

NASTRAN THERMAL ANALYZER

A GENERAL PURPOSE FINITE-ELEMENT HEAT TRANSFER COMPUTER PROGRAM

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SUMMARY

A general purpose heat transfer analysis capability based on the finite-element method has been added to the NASTRAN system. The program not only can render temperature distributions in solids subjected to various thermal boundary conditions, including effects of diffuse-gray thermal radiation, but is fully compatible in capacity and in the finite-element model representation with that of its structural counterpart in the NASTRAN system. The development history of the finite-element approach for determining temperatures is summarized. The scope of analysis capability, program structure, features, and limitations are given with the objective of providing NASTRAN users with an overall view of the NASTRAN Thermal Analyzer.

INTRODUCTION

The NASTRAN Thermal Analyzer is a general purpose heat transfer computer program based on the finite-element approach. It is fully capable of rendering temperature distributions and heat flows in solids subjected to various boundary conditions, including the modes of convection and radiation, in both steady-state and transient problems. This computer program has been achieved by the addition of new capabilities to NASTRAN including new elements and solution algorithms. The resulting Thermal Analyzer is both self-contained and user-oriented but remains totally compatible in capacity and in the finite-element model representation with that of its structural counterpart in the NASTRAN system. Therefore, thermal and structural analyses of large and complex structural configurations utilizing a unified finite-element representation have become a reality.

The purpose of this paper is to provide the NASTRAN users with an overall view of the NASTRAN Thermal Analyzer describing its scope, program structure, features and limitations. The practical needs which motivated the development of this thermal analysis capability are given. A brief survey of early works with regard to heat transfer analysis by the finite-element method is included. Also described are studies conducted at GSFC which investigated element behaviors, obtainable accuracy and solution efficiency of various schemes used in conjunction with the finite-element method, the feasibility of adding thermal analysis capability to the NASTRAN system, and the various approaches for treating problems with emphasis on radiative exchanges.

BACKGROUND OF DEVELOPMENT

The main objective of a research and development program at GSFC, titled the Structural-Thermal-Optical Program (STOP) (ref. 1), is the development of analytical methods and procedures which can yield reliable predictions of thermal deformation and optical degradation in an orbiting spacecraft (ref. 2), or in a simulated environment. This multi-disciplinary project is intended to render analytical services to projects such as LST (Large Space Telescope), SAS (Small Astronomy Satellite), EOS (Earth Observatory Satellite), etc. To predict alignment or optical performance of a large spacecraft-telescope system, NASTRAN is relied upon for structural analysis to compute thermally induced deformations and stresses. For a reliable structural solution, the thermoelastically uncoupled structural analysis requires accurate temperature data as an input to the NASTRAN structural model. Prior to the existence of the NASTRAN Thermal Analyzer, available general purpose thermal analysis computer programs were designed on the basis of the lumped-node thermal balance method (e.g. ref. 3). They were not only limited in capacity but seriously handicapped by incompatibilities arising from the model representations inherent in the two distinct approaches. The intermodel transfer of temperature data was found to necessitate extensive interpolation and extrapolation. This extra work proved not only a tedious and time-consuming process but also resulted in compromised solution accuracy. To minimize such an interface obstacle, the STOP project undertook the development of a general purpose finite-element heat transfer computer program.

Preliminary studies were then conducted at GSFC to investigate the feasibility of achieving such a computer program as an integrated part of the NASTRAN system. When this task began, the theoretical foundation of the finite-element approach had been established for both steady-state and transient thermal field problems (ref. 4-6). Efforts aimed at broadening its scope were found in literature such as an extension to axis-symmetric problems (refs. 7, 8), a consideration of inhomogeneous material using higher order elements (ref. 9), a demonstration of procedure enabling the achievement of an efficient solution by reducing the order of the set of equations (ref. 10), and many special applications (ref. 11-13). All of these studies, however, were limited to thermal conduction with linear boundary conditions. Since radiation is a dominant mode of the heat transfer process in space-oriented applications and introduces a fourth-power nonlinear temperature term in the boundary conditions, the in-house studies were directed principally to include these nonlinear radiation effects. Other investigations into solution feasibility, accuracy and efficiency, and element behaviors in combined modes of heat transfer analysis were also undertaken. The in-house studies consisting of two parallel efforts were:

