ANALYTICAL AND EXPERIMENTAL STUDY OF

TWO CONCENTRIC CYLINDERS COUPLED BY A FLUID GAP

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INTRODUCTION

From a structural point of view a liquid coolant type nuclear reactor onsists of a heavy steel vessel containing the core and related mechanical omponents and filled with a hot fluid. This vessel is protected from the evere environment of the core by a shielding structure, the thermal liner, hich is usually a relatively thin steel cylinder concentric with the reactor essel and separated from it by a gap filled with the coolant fluid. This rrangement leads to a potential vibration problem if the fundamental frequency, r one of the higher natural vibration frequencies, of this liner system is lose to the frequency of some vibration source present in the reactor vessel. he natural vibration frequency of the liner shell vibrating in a vacuum is eadily calculated by generally available techniques; e.g., references 1 and 2. owever, it is felt that the influence of the fluid cannot be ignored since it ay reduce the fundamental frequency by a factor of two to five, and thus may ower it into a range in which strong vibration sources may be present. Some atural frequency data for the case of a cylindrical shell filled with liquid as been presented in reference 3; however, the case of a cylindrical shell oupled to a concentric shell through a thin fluid gap was studied only for the ase of simple support conditions at both top and bottom. (See references 4 nd 5.) The study described here was undertaken to provide information for the hell rigidly clamped at its base and free at the top since this is a better escription of the conditions encountered in typical reactor designs.

The dimensions of shell considered in this report were selected to model he liner used in the Fast Flux Test Facility reactor designed for the U. S. tomic Energy Commission. The scale factor is approximately 1/14, giving ominal dimensions of 0.52 m (20.5 inches) height, 0.43 m (17 inches) diameter, nd 1.6 mm (1/16 inch) thickness. All measurements were made in "customary" nits. The behavior of prototype coolant liquid, sodium, is modeled by water hich has density, compressibility, and viscosity properties that are adeuately representative. In identifying mode shapes, this report will use the onventional designations: n to signify the number of complete waves in the ircumferential direction (n=0 - axisymmetric, n=1 - translation, n=2 - 'ovalling," etc.); and, m to signify the modal characteristic in the longiudinal direction (m=1 - simple cantilever beam type mode, m=2 - a mode with ne nodal circle, etc.).

EXPERIMENTAL SETUP

The experimental model was fabricated by rolling a 1.5 mm (0.058 inch) thick, 0.51 m (20.125 inches) wide steel plate, seam welding, and soldering to a 13 mm (1/2 inch) brass plate which in turn was securely bolted to a 0.15 x 0.43 x 0.97 m (6 x 17 x 38 inch) steel block. Relatively rigid concrete outer cylinders were formed by casting around the cylindrical model utilizing a hard durometer neoprane spacer to obtain the nominal annular gap desired. The concrete was waterproofed and attached to the steel base as shown in Fig. 1.

The average outside diameter of the fabricated shell - .4338 m (17.08 inches) - was determined by taking the average of 36 measurements. The maximum diametral deviation was less than 0.5%. The shell wall thickness - 1.47 mm (0.058 inch) - was more uniform. The annular gap size - 3.8, 6.4, 13.7, 26.3 and 75 mm (0.151, 0.253, 0.538, 1.03 and 2.94 inches) - was determined by measuring the volume of water necessary to fill the annular region between the concrete shell and the steel shell to various levels. For any of the gap sizes used, the maximum deviation from the average value was about 0.10 (10%). The geometric accuracy of the model exceeds the uniformity expected in the prototype. Steel density - 7.5 Mg/m^3 (0.27 lb/cu. in.) - and elastic modulus - 193 GPa (28×10^6 psi) - were determined from test strips of the steel plate used in fabricating the shell by weighing and by a cantilevered beam frequency test.

