

APPLICATIONS OF NASTRAN TO NUCLEAR PROBLEMS

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SUMMARY

The usefulness of NASTRAN for nuclear applications has been investigated. For this purpose the extent to which suitable solutions may be obtained for one physics problem and two engineering type problems is traced. NASTRAN appears to be a practical tool to solve one-group steady-state neutron diffusion equations. Transient diffusion analysis may be performed after new levels that allow time-dependent temperature calculations are developed.

NASTRAN Piecewise Linear Analysis may be applied to solve those plasticity problems for which a smooth stress-strain curve can be used to describe the nonlinear material behavior. The accuracy decreases when sharp transitions in the stress-strain relations are involved. Improved NASTRAN usefulness will be obtained when nonlinear material capabilities are extended to axisymmetric elements and to include provisions for time-dependent material properties and creep analysis. Rigid Formats 3 and 5 proved to be very convenient for the buckling and normal-mode analysis of a nuclear fuel element.

INTRODUCTION

In addition to NASTRAN finding its way into leading U.S. aerospace industries and research institutes, it is also more and more used in other areas of structural analysis such as civil engineering and the automotive industry. In the past much duplication has occurred with respect to the development of finite-element computer programs, but at present it is worthwhile to investigate the applicability of NASTRAN before deciding to write a finite-element routine or to add large extensions to existing programs. NASTRAN is an easy-to-use general-purpose program with assured continuous management and maintenance and is available at a low price.

The domain of nuclear problems is an important field of application outside the area for which NASTRAN was originally developed, and the present paper discusses the use of NASTRAN for some characteristic nuclear engineering and physical problems. The recently released Level 15 allows the solution of physical problems since a functional for neutron diffusion calculations, which shows a close relationship to one used for thermal analysis, was derived. Except for the computation of a neutron-flux distribution, the paper deals with two structural example problems; one discusses the elastic and plastic deformation of nuclear fuel cladding, and the other concerns the normal-mode and buckling analysis of a fuel element.

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The example problems chosen do not cover the whole area of finite-element applications suitable for nuclear research and design nor are all details of them investigated. Therefore, the paper cannot provide sufficient information needed to decide whether to use NASTRAN or not and may only serve as a guideline to more directed research.

ONE-GROUP NEUTRON DIFFUSION

Recently, different authors (refs. 1, 2, and 3) have applied finite-element techniques to solve problems from nuclear physics. The equations used to describe heat transfer and neutron diffusion show a close similarity, and only a few minor modifications of heat transfer programs are necessary to allow neutron diffusion calculations. Functionals for both phenomena will be discussed and NASTRAN alterations to give neutron-flux distributions will be outlined. The results for one particular application are presented.

For the case in which all neutrons are assumed to belong to the same energy (velocity) group, the equation of neutron diffusion reads (ref. 4)

$$-\frac{1}{v} \frac{\partial \phi}{\partial t} + D \frac{\partial^2 \phi}{\partial x_i \partial x_i} - \sigma \phi + S = 0 \quad (1)$$

Here the summation convention is used while v stands for velocity, D for the diffusion coefficient, ϕ for the scalar flux, σ for the macroscopic absorption cross section, and source term S for the rate of production of neutrons per unit volume per second.

The boundary conditions applied to equation (1) involve continuity of flux and current at interfaces, and vanishing of flux at extrapolated boundaries or return current at the actual boundaries. The general form of the boundary conditions reads

$$D \frac{\partial \phi}{\partial n} + \gamma \phi + q = 0 \quad (2)$$

where γ and q are constants. Equations (1) and (2) can be combined to obtain the functional:

$$F = \iiint \left(\frac{1}{v} \frac{\partial \phi}{\partial t} \phi + \frac{D}{2} \frac{\partial \phi}{\partial x_i} \frac{\partial \phi}{\partial x_i} + \frac{\sigma}{2} \phi^2 - S \phi \right) dV + \iint \left(q \phi + \frac{\gamma}{2} \phi^2 \right) dA \quad (3)$$

where the triple integrals are to be extended over the volume V , and the double integrals over the surface A of this volume. Functional F reaches a minimum value with respect to all admissible variations of flux distribution ϕ .

