CONVOLUTIONAL CODE PERFORMANCE IN PLANETARY
ENTRY CHANNELS
Dr. James Modestino
Rensselaer Polytechnic Institute

DR. MODESTINO: I would like to spend some time this after-
noon talking a little bit about the performance of convolutional
codes in a fading channel which would be typical of a planetary
entry mission. What I would like to talk about in particular
is but one small aspect of some on-going work that is being
conducted at RPI under NASA support. I might say at the out-
set that the primary motivation underlying our work has been
in support of Pioneer-Venus, although we do expect that the
results have much more general application to the planetary
entry mission in general.

In the first table (Table 7-1), I have indicated some of
the tasks that have recently been completed. The first task
has been the modeling of the planetary entry channel for com-
munication purposes. Here, we are primarily interested in
representing the scintillation or the turbulent atmospheric
scattering effects experienced on a planetary entry channel.
A second task has been the investigation of the performance of
short constraint length convolutional codes in conjunction with
coherent BPSK modulation and Viterbi maximum liklihood decoding.
The third task has been the investigation of the performance of
selected long constraint length convolutional codes in conjunc-
tion with, again, coherent BPSK modulation but now sequential
decoding. We have been looking at both the Fano and the Jeli-
nek algorithms for sequential decoding. Our interest here has
primarily been in the computation and/or storage requirements
as a function of the fading channel parameters. Finally, we
have been concerned with the comparison of the performance of
the coded coherent BPSK system with that of the coded inco-
herent MFSK system.

TABLE 7-1

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┌─────────────────────────────────────────────────────────────┐
│                  Tasks Recently Completed                    │
│                                                              │
│  • Modeling of the planetary channel for communication       │
│    purposes.                                                 │
│                                                              │
│  • Investigation of the performance of short constraint      │
│    length convolutional codes in conjunction with coherent   │
│    BPSK modulation and Viterbi maximum liklihood decoding.   │
│                                                              │
│  • Investigation of the performance of selected long con-    │
│    straint length convolutional codes in conjunction with    │
│    coherent BPSK modulation and sequential decoding.         │
│                                                              │
│  • Comparison of the performance of coded coherent BPSK      │
│    system with that of coded incoherent MFSK system.         │
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The next table indicates very briefly how we are going to
model the fading channel. The transmitted signal s(t) is ex-
pressed in terms of a complex signal representation. Here u(t)
is the complex envelope of the transmitted signal and it can be
expressed simply in terms of successive translates of a basic
channel signaling wave form, $u_0(t)$. The quantity T_s which appears
here is the basic channel signaling interval. We have, of course,
modulation by the binary information sequence to be transmitted
represented by the sequence $\{x_i\}$ of ± 1 values. We will assume
that the received signal v(t) is again expressed in complex sig-
nal representation. The complex envelope w(t) in this case
looks like that of the transmitted signal except for the presence
of a modulation factor $[\Gamma + a(t)]$ and the addition of a white
Gaussian noise component n(t). The quantity Γ appearing in
the modulation factor can be expressed as $\Gamma = \gamma e^{j\psi}$. Here the
amplitude γ is a fixed deterministic quantity to be specified
while the phase ψ is a random variable uniformly distributed
over $[-\pi, \pi]$. The quantity a(t) is a complex zero-mean Gaus-
sian process which represents diffuse scattering. It is complete-
ly described either in terms of a frequency dispersion function
$\tilde{\sigma}(f)$ or in terms of an autocorrelation function $R_{aa}(\tau)$.

TABLE 7-2

Fading Channel Characterization

- <u>Transmitted</u> <u>Signal</u>

$$s(t) = \text{Re}\{u(t) \, e^{j\omega_o t}\}$$

with

$$u(t) = \sqrt{2E_s} \sum_i x_i u_o(t - i \, T_s)$$

$\{x_i\} \sim$ binary (± 1) information sequence

$u_o(t) \sim$ complex envelope of channel signaling waveform

- <u>Received</u> <u>Signal</u>

$$v(t) = \text{Re}\{w(t) \, e^{j\omega_o t}\}$$

where

$$w(t) = [\Gamma + a(t)] \, u(t) + n(t)$$

Here

$n(t) \sim$ AWGN process with noise spectral density $N_o/2$ watts/Hz.

