NASTRAN POSTPROCESSOR PROGRAM FOR TRANSIENT RESPONSE TO INPUT ACCELERATIONS

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SUMMARY

The description of a transient analysis program for computing structural responses to input base accelerations is presented. A "hybrid" modal formulation is used and a procedure is demonstrated for generating and writing all modal input data on user tapes via NASTRAN. Use of several new Level 15 modules is illustrated along with a problem associated with reading the postprocessor program input from a user tape. An example application of the program is presented for the analysis of a spacecraft subjected to accelerations initiated by thrust transients. Experience with the program has indicated it to be very efficient and economical because of its simplicity and small central memory storage requirements.

INTRODUCTION

Design loads in aerospace subassemblies or components are often specified in terms of induced acceleration at the mounting interface. This concept has been traditionally used to qualify aerospace hardware by subjecting it to prescribed input accelerations on a vibration exciter. Transient analyses of subassemblies for prescribed acceleration inputs at the interface are, therefore, valuable for designing and augmenting vibration tests, and for computing design loads where vibration tests are not practical.

The transient analysis in the current level of NASTRAN (Level 15.1) does not directly provide for input acceleration forcing functions. By using the artifice of placing a large mass (with respect to the total system mass) at the desired acceleration input point, an input force equal to the total mass times the prescribed acceleration will approximate an acceleration input. Theoretically, as the fictitious added mass becomes infinite, the answer becomes exact. But as the mass becomes large, the mass matrix tends to become ill conditioned. Experience has indicated the "fictitious mass" approach is not desirable.

In addition, the current NASTRAN transient analysis allows for initial conditions only in the direct formulation. The modal formulation, which is generally faster and more economical to run, assumes zero initial conditions. In transient analyses of prestrained structures (such as a missile just prior to a burnout transient), the initial conditions become very important in predicting the magnitude of the structural loading.

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The purpose of this paper is to describe a transient analysis program which has been developed to circumvent the abovementioned NASTRAN limitations. This program employs a modal formulation and allows for nonzero initial conditions. It is assumed that the modal input data have been generated and written on user tapes by NASTRAN. Hence, the program is termed a postprocessor program.

A complete derivation of the program theory is presented along with a detailed discussion on the generation and reading of NASTRAN user tapes. This exercise demonstrates the versatility of several of the new modules added to Level 15.1 as well as some of the limitations of user tapes. Finally, an example application of the program to the transient analysis of a spacecraft is presented.

SYMBOLS

[D]	rigid body transformation matrix for ℓ set
{F}	internal members load vector
[I]	identity matrix
[K]	stiffness matrix
[M]	mass matrix
$[\overline{M}_{ii}]$	generalized mass matrix (equation B10)
$[\overline{M}_{ir}]$	coupled flexible body, rigid body mass matrix (equation 5)
[P]	matrix of modal element force vectors
[RB]	expansion of rigid body transformation matrix to g set
{u}	vector of displacement components
{v}	time derivation of modal coordinate vector (equation 10)
$\lceil\beta_i\rfloor$	critical viscous modal damping coefficient matrix
$\lceil\omega_i^2\rfloor$	eigenvalue matrix
$[\phi]$	matrix of modal eigenvectors
$\{\xi\}$	vector of modal coordinates

Subscripts: (See Appendix A)

α	subset of total members in structure
γ	subset of g set

Notation:

[ ]	square or rectangular matrix
$[\ ]^T$	transpose of matrix
⌈ ⌋	diagonal matrix
{ }	column matrix

PROGRAM THEORY

In this section, the theoretical basis for a program to compute the transient response of a structure to acceleration forcing functions is given. The equations of motion are developed in terms of a "hybrid" modal formulation and reduced to a form which makes maximum use of NASTRAN generated eigenvalue data. Numerical solutions to the resulting equations are discussed along with treatment of the initial conditions. Finally, equations are presented for converting the modal response data into transient member loads and grid point accelerations.

In the derivations an attempt has been made to generally utilize the notation presented in the NASTRAN Manuals (references 1, 2, and 3) for ease of reading and implementation of the resulting equations. In particular, the set notation of Appendix A, which is taken from Section 1.7.3 of reference 3, is used throughout; although, the r set has a somewhat different meaning herein. This difference will become apparent in the course of the derivation.

Equations of Motion

Assuming no external loads are acting on the grid points of a structural system, the undamped equations of motion for displacement set $\{u_a\}$ become

$$\left[M_{aa}\right]\left\{\ddot{u}_a\right\} + \left[K_{aa}\right]\left\{u_a\right\} = 0 \tag{1}$$

where $\left[M_{aa}\right]$ and $\left[K_{aa}\right]$ are the reduced mass and stiffness matrices, respectively (see Section 3.5 of reference 2). It is assumed that the system described by equation (1) is not completely constrained against rigid body motions (i.e., it can have from 1 to 6 rigid body degrees of freedom). Equation (1) may be partitioned as follows:

$$\left[\begin{array}{c|c} M_{\ell\ell} & M_{\ell r} \\ \hline M_{\ell r}^T & M_{rr} \end{array}\right]\left\{\begin{array}{c} \ddot{u}_\ell \\ \ddot{u}_r \end{array}\right\} + \left[\begin{array}{c|c} K_{\ell\ell} & K_{\ell r} \\ \hline K_{\ell r}^T & K_{rr} \end{array}\right]\left\{\begin{array}{c} u_\ell \\ \hline u_r \end{array}\right\} = 0 \tag{2}$$

where by definition, the subset $\{u_r\}$ of the displacement vector $\{u_a\}$, if constrained, would be just sufficient to eliminate rigid body motion without introducing redundant constraints. Selection of the subset $\{u_r\}$ is arbitrary and for the present analysis it is chosen to correspond to the input acceleration degrees of freedom (a.d.o.f.), and it is specified on a NASTRAN "SUPORT" Bulk Data Card. It should be noted that by using the r set for input accelerations, the a.d.o.f. are restricted to a maximum of six. This restriction is not a major limitation since the base of many components can be assumed to be rigidly constrained to a plane. The redundant points in the base can thus be assumed to be rigidly attached to a single acceleration input point.

The mathematical problem at hand is to determine the transient response of the $\{u_\ell\}$ subset to prescribed $\{\ddot{u}_r\}$ inputs. A solution using a modal formulation is presented in the following. This approach allows a significant reduction in size of the problem with little loss in accuracy by truncating the number of modes included in the solution.

Modal Coordinate Transformation

The following "hybrid" transformation between modal coordinates (ξ) and physical coordinates (u) is introduced:

$$\begin{Bmatrix} u_\ell \\ \hline u_r \end{Bmatrix} = \left[\begin{array}{c|c} \phi_{\ell i} & D_{\ell r} \\ \hline 0 & I \end{array}\right] \begin{Bmatrix} \xi_i \\ \hline u_r \end{Bmatrix} \tag{3}$$

where $\left[D_{\ell r}\right]$ is the rigid body mode matrix associated with the rigid body motion of the structure in response to displacements of the $\{u_r\}$ coordinates; $\left[\phi_{\ell i}\right]$ is the matrix of eigenvectors of the structure with the $\{u_r\}$ coordinates constrained to zero (see Appendix B); $\left[I\right]$ is the identity matrix; and $\{\xi_i\}$ is the vector of flexible body modal coordinates.

