ANALYSIS OF NONLINEAR STRUCTURES VIA MODE SYNTHESIS

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SUMMARY

An effective procedure for NASTRAN has been developed that permits any number of substructures of any size to be synthesized for the purpose of developing normal modes of vibration of the complete structural system. The technique is extended to permit modal transient analysis of the subdivided system. This latter procedure permits the use of NASTRAN's ability to include nonlinear forces in the problem. The five-phase process is accomplished using standard NASTRAN rigid formats with problem-independent alter packages and DMAP sequences.

INTRODUCTION

The advent of finite-element techniques has led to development of computer programs that perform efficient analyses of large-order structural systems of arbitrary geometry. Yet, for very large problems, it becomes impractical to attempt to develop and analyze a mathematical model of the entire structure. Questions of adequate data management and the cost of the analyses make such efforts difficult. Consequently, techniques have been developed to permit the structural system to be divided into smaller units, or substructures, which may then be modeled separately.

Substructure models, represented by some truncated set of degrees of freedom, are eventually brought together by the application of compatibility requirements at the interfaces. The resultant system representation is analyzed and the behavior of each substructure is determined by a back-substitution process.

The mode synthesis technique has been developed to permit dynamic analysis of such subdivided structures. A truncated set of vibration modes of a component is used as generalized coordinates to approximate dynamic characteristics. The several substructures are coupled by (1) compatibility relationships at the interfaces, and (2) modal transformations relating physical degrees of freedom with the generalized component modal coordinates. Solution of the resultant equations yields system eigenvalues and eigenvectors.

The use of vibration modes alone, however, usually fails to describe adequately the behavior of the system, particularly in the regions of the interior boundaries (interfaces). Consequently, the substructure vibration modes are augmented by other displacement functions. Numerous alternatives will be found in the literature. Hurty (Reference 1) and Craig and Bampton (Reference 2) employ "constraint" modes, defined by unit *displacements* of interface degrees of freedom on a fixed interface boundary. Bamford, *et al* (Reference 3) and Rubin (Reference 4) use "attachment" modes, defined by unit *forces* at each interface degree of freedom. Rubin has shown that this concept may be used for free interface boundaries, as well as fixed, to produce inertia relief deformations in unrestrained substructures. It is this latter arrangement that is employed in the development of mode synthesis via NASTRAN that follows. The technique can be easily modified to use constraint modes and fixed interface boundaries.

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Synthesizing substructures for dynamic analysis via NASTRAN has been discussed previously. The NASTRAN Theoretical Manual (Reference 5) discusses the use of scalar elements and MPC sets to represent substructures. These procedures were reviewed and amplified by Courtney (Reference 6). The current approach exceeds this earlier work by explicitly including attachment modes and by providing automated communication of data between component and system levels. The ability to develop the structure characteristics and acquire results in spatial coordinates enhances the procedure, particularly when considered from a production environment viewpoint.

The synthesis may be performed using any number of independently formulated substructures of any size. No restrictions are placed on the identification schemes used in component model development, since they are not carried over into the synthesis. The technique uses standarized alter packages and DMAP sequences that are virtually independent of the problem characteristics; thus the often present necessity for user specification of many data handling directives is avoided. Control for matrix partitioning and merging are in the form of user-supplied partition vectors.

An extension of the mode synthesis procedure includes the modal transient response analysis for the synthesized system, wherein the required modal data is recovered on a substructure-by-substructure basis. Since the complete system may be quite large, the modal matrix is truncated to include only those degrees of freedom subjected to external loads or required to define the presence of nonlinear forces. Such nonlinear forces are included only after the mode synthesis has been completed. The responses of the system, including internal forces and stresses, if desired, are recovered by back-substitution into each substructure.

The complete analysis process is divided into five sequential phases, with communication between phases accomplished via User Tapes; e.g., INPT, and Problem Tapes (NPTP, OPTP).

SUBSTRUCTURE MODAL ANALYSIS

Phase I of the mode synthesis/transient response analysis using NASTRAN is the development of substructure, or component, modal data. This information is assembled in Phase II for system modal analysis. There are few restrictions and very little additional demands required for this phase over that associated with a standard NASTRAN analysis. It is necessary to give special consideration to those degrees of freedom in the component model that are involved in connection with other portions of the structural system. Additional information is required to direct the preparation of the component modal data for later synthesis.

The full range of NASTRAN's resources may be employed to define a substructure model. Since substructures are independently defined, no special restrictions are placed on identification or numbering schemes.

Components may have any form of boundary condition at their external boundaries; i.e., those boundary grid points not connected to other components. Connection grid points must be unrestrained in those coordinates that are used to join the components.

Since the interface degrees of freedom are, by definition, unrestrained, the collectivity of normal vibration modes cannot correctly define internal loads at the interface of the component. To correct this deficiency, the dynamic modes are augmented by a selected set of static deformations, which, when properly weighted and summed, will provide the internal loads at the interface degrees of freedom. The static modes are generated by applying a unit force at each interface degree of freedom in turn and letting the component deform. If rigid body motion is prevented by external constraints, a straightforward static analysis results. If, however, a rigid body component of motion does result from the unit load at the interface, an inertia relief analysis is required to produce internal loads. For this reason, Phase 1 of the mode synthesis procedure is performed using Rigid Format No. 2, Static Analysis with Inertia Relief, as modified by the alter package of Figure 1.

```
$  PHASE ONE ALTER PACKAGE
ALTER    50, 50
PURGE      KRR,KLR,QR,DM/REACT $
CHKPNT     KRR,KLR,QR,DM  $
ALTER    76
EQUIV      KAA,KLL/REACT $
CHKPNT     KLL $
COND       LBL6,REACT  $
ALTER    78
LABEL      LBL6 $
ALTER    80
COND       LBL7,REACT $
ALTER    84
LABEL      LBL7 $
ALTER    88
EQUIV      PL,PLI/REACT/PO,POI/REACT $
COND       LBL10,REACT $
ALTER    89
LABEL      LBL10  $
ALTER    110, 114
$  VIBRATION MODES
DPD        DYNAMICS,GPL,SIL,USET/GPLD,SILD,USETD,,,,,,,EED,EQDYN/
           V,N,LUSET/V,N,LUSETD/V,N,Z1/V,N,Z2/V,N,Z3/V,N,Z4/
           V,N,Z5/V,N,Z6/V,N,Z7/C,N,0/V,N,Z9  $
SAVE       Z9  $
PARAM      //C,N,ADD/V,N,VIBCASE/V,Y,NCON=0/C,N,1  $  MODES SUBCASE COUNT
READ       KAA,MAA,MR,DM,EED,USET,CASECC/LAMA,PHIA,MI,OEIGS/
           C,N,MODES/V,N,NEIGS/V,N,VIBCASE  $
SAVE       NEIGS $
CHKPNT     LAMA,PHIA,MI,OEIGS $
OFP        LAMA,OEIGS,,,,//V,N,CARDNO  $
SAVE       CARDNO  $
COND       FINIS,NEIGS  $
$  RECOVER FULL MODE VECTOR
SDR1       USET,,PHIA,,,GO,GM,,KFS,,/PHIG,,QGX/C,N,1/C,N,REIG $
CHKPNT     PHIG,QGX $
SDR2       CASECC,CSTM,MPT,DIT,EQEXIN,SIL,,,BGPDT,LAMA,QGX,PHIG,EST,/
           ,OQG2,OPHIG,OES2,OEF2,PPHIG/C,N,REIG $
CHKPNT     PPHIG $
OFP        OPHIG,OQG2,OEF2,OES2,,//V,N,CARDNO  $
SAVE       CARDNO $
$  DYNAMICS PORTION OF GENERALIZED MATRICES
GKAM,      ,PHIA,MI,LAMA,,,,,CASECC/MHH,BHH,KHH,PHH/V,N,Z9/
           C,N,100/C,N,0./C,N,0./C,N,-1/C,N,-1/C,N,-1/V,N,NOC/V,N,FMODE $
$  ATTACHMENT MODE PARTITION OF GENERALIZED MASS AND STIFFNESS MATRICES
MPYAD      UGV,MGG,/AM/C,N,1 $
MPYAD      AM,UGV,/MGEN/C,N,0 $  GENERALIZED MASS (ATTACHMENT)
MPYAD      UGV,KGG,/AK/C,N,1  $
MPYAD      AK,UGV,/KGEN/C,N,0 $  GENERALIZED STIFFNESS (ATTACHMENT)
```

