NASTRAN BUCKLING ANALYSIS OF A THIN-WALLED CYLINDER

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SUMMARY

The computer run time required for a buckling analysis using NASTRAN is considerably greater than for a linear static analysis of the same structure. For this reason it is very important to obtain an optimal model size for the structure being analyzed, such that the model is neither so large as to increase running time unnecessarily, nor so small as to hinder accuracy. This paper studies the problem of critical load accuracy versus model complexity for a thin walled cylinder, using both Level 12 and Level 15 NASTRAN releases.

INTRODUCTION

Buckling is generally the failure mode for thin-walled shells carrying axially compressive loads and NASTRAN has the capability of solving for the critical load of such problems. The problem which initially stimulated interest in this study was the buckling analysis of a thin-walled cone to be used on a NASA spacecraft. NASTRAN analysis yielded a wide range of answers by varying the finite element model used. No analytical solutions could be found for buckling of a cone which could be used to verify the NASTRAN results. Therefore the buckling analysis of a thin-walled cylinder of approximately the same dimensions as the cone was performed to determine the model complexity needed to obtain good agreement between NASTRAN and analytical solutions. The cylinder analyzed is 78.12 cm in diameter by 55.88 cm long with a wall thickness of 0.102 cm.

There have been complaints about the crudeness of the NASTRAN plate element for buckling analysis. (See, for example, ref. 1.) The majority of the runs made for this study used NASTRAN Level 12.1.2 on the IBM 360/95 computer at the Goddard Space Flight Center. These results are compared to Level 15.1.0 which includes a greatly improved plate element for buckling. This improvement was obtained by including an improved algorithm for the bending effects, whereas Level 12 analysis relied primarily on membrane contributions. A procedure was formulated to measure the effect of this improved plate element, as well as verify the advertised decrease in running time for Level 15.

SYMBOLS

K_{aa}	stiffness matrix
K^d_{aa}	differential stiffness matrix
λ	eigenvalue
u	eigenvector
σ_{cr}	critical stress
E	Young's modulus of elasticity
μ	Poisson's ratio
t	cylinder wall thickness
m	number of half sine waves
L	cylinder length
R	cylinder radius
P_{cr}	critical load
F	applied axial compressive load
θ	angle subtended by one element

NASTRAN MODEL AND THEORY

The bulk data required for a NASTRAN buckling analysis (Rigid Format 5) is identical to that required for a static analysis, with the addition of an EIGB card similar to the EIGR card used for normal mode analysis. The steps NASTRAN uses for solving the buckling problem as presented in ref. 2 are:

1. Solve the linear statics problem ignoring differential stiffness and calculate the internal element forces.

2. Use the element forces to obtain the differential stiffness matrix.

3. Find the eigenvalues and eigenvectors from the matrix equation:

$$\left[K_{aa} + \lambda K_{aa}^{d}\right] \quad \{u\} \quad = 0 \tag{1}$$

where the eigenvalue (λ) is the factor by which the arbitrary applied load is multiplied to obtain the critical load and the eigenvector (u) represents the buckling mode shape.

Plate elements were used to model the cylinder. The AXIS data generating program was used to generate the NASTRAN data deck. (See ref. 4.) This program generates the GRID, CQUAD2 and PQUAD2 cards for a shell of revolution. The grid points are sequenced along circumferential rows to obtain the minimum band width and no active columns.

The critical load of the cylinder was obtained for various model sizes. All of these models were only portions of the complete cylinder, thus yielding only axisymmetric buckling modes. The effect of the rest of the cylinder is simulated by constraining to zero certain degrees of freedom along the edge of the model as described in ref. 3. These degrees of freedom are tangential translation, longitudinal rotation and radial rotation. (Degrees of freedom 2, 4, and 6 in Figure 1.)