Modal Equations of Motion

Substituting equation (3) into equation (2); premultiplying by the transpose of the transformation matrix, and using equations (B5), (B6), (B10), and (B11) of Appendix B leads to the following

$$\left[\begin{array}{c|c} \overline{M}_{ii} & \overline{M}_{ir} \\ \hline \overline{M}_{ri} & \overline{M}_{rr} \end{array}\right] \begin{Bmatrix} \ddot{\xi}_i \\ \hline \ddot{u}_r \end{Bmatrix} + \left[\begin{array}{c|c} \overline{M}_{ii}\omega_i^2 & 0 \\ \hline 0 & 0 \end{array}\right] \begin{Bmatrix} \xi_i \\ \hline u_r \end{Bmatrix} = 0 \tag{4}$$

where

$$\left[\overline{M}_{ir}\right] = \left[\overline{M}_{ri}\right]^T = \left[\phi_{\ell i}^T\right]\left[M_{\ell\ell}D_{\ell r} + M_{\ell r}\right] \tag{5}$$

and
$$\left[\bar{M}_{rr}\right] = \left[D^T_{\ell r}M_{\ell\ell}D_{\ell r} + M^T_{\ell r}D_{\ell r} + D^T_{\ell r}M_{\ell r} + M_{rr}\right] \tag{6}$$

The upper partition matrix of equation (4) yields

$$\{\ddot{\xi}_i\} + \left[\omega_i^2\right]\{\xi_i\} = -\left[\bar{M}_{ii}\right]^{-1}\left[\bar{M}_{ir}\right]\{\ddot{u}_r\} \tag{7}$$

Adding viscous modal damping to equation (7) yields the desired equation of motion of the system as

$$\{\ddot{\xi}_i\} + \left[2\beta_i\omega_i\right]\{\dot{\xi}_i\} + \left[\omega_i^2\right]\{\xi_i\} = -\left[\bar{M}_{ii}\right]^{-1}\left[\bar{M}_{ir}\right]\{\ddot{u}_r\} \tag{8}$$

where β_i is the critical viscous damping ratio for the ith mode.

With the exception of the β_i values all of the other coefficient values are easily obtained as output quantities from a NASTRAN normal mode analysis (Rigid Format 3).

Method of Solution

The method of solution used in the program to solve the equations is a standard fourth order Runge-Kutta numerical integration routine with variable step size error control. Use of this subroutine required reduction of equation (8) to first order and generation of the initial conditions in terms of modal coordinates. These procedures are discussed in the following sections.

Reduction to first order equations. - Integration via the Runge-Kutta Subroutine requires the system equations to be a set of first order differential equations of the form

$$\{\dot{y}_j\} = \{f_j(y_1, y_2, \ldots y_n)\} \quad j = 1, 2, \ldots n \tag{9}$$

Equation (8) can be transformed to the form of (9) by introducing the auxiliary variable $\{v_i\}$ where

$$\{\dot{\xi}_i\} = \{v_i\} \tag{10}$$

Using equation (10), equation (8) then leads to

$$\{\dot{v}_i\} = -\left[2\beta_i\omega_i\right]\{v_i\} - \left[\omega_i^2\right]\{\xi_i\} - \left[1/\overline{M}_{ii}\right]\left[\overline{M}_{ir}\right]\{\ddot{u}_r\} \tag{11}$$

Equations (10) and (11) are now a set of equations in the form of equation (9) (as required) and are integrated simultaneously.

Initial conditions. - If the initial conditions are known for each of the $\{u_a\}$ coordinates, then the modal initial conditions can be determined by premultiplying equation (3) by the matrix

$$\begin{bmatrix} \phi_{\ell i}^T & 0 \\ D_{\ell r}^T & I \end{bmatrix}\begin{bmatrix} K_{\ell\ell} & K_{\ell r} \\ K_{r\ell} & K_{rr} \end{bmatrix}$$

using equations (B5), (B6), and (B11); and solving for $\{\xi_i\}$ to obtain

$$\{\xi_i\} = \left[\overline{M}_{ii}\omega_i^2\right]^{-1}\left[\phi_{\ell i}\right]^T\left[K_{\ell\ell} \,\middle|\, K_{\ell r}\right]\begin{Bmatrix} u_\ell \\ u_r \end{Bmatrix} \tag{12}$$

By taking the time derivative of both sides of equation (12), the initial conditions for $\{\dot{\xi}_i\}$ can also be determined in terms of $\{\dot{u}_\ell\}$ and $\{\dot{u}_r\}$ values.

Since the initial conditions are not generally known in terms of the $\{u_a\}$ coordinates, a different approach was taken for the present program. This program assumes

$$\{\dot{v}_i(0)\} = 0 \tag{13}$$

and

$$\{v_i(0)\} = 0 \tag{14}$$

that is, the structure is assumed to be initially in a steady state deformed position. The initial conditions are then computed from equation (11) as

$$\{\xi_i(0)\} = -\left[1/\overline{M}_{ii}\omega_i^2\right]\left[M_{ir}\right]\{\ddot{u}_r(0)\} \tag{15}$$

Equations (14) and (15) thus yield the necessary, initial conditions for numerical integration of equations (10) and (11).

Computed Response Data

Having obtained the transient response of the modal coordinates, it is desirable to transform these variables into transient member loads and accelerations in terms of the physical $\{u_a\}$ coordinates.

The member loads are given by superposition as

$$\{F_\alpha\} = \left[P_{\alpha i}\right]\{\xi_i\} \tag{16}$$

where $\left[P_{\alpha i}\right]$ is a matrix of modal element force vectors for an arbitrary subset α of the total members in the system and $\{F_\alpha\}$ is the total load vector corresponding to the subset α. The modal member load vectors are obtainable as standard output from a NASTRAN Normal Mode Analysis.

Using equation (10), the grid point accelerations are computed from the second time derivative of the upper partition of equation (3) to be

$$\{\ddot{u}_\gamma\} = \left[\phi_{\gamma i}\right]\{\dot{v}_i\} + \left[RB_{\gamma r}\right]\{\ddot{u}_r\} \tag{17}$$

where γ is an arbitrary subset of the g set and $\left[RB_{gr}\right]$ is a merger of the rigid body transformation matrix $\left[D_{\ell r}\right]$ with the o , s , and m sets. The matrices $\left[\phi_{gi}\right]$ and $\left[RB_{gr}\right]$ are generated by NASTRAN and the subject postprocessor program selects the subset γ to be picked up for use in equation (17). The quantity $\{\dot{v}_i\}$ is given by equation (11) and $\{\ddot{u}_r\}$ is a given input vector.

If the quantities $\{u_\gamma\}$ and $\{\dot{u}_\gamma\}$ are also desired, equation (17) can be reduced to a set of first order equations to be integrated simultaneously with equations (10) and (11). For the present program, however, this was not done.

NASTRAN GENERATED INPUT DATA

The rigid body and flexible body modal data necessary for solution of the foregoing equations are easily generated by a NASTRAN Normal Mode Analysis and written out on user tapes. Hence, the present postprocessor program is designed to read the majority of its input directly from the NASTRAN user tapes.

A DMAP alter package is required to generate part of the NASTRAN data and to write the user tapes. Also, interrogation of the user tapes to read the data requires special considerations. Both of these aspects are discussed in the following sections.

DMAP Alter Package for Normal Mode Analysis

A listing of the DMAP alter package is given in Appendix C. A brief discussion and explanation of the most significant statement is given in the following tabulation. Note the use of the new Level 15 modules VEC, UMERGE, and OUTPUT2. (See sections 3,4, 5.2, and 5.3 of ref. 1 and section 3.5 of ref. 2)

DMAP Alter Statement No.	Function
2 and 3	Merge an r x r null matrix with the $D_{\ell r}$ matrix to create a pseudo a x r size rigid body modal matrix $\begin{bmatrix} D_{\ell r} \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} RB_{ar} \end{bmatrix}$
4 through 20	Merge the omitted coordinates, single point constraint coordinates, and the multipoint constraint coordinates, if present, with $\begin{bmatrix} RB_{ar} \end{bmatrix}$ to obtain $\begin{bmatrix} RB_{gr} \end{bmatrix}$
6	Recovery of omitted coordinates $\begin{bmatrix} RB_{or} \end{bmatrix} = \begin{bmatrix} G_{oa} \end{bmatrix}\begin{bmatrix} RB_{ar} \end{bmatrix}$
7 and 8	Merging of omitted coordinates $\begin{bmatrix} RB_{ar} \\ RB_{or} \end{bmatrix} \rightarrow \begin{bmatrix} RB_{fr} \end{bmatrix}$
12 and 13	Merging of SPC constraints assuming they are zero $\begin{bmatrix} RB_{fr} \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} RB_{nr} \end{bmatrix}$
17	Recovery of dependent MPC coordinates $\begin{bmatrix} RB_{mr} \end{bmatrix} = \begin{bmatrix} G_{mn} \end{bmatrix}\begin{bmatrix} RB_{nr} \end{bmatrix}$
18 and 19	Merging of dependent MPC coordinates $\begin{bmatrix} RB_{nr} \\ RB_{mr} \end{bmatrix} \rightarrow \begin{bmatrix} RB_{gr} \end{bmatrix}$
21	Prints $\begin{bmatrix} RB_{gr} \end{bmatrix}$

