ON EIGENVECTORS OF MULTIPLE EIGENVALUES OBTAINED IN NASTRAN

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SUMMARY

In the case of non-multiple eigenvalues, each of the three real eigenvalue extraction methods available in NASTRAN will, for a given type of normalization, give essentially the same eigenvectors. However, this is not so in the case of multiple eigenvalues. This apparent discrepancy is explained and illustrated by considering the example of a NASTRAN demonstration problem that has both multiple and non-multiple eigenvalues.

INTRODUCTION

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The NASTRAN program provides three basic methods for real eigenvalue extraction. In each of these methods, the eigenvectors obtained can be normalized in three different ways (see Appendix).

In the case of non-multiple eigenvalues, each of the three extraction methods will, for a given type of normalization, give essentially the same eigenvectors. However, in the case of multiple eigenvalues, the three extraction methods will, in general, give different eigenvectors even though they may employ the same type of normalization. Furthermore, in this case (of multiple eigenvalues), even a given method using a given type of normalization may yield different eigenvectors under different conditions (e.g., different frequency limits on the EIGR bulk data card) [1][1]. This discrepancy may seem disturbing, but it is explained in this paper where it is shown that the different eigenvectors corresponding to multiple eigenvalues obtained by different methods and under different conditions have certain definite relationships among them and

[1] Numbers in brackets indicate References given at the end of the paper.

that these relationships depend on the number of arbitrary constants that are inherently assumed in the computation of eigenvectors. This is illustrated by considering the example of a NASTRAN demonstration problem that has both multiple and non-multiple eigenvalues.

THEORETICAL BACKGROUND

The basic eigenvalue problem solved in NASTRAN can be formulated as

$$[K_{aa} - \lambda M_{aa}] \{x\} = 0 \tag{1}$$

where K_{aa} and M_{aa} are respectively the stiffness and mass matrices (both of which are real and symmetric) referred to the analysis set [2], λ is a scalar quantity, and $\{x\}$ is a column vector that comprises all the degrees of freedom in the analysis set. Non-trivial solutions for $\{x\}$ in the above equation are possible if and only if the resultant matrix within the brackets is singular [3,4]. The values of λ that satisfy this condition represent the required eigenvalues.

Let λ_ℓ be one of the eigenvalues of Equation (1). The eigenvector corresponding to this eigenvalue can be obtained from the equation by substituting $\lambda = \lambda_\ell$ and solving for $\{x\}$. We thus have

$$[K_{aa} - \lambda_\ell M_{aa}] \{x\}_\ell = 0 \tag{2}$$

where $\{x\}_\ell$ is the required eigenvector.

Let n be the order of the problem and let $x_1, x_2, x_3, \ldots\ldots, x_n$ be the components of $\{x\}$. Equation (2) can then be rewritten as

$$\sum_{j=1}^{n} C_{ij} x_j = 0 , \qquad i = 1,2,3,\ldots\ldots, n \tag{3}$$

where C_{ij} are constants that depend on K_{aa}, M_{aa}, and λ_ℓ.

Equation (3) represents a system of n linear equations in n unknowns. However, not all of these n equations are independent. The exact number of independent equations depends on the multiplicity of the eigenvalue λ_ℓ. If s $(1 \leq s \leq n)$ is the multiplicity of λ_ℓ, then it can be shown that the rank of the resultant matrix in Equation (2) is (n-s) [3,4]. The number of independent equations in (3) is also, therefore, (n-s). Their solution thus involves s arbitrary constants. The total number of eigenvectors available is therefore equal to ∞^s. However, it can be shown that the number of linearly independent eigenvectors is only s [4].

Depending on the multiplicity s, it is useful to distinguish between the following two cases:

Case (a) Non-multiple Eigenvalues (s = 1)

In this case, (n - 1) of the n equations in (3) are independent and their solution involves a single arbitrary constant. An infinite number of eigenvectors are thus available. However, the important thing to note here is that the relative values of the eigenvector components remain invariant.

Case (b) Multiple Eigenvalues ($1<s\leq n$)

In this case, the number of independent equations in (3) is equal to (n - s) and their solution involves s arbitrary constants. In contrast to the case of non-multiple eigenvalues, the most important thing to note in this case is that the relative values of the eigenvector components are not invariant, but depend on the relative values of the involved constants themselves. Also, it is clear that the solution space in this case is much larger than in the case of non-multiple eigenvalues.

