EVIDENCE FOR CHANGES IN THE ANGULAR VELOCITY OF THE SURFACE REGIONS OF THE SUN AND STARS

INTRODUCTION

R. P. Kraft   If we are willing to believe that there is an angular momentum flux from the sun carried by the solar wind, then we can ask the question: Is there any direct observational evidence that the outer layers of the sun have indeed slowed down — that the torque corresponding to this angular momentum flux acting for a long time will lead to a deceleration of the observable outer layers of the sun? Obviously we cannot get such evidence very directly from the sun itself. But by placing the sun in the context of stellar evolution perhaps we can say something if we are willing to believe that the sun is a typical star.

To do this I want to remind you in 5 minutes, if I can, of the relationship between stellar rotation, stellar evolution, and the basic problem of the HR diagram. The HR diagram is the fundamental diagram of stellar astronomy. In it, astronomers plot the effective temperature T_e of a star against its luminosity L, where $L = 4\pi R^2 \sigma T_e^4$, where R is the radius and L is in units of energy (erg/sec). For a large sample of stars in the vicinity of the sun we find that the overwhelming majority (~90 percent) lie in a narrow band, called the *main sequence*, running diagonally across the diagram. The sun is a member of this band. There are a few stars in the lower left-hand corner of the diagram called *white dwarfs*. There are also a few up at the top right-hand side of the diagram (say 5 percent to 10 percent), which are called *giants* and *supergiants*.

The leading task of observational stellar astronomy is to explain why this diagram is not everywhere dense — why there aren't points all over it. Now it is an observed fact that luminosities of main sequence stars are a monotone increasing function of mass. So if we go up along the main sequence we go to stars of larger and larger mass; thus there is a mass luminosity relation along that sequence in which luminosity goes more or less as the third or fourth power of the mass. The interpretation of this observation is that stars somehow arrive on this main sequence, which then represents a locus of points of stars of different mass burning hydrogen in equilibrium — that is, converting hydrogen into helium. The rate at which the nuclear processes take place depends critically upon a high positive power of the central temperature and density, and these in turn depend on the total mass of the star. Consequently, stars high on the main sequence burn out their critical hydrogen supply first, and therefore evolve into the giant region. Theoretical calculations bear this out. One computes an evolving stellar model in which stars start out on the main sequence chemically homogenous, burn up a critical hydrogen supply in the central region, and move rapidly in the HR diagram to the giant domain. Then they may burn helium for a while, or perhaps hydrogen also in shell sources. Eventually they contract, exhaust their nuclear energy sources, and wind up in the degenerate white dwarf domain. Before you get to the main sequence you have a period of gravitational contraction, with stars moving in from the right-hand side of the HR diagram, simply in a state of gravitational contraction. The time scales involved here for the sun, for example, are of the order of 10^8 years for the gravitational contraction. The hydrogen burning lifetime on the main sequence is of the order of 10^{10} years, and subsequent stages — the red giant episode and presumably the dwarfs — spend most of their time on the main sequence.

What this has to do with stellar rotation is simply the issue of whether the e-folding time for significant changes in rotational velocities of stars owing to the action, let us say, of winds similar to the solar wind is short, long, or of the same order as the nuclear or the gravitational time for the star. If it is very much longer, the problem is of no interest. If it's very much shorter

than these critical times, espeically the nuclear time, of course, then one has an interesting problem.

Rotation can be described in relation to the topology of the HR diagram. I draw a roughly vertical line, intersecting the main sequence just above the position of the sun, in the region of what are called the F-type stars. To the left of this line one has the domain of rapid rotation, in which stars have rotational velocities up to a few hundred km/sec. To the right of this line one has the domain of slow rotation. This includes the main sequence and giants. Thus the main sequence from just above the sun on down is a domain of slow rotation, and the upper main sequence above middle F is a domain of rapid rotation.

Figure 1 shows the post main sequence evolutionary tracks from the work of Iben and others. These stars
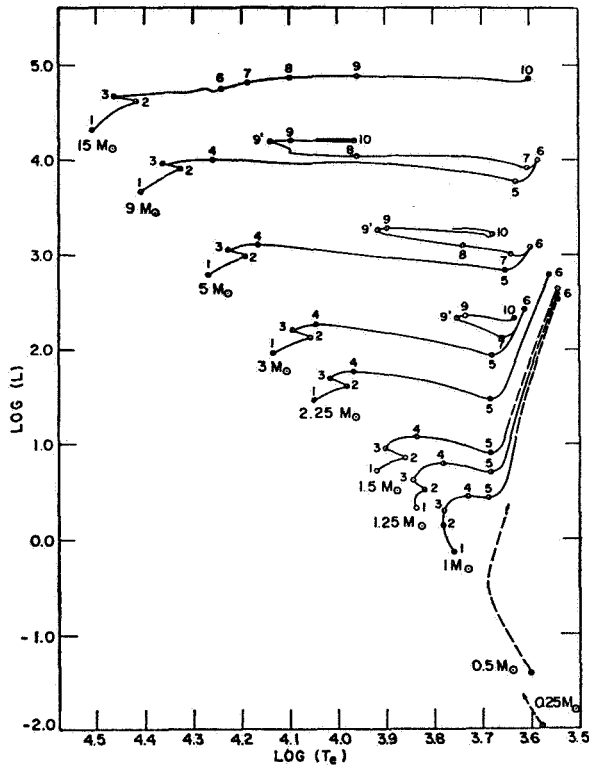

