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3-D Multilateration: A Precision Geodetic Measurement System

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The assessment of earthquake hazards, indication of probable locations for earthquakes, and the eventual possibility of earthquake prediction or premonitory warning have become an important part of the NASA Earth Physics Applications Program. The key to moving toward these goals is believed to be precision monitoring of the near- and far-field strain buildup and release within a few hundred kilometers of active fault zones such as the San Andreas. A system with the capability of determining 1-cm accuracy station positions in three dimensions has been designed using pulsed laser Earth satellite tracking stations coupled with strictly geometric data reduction.

Introduction

The systems analysis and laboratory demonstration described in this article indicate that a new technique of satellite geodesy, to be called *3-D Multilateration*, can determine the relative three-dimensional coordinates of ground stations within 1 cm over baselines of 20–10,000 km. With this high accuracy, several crucial geodetic applications become possible, including earthquake hazards assessment, precision surveying, plate tectonics, and orbital determination.

Achievement of this accuracy can be attained through use of pulsed lasers to measure simultaneously the slant ranges between an ensemble of six or more ground stations and a compact moving retroreflector whose trajectory is known *a priori* only to the accuracy necessary for aiming the lasers. Specifically, the positions of the satellite- or airplane-carried retroreflector are eliminated from the equations which govern determination of station locations. The trajectory of the retroreflector *and* the locations of the stations are determined in the same solution. The numerical analysis has shown that suitably chosen multi-station configurations result in well-conditioned solutions with very small error magnification of the inherent ranging errors.

Laboratory tests have demonstrated that a laser hardware system with a ranging accuracy of 3 cm can be built from commercially available components. By 1975, at the latest, an accuracy level of 1 cm can be achieved. Costs are probably lower than other proposed systems on a sites-occupied and determined basis.

Nature of the Earth Physics Applications

The basic cause of earthquakes is an unrelieved buildup of strain within the Earth. In many situations, this strain is relieved by a gradual creeping between the crustal blocks on different sides of a fault. However, if the creeping is insufficient to relieve the buildup of strain, an earthquake will eventually result. Many geophysicists now believe that a reasonable estimate of the location, and possibly the approximate time, of major earthquakes in known fault zones can be made by relating: (1) the amount of creeping taking place along the fault, and (2) the strain buildup in a large region surrounding the fault.

There is little problem in measuring the creeping along the fault since the motion takes place over relatively short distances. However, in order to evaluate the strain buildup in a large region surrounding the fault, it is necessary to measure precisely the relative motion of points in various parts (including the periphery) of this large region. If this motion is to be evaluated within a period of 1 to 2 yr, it will be necessary to measure distances as large as 1000 km with an accuracy of 3 cm or better. At present, there is no technique which is capable of making such measurements.

The 3-D Multilateration technique appears to provide not only a 1-cm accuracy distance measuring capability, but also determines each station position in *three dimensions.* This allows complete evaluation of the strain field. Furthermore, since the 3-D Multilateration technique will require only 1–5 days to obtain raw data and process this data into station positions, it will be possible to detect the presence of rapidly occurring ground deformations which may give a premonitory indication of earthquakes.

It should be noted that the 3-D Multilateration technique can be used in facets of geophysical analysis not directly related to earthquake prediction. This technique is sufficiently flexible to permit a wide variety of applications, as illustrated in Figure 1.

In summary, if the performance of the 3-D Multilateration technique demonstrated in this article is realized, not only will science be provided with a valuable tool for investigation of tectonic phenomena, but there is a significant possibility that geophysicists will be able to predict the location, and perhaps the approximate time, of future earthquakes.

