COUPLED ROTOR/AIRFRAME VIBRATION PREDICTION METHODS

J. A. Staley
Senior Dynamics Engineer

J. J. Sciarra
Senior Structures Engineer

Boeing Vertol Company
Philadelphia, Pa.

Abstract

The problems of airframe structural dynamic representation and effects of coupled rotor/airframe vibration are discussed. Several finite element computer programs (including NASTRAN) and methods for idealization and computation of airframe natural modes and frequencies and forced response are reviewed. Methods for obtaining a simultaneous rotor and fuselage vibratory response, determining effectiveness of vibration control devices, and energy methods for structural optimization are also discussed. Application of these methods is shown for the vibration prediction of the Model 347 helicopter.

Notation

A - airframe mobility matrix
B - rotor impedance matrix
EI - blade bending rigidity
F - force
GJ - blade torsional rigidity
I - identity matrix
k - rotor frequency multiple, 1, 2, etc.
K - stiffness matrix
M - mass matrix
q - airframe mode generalized coordinate
X - airframe displacements
$\mathcal{m}$ - airframe mode generalized mass
ω - airframe mode natural frequency
ϕ - airframe mode shape (eigen vector)
Ω - rotor frequency
[] - matrix
{} - column vector

Subscripts

A - absorber, airframe
c - cosine component amplitude
H - hub
k - rotor frequency multiple, 1, 2, etc.
n - airframe mode number
o - zero hub motion
R - rotor
s - sine component

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Superscripts

$\dot{}$ - velocity
$\ddot{}$ - acceleration
T - transpose

Prediction of helicopter airframe vibration involves two major problem areas:

- Prediction of rotor vibratory hub loads
- Prediction of airframe dynamic characteristics.

The effects of vibratory hub motion on vibratory hub loads and effects of vibration control devices and resulting airframe fatigue stresses must also be considered.

Methods for independent prediction of vibratory hub loads and airframe dynamic characteristics have been developed previously and are discussed briefly below. Independent determination of rotor vibratory loads and airframe vibratory response to these loads does not account for any interaction between airframe vibratory motion on rotor vibratory loads. One approximate method for accounting for these interactions is to assume that an effective rotor mass is attached to the airframe at the rotor hub. A more direct method is to compute (or measure) the rotor hub impedance and determine compatible vibratory hub loads and hub motions. This method is discussed below. A simple example of compatible rotor load-hub motion is given for a single rotor helicopter with vertical hub motion. In addition, flight test results for the Model 347 helicopter are compared with vibration predictions obtained using a coupled rotor/airframe vibration computer program.

Rotor Vibratory Hub Loads

Methods and digital computer programs have been developed for prediction of rotor vibratory hub loads for constant speed level flight conditions [1,2,3]. Rotor blades are represented by lumped parameter analytical models as indicated in Figure 1. Iteration techniques are used to compute individual blade deflections and aerodynamic and inertia load distributions at integer multiples of the rotor rotating frequency. The total rotating and fixed system rotor vibratory hub loads are obtained by summing individual blade root shears and moments. The vibratory hub loads may be computed assuming no hub motion. If the vibratory hub motions are known, effects of these motions may be included when computing blade aerodynamic and inertia loads.

Airframe Dynamics

Structural Model, Natural Modes and Frequencies, and Forced Response

Finite element methods have been used in the helicopter industry for some time for prediction of airframe dynamic characteristics[4]. As indicated in Figure 2, developing a finite element airframe model consists of:

- Defining nodal data
- Defining elastic properties of members connecting nodes
- Defining mass properties associated with each node.

Nodal data and properties of structural members are used to develop stiffness matrices for individual members. These matrices relate forces at each node to nodal displacements. The stiffness matrices for individual members are superimposed to obtain the stiffness matrix for the entire airframe.

Most of the degrees of freedom are reduced from the airframe gross stiffness matrix. Mass properties are concentrated at the remaining (retained) degrees of freedom. Equations (1) are the airframe equations of motion with the gross stiffness matrix. Equations (3) are the airframe equations of motion, in terms of the reduced stiffness matrix.

$$\begin{bmatrix} M & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \end{Bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} F_R \\ 0 \end{Bmatrix} \quad (1)$$

$$\left[\bar{K}_{11}\right] = \left[K_{11}\right] - \left[K_{12}\right] \left[K_{22}\right]^{-1} \left[K_{21}\right] \quad (2)$$

$$\left[M\right] \left\{\ddot{X}_1\right\} + \left[\bar{K}_{11}\right] \left\{X_1\right\} = \left\{F_R\right\} \quad (3)$$

The solution for natural modes and frequencies is made using the reduced stiffness matrix and the mass matrix associated with the retained degrees of freedom.

The airframe motions are expressed in terms of natural modes:

$$\left\{X_1\right\} = \left[\phi\right] \left\{q\right\} \quad (4)$$

and, after assuming sinusoidal motion with no external forces, Equation (3) becomes:

$$\omega_n^2 \left\{\phi_n\right\} = \left[M\right]^{-1} \left[\bar{K}_{11}\right] \left\{\phi_n\right\} \quad (5)$$

The modal generalized mass is then computed. A value of modal damping is assumed for each mode, and these modal properties are used to compute airframe response to vibratory hub loads:

$$m_n = \left\{\phi_n\right\}^T \left[M\right] \left\{\phi_n\right\} \quad (6)$$

$$\ddot{q}_n + 2\zeta_n \omega_n \dot{q}_n + \omega_n^2 q_n = \left\{\phi_{Rn}\right\}^T \left\{F_R\right\} / m_n \quad (7)$$

Substructures Method

A large saving in computer time can be realized by performing the matrix reduction process on several smaller substructure stiffness matrices instead of on the large stiffness matrix for the entire airframe. In one application, use of the substructures method reduced computer running time from about ten to two hours on an IBM 360/65 computer.