The functional most commonly used for finite-element heat conduction computation is

$$P = \iiint \left(\rho c \frac{\partial \theta}{\partial t} \theta + \frac{\lambda}{2} \frac{\partial \theta}{\partial x_i} \frac{\partial \theta}{\partial x_i} - Q^o \theta \right) dV + \iint \left(\frac{\alpha}{2} \theta_m^2 - \alpha \theta_m \theta + \frac{\alpha}{2} \theta^2 \right) dA \quad (4)$$

which reaches a minimum value with respect to all admissible variations of temperature distribution θ . The symbol Q^o is used for volumetric heat generation rate, ρ for density, and c for specific heat while λ stands for the coefficient of heat conductivity. The coefficient of heat transfer between the structure and its surrounding medium is represented by α and the temperature of the surrounding medium by θ_m . Comparison of expressions (3) and (4) shows that duality of finite-element temperature programs is obtained when for neutron diffusion calculations the following substitutions are made: $\frac{1}{v}$ for ρc ; D for λ ; S for Q^o ; $-\frac{q}{\gamma}$ for θ_m ; and γ for α . Moreover, the heat capacity matrix has to be generated using a coefficient of thermal capacity equal to σ and added to the heat conduction matrix. As yet NASTRAN heat flow capability can be used for stationary problems only and in this way applications to the calculation of neutron-flux distributions have to be restricted to steady-state conditions. The calculations can be performed by carrying out the substitutions mentioned and the insertion of an ALTER package (fig. 1). In Level 15.1.1 the heat capacity matrix cannot be generated directly. However MAT1, MAT2, or MAT3 cards can be used to submit values for the mass density equal to those desired for the thermal capacity and module SMA2 can be executed to obtain the mass matrix. The ALTER package allows the generation of matrix MGG and the addition of it to the heat conduction matrix KGGX. Module SCE2 removes additional matrix elements implied by MGG and undesired in the heat capacity matrix. The method described has been used to determine the flux distribution in the basic area of symmetry in a fuel-moderator mixture located between a square array of cruciform control rods. The rods are black to thermal neutrons and a uniform slowing density has been assumed. Figure 2 shows the geometry together with the idealization and calculated lines of constant neutron flux. The problem has also been investigated by Semenza et al. (ref. 2) and the agreement looks reasonable.

DEFORMATION OF NUCLEAR-FUEL CLADDING

One of the most important problems in the design of nuclear fuel elements is the interactive deformation of fuel pellets and the surrounding cladding (fig. 3). For most of the current cladding and fuel materials, the temperatures at which the loadings are applied will cause creep and plasticity to occur. Moreover, the nuclear irradiation leads to swelling, introduced by helium bubble formation, and embrittlement of the materials; thus, two more complications must be dealt with in analyzing the deformations.

With the exception of contact forces implied by the cladding, the pellet is only loaded thermally. The cladding is subject to

- (1) Differential pressure between coolant on the outside and released fission gas on the inside
- (2) Contact pressure and axial forces due to swelling and thermal deformation of the fuel
- (3) Thermal load caused by heat transfer through the clad wall

The magnitude of these loadings depends on time, neutron dose, and power output of the reactor.

NASTRAN permits the calculation of deformations both elastically and plastically. A static solution could be obtained by use of solid-of-revolution elements. After a choice has been made with respect to the grid points where contact occurs, multipoint-constraint equations could provide expressions for the displacements of internal cladding grid points in terms of external pellet grid points and the undeformed gap. The idealization and the deformed shape are given in figure 4, and the results show close agreement with solutions obtained from other programs (ref. 5).

