

## ORBIT DETERMINATION

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## 1.0 INTRODUCTION

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The effective utilization of the planned GEOS-C satellite for geoidal mapping will require the determination of the satellite orbit with an accuracy of at least a few meters in the radial coordinate. The measurements made by the GEOS-C altimeter can be used both for orbit and geopotential improvement, but only after the instrument has been well checked out and calibrated and techniques have been developed for effectively using altimeter data for orbit determination. Prior to this stage, it will be necessary to have available a satellite ephemeris based on ground tracking data. Inter-satellite tracking data may also be used, but even here there is a link to a ground tracking station. The scope of this paper will be limited to the conventional type of single satellite tracking data.

The determination of an accurate satellite ephemeris is limited by a number of factors, including station position errors, measurement biases, tropospheric and ionospheric refraction, station timing errors, and errors in knowledge of the earth's potential field. If we wish to relate altimeter measurements to the geoid, then we require that the satellite orbit be

determined relative to the earth's center of mass and not relative to, say, some particular station. Intuitively, this would suggest that the satellite orbits need to be determined using at least one revolution of tracking and no arcs shorter than this will be considered. Since the altimeter power is limited to approximately one satellite revolution per day, it is therefore suggested that one revolution is the maximum period for which the most accurate orbit is necessary. Accordingly, we will consider the accuracy of orbits of one revolution, but which may be based on more than one revolution of tracking data.

If there were no errors in the geopotential field (or other forces acting on the GEOS-C satellite), then the accuracy of an estimated orbit would be improved with the addition of tracking data over longer and longer periods of time to the orbit estimation process. In this way, the effects of measurement noise and the various systematic errors are minimized. Conversely, if knowledge of the geopotential field were very poor, then the satellite position (and velocity) could be estimated at each time point if at least three (or six) simultaneous measurements were available. The accuracy limitation in this particular situation is due to measurement errors and station position errors. In practice, the true situation is somewhere between these two extremes, and the most accurate orbit is obtained using some finite arc length which depends on the satellite orbital elements, the number and locations of the tracking stations, the measurement type and accuracy, and the accuracy of the geopotential model used.

Two global networks of tracking stations will be considered in this paper and the accuracy of orbits using their data will be compared in a limited set of simulations. The networks will be compared both separately and together, with the objective of determining the amount of tracking beyond which the addition of more data from more stations produces a negligible orbit improvement. This rather ambitious objective cannot, of course, be definitively answered without an extensive study, but the results obtained do have strong implications on the need for extensive tracking of the GEOS-C satellite, particularly with regard to the need for continuous tracking. Currently existing tracking stations are emphasized.

## 2.0 METHOD OF ANALYSIS

The analysis of orbit determination accuracy was made using a computer program which simulates the normal reduction of satellite tracking data and propagates through this process the expected errors in the data and the station positions, and the model of the geopotential coefficient error. All these errors are propagated into the satellite ephemeris and are then transformed into radial, cross track, and along track components. For the altimeter application, the radial error component is the only one whose accuracy is critical and results only for radial errors will be considered in the following analysis.

With a large number of well distributed tracking stations used in the GEOS-C orbit estimation, individual error sources of measurement biases (for range type measurements), errors in refraction corrections, station position errors, etc., all have small effects if all such errors are independent. Error analysis runs have indeed verified this and for none of the arcs considered did any single systematic error of the expected amplitude have an effect greater than 0.5 meters. In all cases, the dominant error source is geopotential coefficient error. By comparison, all other errors are essentially negligible, with the total radial uncertainty only slightly greater than the geopotential coefficient error effect except at those times when the coefficient errors have very small effects.