The airframe is divided into several substructures, and all but mass and boundary degrees of freedom are reduced from the stiffness matrix of each substructure. The stiffness matrices of the substructures are then merged (superimposed or added just as they are for individual members) to form a stiffness matrix for the entire airframe. Any degree of freedom on the boundaries may be reduced after merging the substructure matrices (Figure 3).

NASTRAN

New developments in finite element analysis have been occurring on a continuous basis. New programs and new structural elements, both dynamic and stress analysis capability, FORTRAN programming capability by the engineer within the finite element program, and greater problem size capability have been developed[5]. NASTRAN (NASA Structural Analysis)[6] is a government developed, maintained, and continually updated finite element program which has apparently provided a solution to the difficulties of developing and maintaining finite element programs by private contractors. NASTRAN is similar to other finite element computer programs except that it generally provides additional capability:

- More types of structural elements
- Common deck for stress and dynamic analysis
- User programming capability
- Transient vibration analysis, buckling, non-linear, and static capability
- Unlimited size capability for mass and stiffness matrices.

For a nominal fee, this program and manuals describing the program and its use are available. NASTRAN provides a standard for airframe dynamic analysis and relieves contractors of some of the problems of maintaining the most up-to-date methods for airframe structural analysis.

Energy Methods for Structural Optimization

One further development related to airframe dynamics is the Damped Forced Response Method[7,8]. The airframe forced response is computed, and structural members with significant strain energy are identified. These members are changed to reduce vibration response for modes with frequencies above and below the rotor exciting frequency. This method is outlined in Figure 4.

Vibration Control Devices

Vibration control devices such as absorbers are often used to reduce vibration in local areas of the airframe. The force output for an absorber may be computed by expressing the vibration as the sum of vibration due to rotor forces and the vibration due to the force output by the absorber.

$$\begin{Bmatrix} X_A \\ X_R \end{Bmatrix} = \begin{bmatrix} A_{AA} & A_{AR} \\ A_{RA} & A_{RR} \end{bmatrix} \begin{Bmatrix} F_A \\ F_R \end{Bmatrix} \quad (8)$$

The absorber force output required to null vibration at the absorber attachment point is

$$\{F_A\} = -\left[A_{AA}\right]^{-1} \left[A_{AR}\right] \{F_R\} \quad (9)$$

The corresponding motions at the rotor hub are

$$\{X_R\} = \left[A_{RR} - A_{RA} A_{AA}^{-1} A_{AR}\right] \{F_F\} \quad (10)$$

The mobility matrices in the above equations may be obtained analytically using computed modal properties (Equations (1) through (7)) or by applying unit vibratory loads to the airframe in a series of shake tests.

This method was applied to prediction of cockpit vibration with a vertical cockpit absorber for the Model 347 helicopter[8]. Analytical and flight test results are compared in Figure 5.

Coupled Rotor/Airframe Analysis

Theory

Any vibratory motion of the rotor hub will change the rotor blade vibratory aerodynamic and inertia forces which are summed to obtain vibratory hub loads. Changes in hub loads will in turn cause changes in vibratory hub motions[9,10].

Airframe Motion is assumed to be related to vibratory hub loads by a mobility matrix for a particular exciting frequency:

$$\begin{Bmatrix} X_{ks} \\ X_{kc} \end{Bmatrix} = \begin{bmatrix} A_{k11} A_{k12} \\ A_{k21} A_{k22} \end{bmatrix} \begin{Bmatrix} F_{ks} \\ F_{kc} \end{Bmatrix} = \left[A_k\right] \begin{Bmatrix} F_{ks} \\ F_{kc} \end{Bmatrix} \quad (11)$$

where

$$\{X_k\} = \{X_{ks}\} \sin k\Omega t + \{X_{kc}\} \cos k\Omega t$$

$$\{F_k\} = \{F_{ks}\} \sin k\Omega t + \{F_{kc}\} \cos k\Omega t$$

The airframe mobility data are airframe responses to unit vibratory hub loads; these data may be obtained analytically by using theoretical modal properties (Equation (4) through (7)), or by conducting an airframe shake test. It is emphasized that these are airframe response characteristics for no blade mass attached to the airframe at the rotor hub. All blade inertia effects will be included in the rotor vibratory hub loads as modified by vibratory hub motion.

In general, six sine and six cosine components of shaking forces and moments exist at each rotor hub; a tandem rotor helicopter would have a total of 24 components of vibratory forces. If only the rotor hub motions are considered, the relationship between hub motion and hub forces is:

$$\underset{24\times1}{\begin{Bmatrix} X_{Hks} \\ X_{Hkc} \end{Bmatrix}} = \underset{24\times24}{\begin{bmatrix} A_{Hk} \end{bmatrix}} \underset{24\times1}{\begin{Bmatrix} F_{ks} \\ F_{kc} \end{Bmatrix}} \tag{12}$$

The vibratory hub loads are assumed to be loads with no hub motion plus an increment of hub loads proportional to hub motion:

$$\underset{24\times1}{\begin{Bmatrix} F_{ks} \\ F_{kc} \end{Bmatrix}} = \underset{24\times1}{\begin{Bmatrix} F_{kso} \\ F_{kco} \end{Bmatrix}} + \underset{24\times24}{\begin{bmatrix} B_{k11} & B_{k12} \\ B_{k21} & B_{k22} \end{bmatrix}} \underset{24\times1}{\begin{Bmatrix} X_{Hks} \\ X_{Hkc} \end{Bmatrix}} \tag{13}$$

$$= \begin{Bmatrix} F_{kso} \\ F_{kco} \end{Bmatrix} + \begin{bmatrix} B_k \end{bmatrix} \begin{Bmatrix} X_{Hks} \\ X_{Hkc} \end{Bmatrix}$$

The coefficients of the B matrix are obtained by making several computations of vibratory hub loads:

- Components of vibratory hub loads are computed assuming no hub motion
- Components of vibratory hub loads are computed assuming a small vibratory hub motion at the frequency for each degree of freedom of hub motion at each rotor
- Changes in sine and cosine components of vibratory hub forces per unit vibratory hub motion in each rotor hub degree of freedom are then computed.

