

# SATELLITE ALTITUDE DETERMINATION UNCERTAINTIES

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## I. INTRODUCTION

The subject of Satellite altitude determination uncertainties will be discussed from the standpoint of the GEOS-C satellite, which is representative of the state of the art of the first half of the decade of the seventies, and also from the longer range viewpoint afforded by the Geopause concept which gives us a glimpse of the possibilities for the latter half of this decade. GEOS-C will be tracked by a number of the conventional satellite tracking systems which have been used with GEOS-I and GEOS-II, which were tracked by range and range rate systems, laser systems having accuracies of the order of a meter, C-band radar systems and the Tranet Doppler system. GEOS-C will also be tracked by two advanced systems; namely, a satellite-to-satellite tracking system and lasers capable of decimeter accuracies which are being developed in connection with the Goddard Earth and Ocean Dynamics Satellite Applications Program (1 - 4). Aspects of satellite-to-satellite tracking and laser tracking are also being discussed in other papers presented at this conference. (11, 17)

The present discussion will focus on methods for short-arc tracking which are essentially geometric in nature. One uses combinations of lasers and collocated cameras. The other method relies only on lasers, using three or more to obtain the position fix. Two typical locales are looked at, the Caribbean area, and a region associated with tracking sites at Goddard, Bermuda and Canada which encompasses a portion of the Gulf Stream in which meanders develop. This latter region, which is of interest for oceanographic, earth dynamics, and practical reasons, will be referred to here simply as the Gulf Stream Meander region.

The discussion is organized in terms of a specific type of GEOS-C orbit which would satisfy a number of scientific objectives including the study of the gravitational field by means of both the altimeter and the satellite-to-satellite tracking system, studies of tides and of the Gulf Stream meanders. This serves to indicate an experimental configuration which is compatible with these several objectives of a program such as that of GEOS-C.

The long-arc tracking of GEOS-C can be considered in terms of satellite-to-satellite tracking and in terms of tracking by means of other systems such as precision laser systems, for example.

For the purposes of the first part of the discussion, two GEOS cases will be considered. The first deals with results of a study conducted by Berbert and Loveless to indicate capabilities in the Caribbean area using a short-arc approach (5). Here the orbital inclination was taken to be  $22^\circ$ , a value which was originally planned for GEOS-C. The results of this study are not, however, affected significantly by this choice since geometrical arrangements similar to those considered here would occur for other inclinations now under consideration. The second case deals with an inclination of  $65^\circ$  which is one of the higher values now being considered for GEOS-C. The final choice will probably lie somewhere between these two values. For this case too, there is interest in a short-arc calibration and validation capability. It is of interest to select a region which will serve as many of the scientific objectives as possible and yet be reasonably practicable to implement too. In order to indicate the kinds of scientific objectives which might be served, a particular typical selection for the orbit of GEOS-C will be discussed.

## II. SHORT-ARC TRACKING OF GEOS-C IN THE CARIBBEAN AREA

The consideration of short-arc and long-arc tracking error budgets can begin with a look at the overall error problem. A typical error breakdown for the GEOS-C altimeter is indicated in Table I (6). Quantities associated with factors other than the orbit errors have an rms value of approximately 3 meters. This leaves 4 meters or so which can be assigned to the calibration process if the 5 meter rms overall accuracy goal is to be met. Allowing 1 or 2 meters for uncertainties associated with the geoid means that the uncertainties associated with the orbit determination process should contribute no more than about 3.5 meters.

A detailed analysis of short-arc tracking using lasers and cameras in the Caribbean area has been conducted by Berbert and Loveless (5). A GEOS-C ground track for the  $22^\circ$  inclination case in the neighborhood of several possible tracking locations in the Caribbean is seen in Figure 1. Elevation angles as functions of time for 4 of these sites for an orbit at a mean height of about 800 nautical miles are seen in Figure 2. The durations of the corresponding tracks above an angle of about  $48^\circ$  are indicated in Figure 3.

Results of an analysis of orbital altitude uncertainties determined by means of geometric error propagation using range and angle data from Antigua are seen in Figure 4. A reasonably conservative value of 2 meters is assumed for the laser range uncertainty and results for various values assumed for the angle uncertainties are indicated by the several curves. Accuracies of a second of arc should be achievable with cameras of the MOTS type, for example.

An analysis of a number of cases involving various combinations of lasers and cameras is summarized in Figure 5. Assumptions underlying these analyses are listed in Table II. The other angle measure accuracies of 100" listed there were those assumed for the laser angles used in the

analyses indicated by the open circles in Figure 5. In all cases, in addition to the altimeter bias uncertainty, uncertainties in orbital, survey, and range measure parameters were also estimated. The triangle corresponds to a similar analysis of a three-laser-only case made for a much larger triangle based on stations at Antigua, Key West, and Panama. It resulted in a value of 4.1 meters, only slightly higher than that for the smaller triangle. As can be seen, a number of cases meet both the basic 4 meter requirement and the 3.5 meter figure obtained by allowing a couple of meters for uncertainties associated with the geoid.

Berbert and Loveless concluded that the 2 laser 2 camera combination was probably the most cost effective in terms of the probabilities of obtaining reasonable amounts of data.

### III. THE SELECTION OF A TYPICAL GEOS-C ORBIT

The GEOS-C altimeter is expected to be of value in connection with studies of the earth's gravitational field and, if sufficient accuracy can be obtained, also in connection with studies of tides and circulation phenomena such as those associated with the Gulf Stream, for example.

#### A. Gravitational Field Studies

Studies of the gravitational field will also be conducted by means of the satellite-to-satellite tracking system. If one begins with the assumption of the value of  $65^\circ$  for the inclination of GEOS-C, and a value of 0.005 or less for the eccentricity to simplify the altimeter design, one is at liberty to adjust the mean altitude or the period within certain limits in the attempt to achieve as many of the scientific goals as possible. Altitudes within one or two hundred kilometers of, say, 900 kilometers are not unreasonable to consider here on the basis of current thinking about GEOS-C choices.

#### 1. Satellite-to-Satellite Tracking Studies of the Gravitational Field

The gravity field experiment conducted with the satellite-to-satellite tracking system can resolve gravitational features only down to a certain size which is a function of the satellite altitude. This function has been studied by Schwartz who presents the relationship shown in Figure 6(7). On this basis, a satellite at an altitude between 900 and 1000 kilometers could resolve gravitational features about six degrees in size if it is tracked from another satellite. There is interest then in achieving a ground track spacing of approximately  $6^\circ$  at the equator for the purposes of the satellite-to-satellite gravitational field experiment. Such a study of the gravitational field will be of great interest intrinsically, and will provide the material for a most valuable comparison with altimeter studies of the gravitational field.

## 2. Altimeter Studies of the Gravitational Field

The altimeter, on the other hand, is capable of finer resolution. Ultimately, a one degree survey is desirable, for example. A mean altitude of about 980 kilometers and a nodal period of about a 105 minutes permits the achieving of both of these objectives. It is characterized by equator crossing spaced about  $26^\circ$  apart between each revolution and separated by about  $6\text{-}1/4^\circ$  each day, as is indicated in Figure 7. Thus, at the end of four days, the equator crossing has moved some  $25^\circ$  and the tracing of the one degree pattern then begins. This takes some 25 days to complete. Since the altimeter cannot operate continuously, due to power limitations, an actual survey of this type would take much longer, on the order of a year, in fact.

Clearly other strategies are possible, e.g., by selecting patterns which would give spacings of  $6^\circ$ ,  $3^\circ$ ,  $1.5^\circ$ , etc. The example sketched here will suffice for the purposes of the present discussions, however. Resonances may be associated with some of these choices. A preliminary look at this point indicates that these will not be unduly severe, however.

With these specific choices in mind, then, one is in a position to consider the problem of short-arc tracking concretely.

### B. Oceanographic Studies

The Atlantic region off the coast of the Northeastern United States is of particular interest from the standpoint of the Gulf Stream meanders as is indicated in Figure 8 which is given by Hansen (8). These features have amplitudes on the order of a meter and hence might be within the capability of an altimeter of the kind to be flown on GEOS-C, or possibly on a spacecraft of the SATS type. Tidal variations in this same region, while not quite as large as those found elsewhere, are nevertheless of considerable size, i.e., of the order of a meter also. This is indicated in Figures 9 through 11, where the certain tidal components are seen (9, 10). This region is also a reasonably attractive one from the standpoint of some of the practicalities of short-arc tracking. Good advantage could be taken of lasers usually available at Goddard, and possibly also at SAO.

An unusually useful system could be obtained by adding lasers at Bermuda and at a Canadian site chosen to be on the same meridian as Bermuda and as far north of Goddard as Goddard is north of Bermuda. This configuration is ideal for precision, short-arc tracking of GEOS-C in the region we are focusing upon. This can be seen readily from an inspection of Figures 12 & 13. Lasers having 10 centimeter accuracy capabilities will, when located at these sites, make it possible to determine the altitude of GEOS-C with relative accuracies of the order of a meter or better over a considerable portion of the region defined by these tracking sites at Goddard, Bermuda, and in Canada. A fourth laser at SAO would provide the important checks on the instrumental biases by providing the redundant information. It would also be most valuable in connection with reducing the impact of the cloud cover problem.

## 1. Gulf Stream Meander Studies

The altimeter tracking patterns are also good for observing the Gulf Stream meanders. Shown in Figure 13 are surface tracks of a  $65^\circ$  orbit with  $6\frac{1}{4}$  daily spacing which was obtained in the earlier discussion. It is seen that the northward and southward going tracks cross the two principal branches of a typical Gulf Stream meander orthogonally, providing almost ideal geometry for studying the behavior of these interesting features. Each ground track seen in Figure 13 will be followed four days later by one removed just one degree from it, hence it will be possible to observe each feature once every four days. This frequency is well matched to the observational needs of a Gulf Stream meander experiment, as can be seen from inspection of Figures 13 and 14(8). The mean wave length of a meander is often of the order of 300 kilometers, as Figures 13 and 14 show. A typical meander moves a distance equal to its own wave length in about a couple of months. This interval might be thought of as a characteristic time constant which can be associated with the Gulf Stream meanders in this sense. Observations every four days are well suited for such an experiment. In fact observations every ten days or so would be most welcome, as Hansen has already pointed out (8). This also allows a margin for gaps in the observing program which might be due to such things as weather conditions or operational factors.

