# 28. Estimation of Automobile-Driver Describing Functions From Highway Tests Using the Double Steering Wheel*

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The automobile-driver describing function for lateral position control was estimated for three subjects from frequency response analysis of straight road test results. The measurement procedure employed an instrumented full size sedan with known steering response characteristics, and equipped with a lateral lane position measuring device based on video detection of white stripe lane markings. Forcing functions were inserted through a servo driven "double steering wheel" coupling the driver to the steering system proper. Random appearing, Gaussian, and transient time functions were used.

The quasi-linear models fitted to the random appearing input frequency response characterized the driver as compensating for lateral position error in a proportional, derivative, and integral manner. Similar parameters were fitted to the Gabor transformed frequency response of the driver to transient functions. A fourth term corresponding to response to lateral acceleration was determined by matching the time response histories of the model to the experimental results. This model parameter also accounted for the driver's direct response to steering wheel reaction torques.

The time histories show evidence of pulse-like nonlinear behavior during extended response to step transients which appear as high frequency remnant power.

## INTRODUCTION

The present entirely manual controlled approach to road vehicle operation demands high performance of the man-vehicle closed-loop system to ensure safe transportation. In this paper we consider the lateral control or steering subsystem alone. Here the prime requirement is to minimize lateral position error relative to a driver selected path on the pavement or other surface, despite disturbances due to mechanical and aerodynamic forces acting on the vehicle. This requirement implies good closed-loop regulation, conventionally accomplished by a combination of vehicle handling qualities and driver responsiveness. Previous researchers have demonstrated

the feasibility of a frequency domain analytical approach to steering subsystem performance using now classical human operator theory (Weir (ref. 1); McRuer and Weir (ref. 2)), and at a previous Annual Manual we have reported initial measurements of driver describing functions obtained under highway conditions using the newly developed "double steering wheel" forcing function injector (Crossman and Szostak (ref. 3)). The present paper reports the results of extended application of this technique, and considers various possible driver models that can be fitted to the data. As pointed out in the previous paper, one can also employ highway curvature to force the steering system, and describing function estimation for this case, together with the relevant psychophysics, will form the subject of a later report.

We have previously shown (ref. 3) that the vehicle yields a zero order path curvature response to steering wheel angle, so that the driver could ideally employ an open-loop control mode

for typical highway geometry.* However, it is clear that this system configuration offers inadequate precision under typical real highway conditions. When traversing a path of approximately constant (or zero) curvature defined by lane markings or other visible features in the highway environment, an additional forcing function is injected partly in visible form, as lane marking deviations, and partly invisible as lateral force on the car due to crosswind and surface irregularity. Hence the driver experiences a mixed compensatory and pursuit tracking task. Some advance information may be obtained from visible surface undulation, permitting a preview mode, and on a well-known road there may be some precognition. In normal circumstances, quickening, via vehicle heading is also available. Thus the total steering task presents a complex generally time-varying mixture of tracking modes, and a driver model based purely on the single input compensatory case is unlikely to yield adequate precision.

Nevertheless, Weir (ref. 1) has surveyed the various possible compensatory loop closures using a nominal operator describing function, and shown that pure lateral position response would be poor, yaw rate response relatively good, and so forth.

As noted by McRuer and Weir (ref. 2) there are significant differences between a laboratory task where stimuli are explicit and the task of car steering with its proprioceptive and motion stimuli, and which has a visual field rich in cues. The gap may be bridges by simulation. Wierwille et al. (ref. 4) and Weir and Wojcik (ref. 5) have reported describing functions estimated in fixed-base automobile simulators, but to our knowledge no genuine field test data have yet been reported.

In the present study we sought to estimate parameters of the random-appearing, Gaussian, and transient open-loop describing functions representing the regular automobile driver responding to unpredictable steering disturbance inputs under essentially compensatory tracking conditions on an actual highway in a standard sedan car.

A leading feature of the results has been the appearance of a low frequency first order gain increase and phase lag corresponding to pure integration down to frequencies on the order of 0.02 Hz. This may be related to the phenomenon of low frequency "phase droop" as noted by McRuer et al. (ref. 6). The extended crossover model for compensatory control tasks accounts for low frequency phase lag occurring with no measured break point in the amplitude ratio, as being directly related to the operator's neuromuscular system response. However, Taylor (ref. 7) found no supporting evidence for the low frequency phase lag in his time domain models with extended record lengths. Both the classical Goodyear study (Cacioppo (ref. 8)) and certain Japanese researchers (Iguchi (ref. 9) and Kobayashi (ref. 10) have represented the human operator as a parameter-adaptive Proportional-Integral-Derivative (PID) controller, and this is particularly attractive in the highway case since, as pointed out by Weir (ref. 1) both proportional and derivative data are directly available to the driver. Response based on a suitable weighted sum of these together with timed repetitions to correct small lateral position errors, seemed initially plausible to us in the light of earlier highway studies, and would indeed generate a PID model form. The specific experiments were planned with this in mind.

## EXPERIMENTAL EQUIPMENT AND DESIGN

Figure 1 shows the experimental configuration in schematic form. Novel features were the use of a special purpose forcing function injector and a lateral position recording device. The "plant" consisted of a standard sedan car whose steering response had been previously determined. The experimental trials were run during a single day on a preselected stretch of test road, all equipment and test subjects being transported to the site in the experimental car.

*The "double steering wheel" forcing function injector.*—This device consisted of a second steering wheel mounted concentrically on the
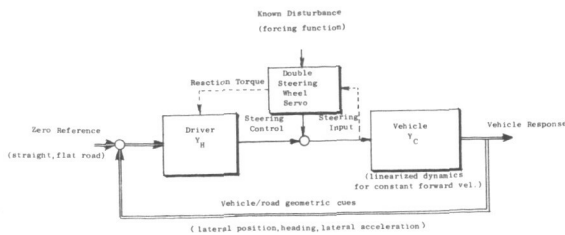
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FIGURE 1.—Compensatory driving task: disturbance injected into the vehicle-driver steering control loop via the double steering wheel.
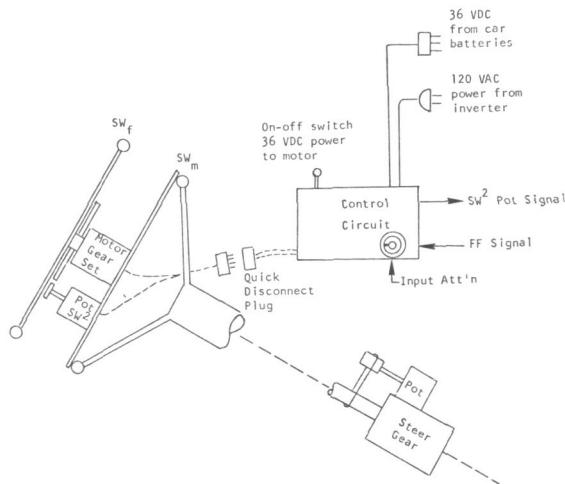


FIGURE 2.—Mechanical layout of the double steering wheel differential drive forcing function system.