EIGENVECTOR COMPUTATION IN NASTRAN

The exact procedure for eigenvector computation in NASTRAN depends on the method used for eigenvalue extraction and is described in detail in the Theoretical Manual [2]. Thus, in the Inverse power method, trial eigenvectors are assumed and iterated until convergence occurs. In the Determinant method, eigenvectors are computed by the method of backward substitution after each of the corresponding eigenvalues has been calculated. In the Givens method, all the eigenvalues are first obtained and then the required number of eigenvectors is computed by repeated use of backward substitution.

The orthogonality of eigenvectors of closely spaced eigenvalues is guaranteed by the procedure employed in the Inverse power method. NASTRAN employs the Gram-Schmidt orthogonalization procedure [4] for the purpose in the case of the Determinant and Givens methods.

Whatever the method employed, the eigenvectors obtained all exhibit the characteristics described in the previous section and also involve the inherent use of one or more arbitrary constants. In general, for a given mode, the involved constants for the computation of the corresponding eigenvector will not be the same in all the three methods. It should also be noted that the user has no control whatever on the selection or the choice of these constants.

EXAMPLE

To illustrate the points made in the paper, we consider the problem of the vibration of a square plate with hinged supports at all the four edges. (This is the same as NASTRAN Demonstration Problem 3-1. See Reference 5 for details.)

The finite element model employed is shown in Figure 1. Only half the plate has been modelled and symmetric boundary conditions used on the center-lir in order to reduce the order of the problem as well as the bandwidth.

The parameters of the model are as follows:

Length = ℓ = 508.0 mm (20.0 in.)

Width = w = 508.0 mm (20.0 in.)

Thickness = t = 25.4 mm (1.0 in.)

Young's modulus = E = 2.06843×10^{11} N/m^2 (3.0×10^7 psi)

Poisson's ratio = ν = 0.3

Mass density = ρ = 2.20197×10^9 Kg/m^3 (206.04393 $lb.-sec^2/in.^4$)

The eigenvalues and eigenvectors for the first six modes of the model were obtained by the Inverse power and Determinant methods.[2] For each method, four separate runs were made with different eigenvalue extraction data. The detailed data for all the eight runs are given in Table 1. Since the number of degrees of freedom in the analysis set (or a-set) in this problem is as large as 590 and only the first few modes were required for the purpose of the present paper, the Givens method was considered unsuitable.

The eigenvalues and natural frequencies obtained in the eight runs (as well as the corresponding theoretical values [6]) are presented in Table 2. It can be seen from this that the third and fourth modes together represent an eigenvalue of multiplicity two, that is, a double root. The other modes yield non-multiple eigenvalues.[3]

Tables 3 through 7 present, for the first six modes, the eigenvector components corresponding to the vertical displacements at points 111 through 121 (along the line AB in Figure 1). The results for the third and fourth modes, which represent a multiple eigenvalue, are included together in Table 5.

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[2] The runs were made on the CDC computer using an improved version of the 15.5 level of NASTRAN.

[3] It is true that no conclusions can be drawn from Table 2 regarding the multiplicity of the eigenvalue for the sixth mode because, in general, at least (m + 1) eigenvalues must be obtained before one can examine the multiplicity of the m^{th} eigenvalue. However, in the present problem, the non-multiplicity of the sixth eigenvalue was confirmed from additional runs with more than six eigenvalues.

An examination of Table 3 (corresponding to the non-multiple eigenvalue of the first mode) reveals that the eigenvector components for all the runs are essentially the same. (Actually, the results for all the runs except Run 3 are identical. The extremely small differences between the results of Run 3 and those of others are so inconsequential as to be practically meaningless.) The results thus show that they are essentially unaffected by differences in the eigenvalue extraction data.

Tables 4, 6 and 7 (corresponding to the non-multiple eigenvalues of the second, fifth and sixth modes respectively) need some explanation. Note that all the eigenvector components given in these three tables are extremely small in magnitude. Actually, the corresponding theoretical values are all zero [6]. For all practical purposes, the differences among the various runs in these tables can, therefore, be considered as quite insignificant.