Similar studies of the Kuroshio current could be conducted by means of lasers similarly placed in Japan and nearby islands such as Iwo Jima.

## 2. Tidal Studies

Tidal studies can also be conducted in this region by means of short-arc tracking. Once each day the GEOS-C altimeter satellite ground track passes through or very close to the Goddard-Bermuda-Canada triangle as is indicated in Figures 13 and 15. At least one of the tracks of the type seen in Figure 15, for example, would occur each day. These tracks are nearly orthogonal to the co-range lines of the semi-diurnal tide as can also be seen from Figure 15. The orbit selected for GEOS-C in the above discussion has the property of moving about  $10.5$  degrees each day relative to the moon. A complete cycle of the semi-diurnal lunar tide is thus observed by GEOS-C about once every 17 days. The daily observations of GEOS-C in the Goddard-Bermuda-Canada triangle thus occur about  $10.5^\circ$  apart in this cycle, and hence provide ideal data for sampling this important tidal component.

## C. Earth Dynamics Studies

The Goddard-Bermuda-Canada triangle also has other uses in connection with the Earth Dynamics side of the Earth and Ocean Dynamic Satellite Applications program (1).

### 1. Polar Motion and UT 1

The Bermuda-Canada leg is suitable for observing polar motion in the manner of the experiment conducted by Smith (11). The Goddard-Bermuda and Goddard-Canada links taken together are also useful for a companion experiment to observe the variations of the earth's rotational rate.

## 2. Gravitational Field Fine Structure

Fine structure in the gravity field should also be deducible from the observations made in this general area, but perhaps somewhat away from the immediate neighborhood of the Gulf Stream meanders.

### IV. LONG-ARC TRACKING

The surveys of the gravitational field over longer arcs will be greatly facilitated by the long-arc satellite-to-satellite tracking of GEOS-C which can be conducted through ATS-F. The accuracy capability of this tracking approach is indicated in Figure 16. In the case looked at here, accuracies of some four meters or better persisted for almost three hours beyond the time interval shown in the Figure before the results deteriorated. It is seen that altitude accuracies in the 3 to 4 meter range can be achieved in this way. This is reasonably comparable to the current estimates of the accuracy of the world-wide geoid obtained from satellite orbit analyses (12). The latter have spatial resolution of the order of  $12^\circ$ , however. Hence, altimeter and satellite-to-satellite tracking surveys even at the  $6^\circ$  resolution level will definitely provide new information. They will of course also provide the extremely valuable independent views which are so important. Satellite-to-satellite tracking may also be useful when combined with precision laser tracking in the Goddard-Bermuda-Canada triangle in making observations in the neighborhood of the amphidromic point in the North Atlantic seen in Figures 9 and 10. Such a region could be a good one in which to make the cross-over point checks which have been proposed by Stanley (13).

### V. THE GEOPAUSE SATELLITE SYSTEM CONCEPT

From the long range point of view the aim is to study sea surface topography at the decimeter level (1, 16). Difficulties in the current state of the art associated with lack of sufficient knowledge of the gravity field prevent this at the present time. The Geopause satellite concept offers the promise of being able to contribute here in connection with the main problem of satellite oceanography, i.e., that of observing the height of the ocean surface relative to the geoid at sub-meter accuracy levels (4). The Geopause spacecraft is conceived of as being in a polar, nearly circular orbit at a distance of about 4.6 earth radii and having a period of about 14 hours in an orbit plane which is both polar and normal to the ecliptic. (Cf. Figure 17.) At this height uncertainties in only a few gravitational harmonic terms correspond to orbit perturbation amplitudes above the decimeter level. The tracking data coverage afforded by the Geopause orbit is ideal for doing the three things necessary for dealing with the orbit determination problem at the decimeter level, i.e., solving

for those remaining environmental parameters which are effective and observable at this level, solving for tracking station locations, and monitoring tracking system biases on a continuing basis. (Cf. Figure 18.) Estimates indicate that the Geopause, tracked by two-centimeter ranging systems from ten selected NASA-affiliated sites for a week, could yield locations of these stations and of the Geopause satellite altitude with decimeter accuracy.

#### 1. Earth Dynamics Experiments

This furnishes the basis for high-resolution polar motion and UT 1 studies and advanced fault motion experiments.

#### 2. Oceanographic Experiments

Tracking from two Geopause satellites separated by a quarter of a revolution to an altimeter spacecraft in a coplanar low-altitude orbit should furnish the basic data for finding two components of the altimeter's position in the Geopause orbit plane with accuracies approaching a decimeter. From these two components one can determine any other two components including, in particular, the radial distance component. (Cf. Figure 19.) This is obtained relative to the coordinate system defined by the Geopause system, and hence relative to the earth's center. Decimeter altimeter data of the type which is anticipated will then give the position of the ocean surface relative to the altimeter spacecraft at this accuracy level. The position of the geoid is determined independently through information gotten from the tracking between Geopause and a coplanar, low-altitude gravity field satellite by means of a range rate system having 0.03 millimeters per second accuracy. (Cf. Figure 20.) A survey of the gravitational field can be completed by this approach in about a couple of months using a drag-free satellite orbiting at an altitude of about 250 km. This will furnish the basic information for determining the position of the geoid at decimeter accuracy with  $2.5^\circ$  spatial resolution. Thus one has, independently, the positions of the geoid and of the ocean surface to submeter accuracies, from which the heights of the ocean surface above the geoid follow directly. A whole new range of oceanographic experiments will thus be opened up. For example, the surface heights just mentioned will provide the basic boundary condition data for use in unlocking problems associated with the general circulation of the oceans. Detailed studies of currents, tides, storm surges, and tsunamis will also then become feasible.

## VI. REFERENCES

1. "Earth and Ocean Dynamics Satellite Applications Program," NASA, Washington, D. C., April 1, 1971 (Preliminary Issue).
2. Siry, Joseph W., "Proposed Earth Physics and Geodesy Programs Including an ATS-GEOS Tracking and Orbit Determination Experiment" letter to NASA Headquarters, J. Naugle and J. Rosenberg with enclosure, August 27, 1969.
3. Felsentreger, T. L., Grenchik, T. J., and Schmid, P. E., "Geodetic Earth Orbiting Satellite (GEOS-C)-Applications Technology Satellite (ATS-F) Tracking Experiment," GSFC X-552-70-96, March 1970.
4. Siry, Joseph W., "A Geopause Satellite System Concept," Presented at the Symposium on the Use of Artificial Satellites for Geodesy of the American Geophysical Union Annual Meeting, Washington, D. C., April 15-17, 1971.
5. Berbert, John H., and Loveless, Fred M., "A Satellite Altimeter Bias Recovery Simulation," GSFC X-550-71-224, May, 1971.
6. Stanley, H. Ray, Private Communication.
7. Schwarz, Charles R., "Gravity Field Refinement by Satellite to Satellite Doppler Tracking," Department of Geodetic Science Report No. 147, Ohio State University Research Foundation, December, 1970.
8. Hansen, Donald V., "Gulf Stream Meanders Between Cape Hatteras and the Grand Banks" Deep-Sea Research and Oceanographic Abstracts, 17, 495-511, 1970.
9. Hendershott, Myrl, and Munk, Walter, "Tides," Annual Review of Fluid Mechanics, 2, 205-224, 1970.
10. Defant, Albert, "Physical Oceanography" Volume II, Macmillan, New York, 1961.
11. Smith, D. E. , these proceedings.

VI. REFERENCES (continued)

12. Gaposchkin, E. M., and Lambeck, K., 1969 Smithsonian Standard Earth (II), SAO Special Report 315, May 18, 1970.
13. Stanley, H. Ray, Private Communication.
14. Vonbun, F. O., "The ATS-F/Nimbus-E Tracking Experiment," Presented at the 48th IAU Symposium, May 9-15, 1971, Morioka, Japan
15. Vonbun, F. O., "Geodetic Satellite Mission and GEOS-C Spacecraft" Space Research XI, Academy Verlag, Berlin 1971, 457-467.
16. NASA, "The Terrestrial Environment: Solid Earth and Ocean Physics" Prepared by MIT for NASA, ERC, April, 1970.
17. Vonbun, F. O., these proceedings.