FIGURE 3.—The double steering wheel.

car's regular steering wheel and linked to it through an angular positioning servomechanism comprising an electric motor, gear train and servo amplifier, their relative angular position being commanded by a voltage reference signal (fig. 2). The driver steered the car via the second (top) steering wheel and the required forcing function was thus introduced as a time-varying angular increment to his steering output. The unloaded dynamic response of the servomechanism to voltage inputs was essentially second order with $f_{co} = 1.8$ cps and $\zeta = 0.5$, but the damping factor was increased under field conditions due to loading by the driver's limbs and the vehicle steering system. Consequently, a true step command voltage input would appear to the driver to yield an overdamped second order approach to a new angular displacement. However, the forcing function $\theta_F$, assumed in the data analysis,
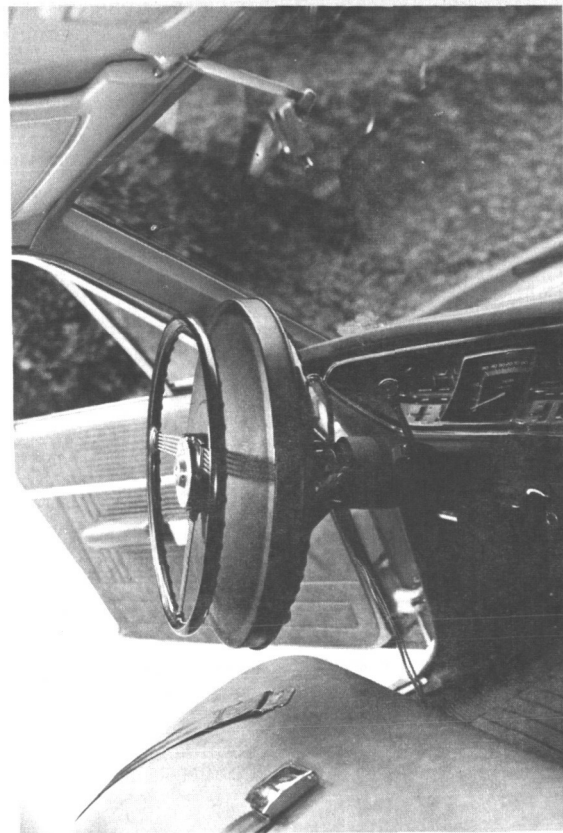
was the measured output rather than the input to the angular positioning servomechanism.

The driver's view of the lower steering wheel was obscured by a cover to prevent direct visual feedback (fig. 3).

*The lateral position recording system.*—An optical sensing system based on a closed-circuit television camera provided a continuous record of the lateral position of the vehicle relative to the painted highway centerline stripe. This was accomplished as follows. A convex mirror reflected an image of the pavement surface into the lens of a commercial TV camera mounted at the rear of the car, the horizontal sweep being aligned with the lane markings while the vertical sweep provided a lateral scan. The video voltage output was filtered to remove the horizontal scan component, leaving a radar-type pulse waveform

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* Fuller descriptions are given in report HFT 66–9 (ref. 11).

with **60 Hz** repetition rate. The peak voltage due to the relatively bright lane marking triggered a rectangular pulse and this in turn caused sampling of a synchronized timebase, providing a voltage level proportional to the lateral displacement of the centerline on the camera screen. This level was maintained between sweeps, and in the absence of the white stripe, by a zero order hold circuit. The resulting signal was recorded by an onboard instrumentation tape recorder.

Since the camera was mounted approximately **12 ft** rearward from the C.G. of the vehicle, the lateral position trace was delayed approximately **0.27** seconds at **30** mph. Interruptions in the painted centerline, bridged by the zero order hold circuit, introduced a further delay averaging **0.20 ± 0.05** sec at **30** mph, for a total time delay of **0.47 ± 0.05** sec. This was corrected during off line data analysis.

**The** test car dynamics and instrumentation.[*]— The test vehicle was a **1965** Ford sedan whose sinusoidal steering response dynamics had been previously determined. This was done at a series of test frequencies by manual application of sinusoidal steering inputs, the inputs (steering wheel angular displacement, speed), and outputs (yaw rate, roll rate, and lateral acceleration) being recorded at different points relative to the vehicle's C.G. In the frequency range of interest in the present context, the vehicle lateral position response to steering wheel angular displacement can be characterized as lagged double integration, the constant of integration being a function of forward speed:

$$y_, = \frac{E_{y_c}}{\Theta_s} \approx \frac{K}{s^2} \cdot \frac{1}{T_c s + 1}$$

where, at **30** mph, $K \approx 2.13$ cm/sec²/deg
$T_c \approx 0.25$ sec.

**The** *test* road and conditions.—The test road was a straight, almost flat, country road in new condition with **11** ft lanes and **5** ft paved shoulders, 8900 ft in length, permitting test runs of from **150** to **180** sec duration. There was some mist, visibility being estimated at **300** ft, and no wind at the time of the test, which was mid-day. There were solid white lane markings between

the road lanes and shoulder and a dashed center line striping.

Subjects and instructions.—All three subjects were Univ. of Calif. students with previous driving experience, but only one (subject C) had had previous experience with the double steering wheel. Their ages ranged from **22** through **40.**

The subjects were requested to keep the vehicle within the right half driving lane and to maintain a constant speed: for subject A and B, **30** mph; for subject C, **40** mph. An auxiliary tachometer was mounted on the dash to the side to display the speed since the cover between the two steering wheels blocked the view of the vehicle's standard speedometer. If necessary, the subject could regain normal control of the cay by simply reaching around the cover and grasping the vehicle's steering wheel.

Experimental design.—The three distinct forcing functions used are referred to below as random appearing (RA), gaussian noise (GN) and transient disturbance (TD). Each subject performed one run with RA and TD, and two with GN, in the order given in table **1.** As noted above, subject C maintained an average speed some **5** to **10** mph faster than the other subjects.

**(1)** Random appearing. The sum of sinusoidal components with the following amplitudes and frequencies (table **2)**

**(2)** Gaussian noise. White noise from an electronic generator [*] passed through a fourth-order band-pass filter [†] with the following settings:

Low pass cut-off frequency = **0.02** cps
High pass cut-off frequency = **0.25** cps
The peak disturbance amplitude was **±90°.**

**(3)** Transient disturbance. This included positive and negative impulses, steps and truncated ramps in random sequence. The bandwidth of the step and impulse inputs were limited by the response of the angular position servo, as follows.