Table 5 (corresponding to the multiple eigenvalue of the third and fourth modes) is interesting. It can be seen that the results given by the runs employing the Determinant method are essentially the same and that they are essentially unaffected by differences in the eigenvalue extraction data. The same thing is not wholly true for the runs employing the Inverse power method because slight variations are noticeable among some of them. (In this connection, it should be emphasized that the association of a particular eigenvector with a particular mode <u>number</u> in this table is irrelevant because the <u>numbering</u> of the modes for this multiple eigenvalue is completely arbitrary.) The most important point to note in this table, however, is that the eigenvectors given by the Inverse power method are completely different from those given by the Determinant method. At first sight, this discrepancy in the results may seem disturbing. However, it can be explained from a theoretical viewpoint and the discrepancy may be shown to be apparent and not real.

Now, since the third and fourth modes involve a multiple eigenvalue of order two, it follows from the theory presented earlier that their eigenvectors involve two arbitrary constants in them and that the relative values of the eigenvector components depend upon the relative values of these two constants. The differences among the various eigenvectors shown in Table 5 are therefore clearly due to the fact that they represent different relative values of the two involved constants.

The theory also shows that, in the case of a multiple eigenvalue of order two, the total number of eigenvectors possible is doubly infinite. The solution space is thus much larger than in the case of non-multiple eigenvalues. The theory further shows that the number of <u>independent</u> eigenvectors in this case is only two. This means that every eigenvector in the solution field can be obtained by the linear combination of any two eigenvectors that are themselves linearly independent. This can be shown to be true in the case of the eigenvectors given in Table 5.

Let $\{x\}_{ij}$ represent any eigenvector in Table 5 where i denotes the mode number and j denotes the run number. It can be seen from the table that there are several eigenvectors that are identical and, therefore, linearly dependent. One such set, for instance, is given by the eigenvectors $\{x\}_{31}$ and $\{x\}_{42}$. Another such set is given by $\{x\}_{32}$ and $\{x\}_{43}$. Yet another such example is the set comprising $\{x\}_{35}$, $\{x\}_{36}$, $\{x\}_{37}$ and $\{x\}_{38}$. A fourth such set is given by $\{x\}_{46}$, $\{x\}_{47}$ and $\{x\}_{48}$.

As an example, consider the eigenvectors $\{x\}_{36}$ and $\{x\}_{46}$ in Table 5 which are clearly linearly independent. It can be shown that every eigenvector in Table 5 can be obtained as a suitable linear combination of these two eigenvectors. Simple algebra shows that, for instance, the eigenvector $\{x\}_{32}$ in Table 5 can be obtained from these two independent eigenvectors by the following (approximate) relationship:

$$\{x\}_{32} = 0.9850782\{x\}_{36} - 0.1917116\{x\}_{46} \tag{4}$$

Similarly, it can be shown that the following (approximate) relationship can be used to obtain the eigenvector $\{x\}_{34}$ from $\{x\}_{36}$ and $\{x\}_{46}$:

$$\{x\}_{34} = 0.9850979\{x\}_{36} - 0.1914862\{x\}_{46} \tag{5}$$

In a like manner, similar linear relationships can be shown to exist between any eigenvector and any two linearly independent eigenvectors in Table 5.

The above discussion and relationships clearly show that the discrepancy in the results presented in Table 5 is thus only apparent and not real.

It is interesting to note that the plate problem considered in this example yields multiple roots at many higher modes also. Thus, for instance, modes 9 and 10 represent a double root. So do modes 12 and 13. Results similar to the above can be expected to occur in these cases also.

CONCLUSIONS

NASTRAN provides three basic methods for real eigenvalue extraction. In the case of non-multiple eigenvalues, each of these three methods will, for a given type of normalization, give essentially the same eigenvectors. However, this is not so in the case of multiple eigenvalues. This apparent discrepancy has been explained and illustrated by considering the example of a NASTRAN demonstration problem that has both multiple and non-multiple eigenvalues.

APPENDIX

The NASTRAN program provides three basic methods for real eigenvalue exraction. These are:

) The Inverse power method,

) The Determinant method, and,

) The Givens (or Tridiagonal) method.

