A COMPARISON OF RELIABILITY AND CONVENTIONAL ESTIMATION OF SAFE FATIGUE LIFE AND SAFE INSPECTION INTERVALS

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SUMMARY

Both the conventional and reliability analyses for determining safe fatigue life are predicated on a population having a specified (usually log normal) distribution of life to collapse under a fatigue test load.

Under a random service load spectrum, random occurrences of load larger than the fatigue test load may confront and cause collapse of structures which are weakened, though not yet to the fatigue test load. These collapses are included in reliability but excluded in conventional analysis.

The theory of risk determination by each method is given, and several reasonably typical examples have been worked out, in which it transpires that if one excludes collapse through exceedance of the uncracked strength, the reliability and conventional analyses gave virtually identical probabilities of failure or survival.

INTRODUCTION

The conventional approach to safe-life estimation envisages a fatigue test which imposes on at least one full-scale structure the equivalent fatigue damaging effect of service loading, according to some regular pattern which restricts, however, the largest load, regularly applied, to some fraction of the virgin strength. Life to collapse is regarded as a statistical variable, of whose population mean the test failure is treated as an estimator. Variability is estimated from other representative experiments in which each member's strength falls to a single lower value (in different lifetimes), which is accounted failure, and the probability density function of life to failure is usually assumed log normal.

Determination of the safe life as a function of desired or acceptable probability of failure requires merely the estimation of the desired percentile of the population, that is, the desired percentile of the distribution of fatigue lives, measured to the point at which each member's strength has fallen to the largest applied load in the test sequence.

Reliability theory, applied to this problem attributes the same strength properties to the population as before, including the decay of strength as fatigue crack growth occurs, but does not assume that collapse occurs when each member's strength has fallen to a common value. Collapse occurs rather when a member of the population meets a load larger than its current strength, and this event would correspond to a conventionally assessed life for that member if the service spectrum were modified or truncated so that all load peaks larger than the fatigue test load were reduced to that value.

The purpose of this study was to present the theory of risk determination for each method and to ascertain by the working of several reasonably typical examples whether the conventional method significantly underestimated the failure risk through ignoring service loads higher than the fatigue test load.

SYMBOLS

b	ratio of maximum fatigue test load to virgin strength or strength at critical crack length
g	ratio of crack propagation time (from detectable to critical size) to total life H
H	population life in hours; a log normal random variable
$\tilde{H}$	population geometric mean life in hours
l	crack length
l_{cr}	"critical" crack length at which strength U has fallen to bU_o
l_d	crack length detectable with certainty
$\tilde{m}(V)$	frequency of occurrence, per hour, of applied load $>V$
n	number of load cycles applied
$p(U)$	probability density function of strength for population
$p(V)$	probability density function for applied load V for some arbitrary time interval

$P(t)$	probability of collapse before time t
$P(n)$	probability of collapse before nth applied load
P_c	probability of collapse in arbitrary time interval
δP_c	probability of collapse in arbitrary time interval of small element of population characterised by its value of H
$r(t)$	risk or risk rate or risk of failure at time t of survivors at time t
$r(n)$	risk or rate of failure at the nth applied load of survivors of (n-1)th load
$R(t)$	reliability at time t or probability of survival to time t
$R(n)$	reliability at nth applied load or probability of survival from first to (n-1)th load
t	time, hours
T_b	safe inspection period for probability of failure p = p percent of gH
U	strength
U_o	virgin strength
V	applied load
σ	standard deviation of $\log H$
ϕ	strength decay function of crack size
ψ	crack propagation (time function)

STATISTICAL MODEL AND SAFE-LIFE ANALYSES

The statistical model used herein is the one used in references 1 and 2, as shown in figure 1 in both normal and logarithmic coordinates, and has the following features:

(1) The population life H is log normally distributed with geometric mean $\tilde{H}$ and variance σ^2, H being the hours in which the strength U is reduced from U_o to bU_o which corresponds to the largest load in the test spectrum.