$\Gamma \triangleq \gamma \, e^{j\psi} \sim \gamma$ fixed deterministic quantity and ψ uniformly distributed over $[-\pi, \pi]$

$a(t) \sim$ complex zero-mean Gaussian process representing diffuse scattering

<u>Frequency</u> <u>Dispersion</u> <u>Function</u>

$$\overset{\sim}{\sigma}(f) = \frac{\sigma_a^2}{2\pi} \frac{B_o}{B_o^2 + f^2}$$

$B_o \sim$ channel coherence bandwidth in Hz.

<u>Autocorrelation</u> <u>function</u>

$$R_{aa}(\tau) = \sigma_a^2 \, e^{-2\pi B_o /\tau/}$$

In our work we have made use of a particularly simple choice for $\overset{\sim}{\sigma}(f)$ as indicated in the slide by the first-order Butterworth spectra. Here the frequency dispersion function $\overset{\sim}{\sigma}(f)$ is completely described in terms of a scale parameter σ_a^2 and a quantity B_O measured in Hertz which we will call the channel coherence bandwidth. The coherence bandwidth B_O, or more precisely its reciprocal, is a measure of the amount of memory on the channel. Thus, in terms of this particular model, there are three quantities we have to specify; the amplitude term γ, the scale parameter σ_a^2 of the diffuse scattering component $a(t)$, and the channel coherence bandwidth B_O. Actually, with respect to this last quantity, it will prove more convenient to specify the dimensionless quantity $B_O T_s$ which represents the coherence bandwidth normalized to the signaling rate of $f_s = 1/T_s$. The appropriate specification of these parameter values, of course, depends heavily upon mission parameters and, in particular, the communications geometry.

I would like to mention at the outset, and I think this was brought out in the previous talk, that some of the theoretical propagation studies result in a channel model which differs somewhat from that which I have described. In particular, the amplitude of the fading signal component as I have described it possesses a Rayleigh-Rice distribution while the propagation studies predict a lognominal distribution. For a number of reasons which I don't really want to get into at this time we have found it much more convenient to make use of the model I have described. In any event, in the regime where the lognormal result can be justified, there is close agreement between the two distributions. Furthermore, it is important that the parameters in the model described here can be related quite easily to the results of the theoretical propagation studies.

In the table below I have indicated some typical channel model parameters. These data are derived from a paper by Woo, et al., from JPL and are for a Venus mission. The quantity L, here is the depth of penetration into the Venusian atmosphere, σ_X^2 is a scale parameter representing the variance of the log-normal amplitude component and B_X is the corresponding band-width of this component. We have developed techniques which allow the parameters B_O, σ_a^2 and γ to be related to σ_X^2 and B_X allowing completion of the table as indicated. Observe that for a depth of penetration of 55 kilometers a value for B_O of 0.146 Hz is appropriate. The location parameter γ and scale parameter σ_a^2 can similarly be determined. The case $\gamma=1.0$ represents the best fit to the theoretical propagation results and we have in addition carried through the case $\gamma= 0$ as somewhat of a worst case.

Table 7-3

Summary of Fading Channel Model Parameters

L*, km	σ_X^2	B_X, Hz	$B_O=\sqrt{2}\,\sigma_X B_X$, Hz	σ_a^2	
				$\gamma=0$	$\gamma=1$
55	0.056	0.436	0.146	1.118	0.112
30	0.018	0.59	0.112	1.037	0.036
10	0.0025	1.02	0.071	1.005	0.005
5	0.007	1.45	0.054	1.001	0.001
1	4×10^{-5}	3.23	0.029	1.000	-

