APPLICATIONS OF NASTRAN SUBSTRUCTURING

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SUMMARY

This paper describes the application of substructuring techniques for two example problems, (i) a square plate and (ii) the static analysis of a frame-wall interaction problem in multistory structures. Presently, multipoint constraint forces are not retrieved in NASTRAN. A DMAP routine for calculating the multipoint constraint forces is also presented herein.

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INTRODUCTION

The use of substructuring techniques in NASTRAN is well documented (references 1 and 2). However, it is felt that example problems involving large degree of freedom (d.o.f.) systems would bring out the advantages of substructuring in greater detail and will be of help to the NASTRAN user community. This is attempted in this paper.

There are several cases where the analyst will be interested in evaluating the multipoint constraint forces - for example, the frame-wall interaction problem in multistory structures or the nuclear fuel pellet-cladding problem in nuclear engineering. These forces are not presently retrieved in NASTRAN. A DMAP routine, based on the Lagrange multiplier technique, is presented herein for the calculation of multipoint forces of constraint. When this DMAP routine is applied for large d.o.f. problems, the computing effort needed is so great as to make it impracticable. The substructuring feature in NASTRAN overcomes this difficulty. This paper uses the substructure partitioning and the Lagrange multiplier technique to retrieve the interaction forces between the shear wall and frame of a multistory structure.

DESCRIPTION OF PROCEDURE

The details of substructure partitioning are explained in the NASTRAN User's Manual (ref. 2) and will not be described here. With reference to static analysis, the method is briefly outlined in the following paragraphs.

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The complete structure is divided into a number of substructures, the boundaries of which may be specified arbitrarily; however, for convenience, it is preferable to make structural partitioning correspond to physical partitioning. Each substructure is first analysed separately, assuming that all common boundaries (joints) with the adjacent substructures are completely fixed. (In NASTRAN, this is called the Phase I operation.) From this analysis, the displacements of all interior points in each substructure with the adjacent substructure boundaries fixed are evaluated. These boundaries are then relaxed simultaneously and the boundary displacements are determined from the equations of equilibrium at the boundary joints (the Phase II NASTRAN operation). Each substructure can now be analysed for boundary displacements. Adding these to the Phase I displacements, (displacements of interior points in each substructure with adjacent boundaries fixed) we get the final displacements. (This is achieved in NASTRAN in Phase III operation.)

The addition of the reduced substructure boundary loads and stiffness matrices to obtain the total boundary load and stiffness matrix for the complete structure, and the partition of the boundary displacement of the complete structure into the boundary displacements of the separate substructures is achieved with the aid of partitioning vectors. The partitioning vector for each substructure is a vector of size $n \times 1$ where n is the total degrees of freedom in the a-set. The various steps in the construction of the partitioning vectors are explained in ref. 2. For cases where all the grid points in the total structure have been numbered distinctly, the partitioning vectors can be formed as follows:

1. Arrange the grid points in the a-set, in ascending sequence. List the connected degrees of freedom at these grid points (the components of the a-set) as scalar point internal indices in ascending numerical sequence starting with 1. This gives the size n of the partitioning vector.

2. The partitioning vector for each substructure is obtained by entering real 1's in all locations where the substructure under consideration has connection components with any other substructure.

The formation of the partitioning vector when one substructure has connection with two or more substructures and when the grid point numbering for the total structure shows discontinuities is illustrated in Example Problem Number 2.

EXAMPLE PROBLEMS

Problem Number 1

The structural problem consists of a square plate with hinged supports on all boundaries. The 10×20 model, as shown in Fig. 1, uses one-half of the structure and symmetric boundary constraints on the midline in order to reduce the order of the problem and the band width by one-half. Because only the bending modes are desired, the inplane deflections and rotations normal to the plane are constrained. This is the same problem as that solved in the NASTRAN demonstration manual (ref. 3).