A single exciter coil was situated close to the inside shell wall with an approximate 5 mm (3/16 inch) air space, and provided a magnetic force over approximately 0.016 m^2 (25 square inches) of the shell. Both a sinusoidal and a wide-band random current signal, controlled by signal generators were utilized during testing. The motion of the top of the test cylinder was monitored by miniature piezoelectric accelerometers cemented to the inside of the cylinder every 30°, with 0° defined to be opposite the center of the above coil. In addition, three accelerometers were cemented at equal spaces along the 0° longitudinal line on the cylinder and a movable accelerometer mounted on a magnet was used to search for node points.

TESTING PROCEDURE

Testing for each water-filled annular gap and the cylinder in air consisted of three phases. First, natural frequencies were determined by exciting the shell with a wide-band random force and inspecting power spectral density plots produced by a Fourier analyzer from the time history signals of several accelerometers. Second, the shell was excited with a sinusoidal current applied to the coil, using a range of frequencies in the vicinity of each of the natural vibration frequencies detected by random excitation. For each natural frequency, f_n, the accelerometer signal in a narrow band about this frequency was processed by the Fourier analyzer to provide a more accurate value of the peak frequency and to establish the RMS acceleration at each accelerometer position. This information was plotted to identify mode shapes corresponding to the natural frequencies. In the third phase of testing, the transfer function between the RMS displacement and peak coil current was plotted at discrete points in a narrow frequency band about each natural frequency so that the equivalent viscous damping ratio could be calculated by the half power point bandwidth method as outlined in reference 6.

ANALYTICAL METHODS

Analytical solutions to the structural problems were obtained by using the NASTRAN finite element analysis program (see reference 7) with corroborating solutions obtained by using the SAP IV code (see reference 8) and through a Rayleigh-Ritz solution (see reference 9). The structural dimensions used for the analytical work were: height (i.e., length) - 0.5112 m (20.125 inches); radius to the mid-surface of the shell - .2160 m (8.505 inches); shell thickness - 1.473 mm (0.058 inch); elastic modulus - 182.7 GPa (26.5 x 10^6 psi); Poisson's ratio - 0.3; and, shell material mass density - 7497.55 kg/m^3 (701 x 10^{-6} lb-sec^2/in.^{4}). Analytical solutions for the coupled fluid-structure problem were obtained from the NASTRAN code only, using the following additional parameters: fluid mass density - 1000.11 kg/m^3 (93.6 x 10^{-6} lb-sec^2/in.^{4}); bulk modulus - 2.07 GPa (0.3 x 10^6 psi); and, gap sizes as noted above. The basic grid, as used in the reported structural finite-element formulations, consisted of 10 divisions vertically and 9 divisions over a quarter of the shell circumferentially (10° segments). Some comparison runs using a finer mesh were made to establish the adequacy of the mesh for the purposes of this study. For example, by using a mesh size with twice as many divisions in each direction, it was established that the basic grid gives results with the minimum frequencies (which were those with the most error) about 2% too high. Similarly, it was found that using a grid with divisions three times larger than the basic grid gives errors of 30 to 50 percent.

The basic grid used in the fluid-structural analyses had 5 divisions vertically, 6 divisions circumferentially (15° segments) and 5 divisions through the fluid in the radial direction. In solving the coupled problem, NASTRAN uses a finite-element representation of the fluid region. Compressible fluid with small motion is assumed to give a linear (acoustic type) formulation. The fluid is also assumed to be irrotational so that a scalar potential function (pressure) can be used as the solution variable in place of the three components of displacement.

VIBRATION IN AIR

Before considering the shell vibrating with a fluid, the shell in a vacuum was studied to establish the significance of variations in the boundary conditions and to establish the degree of correspondence to be expected between analytical and experimental results. Figure 2 presents a summary of these results. In all cases the top boundary of the shell was considered to be free. In comparing analytical results for the fixed base condition (rigidly clamped) to the simply supported base condition (all edge displacements prevented but unrestrained rotation about the tangent), little difference was found. The degree of rotational restraint at the base does not affect the overall results significantly as it is primarily a local flexure condition. Hence it is not considered further. The base boundary condition was then further relaxed by allowing edge motion in the axial direction against an elastic spring restraint, K_Z; using K_Z = infinite, 10^6, 10^5 and 10 units, where each unit is 4.656 kN/m per m of circumference (1 lb/in. / 1.5 in.). The K_Z infinite case corresponds to the simply supported condition, whereas $K_Z = 10$ which is effectively $K_Z = 0$ corresponds to a shear diaphragm boundary condition. The curves corresponding to each of these conditions are shown in the figure. Comparable results obtained by using the SAP IV program gave frequencies about 3% lower; frequencies obtained by the Rayleigh-Ritz method were up to 5% lower.