Figure 1. Phase I alter package.

```
CHKPNT   MGEN,KGEN  $
$  OFF-DIAGONAL PARTITIONS
MPYAD    AM,PHIG,/ML/C,N,0  $  L.L. PARTITION  (MASS)
MPYAD    AK,PHIG,/KL/C,N,0  $  L.L. PARTITION  (STIFFNESS)
TRNSP    ML/MU $  U.R. PARTITION  (MASS)
TRNSP    KL/KU $  U.R. PARTITION  (STIFFNESS)
$  GENERALIZED MATRICES AND CONNECTION PARTITION OF MODES
MERGE    MHH,ML,MU,MGEN,STATIC,/MASS/C,N,-1/C,N,2/C,N,6  $
MERGE    KHH,KL,KU,KGEN,STATIC,/STIFF/C,N,-1/C,N,2/C,N,6  $
MERGE    PHIG,,UGV,,STATIC,/MODES/C,N,1/C,N,2/C,N,2  $
PARTN    MODES,,CDOF/,INTER,,/C,N,1/C,N,2/C,N,0/C,N,2 $
MERGE    QGX,,QG,,STATIC,/QQG/C,N,1/C,N,2/C,N,2  $
CHKPNT   MODES,QQG  $
PARAM    //C,N,NOP/V,Y,PRINT=-1 $
COND     LIST,PRINT $
MATPRN   MASS,STIFF,INTER,,//$
LABEL    LIST  $
$  WRITE MATRICES ONTO USER TAPE -INPT-
PARAM    //C,N,SUB/V,N,KOUNT/V,Y,COMP=0/C,N,1 $
PARAM    //C,N,MPY/V,N,COUNT/V,N,KOUNT/C,N,3 $
COND     FINIS,COUNT  $  EXIT IF SEQUENCE NUMBER IS LESS THAN 0
PARAM    //C,N,SUB/V,N,IOTEST/V,N,COUNT/C,N,1 $
COND     I01,IOTEST  $  JUMP IF FIRST COMPONENT
INPUTT1  /,,,,/C,N,-1  $  REWIND -INPT-
JUMP     I02  $
LABEL    I01  $
OUTPUT1, ,,,,//C,N,-1 $
LABEL    I02  $
OUTPUT1  MASS,STIFF,INTER,,//V,N,COUNT  $
ENDALTER
```

Figure 1. Phase I alter package (concluded).

The DMAP alterations supplied for Phase I add the eigenvalue extraction module and other modules that merge the results of the static and dynamic processes to form the final component mode matrix.

Data required by Phase II is output to a User Tape. These data include the generalized mass and generalized stiffness matrices associated with the merged mode vector matrix and the merged mode vector matrix truncated to include only the interface degrees of freedom. The truncation is directed by a partitioning vector supplied by the user. The Phase I analysis should be checkpointed for later use in recovery activity in Phases III and V.

Additional data required beyond the model description include special control parameters, entered via bulk data card PARAM, and partition vectors, entered via bulk data card DMI. The special parameters required for Phase I include:

1. COMP – The sequence number of the component. Used to position the User Tape before writing component data.

2. NCON – The number of interface degrees of freedom for the component. The number of static load subcases/static modes must agree with NCON.

3. PRINT – A positive value causes printing of the matrices written on the User Tape.

The partition vectors needed are:

1. STATIC – Used to merge static and dynamic modes into one mode vector matrix. M = sum of all modes computed (static and dynamic, including rigid body). An element value of 1 locates a static mode in the merged matrix. There must be exactly the same number of positive elements in the vector as there are static modes.

2. CDOF – Used to identify the interface degrees of freedom. M = g size. Vector contains 1 for elements corresponding to the internal sequence number of all interface degrees of freedom.

The Case Control deck must include (n + 1) subcases, where n is the number of interface degrees of freedom for the component (and, consequently, the number of static load cases). The first n subcases are static cases, with a LOAD reference in each. The last subcase is the modal extraction subcase, with a METHOD card. Figure 2 illustrates a typical Case Control Deck.

```
REQUEST(NPTP,HD,S)   RING IN
REQUEST(INPT,HD,S)   RING IN
ATTACH(N15,NASTRAN15,ID=TONE90970)
REWIND(NPTP,INPT)
MAP(OFF)
N15.ATTACH
RETURN(NPTP,INPT)
EXIT.
DMP(140000)
RETURN(NPTP,INPT)
(7-8-9)
ID       PHASE,    ONE
APP      DISP
SOL      2,1
CHKPNT YES
TIME     2
$    (PHASE ONE ALTER PACKAGE GOES HERE)
CEND
TITLE=DEMONSTRATION OF PHASE ONE DECK SET-UP
   SPC = 173
   MPC = 4177
OUTPUT
   SET 37 = 201 THRU 287
     VECTOR = ALL
     SPCFORCES = 37
$  THIS EXAMPLE ASSUMES FOUR CONNECTION
$   DEGREES OF FREEDOM.
SUBCASE  100
   LOAD = 57
SUBCASE  200
   LOAD = 371
SUBCASE  300
   LOAD = 66
SUBCASE 400
   LOAD = 331
SUBCASE  500
   METHOD = 850
BEGIN BULK
$  (BULK DATA DECK GOES HERE)
ENDDATA
(6-7-8-9)
```

Figure 2. Typical deck setup for Phase I.

