130135

STAR

SPACE SYSTEMS

FINAL REPORT
FOR
LUNAR LIBRATION POINT FLIGHT DYNAMICS STUDY
(MAY 1968 - NOV 1968)
CONTRACT NO: NAS 5-11551

Prepared by
GENERAL ELECTRIC COMPANY

for

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

N73-16836

(NASA-CR-130135) LUNAR LIBRATION POINT FLIGHT DYNAMICS STUDY Final Report, May - Nov. 1968 (General Electric Co.) 132 p HC $8.75 CSCL 22A G3/30 Unclas 54754

GENERAL ELECTRIC

FINAL REPORT

FOR

LUNAR LIBRATION POINT FLIGHT DYNAMICS STUDY

(MAY 1968 — NOVEMBER 1968)

CONTRACT NO. NAS 5—11551

GODDARD SPACE FLIGHT CENTER

CONTRACTING OFFICER: C.W. TROTTER
TECHNICAL MONITOR: ROBERT GROVES

PREPARED BY

GENERAL ELECTRIC

SPACE SYSTEMS ORGANIZATION

Valley Forge Space Center

P. O. Box 8555 • Philadelphia, Penna. 19101

STUDY MANAGER — DR. J.D. PORTER

FOR

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

ABSTRACT

This study evaluated two satellite concepts (Halo - a satellite "orbiting" the L_2 point, and Hummingbird - a satellite hovering near L_2) for a lunar libration point (L_2) satellite to be used as a tracking and communications link with the far side of the moon. Study areas included flight dynamics, communications, attitude control, propulsion, and system integration. On the basis of these studies, both the concepts were proved feasible. However, the Halo was shown to be the better concept. The Halo concept should be investigated in more detail, and technology studies in the areas of multiple feed antennas and specific attitude control techniques should be initiated.

TABLE OF CONTENTS

TABLE OF CONTENTS (Cont'd)

LIST OF ILLUSTRATIONS

SECTION 1

INTRODUCTION AND SUMMARY

This is the final report on the Lunar Libration Point Flight Dynamics Study as specified in Contract NAS5-11551.

The program objective was to obtain preliminary information on the problems involved with communications and flight dynamics for a lunar libration point relay satellite system for use as a tracking and communications link between the far side of the moon and an earthbound tracking system for two types of satellites:

a. Halo type, which orbits the L_2 point while continuously visible from Earth.

b. Hummingbird type, which hovers near the L_2 point and is also continuously visible from Earth.

On the basis of these studies, the feasibility of each concept was to be evaluated and the better concept selected as the preferred system.

The study was performed by determining the system requirements on the basis of flight dynamics and communications studies, and by synthesizing and evaluating system concepts for both the Halo and Hummingbird satellites. In order to synthesize the concepts, studies in the attitude control and propulsion areas were required. Other subsystem models, such as the power and structural subsystems, were utilized as necessary. The study guidelines are given in the following list:

a. 1971 State of the Art

b. Three Year Lifetime

c. NASA Supplied List of Launch Vehicles

d. Use of Apollo Communication Modes as the Desired Communications Links

e. Use of Apollo CSM and LM as Typical Lunar Orbiting and Lunar Surface Vehicles.

Both types of satellites were deemed feasible. However, on the basis of lower system weight and cost, and no increase in system complexity, the Halo satellite was selected as the preferred system.

It was recognized early in the study that the major trade-off area was whether the more difficult attitude control problem of the Halo orbiter would outweigh the increased propulsion requirements of the Hummingbird. The tradeoff went in favor of the Halo, however, because the selection of a dual beam, single antenna for both concepts simplified the Halo attitude control requirements to the point of equivalency with the Hummingbird; also the larger propulsion requirements of the Hummingbird tipped the scales in the Halo's favor.

The studies in the areas of flight dynamics, communications, attitude control, propulsion, and systems integration and evaluation will be described in the following sections as well as the conclusions and recommendations.

SECTION 2

FLIGHT DYNAMICS STUDIES

The major objectives of the flight dynamics studies were to determine the ΔV requirements for orbit injection and orbit maintenance and to determine the relationship between tracking accuracy and station-keeping ΔV requirement.

The areas described in the following sections are transfer trajectories, launch windows, equations of motion around L_2, stabilization requirements and orbit simulation.

Many flight modes such as transfers are similar for both the Halo and Hummingbird concepts. Rather than repeat the discussions, this section will be described along the flight mode lines rather than orbiter type. Unless otherwise noted, the following material pertains to both the Halo and Hummingbird concepts.

2.1 TRANSFER TRAJECTORIES

At the present time a satellite in orbit about or in the vicinity of the L_2 lunar libration point must by necessity originate from the Earth. Thus, an investigation of the possible transfer trajectories between the Earth and L_2 was carried out. Since there exists an infinite number of possible transfers, it was decided to determine only the existence and characteristics of some reasonable transfers without recourse to any optimization procedure. Reasonable transfers here refer to practical flight times and allowable velocity impulses. As a start in understanding the transfer trajectory problem, calculations were restricted initially to the Earth-Moon plane. These results were then applied as a first approximation for the calculation of out-of-plane transfers. Out-of-plane transfers are necessary since an Earth launch into the Moon's orbital plane is possible only under very restrictive conditions. It was assumed that a launch vehicle will initially place the satellite in a low-Earth parking orbit. The launch window problem is then involved in blending the timing and geometry constraints of the launch site location and the injection into the transfer trajectory.

2.1.1 TRAJECTORY CALCULATIONS

Transfer trajectories were computed by numerically integrating the restricted 3-body equations of motion expressed in a rotating coordinate system. The model used was one in which the Earth and Moon revolve in circular orbits about their barycenter. The coordinate system employed was centered at the Moon with the X-axis along the Earth to Moon line direction, the Z-axis perpendicular to the Earth-Moon plane, and the Y-axis in the plane so as to form a right-handed system. The equations of motion in this system are:

$$\ddot{X} = 2\dot{Y} + X\left(1 - \frac{1-\mu}{R_1^3} - \frac{\mu}{R_2^3}\right) + (1-\mu)\left(1 - \frac{1}{R_1^3}\right)$$

$$\ddot{Y} = -2\dot{X} + Y\left(1 - \frac{(1-\mu)}{R_1^3} - \frac{\mu}{R_2^3}\right)$$

$$\ddot{Z} = -Z\left(\frac{(1-\mu)}{R_1^3} + \frac{\mu}{R_2^3}\right)$$

where $R_1^2 = (X+1)^2 + Y^2 + Z^2$

$$R_2^2 = X^2 + Y^2 + Z^2$$

R_1 is the distance of the vehicle from the center of the Earth and R_2 is the distance from the center of the Moon. An important parameter, the only constant of integration of these equations, is the Jacobi constant given by:

$$C = \Omega - V^2$$

where $\Omega = (X + 1 - \mu)^2 + Y^2 + \frac{2(1-\mu)}{R_1} + \frac{2\mu}{R_2}$

The units used in these equations are such that the unit of mass is equal to the sum of the masses of the Earth and Moon and the unit of distance is equal to the Earth-Moon distance. Also, the unit of time is chosen such that the angular velocity of the Earth-Moon line is equal to 1. In this system of units the parameter $\mu = 0.01215$ and is equal to the ratio of the mass of the Moon to the sum of the masses. The relation of these units to the metric system is given below:

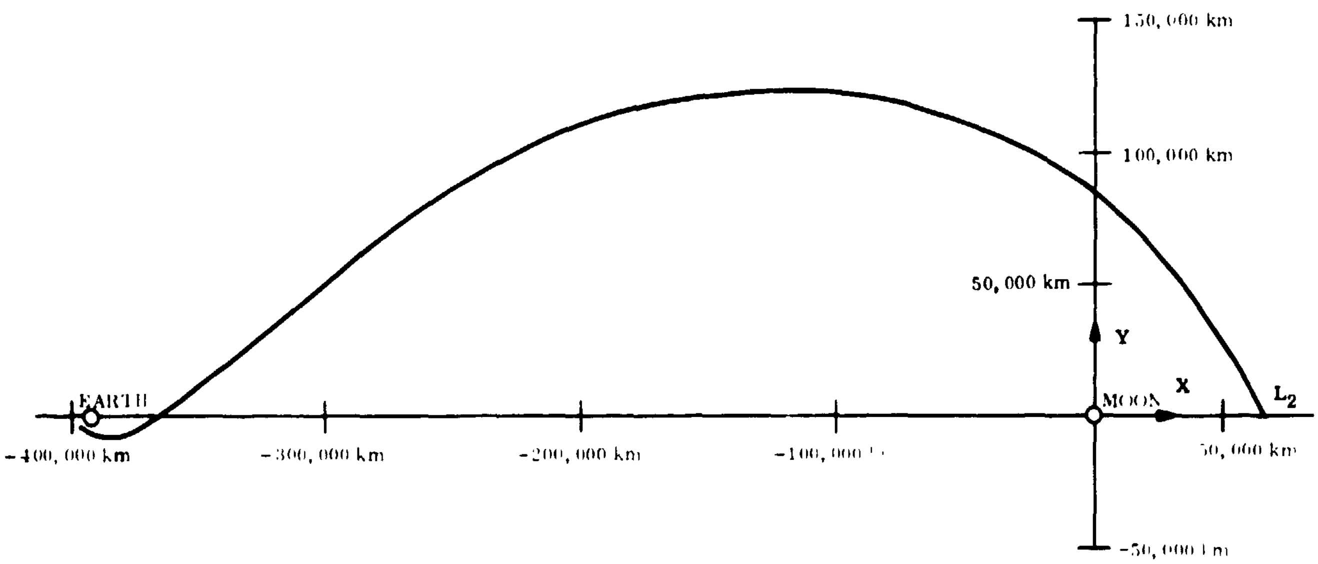
Distance Unit	384,400 km
Time Unit	4.34838 days
Velocity Unit	1.02316 km/sec
Acceleration Unit	2.72334×10^{-3} m/sec^2

The numerical integration was performed by utilizing a Runge-Kutta procedure, since this allowed a variable integration step size depending on the vehicle's position in the Earth-Moon system. It was assumed that transfer trajectories originated at a distance from Earth corresponding to a parking orbit altitude of 185.2 km and ended by passing through the L_2 location. The retro velocity requirement at L_2 is then quoted simply as the vehicle's velocity at L_2. In actuality the vehicle will not come to rest at L_2, but will either orbit it or will be forced to remain at some off-set point. The difference between the true and the simplified terminal velocities, however, will be small. In practice, some transfer trajectories were calculated by starting at the L_2 point and traveling toward the Earth. This was done in order to simplify the interaction calculations which were necessary to match end conditions. Due to the symmetry of the rotating coordinate system, time reversal trajectories were obtained by mirroring in the X-Z plane.

2.1.2 IN-PLANE TRAJECTORIES

The first types of transfer trajectories investigated were ones which lie in the Earth-Moon plane and allow two-dimensional calculations. Two modes of transfers were studied: direct, and close lunar fly-by. In the direct mode, trajectories are free-flown from an Earth parking orbit to the L_2 point, while in the other mode, a close lunar pass is made on the Earth outbound leg and a velocity impulse is applied to the vehicle while near the Moon to reduce the velocity requirements at L_2.

Direct mode transfers were computed by varying the velocity vector in the vicinity of the Earth, assuming a tangential impulse until a trajectory passed through the L_2 point. An example of direct mode transfer is shown in Figure 2-1 for a trip time of four days and for a velocity requirement of 1230 m/sec at L_2. This velocity is typical of acceptable direct mode transfer trajectories. In fact, the velocity requirement at the L_2 point assuming a massless Moon is 1054 m/sec.



C = 1.7265

TRIP TIME 4 DAYS

ΔV_{L_2} 1230 M/SEC

Figure 2-1. Earth to L_2 Transfer Trajectory in Rotating Coordinate System

Fly-by mode transfer trajectories were calculated by computing transfers between the L_2 point and the Moon and between the Earth and the Moon. Position and velocity of the vehicle for the different legs in the vicinity of the Moon were then studied in order to determine the required velocity impulse.

Transfer trajectories from the L_2 point to the Moon were constructed by varying the magnitude of the velocity vector at L_2 and determining the closest approach distance at the Moon (perilune distance). Figure 2-2 shows the minimum radius at the Moon and trip time as a function of direction for a fixed velocity at L_2 of 102.3 m/sec. Higher velocities will lie below this curve with lower velocities shown above it. By inspecting various velocity curves of the Figure 2-2 type, there are found to be two transfer trajectories which give a minimum ΔV at L_2 in order to satisfy a given perilune distance.

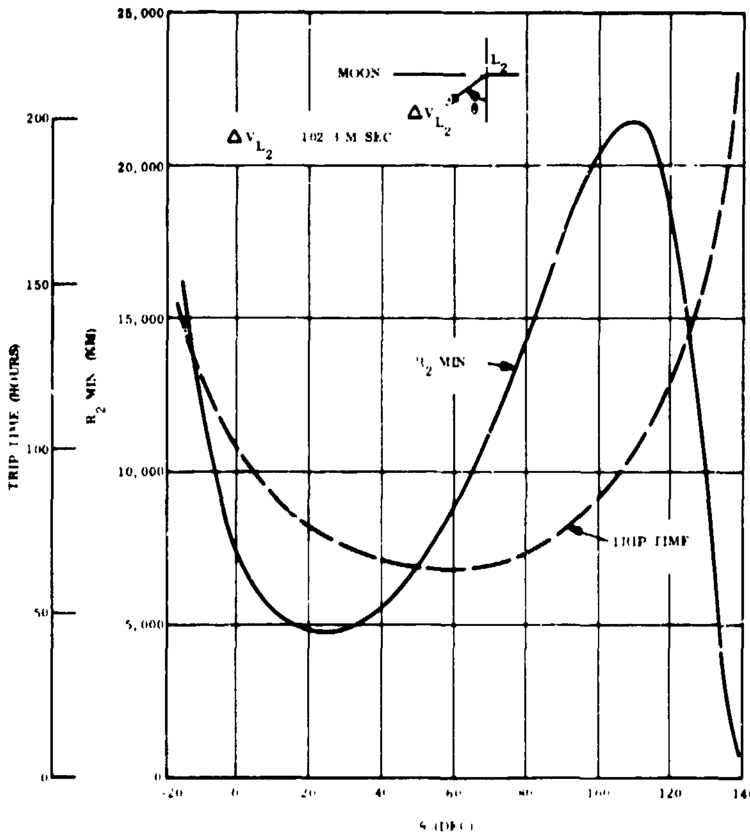


Figure 2-2. L_2 to Moon Transfer Trajectories

These were characterized by the terms "slow" and "fast" due to their difference in trip times. They are shown in Figure 2-3 where the perilune radius is plotted as a function of minimum ΔV at L_2. This minimum ΔV was found by varying the direction of the velocity vector at L_2 for a fixed perilune radius. An interesting aspect of the "slow" L_2-Moon transfer is the property that the vehicle never crosses the Earth-Moon line behind the Moon, thus satisfying the occultation problem.

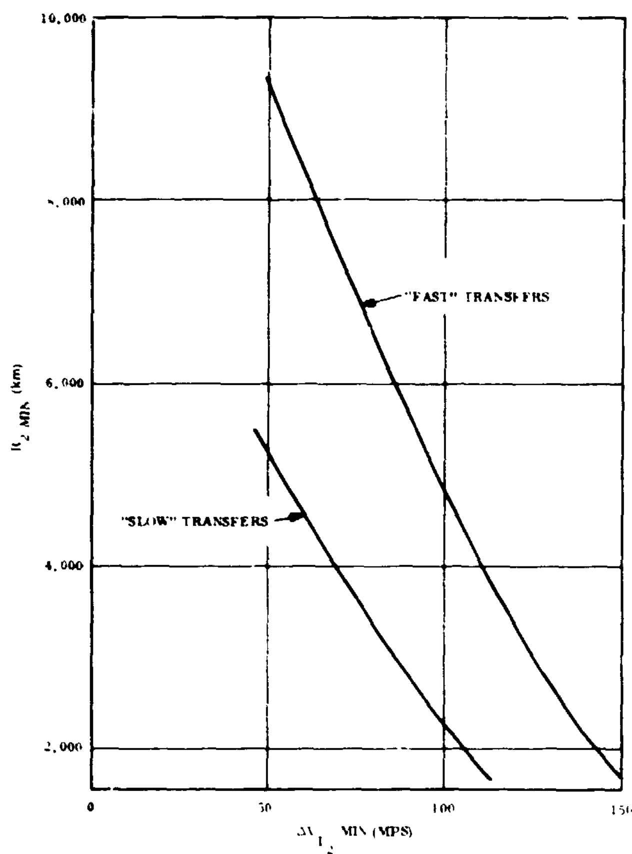


Figure 2-3. L_2 to Moon Transfer Trajectories

Next, Earth to Moon fly-by trajectories were constructed and studied. These transfers originated from a simulated 185.2 km circular parking orbit about the Earth. A tangential impulse at Earth was assumed and the velocity magnitude and injection radius position were varied in order to obtain transfers. Some typical Earth to Moon transfers are shown in Figure 2-4 for a fixed velocity magnitude at Earth. These trajectories and others were used to determine the vehicle's velocity in the vicinity of the Moon.

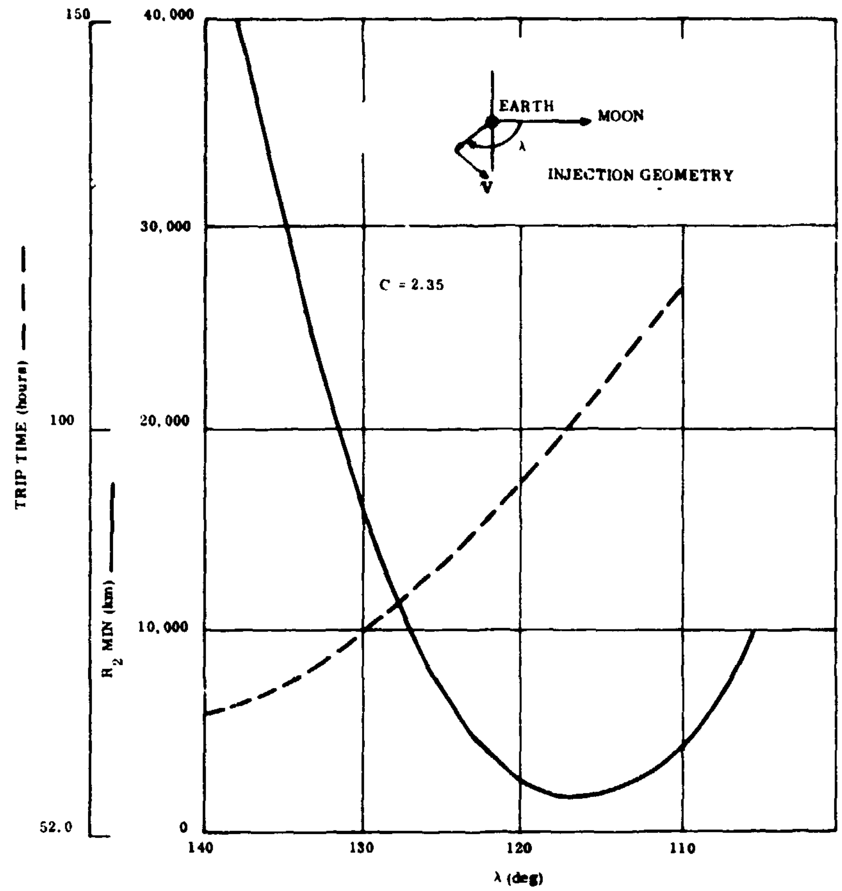


Figure 2-4. Earth to Moon Transfer Trajectories

Finally these two transfers were matched in the vicinity of the Moon. The procedure for doing this was as follows. Representative transfers of the "fast" and "slow" L_2 to Moon types were found which had a perilune miss distance of 185.2 km. A tangential velocity impulse was then applied at this point in order to obtain the final desired perigee condition 185.2 km closest to approach at the Earth. An iteration on the magnitude of this impulse had to be made in order to obtain the end conditions. The Earth-to-Moon transfers provided initial starting values for the iteration. A minimum velocity was then found by varying the point of application in the trajectory near the perilune point. An absolute minimum velocity impulse might be found if non-tangential impulses were allowed, however, this would result at the most in only a few percent decrease.

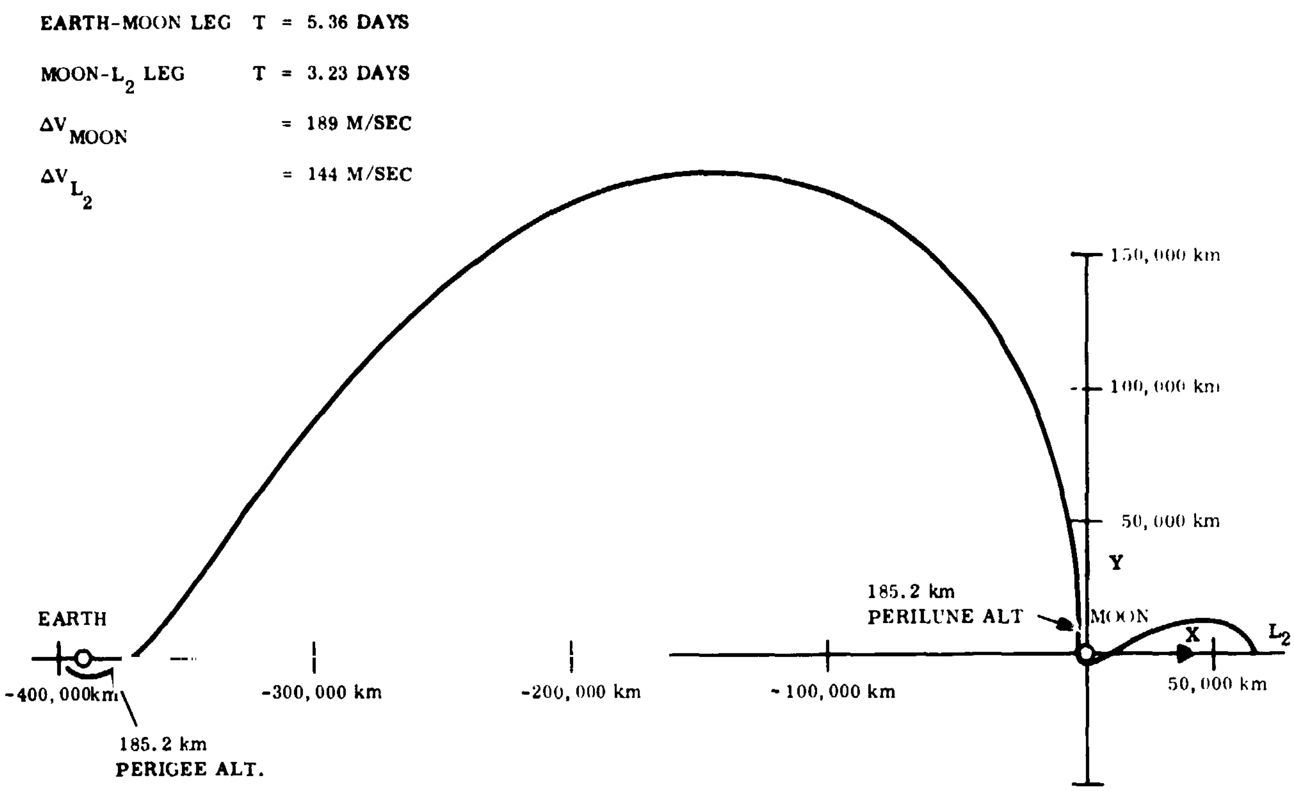


Figure 2-5. Earth to L_2 Transfer Trajectory "Fast" L_2-Moon Transfer

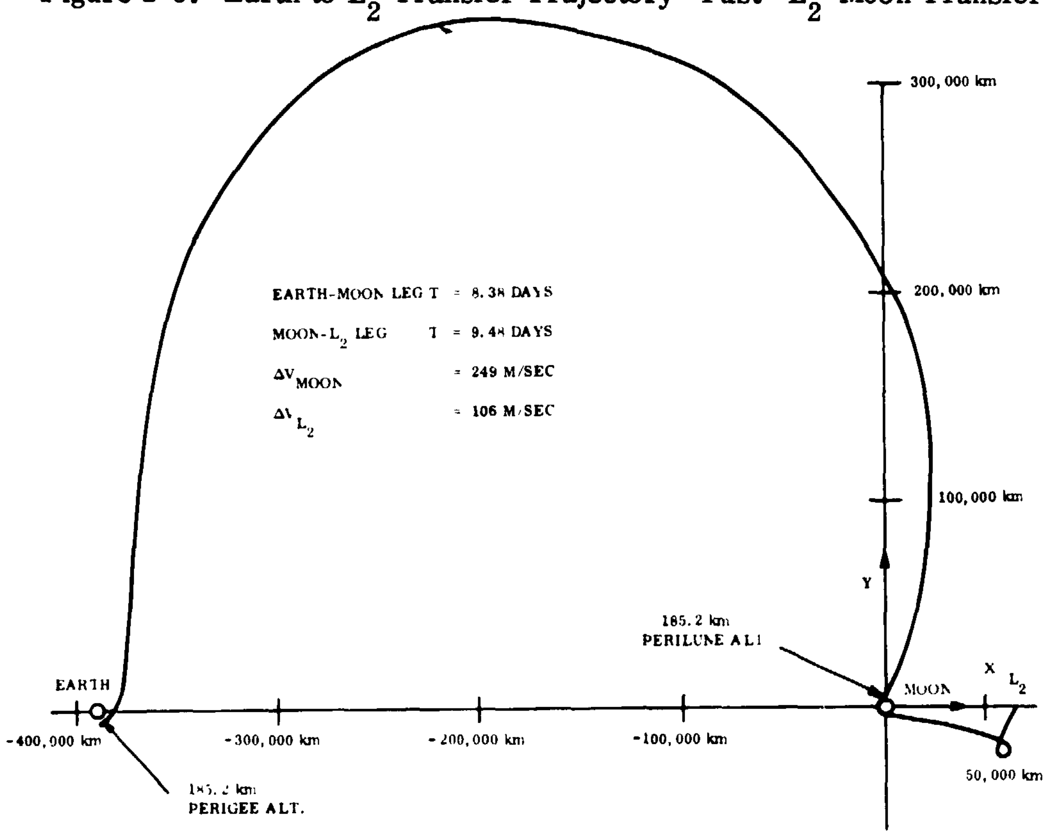


Figure 2-6. Earth to L_2 Transfer Trajectory "Slow" L_2-Moon Transfer

Two sample complete Earth to L_2 point transfers using close lunar fly-by are shown in Figures 2-5 and 2-6 for the "fast" and "slow" cases. In Figure 2-5 ("fast" Moon-L_2 transfer) the total Earth to L_2 trip time is 8.59 days; 5.36 days for the Earth to Moon leg and 3.23 days for the Moon to L_2 leg. The total required ΔV is 333 m/sec; 189 m/sec in the vicinity of the Moon and 144 m/sec at L_2. In Figure 2-6 ("slow" Moon-L_2 transfer) the total Earth to L_2 trip time is 17.86 days; 8.38 days for the Earth to Moon leg and 9.48 days for the Moon to L_2 leg. The total required ΔV is 353 m/sec; 247 m/sec in the vicinity of the Moon and 106 m/sec at L_2. Thus, comparing the direct and close lunar fly-by modes, there is a sizable reduction in ΔV requirement in employing the latter mode.

So far, only transfer trajectories which lie in the Earth-Moon plane have been investigated. However, in order to obtain a reasonable launch window, transfer trajectories out of the plane must be flown. The launch window problem will now be introduced and discussed.

2.1.3 LAUNCH WINDOWS

The problems involved in obtaining a launch window for Earth to L_2 transfer trajectories lie in the following four facts:

a. Launch site is situated at a fixed latitude on a rotating Earth.

b. Moon and L_2 point are in an orbit inclined to the Earth's equatorial plane.

c. The position of the Moon relative to the ascending node of the Moon's orbital plane on the equatorial plane moves through 360 degrees during one month.

d. The inclination of the Moon's orbital plane relative to the Earth equatorial plane varies from about 18.5 degrees to 28.5 degrees over an 18-1/2 year cycle.

A launch window is available, however, twice a day every day of the month under the following three assumptions:

a. Variable launch azimuth

b. Utilization of an Earth parking orbit

c. Variable inclination of the trans-lunar orbit

A launch period is obtained by determining those periods during the day when a launch plane can be achieved which includes both the launch site and the Moon at lunar fly-by. This implies that this plane includes the Earth-Moon line at fly-by. This is an approximation; however, actual trajectory calculations show it to be a very good one. A variable launch plane is obtained by adjusting the launch azimuth and the variable time in Earth parking orbit allows two launch periods during the day, one corresponding to a short coast in orbit and the other to a long coast in orbit. During each day of the month the Moon will be in a different position in its orbit, thus causing a variation in each of the launch periods and in the required inclination of the near-Earth transfer trajectory relative to the Moon's orbital plane (i_{TR}).

As an example of possible launch windows, numerical calculations were carried out for the following conditions.

a. Launch site is Cape Kennedy
b. Launch azimuth range is 90^{o} to 115^{o}
c. Inclination of Moon's orbital plane is 26^{o} (1972)

Results are presented in Figures 2-7 and 2-8. In Figure 2-7 the two launch periods (Δt_1, Δt_2) are shown as a function of the position of the Moon at lunar fly-by. The angle β is the central angle of the Earth-Moon line measured from the ascending node of the Moon's orbital plane on the Earth equatorial plane. Also shown in the plot is the waiting time between the two launch periods (Δt_1 $-t_2$). As can be seen, each launch period varies from 1.3 to 4.6 hours over the month. A total constant launch period of 5.9 hours is possible each day of the month. The waiting time between launch periods varies from 2.1 hours to 9 hours over the month. Figure 2-8 presents the transfer trajectory inclination (i_{TR}) near the Earth as a function of position of the Moon at lunar fly-by over the month. In order to make use of the total possible launch windows, inclinations of -63^{o} to $+63^{o}$ relative to the moon's orbital plane must be available. However, the smallest inclination that can be used has an absolute magnitude of 2.5 degrees, the difference between the launch site latitude and the inclination of the Moon's orbital plan in 1972. Thus, it is seen that transfer trajectories which travel out of the Moon's orbital plane must be used in order to obtain a launch windows.