The results obtained experimentally for the shell vibrating in air (i.e., effectively in a vacuum) are shown as data points in Figure 2. Although the experimental setup was intended to simulate a fixed condition (K_Z = infinite) at the base, it can be seen that effectively it is behaving as a shell supported by elastic springs in the axial direction with $K_Z = 10^5$. Note that for both m = 1 and m = 2, the modes corresponding to the lower n values were not detected. Difficulties were encountered in this regard and explanations posed here and elsewhere are not totally satisfactory. Efforts to get response of these modes included the use of the local electromagnetic coil described previously as well as acoustic excitation at various amplitude levels applied by a loud speaker. Fourier analyzer processing of the acceleration response to a random excitation did show a frequency at 140 Hz which probably corresponds to the m = 1, n = 3 mode, but no corresponding response was generated wh a single harmonic excitation at this frequency was applied. Frequency response analysis performed by the NASTRAN program indicated that the modes which failed to respond are of somewhat lesser intensity than modes at neighboring frequencies with high n values, but this difference was not large enough to justify the difficulties encountered in detecting these modes. One possible explanation is in the relation between flexural strain energy (associated with high n values) and membrane strain energy as discussed by Croll. (See referenc 10.) The masking effect of the higher n value modes at frequencies close to that of the low n value mode may also be the difficulty. This is particularly suspect in the tests using the electromagnetic exciter since with a pure harmonic current applied to the device the resulting force function carries higher harmonic frequencies of an amplitude up to 20% of the fundamental harmonic. A factor which may be totally responsible for the difficulty of experimentally detecting these low n value modes is the geometrical imperfections of the shell as fabricated. NASTRAN is suited to the evaluation of this effect and a study to perform such an evaluation is contemplated. Finally, it should be noted that the frequencies of the low n value mode shapes are highly dependent on the stiffness K_Z, as clearly shown in Fig. 2, so that the relatively undeterminable and possibly nonlinear nature of this restraint may greatly reduce the sharpness of the associated response. However, for the modes that gave a clear response, increasing the driving force by a factor of four lowered the peak response frequency by at most 1/2%; hence, support nonlinearity appears to be negligible.

VIBRATION IN WATER

The vibration frequencies of the shell with a fluid gap were first approxnated by applying the added mass factors, as described in reference 4 for nfinitely long shells, to the frequencies obtained for the in air vibration ase. A comparison between the experimental results and these extrapolations s shown in Fig. 3. This method appears to give frequency predictions that are omewhat low but generally within 10% of the experimental values.

Figure 4 shows the frequencies predicted by the NASTRAN program for the hell vibrating with four selected water gaps, and with the in-vacuum soluions shown for comparison. These frequencies are compared with the experientally measured frequencies in Figure 3. The directly computed values are etween 20 to 30 percent higher than the experimental results; however, by omparing results for problems run with several different models, it is xpected that the mesh size used to generate the data in Figure 3 will give requencies which are between 20% and 35% too high. The mesh size used for enerating the data consisted of 5 divisions vertically, 5 divisions radially hrough the fluid, and 15° divisions circumferentially along the shell. owever, for one particular fluid gap size, the mesh divisions were reduced n steps down to 10 by 5 by 6°, respectively. The number of divisions adially through the fluid was also varied, but this did not affect the esults significantly; thus, the 5-division model can be considered fine nough for accurate results. This somewhat refined analysis gives results hat are quite adequate for design purposes. However, the cost of running a eries of fluid-solid interaction problems through NASTRAN, particularly with much finer mesh than used here, would be prohibitive. The runs involving he (5, 5, 15°) mesh required about 1 minute of IBM S/370-195 CPU time; the 10, 5, 6°) case required 8 minutes. Both were run with NASTRAN level 15.1 sing 180° of the shell modeled for analysis.