Phase II of the mode synthesis analysis involves generation of system dynamic characteristics throu synthesis of component information. This is performed using Rigid Format 3, Normal Mode Analys modified by the alter package of Figure 3. Direction is required from the user regarding compatibili relations among the various components, and selection of component generalized coordinates. Us attention to certain aspects of internal data storage arrangements is required; in most cases, howev this is a trivial consideration. Since the activity involves operations on generalized, rather th physical, coordinates, the intuition and understanding of the user become much more necessary th in a normal study. Further, component coupling is accomplished by imposing transforms (develop from physical coordinate relationships) on the generalized coordinates. This requires the use of sor mathematical sleight-of-hand that can lead to confusion unless the user is alert.

```
$  PHASE TWO ALTER PACKAGE
ALTER   6,74
$  STORE COMPONENT PARTITIONING VECTORS ON TEMPORARY DISK FILE
OUTPUT2   POS1,LOC1,TRUNK1,,//C,N,-1/V,Y,HOLD=11 $
OUTPUT2   POS2,LOC2,TRUNK2,,//C,N,0/V,Y,HOLD $
$  INSERT ADDITIONAL -OUTPUT2- STATEMENTS
$  UNTIL ALL COMPONENTS ARE TREATED
INPUTT2   /,,,,/C,N,-1/V,Y,HOLD $
PARAM     //C,N,NOP/V,N,TRUE=-1 $
$  PARAMETER K IS COMPONENT LOOP COUNTER
PARAM     //C,N,SUB/V,N,K/V,Y,NCOMP=1/C,N,1   $
PURGE     SUMK,SUMM,SUMC/TRUE  $
INPUTT1   /,,,,/C,N,-1 $
JUMP      TOP1  $
INPUTT2   /SUMK,SUMM,SUMC,,/C,N,0/V,Y,XTRA=13 $
$  LOOP OVER ALL SUBSTRUCTURES, TRUNCATING COMPONENT DATA AND
$  MERGING TO FORM UNCOUPLED SYSTEM MATRICES
LABEL     TOP1  $
INPUTT2   /CP,RP,TRUNK,,/C,N,0/V,Y,HOLD $  PARTITION VECTORS
INPUTT1   /MASS,STIFF,INTER,,/C,N,0 $  COMPONENT DATA
PARTN     MASS,TRUNK,/MM,,,/C,N,-1/C,N,2/C,N,6 $
PARTN     STIFF,TRUNK,/KK,,,/C,N,-1/C,N,2/C,N,6 $
PARTN     INTER,TRUNK,/CC,,,/C,N,1/C,N,2/C,N,2 $
MERGE,    ,,,CC,CP,RP/CCX/C,N,1/C,N,2/C,N,2  $
ADD       SUMC,CCX/CCXX $
EQUIV     CCXX,SUMC/TRUE  $
MERGE,    ,,,MM,CP,/MMX/C,N,-1/C,N,2/C,N,6  $
ADD       SUMM,MMX/MMXX  $
EQUIV     MMXX,SUMM/TRUE  $
MERGE,    ,,,KK,CP,/KKX/C,N,-1/C,N,2/C,N,6  $
ADD       SUMK,KKX/KKXX $
EQUIV     KKXX,SUMK/TRUE $
PARAM     //C,N,SUB/V,N,K/V,N,K/C,N,1 $DECREMENT COUNTER
COND      END1,K $
REPT      TOP1,20 $
LABEL     END1 $
CHKPNT    SUMC,SUMM,SUMK $
PARAM     //C,N,NOP/V,Y,PRINT=-1 $
```

Figure 3. Phase II alter package.

```
COND      PRNT1,PRINT $
MATPRN    SUMC,SUMM,SUMK,,//$ PRINT SYSTEM UNCOUPLED MATRICES
LABEL     PRNT1 $
PARAM     //C,N,NOP/V,N,NSKIP=0 $
GP4       CASECC,GEOM4,EQEXIN,SIL,GPDT/RGG,,USET,/
          V,N,LUSET/V,N,MPCF1/V,N,MPCF2/V,N,SINGLE/V,N,OMIT/
          V,N,REACT/V,N,NSKIP/V,N,REPEAT/V,N,NOSET/V,N,NOA/V,N,NOL $
SAVE      MPCF1,MPCF2,SINGLE,OMIT,REACT,NSKIP,REPEAT,NOSET,NOA,NOL $
CHKPNT    RGG,USET  $
$  COMPATIBILITY RELATIONS YIELD REDUCTION TRANSFORM
VEC       USET/VF/C,N,G/C,N,COMP/C,N,F $
PARTN     RGG,VF,/RG,,,/C,N,1/C,N,2/C,N,2  $
MPYAD     RG,SUMC,/A/C,N,0  $  COMPATIBILITY IN GENERALIZED COORDS.
VEC       USET/VO/C,N,F/C,N,COMP/C,N,0  $
PARTN     A,VO,/AI,,AD,/C,N,1/C,N,2/C,N,2/C,N,0/C,N,1  $
SOLVE     AD,AI/B/C,N,0/C,N,-1 $
COND      PRNT2,PRINT  $
MATPRN    A,B,,,//$
LABEL     PRNT2  $
SMP2      USET,B,SUMM/MAA $ REDUCED SYSTEM MASS MATRIX
SMP2      USET,B,SUMK/KAA  $  REDUCED SYSTEM STIFFNESS MATRIX
COND      PRNT3,PRINT  $
MATPRN    MAA,KAA,,,//$
LABEL     PRNT3 $
$  CONTINUE WITH STANDARD ANALYSIS METHODS USING -MAA- AND -KAA-
ALTER   80,80
FBS       LLL,ULL,KLR/DM/C,N,1/C,N,-1/C,N,2/C,N,2  $
ALTER     95,  112
MPYAD     B,PHIA,/DEPEND/C,N,0  $ RECOVER DEPENDENT COORDS
MERGE     PHIA,DEPEND,,,,VO/FULL/C,N,1/C,N,2/C,N,2 $
OUTPUT2   LAMA,MI,FULL,,//C,N,0/V,Y,HOLD  $
$  PRINT ON DEMAND
PARAM     //C,N,NOP/V,Y,VECTOR=-1 $
COND      PRNT4,VECTOR $
MATPRN    FULL,,,,//$
LABEL     PRNT4  $
ENDALTER
```

Figure 3. Phase II alter package (concluded).

All component data required for mode synthesis is, at the beginning of Phase II, contained on a Jser Tape in a sequential manner. In most cases, the number of component generalized coordinates modes) to be used in the synthesis task will be less than the number provided through the individual omponent analyses of Phase I. The user identifies the modes to be used by supplying partitioning ectors for each component.

