# SPACECRAFT REORIENTATION VIA SLEWING ABOUT NONORTHOGONAL AXES 

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I will be presenting a synopsis of a mathematical technique which was developed to meet the demanding requirements for precise spacecraft reorientations. By a reorientation I mean either a change in pointing which requires slews about two axes, or a total change in attitude which requires motion about three axes.

Normally it is assumed that these slews take place about orthogonal axes. Thus a yaw of $90^{\circ}$ followed by a pitch of $90^{\circ}$, followed in turn by a roll of $90^{\circ}$, will reorient the spacecraft so that it is pointing vertically and the sunshade is parallel to the desk.

In reality, however, the motion is controlled via devices mounted on the spacecraft; and these devices may be slightly skewed, due to mounting imperfections, launch stress, thermal bending. Thus, a misalignment of the first axis actually causes the spacecraft to move like this, a misalignment of the second axis adds to the error like so, and finally the third axis may also be skewed so that the final attitude looks like this.

I think you can see by this simple demonstration that for precise reorientations we can no longer afford the computation luxury of orthogonal axes. Even if the misalignments are small, they can accumulate and even grow. In the example just shown, a one-minute error in each axis would have caused a pointing error of over $31 / 2$ minutes, and unless the errors at this attitude are nulled out the next reorientation will be even worse. The elimination of the errors, however, will require additional commands and loss of valuable time; and if the errors are large an attitude determination may be required.

To solve this problem of skewed axes, a mathematical procedure was developed which determines the exact slews relative to any known axes. The axes may be completely arbitrary; their coordinates merely reside on a data base which can be changed at any time without affecting the software.

Given the desired reorientation, the procedure then determines all of the two-legged or three-legged slews which will accomplish the reorientation. For example, there are 12 possible permutations of three axes, and if they are orthogonal each always has two solutions. The nonorthogonal case, on the other hand, gives some surprising results, including the nonexistence of a solution: Consider a large reorientation where the three axes are nearly collinear. In this connection, it can be shown that every three-axis reorientation can be accomplished by three successive slews, if and only if the middle axis is perpendicular to the other two.

The development of this procedure was initiated to meet the precise requirements of the OAO Copernicus spacecraft, and it is performing this function quite adequately even though two of the axes lack orthogonality by $21 / 2$ minutes.

Even though it was small misalignments which motivated the investigation, the resulting procedure is quite general and may have many other applications. For example, in some cases it may be possible to relax the alignment requirements during installation. In fact, since the procedure is completely general it may be useful when the axes are skewed on purpose.

Many control systems are presently being designed around a skewed concept - one such system mounts the sensors on the six nonparallel sides of a dodecahedron; as in Figure 1. The advantages of such a design are threefold: It is redundant - any three can fail; failures or false signals can be detected by majority logic; and it is more precise.

In Figure 2 we have a few examples of the various control configurations and their applications, and although there are many variations, each is based on a geometric figure having symmetry. However, the procedure I have been describing is flexible enough so that even symmetry is not required. The most sophisticated control system ever launched into space is based on such a principle. This system uses a triad of semicircular canals which for some reason or another are nonorthogonal (see Figure 3).


Figure 1. Dodecahedron mounting.

| AXES | DESIGN GEOMETRY | APPLICATION |
| :---: | :--- | :--- |
| 3 | CUBE (orthogonal) | OAO - GYROS |
| 4 | PYRAMID (nonorthogonal) | HEAO - REACTION WHEELS |
| 4 | OCTAHEDRON (nonorthogonal) |  |
| 6 | DODECAHEDRON (nonorthogonal) | HEAO - GYROS |
| 6 | CONE (nonorthogonal) | IUE - GYROS |

Figure 2. Control configurations.


Figure 3. The human ear.