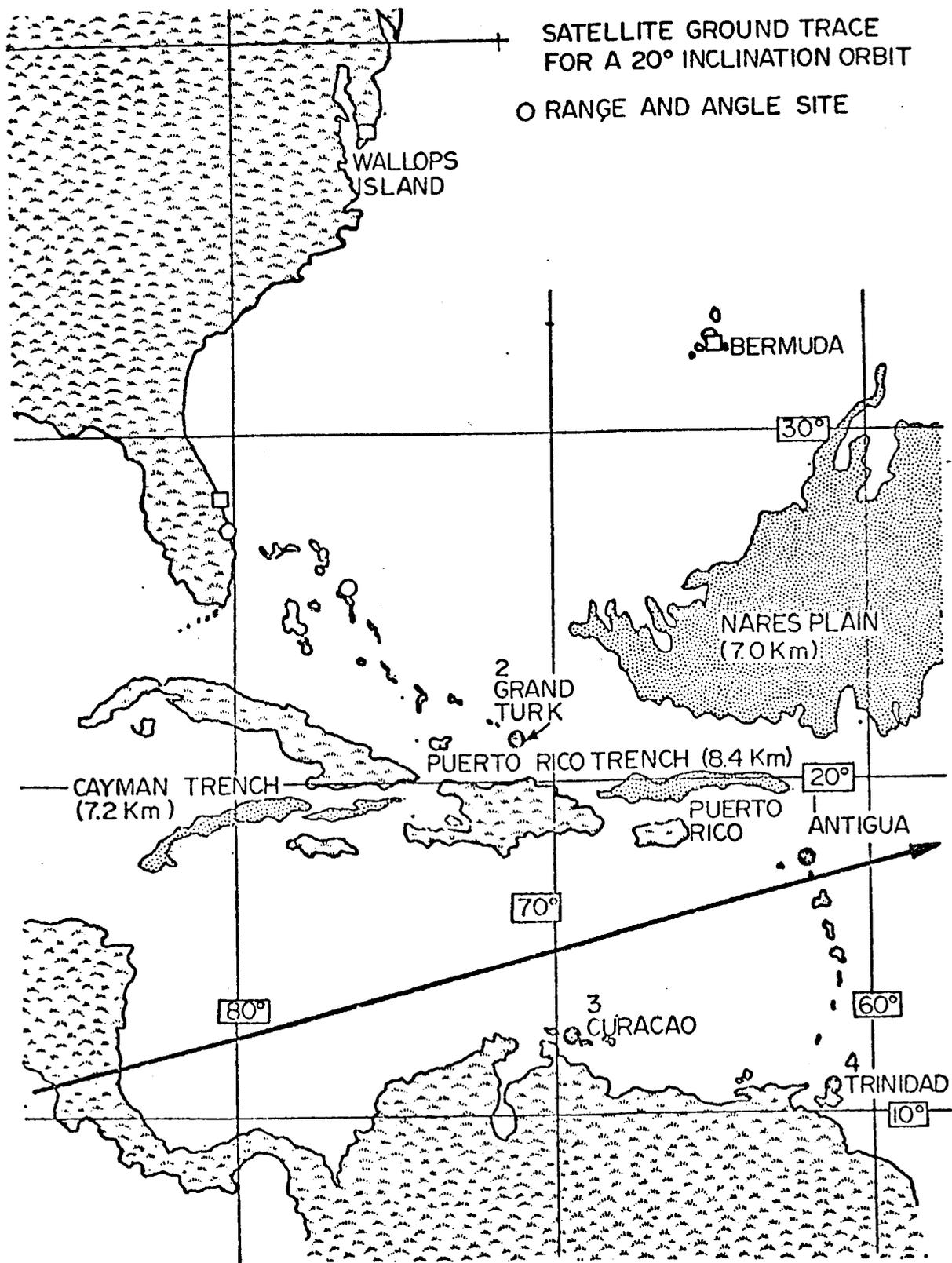


Figure 1

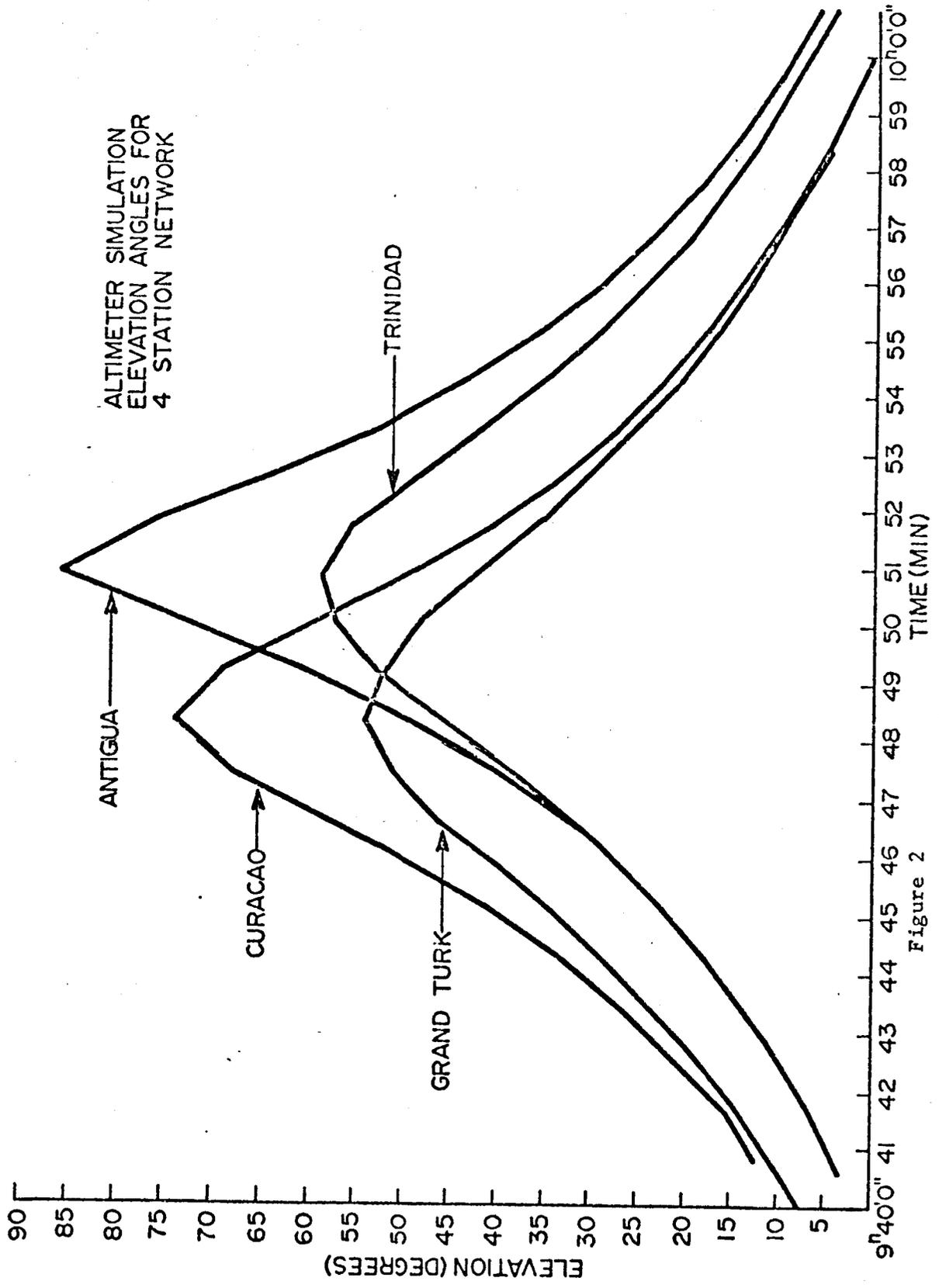


Figure 2

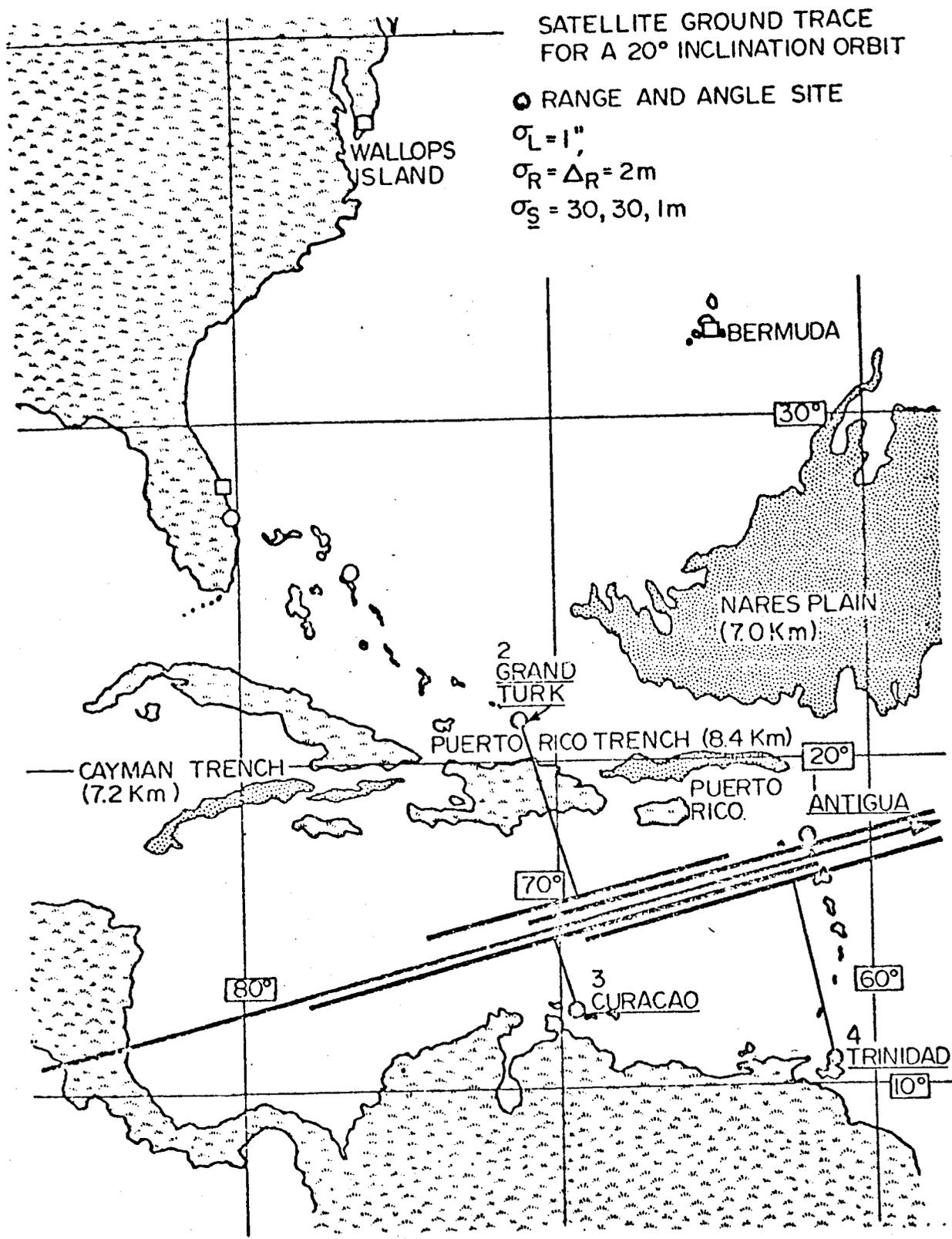


Figure 3

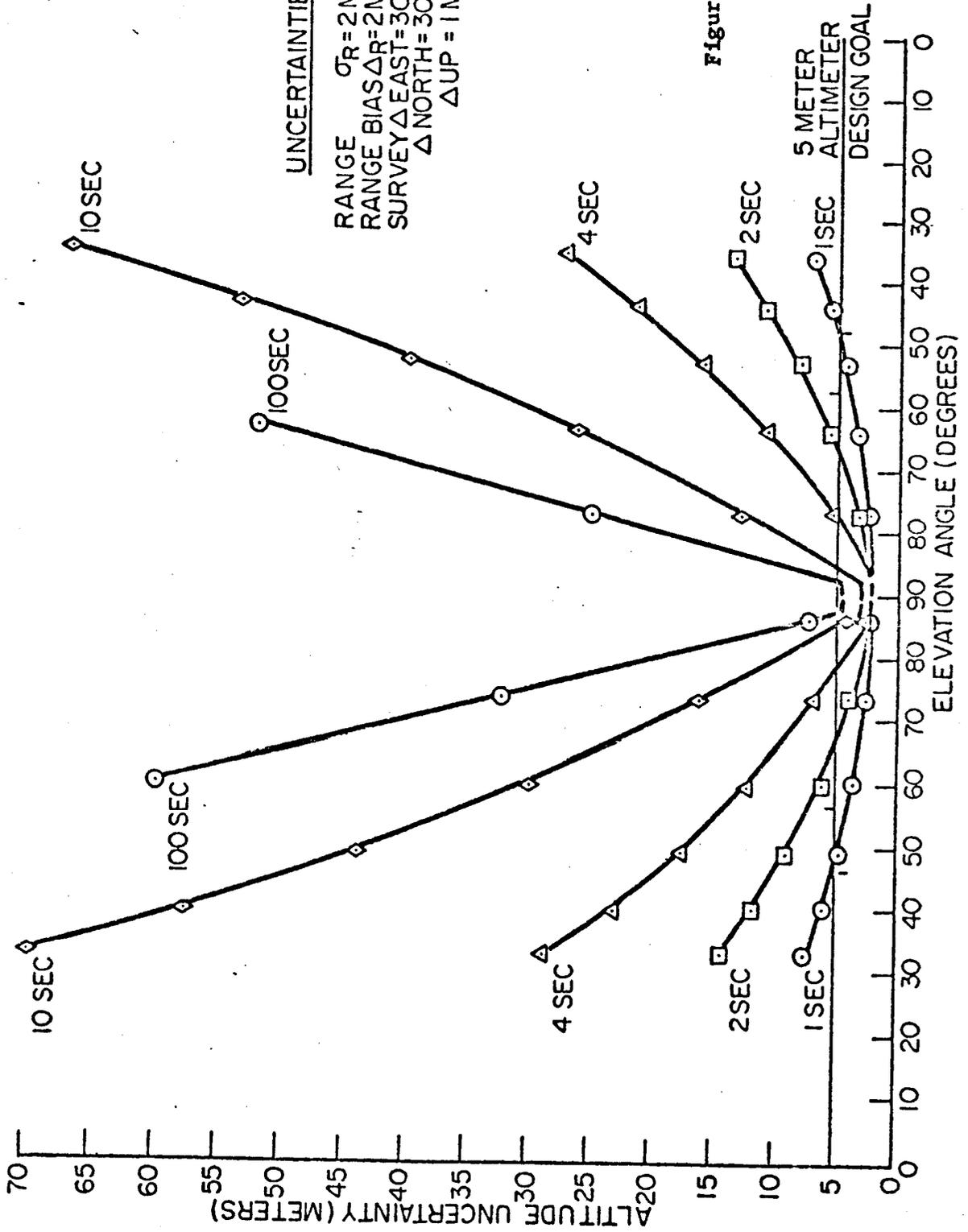
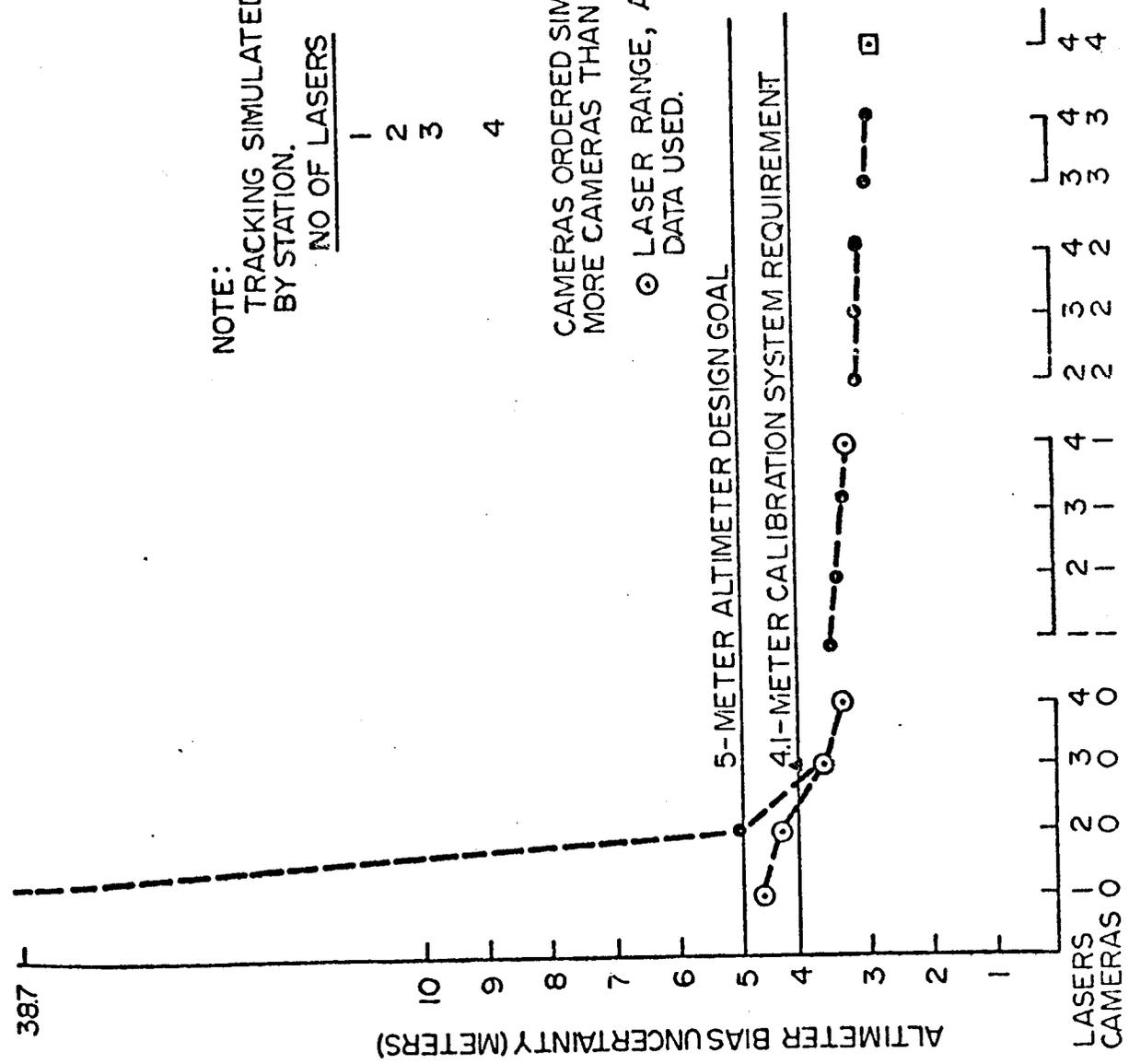


Figure 4

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ALTIMETER BIAS UNCERTAINTY (METERS)



NOTE:  
TRACKING SIMULATED FOR LASERS WAS ORDERED  
BY STATION.

NO OF LASERS	STATIONS
1	ANTIGUA
2	ANTIGUA, GRAND TURK
3	ANTIGUA, GRAND TURK, CURACAO
4	ANTIGUA, GRAND TURK, CURACAO, TRINIDAD

CAMERAS ORDERED SIMILARLY. THERE WERE NEVER  
MORE CAMERAS THAN LASERS.