Figure 1. *Post main sequence evolutionary tracks showing progression from the main sequence (starting points on the left) toward the red giant stage for different solar masses [adapted from Iben and others].*

have exhausted their critical hydrogen supply and have moved on to the right in the HR diagram to become giants. Essentially their radii swell up at an almost constant luminosity. One can understand the slow rotation for these stars if angular momentum is essentially con-

served, because the radii of the giants are much larger on the average than the radii of the main sequence stars.

Referring to the main sequence, figure 2 shows the distribution of mean rotational velocities along the main sequence. $V_{\sin i}$ refers to the mean value (over a large sample of stars) of the rotational velocity projected on the line of sight (recall that a spectrograph measures only the radial component of motion); V refers to the
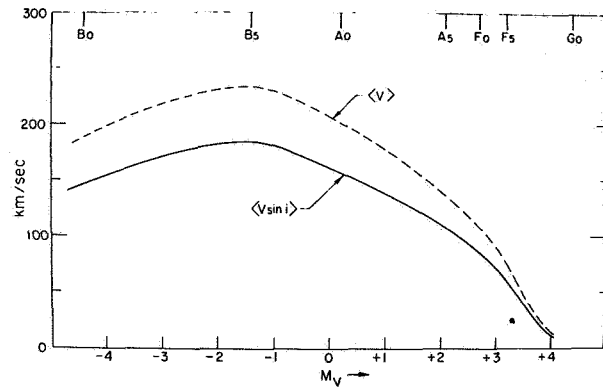

Figure 2. *Distribution of mean line-of-sight rotational velocities as a function of absolute magnitude as a representation of luminosity together with the mean rotation based upon the assumption of a random distribution of rotational directions.*

mean true value, assuming a random orientation of rotational axes. The abscissa is absolute magnitude, a logarithmic measure of luminosity. Higher luminosity stars are to the left. The rotations go up to a few hundred km/sec on the average, reach a maximum where stars have temperatures of the order of 15,000° or 20,000° ($M_V = -1$), and then decline rapidly in middle F spectral class, near $M_V = +\beta$. The sun on this diagram would be at $M_V = +4.7$, and the rotations there on the average along the main sequence become very slow. This region between $M_V = +2.5$ and $M_V = +4$ is the "break" in rotations on the main sequence that I will be returning to several times in this talk.

Figure 3 shows the same thing in terms of specific angular momentum $J = J/M_e$, where M_e is the mass of a star. The momentum J is shown as a function of M_e along the main sequence, assuming that the stars are rigid rotators. It is seen that J varies roughly as $M_e^{2/3}$ down to masses just a little larger than the sun's; then J takes a sharp drop. Thus the stars of solar rotation have angular momentum densities that are very low in contrast to the stars higher up on the main sequence, if we take the surface rotational velocity as characteristic of the total rotation. The figure also shows that if you
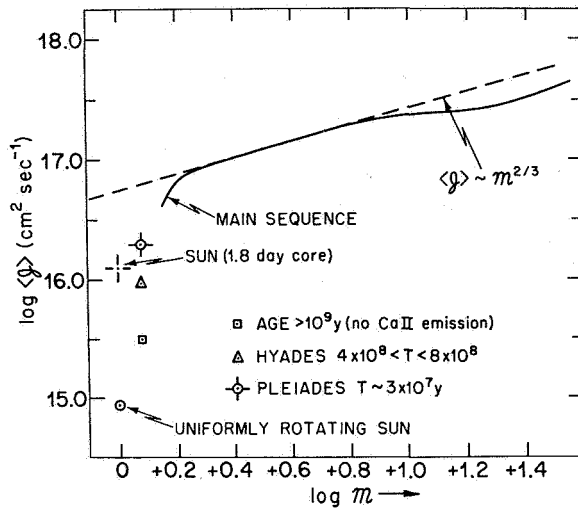
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Figure 3. *Specific angular momentum versus absolute magnitude for two solar models, the Pleiades, Hyades, and* Ca II *emission-free objects.*

Now, Wilson showed from an extensive amount of work that one index of stellar chromospheric activity was correlated with stellar age; that is, when stars first arrive on the main sequence after gravitational contraction, they give evidence for a large degree of chromospheric activity as measured by the strength of the ionized calcium (Ca II) emission at the bottom of the K-line, which, as you know, arises in the chromosphere. The body of evidence Wilson assembled strongly suggests, and indeed I think proves that, as a star sits on the main sequence burning up its hydrogen supply, its chromosphere tends to die away with time, in the sense that the Ca II emission gradually dies away with time. The first work was based on stars of the general field in the vicinity of the sun. Later, Wilson confirmed the result by studying the stars in galactic cluster where age dating can be obtained from the nuclear time of the turnoff on the main sequence. Young clusters like the Pleiades, which has an age of 3×10^7 years, were found to have strong Ca II emission. As you go to the Hyades with an age of 5×10^8 years, the Ca II emission weakens. When you look at typical field stars with Ca II emission, the strengths are similar to those of the Hyades or are weaker. As you go on toward stars that don't have any Ca II emission, they are the ones that on the average have the oldest nuclear ages.