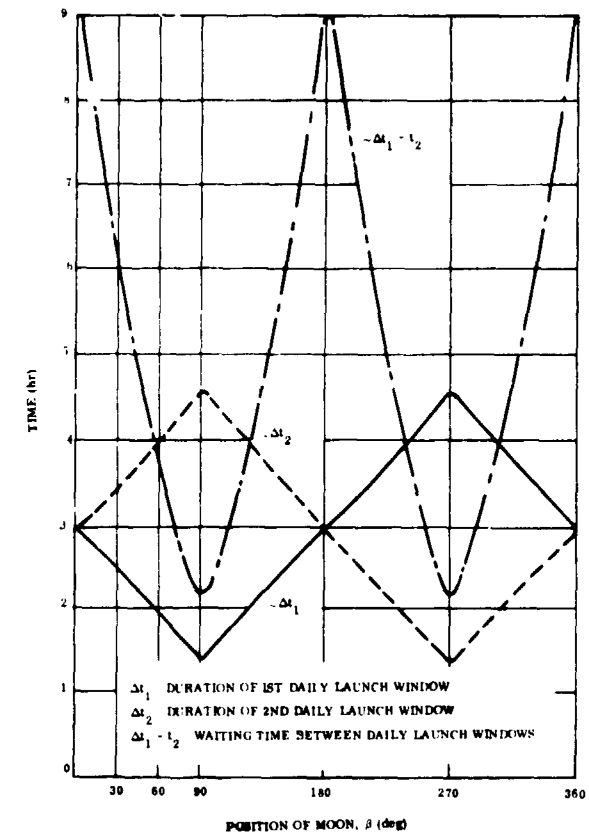


Figure 2-7. Launch Times vs Moon Position

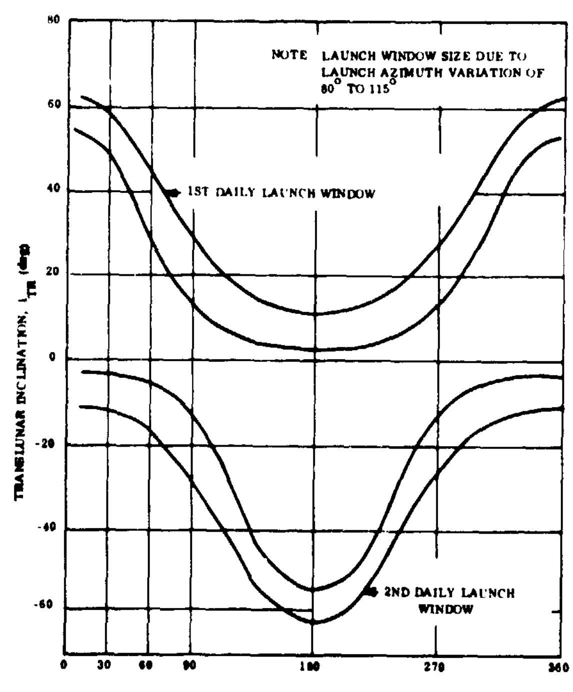


Figure 2-8. Transfer Trajectory Inclination vs Moon Position

2.1.4 OUT-OF-PLANE TRANSFER TRAJECTORIES

During this portion of the study, it was desired to compute representative transfer trajectories which lie out of the Moon's orbital plane. No effort was made to exhaust the total possibility of this type of transfer but only to show the existence of applicable ones. Also, due to the large savings in velocity requirements, only the close lunar fly-by mode transfers were investigated.

The method of computing matched transfer legs was similar to that used for the in-plane trajectories. Transfers were calculated from the L_2 point to the vicinity of the Moon with the velocity vector at L_2 now having some elevation angle with respect to the Moon's orbital plane. For a fixed elevation angle, the velocity magnitude was varied until a perilune miss distance of 185.2 km was obtained. The in-plane velocity angle was that found from the previous studies for the "fast" L_2-Moon transfer. Relations between the velocity elevation angle at L_2, the velocity magnitude at L_2, and the radius of closest approach at the Moon are presented in Figure 2-9. It can be seen from the upper plot in Figure 2-9 that a perilune altitude of 185.2 km ($R_{2_{MIN}}$ = 1922 km) can

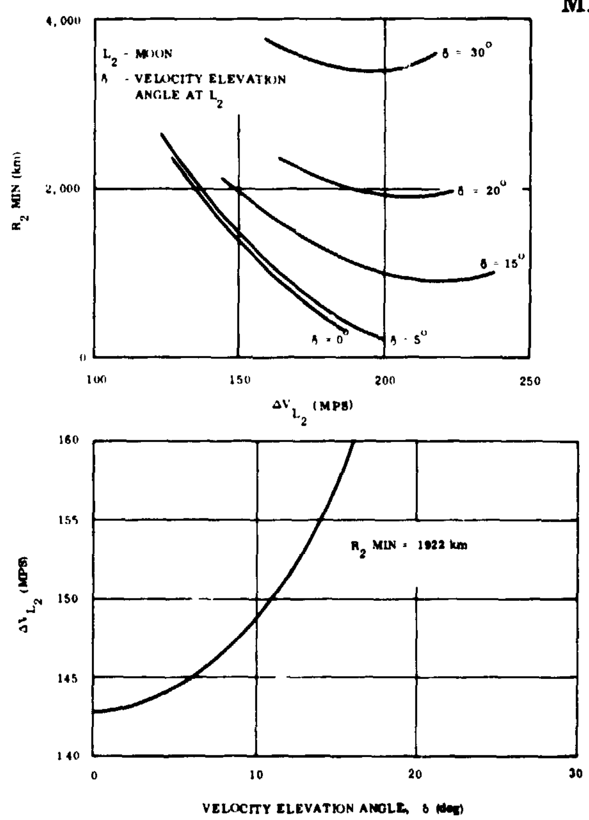


Figure 2-9. L_2-Moon Out-Of-Plane Transfers

not be achieved if the velocity elevation angle is greater than about 20 degrees. The velocity at L_2 (ΔV_{L_2})as a function of elevation angle for this perilune altitude is shown in the lower plot.

Moon to Earth transfers were then computed by applying a tangential velocity impulse in the vicinity of the Moon, at or near the perilune point. Parameters of actual computed patched trajectories are presented in Table 2-1. The perigee altitude at the Earth was held at 185.2 km. This table indicates that transfers with high near-Earth inclinations (i_{TR}) can easily be found. For example, the transfer with i_{TR} = 54.31 degrees requires a total velocity impulse of 370.5 m/sec, compared with the required impulse of 333 m/sec for the in-plane transfer (Figure 2-5), an increase of only 11 percent.

The results shown in Table 2-1 are for positive velocity vector elevation angles at L_2. However, due to the symmetry of these transfers with respect to the X - Y plane, identical results except for negative near-Earth inclinations ($-i_{TR}$) are obtained for negative elevation angles.

2.2 EQUATIONS OF MOTION

In the restricted problem of three bodies, the equations of motion of the third body (considered to have negligible mass) in a coordinate system rotating with the primary bodies are (Ref 13, p. 261):

$$\ddot{X} - 2\dot{Y} = \frac{\partial \Omega}{\partial X} = \frac{\partial \Omega}{\partial r_1} \frac{\partial r_1}{\partial X} + \frac{\partial \Omega}{\partial r_2} \frac{\partial r_2}{\partial X}$$

$$\ddot{Y} + 2\dot{X} = \frac{\partial \Omega}{\partial Y} = \frac{\partial \Omega}{\partial r_1} \frac{\partial r_1}{\partial Y} + \frac{\partial \Omega}{\partial r_2} \frac{\partial r_2}{\partial Y}$$

$$\ddot{Z} = \frac{\partial \Omega}{\partial Z}$$

Table 2-1. Sample Trajectories

$\delta = 5^{o}$ $\Delta V_{L_2} = 145.3$ m/sec	Sample Number			
	1	2	3	4
$\Delta\alpha$ (deg)	-29.5	-15.8	-1.2	13.5
ΔV_M (m/sec)	196.5	193.5	193.5	196.7
ΔV_T (m/sec)	341.8	338.8	338.8	342.0
Δt_1 (days)	3.22	3.22	3.22	3.22
Δt_2 (days)	5.19	5.34	5.50	5.67
Δt_T (days)	8.41	8.56	8.72	8.89
i_{TR} (deg)	20.61	19.53	18.46	17.47

$\delta = 10^{o}$ $\Delta V_{L_2} = 149.8$ m/sec	Sample Number			
	1	2	3	4
$\Delta\alpha$ (deg)	-14.7	0	14.2	28.5
ΔV_M (m/sec)	220.7	215.1	215.0	219.2
ΔV_T (m/sec)	370.5	264.9	364.8	369.0
Δt_1 (days)	3.21	3.21	3.22	3.22
Δt_2 (days)	5.26	5.47	5.68	5.87
Δt_T (days)	8.47	8.68	8.90	9.09
i_{TR} (deg)	54.31	48.25	43.69	40.11

where:

$\Delta\alpha$: central angle from perilune point

ΔV_M : velocity impulse at Moon

ΔV_T : total velocity impulse

Δt_1 : L_2-Moon trip time

Δt_2 : Moon-Earth trip time

Δt_T : total trip time

i_{TR} : inclination of transfer trajectory near Earth where

$$\Omega = (1 - \mu)\left(\frac{1}{r_1} + \frac{r_1^{\,2}}{2}\right) + \mu\left(\frac{1}{r_2} + \frac{r_2^{\,2}}{2}\right) - \frac{1}{2} \quad \mu\,(1 - \mu)$$

r_1 = distance from first body (mass $1 - \mu$)

r_2 = distance from second body (mass μ)

The "libration points" are those equilibrium points where

$$\frac{\partial\Omega}{\partial X} = \frac{\partial\Omega}{\partial Y} = \frac{\partial\Omega}{\partial Z} = 0$$

For the collinear points, $Y = Z = 0$. Therefore

$$\frac{\partial r_1}{\partial Y} = \frac{\partial r_2}{\partial Y} = 0 \text{ and } \frac{\partial r_1}{\partial X} = \frac{\partial r_2}{\partial X} = 1$$

The L_2 point is located along the X-axis at a distance $r_2 = \rho$ beyond the smaller body such that

$$\frac{\partial\Omega}{\partial r_1} + \frac{\partial\Omega}{\partial r_2} = 0$$

or

$$(1 - \mu)\left(\frac{1}{r_1^{\,2}} - r_1\right) + \mu\left(\frac{1}{r_2^{\,2}} - r_2\right) = 0$$

Since $r_1 = 1 + r_2 = 1 + \rho$, then

$$(1 - \mu)\left[\frac{1}{(1 + \rho)^2} - 1 - \rho\right] + \mu\left(\frac{1}{\rho^2} - \rho\right) = 0$$

or

$$1 - \mu + \rho - \frac{1 - \mu}{(1 + \rho)^2} - \frac{\mu}{\rho^2} = 0$$

For the Earth-Moon system, μ = .012150. With this value, the above equation has a positive real root at ρ = 0.16782991.

With a value of 384,400 km for the Earth-Moon distance, the distance at L_2 from the Moon is 64,513.8 km.

If the equations of motion are linearized around the L_2 point, then in a rotating coordinate system centered at L_2, we have

$$\ddot{X} - 2\dot{Y} - (1 + 2A)\, X = 0$$

$$\ddot{Y} + 2\dot{X} - (1 - A)\; Y = 0$$

$$\ddot{Z} + AZ = 0$$

where

$$A = \frac{1 - \mu}{r_1^{\,3}} + \frac{\mu}{r_2^{\,3}}$$

The solutions of these equations are:

$$X = \frac{1}{\alpha_1^{\,2} + \alpha_2^{\,2}} \left\{ \left[\alpha_1^{\,2} + A + 3) \cosh\alpha_1 t - (\alpha_2^{\,2} - A - 3)\; (\cos\alpha_2 t) \right] X_o \right.$$

$$+ 2(1 - A)\; \left(\frac{1}{\alpha_1} \sinh\alpha_1 t - \frac{1}{\alpha_2} \sin\alpha_2 t\right)\; Y_o$$

$$+ \left(\frac{\alpha_1^{\,2} + A - 1}{\alpha_1} \sinh\alpha_1 t + \frac{\alpha_2^{\,2} - A + 1}{\alpha_2} \sin\alpha_2 t \right) \dot{X}_o$$

$$\left. + 2\; (\cosh\alpha_1 t - \cos\alpha_2 t)\; \dot{Y}_o \right\}$$

$$Y = \frac{1}{\alpha_1^2 + \alpha_2^2} \left\{ -2(1+2A)\left(\frac{1}{\alpha_1} \sinh \alpha_1 t - \frac{1}{\alpha_2} \sin \alpha_2 t\right) X_o \right.$$

$$+ \left[(\alpha_1^2 - 2A + 3) \cosh \alpha_1 t + (\alpha_2^2 + 2A - 3) \cos \alpha_2 t \right] Y_o$$

$$- 2 (\cosh \alpha_1 t - \cos \alpha_2 t) \dot{X}_o$$

$$\left. + \left(\frac{\alpha_1 - 2A - 1}{\alpha_1} \sinh \alpha_1 t + \frac{\alpha_2^2 + 2A + 1}{\alpha_2} \sin \alpha_2 t \right) \dot{Y}_o \right\}$$

$$Z = (\cos \alpha_3 t)\, Z_o + \frac{1}{\alpha_3} (\sin \alpha_3 t)\, \dot{Z}_o$$

where α_1 and α_2 are the magnitudes of the real and imaginary roots, respectively, of

$$S^4 - (A - 2) S^2 + (1 + 2A)(1 - A) = 0$$

and

$$\alpha_3 = \sqrt{A}$$

For the L_2 point with $\mu = .012150$:

$A = 3.1904366$ (The unit of time is $\frac{1 \text{ sidereal month}}{2\pi}$)

$\alpha_1 = 2.15868$

$\alpha_2 = 1.86265$

$\alpha_3 = 1.78618$

With these values, the solution in the X-Y plane is:

$$X = (1.3347X_o + .2460\dot{Y}_o) \cosh \alpha_1 t - (.2496Y_o - .3904\dot{X}_o) \sinh \alpha_1 t$$

$$- (.3347X_o + .2460\dot{Y}_o) \cos \alpha_2 t + (.2893Y_o + .0845\dot{X}_o) \sin \alpha_2 t$$

$$Y = (.1573Y_o - .2460\dot{X}_o) \cosh \alpha_1 t - (.8412X_o + .1551\dot{Y}_o) \sinh \alpha_1 t$$

$$+ (.8427Y_o + .2460\dot{X}_o) \cos \alpha_2 t + (.9749X_o + .7166\dot{Y}_o) \sin \alpha_2 t$$

For small values of t, the solution can be written (to terms in t^4):

$$X = \left[1 + (1 + 2A)\, t^2/2 - (1 + 2A)\, t^4/24\right] X_o + \left[(1 - A)\, t^3/3\right] Y_o$$

$$+ \left[t - (3 - 2A)\, t^3/6\right] \dot{X}_o + \left[t^2 - (2 - A)\, t^4/12\right] \dot{Y}_o$$

$$Y = -\left[(1 + 2A)\, t^3/3\right] X_o + \left[1 + (1 - A)\, t^2/2 - (1 - A)(3 + A)\, t^4/24\right] Y_o$$

$$- \left[t^2 - (2 - A)\, t^4/12\right] \dot{X}_o + \left[t - (3 + A)\, t^3/6\right] \dot{Y}_o$$

If the third body is subject to constant external accelerations, the equations of motion are:

$$\ddot{X} - 2\dot{Y} - (1 + 2A) X = a_x$$

$$\ddot{Y} + 2\dot{X} - (1 - A) Y = a_y$$

$$\ddot{Z} + AZ = a_z$$

For zero initial conditions, the solution of these equations is:

$$X = \frac{1}{\alpha_1^2 + \alpha_2^2} \left\{ \left[\frac{\alpha_1^2 + A - 1}{\alpha_1^2} (\cosh \alpha_1 t - 1) - \frac{\alpha_2^2 - A + 1}{\alpha_2^2} (\cos \alpha_2 t - 1) \right] a_x \right.$$

$$\left. + \left[\frac{2}{\alpha_1} \sinh \alpha_1 t - \frac{2}{\alpha_2} \sin \alpha_2 t \right] a_y \right\}$$

$$Y = \frac{1}{\alpha_1^2 + \alpha_2^2} \left\{ \left[-\frac{2}{\alpha_1} \sinh \alpha_1 t + \frac{2}{\alpha_2} \sin \alpha_2 t \right] a_x \right.$$

$$\left. + \left[\frac{\alpha_1^2 - 2A - 1}{\alpha_1^2} (\cosh \alpha_1 t - 1) - \frac{\alpha_2^2 + 2A + 1}{\alpha_2^2} (\cos \alpha_2 t - 1) \right] a_y \right\}$$

$$Z = \frac{1}{\alpha_3^2} (1 - \cos \alpha_3 t)\, a_z$$

The Z equation is uncoupled from X and Y and can be treated separately. In order for the vehicle to be in view of the Earth, it must be displaced at least 3100 km from the L_2 point as shown in Figure 2-10. This can be accomplished within the scope of the mission by maintaining the vehicle in an offset position (Hummingbird, References 1 and 3) or allowing the vehicle to "orbit" the L_2 point (Halo orbit) but not coming closer to it than 3100 km. In order to determine the nature of these orbits, and to establish the ΔV requirements, one must consider the free motion of a vehicle in the region of L_2.

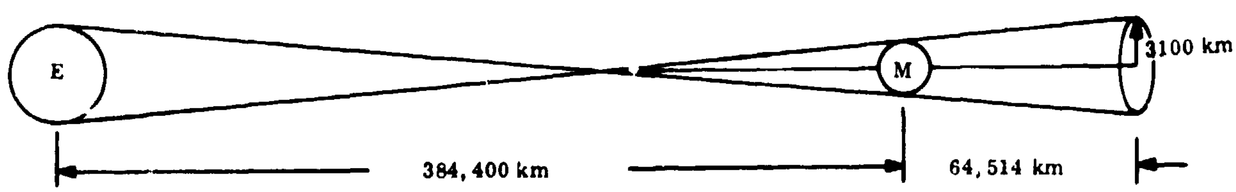


Figure 2-10. Earth-Moon-L_2 Geometry

2.2.1 HUMMINGBIRD

In the Hummingbird concept, the vehicle is maintained at a fixed location, displaced a minimum of 3100 km from the L_2 point in a direction perpendicular to the Earth-Moon line so as to insure line-of-sight at all times from the Earth. For this configuration to be maintained, continuous thrusting must be supplied. The thrust requirement may be obtained from the equations of motion of the vehicle subject to constant external accelerations.

In lunar units, an offset of 3100 km is

$$\frac{3100}{384,400} = 0.00806$$

and $A = 3.19$ (for L_2)

For the vehicle to be stationary in the rotating coordinate system

$$\ddot{X} = \ddot{Y} = \ddot{Z} = \dot{X} = \dot{Y} = \dot{Z} = 0$$

Therefore, letting $X = 0$, the required thrust for an offset in the Y direction is

$$a_Y = (A - 1)Y_{offset} = 2.19 \times 0.00806 = 0.0177 \text{ lunar acceleration units}$$

$$= 4.8 \times 10^{-5} \text{ m/sec}^2$$

For an offset in the Z direction, the required thrust is

$$a_z = A\ Z_{offset} = 3.19 \times 0.00806 = 0.0257 \text{ lunar acceleration units}$$

$$= 7.0 \times 10^{-5} \text{ m/sec}^2$$

An offset in the Y direction (in the Earth-Moon plane) requires the least thrust and is therefore preferred. Since the offset from the L_2 point is small compared with the distance of L_2 from the Moon (64,500 km), the distance of the vehicle from the Moon (and from the Earth) is practically unchanged, and the use of the linearized equations is justified.

2.2.2 HALO

If the values of X, Y, $\dot{X}$ and $\dot{Y}$ at some time are such that coefficients of the hyperbolic terms vanish, the motion of the vehicle in the X-Y plane will be along an ellipse centered at L_2 with its major axis in the Y direction. The ratio of the maximum excursion in the Y direction to the maximum excursion in the X direction is 2.9126 and the period is 14.67 days. The motion in the Z direction is simple harmonic with a period of 15.30 days. If the initial conditions are chosen so that only the periodic modes are excited and the maximum Y and Z excursions are each greater than 3100 km and approximately 90^o out of phase, the vehicle will orbit around the L_2 point and will be continuously visible from the Earth. Since the period of the out-of-plane motion is slightly different from the period on the in-plane motion, the projection of the path on a plane perpendicular to the Earth-Moon line will be a Lissajous figure; that is, the initially circular trace will become elliptical and the line-of-sight to the Earth will be interrupted by the Moon for some portion of each orbit. This can be prevented by thrusting at periodic intervals so as to force the two periods to be equal and to maintain the 90-degree phase relationship.

The phase difference between the two motions accumulating during one-half an orbit is:

$$\Delta\phi = \frac{\alpha_2 - \alpha_3}{\alpha_2} \pi = 0.129 \text{ radians}$$

If the phase control is accomplished by adjusting the out-of-plane motion, the impulse required to correct this phase difference is

$$\Delta V = \Delta\phi \cdot \alpha_3 \cdot Z_{max}$$

The impulse is applied in the Z direction at the point at maximum Z amplitude. For a Z_{max} of 3100 km, the required impulse is 1.90 m/sec each one-half period or 94 m/sec/yr.

Period control of the X-Y motion is more complex due to the presence of the divergent mode. Let it be assumed that corrections are made every one-half orbit, and the resulting change in the half-period is ΔT. Also, let $X(0)$, $Y(0)$, $\dot{X}(0)$, $\dot{Y}(0)$ be the X and Y positions and velocities just after a correction, and $X(\frac{T}{2} + \Delta T)$, $Y(\frac{T}{2} + \Delta T)$, $\dot{X}(\frac{T}{2} + \Delta T)$, $\dot{Y}(\frac{T}{2} + \Delta T)$ be the positions and velocities just before the next correction. (T is the natural X-Y period.) The conditions for periodicity are then:

$$X(\frac{T}{2} + \Delta T) = -X(0), \quad Y(\frac{T}{2} + \Delta T) = -Y(0)$$

$$\dot{X}(\frac{T}{2} + \Delta T) + \Delta\dot{X} = -\dot{X}(0), \quad \dot{Y}(\frac{T}{2} + \Delta T) + \Delta\dot{Y} = -\dot{Y}(0)$$

If the solutions to the linearized equations of motion are substituted into the first set of the above equations, the resulting equations can be solved for $\dot{X}(0)$, $\dot{Y}(0)$, $\dot{X}(\frac{T}{2} + \Delta T)$ and $\dot{Y}(\frac{T}{2} + \Delta T)$ in terms of $X(0)$ and $Y(0)$. If these are substituted into the second set, the required velocity changes can be solved. The resulting expressions are:

$$\Delta\dot{X} = -22.06 \ \Delta T \cdot X(0)$$

$$\Delta\dot{Y} = -2.60 \ \Delta T \cdot Y(0)$$

The position in the orbit can be expressed as follows:

$$X = X_{max} \cos\theta = 0.343 \ Y_{max} \cos\theta$$

$$Y = -Y_{max} \sin\theta$$

where $\theta = \alpha_2(t - t_o)$ and t_o is the time at which the vehicle crosses the X axis. The velocity requirement is then

$$\Delta\dot{X} = \left[-7.57 \ \Delta T \cos\theta\right] Y_{max}$$

$$\Delta\dot{Y} = \left[2.60 \ \Delta T \sin\theta\right] \ Y_{max}$$

or

$$\Delta V = \sqrt{\Delta \dot{X}^2 + \Delta \dot{Y}^2} = \sqrt{57.36 \cos^2 \theta + 6.76 \sin^2 \theta}\ \Delta T \cdot Y_{max}$$

It can be seen that ΔV is a minimum when $\theta = 90^{\circ}$, that is, when the vehicle crosses the Y axis. Therefore the optimum point to adjust phase is when Y is maximum and X is zero. ΔT for one-half period is about 7.5 hours.

The required impulse is in the Y direction and for a Y amplitude of 3100 km is 1.52 m/sec every one-half orbit or 73 m/sec/yr. If it is considered undesirable for the vehicle to be in a divergent condition (due to the possibility of failing to perform a correction) a method of phase control can be used in which the direction of the impulse is always such that $\Delta \dot{X} = -0.63024\ \Delta \dot{Y}$, or at an angle of 122.22° or -57.78° from the X axis. In this case, the divergent mode is not excited and the only permanent effect of the impulse will be on the phase and amplitude of the oscillatory motion. If the impulse is applied about 12 hours <u>before</u> the time of maximum Y excursion, only the phase will be affected, the amplitude remaining unchanged. The impulse necessary to make the X-Y period equal to the Z period is about 1.8 m/sec every one-half orbit or about 85/m/sec/yr for a maximum Y excursion of 3100 km. If the impulse is made in the same direction but 12 hours before the time of maximum X excursion, the amplitude only will be changed. For a 1 m/sec impulse, the change in the maximum Y excursion will be 233 km.

The corrections for maintaining the X-Y and Z periods equal require a significant amount of fuel over the expected lifetime of the vehicle. The corrections make up the major portion of the total orbital maintenance requirement, since it appears that the ΔV required for orbit stability is about an order of magnitude less. It is of interest, therefore, to investigate the possibility of reducing the requirement for period control. It has been found that if the amplitude of the halo orbit in the X-Y plane is increased, the X-Y and Z periods become more equal, due to non-linear effects.

Using a computer program which numerically integrates the exact, restricted three-body equations, a family of orbits around L_2 have been found for which the X-Y and Z periods are exactly equal, that is, the orbits are truly periodic in three dimensions, and therefore would require no period control. (The orbits being unstable, however, active control would still be necessary for stability.) The maximum Y displacement of these orbits is from 33,000 km to 45,000 km depending on the maximum Z displacement; 100% Earth visibility is achieved at all times. The major disadvantage is that the maximum range from the Moon is about 80,000 km or 15,000 km larger than the maximum range with the smaller orbits. Transmission losses would, therefore, be greater. In addition, in order to maintain a more or less constant angular separation between the Earth and the Moon as viewed from the orbit, the maximum Z excursion would also have to be on the order of 35,000 km to 40,000 km. From a preliminary analysis, it appears that the insertion ΔV requirements for such an orbit would be substantially larger than for a small orbit, since indirect transfers (transfers using a close lunar fly-by) could probably not be used to good advantage.

The solution of the linearized equations of motion in two dimensions has the form:

$$X = A_1 e^{\alpha_1 t} + A_2 e^{-\alpha_1 t} + A_3 \cos \alpha_2 t + A_4 \sin \alpha_2 t$$

$$Y = B_1 e^{\alpha_1 t} + B_2 e^{-\alpha_1 t} + B_3 \cos \alpha_2 t + B_4 \sin \alpha_2 t$$

where

$$A_1 = 1.3374 X_o - 0.2486 Y_o + 0.3904 \dot{X}_o + 0.2460 \dot{Y}_o$$

and

$$B_1 = -0.6302 A_1$$

It can be seen that A_1, since it is the coefficient of an increasing exponential term, determines the rate of divergence, while the other terms are all bounded. This suggests the possible control strategy of changing $\dot{X}$ and $\dot{Y}$ to make $A_1 = 0$. For a given ΔV magnitude, the maximum change in A_1 is obtained when $\Delta\dot{Y} = 0.6302\,\Delta\dot{X}$.

The control strategy is, therefore,

$$\Delta\dot{X} = \frac{1}{0.6302}\,\Delta\dot{Y} = -K_1 A_1 \quad .$$

To insure stability, $\dfrac{1 - e^{-\alpha_1 T}}{0.3904} < K_1 < \dfrac{1 + e^{-\alpha_1 T}}{0.3904}$, where T is the interval between corrections.

In order to implement this method of control, it is necessary to be able to estimate the four state variables: X, Y, $\dot{X}$ and $\dot{Y}$, from Earth-based tracking data. Since these estimates will, in general, be subject to some errors, some residual divergence will remain after each correction. A knowledge of the capability of the tracking and data processing system is therefore necessary to predict the stationkeeping fuel required to insure stability. This problem is discussed in Section 2.3.

2.2.3 NONLINEAR EFFECTS

The linearized analysis presented so far is only an approximation to the actual motion around the libration point. A more accurate treatment of the problem should consider such effects as eccentricity of the lunar orbit, solar gravitational perturbations, and non-linear terms in the Earth-Moon gravity field.

2.2.3.1 Eccentricity Effects

The libration points represent equilibrium solutions of the equations of motion for cases where the primary bodies move in elliptical orbits as well as circular orbits. Consequently, the effect of eccentricity of the lunar orbit on motion near a libration point is relatively small and does not affect the general nature of the solution.

Using a first-order analysis, Farquhar (Ref 12) obtains additional periodic terms representing the effect of eccentricity. The largest of these terms is about 10% of the amplitude of the halo orbit, or a little more than 300 km for a Y amplitude of 3100 km. This is the largest perturbation for an orbit of this size, since it is an order of magnitude larger than the effects of solar gravitational perturbation and nonlinear terms in the Earth-Moon gravity field. Periodic fluctuations in position of several hundred kilometers can be expected also for the Hummingbird concept, due to lunar orbit eccentricity.

2.2.3.2 Solar Perturbation Effects

The effect of solar gravitation on the motion of a vehicle near a libration point has been studied by a number of investigators. A popular device for performing this analysis has been to assume that the Earth and Moon move in circular orbits around their barycenter, which in turn moves in a circular orbit around the sun (The so-called Very Restricted Problem of Four Bodies, References 1, 4, and 5). Although this model is attractive because of its simplicity, its value is dubious because it completely neglects the indirect effect of the Sun on the Moon. It has been shown (References 6 and 7) that in the linear approximation, the direct effect of the sun on a vehicle at a collinear libration point (L_1, L_2 and L_3) is completely cancelled by the indirect effect of the solar perturbation of the Sun on the Moon. Consequently, an analysis based on the use of a circular orbit for the Earth and Moon will usually result in erroneous conclusions concerning the nature and magnitude of the solar effect. In particular, such an analysis predicts a large periodic solar perturbation (about 10^{-5}g) with a period of one-half a synodic month (14.765 days), whereas an analysis which includes the indirect effect shows that no solar perturbation at all exists with this period for a vehicle either at a libration point or moving freely around it. The net acceleration of a body at a libration point due to the presence of the sun consists only of periodic terms which are of second order or higher in the ratio of the Moon's distance to the Sun's distance (as described by Nicholson, Reference 7). For the L_2 point, the maximum magnitude of the principal portion of this acceleration is about 2.3×10^{-8} m/sec^2 (about 2×10^{-9}g). Two periods are present, one equal to a synodic month and the other equal to one third of a synodic month. If the initial conditions are chosen so that the natural modes are not excited, the forced motion resulting from this acceleration has a maximum amplitude of about 1 km. If the vehicle is displaced from the L_2 point, as is the case for a far-side communications relay, the first-order solar perturbation terms will no longer completely cancel, and there will be a residual acceleration due to the Sun whose magnitude will be proportional to the distance of the vehicle from the L_2 point. The magnitude will always be less than 1/200 of the acceleration due to the Earth-Moon gravity field. Farquhar (Reference 12) obtains an effect of about 1.2% of the halo orbit amplitude due to solar perturbation.

For a fixed displacement (the Hummingbird concept), the period of the first-order solar perturbation will be one-half a synodic month or 14.765 days. Since this is very close to the period of the in-plane free motion (14.669 days), a near-resonant condition exists which, if uncontrolled, would result in fluctuations of several hundred kilometers around the desired location, even though the disturbing acceleration is small.

In order to maintain the vehicle on station, the guidance system must control: (1) the tendency toward divergent motion due to the basic instability of the L_2 point and (2) the buildup of oscillations due to the solar perturbation.