Plasticity calculations of cladding deformations as a result of mechanical loading have been performed using Piecewise Linear Analysis. Because this feature is as yet not applicable for solid-of-revolution elements, the cladding wall has been idealized by parallel plate elements (fig. 5). In using this model, all axial displacements were restrained and all elements were kept parallel during deformation while interelement distances remained constant. Moreover all collinear grid points remained collinear and on the same radii. All these restrictions could easily be accomplished by multipoint and single point constraints leading to a number of independent degrees of freedom equal to 1. The application of this model means that radial stresses are neglected. These, however, proved to be of minor importance and are partly counterbalanced by the assumption of constant interelement distances.

During each pass through the Piecewise Linear Analysis loop a new value for the stress-strain slope is estimated by using

$$E^i = \frac{\sigma_a^{i+1} - \sigma_a^i}{\epsilon_a^{i+1} - \epsilon_a^i} \quad (5)$$

where σ_a^i and ϵ_a^i are the uniaxial equivalences of stress and strain obtained after application of the i th load increment and

$$\epsilon_a^{i+1} = \epsilon_a^i + \frac{\Delta\alpha_{i+1}}{\Delta\alpha_i} (\epsilon_a^i - \epsilon_a^{i-1}) \quad (6)$$

$$\sigma_a^{i+1} = F(\epsilon_a^{i+1}) \quad (7)$$

where $\Delta\alpha_i$ denotes the load increment during the i th load factor and F denotes a user-supplied tabular function of stresses and strains.

If a strain-hardening law is applied, the model requires that within a small increase of load, all elements deform beyond the elastic limit. The overall rigidity decreases considerably in the plastic stage and a load increment beyond the yielding point causes a strain increment which may be many times greater than that registered during the elastic deformation.

This is why the use of expression (6) to predict the strain due to the next load factor can lead to greatly underestimating the strain and, in such cases, the value used for E^i will be much too large. The present levels of NASTRAN do not contain a check on the accuracy of E^i . For the model described, acceptable results could only be obtained by choosing the piecewise linear factors in such a way that each of the nonlinear elements will be loaded up to the yielding point for one particular load factor.

This approach is of course very laborious and not feasible for structures with many nonlinear elements. However, the accuracy can be improved if the relationship between stresses and strains can be described by a smooth curve and if small load increments are used.

Thermal stresses are an important factor in the analysis of the cladding. When these stresses alternate, as is the case during startups and shutdowns of the reactor, they become of even more importance and a stepwise increase of the cladding diameter, called "thermal ratchetting," may be observed.

In Piecewise Linear Analysis (PLA), thermal loading can be taken into account by generating the relevant load vector separately and adding it stepwise to the mechanical loading after the mechanical loading has been fully accumulated during repeated execution of the PLA loop. The deviating values of E^i and the alternate direction of strain increments for elements compressed by the thermal strain could be accomplished by application of Direct Table Input. These tables can substitute the regular element summary, connection, and property tables for nonlinear elements, but once again the approach proved to be laborious and useless for practical applications. NASTRAN did not allow creep calculations or the consideration of time-dependent material properties introduced by the influence of nuclear irradiation.

BUCKLING AND NORMAL-MODE ANALYSIS OF FUEL ELEMENT

In many types of nuclear reactors, the heat generated by nuclear fission is removed by a gaseous or liquid coolant flowing longitudinally along the fuel rods. Considering the buckling of these rods when subjected to the flowing environment, it does not matter whether the fluid is assumed to flow through or along the tube, and the critical value of the fluid velocity can be found by solving the differential equation for steady flow of the fluid through the tube:

$$EI \frac{\partial^4 z}{\partial x^4} + \rho v^2 \frac{\partial^2 z}{\partial x^2} = p$$