Now, the other thing Wilson was able to show is that, as you go up the main sequence, the Ca II emission rather abruptly disappears in middle F. So among main sequence stars, Ca II emission is confined to those from just above the sun on downwards; the ones above middle F do not show it. Now it is remarkable that the main sequence location where Ca II emission disappears is the *same* as that where the rotational velocities take a nosedive. It is also the same place where stars begin, as one passes down the main sequence, to develop an outer convection zone owing to the mid-ionization of hydrogen.

The picture that develops from all this is essentially the following: One imagines that a star arrives on the main sequence after gravitational contraction. One postulates that if the star arrives on the main sequence below middle F then, because the outer part of the star has a well-developed hydrogen convection zone, there is acoustic heating of the chromosphere and of the corona; therefore, a wind exists owing to coronal evaporation, which then is capable, if coupled to a magnetic field, of carrying away angular momentum. On the other hand, if the star arrives on the main sequence somewhat above middle F the convection zone disappears — that is, the structure of the star does not permit an external hydrogen

put all the angular momentum in the solar system back into the sun, of course, you get up to the extension of the specific angular momentum line for the more luminous stars. On the other hand, if you restored the angular momentum of the sun by assuming it had a rapidly rotating core, then you would also restore the sun back up to the relation $J \sim M_e^{2/3}$. It should be remembered, of course, that the relation $J \sim M_e^{2/3}$ is statistical in character because, at a given M_e, stars do, in fact, have a very wide range of true rotational velocities.

One may ask first of all, why are all stellar rotations so slow, even those of the upper main sequence. If you try to contract a piece of the interstellar medium with the differential galactic rotation in it and a mass equal to one solar mass, and ask that angular momentum be conserved, you will wind up with a surface rotational velocity exceeding the speed of light when the mass has solar dimensions. So somewhere in the early stages of contraction you've got to get rid of a lot of angular momentum, and even the most rapidly rotating main sequence stars have a velocity of only a few hundred km/sec. Thus only about 1 percent of the initial angular momentum winds up even in the most rapidly rotating star, and there has to be a lot of angular momentum dumped in the early stages of stellar evolution. In addition, after you dispose of most of the angular momentum, there is finally the question of the origin of the break on the main sequence in early F where rotation takes a nosedive and gets extremely slow. And one would imagine that this might have something to do with the existence of an angular momentum flux in the solar wind.

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convection zone — in which case there is no acoustic hearing, no corona, no wind, and therefore there is no torque exerted by outflowing gas. A few years ago, I looked for observational confirmation of this picture. One predicts that if you looked at field stars below middle *F* with strong Ca *II* emission or looked at stars in young galactic clusters below middle *F* you would on the average find them rotating more rapidly than older stars like the sun. In other words, what one would look

for is a systematic decay of rotational velocity of the surface regions of stars with advancing age. Since the clusters give a nuclear age-dating of the stars, you simply assemble them into age groups, observe them for rotation, and see if in fact the mean rotation systematically declines with advancing age.