○ LASER RANGE, AZIMUTH & ELEVATION ANGLE  
DATA USED.    ▲ ANTIGUA, KEY WEST, PANAMA

Figure 5

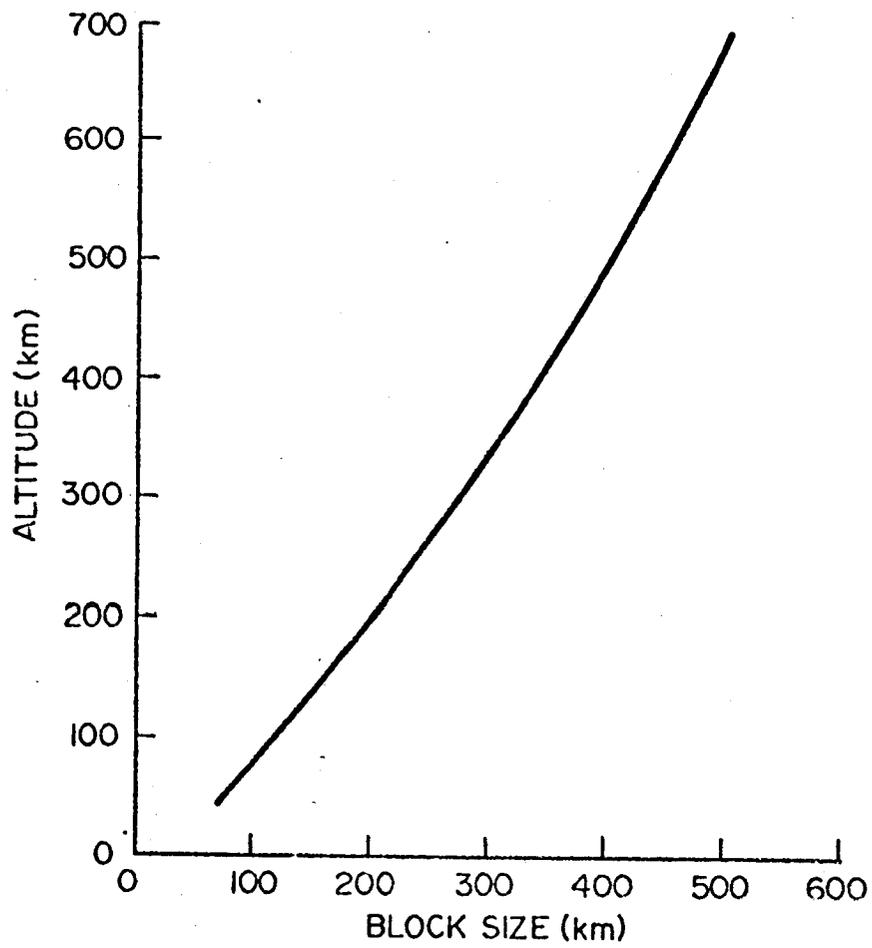


Figure 6



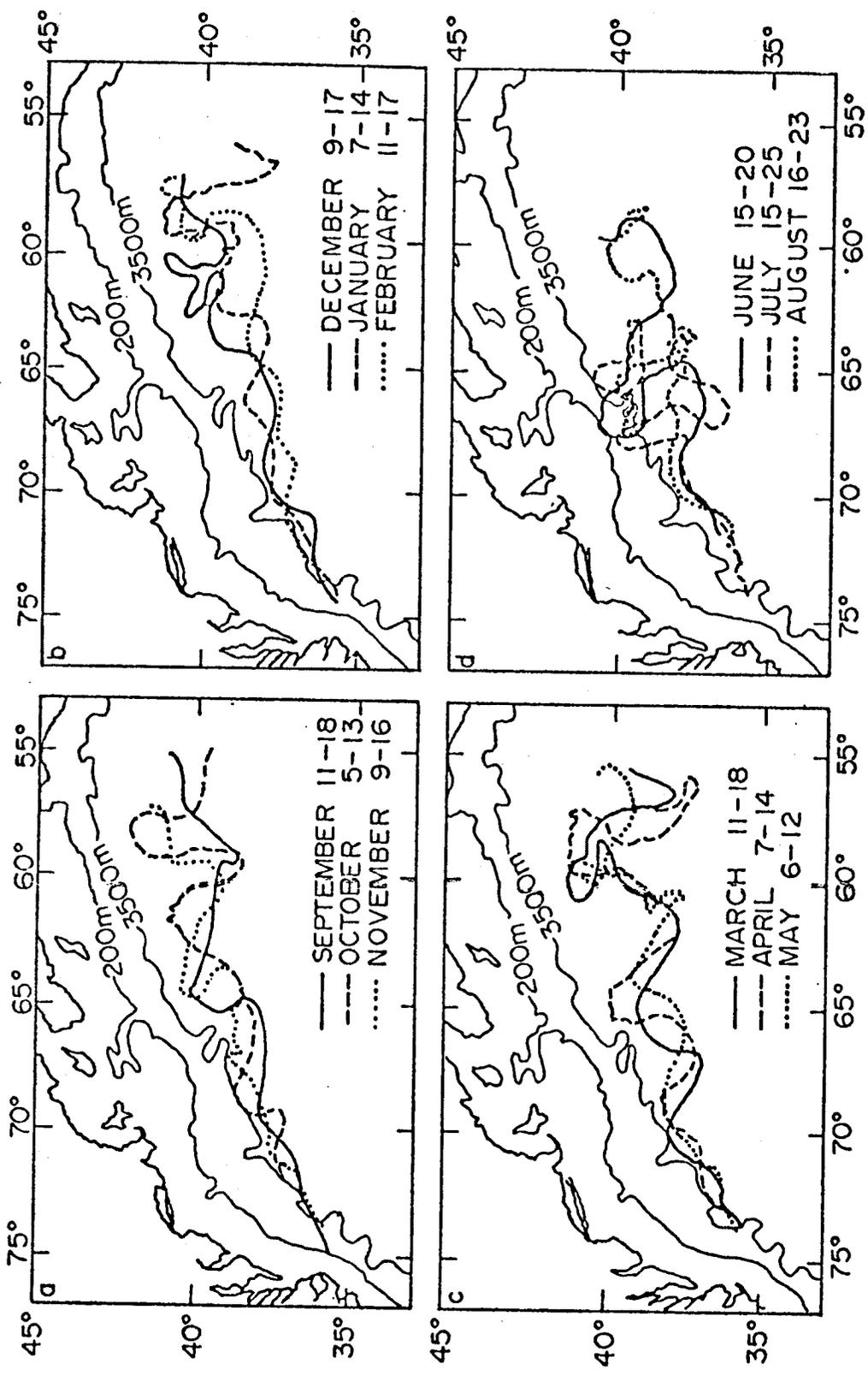


Figure 8

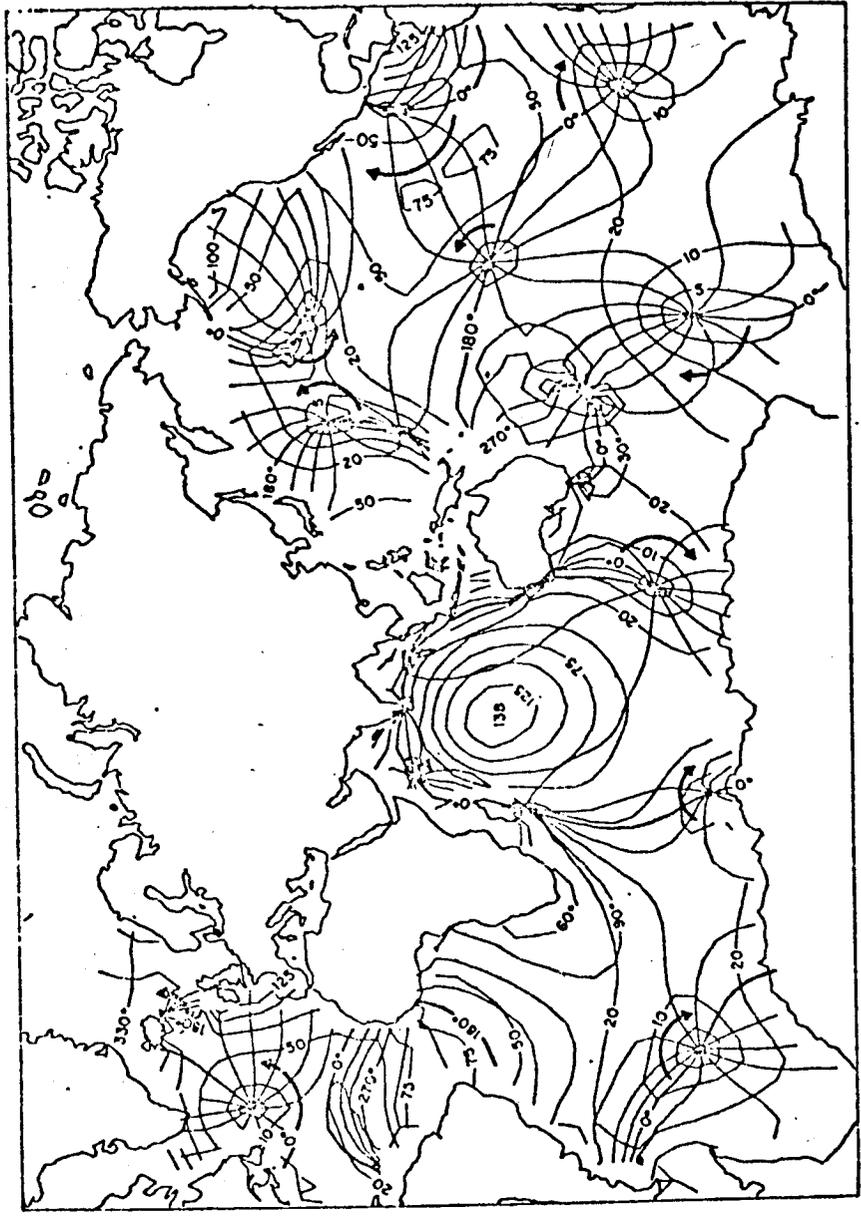


Figure 9  
7-18

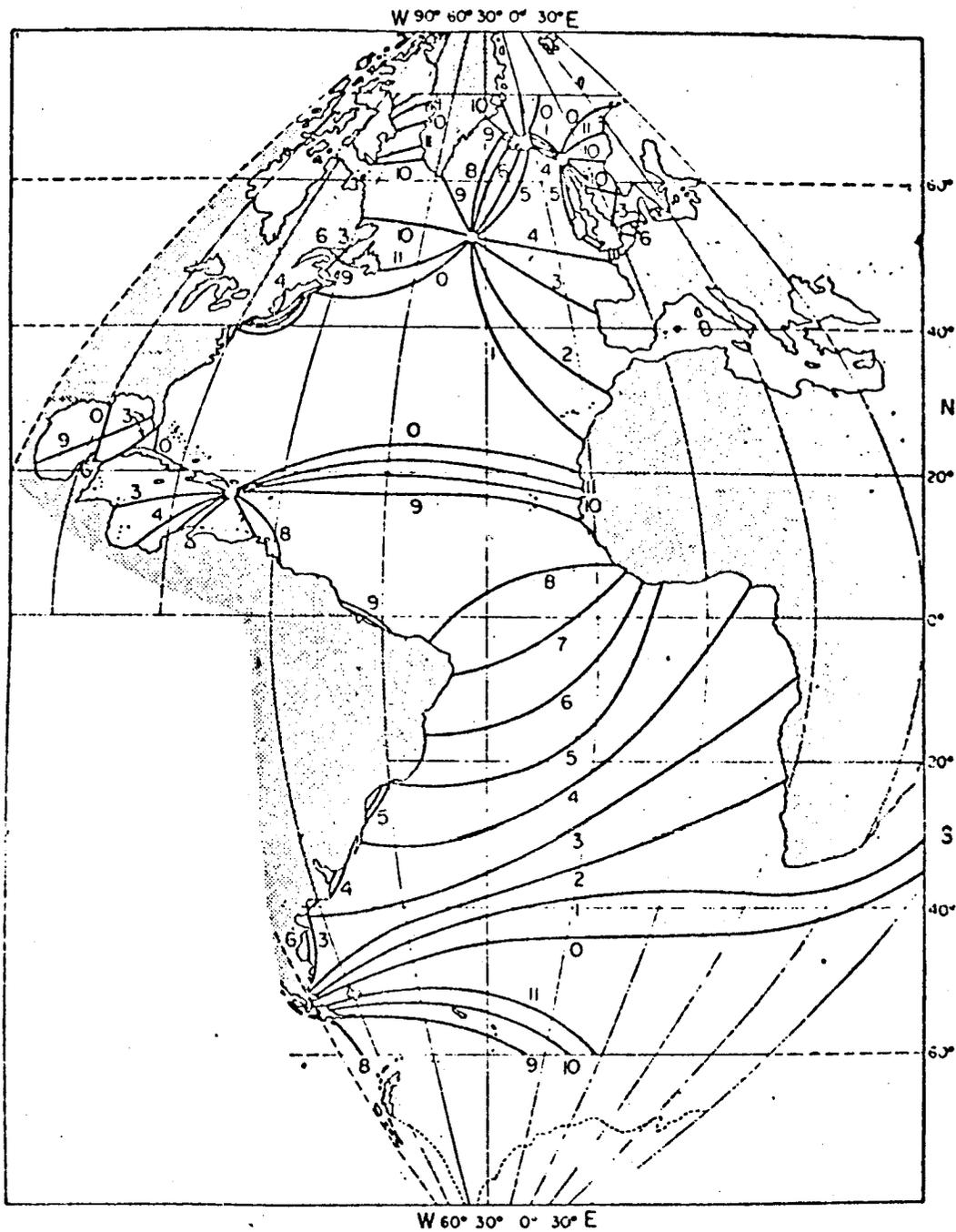
