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23. A Bayesian Model for Visual Space Perception

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A model for visual space perception is proposed that contains desirable features in the theories of Gibson (ref. 1) and Brunswik (ref. 2). This model is a Bayesian processor of proximal stimuli which contains three important elements: an internal model of the Markov process describing the knowledge of the distal world, the a priori distribution of the state of the Markov process, and an internal model relating state to proximal stimuli. The universality of the model is discussed and it is compared with signal detection theory models. Experimental results of Kinchla are used as a special case.

INTRODUCTION

There have been two broad theories of visual perception expounded in the past several decades; they are usually attributed to Gibson (ref. 1) and Brunswik (ref. 2). Gibson seems to be the first to present a unified picture of all the various cues about the three-dimensional world that are available from the two-dimensional retinal image, especially motion cues and cues of the gradient of texture. Gibson's theory is a deterministic one, and it is his thesis that the visual stimulus contains enough information to provide veridical perception of distal objects. However, Gibson's theory does not account for the fact that perceptual responses are inherently stochastic.

Brunswik (ref. 2), on the other hand, proposes the perceptual theory of probabilistic functionalism. Unlike Gibson, he assumes that the perceptual process is random, and that people are continually learning about the validity of the cues of the proximal stimuli. Brunswik proposed that massive ecological surveys be taken to determine the correlations between the range of stimuli and the perceptual response. He, unlike Gibson, felt that a detailed examination of the proximal stimulus would not be a worthwhile course of action because the real scenes observed everyday do not contain these limited number of stimuli. The major criticism of Brunswik's approach seems to be that there is no hope or interest in finding the stimuli which determine the various responses. (See Hochberg (ref. 3) for a further discussion of Gibson's and Brunswik's theories.)

In this paper we present a model of visual space perception which has been influenced by Gibson and Brunswik. Briefly, it models visual space perception by a Bayesian processor which operates on the proximal stimuli; the percept is a conditional probability density function. The model has the advantage that the many stimuli noted by Gibson are incorporated, yet it describes the randomness and ambiguities observed in experiments.

This paper contains a review of the very basic concepts of Bayesian estimated theory; a description and discussion of visual space perception model; a notation of the similarities and differences to signal detection theory in psychophysics; and an application of the model to some experimental results by Kinchla and Allen (ref. 4).

REVIEW OF BAYESIAN ESTIMATION THEORY

In thissection we give a very brief review of some important results in the field of estimation theory. A discussion of the results for estimating constant but unknown parameters is presented first, and a treatment for dynamic systems Markov processes) follows. Jazwinski (ref. 5) and Nahi (ref. 6) are general references.

Let us denote by X the (column) vector of variables which we are trying to estimate, and let $z(t)$ be a vector of "measurements" or variables observed at time t. Let $Z(t)$ be thecollection of all known information up through time t, i.e., the a priori distribution and the set of all measurements taken between t_0 and t; $p(x)$ represents the a priori probability density function of the vector X at t_0. In concept, the Bayesian processor does nothing more than find the conditional distribution of X from the measurements z and the a priori distribution. This is shown schematically in figure 1.

Parameter Estimation

The Bayesian processor requires the joint probability density function (PDF) $p(x,z)$. Many times this is not available directly and must be calculated from a measurement equation

$$z=h(x,v) \tag{1}$$

and the PDF $p(x,v)$, where v is a vector of observation disturbances. The PDF of x conditioned on one observation z and the a priori distribution, i.e., $Z=\{p(x),z\}$ is given by Bayes' Rule

$$p(x|Z)=\frac{p(x,z)}{p(z)}=\frac{p(z|x)p(x)}{p(z)} \tag{2}$$

where $p(z)$ is the scale factor found by integrating the numerator with respect to x. This conditional PDF can be used to find a variety of estimates, e.g., conditional mean, conditional median, or conditional mode.

Figure 2 shows an example of an a priori probability density function for a scalar x and examples of conditional probabilities that result from particular realizations of z.

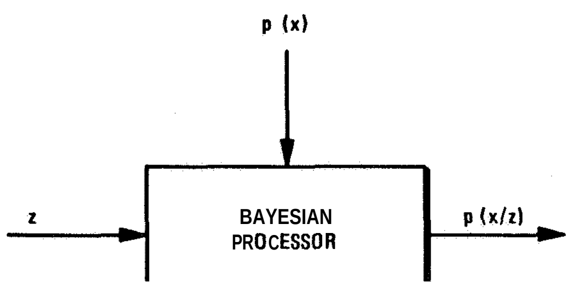

FIGURE 1.—Inputs and output of a Bayesian processor.