Consider first the field emission-line stars shown by Wilson to be younger than those without emission. In figure 4, these are compared in a pseudoHR diagram.
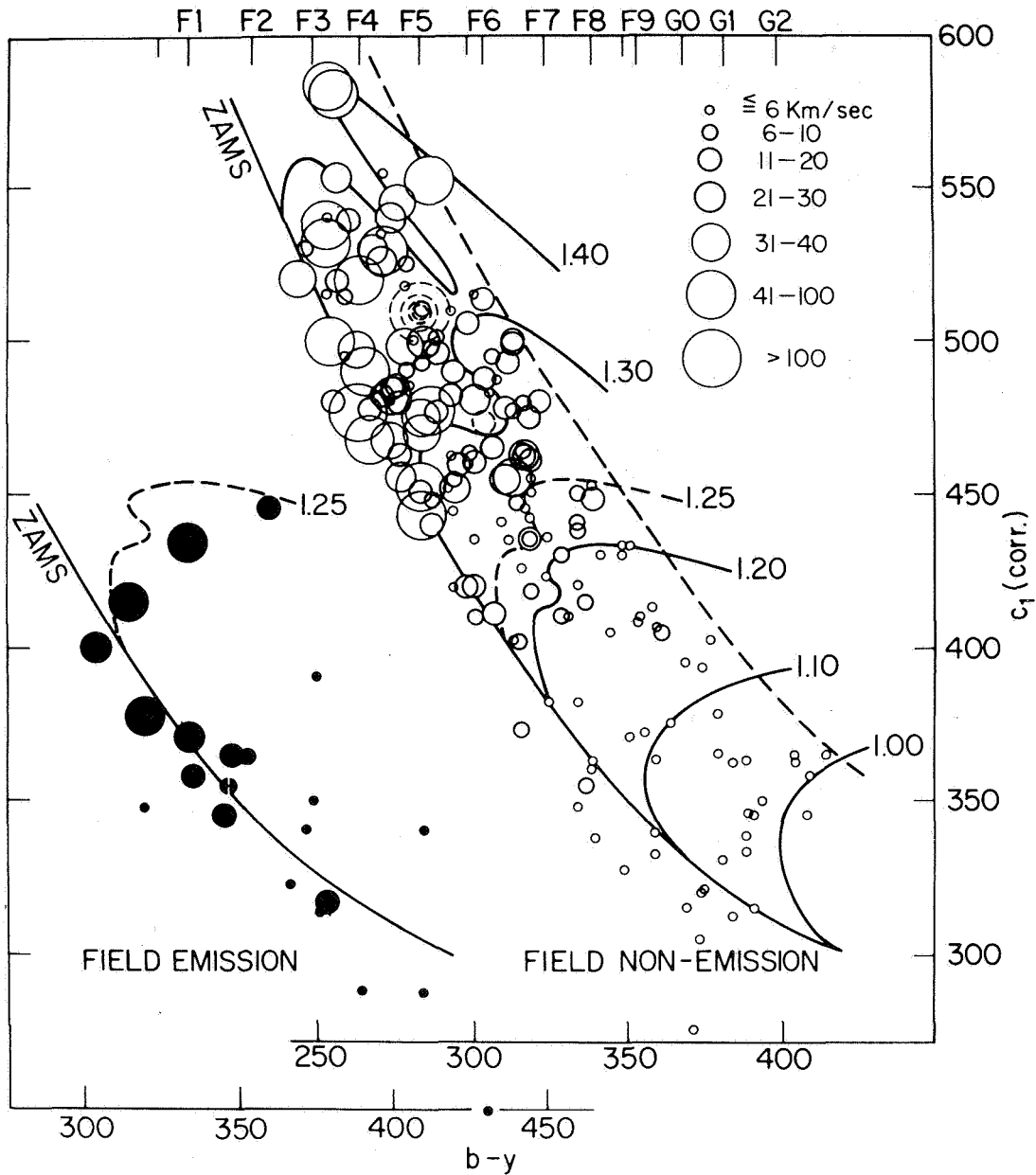

Figure 4. *Strömgren diagram representation of the HR diagram for both field emission and field emission-free stars of variable solar mass [from Ap. J., Vol. 150, 1967, p. 551].*

The color (b-y) increases with decreasing surface temperature; the quantity C is related to the luminosity in the sense that luminosity increases upward. Filled circles refer to emission-line stars and open circles to nonemission line stars. Rotational velocities are proportional to the sizes of the dots. The break in rotation corresponding to stars of middle F type is apparent. Figure 4 shows what happens in clusters. In the Pleiades there are large rotations and in the Hyades intermediate rotations. The field stars have emission lines of strength similar to the Hyades; both have rotations much larger than the non-emission-line old stars in the solar vicinity.

This finding is summarized in figure 5. If at a given mass you make a cut in the rotational velocity, the Pleiades are highest, field emission and Hyades stars are next, and then the field nonemission stars have the lowest velocities. Figure 6 shows the theoretical justification for the statement that the existence of the convection zone is related to the production of a stellar wind. The thickness of the outer hydrogen convection zone in main sequence stars is given as a function of surface temperature from work of Baker. Alpha is the ratio of mixing length to scale height and is only of interest to people who worry about convection theory. In any event, you see that the zone gets thinner and thinner as you go up the main sequence. Eventually, the convection zone disappears and one would then argue there is
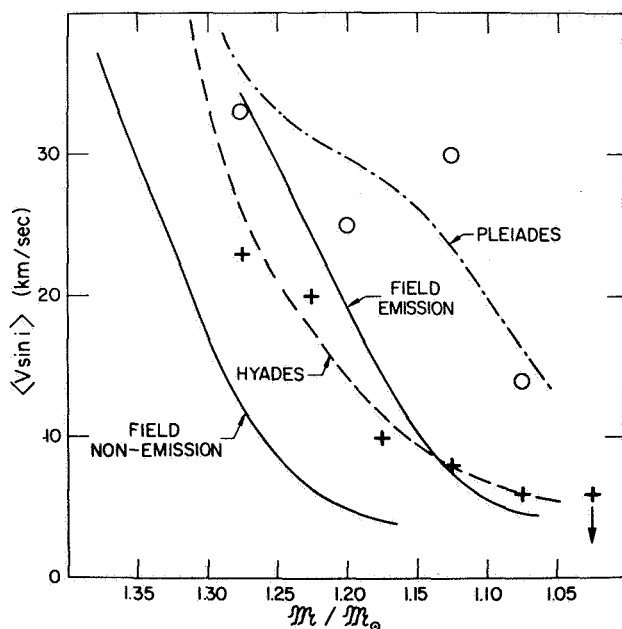
Figure 6. *Normalized hydrogen convection zone thickness as a function of surface temperature, T_e. α is the ratio of mixing length to scale height (adapted from Baker).*

no acoustic heating, no corona, and therefore no wind for stars above middle F.