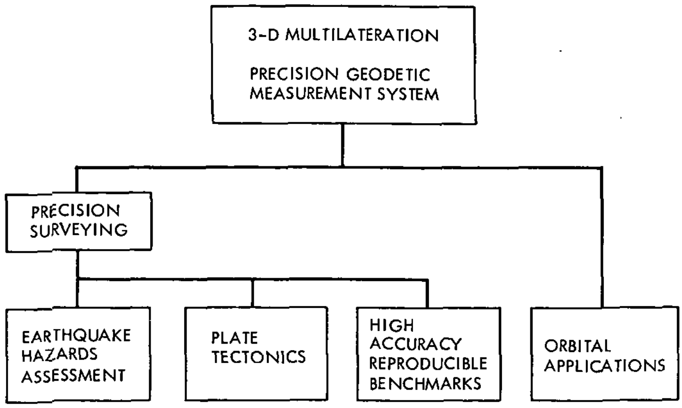

Figure 1. Applications of 3-D Multilateration

The Requirement for 1-cm Position Determinations

Fault motions along the San Andreas are in the range of 3 cm per year, as measured near the fault zones. Tectonic plate motions might be as high as 5–10 cm per year. If we assume that 2 cm per year is a reasonable motion threshold in creep determinations, then it can be easily calculated that the accuracy with which station positions can be determined (expressed in centimeters) is also the number of years over which sites must be monitored in order to prove the motion is real, and obtain a safe quantitative measure of it (assuming a 2:1 motion-threshold to error-level ratio).

For this reason, an accuracy of 5 cm or better is required in these applications, and 1 cm is a highly desirable goal. Systems for station position measurement which fail to yield all three dimensional components, or which cannot be refined to approach 1 cm, are weak in this context of earthquake hazards estimation and precision surveying. For the purpose of the system reported here, 1 cm is the assumed goal, and it is shown that this can be closely approached with equipment and methods of data analysis which currently exist and can be assembled into the 3-D Multilateration hardware–software system.

Description of the 3-D Multilateration Technique

The 3-D Multilateration technique can be implemented through use of a number of ground stations which simultaneously transmit laser pulses to compact reflectors on a moving vehicle, e.g., an airplane or a satellite. Each station evaluates station-to-vehicle range by measuring the time interval between transmission of the original pulse and reception of the reflected pulse. Simultaneous range measurements are then processed so as to yield relative station locations in three dimensions.

The geometric coordinate system used for this system is a relative coordinate system, in which Station 1 is placed at the origin, arbitrary placement of Station 2 fixes the X axis, and arbitrary location of Station 3 defines the X–Y plane of the coordinate system (See Figure 2).

This coordinate system can be linked to the inertial geocentric coordinate system, if desired, via the geographic coordinates of the first three adopted stations. Errors in this transformation, however, have no bearing whatsoever on the *relative* station position determinations necessary for the tectonic applications.

Conceptually, only two stations are required for determination of the baseline between them. In this case, the inertial trajectory in Earth-centered coordinates must be computed and modeled over days, weeks, and even months to permit the data reduction. That is, in the existing dynamic laser tracking systems, several passes of the satellite over two stations are required to permit adequate determination, and these determinations must be done by tying the satellite passes together in geocentric coordinates. The difficulty of modeling spacecraft behavior to sub-meter accuracy levels is well known, and probably limits the dynamic laser systems to accuracies of 20 to 50 cm.

Solving equations simultaneously for both station and satellite positions, thereby eliminating dependence upon trajectory calculation, turns out to be possible if six or more stations range simultaneously. Conceptually, the reason is as follows. Assume six stations range to four reflector positions (not necessarily on the same trajectory). This yields 24 equations which are equal to the number of trajectory parameters (4×3) plus the station positions in the above coordinate system ($6 \times 3 - 6$). It turns out, in detailed theoretical analysis, that six stations ranging to four or more points (not in the same plane) are sufficient to solve the 24 simultaneous quadratic equations. It is only necessary that the stations be reasonably located (not in a straight line,

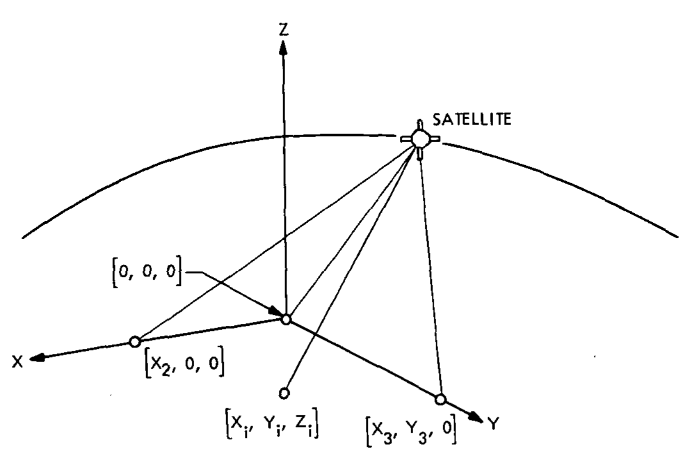

Figure 2. Geometric coordinate system

for example), and that the reflector points not be in a plane (using parts of two different spacecraft passes, for example). Redundant observations contribute to a least squares solution of improved quality, and, in actual operation, thousands of observations may be combined. Since geometric systems are reversible in principle, four stations viewing six trajectory points should also work. This is the case, but only if the four stations are well out of a plane (are intercontinental, for example). This and other special cases noted in Reference 1 are not discussed further here in the interest of brevity.