(2) Crack propagation in each member is scaled to the member's potential life to failure H under the specified test history and follows the expression

$$\frac{l}{l_{cr}} = \psi\left(\frac{t}{H}\right)$$

(3) Strength is related to crack size; thus,

$$\frac{U}{U_o} = \phi\left(\frac{l}{l_{cr}}\right)$$

and the condition (1) gives $\phi(1) = b$.

(4) Whereas crack propagation is governed by condition (2), failure is governed by the frequency of occurrence $\tilde{m}(V)$ per hour, of service loads exceeding V, or in nondimensional terms, the frequency $\tilde{m}(V/U_o)$ of service loads greater than V/U_o.

In conventional analysis, a safe life for a probability of failure p is merely the p percentile of the variable H. Insofar as H is the time at which U falls to U_o, it is independent of the shape of the crack propagation curve and is only dependent on the time H at which $l = l_{cr}$.

The calculation of failure by the reliability approach requires the following definitions (refs. 4 and 5):

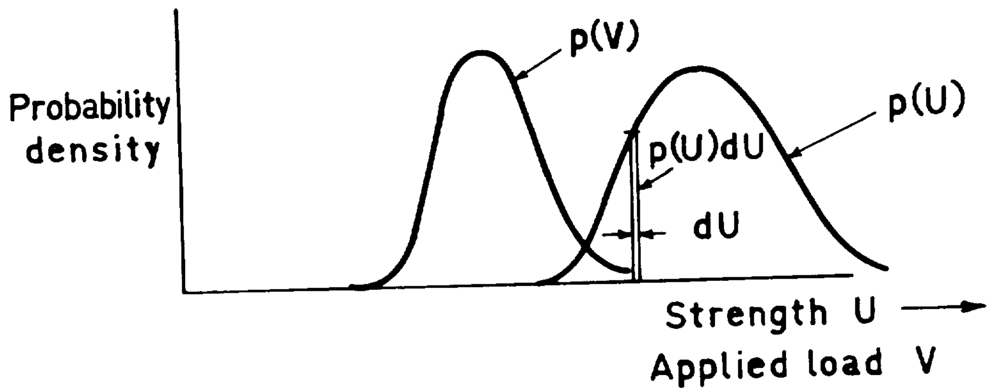
P(t)	probability of fracture before time t
R(t)	reliability at time t or probability of survival to time t
r(t)	risk or rate of failure at time t of survivors to time t, $\frac{1}{R(t)}\frac{dP(t)}{dt}$

and the expression

$$R(t) = e^{-\int_0^t r(t)\,dt} \tag{1}$$

Or, alternatively, the probability of failure before a given time, the reliability and the risk (hazard rate or force of mortality) may be expressed as a function of number of cycles n, as P(n), R(n), and r(n). In this case r(n) dn is the probability of failure in dn cycles of members surviving at n cycles, so that (with dn = 1), r(n) is the probability of failure per cycle of members which survive to the nth cycle.

If the probability density functions of strength and of load occurring in some arbitrary time are p(U) and p(V), respectively, as shown in the following diagram,



then the probability of collapse in this arbitrary time is the probability that a load V falls on a structure of strength U less than V. For a load lying between V and V + dV, occurring with probability p(V) dV, its contribution to the probability of collapse is

$$p(V)\, dV \int_{U=0}^{U=V} p(U)\, dU \tag{2}$$

and the total probability of collapse is

$$P_c = \int_{V=0}^{V=\infty} p(V) \int_{U=0}^{U=V} p(U)\, dU\, dV \tag{3}$$

Or, alternately, if there is considered an element of the population of structures lying between U and U + dU, the probability of a structure having a strength in this interval being p(U) dU, its contribution to the probability of collapse is

$$p(U)\, dU \int_{V=U}^{V=\infty} p(V)\, dV \tag{4}$$

so that the total probability of collapse is also

$$P_c = \int_{U=0}^{U=\infty} p(U) \int_{V=U}^{V=\infty} p(V)\, dV\, dU \tag{5}$$