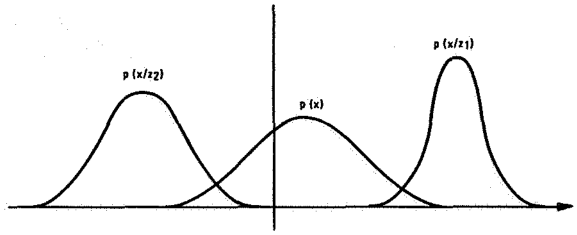

FIGURE 2.—Examples of a priori and conditional probability density functions.

Linear regression is a Bayesian estimation technique which uses the first two moments of $p(x,z)$. For X and z of zero mean, the minimum variance linear estimate $\hat{x}$ is

$$\hat{x}=E(xz^T)E(zz^T)^{-1}z \tag{3}$$

where the superscript T denotes transpose.

Markov Processes

A Bayesian processor for a Markov process can be found when observations related to the state of the process are available. In general the processor computes $p[x(t_1)|Z(t_2)]$ where $x(t_1)$ is the state of the process at time t_1, and

$$Z(t_2)=\{p[x(t_0)],\ z(s),\ t_0\leq s\leq t_2\}.$$

This corresponds to predicting, filtering, and smoothing when t_1 is greater than, equal to, and less than t_2, respectively.

The Bayesian processor requires the following items to carry out the computations of the conditional PDF. The discussion will be limited to continuous-state, continuous-time situations with the knowledge that the general concepts hold for discrete-state and/or discrete time models.

(1) *The state* equation.—This is an equation which describes the evolution of the state of the Markov process with time. This may be written

$$\frac{dx(t)}{dt}=\dot{x}(t)=f(x(t),\ w(t),\ t) \tag{4}$$

where $x(t)$ is the state at time t, and $w(t)$ a random forcing function (sometimes called the process noise). For equation (4) to be a Markov process with state $x(t)$, $w(t_1)$ and $w(t_2)$ must be independent for $t_1\neq t_2$. It is also referred to as white noise since its power spectral density function is a constant, that is, it contains equal amounts of power at all frequencies. The prescription of $p(w(t))$ is part of the specification for the state equation.

(2) The *a* priori distribution.—The a priori distribution of $x(t_0)$ is required not only to describe the state before any observations are taken but it is the "initial condition" for the computation of the conditional distribution. The state equation (4) and the a priori distribution can be used to calculate $p[x(t)]$ the a priori distribution of the state for $t > t_0$, i.e., without the benefit of any observations. In general, this requires a solution to a partial differential equation, although the linear-Gaussian ease can be solved with a vector and matrix differential equation for the mean and covariance (Jazwinski (ref. *5*)).

(3) The measurement equation.—The measurement equation describes the instantaneous relationship between the observations $z(t)$ and the state. It may be written as in equation (1) but now $z(t)$ is a vector of time functions:

$$z(t)=h(x(t),\ v(t),\ t). \qquad (5)$$

The observation noise $v(t_1)$ is independent of $v(t_2)$ if $t_1 \neq t_2$; $p[v(t)]$ is needed to complete the description of the measurement equation.

The following example may help to solidify the Bayesian concepts. Suppose we have a linear RC circuit being driven by white noise $w(t)$. We measure the voltage across the capacitor with a voltmeter which gives erroneous readings, and wish to find the probability density function of the capacitor voltage as a function of time. In this case $x(t)$ is the capacitor voltage, $w(t)$ is the input voltage to the circuit, $z(t)$ is the voltmeter reading and $v(t)$ is the source of error in the voltmeter. The result of the calculating $p[x(t)|Z(t)]$ continuously might appear as shown in figure 3 which shows a particular realization for $x(t)$, $z(t)$, and $p[x(t)|Z(t)]$ at several time instants. This conditional PDF can be used to derive an estimate for the voltage at any time.