Table 1 gives the mean rotation as a function of age. One has first the type of star, the mean age, the mean rotational velocity, and the specific angular momentum J assuming uniform rotation. The value of J corresponding to $J \sim M_e^{2/3}$ is also given. According to Dicke, the

Figure 5. *Line-of-sight rotational surface speeds as a function of mass normalized to the sun [from Ap. J., Vol. 150, 1967, p. 551].*
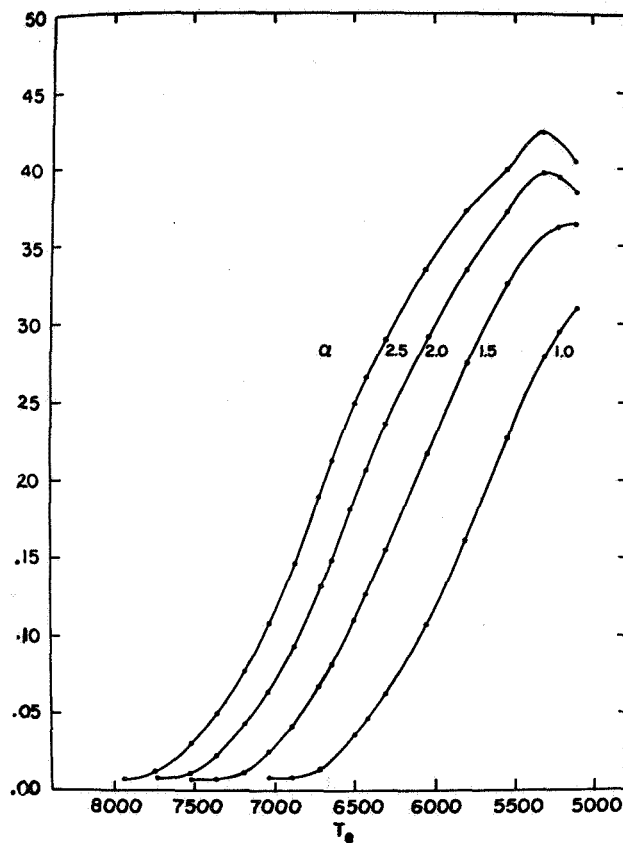
Table 1. $\langle v \rangle$ *as a function of age for stars of mass M = 1.2 $M_\ominus$ and for the sun*

Star	Approximate average age (yr)	$\langle v \rangle$ (km/sec)	log J
Field nonemission	3×10^9	6	15.5
Hyades cluster	4×10^8	18	16.0
Pleiades cluster	3×10^7	39	16.3
Sun (uniform)	5×10^9	2	15.0
Sun (1.8 day core)	5×10^9	2	16.2

torque exerted by the wind has acted only to slow down the external convection zone and perhaps a small portion of the radiative region. If the Dicke model is correct, it predicts a certain relationship between the abundances of Li and Be; see Dicke, p. 290.

Olin Wilson has shown that the Ca II emission dies away with time when a star occupies a certain position on the main sequence. Now, this is essentially a statistical statement, averaged over time. There are two ways in which the Ca II emission in a star might change. One is the very short-term secular variation that might be induced by stellar rotation. That is, if the emission comes from a few localized plages on the surface, then you might expect it to be modulated by stellar rotation. Essentially nothing about that is known from stellar astronomy observations.

A second kind of modulation of the Ca II emission might come from solar cycles — that is, the stellar analogs of what the solar cycle is — since we know that the strength of the Ca II emission integrated over the solar surface would certainly be a function of phase in the solar cycle.

R. P. Kraft   I think we could allow now about 5 to 10 min on discussion of these two talks on the observational side of stellar astronomy.

J. V. Hollweg   In the sun the Ca emission is closely related to supergranulation and, although you spoke of acoustic heating I think these emissions are also consistent with heating by supergranulation driven Alfvén waves, which I discussed this morning.

R. P. Kraft   Yes, I realize there's been quite a bit of discussion about these points. I didn't mean to imply acoustical heating was in question. Also, lack of time prohibited any real discussion about the problem of the relation between solar Ca emission as it would be seen integrated over the whole disk and Ca emission in stars, which is, after all, the only way you can observe stars. That's a rather difficult question that sometimes solar physicists also balk at answering, answering how much comes from plages and — well, they don't always want to tell you exactly.

A. Barnes   I was wondering whether there are any observations of Ca enhancement in stars that have detectable magnetic fields.

O. C. Wilson   Offhand, I can't think of any, because the stars in which magnetic fields have been found are all earlier types — that is, up to F in general.