The coupled rotor/airframe solution for compatible rotor hub motions and rotor hub loads is obtained by substituting Equation (13) in Equation (12) and solving for vibratory hub motions:

$$\begin{Bmatrix} X_{Hks} \\ X_{Hkc} \end{Bmatrix} = \begin{bmatrix} [I]-[A][B] \end{bmatrix}^{-1} [A] \begin{Bmatrix} F_{kso} \\ F_{kco} \end{Bmatrix} \tag{14}$$

Once a solution for Equation (14) is obtained, the total vibratory hub loads may be computed using Equation (13) and the vibration for the entire airframe may be computed using Equation (11).

Single Rotor Example

Figure 6 shows a simple example of the coupled rotor airframe method applied to a single rotor helicopter vertical vibration analysis. Hub vertical vibration response and the vertical vibratory hub loads are computed at a frequency of four times rotor speed (4/rev). The airframe is represented by its rigid body vertical mode and one flexible mode. Figure 6b shows airframe mobilities vs flexible mode natural frequency for 4/rev vertical hub forces. Hub vertical shaking forces vs hub vertical motion are shown in Figure 6c. The vibratory hub loads are seen to vary approximately linearly at least up to .005 inches of motion at the 4/rev frequency. Figures 6d and 6e show compatible rotor hub vertical vibration amplitudes and rotor hub shaking forces vs flexible mode natural frequency.

For this example, the rotor vibratory hub motions and forces both peak when the flexible mode natural frequency is just above the rotor hub force exciting frequency. This is not a general result, but depends upon the relationships between hub shaking forces and hub motions.

Coupled Rotor/Airframe Analysis Computer Program (D-65)

Figure 7 shows the flow-diagram for the Boeing Vertol D-65 Coupled Rotor/Airframe Analysis computer program. This program links three major computer programs[10]:

- Trim analysis program A-97
- Rotor vibratory hub loads analysis program D-88
- Airframe forced response analysis program D-96.

Compatible fuselage motions and vibratory hub loads are obtained using this program with the method discussed above. In its current state, the D-65 program computes three vibratory rotor forces and three vibratory rotor moments at each rotor for either single or tandem rotor helicopters. Response to translational and rotational vibratory hub forces is computed for the airframe, but compatibility of hub forces and motions is satisfied for hub translational degrees of freedom only in the current version of the program. The program will be modified in the near future to provide compatibility for hub rotational degrees of freedom.

Analysis vs Test Results for the Model 347 Helicopter

The D-65 coupled rotor/airframe program was used to predict Model 347 flight vibration levels. Figure 8 shows the model used to predict airframe dynamic characteristics. Figure 9 compares predicted vertical and lateral cockpit vibration levels vs vibration levels measured in flight. Vertical vibration levels are in reasonably good agreement at high airspeeds where vibration levels may become significant. Lateral vibration levels are higher than predicted.

Conclusions

Methods have been developed independently for prediction of rotor vibratory hub loads and airframe dynamic characteristics. Methods are available for including effects of vibration control devices on airframe vibration and for optimizing the airframe structure. The substructure method is available for minimizing computer running time in analysis of airframe structures, and NASTRAN now provides a common finite element structural analysis program available to all aerospace contractors. Rotor hub vibratory motions can modify rotor hub vibratory forces acting on the airframe. A linear coupled rotor/airframe analysis method provides an approach for determining compatible hub motions and hub shaking forces. This method should be studied further to determine its validity. A method of this type should be considered in applications of NASTRAN for prediction of helicopter vibration; the user programming feature in NASTRAN should permit a coupled rotor/airframe solution of this type within NASTRAN.

Figure 10 shows a scheme for solving for rotor trim, rotor forces with no hub motion, and the rotor impedance matrix using a rotor analysis program. NASTRAN would be programmed to use these mobilities and the rotor analysis results to solve for compatible rotor/airframe loads and motions. The NASTRAN airframe analysis could include airframe installed vibration control devices either in the initial airframe analysis or in the coupled rotor/airframe solution. Finally, results of these analyses could be used to determine optimum changes to the airframe structural elements for minimizing airframe vibration.