2.2.3.3 Nonlinear Gravitational Effects

In the region of linearity, the periods of both in-plane and the out-of-plane motion are independent of the size of the orbit. If, however, the size of the orbit becomes large enough so that the departure from linearity becomes significant, the periods will depend to some extent on the amplitude of the motion. For displacements as large as 3100 km, the nonlinear acceleration terms become on the order of a few percent of the linear terms.

The principal effect of the second-order terms in a power series expansion of the gravity field is to shift the center of the halo orbit towards the Moon by an amount proportional to the square of the halo orbit amplitude. For maximum Y and Z excursions of 3100 km, the shift in the orbit is about 70 km. A third-order solution shows that the X-Y period of the orbit is increased and the Z period is decreased, again by an amount proportional to the square of the orbit amplitude. For a 3100 km orbit, the X-Y period increase is about two minutes, and the Z period decrease is about nine minutes. (Nominal halo orbit period is about 15 days.)

It can be concluded that, although some of these effects are appreciable, they do not cause any significant increase in stationkeeping propulsion requirements above that predicted by the linearized analysis. For an actual mission, of course, these higher order terms would have to be taken into account in determining the vehicle state and computing the required correction.

2.3 TRACKING ACCURACY STUDY

2.3.1 TECHNICAL APPROACH

A prediction accuracy program has been developed by utilizing existing error analysis subroutines and adding functions peculiar to the lunar libration point study. This program was then used to perform a tracking accuracy study in order to determine the accuracy with which the state of a vehicle near the L_2 libration point can be estimated from Earth-based tracking data. The results of this study were used to determine the effect of tracking uncertainties on stationkeeping scheduling and fuel requirements.

Inputs to the program are:

a. Initial state error covariance matrix

b. Tracking stations location

c. Earth rotation rate and Moon's mean motion

d. Frequency of observations

e. Measurement error variance

f. Frequency of print-out

Output of the program is:

a. Time

b. Tracking data (range, range rate, azimuth, elevation)

c. State error covariance matrix at specified time

d. Standard deviations in state errors

e. Correlation matrix

Tracking data was processed by means of a Kalman-Schmidt linear filter which handles each data point sequentially.

The standard equations used were:

$$P\,(t_n + 1) = \Phi\,(t_{n+1}, t_n)\, P'\,(t_n)\, \Phi^T\,(t_{n+1}, t_n)$$

$$K = P\,(t_{n+1})\, G^T \left[G\, P\,(t_{n+1})\, G^T + \sigma^2 \right]^{-1}$$

$$P'\,(t_{n+1}) = \left[I - KG \right] P\,(t_{n+1})$$

where

$P\,(t_{n+1})$	=	covariance matrix at t_{n+1} (before observation)
$P'\,(t_{n+1})$	=	covariance matrix at t_{n+1} (after observation)
G	=	vector of partials of observables with respect to state variables
I	=	identity matrix
K	=	linear filter matrix
Φ	=	state transition matrix between times of observation
σ^2	=	variance of random error in observable

The mathematics was simplified by the use of a three-dimensional restricted three-body model with the primary coordinate system being a rotating one centered at the Moon. Since the tracking stations are on a rotating Earth, the appropriate rotation transformations were developed to calculate nominal tracking data from the motion of a vehicle at or near L_2. Linearized equations of motion for the vehicle were used and thus the vehicle state was given in closed form as a function of the initial state and elapsed time. The state transition matrix was formed from this linearized solution and was a function of elapsed time only. Since all calculations were in closed form, and no numerical integrating was required, computer processing time was quite small.

The state covariance matrix calculated was the uncertainty in the vehicle's state vector (position and velocity) as estimated from tracking data obtained from Earth ground stations. For the case of the Hummingbird, imposed accelerations were included in the vehicle's state vector and the effect of their uncertainty on position and velocity was determined. An accurate estimation is possible due to the variable geometry caused by relative vehicle and tracking station motion and to the unstable vehicle dynamics.

It was assumed in the data processing that the only source of error was unbiased, uncorrelated, random noise on the observations. Sources of error which were not factored in were: observation bias errors, station location errors, and model errors.

2.3.2 RESULTS OF TRACKING ACCURACY PROGRAM

An orbit determination error analysis was performed utilizing the tracking accuracy program in order to determine the accuracy with which the vehicle state can be estimated from Earth-based tracking data. The following assumptions were made:

a. Range rate was observed from a tracking station at a latitude of 35^{o}N.

b. Observations were made at intervals of 1.04 hours until 10 observations had been processed. The resulting covariance matrix was then propagated ahead 14.6 hours with no observations being made. This sequence was repeated until four tracking passes were accumulated.

c. The standard deviation of the noise on a single observation was 0.01 meter/second.

d. The vehicle was assumed to be moving freely near L_2 under the influence of gravitational forces only (Halo orbit).

2.3.2.1 HALO

Two cases were considered. In Case 1, the initial covariance matrix was chosen to be very large. This was done to approximate a situation where the orbit estimation was made from the tracking data alone with essentially no a priori information concerning the vehicle state. The actual variances used corresponded to standard deviations of 2720 km in all three position coordinates and 7.24 meters/second in all three velocity components. All correlations were assumed zero; that is, the initial covariance matrix was diagonal. For Case 2, the initial covariance matrix was the covariance matrix from Case 1 after four tracking passes with variances of 0.01 m^2/sec^2 (corresponding to velocity errors of 0.1 m/sec) added to the three velocity diagonal elements. The resulting covariance matrix then represents the knowledge of the trajectory state after a correction has been made, i.e., no position information is lost, but knowledge of velocity is degraded due to execution errors in performing the correction.

The results of the analysis are shown in Tables 2-2 and 2-3. The first six columns give the standard deviations of the errors in the three position and three velocity coordinates just before and just after each tracking pass. The seventh column is the standard deviation of the minimum velocity required to correct the divergence and is, therefore, a measure of the uncertainty in the estimate of the divergent mode. It can be seen that only two tracking passes are required to recover the information lost during a correction. After four passes, the uncertainty in the estimate of the velocity required to correct the

Table 2-2. Errors in Estimates of State Variables, Case 1

Time (hrs)	No. of Observations	Standard Deviations						
		σ_X	σ_Y	σ_Z	$\sigma_{\dot{X}}$	$\sigma_{\dot{Y}}$	$\sigma_{\dot{Z}}$	σ_{V_c}
		(Kilometers)			(Meters/Second)			
0	0	2.20	2720	2720	7.24	7.24	7.24	22.4
10.4	10	400	1270	2300	0.114	4.13	6.30	1.70
25.0	10	395	1190	2210	0.166	4.46	6.93	2.31
35.5	20	185	54	72	0.044	1.89	3.19	0.45
50.0	20	183	148	228	0.058	1.80	3.02	0.62
60.5	30	20.6	20.9	26.6	0.012	0.21	0.34	0.068
75.0	30	20.1	30.4	40.5	0.014	0.19	0.31	0.093
85.5	40	9.1	15.7	17.3	0.009	0.088	0.144	0.040
100.0	40	8.8	19.5	22.4	0.008	0.074	0.127	0.054

Table 2-3. Errors in Estimates of State Variables, Case 2

Time (hrs)	No. of Observations	Standard Deviations						
		σ_X	σ_Y	σ_Z	$\sigma_{\dot{X}}$	$\sigma_{\dot{Y}}$	$\sigma_{\dot{Z}}$	σ_{V_c}
		(Kilometers)			(Meters/Second)			
	0	9.1	15.7	17.3	0.103	0.135	0.177	0.110
10.4	10	7.8	16.0	20.9	0.011	0.102	0.166	0.048
25.0	10	7.5	20.2	27.0	0.025	0.085	0.145	0.064
35.5	20	5.3	15.8	24.4	0.008	0.045	0.113	0.038
50.0	20	5.2	17.5	28.3	0.010	0.033	0.091	0.051
60.5	30	2.7	10.6	17.0	0.006	0.022	0.069	0.025
75.0	30	2.7	11.0	18.4	0.007	0.019	0.061	0.033
85.5	40	1.8	6.8	12.3	0.005	0.018	0.055	0.016
100.0	40	1.8	6.8	12.9	0.006	0.018	0.051	0.022

divergence is 0.016 m/sec, that is, if a correction is made at that time based on the current estimate of the vehicle state, an error of 0.016 m/sec will remain due to orbit estimation errors. If it is assumed that errors of 0.04 m/sec, randomly oriented, are made in executing phase corrections, the expected component of error in the direction giving the maximum divergence will be 0.02 m/sec. This error combined with the orbit estimation error is $\sqrt{0.02^2 + 0.016^2} = 0.026$ m/sec. Since the divergence grows exponentially with a time constant of very nearly two days, the error will grow, in a four day period, to $0.026\ e^2$ or about 0.2 m/sec with these assumptions. The ΔV rate required for stationkeeping is then 0.05 m/sec/day, which is only 20% of the requirement for period control.

2.3.2.2 Hummingbird

A similar investigation was made for the Hummingbird concept. In this case, since there is some uncertainty introduced due to error in thrust magnitude and direction, the effect of estimating acceleration, as well as position and velocity, was included. It was found that under these conditions range rate tracking alone was insufficient to determine the vehicle state adequately. It is concluded that both range and range rate tracking are required for the Hummingbird case in order to provide orbit estimation of sufficient accuracy to insure orbit stability. With range rate measurements accurate to 0.01 m/sec, and range measurements accurate to 1 km, the ΔV rate required for stationkeeping is estimated to be 0.1 to 0.2 m/sec/day.

2.4 HALO ORBIT SIMULATION

In order to confirm the conclusions concerning the magnitude of the velocity corrections required for orbit maintenance, a simulation of the orbit maintenance procedure was performed. A typical Halo orbit trajectory orbiting the L_2 point was generated using an "N-body" trajectory program which integrates the exact equations of motion in an Earth-centered equatorial system. An integration interval of two hours was used. Positions of the Sun and Moon were obtained from an ephemeris tape based on the JPL ephemeris tape system. Use of the N-body program provided "real world" conditions in that the effects of solar perturbation, non-linearity and lunar eccentricity were included. Velocity corrections were made at intervals of four to six days to control the amplitude and phase of the in-plane and out-of-plane motions, and to suppress the divergence resulting from the unstable nature of the equilibrium at the L_2 point. The corrections were calculated using the solutions to the linearized equations of motion. In order to simulate the effect of orbit determination uncertainties and maneuver errors, a velocity error of about 0.05 meters/second was added to each correction. The direction of the velocity error was chosen to approximate a "worst case" condition, that is, the direction resulting in the most rapid divergence.

The starting point of the trajectory (in the rotating coordinate system) was

$$\bar{r}_{initial} = \begin{pmatrix} -2000 \text{ km} \\ 0 \\ 2000 \text{ km} \end{pmatrix}$$

That is, 2000 km above the Moon's orbital plane and 2000 km from L_2 toward the Moon. This was assumed to be the point at which the final retro maneuver was performed to inject the vehicle into the halo orbit from the transfer trajectory. Since this point does not lie on the desired halo orbit, it simulates the effect of guidance errors during the Earth-L_2 transfer.

NOTE:

It can be shown that aim point errors of several thousand kilometers due to midcourse guidance errors can be accommodated by adjusting the retro maneuver so as to place the vehicle on a path which approaches the desired halo orbit asymptotically. (A second maneuver is usually required to establish the proper phase relationship between the in-plane and out-of-plane motions.) Of course, the vehicle's position and velocity at the time of the retro maneuver must be accurately known; this will always be the case since there will be adequate time for tracking and orbit determination between the last midcourse correction and orbit injection.

The initial velocity (relative to L_2) was chosen so as to yield an orbit with maximum excursions of about 3500 km in both the Y and Z directions. Since the magnitude of the retro maneuver (for indirect transfers) is about 150 meters/second, and attitude errors during injection may be as large as 1/2 degree, a velocity error of 1.5 meters/second was added to the initial velocity to simulate the retro maneuver execution error. After four days, a correction was made to establish the proper phase relationship between the in-plane and out-of-plane motions, and to nullify the divergence resulting from the initial velocity error. This process was continued with corrections being made after 4, 8, 14, 18, 22, 27, 31 and 35 days. The resulting orbit is shown in Figures 2-11 through 2-14. Figures 2-11 and 2-12 show the projections of the orbit on the X-Y plane and the Y-Z plane respectively for the first 20 days. Figures 2-13 and 2-14 show the projections from 20 days after injection to 37 days. The dotted portions of the trace shows the path the vehicle would have taken if the corrections had not been made. The arrows show the direction in which the correction was made. The magnitude of each correction and the purpose of the correction are listed in Table 2-4. The table also shows the amount of divergence remaining after each correction, expressed in terms of the minimum velocity which would be required to remove the divergence. This error is a combination of the "noise" added to simulate orbit determination errors, and the errors resulting from the use of linear theory in computing the corrections, thereby neglecting non-linear effects, lunar eccentricity and solar perturbation. The magnitude of the errors resulting from the use of a linear model was estimated to be about 0.05 to 0.1 meter/second. In actual practice, these errors could be eliminated by the use of a more exact model in computing the corrections. Even with these errors, the sum of the velocity corrections is 9.3 meters/second for 31 days (the first correction, four days after injection, is not included since it is atypical), or about 0.30 meter/second/day. Allowing for the fact that no special attempt was made to optimize the times at which the corrections were made, this figure is in good agreement with the requirement of 0.25 meters/second/day for period control, obtained from linear theory.

The generation of this trajectory demonstrated the feasibility of stabilizing and maintaining a suitable "halo" orbit under "real world" conditions and allowing for orbit determination errors.

Table 2-4. Typical Correction Requirements

Time of Correction (days after injection)	Magnitude of Correction (Meters/second)	Purpose of Correction	Divergence Remaining after Correction (Meters/second)
4	9.10	divergence, Z phase, Z amplitude	0.19
8	1.38	divergence	0.04
14	.87	divergence	0.17
18	1.33	divergence	0.02
22	1.97	Y amplitude, Z phase	0.03
27	0.99	divergence, Y phase Z amplitude	0.15
31	1.53	divergence, Z phase	0.01
35	1.21	Y phase	

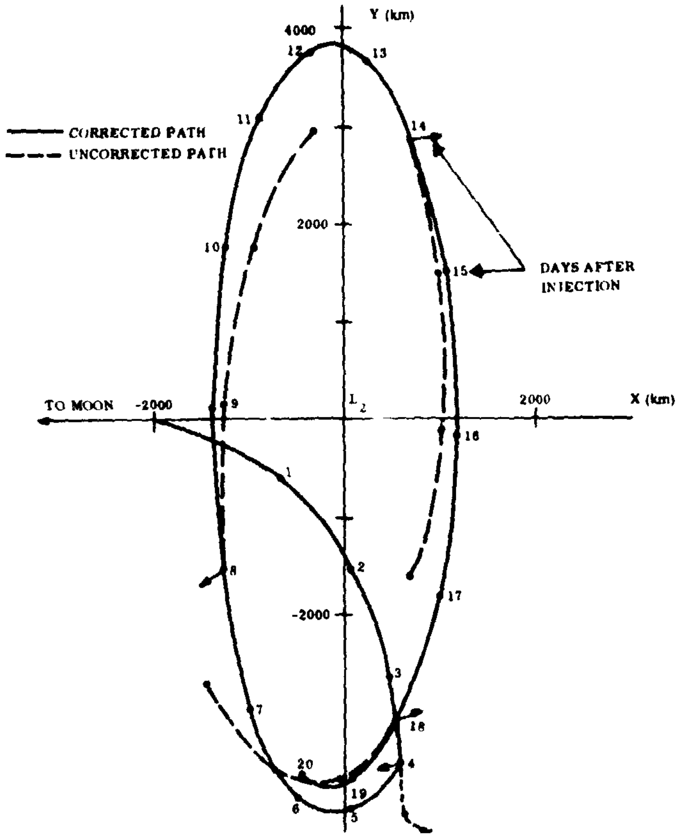


Figure 2-11. Ha.. Orbit, X-Y Projection, 0 to 20 Days

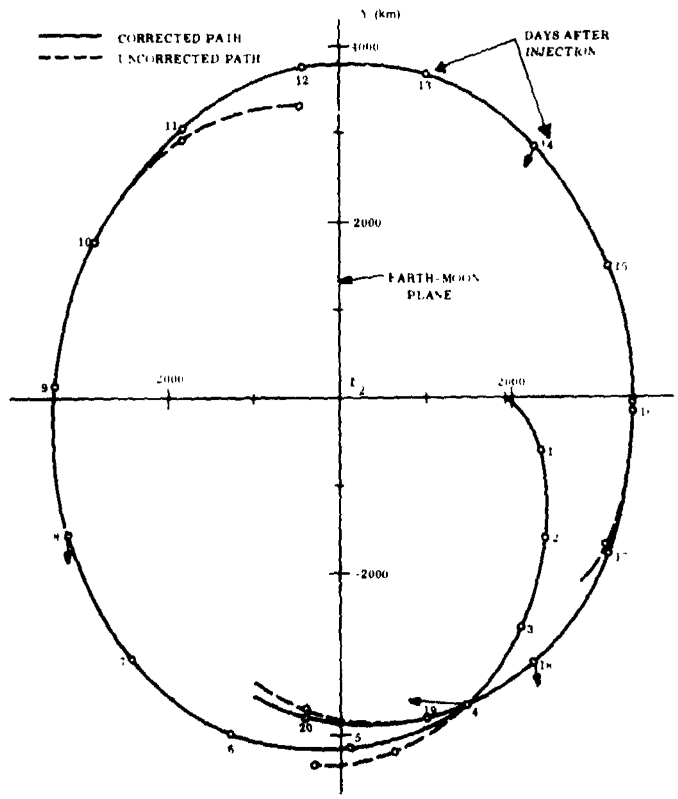


Figure 2-12. Halo Orbit, Y-Z Projection, 0 to 20 Days

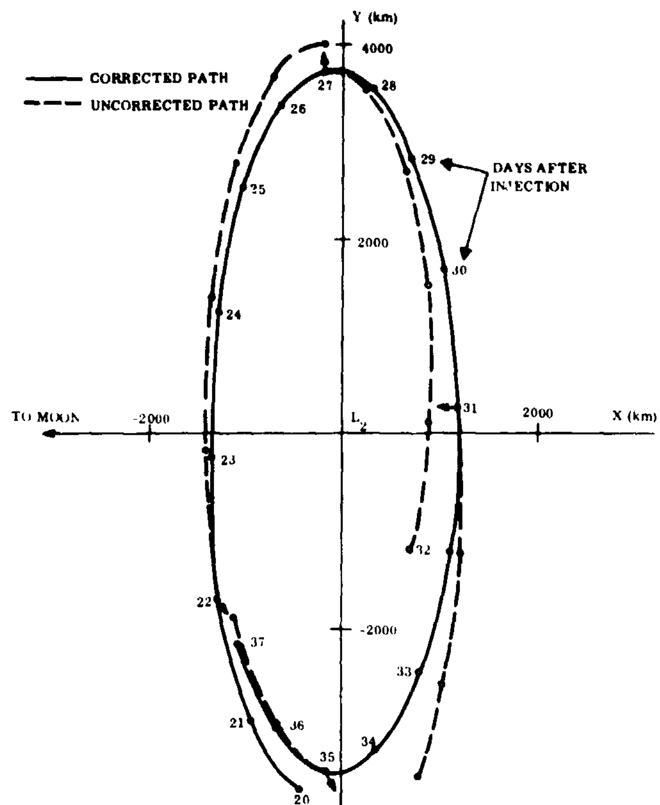


Figure 2-13. Halo Orbit, X-Y Projection, 20 to 37 Days

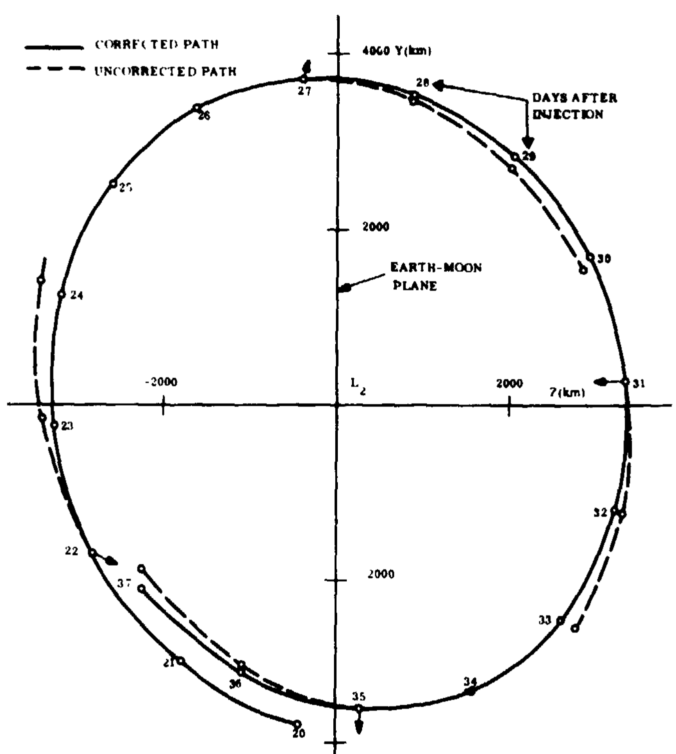


Figure 2-14. Halo Orbit, Y-Z Projection, 20 to 37 Days

SECTION 3

COMMUNICATIONS STUDIES

The objective of the communications portion of this study is to determine for a satellite in the vicinity of the lunar libration point what is necessary to provide the following:

a. Relay of the Apollo Unified S-Band telecommunications and tracking signals between:

   1. Lunar bases on the far side of the moon

   2. Lunar base and lunar orbiting spacecraft

   3. Lunar base or lunar orbiting spacecraft and earth

b. Commands, range and range rate tracking, and telemetry for the lunar libration point satellite itself.

3.1 LINK ANALYSES

3.1.1 APPROACH

Introduction of a relay at the lunar libration satellite (LLS) will make compound links of both the up and the down Apollo Unified S-Band links. Since the Manned Space Flight Network ground stations that will be used when the vehicles are at lunar distances have 25.90-meter diameter antennas, high power (10KW) transmitters, and low noise (33°K) receivers, the space-to-space legs of the compound links are more difficult than the earth-to-space or space-to-earth links. In order to see what a straightforward design of these links requires, the design parameters will be determined initially for an LLS which would provide the same quality signals to the command service module (CSM) and to the MSFN, assuming that the MSFN-to-LLS and LLS-to-MSFN are perfect links. It will be seen that such a design has an excessively large antenna and requires a very large amount of electronic circuits for antenna-beam tracking of the Apollo Terminals. If a single antenna with a beamwidth large enough to cover both the Earth and the Moon is used, it leads to a communication capability for only low-rate data and voice (Reference 8). Attention will then be concentrated on a system where the LLS has a single antenna with two beams, one just large enough to cover the lunar disc and one the earth. This system will provide most of the Unified S-Band services. It will be shown to be quite feasible.

Table 3-1 indicates a $435^{\circ}K$ equivalent noise temperature for the down link which results from an antenna noise temperature of $240^{\circ}K$, a receiver with a 2 dB noise figure, and circuit losses of 3 dB. The 3 dB figure for the circuit losses is felt to be reasonable because, in addition to the usual line losses, there will be significant loss due to the complex feed system necessary for the multiple narrow beams required. It is seen that 44.1 dB of gain must be provided by the receiving antenna in order to maintain the quality of the down link at the same level. A 44.1 dB antenna has a half-power beamwidth of 1.0°; at the down link frequency of 2.2875 GHz, a 30-foot diameter antenna would be required to achieve this gain.

Insertion and erection of a 9.14-meter diameter antenna in orbit at the lunar libration point would be a very expensive task. The cost would be further escalated because hundreds of pounds of complex electronics would be required to track the Apollo vehicles or bases due to the narrow beamwidth. There would also be a serious operational problem of signal acquisition with narrow-beam antennas on each terminal. For these reasons, attention in this study has been concentrated on a system having an antenna beam just wide enough to cover the moon, as this will provide service almost everywhere on the far side of the moon.

For the sake of completeness a differential analysis of the up-link is included in Table 3-2. It assumes that the 30-foot diameter antenna can be used for transmission to the MSFN. Again assuming circuit losses of 3 dB, it is seen that the quality of the link will be maintained if the transmitter power is 25 watts, which is a reasonable amount.

3.1.2 GEOMETRY AND CARRIER FREQUENCIES

Figure 3-1 indicates the geometry of the Apollo Unified S-Band communication links. The existing up-link from the MSFN directly to the CSM has a carrier frequency $f_1 = 2.10640625$ GHz, and the existing PM down-link has a carrier frequency $f_2 = 240/221\ f_1 = 2.2875$ GHz.

Table 3-1. Differential Analysis of Down-Link

Parameter	CSM-to-MSFN	CSM-to-LLS	dB Difference in Power for Links
Maximum Range	298,000 Km	70,000 Km	-15.1
Receiving Antenna Gain	53.0 dB	44.1 dB	8.9
Receive Circuit Losses	0 dB	3.0 dB	3.0
Equivalent Noise Temperature	209°K	435°K	3.2
			0

MSFN parameters taken from Reference 9.

Table 3-2. Differential Analysis of Up-Link

Parameter	MSFN-to-CSM	LLS-to-CSM	dB Difference in Power for Links
Maximum Range	298,000 Km	70,000 Km	-15.1
Transmitting Antenna Gain	52.0 dB	43.5 dB	8.5
Transmit Circuit Losses	0 dB	3.0 dB	3.0
Transmitter Power	40.0 dBW	13.9 dBW	26.1
Receiving Antenna Gain*	0 dB	23.3 dB	-23.3
Receive Circuit Losses	6.2 dB	7.0 dB	0.8
			0

*Assumes omni-directional antenna on the MSFN-to-CSM link and hi-gain antenna on the LLS-to-CSM link.

MSFN and CSM parameters taken from Reference 10.

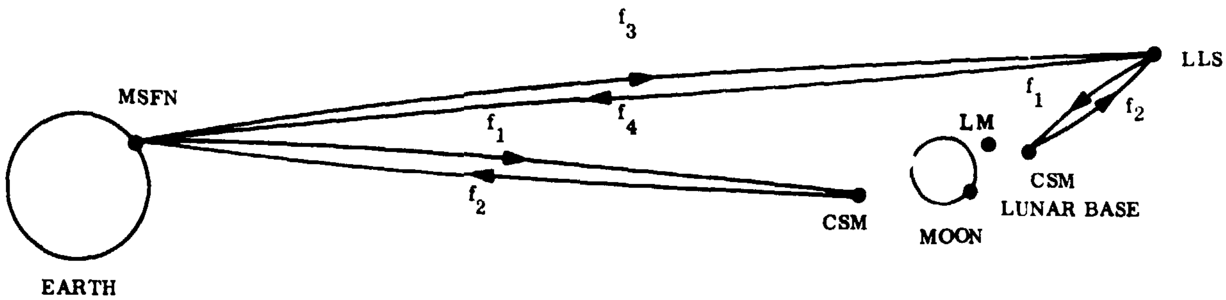


Figure 3-1. Apollo Unified S-Band Communication Links

When the CSM is occulted by the moon, its communications will be relayed via the LLS. To avoid alterations in the CSM receiver design, the carrier frequencies received and transmitted by the CSM are maintained at f_1 and f_2, respectively. In order to maintain the ability to determine range rate from doppler frequency shifts, the LLS should be designed to coherently translate the frequencies, so

$$f_1 = k_1 f_3$$

$$\text{and } f_4 = k_2 f_2$$

To avoid confusion, only the communication links to one CSM are shown in Figure 3-1. Service will be provided to other vehicles and lunar bases in the same fashion. For example, suppose the up and down carrier frequencies for a lunar module (LM) are f_5 and f_6 = 240/221 f_5, respectively. When relay via the LLS is necessary, the carrier frequencies for the MSFN-to-LLS and the LLS-to-MSFN links will be taken as:

$$f_7 = \frac{1}{k_1} f_5$$

$$\text{and } f_8 = k_2 f_6$$

Observe that the function of the LLS transponder is simply to translate in frequency the incoming signal and repeat it. This simple configuration is possible since all Apollo vehicles and terminals operating in space at the same time will be assigned different carrier frequencies. Thus, translation can be arranged so that all signals are in distinct bands, even though several transmissions may be occurring simultaneously. The design of the transponder is further discussed in Section 3.2.

3.1.3 SIGNAL FLOW

Relay of up-link signals from MSFN to the CSM, LM, or lunar base is achieved at the LLS simply by receiving the signal from the antenna feed directed toward the earth, translating it in frequency, and transmitting it from another feed on the same antenna directed toward the moon. The down-link signals would be handled in a similar fashion by the use of diplexers in the antenna feeds.

In order for one occulted terminal to communicate with another occulted terminal, the signal will be passed from the LLS down to the MSFN where it will be translated to the frequency appropriate to the up-link for the receiving base and transmitted to the LLS where it will be relayed to the destination. Although this procedure may at first appear to be quite complicated, it in fact has many advantages, viz.:

a. The transponder design is kept very simple. No modulation, demodulation, or switching is required. For this reason, the transponder can be designed to handle many lunar terminals efficiently.

b. The MSFN can monitor all transmissions within the system.

c. No additional frequency spectrum is needed for this type of communication.

d. The problem of a CSM, LM, or lunar base receiving simultaneously from multiple sources is avoided.

e. CSM, LM, and lunar base equipment design need not be altered to permit this type of communication.

Communication between an occulted terminal and a non-occulted terminal is accomplished by having the occulted terminal communicate with the MSFN via the LLS and having the other terminal communicate directly with the MSFN. In every case it would be the burden of the MSFN to determine the destination of the signal it receives and then to transmit it at the appropriate frequency (either to the LLS for relay or to the destination directly).

3.1.4 TELEMETRY, TRACKING, AND CONTROL FOR THE LUNAR LIBRATION SATELLITES

The necessary telemetry, tracking and control for the LLS should be provided so that these functions can be achieved independent of any Apollo Unified S-Band signals. Operation at some S-Band frequency close to the Apollo signals is recommended so that the same antenna and feed can be used for all earth-signals at the LLS.

Since it is necessary to know accurately the range and range rate of the LLS, a pseudo-random noise (PN) ranging code is transmitted to the LLS where it is coherently frequency translated and returned to the tracking station. Standard command and telemetry links are envisioned for the LLS.

3.1.5 POWER BUDGET

From the parameters of the Apollo Unified S-Band system and those assumed for the lunar libration satellite can be calculated S/N_o (the ratio of received signal power to noise power spectral density) for each leg of each communication link. By appropriately combining these for the two legs of a compound link, the effective S/N_o at the receiver can be calculated. Test measurements have established the value of S/N_o for threshold operation of the several services of the various PM modes. Comparison of these values then will reveal the circuit margin anticipated in each case.

3.1.6 S/N_o REQUIRED FOR SERVICES IN VARIOUS MODES

Measured data has been obtained (Reference 11) that indicates the value of S/N_o for operation at threshold for each service of the various modes. These values, along with the criteria for services, are tabulated in Table 3-3 for the Up-link and in Table 3-4 for the PM Down-link.