*L is depth of penetration into Venusian atmosphere

Figure 7-32 indicates some typical results. In this case we consider a constraint length K=6 code with rate R=1/3. The location parameter $\gamma= 1.0$ and $\sigma_a^2 = 0.1$ which would correspond approximately to the top line of the preceding table
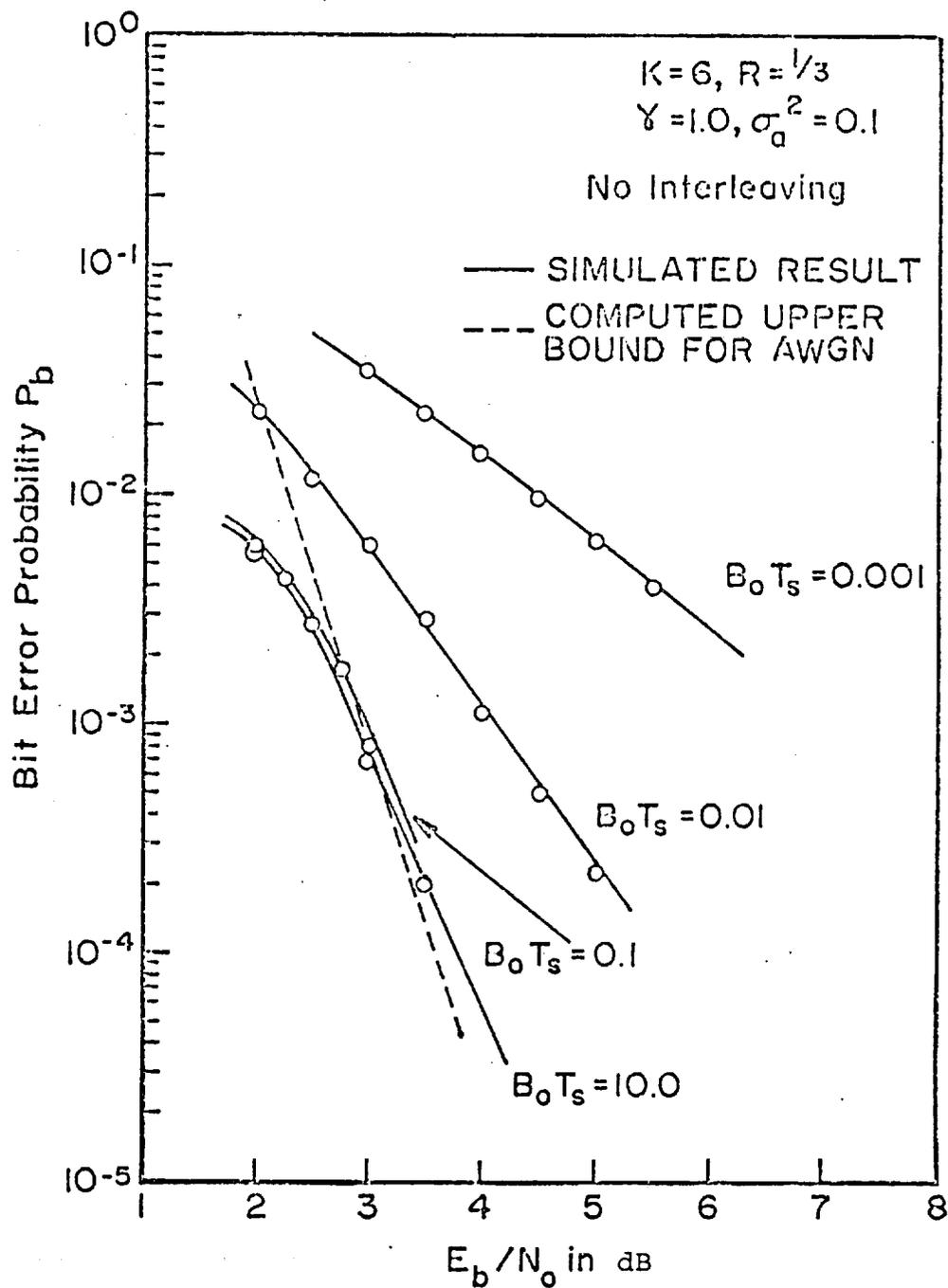
Simulated Performance of K=6, R=1/3
Code for Different Values of $B_0 T_s$
and with $\gamma = 1.0$, $\sigma_a^2 = 0.1$

Figure 7-32

indicating a depth of penetration into the Venusian atmosphere of 55 kilometers. The resulting bit error probability P_b as a function of E_b/N_0 is indicated for several values of $B_0 T_s$. If $B_0 T_s$ is small this would indicate considerable channel memory while large values of $B_0 T_s$ indicate little or no channel memory. The dotted line illustrated in this figure represents the performance that would be obtained on the additive white Gaussian noise (AWGN) channel. It represents a computed upper bound which we know to be extremely tight on the tails. As the figure indicates, the presence of memory on the channel results in severe degradation in performance over that which would have been obtained on the AWGN channel.

