THE DYNAMIC ANALYSIS OF SUBMERGED STRUCTURES

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SUMMARY

Methods are described by which the dynamic interaction of structures with surrounding fluids can be computed by using finite element techniques. In all cases, the fluid is assumed to behave as an acoustic medium and is initially stationary. Such problems can be solved either by explicitly modeling the fluid (using pressure or displacement as the basic fluid unknown) or by using decoupling approximations which take account of the fluid effects without actually modeling the fluid.

INTRODUCTION

Recently there has been a growing interest in solving problems in which structures interact with fluids. Within the Navy, for example, application areas include underwater vibrations, ship silencing, shock response of ships, underside slamming, flow-induced vibrations, and the motions of liquids and gases in containers. The last two problem areas are also of interest to the aerospace community.

Concern here is with the dynamic structural response of submerged structures, including the determination of natural frequencies and the response of submerged structures to sinusoidal and general transient excitations. In the latter category, a very important application is the shock response of submarines to underwater explosions.

In all problems considered here, the structure is assumed to be initially at rest with respect to the fluid; i.e., there is no fluid flow initially. Also, the fluid is assumed to behave as an acoustic medium in which each material point undergoes only infinitesimal displacement from the static equilibrium position.

Such problems were apparently first formulated by using finite elements by Zienkiewicz and Newton (refs. 1 and 2). Choosing pressure as the basic fluid unknown, they applied a variational process to obtain the finite-element equations for the fluid. These were then coupled to the usual structural

equations in a consistent manner. The application of Zienkiewicz's approach with NASTRAN (refs. 3 and 4) is straightforward but requires program modifications. In fact, a similar approach was adopted in the development of the NASTRAN hydroelastic analyzer (refs. 4 and 5). This latter capability is restricted to the case of fluids contained in axisymmetric containers, although the motions of the coupled fluid-structure system need not be axisymmetric.

Another approach to the interaction problem uses fluid displacements (rather than pressures) as the fundamental unknowns (ref. 6). In brief, this approach converts standard NASTRAN elements into "mock" fluid elements by defining (in three dimensions) the 6 x 6 material matrix $\underline{G}$ to be

$$\underline{G} = k \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{1}$$

In equation (1), the bulk modulus k is given by

$$k = \rho c^2 \tag{2}$$

where ρ and c are the unperturbed mass density and sonic speed, respectively, of the fluid. To the NASTRAN user, Kalinowski's displacement approach (ref. 6) has the advantages of using a standard version of the program and of generating symmetric matrices (in contrast to Zienkiewicz's approach which generates nonsymmetric matrices). However, Kalinowski's method has the disadvantage of requiring three unknowns per fluid point rather than one (pressure). This penalty affects adversely both the order and the bandwidth of the assembled matrices.

The purpose of the remainder of this paper is to describe two additional methods for solving fluid-structure interaction problems. Both use only standard versions of NASTRAN.

A PRESSURE ANALOG METHOD

All approaches described in this paper treat the fluid as an acoustic medium: a compressible, inviscid fluid which is initially stationary and undergoes only small amplitude motion, and whose pressure satisfies the wave equation

$$\nabla^2 p = \ddot{p}/c^2 \tag{3}$$

where overdots denote partial differentiation with respect to the time t. The boundary condition at the fluid-structure interface can be obtained from momentum and continuity considerations:

$$\frac{\partial p}{\partial n} = -\rho \ddot{u}_n \tag{4}$$

where n is the unit outward normal from the solid at the fluid-solid interface, and ρ is the fluid mass density. A special case of equation (4) occurs at rigid walls where

$$\frac{\partial p}{\partial n} = 0 \tag{5}$$

At a free surface, in the absence of surface waves, the boundary condition is simply

$$p = 0 \tag{6}$$

The goals here are to solve fluid-structure interaction problems in which the fluid is described by equations (3) to (6) and to use only standard capabilities available in structural analysis codes such as NASTRAN.

In classical elasticity theory (ref. 7), the x component of the momentum equation is

$$\sigma_{xx,x} + \sigma_{xy,y} + \sigma_{xz,z} + \rho f_x = \rho \ddot{u} \tag{7}$$

where ρ is the mass density of the (solid) medium, u is the x component of displacement, f_x is the x component of body force, and the partial differentiation with respect to a coordinate is abbreviated with commas:

$$(\ )_{,x} \equiv \frac{\partial}{\partial x}(\ ) \tag{8}$$

In programs such as NASTRAN, the stress and strain components are generally described with vector notation as follows:

$$\underline{\sigma} = \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix}, \quad \underline{\varepsilon} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} u_{,x} \\ v_{,y} \\ w_{,z} \\ u_{,y} + v_{,x} \\ v_{,z} + w_{,y} \\ w_{,x} + u_{,z} \end{Bmatrix} \tag{9}$$

where u, v, and w are the three Cartesian displacement components. Thus, for linear materials, Hooke's law can be written in terms of a systematic 6 x 6 material matrix $\underline{G}$, where

$$\underline{\sigma} = \underline{G}\,\underline{\varepsilon} \tag{10}$$

By substituting the strain definitions (9) into (10) and (10) into (7),

the wave equation (3) can be obtained if

$$u = p \tag{11}$$

$$v \equiv w \equiv 0 \tag{12}$$

$$\left.\begin{aligned} G_{14} = G_{16} = G_{46} = 0 \\ G_{11} = G_{44} = G_{66} = \rho c^2 \end{aligned}\right\} \tag{13}$$

$$f_x = 0 \tag{14}$$

In other words, the x component of displacement u can represent fluid pressure p if v and w are fixed everywhere in the fluid, if no body forces are applied, and if six of the material constants are as prescribed in equation (13). Although the other 15 constants can be arbitrarily chosen, it is convenient to choose them so that $\underline{G}$ is invariant under a coordinate system rotation (i.e., $\underline{G}$ is isotropic). A necessary and sufficient condition for $\underline{G}$ to be isotropic is that it have the general form

$$\underline{G} = \begin{bmatrix} \lambda+2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda+2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix} \tag{15}$$

where λ and μ are the Lamé constants (ref. 7). The only $\underline{G}$ matrix satisfying both equations (15) and (13) is

$$\underline{G} = \rho c^2 \begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 \\ -1 & 1 & -1 & 0 & 0 & 0 \\ -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{16}$$

With $\underline{G}$ isotropic, there is no need to be concerned about using finite elements whose material matrices are based on some local element coordinate system.

To summarize, the fluid is modeled with standard elastic finite elements (e.g., QDMEM in 2-D and general solids in 3-D) having material properties given by equation (16), where fluid pressure is represented by the x displacement u.

Boundary Conditions

Interface Condition

The boundary condition that the pressure p must satisfy at a fluid-solid interface is given by equation (4). To evaluate the left-hand side of equation (4), compute the directional derivative of p in the direction of the unit outward normal $\underline{\nu}$ from the fluid at a surface point. Replacing p with its structural analog u yields

$$\frac{\partial u}{\partial \nu} = \underline{\nabla} u \cdot \underline{\nu} = u,_x \nu_x + u,_y \nu_y + u,_z \nu_z \tag{17}$$

By using the constitutive equations (10) and (16) with equation (17) yields

$$\frac{\partial u}{\partial \nu} = \frac{1}{\rho c^2} (\sigma_{xx} \nu_x + \sigma_{xy} \nu_y + \sigma_{xz} \nu_z) \tag{18}$$

where the parenthetical expression is equal to the x component of the stress vector $\underline{T}^{(\nu)}$ acting on a surface with unit outward normal $\underline{\nu}$ (ref. 7). Hence,

$$\frac{\partial u}{\partial \nu} = \frac{T_x^{(\nu)}}{\rho c^2} \tag{19}$$

If the surface is discretized by a finite number of grid points, the surface traction $T_x^{(\nu)}$ can be replaced by its lumped equivalent

$$T_x^{(\nu)} = \frac{F_x}{A} \tag{20}$$

where F_x is the x component of the force applied to a particular point (on the surface with outward unit normal $\underline{\nu}$) to which the area A has been assigned. Hence, from equations (19) and (20), the final expression for the directional derivative is obtained as

$$\frac{\partial u}{\partial \nu} = \frac{F_x}{\rho c^2 A} \tag{21}$$

Since all symbols in equation (21) refer to the fluid, including u which represents the pressure p, merely combine equation (21) with the boundary condition of equation (4) at the interface to obtain the lumped interface condition

$$F_x^p = (\rho c)^2 A \ddot{u}_n \tag{22}$$

in which $\underline{n} = -\underline{\nu}$ and the superscript p has been placed on F_x to emphasize that the "force" is applied to the pressure variable. That is, if the outward normal component of structural acceleration at a point is $\ddot{u}_n$, the effect on the fluid pressure is that of a "force" given by equation (22) applied to the fluid

variable p. In equation (22), ρ and c refer to the fluid.

Equation (22) provides the influence of the structural motion on the pressure. The inverse relationship (that of fluid pressure on the structural motion) is obtained merely by applying a normal force to the structural point equal to pA.