The model is divided into five substructures. (This is not the best division of the problem; however, since the purpose herein is to demonstrate the use of identical substructures and the second stiffness reduction in Phase II, no attempt is made to choose the best subdivision.) The a-set points consist of (i) points on the boundaries of the substructures, 12 thru 22, 88 thru 98, 154 thru 164, 220 thru 230 and (ii) additional points in each substructure needed to define the dynamic response (this is largely based on the analyst's judgement), 55, 60, 65, 121, 126, 131, 187, 192, and 197. Note that Phase I runs are made only for two substructures, substructure 1 and substructure 2 (Sub-5 is identical to Sub-1; Sub-3 and Sub-4 are identical to Sub-2). There are 53 a-set points with 3 d.o.f. per grid point (Total d.o.f. = 159). Applying the boundary condition $y = 0$ along $X = 0$, 7 d.o.f. are eliminated; applying the condition $u_z = \theta_x = 0$ along $X = 10$, 14 d.o.f. are eliminated; this leaves 138 d.o.f. in a-set. Since all the grid points in the boundaries are not needed for reasonably satisfactory dynamic response of the structure, a second stiffness reduction is done in Phase II. The grid points omitted are 13 thru 16, 18 thru 21, 89 thru 92, 94 thru 97, 155 thru 158, 160 thru 163, 221 thru 224, and 226 thru 229 (total of 32 points each of 3 d.o.f.). There are thus only 42 d.o.f. in the final solution of the pseudo-structure in Phase II. The natural frequency comparisons with and without the second stiffness reduction of Phase II is given in Table 1.

Problem Number 2

This problem deals with the analysis of a multistory structure. The shear wall and frame are treated as separate structures and they are discretized and divided into substructures as shown in Figures 2 through 5. The shear wall is divided into 30 substructures (three for each story). The frame is divided into 10 substructures. Phase I analysis is performed for 7 of the 30 substructures of the shear wall and 2 of the 10 substructures of the frame (due to the repetitive geometry, it is enough if 3 substructures of shear wall and 1 of frame are analysed for Phase I; however, to reduce the a-set points, the former approach is used).

Substructure 1 has connection points with substructures 2 and 4; substructure 4 has connection points with substructures 1, 2, 5, and 7. The grid point numbering for the total structure is available, even though it is not continuous serially. Under these conditions, the partitioning vectors for all the substructures can be formed as shown below.

(1) Arrange the grid points in the boundaries in ascending sequence: 49-56, 105-112, 161-168, 217-224, 273-280, 329-336, 385-392, 441-448, 497-504, 1749-1756, 1805-1812, 1861-1868, 1917-1924, 1973-1980, 2029-2036, 2085-2092, 2141-2148, 2197-2204, 2905-2907, 2912-2914, 2919-2921, 2926-2928, 2933-2935, 2940-2942,

2947-2949, 2954-2956, 2961-2963, 2968-2970, 3105-3107, 3112-3114, 3119-3121, 3126-3128, 3133-3135, 3140-3142, 3147-3149, 3154-3156, 3161-3163, 3168-3170. Since each point has 2 degrees of fredom (u and v), the components of the a-set are listed as scalar point internal indices in ascending numerical sequence starting with 1 as follows:

A-Set grid point	Scalar point internal index
49	1
	2
50	3
	4
51	5
	6
⋮	⋮
1749	145
	146
1750	147
	148
⋮	⋮
2905	289
	290
2906	291
	292
⋮	⋮
3105	349
	350
3106	351
	352
⋮	⋮
3170	407
	408

(2) The partitioning vector for each substructure is obtained by entering real 1's in all locations where the substructure under consideration has connection components with any other substructure. The partitioning vectors for 2 sample substructures is shown on the following pages (size of partitioning vectors 408×1):

Scalar point Internal Index	Substructure 1	Substructure 4
1	1.0	1.0
2	1.0	1.0
3	1.0	1.0
4	1.0	1.0
5	1.0	1.0
6	1.0	1.0
7	1.0	1.0
8	1.0	1.0
9	1.0	1.0
10	1.0	1.0
11	1.0	1.0
12	1.0	1.0
13	1.0	1.0
14	1.0	1.0
15	1.0	1.0
16	1.0	1.0
17		1.0
18		1.0
19		1.0
20		1.0
21		1.0
22		1.0
23		1.0
24		1.0
25		1.0
26		1.0
27		1.0
28		1.0
29		1.0
30		1.0
31		1.0
32		1.0
33		
34		
35		
36		
37		
38		
39		
40		
41		
42		
43		
44		
45		
46		
47		
48		

