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The goals of satellite altimetry are to achieve a standard deviation accuracy of less than $\pm$ 1 meter (for Geodesy) and $\pm$ 0.1 meter (for Physical Oceanography) when operated over ocean.

Recognition and reduction to a minimum of every possible source of error is mandatory if these goals are to be reached.

Antenna/Satellite altitude errors can generate significant bias errors on altitude measurements. Whether precise antenna pointing (or equivalently) satellite attitude control is required to reduce the residual (unknown) bias errors depends on the altimeter design implemented.

Specifically, our analysis shows that of the three basic types of Pulsed Radar Altimeter design:

The "Pulse Width Limited Altimeter" design results in negligible residual altitude bias error, $e_h(\phi_E)$, if the antenna 3 db beam width $\theta_A \geq 5\phi_{MAX}$ and $\theta_A \geq 10\theta_T$, where ϕ_{MAX} = Satellite Maximum Attitude Error with respect to Nadir and $\theta_T = 2\sqrt{\frac{cT}{h}}$ = the pulse beamwidth (i.e. the angle subtended by the area illuminated by the pulse at Nadir);

E.

. The "Beamwidth Limited Altimeter" design, which occurs when $\theta_A < \phi_M$, $\theta_A < \theta_T$, will require antenna pointing to an accuracy of about ± 1 milliradian to reduce the residual altitude bias error, $e_h(\phi_E)$, to an acceptable level;

. Between these extremes, the "Antenna Effects Altimeter" design, will require antenna pointing to arrive at an acceptable residual bias error, $\theta_A < \frac{4}{3}\phi_M$. If $\theta_A > \frac{4}{3}\phi_{MAX}$, then two siutably positioned samples of the average return waveform will measure the attitude error, ϕ_E , well enough to reduce the residual error, $e_h(\phi_E)$, to an acceptable level.

The two statements, "negligible residual altitude bias error" and "acceptable residual altitude bias error" are certainly not quantitative, however, they do have a quantitative meaning in this paper. "Negligible residual altitude bias error" means that the residual uncertainty of this bias error is on the order of one-tenth the total specified error performance of the satellite altimeter system. "Acceptable residual altitude bias error" means that this error combined with all the other system errors still allows the satellite altimeter system to meet the specified error performance.

After a narrative and pictorial description of each of the three types of altimeter design and the source and form of the altitude bias errors arising from Antenna/Satellite attitude errors in each design type a quantitative comparison of the three systems is made in a typical satellite altimeter application.

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Pulsewidth Limited Altimetry - The essential features of this type
of altimeter design are shown in Figure 1. In Figure 1a, a side view
of the pulse altimeter geometry is shown. Note that the antenna 3 db
beamwidth, θ_A, is larger than the maximum satellite attitude error
ϕ MAX. Also note that θ_A is much larger than the pulse beamwidth,
θ_T. Note that the pulse beamwidth, θ_T, is defined as the angle sub-
tended by the radar area illuminated at Nadir, N, by the transmitted
pulse of duration T. The pulse beamwidth is thus:

1)
$$\theta_T \cong 2\sqrt{\frac{cT}{h}} \quad \text{radians}$$

where T = pulse duration in seconds

C = speed of light

h = altitude

Figure 1b shows a plan view of the radar area illuminated at time
T as a circular spot with radius $r(T)$.

2)
$$A(T) = \Pi r^2(T)$$

and from geometry with $h \gg \frac{cT}{2}$;

3)
$$r^2(T) = cTh$$

Also shown in Figure 1b is the radar area illuminated at a time NT
after the first return from Nadir as a thin ring with an effective
radius of,

4)
$$r_E(NT) = \frac{1}{2}\left[r(NT) + r\left[(N-1)T\right]\right]$$

and a thickness of,

5)
$$\Delta r(NT) = r(NT) - r\left[(N-1)T\right].$$

The radar area illuminated at time NT is,

6)
$$A(NT) = 2\pi r_E(NT)\Delta r(NT).$$

Equation 6) reduces to,

7)
$$A(NT) = \pi c T h\left[1 + \left(\frac{2N-1}{2}\right)\frac{cT}{2}\right].$$

The range R(NT) from the altimeter to the ring illuminated at NT has increased from the altitude, h, to an effective range of,

8)
$$R_E(NT) = h + \left(\frac{2N-1}{2}\right)\frac{cT}{2}$$

And, since the average power returned to the altimeter at NT is directly proportional to the radar illuminated area and inversely proportional to the fourth power of the range to the illuminated area, the average power returned at NT is:

9)
$$\overline{P_R(NT)} = \left[\frac{P_T \; G^2(\phi)\lambda^2\sigma^2(\phi)}{(4\pi)^3}\right]\frac{A(NT)}{R_E^4(NT)}$$

Where P_T = Transmitted Power

$G(\phi)$ = Antenna Gain Vs. Angle From Antenna Boresight

λ = Transmitted Wavelength

$\sigma^0(\phi)$ = Average Radar Backscattering Cross-section Vs. Incidence Angle of the Illuminated Surface.

Carrying out the indicated operations on Area and Range, equation 9) reduces to:

10)
$$\overline{P_R(NT)} = \left[\frac{P_T G^2(\phi)\lambda^2\sigma^0(\phi)}{(4\pi)^3}\right]\frac{\pi c T}{h^3}\left[1 - 3\left(\frac{2N-1}{2}\right)\frac{cT}{2h}\right]$$

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The solid curve of Figure 1c shows the waveform of the power return with no antenna/attitude error. The ramp rises from zero to maximum at time T and decays according to equation 10 after time T. The effect of the off center antenna gain due to a small error ϕ_E, on this average waveform is to decrease the peak, at T, and to decrease the rate of trailing edge fall off slightly. This is shown by the dashed line trailing edge where the peak value has been normalized so the trailing edge effect will stand out.

Because the Satellite Altimeter over ocean is expected to measure altitude to the instantaneous mean sea level averaged over the illuminated area A(T) then optimum tracking should be insensitive to variations in surface roughness.