DMAP Alter Statement No.	Function
22 and 23	Write $[RB_{gr}]$ on a user tape
24 and 25	Change the eigenvalue problem from "a" size to "ℓ" size in agreement with equation (B9)
26 and 27	Change the checkpointed modal matrix from "a" size to "ℓ" size
29	Merging of the "r" coordinates constrained to zero with the "ℓ" size modal matrix $\begin{bmatrix}\phi_{\ell i}\\ \hline 0\end{bmatrix} \rightarrow [\phi_{ai}]$
30 and 31	$[\overline{M}_{ir}] = [\phi_{\ell i}]^T[M_{\ell\ell}D_{\ell r} + M_{\ell r}]$
32	Prints $[\overline{M}_{ir}]$
33	Writes $[\overline{M}_{ir}]$ on a user tape
35 and 36	Write the modal deflections (OPHIG), the modal SPC forces (OQG1), and the modal element forces (OEF1) on a user tape

Interrogation of User Tapes

When using NASTRAN user tapes for input to postprocessor programs, the analyst must read the tape and selectively extract the required input from the totality of data present. This task requires either a prior knowledge of the format used in writing the tape or interrogation of the tape to see how it is written.

Unfortunately, the user tapes generated by NASTRAN are written in unformated binary (i.e. with a mixture of integer, floating point, and alphanumeric formats). In addition, some of the data is packed (i.e., zeros omitted). This randomness eliminates a prior knowledge of the format.

Interrogation of the tapes using standard tape dump routines is also somewhat futile since these programs read and print all data in a single format. To illustrate this problem, 19 records of a typical user tape, written to an E format, are listed in figure 1. Obviously, much of the data given is meaningless.

This interrogation problem was circumvented for the present program by writing a special tape dump program for the Langley Research Center CDC 6000 series computer to read and print the mixed format. The program logic was patterned after the NASTRAN module TABPRT and a listing is given in Appendix D. Using the Appendix D program, the same tape used to generate figure 1 was again read and the results are given in figure 2. From this improved interrogation the analyst can easily find where desired data are located and adjust the read statements in the postprocessor program accordingly.

From the foregoing discussion, it is apparent that a postprocessor program must be dynamic. That is, the input read statements must be continually changed to fit each new problem after interrogation of the user tape.

EXAMPLE APPLICATION

The subject transient analysis program was developed in support of the Viking Project, which has a mission to soft-land a scientific payload on the surface of Mars in 1976. In particular, this program was intended to provide transient loads and accelerations in the Viking Dynamic Simulator (VDS) shown in figure 3 for input acceleration transients at the base of the Centaur truss adaptor. The VDS is a dummy spacecraft, which is dynamically similar to the actual Viking spacecraft, and will be flown on a proof (or test) flight of a new Titan D-1T Centaur launch vehicle configuration in 1974. This launch vehicle will be used for the Viking mission and is shown in figure 4 along with the Viking spacecraft.

Several discrete transient events induce high loads into the VDS with the more prominent of these being Titan Stage 0 Ignition, Titan Stage 1 Shutdown, and the Centaur Main Engine Cutoffs. All of these events were analyzed in detail using the subject program and some of the results from the Titan Stage 0 Ignition were selected as a typical illustration of input and output data.

Six degree-of-freedom acceleration inputs into the base of the VDS were determined analytically from a transient loads analysis of the actual Viking configuration as depicted in figure 5. Input to the transient analysis of the actual Viking configuration was based on measured force transients from previous Titan launches. The base of the VDS was constrained to a plane and the six components of acceleration were input at a single grid point in the center of the base.

A typical set of input translational components of acceleration are shown in figure 6. The longitudinal (or Z) component is seen to be the most significant and it starts at 1g (9.81 m/sec^2) to represent the initial gravity load on the vehicle resting on the launch pad. This gravity loading causes an initial condition on the modal coordinates $\{\xi_i(0)\}$ as indicated in equation (16). The near sinusoidal oscillation of the Z component after 0.5 second is attributed to excitation of a longitudinal mode of the vehicle by the initial thrust transient.

Using the input acceleration (see figure 6) and the modal data (for 18 modes) from the NASTRAN analysis, selected loads for VDS members and accelerations were computed using a viscous damping model which is a function of the modal frequencies. A typical load-versus-time response is shown for a member of the Viking Spacecraft Adaptor in figure 7. Similarly, the translational terms of the acceleration computed for the top of the VDS, are shown in figure 8.

A typical computer run for the example problem on the Langley Research Center CDC 6600 Computer required a storage of $53\ 000_8$ and 200 CPU seconds for execution (including time for generation of 31 output plots). A comparable NASTRAN transient analysis would require at least 120% increase in storage, and considerable increase in CPU time and calls to the operating system. Postprocessor programs thus are seen to offer potential economic savings in addition to special purpose capability.

CONCLUDING REMARKS

The transient analysis program described herein yields a simple, convenient, and economical approach for treating input accelerations and modal initial conditions. Other than the limitation of six on the maximum number of input acceleration components, the program is applicable to a broad spectrum of structural applications.

The fact that such a postprocessor program could be simply written to interface with NASTRAN demonstrates the expanded utility of NASTRAN via the new Level 15 utility modules and user tape option. Tailor-made programs such as the present one can be designed to be very efficient in comparison to NASTRAN. Thus, the authors would encourage further additions and refinement of postprocessor convenience modules rather than expanded capability and complexity of NASTRAN. In particular the formats for writing user tapes, so that they may be easily read by postprocessor programs, should be given prime consideration.

APPENDIX A

THE NESTED VECTOR SET CONCEPT USED TO REPRESENT COMPONENTS OF DISPLACEMENT

In constructing the matrices used in the Displacement Approach, each row and/or column of a matrix is associated closely with a grid point, a scalar point, or an extra point. Every grid point has 6 degrees of freedom associated with it, and hence 6 rows and/or columns of the matrix. Scalar and extra points only have one degree of freedom. At each point (grid, scalar, extra) these degrees of freedom can be further classified into subsets, depending on the constraints or handling required for particular degrees of freedom. (For example, in a two-dimensional problem all z degrees of freedom are constrained and hence belong to the s (single-point constraint) set.) Each degree of freedom can be considered as a "point," and the entire model is the collection of these one-dimensional points.

Nearly all of the matrix operations in displacement analysis are concerned with partitioning, merging, and transforming matrix arrays from one subset of displacement components to another. All the components of displacement of a given type (such as all points constrained by single-point constraints) form a vector set that is distinguished by a subscript from other sets. A given component of displacement can belong to several vector sets. The mutually exclusive vector sets, the sum of whose members are the set of all physical components of displacements, are as follows:

u_m points eliminated by multipoint constraints

u_s points eliminated by single-point constraints

u_o points omitted by structural matrix partitioning

u_r points to which determinate reactions are applied in static analysis,

u_ℓ the remaining structural points used in static analysis (points left over)

u_e extra degrees of freedom introduced in dynamic analysis to describe control systems

The vector sets obtained by combining two or more of the above sets are (+ sign indicates the union of two sets)

$u_a = u_r + u_\ell$, the set used in real eigenvalue analysis

$u_d = u_a + u_e$, the set used in dynamic analysis by the direct method

$u_f = u_a + u_o$, unconstrained (free) structural points

$u_n = u_f + u_s$, all structural points not constrained by multipoint constraints

$u_g = u_n + u_m$, all structural (grid) points including scalar points

$u_p = u_g + u_e$, all physical points

In dynamic analysis, additional vector sets are obtained by a modal transformation derived from real eigenvalue analysis of the set u_a. These are