Scalar point Internal Index	Substructure 1	Substructure 4
49		
50		
51		
52		
.		
.		
.		
289	1.0	
290	1.0	
291	1.0	
292	1.0	
293	1.0	1.0
294	1.0	1.0
295		1.0
296		1.0
297		1.0
298		1.0
299		1.0
300		1.0
301		
302		
303		
304		
.		
.		
.		
408		

The a-set points for the frame are 5, 9, 13, 16, 20, 24, 27, 31, 35, 38, 42, 46, 49, 53, 57, 60, 64, 68, 71, 75, 79, 82, 86, 90, 93, 97, 101, 104, 108, 112. Since there are 3 d.o.f. per grid point, (u, v, and θ_z) the a-set components total 90. The partitioning vectors for the substructures of the frame (size 90×1) can be formed easily.

Since it is of interest to know how the frame and the wall acting alone will resist the lateral wind load, the frame and the shear wall are analysed separately at first.

This example will also be used to illustrate the use of multiple level substructuring. The multistory structure is to be analysed for different first story heights of 12 ft, 13 ft, 15 ft, and 20 ft. In order that the entire calculations are not to be repeated, a Phase II (Initial) run is made where substructures 4 thru 30 of the shear wall are combined into a "super-substructure"; so also substructures 2 thru 10 of the frame. The Phase II (Final) run consists of combining the first story substructures to the super-substructures of shear wall and frame, respectively. The data recovery of substructures of interest is achieved in Phase III. For a different first-story height of the multistory structure, the Phase I run for the substructures of the first story and Phase II (Final) runs are repeated with the necessary Phase III runs.

The stiffness matrices with respect to the active degrees of freedom of the wall and frame, respectively, are merged by means of vector of size 498×1. The interaction of the wall and frame is studied using multipoint constraint equations; the conditions to be satisfied being (i) u and v displacements at corresponding points of wall and frame are equal and (ii) θ_z of frame at connection points with the wall should be equal to the fictitious θ_z values of the wall obtained by dividing the difference of the vertical displacements at the two ends of the left wall at each floor level by the width.

The value of the maximum displacement for wall alone, frame alone, and frame-wall interaction for the case of the lateral wind load is given in Table 2. It is to be pointed out that without substructuring, each of the cases investigated would have involved considerably more computing effort. For example, the frame-wall interaction problem has a total of 3210 degrees of freedom. In a direct analysis of the total structure the stiffness matrix of 3210×3210 has to be decomposed; whereas in substructuring, 6 substructures in each of which the size of matrix does not exceed 112×112, 2 of size 30×30, 1 of size 54×54, and 1 of size 468×468 are solved.

As seen from Table 2, for solving six different problems, a total time of about 900 sec is only needed while using substructuring techniques whereas for the solution of one shear-wall problem alone, about 2450 sec is needed without substructuring. The total time for solving all the cases without substructuring will be exhorbitantly high (the bulk of the time is spent on decomposition of the large stiffness matrix). It should be mentioned that this wide discrepancy in time with and without substructuring is largely due to the repetitive nature of the structure geometry of this problem and also that Phase III runs are performed only at the portion of interest in the structure. Nonetheless, time savings are bound to result, in general, with the use of substructuring.

EVALUATION OF MULTIPOINT CONSTRAINT FORCES

In NASTRAN, the multipoint constraint forces are not retrieved. A DMAP program is written here to retrieve these forces. The theoretical basis for this DMAP routine lies in the use of the Lagrange multiplier technique.

From the minimum potential energy principle, we have the functional

$$\pi_P = 1/2 \int_V \{\epsilon\}^T [E] \{\epsilon\} \, dV - \int_{s_\sigma} \Delta \, \bar{T} \, dS$$

where s_σ is the surface upon which the tractions $\bar{T}$ are prescribed.