Figure 1d shows the dispersive effect of sea state on the rising portion of the radar return. The solid line curve shows the waveform expected from a gaussian distribution of radar backscattering area about mean sea level with a standard deviation σ_z. The significant wave height $\overline{H}_{1/3} = 4 \sigma_z$, so a sea state with $H_{1/3} = 4/3 \, CT$ would give radar return starting at about $t = -T$ and display the form of a probability distribution function until the pulse trailing edge has penetrated to $3\sigma_z$ below the mean sea level which occurs at $t = 2T$. The dashed line of Figure 1d shows the return from ocean with significant wave heights of approximately CT/100. Note that the rising portion of both returns is symetrical about the time point $t = T/2$ so that a split gate energy tracker which balanced the average energy seen in the early

gate, E, with one half the average energy seen in a later gate, L, will position the early gate to start at t = 0 and end at t = T for any sea state so long as the separation between gates is large enough to not see dispersion effects. The position and separation of the gates for such a tracker are shown above the waveform of Figure 1d. The idea of time separated tracking gates to give mean sea level tracking independent of sea state was first advanced by George Bush of Applied Physics Laboratory/John Hopkins University. We are indebted to him and to Professor Willard Pierson of New York University who has showed that of a number of possible tracking laws modeled, this one is the least sensitive to expected sea states and departures of the surface distribution from gaussian.

With the split gate energy tracker of Figure 1d, the altitude error arising from Antenna/Satellite attitude error has the form shown in Figure 1e. Note that with $\phi_E = 0$ there will be a slight negative error proportional to the pulse width and the ratio of antenna beam width to pulse beamwidth. If the antenna were always pointed exactly at Nadir, the bias error would constant at:

(11)
$$\varepsilon_h(\phi_E=0)=-k_o\left(\frac{\theta_T}{\theta_A}\right)\frac{cT}{2} \quad .$$

Bias error reduction would simply consist of adding this precomputed (or measured) error to all altitude reading which would result in a residual altitude bias error, $e_h(\phi_E)$, equal to zero. If the Antenna/Satellite attitude maximum error, ϕ_M, is small compared to θ_A, as shown in Figure 1e, then the bias error at $\phi_E=\phi_M$

change is small. The form of the error function, $\varepsilon_h(\phi_E)$, is:

$$(12) \qquad \varepsilon_h(\phi_E) \doteq \left[\left(\frac{\phi_E}{\theta_A}\right)^2 - k_o\right] \frac{cT}{2} \left(\frac{\theta_T}{\theta_A}\right)^* .$$

Optimum error reduction in this case might consist of simply adding the average of the errors at $\phi_E = 0$ and $\phi_E = \phi_{MAX}$ which limits the residual bias uncertainty to:

$$(13) \qquad |e_h(\phi_E)| \leq \frac{1}{2}\left(\varepsilon_h(\phi_M) - \varepsilon_h(o)\right)$$

with the probability density between these limits dependent on the probability distribution of ϕ_E.

Figure 1f shows essentially the limits of altitude tracking error as a function of sea state for the split gate tracker of Fig. 1d. If the distribution of area above and below mean sea level is symmetrical and if the average radar backscattering cross-section is the same for every unit of area, then there will be zero error from sea state. If the distribution of area is not symmetrical and/or if the area below mean sea level (troughs) gives a larger radar return on the average than the area above mean sea level (crests) this would generate a positive error increasing as a function of wave height.

*We use ko = 0.225 which was obtained from an "empirical" fit to many computer solutions for tracking error versus attitude error with Altitude, Antenna beamwidth and pulsewidth varied over a wide range.

If the converse (E/M crests > troughs) were true the error would be negative proportional to wave height. To get a feeling for the magnitude and form of tracking error arising from E/M troughs > crests, a linearly weighted backscattering with crests giving $0.75 \ \sigma_{MSL}^{o}$ and troughs giving $1.25 \ \sigma_{MSL}^{o}$. This operation has the effect of shifting the Radar Altimeter observed Mean Sea Level to $e_h(H_{1/3})$ below the Geometric Mean Sea Level. The approximate equation for $\varepsilon_h(H_{1/3})$ obtained from analysis of the altimeter tracking error vs. sea state buildup is:

$$(14) \qquad \varepsilon_h(H_{1/3}) \doteq 0.025 \sqrt{\frac{H_{1/3}}{cT}} \left[\frac{cT}{2}\right] \quad \text{METERS}$$

If the significant wave height can be hind cast to $\pm 20\%$, then reduction of this error source to a residual sea state bias error, $e_h(H_{1/3})$, would give a final uncertainty of:

$$(15) \qquad e_h(H_{1/3}) = \frac{\partial \left[\varepsilon_h(H_{1/3})\right]}{\partial H_{1/3}} \times \left[\pm 0.2 H_{1/3}\right] = 0.125 \sqrt{cT H_{1/3}} \ \text{CM.}$$

Solution of 15 for CT = 30 meters, $H_{1/3}$ = 30 meters gives a residual error of $\varepsilon_h(H_{1/3})$ = 3.75 cm.

Ben Yaplee's experimental data on the differential radar backscattering cross-section versus surface depth indicates a linear increase in σ^o from crests to troughs.

Lee Miller's* analysis of Yaplee's data gives the following equation for the variation of radar cross-section about MSL:

*Second Quarterly Report on Contract No. NAS6-1952, Research Triangle Institute North Carolina; L. S. Miller.

(16)
$$\sigma^0(z) = \sigma^0(z=0)\left[1 - m\frac{z}{\sigma_z^0}\right]$$

Where the value of m lies between:

m = 0.141 for 20 knot winds

and

m = 0.185 for calm seas.

Assuming that these are essentially correct, the residual bias error on altitude due to sea state, $e_h(H_{1/3})$, given either 20% $H_{1/3}$ measurements or hind casts will be no greater than ± 5 cm, with a standard deviation of about ±2 cm, which would probably be an acceptable part of the error budget for even a ± 10 cm satellite radar altimeter.