We see the key point, then, that simultaneous ranging from six or more stations frees laser tracking from the limiting error source of current dynamical approaches. The trajectory need be known only well enough to acquire the spacecraft for tracking, and the limiting accuracy achievable improves from 20 cm to under 1 cm in principle. 3-D Multilateration provides:

(1) The relative three-dimensional coordinates of all six stations utilized in the operational configuration.

(2) The equally precise positions of the moving retroreflector in the adopted coordinate system.

Given six ground stations (Figure 3) which make a sequence of four or more simultaneous strikes over two or more satellite passes separated in time by any arbitrary duration, a mathematically and numerically stable solution for the determination of relative three-dimensional station locations exists, and can be obtained without any knowledge whatsoever of the satellite position.

Hardware System and Accuracy Demonstration

The hardware subsystem for measuring station-vehicle ranges is identical for all station configurations. In order to attain high accuracy range measurements, the subsystem utilizes a new type of pulsed laser. This laser is a mode-locked, Q-spoiled ruby laser, and has the capability of emitting light pulses of very short duration (0.1 ns or less). The subsystem employs a tracking mount to aim transmitting and receiving telescopes at the vehicle. A measurement is made of the time required for a laser pulse to make a round-trip flight from the station to the vehicle-borne retroreflector. This time, measured with a resolution of 0.1 ns, is used in conjunction with an atmospheric model to calculate the range to the vehicle.

Each ground station also contains an X–Y tracking mount for steering the two telescopes, a small computer to direct the tracking mount toward the satellite, timing circuitry, recording equipment, and power supplies. In order to satisfy the requirement for "simultaneous" ranging, the clocks at each station must be synchronized to 3 μs; such synchronization is easily achievable using low-cost components. Synchronization of laser firing to 1 ms is adequate and well within the state of the art; variations of laser firing within this range are compensated for by time-tagging range measurements,

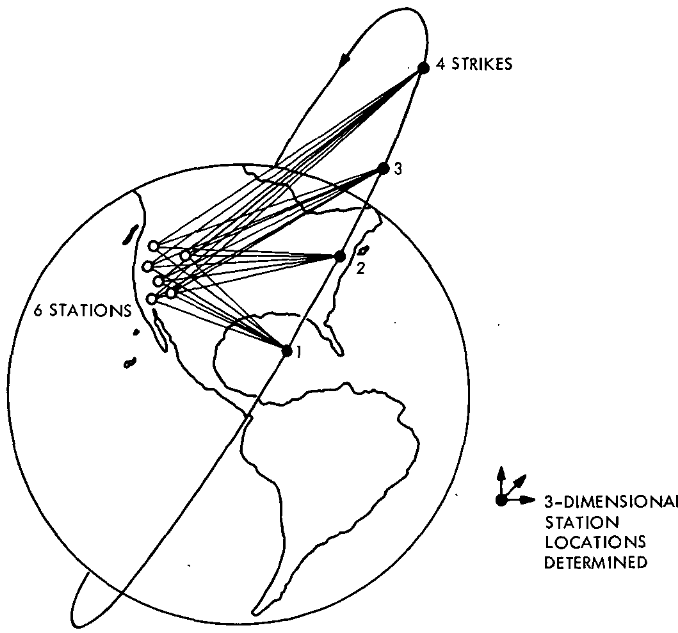

Figure 3. Six-station solution

and interpolating between successive measurements so as to obtain effective simultaneity among stations.

A demonstration ranging system was assembled using commercially available components in order to evaluate the measurement errors produced by current hardware. This system simulated a long distance ranging system over short path lengths by using attenuated return signals. The results obtained with this system show that hardware-related errors in range measurement can be made acceptably small, typically less than 2 cm. Future systems, fabricated circa 1975, can be expected to operate at satellite distances with 1-cm ranging accuracy.