A MODEL FOR VISUAL SPACE PERCEPTION

In this section we present a probabilistic model for visual space perception that exhibits important characteristics of the human response:

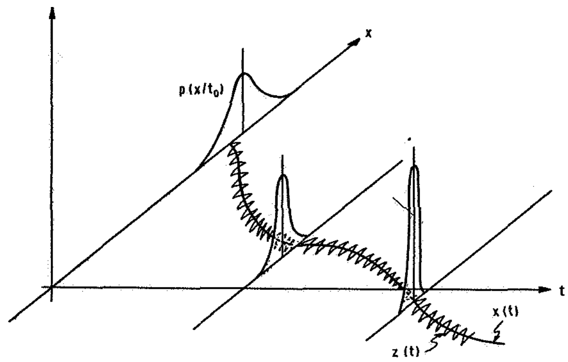

FIGURE 3.—Sketch of conditional probability density function.

(1) On any one trial observers may express uncertainty about objects' relationships (e.g., distance).

(2) Repeated presentation of stimuli results in a distribution of responses. These may fall into multiple categories if the stimulus is ambiguous (for example, the Necker Cube).

The proposed model for visual space perception is shown in figure 4. The central element in this model is a Bayesian processor consisting of a state equation, an a priori distribution, and a measurement equation. The processor operates on $z(t)$, which is the vector of stimuli as modified by the sensory processes, and yields a conditional probability density function for the state. There is some justification for calling this conditional PDF the percept because it recognizes the possibility of uncertainty of interrelationships between objects after observations have been made.

The state equation describes the behavior of the perceiver's knowledge of the distal world. The process noise is used to account for the two types of uncertainties that the perceiver has about the distal world. The first type is the imprecise modeling of distal constraints, e.g., assuming constant velocity motion when an object is actually accelerating. The second type is the degradation of certainty due to imperfect memory process.

The sensory process and/or response processes shown in figure 4 must be of a random nature to yield different responses in different trials to the same proximal stimulus if the parameters of the Bayesian processor remain constant. The determination of these two processes is similar to a problem faced by early users of signal detection

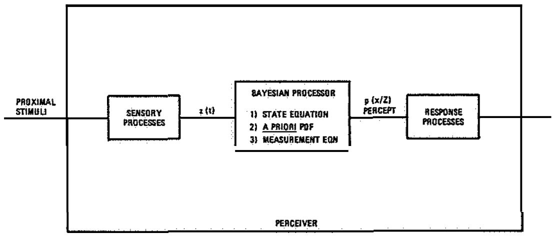

FIGURE 4.—A Bayesian model for visual space perception.

theory in psychophysics, and we postpone discussion on sensory and response processes until the section which compares the Bayesian processor with signal detection theory.

DISCUSSION OF THE MODEL

Rationale for the Model

There are four major reasons for postulating the model described above. The first is that the model is probabilistic: it describes the uncertainty on any one trial through a conditional PDF (the percept), and the ensemble of percepts which may exhibit ambiguous responses. These are two major characteristics of visual space perception.

A second reason for choosing a model of this type is that it provides the capability for modeling perception of state variables of which there is no direct observation. An example of this is the perception of velocity even though only the instantaneous position is contained in the proximal stimulus.

A third advantage of this model is that it provides the capability for modeling space perception in a time-varying dynamic environment. Arbitrary motion of objects relative to the observer or to other objects is allowed, and the static scene used in so many studies (e.g., size-distance, shape-slant) is just a special case. Whether or not the distal stimulus changes, the conditional distribution of the internal state will exhibit transient behavior.

The fourth reason for choosing this model is the possibility for unifying many of the concepts and experimental results in visual space perception. The model is general and perhaps should be considered as a framework for organizing the important variables and their interplay. It is not necessary that all three parts of the model receive equal attention, and in fact, portions of the model might be ignored in certain situations. See the discussion below on how the model might be applied in some specific situations.

Relationship to Other Work

There are two aspects of the proposed model that are related to prior work in human information processing: Bayesian processing, and internal models. Bayesian models have been used to describe decision making (see, for example, Edwards (ref. 7)). It has been found that humans accumulate information at approximately one half the rate of an optimal Bayesian processor in discrete observations. The same effect can be realized in visual space perception by postulating noise sources to reduce the rate of accumulation of information. The primary difference between the Bayesian model for space perception and the Bayesian decision maker is that the former is more of a reflex action (Brunswick (ref. *2*)) whereas the latter involves conscious deliberation and thought.

The concept of an internal model of the distal stimulus has been found useful in other models of information processing (e.g., Carbonell (ref. 8), Smallwood (ref. 9), and Carbonell, Ward, and Senders (ref. 10)). In these papers the internal model is used for extrapolation, that is, to describe the time evolution of the observer's knowledge in the presence of null stimuli; this function is performed by the state equation in the visual space perception model. Another role of the internal model is to relate distal to proximal stimuli, and this is accomplished through the measurement equation in the space perception model.