The key to the mode synthesis task is the compatibility relationship among the interface oordinates, expressed in the following equation:

$$S_1 X_c^{(1)} + S_2 X_c^{(2)} + \ldots\ldots + S_j X_c^{(j)} = 0 \tag{1}$$

where $X_c^{(i)}$ are the interface coordinates for component i and S_i is a transformation employed to express the interface motions of all components in a common coordinate system. In matrix form, thi becomes

$$[S_1 \vdots S_2 \vdots \dots \vdots S_j] \begin{Bmatrix} X_c^{(1)} \\ \cdots \\ X_c^{(2)} \\ \cdots \\ X_c^{(j)} \end{Bmatrix} = 0 \qquad (2$$

To represent this relation in terms of component generalized coordinates, a modal transformatior is imposed, which yields

$$[S_1 \vdots S_2 \vdots \dots \vdots S_j] \begin{bmatrix} \phi_c^{(1)} & & & \\ & \phi_c^{(2)} & & \\ & & \ddots & \\ & & & \phi_c^{(j)} \end{bmatrix} \begin{Bmatrix} q^{(1)} \\ \cdots \\ q^{(2)} \\ \cdots \\ \vdots \\ \cdots \\ q^{(j)} \\ \cdots \end{Bmatrix} = 0 \qquad (3)$$

For simplicity, the compatibility equation (Eq. 3) is rewritten as

$$[A] \; \{q\} = 0 \qquad (4)$$

Motion at the interfaces is redundantly defined in the above equation. A set of dependent coordinates must be identified such that they are equal in number to the excess interface coordinates, which, in turn, are equal in number to the compatibility equations. The choice is essentially arbitrary, except that for a singular system an appropriate set of rigid-body modes must be retained as independent coordinates for reasons explained below. This set must be adequate to supply rigid-body motion in all directions that are singular in the complete structural system. The compatibility equation is partitioned into dependent and independent coordinates:

$$[A_I \vdots A_D] \begin{Bmatrix} q_I \\ \cdots \\ q_D \end{Bmatrix} = 0 \qquad (5$$

The relationship between dependent and independent coordinates is thus

$$\{q_D\} = -[A_D]^{-1} [A_I] \{q_I\} \tag{6}$$

It is apparent from Eq. 6 that the dependent coordinates must be chosen so that $[A_D]$ is nonsingular, and hence invertible. The full set of generalized coordinates can now be defined in terms of the independent coordinates.

$$\left\{\begin{matrix} q_I \\ \cdots \\ q_D \end{matrix}\right\} = \begin{bmatrix} I \\ \cdots\cdots\cdots\cdots \\ -[A_D]^{-1}[A_I] \end{bmatrix} \{q_I\} \tag{7}$$

This equation supplies the necessary coupling of component generalized coordinates required to define the behavior of the full structural system. Letting

$$[\beta] = \begin{bmatrix} I \\ \cdots\cdots\cdots\cdots \\ -[A_D]^{-1} [A_I] \end{bmatrix} \tag{8}$$

the uncoupled component generalized mass and stiffness matrices are transformed into coupled system matrices. If the assembled uncoupled matrices are $[\mathfrak{m}]$ and $[k]$ where

$$[\mathfrak{m}] = \begin{bmatrix} \mathfrak{m}^{(1)} & & & \\ & \mathfrak{m}^{(2)} & & \\ & & \ddots & \\ & & & \mathfrak{m}^{(j)} \end{bmatrix}$$

$$[k] = \begin{bmatrix} k^{(1)} & & & \\ & k^{(2)} & & \\ & & \ddots & \\ & & & k^{(j)} \end{bmatrix}$$

then application of the transformation results in

$$[M] = [\beta]^T [m] [\beta]$$

$$[K] = [\beta]^T [k] [\beta]$$

where [M] and [K] are coupled system mass and stiffness matrices in terms of the independent set of generalized coordinates. These are used in the eigenvalue extraction to acquire system model data. The mode vectors obtained are the system vibration modes, expressed in terms of the independent component generalized coordinates.

The system mode vectors are expanded to include the dependent generalized coordinates and stored, along with the eigenvalue table and system generalized mass matrix, on another User Tape. These data are accessed in Phase III for purposes of recovering mode data in component physical coordinates.

Each component generalized coordinate is represented in the NASTRAN analysis by a scalar point. In addition, the interface degrees of freedom are mapped on a one-to-one basis on another set of scalar points. It is advantageous to employ a numbering sequence that can be used to distinguish easily between those scalar points representing component generalized coordinates and those representing interface degrees of freedom. The numbering scheme for the scalar points is arbitrary; however, partitioning vectors described below must be compatible with the internal sequence of points; thus, it is desirable that some logical numbering scheme be employed to facilitate bookkeeping. Both component mode and component interface degree of freedom internal sequences cannot be altered from the Phase I condition. Regardless of what scalar point numbering scheme is employed, compatibility with these sequences must be maintained. Note that, if necessary, SEQGP cards can be used to resequence scalar points.

The definition of interface compatibility relationships (Eq. 2) is introduced by means of multi-point constraint equations. It is the responsibility of the user to verify the coordinate relationships defined by the MPC equations, including any necessary coordinate transformations. The MPC cards supply these relationships in terms of the scalar point equivalents of the interface degrees of freedom. The relationships defined must be sufficient to eliminate all redundancy in describing interface motion. To illustrate, if there are n components interfacing at a grid point, then there must be (n - 1) MPC equations supplied for every direction in which motion can occur at that grid point. Within that requirement, the choice of dependent degrees of freedom is arbitrary, since the actual synthesis activity does not consider the distinction. The scalar points used as *independent* coordinates in the MPC equations are placed in the S-set by SPC cards. This is done solely for the purpose of internal bookkeeping and does not impose any real constraint on the actual structure. The transform to generalized coordinates (Eq. 3) is performed automatically.

The division of component generalized coordinates into independent and dependent sets, as required by Eq. 5, is done by specifying the dependent set on OMIT cards. The number of coordinates omitted must be identical to the number of interface dependency equations defined by MPCs. The specification is in terms of the scalar points representing the generalized coordinates. Based on this set selection, NASTRAN automatically generates the coupled system matrices required by the eigenvalue module.

If the full structure being analyzed exhibits singularities, temporary supports should be specified via a SUPORT card. The scalar points that should be supported in this manner are those that represent t of component rigid-body modes sufficient to define rigid-body motion of the full structure. Note t the number of supports used must be equal to the number of rigid-body modes of the full system. e normal choice of temporary support location will be the rigid-body modes of a single component.

It is recommended that normalization of eigenvectors be done to a unit generalized mass to ntain a recognizable standard through the transformation back to physical coordinates.

Special parameters required for this phase are:

NCOMP – The number of components in the synthesis (and for which data have been placed on a User Tape).

PRINT – A positive value will cause the printing of intermediate matrices.

VECTOR – A positive value will cause the printing of a matrix containing mode vectors in component generalized coordinates.

e partition vectors required are:

TRUNKi – Identifies modes of component i present on the User Tape that are to be deleted (i.e., not used for the synthesis). M = number of component i modes computed and stored on the User Tape. An element corresponding to a mode to be deleted has value of 1.

POSi – Locates component modes in uncoupled system matrix. M = sum of all component modes to be used in the synthesis. Element has a value of 1 when corresponding to a mode of component i. (The sum of all POSi vectors is a unity vector.)

