TWO-DIMENSIONAL FINITE-ELEMENT TEMPERATURE VARIANCE ANALYSIS

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The finite-element method (FEM), already used in structural analysis, can be extended to thermal analysis. What I have done is to apply this method to thermal analysis and to formulate a variance analysis of the temperature results. This variance analysis determines the sensitivity of predicted temperatures to uncertainties in input variables.

Conceptually, FEM involves the division of the continuum into a finite number of elements–triangles, for example (see Figure 1). The temperature field within each triangle is described in terms of the temperatures at the vertices. By the use of the variational principle, the integral equation describing thermal potential energy is minimized, yielding a system of algebraic equations. This system may be solved by a computer to yield the desired solution matrix of predicted temperatures.

FEM already has been applied to conductive heat transfer problems, but one important area, essential to space applications, which has received little attention is radiation. I have been able to formulate the radiation equations for two-dimensional elements and have written a computer program for generalized two-dimensional heat transfer which includes this radiation capability. Currently, this program handles 100 finite elements; soon it will be expanded to 300 elements.

The availability of a working thermal finite-element computer program led to an analysis of the effect of input data errors upon the computed results. A temperature variance analysis was undertaken. This study resulted in a sensitivity computer program which uses information about the initial thermal parameters and their associated errors to produce an analysis of the overall temperature variance of the system.

Consider, for example, the simple geometry shown in Figure 2. FEM was applied to this symmetric problem, and the computer program produced the temperature predictions shown in Figure 3 for the variable nodes 1 to 5. Then the sensitivity computer program was used, with initial and fixed temperature errors of 5, 10, and finally 15 percent assumed. Figure 3 contains the predicted results. For example, the error in the predicted temperature of node 1 may be up to 13.02 K for the 5 percent case. One may draw the conclusion that in order to have less than a 23.0-K error in all predicted temperatures, it is necessary that each of the initial temperature specifications have an error of 10 percent or less.

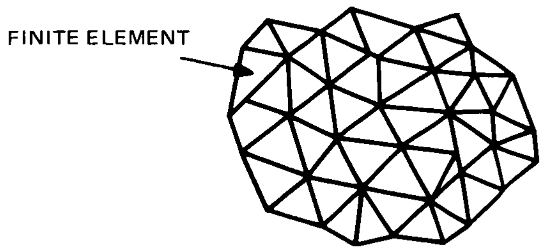
In addition, the sensitivity program produced information showing the relative influence of each thermal parameter upon each predicted temperature. For example, for the sample problem in the 10 percent case, the sensitivity program yielded the values shown in Figure 4.

The following conclusions can be drawn:

(1) In general, all predicted temperatures were influenced most by the values of those temperatures along the fixed boundaries. Errors in thermal simulations, therefore, can be reduced by more accurate specification of these temperatures.

(2) In particular, the temperature error of fixed node 8 greatly affected the temperatures of nodes 1 and 2, affected somewhat node 3, but hardly affected nodes 4 and 5 at all. Likewise, the predicted temperature at node 3 was most affected by the error of node 9.

The sensitivity analysis program can be used by the thermal analyst to determine which input parameters contribute most to error in temperatures predicted by the FEM. It can show him how various levels of accuracy in the input data affect the accuracy of the final answers. It is thus a powerful analytical tool that assists one in deciding exactly how close each parameter must be calculated or measured in order to achieve effective results. It also can be used by the spacecraft designer to pinpoint weak links in a thermal design.



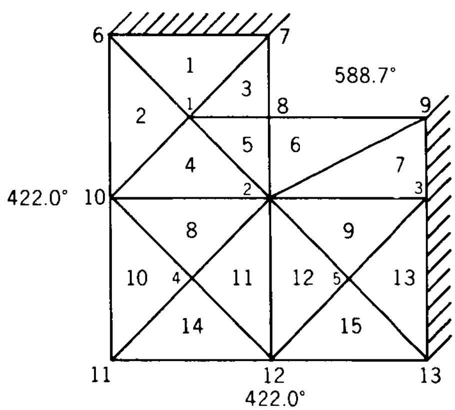
INTEGRAL EQUATION OF THERMAL POTENTIAL ENERGY

$$X = \iint_A \left[\frac{1}{2} k_x \left(\frac{\partial T}{\partial x} \right)^2 + \frac{1}{2} k_y \left(\frac{\partial T}{\partial y} \right)^2 - QT + \rho c \frac{\partial T}{\partial t} T \right] dx\, dy$$

$$+ \int_\Gamma qT \, d\Gamma + \int_\Gamma \frac{1}{2} hT^2 - T_\infty T \, d\Gamma$$

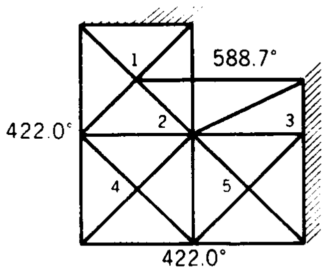
$$+ \int_\Gamma \sigma F_{\Gamma s} \left[\frac{1}{5} \varepsilon T_\Gamma^5 - \alpha T_s^4 T_\Gamma \right] d\Gamma$$

Figure 1–Application of FEM to thermal analysis.



INITIAL TEMPERATURE = 310.9 K
$K_x = K_y = 1.566$ W/m-K $\quad \rho c = 6.706 \times 10^4$ J/m^3-K
$\Delta K = 0.1566$ (10%) $\quad \Delta\rho c = 6.706 \times 10^3$ (10%)

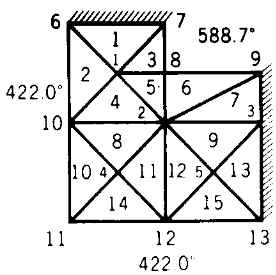
Figure 2–Heat pipe problem–quarter of a symmetric pipe.



RESULTS

i	T_i (t=0.0) °K	T_i (t=1.0) °K	ΔT_i for 5% case K°	ΔT_i for 10% case K°	ΔT_i for 15% case K°
1	310.9	482.8	13.02	22.91	33.41
2	310.9	453.5	9.30	17.50	25.93
3	310.9	471.7	10.27	18.84	27.75
4	310.9	421.4	10.75	20.49	30.43
5	310.9	425.1	9.31	17.43	25.78

Figure 3—Comparison of ΔT_i for initial variance of cases 5, 10, and 15 percent.



RESULTS

s_{ij}	T_1	T_2	T_3	T_4	T_5
K_{y7}	.00	.04	10.90	.00	.07
K_{y9}	.00	.37	2.59	.00	1.52
ρc_9	.00	.08	.48	.00	.20
$T_{(t+\Delta t)1}$	43.65	3.94	.07	.01	.00
$T_{(t+\Delta t)5}$	.05	4.97	3.63	.01	21.75
$T_{(t+\Delta t)8}$	81.06	78.20	1.45	.29	.18
$T_{(t+\Delta t)9}$	.26	27.10	197.84	.04	2.21
$T_{(t+\Delta t)12}$	.12	13.13	2.82	108.08	112.82
$T_{(t)9}$	.35	37.17	1.96	.14	.06
$T_{(t)13}$	.00	.15	22.67	.01	7.92

Figure 4—Typical $[(\partial T/\partial s_{ij})\Delta s_{ij}]^2$ values.