Here z stands for the transverse displacement, v for fluid velocity, and EI for the bending stiffness of the tube. This agrees (ref. 6) with the equation for a pipe carrying a lateral load p and an axial load of ρv^2 . When λ is used for the length it means that buckling of the pipe will occur when

$$\rho v^2 = EI \left(\frac{\pi}{\lambda} \right)^2$$

therefore, the critical velocity is

$$v = \frac{\pi}{\lambda} \sqrt{\frac{EI}{\rho}}$$

NASTRAN allows accurate¹ determination of the buckling load, and therefore the critical velocity, when the pipe is idealized by only a few collinear BAR elements. Therefore, this program may also be used for the computation of critical velocities in complete fuel elements. In order to reduce the number of degrees of freedom, a NASTRAN fuel-element analysis has been performed by use of an artificial element with fewer fuel rods than usual but with a length-to-width ratio in agreement with existing fast-breeder reactor designs. The element considered contains 37 fuel rods arranged in a hexagonal pattern with an 8-mm pitch. The 6-mm-diameter rods occupy the full 800-mm length of the element.

To support the rods in the transverse directions, the element is equipped with four intermediate honeycomb-like grids; thus, all transverse displacements are equated at grid locations. The element is assumed to be clamped in the lower element supporting plate, whereas only longitudinal displacements are possible in the upper plate.

Every fuel rod is idealized by 10 collinear BAR elements of equal length while each side of the hexagonal prismatic container is modeled by a rectangular pattern of QUAD1 elements with 10 elements over the length and four elements over the width. Figure 6 gives the undeformed element together with normal modes 1, 2, and 5. The first four modes show vibrations of the fuel rods and mode 5 represents the impact of the lowest container frequency on the behavior of the fuel element also. When liquid sodium is used for cooling, the critical velocity is found to be 5.7 m/sec. Matrix partitioning has been applied to obtain 195 degrees of freedom in the a-set. For the problem concerned, this figure is too high to produce a significant reduction in CPU time. Checkpointing with restarting proved to be profitable when other eigenvalue ranges of interest had to be investigated.

¹0.16 percent deviation for three elements.

CONCLUDING REMARKS

The use of NASTRAN for the solution of three nuclear problems has been investigated. In view of the fact that NASTRAN was developed for solution of problems in aerospace structures, it is not astonishing that it is somewhat limited in the scope of nuclear problems that it will solve and requires more user effort than special-purpose nuclear programs. In summary, the conclusions from the analysis of the chosen examples can be stated as follows:

1. Steady-state one-group neutron-flux distributions can be computed very conveniently by the use of NASTRAN. One of the next official levels featuring transient-temperature calculations will also permit the solution of time-dependent diffusion equations. The applicability to multigroup diffusion and other problems from nuclear physics may be investigated.

2. At this time, Rigid Format 6 seems unsuitable when a strain-hardening law is used and alternating thermal loads are involved. In order to improve the present approach used in Piecewise Linear Analysis (PLA) it may be advisable to repeat the calculation in those cases where a large deviation between the estimated and actual strains occurs. Reference 7, 8, or 9 may be useful if an alternative approach is shown to be desirable. Other areas of interest for nuclear users will be to have PLA available for solid-of-revolution elements also and to be able to apply NASTRAN for creep calculations.

3. Buckling and normal-mode analysis can be applied directly to nuclear problems and to permit calculations of critical fluid velocities. The use of matrix partitioning reduces the number of independent degrees of freedom without loss of accuracy. More emphasis on creep applications is also desirable for these rigid formats, and a provision to perform creep-buckling analysis would mean a considerable improvement.

The criticism expressed did not show to full advantage the many important features such as checkpointing, substructuring, and multipoint constraints which are not found in any known computer code for nuclear use. These features give NASTRAN the lead over its competitors. The general and flexible applicability of NASTRAN constitutes its main power, and especially when more effort can be spent to improve nonlinear material capabilities, NASTRAN will have a large potential in the area of nuclear research and design.