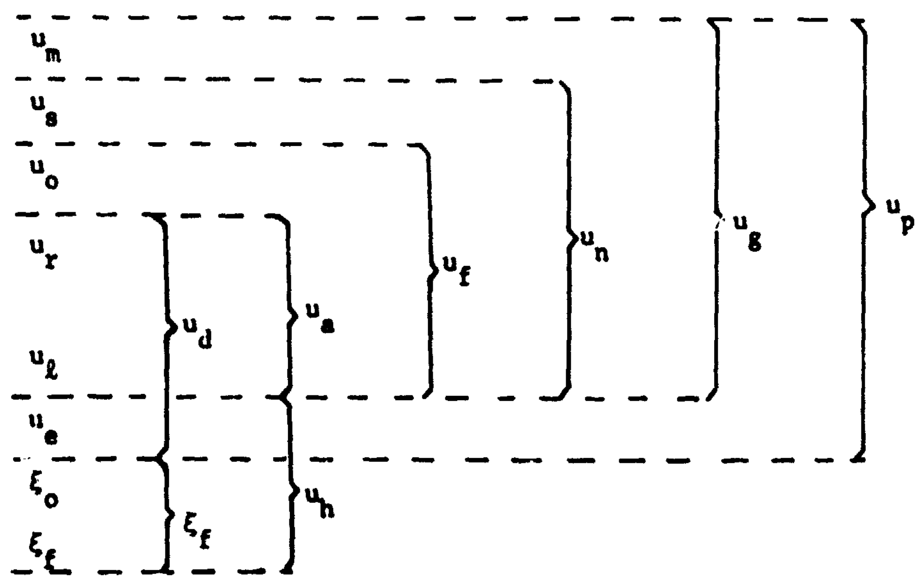
ξ_o rigid body (zero frequency) modal coordinates

ξ_f finite frequency modal coordinates

$\xi_i = \xi_o + \xi_f$, the set of all modal coordinates

One vector set is defined that combines physical and modal coordinates. The set is $u_h = \xi_i + u_e$, the set used in dynamic analysis by the modal method.

The nesting of vector sets is depicted by the following diagram:



The data block USET (USETD in dynamics) is central to this set classification. Each word of USET corresponds to a degree of freedom in the problem. Each set is assigned a bit in the word. If a degree of freedom belongs to a given set, the corresponding bit is on. Every degree of freedom can then be classified by analysis of USET. The common block /BITPØS/ relates the sets to bit numbers.

APPENDIX B

MODAL PROPERTIES

In this section, several identities relating to both the rigid body modes and the flexible body modes are presented. Although these identities are perhaps familiar, they are included herein for completeness and continuity of notation.

Rigid Body Modal Properties

For periodic motion of frequency ω, equation (2) reduces to the eigenvalue equation

$$\left[\begin{bmatrix} K_{\ell\ell} & K_{\ell r} \\ K_{\ell r}^T & K_{rr} \end{bmatrix} - \omega^2 \begin{bmatrix} M_{\ell\ell} & M_{\ell r} \\ M_{\ell r}^T & M_{rr} \end{bmatrix}\right] \begin{Bmatrix} u_\ell \\ u_r \end{Bmatrix} = 0 \tag{B1}$$

The solution of equation (B1) yields the natural frequencies and the corresponding natural modes of the system. For the rigid body modes corresponding to $\omega = 0$, equation (B1) reduces to

$$\begin{bmatrix} K_{\ell\ell} & K_{\ell r} \\ K_{\ell r}^T & K_{rr} \end{bmatrix} \begin{Bmatrix} u_\ell \\ u_r \end{Bmatrix}_{rig.} = 0 \tag{B2}$$

Since the rigid body mode matrix $\left[D_{\ell r}\right]$ relates the rigid body motions $\{u_\ell\}_{rig.}$ in terms of $\{u_r\}_{rig.}$, the following transformation may be written:

$$\begin{Bmatrix} u_\ell \\ u_r \end{Bmatrix}_{rig.} = \begin{bmatrix} D_{\ell r} \\ I \end{bmatrix} \{u_r\}_{rig.} \tag{B3}$$

where $\left[I\right]$ is the identity matrix. Using equation (B3), equation (B2) gives

$$\begin{bmatrix} K_{\ell\ell} & K_{\ell r} \\ K_{\ell r}^T & K_{rr} \end{bmatrix} \begin{bmatrix} D_{\ell r} \\ I \end{bmatrix} \{u_r\}_{rig.} = 0 \tag{B4}$$

For arbitrary $\{u_r\}_{rig.}$ displacements, it follows from the partitions of (B4) that

$$\left[K_{\ell\ell}\right]\left[D_{\ell r}\right] + \left[K_{\ell r}\right] = 0 \tag{B5}$$

and

$$\left[K_{\ell r}^T\right]\left[D_{\ell r}\right] + \left[K_{rr}\right] = 0 \tag{B6}$$

Equations (B5) and (B6) thus yield two important identities relating the rigid body modes and the partitions of the stiffness matrix.

It should also be noted that solving equation (B5) for the rigid body mode matrix yields

$$\left[\bar{D}_{\ell r}\right] = -\left[K_{\ell\ell}\right]^{-1}\left[K_{\ell r}\right] \tag{B7}$$

which is consistent with equation (41) in Section 3.5 of reference 2 and is the equation used in NASTRAN Rigid Format 3 to compute the rigid body mode matrix.

Flexible Body Modal Properties

By definition of the $\{u_r\}$ degrees-of-freedom, introduction of the constraint

$$\{u_r\} = 0 \tag{B8}$$

eliminates rigid body motion leaving only flexible body motion. Using equation (B8), the upper partition of equation (B1) yields the following eigenvalue equation for the flexible body modes:

$$\left[K_{\ell\ell} - \omega^2 M_{\ell\ell}\right]\{u_\ell\} = 0 \tag{B9}$$

The modal matrix $\left[\phi_{\ell i}\right]$ of the i eigenvectors of equation (B9) is shown in reference 4 to satisfy the following orthogonality relationships:

$$\left[\phi_{\ell i}\right]^T\left[M_{\ell\ell}\right]\left[\phi_{\ell i}\right] = \left[\bar{M}_{ii}\right] \tag{B10}$$

$$\left[\phi_{\ell i}\right]\left[K_{\ell\ell}\right]\left[\phi_{\ell i}\right] = \left[\bar{M}_{ii}\omega_i^2\right] \tag{B11}$$

The above equations provide the foundation for modal formulation.

APPENDIX C

DMAP ALTER PACKAGE

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ALTER    81
VEC      USET/VA/C,N,A/C,N,L/C,N,R $
MERGE    DM,,,,,VA/RBA/C,N,0/C,N,2/C,N,2 $
EQUIV    RBA,RBF/OMIT $
COND     LBOM,OMIT $
MPYAD    GO,RBA,/RBO/C,N,0/C,N,1/C,N,0/C,N,1 $
VEC      USET/VF/C,N,F/C,N,A/C,N,O $
MERGE    RBA,RBO,,,,VF/RBF/C,N,0/C,N,2/C,N,2 $
LABEL    LBOM $
EQUIV    RBF,RBN/SINGLE $
COND     LBSS,SINGLE $
VEC      USET/VN/C,N,N/C,N,F/C,N,S $
MERGE    RBF,,,,,VN/RBN/C,N,0/C,N,2/C,N,2 $
LABEL    LBSS $
EQUIV    RBN,RBG/MPCF2 $
COND     LBMM,MPCF2 $
MPYAD    GM,RBN,/RBM/C,N,0/C,N,1/C,N,0/C,N,1 $
VEC      USET/VG/C,N,G/C,N,N/C,N,M $
MERGE    RBN,RBM,,,,VG/RBG/C,N,0/C,N,2/C,N,2 $
LABEL    LBMM $
MATGPR   GPL,USET,SIL,RBG//C,N,R/C,N,G $
OUTPUT2, ,,,,//C,N,-1/C,N,11 $
OUTPUT2, RBG,,,,//C,N,0/C,N,11 $
ALTER    89,89
READ     KLL,MLL,,,EED,,CASECC/LAMA,PHIL,MI,OEIGS/
ALTER    91,91
CHKPNT   LAMA,PHIL,MI,OEIGS $
ALTER    93
UMERGE   USET,PHIL,/PHIA/V,N,MAJOR=A/V,N,SUBO=L/V,N,SUB1=R $
MPYAD    MLL,DM,MLR/TMP/C,N,0/C,N,1/C,N,1/C,N,1 $
MPYAD    PHIL,TMP,/MIR/C,N,1/C,N,1/C,N,0/C,N,1 $
MATPRN   MIR,,,,// $
OUTPUT2, MIR,,,,//C,N,0/C,N,11 $
ALTER    105
OUTPUT2, OPHIG,OQG1,OEF1,,//C,N,0/C,N,11 $
OUTPUT2, ,,,,//C,N,-9/C,N,11 $
ENDALTER
```

APPENDIX D

MULTI-FORMAT TAPE DUMP PROGRAM