The collection of multipoint constraint equations can be written in the form

$$\begin{bmatrix} C \end{bmatrix} \begin{Bmatrix} \Delta \end{Bmatrix} = 0$$

To account for such constraints, we invoke the method of Lagrange multiplers, and defining the vector $\{\lambda\}$ of these multipliers, we have the augmented functional

$$\pi_P = \left\{\frac{\Delta}{2}\right\}^T \begin{bmatrix} K \end{bmatrix} \left\{\Delta\right\} - \left\{\Delta\right\}^T \left\{P\right\} + \left\{\Delta\right\}^T \begin{bmatrix} C \end{bmatrix}^T \left\{\lambda\right\}$$

After applying the first necessary conditions, we have

$$\left[\begin{array}{c|c} K & C^T \\ \hline C & O \end{array}\right] \begin{Bmatrix} \Delta \\ \lambda \end{Bmatrix} = \begin{Bmatrix} P \\ O \end{Bmatrix}$$

This equation can now be solved for $\left\{ \begin{matrix} \Delta \\ \lambda \end{matrix} \right\}$. Note that the system of equations, in general, is not now positive definite, and hence the unsymmetric decomposition routine of NASTRAN has to be used for the solution. From the stand point of units, λ's have the unit of lb/in. or in-lb/in. depending on whether the particular multipoint constraint equation equates displacements or rotations. This discloses that from a purely physical standpoint, the λ's represent the average value of the distributed force or moment needed to satisfy the multipoint constraint equation.

For this problem the a-set stiffness matrix for the wall is of size 408 × 408; that for the frame is 90 × 90; and there are 30 multipoint constraint equations. Thus the augmented matrix is the size 528 × 528. An unsymmetric decomposition of this matrix on CDC 6600 machine with 140K (octal) storage will require about 18.5 minutes. Since this is very expensive, an alternative formulation is used herein. The 408 × 408 a-set stiffness matrix of the wall is reduced to 20 × 20 (retaining only the d.o.f. at each connection point with the frame); the 90 × 90 a-set stiffness matrix of the frame is reduced to 30 × 30 (retaining only the d.o.f. at each connection point with the wall); thus, with 30 multipoint equations, the augmented matrix of size 80 × 80 need only be unsymmetrically decomposed. The reduction of a-set stiffness and load matrices and the solution of the augmented matrix took only about 88 sec on CDC 6600 with 140K (octal) storage. The DMAP package for this frame-wall interaction problem, including the stiffness reductions mentioned above, is given in the Appendix.

Even though the procedure described herein for the calculation of multipoint constraint forces is general and can be used for problems that do not involve and/or necessitate substructuring techniques, it has to be emphasized that for large problems, the method can be used only with substructuring. Even then, an additional stiffness reduction would considerably shorten the computing effort. This is because unsymmetric decomposition of large matrices will involve unacceptably high computing costs.

CONCLUDING REMARKS

The NASTRAN substructuring techniques have been applied for the solution of a static and a dynamic problem. In both problems, substructuring is found to result in considerable saving of computing effort. The multiple level substructuring technique, which facilitates the efficient reanalysis of the structure when only a portion of the structure is modifed, has been applied for a frame-wall interaction problem. In NASTRAN, the multipoint constraint forces are not presently retrieved. A DMAP routine for retrieving the multipoint constraint forces has been written and has been successfully used in calculating the interactive forces between the frame and shear wall of a multistory structure.

REFERENCES

1. MacNeal, R. H., ed.: The NASTRAN Theoretical Manual, NASA SP-221, Sept. 1970 (Revised 1972).

2. McCormick, C. W., ed.: The NASTRAN User's Manual, NASA SP-222, Sept. 1970 (Revised 1972).

3. NASTRAN Demonstration Problem Manual, NASA SP-224, Sept. 1970.

APPENDIX

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SECOND STIFFNESS REDUCTION IN PHASE 2 TO RETRIEVE MPC FORCES
LAGRANGE MULTIPLIER TECHNIQUE


     N A S T R A N   S O U R C E   P R O G R A M   C O M P I L A T I O N
DMAP-DMAP INSTRUCTION
 NO.