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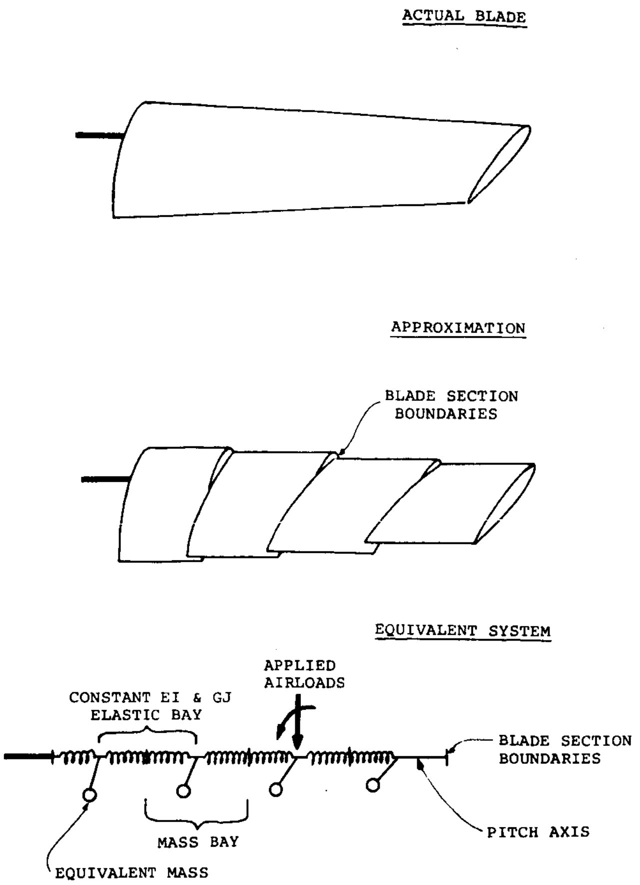