Beamwidth Limited Altimetry - Figure 2, shows the significant features of Beamwidth Limited Satellite altimetry. In comparing Figures 2a & 2b with Figures 1a & 1b, note that the antenna beamwidth, θ_A, is less than the maximum satellite attitude error ϕ_M and much less than the pulse beamwidth, θ_T, and therefore the total area ii.uminated at Nadir is reduced to only that area subtended by θ_A. This is the defining feature of beamwidth limited altimetry. Note also in Figure 2b that even at fairly small angles off Nadir the portion of the expanding ring area does not completely fill the area subtended by θ_A. This fact will cause a decrease in the peak amplitude of the return off Nadir compared to that at Nadir and also a time dispersion (i.e. a widening) of the return pulses as shown in Figure 2c.

This return pulse widening off Nadir could easily be confused with the expected time dispersion due to increasing wave heights (Figure 2d) which has been proposed as an absolutely foolproof method of measuring sea state directly. Figures 2c and d show that for both Altimetry and Sea State measurement by return pulse time dispersion, it would be necessary to point the antenna very accurately toward Nadir in the beamwidth limited type of altimeter.

Selection of a tracking law for beamwidth limited altimetry is illustrated in Figure 2d. The average return waveshape will be an almost symmetrical pulse with a width equal to the transmitted pulse T for low sea states and a larger width for higher sea states. The tracking law selected is shown above the return waveform as an adjacent split gate energy tracker. The slight assymmetry arises from the fast rise from zero, asymtotic approach to maximum value of the leading edge with just the opposite occurring on the trailing edge. This causes a constant tracking bias error whose magnitude is a function of how beamwidth limited the design actually is, (i.e. on the ratio θ_A/θ_T). An approximate equation for this bias is:

$$(17) \qquad \varepsilon_h(\phi_E=0) \doteq 0.31 \frac{cT}{2} \left(\frac{\theta_A}{\theta_T}\right)^2 \quad \text{METERS}$$

The Altitude Tracking Error arising from Antenna/Satellite attitude error for beamwidth limited altimetry is shown in Figure 2e. Note that because the return is centered about the area illuminated at the error angle, the tracked range to that return will increase directly with altitude, h, and directly with the square of the error angle, ϕ_E.

The equation for tracking error versus pointing error is thus:

(18) $$\epsilon_h(\phi_E) = {}_h(\phi_E = 0) + \frac{1}{2}h\phi_E^2$$

To further illustrate the absolute requirement for accurate antenna pointing in beamwidth limited altimetry (or laser altimetry) put a typical value of h = 1000 KM for satellite altitude and an exceptional satellite attitude control capability of $\pm$ 15 arc minutes 3σ with respect to local vertical into equation 18. The tracking error is bounded at $\pm$ 20 meters! With this type of altimetry, perhaps the only answer is to point the antenna as nearly as possible to the vertical, set up an oscillation about this direction and take the nearest altitudes observed as the best actual altitude to mean sea level.

The tracking error versus sea state of beamwidth limited altimetry has essentially the same form as that for pulse width limited altimetry as shown in Figure 2f and for the same reasons. That is, if the radar sea surface is symmetrical about mean sea level there will be zero altitude error versus wave height but if troughs give back more average radar return than crests (as seems likely) then the time error tracked will be in the positive direction and proportional to some function of the ratio of significant wave height to the radar pulse length, This is reasonable because the percentage distortion due to sea state will be less for long radar pulse lengths, CT, than for short ones. To translate altitude time errors into altitude errors, use the factor CT/2 (the basic pulse radar range resolution capability) to arrive at

an equation for tracking error versus sea state of the form:

$$(19) \qquad \epsilon_h(H_{1/3}) = k\frac{cT}{2}\left(\frac{H_{1/3}}{cT}\right)^A$$

where the exponent A is probably a fraction between 1/4 and 1/2.

As in the case of pulsewidth limited altimetry, a 20% accurate $H_{1/3}$ by hind cast or measurement will probably suffice to reduce the residual uncertainty due to sea state, $e_h(H_{1/3})$ to less than $\pm$ 5 cm with a standard deviation less than $\pm$ 2 cm.

Antenna Effects Radar Altimetry - As shown in Figure 3, antenna effects altimetry includes the design options which lie between strictly pulsewidth limited and strictly beamwidth limited altimetry designs. As shown in Figure 3a, the antenna beamwidth θ_A is on the order of the maximum satellite attitude error, ϕ_M, and the pulse beamwidth, θ_T, however, a point can be made here:

.If $\phi_M \leq \frac{5}{8}\theta_A$, then antenna pointing will not be required to achieve acceptable residual altitude bias errors arising from antenna/satellite attitude errors because these errors can be measured directly from suitable sampling of the radar return waveform.

Figure 3b illustrates how this extreme sensitivity of the return waveshape with respect to pointing error, ϕ_E, comes about.

Note that when the antenna is pointed directly at Nadir, $\phi_E = 0$, the area illuminated from $0 \leq t \leq T$ is centered on the gain center of the antenna. Because the increasing area is weighted by decreasing antenna gain, the linear buildup of area will result in a return leading edge resembling an RC step response until $t = T$, as shown in Figure 3c - solid curve.

Also note that after reaching a peak at $t = T$, the return falls off in an RC time constant fashion.

Now refer back to Figure 3b and the effective antenna contour when $\phi_E = \theta_A/2$. Note that the portion of the increasing area illuminated from $0 \leq t \leq T$ is less than 1/2 contained within the effective beamwidth and that less than 1/2 A(T) which is contained is illuminated with about 2 db less than boresite antenna gain. This results in a return rise time resembling an RC response to a ramp input until $t + T$. Note in Figure 3c (the dashed curve) that the amplitude of the return at $t = T$, when $\phi_E = 1/2 \theta_A$, is about 1/4 the amplitude reached @ $t = T$ for $\phi_E = 0$. Also note that for $t > T$ the return is almost flat so that the average return waveform for $\phi_E = \theta_A/2$ resembles the average return waveform for strictly pulsewidth limited altimetry; thus, the split gate energy tracker shown in Figure 3d which would track with almost zero error for a pulsewidth limited antenna design will also track with near zero error at $\phi_E = \theta_A/2$ in the antenna effects altimeter design, (see the error curve of Figure 3e).