Figure 4 indicates the schematic of hardware components in the system. These can be assembled from commercially available components, packaged compactly, and placed in a transportable van as indicated in Figure 5.

Analysis of System Errors

It is emphasized that the system errors caused by the satellite, Earth constants, and orbital perturbations do *not* enter into the process. In fact, since the proposed techniques are independent of the location of the retroreflector, the only error sources which enter into the range measurement are:

(1) Bias error due to atmospheric delay.

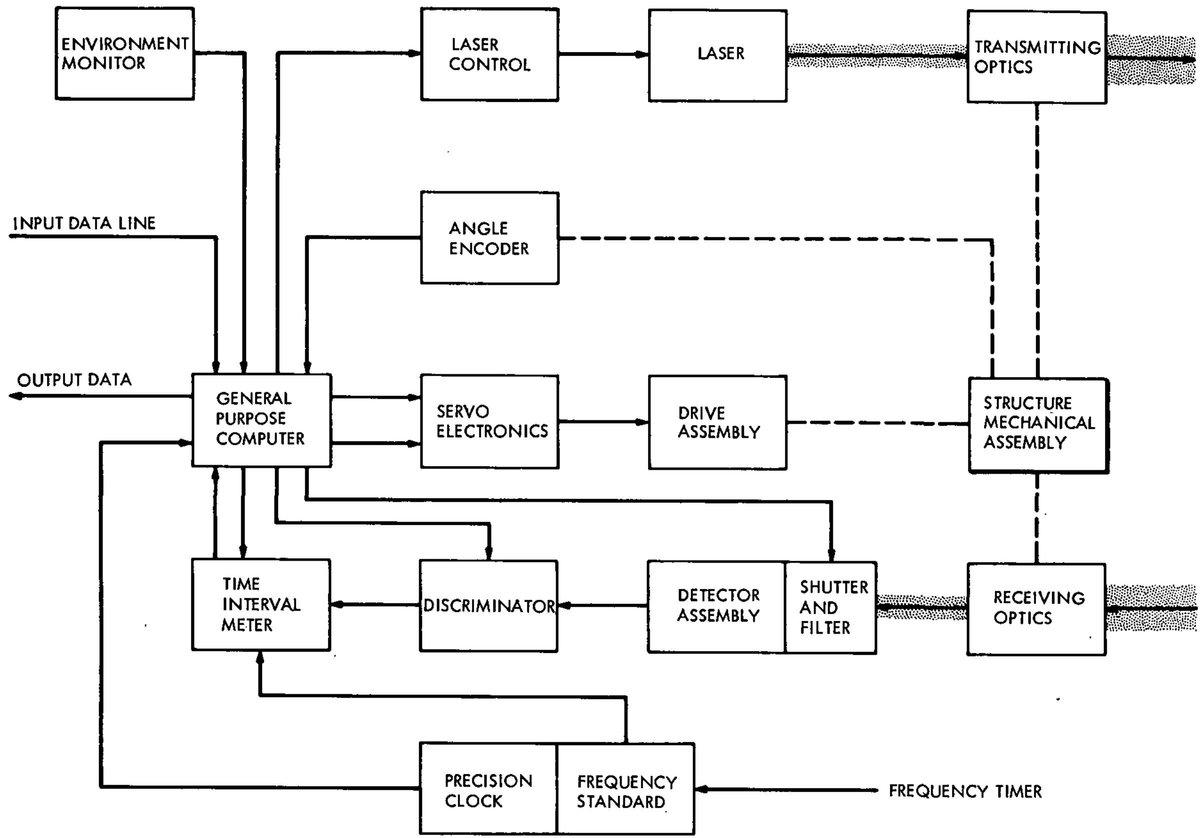

Figure 4. Schematic of laser ranging station hardware

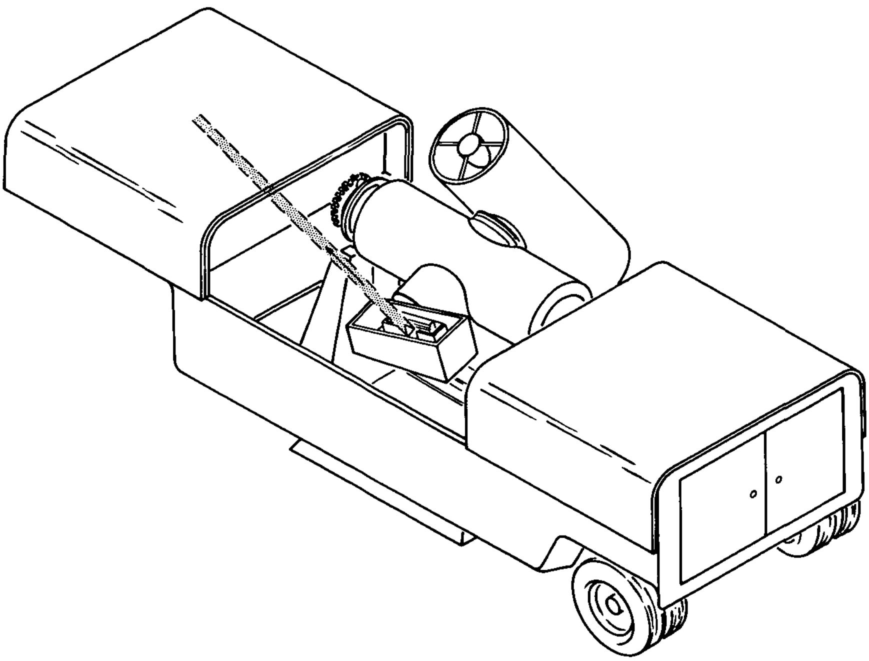

Figure 5. Artist's conception of laser tracking station

(2) Random error due to atmospheric turbulence.

(3) Random equipment measurement error.

(4) Equipment bias error.

It is shown in Reference 1 that a ranging subsystem can be fabricated in which the net effect of these errors will lead to ranging accuracy of 1 cm.

Careful sensitivity analyses of the system have been made. In addition, a detailed Monte Carlo numerical simulation of the system was conducted, in which the raw range measurements were corrupted by random noise, and the station parameters were solved for various configurations and data sets. The conclusions appear in detail in Reference 1, and include:

(1) For a single data set consisting of two satellite passes at different altitudes (such as 500 and 750 km), with 50 ranging points taken to each, the standard deviation of error in station coordinates ranged from one to four times the random range error (error magnifications of 1 to 4 times).

(2) For 10 passes as above, in a combined solution, the error in station coordinate determinations ranged from 0.1 to 1.0 times the ranging error.

We can conclude from this that, given a 1-cm ranging system, station positions of 3- to 5-cm accuracy can be obtained on each clear-weather day of observation. For 10 such days, which would be a normal site-occupancy time, accuracy of 0.3 to 1.0 cm would be obtained. No other proposed system of station position measurement has as much promise of reaching such high accuracy.