The model for geopotential coefficient error is based upon the differences between two gravity models which are basically independent. These models are the Smithsonian Astrophysical Observatory M1 model [Lundquist and Veis, 1966] and the Johns Hopkins Applied Physics Laboratory 3.5 model [Guier and Newton, 1965]. It has been shown [Martin and Roy, 1971] that 25% of the differences between these two models produces a quite valid estimate of the geopotential coefficient error effects on a short arc data reduction using the SAO 1969 Standard Earth gravity model [Gaposchkin and Lambeck, 1969]. Some care must be exercised in the interpretation of results when using this set of differences as an error model, since it can have only a statistical interpretation. However, because of the success in predicting GEOS-B errors, and the relatively small altitude and inclination differences between GEOS-B and the planned GEOS-C, the peak amplitudes and locations of peaks and minima should be reliable with a reasonable degree of confidence.

### 3.0 GEOS-C SIMULATIONS

The GEOS-C satellite is presently planned for a  $115^\circ$  inclination and an altitude of 500 nm. Ground tracks for three revolutions of such a satellite are shown on Figure 1. Also shown on Figure 1 are the locations of 12 Doppler measurement sites and 8 range measurement sites. The geodetic locations of these stations are shown in Table 1. The Doppler stations are representative of existing Navy Doppler sites, and the range measurement sites are representative of existing C-band radar and laser measurement sites.

The coverage provided by the Doppler sites is shown in Figure 2 for the stations tracking down to  $5^\circ$  elevation angles. For this set of stations, the coverage has good geographic distribution, and would appear to provide satellite coverage for greater than 50% of the time. Coverage provided by C-band radar sites is rather heavily concentrated along the United States east coast and provides little tracking at the high latitudes. Laser trackers are, in most cases, mobile and can be located on most land areas.

Simulations were performed for the Doppler network with tracking from all stations when the satellite was above  $5^\circ$  elevation angle. Arc lengths of 2, 4, and 6 hours were simulated with the 2 hour arc falling in the middle of the 4 hour arc and the 4 hour arc in the middle of the 6 hour arc. A frequency bias was assumed to be adjusted for each Doppler pass of each

GEODETIC LOCATIONS  
TABLE 1

DOPPLER SITES

	$\phi$	$\lambda$ (E)
LASHAM, ENGLAND	51° 11' 10"6	358° 58' 30"5
SAO JOSE DOS CAMPOS, BRAZIL	-23° 13' 01"7	314° 07' 50"6
SAN MIQUEL, PHILIPPINES	14° 58' 57"8	120° 04' 26"0
SMITHFIELD, AUSTRALIA	-34° 40' 31"4	138° 39' 12"4
MISAWA, JAPAN	40° 43' 04"6	141° 20' 04"7
ANCHORAGE, ALASKA	61° 17' 02"0	210° 10' 37"5
THULE, GREENLAND	76° 32' 18"6	291° 13' 46"7
SOUTH POINT, HAWAII	21° 31' 26"9	202° 00' 00"6
LOS CRUCES, NEW MEXICO	32° 16' 43"8	253° 14' 48"3
HOWARD COUNTY, MARYLAND	39° 09' 47"8	283° 06' 11"7
MCMURDO SOUND, ANTARCTICA	-77° 50' 51"7	166° 40' 25"3
PRETORIA, SOUTH AFRICA	-25° 56' 46"1	28° 20' 53"0

RANGE MEASURING SITES

CARNARVON, AUSTRALIA	-24° 53' 47"5	113° 43' 02"1
KOUROU, FRENCH GUIANA	5° 06' 46"3	307° 29' 19"5
SANTIAGO, CHILE	-33° 00' 00"0	289° 00' 00"0
ANTIGUA, BRITISH W. INDIES	17° 08' 37"6	298° 12' 25"8
MERRITT ISLAND, FLORIDA	28° 25' 29"0	279° 20' 07"5
BERMUDA	32° 20' 52"8	295° 20' 47"6
WALLOPS ISLAND, VA.	37° 51' 36"8	284° 29' 25"9
WHITE SANDS, NEW MEXICO	32° 21' 28"8	253° 37' 47"9