In Figure 3e, note that the error curve is negative for pointing errors between $\phi_E = 0$ and ϕ_E just less than $\theta_A/2$. At and beyond $\phi_E = \theta_A/2$ the tracking error is positive and growing exponentially. An approximate equation for altitude error vs. antenna/satellite attitude error is:

$$(20) \qquad \varepsilon_h(\phi_E) \doteq \left[\left(\frac{\phi_E}{\theta_A}\right)^2 - 0.225\right] \frac{cT}{2}\left(\frac{\theta_T}{\theta_A}\right) \quad \text{METERS}$$

One feature of the action of the split gate energy tracker shown here is that as ϕ_E approaches θ_A the tracker cannot acquire or track the radar returns. This is simply because this tracker requires an initial rapid rise on the order of T or it cannot find a balance point. It is this feature which minimizes the attitude error generated tracking error when compared to the strictly beamwidth limited altimeter design and it also serves as an indicator that $\phi_M < \theta_A$ if an altimeter with this tracking rule does acquire and track.*

*Note – The Skylab Altimeter falls in the category of an antenna effects altimeter design with $\theta_A \doteq 1.4^o$, θ_T (T = 100Nsec) $\doteq$ 0.95^o and ϕ_M which initially may be as high as $\pm 2^o$. This is the reason an initial antenna alignment mode is included in the altimeter experiment. This initial on-orbit antenna alignment consists of a "spiral scan" which settles into a square about the point where the peak of the radar return is maximized then is shut off because, once found, the satellite attitude control will maintain this pointing direction within the requirements of the immediately following altimeter experiment(s).

Dispersion due to increased wave heights for antenna effects altimetry, (see Figure 3d & f) would not be symmetrical in time about $t = T/2$ even with an E/M ocean symmetrical about the mean sea level. We have not yet found the error curves for sea state for any particular antenna effects altimeter design but it would not be surprising if it had the form of the curve shown in Figure 3f which shows an increasingly negative altitude error as sea state builds up to $H_{1/3} = \frac{4}{3}cT$.

This is pointed out as a problem area because if the form and magnitude of the error curve for a symmetrical E/M ocean were just right, then (as shown in Figure 3f), the error curve for the expected asymmetrical E/M ocean could lie on zero. We don't want any such thing to happen by dumb luck although it would be perfect if it could be made to happen by design based on knowing what we were doing. At any rate, with proper care in our experiments, we will one day know what the altitude error vs. sea state should be for any altimeter design and the process of reducing that error to an acceptable level will depend on hind cast and/ or measured sea state as discussed before.

Comparison of the three types of Altimeter Systems in a typical satellite application is shown in Figures 4 and 5. Each $T = 100$ Nsec altimeter system is assumed to operate in a manner which allows 5000 independent returns per second to be tracked. Under this assumption, the standard deviation jitter error, σ_h, of altitude readouts averaged over one second will be approximately ± 22 cm for both the "Pulse Width Limited" and the "Antenna Effects" Altimeters, but will only be about ± 2 cm for the "Beamwidth Limited" design.

F

This difference of 10/1 in jitter error performance comes about entirely from the fact that the returns seen by the beamwidth limited adjacent split gate tracker are almost unity correlated in each gate even though the amplitude distribution of the returns is Rayleigh (or Exponential) distributed depending on whether envelope (or Square Law) detection is employed. Compare this to the split gate energy tracker of the "Pulsewidth" and "Antenna Effects" altimeters where there is essentially complete decorrelation of the returns between the "early" and the "late" gates.

In this comparison, we are looking at altimeter designs which would be satisfactory for a Satellite Altimeter System whose goal was to achieve an overall one sigma accuracy of 50 cm on the position of mean sea level over the Geoid.

A reasonable choice of orbit parameters is given in Figure 4a as a nearly circular, 825 kilometer, nearly polar orbit.

Given a satellite attitude control capability of one degree about the vertical Figure 4b shows the altitude error range expected for the "Pulsewidth Limited" altimeter design to be from -23.5 cm at $\phi_E = 0$ to -22.5 cm @ $\phi_E = 1^O$. The residual uncertainty, $e_h(\phi_E)$, obtained from merely assuming -23 cm error regardless of ϕ_E actual is contained within $\pm$ 0.5 cm, which is a negligible part of a system with a $\pm$ 50 cm one sigma error budget as stated in Figure 5a.

The altitude error due to satellite attitude error, $\varepsilon_h(\phi_E)$, for the "Antenna Effects" altimeter of Figure 4c is (from equation 20) minus about 1.2 meters @ $\phi_E = 0$ and goes to plus about 0.2 meters @ $\phi_E = 1^0$. Error reduction to an uncertainty of about ± 20 cm maximum residual uncertainty can be done (as shown in Figure 5b) by obtaining a measure of the error angle ϕ_E from the difference, Δ, between the averaged sample voltages V_1 and V_2 taken at the sample times S_1 and S_2. This level of maximum residual bias uncertainty is almost negligible in a ± 50 cm one sigma system error budget.

Figure 4d shows the error for a (reasonable?) "Beamwidth Limited" Altimeter design as going from positive 0.825 Meters at $\phi_E = 0$ to positive 126 meters at $\phi_E = 1^0$. This shows the absolute necessity for antenna pointing control to as close as possible to the Nadir.

Figure 5c shows the error reduction possible if the antenna can be pointed and maintained within 10% of its beamwidth with respect to Nadir. In this case, the residual uncertainty due to $\phi_A \leq \theta_A/10$ is:

$$e_h(\phi_A) \leq \pm 11.5 \text{ CM.}$$

As stated earlier, this might be done by a hill climb routine which continuously searched for minimum tracked altitude. The altitude record would look like a cycloid and with proper processing might yield acceptable altitude best estimates.

A more certain method of pointing control is by "Time Difference Monopulse".* The geometry and timing of this system is shown in Figure 5c. The system can consist of a four hour monopulse feed which creates one sum beam, Σ , and four difference beams $\Delta_1, \Delta_2, \Delta_3, \& \Delta_4$ separated in foresite by $1/2\ \theta_\Sigma$ as shown. The returns thru each beam are individually centroid tracked. When the tracked Δ beam returns all lag the tracked Σ beam return equally in time the Σ beam is pointed at Nadir. If a pointing error exists, the direction and amount of the error in the $\Delta_1 - \Delta_2$ plane is measured by the difference in tracked return time, $t\Delta_1 - t\Delta_2 = k\phi_{1-2}$, and by steering to null this differ-ence the error is corrected out. The same is true for an error developed in the $\Delta_3 - \Delta_4$ direction.