The easiest way to combat the effects of the channel memory is by the use of some form of interleaving. In Figure 7 - 33 we indicate the performance obtained with a very simple square block interleaver for the same code and channel parameters. Here, again, the dotted line represents performance on the AWGN channel. We see that using a 20 x 20 interleaver with $B_0 T_s = 0.001$ we can obtain performance relatively close to that predicted by the AWGN results. The solid line, here, is labeled "limiting case of zero channel memory," and represents a large $B_0 T_s$ value say 10.

It is clear then that some form of interleaving is required to combat the memory of the channel. On the basis of a large number of simulation results it has been concluded that the amount of interleaving required is quite insensitive to the code constraint length and/or rate. In Table 7 - 4 we indicate in tabular form the required interleaver size as a function of $B_0 T_s$ to achieve performance within a few tenths of a db of the limiting case of zero channel memory.
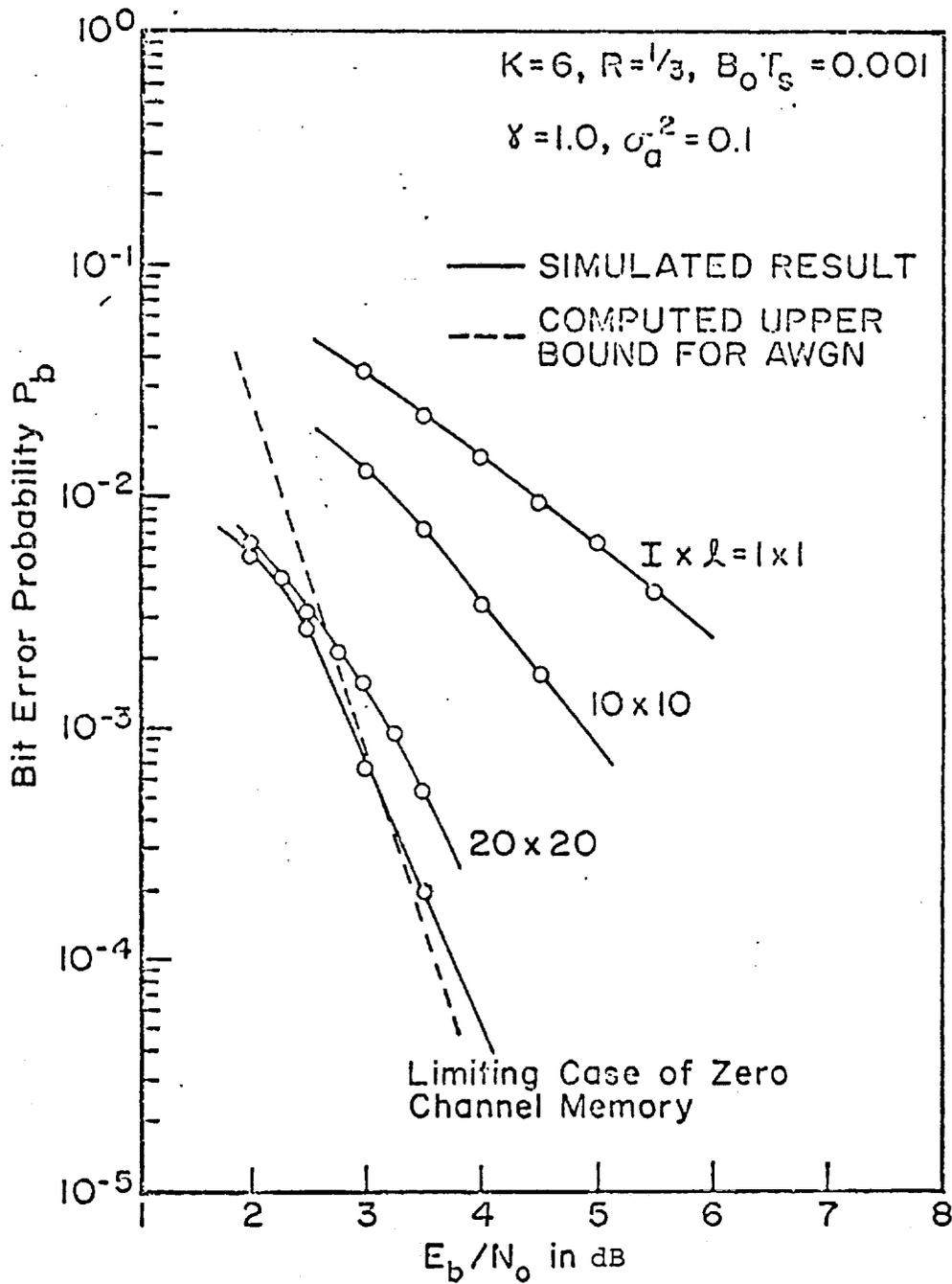
Effects of Block Interleaving for
K = 6, R = 1/3 Code With $B_o T_s$ = 0.001
and γ = 1.0, σ_a^2 = 0.1