It is interesting to note that the concept of an internal model may have a physiological basis. In discussing the orienting reflex, Sokolov (ref. **11)** gives some justification for a neuronal model of the stimulus. The orienting reflex is activated when a mismatch occurs between the afferent signals and the extrapolations of the model. On the behavioral level, Berlyne (ref. **12)** uses the concept of the novelty of a stimulus as a determinant of arousal and exploratory behavior. The very notion of novelty implies a standard for comparing stimuli, i.e., an internal model.

Potential Applications of the Model

The overall purpose of the model is that of predicting visual space perception with some degree of certainty, and that of providing insight into the perceptual process. Very little has been done in the way of experimental verification of this model, so at this point in time no definite statements can be made concerning the constancies of the model nor the range of validity. Corroboration of the model with previously published results is usually difficult because of the lack of proper data presentation to evaluate the model.

However, there are several situations in which the model may prove valuable. One of these is the problem of determining the relative strengths of visual cues and higher order variables in perception, especially in an environment which is more or less uncontrolled (Brunswick (ref. **2**)) but which may be measured. This includes the possibility of taking observer motion into account. If the parameters of the model are sensitive indicators, then it might be useful as a descriptor of changes in perceptual response in learning, attention, and other studies.

Application of the Model

The model as presented is limited to visual space perception, i.e., the perception of position, orientation, and motion of objects relative to the observer. The studies of figure, form, etc. cannot be treated in this formulation because they imply spatial processing of the proximal stimulus as opposed to the temporal processing considered here.

The model is general, and one of the major criticisms of it is that there are too many degrees of freedom to provide useful quantitative insight into the perceptual process. Yet the model is simple in concept, and we feel that judicious use of the general framework will be fruitful. Some specific applications have already been carried out by other investigators and one of these (on movement perception) will be discussed in detail.

The basic procedure in determining the form of the model is to first derive an estimate of the conditional PDF of the internal state, and from these experimental data, determine an a priori distribution, a state equation, and a measurement equation which would produce the measured conditional PDF. We note here that the results of this type of analysis will not, in general, yield unique results. First, the number and choice of internal state variables must be determined, that is, the internal state variables might be a subset of the distal state variables, but which subset? Second, there may be intervening parameters incompletely specified for a given input/output function. For example, their sum or product is a known quantity, but no other equations exist.

The freedom in choosing the state equation and measurement equation is immense, yet simplifications in each application can be made to reduce the choice to a manageable level. Care must be exercised, however, since as Brunswik (ref. **2**) points out, perception is an irrational process, and assumptions which are reasonable on the basis of physical laws may be inappropriate. However, some general guidelines can be used. In situations involving the (perhaps) complex motion of a simple object, the emphasis should be placed on the state equation since there is no complexity in determining what parameters make up the proximal stimuli. For studies of depth perception, size estimation, and orientation from static scenes, the state equation can be ignored, and the effort concentrated on the measurement equation which contains a myriad of cues (Gibson (ref. 1)) and functional relationships between the proximal stimulus and the elements of the internal state vector. There are many situations, however, where both the state and measurement equations are necessary, as would be the case for depth perception when there is relative motion between the object and the observer.

The a priori distribution represents such effects as experimental set and previous knowledge of the state variables, e.g. "familiar size" in depth perception experiments. (Whether familiar size is beneficial in depth perception or not is another matter.) The conditional PDF may also undergo abrupt changes if discrete (rather than continuous) measurements are incorporated. This situation arises when a subject is given some feedback on his errors in perception, leading to improved performance, reductions of illusions, etc.

COMPARISON WITH STATISTICAL DECISION THEORY MODELS IN PSYCHOPHYSICS

The main purpose of the application of detection theory in psychophysics is to separate the detectability of a signal from the decision processes of the observer (Green and Swets (ref. **13**)). The Bayesian model of perception is an attempt to model perceptual behavior and to examine the interactions of visual cues and the internal models. In most of the experiments using signal detection theory (SDT), the signal is a single point in signal (state) space, whereas we are dealing with a continuum. Moreover, the stimuli are usually discrete time presentations whereas we are allowing for a continuous presentation of signal and noise.