Maximum reduction of residual error could be obtained post flight if the differences $k\phi_{1-2} \& k\phi_{3-4}$ were included with each altitude report.

* Invention of Dr. Kiyo Tomiyasu, General Electric Co., Satellite System Operation, King of Prussia and Tom Godbey.

Conclusions - Prior presentations and papers on Satellite Altimetry presented either the Pulsewidth Limited or the Beamwidth Limited altimeter designs, the former as presenting "no problem" with reasonable satellite attitude control, the latter as having only two problems - that of satellite attitude measurement and control and that of maintaining boresite of the large antenna with respect to the satellite. The claims for Pulsewidth Limited Altimetry have been verified. The problems of Beamwidth Limited Altimetry probably can't be solved with Satellite Attitude Control, but appear soluable with antenna pointing in which the radar seeks and maintains Nadir.

The satellite altimeter systems presently being built and/or being conceived are of the Antenna Effects type which do have a Satellite Attitude Control problem.

It is hoped that this review and examination of the basic altimeter design choices available will stimulate and challenge satellite altimeter system designers to re-examine the "practical limitations of satellites." These mundane matters are forcing us into Antenna Effects Altimetry designs.

Is the added complexity compared to Pulsewidth Limited Altimetry justified? Should we go all the way to the sophistication and complexity of Beamwidth Limited Altimetry and what would be the added capability of this type of design?
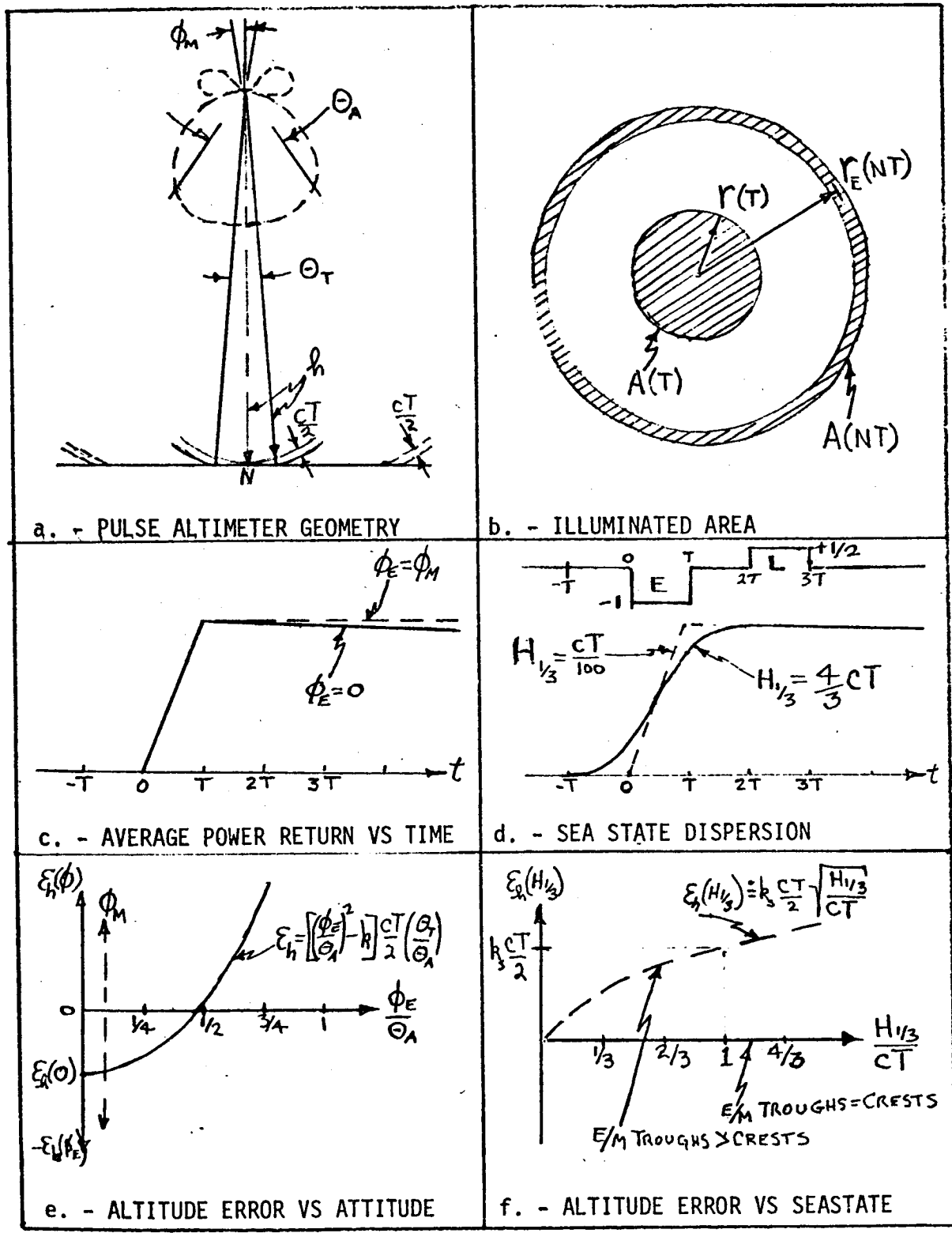
a. - PULSE ALTIMETER GEOMETRY