In the example of concern, in which strength U is distributed as a function of H and also decreases with time, the calculation is most readily made by taking small elements p(H) dH of the population characterised by their values of H and using equation (4) to find the contribution to the probability of failure by each element; that is,

$$\delta P_c = p(H)\ dH \int_{V=U}^{V=\infty} p(V)\ dV$$

$$= p(H)\ dH \left[Pr(V > U)\right] \tag{6}$$

It will be noted that U is a function of time $U(t) = U_0\phi\left\{\psi(t/H)\right\}$ so that

$$\delta P_c = p(H)\ dH \left[Pr(V > U(t))\right]$$

$$= p(H)\ dH \left\{1 - e^{-\int_0^t r(t)dt}\right\}$$

from equation (1), r(t) being the risk function for this element,

$$\delta P_c = p(H)\ dH \left\{1 - e^{-\int_0^t \tilde{m}(V > U(t))dt}\right\} \tag{7}$$

where $\tilde{m}(V > U(t))$ is the frequency per hour with which the applied load exceeds the element's strength U(t).

The total probability of collapse is

$$P_c = \int_{H=0}^{H=\infty} p(H) \left\{1 - e^{-\int_0^t m(V > U(t))dt}\right\} dH \tag{8}$$

which is identical, allowing for a difference in notation, with the expression

$$P(t) = \int_{F(U)=0}^{F(U)=1} \left\{1 - e^{-\int_0^t n(U)dt}\right\} dF(U)$$

of reference 2 (p. 29).

INSPECTION INTERVAL ANALYSIS

Safety may be achieved in an inspectable structure if the critical crack length l_{cr}, at which strength falls to a selected unsafe value, is larger than the crack length detectable with certainty l_d. The time remaining in which a crack propagates from l_d to l_{cr} (and strength to $U = bU_0$) is some fraction of the life H, say gH, and with this model g is a constant for all members of the population. Thus gH is log normally distributed with median $g\tilde{H}$ and variance σ^2.

In conventional analysis, the critical length l_{cr} is the same for all members of the population, and the unsafe value of strength is equated to bU_0, the highest load in the fatigue test programme. A safe operating period after inspection T_b for a probability of failure p in the interval is the p percentile of the variable crack propagation time gH, since it can readily be seen that only p percent of cracks can propagate from l_d to l_{cr} in a time less than T_b. This result assumes that all structures are cracked to just below l_d at the inspection date, and it is seen to be independent of the shape of the crack propagation curve for cracks smaller than l_d.

In reliability analysis, structures may be considered to be cracked to just below l_d at the beginning of the propagation time but to reach a failure state governed by load exceeding strength. Where the safe lives, as calculated by reliability and conventional methods, coincide it is concluded that this will imply a coincidence of the values of safe inspection intervals.

APPLICATION OF THE THEORY TO TYPICAL EXAMPLES

Example A(1) represents a military aircraft situation where the structures are subjected to the manoeuvre load spectrum (curve A of fig. 2) in which limit load is exceeded once per 100 hours, crack size l/l_{cr} is a power function of t/H, the decay of strength with crack size conforms to the laws of fracture mechanics, and the standard deviation σ is 0.167.

Example A(2) represents the same situation as example A(1) except that the standard deviation σ is $0.167\sqrt{2}$.

Example B(1) represents a civil aircraft situation where the structures are subjected to the gust spectrum (curve B of fig. 2) in which three-fourths of limit load is exceeded once in 5000 hours, crack propagation follows figure 28 of reference 3, the decay of strength is a linear function of crack length, and the standard deviation is 0.17.

Example B(2) represents the same situation as example B(1) except that (perhaps unrealistically) crack growth is assumed linear from zero time up to failure.

Constants used in the various calculations are listed in table I, and the results of the calculations are shown in figure 3 where probability of failure or survival is plotted against life in hours. Calculations for the conventional analysis have been made with the same computer programme by truncating the load spectra at bU_O.

DISCUSSION OF RESULTS OF THE ANALYSIS

Examination of figure 3 shows that for the Civil example B(1) both methods of analysis gave virtually identical results within the computed range from $p = 0.001$ to $p = 0.999$. In the computations the distribution of H was divided into its 0.1 percentile. If results are desired for $p < 0.001$ these can readily be obtained by computing with smaller elements of the distribution of H. For example A(1), both methods gave virtually identical probabilities of failure for lifetimes longer than 3000 hours, but for shorter lifetimes the reliability method gave higher probabilities than the conventional method. It is appreciated that the reliability method of analysis included, whereas the conventional method excluded, the risk of failure from loads exceeding the virgin strength (whether of uncracked structure or of cracked but yet unweakened structure).