Table 3-3. S/N_o Required and Margins for Services in the Various Modes of the Apollo Unified S-Band Up-Link

Mode	Service	Quality	Threshold S/N_o dB	Nominal Margin CSM-Hi-Gain dB	LM-Steerable dB	Lunar Base dB
1	Carrier	12 dB loop S/N	53.9	5.3	-0.9	10.4
	PRN					
2	Carrier	12 dB loop S/N	51.3	7.9	1.7	13.0
	Voice	90% intelligibility	55.5	3.7	-2.5	8.8
	Voice	70% intelligibility	52.0	7.2	1.0	12.3
3	Carrier	12 dB loop S/N	51 3	7.9	1.7	13.0
	Updata	6.7×10^{-6} SBER	58.4	0.8	-5.4	5.9
4	Carrier	12 dB loop S/N	54.4	4.8	-1.4	9.9
	Voice	90% intelligibility	59.5	-0.3	-6.5	4.8
	Voice	70% intelligibility	54.8	4.4	-1.8	9.5
	PRN					
5	Carrier	12 dB loop S/N	54.4	4.8	-1.4	9.9
	Updata	6.7×10^{-6} SBER	64.2	-5.0	-11.2	0.1
	PRN					
6	Carrier	12 dB loop S/N	45.9	13.3	7.1	18.4
	Voice	90% intelligibility	59.5	-0.3	-6.5	4.8
	Voice	70% intelligibility	54.8	4.4	-1.8	9.5
	Updata	6.7×10^{-6} SBER	66.2	-7.0	-13.2	-1.9
	PRN					
7	Carrier	12 dB loop S/N	46.8	12.4	6.2	17.5
	Voice	90% intelligibility	58.7	0.5	-5.7	5.6
	Voice	70% intelligibility	53.8	5.4	-0.8	10.5
	Updata	6.7×10^{-6} SBER	62.2	-3.0	-9.2	2.1
8	Carrier	12 dB loop S/N	51.3	7.9	1.7	13.0
	Voice	90% intelligibility	56.0	3.2	-3.0	8.3
	Voice	70% intelligibility	51.5	7.7	1.5	12.8

Margins for worst case Apollo parameters are the same for the CSM-Hi-Gain, but 0.5 dB lower for both the ML-Steerable and Lunar Base.

Table 3-4. S/N_o Required and Margins for Services in the Various Modes of the Apollo Unified S-Band Down-Link

Mode	Service	Quality	Threshold S/N_o dB	Nominal Margin CSM-Hi-Gain dB	LM-Steerable dB	Lunar Base dB
1	Carrier	12 dB loop S/N	33.6	32.0	29.8	37.3
	51.2 Kbps TLM	10^{-6} BER	65.6	0.0	-2.2	5.3
	Voice	90% intelligibility	61.8	3.8	1.6	9.1
	Voice	70% intelligibility	56.8	7.0	6.6	14.1
2	Carrier	12 dB loop S/N	23.9	41.7	39.5	47.0
	51.2 Kbps TLM	10^{-6} BER	66.1	-0.5	3.7	4.8
	Voice	90% intelligibility	61.8	3.8	1.6	9.1
	Voice	70% intelligibility	56.8	7.0	6.6	14.1
	PRN	99.6% acq. in 9 sec.	38.3	27.3	25.1	32.6
2B	Carrier					
	51.2 Kbps TLM					
	Voice/biomed	90% intell/acceptable	64.3/?	1.3/?	-0.9/?	6.6/?
	PRN					
3	Carrier	12 dB loop S/N	33.9	31.7	29.5	37.0
	1.6 Kbps TLM	10^{-6} BER	56.7	8.9	6.7	14.2
	Voice	90% intelligibility	54.6	11.0	8.8	16.3
	PRN	99.6% acq. in 9 sec.	38.3	27.3	25.1	32.6
3B	Carrier					
	1.6 Kbps TLM					
	Voice/biomed	90% intell/acceptable	57.6/?	8.0	5.8	13.3
	PRN					
4	Carrier	12 dB loop S/N	33.6	32.0	29.8	37.3
	1.6 Kbps TLM	10^{-6} BER	56.1	9.5	7.3	14.8
	Voice	90% intelligibility	54.7	10.9	8.7	16.2
	Voice	70% intelligibility	49.7	15.9	13.7	21.2
5	Carrier	12 dB loop S/N	35.8			
	1.6 Kbps TLM	10^{-6} BER	47.2	29.8	27.6	35.1
				18.4	16.2	23.7
6	Carrier	12 dB loop S/N	31.3	34.3	32.1	39.6
	Key	99.6% acq. in 9 sec.	30.5	35.1	32.9	40.4
7	Carrier	12 dB loop S/N	29.4	36.2	34.0	41.5
	PRN	99.6% acq. in 9 sec.	38.3	27.6	25.4	32.6
8	Carrier	12 dB loop S/N	34.2	31.4	29.2	36.7
	1.6 Kbps TLM	10^{-6} BER	51.2	14.4	12.2	19.7
	Back-up Voice	90% intelligibility	63.3	2.3	0.1	7.6
	Back-up Voice	70% intelligibility	49.7	15.9	13.7	21.2
9	Carrier	6 dB loop S/N	30.2	35.4	33.2	40.7
	1.6 Kbps TLM	10^{-6} BER	47.8	17.8	15.6	23.1
	PRN	99.6% acq. in 9 sec.	38.3	27.3	25.1	32.6

Margins for the worst case Apollo parameters are 0.9 dB lower for the CSM-Hi-Gain, 1.8 dB for the LM-Steerable, and 1.1 dB lower for the Lunar Base.

3.1.7 SYSTEM PARAMETERS

The pertinent Apollo Unified S-Band parameters are listed in Table 3-5, and those assumed for the LLS in Table 3-6.

The critical links are the space-to-space links: Moon-to-LLS and LLS-to-Moon. The parameter values selected for these links are felt to be realistic for a carefully designed system using 1971-72 technology. The antenna gain for these links has been deliberately selected to be the gain at the 4-dB points of an antenna beam that just covers the moon from the position of the LLS, because this is the maximum gain for which a simple non-tracking transponder is possible.

Implementation of components to yield these parameter values is discussed in Sections 3.2 and 3.3.

3.1.8 S/N_o FOR A COMPOUND LINK

The signal-to-noise density ratio at the receiver of a compound link will be determined here in terms of the signal-to-noise density ratios of the individual links and the improvement factor due to the transponder.

Let the signal-to-noise density ratio in the first leg be $S_1/N_{o1} = K_1$. Let the improvement factor be α, i.e., the signal-to-noise density ratio out of the transponder is αK_1. Assuming the input to the second leg is all signal, let the signal-to-noise density ratio in the second leg be $S_2/N_{o2} = K_2$, where

$$S_2 = A(\alpha S_1 + N_{o1} B)$$

where A is the amplification due to the transponder and B is the transponder bandwidth. The ultimate signal-to-noise density ratio at the receiver is:

Table 3-5. Apollo Unified S-Band Parameters

Parameter	MSFN Station	CSM	LM
Transmitter Power	10 Kw	11.2 W 12.6 W	20 W 0.72 W
Transmitting Antenna Gain	52.0 dB	26.7 dB 0 dB	32.0 dB 20.5 dB -3.0 dB
Receiving Antenna Gain	53.0 dB	23.3 dB 0 dB	31.2 dB 16.5 dB -3.0 dB
Transmit Circuit Losses	0 dB	7.0 dB 6.2 dB	8.9 dB 5.1 dB 4.9 dB
Receive Circuit Losses	0 dB	7.0 dB 6.2 dB	9.8 dB 5.9 dB 5.7 dB

Source: Reference 10.

Table 3-6. Lunar Libration Satellite Parameters

Parameter	MSFN-to-LLS	LLS-to-MSFN	LLS-to-Moon	Moon-to LLS
Antenna Gain	31.6 dB	31.0 dB	31.0 dB	31.6 dB
Polarization Losses	0.7 dB	0.7 dB	0.5 dB	0.5 dB
Transmit Circuit Losses	---	2.0 dB	1.5 dB	---
Receive Circuit Losses	3.0 dB	---	---	1.0 dB
Receiver Noise Figure	11.0 dB	---	---	2.0 dB
Transmitter Power (Apollo Signals)	---	5 W	50 W	---
Transmitter Power	---	5 W	---	---

$$K = \frac{\alpha A S_1}{A N_{o1} + N_{o2}} = \frac{\alpha A S_1}{A N_{o1} + S_2/K_2}$$

$$= \frac{\alpha A S_1}{A N_{o1} + \dfrac{\alpha S_1 + A N_{o1} B}{K_2}}$$

$$= \frac{\alpha S_1 K_2}{K_2 N_{o1} + \alpha S_1 + N_{o1} B}$$

$$= \frac{\alpha K_1 K_2}{\alpha K_1 + K_2 + B}$$

3.1.9 S/N_o AVAILABLE FOR LINKS RELAYED VIA THE LUNAR LIBRATION SATELLITE

3.1.9.1 Up-Link

As Table 3-7 shows, the MSFN-to-LLS leg of the up-link is a very strong link. For simplicity, the carrier frequency of this link is assumed to be the same as that of the direct down-link, or $k_1 = 221/240$. The calculations will not change significantly whatever the actual value is. Hence, the S/N_o at the CSM, LM, or lunar base receiver will be essentially that of the LLS-to-Moon link. The calculations of these values are indicated in Table 3-8.

3.1.9.2 PM Down-Link

The calculations of the S/N_o for the LLS-to-MSFN leg of the down-link are given in Table 3-9. The LLS-MSFN leg is shown to be a strong link, but it does cause the S/N_o for the overall down-link to be somewhat lower than that for the space-to-space leg. The calculations of the S/N_o for the CSM, LM, and lunar base-to-LLS leg of the down-link are given in Table 3-10. The S/N_o for the overall down-link is given at the bottom of Table 3-10. Again, for simplicity, the carrier frequency of the LLS-to-MSFN link is assumed to be the

Table 3-7. S/N_o for MSFN-to-LLS Link

Transmitter Power	40.0 dBW
Transmitting Antenna Gain	53.0 dB
Transmit Circuit Losses	0 dB
Effective Radiated Power	93.0 dBW
Max Range 460,000 Km	
Carrier Frequency 2.2875/2.2825 GHz	
Dispersion Loss	212.9 dB
Polarization Loss	0.7 dB
Receiving Antenna Gain	31.6 dB
Receive Circuit Losses	3.0 dB
Signal Power Available at Receiver, S	-92.0 dBW
Receiver Noise Figure 11 dB	
Receiver Noise Temperature 3360°K	
Antenna Noise Temperature 290°K	
Equivalent Noise Temperature 3650°K	
Noise Power Density	-193.0 dBW
S/N_o	101.0 dB

Table 3-8. S/N_o for the LLS-to-Moon Link

	CSM-Hi-Gain dB(W)	LM-Steerable dB(W)	Lunar Base db(W)
Transmitter Power	17.0	17.0	17.0
Transmit Circuit Losses	1.5	1.5	1.5
Transmitting Antenna Gain	31.0	31.0	31.0
Effective Radiated Power	46.5	46.5	46.5
Dispersion Loss	195.8	195.8	195.8
Polarization Loss	0.5	0.5	0.5
Receiving Antenna Gain	23.3	16.0	31.2
Receive Circuit Losses	7.0	5.9	9.8
Signal Power Received, S	-133.5	-139.7	-128.4
Equivalent Noise Power Density, *N_o	-192.7	-192.7	-192.7
S/N_o for LLS-to-Moon Link	59.2	53.0	64.3
S/N_o for Overall Up-Link	59.2	53.0	64.3
* Receiver Noise Figure	11.3 dB	11.3 dB	11.3 dB
* Receiver Noise Temperature	3630°K	3630°K	3630°K
* Antenna Noise Temperature	100°K	100°K	100°K
* Equivalent Noise Temperature	3880°K	3870°K	3890°K

Transponder Bandwidth Assumed to be 16 MHz

Table 3-9. S/N_o for LLS-to-MSFN PM Link

	Nominal
Transmitter Power 5W	7.0 dBW
Transmit Circuit Losses	2.0 dB
Transmitting Antenna Gain	31.0 dB
Effective Radiated Power	36.0 dBW
Max Range 460,000 Km	
Carrier Frequency 2.10640625/2.101802 GHz	
Dispersion Loss	212.2 dB
Polarization Loss	0.7 dB
Receiver Antenna Gain	52.0 dB
Line Losses	0 dB
Signal Power Available at Receiver, S	-124.9 dBW
Receiver Noise Temperature 33°K	
Antenna Noise Temperature 125°K	
Equivalent Noise Temperature 158°K	
Noise Power Density, N_o	-206.6 dBW
S/N_o	81.7 dB

Table 3-10. S/N_o for the Moon-to-LLS PM Link

	CSM-Hi-Gain dB(W)	LM-Steerable dB(W)	Lunar Base dB(W)
Transmitter Power	10.5	13.0	13.0
Transmit Circuit Losses	7.0	5.1	8.9
Transmitting Antenna Gain	26.7	20.0	32.0
Effective Radiated Power	30.2	27.9	36.1
Dispersion Loss	196.5	196.5	196.5
Polarization Loss	0.5	0.5	0.5
Receiving Antenna Gain	31.6	31.6	31.6
Receive Circuit Losses	1.0	1.0	1.0
Signal Power Received, S	-132.2	-138.5	-130.3
Equivalent Noise Power Density, * N_o	-202.4	-202.4	-202.4
S/N_o for LLS-to-Moon Link	66.2	63.9	72.1
S/N_o for Overall Down-Link	65.6	63.4	70.9

* Receiver Noise Figure	2.0 dB	2.0 dB	2.0 dB
Receiver Noise Temperature	170°K	170°K	170°K
Antenna Noise Temperature	240°K	240°K	240°K
Equivalent Noise Temperature	420°K	420°K	420°K

Transponder Bandwidth Assumed to be 16 MHz same as that of the direct up-link, or k_4 = 221/240. No significant change in the calculations will occur for a slightly different carrier frequency.

It is assumed in these calculations that the LLS is relaying only one signal at a time. If two or more were to be handled simultaneously, the power transmitted by the LLS would be divided among them. The resultant degradation in the overall S/N_o would be slight (a fraction of a dB) for the down-link, but appreciable (several dB's) for the up-link. Thus it is desirable to program up-link transmissions so that there is communication with only one terminal at a time.

3.1.10 MARGINS FOR SERVICES IN VARIOUS MODES

The difference between the available S/N_o and the S/N_o required for threshold represents the margin with which a service is (or is not) provided. The margins for the services of the up-link modes are listed in Table 3-3. Margins for the services of the PM down-link modes are listed in Table 3-4.

No calculations have been included on the FM down-link modes because no test data was available from which their threshold S/N_o could be derived. The FM modes provide basically two services: (1) replay of data stored while the Apollo vehicles are behind the moon and (2) real-time television. Successful operation of an LLS relay would obviate the need for the first function. Based on the differential link analysis and on the data presented in Reference 1, it is estimated that relay of TV through the LLS would result in an acceptable picture when transmission is from the erectable antenna on the lunar surface, but a sub-marginal picture when transmission is from the CSM hi-gain antenna. Note that the transponder design is capable of handling the FM modes without modification.

3.2 TRANSPONDER CHARACTERISTICS

Figure 3-2 is a functional diagram of the Lunar Libration Satellite transponder. The transponder performs the following functions:

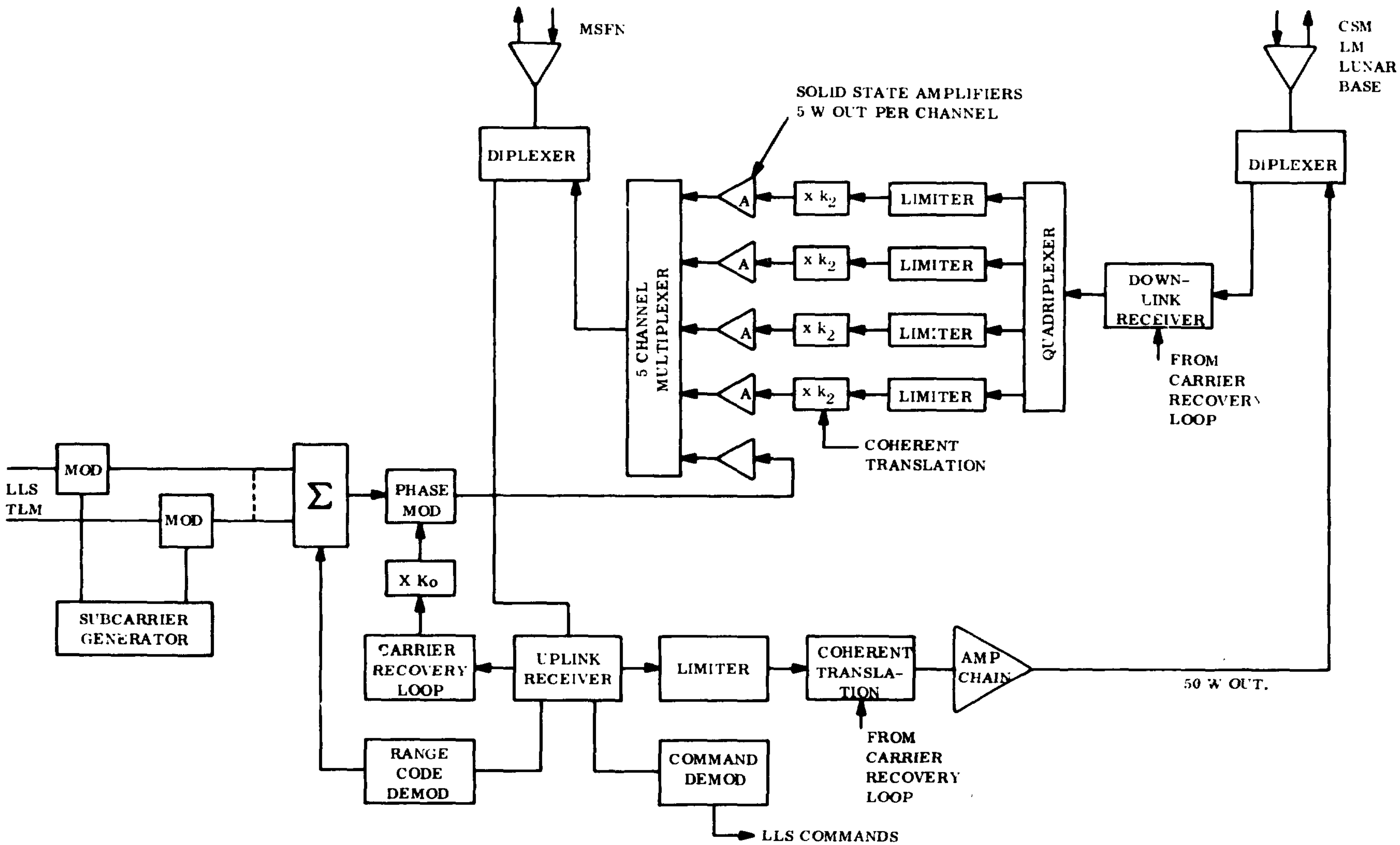


Figure 3-2. Block Diagram of Lunar Libration Satellite Transponder

a. Command receiver for satellite housekeeping.

b. Telemetry transmitter for satellite monitoring, housekeeping and ranging.

c. Data relay from Manned Space Flight Network to Apollo mission spacecraft.

d. Data relay from Apollo mission spacecraft to Manned Space Flight Network.

It is assumed that four 4MHz spacecraft-to-earth data channels may operate simultaneously, but that only a single 4MHz earth-to-spacecraft link will be operative at any one time. Four separate links will be available, however. Ranging and telemetry data from LLS-to-earth will occupy another 4MHz down-channel. The satellite up-link transmitter will product a total rf power output of 50 watts. The satellite down-link will contain two channels, one for LLS ranging and TM data, and one for Apollo data. The two channels will employ a common 10-watt power amplifier. Phase coherence of the up-link and down-link carriers will be preserved in the translational processes. On-board telemetry and the ranging code will be transmitted from the LLS to earth on a carrier that is phase-locked to the up-link carrier.

3.2.1 FIFTY WATT LLS TO APOLLO TRANSMITTER

The bandwidth and power requirements here indicate use of a traveling wave tube amplifier. A good estimate of the properties of such an amplifier can be gained from the following specifications for the Varian X-1250 TWTA:

VARIAN/EIMAC X-1250 TWTA

Frequency	2.2 to 2.3 GHz
RF Power Output	56 watts
Total Power Input	175 watts
Efficiency	32%
Length	11-1/2 inches (0.2921 m)
Width	5-3/8 inches (0.1365 m)
Height	2-7/8 inches
Weight	7.8 pounds
Input Voltage	25-50 vdc
Temperature	$-10^{\circ}C$ to $+70^{\circ}C$
Gain	30 dB

3.2.3 RECEIVER PREAMPLIFIERS

Transistor and tunnel diode amplifiers operating at 2.2 to 2.3 GHz have not achieved noise figures better than 3.5 to 4.0 dB. Uncooled parametric amplifiers achieve noise figures as low as 1.3 dB (Micromega R-1108). Units designed for spacecraft environment and provided with solid state pump power sources can be expected to exhibit noise figures of 2.0 dB or slightly less.

3.2.4 10-WATT DOWN-LINK TRANSMITTER

A 10-watt traveling wave tube amplifier is estimated to occupy a volume 1.5" x 5.5" x 14" and to weigh 6.5 lbs.

3.3 ANTENNA CHARACTERISTICS

The recommended antenna configuration for either the halo or hummingbird orbits is a single parabolic reflector (3.5 meters in diameter) with multiple antenna beams. The preferred antenna is a compromise between smaller, lower gain antennas which would limit the available service and larger, higher gain antennas which would require complex electronics for tracking.

In order to avoid the complexities of tracking, the antenna beam must be broad enough to cover the angular extent of the Moon. The mean angular size of the Moon is 3.1 degrees and has extremes of 2.86 and 3.4 degrees corresponding to apogee and perigee of the Moon. The angular size varies negligibly with satellite offset for offsets from 3,100 to 4,500 km. The optimum antenna gain to provide this coverage is realized by choosing the 4 dB beamwidth equal to the Moon size from maximum range and allowing ± 0.1 degree for attitude control error. The optimum antenna would thus have a 3.06 degree, 4 dB beamwidth which corresponds to a 2.66 half-power beamwidth. A 3.5 m diameter parabola would satisfy this requirement and, if 55 percent efficient, would have a peak gain of 35.7 dB. This antenna would provide 31.7 dB gain at the edge of the Moon at maximum range and 30.2 dB at minimum range.

The difference in gain compensates for the difference in path loss for the two ranges and, thus, represents the maximum link capability for a single, non-tracking, antenna beam.

The links to the Earth would require an additional antenna beam angularly displaced from the beam for the Moon links. The amount of beam displacement required varies with the satellite offset as shown in Figure 3-3.

It is desirable for the feeds which generate the two beams to be physically and electrically separate. The feed design and focal length-to-diameter ratio may be chosen in conjunction to meet the beam displacement requirements for satellite offsets of 4000 km or more. The feed development and packaging of the polarizers and circuitry become simpler for the larger offsets, but it remains to be determined whether these savings counterbalance the additional fuel required to maintain the larger offsets.

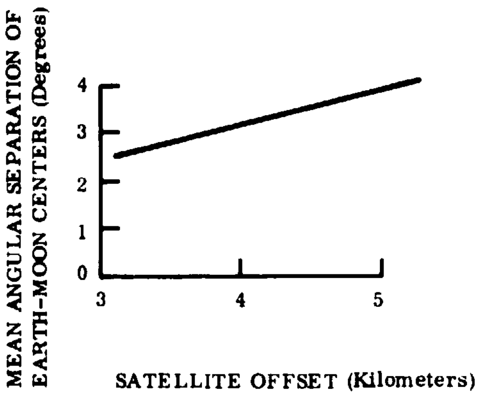


Figure 3-3. Beam Displacement vs Satellite Offset

The feed for the Moon-directed beam would be placed on the reflector axis so that maximum antenna performance will be realized for the critical LM and CSM links. The feed for the Earth-directed beam would be located off axis. For the hummingbird, the antenna axis would be pointed at the Moon center at all times. The center of the Earth-directed beam would migrate somewhat as the Earth-Moon distance changed, but the Earth would always be within the 3 dB beamwidth. For the halo orbit, the antenna axis would again be pointed at the Moon center and the Earth-directed beam would precess around the axis at the LLS orbital rate. This could be done by rotating the feed or by electronically switching between a cluster of feeds. Of these alternates, mechanical rotation appears to be the most attractive.

SECTION 4

ATTITUDE CONTROL STUDIES

Attitude control must be provided during the midcourse correction and lunar orbit insertion, as well as during the communication and stationkeeping phases of the mission. Guidance considerations require that the thrust vector be aligned to within ± 0.5 degrees during midcourse correction and lunar orbit insertion. Communications require that the antenna be aligned to the Moon to within 0.5 degrees, but a goal of 0.1° has been selected for the study. An attitude control concept for the midcourse correction and lunar orbit insertion is described. Several feasible attitude control concepts during the communications and stationkeeping phases of the mission are described for each of the two proposed orbits. The initial stabilization procedure is specified for each concept. Disturbance torques are estimated and controller requirements are determined. System block diagrams and weight and power summaries for each concept are also included.

4.1 ATTITUDE CONTROL DURING MIDCOURSE CORRECTION AND LUNAR ORBIT INSERTION

The attitude control system proposed during the orbit transfer phase of the mission will be similar to the system that has proved itself on the Mariner, Surveyor, and Lunar Orbiter missions. This system uses sun sensors to control two axes to the sun, and a star tracker to control the third axis to the star canopus. Control torque is provided by a pneumatic system. Rate information is provided by a body mounted three axis gyro package. The gyros would be in the rate mode. This orientation is maintained throughout the transfer orbit except during midcourse correction and lunar orbit insertion. At this time, control is transferred from the sun sensors and the star tracker to the body mounted gyro package that has been switched to the rate and position mode. The vehicle is then slewed by precessing the gyros one axis at a time to the orientation required for each rocket burn. Because of the increase in disturbance torque due to rocket misalignment, control torque during rocket burn is provided by vectoring of the rocket thrust. Guidance considerations require that the thrust vector be aligned to within ± 0.5 degrees.

4.2 ATTITUDE CONTROL CONCEPTS DURING THE COMMUNICATION AND STATION-KEEPING PHASES OF THE MISSION

Several feasible attitude control concepts were selected for each of the two orbits proposed.

4.2.1 HUMMINGBIRD ORBIT

In this orbit, the satellite would lead the Moon around the Earth and always remain in the Moon orbital plane. The satellite position would be maintained so as to present a constant geometry between the Earth, the satellite, and the Moon.

The first concept uses an internal constant speed momentum wheel as a gyroscope for orientation control. The spin axis of the momentum wheel is maintained normal to the Earth/Moon plane. This provides control about two axes of the vehicle. Control about the third axis, which is parallel to the momentum wheel spin axis, is obtained by modulating a flywheel, whose spin axis is parallel to the spin axis of the momentum wheel, using an error signal from an Earth sensor operating in the infrared range of the spectrum. Precession control to align and maintain the rotor axis normal to the Earth/Moon plane will be performed by reaction jets. Jet actuation will be based on error signals from the Earth sensor and the Canopus star tracker. A nutation damper will be provided to damp out any coning motion induced by jet actuation or disturbance torques. The communications antenna would be rigidly attached to the vehicle with its symmetrical axis pointed to the Moon. A secondary feed would be rotated off this axis at some fixed angle to point to the Earth. A solar array, for power collection, will be mounted to each end of the vehicle on the axis perpendicular to the Earth/Moon plane. The arrays will be rotated about this axis so that they always face the sun. Because the Earth/Moon plane is inclined only five degrees with respect to the ecliptic plane, the normal to the solar array will never be misaligned more than five degrees with respect to the sun line. To provide period and stabilization control to maintain the hummingbird orbit, a thruster will be mounted to the vehicle so that it lies in the Earth/Moon plane and is perpendicular to the Earth/Moon line. To minimize disturbance torques, the line of action of the thruster must be as close as possible to the center of the mass of the vehicle. Calculations show that this moment arm should be kept to within 0.00254 meter (0.1 inch). To accomplish this, some form of thrust vector control about two axes must be provided. For orbit stabilization control, it would be advisable to have a separate thruster aligned along the yaw axis acting through the center of mass with the moment arm not to exceed 0.00635 meter (0.25 inch).

Initial stabilization is accomplished by using the body mounted gyro package that has been switched to the rate and position mode to orient the vehicle so that the spin axis of the flywheel is perpendicular to the orbit plane. The vehicle is then spun about this axis to establish an angular momentum vector to maintain this orientation until the flywheel can be spun up. Since this will probably be a minimum moment of inertia axis, the nutation damper should be caged prior to spin up to minimize cone angle build up. The flywheel is then energized. As the flywheel spins up, the vehicle rate will decrease until it reaches zero. It will be maintained at zero by the pitch jets. The nutation damper can now be uncaged to remove any cone angle that may have built up. The flywheel should be brought to top speed as quickly as possible. If the vehicle is not exactly symmetrical about the spin axis of the flywheel, this axis as the flywheel spins up temporarily becomes an intermediate moment of inertia axis which is unstable and will cause a cone angle build up. Therefore, it is necessary to pass through this region as quickly as possible. When the flywheel reaches top speed, the sun is acquired as a reference with the pitch axis. Using the body mounted gyros, the vehicle is rotated about the pitch axis until the Earth sensor points to the Earth. Control is then switched to the Earth sensor and the Earth is acquired. Once the Earth is acquired, any roll and yaw errors can be reduced by precessing the vehicle using error signals from the Earth sensor and the canopus tracker. The solar array can then be extended and the antenna erected. A block diagram of this stabilization system is given in Figure 4-1.

The second concept would have a three axis active control system. Two axes of the vehicle would be controlled to the Earth using an Earth sensor; the other axis would be controlled to the star canopus using a star tracker. The actuators would be flywheels with jet unloading. The communications antenna, the solar arrays, and the period and stabilization control thruster would require the same mounting as in concept No. 1.

Initial stabilization is accomplished by using the body mounted gyro package that has been switched to the rate and position mode to orient the vehicle so that the Earth sensor is pointing to the Earth and the canopus tracker pointing to the star canopus. Control is then switched to the Earth sensor and the canopus tracker, and the Earth and star canopus are then acquired. The solar array can then be extended and the antenna erected. A block diagram of this concept is given in Figure 4-2.

4.2.2 HALO ORBIT

In this orbit, the satellite would circle the libration point. The satellite/libration point plane would be inclined 71 degrees to the Earth/Moon plane. The period of rotation about the libration point would be approximately 15 days. The distance from the libration point would be sufficient to allow the satellite to view the Earth at all times. With this orbit, unlike the Hummingbird orbit, the satellite will move out of the Earth/Moon plane as it circles the libration point. As viewed from the Earth, this excursion out of the Earth/Moon plane will be approximately $\pm 0.5^{\circ}$.