```
      PROGRAM A3930 (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE1)
C*********************************************************************C
C                                                                     C
C     THIS PROGRAM READS A N Y  TAPE WITH VARIABLE LENGTH BINARY RECORDS  C
C     REQUEST TAPE MUST BE LABELED AS TAPE1                           C
C     THE PROGRAM DETERMINES THE MODE (REAL,INTEGER OR ALPHA) OF EACH WORD C
C     ON THE TAPE.                                                    C
C     THE PROGRAM  THEN PRINTS EACH RECORD IN THE CORRECT FORMAT.     C
C                                                                     C
C*********************************************************************C
      DIMENSION NN(513),FM(3),FMT(10)
      DATA FM /7H,A10,5X , 6H,E15.7 , 7H,I10,5X /
      DATA FMT(1),FMT(10),CPAREN /4H(11X , 1H) , 1H) /
      REWIND 1
      WRITE(6,1)
    1 FORMAT(1H1)
      NREC=0
    2 ICNT=IVAR(1,NN,0,513)
      IF(ICNT.EQ.0) STOP
      IF(ICNT.LT.0) GO TO 7
      NREC=NREC+1
      WRITE(6,3) NREC
    3 FORMAT(1H ,*RECORD*I4)
      DO 6 I=1,ICNT,8
      DO 4 J=1,8
      J2=I+J-1
      CALL WHAT(NN(J2),NTYPE)
      FMT(1+J)=FM(NTYPE)
      IF(J2.EQ.ICNT) GO TO 5
    4 CONTINUE
      GO TO 6
    5 FMT(2+J)=CPAREN
    6 WRITE(6,FMT) (NN(J),J=I,J2)
      GO TO 2
    7 WRITE(6,8)
    8 FORMAT(1H ,30H****P A R I T Y  E R R O R****)
      STOP
      END
```

```
      SUBROUTINE WHAT(N,NTYPE)
C     THIS SUBROUTINE DETERMINES THE MODE OF  N
C        NTYPE=1  IF N IS ALPHA
C        NTYPE=2  IF N IS REAL
C        NTYPE=3  IF N IS INTEGER
      INTEGER BB
      DATA BB/4H0000/
      CALL RSHFT(BB,-24)
      CALL RSHFT(BB, 24)
      IF(IABS(  N  ).LE.9999999999) GO TO 140
      M=N
      CALL RSHFT( M ,-24)
      CALL RSHFT( M , 24)
      IF( M .NE.BB) GO TO 100
      M=N
      CALL RSHFT( M ,54)
      IF( M .LT.1.OR. M .GT.47) GO TO 100
      M=N
      CALL RSHFT( M ,-6)
      CALL RSHFT( M ,54)
      IF( M .LT.1.OR. M .GT.47) GO TO 100
C     ALPHA        1
      NTYPE=1
      GO TO 4
C     REAL         2
  100 NTYPE=2
      GO TO 4
C     INTEGER      3
  140 NTYPE=3
    4 RETURN
      END
```

REFERENCES

1. McCormick, Caleb W., ed.: The NASTRAN User's Manual (Level 15). NASA SP-222(01), June, 1972.

2. MacNeal, Richard H., ed.: The NASTRAN Theoretical Manual (Level 15). NASA SP-221(01), April, 1972.

3. Anon.: The NASTRAN Programmer's Manual. NASA SP-223(01), September, 1972.

4. Hurty, Walter C. and Rubinstein, Moshe F.: Dynamics of Structures. Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1964, pp. 121-123.

```
$NPUT
NRCRDS  =  19.
NFILES  =  1.
NODE    =  0.
IPRTBIN =  1.
IPRTBCD =  0.
NUMPRTS =  0.
ISTRPRT =  1.
$END
*** FILE      1  RECORD     1 IS BINARY AND CONTAINS      1 WORDS. ***
BINARY REC
       0.
*** FILE      1  RECORD     2 IS BINARY AND CONTAINS      3 WORDS. ***
BINARY REC
       0.            0.           0.
*** FILE      1  RECORD     3 IS BINARY AND CONTAINS      1 WORDS. ***
BINARY REC
       0.
*** FILE      1  RECORD     4 IS BINARY AND CONTAINS      7 WORDS. ***
BINARY REC
      9.99009844-25     4.40421431E+52     1.15759000-174     -8.25896360E+65     3.33660815-185     -8.41192959E+70
      1.42361948-184
*** FILE      1  RECORD     5 IS BINARY AND CONTAINS      1 WORDS. ***
BINARY REC
       0.
*** FILE      1  RECORD     6 IS BINARY AND CONTAINS      2 WORDS. ***
BINARY REC
      2.41257557+175    2.41257557+175
*** FILE      1  RECORD     7 IS BINARY AND CONTAINS      1 WORDS. ***
BINARY REC
      -0.
*** FILE      1  RECORD     8 IS BINARY AND CONTAINS      1 WORDS. ***
BINARY REC
       0.
*** FILE      1  RECORD     9 IS BINARY AND CONTAINS      1 WORDS. ***
BINARY REC
       0.
*** FILE      1  RECORD    10 IS BINARY AND CONTAINS      2 WORDS. ***
BINARY REC
      2.54743304E-01    5.72023741-146
*** FILE      1  RECORD    11 IS BINARY AND CONTAINS      1 WORDS. ***
BINARY REC
      -0.
*** FILE      1  RECORD    12 IS BINARY AND CONTAINS      1 WORDS. ***
BINARY REC
       0.
*** FILE      1  RECORD    13 IS BINARY AND CONTAINS      7 WORDS. ***
```

Figure 1. - User Tape Interrogation Using Standard Tape Dump Program and Writing to an E Format