The effect of biases was studied briefly, with the conclusion that biases tend to cancel out due to the varieties of geometry obtained over different data sets, and that the system is therefore stable in the presence of bias-type errors. Detailed study of bias plus random errors in the ranging systems will be conducted in the coming months, but the results are not expected to cause any changes of consequence within the conclusions stated in this article.

Comparison With Other Techniques

There are three other techniques which are comparable to the 3-D Multilateration technique:

(1) The Geodolite.

(2) The Goddard Laser Ranging System.

(3) Very Long Baseline Interferometry.

Spectra-Physics of Mountainview, California, manufactures a laser-ranging device known as the *Geodolite*, which is capable of measuring the distance between points on the Earth's surface. The Geodolite can measure only line-of-sight distances, but provides excellent accuracy over its severely constrained range. For example, the U. S. Geological Survey (USGS) has used a Geodolite to measure baselines of 30 km with accuracies of 1 cm or better; however, in order to achieve this accuracy, the USGS combined a number of partially redundant measurements from different benchmarks, and then corrected these raw measurements with temperature and humidity data obtained by overflying the baselines with a helicopter. In summary, although the Geodolite appears to be an excellent device for obtaining high accuracy distance measurements over short baselines, it is not an acceptable tool for measuring the three-dimensional components of distance over long baselines. (Although it would be conceptually possible to augment the Geodolite's measurements with measurements of vertical motion provided by tiltmeters or gravimeters, such systems are not feasible in practice.)