The first halo orbit concept is similar to the first concept considered for the Hummingbird orbit; but, since the satellite in circling the libration point no longer maintains a fixed geometry relative to the Earth and Moon, two changes must be made. Because of the excursion of the satellite out of the Earth/Moon plane of 0.5°, if the spin axis of the constant speed flywheel were maintained normal to the Earth/Moon plane, this would represent an attitude error of $\pm 0.5^{\circ}$. Since the pointing accuracy requirement is $\pm 0.1^{\circ}$, this would be unacceptable. Therefore, the spin axis of the flywheel must be processed $\pm 0.5^{\circ}$ every 15 days. Due to the changing geometry of the Earth and the Moon, if the antenna is to remain rigidly attached to the vehicle, the symmetrical axis of the antenna

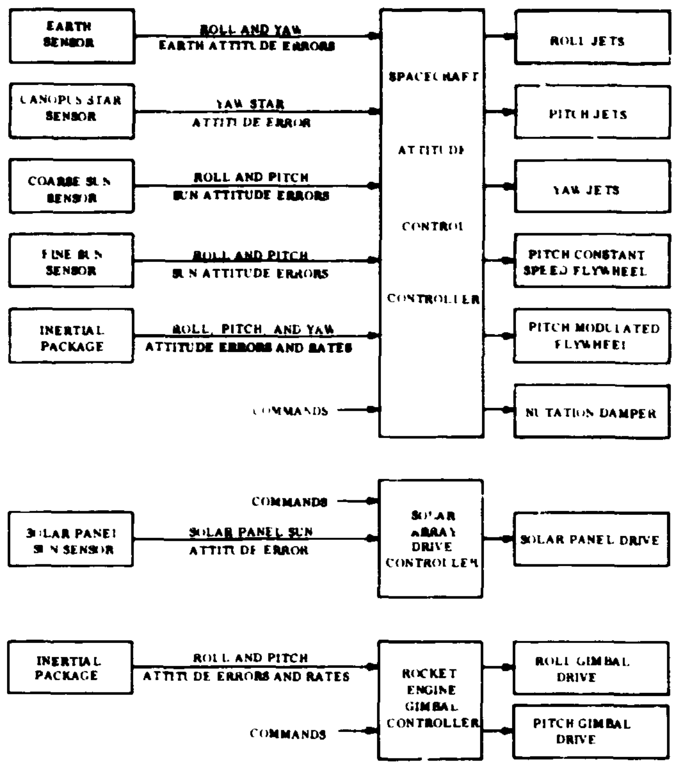


Figure 4-1. Hummingbird and Halo Orbit Stabilization and Control Elock Diagram (Concept No. 1)

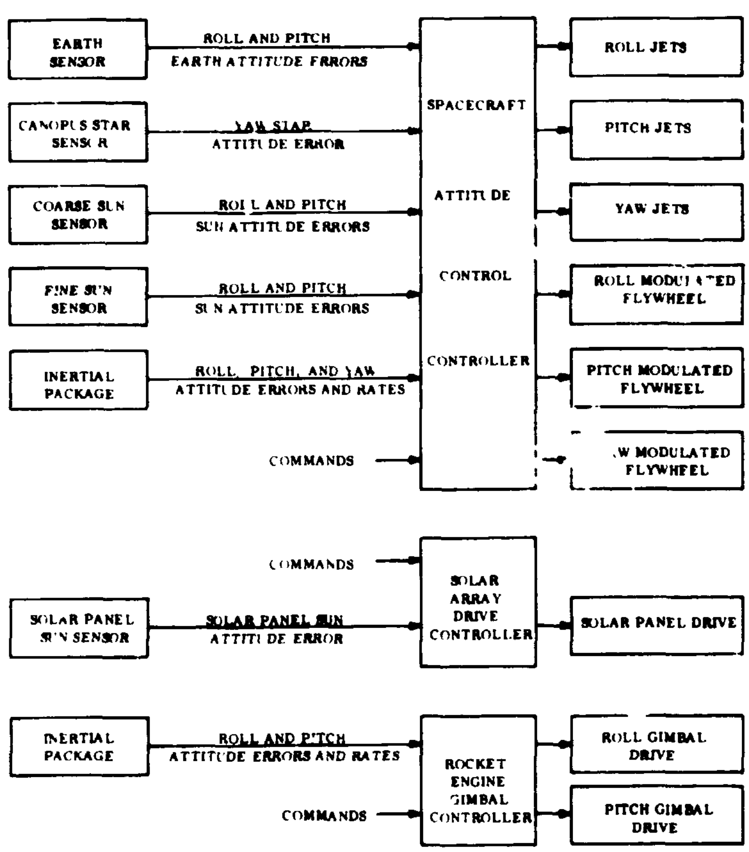


Figure 4-2. Hummingbird and Halo Orbit Stabilization and Control Biock Diagram (Concept No. 2)

must be pointed to the Earth and the secondary feed pointed to the Moon. This secondary feed would be maintained at some fixed angle relative to the symmetrical axis of the antenna, but would have to rotate around it every 15 days. If the antenna could be gimballed about two axes, then the symmetrical axis of the antenna could be pointed at the Moon and the secondary feed pointed to the Earth.

Period and stabilization control will also be required to maintain the Halo orbit. Period control thrusts will be required every seven days when the vehicle crosses the Earth/Moon plane. The duration of the thrusts could be as long as one day. The thrusts will alternate from the positive roll to the negative roll axis. To minimize disturbance torques, the line of action of the thruster must be as close as possible to the center of mass of the vehicle. For orbit stabilization control, a thrust is required in the Earth/Moon plane. 30 degrees off the Earth/Moon line. This can be accomplished with a thruster mounted in the roll/yaw plane of the vehicle, 30 degrees off the yaw axis. The moment arm of these thrusters should not exceed one-quarter inch.

Initial stabilization would be accomplished like concept No. 1 of the Hummingbird orbit.

The second Halo orbit concept is similar to the second concept considered for the Hummingbird orbit, but the communications antenna and the period and stabilization control thrusters would require the same mounting as in concept No. 1 for the Halo orbit.

4.2.3 DISTURBANCE TORQUES

The two major disturbance torques are due to solar pressure and orbit control. The disturbance torque calculations and associated control requirements are given in Appendix I.

4.2.3.1 Solar Pressure

If there is an offset between the center of pressure and the center of mass, solar pressure disturbance torques will be developed. The vehicles for both the Hummingbird and Halo orbits will require solar arrays with large cross-sectional areas. Therefore, the solar array, rather than the vehicle cross section, will be assumed to have predominate effect. It can be easily shown that a component of the moment arm between the center of pressure and center of mass along the pitch axis will cause torques about both the roll and yaw axes. These torques are cyclic over a period of one year. A component of the moment arm along in axis in the orbit plane and perpendicular to the fun line will cause torques about the pitch axis. These torques are accumulative.

4.2.3.2 Orbit Control

Hummingbird Orbit - If there is a moment arm between the thrust vector and the center of mass, disturbance torques will be developed. To maintain the orbit of the Hummingbird requires that a constant thrust be developed along the positive roll axis of the vehicle at all times. A component of the moment arm between the thrust vector and the center of mass along the pitch axis will cause a torque about the yaw axis. These torques are cyclic over a period of one orbit of the Earth. A component of the moment arm along the yaw axis will cause torques about the pitch axis. These torques are accumulative.

Halo Orbit - Orbit control thrusts will be required every seven days when the vehicle crosses the Earth/Moon plane. The duration of the thrusts could be as long as one day. The thrusts will alternate from the positive roll to the negative roll axis. A component of the moment arm along the pitch axis will cause torques about the yaw axis. These torques are cyclic over a period of one orbit of the Earth. A component of the moment arm along the yaw axis will cause torques about the pitch axis. Since these torques will be caused by two separate jets, they will have different moment arms. Therefore, it is possible that these torques could add or subtract from each other. However, the worst case should be considered for sizing of the flywheels.

4.3 HUMMINGBIRD RESULTS

4.3.1 CONCEPT NO. 1

To minimize the disturbance torque due to a moment arm between the thrust vector and the center of mass, the ion engine used for orbit control must have thrust vector control about two axes. For orbit stabilization control, it would be advisable to have a separate thruster aligned along the yaw axis acting through the center of mass instead of using the roll axis ion engine. The moment arm for this thruster should not exceed 0.00635 meter (0.25 inches).

Since the ion engine used for orbit control will be thrust vector controlled, it is expected that the moment arm between the thrust vector and the center of mass can be maintained to within (0.00254 meters (0.1 inch). It is recommended that a constant speed flywheel with an angular momentum storage capacity of 100 lb/ft/sec be used. This would require making precession control corrections on an hourly basis. The maximum impulse that could be imparted from the precession control jets to the vehicle at one time to prevent the attitude error from exceeding 0.1 degree due to the control action would be 0.087 lb/sec. The duration of this pulse should not exceed 0.00175 I_{xz} sec. The total impulse that would be required for control action due to disturbance torques is 656 lb/sec/year. If a gas with an I_{sp} = 110 sec is used, the weight of the gas would be 5.97 lb/year. The impulse required during constant speed flywheel spin up is 100 lb/sec. This would require 0.91 lbs of gas.

It is recommended that couples be used for all control jets to avoid disturbing the orbits with translational thrusts.

For control about the pitch axis, it is recommended that a modulated flywheel with an angular momentum storage capacity of 2 lb/ft/sec be used. The flywheel should unload only 25 percent of maximum momentum (0.5 lb/ft/sec) with a torque not exceeding 0.01 ft/lb. Approximately seven unloadings per day would be required due to the disturbance torque if they act about the pitch axis.

4.3.2 CONCEPT NO. 2

4.3.2.1 Roll, Pitch and Yaw Flywheel Sizing

4.3.2.1.1 Solar Pressure Torques

Solar pressure torques are cyclic over a period of one year about the roll and yaw axes;

they are also accumulative in pitch. To keep from unloading the wheels in the roll and yaw axes due to solar pressure torques would require flywheels with sufficient angular momentum storage to absorb the solar pressure disturbance torque for one-half year.

$$H_w = 0.0695 \text{ ft/lb/sec/day} \left(\frac{365}{2}\right) \text{ days} = 12.7 \text{ ft/lb/sec}$$

This does not seem advisable since the weight of the gas required to remove the angular momentum due to solar pressure is at most 0.115 lb/yr. Therefore, it would seem much more practical to use a 2 lb/ft/sec modulated flywheel on all three axes, and unload them 25% every 7.2 days.

4.3.2.1.2 Orbit Control Torques

The disturbance torques due to the misalignment of the thruster for orbit control are cyclic over a period of one orbit of the Earth in the roll and yaw axis; they are also accumulative in pitch.

To keep from unloading the wheels in roll and yaw would require a minimum of

$$H_w = 3.525 \text{ ft/lb/sec/day (14 days)} = 49.4 \text{ lb/ft/sec}$$

of momentum storage in the roll and yaw wheels if we assume a moment arm of 0.1 inch.

If, instead, we used a 2 lb/ft/sec modulated wheel on all three axes, the number of unloadings per day would be as shown in Appendix I.

4.3.2.2 Conclusions - Conept No. 2

The ion engine on the roll axis used for orbit control must be thrust vector controlled about two axes to minimize the disturbance torque due to a moment arm between the thrust vector and the center of mass. For orbit stabilization control, it would be advisable to have a separate thruster aligned along the yaw axis acting through the center of mass instead of using the roll axis ion engine. The moment arm for this thruster should not exceed 0.00635 meter (0.25 inch).

Since the ion engine used for period control will be thrust vector controlled, it is expected that the moment arm between the thrust vector and the center of mass can be maintained to within 0.00254 meter (0.1 inch).

It is recommended that a modulated flywheel with an angular momentum storage capacity of 2 lb/ft/sec be used for control about each axis. They should be unloaded only 25% of maximum momentum (0.5 lb/ft/sec) with a torque not exceeding 0.01 ft/lb. Approximately seven unloadings per day would be required due to the disturbance torques.

It is also recommended that couples be used for all control jets to avoid disturbing the orbits with translational thrusts. The total control impulse that would be required due to disturbance torques is 656 lb/sec/year. If a gas with an I_{sp} = 110 sec is used, the weight of the gas would be 5.97 lb/year.

4.4 HALO RESULTS

4.4.1 CONCLUSIONS - CONCEPT NO. 1

To minimize the disturbance torque for ΔV corrections due to a moment arm between the thrust vector and the center of mass, the moment arm should not exceed one-quarter inch.

It is recommended that a constant speed flywheel with an angular momentum storage capacity of 100 lb/ft/sec be used. This would require making precession control corrections on an hourly basis if the ΔV correction is imparted over a period of one full day. The maximum impulse that could be imparted from the precession control jets to the vehicle at one time to prevent the attitude error from exceed 0.1 degree due to the control action would be 0.087 lb/sec. The duration of this pulse should not exceed 0.00175 I_{xz} second.

For control about the pitch axis, it is recommended that a modulated flywheel with an angular momentum storage capacity of 2 lb/ft/sec be used. It should be unloaded only 25% of maximum momentum (0.5 lb/ft/sec) with a torque not exceeding 0.01 ft/lb. Approximately 6.5 unloadings will be required during each ΔV correction if the disturbance torque acts about the pitch axis.

It is recommended that couples be used for all control jets to avoid disturbing the orbits with translational thrusts.

The total impulse that would be requi ·ed for control action due to disturbance torques and precession control for satellite motion out of the Earth/Moon plane is 136.1 lb/sec/year. If a gas with an I_{sp} of 110 sec is used, this would require 1.25 lb/year of gas. The impulse required for constant speed flywheel spin up is 100 lb/sec. This requires 0.91 lb of gas.

4.4.2 CONCEPT NO. 2

4.4.2.1 Roll, Pitch and Yaw Flywheel Sizing

4.4.2.1.1 Solar Pressure Torques

Solar pressure torques are cyclic over a period of one year about the roll and yaw axes; they are also accumulative in pitch. To keep from unloading the wheels in the roll and yaw axes due to solar pressure torques would require flywheels with sufficient angular momentum storage to absorb the solar pressure disturbance torque for one-half year.

$$H_w = 0.03456 \text{ ft/lb/sec/day} \left(\frac{365}{2}\right) \text{days} = 6.35 \text{ ft/lb/sec}$$

This does not seem advisable since the weight of the gas required to remove the angular momentum due to solar pressure is at most 0.058 lb/year. Therefore, it would seem much more practical to use a 2 lb/ft/sec modulated flywheel and unload it 25% every 14.4 days.

4.4.2.1.2 Orbit Co· 'rol Torques

As mentioned before, orbit control thrusts will be required every seven days of the halo orbit. These thrusts will alternate from the positive roll to the negative roll axis. These thrusts could produce torques about the yaw axis or the pitch axis depending on where the moment arm is located. The torques produced by the positive roll jet on the yaw axis would cancel each other every one half orbit of the Earth, as would the torques produced by the negative roll jet.

To keep from unloading the wheels in the roll and yaw axes due to period contol torques, would require a minimum of

(1" M.A.)	H_w	=	13 lb/ft/sec
(1/4" M.A.)	H_w	=	3.24 lb/ft/sec
(0.1" M.A.)	H_w	=	1.3 lb/ft/sec

of momentum storage in the roll and yaw wheels. To the above numbers must be added 0.5 lb/ft/sec which is the angular momentum that would be absorbed by the wheels due to solar pressure over 14 days. The angular momentum required for the 0.1 inch moment arm falls within the 2 lb/ft/sec flywheel recommended previously.

If the 2 lb/ft/sec flywheel were used with the other two momentum arms, the number of flywheel unloadings required would be that shown in Appendix I.

4.4.2.2 Conclusions - Concept No. 2

To minimize the disturbance torques for ΔV corrections due to a moment arm between the thrust vector and the center of mass, the moment arm should not exceed one-quarter inch. The ΔV correction should be imparted over a period of not less than one hour so as not to exceed 0.1 degrees attitude error.

It is recommended that a modulated flywheel with an angular momentum storage capacity of 2 lb/ft/sec be used for control about each axis. The flywheels should be unloaded only 25% of maximum momentum (0.5 lb/ft/sec) with a torque not exceeding 0.01 ft/lb. Approximately 6.5 unloadings will be required during each ΔV correction.

It is recommended that couples be used for all control jets to avoid disturbing the orbits with translational thrusts.

4.5 CONCLUSIONS

The weight and power summaries for both concepts are given in Tables 4-1 and 4-2. It can be seen that the weight range is from 103 to 111 pounds and the power requirements vary from 91 to 107 watts. Both concepts are feasible and comparable in weight, power and complexity; either would be satisfactory for Halo or Hummingbird stabilization control.

Table 4-1. Hummingbird and Halo Orbit Attitude Control Subsystem Weight and Power Summary (Concept No. 1)

Component	Number Required	Weight/System lbs	Power/System watts	Development Status
Coarse Sun Sensor	2	1.8	---	OAO
Fine Sun Sensor	2	2.3	---	OAO
Canopus Star Sensor	1	12.0	8.0	Mariner
Earth Sensor	1	0	3.0	Barnes or Quantic (radiometric balance)
Inertial Package	1	7.5	14.5	OAO RAPS Package
Inertial Package Electronics	1	13.7	17.3	OAO RAPS Package
Control Electronics	1	20.0	20.0	To be developed
Solar Array Drive	1	12.0	14.0	Nimbus B
Pitch Axis Modulated Flywheel	1	10.0	5.0	OAO
Nutation Damper	1	8.0	---	Sperical Pendulum Damper
Pitch Axis Constant Speed Flywheel	1	20.0	25.0 ave. (72.0 starting)	
		111.3 (50.4856 kg)	106.8	

Table 4-2. Hummingbird and Halo Orbit Attitude Control Subsystem Weight and Power Summary (Concept No. 2)

Component	Number Required	Weight/System lbs	Power/System watts	Development Status
Coarse Sun Sensor	2	1.8	---	OAO
Fine Sun Sensor	2	2.3	---	OAO
Canopus Star Sensor	1	12.0	8.0	Mariner
Earth Sensor	1	4.0	3.0	Barnes or Quantic (radiametric balance)
Inertial Package	1	7.5	14.5	OAO RAPS Package
Inertial Package Electronics	1	13.7	17.3	OAO RAPS Package
Control Electronics	1	20.0	20.0	To be developed
Solar Array Drive	1	12.0	14.0	Nimbus B
Modulated Flywheels	3	30.0	15.0	OAO
		103.3 (46.8568 kg)	91.8	

SECTION 5

PROPULSION STUDIES

5.1 INTRODUCTION

This report documents the study results of propulsion systems required for the Lunar Libration Point Satellite. Spacecraft propulsion for this mission is required for the performance of the following functions:

a. Midcourse correction and lunar orbit injection

b. Lunar orbit maintenance

c. Spacecraft attitude control

Typical propulsion systems for each of these functions are selected, described and evaluated using a figure of merit comparison. Additionally, launch vehicle cost and payload to lunar transfer trajectory data (as supplied by NASA) are presented.

5.2 LAUNCH VEHICLES FOR EARTH ORBIT AND LUNAR TRANSFER TRAJECTORY

The objective of the launch vehicle study was the selection and tabulation of boosters and upper stage combinations having a broad range of payload capability into a lunar transfer trajectory. The launch trajectory assumed an Eastern Test Range injection into a 185.2-kilometer parking orbit. This was followed by an insertion into the lunar transfer trajectory which required a total inertial velocity of 10, 942.32 m/sec.

The selection of boosters for the tabulation was restricted to the Delta, Atlas, and Titan families with various applicable upper stages. Included in the tabulation are various proposed launch vehicle configurations potentially available within the 1973-75 time period. Table 5-1 contains the launch vehicle tabulation along with the respective payload capability and approximate recurring cost data.

Table 5-1. Launch Vehicle Capabilities Summary

Launch Vehicle	Escape Payload* (lbs)	Replacement Costs ($M)
TAT-Delta - 3 Castors - FW 4	380	3.01
TAT-Delta - 3 Castors - TE 364	470	3.07
SLV3A - Burner 2	625	4.3
TAT-Delta - 6 Castors - TE 364	710	3.36
TAT-Delta - 9 Castors - TE 364	830	3.62
TAT-Delta - 3 Castors - HOSS** - TE 364	1150	3.76
*** TAT-Delta - 6 Castors - HOSS	1280	4.05
TAT-Delta - 9 Castors - HOSS	1480	4.31
Titan 3X - Agena	2300	8.6
SLV3C - Centaur	2800	10.8
SLV3X - Centaur	4800	10.8
Titan 3C	5000	17.2
Titan 3D - Centaur	12500	16.8

*ETR launch with 185.2 km parking orbit

**HOSS - Hydrogen Oxygen Second Stage

***Proposed class of launch vehicles

5.3 PROPULSION SYSTEM CHARACTERISTICS

The objective of this section is to describe the general configuration, performance, weight and power requirement characteristics for three types of propulsion systems which are the most likely candidates for fulfillment of one or more of the following spacecraft functions:

a. Midcourse correction and lunar orbit injection

b. Lunar orbit maintenance

c. Spacecraft attitude control

5.3.1 BIPROPELLANT PROPULSION SYSTEMS

Bipropellant propulsion systems are used for all thrust levels down to a practical minimum of 2 lb_f. Most current state-of-the-art low thrust systems, approximately 1000 lb_f and below, utilize a hypergolic propellant combination of monomethyl hydrazine (MMH) as the fuel and nitrogen tetraoxide (N_2O_4) as the oxidizer; such fuels operate at an oxidizer to fuel mixture ratio (by weight) of 1.6. Vacuum specific impulses of 285 to 295 lb_f-sec/lb_m are typically achieved for this propellant combination. MMH has a propellant density of 54.4 lb_m/ft^3 while N_2O_4 has a density of 90.5 lb_m/ft^3. This propellant combination has the advantage of occupying equal propellant volumes (equal size propellant tanks) at the operating mixture ratio and has a high overall bulk propellant specific gravity of 1.23. MMH freezes at a temperature of -63°F and boils at 189°F while N_2O_4 freezes at 11.8°F and boils at 70°F. A schematic of a typical bipropellant propulsion system is shown in Figure 5-1.

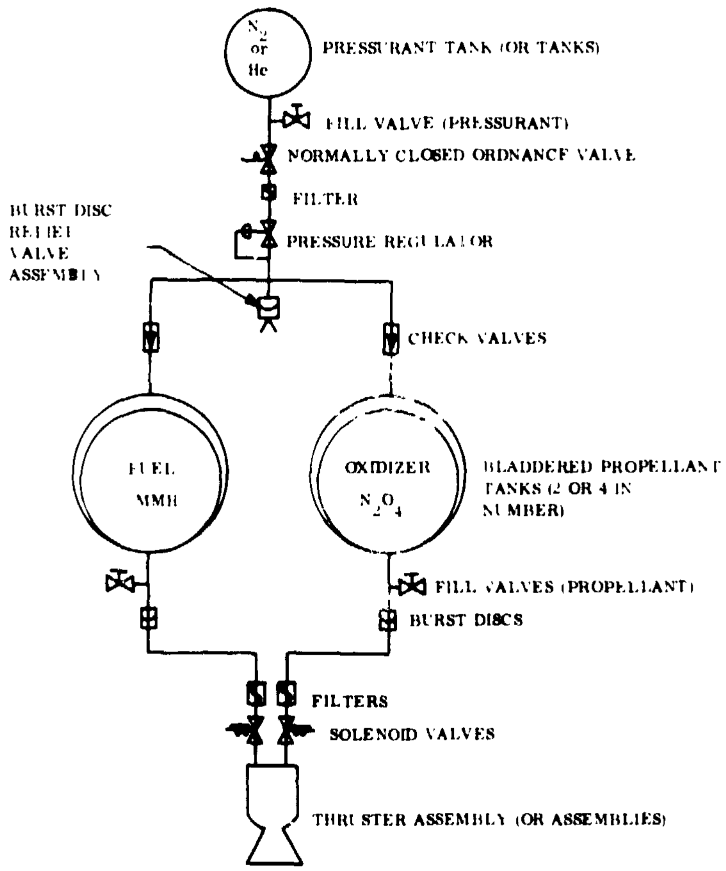


Figure 5-1. Typical Bipropellant Propulsion System Schematic

A curve of bipropellant system weight plotted as a function of total impulse is presented in Figure 5-2. A least-square curve fit of this plot results in the following mathematical equation for propulsion system weight:

$$W_{P.S.} = C_1 + C_2\left(\frac{I_T}{10^3}\right) + C_3\left(\frac{I_T}{10^3}\right)^2 + C_4\left(\frac{I_T}{10^3}\right)^3 + C_5\left(\frac{I_T}{10^3}\right)^4 + N(W_{TH})$$

where $W_{P.S.}$ = propulsion system weight - lbs

I_T = total impulse required - lb-sec

N = number of thrusters required

W_{TH} = weight of each thruster (determined from Figure 5-3) - lbs

C_1 = 14.49

C_2 = 5.755

C_3 = -0.4179×10^{-1}

C_4 = 0.3077×10^{-3}

C_5 = -0.4626×10^{-6}

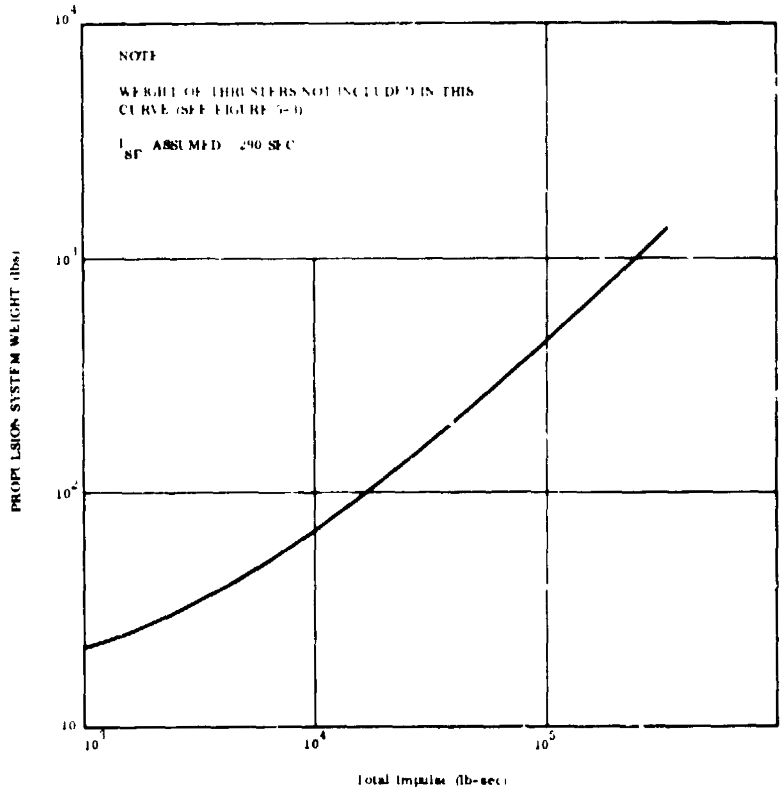


Figure 5-2. Bipropellant (MMH/N_2O_4) Propulsion System Weight vs Total Impulse

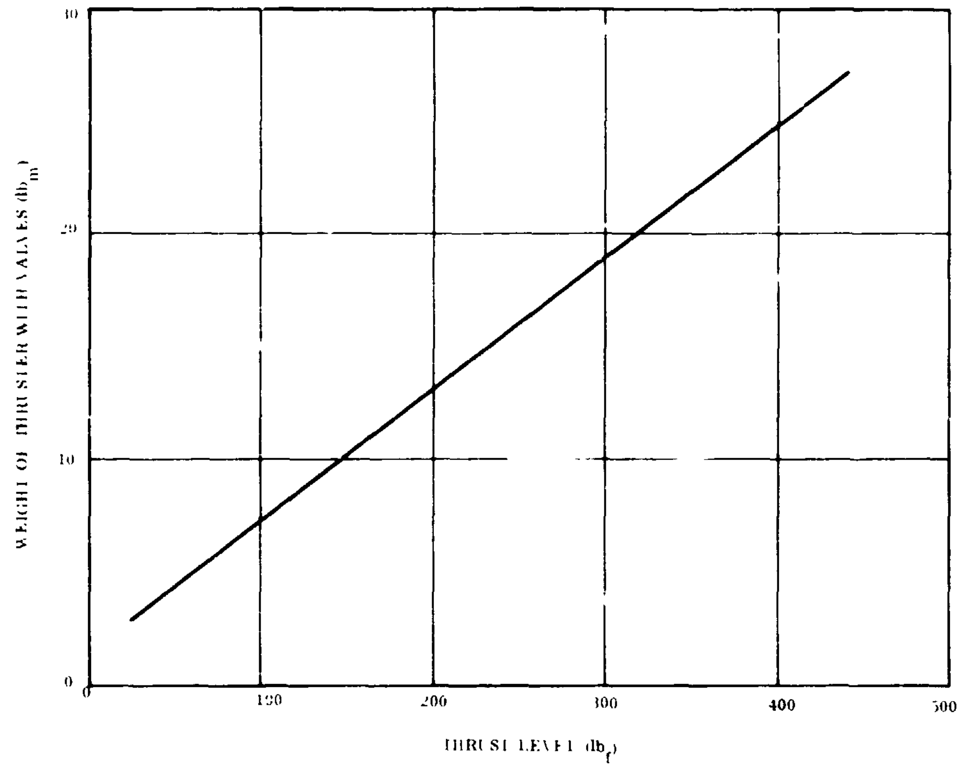


Figure 5-3. Bipropellant (MMH/N_2O_4) System Thruster Weight vs Thrust Level

Power requirements for a bipropellant system are minimal. Each solenoid valve can be estimated to draw approximately 10 watts of electrical power. This results in a maximum estimated power requirement of 50 watts for the system.

5.3.2 MONOPROPELLANT PROPULSIONS SYSTEMS

Monopropellant hydrazine propulsion systems have been designed throughout the thrust range of 0.05 lb_f to 500 lb_f. Hydrazine thrusters utilize Shell 405 as the spontaneous catalyst which decomposes the hydrazine into 1800°F gases consisting of ammonia, nitrogen and hydrogen. Hydrazine (N_2H_4) has a propellant density of 63 lb/ft^3, a freezing temperature of 35.1°F and a boiling temperature of 236°F. A schematic of a typical monopropellant propulsion system is shown in Figure 5-4.