DAMPING

For purposes of determining the equivalent viscous damping ratios, the frequency-response curve was first determined in the 0.1 - 0.5 g acceleration range and then redetermined at the highest acceleration level compatible with the available equipment. The ratio of acceleration levels was at least two and usually five to ten. The damping ratio variation with amplitude level was in the 10 to 40 percent range. Figure 5 shows selected average values of damping for the shell vibrating with various water gaps and in air. Clearly, the smaller fluid gaps are associated with higher damping ratios than those measured in air, typically twice as large. Generally, larger damping is associated with smaller water gaps. For those few cases which do not follow this later trend, the frequency response curves probably were distorted (broadened) by superposition of the response corresponding to an adjacent natural frequency which would make the half power bandwidth method inapplicable.

CONCLUDING REMARKS

Correspondence of experimental and analytical results is within acceptabl limits for design purposes. Several vibration modes corresponding to solutions with low n values eluded experimental detection. The significance of these modes in design, as well as the reasons for the difficulty in detecting them experimentally, are not clear. The feasibility of using a coupled fluid-elastic finite-element analysis to solve vibration problems involving shells containing a fluid has been demonstrated; however, the computer time costs incurred with the present methods prohibit extensive application of this technique.

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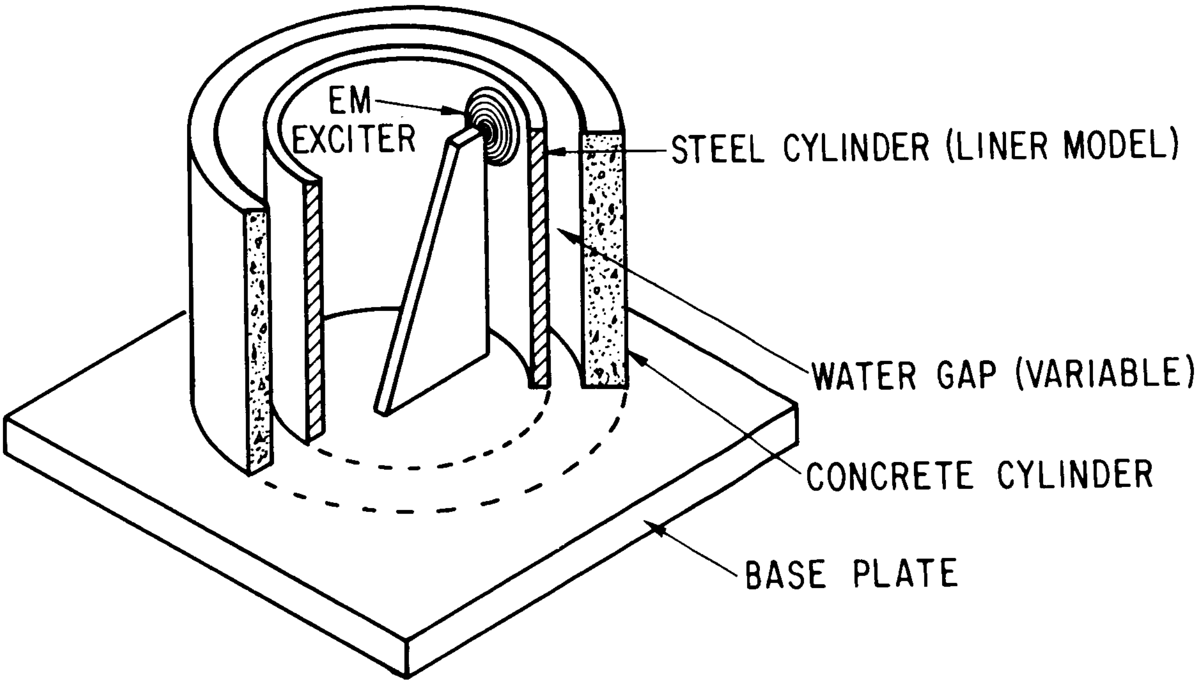

Figure 1. Experimental setup schematic.