ANALYTICAL SOLUTION

The analytical critical stress of a thin walled cylindrical shell under an axial compressive load is given in ref. 5 as

$$\sigma_{cr} = \frac{E}{12(1 - \mu^2)}\left[\frac{\pi t m}{L}\right]^2 + E\left[\frac{L}{\pi R m}\right]^2 \tag{2}$$

The number of half sine waves the cylinder buckles in is given by m in the above equation. Only the number of half sine waves which produces a minimum σ_{cr} is of interest, the others being fictitious numbers the cylinder will never see. This minimum occurs when

$$m = \frac{L}{\pi} \sqrt[4]{\frac{12(1 - \mu^2)}{R^2 t^2}} \tag{3}$$

Assuming the cylinder being analyzed is made of aluminum yields:

$$m \cong 16$$

$$\sigma_{cr} = 1.0859 \times 10^8 \text{ newton/meter}^2$$

$$P_{cr} = 2.7076 \times 10^5 \text{ newtons}$$

This critical load (P_{CR}) was applied as a static load to the finite element model so that the eigenvalues would approach unity as the accuracy increased. The value of the force (F) applied to each grid point (See Figure 1) is calculated from

$$F = \frac{1}{2}\left(\frac{\theta}{360}\right) P_{cr} \tag{4}$$

MODEL VARIABLES AND RESULTS

The model parameters which were varied are: the number of longitudinal elements, the number of circumferential elements, the length of the cylinder, the end fixity of the cylinder and the type of finite elements used. The eigenvalue (λ) for each model was obtained by using NASTRAN.

Longitudinal Elements

The accuracy of the solution is a function of the number of longitudinal elements, because the more elements per half sine wave the closer the actual buckling mode can be approximated. The purpose here is to perform a systematic variation of model complexity to obtain a curve of accuracy as a function of model complexity. Models containing 20, 40, 60, 80 and 120 longitudinal elements were analyzed. All these models were one element wide. Figure 2 plots the eigenvalue of each buckling mode for the various models as a function of the number of half sine waves. The analytical solution from equation (2) is also plotted. As was stated before, the only points on these curves which have real meaning are the minimum points. These minima are plotted to obtain the curve of primary interest (Figure 3). The number of elements per half sine wave required to obtain a given accuracy can be read directly from the graph. Also, the increased accuracy obtained from Level 15 can be seen.

Circumferential Elements

A model more than one finite element wide, as has been used in ref. 1, does not necessarily increase the accuracy of the critical load obtained, because we are dealing with an axisymetric phenomenon. To study this area various models of one, two and four circumferential elements were investigated. Table 1 clearly shows that the accuracy is <u>not</u> a function of the number of circumferential elements, but only a function of the number of longitudinal elements.

Length

Equations (1) and (2) show that the critical stress, and therefore the critical load, is not a function of length. Table 2 shows how this parameter was varied in the NASTRAN model. The length of the 80-element, single-row model was cut in half and then in half again, keeping the same element size. The theoretical solution was verified — the critical load is independent of length. This allows one to decrease the degrees of freedom of the axisymmetric model and still obtain the same accuracy as a full length model.

End Fixity

Experimental solutions show that the critical load is independent of end fixity for long cylinders. (See ref. 6.) Ideally, failure of a pinned-pinned cylinder will occur simultaneously at each half sine wave along the length, but for a fixed-fixed cylinder the failure will occur near the center where the amplitude of the half sine wave is the same as that for a pinned-pinned condition as shown in Figure 4.

Table 3 shows the results of varying the end fixity of the NASTRAN model. The variation in critical load is less than one percent. This fact relieves the analyst of the burden of accurately approximating the actual end conditions of a thin-walled cylinder experiencing buckling.

Type of Finite Elements

The use of triangular plates instead of quadrilateral plates will double the number of finite elements while keeping the degrees of freedom and the bandwidth, and therefore the decomposition time, constant. In the problem being studied this was done by dividing diagonally the quadrilateral plates, as shown in Figure 5. This substitution increased the accuracy of the solution as shown in Table 4. In the case being studied quadrilateral plates were more convenient, however, because the AXIS program generates quadrilateral plate elements.

LEVELS OF NASTRAN

As was shown earlier in Figure 1, the improved plate element of Level 15 offers a great improvement in the accuracy of the critical loads obtained using Rigid Format 5. This allows the user to decrease the model size and still obtain the same accuracy as Level 12. Another great improvement in Level 15 is the decrease in running time for buckling problems. Running time for a finite-element model with a given bandwidth and number of degrees of freedom is reduced by a factor between 3 and 4, as shown in Table 5.

CONCLUDING REMARKS

Many things can be done to reduce the computer time required to do a NASTRAN buckling analysis of a thin-walled cylinder. The optimal number of elements per half sine wave can be obtained from the graph of accuracy versus number of elements. For axisymmetric buckling no increase in accuracy is obtained by increasing the number of rows of elements modeled. The critical load was found to be independent of length, allowing a reduction in model size. The use of triangular plate elements is recommended for improved accuracy, unless a data-generating program that produces quadrilateral plate elements is being used. Finally, Level 15 offers an increase in accuracy and a decrease in running time over the previous Level 12 NASTRAN.