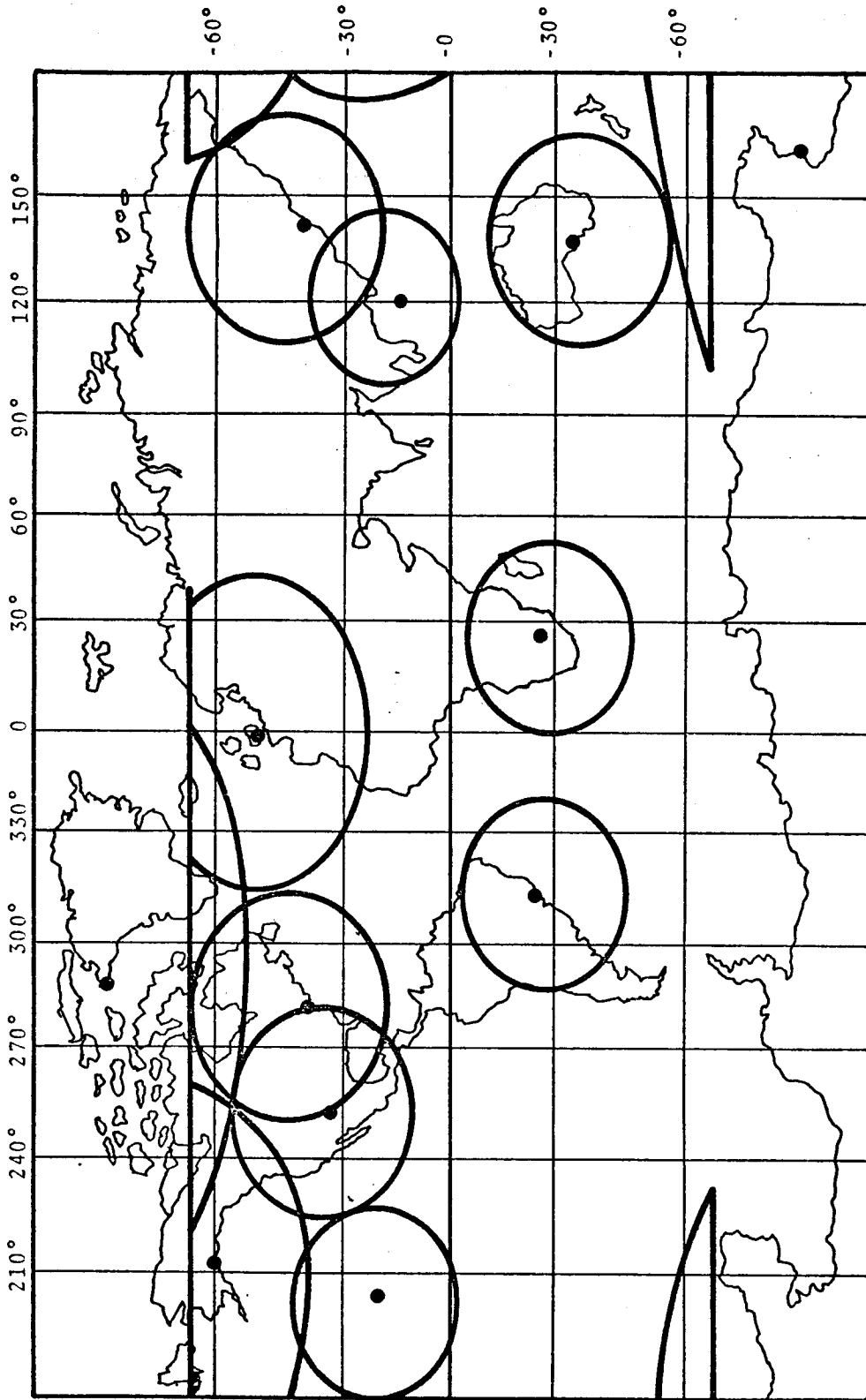
station, with negligible a priori knowledge of the bias. Refraction errors were ignored, but station position errors of 5 meters in each coordinate were propagated.

For the range tracking network, simulations were made for the same 2 hour arc as was the 2 hour Doppler simulation with tracking also down to 5° elevation angle. Each station was considered to have a range bias of 2 meters which was not adjusted but whose effect was propagated through the data reduction. Station position errors of the same magnitude as the Doppler station position uncertainties were propagated.