(a) Step-wide disturbances with ±65° and ± **130"** amplitude, rise time respectively of 0.2 and **0.5** sec

(b) Impulse excursions of **40"** to 60" with approximately **0.3** sec duration and symmetrical waveshape (**0.15** sec rise, **0.15** sec fall)

(c) Ramp disturbances at ±60°/sec., terminating at ±**65°** and ± **130".**

TABLE *1. — Test Run Sequence*

| Run number | Subject | Duration | Type of forcing function | Peak amplitude double steering wheel | Average speed, mph |
|---|---|---|---|---|---|
| 1 | A | 182 sec | RA: (Mixed sinusoid) | ± 120" | 29 |
| 2 | B | 171 sec | | | 33 |
| 3 | C | 159 sec | | | 34 |
| 4 | A | 181 sec | GN : (Gaussian noise) | ± 90" | 31 |
| 5 | A | 180 sec | | | 30 |
| 6 | B | 181 sec | | | 32 |
| 7 | B | 167 sec | | | 33 |
| 8 | C | 156 sec | | | 36 |
| 9 | C | 140 sec | | | 39 |
| 10 | A | 191 sec | TD: (A mixture of step, | ±65° | 30 |
| 11 | B | 180 sec | ramp, and impulse | and 130" peak to peak | 32 |
| 12 | C | 144 sec | inputs) | swings | 40 |

TABLE 2.—*Forcing Functions Used in the Experiment* *

| | Normalized amplitude | Frequency | |
|---|---|---|---|
| | | Rad/sec | Cycles/sec |
| 1 | 1.0 | 0.070 | 0.0111 |
| 2 | 1.0 | .157 | .0250 |
| 3 | 1.0 | .393 | .0625 |
| 4 | 1.0 | .602 | .0957 |
| 5 | 1.0 | .969 | .1540 |
| 6 | 1.0 | 1.490 | .2370 |
| 7 | 1.0 | 2.540 | .4040 |
| 8 | 0.1 | 4.030 | .6410 |
| 9 | 0.1 | 7.570 | 1.2100 |

* Peak amplitude = ±120°
Note: This is identical with the STI forcing function.

## DATA ANALYSIS

The data analyzed were sample time histories of the following variables recorded on instrumentation tape:

*(1)* $\theta_F$ The measured angular output of the forcing function injector

*(2)* $\theta_s$ The angular displacement of the steering wheel, as sensed by a potentiometer coupled to the steering shaft

**(3)** $e_P$ The lateral position of the white stripe relative to the vehicle, sensed by the TV camera system

**(4)** $\theta_D$ The driver's output to the second steering wheel, obtained during playback by summing $\theta_F$ and $\theta_s$, before further processing.

To remove components above the Nyquist frequency, the above signals were passed through a linear phase second order low pass filter with cutoff frequency *1.8* Ha, and damping constant $\pi/4$. They were then digitized at 2.5 samples/sec with *12* bit precision.

Spectral response estimates with RA forcing functions were obtained from the digital data using two distinct techniques.

*(1)* A fast Fourier transform routine identified as BMDX92, part of a biomedical data processing package (Massey (ref. **13**)) running on the Berkeley CDC *6400* computer.

*(2)* A routine based on the recently described Gabor transform principle (Crossman and Delp (ref. 14)), and running on the laboratory PDP–8 computer.

Estimates formed by the two methods agreed quite closely where they could be validly compared.

TD forcing function data could only be analyzed by the Gabor transform technique. The GN data did not yield satisfactory results with either technique, due probably to inadequate run length, and are not considered further here.

## RESULTS

### Mixed Sinusoidal Input Driver Describing Function

Estimates $\hat{H}_F$ and $\hat{Y}_H$ respectively relating system (closed-loop) angular response to forcing function angle and driver response to lateral position error were formed once for each experimental run, and are presented in Bode plot form in figures 4, *5,* and *6.*
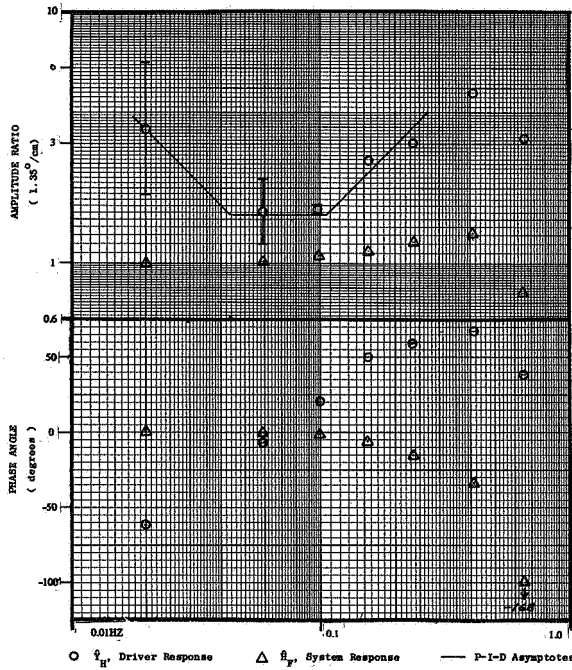
FIGURE 4.—Driver frequency response to lateral position error and system (closed-loop) response to mixed sinusoidal disturbance (run No. 1, subject A).
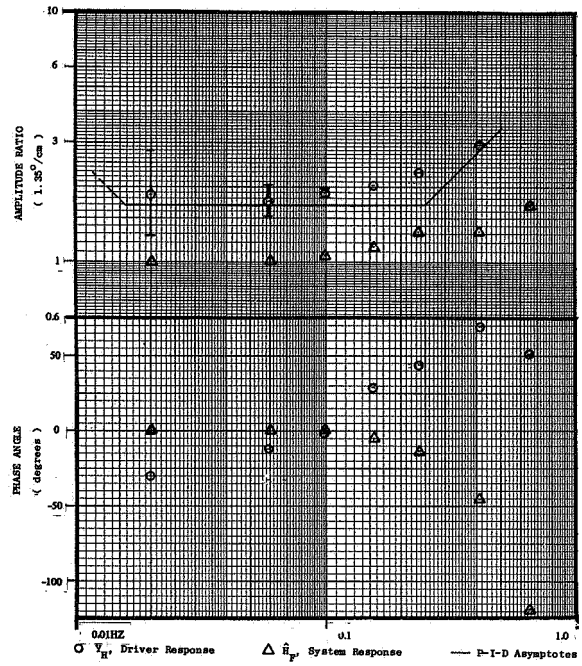


FIGURE 6.—Driver frequency response to lateral position error and system (closed-loop) response to mixed sinusoidal disturbance (run No. 3, subject C).
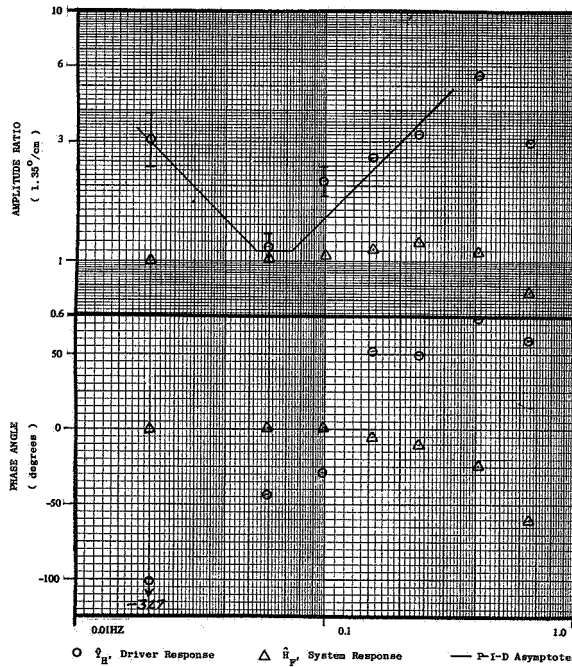


FIGURE 5.—Driver frequency response to lateral position error and system (closed-loop) response to mixed sinusoidal disturbance (run No. 2, subject B).