  1  BEGIN     $

  2  $

  2  $    KGGA,PGA  ARE STIFF AND LOAD MATRICES OF WALL (SIZE 408X408)

  2  $    KGGB,PGB  ARE STIFF AND LOAD MATRICES OF FRAME (SIZE 90X90)

  2  $    SINCE THERE IS  NO LOAD ON THE WALL,THE PGA VECTOR WILL NOT BE USED

  2  $    KG1,KG2 ARE REDUCED STIFF MATRICES OF WALL(20X20) AND FRAME(30X30)

  2  $    KGG AND PG ARE STIFF AND LOAD MATRICES FOR FRAME-WALL COMBINED

  2  $    CMAT IS THE MATRIX OF MULTI-POINT CONSTRAINT EQUATIONS

  2  $    STIF AND LOAD ARE THE AUGMENTED STIFFNESS AND LOAD MATRICES

  2  $    DELTA IS VECTOR OF DISPLACEMENTS,LAMBDA IS VECTOR OF MPC FORCES

  2  $

  2  INPUTT1  /,,,,/C,N,-1/C,N,0/C,N,FRAMWALL $

  3  INPUTT1  /,,,,/C,N,-1/C,N,1/C,N,USERTP1 $

  4  INPUTT1  /KGGA,PGA,,,/C,N,16/C,N,1 $

  5  INPUTT1  /KGGB,PGB,,,/C,N,29 $

  6  SMP1     USET1,KGGA,,,/GOA,KG1,KGOOA,LOOA,UOOA,,,,, $

  7  SMP1     USET2,KGGB,,,/GOB,KG2,KGOOB,LOOB,UOOB,,,,, $

  8  SSG2     USET2,,,,GOB,,PGB/,PUB,,PG2 $

  9  MERGE    KG1,,,KG2,VECT3,/KGG/C,N,-1/C,N,2/C,N,6 $

 10  MERGE,    ,PG2,,,,VECT3/PG/C,N,1/C,N,2/C,N,2 $

 11  TRNSP    CMAT/CTRNSP $

 12  MERGE    KGG,CTRNSP,CMAT,,VECT4,/STIF/C,N,-1/C,N,2/C,N,6 $

 13  MERGE    PG,,,,,VECT4/LOAD/C,N,1/C,N,2/C,N,2 $

 14  SOLVE    STIF,LOAD/SOLVECT/C,N,0/C,N,1/C,N,2/C,N,2 $

 15  PARTN    SOLVECT,,VECT4/DELTA,LAMBDA,,/C,N,1/C,N,2/C,N,2/C,N,2 $

 16  MATPRN   DELTA,LAMBDA,,,// $

 17  END      $

       **NO ERRORS FOUND - EXECUTE NASTRAN PROGRAM**
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S O R T E D   B U L K   D A T A   E C H O