Although this paper dealt with a thin-walled cylinder, which is easily solved by analytical methods, the conclusions reached are applicable to other problems which are more difficult to solve analytically. These problems include stiffened cylinders, thin-walled cones and cylinders with variable wall thickness.

REFERENCES

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Table 1: λ vs Model Complexity

$$\lambda = \frac{\text{NASTRAN Critical Load}}{\text{Theoretical Critical Load}}$$

No. of Axial Elements	No. of Circumferential Elements		
	1	2	4
20	2.013	2.013	2.013
40	1.252	1.252	1.252
60	1.112	1.113	—
80	1.062	—	—

Table 2: Variation in Length

Length, (cm)	No. of Elements	P_{cr}, (kilonewton)
55.88	80	287.5
27.94	40	287.5
13.97	20	287.5

Table 3: Variation in End Fixity (60 Element Model)

Lower End Fixity	Upper End Fixity	P_{cr}, kilonewton
Free	Free	303.0
Pinned	Free	301.3
Pinned	Pinned	301.1
Fixed	Free	303.1
Fixed	Pinned	301.4
Fixed	Fixed	303.2

Table 4: λ vs Type of Elements

$$\lambda = \frac{\text{NASTRAN Critical Load}}{\text{Theoretical Critical Load}}$$

No. of Elements	Type of Elements	
	Quad.	Tria.
40	1.102	1.037
60	1.048	1.028
80	1.028	1.012

Table 5: CPU Time vs Level (IBM 360-95)

No. of Elements	CPU Time, sec		Ratio
	Level 12	Level 15	
20	132	35	3.8
40	250	70	3.6
60	430	124	3.5
80	556	178	3.1