The 2 hour arc was also simulated with both the range and Doppler networks tracking. Weights for the two data types were chosen in such a way that each network was given approximately equal weight. For the same data rate, this requires that a Doppler sigma of 3cm/sec correspond to about a 12 meter range sigma.

For all simulations, the effects of the geopotential coefficient model error discussed above were propagated into the satellite orbit and the radial component computed. Station position and measurement bias errors produced effects which were, in general, negligible when compared to the geopotential error and will consequently be ignored in the discussion below. With the geopotential error above considered, it will be meaningful to consider the estimated error including sign, rather than as just a sigma. In this manner, expected correlations between errors at different spatial locations can be demonstrated.





DOPPLER TRACKING COVERAGE OF GEOS-C

5° CUTOFF

FIGURE 1

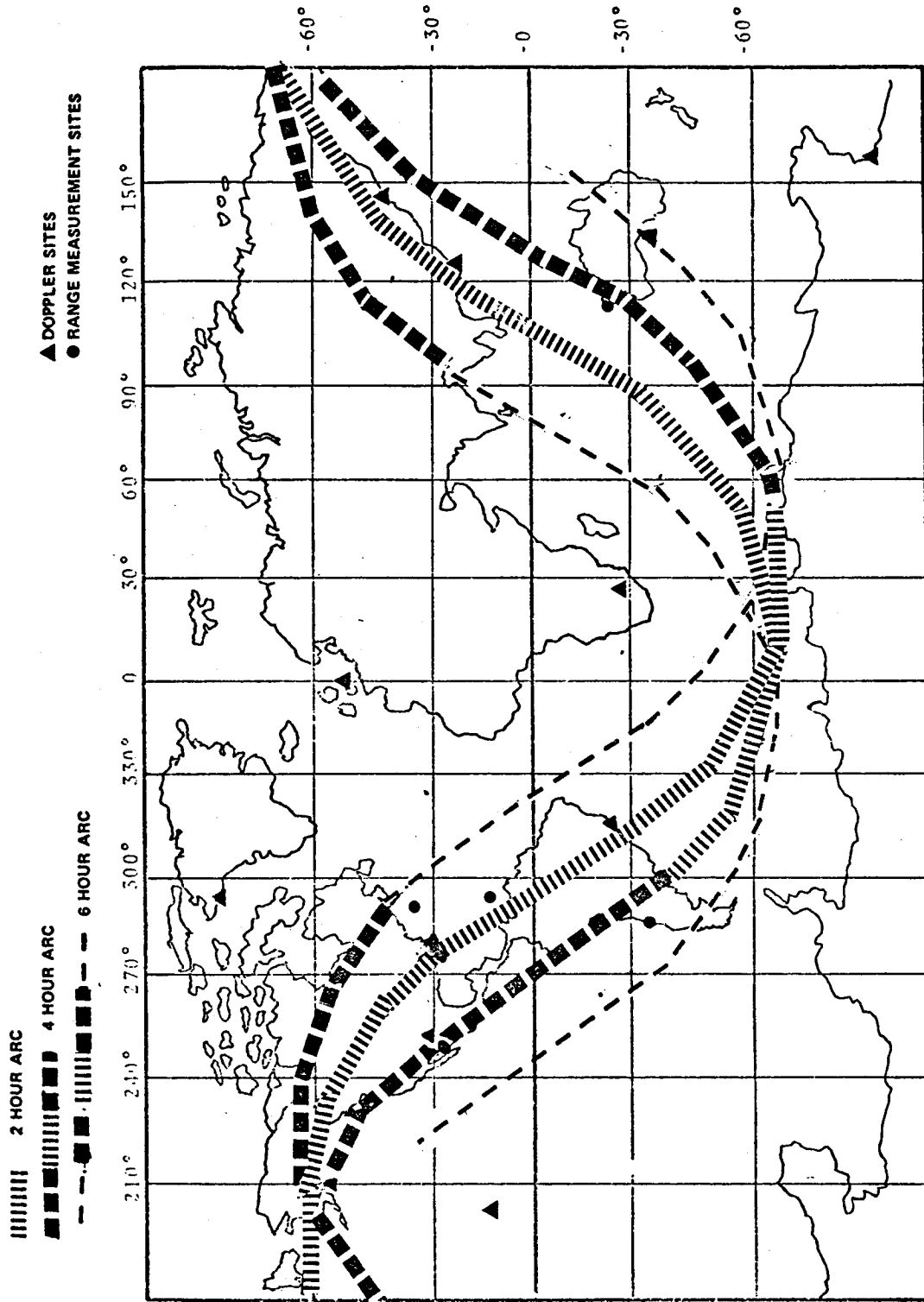


FIGURE 2

#### 4.0 ANALYSIS OF RESULTS

For the 2, 4, and 6 hour arcs using Doppler tracking, the effects of the geopotential model error are shown in Figure 3. The 4 and 6 hour orbits appear to be affected in the overlap period by about the same amount, approximately  $\pm 7m$ . Peak errors occur, for the most part, during periods of limited or no tracking. The error tends to be minimum (i.e., cross zero) during periods of overlapping tracking.

The 2 hour Doppler orbit is affected somewhat less during portions of the arc than are the 4 and 6 hour arcs, apparently indicative that the geopotential model errors can be more absorbed in the orbital elements. The times of minimum error are, however, approximately the same.

The geopotential model error effect for the 2 hour arc is also shown on Figure 4 on an expanded scale. On the same graph is shown the geopotential model error effect on the range tracking network only, and also the geopotential error effect on the combined Doppler plus range orbit. The range orbit error is larger than the Doppler orbit error near the beginning of the arc, but the first tracking is approximately 7 minutes after epoch. However, the maximum orbit error during the tracking period is still at the beginning of track.

The range tracking is heavily concentrated during the 10-30 minute period. There is then a 35 minute break before the