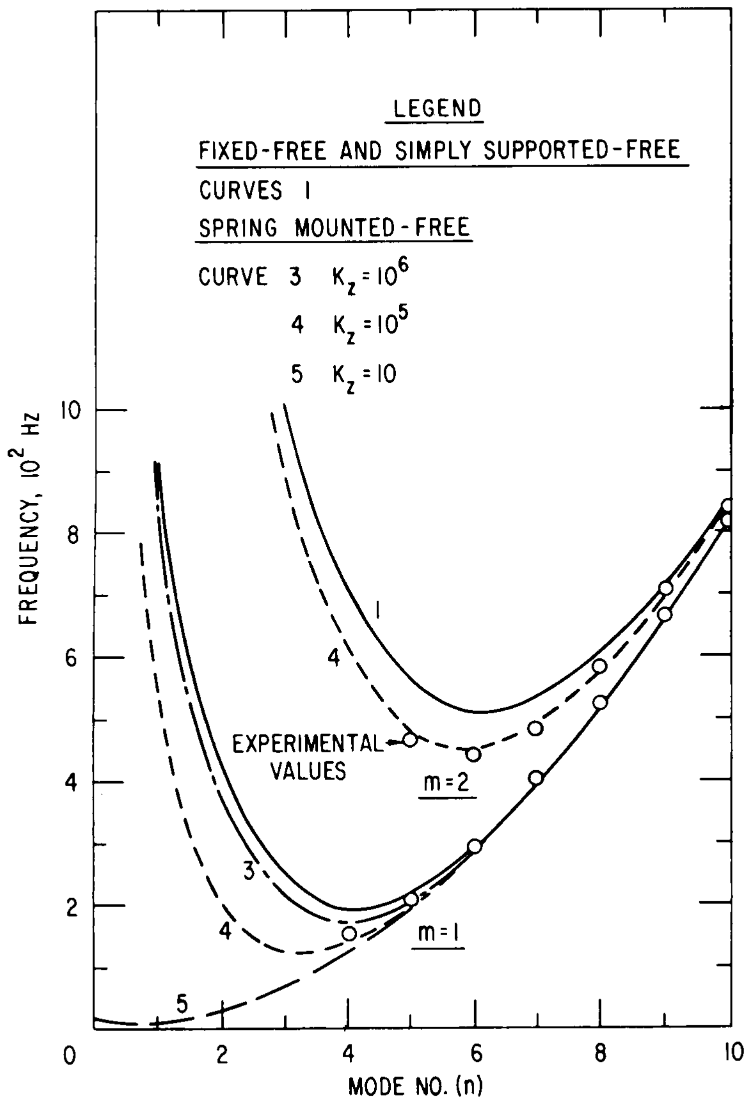

Figure 2. Comparison of experimental and predicted vibration frequencies for the shell without fluid