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NASTRAN EXECUTIVE CONTROL DECK ECHO

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TD      STEADY ONE GROUP NEUTRON DIFFUSION
APP     DISPLACEMENT
SQL     1,0
DIAG 1,14
ALTER 22,22
$ SKPMGG = 1 WILL AVOID PUPPING OF MGG
$ AND ENABLE EXECUTION OF SMA2
PARAM   //C,N,NOP/V,N,SKPMGG=1 $
ALTER 37
$ ADD THE DIFFUSIVITY MATRIX TO THE ABSORPTION MATRIX
ADD     KGGX,MGG/KGG1/C,Y,ALPHA=(1.0,0.0)/C,Y,BETA=(1.0,0.0) $
CHKPNT  KGG1 $
ALTER 44,44
EQUIV   KGG1,KGG/NOGENL $
ALTER 47,47
SMA3    GE1,KGG1/KGG/V,N,IUSET/V,N,NOGENL/V,N,NOSIMP $
ENDALTER
TIME    2
CEND
    
```

Figure 1.- Executive Control Deck for neutron diffusion calculations.

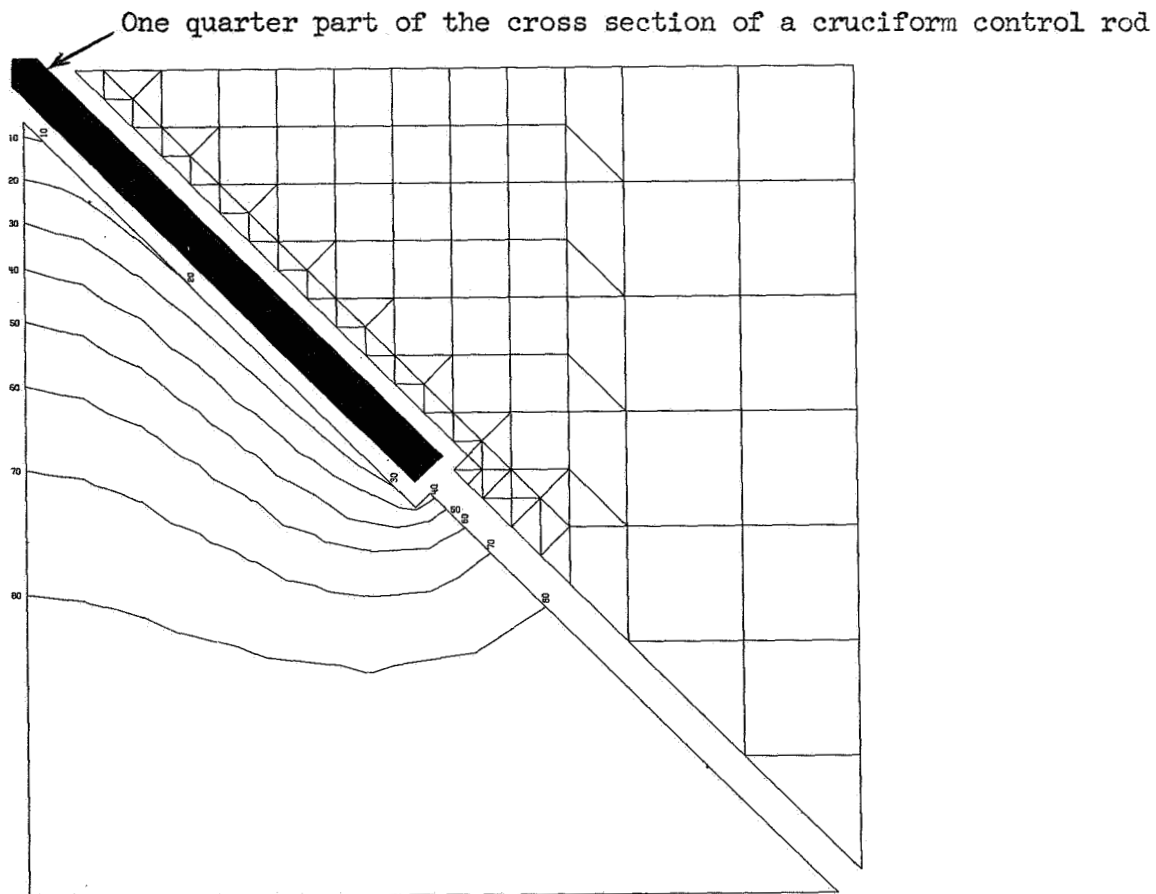
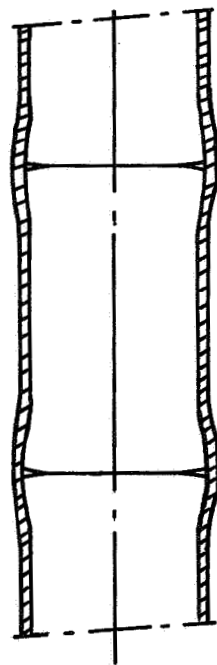
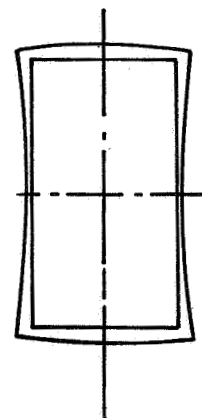