- (1) the structural version of NASTRAN was employed directly to solve heat transfer problems by utilizing mathematical analogies together with manipulations of available elements and solution routines (refs. 14, 15). The concept of treating the radiative fluxes as nonlinear loads was demonstrated successfully for simple radiative problems. The solution scheme, however, was restricted in that all nonlinear radiation problems had to be solved using the direct transient integration algorithm available in the program.

- (2) the derivation of heat elements including radiative effects from the governing equation and boundary conditions approached via a variational principle, and studies centered on heat elements with associated different solution schemes (ref. 16). Two distinct approaches to nonlinear radiative problems were investigated and studies of radiative exchanges between element surfaces of known and unknown temperatures were included. A direct steady-state solution via the consistent linearization method and using an iterative process proved to be able to yield a solution more efficient than using a transient route. The other approach, the direct energy distribution method, which had been illustrated independently in a simple problem (ref. 17), in fact, shares the same theoretical basis as that employed by a direct use of the structural version of NASTRAN (ref. 14, 15). A prototype computer program based on the direct energy distribution method was coded (ref. 18), and studies of element behaviors and solution accuracy were also conducted.

While the NASTRAN System Management Office at Langley Research Center planned the extension of the NASTRAN system to include linear conductive heat transfer analysis capability only, GSFC-STOP was seeking the implementation of a full-fledged finite-element heat transfer computer program. The software capability was finally developed and implemented by the MacNeal-Schwandler Corporation under a subcontract from Bell Aerospace Company. It must be stressed, however, that a cooperative financial and technical effort between these two NASA centers made possible the emergence of this vital new capability.

SCOPE OF ANALYSIS CAPABILITY

The NASTRAN Thermal Analyzer is capable of solving both steady-state and transient heat transfer problems in large size systems of arbitrary configuration. The governing heat equation together with the boundary conditions may be expressed, following the finite-element technique, in matrix form as

$$[B] \{\dot{u}\} + [K] \{u\} = \{P\} + \{N\} \quad (1)$$

where

$\{u\}$ = vector of grid point temperatures

$\{\dot{u}\}$ = rate change of temperature vector

$[K]$ = thermal conductance matrix

$[B]$ = thermal capacitance matrix

$\{P\}$ = vector of grid point heat fluxes

$\{N\}$ = vector of temperature-dependent nonlinear heat fluxes

With appropriate interpretation of the matrices appearing in Eq. (1), three basic types of problem can be identified. They are distinguished by the numerical solution algorithms required for their solutions.

(1) Linear Steady-State Problems

Letting $\{\dot{u}\} = \{N\} = \{O\}$ and $[K] = [K_1]$, Eq. (1) is reduced to a linear steady-state equation of the form

$$[K_1] \{u\} = \{P\}$$

where $[K_1]$ is a constant thermal conductance matrix. This equation is analogous to the basic linear static analysis of the structural version and is treated by the solution algorithm present in Rigid Format 1, (Ref. 19).

(2) Nonlinear Steady-State Problems

Letting $\{\dot{u}\} = \{O\}$ and $[K] = [K_2]$, Eq. (1) has the form

$$[K_2] \{u\} = \{P\} + \{N\}$$

where $[K_2]$ may be temperature-dependent. The permitted nonlinearities may arise from temperature-dependent conduction and convection properties as well as diffuse radiation. A new iterative solution algorithm, which includes test for convergence, has been added to the program for this type of problem.

(3) Linear and Nonlinear Transient Problems

Equation (1) directly represents the general transient heat transfer problem in which $\{P\}$ is allowed to be a time-dependent heat flux vector. The permitted nonlinearities may arise from the effects of radiation or from user specified nonlinear elements. The new solution algorithm accommodated for this type of problem is implicit and is a combination of forward and backward differencing embracing a free parameter β that allows the user to select the amount of each. While a default value exists for the parameter which assures stability in linear problems, an override option is provided which allows the user to trade stability for efficiency. An option is also available which permits the user to linearize the effects of radiation.