R. P. Kraft   It's rather a very interesting point that if you put out of consideration binary stars among the stars in the domain of the HR diagram, where the rapid rotators lie, I think you can make a pretty good case for the statement that the only stars in slow rotation in that part of the HR diagram are in fact the stars with magnetic fields.

C. P. Sonett   Could you give some kind of idea what the bound is? Because, if I recall correctly, there's a limit imposed by line broadening turbulence and rotation.

R. P. Kraft   That's a very difficult question, one of the things that make it very hard to determine stellar rotations, when down to solar rotational velocities; there is a great deal of discussion in non-LTE circles about the meaning of turbulence, but it is true that there are other sources of line broadening that become competitive with rotation in the range less than 5 km/sec. So trying to disentangle rotation from other effects does begin to be difficult.

C. P. Sonett   How small a magnetic field can you determine?

R. P. Kraft   The field strengths that can be determined by the longitudinal Zeeman effect are limited to about 500 gauss. Under that one can't really say anything with certainty. That's about the difference between the left- and right-hand circularly polarized spectra, leading to about 1 μ on a typical plate. Under that, you can't say much for that kind of a field.

J. M. Beckers   It also calls for a variation of calcium emissions which could be stellar rotations. From the emission and principal possibilities and various velocities that have been observed, can you see any variations? ·

O. C. Wilson   I only get to the observatory maybe three or four nights each month, and also there are gaps due to weather and so on, so I cannot say. There are some variations that take place occasionally in the course of days or certainly weeks, but I

don't have a dense enough array of points so far to establish any periodicity. The only things that look as if they might be periodic are these long-term effects that Kraft showed on the screen here.

R. P. Kraft   Some years ago the Russian astronomer Chiginov discovered sinusoidal variations in brightness of a dwarf *M* type star with Ca and H emission lines (one would judge it to be a young *M* star); the variations in amplitude sometimes also show a phase shift. Similar variations in integrated broadband photometry have been found in other dwarf *M* type stars with emission lines, but not in dwarf *M* type stars without emission lines. This has been interpreted as a rotational modulation of a spotted surface. The periods tended to be of the order of a few days and the rotational velocities are not unreasonable — that is, 10 or 15 km/sec. But, of course, it remains to be seen whether that is the correct interpretation of the effect.

D. S. Intriligator   When you showed the pairs of the possible stellar cycles, in each case they weren't necessarily unrelated; it's just they were out of phase. If that is true, that's quite interesting.

O. C. Wilson   That's correct. In the 1961 Cygnus pair, I think there is just an out-of-phase relationship. In the other, I could not see anything (well, with four points) but random scatter in the case of the faint member.

D. S. Intriligator   Unless it's just out of phase the other way.

O. C. Wilson   Well, it went up and went down, then went up.

L. Mestel   I would like to ask for confirmation from the floor on the following. Fitting the observed velocities for the young cluster and the sun does imply a nonlinear dependence of the rate of loss of angular momentum on the angular velocity. I seem to remember Conti was quoted as saying this some years back. It would be very interesting in that it would, I think, give a broad hint of the magnetic field that is doing the braking, and, therefore, its strength should depend on the angular velocity and so one obtains a braking rate that is not linear.

M. M. Conti   The only data we have is for the Hyades, the Pleiades, and the sun, where we have the ages. If these are plotted logarithmically one obtains a reasonable linear relation of just about three points and two of them at one end.

L. Mestel   What is the index?

M. M. Conti   I'm sorry, I don't remember the number.

R. Dicke   I suppose you want to hear a comment on the initial rotation of 1.0 M_Θ mass stars; how much is known about that. I understand it's very little.

R. P. Kraft   Well, it is true that all this work on rotations was done at about 1.2 M_Θ. You see, the decline in rotation along the main sequence is very sharp, and you soon get down to where these troubles exist that Sonett was talking about; there are all kinds of observational reasons the stars are getting fainter, and you have to go higher dispersions to resolve the rotational velocity. Well, all the light is disappearing. You disperse the spectrum out still more and with less and less light. So you know less and less about stellar rotations, the fainter you go. And the difference even between stars of 1.2 M_Θ and of one is exceedingly difficult to handle. So we really don't know as much as we should.

COMMENTS

B. Durney   The angular velocity of the sun is calculated as function of time assuming that the initial angular momentum is given by extrapolating the main sequence values of stars with mass greater than 1.2 M_Θ [*Kraft*, 1968]. The angular momentum loss is assumed to be proportional to $\omega(\omega/\Omega_\Theta)^n$ with $n = 2$ where ω is the angular velocity and Ω_Θ is the sun's angular velocity. Two cases are considered: (1) rigid rotation; and (2) mixed outer shell coupled with a rapidly rotating interior only by viscosity [*Dicke*, 1970c]. A comparison is made between the computed values of ω and the observed angular velocities of 1 M_Θ stars [*Kraft*, 1967]. With the help of the simplest

approximations the time dependence of the magnetic field is estimated for the case of rigid rotation and found identical to that obtained by *Skumanich* [1971] from the variations of the intensity of Ca *II* emission with age.