As a further comparison of the two techniques, we next formulate the signal detection problem as a special case of the Bayesian processor. Although the majority of the work seems to have been done in psychoacoustics (see, for example, Swets (ref. 14)), the original application was in visual detection (Tanner and Swets (ref. **15**)). This exercise will also point up the need for random elements in the sensory and response processes in the Bayesian perception model.

Let x be the signal level which has a probability p of being X and $(1-p)$ of being zero. An observation z of signal x plus noise v is made:

$$z=x+v. \tag{6}$$

Using this information and Bayes' Rule the a posteriori probability density function for x becomes

$$p(x|z)=\frac{p(z|x)p(x)}{p(z)} \tag{7}$$

$$=\frac{p_v(z-x)[(1-p)\delta(x)+p\delta(x-X)]}{(1-p)p_v(z)+p_v(z-X)} \tag{8}$$

where $p_v(\ )$ is the PDF of the observation noise v. Because the a priori distribution of the signal level is limited to two values, the a priori PDF of x is a pair of impulses at $x=0$ and $x=X$, of areas $(1-p)$ and p respectively. The a posteriori PDF remains at two impulses, but of modified areas depending on the distribution of the noise and the observed value of z.

The SDT model assumes that the decision is based on the likelihood ratio * being greater or less than some criterion level. Because the response process is perfect, there must be some source of uncertainty in the model to account for incorrect responses when the proximal stimulus consists of a small signal and no noise. The SDT model, therefore, assumes an internal source of noise between the proximal stimulus, which is the block labeled "sensory processes" in figure **4**, in addition to the ideal response process.

As shown in Green and Swets (ref. 13), and as can be easily derived from equation (8), the ratio of a posteriori probabilities and the likelihood ratio are related by

$$\frac{P(x=X|z)}{P(x=0|z)}=\frac{p}{1-p}\ell(z). \tag{9}$$

Since these are monotonically related, the likelihood criterion and the ratio of a posteriori probabilities are equivalent statistics upon which decisions can be based (Green and Swets (ref. **13**)). The most important point, however, is that the wealth of experience in using SDT techniques may be used to great advantage in experimental verification of the model proposed in this paper if the response processes are assumed ideal, and the sensory processes introduce uncertainty. In other words, we can use SDT to help identify and measure the conditional PDFs of the perceptual process.

Perfect response may not always be the most accurate model, however, since the criterion level of the observer may change from trial to trial, and optimal thresholds involve precisely defined probabilities and entries in the payoff matrix which can hardly be expected to be realized internally by the observer (Swets (ref. **14**, ch. **4**)). In addition to these noisy decision processes, faulty memory of the signal will produce behavior not predicted by the model. This can easily be

*The likelihood ratio for this problem is $\ell(z)=p(z|x)=X)/p(z|x=0)$.

modelled with the Bayesian processor by defining the signal level to come from the following distribution

$$x = \begin{matrix} 0 & \text{with prob. } 1-p \\ X+m & \text{with prob. } p \end{matrix}$$

where m is a memory "error" drawn from $p_m(\ )$. The a posteriori distribution of x conditioned on an observation z is then

$$p(x|z) = \frac{p_v(z-x)[(1-p)\,\delta(x) + p \cdot p_m(x-X)]}{p(z)} \quad (10)$$

APPLICATION OF THE MODEL TO VISUAL MOVEMENT PERCEPTION

Experimental Apparatus

In this section we apply a special case of the Bayesian model of visual space perception to the problem of absolute movement perception and use data reported in Kinchla and Allan (ref, **4**). The experiments involve the presentation of clearly visible tungsten sources subtending an angle of 0.036° energized for 0.1 sec. A *stimulus* pattern (Xi) is the successive presentation of two lights with intervening time intervals of 0.5, 1.0, 1.5 or 2.0 sec. The movement *(m,)* of the stimulus pattern is the angular displacement of the second light relative to the first. Stimulus patterns were presented in pairs (with equal probability) to well trained observers familiar with the patterns. This allows the use of SDT to remove the effects of response thresholds from the data.

The Bayesian Model

We now proceed to specify the form of the model for the experiment described above. Four sources of uncertainty will be accounted for:

(1) Unperceived eye movement

(2) Recorded but unexecuted eye movement commands (efferent copy)

(3) Memory uncertainty

(4) Sensory noise.

State equation.—Rather than deal with an explicit form of the state equation, we will be concerned with the conditional distributions which it describes, and these will be considered shortly.

A priori distribution.—The experiment was performed in the dark with almost no information about the position of the first light. Thus we will assume that the a priori position of the light is $N[0, \infty]$.