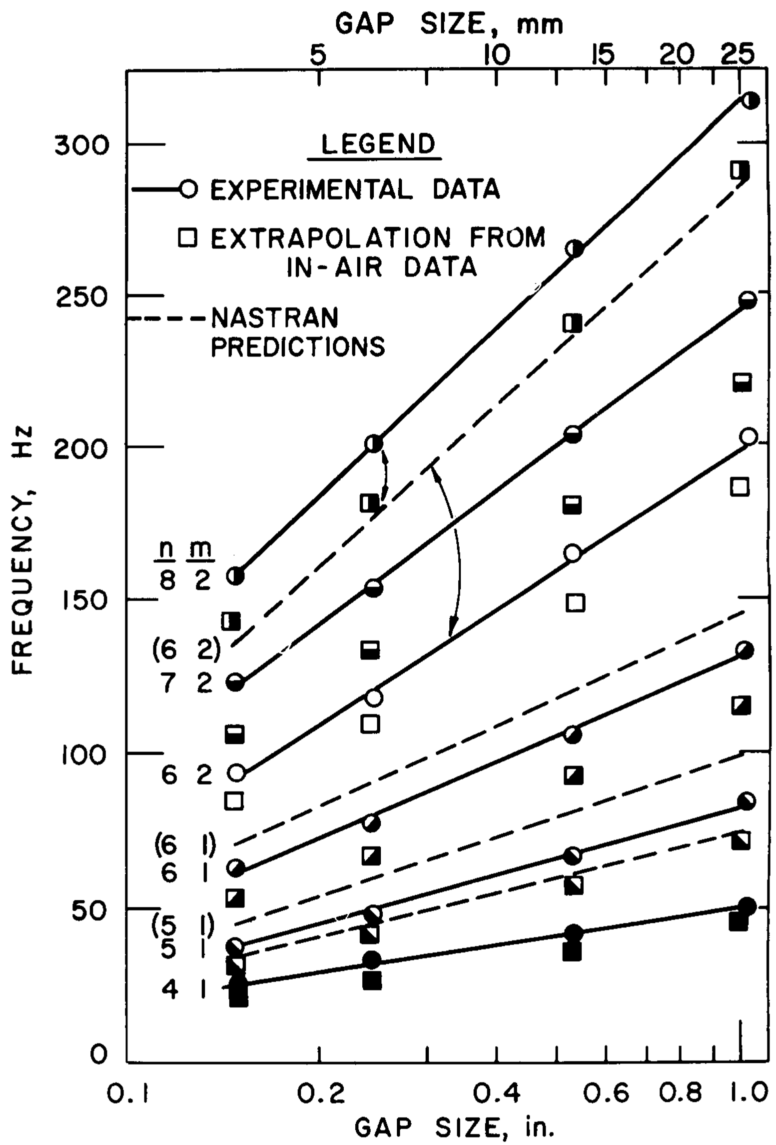

Figure 3. Comparison of experimental and predicted vibration frequencies for the shell with a fluid filled gap