LOCi – Locates interface degrees of freedom for component i in truncated uncoupled mode vector matrix. M = sum of all interface degrees of freedom. Element has a value of 1 when corresponding to a degree of freedom of component i. (The sum of all LOCi vectors is a unity vector).

ferences for MPC, SPC, and eigenvalue METHOD must be present in the Case Control Deck, along h the titles, etc. None of the usual output options is available in this phase. A sample case control k is given in Figure 4.

RECOVERY OF SYSTEM MODE SHAPES

e system modal information in physical coordinates can be recovered on a nponent-by-component basis using the DMAP sequence of Figure 5, the problem tapes generated in ise I, and the User Tape created in Phase II. The User Tape contains not only the modal data but) the partitioning vectors supplied in Phase II to define the positioning of the component eralized coordinates. These are applied to the system mode vector matrix to truncate it to the eralized coordinates for a selected component, $[q_s^{(j)}]$. The portion of the system mode shape for j^{th} component $[\phi_s^{(j)}]$ is expressed in the component global coordinate system by means of k-substitution, using the component mode matrix $[\phi_c^{(j)}]$, as a transformation:

$$[\phi_s^{(j)}] = [\phi_c^{(j)}]\,[q_s^{(j)}] \tag{9}$$

ransient analysis is to be performed, these data are stored on a User Tape for use in Phase IV. Once the dal amplitudes of the physical coordinates have been determined, full use of NASTRAN's output ion is available; this option permits printing and/or punching of displacements, constraint forces, ernal forces, and internal stresses.

```
REQUEST(UT1,*PF)
REQUEST(INPT,HD,S,VSN=XXXXX)
REWIND(UT1,INPT)
ATTACH(N15,NASTRAN15,ID=TONE90970)
MAP(OFF)
N15.ATTACH
CATALOG(UT1,YOURPERMFILE,ID=YOURZZZZZ)
RETURN(INPT)
EXIT.
DMP(140000)
RETURN(INPT)
CATALOG(UT1,YOURPERMFILE,ID=YOURZZZZZ)
(7-8-9)
ID        PHASE,   TWO
APP       DISP
SOL       3,0
TIME      2
$(PHASE TWO ALTER PACKAGE GOES HERE)
CEND
TITLE=DEMONSTRATION OF PHASE TWO DECK SET-UP
      MPC = 381
      SPC = 16
      METHOD = 517
BEGIN BULK
$(BULK DATA DECK GOES HERE)
ENDDATA
(6-7-8-9)
```

Figure 4. Typical deck setup for Phase II.

```
$  PHASE THREE DMAP SEQUENCE
BEGIN    $
INPUTT2   /,,,,/C,N,-1/V,Y,HOLD=11 $
PARAM    //C,N,SUB/V,N,M/V,Y,COMP/C,N,1   $
PARAM     //C,N,MPY/V,N,OVER/V,N,M/C,N,3  $
INPUTT2   /CP,RP,TRUNK,,/V,N,OVER/V,Y,HOLD   $
INPUTT2   /LAMA,MI,FULL,,/C,N,-5/V,Y,HOLD   $
PARTN     FULL,,CP/,COMPMODE,,/C,N,1/C,N,2/C,N,0/C,N,2 $
PARTN     MODES,TRUNK,/MODEA,,,/C,N,1/C,N,2/C,N,2 $
PARTN     QQG,TRUNK,/QA,,,/C,N,1/C,N,2/C,N,2   $
MPYAD     MODEA,COMPMODE,/PHIGG/C,N,0   $
MPYAD     QA,COMPMODE,/QQGG/C,N,0   $
SDR2   CASECC,CSTM,MPT,DIT,EQEXIN,SIL,,,BGPDT,LAMA,QQGG,
              PHIGG,EST,/,OQ,OPHI,OS,OF,PPHIG/C,N,REIG   $
OFP    OPHI,OQ,OF,OS,,//V,N,CARDNO   $
SAVE    CARDNO  $
PARAM     //C,N,SUB/V,N,MM/V,N,M/C,N,1 $
COND      OUT1,MM   $ JUMP IF FIRST SUBSTRUCTURE
INPUTT1   /,,,,/C,N,-1/V,Y,INP1=1 $
JUMP      OUT2  $
LABEL     OUT1  $
OUTPUT1,     ,,,,//C,N,-1/V,Y,INP1   $
LABEL     OUT2  $
OUTPUT1    PHIGG,,,,//V,N,M/V,Y,INP1   $
END   $
```

Figure 5. Phase III DMAP sequence.

Since this phase uses a DMAP approach, only output requests have meaning in the Case Control Deck and no bulk data are needed. The restart dictionary generated in Phase 1 is used, to which is added a last card reading:

n, RE-ENTER AT DMAP SEQUENCE NUMBER 1

where n is the appropriate sequence number (see Section 2.2.1 of the NASTRAN User's Manual). A typical deck setup is given in Figure 6.

```
REQUEST(OPTP,HD,S,VSN=YYYYY)
REQUEST(INP1,HD,S)    RING IN
ATTACH(UT1,YOURPERMFILE,ID=YOURZZZZZ)
REWIND(OPTP,INP1,UT1)
ATTACH(N15,NASTRAN15,ID=TONE90970)
MAP(OFF)
N15.ATTACH
RETURN(OPTP,INP1)
EXIT.
DMP(140000)
RETURN(OPTP,INP1)
(7-8-9)
ID      PHASE,    THREE
APP       DMAP
TIME      2
$(PHASE THREE DMAP SEQUENCE GOES HERE)
$(PHASE ONE RESTART DICTIONARY GOES HERE)
CEND
TITLE=DEMONSTRATION OF PHASE THREE DECK SET-UP
$  SPC AND MPC REFERENCES ARE THE SAME AS FOR PHASE ONE
   SPC = 173
   MPC = 4177
OUTPUT
   SET 16 = 1,3,4 THRU 16, 30,35
   SET 31 = 106 THRU 135, 187 THRU 194
     VECTOR = ALL
     SPCFORCES = 16
     ELSTRESS = 31
BEGIN BULK
ENDDATA
(6-7-8-9)
```

Figure 6. Typical deck setup for Phase III.

MODAL TRANSIENT RESPONSE ANALYSIS

The transient response analysis via NASTRAN mode synthesis requires user direction to merge the required modal data from the component partitions and then perform the response analysis. Since the system being analyzed may be quite large, only those degrees of freedom involved in defining external loadings or associated with extra points are used. Rigid Format 12, Modal Transient Analysis, modified by the alter package of Figure 7, is used.