. 1 ..	2 ..	3 ..	4 ..	5 ..	6 ..	7 ..	8 ..	9 ..	10
DTI	USET1	0	0	408	0	500	0	0	
DTI	USET1	1	496	496	116	116	116	116	+101
+101	116	116	116	116	116	116	116	116	+102
+102	116	116	496	496	116	116	116	116	+103
+103	116	116	116	116	116	116	116	116	+104
+104	116	116	496	496	116	116	116	116	+105
+105	116	116	116	116	116	116	116	116	+106
+106	116	116	496	496	116	116	116	116	+107
+107	116	116	116	116	116	116.	116	116	+108
+108	116	116	496	496	116	116	116	116	+109
+109	116	116	116	116	116	116	116	116	+110
+110	116	116	496	496	116	116	116	116	+111
+111	116	116	116	116	116	116	116	116	+112
+112	116	116	496	496	116	116	116	116	+113.
+113	116	116	116	116	116	116	116	116	+114
+114	116	116	496	496	116	116	116	116	+115
+115	116	116	116	116	116	116	116	116	+116
+116	116	116	496	496	116	116	116	116	+117
+117	116	116	116	116	116	116	116	116	+118
+118	116	116	116	116	116	116	116	116	+119
+119	116	116	116	116	116	116	116	116	+120
+120	116	116	116	116	116	116	116	116	+121
+121	116	116	116	116	116	116	116	116	+122
+122	116	116	116	116	116	116	116	116	+123
+123	116	116	116	116	116	116	116	116	+124
+124	116	116	116	116	116	116	116	116	+125
+125	116	116	116	116	116	116	116	116	+126
+126	116	116	116	116	116	116	116	116	+127
+127	116	116	116	116	116	116	116	116	+128
+128	116	116	116	116	116	116	116	116	+129
+129	116	116	116	116	116	116	116	116	+130
+130	116	116	116	116	116	116	116	116	+131
+131	116	116	116	116	116	116	116	116	+132
+132	116	116	116	116	116	116	116	116	+133
+133	116	116	116	116	116	116	116	116	+134
+134	116	116	116	116	116	116	116	116	+135
+135	116	116	116	116	116	116	116	116	+136
+136	116	116	116	116	116	116	116	116	+137
+137	116	116	116	116	116	116	116	116	+138
+138	116	116	116	116	116	116	116	116	+139
+139	116	116	116	116	116	116	116	116	+140
+140	116	116	116	116	116	116	116	116	+141
+141	116	116	116	116	116	116	116	116	+142
+142	116	116	116	116	116	116	116	116	+143
+143	116	116	116	116	116	116	116	116	+144
+144	116	116	116	116	116	116	116	116	+145
+145	116	116	116	116	116	116	116	116	+146
+146	116	116	116	116	116	116	116	116	+147
+147	116	116	116	116	116	116	116	116	+148
+148	116	116	116	116	116	116	116	116	+149
+149	116	116	116	116	116	116	116	116	+150
+150	116	116	116	116	116	116	116	116	+151
+151	116	116	ENDREC						
DTI	USET2	0	0	90	0	500	0	0	
DTI	USET2	1	116	116	116	116	116	116	+201
+201	496	496	496	116	116	116	116	116	+202
+202	116	496	496	496	116	116	116	116	+203
+203	116	116	496	496	496	116	116	116	+204
+204	116	116	116	496	496	496	116	116	+205
+205	116	116	116	116	496	496	496	116	+206
+206	116	116	116	116	116	496	496	496	+207
+207	116	116	116	116	116	116	496	496	+208
+208	496	116	116	116	116	116	116	496	+209
+209	496	496	116	116	116	116	116	116	+210
+210	496	496	496	116	116	116	116	116	+211
+211	116	116	116	116	ENDREC				
ENDDATA									

Table 1. Natural Frequency Comparisons, CPS.

Mode	Theoretical	NASTRAN 10 x 20	5 Substructures (No stiffness reduction in Ph. 2)	5 Substructures (Stiffness reduction in Ph. 2)
1	0.9069	0.9056	0.9063	0.9068
2	2.2672	2.2634	2.2707	2.2812
3	4.5345	4.5329	4.6070	4.6629
		Total (Phase 1 and Phase 2) time on CDC 6600 machine	258.5 sec	238.5 sec

Table 2. Value of maximum displacement, in. ft., due to lateral wind load of 1 kip/sq. ft.

	Frame Alone	Shear-wall Alone	Frame-Wall interaction problem
Case 1-First story height = 12'	0.051145	0.019336	0.018901
Case 2-First story height = 13'	0.052557	0.019855	0.019448
(Total time for Ph. 1 and Ph. 2 on CDC 6600)	50 + 30 = 80 sec	290 + 220 = 510 sec	160 + 140 = 300 sec

Total time on CDC 6600 for solving the problem of shear wall alone (with a First story height of 13') without the use of substructuring but using omit d.o.f. } 2450 sec.

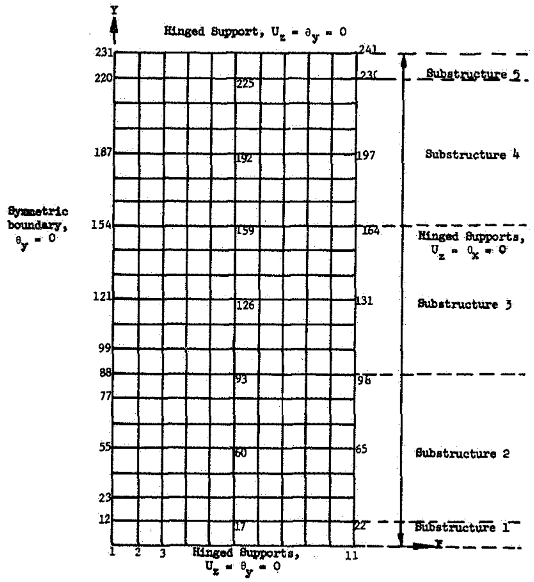


Figure 1. 10 x 20 Half plate model.