```
BINARY REC
      0.                0.                0.                0.                0.                0.
      0.
*** FILE      1  RECORD      14 IS BINARY AND CONTAINS      1 WORDS. ***
BINARY REC
     -0.
*** FILE      1  RECORD      15 IS BINARY AND CONTAINS      1 WORDS. ***
BINARY REC
      0.
*** FILE      1  RECORD      16 IS BINARY AND CONTAINS      7 WO   ***
BINARY REC
  2.54743304E-01    5.72023741-146      0.                0.                0.                0.
      0.
*** FILL      1  RECORD      17 IS BINARY AND CONTAINS      1 WORDS. ***
BINARY REC
     -0.
*** FILE      1  RECORD      18 IS BINARY AND CONTAINS      1 WORDS. ***
BINARY REC
      0.
*** FILE      1  RECORD      19 IS BINARY AND CONTAINS    146 WORDS. ***
BINARY REC
      0.                0.                0.                0.                0.            8.08468733E+02
      0.                0.                0.                0.                0.                0.
      0.                0.                0.                0.                0.                0.
      0.                0.                0.                0.                0.                0.
      0.                0.                0.                0.                0.                0.
      0.                0.                0.                0.                0.                0.
      0.                0.                0.                0.                0.                0.
      0.                0.                0.                0.                0.                0.
      0.                0.          9.29446985+131    1.49136233E-22    2.16410409+190    4.38648832-120
  2.10160831-117    1.44733759-269   -7.17196054E+57   -2.79367800E+77   -7.44354977E+57    6.41157294E-25
  7.89205938E+92    1.17649375E-50    6.79931385E-25    7.36131288-231   -7.17383055E+57   -7.17383055E+57
 -7.17383055E+57   -7.17383055E+57   -7.17383055E+57   -7.17383055E+57   -7.17383055E+57   -7.17383055E+57
 -7.17383055E+57   -7.17383055E+57   -7.17383055E+57   -7.17383055E+57   -7.17383055E+57   -7.17383055E+57
 -7.17383055E+57   -7.17383055E+57   -7.17383055E+57   -7.17383055E+57   -1.54824113E+68   -4.69837902+204
 -1.42785140+200   -8.24407186E+67    1.95028376-194   -2.37872618+197    2.05589076+278   -7.17383055E+57
 -7.17383055E+57   -7.17383055E+57   -7.17383055E+57   -7.17383055E+57   -7.17383055E+57   -7.17383055E+57
 -7.17383055E+57   -7.17383055E+57   -7.17383055E+57   -7.17383055E+57   -7.17383055E+57   -7.17383055E+57
 -7.17383055E+57   -7.17383055E+57   -7.17383055E+57   -7.17383055E+57   -7.17383055E+57   -7.17383055E+57
 -7.17383055E+57   -7.17383055E+57   -7.17383055E+57   -7.17383055E+57   -7.17383055E+57   -7.17383055E+57
 -1.71147519E+70   -5.63721099E+19   -2.29734750E+63   -2.56722218E+66    4.14837723+104    5.91089639E+93
  9.31443422E+13    7.58432902-197    3.78610085E+71    6.97155753-272    2.16655311E+17   -7.17383055E+57
 -7.17383055E+57   -7.17383055E+57   -7.17383055E+57   -7.17383055E+57   -7.17383055E+57   -7.17383055E+57
 -7.17383055E+57   -7.17383055E+57   -7.17383055E+57   -7.17383055E+57   -7.17383055E+57   -7.17383055E+57
 -7.17383055E+57   -7.17383055E+57   -7.17383055E+57   -7.17383055E+57   -7.17383055E+57   -7.17383055E+57
 -7.17383055E+57   -7.17383055E+57
```

Figure 1. - Concluded