Figure 1. Rotor Blade Analytical Model

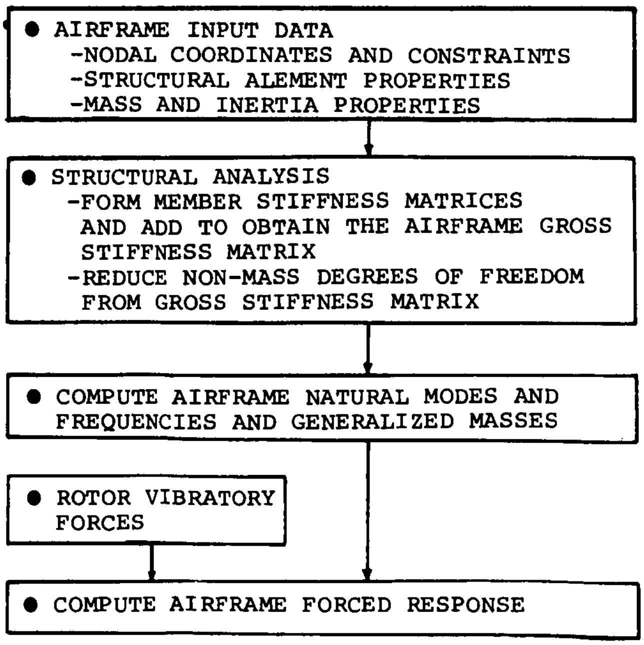


Figure 2. Uncoupled Airframe Dynamic Analysis

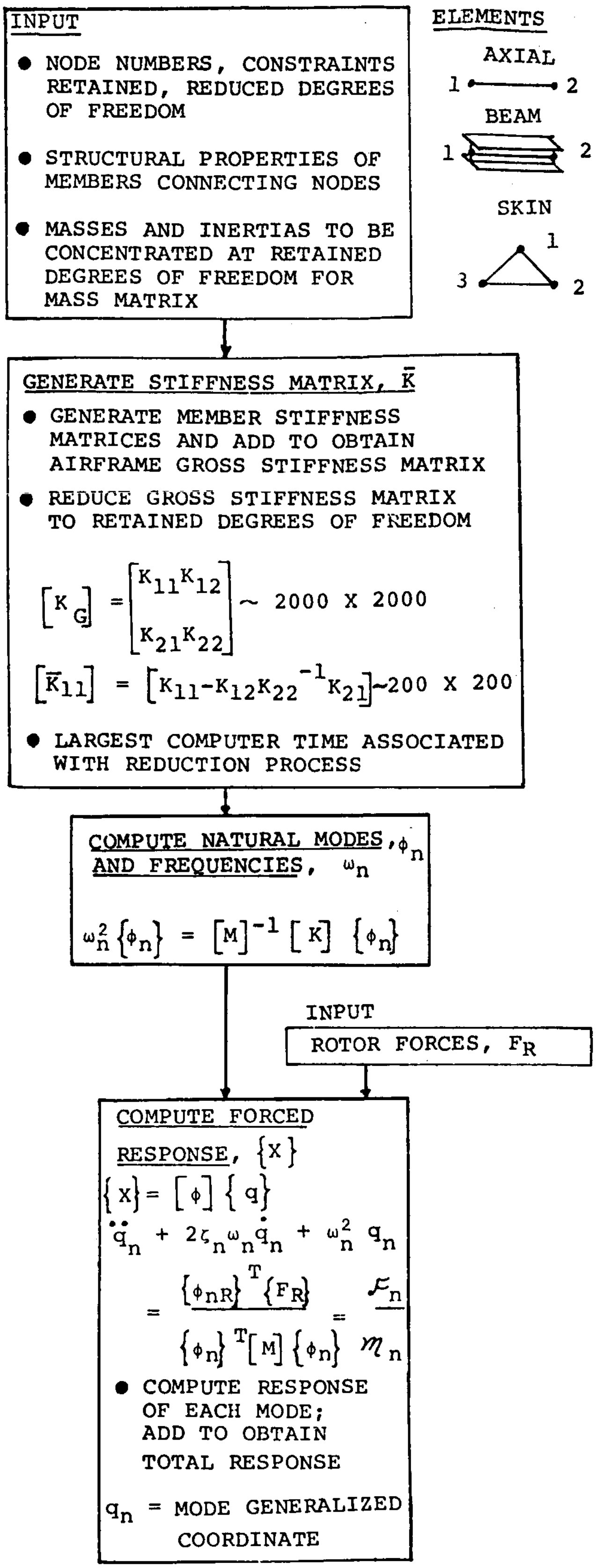


Figure 2. Continued

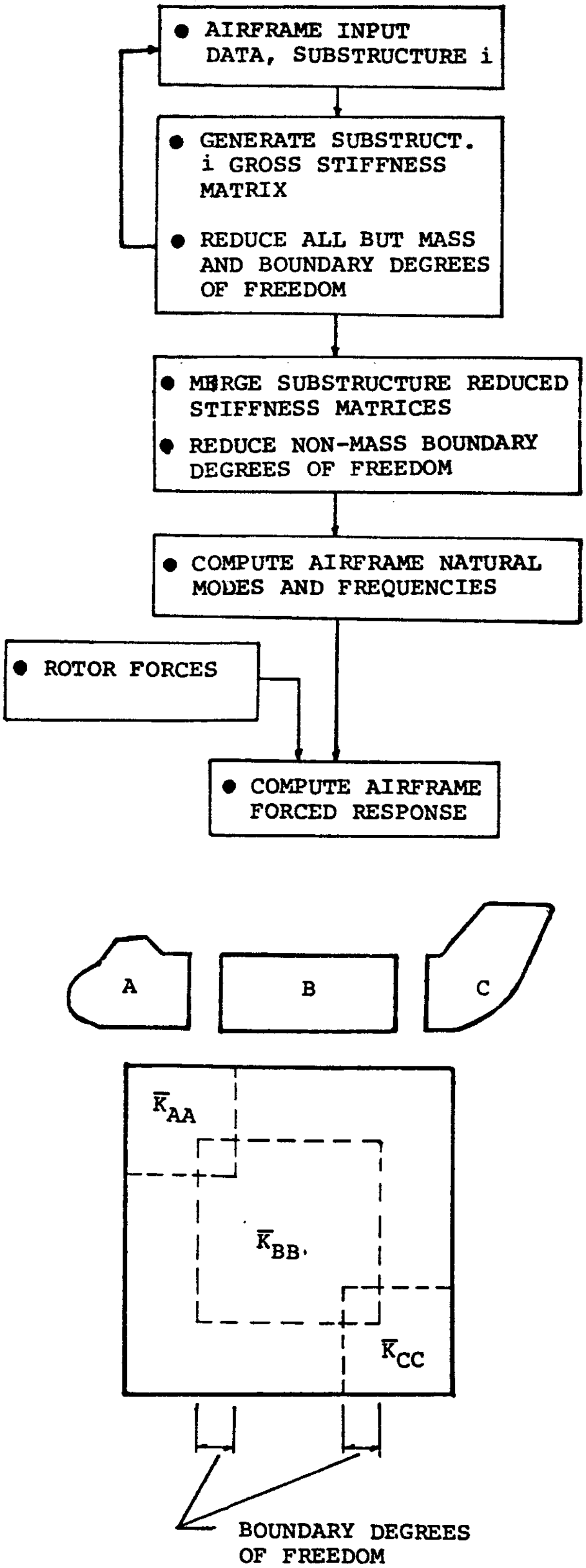


Figure 3. Substructure Method for Generating Airframe Reduced Stiffness Matrix

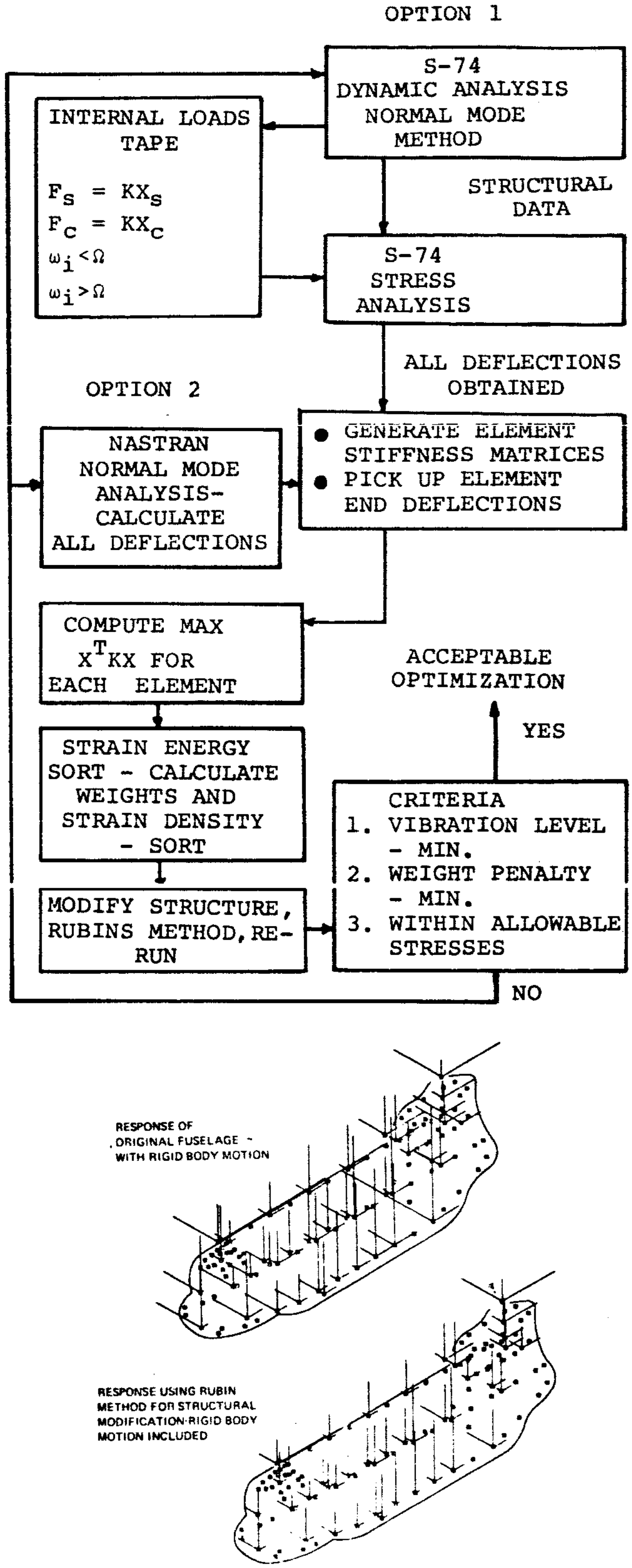


Figure 4. Damped Forced Response Method for Airframe Optimization

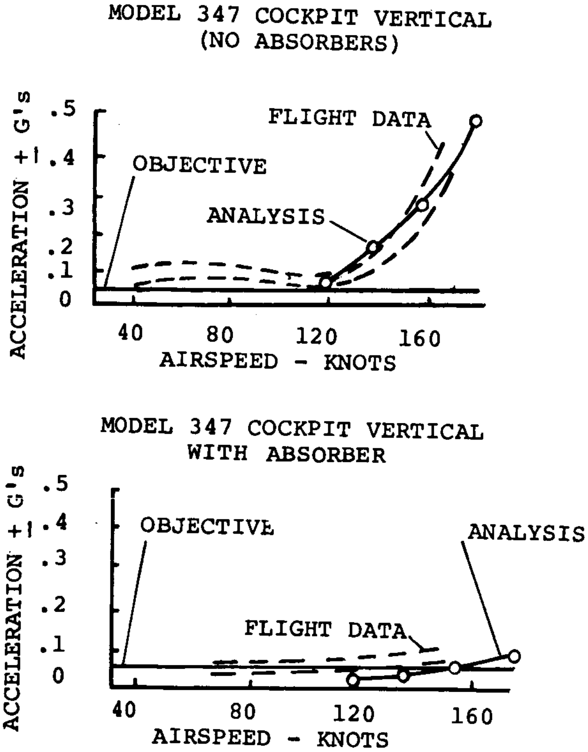
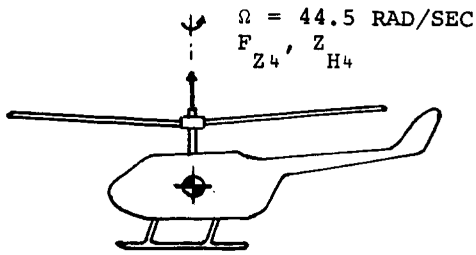


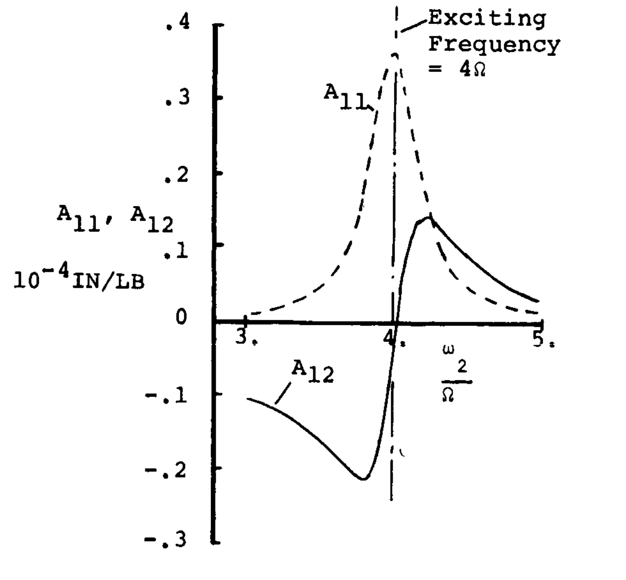
Figure 5. Predicted Vs. Measured Cockpit vibration Reduction with a Vertical Cockpit Absorber