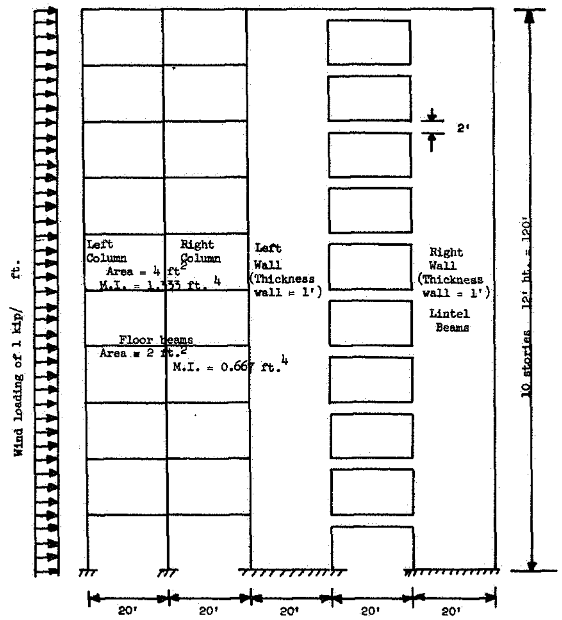
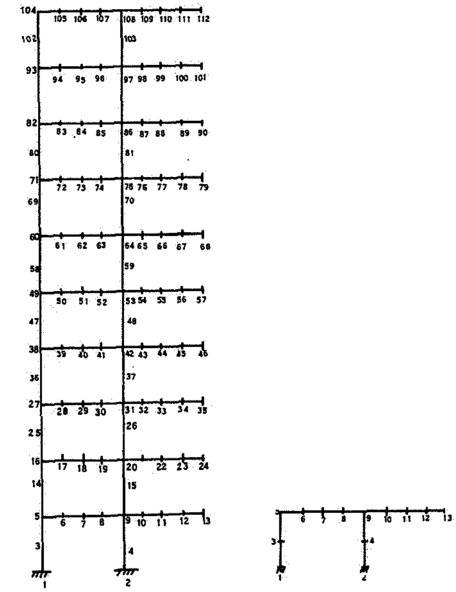


Figure 2. Multistory frame and shear wall.



(a) Discretization. (All C-Bar elements) (b) Substructure 1

Figure 5. Frame details

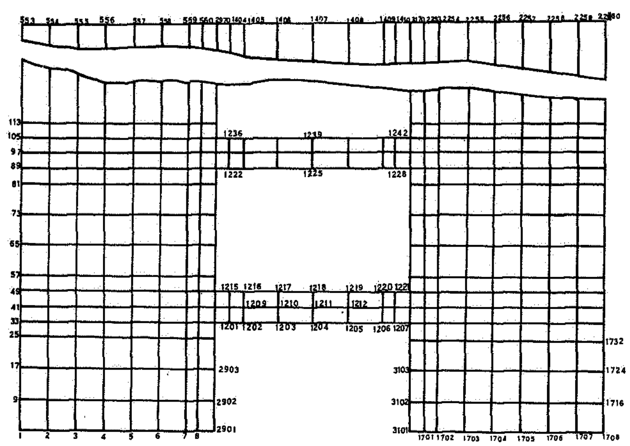


Figure 3. Shear wall Discretization (All elements are CQDMEM)

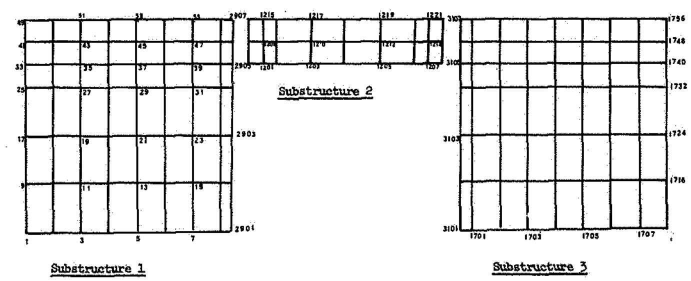


Figure 4. Division of First story into Substructure