The second technique was pioneered by Goddard Space Flight Center. It employs laser ranging between two stations and a satellite retroreflector. Three-dimensional station locations are deduced from processing data over long arcs of weeks and months. This is the "dynamic system" referred to above. Polar motion, possibly universal time, and long period orbital information can be secured in addition to station locations. Current accuracies range in the 0.3- to 2.0-m range depending upon which parameter is considered, with station locations tending toward accuracy levels of 1 to 2 m. As has been noted above, long period orbit determination and modeling restricts the accuracy of station parameter solutions. It is doubtful if the orbit determination capabilities can be sufficiently improved in the foreseeable future to break the 10-cm accuracy level. This is precisely why the geometric techniques are being considered in the context of station location solutions.

Very Long Baseline Interferometry (VLBI) can deduce station locations in three dimensions by measuring differential time of arrival at two or more stations of identical random radio signals from extragalactic radio sources.

The ability to convert VLBI observations into accurate geophysical measurements is limited mainly by Earth atmospheric and charged-particle uncertainties. Dual-frequency VLBI has promise for charged-particle calibrations, and water vapor radiometry are promising for atmospheric wet-component calibration.

Recent VLBI experiments have demonstrated a formal instrumental precision of 5 cm for measurement of two components of a 16-km baseline using 4 h of data. Caution is required in extrapolating these 5-cm results to longer baselines as the systematic atmospheric and charged-particle errors may dominate. Further experiments are planned, and 3-cm or better three-dimensional station location determination accuracy is forecast for the 1975 time frame.

Advantages of this method include: (1) ability to develop 3 cm or better station location accuracy in geocentric coordinates, independent of the separation between stations, and relative to a nearly invariant extragalactic set of sources. (2) The system is virtually weather-independent. (3) Portable stations are feasible (ARIES System: Astronomical Radio Interferometric Earth Surveying). (4) There is no requirement for orbiting reflectors, or to transmit any radio or light signals from the Earth. VLBI is entirely passive, using only natural radio signals.

Possible disadvantages include dependence upon experimental frequency and digital recording systems, the larger (than optical system) dependence on charged-particle and atmospheric corrections, the need for time and polar motion calibrations (in the geocentric system), and possible reliance on major radio tracking facilities. None of these constraints appears to be critical, and the method has great promise.

Implementation and Operational Considerations

The dry atmosphere of the Earth can be calibrated to 1 cm or better, and the hardware will be operational at about the 1-cm level. Weather imposes a constraint until the number of operational stations goes above 10 or 12, as it is necessary to have at least six stations viewing simultaneously. For employment in a favorable climate, over a local area (several hundred kilometers), analysis indicates that eight stations is sufficient to guarantee substantial data returns.

The main implementational disadvantage of the system is that approximately eight stations must be constructed. Reference 1 includes cost estimates for station procurement in lots of four to eight. This estimates that the station cost is one-half to one-third that of competing systems. Operation is intended to be semi-automatic, to keep operational costs down. If these difficult cost and operation constraints can be met, total system cost should be at least comparable to other competing techniques. On a per site-occupied basis, even including substantial transportation costs, the price is probably less than any currently considered system. This arises from the larger number of stations.

As was seen in Figure 5, the system is carried in a single trailer, to be emplaced on prepared sites. An eight-station system should be able to comfortably occupy 50 to 80 sites, twice per year, at a reasonable transportation and manpower cost. This would create a substantial number of precision surveying points suitable for a major geophysical monitoring program as described above.

Conclusions

The study described here, and detailed in Reference 1, has shown that a geometric, multi-station, laser tracking system is feasible and will deliver cost effective, 1-cm accuracy, three-dimensional station position information suitable for major advances in earthquake hazard estimation and geophysics.

The key point of the systems proposal is obtaining freedom from errors in Earth satellite trajectory computations, which currently limits existing two-station laser systems to an accuracy of 20 to 100 cm. The price of this freedom and markedly increased accuracy is six or more stations simultaneously operational. The cost and complexity of such networks does not appear to be prohibitive, and is, on a site-occupied basis at least, quite competitive with existing laser systems and proposed very long baseline radio interferometry approaches.

The proposed 3-D Multilateration system is the only system currently analyzed which has been shown to be capable of 1 cm and lesser errors in three-dimensional station coordinate determinations, which can be operational in the 1975 time frame.

Reference

1. Escobal, P. R., et al., *3-D Multilateration: A Precision Geodetic Measurement System,* Preliminary Document TM 391-340, July 5, 1972 (JPL internal document).