIZED COEFFICIENTS IN UNITS OF 10^{-6}