The well-known *Dicke and Goldenberg* [1967] experiment measuring the difference in flux between the equator and the pole at the limb of the sun has been interpreted by *Dicke* [1970*a, b, c*] as resulting from an oblateness of the surfaces of constant density which define the limb of the sun. For vanishing magnetic and velocity fields, surfaces of constant potential and density coincide. The oblateness of surfaces of constant potential has been attributed by Dicke to a gravitational quadrupole moment caused by a rapidly rotating core. It is well established [*Kraft*, 1967, 1968] that stars with mass larger than $1.2\, M_\odot$ arrive at the main sequence in rapid rotation and that stars with convection zones are subsequently slowed down by the torque exerted by the solar wind. The angular momentum loss of the sun from this torque is given by *Weber and Davis* [1967]:

$$\frac{dJ}{dt} = \frac{2}{3}\, r_a^2\, \omega\, \frac{dM}{dt} \tag{1}$$

where ω is the angular velocity, r_a the Alfvénic point, and dM/dt the solar wind mass flow.

It will be assumed that the angular momentum of the sun when it arrived at the main sequence is given by extrapolating the angular momentum of stars without convection zones [*Kraft*, 1968; *Dicke*, 1970*b*]. Assuming uniform rotation, the present solar wind torque slows down the sun with an e-folding time of about 10^{10} years. Thus, if ω is independent of r, equation (1) can be written

$$\frac{d\omega}{dt} = -\alpha\omega, \qquad \alpha = \frac{1}{4\pi}\, \frac{r_a^2\,(dM/dt)}{\int \rho r^4\, dr} \tag{2}$$

with $\alpha(t = 5 \times 10^9 \text{ yr}) \sim 10^{-3}$, where the unit of time is 10^7 yr. There is little doubt that the sun's angular momentum loss was much larger in the past. The Alfvénic point r_a and presumably also to some lesser extent the mass flow (the properties of the convection zone have not changed significantly in 5×10^9 yr) depend on the average solar magnetic field at the surface. The sun's magnetic field is generated by a dynamo mechanism [*Leighton*, 1969; *Parker*, 1970] driven by differential rotation, which in itself is the result of the interaction of rotation with convection [*Durney*, 1971]. Thus, ultimately the torque exerted by the solar wind is a function of the angular velocity [*Spiegel*, 1968, 1971]. Following Spiegel we write:

$$r_a^2\, \frac{dM}{dt} \propto \left(\frac{\omega}{\Omega_\odot}\right)^n \tag{3}$$

The proportionality factor can be evaluated from the present known angular momentum loss of the sun. Equation (2) can thus be written:

$$\frac{d\omega}{dt} = -10^{-3}\omega \left(\frac{\omega}{\Omega_\odot}\right)^n \tag{4}$$

As described earlier, the initial value for the angular momentum (J_O) will be assumed to be given by extrapolating the main sequence values of stars with mass greater than $1.2\, M_\odot$ [*Kraft*, 1968]. This gives $\omega(t = 0) = 65\, \Omega_\odot$. Table 1 gives ω as function of time

Table 1. *Angular velocity as function of time*

$t(10^7)$ years	$n = 1$		$n = 2$		$n = 3$	
	$\omega \times 10^{-5}$ (a) Rigid rotation	$\omega \times 10^{-5}$ (b) Rotating shell coupled with the core	$\omega \times 10^{-5}$ (a) Rigid rotation	$\omega \times 10^{-5}$ (b) Rotating shell coupled with the core	$\omega \times 10^{-5}$ (a) Rigid rotation	$\omega \times 10^{-5}$ (b) Rotating shell coupled with the core
0.1	18.5	8.12	13.7	6.95	4.3	2.6
1	17.5	7.64	6.1	3.66	2	1.5
3	15.6	6.77	3.6	2.32	1.4	1.09
10	11.3	4.88	2	1.39	0.92	0.81
40	5.2	2.36	1	0.84	0.58	0.61
100	2.5	1.35	0.64	0.67	0.43	0.53
200	1.3	0.95	0.45	0.58	0.34	0.49
300	0.91	0.8	0.37	0.54	0.3	0.46
400	0.69	0.72	0.32	0.51	0.27	0.44
500	0.56	0.68	0.287	0.49	0.25	0.43

(in units 10^7 yr) for $n = 1, 2, 3$. The column "rotating shell coupled with the core" was computed by assuming that the outer shell is coupled with the core only by viscosity. In this outer shell the angular velocity is constant across the convection zone and varies as $1/r^2$ for $r_{con} > r > 0.54\ R_\Theta$ where r_{con} is the inner radius of the convection zone. The equation for ω (the angular velocity at the surface) is now [*Dicke*, 1970c).

$$\frac{d\omega}{dt} = -2.5 \times 10^{-3}\ \omega \left(\frac{\omega}{\Omega_\Theta}\right)^n$$