The closed-loop system shows good regulation with near unity gain and zero phase at low frequency ($\leq 0.3$ Hz). The slight rise in amplitude ratio at middle frequencies (0.2 to 0.4 Hz) may be ascribed to driver response to position error resulting from his lagging response to high frequency disturbances. The expected high frequency rolloff is not clearly delineated due to decreasing coherence at frequencies above 0.5 Hz, but a system cutoff frequency in the range 0.6 to 0.8 Hz can be extrapolated from the data. This general pattern of behavior appears to be consistent with one's subjective experience of steering.

Plots of $\hat{Y}_H$, the driver's angular response to lateral position error are presented because lateral position is the state variable of prime importance in system operation and will usually be treated as the system output, for instance in congested traffic and on narrow roads. The other main state variable directly sensed by the driver is relative heading, $\gamma$ which under normal highway conditions at constant speed is the first derivative of lateral position:

$$\gamma \approx \frac{1}{V}\frac{de,}{dt}.$$

The estimated phase of $\hat{Y}_H$ has been corrected for estimated delay in the recording of $e_,$, the correction being enough to make the frequency response of the vehicle ($\Theta_s/E_p$, not shown) agree with previously determined phase response.

The Bode plots (figs. 4 and 5) for subjects A and B respectively, show tho presence of integral, proportional, and derivative compensation with varying break frequencies. Subject C (fig. 6) does not show evidence of integrator action, nor is the derivative action strongly evident; however, he does show high average amplitude ratio. The first order slope at high frequencies can be readily interpreted to mean that in this frequency range the subject responds proportionally to relative heading or else to lateral velocity. The low frequency rise in amplitude ratio, described above as integral control action, indicates that the subject nulls out accumulated position error relatively slowly. The phase data are consistent showing a considerable lag ($-40°$ to $-60''$) with decreasing frequency for all three subjects. Further evidence for this type of compensation in a quasi-linear describing model is presented below.

### Transient Input Describing Functions

The data for runs No. 10, 11, and 12 were filtered and digitized at 5 Hz and analyzed using the Gabor transform technique; this centers a Gaussian weighted "data window" at the onset or center of the transient disturbance and forms response estimates as the ratio of Fourier transform of the input and output series, utilizing the fact that the Fourier transform of a Gaussian time function is also a Gaussian frequency function. The transient input is thus convolved with the Fourier transform of the normal curve. The theoretically infinite integration period required for a Fourier transform is adequately approximated by the $\pm 3\sigma$ limits of the data window. For transient inputs with finite rise time, departure from the ideal spectrum begins at a frequency greater than $\sim 0.8/T_R$, where $T_R$ is, for example, rise time of a step input. Since the high frequency response of the double steering wheel was limited (discussed earlier), the forcing transients $\theta_F$ as

actually applied had a rise time in the order of 0.8 sec; the variance of the spectral estimates was significantly increased above 1 Hz.

The transfer from $E_,$, the lateral position error to driver output, $\Theta_D$, was obtained from the ratio

$$\hat{Y}_H = \frac{\check{\Theta}\mathcal{B}}{\bar{E}_p}.$$

The time history used as input to this transformation was obtained by averaging several step responses of each subject. The steps were selected such that sufficient time separated the transients so that low frequencies could be resolved, and only suprathreshold events were used, i.e., where enough lateral position error had occurred to provoke a driver response. This time averaging process tends to smooth the data by obliterating the driver's high frequency output leaving only the gross response (see fig. 7). This point is discussed further below.

The time averaged responses are shown in figures 8 through 10 for step disturbances and figure 11 for truncated ramp disturbances, and the results of the averaged transient input analysis are sketched in the Bode plots of figures 12 through 14 for each subject. Again three-term controller response yields an adequate driver
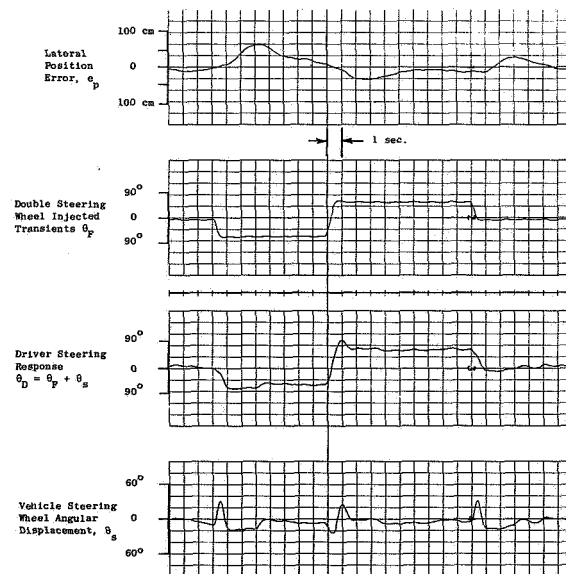


FIGURE 7.—Typical time responses due to transient disturbances (subject B, run No. 11, events 9, 10, 11 used in time average, figs. 12 through 15).
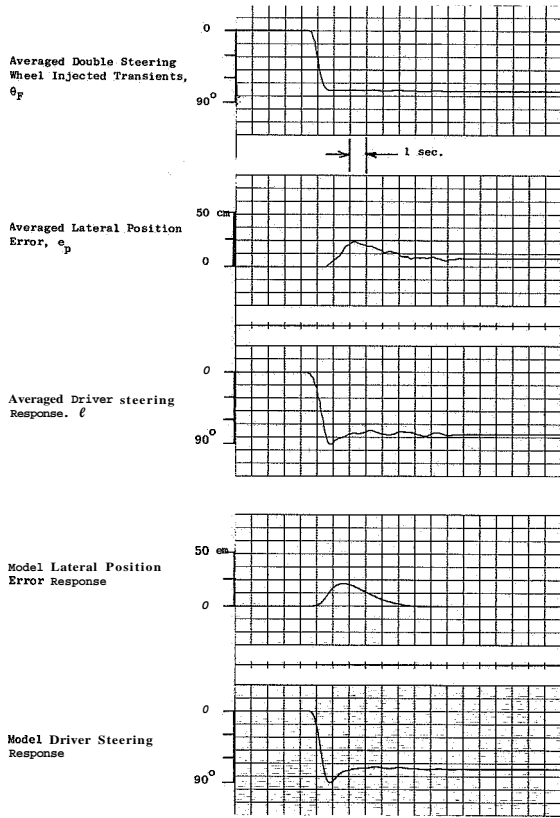
FIGURE 8.—Run No. 10, time average response for subject A and P-I-D model response to time average $\theta_F$ (3 steps).
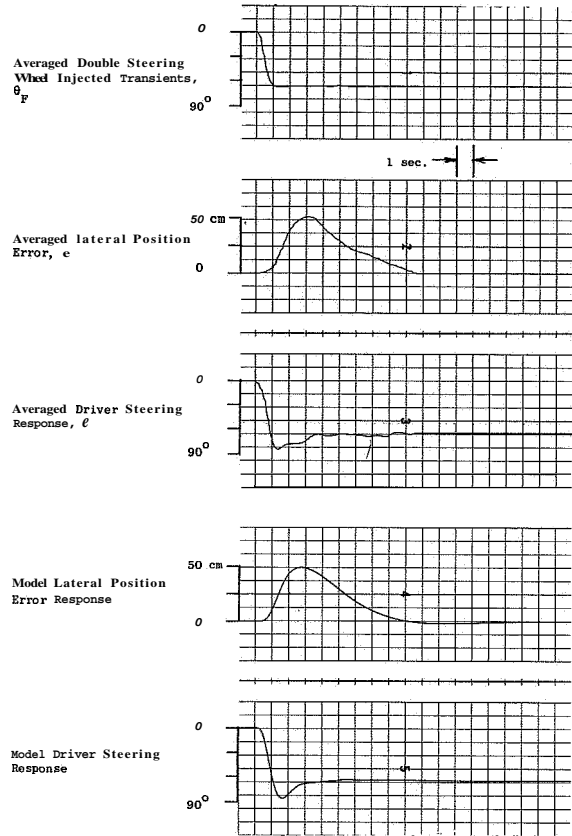