Measurement equation.—At the time of the first light we assume that the stimulus received by the Bayesian processor is

$$z(0) = x(0) + v(0) \quad (12)$$

where $x(0)$ is the "position" of the light in a stimulus domain and $v(0)$ is sensory noise assumed to be $N[0,\sigma_v^2]$. At the time of the second light we bave

$$z(T) = x(T) + v(T) + w_e(T) \quad (13)$$

where $x(T)$ is the position of the light at time T, $v(T)$ is sensory noise $N[0,\sigma_v^2]$, and $w_e(T)$ is the change in stimulus due to unperceived eye movements, with distribution $N[0,\sigma_e^2(T)]$.

Conditional Distributions

We now present the calculations that would be carried out under the conditions described above. The general form of these calculations may be found in Nahi (ref. **6**) or Jazwinski (ref. *5*).

Because the a priori variance of the initial position is very large (specifically, much larger than σ_v^2), it can be shown that the conditional distribution of $x(0)$ just after the first light ($t=0^+$) is $N[z(0),\sigma_v^2]$. Just prior to the second light the conditional distribution of $x(0)$ is $N[z(0)+u(T),\sigma_v^2+\sigma_{\text{mem}}^2(T)]$ where $u(T)$ is the effect of unexecuted eye movement commands and $\sigma_{\text{mem}}(T)$ is the increase in uncertainty due to faulty memory.

The stimulus received by the processor at time T can be expressed by

$$z(T) = x(0) + m + v(T) + w_e(T) \quad (14)$$

where m is the movement m_i or m_j. Thus, when $z(T)$ arrives, the Bayesian processor must decide whether it comes from a Gaussian distribution with variance $2\sigma_v^2+\sigma_e^2(T)+\sigma_{\text{mem}}^2(T)$ and mean either $z(0)+u(T)+m_i$ or $z(0)+u(T)+m_j$.

To evaluate the parameters of the model from experimental data, we assume that the ideal decision making portion of the model behaves as follows. Let R_j denote the response "stimulus

S_j," and the probability of this response conditioned on S_j is

$$P(R_j|S_j)=P[z(T)>z(0)+u(T)+m_i+\beta|m=m_j] \quad (15)$$

where β is the threshold criterion relative to the smaller mean of the conditional distributions of $z(T)$. Using equation (14) this becomes

$$P(R_j|S_j)=P[x(0)-z(0)+v(T)+w_e(T)-u(T)>\beta-(m_j-m_i)] \quad (16)$$
$$=1-F[Z(R_j|S_j)]$$

where $F(\ )$ is the cumulative distribution function for a normalized Gaussian variable, and $Z(R_j|S_j)$ is given by

$$Z(R_j|S_j)=\frac{\beta-(m_j-m_i)}{\sqrt{2\sigma_v^2+\sigma_u^2(T)+\sigma_{\text{mem}}^2(T)+\sigma_e^2(T)}} \quad (17)$$

Similarly, the probability of response R_j conditioned on stimulus S_i is

$$P(R_j|S_i)=P[z(T)>z(0)+u(T)+m_i+\beta|m=m_i] \quad (18)$$
$$=1-F[Z(R_j|S_i)]$$

where

$$Z(R_j|S_i)=\frac{\beta}{\sqrt{2\sigma_v^2+\sigma_u^2(T)+\sigma_{\text{mem}}^2(T)+\sigma_e^2(T)}} \quad (19)$$

At this point we specify the functional form for the time-dependent variances. In particular it is assumed that

$$\sigma_u^2(T)+\sigma_{\text{mem}}^2(T)+\sigma_e^2(T)=\Phi T \quad (20)$$

where Φ is a constant. This form assumes that the variables are drawn from random walk processes, but as Kinchla and Allan (ref. 4) point out, there is little if any experimental data on the variances of these variables, although Carbonell et al. (ref. 10) have found this form to be of some value in other contexts.

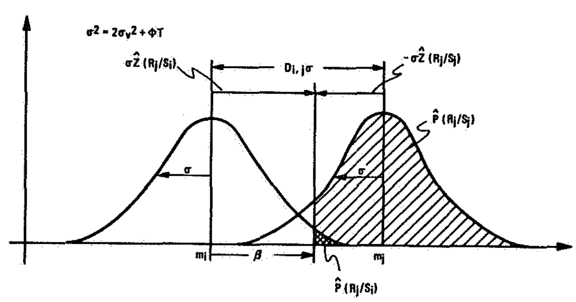

FIGURE 5.—Equivalent detection problem.