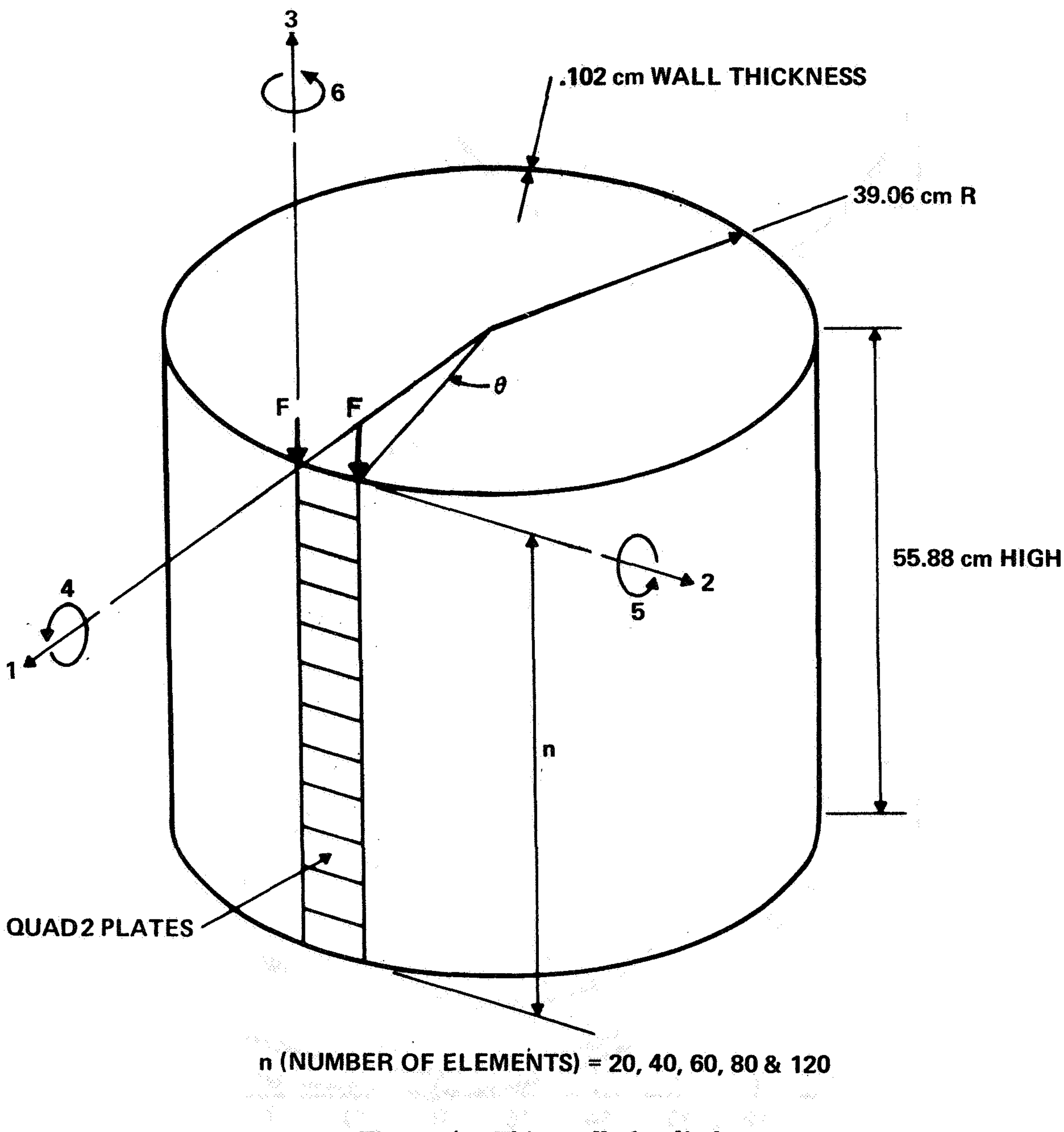

Figure 1.- Thin-walled cylinder.