```
RECORD  1
              3
RECORD  2
             10          18          72
RECORD  3
              7
RECORD  4
        NAST        RAN         FORT        TAP         E ID        COD         E -
RECORD  5
              2
RECORD  6
        XXXX        XXXX
RECORD  7
             -1
RECORD  8
              0
RECORD  9
              2
RECORD 10
        OPHI         6
RECORD 11
             -1
RECORD 12
              7
RECORD 13
            101           0       10656           0           0           0           0
RECORD 14
             -2
RECORD 15
              7
RECORD 16
        OPHI         6                10          18          72           1           1
RECORD 17
             -3
RECORD 18
            146
RECORD 19
             21           7           0           1           1  8.0846873E+02    2           0
              1           0           0           0           0           0       0           0
              0           0           0           0           0           0       0           0
              0           0           0           0           0           0       0           0
              0           0           0           0           0           0       0           0
              0           0           0           0           0           0       0           0
         =    0   ====    0   VIKI        NG D        YNAM        IC S        INUL        ATOR
                                 A        NALY        TICA        L DY        NAMI        CS
                              LKC         -GDC        -VDS        NOD         EL 8        -3-7
        2
                              EXT         . OF        IST         ROn         T WI        TH U
        PDAT        ED V      SCA         AND         PLLA
```

Figure 2. - User Tape Interrogation Using Tape Dump Program of Appendix D

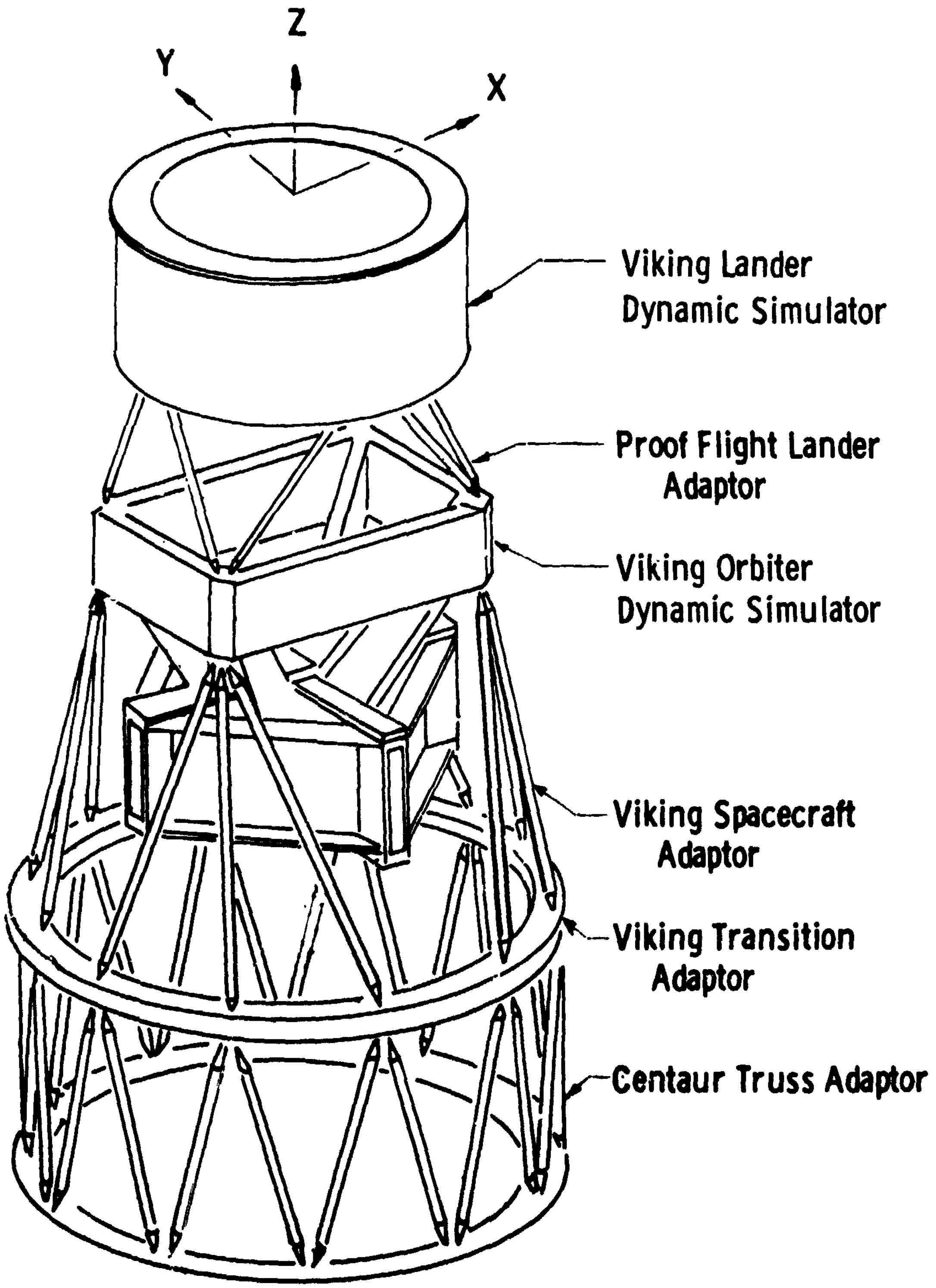

Figure 3. - Viking Dynamic Simulator

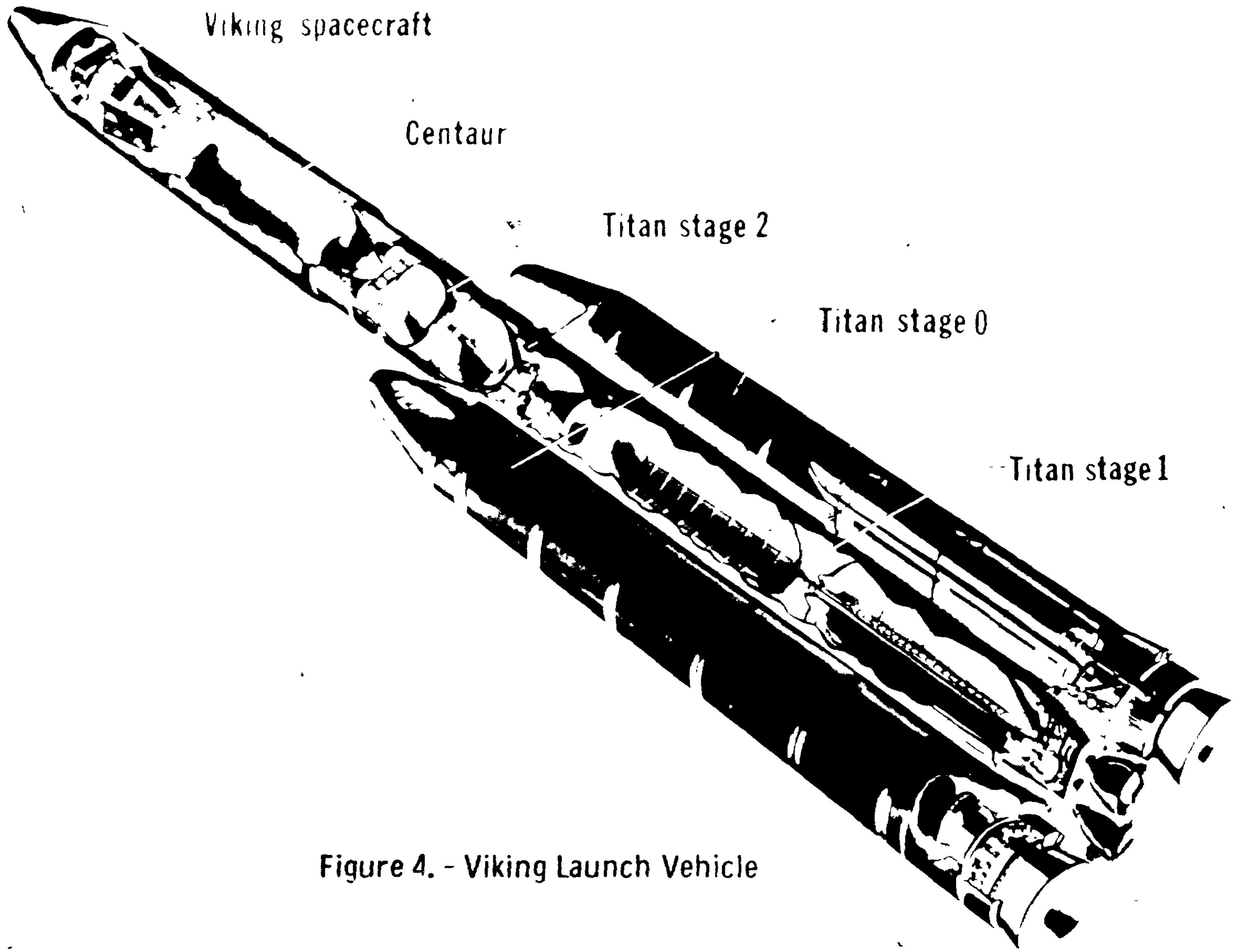

Figure 4. - Viking Launch Vehicle

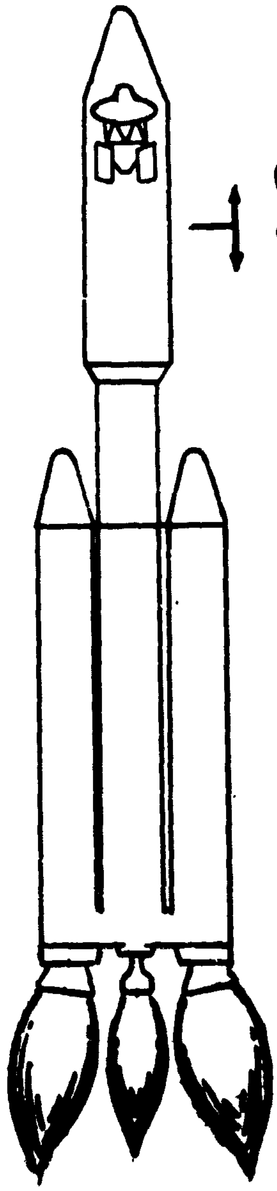

Figure 5. - Viking Configuration Used to Determine Input Acceleration-Time Histories

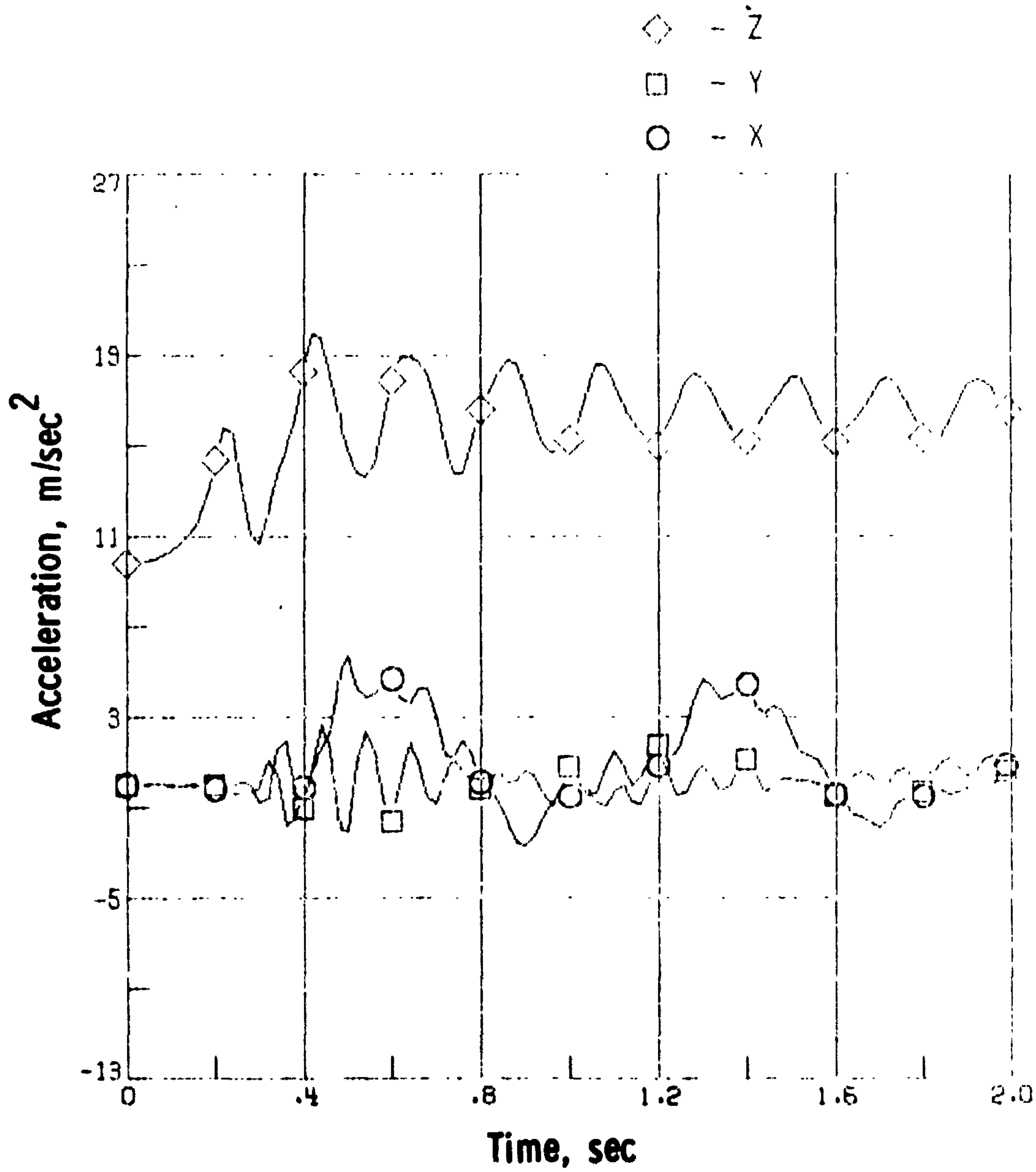

Figure 6. - Translational Accelerations Used As Input

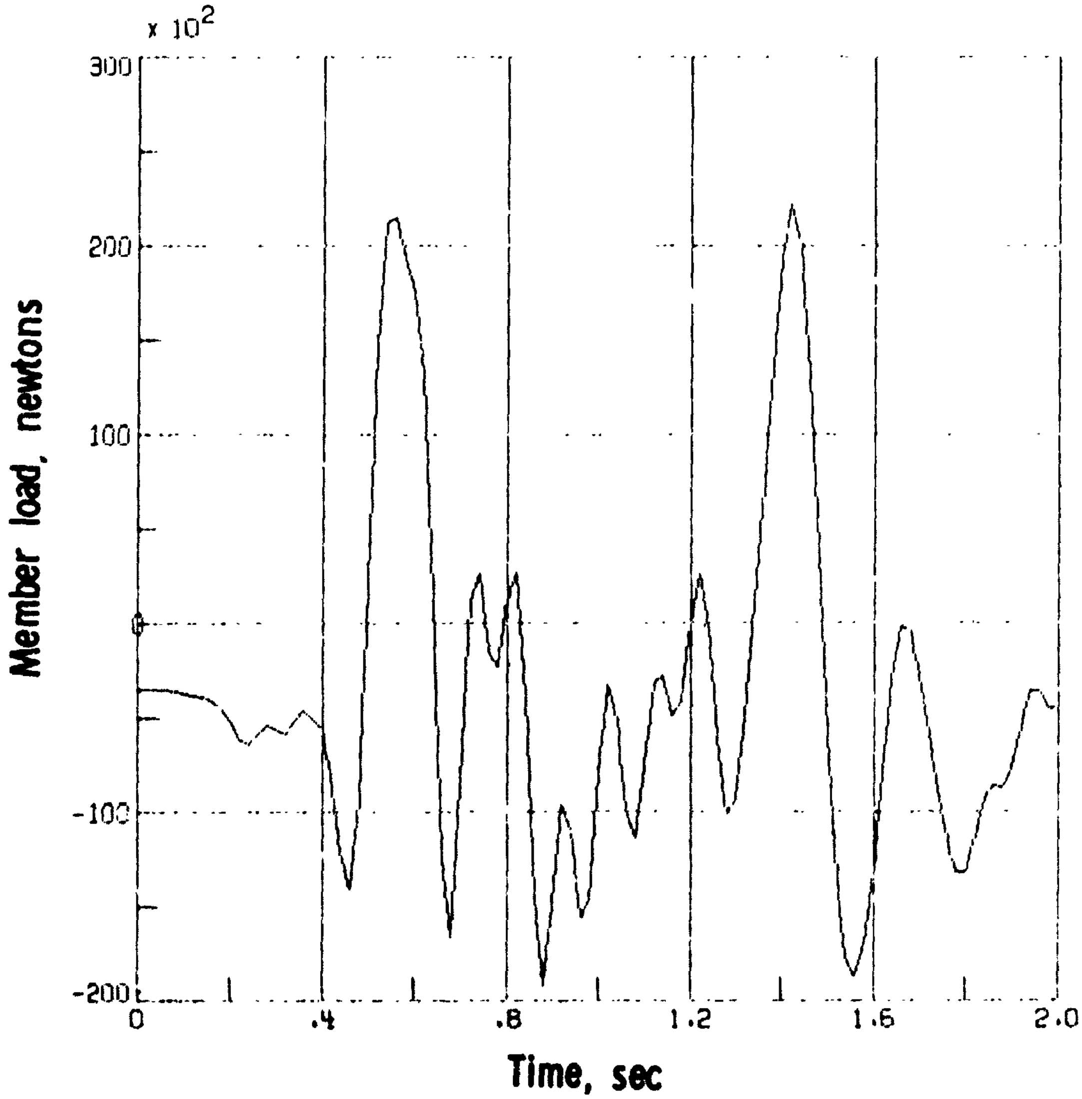

Figure 7. - Typical Load Versus Time for a Member of the Viking Spacecraft Adaptor Truss

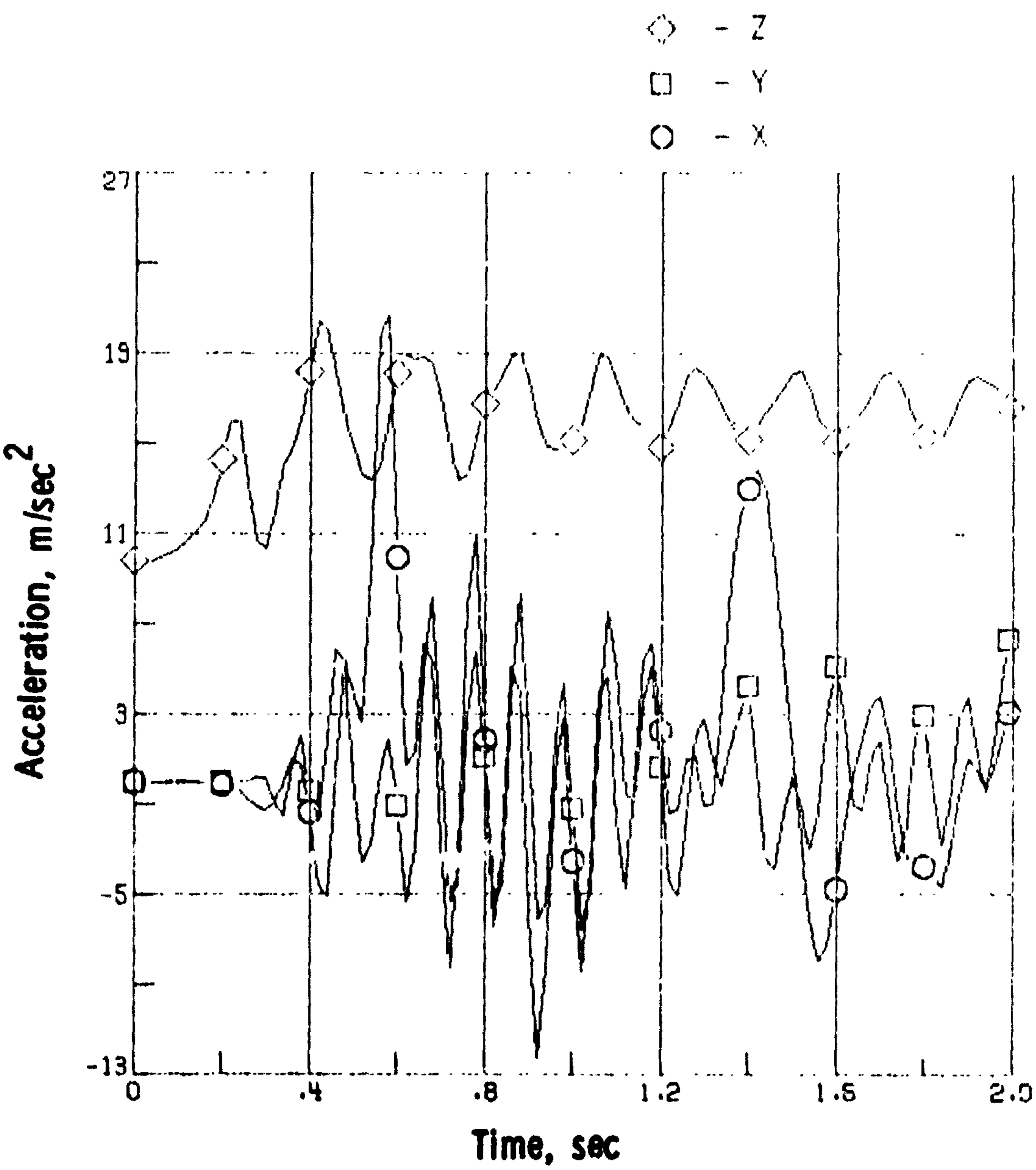

Figure 8. - Output Acceleration at the Top of the Viking Spacecraft

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