Figure 5 shows the equivalent detection problem. The "discriminability index" (Kinchla and Allan, ref. 4) is the normalized distance between distributions, and is given by

$$D_{j,i}=Z(R_j|S_i)-Z(R_j|S_j) \quad (21)$$

$$=\frac{m_j-m_i}{\sqrt{2\sigma_v^2+\Phi T}}. \quad (22)$$

The parameters of the model are found by observing the empirical probabilities $\hat{P}(R_j|S_i)$ and $\hat{P}(R_j|S_j)$, looking up the corresponding values of Z in a normal probability table, and then

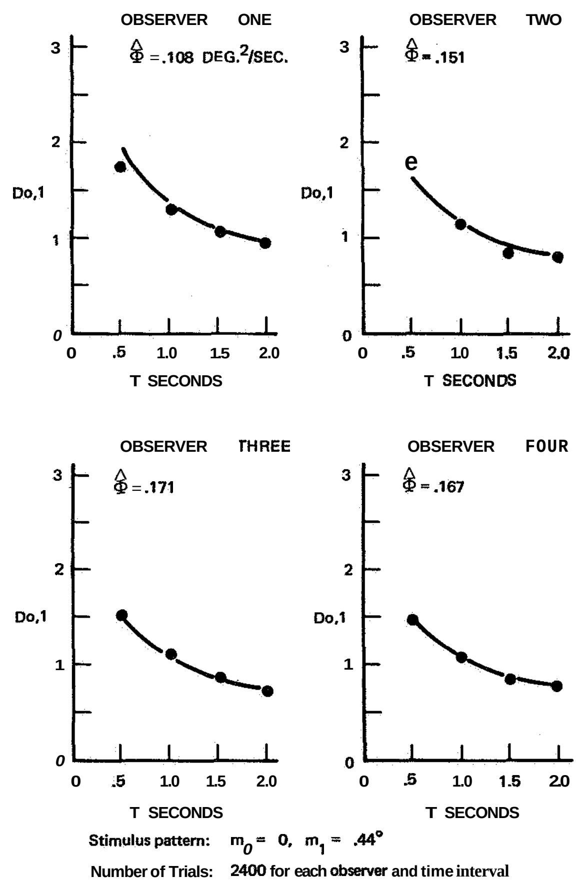

FIGURE 6.—Data from Kinchla and Allan (ref. 4) for stationary judgement task (stimulus pattern: $m_0=0$, $m_1=0.44°$; number of trials: 2400 for each observer and time interval).

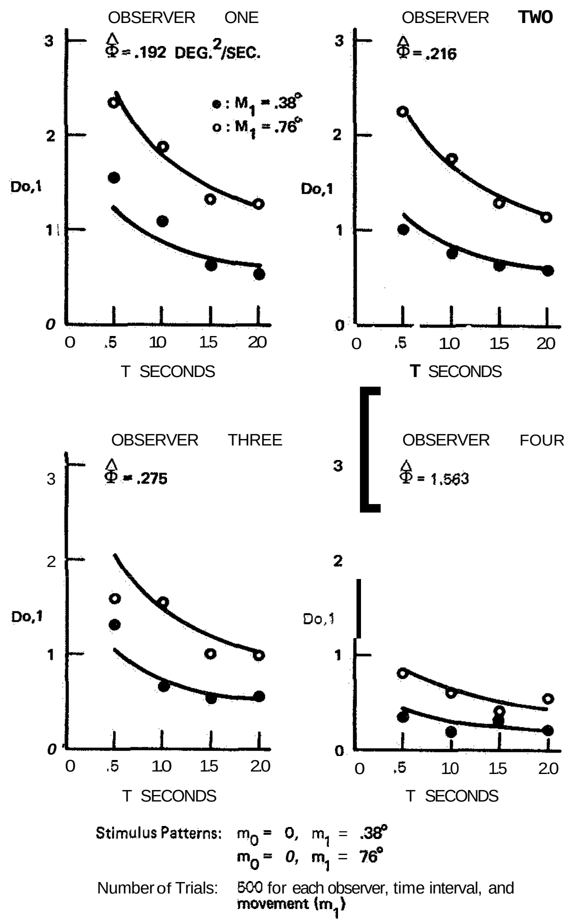

FIGURE 7.—Data from Kinchla and Allan (ref. 4) for stationary judgement task (stimulus patterns: $m_0=0$, $m_1=0.38°$, $m_0=0$, $m_1=76°$; number of trials: 500 for each observer, time interval, and movement m_1).