The Inverse power and Determinant methods are "tracking" methods and as uch are efficient in those cases where only a few of many eigenvalues are deired. On the other hand, the Givens method is a transformation method that is fficient only in those cases where all eigenvalues or a high proportion of all igenvalues are required [2].

In each of the three methods, the eigenvectors obtained can be normalized n any one of three different ways. The three types of normalization (NØRMS) vailable are [1]:

) normalizing to unit value of the generalized mass (MASS),

) normalizing to unit value of the largest component in the analysis set (MAX), and,

) normalizing to unit value of a specified component in the analysis set (PØINT).

REFERENCES

1. The NASTRAN User's Manual (Level 15.0), NASA SP-222(01), May 1973.

2. The NASTRAN Theoretical Manual (Level 15.0), NASA SP-221(01), December 1972.

3. Hohn, F. E., Elementary Matrix Algebra, Second Edition, The Macmillan Company, New York, 1967.

4. Hildebrand, F. B., Methods of Applied Mathematics, Second Edition, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1965.

5. NASTRAN Demonstration Problem Manual (Level 15), NASA SP-224(01).

6. Harris, C. M., and Crede, C. E., Ed., Shock and Vibration Handbook, Volume 1, McGraw-Hill Book Company, Inc., New York, 1961.

Table 1. Eigenvalue Extraction Data* for the NASTRAN Runs

Run no.	Eigenvalue extraction method	Normalization method	Frequency limits (Hz)		Number of desired roots
			f_1	f_2	
1	Inverse power	MAX	0.89	5.00	3
2			0.89	5.00	4
3			0.89	6.50	5
4			0.89	8.00	6
5	Determinant	MAX	0.89	5.00	3
6			0.89	5.00	4
7			0.89	6.50	5
8			0.89	8.50	6

* See Reference 1 for details.

Table 2. Eigenvalues and Natural Frequencies

Mode no.	NASTRAN results (Runs 1 through 8 --- See Table 1)		Theoretical values (Reference 6)	
	Eigenvalues $(rad/sec)^2$	Natural frequencies (Hz)	Eigenvalues $(rad/sec)^2$	Natural frequencies (Hz)
1[a]	3.237408E+01	9.055634E-01	3.246970E+01	9.068997E-01
2[a]	2.022407E+02	2.263364E+00	2.029356E+02	2.267249E+00
3[a]	8.111597E+02	4.532870E+00	8.117425E+02	4.534499E+00
4[b]	8.111597E+02	4.532870E+00	8.117425E+02	4.534499E+00
5[c]	1.352052E+03	5.852169E+00	1.371845E+03	5.894848E+00
6[d]	2.355330E+03	7.724066E+00	2.345936E+03	7.708648E+00

[a] Obtained by all runs.

[b] Obtained by all runs except Runs 1 and 5.

[c] Obtained by Runs 3, 4, 7 and 8.

[d] Obtained only by Runs 4 and 8.

Table 3. Eigenvector Components* Corresponding to Vertical Displacements for Mode 1

Eigenvalue = 3.237408E+01 $(rad/sec)^2$ Natural frequency = 9.055634E-01 Hz

Grid point (see Figure 1)	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8
111	1.000000E+00	1.000000E+00	1.000000E+00	1.000000E+00	1.000000E+00	1.000000E+00	1.000000E+00	1.000000E+00
112	9.876883E-01	9.876883E-01	9.876883E-01	9.876883E-01	9.876883E-01	9.876883E-01	9.876883E-01	9.876883E-01
113	9.510565E-01	9.510565E-01	9.510565E-01	9.510565E-01	9.510565E-01	9.510565E-01	9.510565E-01	9.510565E-01
114	8.910065E-01	8.910065E-01	8.910064E-01	8.910065E-01	8.910065E-01	8.910065E-01	8.910065E-01	8.910065E-01
115	8.090170E-01	8.090170E-01	8.090168E-01	8.090170E-01	8.090170E-01	8.090170E-01	8.090170E-01	8.090170E-01
116	7.071068E-01	7.071068E-01	7.071065E-01	7.071068E-01	7.071068E-01	7.071068E-01	7.071068E-01	7.071068E-01
117	5.877853E-01	5.877853E-01	5.877850E-01	5.877853E-01	5.877853E-01	5.877853E-01	5.877853E-01	5.877853E-01
118	4.539905E-01	4.539905E-01	4.539902E-01	4.539905E-01	4.539905E-01	4.539905E-01	4.539905E-01	4.539905E-01
119	3.090170E-01	3.090170E-01	3.090168E-01	3.090170E-01	3.090170E-01	3.090170E-01	3.090170E-01	3.090170E-01
120	1.564345E-01	1.564345E-01	1.564344E-01	1.564345E-01	1.564345E-01	1.564345E-01	1.564345E-01	1.564345E-01
121	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	Inverse power method				Determinant method			