(a) Single Rotor Helicopter Vertical Vibration

Figure 6. Coupled Rotor/Airframe Analysis for a Single Rotor Helicopter Vertical Vibration

$$\begin{Bmatrix} Z_{H4s} \\ Z_{H4c} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{Bmatrix} F_{Z4s} \\ F_{Z4c} \end{Bmatrix} \qquad A_{22} = A_{11} \qquad A_{21} = -A_{12}$$

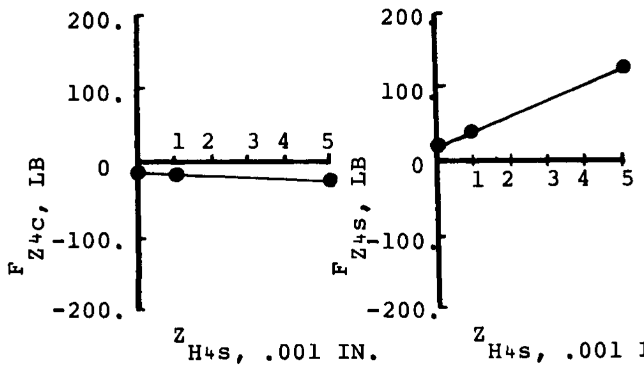


(b) Airframe·Hub Mobilities

$$F_{Z4} = F_{Z4c} \cos 4\Omega t + F_{Z4s} \sin 4\Omega t$$

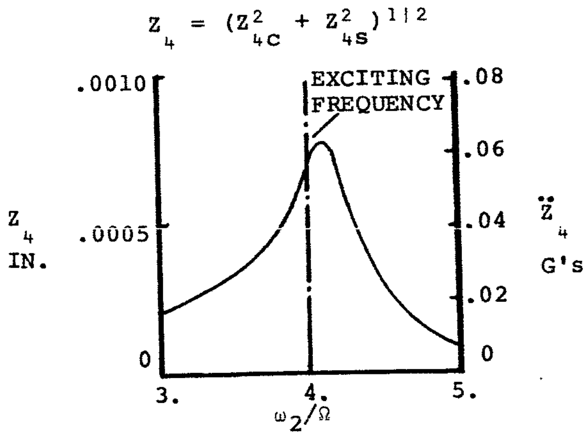
$$Z_{H4} = Z_{H4c} \cos 4\Omega t + Z_{H4s} \sin 4\Omega t$$