The probability of such overload failures can readily be derived from the frequency of exceedance of U_O for example A, namely once per million hours. This probability of overload failures is plotted as a dashed line in figure 3; the reliability calculation closely approximates this curve at low probabilities of failure.

The result for example A(2) is similar to that for example A(1), except that, because of the larger scatter, the reliability calculation assessed a given probability to have been reached in a slightly shorter lifetime; for example, a probability of failure of 0.002 was reached in 1500 hours by reliability analysis and in 1750 hours by conventional analysis with the corresponding scatter factors being $5\frac{1}{3}$ and $4\frac{4}{7}$, respectively. Again, at a probability of failure of 0.001, the major contribution was overload failure through loads greater than the virgin strength.

Reliability analysis provides a rigorous method for validating the conventional methods of safe life and inspection interval analyses which are based upon a seemingly arbitrary choice of the value of unsafe strength, this choice having been made by choosing what is to be the highest load in the fatigue test programme on the representative structure to estimate mean life. The conventional analysis is vastly less time consuming than the reliability analysis, since it involves a simple slide-rule calculation rather than a complex digital computer programme run.

Examples A(1), A(2), and B(1) were constructed to represent closely conditions existing in military and civil aircraft situations. For the most part the reliability analysis validates the simpler conventional analysis. For the military type of spectrum and at short lives, the probability of failure is dominated by loads exceeding the uncracked strength; when these are added to the conventional analysis, the result agrees closely with the reliability methods.

Example B(2) represents an artificial extreme example of a structure assumed to have linearly decaying strength from zero time up to failure. Nevertheless, here again, at probabilities of failure less than 20 percent, the corresponding lifetimes were virtually identical with those for a more usual strength-decay curve, or, indeed, for the step-function strength decay curve which is implicit in the conventional analysis.

The examples that have been discussed have not considered the case of a long period of detectable crack propagation during which the strength does not decay below virgin strength. Here inspection will not prevent failures from exceedance of the virgin strength, but will weed out cracked structures before they become weakened.

CONCLUSIONS

For a range of conditions which are typical of military and civil aircraft structures and load histories, reliability analysis validates the much simpler conventional methods of safe life and inspection interval analysis.

The reliability method, ipso facto, includes the probability of failure through loads exceeding the virgin strength – a factor which is inevitable by any fatigue analysis, inspection schedule, or safe-life determination.

Where there is a long detectable crack propagation time without diminution of the structural strength, inspection will weed out cracked structures before they become weakened but will not prevent failures from loads exceeding the virgin strength.

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TABLE I

CONSTANTS USED IN EXAMPLES A(1) AND A(2) (MILITARY) AND EXAMPLES B(1) AND B(2) (CIVIL) SAFE-LIFE ANALYSIS

Constant in calculation	Example A (military aircraft)	Example B (civil aircraft)
$\tilde{H}$ = Median population life	8000 hours	25000 hours
σ = Standard deviation	0.167 for A(1) $0.167\sqrt{2}$ for A(2)	0.17
bU_o = Highest test load	$0.67U_o$	$0.5U_o$
$l/l_{cr} = \psi(h/H)$	$(t/H)^{9.0}$	B(1): 0 for $t/H<0.6$ $t/H - 0.6$ for $0.6<t/H<0.97$ $-20+21t/H$ for $t/H>0.97$ B(2): $l/l_{cr} = t/H$
$U/U_o = \phi\left(l/l_{cr}\right)$	1 for $l/l_{cr}<0.44$ $0.67\sqrt{l_{cr}/l}$ for $l/l_{cr}>0.44$	$1 - l/2l_{cr}$
$U/U_o = \phi\left\{\psi(t/H)\right\}$	1 for $t/H<0.91$ $0.67(H/t)^{4.5}$ for $t/H>0.91$	B(1): 0 for $t/H<0.6$ $1.3 - 0.5t/H$ for $0.6<t/H<0.97$ $11 - 10.5t/H$ for $t/H>0.97$ B(2): $1 - 0.5t/H$
$\tilde{m}\left(V/U_o\right)$	$10^6 - 12V/U_o$	$10^{4.3} - 15V/U_o$