PROGRAM STRUCTURE

The NASTRAN Thermal Analyzer has been designed to perform thermal analysis utilizing input and output formats compatible with those of the structural version. This

capability has been achieved by making maximum use of existing elements and system capabilities needed to satisfy the unique requirements posed by thermal problems. All input quantities as well as output displays are physically meaningful to users in the thermal field.

Features such as nonlinear and scalar elements, multipoint constraints, transfer functions, DMAP, etc. are automatically inherited from NASTRAN. The user, therefore, is provided with a powerful tool to include effects not normally modeled by other elements as long as those effects can be described by zero or first order system of equations

The essential points which characterize this thermal analyzer are summarized as follows:

(1) Geometry Description

The body to be analyzed is idealized as an assemblage of appropriate finite elements. NASTRAN grid, scalar, and extra points remain valid for use in this description. However, only one degree-of-freedom is associated with a grid point because of the nature of the scalar temperature field problem.

(2) Types of Elements

The program contains elements in three general categories:

- (a) Heat conduction elements – The constant gradient line, triangle and tetrahedra are the basic elements and utilize linear temperature gradients in one, two and three dimensions, respectively. All elements share common descriptions with their structural counterparts and are summarized in Table 1. Quadrilateral elements are composed of overlapping triangles, and wedges and hexahedra from sub-tetrahedra. Solid-of-revolution elements of triangular and trapezoidal cross-section are available for analyzing axisymmetric problems. Lumped capacitance matrices are used with all conduction elements to account for the effect of heat storage.
- (b) Boundary elements – XHBDY elements are used to define surface heat flux input and to describe convective and radiative exchanges between boundary elements. More than one boundary element can be connected to any one heat conduction element to account for multiple boundary heat inputs, including convection and radiation. A special cylindrical element with an arbitrary user specified elliptic cross-section is included for convenience in treating vector heat inputs. With this one dimensional element the effects of directional radiant heat inputs are automatically included in that peripheral energy inputs over the lateral surfaces are integrated internally.
- (c) Special elements – Several element types are available for the indirect inclusion of effects which cannot be otherwise modeled if only heat conduction

elements together with boundary elements were used. These special elements may be employed to enhance the analysis capability of the program. Scalar elements, nonlinear elements, transfer functions, and direct matrix inputs are included in this category. In heat transfer problems, scalar spring elements are analogous to thermal conductors and scalar dampers to thermal capacitance.

(3) Material Properties

Isotropic and anisotropic material behaviors are included. The treatment of nonlinearities arising from temperature-dependent thermal conductivity and convective film coefficient requires an iterative solution algorithm and this has been automated for steady-state problems only. The process involves the supply of temperature-dependent data in the form of tabulated functions which are interrogated at the beginning of each iterative step. In transient cases, however, the user must rely on the use of nonlinear elements to treat the nonlinear effects as additional thermal loads that are evaluated at the previous integration time step.

(4) Constraints and Partitions

Constraint and partitioning features of the structural version remain valid. Single-point constraints are used for the specification of prescribed temperatures, and multipoint constraints are used to describe known temperature dependency between temperature degrees of freedom. The omitted degree-of-freedom capability employs the well-known Guyan reduction technique to reduce the problem size of the solution set.

(5) Boundary Conditions, Initial Conditions, and Thermal Loading

Convective exchange along the boundary can be specified between a surface and an ambient zone of known temperature, or between two or more surfaces or zones of unknown temperatures. Constant temperatures are specified directly by using constraints while temperatures which are arbitrary functions of time are specified indirectly by using simple modeling procedures which avoid unnecessary retriangularization when solving transient problems. Arbitrary initial temperature distributions can be specified in transient analysis.