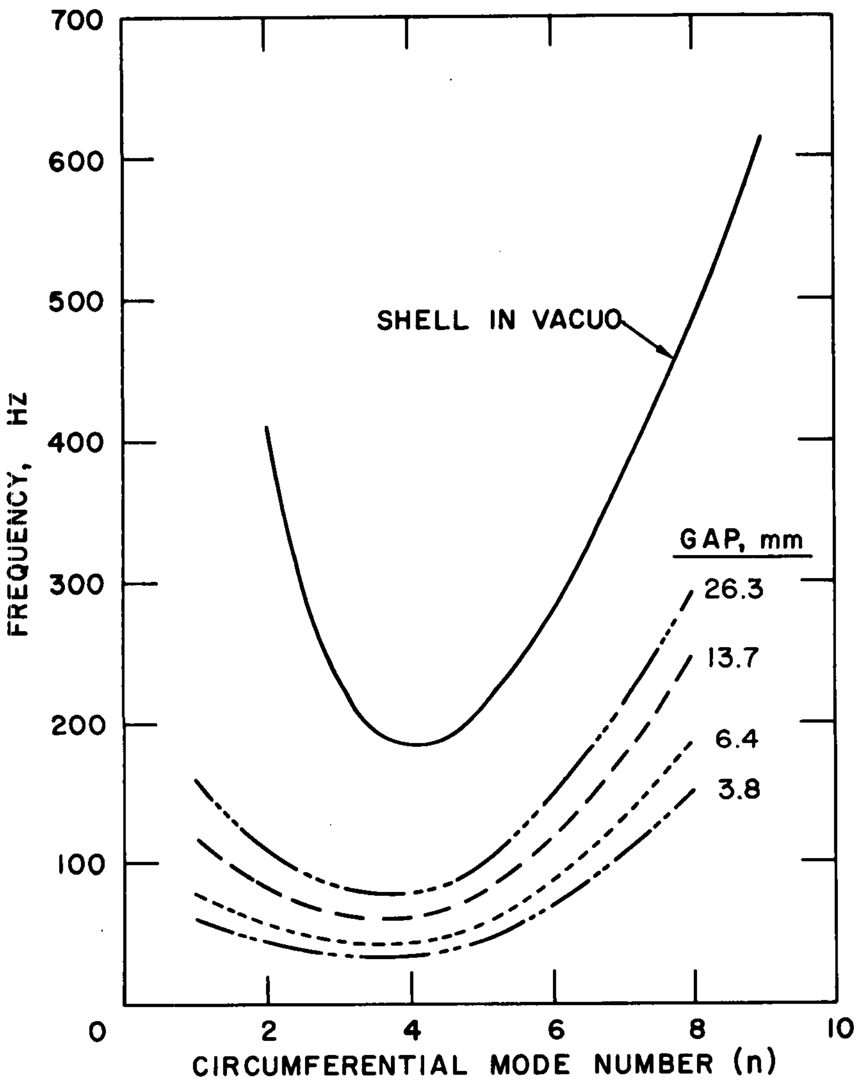

Figure 4. Computed m = 1 type vibration frequencies for the shell with and with- without fluid.

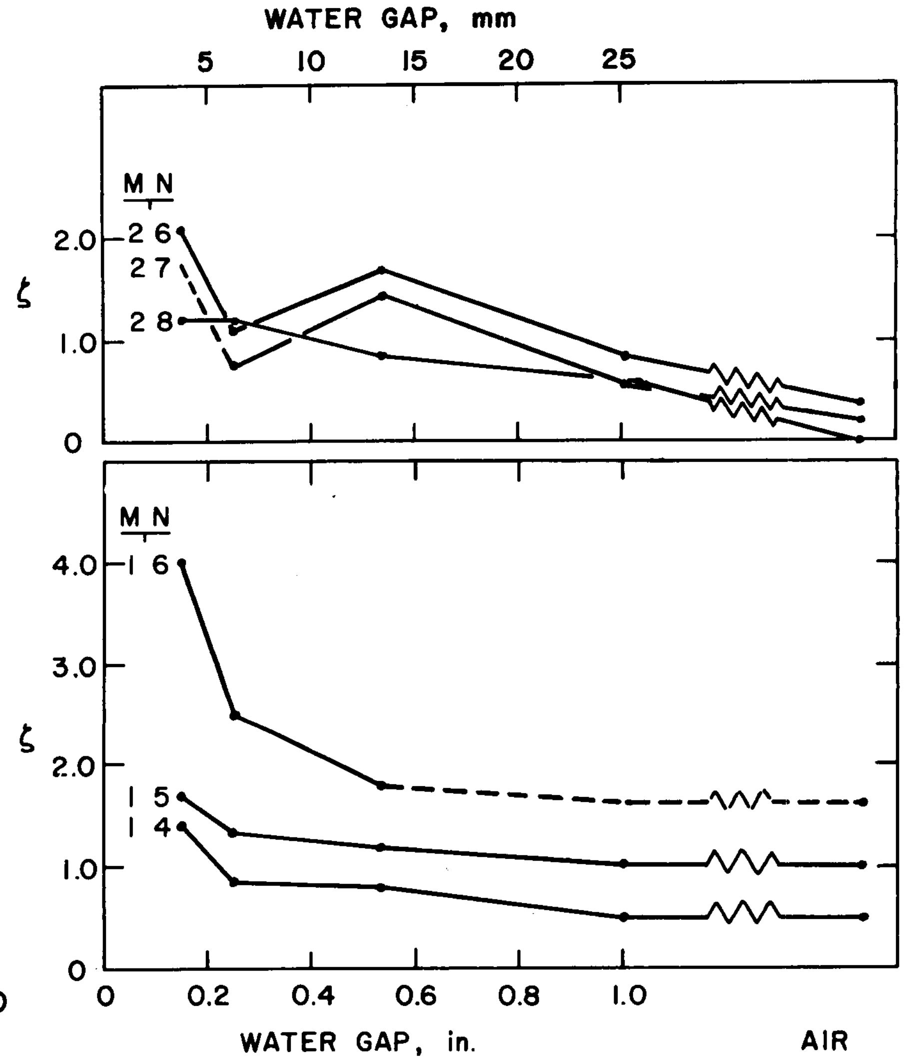

Figure 5. Experimentally determined damping coefficients.