The system mode vector matrices for each substructure, written on a User Tape in Phase III, are partitioned to extract the portions associated with externally loaded degrees of freedom. If extra points are to be added at this stage (e.g., for nonlinear force definitions), the degrees of freedom needed to define the relationship of the extra points to the physical structure must also be included. These truncated mode vector sets are merged to form a single truncated system mode matrix. The system modes to be used in the response analysis are selected by the user via frequency limits or mode count.

```
$  PHASE FOUR ALTER PACKAGE
ALTER  23,42
ALTER   48,  91
$  STORE SUBSTRUCTURE LOAD AND ASSIGN PARTITION VECTORS
INPUTT1  /,,,,/C,N,-1/V,Y,INP1=1  $
OUTPUT1  LOAD1,ASSIGN1,,,//V,Y,NCOMP=1/V,Y,INP1 $
OUTPUT2  LOAD1,ASSIGN1,,,//C,N,-1/V,Y,TEMP=12 $
$  INSERT ADDITIONAL -OUTPUT2- STATEMENTS
$  UNTIL ALL COMPONENTS ARE TREATED
$  TEMPORARY DISK FILE (TEMP = 12) TO REDUCE TAPE MANIPULATIONS
OUTPUT1  LOAD2,ASSIGN2,,,//C,N,0/V,Y,INP1 $
OUTPUT2  LOAD2,ASSIGN2,,,//C,N,0/V,Y,TEMP $
$  CONTINUE UNTIL ALL SUBSTRUCTURES HAVE BEEN TREATED
INPUTT1   /,,,,/C,N,-1/V,Y,INP1 $
INPUTT2   /,,,,/C,N,-1/V,Y,TEMP  $
PARAM    //C,N,NOP/V,N,TRUE=-1 $
PARAM    //C,N,SUB/V,N,K/V,Y,NCOMP/C,N,1 $
PURGE    PHIA,MAA/TRUE  $
JUMP     TOPLOOP  $
INPUTT1  /PHIA,MAA,GO,GM,KFS/C,N,0/V,Y,INP1  $ DUMMY READ
LABEL    TOPLOOP $
INPUTT1  /SUBMODE,,,,/C,N,0/V,Y,INP1  $
INPUTT2  /LOAD,ASSIGN,,,/C,N,0/V,Y,TEMP  $
$  TRUNCATE MODE VECTOR TO LOADED AND E.P. REFERENCED DOF ONLY.
PARTN    SUBMODE,,LOAD/,LOADMODE,,/C,N,1/C,N,2/C,N,0/C,N,2  $
$  ASSEMBLE SYSTEM MATRIX
MERGE,   ,LOADMODE,,,,ASSIGN/LDMD/C,N,1/C,N,2/C,N,2  $
ADD      PHIA,LDMD/NEWMODE  $
EQUIV    NEWMODE,PHIA/TRUE $
PARAM    //C,N,SUB/V,N,K/V,N,K/C,N,1  $
COND     ENDLOOP,K $
REPT     TOPLOOP,20 $
LABEL    ENDLOOP  $
CHKPNT   PHIA  $
INPUTT2  /LAMA,MI,,,/C,N,-5/V,Y,HOLD=11  $
CHKPNT   LAMA,MI  $
$  MODE MATRIX CONTAINING ONLY NEEDED DOF HAS NOW BEEN FORMED.
$  CONTINUE STANDARD ANALYSIS
ALTER   94,94
ALTER   98,103
ALTER   125
EQUIV    UHVT,UHVTX/NOUE/PPT,PPG/NOUE  $
COND     EOFF,NOUE  $
PARTN    UHVT,,EP/UHVTX,,,/C,N,1/C,N,2/C,N,2  $
LABEL    EOFF  $
$  OUTPUT SOLUTION SET RESPONSE HISTORY AND LOAD HISTORY
PARAM    //C,N,MPY/V,N,SKIP/V,Y,NCOMP/C,N,2 $
OUTPUT1   UHVTX,PPT,,,//V,N,SKIP/V,Y,INP1  $
ENDALTER
```

Figure 7. Phase IV alter package.

Phase IV is similar to Phase II in that only scalar points are used. The scalar points represent all aded coordinates and all interface coordinates for extra points, if any. The numbering scheme for e scalar points is arbitrary; however, the partitioning vector described below must be compatible with e internal sequence of each component, which cannot be altered. Some logical number scheme to cilitate bookkeeping is obviously desirable. SEQGP cards may be used to resequence scalar points.

Note that no boundary conditions are specified. They are implicit in the modal information that ill be used in the analysis. All system data are entered in terms of the physical degrees of freedom or, ore exactly, their scalar point equivalents. NASTRAN automatically performs the required ansformation to modal coordinates.

Standard NASTRAN methods for defining external loads sets are used. The loads are applied to e scalar points representing the appropriate physical degrees of freedom. It is the user's sponsibility to ensure that this assignment is correctly specified as no internal check can be made.

The interactions between any extra points and the structure are defined by supplementary atrices added to structural mass, damping, and stiffness matrices, expanded to accommodate the ditional degrees of freedom. These matrices are input directly via DMIG bulk data cards or formed om transfer functions entered via TF bulk data cards.

There is but one special parameter required for this phase: NCOMP – the number of bstructures in the system. Partition vectors required include:

LOADi – Identifies the degrees of freedom in substructure i to be used in the transient response analysis. M = g size of the substructure. The element corresponding to the coordinate to be used is given a value of 1. Degrees of freedom to be loaded by external forces or that are to be referenced by extra points are included.

ASSIGNi – Identifies the portion of retained degrees of freedom of substructure i in the assembled system representation. M = the sum of all coordinates in the structure to be employed in the transient analysis. The element corresponding to a coordinate from substructure i has a value of 1.

EP – Identifies position of extra points in augmented modal matrices. M = number of modes used in transient analysis plus number of extra points. This vector is required only if extra points are used. Further, extra points are always sequenced last in such matrices. A value of 1 indicates position of extra point degree of freedom.

ie Case Control deck includes all references required to perform a transient response analysis. In dition, output requests for solution set responses as well as applied and nonlinear force histories are ade. A typical deck setup is illustrated in Figure 8.

RECOVERY OF PHYSICAL RESPONSES

he response of the entire structural system can be recovered on a substructure-by-substructure basis / using the DMAP sequence of Figure 9, the Phase I Problem Tapes, and the Phase IV User Tape. The lution set response matrix generated in Phase IV is pre-multiplied by the appropriate partition of the stem mode shape matrix to form the substructure response matrix. Then by using standard NASTRAN covery methods, the full complement of output options may be exercised.