Several options are available to users for the specification of thermal flux input. Steady or time varying scalar heat flux can be described at the element and/or grid point level. Vector heat flux, such as that of radiant flux from a distant source, is described by specifying the flux intensity and vector direction. Both the flux intensity and vector direction may be time dependent and this allows, for example, the automated accommodation of rotating bodies in a vector heat flux field. Volumetric heat generation can be specified at the element level.

Required input data for diffuse-gray thermal radiation problems consists of the Stefan-Boltzmann's constant, reference absolute temperature, thermo-physical properties

and an array of exchange coefficients together with a list which identifies the elements that are radiatively interacting. The array of radiative exchange coefficients AF is a symmetric matrix according to the reciprocity rule. It is the product of the emitting surface area A and the geometric view factor F between the emitting and receiving surfaces. The Thermal Analyzer accepts radiation data via card or tape.

(6) Graphics Capability

The structural plotting feature for data checkout is available. Included options are orthotropic, perspective or stereoscopic projection capabilities. Time-history data of element heat flux, thermal loads, and temperatures at grid points can all be graphically shown in x-y plots.

(7) Integrated Thermo-Structural Analysis

The thermo-elastically uncoupled thermal and structural analyses are performed in two passes through the NASTRAN System, but may be made to appear as a single continuous run by the use of computer system control language. In the case of transient analysis, temperature distributions computed by the NASTRAN Thermal Analyzer are recorded on magnetic tape or punched cards at predetermined time intervals and, subsequently, employed for static structural analysis. Back-to-back thermal and structural analyses require that the grid point locations must be identical in both models on a point-to-point basis.

A VIEW FACTOR GENERATION COMPUTER PROGRAM

In computing temperatures involving radiative interchanges between surfaces, the geometric view factors between any two radiatively active element surfaces are necessary to form the exchange coefficients as input to the Thermal Analyzer. In an in-house STOP project effort, GSFC has developed an IBM-360 program named "VIEW" which computes the view factors and the required exchange coefficients between radiating boundary elements. VIEW has compatible input-output formats with the Thermal Analyzer and possesses other programming features similar to those of NASTRAN. A detailed description of this program is presented in an associated paper titled "VIEW - A Modification of the RAVFAC View Factor Program for Use with NASTRAN Level 15."

FURTHER STUDY IN EFFECT OF VIEW FACTOR COMPUTATIONS

In studying accuracy, one of the factors that might influence the result of solution involving radiative exchanges between elements is the method that computes the amount of net radiant energy on the element level which is then evenly distributed to its vertices. This approximation involves the computation of view factors which are computed on the element surface to element surface basis associated with the element temperature which is an average of those temperatures at the vertices. An alternate approach is to form an isothermal area of the temperature at the grid point by dividing the origin elements into subelements and assembling the subelements from the adjoining elements to that grid

point while subelements are formed by connecting the centroid to each midpoint on the side of an element. The difference of the view factor results from these two approaches is evident in view of a demonstration of the view factor F_{A-B} between two shaded areas as shown in Figure 1.

A direct computation of F_{A-B} gives 0.23344, the result obtained from a summation of one-third of the view factors computed on the basis of element surface to element surface is 0.12471, and the result obtained from summing up the view factors on the basis of subelement to subelement following the rule of the view factor algebra is 0.233989. It is to be noted, however, that these computations took into consideration the view factor alone. A study combining the effects of temperatures and view factors which involves a modification of a prototype computer program to assess the quantitative influence to the solution accuracy is in progress and will be reported separately.

ILLUSTRATIVE PROBLEM

Since the delivery of the IBM-360 NASTRAN Thermal Analyzer to GSFC in June of 1972, the system check-out phase has proceeded. A problem was designed to demonstrate program capabilities and features including: inter-element and inter-program (with regard to VIEW program) compatibilities, coordinate transformations, combined thermal modes of operation, vector heat flux input description, and the application of the different solution algorithms. The problem is that of a fin-like bent plate, whose dimensions, thermophysical properties and finite-element model are shown in Figure 2 where the input parameters with the subscript t refer to the tube and those without any refer to the plate. The underside of the plate is insulated thermally, and the upper face is exposed to a directional heat flux of S. The flowing fluid has a temperature $T_{in} = 1200$ K at the entrance and a linear temperature drop of 60 K across the tube which has a wall thickness $\tau = 1$ cm and an outer radius $r_0 = 20$ cm. Determine temperature distributions in the plate for the following cases:

- (1) The upper face dissipates heat to the surroundings of $T_{air} = 300$ K with a convective film coefficient $h_{air} = 1.135 \text{ W/cm}^2 \text{ } ^\circ\text{K}$.
- (2) A radiative enclosure of a temperature T_∞ replaces the convective environment in the preceding case, and radiative interchanges between all surface elements are taken into consideration. The solution is approached by the direct nonlinear steady-state solution algorithm.
- (3) The plate has a uniform temperature at $T_0 = 300$ K initially. The temperature response of the plate is determined by the nonlinear transient solution algorithm. The process starts with an activation of the flow into the tube which has identical tube-fluid conditions as previously described.

In view of the length that would be required to embed the complete meaningful computer print out of this problem, which consists of detailed card descriptions of the control and input data decks and the output results, solutions are not included in this report

but have been prepared in a separately bound copy (ref. 20) which will be made available to the audience of this colloquium as well as to any interested reader who will make a request to the authors.

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TABLE-1
HEAT CONDUCTION ELEMENTS

DIMENSION	TYPE	ELEMENTS
1-D	Linear	BAR, R ϕ D, C ϕ NR ϕ D, TUBE
2-D	Membrane	TRMEM, TRIA1, TRIA2, QDMEM, QUAD1, QUAD2
	Solid of Revolution	TRIARG, TRAPRG
3-D	Solid	TETRA, WEDGE, HEXA1, HEXA2

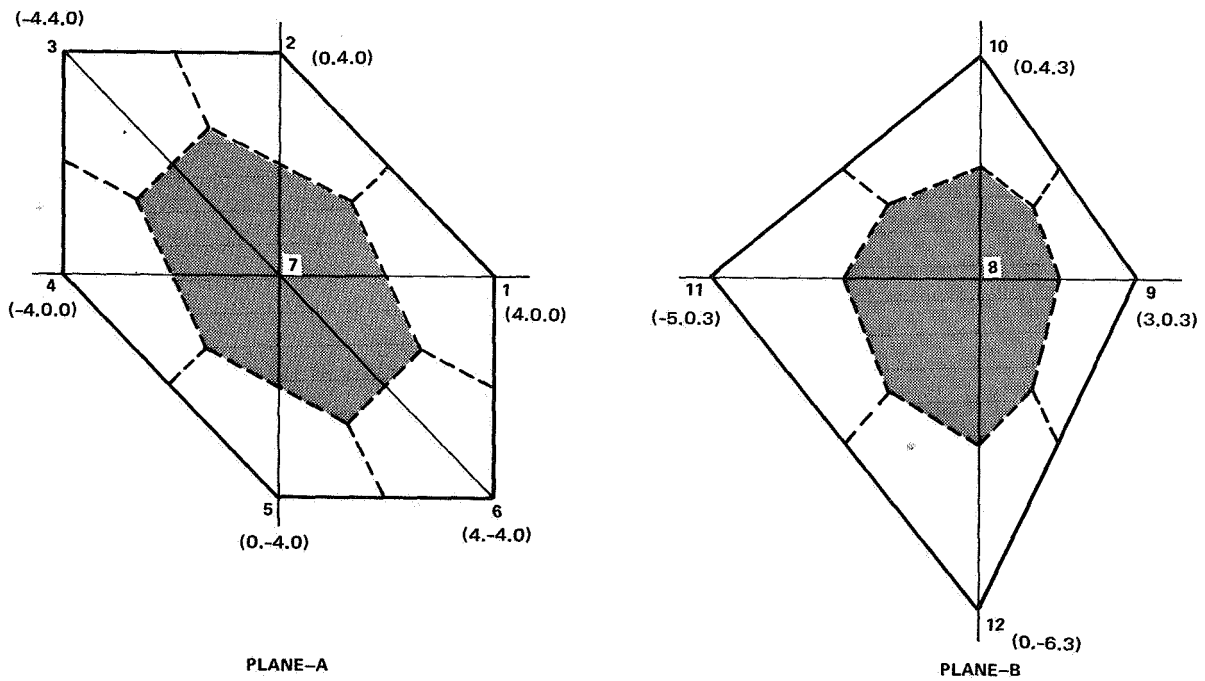


Figure 1. Sub-elements joined at two common vertices to form two isothermal radiative surfaces.