Figure 2.- Idealization and lines of constant neutron flow in a fuel-moderator mixture area between cruciform control rods.



(a) Interactive deformation.



(b) Thermally expanding pellet.

Figure 3.- Pellet and cladding.

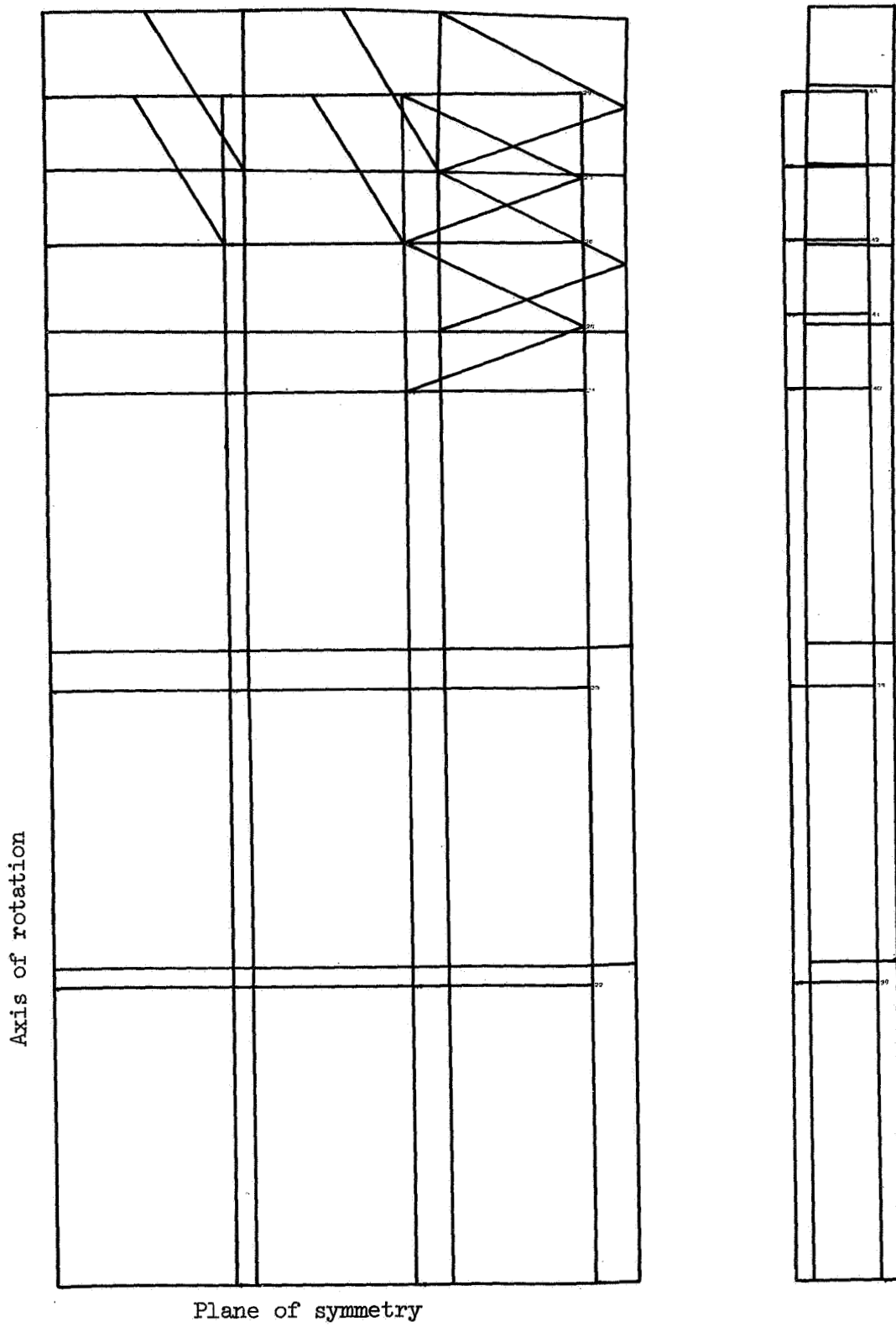


Figure 4.- Computed interactive deformation of pellet and cladding.

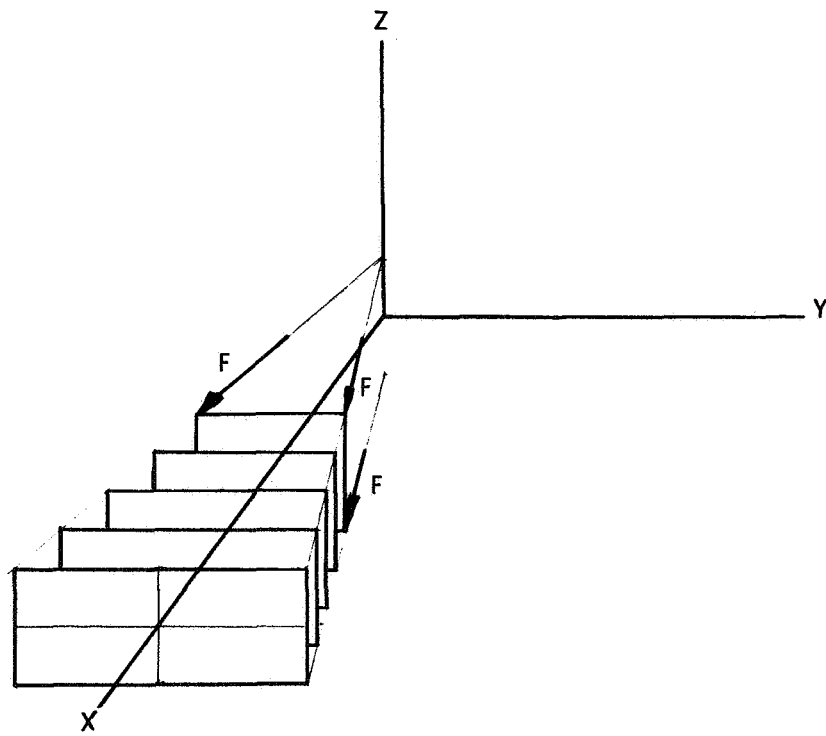


Figure 5.- Idealization of a tube for Piecewise Linear Analysis.



Figure 6.- Normal modes of nuclear fuel element.