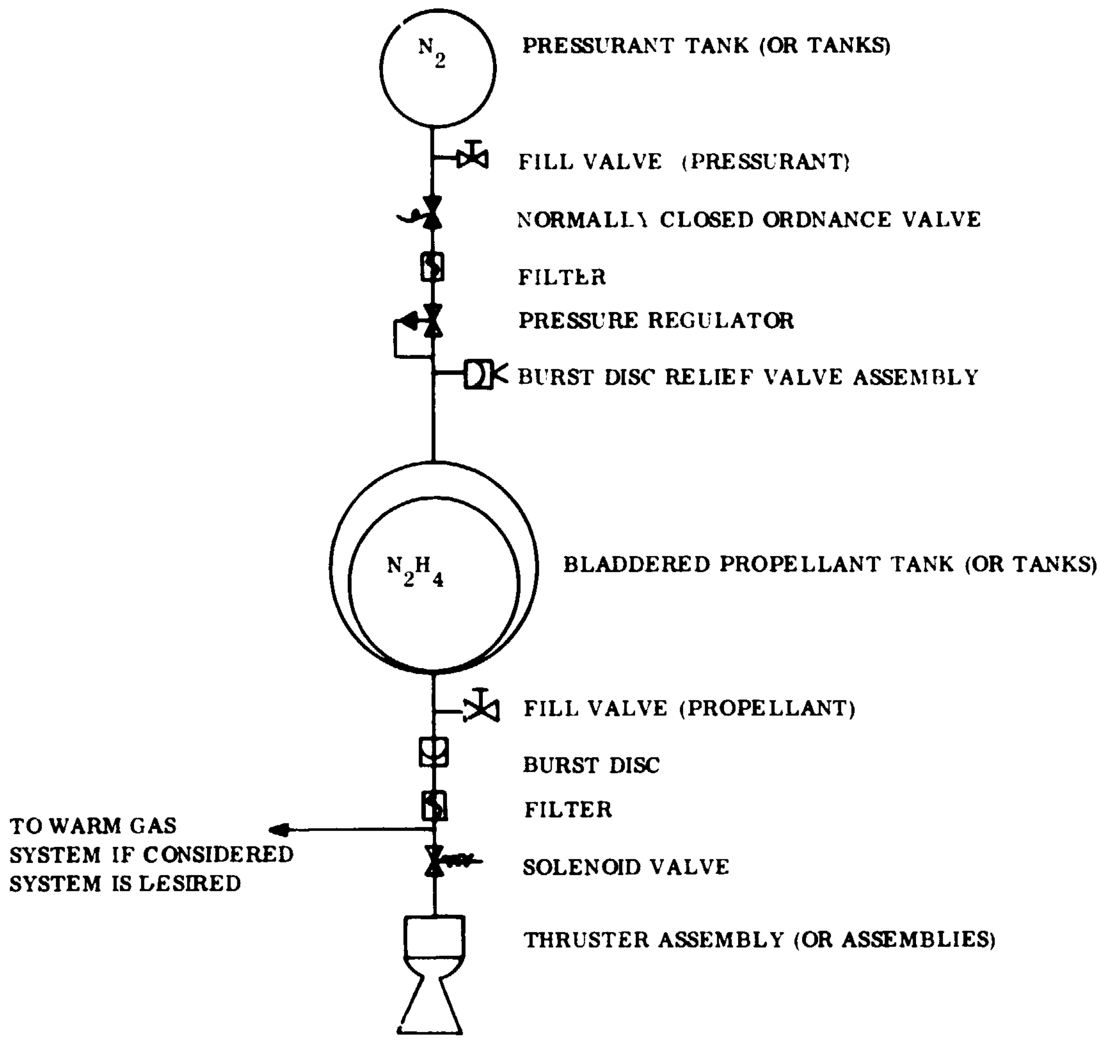


Figure 5-4. Typical Monopropellant Propulsion System Schematic

A curve of monopropellant system weight plotted as a function of total impulse is presented in Figure 5-5. A least-square curve fit of this plot results in the following equation for system weight:

$$W_{P.S.} = C_1 + C_2\left(\frac{I_T}{10^3}\right) + C_3\left(\frac{I_T}{10^3}\right)^2 + C_4\left(\frac{I_T}{10^3}\right)^3 + C_5\left(\frac{I_T}{10^3}\right)^4 + N(W_{TH})$$

where $W_{P.S.}$ = propulsion system weight - lbs

I_T = total impulse required - lb-sec

N = number of thrusters required

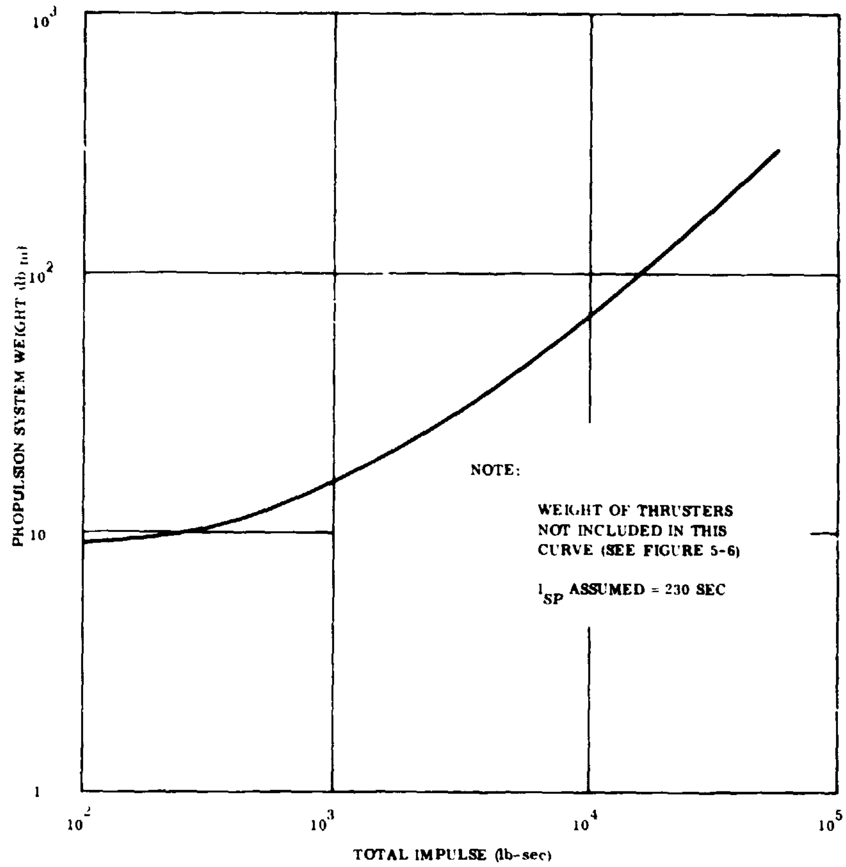


Figure 5-5. Monopropellant (N_2H_4) Propulsion System Weight vs Total Impulse

W_{TH} = weight of each thruster - lbs (determined from Figure 5-6)

C_1 = 7.049

C_2 = 8.967

C_3 = -0.525

C_4 = 0.2866×10^{-1}

C_5 = -0.3897×10^{-3}

Power requirements are limited to solenoid valves each of which requires approximately 10 watts of electrical power. This results in a maximum estimated power requirement for the system of 50 watts.

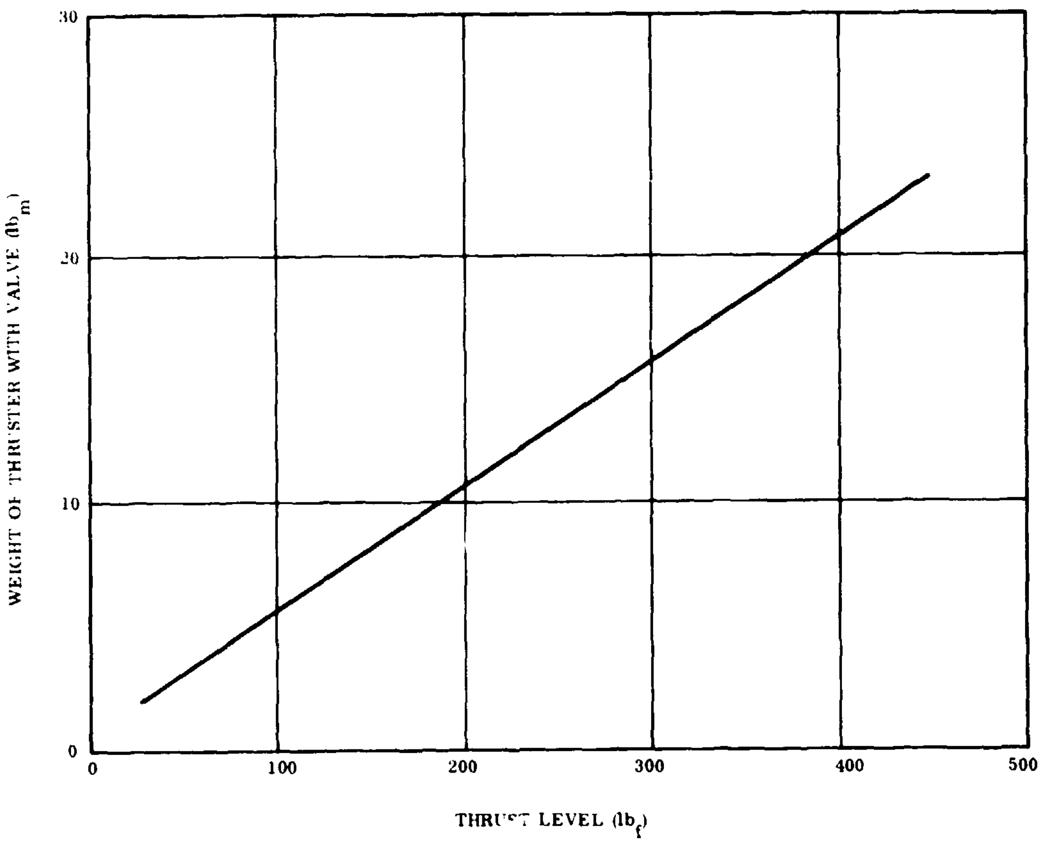


Figure 5-6. Monopropellant (N_2H_4) System Thruster Weight vs Thrust Level

Another monopropellant system not considered in this study but which may be available by the 1973-75 time period is the one using a propellant blend consisting of 76% N_2H_4 and 24% $N_2H_5NO_3$. The advantages of this propellant over hydrazine are a specific gravity of 1.106, a freezing temperature of $+2^{\circ}F$ and a propulsion system specific impulse in the range of 250 to 255 seconds.

5.3.3 ION ENGINE

There are only two fully developed flight systems at this time. One system is a 5-20 micro lb unit built by Electro-Optics for NASA's ATS-D flight. The second system is in final testing by NASA-Lewis for the SERT-II which is scheduled for some time in 1969.

EOS Micro-Thrust System - Thrust level at 5 to 20 micro lb is too low for use in the Lunar Libration Spacecraft. This is a cesium contact ion engine for which the thrust can be varied in 5 micro lb steps from 5 to 20 micro lb. The power level is 34 watts total at 20 micro lb. The total energy capability is low since this was one of the experiments flown on ATS-D and presumbaly will also be on board ATS-E.

SERT-II ION Engine System- The SERT-II ion engine has a thrust of 6.2 m lb at a total power input of 900 to 1000 watts. This thrust level is approximately that required for the Hummingbird-Lunar stationkeeping application. The engine is a mercury-electron-bombardment thruster with a 15-cm diameter discharge chamber, a mercury plasma-bridge neutralizer, and a pressurized mercury propellant tank. The tank carries 30.8 lb of mercury, a nine month supply, although program goal is continuous operation for six month. The total system loaded weight 40 lb plus 5 to 10 lb for the mechanical gimbal system. The mechanical gimbal operates in two planes to $\pm 10^{\circ}$. Pointing accuracy only requires control to the nearest degree.

A complete prototype thruster has been operated for 1000 hours without failure. Additional certification testing is in progress. Separate key components of the thruster have been tested from 2000 to 3400 hours. As a consequence, it is estimated that the thruster system has a potential life of 10,000 hours. A three year life requirement for the Hummingbird application amounts to 26,280 hours. Figure 5-7 shows some detail as to the assemblage of the thruster. Two complete units will be flown in SERT-II.

5.3.3.1 Hummingbird System Size

The Hummingbird thruster would have a thrust of 6 m lb with a specific impulse of 4550 to 4650 sec. Total hardware weight would be about 42 lb including two thrusters at 65 to 70 lb each, a main propellant tank 10 inches in diameter and weighing 6 lb; a power conditioning and control panel 20" x 20" x 4" and weighing about 18 lb; plus a support structure. The thruster would be cylinders 9" x 9" in size. By 1971 electrical thrust vector control may be possible at no increase in weight.

5.3.3.2 Major Problems

The only major problem would be whether or not electrical gimballing will be proven out by 1971. This type of thruster, being multi-apertured, is not so readily controlled as

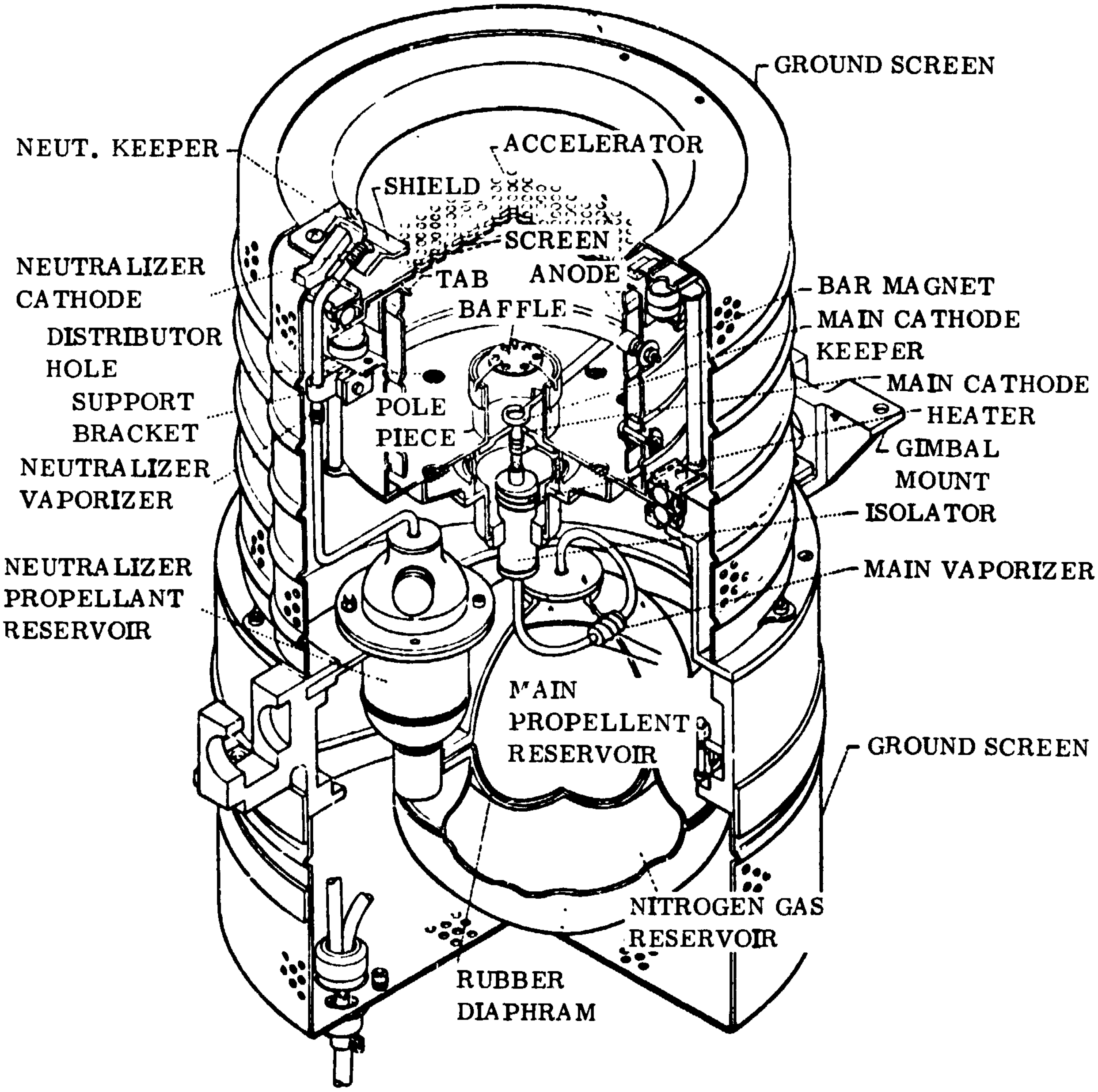


Figure 5-7. SERT II Thruster System, Anode Diameter 15 Centimeters

are "slit" and "button" type thrusters for thrust vector control. Micro thrusters of 10 micro lb and the ATS-D 5 to 20 micro lb thrusters are easily thrust vector controlled (TVC) by segmenting the accelerating electrode. "Slit" thrusters of 0.3 to 0.8 m lb can also be electrically thrust vector controlled. NASA-Lewis is currently engaged in research to determine if a successful electrical approach can be developed for TVC. Electro-Optics (EOS) is exploring a thermal shift of position of the accelerating electrode screen as a means of achieving TVC.

Extension of total life from 10,000 hours to 26,280 hours sounds formidable but Lewis feels this can be easily accomplished.

5.4 MISCOURSE CORRECTION AND LUNAR ORBIT INJECTION

5.4.1 REQUIREMENTS

The spacecraft propulsion system requirements for a lunar transfer trajectory terminating in an orbit near the L_2 location were assumed to be those associated with a near optimum trajectory mode as defined in the mid-term report. The mode chosen was a close lunar fly-by which requires that this velocity impulse must be imparted within a 10-minute time interval for maximum propulsion effectiveness. Also assumed was the use of a "fast" trajectory which requires 8.57 days for accomplishing the Earth to L_2 transfer.

The mission velocity impulse requirements for the trajectory to L_2, as shown in Table 5-2, total 375.51 m/sec. This value was used for all subsequent propulsion system weight calculations.

Table 5-2. Velocity Impulse Requirements Summary, Moon to L_2

Propulsion Requirement	Velocity Impulse (m/sec)
Earth-Moon Midcourse Correction	36.57
Velocity Impulse Near Moon	190.80
Moon-L_2 Midcourse Correction	6.096
Orbit Establishment Near L_2	142.03
Total	375.49

Figure 5-8 presents a plot of propellant requirements as a function of needed velocity impulse for a typical bipropellant and monopropellant system. As shown in the figure, the weight of propellant required to impart a velocity impulse of 375.49 m/sec to a spacecraft will consume approximately 14% of the spacecraft weight using a bipropellant propulsion system and approximately 18% using a monopropellant propulsion system. These percentage values represent reasonable weight allocation requirements for this propulsion function. Therefore, these two types of propulsion systems will be compared in the subsequent tradeoff studies for performing the function of lunar orbit injection.

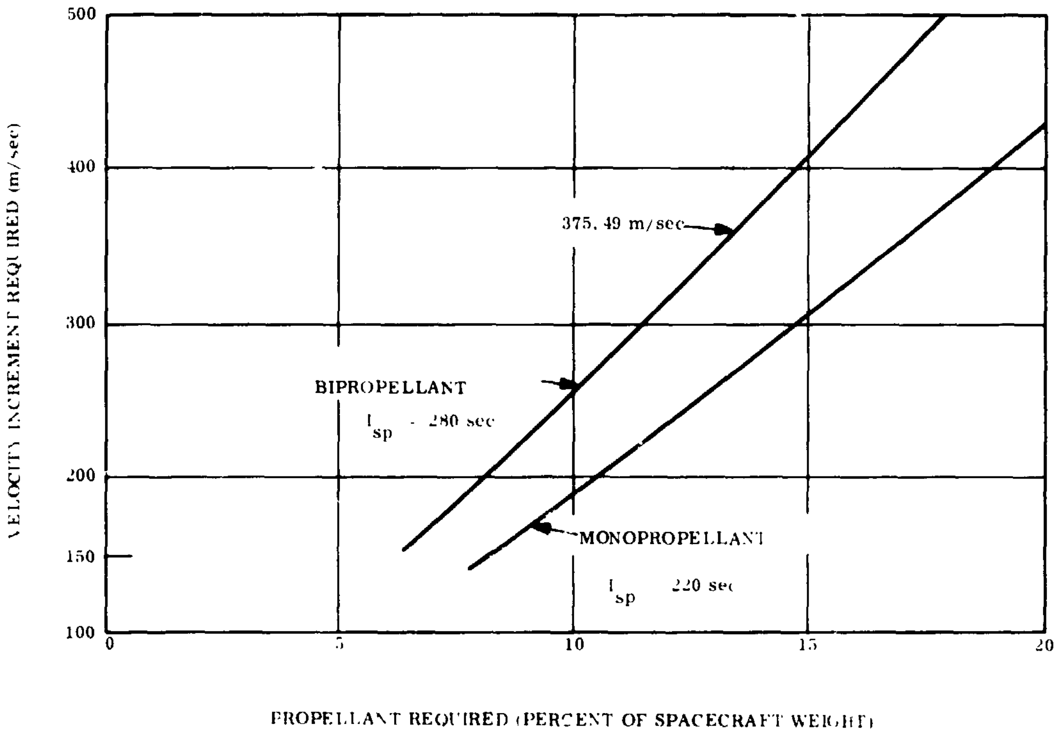


Figure 5-8. Mid-course Correction and Lunar Orbit Injection Propellant Requirements

5.4.2 HALO ORBIT

Based on subsystem weight estimates, a spacecraft weight in lunar transfer trajectory of 1200 lbs was assumed for the Halo orbit concept. Using this weight plus the velocity impulse requirement, the total impulse that the propulsion system must deliver can be calculated as follows:

$$I_T = \Delta V\left(\frac{W}{g}\right) = 375.49 \text{ m/sec } \frac{1200 \text{ lbs}}{9.81 \text{ m/sec}^2}$$

$$= 47,400 \text{ lb-sec}$$

A constraint on the propulsion system is that the velocity impulse at the Moon be delivered within 10 minutes. The impulse requirement for this fly-by is:

$$I_T = 190.8 \times \frac{1200}{9.81} = 23,400 \text{ lb-sec}$$

Therefore, the minimum thrust level required is:

$$F = \frac{I_T}{\text{TIME}} = \frac{23,400 \text{ lb-sec}}{600 \text{ sec}} \approx 40 \text{ lbs}$$

Two types of propulsion systems, bipropellant and monopropellant, were evaluated for this function. The characteristics of each were determined using Section 5.3 of this report. Assuming specific impulses of 280 seconds and 220 seconds, propellant weights of 170 lbs and 216 lbs for the bipropellant and monopropellant system, respectively, are required to perform this function. The bipropellant system has the advantage of lesser complexity and of proving propellant commonality for other spacecraft functions requiring low thrust levels. Halo orbit propulsion system weights and performance levels of the two systems are determined from the Section 5-3 data are summarized in Table 5-3.

5.4.3 HUMMINGBIRD ORBIT

A spacecraft weight of 1300 lbs in lunar transfer trajectory was assumed for the Hummingbird orbit concept. The total impulse required for this spacecraft weight was calculated to be 49,800 lb-sec. The thrust level required for delivering the lunar fly-by velocity impulse within 10 minutes is approximately 55 lbs. Bipropellant and monopropellant systems requiring 180 to 230 lbs of propellant respectively were chosen for evaluation. Hummingbird orbit propulsion system weights and performance levels for the two considered systems are summarized in Table 5-3.

Table 5-3. Spacecraft Propulsion System Summary

Propulsion Function	ΔV Req'd (ft/sec)	Spacecraft Weight (lbs_M)	Total Impulse (lb_f-sec)	System No. 1 Propulsion Type	System No. 1 Thrust (lbs_F)	System No. 1 Specific Impulse (sec)	System No. 1 Propellant Weight (lbs_M)	System No. 1 System Weight (lbs_M)	System No. 2 Propulsion Type	System No. 2 Thrust (lbs_F)	System No. 2 Specific Impulse (sec)	System No. 2 Propellant Weight (lbs_M)	System No. 2 System Weight (lbs_M)
HALO ORBIT													
1. Lunar Orbit Injection	375.51	1200	47,400	Bipropellant	40	280	170	245	Monopropellant	40	220	216	500
2. Lunar Orbit Maintenance (3 yrs)	255.11	1000	26,000	Monopropellant	0.1	200	130	200	Monopropellant	0.1	200	130	
3. Attitude Control (3 yrs)	--	1000	2,000	Monopropellant	0.1	125	16		Monopropellant	0.1	125	16	
								Total 445 lbs					Total 500 lbs
HUMMINGBIRD ORBIT													
1. Lunar Orbit Injection	375.51	1300	49,800	Bipropellant	55	280	180	260	Monopropellant	55	220	230	360
2. Lunar Orbit Maintenance (3 yrs)	4541.52	1100	510,000	Ion	5.4×10^{-3}	4500	114	155	Ion	5.4×10^{-3}	4500	114	155
3. Attitude Control (3 yrs)	--	1100	3,000	Monopropellant	0.1	125	24	45	Monopropellant	0.1	125	24	
								Total 460 lbs					Total 515 lbs

5.5 LUNAR ORBIT MAINTENANCE

5.5.1 HALO ORBIT

Stationkeeping for the spacecraft in a Halo orbit requires imparting a velocity impulse of 1.76 m/sec every one-half orbit for the purpose of maintaining orbit period control. Assuming a three year spacecraft mission design life, the total velocity impulse required is 255.11 m/sec. In the previous section the assumed spacecraft weight was given as 1200 lbs. However, approximately 200 lbs of propellant were expended for the functions of midcourse correction and lunar orbit injection. Therefore, the spacecraft weight in Halo orbit is estimated to be 1000 lbs. Thus, the total impulse required for the propulsion system is:

$$I_T = 837 \times \frac{1000}{32.2} = 26{,}000 \text{ lb-sec}$$

Because of the precision antenna pointing accuracy tolerance requirements, the orbit period control function must be achieved using a propulsion system operating at a relatively low thrust level. A value of 0.1 lb thrust was chosen as the approximate maximum level which will fulfill the spacecraft requirements. Thruster firing durations of 30 minutes every 7-1/2 days are required at this thrust level to control the orbit period.

At this thrust level and firing duration, hydrazine thrusters exhibit superior performance at minimum power requirements when compared to other applicable propulsion systems utilizing cold gas or electrically heated gas. A Hamilton Standard designed hydrazine thruster which operates at 0.05 to 0.1 lb thrust is undergoing development and is scheduled to fly on a military spacecraft before 1970. The thruster delivers a steady state specific impulse of 200 seconds and requires less than five watts of electrical power to operate the flow control solenoid valve. The weight of the thruster with valve is approximately 0.2 lbs. Because the hydrazine thruster performance, weight and power requirements characteristics associated with this low thrust level are difficult to equal with other types of propulsion systems, only the hydrazine system was considered for the propulsion system tradeoffs for supplying orbit maintenance to the Halo concept spacecraft.

The total impulse requirement for orbit maintenance, as calculated earlier in this section,is 26,000 lb-sec for a three year mission. Using a specific impulse of 200 sec, the weight of hydrazine propellant required is 130 lbs. The propulsion system weights, as combined with other required propulsion functions, are summarized in Table 5-3.

5.5.2 HUMMINGBIRD ORBIT

Stationkeeping for a spacecraft in a Hummingbird orbit requires the imparting of a continuous acceleration of 4.8×10^{-5} m/sec^2. The spacecraft weight in Hummingbird orbit was previously assumed to be 1300 lbs in a lunar transfer trajectory. As shown in Section 5.4 of this report, a propellant weight of approximately 250 lbs is required to position the spacecraft at the Hummingbird station. Therefore the spacecraft weight for the purpose of stationkeeping is approximately 1100 lbs.

The continuous thrust level of the propulsion system is calculated as follows:

$$F = \frac{1100}{32.2} (1.575 \times 10^{-4})$$

$$= 5.5 \times 10^{-3} \text{ lbs}$$

Assuming a three year spacecraft mission design life, the total impulse required from the propulsion system is 510,000 lb-sec. The total velocity impulse required is 14,900 fps. Using Figure 5-9, which presents the propellant required plotted as a function of propulsion system delivered impulse, the following table can be generated:

Propulsion Type	Specific Impulse (sec)	Propellant Required (% of Spacecraft Weight)
Ion Engine	4500	10
SPET	1200	32
Colloid Engine	1000	37

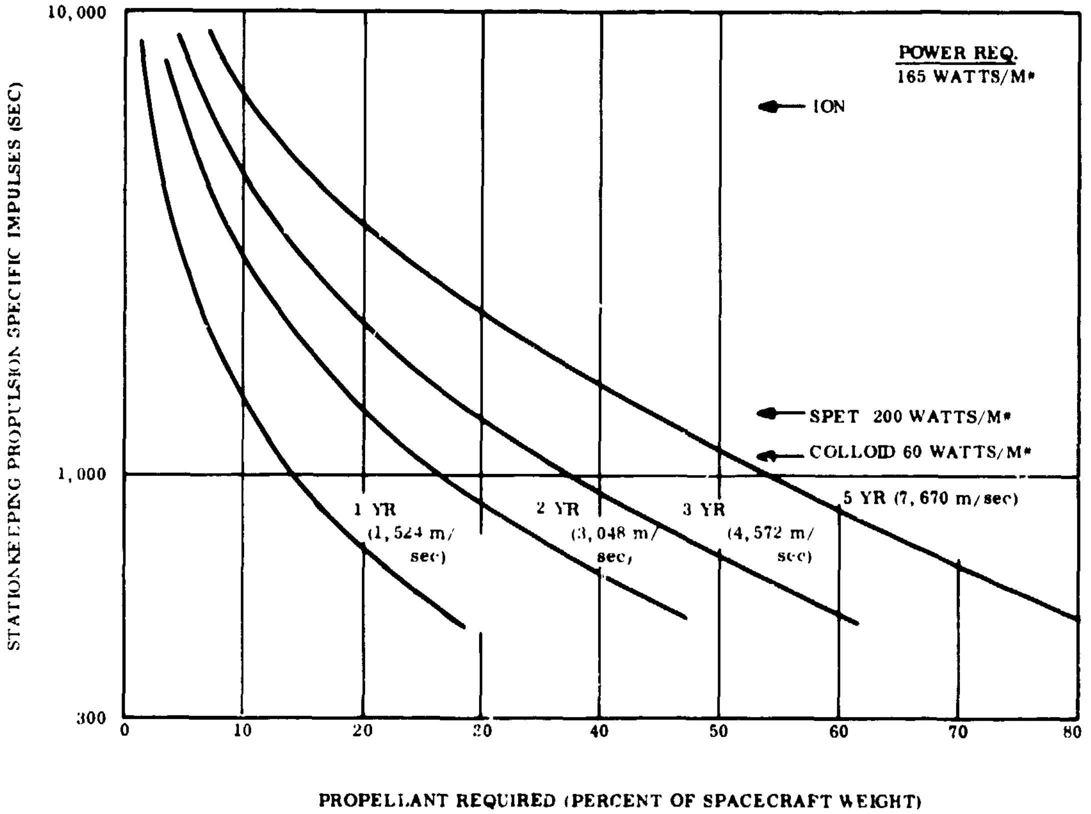


Figure 5-9. Propellant Required (Percent of Spacecraft Weight) as a Function of Propulsion System Delivered Impulse

For the three year mission, the ion engine offers a significant weight advantage over both the SPET and Colloid engines. Additionally, an ion engine capable of delivering a thrust in excess of six millipounds has been built and tested; the largest SPET and Colloid engine built have thrust capabilities in the range of 1 to 50 micropounds . Therefore, for the purpose of this study, only the ion engine will be considered for the propulsion device to supply the station maintenance required by the Hummingbird orbit.

Table 5-3 presents the propulsion system weights required for delivery of a velocity impulse of 4,541.52 m/sec to the spacecraft. Assuming use of a backup thruster to insure three year life, the total weight of the ion engine propulsion system is estimated to be 181 pounds.

