THE CALCULATION OF EFFICIENT HIGH PRECISION ORBITS BY OPTIMUM MATCHING OF THE FORMULATION AND NUMERICAL INTEGRATOR

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The objective of this research was to find improved computer algorithms to compute orbits, i.e., orbital trajectories, and the improvements we were seeking were in the areas of either efficiency or accuracy. The goals of this work, as well as some applications, are outlined in Figure 1.

For example, we'd like to increase the speed at which we can compute an orbital path, at a given accuracy level, or we'd like to increase the maximum accuracy levels we can currently achieve with our present technology. We'd expect this type of research to have direct applications in the areas of operational orbit determination in an environment where we routinely compute the trajectories of 30 or 40 satellites for the purposes of maintaining tracking or reducing experimental data. A more efficient algorithm could lead to significant reductions in the amount of computer time used for this operation. As to applications in the area of satellite geodesy, as you know we are in the process of trying to reduce very high precision tracking data such as laser data or very long baseline interferometry. Therefore, we'd like to improve the maximum accuracy level that we can achieve from the algorithms we now have. And finally, in the mission planning area, typical computer programs which perform mission planning operations (such as lifetime studies or maneuver analysis) go through the process of computing orbits over many resolutions; and as a result take up a considerable amount of computer time. We would expect a more efficient algorithm to improve this operation also.

- RESEARCH GOAL
  - —TO DEVELOP EFFICIENT, HIGH ACCURACY ORBIT COMPUTATION PROCESSES FOR EARTH SATELLITES BY OPTIMUM SELECTION OF THE ANALYTICAL FORMULATION AND INTEGRATION METHOD
  - —AN OPTIMUM METHOD MINIMIZES THE NUMBER OF PERTURBATIVE ACCELERATION COMPUTATIONS FOR A GIVEN ACCURACY
- APPLICATIONS
  - —OPERATIONAL ORBIT DETERMINATION
  - —SATELLITE GEODESY
  - —MISSION PLANNING

Figure 1. Calculation of precision satellite orbits with nonsingular elements.

The approach we are taking is trying to find an optimum matching of formulation, that is, equations which describe the motion of an artificial satellite, and the numerical process by which we solve these equations.

Figure 2 shows an example of what we are talking about. The top equation is the equation of motion of a satellite in the classical form. This is the typical form that you find in most computer programs existing today.

The second equation is a new equation that we're trying to calibrate against the first one. This particular equation is consistent with the first in that they both describe the motion of the satellite. The second equation, though, is especially "tuned" for specific types of satellites. In particular, it is suited for satellite of the geosynchronous type; or satellites which have a low eccentricity inclination, such as many of the geodetic satellites.

During the course of our research, we coupled these two formulations with optimum numerical methods, and performed experiments to see what happened in terms of overall accuracy and efficiency.

Figure 3 indicates some of the results of the experiments we performed. First of all, note that we're plotting error against cost. The number of function evaluations is directly proportional to computer time. So you can see that as more computer time is used the errors decrease. And you can also see that the curves level off, which means that we do achieve a maximum accuracy boundary at a certain point.

The dotted lines represent the results of the computation of an orbit of the synchronous type, for 28 revolutions over all cost and accuracy ranges. And we can see that over the total accuracy spectrum – from a kilometer accuracy down to a centimeter accuracy – the curve representing the new method (the dots) preformed significantly better than the classical method.

I COWELL: $\ddot{\vec{x}} = \frac{-\mu \vec{x}}{|x|^3} + \vec{P}(t, \vec{x})$

II VARIATION OF PARAMETER (GAUSSIAN FORM):

$$\dot{\vec{\alpha}} = \frac{\partial \vec{\alpha}}{\partial t} + \frac{\partial \vec{\alpha}}{\partial \dot{\vec{x}}} \cdot \vec{P}(t, \vec{x})$$

WHERE $\vec{\alpha}$ IS A NON-SINGULAR SET OF ORBITAL ELEMENTS, e.g. EQUINOCTIAL

a	$p = \tan \frac{i}{2} \sin \Omega$
$h = e \sin (\omega + \Omega)$	$q = \tan \frac{i}{2} \cos \Omega$
$k = e \cos (\omega + \Omega)$	$\lambda = M + \omega + \Omega$

(BROUCKE AND CEFOLA, 1972)

Figure 2. Formulations.