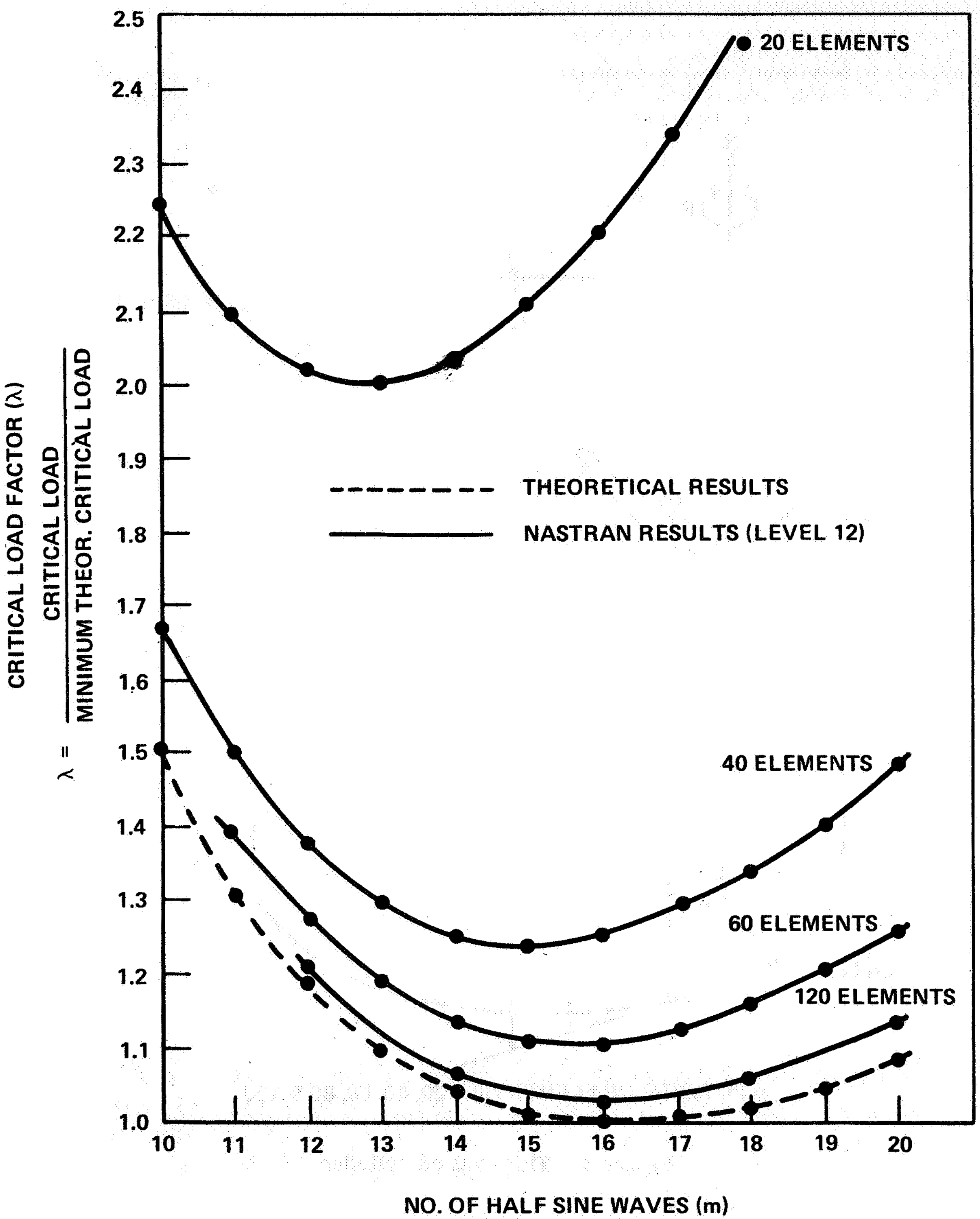

Figure 2.- Variation of λ with number of half-sine waves.