b. - ILLUMINATED AREA

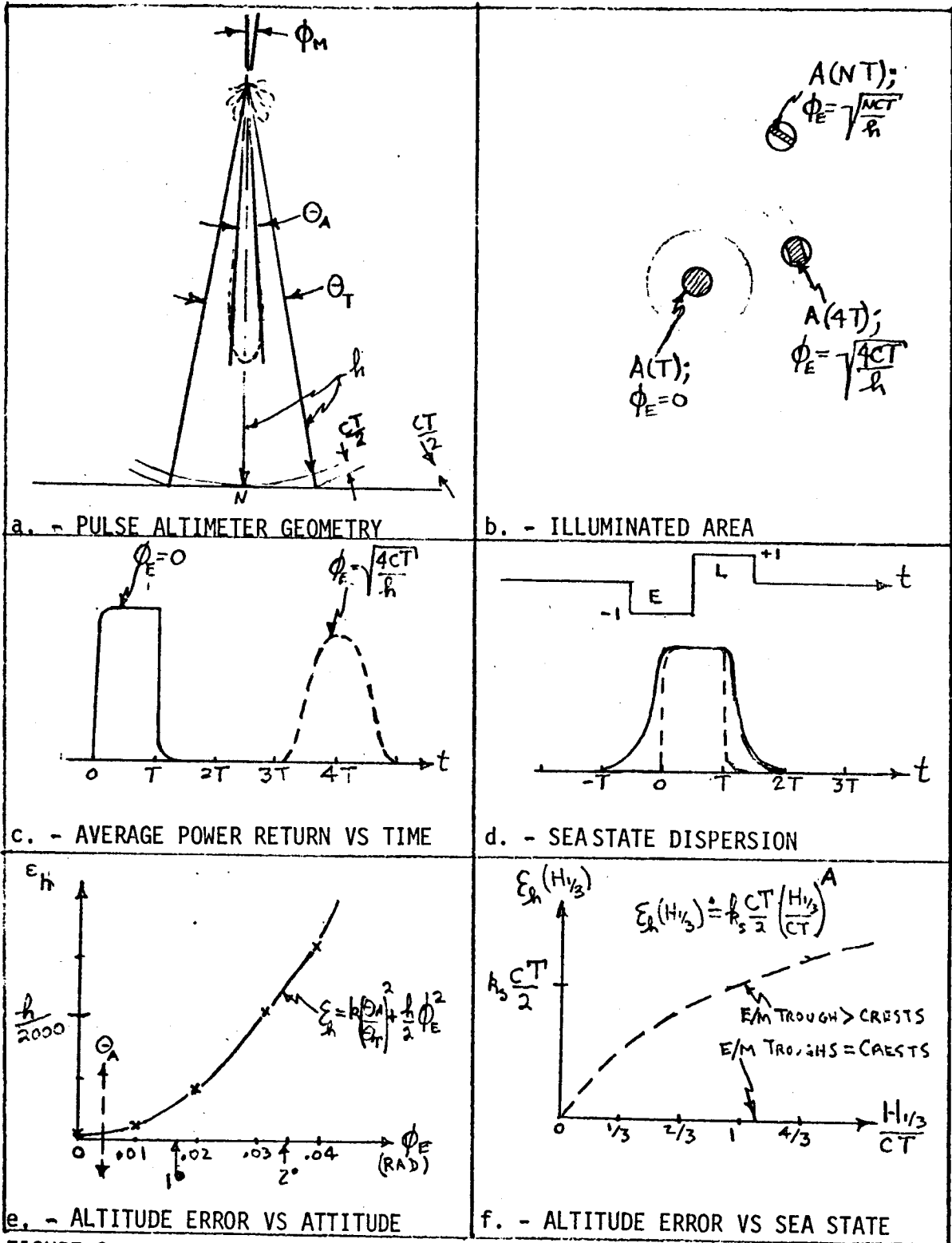c. - AVERAGE POWER RETURN VS TIME

d. - SEA STATE DISPERSION

e. - ALTITUDE ERROR VS ATTITUDE

f. - ALTITUDE ERROR VS SEASTATE

FIGURE 1. - PULSEWIDTH LIMITED SATELLITE RADAR ALTIMETRY.

a. - PULSE ALTIMETER GEOMETRY

b. - ILLUMINATED AREA

c. - AVERAGE POWER RETURN VS TIME

d. - SEA STATE DISPERSION

e. - ALTITUDE ERROR VS ATTITUDE

f. - ALTITUDE ERROR VS SEA STATE

FIGURE 2. - BEAMWIDTH LIMITED SATELLITE RADAR ALTIMETRY.