* The eigenvalue analyses were done by using the British system of units. However, the eigenvector components obtained represent relative values and thus are independent of the system of units employed.

Table 4. Eigenvector Components* Corresponding to Vertical Displacements for Mode 2

Eigenvalue = 2.022407E+02 $(rad/sec)^2$ — Natural frequency = 2.263364E+00 Hz

Grid point (see Figure 1)	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8
111	8.582833E-06	8.582833E-06	-8.804327E-07	2.867177E-11	7.498964E-14	7.498964E-14	7.498964E-14	7.498964E-14
112	8.476104E-06	8.476104E-06	-8.336322E-07	2.821834E-11	6.467638E-14	6.467638E-14	6.467638E-14	6.467638E-14
113	8.158775E-06	8.158775E-06	-7.022221E-07	2.688187E-11	3.426100E-14	3.426100E-14	3.426100E-14	3.426100E-14
114	7.639303E-06	7.639303E-06	-5.112464E-07	2.474057E-11	-1.083766E-14	-1.083766E-14	-1.083766E-14	-1.083766E-14
115	6.931394E-06	6.931394E-06	-2.964310E-07	2.192269E-11	-6.537558E-14	-6.537558E-14	-6.537558E-14	-6.537558E-14
116	6.053466E-06	6.053466E-06	-9.654166E-08	1.860159E-11	-1.164175E-13	-1.164175E-13	-1.164175E-13	-1.164175E-13
117	5.027983E-06	5.027983E-06	5.486849E-08	1.497935E-11	-1.438612E-13	-1.438612E-13	-1.438612E-13	-1.438612E-13
118	3.880710E-06	3.880710E-06	1.365259E-07	1.122344E-11	-1.422880E-13	-1.422880E-13	-1.422880E-13	-1.422880E-13
119	2.639977E-06	2.639977E-06	1.437853E-07	7.447627E-12	-1.153687E-13	-1.153687E-13	-1.153687E-13	-1.153687E-13
120	1.335955E-06	1.335955E-06	8.931819E-08	3.706543E-12	.467638E-14	.467638E-14	.467638E-14	.467638E-14
121	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	Inverse power method				Determinant method			

* See note given under Table 3.

Table 5. Eigenvector Components* Corresponding to Vertical Displacements for Modes 3 and 4