FIGURE 9.—Run No. 11, time average response for subject B and P-I-D model response to time average $\theta_F$ (6 steps).

model. Intuitively, the driver, after responding to the torque input of the double steering wheel and correcting his heading to avoid immediately leaving the roadway, must then null out any accumulated lateral position error duplicating the response of an integral controller. The increasing phase lag at low frequency also supports this view.

Agreement between transient and sinusoidal driver response is apparent from table **3** where the parameters of proportional/integral/derivative control determined from the Bode plots are tabulated. These values were obtained for each case by application of Ausman's unfactored polynomial method.

$$Y_H = \frac{\theta_D}{E_p}$$

$$= \left[ K_p + \frac{K_I}{s} + K_D s + K_A s^2 \right] \frac{e^{-\tau s}}{Ts+1} \bigg|_{s=j\omega<0.5 \text{ Hz}}$$

As a check against the loss of information by time averaging, each step used in the average response for subject C was Gabor transformed and plotted. Figure **15** shows these values with the time average values. The vector average of the transfers of these steps was computed at each frequency and found to coincide with the time average transfer. A fit of PID parameters to each transfer was made and the values are given in table **3.** This indicates a great deal of variability from one step response to another by each subject. Therefore, the Bode plots of figures **12** through **14** can be said to indicate only a central tendency for the driver.

## DISCUSSION

The results presented above contain the major substantive outcome of the study. However, subjective reports of the subjects together with time
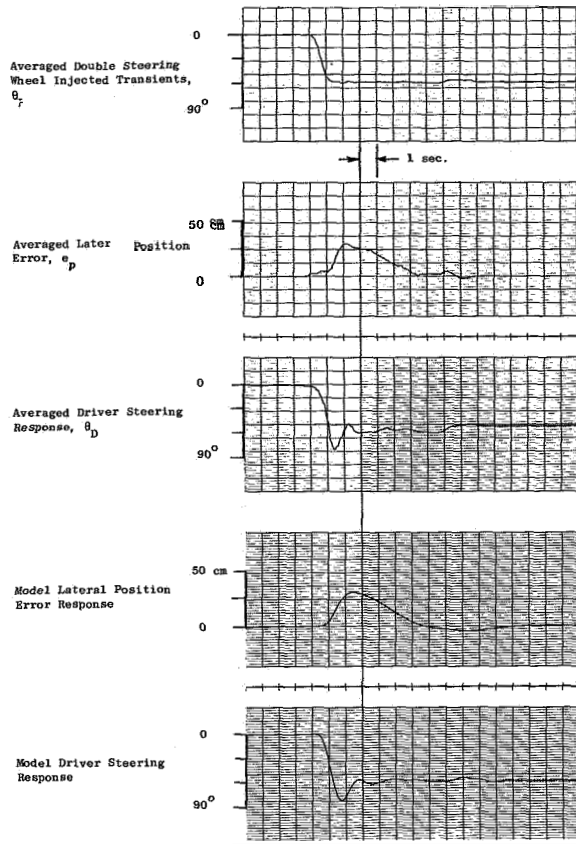
**FIGURE 10.**—Run No. 12, time average response for subject C and P-I-D model response to time average $\theta_F$ (3 steps).

**FIGURE 11.**—Run No, 10, time average responses for subject A and P-I-D model response to time average $\theta_F$ (2 ramps).

domain modelling of the driver's feasible control policy may throw further light on the validity and generality of the results presented.

## Subjects Learned Response to the Forcing Function Injector

Since most drivers sense the torque being applied during steering corrections, it would be natural for them to correct their steering responses if unexpected "feel" were introduced by the experimental double steering wheel inputs. From subject comments, it appears that they are sensitive to larger than normal reaction torques and respond to them. The time histories of transient responses show an initial proportional response with small lag ($\approx 0.25$ sec) and some overshoot. This fast response cannot possibly be ascribed to visual feedback, since the vehicle response is at best that of a lagged single inte-
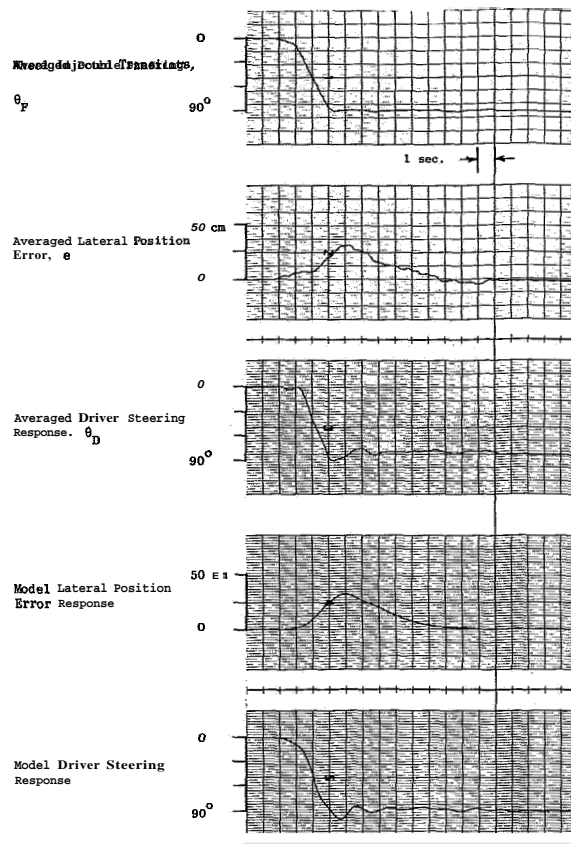
grator. Therefore, the driver clearly responded directly to the reaction torque at his hands created by the forcing function injector's angular displacement.