5.6 ATTITUDE CONTROL

Attitude control propulsion is required for this spacecraft to perform the function of unloading reaction wheels. The total impulse requirement for this function is relatively small; the requirement was estimated at approximately 2000 lb-sec for the Halo orbit spacecraft and 3000 lb-sec for the Hummingbird orbit spacecraft. Thrust level requirements were estimated to be in the range of 0.05 to 0.1 lbs, making the choice of a hydrazine system the most attractive approach. Operating at this thrust level and in a pulse mode, an average specific impulse of 125 seconds was assumed in determining the weight of propellant required. Table 5-3 summarizes the propulsion system weights and performance for performing attitude control on the Halo and Hummingbird spacecraft.

5.7 FIGURE OF MERIT PROPULSION SYSTEM TRADEOFF

The following figure of merit model was used in the propulsion system tradeoff conducted on the Lunar Libration Point Satellite:

$$\mathrm{FOM} = \frac{I_t R}{\left[W_1 + W_2 + W_3\right] Q + \left[D + NR\right]} \qquad \left(\frac{\text{lb-sec}}{\$}\right)$$

where I_{tot} = total impulse, lb-sec

R = reliability for required firing duty cycle

W_1 = hardware weight, propulsion

W_2 = equivalent power weight

W_3 = propellant weight

Q = dollar value of one pound in specific spacecraft

D = non-recurring (development) cost

R = recurring cost per system

N = number of systems required

Q is a factor which considers other subsystems, the mission, the booster, and possibly the national needs reflected in the mission. A pound may change the mission lifetime, change the booster, curtail other functions, etc.

An attempt to evaluate Q on the basis of launch vehicle costs was made as shown in Figure 5-10. Launch vehicle costs used were those presented in Table 5-1. The slope of the cost line in the vicinity of a 1200-lb spacecraft was determined at \$700 per pound, and that of a 1300-pound spacecraft was also \$700 per pound. These two values of Q were then used for evaluation of the Halo orbit and Hummingbird orbit spacecraft.

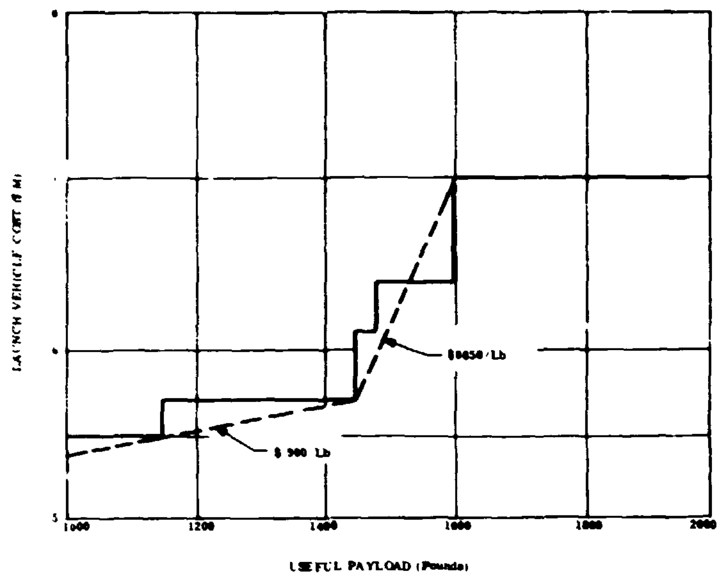


Figure 5-10. Launch Vehicle vs Payload to Lunar Transfer Injection

Table 5-5 summarizes the values used for calculating the figure of merit for the two spacecraft:

Table 5-5. Propulsion System Figure of Merit Parameters

Factor	Halo Spacecraft		Hummingbird Spacecraft	
	System No. 1	System No. 2	System No. 1	System No. 2
I_T	75,400	75,400	562,800	562,800
R	0.97	0.97	0.95	0.95
W_1	129	138	154	158
W_2	20	20	240	240
W_3	316	362	318	368
Q	700	700	700	700
D	2,000,000	1,500,000	4×10^6	3.5×10^6
R	325,000	250,000	5×10^5	4.25×10^4
N	4	4	4	4

Development and recurring cost data were based on ROM values obtained from such propulsion vendors as Rocketdyne, Marquardt, Aerojet-General, Hughes, and EOS. The number of systems required was estimated to be four: prime, spare, qualification, and engineering units. Table 5-6 summarizes the types of propulsion systems evaluated and the respective figure of merit values calculated for each:

Table 5-6. Propulsion System Figure of Merit Results

Function	Halo Spacecraft		Hummingbird Spacecraft	
	System No. 1	System No. 2	System No. 1	System No. 2
Lunar Orbit Injection	Bipropellant	Monopropellant	Bipropellant	Monopropellant
Lunar Orbit Maintenance	Monopropellant	Monopropellant	Ion	Ion
Attitude Control	Monopropellant	Monopropellant	Monopropellant	Monopropellant
Total Propulsion Weight	445	500	460	515
Figure of Merit	0.0202	0.0255	0.824	0.0934

The figure of merit calculation for the 1200-lb Halo spacecraft has a higher value for the heavier all-monopropellant propulsion system (System No. 2) as compared to a mixed bipropellant/monopropellant system (System No. 1). This is also true for the 1300-lb Hummingbird spacecraft, although the magnitude of the figure of merit difference is significantly less for the heavier spacecraft. However, if the weight of the Hummingbird spacecraft approaches or exceeds 1600 lbs, the value of Q should be readjusted, bearing in mind that a 65-lb weight savings for the bipropellant/monopropellant system could mean use of a lower cost launch vehicle. The plot contained in Figure 5-10 shows that the 1200-lb Halo spacecraft is at the lower end of a particular launch vehicle's capability which extends to a value 1450 lbs. Therefore, the payload penalty of 55 lbs attributed to monopropellant propulsion for the Halo spacecraft is probably insignificant.

5.8 SUMMARY AND CONCLUSIONS

The preceding sections have described propulsion systems capable of providing the following Halo and Hummingbird orbit concept spacecraft functions:

a. Midcourse correction and lunar orbit injection

b. Lunar orbit maintenance

c. Spacecraft attitude control

Two types of systems were described, and weight estimates made for each of the two orbit concepts. A figure of merit rating was then made for each of the two systems based on a propulsion system figure of merit model.

The following conclusions regarding types of propulsion were made as a result of this study.

5.8.1 HALO ORBIT - 1200 LB SPACECRAFT

- An integrated monopropellant hydrazine propulsion system utilizing thrusters of several sizes appears to be the most attractive system for supplying all three of the required spacecraft functions.

5.8.2 HUMMINGBIRD ORBIT - 1300 LB SPACECRAFT

- An ion engine is the only reasonable choice for the propulsion system to supply the function of lunar orbit maintenance for the spacecraft.

- An integrated monopropellant hydrazine propulsion system utilizing two sizes of thrusters has the highest figure of merit rating for supplying the remaining two spacecraft functions. However, as the spacecraft weight approaches or exceeds 1600 pounds, the weight savings attributed to a combination bipropellant-monopropellant system may favor use of this system over the heavier integrated monopropellant propulsion system.

SECTION 6
SYSTEM INTEGRATION AND EVALUATION

The goal of the system integration and evaluation work was to synthesize systems for both the Halo and Hummingbird concepts and to select one of these as the preferred system.

The Halo and Hummingbird systems were synthesized from the selected subsystems which were described by mathematical models. The systems were compared and the preferred system selected. This section details the subsystem modeling, the system synthesis, and evaluation.

6.1 SUBSYSTEM MODELING

The spacecraft can be synthesized from the following subsystems:

a. Antenna

b. Transponder/TT&C

c. Electrical Power

d. Thermal Control

e. Midcourse/Insertion Propulsion

f. Orbit Maintenance/Stabilization Propulsion

g. Attitude Control

h. Structures

The mathematical models for weight, fabrication costs and engineering development costs for four of these subsystems (Antenna, Electrical Power, Thermal Control and Structures) are given in Appendix II. These models were developed by General Electric, under General Electric discretionary funds, and represent estimates on weight and costs at the subsystem level. The models have been used in connection with Contract NAS3-9708 for NASA-Lewis. All models are based on 1971 technology and are assumed to have a three-year lifetime. The remaining subsystems' characteristics have been estimated by the cognizant subsystem engineer on the study. The components of the transponder are also given in Appendix II.

6.2 SYSTEM SYNTHESIS

Using t e subsystem models, the subsystem engineers' estimates, and the power requirements given in Table 6-1, it was possible to synthesize spacecraft for both the Halo and Hummingbird concepts. The weight and relative cost figures are given in Table 6-2 for both systems designed for a three-year lifetime. By comparing the weights of each system to the payload characteristics of the candidate boosters, it can be seen that the Halo Orbiter can be boosted into orbit with a TAT Delta +3 castors + HOSS; the Hummingbird requires the 6-Castor version. The relative summation of the subsystem costs for each concept is given in Table 6-3.

6.3 SYSTEM EVALUATION

In order to compare the two concepts, certain parameters must be evaluated and combined into a figure of merit to provide a common scale for comparison. The major parameters are the weight, fabrication costs, engineering development costs, complexity and size. By assigning relative weight factors for the spacecraft, ground, and user, and weighting factors for the subsystems and major parameters for each, the figures of merit for both Halo and Humming rd could be obtained as shown in Table 6-4.

The procedures in this case can be substantially simplified, however, due to the commonality of many spacecraft subsystems and ground and user requirements. Those elements that are common can be eliminated from the evaluation and, hence, to first order only the spacecraft's major parameters themselves can be compared. The major parameters are given in Table 6-5.

Table 6-1. Power Requirements

Electrical Power	Power (W) Halo	Power (W) Hummingbird
Transponder	320	320
Stationkeeping Prop. System	65	870
Attitude Control	100	100
TOTAL	485	1290

Table 6-2. Weight and Cost Comparison

Subsystem	Weight (lb)		Relative Fabrication Costs ($)		Relative Development Cost ($)	
	Halo	Hummingbird	Halo	Hummingbird	Halo	Hummingbird
Antenna	21	21	.037	.037	.048	.048
Transponder/TT&C	17	17	.104	.104	.075	.075
Electrical Power without Harness	194	307	.252	.605	.384	.705
Thermal Control	26	26	.004	.004	.011	.011
Midcourse/Insert. Prop.	500	360	.104	.187	.086	.229
Stationkeeping Prop.		155	.083	.166	.046	.229
Attitude Control	126	134	.384	.384	.120	.120
Structures & Harness	200	227	.031	.036	.030	.033
TOTALS	1084 (Est. 1100)	1237 (Est. 1300)	1.000	1.530	1.000	1.450

Table 6-3. Relative Costs

Program Costs	Relative Cost ($) Halo	Hummingbird
Launch Vehicle	.274	.296
Sat. Fabrication	.088	.134
Sat. Development	.638	.921
TOTAL	1.000	1.370

Table 6-4. Evaluation Procedure

SYSTEM		"HALO"	"HUMMING3'RD"
SPACECRAFT	W_S		
SUBSYSTEMS			
ANTENNA	W_{SS1}		
WEIGHT	W_{P11}	R_{HALO}	$R_{HUMMINGBIRD}$
FAB. COST	W_{P12}		
DEV. COST	W_{P13}		
COMPLEXITY	W_{P14}		
SIZE	W_{P15}		
TRANSPONDER	W_{SS2}		
WEIGHT	W_{P21}		
.			
.			
SIZE	W_{P25}		
ELECTRICAL POWER			
.			
.			
.			
.			
.			
LAUNCH VEHICLE			
FAB. COST	W_{PM21}		
DEV. COST			
COMPLEXITY			
GROUND	W_G		
TRACKING REQUIREMENTS	W_{GG1}		
SYSTEM MODIFICATIONS	W_{GG2}		
USER	W_U		
TRACKING REQUIREMENTS	W_{UU1}		
SYSTEM MODIFICATIONS	W_{UU2}		

$$\text{FIGURE OF MERIT} = W_S\left[\sum_{i=1}^{M_1} W_{SS_i} \sum_{J=1}^{M_2} W_{P_G}\, R\right] + W_G\left[\sum_{k=1}^{M_3} W_{GG_k}\, R\right] + W_U\left[\sum_{\ell=1}^{M_4} W_{UU_\ell}\, R\right]$$

Table 6-5. System Comparison

Major Parameter	Halo	Hummingbird
Weight (lb)	1084	1237
System Fabrication Cost-Relative (1000$) (Including launch vehicle)	1.0	1.24
Engineering Dev. Cost-Relative (1000$)	1.0	1.45
Complexity (Relative)	---	---
Size (Relative)	---	Larger

The first three elements have been described in the preceding sections. The basis for the evaluation of the complexity factor is that while all other subsystems are common or equivalent, the Hummingbird requires almost three times the electrical system as well as a more complex propulsion system than that selected for the Halo orbiter. However, the rotating dual feed requirement on the Halo antenna is more complex than the fixed dual antenna feed on the Hummingbird. Due to the much larger solar array requirements (222 sq ft vs 82 sp ft), the Hummingbird spacecraft would be larger than the Halo.

Regardless of the weighting factors, it can be seen that the Halo spacecraft concept is superior since it exceeds the Hummingbird spacecraft concept in each major parameter.

SECTION 7

CONCLUSIONS AND RECOMMENDATIONS

The major conclusions and recommendations are given in the following lists.

7.1 CONCLUSIONS

- Transfers are possible every day.
- The stabilization/maintenance requirements are compatible with tracking capabilities.
  - Halo - Range Rate Tracking
    ΔV for Phase Control = 0.23 m/sec/day
    ΔV for Stability = 0.05 m/sec/day Corrections 2 to 4 days
  - Hummingbird - Range & Range Rate Tracking required
    ΔV = 0.1 to 0.2 m/sec/day
- The dynamics model and the maintenance strategy have been verified by a sample orbit.
- Both Halo and Hummingbird concepts are feasible from a flight dynamics point of view.
- Most of the communications links can be satisified using one 11-1/2-ft antenna with dual feeds.
- Attitude control can be maintained using either a dual spinner or three-axis active concept.
- The optimum propulsion system for the Halo orbiter is an all-monopropellant system, whereas the optimum for the Hummingbird is a monopropellant-ion mix.
- Both concepts are feasible, but the Halo concept is superior on a weight and cost basis.

7.2 RECOMMENDATIONS FOR FUTURE STUDIES

- Utilization of a Libration Point Satellite for detailed lunar gravitational field studies.

- Detailed technology studies
    - Dual feed antenna
    - Apollo communications subsystem improvements
    - Specific control system designs in more detail
- System studies
    - Phase B of the Halo concept
    - Complexity/reliability quantification

Conclusions in each study area have already been given in the appropriate section.

SECTION 8

REFERENCES

1. Dunn, G. L., "A Libration Point Satellite Relay for Lunar Farside Communication," General Electric Company, GE TIS 63SPC-5, April 1963.

2. Farquhar, R. W., "Stationkeeping in the Vicinity of Collinear Libration Points with an Application to a Lunar Communications Problem," American Astronautical Society Paper 66-132, July 1966.

3. Vonbun, F. O., "A Hummingbird for the L_2 Lunar Libration Point," Goddard Space Flight Center X-document, X-507-67-167, April 1967.

4. Richards, P. B., "Approximate Effect of the Sun on Satellites near the Interior Earth-Moon Libration Point," AIAA Paper 65-512, 1965.

5. deVries, J. P. and Pauson, W. M., "Study of Motion near Libration Points in the Earth-Moon System," General Electric Co., King of Prussia, Pa., Space Sciences Laboratory, GE TIS R62SD5, 1962.

6. Colombo, G., "The Stabilization of an Artificial Satellite at the Inferior Conjunction Point of the Earth-Moon System, "Smithsonian Contrib., Astrophys, Vol. 6, pp 213-222, 1961.

7. Nicholson, F. T., "Effect of Solar Perturbation on Motion near the Collinear Earth-Moon Libration Points," AIAA Journal, Vol. 5, No. 12, December 1967, pp 2237-2241.

8. Schmid, P. E., "Lunar Far-Side Communication Satellites," Goddard Space Flight Center, X-507-67-328, July 28, 1967.

9. Moorehead, R. W. and Arndt, G. D., "The Apollo Unified S-Band Telecommunications System," Proceedings of the 1968 National Telemetering Conference, pp. 251-261.

10. NASA Manned Spacecraft Center Internal Note MSC-EB-R-67-1. "A Performance Analysis of the Apollo Unified S-Band Communications System for a Typical Lunar Mission", by C. T. Dawson, G. E. Arndt, and R. W. Moorehead, May 1, 1967.

11. Motorola, Inc. Military Division, Task III, Final Report, Apollo Unified S-Band System Block II Compatibility Test Program, Contract NAS9-2563.

12. Farquhar, R. W., "The Control and Use of Libration Point Satellites," Stanford University Department of Aeronautics and Astronautics, Report No. 350, July 1968.

13. Brouwer, D. and Clemence, G. M., Methods of Celestial Mechanics, Academic Press, 1961.

APPENDIX I

ATTITUDE CONTROL

1.1 CONTROLLER REQUIREMENTS

1.1.1 HUMMINGBIRD ORBIT

1.1.1.1 Disturbance Torque Magnitude

1.1.1.1.1 Solar Pressure

Assume 1000 watts solar array $\left\{\begin{array}{l}\text{320 Communications}\\ \text{100 Attitude Control}\\ \text{580 Period Control}\end{array}\right.$

at 10 watts/ft^2

Area = 100 ft^2

Solar pressure = 9.65×10^{-8} #/ft^2

Assume center of pressure of mass

offset of 1 inch

Torque = 9.65×10^{-8} #/ft^2 (100 ft^2) (1/12 ft) = 8.04×10^{-7} # ft

Time = 3600 sec/hr (24 hr/day) = 86,400 sec/day

Angular Momentum = .0695 ft # sec/day

1.1.1.1.2 Orbit Control

Acceleration Required = 4.8×10^{-5} meter/sec^2 x 3.281 ft/m = 15.75×10^{-5} ft/sec^2

Assume 1000 lb spacecraft

Thrust = 15.75×10^{-5} ft/sec^2 (1000 #/32.2 ft/sec^2) = 4.89×10^{-3} lbs

Assume a 1 inch moment arm between thrust vector and center of mass

Torque = 4.89×10^{-3} lbs (1/12 ft) = 4.08×10^{-4} ft #

Angular Momentum = 4.08×10^{-4} ft # (86,400 sec/day) = 35.25 ft # sec/day

Assuming one-quarter inch moment arm between thrust vector and center of mass

T_d = 4.89×10^{-3} # (1/4 x 12 ft) = 1.02×10^{-4} ft #

$H_d = 1.02 \times 10^{-4}$ ft # $(8.6400 \times 10^4$ sec/day$) = 8.82$ ft # sec/day

Assuming 0.1 inch moment arm between thrust vector and center of mass

$T_d = 4.89 \times 10^{-3}$ # (0.1/12 ft) $= 4.08 \times 10^{-5}$ ft #

$H_d = 4.08 \times 10^{-5}$ ft # $(8.6400 \times 10^4$ sec/day$) = 3.525$ ft # sec/day

1.1.1.2 Concept #1

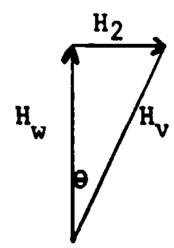
1.1.1.2.1 Constant Speed

Flywheel Size - A component of the moment arm along the pitch axis will cause the spin axis of the flywheel to precess. To prevent the flywheel from precessing more than 0.1 degree, the flywheel must hav the following angular momentum relationship due to the disturbance angular momentum.

$\tan \Theta = H_d/H_w$

$H_w = H_d/\tan \Theta$

$\Theta_{max} = 0.1$ deg



$$H_w = H_d/0.1/57.3 = 573\ H_d$$

1.1.1.2.1.1 Solar Pressure Torques

Since solar pressure torques about the roll and yaw axes are cyclic over a period of one year, it would require a flywheel with enough angular momentum storage so that it would not precess more than 0.1 degree in one half year to keep from expanding gas.

$$H_w = 573\ (.0695 \text{ ft \# sec/day})\ (365/2 \text{ days}) = 7268 \text{ ft \# sec}$$

However, this is impractical, so the angular momentum required if corrections are made at periodic intervals was calculated. With a roll sensor both the roll and yaw errors can be sensed every half orbit but 90 degrees out of phase. Therefore, either a roll or a yaw correction every quarter of an orbit could be made. It would require a flywheel with enough angular momentum storage so that it would not precess more than 0.1 degree in one half orbit.

$$H_w = 573 \; (.0695 \text{ ft \# sec/day}) \; (7 \text{ days}) = 278.8 \text{ \# ft sec}$$

To make corrections more often than every quarter of an orbit would require both a roll and yaw sensor. The flywheel size required to keep from precessing more than 0.1 degree per day is

$$H_w = 573 \; (.0695 \text{ ft \# sec/day}) - 37.8 \text{ \# ft sec/day}$$

1.1.1.2.1.2 Orbit Control Torques

Since disturbance torques about the yaw axis due to the misalignment of the thruster for orbit control are cyclic over a period of one orbit of the Earth, it would require a flywheel with enough angular momentum storage so that it would not precess more than 0.1 degree in one half orbit or 14 days to keep from expending gas.

1" moment arm

$$H_w = 573 \; (35.25 \text{ ft \# sec/day}) \; (14 \text{ days}) = 282{,}775 \text{ ft \# sec}$$

1/4" moment arm

$$H_w = 573 \; (8.82 \text{ ft \# sec/day}) \; (14 \text{ days}) = 70{,}754 \text{ ft \# sec}$$

0.1" moment arm

$$H_w = 573 \; (3.525 \text{ ft \# sec/day}) \; (14 \text{ days}) = 28{,}278 \text{ ft \# sec}$$

Flywheel angular momentum required to make correction on a daily basis:

1" moment arm

$$H_w = 573 \; (35.25 \text{ ft \# sec/day}) = 20{,}198 \text{ ft \# sec}$$

1/4" moment arm

$$H_w = 573 \; (8.82 \text{ ft \# sec/day}) = 5{,}054 \text{ ft \# sec}$$

0.1" moment arm

$H_w = 573$ (3.525 ft # sec/day) = 2,020 ft # sec

Flywheel angular momentum required to make corrections on an hourly basis:

1" moment arm

$H_w = 842$ ft # sec

1/4" moment arm

$H_w = 211$ ft # sec

0.1" moment arm

$H_w = 84$ ft # sec

1.1.1.2.2 Precession Thruster Size (Assuming a 2 Ft Moment Arm)

To prevent the attitude errors from exceeding 0.1 degree, the maximum impulse that can be imparted to the vehicle is calculated as follows:

The equations describing the motion of a rigid symmetrical body with a constant speed flywheel and a pulse of torque applied about the roll axis

$$\dot{\omega}_x = (I_y - I_z)/I_{xz} \ \omega_y \ \omega_z + H_w/I_{xz} \ \omega_z + T_x/I_{xz} - T_x/I_{xz} \ \mu \ (t - k)$$

$$\dot{\omega}_y = 0$$

$$\dot{\omega}_z = (I_x - I_y)/I_{xz} \ \omega_x \ \omega_y - H_w/I_{xz} \ \omega_x$$

Since the vehicle is non spinning, $\omega_y = 0$.

Using Laplace transformations and assuming zero initial conditions,

$$S^2 \ \phi(S) - \frac{H_w}{I_{xz}} \ S \ \psi(S) = \frac{T_x}{I_{xz}} \ \frac{(1 - e^{-kS})}{S}$$

$$\frac{H_w}{I_z} S \phi(S) + S^2 \psi(S) = 0$$

$$\phi(S) = \frac{T_x/I_{xz} \ (1 - e^{-kS})}{S \ (S^2 + \frac{H_w^2}{I_{xz}^2})}$$

$$\psi(S) = \frac{\frac{-T_x H_w}{I_{xz}^2} \ (1 - e^{-kS})}{S^2 \ (S^2 + \frac{H_w^2}{I_{xz}^2})}$$

$$\phi(t) = \frac{T_x I_{xz}}{H_w^2} \ (1 - \cos \frac{H_w}{I_{xz}} t) - \frac{T_x I_{xz}}{H_{wy}^2} \ (1 - \cos \frac{H_w}{I_{xz}} \left[t - k\right] \mu \ (t - k)$$

$$\psi(t) = \frac{T_x I_{xz}}{H_w^2} \ (\frac{H_w}{I_{xz}} t - \sin \frac{H_w}{I_{xz}} t) + \frac{T_x I_{xz}}{H_w^2} \ (\frac{H_w}{I_{xz}} \left[t - k\right] - \sin \frac{H_w}{I_{xz}} \left[t - k\right] \mu \ (t - k)$$

$t \leq k$

$$\phi(t) = \frac{I_x I_{xz}}{H_w^2} \ (1 - \cos \frac{H_w}{I_{xz}} t)$$

$$\psi(t) = - \frac{T_x I_{xz}}{H_w^2} \ (\frac{H_w}{I_{xz}} t - \sin \frac{H_w}{I_{xz}} t)$$

$t > k$

$$\phi(t) = \frac{T_x I_{xz}}{H_w^2} \ (\cancel{1} - \cos \frac{H_w}{I_{xz}} t) - \frac{T_x I_{xz}}{H_w^2} \left(\cancel{1} - \left[\cos \frac{H_w}{I_{xz}} t \cos \frac{H_w}{I_{xz}} k + \sin \frac{H_w}{I_{xz}} t \sin \frac{H_w}{I_{xz}} k\right]\right)$$

$$\psi(t) = -\frac{T_x I_{xz}}{H_w^2}\left(\cancel{\frac{H_w}{I_{xz}}}\, t - \sin\frac{H_w}{I_{xz}}\, t\right) + \frac{T_x I_{xz}}{H_w^2}\left(\cancel{\frac{H_w}{I_{xz}}}\, t - \frac{H_w}{I_{xz}}\, k - \left[\sin\frac{H_w}{I_{xz}}\, t \cos\frac{H_w}{I_{xz}}\, k - \cos\frac{H_w}{I_{xz}}\, t \sin\frac{H_w}{I_{xz}}\, k\right]\right)$$

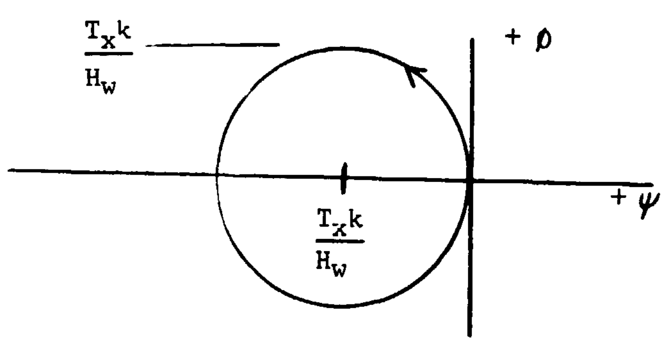
if $\frac{H_w}{I_{xz}}\, k < \frac{10}{57.3}$

$$k < \frac{10}{57.3}\ \frac{I_{xz}}{H_w}$$

$$\phi(t) = \frac{T_x k}{H_w}\ \sin\frac{H_w}{I_{xz}}\, t$$

$$\psi(t) = \frac{T_x k}{H_w}\ \left(1 - \cos\frac{H_w}{I_{xz}}\, t\right)$$

Therefore, if $k < \frac{10}{57.3}\ \frac{I_{xz}}{H_w}$ the motion of the vehicle would be the same as that due to an impulse.



where $T_x k = \frac{0.1}{57.3}\ H_w = 1.75 \times 10^{-7}\ H_w$ # ft sec (assuming a 2 ft moment arm)

$$I = \frac{1.75 \times 10^{-3}}{2\text{ ft}}\ H_w \text{ \# ft sec} = 8.7 \times 10^{-4}\ H_w \text{ \# sec}$$

The motion would be a precession of the spin axis of the flywheel and a coning motion whose amplitude would be equal to the precession angle. This coning motion would be damped out by the nutation damper. Additional pulses should not be allowed for several time constants of the nutation damper.

1.1.1.2.3 Pitch Control Flywheel Size

1.1.1.2.3.1 Solar Pressure

$$f = \frac{0.5 \text{ \# ft sec/unloading}}{.0695 \text{ \# ft sec/day}} = 7.2 \text{ days/unloading}$$

1.1.1.2.3.2 Orbit Control

1" moment arm

$$f = \frac{35.25 \text{ ft \# sec/day}}{0.5 \text{ ft \# sec/unloading}} = 70.5 \text{ unloadings/day}$$

1/4" moment arm

$$f = \frac{8.82 \text{ ft \# sec/day}}{0.5 \text{ ft \# sec/unloading}} = 17.6 \text{ unloadings/day}$$

0.1" moment arm

$$f = \frac{3.525 \text{ ft \# sec/day}}{0.5 \text{ ft \# sec/unloading}} = 7.05 \text{ unloadings/day}$$

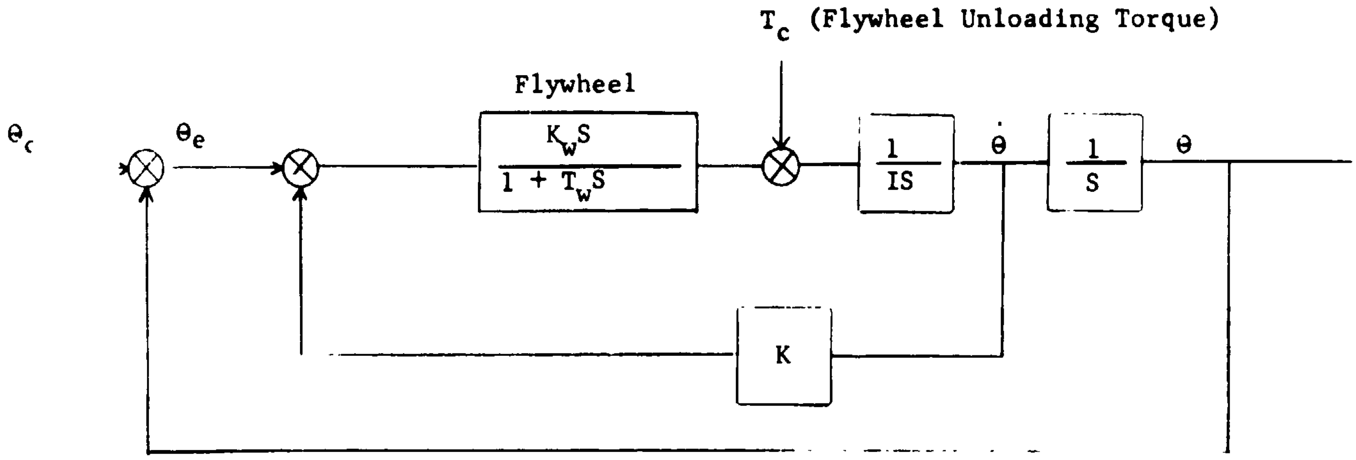
1.1.1.2.4 Pitch Thruster Size (Assume a 1 Ft Moment Arm)

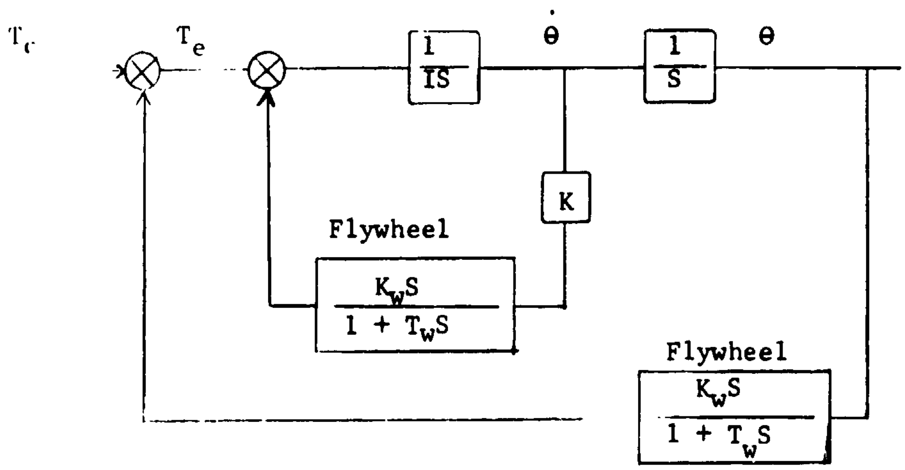
If we use a 0.01# thruster with an I_{sp} = 110 sec, the on time would be

$$t = \frac{0.5 \text{ \# ft sec}}{0.01 \text{ \# (1 ft)}} = 50 \text{ sec}$$

This torque should be low enough so that we do not exceed 0.1 degree with the unloading transient as shown by the following analysis.