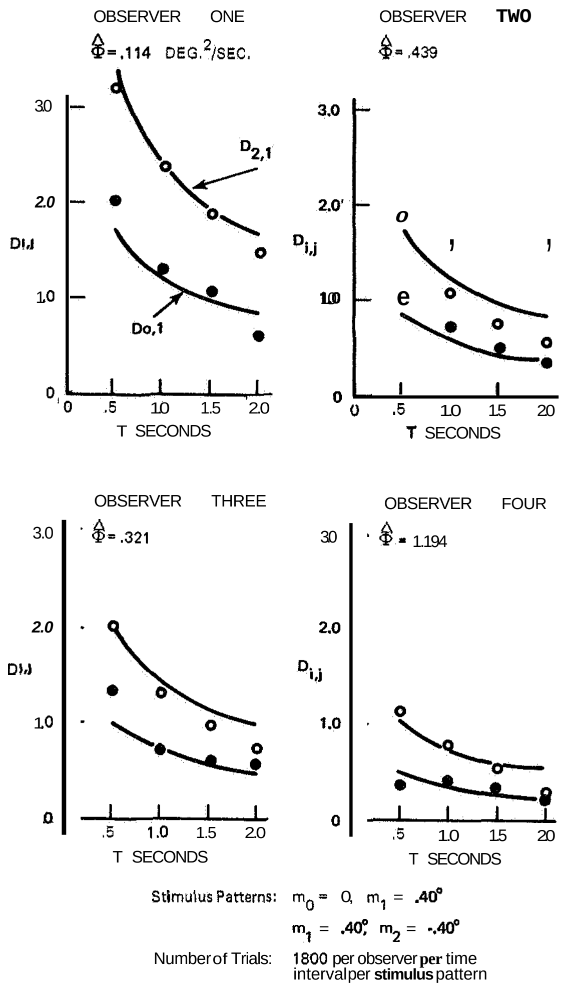

FIGURE 8.—Data from Kinchla and Allan (ref. 4) for stationary judgment and movement discrimination tasks (stimulus patterns: $m_0=0$, $m_1=0.40°$, $m_1=0.40°$, $m_2=-0.40$"; number of trials: 1800 per observer, per time interval, per stimulus pattern).

finding $\hat{D}_{j,i}$ from equation (21). Those values of $\hat{\sigma}_v^2$ and $\hat{\Phi}$ which give closest agreement in the least squares sense via equation (22) are the parameters of the model.

Experimental Results

Figures 6, 7, and 8 show data from three experiments. The first two are stationary judgment tasks with $m_0=0$ and $m_1=0.44°$ in experiment 1; $m_0=0$ and $m_1-0.38°$ or 0.76" in experiment 2. Experiment 3 contained both stationary and directional judgment tasks for which $m_1=40°$, $m_2=-0.40$" in one pair of stimulus patterns, and $m_1=0$, $m_2=-0.40$" for the other pattern. These curves are found by minimizing sum of squared discrepancies between the observed statistic and that predicted by the model. In all cases the observation noise σ_v^2 was negligible and the curves are determined by one parameter $\hat{\Phi}$. The model accounts for at least 90 percent of the variance of $\hat{D}$ in all cases, and typically accounts for 95 percent. The behavior of Φ for the four observers is shown in table 1. Note that there are individual differences for $\hat{\Phi}$ between subjects (which might be the basis for evaluating certain skills), as well as some variation within subjects. It is not known what time interval elapsed between experiments.

TABLE 1.—*Estimates of Φ in Three Experiments (Kinchla and Allan, ref. 4)*

	Subject			
Experiment	1	2	3	4
1	0.108	0.151	0.171	0.167
2	.192	.216	.275	1.563
3	.114	.439	.321	1.194

CONCLUSIONS

We have proposed a model for visual space perception in which a Bayesian processor provides the percept, a conditional probability distribution of the knowledge of distal objects. This model incorporates desirable qualities of the theories of Gibson and Brunswik and has the potential for unifying many of the concepts and results in visual space perception. Signal detection theory may be considered as a special case of the model, but its primary power lies in its ability to extract percepts independently of response thresholds. The model was applied to the problem of visual movement perception and its parameters evaluated with previously published data.

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