From this evidence, it would seem necessary to evaluate the driver's response $Y_{H2}$ to torque variation, as well as his response $Y_{H1}$ to road/vehicle geometric cues.

The reaction torque of the steering wheel, $\tau_{fb}$, is generally proportional to the angular acceleration of the car, which at small angles is itself proportional to lateral acceleration for a given forward speed. The driver estimates a steering wheel torque, $\tau_{est}$ necessary to maintain the steering angular position decided on the basis of visual cues, and follows up the estimated torque $\tau_{est}$ by closed loop control. However, his estimates are imperfect particularly under dynamic conditions, and there will be a residual "error" torque tending to force the steering
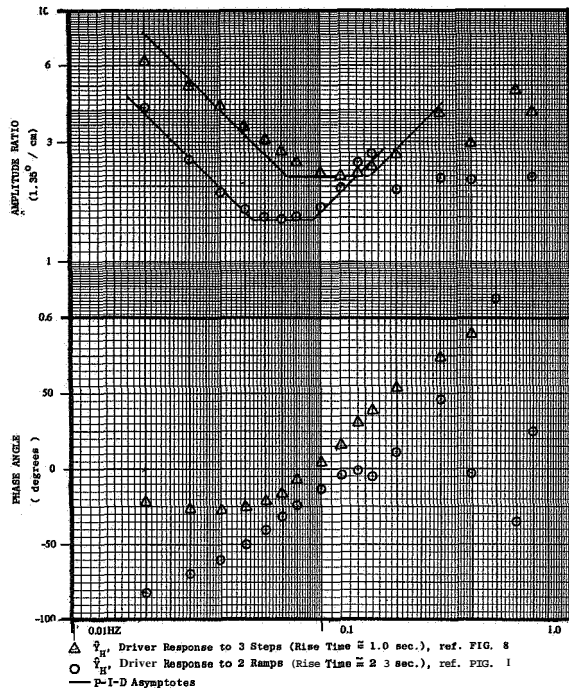
FIGURE 12.—Driver frequency response to lateral position error from Gabor transform analysis of time averaged transient disturbance responses (run No. 10, subject A).



FIGURE 13.—Driver frequency response to lateral position error from Gabor transform analysis of time averaged transient disturbance responses (run No. 11, subject B).
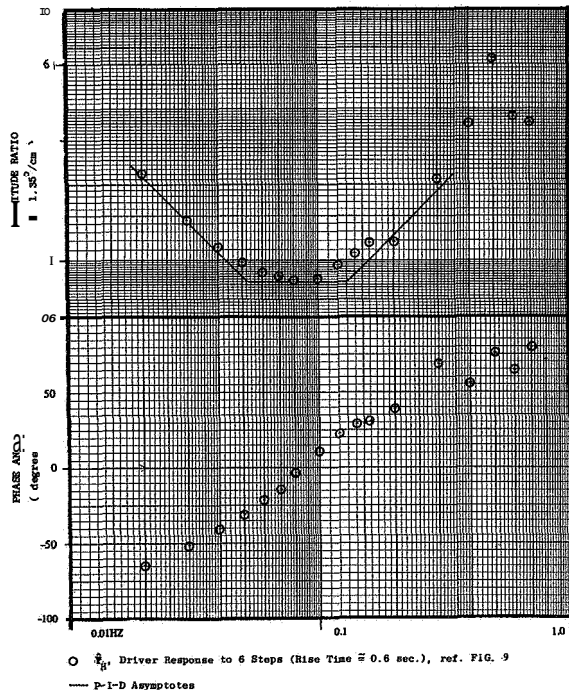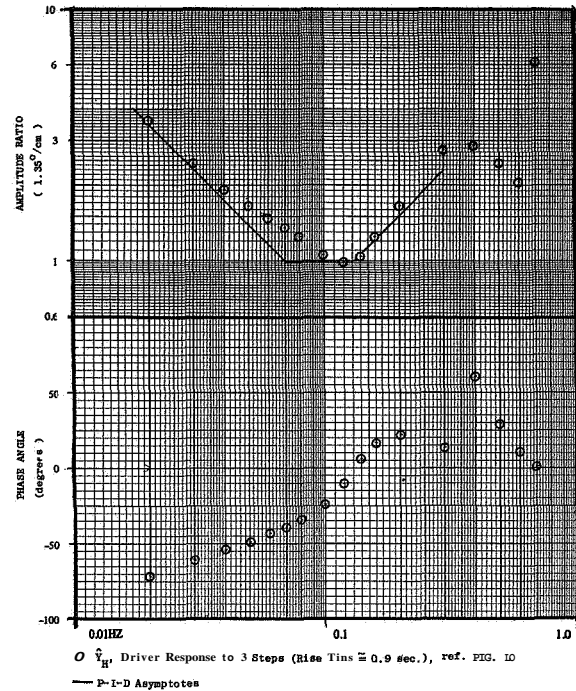


FIGURE 14.—Driver frequency response to lateral position error from Gabor transform analysis of time averaged transient disturbance responses (run No. 12, subject C).

system. The resulting control flow diagram is shown in figure **16.** The results given must therefore be interpreted in the light of the existence of two linked control subsystems.

It was observed that as a subject becomes more familiar with the double steering wheel torques produced by a given forcing function, he can diatort the effect of the forcing function on steering output and road position by passively allowing the top ateering wheel to displace backwards. His arms, torso, and the inertia of the wheel respond like a soft spring. He might also choose to react to changing outer feedback loop error signals by attempting to null out all effects of the forcing function while controlling to the desired steering wheel torque level. He would maintain this torque by backing off with the forcing function utilizing his torque feedback loop so that the net response corrects the outer loop error.

These problems could possibly have been overcome by adopting zero force feedback system for the double steering wheel system using a vehicle equipped with modified power steering.

TABLE 3.—*Proportional, Integral, Derivative Acceleration Model Parameters for Frequency Response of Subjects to Lateral Position Error*

| | | | | | Parameters | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | Proportional | Integral | Derivative (heading) | Lateral acceleration |
| | | Run no. | Subject | Frequency response, fig. no. | Time response, fig. no. | $K_P$, deg/cm | $K_I$, $\dfrac{\text{deg/sec}}{\text{cm}}$ | $K_D$, $\dfrac{\text{deg}}{\text{cm/sec}}$ | $K_A$,* $\dfrac{\text{deg}}{\text{cm/deg}^2}$ |
| Double steering wheel injected forcing function | Mixed sinusoids | 1 | A | 4 | ... | 2.07 | 0.57 | 3.1 | ........ |
| | | 2 | B | 5 | ... | 1.5 | 0.51 | 3.2 | ........ |
| | | 3 | C | 6 | ... | 2.3 | 0.23 | 1.5 | ........ |
| | Averaged transients | 10 | A | 12 | 8 | 2.96 | 1.35 | 2.96 | 1.9 |
| | | 10 | A | 12 | 11 | 1.98 | 0.66 | 3.5 | 0 |
| | | 11 | B | 13 | 9 | 1.1 | 0.36 | 1.35 | 0.7 |
| | | 12 | C | 14 | 10 | 1.35 | 0.62 | 1.66 | 0.28 |
| | Individual transients | 12–9 | C | 15 | ... | 1.35 | 0.62 | 1.9 | ........ |
| | | 12 | C | 15 | ... | 0.9 | 0.97 | 1.08 | ........ |
| | | 12–14 | C | 15 | ... | 0.9 | 0.35 | 1.08 | ........ |