a. - PULSE ALTIMETER GEOMETRY

b. - ILLUMINATED AREA

c. - AVERAGE POWER RETURN VS TIME

d. - SEASTATE DISPERSION

e. - ALTITUDE ERROR VS ATTITUDE
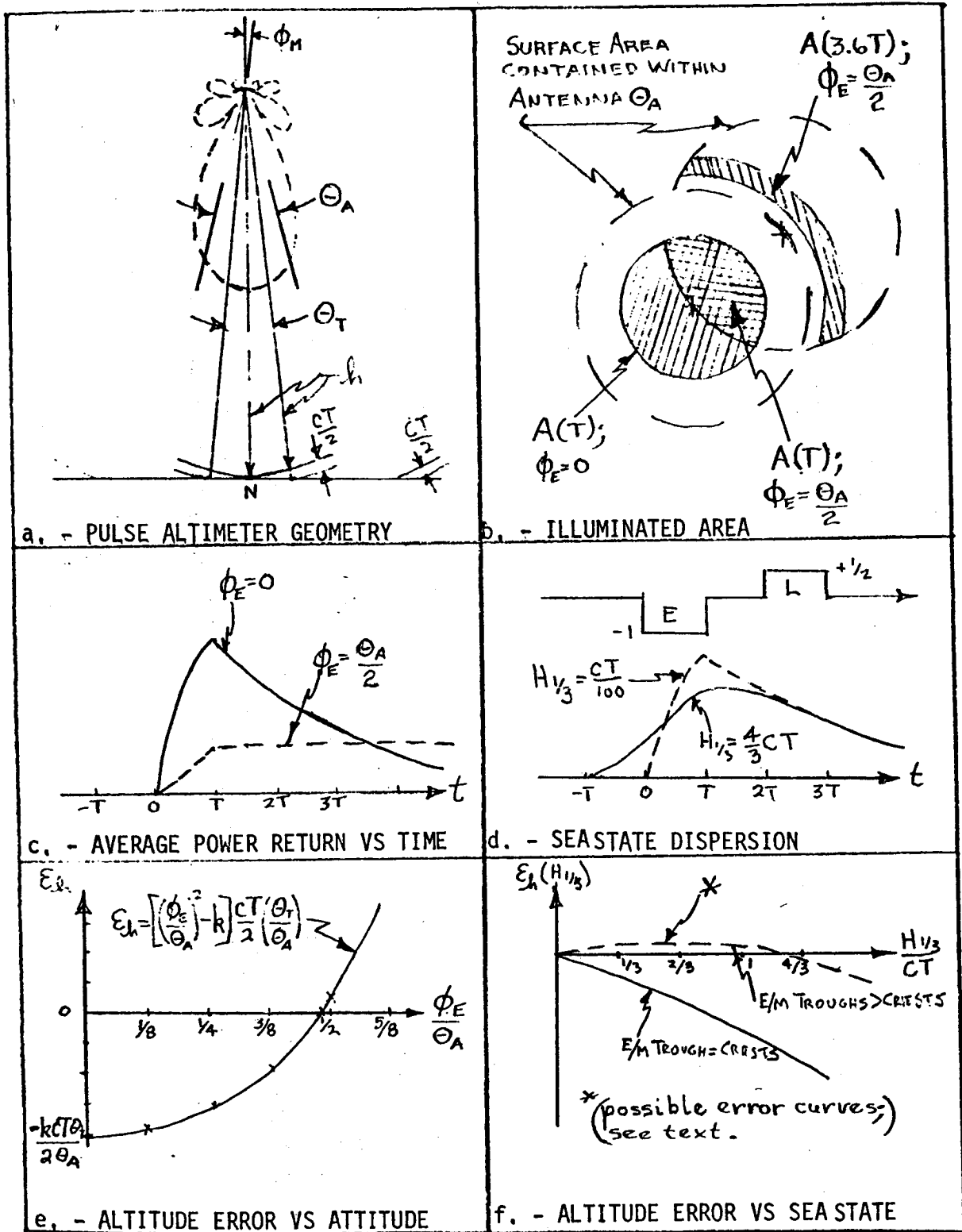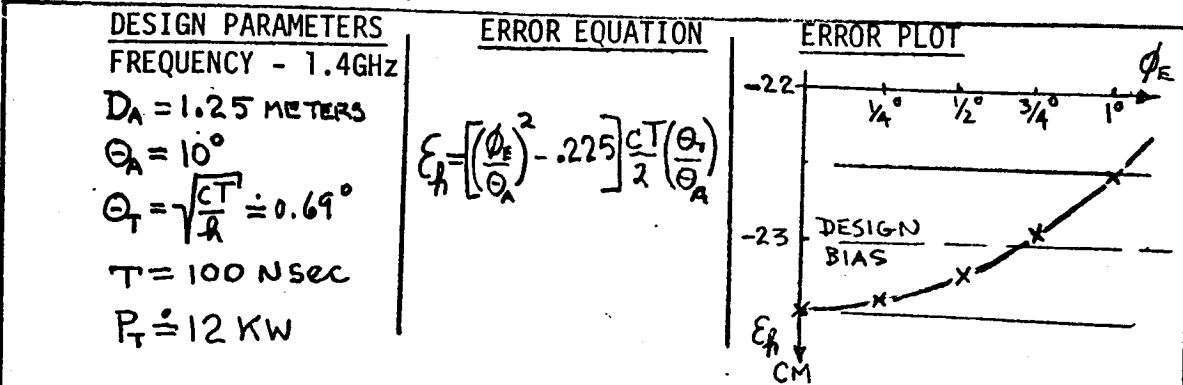
f. - ALTITUDE ERROR VS SEA STATE

FIGURE 3. - ANTENNA EFFECTS SATELLITE RADAR ALTIMETRY.

ALTITUDE = 825 ± 50 KILOMETERS

INCLINATION - NEAR POLAR ORBIT
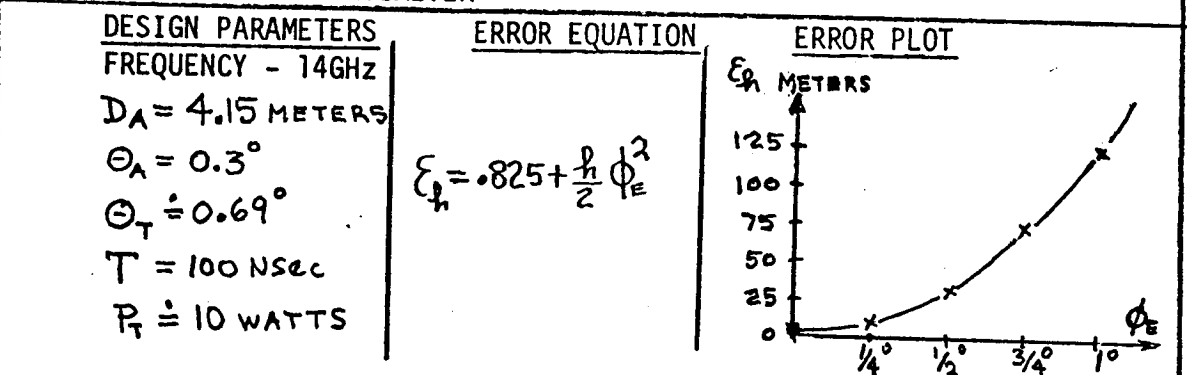
MAXIMUM ATTITUDE ERROR ABOUT NADIR = 1^0

a. SATELLITE PARAMETERS.

DESIGN PARAMETERS	ERROR EQUATION	ERROR PLOT
FREQUENCY - 1.4GHz $D_A = 1.25$ METERS $\Theta_A = 10^0$ $\Theta_T = \sqrt{\frac{cT}{h}} \doteq 0.69^0$ $T = 100$ NSec $P_T \doteq 12$ KW	$\mathcal{E}_h = \left[\left(\frac{\phi_E}{\Theta_A}\right)^2 - .225\right]\frac{cT}{2}\left(\frac{\Theta_T}{\Theta_A}\right)$	

b. PULSEWIDTH LIMITED ALTIMETER.