The final set of matrix equations takes the form

$$\underline{M}\ddot{\underline{U}} + \underline{K}\underline{U} = \underline{F} \tag{23}$$

where, with the subscripts s and f denoting solid and fluid,

$$\underline{U} = \begin{Bmatrix} \underline{u} \\ \underline{p} \end{Bmatrix} \quad \underline{M} = \begin{bmatrix} \underline{M}_{ss} & \underline{0} \\ \underline{M}_{fs} & \underline{M}_{ff} \end{bmatrix} \quad \underline{K} = \begin{bmatrix} \underline{K}_{ss} & \underline{K}_{sf} \\ \underline{0} & \underline{K}_{ff} \end{bmatrix} \tag{24}$$

In equation (23), the vector of unknowns $\underline{U}$ includes both the structural displacements and the fluid pressures. In equation (24), $\underline{M}_{fs}$ is a matrix which is assembled by placing the term $-(\rho c)^2 A$ in the row corresponding to each interface p variable and the column corresponding to the associated structural normal displacement. Similarly, $\underline{K}_{sf}$ is a matrix which is assembled by placing +A in the row corresponding to each surface normal displacement and the column corresponding to the associated p. In NASTRAN, both $\underline{M}_{fs}$ and $\underline{K}_{fs}$ can be inserted by using direct matrix input (DMIG cards).

Infinite Fluids

The foregoing derivations apply directly to a wide variety of fluid-solid systems of finite extent. For structures in "infinite" fluids, interface disturbances travel far without reflection so that radiation damping may be significant. Although one should generally consider other approaches to solve such problems, approximations involving the current method can also be derived.

The approach here is to truncate the fluid model at a distance "sufficiently far" from the structure and apply the radiation condition (ref. 8) there:

$$\frac{\partial p}{\partial n} = -\frac{1}{c}\frac{\partial p}{\partial t} \tag{25}$$

This condition has also been referred to as the nonreflecting boundary condition (ref. 1) or the wave-absorbing boundary condition (refs. 6 and 9).

Equation (25) can be applied with NASTRAN by combining with equation (21), where again the pressure p is replaced by its structural analog u. Hence, the radiation condition becomes

$$F_x^p = - \rho c A \dot{p} \tag{26}$$

In other words, a grounded scalar dashpot with damping constant ρcA should be connected to each p variable.

Numerical Example

These concepts can be illustrated by considering the two-dimensional problem of a steel ring vibrating in water (ref. 10). The ring has a radius of ten inches and a radial thickness of one inch. The inside is evacuated. The fluid region is a circular annulus with an outer radius of 32.6 inches. The outer surface is a "free" surface (where p=0). The problem was solved (ref. 10) by modeling with NASTRAN a 90° sector comprised of eight BAR elements representing the ring and an 8x6 mesh of QDMEM elements representing the fluid. Typical results for the in-fluid natural frequencies (in radians/second) are as follows:

Fourier Harmonic	NASTRAN frequency	Analytical frequency
0	4333.	4364.
2	231.6	226.1
4	1486.	1432.
6	3884.	3724.

The analytical results were derived by Schroeder and Marcus (ref. 11) based on a method developed by Junger (ref. 12). Calculations reported in Ref. 10 indicate that grid refinement does result in convergence to the analytical results.

DECOUPLING APPROXIMATIONS

For problems in which a structure interacts with a fluid of infinite extent, it may be expensive to model enough fluid to produce satisfactory results, even if the radiation condition (eqs. (25) and (26)) is applied at some arbitrary outer surface. Since the interest is in the structure rather than in the fluid, various schemes can also be used that approximate the fluid effects without actually modeling the fluid (refs. 13 and 14). These schemes, generally referred to as decoupling approximations, result in an analytical expression (in differential equation form) describing the relationship between the fluid pressure at the interface and the interface motion.

For certain dynamics problems in which the source of the disturbance is in the fluid rather than in the structure (e.g., underwater explosions), the decoupling approximation supplies only the radiation pressure. However, the

other contributors to the total dynamic pressure (the incident free field pressure and the scattered pressure) can be computed as if the structure were rigid and stationary and thus comprise the usual right-hand-side forcing function.

One decoupling approximation which has been used with NASTRAN was developed (but not yet published) by Dr. Hansen Huang of the Naval Research Laboratory. For axisymmetric cylinders modeled with conical shell elements, the interface pressure-motion equation is approximated by

$$\dot{p}_n + (cn/r)p_n = \rho c \ddot{w}_n \tag{27}$$

where p_n is the n^{th} Fourier harmonic of the radiation pressure

c is the speed of sound in fluid

ρ is the mass density of fluid

r is the radius of cylinder

w_n is the n^{th} Fourier harmonic of the outward normal component of shell displacement

Equation (27) is equivalent to the so-called doubly asymptotic approximation of Geers (ref. 13), which was formulated for more general three-dimensional situations.