* The parameter $K_A$ accounts for the driver response to a variety of feedback cues including steering wheel reaction torque as well as lateral acceleration.
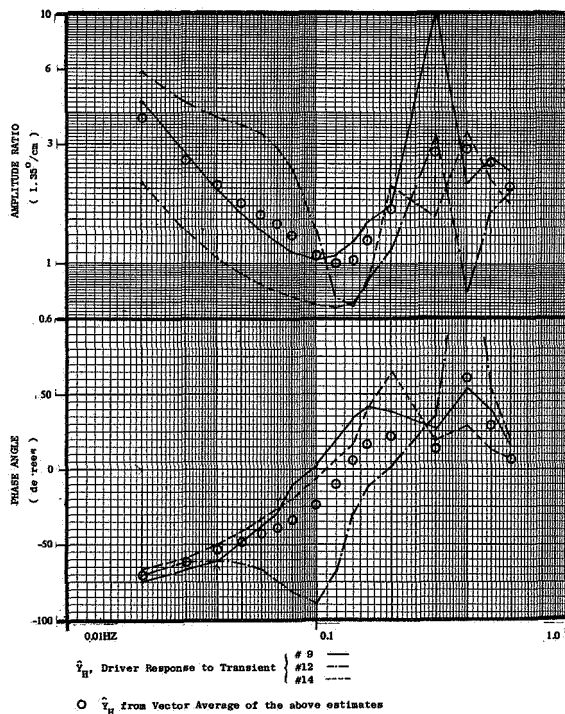


$\hat{Y}_H$, Driver Response to Transient $\begin{cases} \#9 \text{ ——} \\ \#12 \text{ ———} \\ \#14 \text{ ----} \end{cases}$

O $\hat{Y}_H$ from Vector Average of the above estimates

FIGURE 15.—Frequency response and vector average response from Gabor analysis of driver response to lateral position error due to three transient disturbances (run No. 12, subject C).

In the present experiment the effect was reduced as far as possible by using two subjects unfamiliar with the double steering wheel and by the use of random appearing forcing functions which slowed down the learning process.

Subsequent discussion will also indicate that the possible effects on the results occur at the upper limit of the resolvable spectral measurements at the frequency associated with response to lateral acceleration and thus have little effect on the low frequency parameter estimation.

### A Comprehensive Frequency Domain Model of the Vehicle/Driver System for Straight Road Driving With Injected Angular Steering Disturbances

A complete model of the vehicle/driver system has been developed on the assumption that the driver will utilize the simplest forms of available feedback signals for his control actions, and thus will not need to develop any lead compensation nor signal differentiation when the derivative signals are available directly. Thus all driver responses have been represented as proportional to directly observable signals, including:
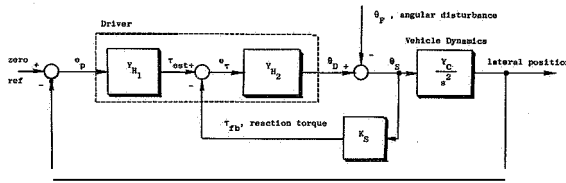
**FIGURE** 16.—Reaction torque compensation modeled as an inner loop response of the driver.
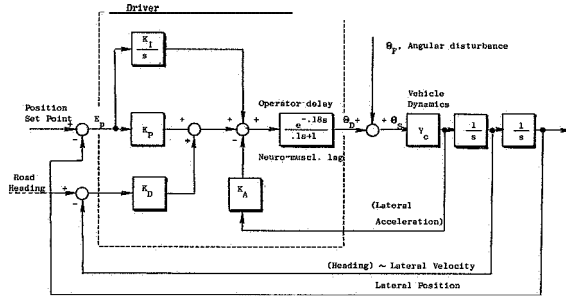


**FIGURE** 17.—Driver model for straight road steering control (constant speed) with injected angular disturbances.

(1) Lateral position error, the difference from actual car position to lane set point.

(2) Lateral position error when present over long periods (or with low frequency).

**(3)** Heading error (between car trajectory and road).

(4) A differential level of car response feedback directly estimated by the driver in several forms:

    (a) Lateral (and angular) acceleration

    (b) Car body roll angle

    (c) Steering torque feedback.

The model is shown in figure 17. The parameter for $K$, $K_I$, and $K_D$ are tabulated in table 3. The $K_A$ term is estimated by matching model and averaged time history responses, $\theta_D$ and e. Further clarification and discussion of the various loops shown in figure 17 follows.

Integral control term.—This permits precise positioning over relatively long periods.

Lateral acceleration response.—This control mode is modelled as a response to the car's lateral acceleration. However, as shown above, it may in reality reflect a closed-loop response to torque disturbance sensed at the steering wheel. This response has low resolution since the driver must first estimate the "correct" torque associated

with the desired steering angle before attempting to null out deviations from it resulting from forcing function injector action. Since steering wheel torque is highly correlated with angular and lateral acceleration of the vehicle the two response modes (aceeleration and torque) cannot be separately modelled.

We may note the driver's modelled response to forcing angle alone, without considering lateral position or heading errors, whichever cue is used (acceleration or torque). Initial driver response to the lateral acceleration (or torque) is modelled as

$$\frac{\theta_D}{\theta_F} \approx K_A Y_C.$$

The driver can apparently eliminate a large portion of the forcing function amplitude without resorting to outer loop feedback. This may be better described as feedforward regulation, since the driver estimates and cancels the sensed disturbance before it has had an opportunity to disturb the plant.

Reaction time and neuromuscular lag terms.— The reaction time and neuromuscular lag terms were estimated from the time histories, with some uncertainty. However, the values are common values found in the literature. For an equivalent control task and forcing function bandwidth, classical data on test pilots suggest a time delay of 0.18 sec (McRuer and Krendel (ref. 15)). The neuromuscular lag time constant is less sharply defined, ranging from 0.1 to 0.2 sec. But a close examination of test run No. 1 subject A's response to step inputs shows an overall lag from $\theta_F$ to $\theta_D$ of 0.25 to 0.30 sec, as measured during the initial signal rise period. Thus a physical response lag time constant of 0.275 to $0.18 \approx 0.10$ sec appears to be a good approximation on an average basis.

This estimation has an effect on the determination of the lateral acceleration response parameter. For example, when the lag term was increased to $1/(0.2s+1)$, the only real change required in the model was to double the lateral acceleration control gain $K_A$.

Figures 8 through 10 show the time history response of the model to the time averaged "step" transients for subjects **A,** B, and **C,** respectively. **A** truncated ramp response for subject C is shown in Figure 11. It was observed that the driver

corrected lateral position error only to within a tolerance threshold, while the linear model nulled out the error completely by the $K_I/s$ compensation. Excessive integral action may result in overshoot in the lateral position response. Also, the actual time averaged responses indicate possible pulse-like behavior (0.5 to **0.9** Hz). This may account for part of the spectral variation shown in figures **12** through **15** for that frequency range.