$$\begin{Bmatrix} F_{Z4s} \\ F_{Z4c} \end{Bmatrix} = \begin{Bmatrix} 19.6 \\ -16.5 \end{Bmatrix} + \begin{bmatrix} 20.77 & 1.33 \\ -1.33 & 20.77 \end{bmatrix} \times 10^3 \begin{Bmatrix} Z_{H4s} \\ Z_{H4c} \end{Bmatrix}$$

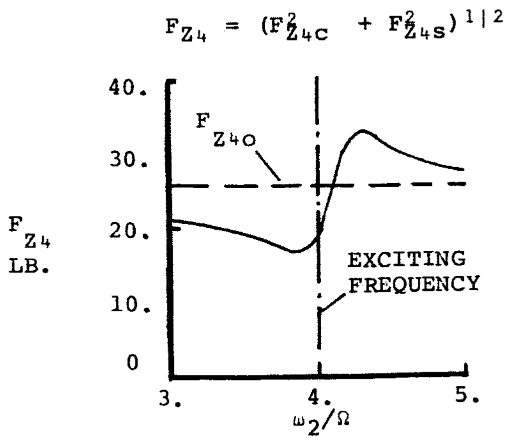


(c) Hub Forces Vs. Hub Motion

Figure 6. Continued



(d) Vibratory Hub Motion



(e) Vibratory Hub Force

Figure 6. Continued