Eigenvalue = 8.111597E+02 $(rad/sec)^2$ Natural frequency = 4.532870E+00 Hz

Mode no.	Grid point (see Figure 1)	Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8
3	111	1.000000E+00	-3.591128E-01	1.000000E+00	-3.588908E-01	-1.699369E-01	-1.699370E-01	-1.699370E-01	-1.699370E-01
	112	9.536365E-01	-2.786022E-01	9.536365E-01	-2.783910E-01	-9.874144E-02	-9.874148E-02	-9.874151E-02	-9.874146E-02
	113	8.231105E-01	-5.563910E-02	8.231104E-01	-5.545871E-02	9.802834E-02	9.802830E-02	9.802828E-02	9.802832E-02
	114	6.322864E-01	2.581424E-01	6.322863E-01	2.582780E-01	3.736201E-01	3.736201E-01	3.736201E-01	3.736201E-01
	115	4.152396E-01	5.893736E-01	4.152395E-01	5.894585E-01	6.616324E-01	6.616324E-01	6.616324E-01	6.616324E-01
	116	2.090134E-01	8.590667E-01	2.090132E-01	8.591037E-01	8.906449E-01	8.906449E-01	8.906449E-01	8.906449E-01
	117	4.579713E-02	1.000000E+00	4.579698E-02	1.000000E+00	1.000000E+00	1.000000E+00	1.000000E+00	1.000000E+00
	118	-5.377633E-02	9.715794E-01	-5.377648E-02	9.715579E-01	9.532895E-01	9.532895E-01	9.532895E-01	9.532895E-01
	119	-8.476043E-02	7.689300E-01	-8.476054E-02	7.689038E-01	7.466009E-01	7.466009E-01	7.466009E-01	7.466009E-01
	120	-5.856040E-02	4.242319E-01	-5.856046E-02	4.242148E-01	4.097170E-01	4.097170E-01	4.097170E-01	4.097170E-01
	121	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	111		1.000000E+00	-3.591128E-01	1.000000E+00		1.000000E+00	1.000000E+00	1.000000E+00
	112		9.536365E-01	-2.786022E-01	9.536270E-01		9.458695E-01	9.458695E-01	9.458695E-01
	113		8.231105E-01	-5.563910E-02	8.230750E-01		7.939267E-01	7.939267E-01	7.939267E-01
	114		6.322864E-01	2.581424E-01	6.322146E-01		5.732738E-01	5.732738E-01	5.732738E-01
	115		4.152396E-01	5.893736E-01	4.151303E-01		3.254212E-01	3.254212E-01	3.254212E-01
	116		2.090134E-01	8.590667E-01	2.088751E-01		9.540108E-02	9.540110E-02	9.540107E-02
	117		4.579713E-02	1.000000E+00	4.564668E-02		-7.782729E-02	-7.782727E-02	-7.782730E-02
	118		-5.377633E-02	9.715794E-01	-5.391728E-02		-1.695951E-01	-1.695951E-01	-1.695951E-01
	119		-8.476043E-02	7.689300E-01	-8.486974E-02		-1.745788E-01	-1.745788E-01	-1.745788E-01
	120		-5.856040E-02	4.242319E-01	-5.862008E-02		-1.075995E-01	-1.075995E-01	-1.075995E-01
	121		0.0	0.0	0.0		0.0	0.0	0.0
		Inverse power method				Determinant method			

* See note given under Table 3.

Table 6. Eigenvector Components* Corresponding to Vertical Displacements for Mod

Eigenvalue = 1.352052E+03 $(rad/sec)^2$ Natural frequency = 5.852169E+00 Hz

Grid point (see Figure 1)	Run 3	Run 4	Run 7	Run 8
111	-5.699912E-07	-1.843495E-06	-2.183520E-12	-7.436551E-12
112	-6.080192E-07	-1.758002E-06	-2.089282E-12	-6.985661E-12
113	-7.113474E-07	-1.517318E-06	-1.771980E-12	-5.644270E-12
114	-8.501140E-07	-1.165451E-06	-1.135621E-12	-3.503232E-12
115	-9.820411E-07	-7.652404E-07	-2.059582E-13	-8.178176E-13
116	-1.061947E-06	-3.849968E-07	7.125514E-13	1.746165E-12
117	-1.052001E-06	-8.408051E-08	1.267204E-12	3.572532E-12
118	-9.304697E-07	9.946076E-08	1.520344E-12	4.248741E-12
119	-6.970452E-07	1.565069E-07	1.396781E-12	3.669778E-12
120	-3.735721E-07	1.080931E-07	8.658139E-13	2.111212E-12
121	0.0	0.0	0.0	0.0
	Inverse power method		Determinant method	

* See note given under Table 3.

Table 7. Eigenvector Components* Corresponding to Vertical Displacements for Mode 6

Eigenvalue = 2.355330 $(rad/sec)^2$ Natural frequency = 7.724066E+00 Hz

Grid point (see Figure 1)	Run 4	Run 8
111	-4.899823E-10	-6.518056E-12
112	-4.359024E-10	-5.832685E-12
113	-2.854791E-10	-3.934395E-12
114	-7.157563E-11	-1.252190E-12
115	1.590946E-10	1.593764E-12
116	3.561466E-10	3.982927E-12
117	4.764730E-10	5.414795E-12
118	4.937036E-10	5.612997E-12
119	4.039232E-10	4.588831E-12
120	2.265359E-10	2.571805E-12
121	0.0	0.0
	Inverse power method	Determinant method

* See note given under Table 3.