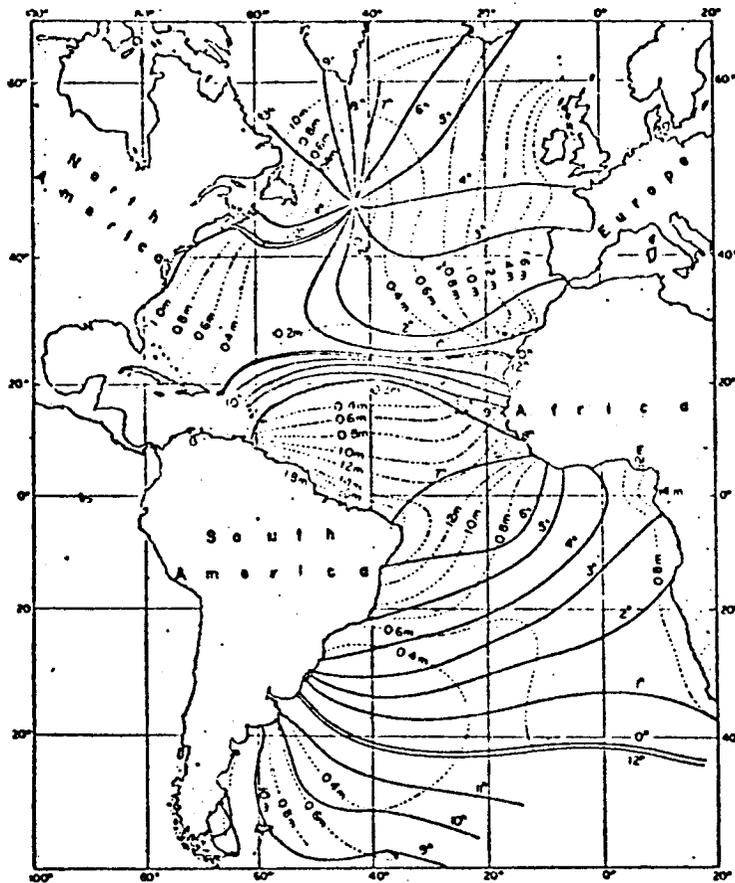


Figure 10  
7-19



Theoretical tides of Atlantic Ocean. Full lines: co-tidal lines referred to moon-transition through meridian of Grw., dashed lines: co-range lines of the semi-diurnal tide  $M_2$  in m (according to Hansen).