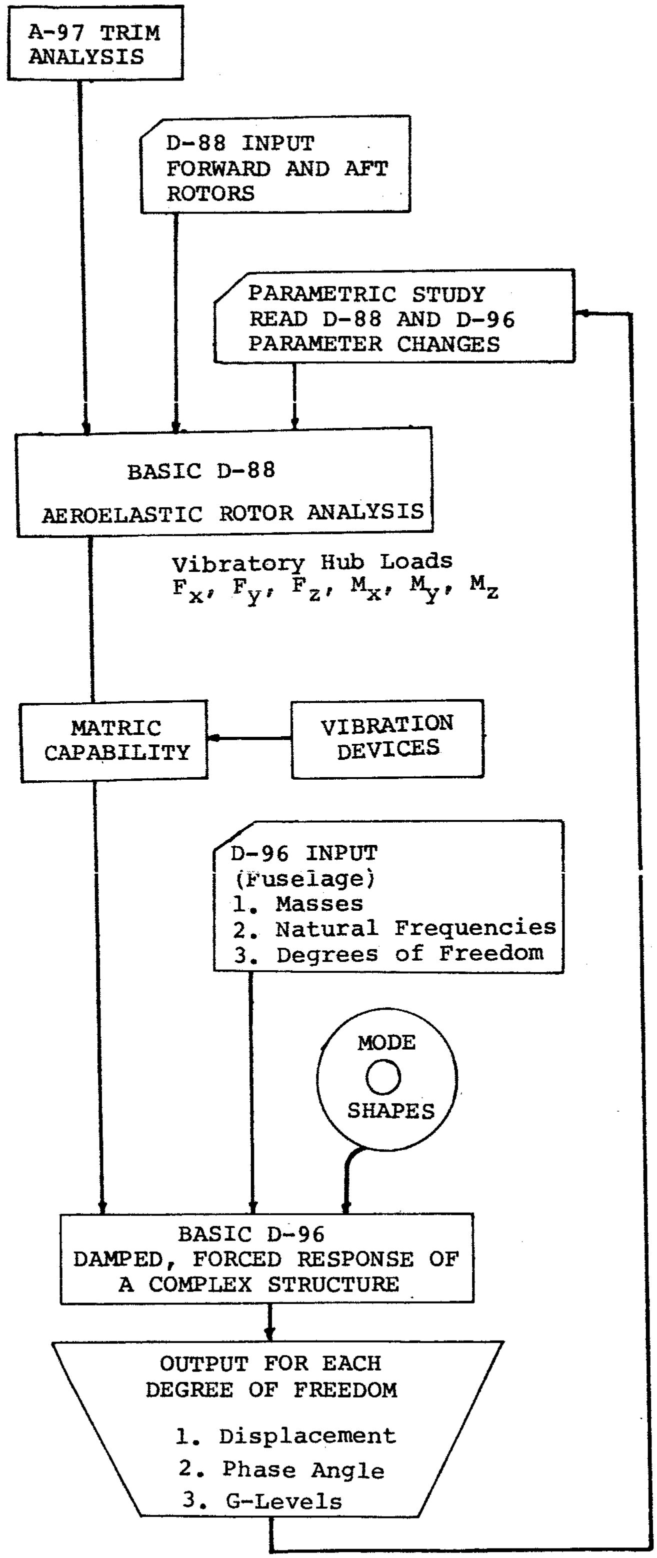


Figure 7. D-65 Coupled Rotor/Airframe Program Flow Diagram

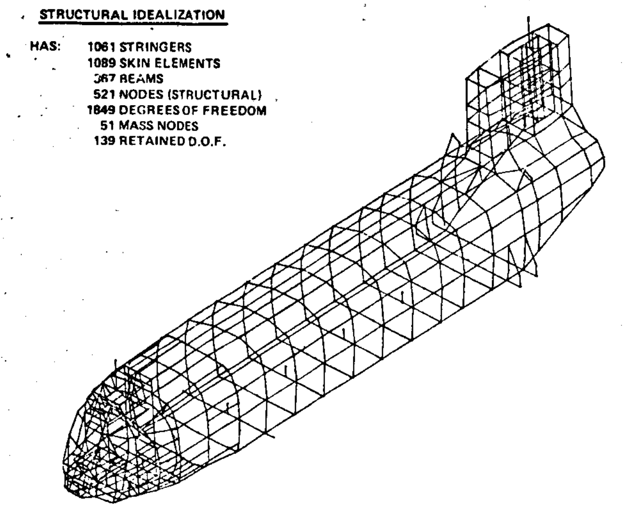


Figure 8. Model 347 Airframe Dynamic Model

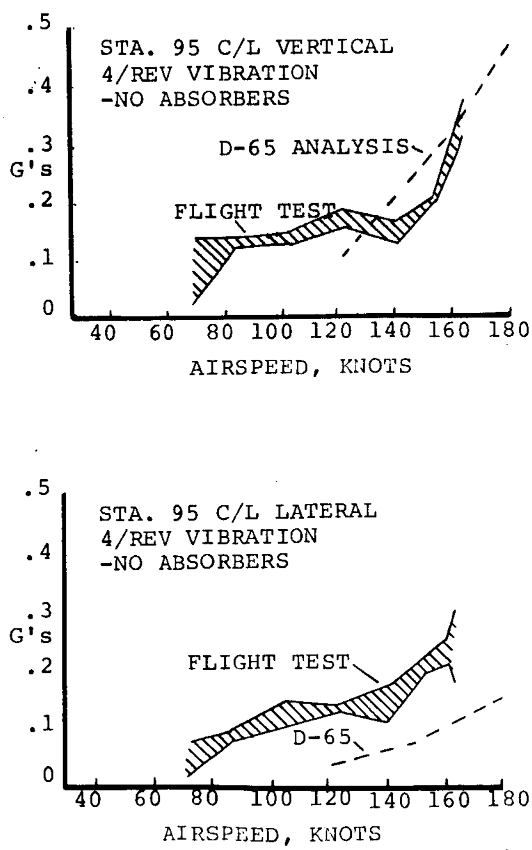


Figure 9. Model 347 Flight Data Vs. D-65 Coupled Rotor/Airframe Analysis Results

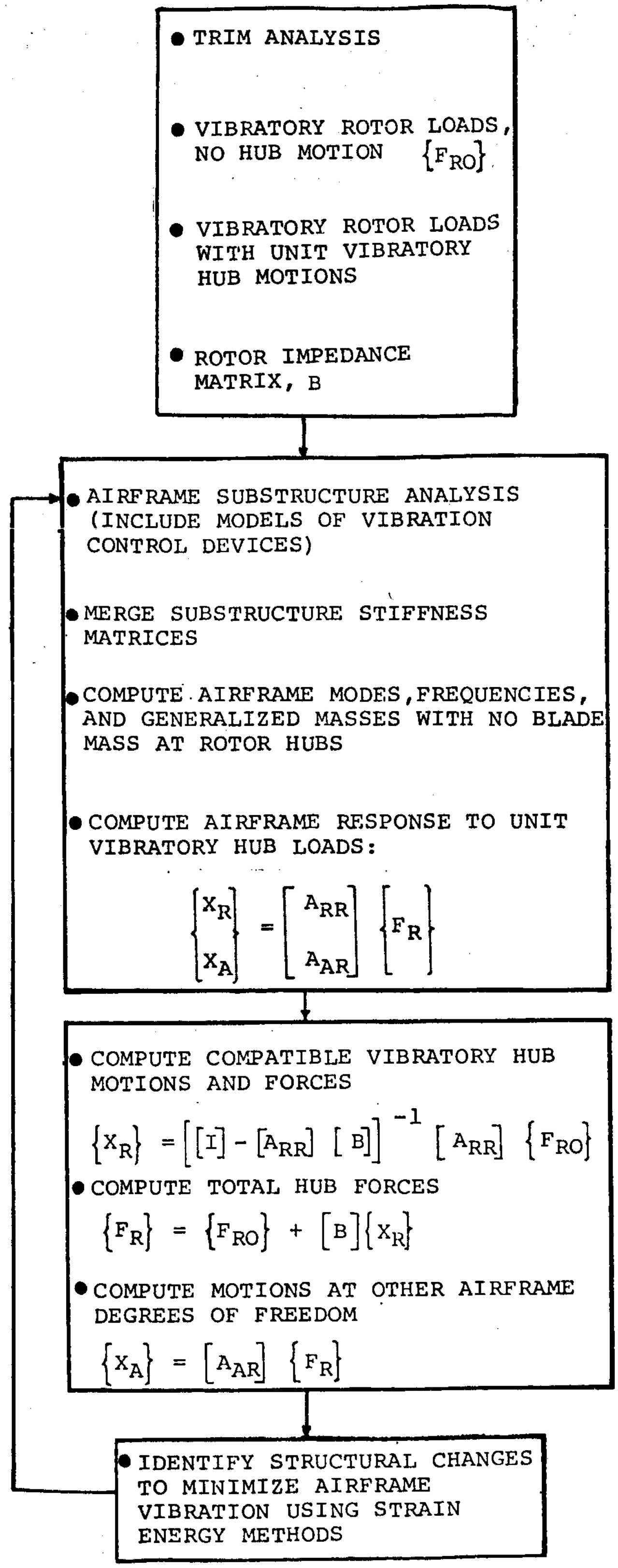


Figure 10. Coupled Rotor/Airframe/NASTRAN Analysis