This phase is executed in a restart mode; consequently, the mathematical model data are available om the OPTP. (The restart dictionary must be augmented as for Phase III.) The additional

```
REQUEST(INP1,HD,S,VSN=PPPPP)    RING IN
ATTACH(UT1,YOURPERMFILE,ID=YOURZZZZZ)
REWIND(INP1,UT1)
ATTACH(N15,NASTRAN15,ID=TONE90970)
MAP(OFF)
N15.ATTACH
RETURN(INP1)
EXIT.
DMP(140000)
RETURN(INP1)
(7-8-9)
ID     PHASE,    FOUR
APP        DISP
SOL        12,3
TIME       2
$(PHASE FOUR ALTER PACKAGE GOES HERE)
CEND
TITLE=DEMONSTRATION OF PHASE FOUR DECK SET-UP
$  K2PP AND NONLINEAR REFERENCES NOT REQUIRED
$  FOR LINEAR PROBLEMS.
   K2PP = EXTRA
   NONLINEAR = 5
   DLOAD = 75
   TSTEP = 70
OUTPUT
   SET 784 = 1 THRU 12,501 THRU 508
   SET 208 = 1 THRU 6
   SET 611 = 1 THRU 12
   SET 1800 = 117,159,170,182,197
   SET 960 = 960 THRU 969
     NLLOAD = 960
     SDISPLACEMENT = 784
     SVELOCITY = 208
     SACCELERATION = 611
     OLOAD = 1800
BEGIN BULK
$(BULK DATA DECK GOES HERE)
ENDDATA
(6-7-8-9)
```

Figure 8. Typical deck setup for Phase IV.

information required consists solely of a partition vector, DISCARD, required to identify the syste modes not employed in the transient analysis. M = number of system modes originally found Phase II. Excluded modes are identified by a corresponding element value of 1.

Because this phase employs a DMAP approach, no references to boundary conditions, etc., a needed. Titling and output requests are chosen as the user wishes for each substructure. Outp selections are for physical degrees of freedom and associated functions. A sample deck setup is sho in Figure 10.

NASTRAN MODE SYNTHESIS SAMPLE PROBLEM

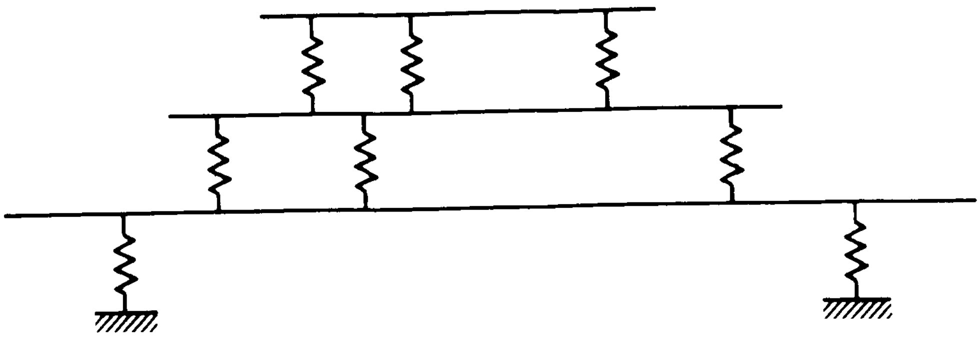
The mathematical model of Figure 11 has been developed as an aid to demonstrating the procedu associated with NASTRAN mode synthesis. While the system is small, it does provide the opportur

```
$  PHASE FIVE DMAP SEQUENCE
BEGIN    $
PARAM     //C,N,SUB/V,N,MARK/V,Y,COMP=1/C,N,1   $
INPUTT1    /,,,,/C,N,-1/V,Y,INP1=1   $
INPUTT1    /SUBMODE,,,,/V,N,MARK/V,Y,INP1   $
PARTN     SUBMODE,  DISCARD,  /MODEMAT,,,/C,N,1/C,N,2/C,N,2   $
INPUTT1    /UHVTX,PPT,,,/C,N,-5/V,Y,INP1   $
MPYAD     MODEMAT,UHVTX,/UDV1T/C,N,0/C,N,1/C,N,1/C,N,1   $
CASE      CASECC,/CASEXX/C,N,TRAN/V,N,REPEAT=-1/V,N,NOLOOP $
DPD      DYNAMICS,GPL,SIL,USET/GPLD,SILD,USETD,,,,,,,,EQDYN/
             V,N,LUSET/V,N,LUSETD/V,N,NOTFL/V,N,NODLT/V,N,NOPSDL/
             V,N,NOFRL/V,N,NONLFT/V,N,NOTRL/V,N,NOEED/
             C,N,0/V,N,NOUE   $
VEC       USETD/VD/C,N,P/C,N,D/C,N,COMP   $
PARTN     UDV1T,,VD/UDSET,,,/C,N,1/C,N,2/C,N,2   $
SDR1      USETD,,UDSET,,,GO,GM,,KFS,,/U,,QP/C,N,1/C,N,DYNAMICS    $
SDR2      CASEXX,CSTM,MPT,DIT,EQDYN,SILD,,,BGPDT,PPT,QP,UDV1T,EST,XYCDB/
              OP,OQ,OU,OS,OF,PUGV/C,N,TRANRESP   $
CHKPNT     PUGV   $
SDR3     OP,OQ,OU,OS,OF,/OP2,OQ2,OU2,OS2,OF2,$
OFP       OU2,OP2,OQ2,OF2,OS2,//V,N,CARDNO $
END    $
```

Figure 9. Phase V DMAP sequence.

```
REQUEST(OPTP,HD,S,VSN=YYYYY)
REQUEST(INP1,HD,S,VSN=PPPPP)
REWIND(OPTP,INP1)
ATTACH(N15,NASTRAN15,ID=TONE90970)
MAP(OFF)
N15.ATTACH
RETURN(OPTP,INP1)
EXIT.
DMP(140000)
RETURN(OPTP,INP1)
(7-8-9)
ID    PHASE,   FIVE
APP       DMAP
TIME      2
$(PHASE FIVE DMAP SEQUENCE GOES HERE)
$(PHASE ONE RESTART DICTIONARY GOES HERE)
CEND
TITLE=DEMONSTRATION OF PHASE FIVE DECK SET-UP
OUTPUT
   SET 23 = 1,3,26,18,12 THRU 16
   SET 38 = 38,24,36
   SET 90 = 12,18,24,36,60,66
   SET 91 = 12,18,24,36,60
   SET 92 = 12,18,24,36
   SET 187 = 122,132
     DISPLACEMENT = 90
     VELOCITY = 91
     ACCELERATION = 92
     SPCFORCES = 187
     ELSTRESS = 23
     ELFORCE = 38
BEGIN BULK
$(BULK DATA DECK ADDITIONS GO  HERE)
ENDDATA
(6-7-8-9)
```

Figure 10. Typical deck setup for Phase V.