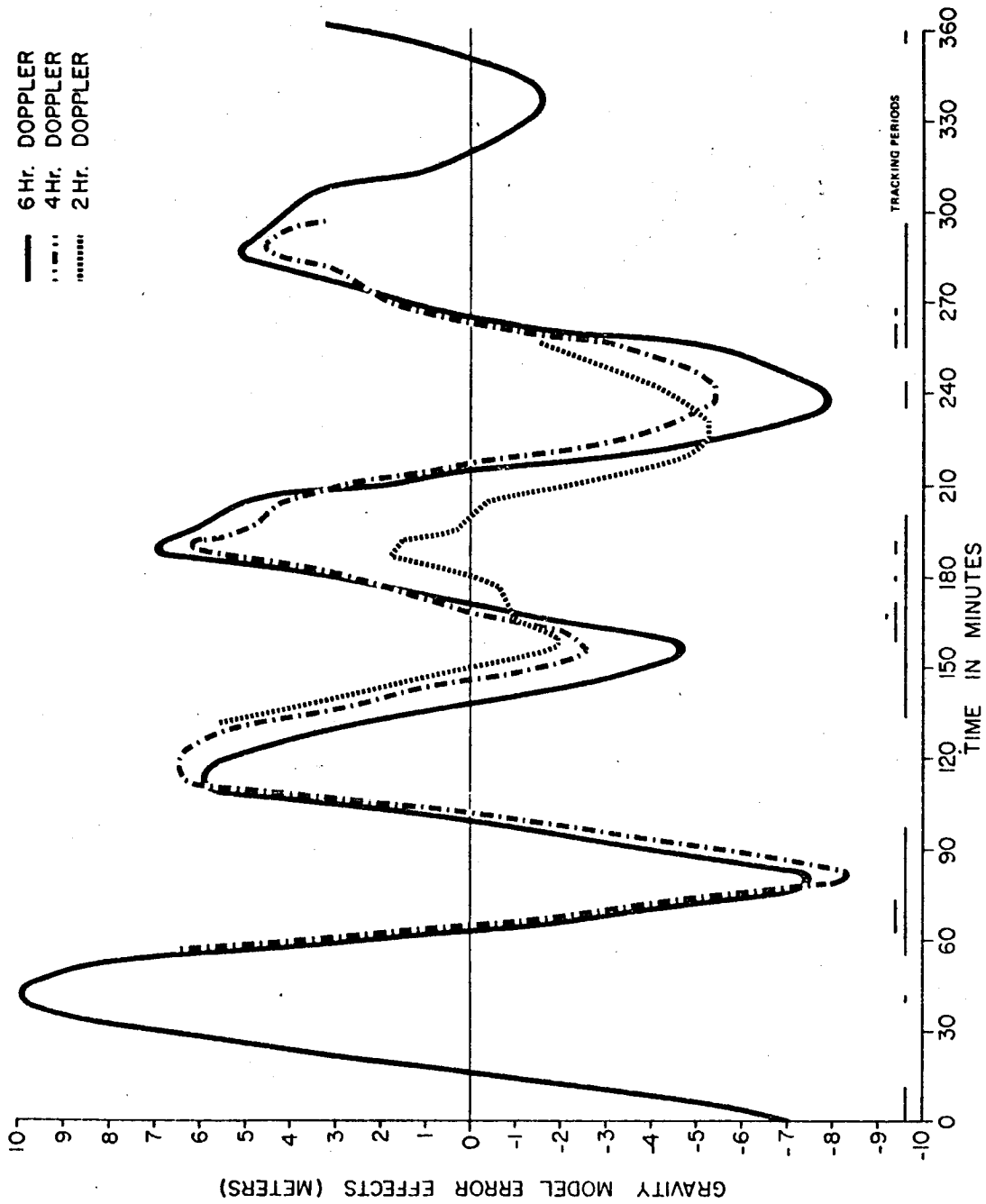


FIGURE 3 Geopotential Model Error Effects on Doppler Determined Orbits

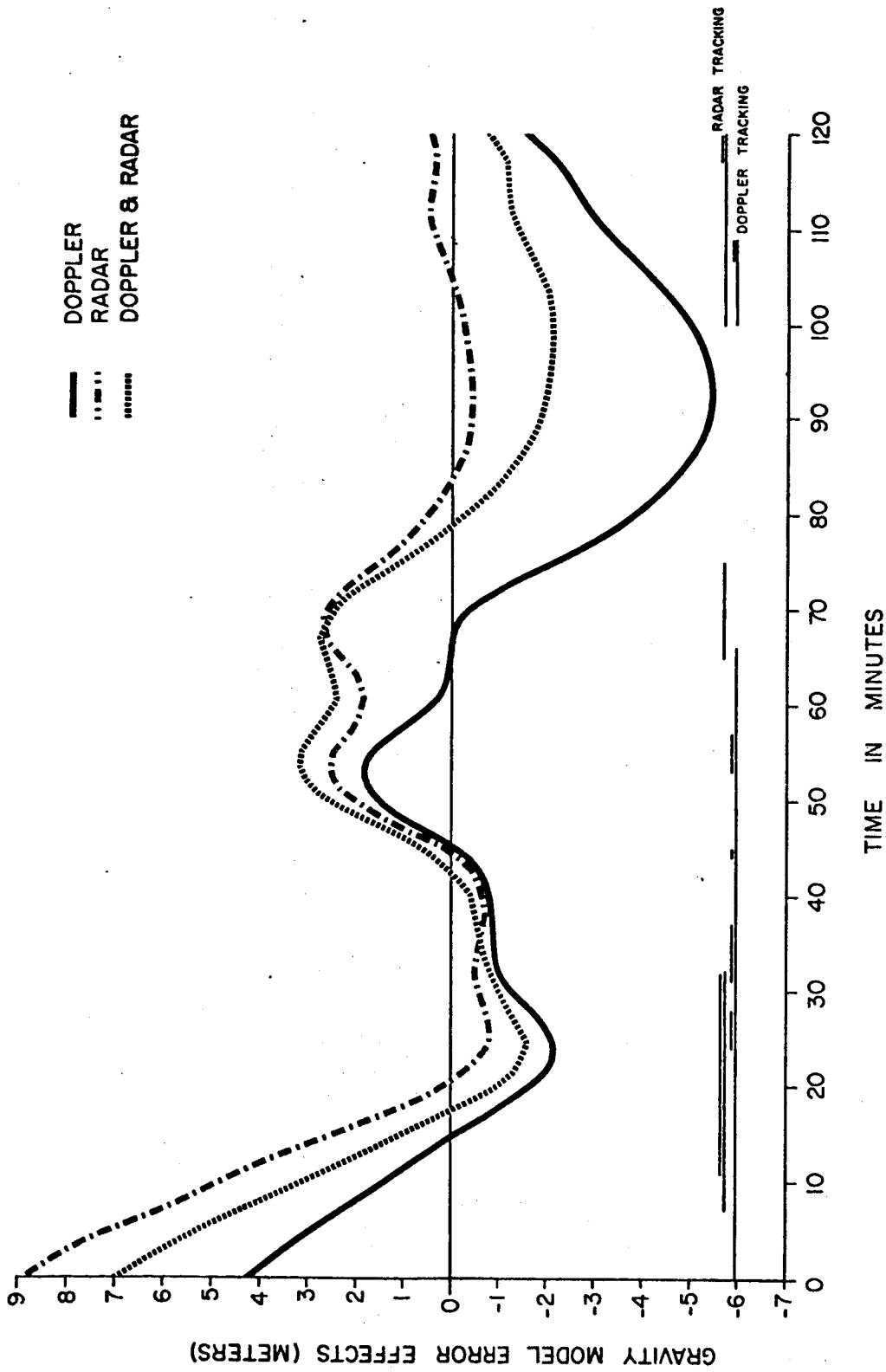


FIGURE 4 Geopotential Model Error Effects on Two Revolution Doppler and Radar Orbits

satellite is seen by Carnarvon, and another 25 minute gap before the satellite is picked up by Santiago. During this time, including the tracking gaps, the maximum orbit radial error only slightly exceeds 2 meters.

As might be expected, the orbital error for the range-Doppler solution is intermediate between that of the range only and Doppler only solutions throughout most of the arc. Unfortunately, the model error effects tend to have the same sign on both the range and Doppler solutions, so the combined solution is always worse than one of the solutions.

The similarity of the geopotential model error effects for the different tracking periods and, to a lesser extent different tracking systems, is indeed striking, and suggests