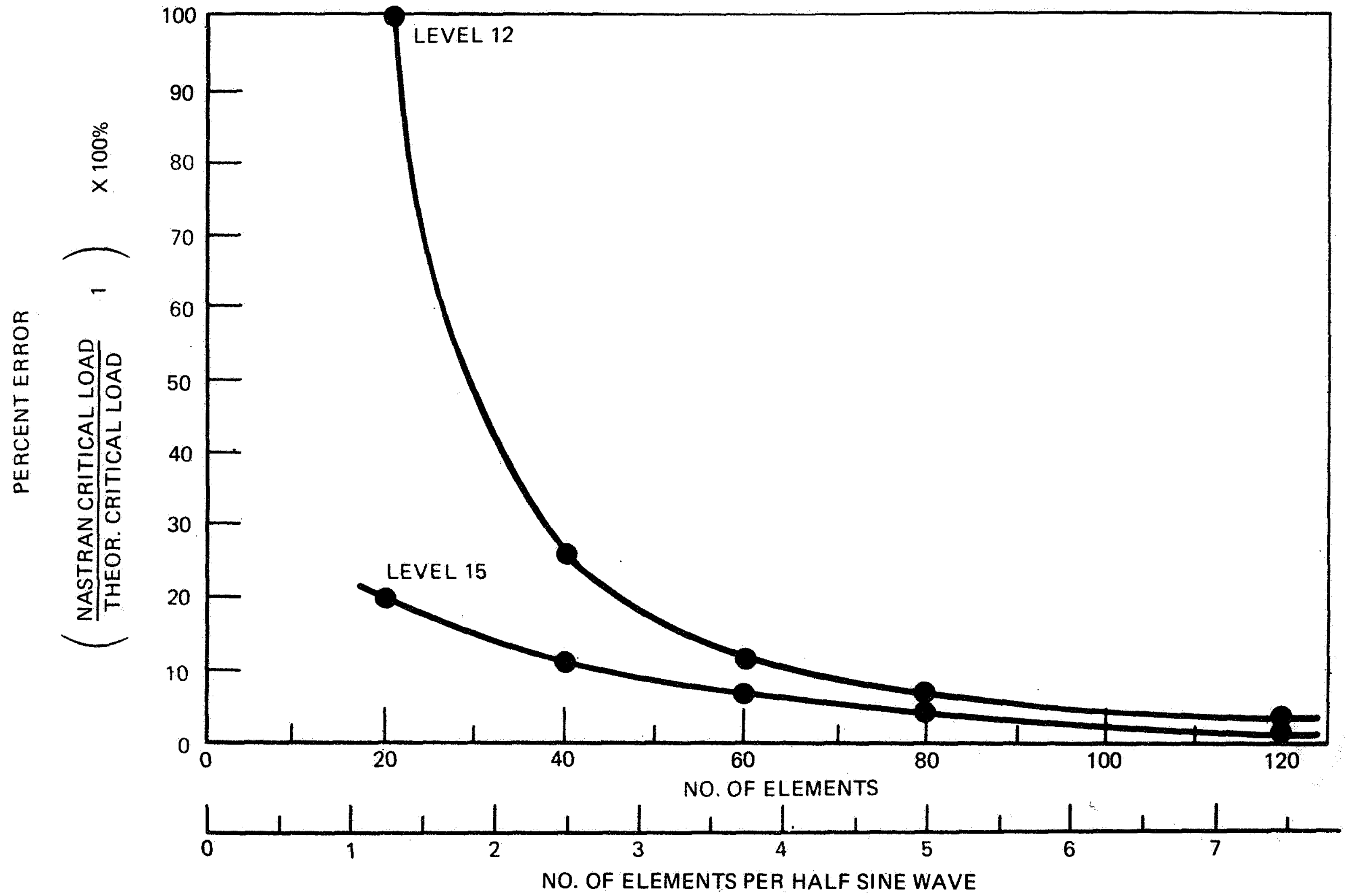

Figure 3.- Comparison of accuracy and model complexity.

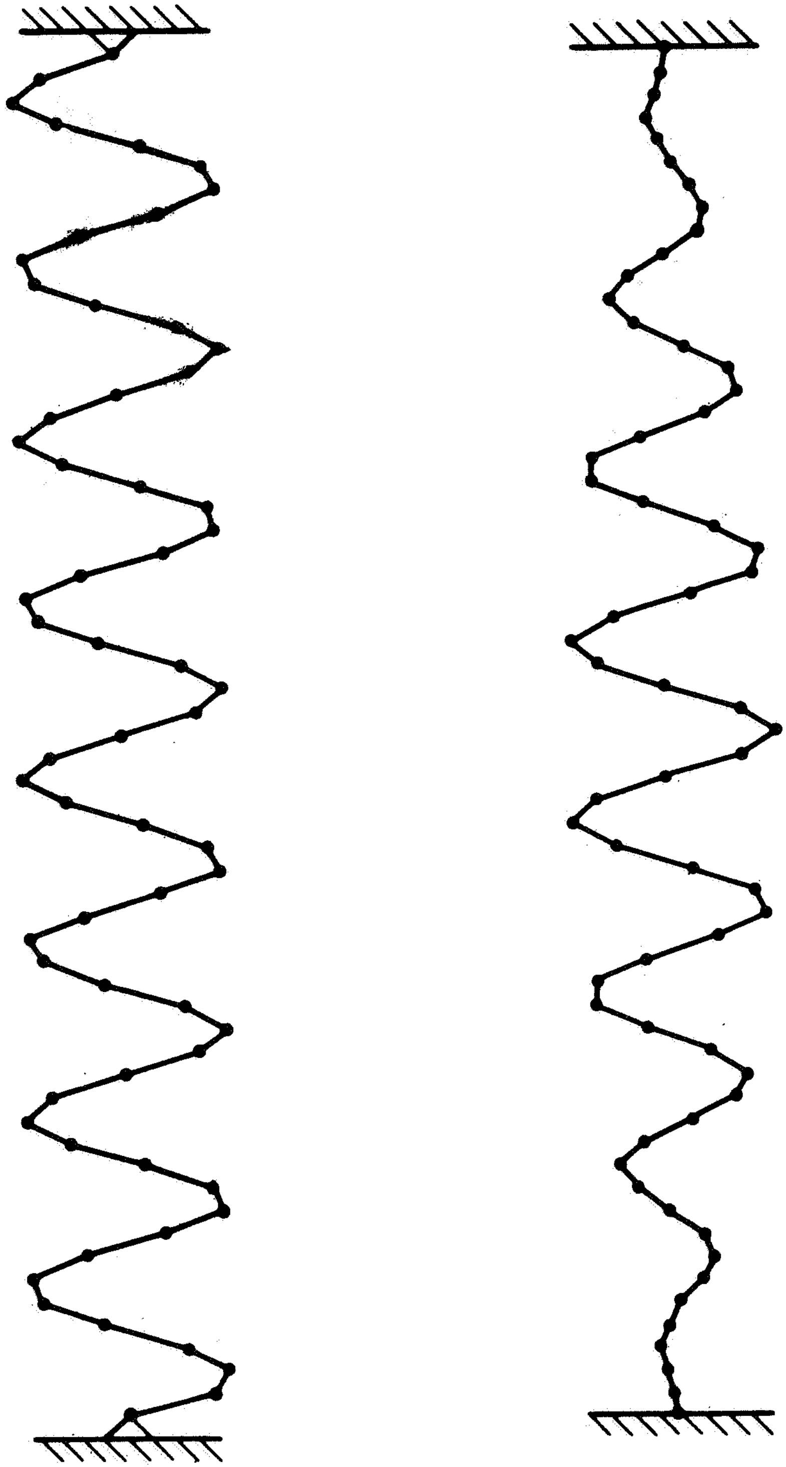

Figure 4.- Buckling mode shapes (60 element cylinder).