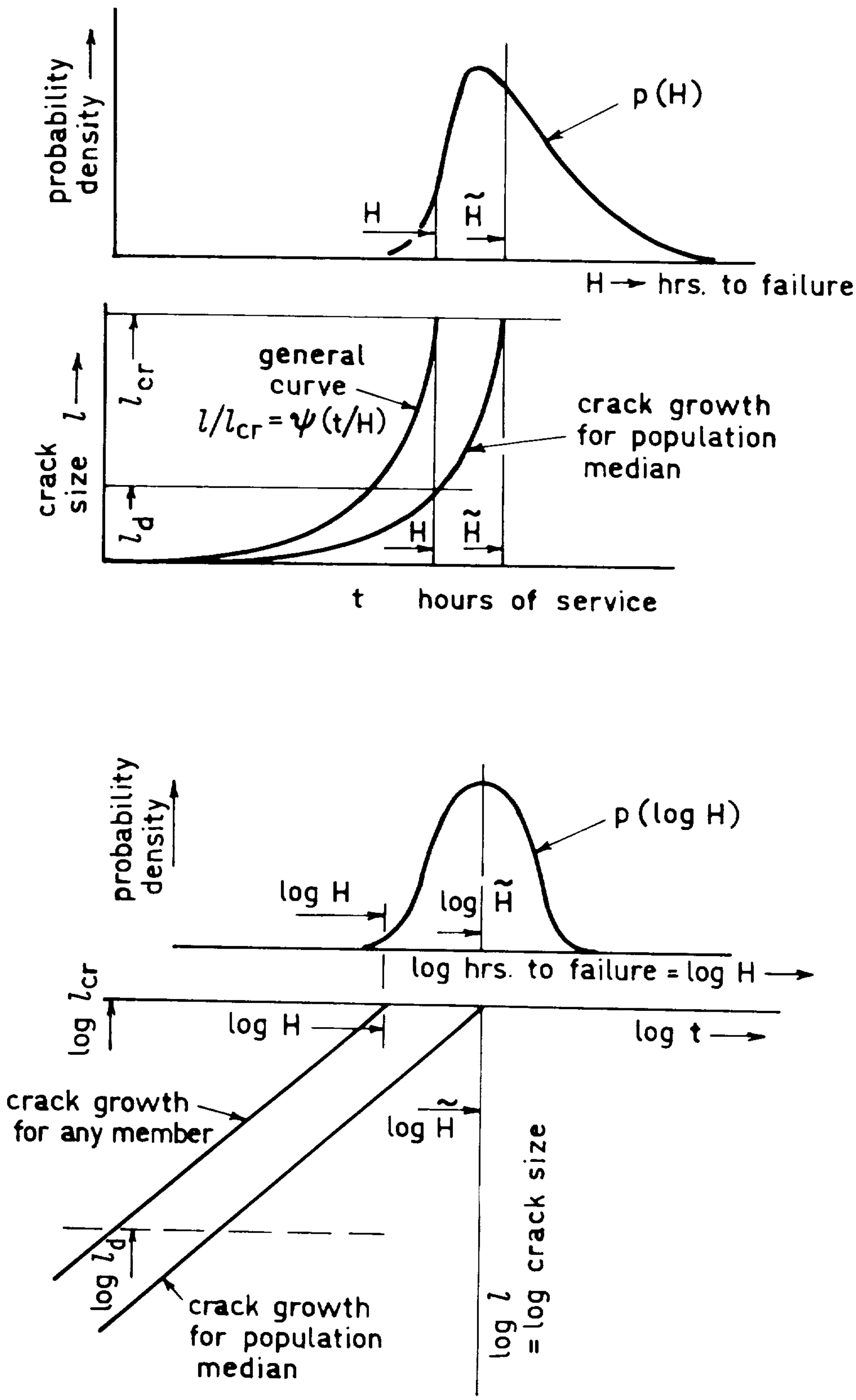

Figure 1.- Crack growth and failure distribution model.