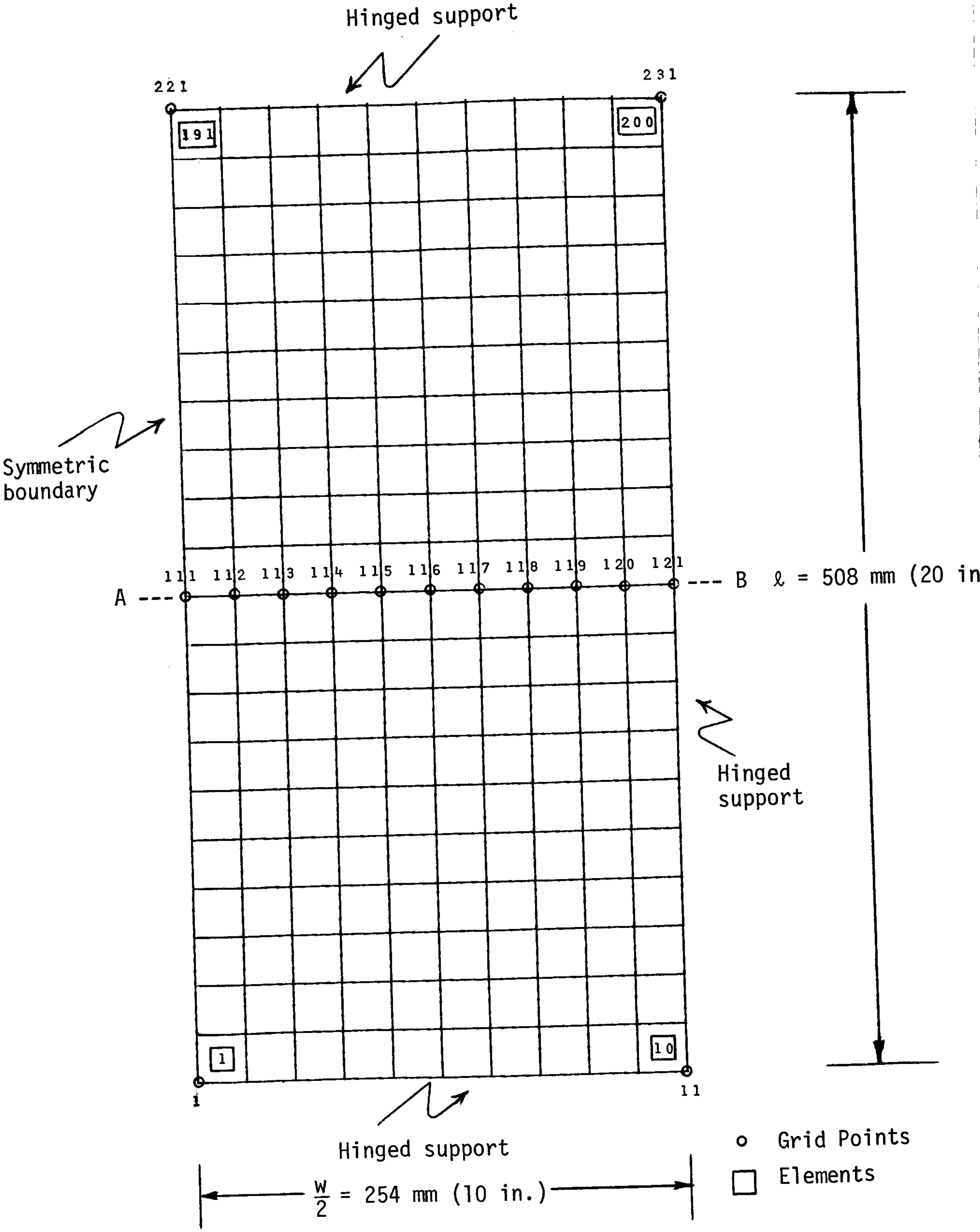

Figure 1. Finite element model of square plate in the Example.