that the reduction in orbit error through the use of more tracking is not easily accomplished. It also suggests that the comparison of orbits generated using different tracking systems but the same geopotential model will be a very poor measure of the actual orbit accuracy.

The extrapolation of the range determined orbit for an excess of 30 minutes without a serious increase in orbit error shows that the orbit error need not grow excessively without continuous tracking. Combining this conclusion with the result that the Doppler orbits are minimum during simultaneous track would suggest that some period of concentrated tracking

combined with some amount of global tracking is adequate for a well determined and accurate global orbit over a single revolution.

It should also be noted that the one revolution solutions, at least for the particular tracking periods used, is significantly less affected by geopotential coefficient error than are multi-revolution solutions.

## 5.0 CONCLUSIONS

Because of the limited nature of the simulations, conclusions drawn must be considered tentative until additional arcs are investigated and the geopotential model error is more fully validated. The conclusions regarding the amount and type of ground tracking which produces the orbit with the minimum radial error may be summarized as:

1. Minimum orbit error tends to occur during periods of simultaneous ground tracking.
2. Single revolution solutions would be expected to have less error than multiple revolution solutions.
3. No type of tracking instrument has any strong advantage over another type, given a sufficient amount of data.
4. Continuous tracking is not necessary for accurate orbits.
5. With good tracking geometry, radial errors of approximately 2 meters or less appear possible.
6. The Doppler system appears capable of approximately 5 meter height accuracies on a global scale.

### ACKNOWLEDGMENT

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