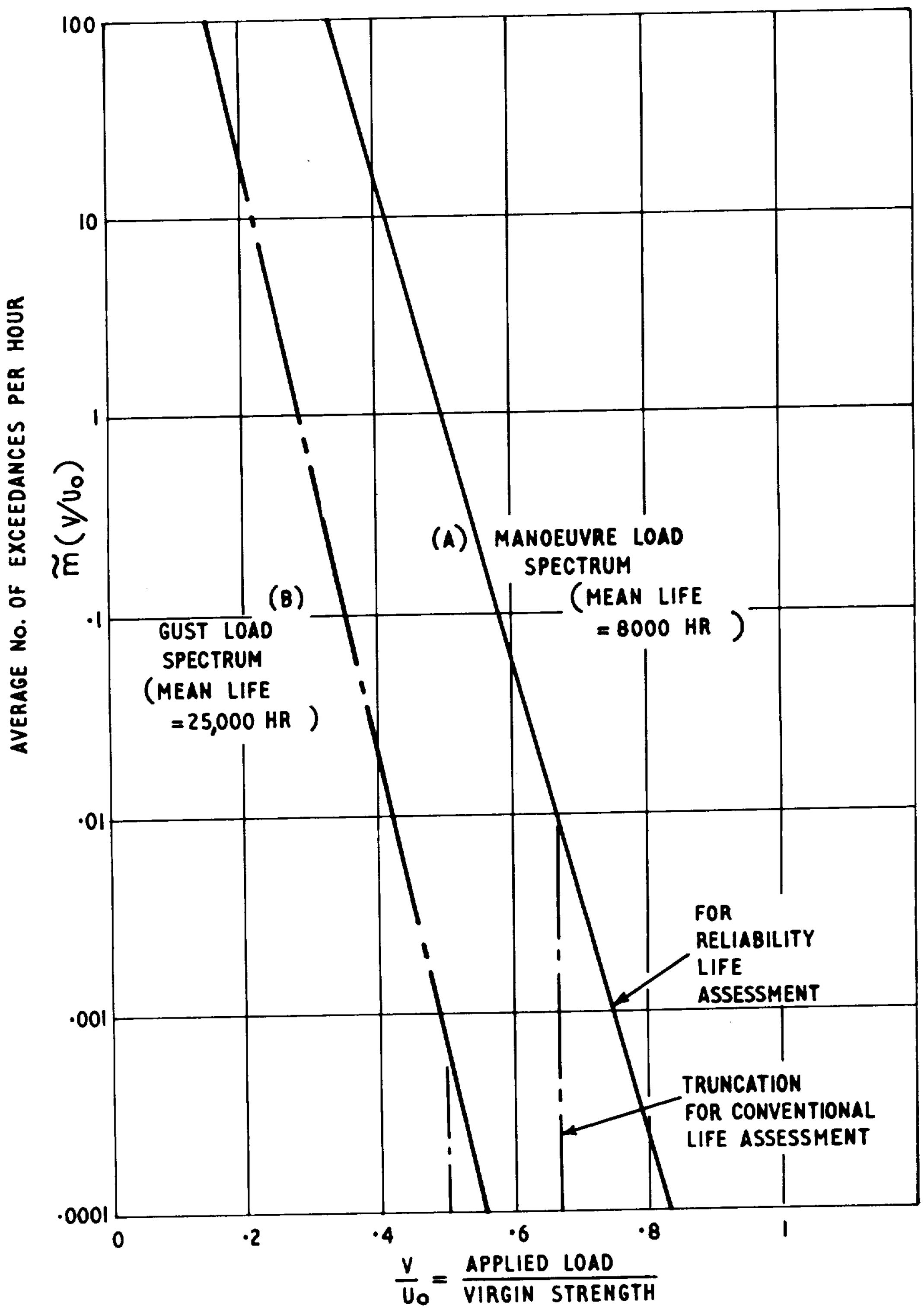

Figure 2.- Typical load exceedance curves.