(5)

$$-\frac{4\sqrt{2\pi\nu_c}\ \rho_c r_c^2}{M_{r_c}}\ 1.78 \times 10^7 \int_0^t \frac{d\omega/d\tau}{(t-\tau)^{1/2}}\ d\tau$$

where the subindex "c" refers to values at the core surface ($r_c = 0.54\ R_\Theta$) and M_{r_c} is the mass contained in the outer shell ($r > 0.54\ R_\Theta$). The factor 1.78×10^7 is introduced by the time unit of 10^7 yr. Equation (5) was solved by Dicke for a torque that depends exponentially on time. In the present case, it can be solved numerically by iteration. The initial angular velocity at the surface was taken equal to $28.5\ \Omega_\Theta$, and $\omega(r, t = 0) = \omega(r_c, 0)$ for $r < r_c$. With these values of ω the angular momentum at $t = 0$ is equal to J_0.

For the case of rigid rotation and with the help of very crude approximations the time dependence of the magnetic field can be estimated as follows: In the first approximation the mass flow $C = dM/dt$ will depend on the total amount of mechanical heating absorbed by the corona. The corona is heated predominantly by shock waves [*Osterbrock*, 1961; *Kopp*, 1968] having their origin in the convection zone which has not changed much during the sun's lifetime. We assume therefore that the mass flow C is independent of time. Even for modest magnetic fields the Alfvénic point is at much larger distance than the sonic point and therefore the velocity at the Alfvénic point u_a should not depend strongly on B; we assume thus that also $u_a \sim$ constant.

Now, the Alfvénic point is defined by

$$\frac{4\pi\rho_a u_a^2}{B_a^2} = \frac{4\pi C^2}{B_a^2 r_a^4 \rho_a} = \frac{4\pi C^2}{B_\Theta^2 r_\Theta^4 \rho_a} = 1$$

(6)

where B_Θ is the magnetic field at the base of the corona and r_Θ is the radius of the sun. From equation (6), and from the assumptions that C and u_a are in the first approximation independent of time we obtain

$$r_a{}^2 \propto \frac{1}{\rho_a} \propto B_\Theta{}^2$$

Equation (3) (with $n = 2$) shows that $r_a{}^2 \propto \omega^2$ and therefore

$$B_\Theta \propto \omega \tag{7}$$

From equation (4) ($n = 2$) and from the initial value of ω ($\omega \sim 65\ \Omega_\Theta$) it is easily seen that ω will be proportional to $1/t^{1/2}$ for $t \gg 10^6$ yr. Therefore,

$$B_\Theta \propto 1/t^{1/2} \tag{8}$$

for $t \gg 10^6$ yr. This is exactly the dependence of the magnetic field on time found by Skumanich from the observed variations of Ca II emission with age.

The dependence of the magnetic field on angular velocity given by equation (7) differs from that suggested by *Cowling* [1965]. However, even the dependence of differential rotation on ω is not simple [*Durney*, 1971]. In relation to the magnetic field and keeping in mind Leighton's model for the solar cycle, differential rotation and the "tilt" of the axis of the sunspot groups depend on ω. It is unlikely, however, that the strong dependence of ω on r required by Leighton's model is real [*Durney*, 1971]. Perhaps the concentration of the magnetic flux by convection as suggested by *Weiss* [1964] should be included explicitly. The above considerations suggest that a reliable theoretical estimate of the dependence of B on ω is difficult.

We discuss now the dependence of the angular velocity on time as given in table 1. Consider the case $n = 2$. For the Hyades ($t = 3$) $\omega \sim 1.4 \times 10^{-5}$ [*Kraft*, 1967] and even the case of "rotating shell coupled with the core" gives too large an angular velocity. The angular velocity for the Pleiades ($t = 40$) is about 0.84×10^{-5} (in perfect agreement with case 2). However, its age is the subject of some discussion and could be as large as $t = 90$ [*van den Heuvel*, 1969]. The perfect agreement between the angular velocity of the sun and the case of rigid rotation for $n = 2$ is, of course, accidental. It is easy to understand on the other hand why case 2 gives angular velocities for $t = 500$ that are too large. Initially the viscous stresses at the core surface ($r = r_c$) are negligible and only the outer shell is slowed down, greatly reducing the angular momentum loss. At $t \sim 20$ the torque exerted by the core becomes significant; from then on the angular velocity decreases slowly. It should be noted from table 1 that the angular velocity decreases initially very fast. For the case of rigid rotation and $n = 2$, the sun is already rotating in about 2 days for $t = 3 \times 10^7$ yr. Even if during the Hayashi phase the primordial magnetic field was destroyed, since the star is convective, a dynamo mechanism would have generated a magnetic field that could have initially coupled the inner core ($r < 0.54\ R_\Theta$) with the outer shell. The sun could have rapidly lost angular momentum without a large amount of lithium destruction. Since it was likely that the magnetic field was subsequently expelled from the interior by magnetic buoyancy (enhancing possibly the initial angular momentum loss) the core and the outer shell could have decoupled giving rise to the observed lithium depletion [*Danziger*, 1969; *Wallerstein and Conti*, 1969].

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