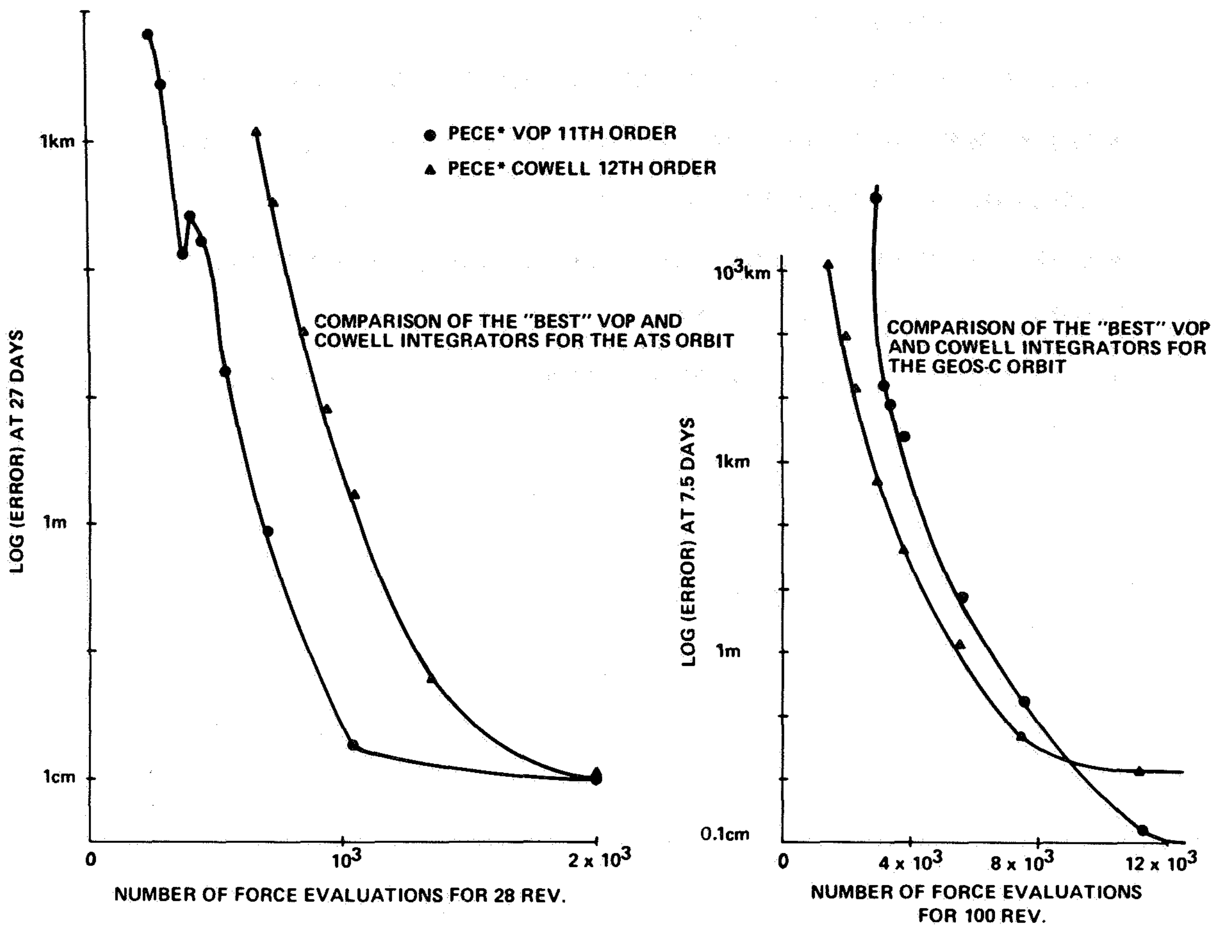

Figure 3. Comparison of the VOP and Cowell integrators.

This means that if you ask for a given accuracy anywhere between a kilometer and a centimeter, you are going to do better efficiency-wise with the new method.

On the other hand, we tried the same processes on a geodetic type orbit (a two-hour satellite, which is much closer). In this case we did not find any improvement using the "variation of parameter" (VOP) approach, as we are calling these new algorithms. But we did find this: that when you ask for very high accuracy down in the submeter region, the new method is able to achieve it, and the classical method is not.

And this was the significant result. We are saying that we can achieve accuracies that were not obtainable before by classical techniques.

Figure 4 shows some of the conclusions that we drew from this particular set of experiments: First, that there is no "best" method available for all accuracies for all orbits. And so we are developing a system to be responsive to that type of requirement. We are making them flexible so that we can use either a new orbit theory or a new numerical process to meet the particular problem at hand.

- NO "BEST METHOD" FOR ALL ORBITS AND ACCURACIES IS AVAILABLE
- PECE* VOP IS SUPERIOR (50%) OVER COWELL FOR GEOSYNCHRONOUS ORBITS
- PECE* COWELL IS SUPERIOR (10-20%) FOR CLOSE EARTH SATELLITES WITH LARGE GEOPOTENTIAL PERTURBATIONS
- STABILITY ANALYSIS INDICATES THE POSSIBILITY OF EXPLORING HIGHER ORDER INTEGRATION METHODS WITH VOP
- POSSIBILITY OF USING AVERAGED EQUATIONS FOR DEFINITIVE ORBIT DETERMINATION

Figure 4. Conclusions.

The new VOP method was superior to the classical method for the geosynchronous orbit. However, it wasn't so for the geodetic satellite, the two-hour satellite. However, again, it was able to achieve accuracies well below the classical boundary.

I also want to point out two byproducts. We discovered that from the stability analysis we performed, our warehouse of numerical methods didn't contain methods that would probably be optimum for this new formulation. So we are looking for new numerical methods that we feel will make the VOP formulation even more efficient.

And finally, this particular new formulation is amenable to numerical averaging, and we feel this can also lead to significant efficiency improvements.