Pitch Axis Control Loop





where

$K = 10$ sec

$K_w = 2400$ # ft sec/rad

$T_w = 50$ sec

$I = 100$ slug-ft^2

Inner Loop Transfer Function

$$\frac{\dot{\theta}(S)}{T_e(S)} = \frac{\frac{1}{IS}}{1 + \frac{KK_w}{I(1+T_wS)}} = \frac{\cancel{I}\,(1+T_wS)}{\cancel{I}\,S\left[I\,(1+T_wS)+KK_w\right]} = \frac{(1+T_wS)}{S(I+KK_w)\left(1+\frac{T_wI}{I+KK_w}\,S\right)}$$

Closed Loop Transfer Function

$$\frac{\theta(S)}{T_c(S)} = \frac{\dfrac{(1+T_wS)}{S^2\,(I+KK_w)\left(1+\frac{T_wI}{I+KK_w}\,S\right)}}{\left[1+\dfrac{K_w}{S(I+KK_w)\left(1+\frac{T_wI}{I+KK_w}\,S\right)}\right]} = \frac{(1+T_wS)}{S\left[S(I+KK_w)\left(\frac{1+T_wI}{I+KK_w}\,S\right)+K_w\right]}$$

$$\frac{\theta(S)}{T_c(S)} = \frac{(1 + T_w S)}{S\left[T_w I S^2 + (1 + KK_w)\, S + K_w\right]} = \frac{(S + \frac{1}{T_w})}{IS\left[\left(S + \frac{I + KK_w}{2T_w I}\right)^2 + \frac{K_w}{T_w I} - \left(\frac{I + KK_w}{2T_w I}\right)^2\right]}$$

$$\frac{\theta(S)}{T_c(S)} = \frac{\frac{1}{I}\,(S + \frac{1}{T_w})}{S\left(S + \left(\frac{I + KK_w}{2T_w I}\right) + 1/2\sqrt{\left(\frac{I+KK_w}{T_w I}\right)^2 - \frac{4K_w}{T_w I}}\right)\left(S + \left(\frac{I+KK_w}{2T_w I}\right) - 1/2\sqrt{\left(\frac{I+KK_w}{T_w I}\right)^2 - \frac{4K_w}{T_w I}}\right)}$$

$$\theta(S) = \frac{\frac{T_c}{100}\,(S + \frac{1}{50})}{S^2\,\left(S+2.41 + 1/2\sqrt{(4.82)^2 - \frac{4(2400)}{5000}}\right)\left(S + 2.41 - 1/2\sqrt{(4.82)^2 - \frac{4(2400)}{5000}}\right)}$$

$$\theta(S) = \frac{\frac{T_c}{100}\,(S + .02)}{S^2\,(S + 4.8)\,(S + .1)}$$

General Solution

$$\frac{S + A_o}{S^2\,(S + \alpha)\,(S + \beta)} = \frac{1}{\alpha\beta}\quad 1 + A_o\,t - \frac{A_o\,(\alpha + \beta)}{\alpha^2\,\beta^2} + \frac{1}{\beta - \alpha}\left[\frac{A_o - \alpha}{\alpha^2}\,e^{-\alpha t} - \frac{A_o - \beta}{\beta^2}\,e^{-\beta t}\right]$$

Time Response

$$\theta(t) = \frac{T_c/100}{4.8(.1)}\left[1 + .02\,t\right] - \frac{.02\,(4.8+.1)\,T_c/100}{(4.8)^2\,(.1)^2} + \frac{T_c/100}{-.47}$$

$$\left[\frac{-4.78}{23}\,e^{-4.8t} - \frac{-.08}{.01}\,e^{-.1t}\right]$$

$$\theta(t) = .0208\ T_c + 4 \times 10^{-4}\ T_c t - .0435\ T_c + 4.35 \times 10^{-5}\ T_c e^{-4.8t} + .16\ T_c e^{-.1t}$$

$$\theta_{t = 0} = .1373\ T_c \text{ rad}$$

$$\theta_{t = 50 \text{ sec}} = .1373\ T_c + .02\ T_c + .00112\ T_c = -.002\ T_c \text{ rad}$$

$$0.1^{\circ} = \frac{0.1}{57.3} = \frac{1}{573} = .00175 \text{ rad}$$

$$T_c \quad .01 \text{ ft \#}$$

1.1.1.2.5 Gas Required

Assume a 2 ft moment arm

$I_{sp} = 220$ sec for thrust $> 3\#$

$I_{sp} = 150$ sec for thrust $\geq 0.1\# \leq 3\#$

$I_{sp} = 110$ sec for thrust $< 0.1\#$

1.1.1.2.5.1 Solar Pressure Torques

$$\text{Impulse} = \frac{.0695 \text{ ft \# sec/day } (365\ \frac{\text{days}}{\text{yr}})}{2 \text{ ft}} = 12.7\# \text{ sec/yr}$$

$$I_{sp} = 220 \quad w = \frac{12.7\# \text{ sec/yr}}{220 \text{ sec}} = .058\#/\text{yr}$$

$$I_{sp} = 150 \quad w = \frac{12.7}{150} = .085\#/\text{yr}$$

$$I_{sp} = 110 \quad w = \frac{12.7}{110} = .115\#/\text{yr}$$

1.1.1.2.5.2 Orbit Control Torques

1" moment arm

$$\text{Impulse} = \frac{35.25 \text{ ft \# sec/day } (265 \text{ days/yr})}{2 \text{ ft}} = 6{,}433\# \text{ sec/yr}$$

$I_{sp} = 220 \quad w = \dfrac{6,433\# \text{ sec/yr}}{220 \text{ sec}} = 29.2\#/\text{yr}$

$I_{sp} = 150 \quad w = 42.9\#/\text{yr}$

$I_{sp} = 110 \quad w = 58.5\#/\text{yr}$

1/4" moment arm

$$\text{Impulse} = \frac{8.82 \text{ ft } \# \text{ sec/day } (365 \text{ days/yr})}{2 \text{ ft}} = 1,610\# \text{ sec/yr}$$

$I_{sp} = 220 \quad w = 7.32\#/\text{yr}$

$I_{sp} = 150 \quad w = 10.73\#/\text{yr}$

$I_{sp} = 110 \quad w = 14.64\#/\text{yr}$

0.1" moment arm

$$\text{Impulse} = \frac{3.525 \text{ ft } \# \text{ sec/day } (265 \text{ days/yr})}{2 \text{ ft}} = 643.3\# \text{ sec/yr}$$

$I_{sp} = 220 \quad w = 2.92\#/\text{yr}$

$I_{sp} = 150 \quad w = 4.29\#/\text{yr}$

$I_{sp} = 110 \quad w = 5.85\#/\text{yr}$

1.1.1.2.5.3 <u>Constant Speed Flywheel Spin Up</u>

As the constant speed flywheel is spun up, the angular momentum imparted to the vehicle must be removed by the pitch axis jets. This would require the following amount of gas if a one foot moment arm is assumed

$$\text{Impulse} = \frac{H_w \# \text{ ft sec}}{1 \text{ ft}} = H_w \# \text{ sec}$$

If I_{sp} = 110 sec $\quad w = \frac{H_w \text{ \# ft sec}}{1 \text{ ft } (110 \text{ sec})} = \frac{H_w}{110}$ #

1.1.2 HUMMINGBIRD ORBIT

1.1.2.1 Disturbance Torque Magnitude

1.1.2.1.1 Solar Pressure

Assume 500 watts solar array at 10 watts/ft^2 { 320 Communications, 100 Attitude Control, 80 Miscellaneous }

Area = 50 ft^2

Solar pressure = 9.65×10^{-8} #/ft^2

Assume center of pressure and center of mass offset of 1"

Torque = 9.65×10^{-8} #/ft^2 (50 ft^2)($\frac{1}{12}$ ft) = 4.02×10^{-7} ft #

Angular Momentum = 4.02×10^{-7} ft # (8.6400×10^{-4} sec/day) = .0356 ft # sec/day

1.1.2.1.2 Orbit Control

Assume 1000# spacecraft

ΔV required = 1.52 $\frac{\text{meters}}{\text{sec}}$ / impulse x 3.281 ft/m = 5 ft/sec/impulse

One impulse required every seven days

$$F = m\,a$$

$$a = \frac{F}{m}$$

$$V = \frac{F}{m}\,t$$

Impulse = F t = m V = $\frac{1000\#}{32.3 \text{ ft/sec}^2}$ (5 ft/sec) = 155# sec

If we assume a 1" moment arm between thrust vector and CM

Angular Momentum = 155# sec ($\frac{1}{12}$ ft) = 13# ft sec/ΔV correction

1/4" moment arm

$$H = 155 \# \text{ sec } (\frac{1}{4(12)} \text{ ft}) = 13\# \text{ ft sec}/\Delta V \text{ correction}$$

0.1" moment arm

$$H = 155\# \text{ sec } (\frac{0.1}{12} \text{ ft}) = 1.3\# \text{ ft sec}/\Delta V \text{ correction}$$

1.1.2.2 <u>Concept #1</u>

1.1.2.2.1 <u>Constant Speed Flywheel Sizing</u>

A component of the moment arm along the pitch axis will cause the spin axis of the flywheel to precess. To prevent the flywheel from precessing more than 0.1 degree, the flywheel must have the following angular momentum relationship to the disturbance angular momentum. (Same as derived for the Hummingbird orbit)

$$H_w = 573 \; H_d$$

1.1.2.2.1.1 <u>Solar Pressure Torques</u>

As with the Hummingbird orbit, the solar pressure torques about the roll and yaw axes are cyclic over a period of one year. Since the attitude error must not be greater than 0.1 degree, to keep from expending gas, the flywheel must be sized so that it will not precess more than 0.1 degree in one half year.

$$H_w = 573 \; (.03456 \text{ ft \# sec/day}) \; (\frac{365}{2} \text{ days}) = 3634 \text{ ft \# sec}$$

Since this is impractical, the flywheel angular momentum required is calculated if corrections are made on a weekly or on a daily basis.

(Weekly) $H_w = 573 \; (.02456 \text{ ft \# sec/day}) \; (7 \text{ days}) = 139\# \text{ ft sec}$

(Daily) $H_w = 573 \; (.03456 \text{ ft \# sec/day}) = 18\# \text{ ft sec}$

1.1.2.2.1.2 Orbit Control Torques

Orbit control thrusts will be required every seven days of the Halo orbit when the vehicle crosses the Earth/Moon orbit plane. The duration of these thrusts could be as long as one day. The thrusts will alternate from the positive roll to the negative roll axis. These thrusts could produce torques about the yaw axis or the pitch axis depending on where the moment arm is located. Yaw torques will precess the vehicle about the roll axis, and pitch torques will generate angular momentum about the pitch axis which must be absorbed by the modulated pitch flywheel. The torques produced by the positive roll jet on the yaw axis would cancel each other every one half orbit of the Earth as would the torques produced by the negative roll jet. To keep from expending gas, the constant speed flywheel would have to be sized so that it will not precess more than 0.1 degree due to each period control thrust.

1" moment arm

H_w = 573 (13# ft sec) = 7450# ft sec

1/4" moment arm

H_w = 573 (3.24# ft sec) = 1860# ft sec

0.1" moment arm

H_w = 573 (1.3# ft sec) = 745# ft sec

Since these wheel sizes do not seem practical, then it would seem advisable to size the thrust for orbit control to require a full day to obtain the necessary ΔV. The flywheel could then be sized to require a correction every hour for twenty-four hours every seven days.

1" moment arm

H_w = 310# ft sec

1/4" moment arm

$$H_w = 77.5\# \text{ ft sec}$$

0.1" moment arm

$$H_w = 31\# \text{ ft sec}$$

1.1.2.2.2 Pitch Control Flywheel Size

For both the solar pressure and orbit control disturbance torques, if the moment arm lies along the yaw axis, pitch torques will be produced. Both of these torques are accumulative.

If we assume the use of a 2# ft sec modulated flywheel for pitch control and unload only 25 percent of maximum momentum or 0.5# ft sec, the frequency of momentum unloading for the disturbance torques is calculated below.

1.1.2.2.2.1 Solar Pressure

$$f = \frac{0.5\# \text{ ft sec/unloading}}{.03456 \text{ ft \# sec/day}} = 14.4 \text{ days/unloading}$$

1.1.2.2.2.2 Orbit Control

1" moment arm

$$f = \frac{13\# \text{ ft sec/ V correction}}{0.5\# \text{ ft sec/unloading}} = 26 \text{ unloadings/}\Delta\text{V correction}$$

1/4" moment arm

$$f = \frac{3.24\# \text{ ft sec/ V correction}}{0.5\# \text{ ft sec/unloading}} = 6.48 \text{ unloading/}\Delta\text{V correction}$$

0.1" moment arm

$$f = \frac{1.3\# \text{ ft sec/ V correction}}{0.5\# \text{ ft sec/unloading}} = 2.6 \text{ unloading/}\Delta\text{V correction}$$

The number of flywheel unloadings specified under Orbit Control will be required for the duration of the ΔV correction, which could be as long as one day every seven days. Therefore, with a 1/4" moment arm and a ΔV duration of one day, there would be 6.5 flywheel unloadings during the period of the day. As calculated in section 1.1.1.2.4, the torque should not exceed .01 ft # so as not to exceed 0.1 degree with the unloading transient.

1.1.2.2.3 <u>Gas Required</u>

Assume a 2 ft moment arm

I_{sp} = 220 sec for thrust $>$ 3#

I_{sp} = 150 sec for thrust $\geq$ 0.1# $\leq$ 3#

I_{sp} = 110 sec for thrust $<$ 0.1#

1.1.2.2.3.1 <u>Solar Pressure Torques</u>

$$\text{Impulse} = \frac{.03456 \text{ ft \# sec/day } (365 \text{ days/yr})}{2 \text{ ft}} = 6.4\text{\# sec/yr}$$

$$I_{sp} = 220 \quad w = \frac{6.4\text{\# sec/yr}}{220 \text{ sec}} = .029\text{\#/yr}$$

$$I_{sp} = 150 \quad w = .0427\text{\#/yr}$$

$$I_{sp} = 110 \quad w = .0582\text{\#/yr}$$

1.1.2.2.3.2 <u>Orbit Control Torques</u>

1" moment arm

$$\text{Impulse} = \frac{13\text{\# ft sec}/\Delta V \text{ correction } (52\,\Delta V \text{ corrections/yr})}{2 \text{ ft}} = 338\text{\# sec/yr}$$

$$I_{sp} = 220 \quad w = 1.54\text{\#/yr}$$

$I_{sp} = 150 \quad w = 2.25\#/yr$

$I_{sp} = 110 \quad w = 3.08\#/yr$

1/4" moment arm

$$\text{Impulse} = \frac{3.24\# \text{ ft sec}/\Delta V \text{ corrections } (52\Delta V \text{ corrections/yr})}{2 \text{ ft}} = 338\# \text{ sec/yr}$$

$I_{sp} = 220 \quad w = .383\#/yr$

$I_{sp} = 150 \quad w = .56\#/yr$

$I_{sp} = 110 \quad w = .765\#/yr$

0.1" moment arm

$$\text{Impulse} = \frac{1.3\# \text{ ft sec}/\Delta V \text{ correction } (52\Delta V \text{ corrections/yr})}{2 \text{ ft}} = 33.8\# \text{ sec/yr}$$

$I_{sp} = 220 \quad w = .154\#/yr$

$I_{sp} = 150 \quad w = .225\#/yr$

$I_{sp} = 110 \quad w = .308\#/yr$

1.1.2.2.3.3 Precession Control to Correct for Satellite Motion Out of the Earth/Moon Orbit Plane

As the satellite circles the libration point, it moves out of the Earth/Moon orbit plane $\pm$ 0.5 degree as viewed from the Earth. Since the allowable error is only $\pm$ 0.1 degree, the spin axis of the flywheel must be precessed to correct for this error. Effectively, the spin axis must be precessed 2 degrees every 14 days.

$H_c = H_w \tan \theta$

$$H_c = \frac{1/7 \text{ deg/day}}{57.3 \text{ deg/rad}} \quad H_w = 2.5 \times 10^{-3} \; H_w \text{ ft \# sec/day}$$

$$\text{Impulse} = \frac{2.5 \times 10^{-3} \; H_w \text{ ft \# sec/day (265 days/yr)}}{2 \text{ ft}} = .455 \; H_w \text{ \# sec/yr}$$

$$\text{If } I_{sp} = 110 \text{ sec} \quad w = \frac{.455 \; H_w \text{ \# sec/yr}}{1110 \text{ sec}} = 4.15 \times 10^{-3} \; H_w \text{ \#/yr}$$

1.1.2.2.3.4 Constant Speed Flywheel Spin Up

As the constant speed flywheel is spun up, the angular momentum imparted to the vehicle must be removed by the pitch axis jets. This would require the following amount of gas if a 1 ft moment arm is assumed.

$$\text{Impulse} = \frac{H_w \text{ \# ft sec}}{1 \text{ ft}} = H_w \text{ \# sec}$$

$$\text{If } I_{sp} = 110 \text{ sec} \quad w = \frac{H_w \text{ \# ft sec}}{1 \text{ ft (110 sec)}} = \frac{H_w}{110} \text{ \#}$$

APPENDIX II

SUBSYSTEM MODELS

The subsystem models for the Antenna, Thermal Control, Electrical Power, and Structures are given in the following charts and sections. The Transponder components are also listed.

ANTENNA

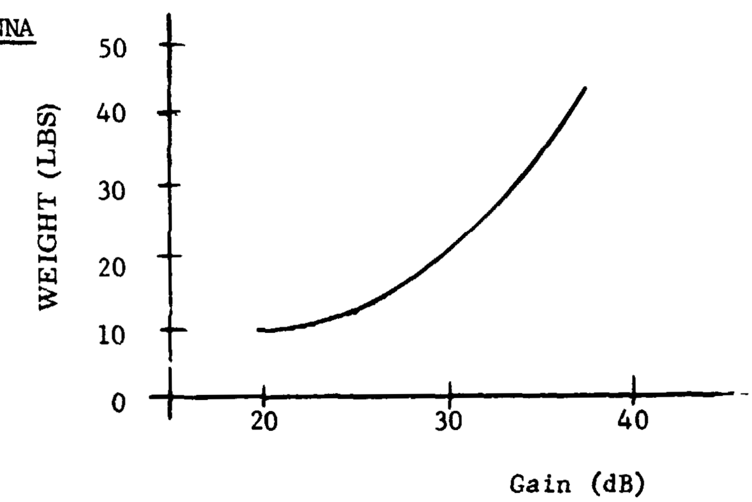


Figure AII-1. Antenna Weight vs Gain

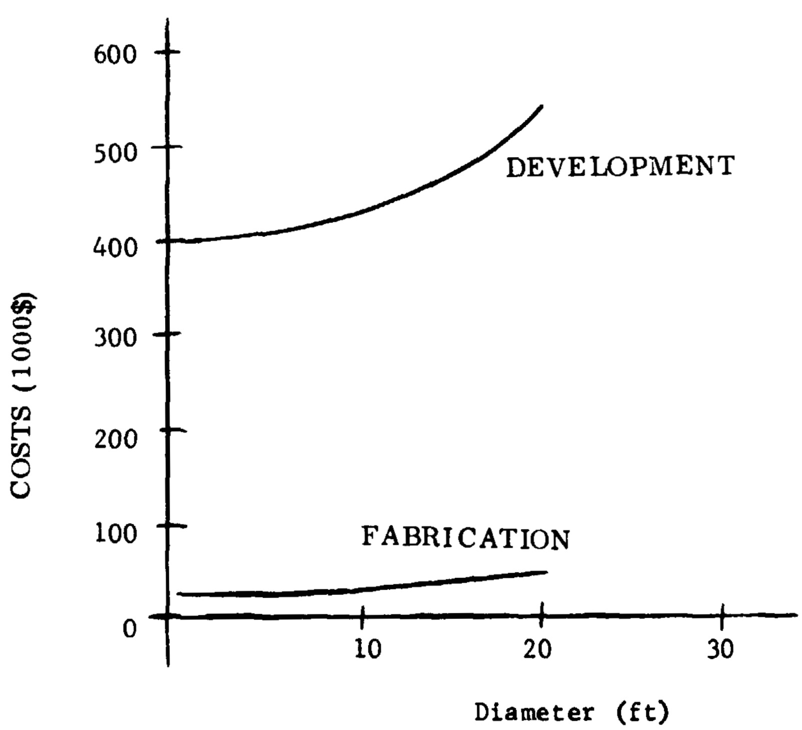


Figure AII-2. Antenna Costs vs Diameter

THERMAL CONTROL FACTORS

QQ(1) = Transponder/TT&C dissipation (W) = 245.

QQ(2) = Power conditioner input (W)*.240 = 76.8

QQ(3) = Attitude control/stationkeeping input (W) = 165.

Thermal Control Area Factors are:

AA(1) = .0176

AA(2) = .0337

AA(3) = .042

Thermal Control Area = ATC

ATC = QQ(1)*AA(1) + QQ(2)*AA(2) + QQ(3)*AA(3) = 13.84

Thermal Control Weight Factor = 1.85

Thermal Control Weight = WTC

WTC = ATC*1.85 = 26.

Thermal Control Fabrication Cost = CTC1

CTC1 = 1000.*(EXP(2.3*(1.28 + 1.19*(ALOG(WTC)/2.3-2.)))) = 3.98K

Thermal Control Development Cost = CTC2

CTC2 = 1000.*(EXP(2.3*(2.4 + .7*(ALOG(WTC)/2.3-2.)))) = 98.K

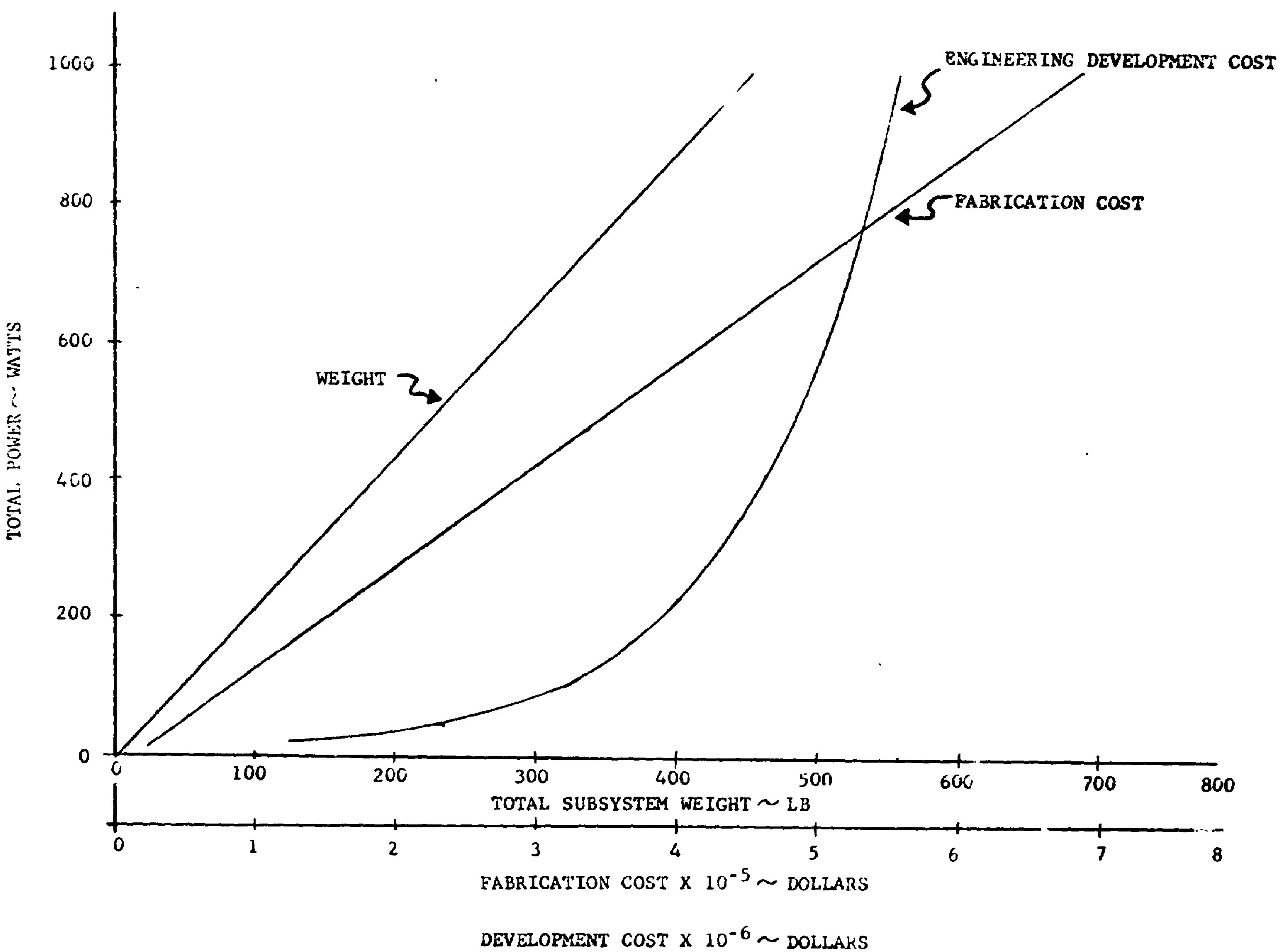


Figure AII-3. Electrical Power Subsystem Weight and Costs

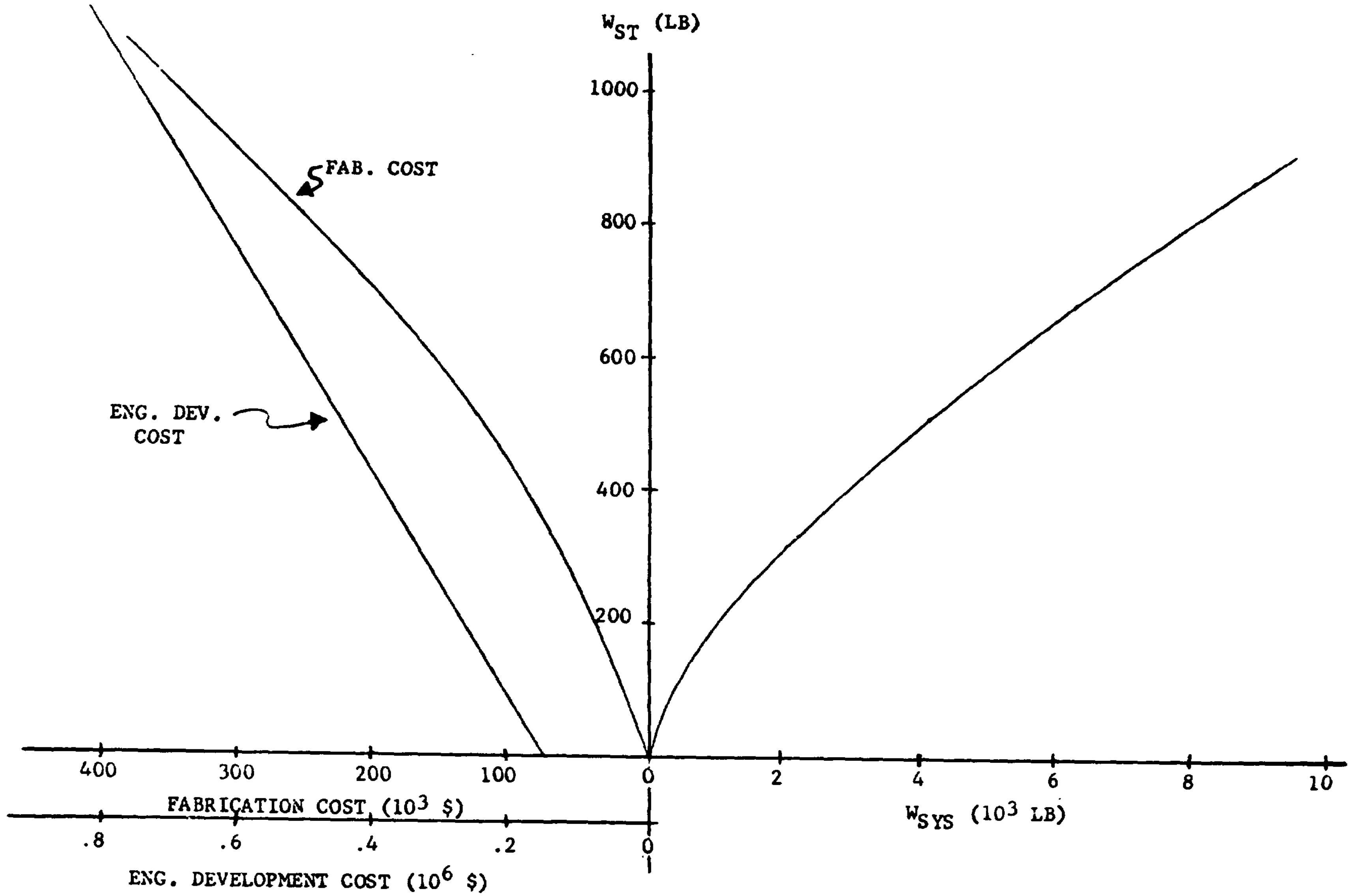


Figure AII-4. Structural Weights and Costs

TRANSPONDER/TT&C

Consists of the following:

1 50 W Up Link Transmitter (TWT)

1 5 W Down Link Transmitters (Solid state)

1 Up Link Receiver

1 Down Link Receiver

1 50 W Diplexer

1 25 W Diplexer

1 20 W Quadruplexer

1 Range Demodulator

1 Command Demodulator

Telemetry SCO's and Modulators (as required)

Estimated Physical Parameters:

Weight	17 lb
Volume	432 in^3
Input Power	320 W

Estimated Costs:

	Fabrication	Development
Receiver-Exciter	40K	205K
50 W TWT	45	360
TT&C	40	95
	125K	660K