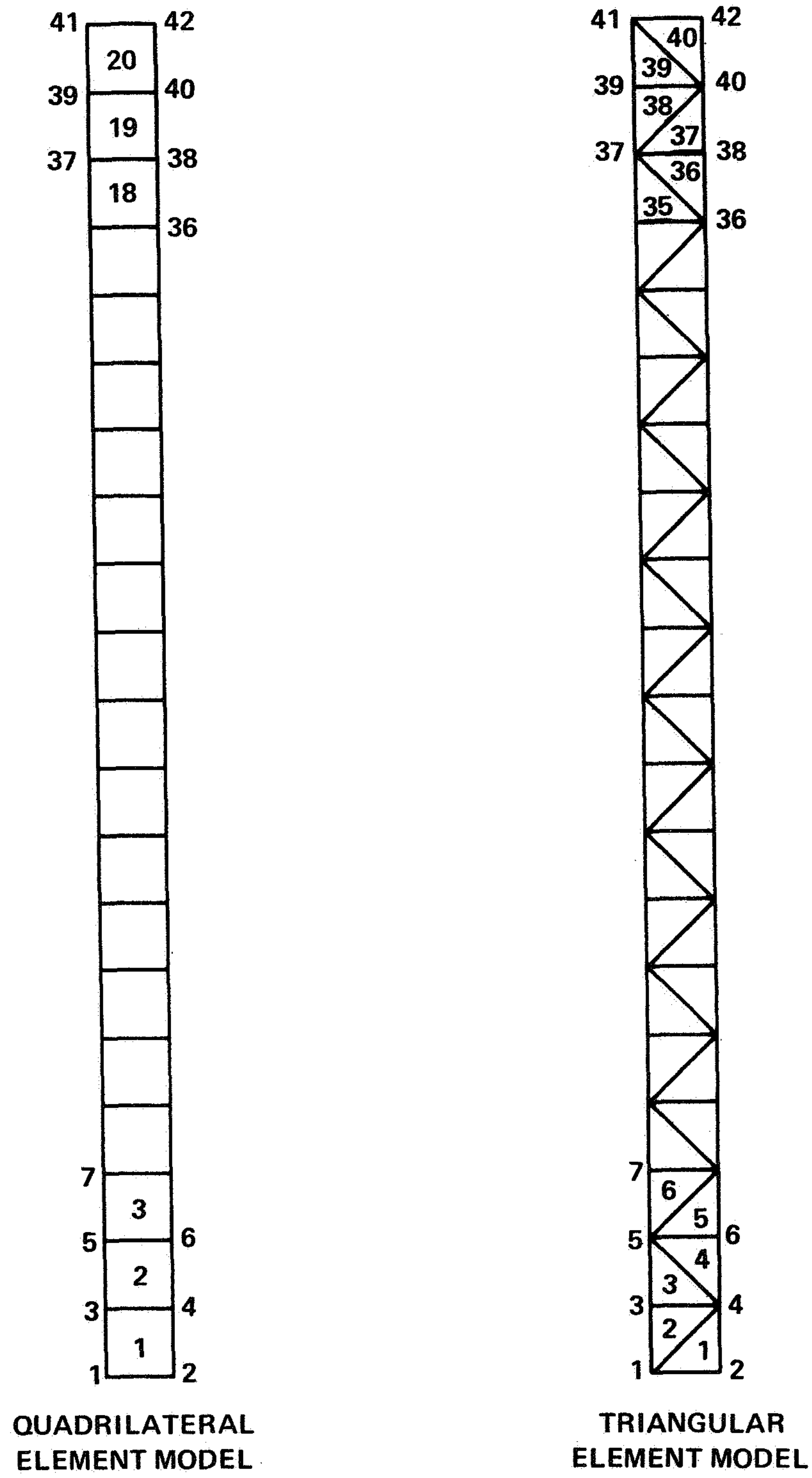

Figure 5.- Finite-element model.