Figure 7-33

TABLE 7-4

$B_o T_s$	Required $1 \times \ell$
.1	1 x 1
.01	10 x 10
.001	100 x 100

Summary of Interleaving Requirements as a
Function of $B_o T_s$ to Obtain
Performance Within a Few
Tenths of a db of Limiting Performance

In the simulation results reported so far we have assumed infinite quantization of the receiver output. Typical performance as a function of the number Q of quantization levels allowed at the receiver output is illustrated in Figure 7-34. We see that Q=8 level quantization results in performance within a few tenths of a dB of the performance with infinite level quantization.

It would appear at this point that, if we were to make use of the simple interleaver structures described here and Q=8 level receiver output quantization, performance within a few tenths of a dB of that predicted for the AWGN channel can be achieved. Unfortunately, the results have all assumed perfect phase tracking and, of course, this need not be the case. Since we are considering a coherent BPSK system we must address the effects of imperfect phase tracking. Recall that in the case of amplitude fading along, the channel memory really bothered us. If we look now at the case of phase tracking it is possible that we can exploit the channel memory to estimate the signal phase. In particular, with appreciable channel memory (i.e., $B_o T_s \ll 1$)
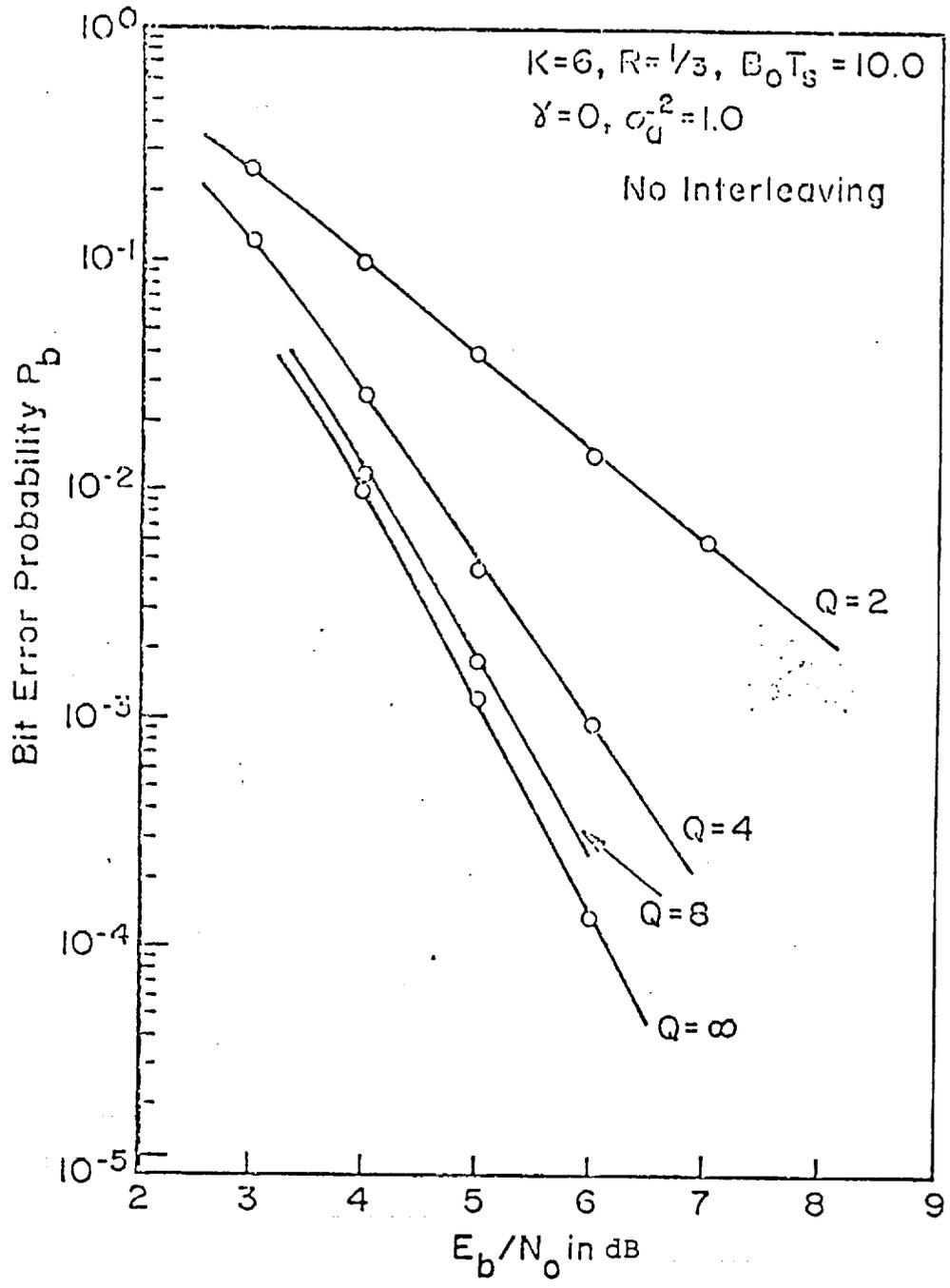
Effects of Quantization on K=6,
R=1/3 Code with $B_0 T_s$ = 10.0
and $\gamma = 0$, σ_a^2 = 1.0