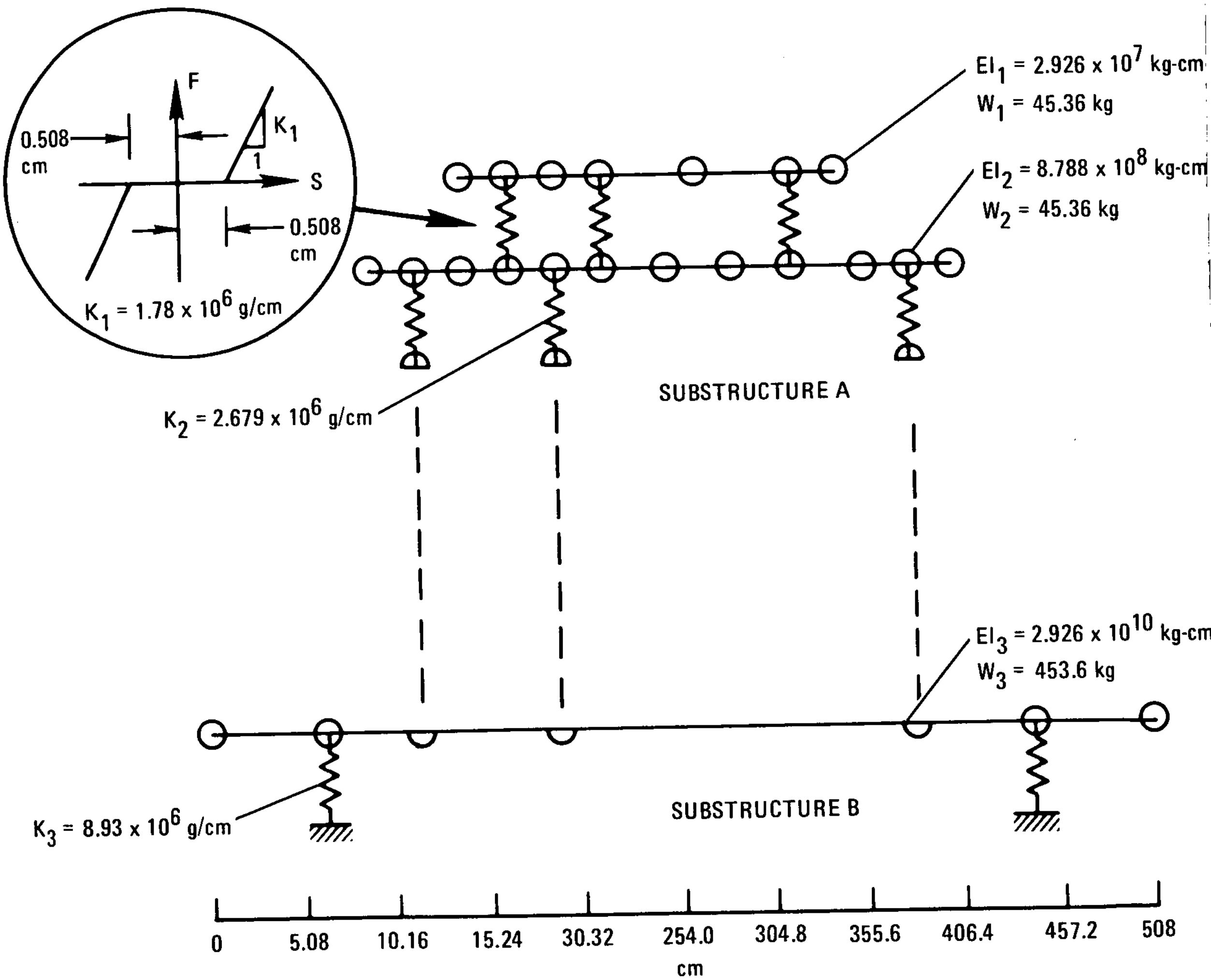


Figure 11. Sample problem mathematical model.

to demonstrate the major flexibilities of the technique. The problem exhibits a redundant interfa both free-free and constrained component modes, and nonlinear elements.

The system to be analyzed consists of three uniform elastic beams in parallel, connected spring elements. The springs connecting the upper beam to the middle one are nonlinear and exhil dead zone about the null deflection point. Excitation is provided by a sinusoidal force acting vertically at the end of the lowest beam.

For demonstration purposes, the structure is modeled as two components (see Figure 11). The first, identified as substructure A, consists of the upper and middle beams and the springs between middle beam and the lower beam. The second component, substructure B, is composed of the lower beam and its support springs. The component modes for substructure A are developed for the linear portions of the structure only – the nonlinear portions are added when the transient studies are performed.

The first user judgment comes immediately when he must decide which component modes to use in the synthesis. For the transient response study to follow, the 30 Hz sinusoidal applied force suggests the desirability of good system definition up to about 100 Hz. Based on this objective, a nominal cutoff of 300 Hz for component modes was chosen. For substructure A this presents no problem – the 14 lowest frequency modes plus the three static modes are selected. For substructure B, however, this criterion conflicts with the distribution of modal frequencies. Five vibration modes exist for substructure B that have frequencies under 300 Hz. Adding the three static modes gives a total of eight. However, the substructure only has seven independent degrees of freedom. The solution of the conflict was chosen to be a violation of the 300 Hz component mode criterion. For demonstration purposes, still another mode was deleted; thus the three static modes and three dynamic modes with a highest frequency of 33.7 remain. The resulting system modal frequencies did not show significant error until the 11th elastic mode (70 Hz), where just under 1% error in frequency is observed. However, some degradation of response accuracy results from this compromise modeling technique.

The nonlinear elements between upper and middle beams are integrated with the elastic structure definition by addition of extra points and use of a direct input matrix.

Comparison of the results obtained via mode synthesis with those acquired from the standard modal transient study indicates excellent correlation for component A (see Table 1), with errors limited to under 1% for displacements and accelerations and under 3% for internal loads. The effect of the severe truncation of the substructure B mode set is apparent when comparing results with the standard study (Table 1). Displacement and acceleration values are quite good, with errors of less than 1%, even though the error terms are substantially larger than those for substructure A. Bending moments and shears for the selected beam element are in error by 10.7%, because the limited number of elastic modes used in the study was inadequate to give good representation. For the more general case, with a great many more degrees of freedom in the substructure, it will be possible to employ an adequate number of substructure modes to provide sufficient description of elastic behavior.

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Table 1. Summary of peak responses.

Response Quantity				
Grid or Element ID	**Quantity**	**Standard Analysis**	**Synthesized Analysis**	**Error %**
101	DISP	-4.29	-4.29	<0.01
201	DISP	-3.10	-3.10	<0.01
301	DISP	1.11	1.11	0.22
101	ACC	5.84×10^4	5.84×10^4	0.02
201	ACC	7.12×10^4	7.13×10^4	0.06
301	ACC	1.65×10^4	1.66×10^4	0.59
51	FORCE	1.43×10^4	1.44×10^4	0.28
53	FORCE	4.58×10^4	4.58×10^4	0.02
3	$(BM)_A$	1.49×10^5	1.48×10^5	0.29
3	$(BM)_B$	-6.60×10^4	-6.60×10^4	0.06
3	SHEAR	1.16×10^4	1.15×10^4	0.8
10	$(BM)_A$	4.10×10^5	3.97×10^5	2.95
10	$(BM)_B$	4.58×10^5	4.72×10^5	2.95
10	SHEAR	6.65×10^3	6.78×10^3	1.9
31	$(BM)_A$	0	0	-
31	$(BM)_B$	1.31×10^6	1.17×10^6	10.
31	SHEAR	-5.25×10^4	-4.69×10^4	10.