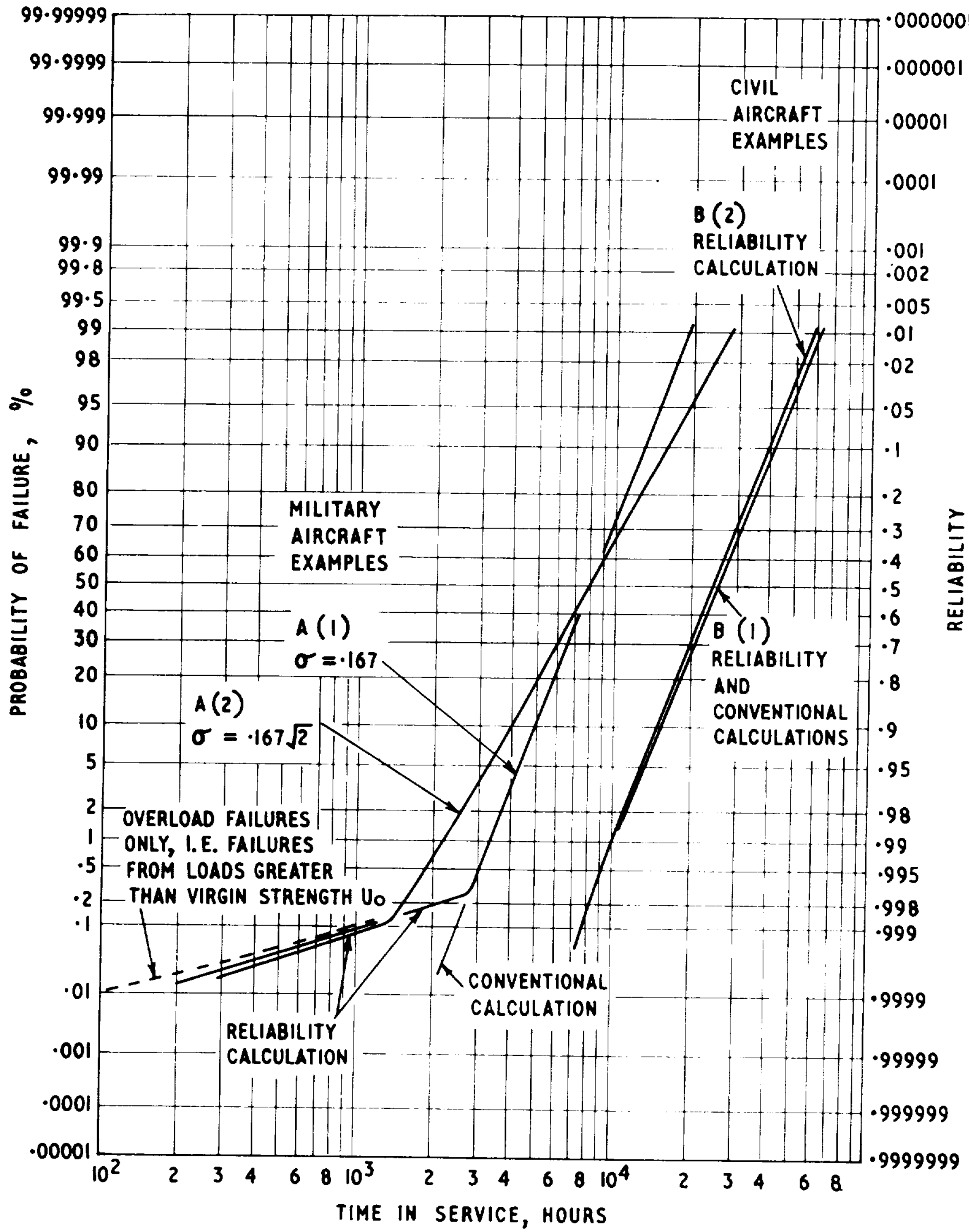

Figure 3.- Probabilities of failure and survival calculated by reliability and conventional analyses.

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