## CONCLUSIONS

The double steering wheel forcing function injector is a viable experimental tool for producing a known disturbance in the driver/vehicle steering control loop permitting measurement of low and mid frequency compensation by the driver.

Using this method, the results have indicated an integral control-like action in response to a transient disturbance which is used by the driver to reduce the vehicle lateral position error to within his tolerance limits.

The frequency and response analysis also strongly supports earlier conjectures that feedback cues related to the first time derivative of lateral position viz. relative heading or lateral velocity, and the second derivative, viz. lateral acceleration or steering torque reaction are used by the driver in steering the vehicle. This assumption yielded good models of driver response both to random appearing mixed sinusoidal disturbances and to transient disturbances. Drivers also learned to respond directly to reaction torque from the angular displacement injected into the steering control loop. This state variable is highly correlated with lateral acceleration and was not distinguished from it in the model.

Results were unsatisfactory above about **1** Hz. This may have been due to unwise selection of a relatively low sampling rate in digitization, and further processing of the data may yield improved high frequency estimates. It may also have been due to genuine driver remnant, but this cannot confidently be estimated from the data analyzed to date. The time histories show evidence of pulse-like nonlinear behavior during extended response to step transients which could appear as high frequency remnant power.

Further experimentation will further define the high frequency response of the driver and seek to delineate evidence of pulse-like corrections (minimum effort compensation) which are an equally viable explanation of the integral control effect observed in the tests. This would lead to a nonlinear model of possible driver steering behavior in which an initial quasilinear response to a transient disturbance is followed by timed steering wheel pulses whenever the projected path of the vehicle exceeds some intrinsic threshold for allowable lateral position error.

## SYMBOLS

Time functions are in lower case; frequency functions in upper case; and $\wedge$ indicates experimental estimate.

$\theta_F$ angular displacement of the forcing function injector

$\theta_s$ angular displacement of the vehicle steering wheel

$\theta_D = \theta_F + \theta_s$ angular displacement of the second steering wheel, i.e., driver output

$e_p$ lateral position error, i.e., vehicle displacement from white stripe

$Y_{H_1} = \dfrac{\Theta_D}{E_p}$ driver response to lateral position error

$Y_{H_2} = \dfrac{\Theta_D}{E_\tau}$ driver response to the torque imbalance generated by the forcing function injector

$H_F = \dfrac{\Theta_D}{\Theta_F}$ system response to forcing function

$Y_C = \dfrac{s^2 E_p}{\Theta_s}$ vehicle lateral position response to angular displacement of the regular steering wheel

$Y_C \approx \dfrac{K}{s^2(T_c s + 1)}$ where

$$\left.\begin{array}{l} K \approx \dfrac{2.13 \text{ cm/sec}^2}{\text{degree}} \text{ at 30 mph} \\[2ex] K \approx \dfrac{2.93 \text{ cm/sec}^2}{\text{degree}} \text{ at 40 mph} \end{array}\right\} \text{ up to 0.5 Hz}$$

$T_c =$ vehicle response lag

$V$ vehicle forward velocity

$\gamma = \dfrac{s}{V} e_p$    relative heading of car — lateral velocity for constant $V$

$K_s = \dfrac{\tau_{fb}}{e_r}$    vehicle model self-aligning torque feedback constant

$$\dfrac{451 \text{ cm gm}}{\text{degree}} \text{ at } 30 \text{ mph}$$

$$\approx \dfrac{634 \text{ cm gm}}{\text{degree}} \text{ at } 40 \text{ mph}$$

$\tau_{fb}$    steering feedback torque from steering gear

$\tau_{est}$    the driver estimated steering wheel torque

$e_\tau = \tau_{est} - \tau_{fb}$    torque imbalance (error)

$K_P$    driver model response to lateral position error

$K_I$    driver model response to integrated lateral position error

$K_D$    driver model response to heading angle or lateral velocity

$K_A$    driver model response to lateral acceleration or steering torque

$T$    neuro-muscular time constant

$T_R$    rise time of transient input

$s$    laPlace domain operator

## REFERENCES

1. WEIR, D.: Closed-Loop Directional Control of Automobiles. IEEE Eighth Annual Symposium on Human Factors in Electronics, 1967.
2. McRUER, D.; AND WEIR, D. H.: Theory of Manual Vehicular Control. IEEE Trans., vol. MMS–IO, Dec. 1969.
3. CROSSMAN, E. R. F. W.; AND SZOSTAK, H.: Man-Machine Models for Car Steering. Fourth Annual NASA/University Conference on Manual Control, NASA SP–192, 1968.
4. WIERWILLE, W. W.; ET AL.: An Experimental Study of Human Operator Models and Closed-Loop Analysis Methods for High-speed Automobile Driving. IEEE Trans., vol. HFE–8, Sept. 1967.
5. WEIR, D.; AND WOJCIK, C. K.: The Measurement of Driver Describing Function in Simulated Steering Control Tasks. Seventh Annual NASA/University Conference on Manual Control, Univ. of Southern California (Los Angeles), 1971.
6. McRUER, D.; ET AL.: Human Pilot Dynamics in Compensatory Systems. AFFDL TR 65–15, 1965.
7. TAYLOR, L. W.: A Look at Pilot Modelling Techniques at Low Frequencies. Sixth Annual NASA/University Conference on Manual Control, 1970.
8. CACIOPPO, A. J.: Pilot Information Utilization: A Study in Human Response Dynamics. GER–7686A, Goodyear Aircraft Corp., 1956.
9. IGUCHI, M.: Man-Machine Systems. Kyoritsu Publishing Co. (Tokyo), 1970.
10. KOBAYASHI, M.: Research on Characteristics of Human Control Behavior. Thesis, M.E., Univ. of Tokyo, 1967.
11. CROSSMAN, E. R. F. W.; SZOSTAK, H.; AND BECHARD, J.: A Device for Continuously Recording the Lateral Position of a Vehicle on the Highway by Optically Sensing Its Distance from Painted Lane Markings. HFT Group Working Paper 66–9, Univ. of California, Berkeley, 1966.
12. SZOSTAK, H.: Investigation of the Dynamic Steering Response of a 1965 Ford Sedan Test Vehicle. HFT Group Working Paper 67–8, Univ. of California, Berkeley, 1967.
13. MASSEY, F.; AND JENRICH, R.: BMDX92, Time Series Spectrum Estimation. UCLA, Los Angeles, 1966.
14. CROSSMAN, E. R. F. W.; AND DELP, P.: Application of Gabor's Elementary-Signal Theorem to Estimation of Nonstationary Human Spectral Response. Fifth Annual NASA/University Conference on Manual Control. NASA SP–215, 1969.
15. McRUER, D.; AND KRENDEL, E. S.: Dynamic Response of Human Operators. WADC–TR–56-524, Wright-Patterson AFB, Oct. 1957.