Figure 11

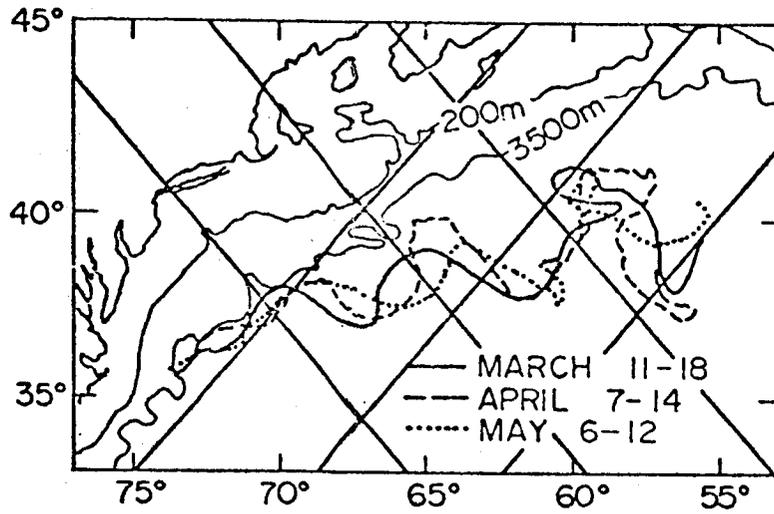


Figure 12

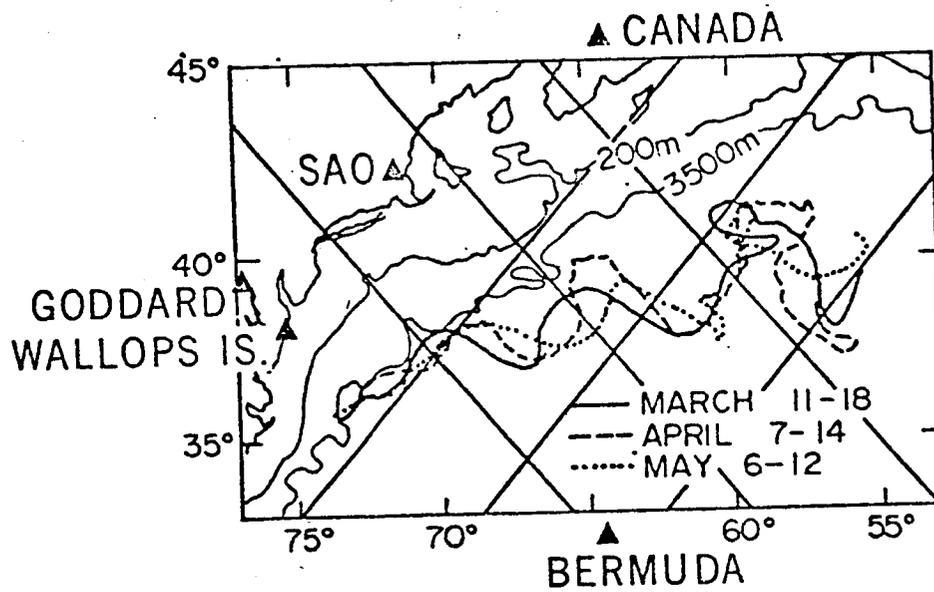


Figure 13

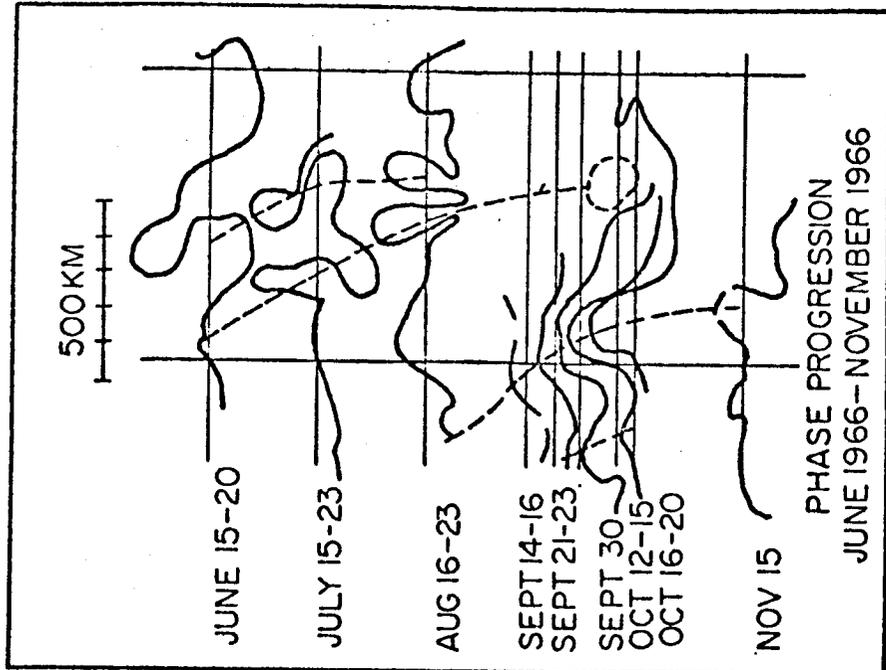
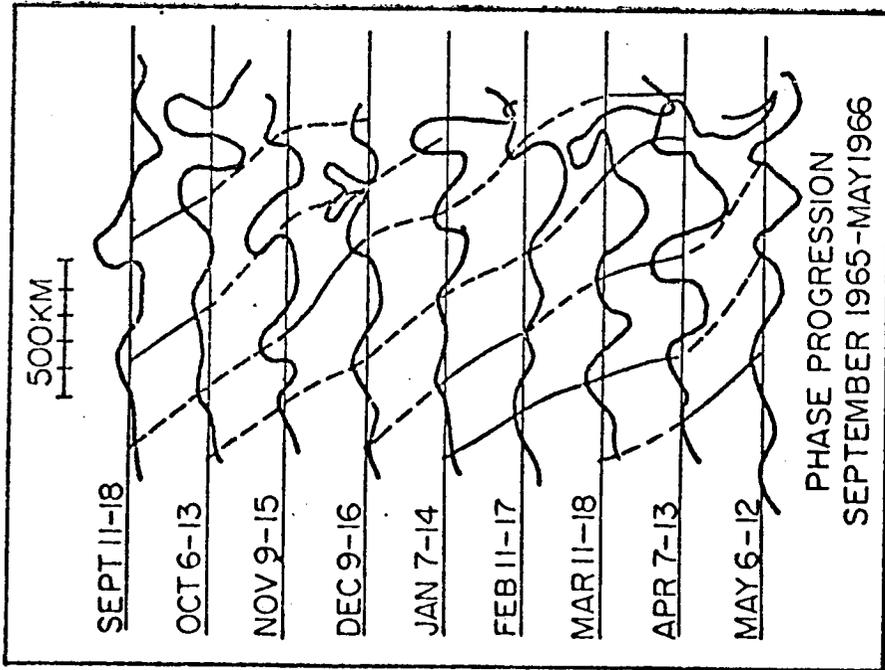
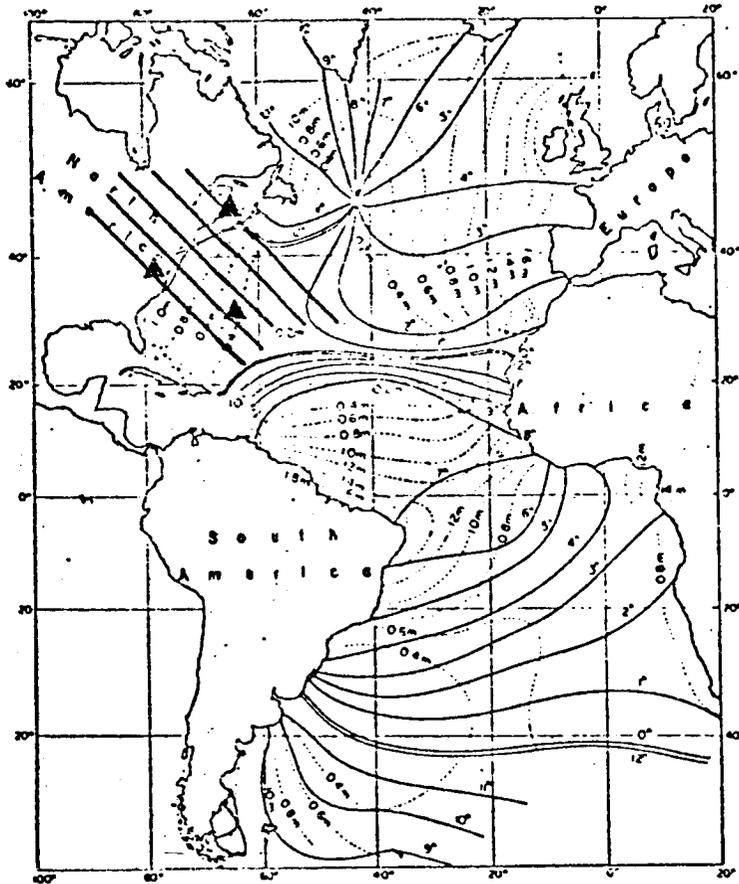


Figure 14



Theoretical tides of Atlantic Ocean. Full lines: co-tidal lines referred to moon-transition through meridian of Grw., dashed lines: co-range lines of the semi-diurnal tide  $M_2$  in m (according to Hansen).