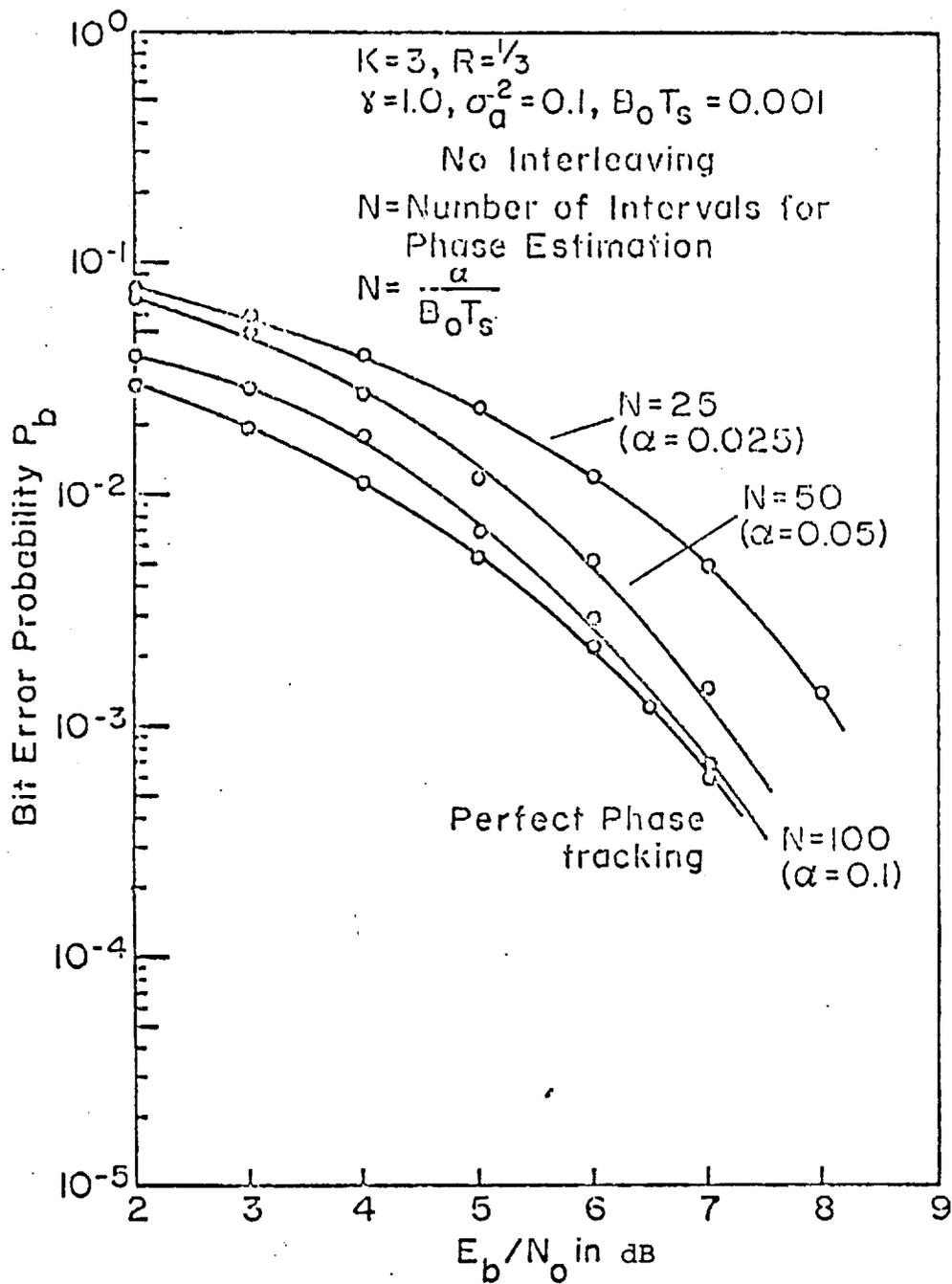Figure 7-34

the channel changes very little over many successive signaling intervals. It is possible then to make use of past receiver outputs to estimate the phase during the next signaling interval and use it for coherent local oscillator injection. Typical performance obtained with such a phase estimation scheme is illustrated in Figure 7-35. In this case the constraint length K=3 the rate R=1/3 and $B_0 T_s = 0.001$. The quantity N is the number of past signaling intervals used for phase estimation. We expect the received signal phase to change very little over a number of channel signaling intervals which is approximately $1/B_0 T_s$. As a result, the curves in this figure are parameterized by $N = (\alpha/B_0 T_s)$ where $0 < \alpha \leq 1$ represents the fraction of the total possible signaling intervals used for phase estimation. The phase estimator utilizes the in-phase and quadrature matched filter outputs during N past intervals to predict the phase during the next signaling interval. We see from the figure the performance obtained with N=25, 50 and 100 compared with that which we would have obtained with perfect phase tracking. With N=100 (i.e., $\alpha = 0.1$) it is possible to obtain performance which is again within a few tenths of a dB of that obtained on the AWGN channel.