DESIGN PARAMETERS	ERROR EQUATION	ERROR PLOT
FREQUENCY - 7GHz $D_A = 1.25$ METERS $\Theta_A = 2^0$ $\Theta_T \doteq 0.69^0$ $T = 100$ NSec $P_T \doteq 2$ KW	$\mathcal{E}_h = \left[\left(\frac{\phi_E}{\Theta_A}\right)^2 - .225\right]\frac{cT}{2}\left(\frac{\Theta_T}{\Theta_A}\right)$	

c. ANTENNA EFFECTS ALTIMETER

| DESIGN PARAMETERS	ERROR EQUATION	ERROR PLOT
FREQUENCY - 14GHz $D_A = 4.15$ METERS $\Theta_A = 0.3^0$ $\Theta_T \doteq 0.69^0$ $T = 100$ NSec $P_T \doteq 10$ WATTS	$\mathcal{E}_h = .825 + \frac{h}{2}\phi_E^2$	
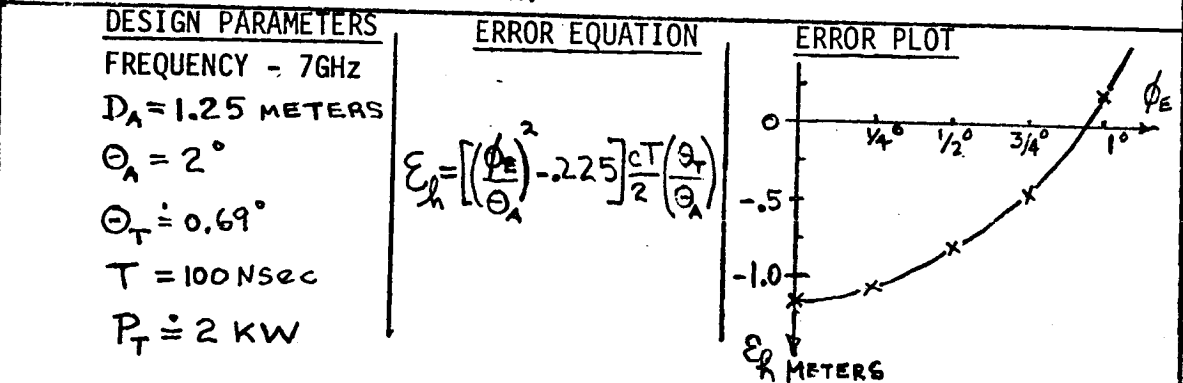
d. BEAMWIDTH LIMITED ALTIMETER

FIGURE 4. QUANTITATIVE COMPARISON OF ALTIMETER DESIGNS.

HOW - SUBTRACT-23CM DESIGN BIAS
$\overline{\text{RESULT}}$ - $e_h(\phi_E) \leq \pm 0.5$CM
JITTER ERROR, $\sigma_h \doteq \pm 22$CM
ANTENNA STEERING NOT REQUIRED

a.   -   PULSEWIDTH LIMITED ALTIMETER - REDUCTION BY DESIGN

HOW - SAMPLE RETURN WAVEFORM
$\overline{\text{AT}}$ OPTIMUM SELECTED POINTS
S_1 and S_2.   OBTAIN $=\overline{V}_1 - \overline{V}_2$.
FIND ϕ_E FROM $\phi_E = f(\Delta)$.
REDUCE $\varepsilon_h(\phi_E)$ TO $e_h(\phi_E)$.

$\underline{\text{RESULT}}$ - $e_h(\phi_E) \leq \pm 20$CM
JITTER ERROR, $\sigma_h \doteq \pm 22$CM
ANTENNA STEERING NOT REQUIRED
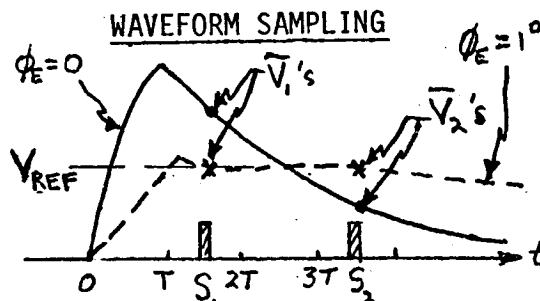
WAVEFORM SAMPLING

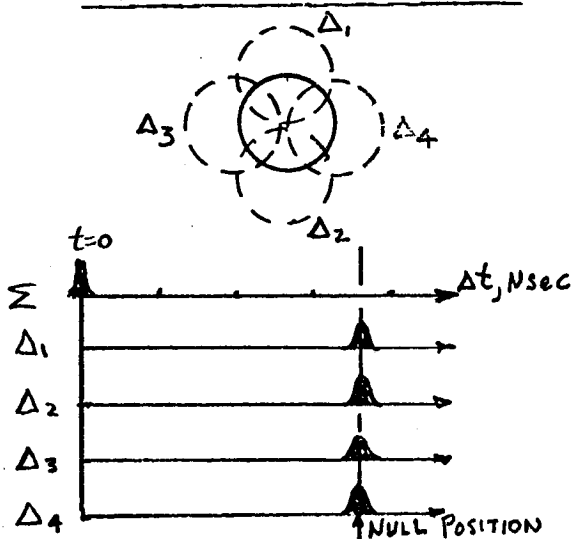$\Delta \doteq \overline{V}_1 - \widetilde{V}_2$

$\phi_E = f(\Delta)$

b.   -   ANTENNA EFFECTS ALTIMETER - REDUCTION BY WAVEFORM SAMPLING.

HOW - HILL CLIMB FOR MINIMUM
$\overline{\text{TRACKED}}$ ALTITUDE OR USE
"TIME DIFFERENCE MONOPULSE"
AS SHOWN AT RIGHT TO
MAINTAIN Σ BEAM WITHIN
$\pm\theta_{A/10} = \pm.03°$ OF NADIR.

$\underline{\text{RESULT}}$ - $e_h(\phi_E) \leq \pm 11.5$CM
JITTER ERROR, $\sigma_h \doteq \pm 2$CM
PRECISION ANTENNA STEERING
AND KNOWLEDGE OF ϕ_E IS
REQUIRED.

"TIME DIFFERENCE MONOPULSE"

c.   -   BEAM WIDTH LIMITED ALTIMETER - REDUCTION BY ANTENNA STEERING.

FIGURE 5.   -   REDUCTION OF $\varepsilon_h(\phi_E)$ TO $e_h(\phi_E)$.