Figure 15

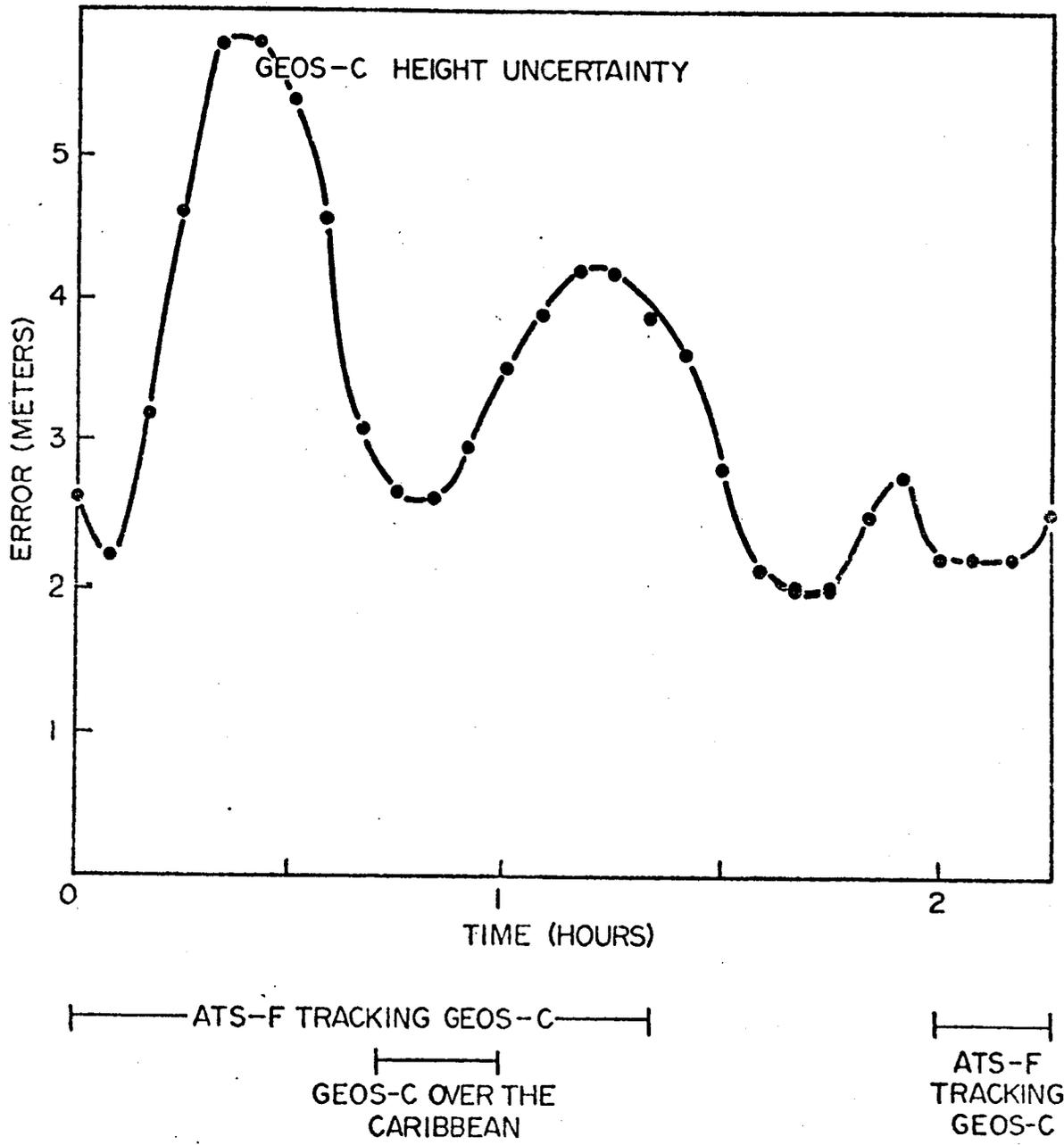


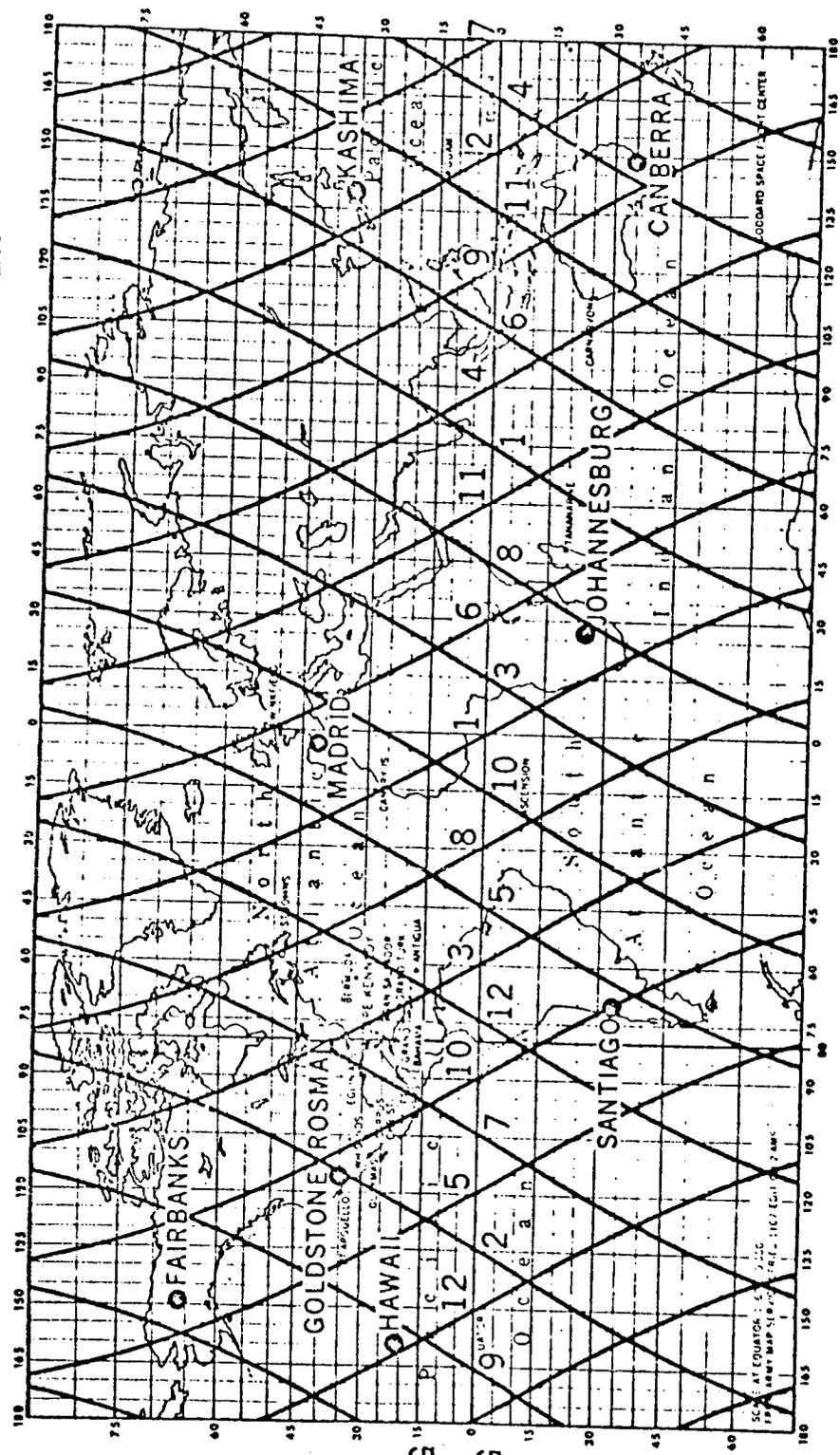
Figure 16  
7-25

# A GEOPAUSE SATELLITE SYSTEM CONCEPT

## THE GEOPAUSE SATELLITE ORBIT

- PERIOD  $\sim$  14 HOURS,  $A \sim 4.6$  EARTH RADII
- POLAR, NORMAL TO ECLIPTIC
- NEARLY CIRCULAR
- FEW GRAVITY TERM UNCERTAINTIES CORRESPOND TO PERTURBATIONS OVER A DECIMETER
- IN THIS SENSE, THE ORBIT IS NEAR THE GEOPOTENTIAL BOUNDARY, i.e., THE GEOPAUSE

# GEOPAUSE SUBSATELLITE TRACKS DURING ONE WEEK

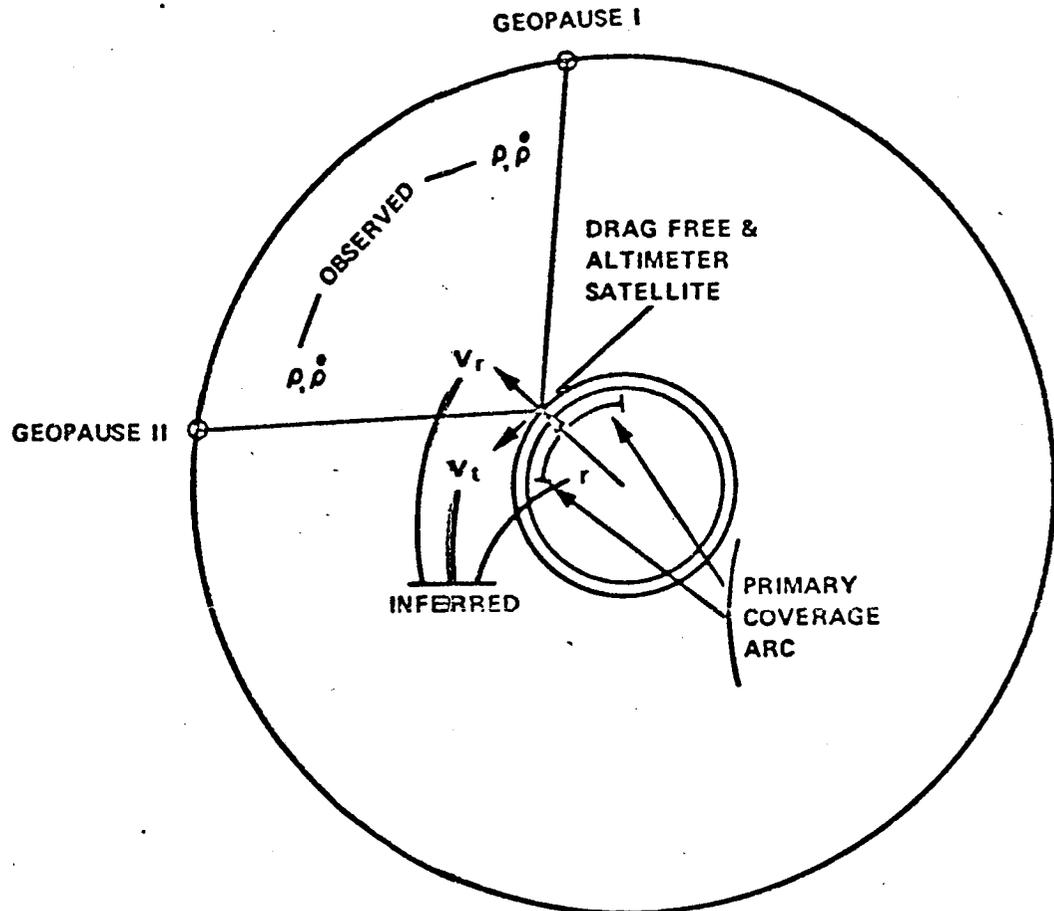


ASCENDING  
NODE NO.  
DESCENDING  
NODE NO.

● TYPICAL FUNDAMENTAL STATIONS

Figure 18

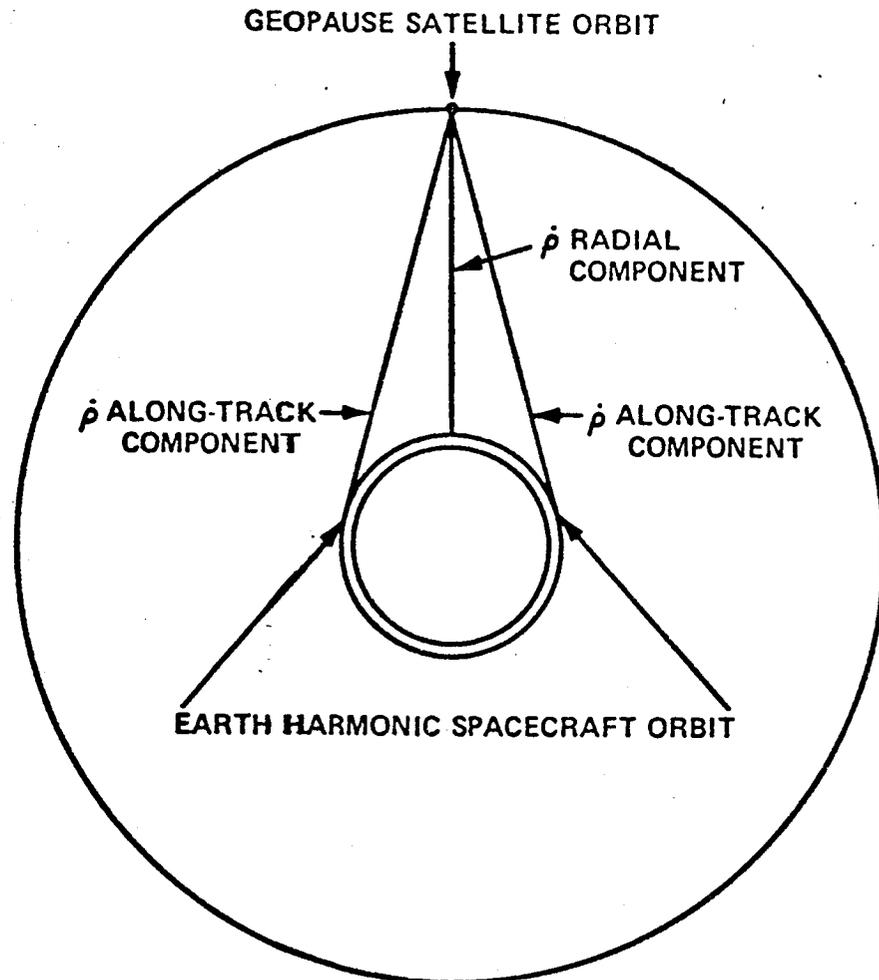
# GEOPAUSE DRAG FREE & ALTIMETER SPACECRAFT SATELLITE-TO-SATELLITE TRACKING



PRIMARY COVERAGE ARC MOVES ABOUT  $24^\circ$  LONGITUDE  
AND  $40^\circ$  IN LATITUDE EACH REVOLUTION

Figure 19

# A GEOPAUSE SATELLITE SYSTEM CONCEPT



SATELLITE-TO-SATELLITE RANGE RATE TRACKING GEOMETRY FOR VARIOUS RELATIVE POSITIONS OF THE GEOPAUSE AND COPLANAR EARTH HARMONIC SPACECRAFT

Figure 20

Table I

GEOS-C Mission Altimeter Evaluation

Satellite Altimeter System Measurement Error Source	Error (m)
Altimeter Instrumentation	2
Refraction	0.2
Reflection from Waves	0.5
Spacecraft Attitude	2
Root Sum Square	2.9
Calibration Error Allocation	4.1
Altimeter System	5
Evaluation Goal	

Table II

GEOS-C Mission Altimeter Evaluation Analysis

Assumptions	A Priori Uncertainties	Noise rms
Recovered Quantities		
Range Measures	2 m	2 m
Altimeter Height Measures	100 m	10 m
Orbit - R&V Components	1 km, 1 km/sec	
Station Positions - E, N, V Components	30, 30, 1 m	
Other Quantities		
Camera Angle Measures		1"
Other Angle Measures		100"