The conclusions to be drawn from these simulation studies are summarized in Table 7-5. Finally, Table 7-6 indicates the future work to be performed under this program.

Thank you.

MR. GRANT: The last speaker of this session is Dr. Thomas Croft of Stanford University. Dr. Croft is a Senior Research Associate in the Center for Radar Astronomy and a member of the radio science teams for the Pioneer Venus and Mariner-Jupiter -Saturn missions.

Effects of Imperfect Phase Tracking

for K=3, R=1/3 Code With $\gamma = 1.0$

$\sigma_a^2 = 0.1$ and $B_0 T_s = 0.001$

Figure 7-35

Table 7-5

Conclusions

- Even in the absence of phase tracking errors some degree of interleaving is required to combat time correlated fading of channel.

- Simulation results have indicated only modest amounts of interleaving are required to approach performance of memoryless channel.

- Additional propagation results are required particularly on the phase perturbation process.

- More recent results have indicated the definite superiority of noncoherent MFSK system when phase tracking errors are considered.

Table 7-6

Future Work

- Additional Modeling of Phase Tracking Errors in Coherent BPSK System

- Investigate the Performance of Coded Incoherent MFSK System

- Investigate the Performance of Coded PCM/FM System

- Explore the Desirability and/or Feasibility of Concatenated Coding Schemes

- Investigate the Frequency Tracking and/or Acquisition Problem Associated with PCM/FM