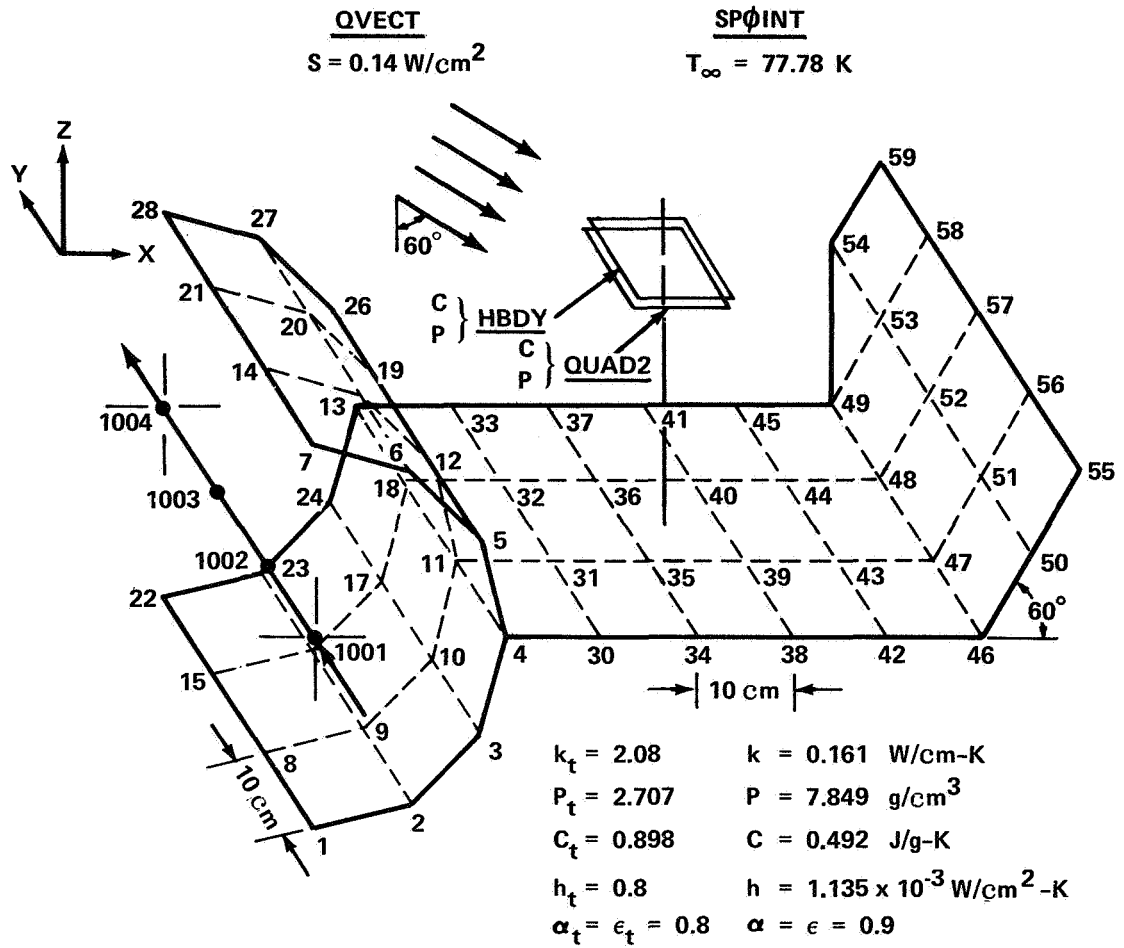


Figure 2. Finite-element model of a bent fin-tube configuration with boundary conditions.