To illustrate how approximations such as equation (27) are applied with NASTRAN, consider a cylindrical shell subjected to an underwater shock loading (ref. 15). In the absence of radiation pressure p, the usual transient matrix equation would apply

$$\underline{m}\ddot{\underline{u}} + \underline{k}\underline{u} = \underline{f}(t) \tag{28}$$

where the forcing function f consists of the sum of the incident and scattered pressures. The radiation pressure, which depends on the shell motion, supplies an additional load on the structure so that the equilibrium equations for the structure, from equation (28), become

$$\underline{m}\ddot{\underline{u}} + \underline{k}\underline{u} + \underline{h}\underline{p} = \underline{f}(t) \tag{29}$$

where $\underline{p}$ is the vector of radiation pressures at each point, and $\underline{h}$ is a matrix of area factors converting pressure to force.

Because of the mutual dependence of the shell motion and the radiation pressure on each other, equations (27) and (29) must be solved simultaneously. This solution is accomplished in NASTRAN by defining a new set of scalar degree of freedom representing the radiation pressure at each surface point. The set of all such pressures is grouped into the vector $\underline{p}$. Thus, to NASTRAN, the vector of unknowns (displacements and pressures) is

$$\underline{U} = \begin{Bmatrix} \underline{u} \\ \underline{p} \end{Bmatrix} \tag{30}$$

After equation (27) has been written at each point and combined with equation (29), the resulting matrix equation which NASTRAN integrates is of the usual form

$$\underline{M}\ddot{\underline{U}} + \underline{B}\dot{\underline{U}} + \underline{K}\underline{U} = \underline{F}(t) \tag{31}$$

The contributions to $\underline{M}$, $\underline{B}$, and $\underline{K}$ corresponding to the $\underline{u}$ variables are computed directly by NASTRAN based on geometry and material properties. The contributions to $\underline{M}$, $\underline{B}$, and $\underline{K}$ corresponding to the radiation pressure variables ($\underline{p}$) are the coefficients appearing in equation (27). Hence, they are computed externally to NASTRAN and input directly (using the TF bulk data card) into the program. Thus, the matrices $\underline{M}$, $\underline{B}$, $\underline{K}$, and $\underline{F}$ in equation (31) take the unsymmetric form

$$M = \begin{bmatrix} \underline{m} & 0 \\ \underline{d} & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ 0 & \underline{a} \end{bmatrix} \quad K = \begin{bmatrix} \underline{k} & \underline{h} \\ 0 & \underline{b} \end{bmatrix} \quad F = \begin{Bmatrix} \underline{f}(t) \\ 0 \end{Bmatrix} \tag{32}$$

NASTRAN integrates equations (32) by using the Newmark-Beta finite-difference algorithm, which is unconditionally stable if equation (32) is stable.

Unfortunately, specific numerical results are not yet available for general publication. However, early success with decoupling approximations has convinced the authors that such approaches have great promise as an alternative to the explicit finite element modeling of fluids of "infinite" extent.

GENERAL REMARKS

Two methods by which NASTRAN can be applied without modification to the solution of fluid-structure interaction problems have been described.

The first, the structural analog, is the "lumped" equivalent of the consistent formulation described by Zienkiewicz and Newton (refs. 1 and 2). However, the lumped approach has the advantage for the NASTRAN user that it can be applied without program modification. Otherwise, the two pressure formulations have similar characteristics, including nonsymmetric matrices. This is in sharp contrast to displacement approaches (ref. 6) which assemble symmetric matrices, but at the expense of having three times the number of fluid unknowns.

It should be emphasized that although the analog method was developed specifically for an acoustic fluid, the same technique could be applied to a wide variety of problems in mathematical physics, including heat conduction (ref. 16), the Helmholtz equation, electrical or magnetic potential problems, the torsion of prismatic bars, potential fluid flow, or seepage through porous media. In the present context of fluid-structure interaction problems, the calculation of added mass matrices can be accomplished directly, since it

involves solving Poisson's equation in the fluid region where the source terms occur only at the fluid-structure interface. Such equations can also be solved with standard three-dimensional steady-state heat conduction codes such as those contained in NASTRAN Level 15.5 (ref. 4), the Navy's thermo-structural NASTRAN (refs, 17 and 18), or the CINDA thermal analysis program (ref. 19).

For infinite media, the explicit modeling of the fluid is often uneconomical compared with some of the competitive methods. For fluid-structure interaction problems, the use of decoupling approximations provides an attractive alternative. For other general field problems (e.g., potential flow), integral equation techniques are widely used (ref. 20).

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