RIGID FORMAT ALTER PACKETS FOR THE ANALYSIS OF ELECTROMAGNETIC FIELD PROBLEMS

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SUMMARY

NASTRAN has been used to solve two types of electromagnetic field problems. The diffusion equation and the boundary conditions valid for problems of these kinds together with a replacing potential energy function have been given. The extent to which an analogy with finite element displacement and temperature approaches holds is indicated. The outputting of complex quantities is made possible after adjustment of standard rigid format 1 input data blocks to module SDR2. The applications made involve the study of the proximity effect in a system of three parallel conductors and the analysis of the magnetic field in the vicinity of the points of contact in circuit breakers.

INTRODUCTION

Obviously the knowledge of electromagnetic field distributions is of prime interest in Electric Power Engineering.
Transient magnetic fields present in conducting materials cause the introduction of eddy currents in such materials. Energy losses leading to temperature increases result from these currents. The most familiar application of magnetic fields is in transformers where they determine the modification of current and voltage levels. They have also proven to determine the design of advanced types of power generators such as fusion reactors and MHD generators.

These and many other reasons explain why the analysis of electromagnetic problems is as important as it is. After finite difference techniques were used for these problems originally we now observe more and more interest in finite element methods. Among the applications of finite element techniques the works of Chari and Silvester (Ref.|1|), Chari (Ref.|2|), Silvester and Rafinejad (Ref.|3|) and Donea et al (Ref.|4|) have to be mentioned.

Rather than writing a stand-alone finite element code for the exclusive study of electromagnetic field problems it was felt worthwhile to investigate the applicability of the NASTRAN system.

When studying the equations and boundary conditions describing the problems involved it shows out there are many similarities with the ones NASTRAN is designed to deal with. The equation of balance valid here is identical to the equation of heat balance in solids except for the unknown quantity which is a vectorial instead of a scalar. When considering harmonic oscillations the transient term in the equation of balance may be transformed to a stationary one and a diffusion equation is obtained. In case the unknown field was a scalar quantity the solution to the magnetic field problem would be similar to the one valid for one-group neutron diffusion.

A NASTRAN solution to the neutron problem is reported in Ref. |5|. Electromagnetic field problems discussed in the present work mean a class of other nonstructural applications of NASTRAN and the approach applied here in fact means an extension of the research discussed in this reference.

FORMULATION OF THE ELECTROMAGNETIC FIELD EQUATIONS

Electromagnetic problems are described by Maxwell's equations. Without going into details these relations will be restated here for completeness.

Placing an electrically conducting body in a harmonically oscillating magnetic field $\overline{B}$ will cause eddy currents to be introduced in it. The relation between $\overline{B}$ and the electric field $\overline{E}$ introduced is given by Faraday's law of induction:

$$\nabla \times \overline{E} = - \frac{\partial \overline{B}}{\partial t} \tag{1}$$

Current density $\overline{J}$ is obtained from Ohm's law:

$$\overline{J} = \sigma \overline{E} \tag{2}$$

where σ denotes the conductivity.

These currents generate a magnetic field which interferes with the original one. When restricting ourselves to slowly changing systems ($f \leq 100$ Hz) we may use the relation:

$$\nabla \times \overline{B} = \mu \overline{J} \tag{3}$$

where μ stands for the permeability of the conductor.

We introduce the concepts of the magnetic vector potential $\overline{A}$ which is related to $\overline{B}$ by:

$$\overline{B} = \nabla \times \overline{A} \tag{4}$$

and the one of the electric potential ψ defined by:

$$\bar{E}_{el} = - \nabla\psi \tag{5}$$

$\bar{E}_{el}$ indicates the electric field present when magnetic influences are omitted.
By definition

$$\nabla . \bar{A} = 0 \tag{6}$$

Moreover

$$\bar{J} = - \sigma \frac{\partial \bar{A}}{\partial t} + \bar{J}_e \tag{7}$$

where $\bar{J}_e$ represents the current caused by the electric potential.
Equations (1) ... (5) can be combined to obtain the diffusion equation

$$\nabla^2 \bar{A} = \mu\sigma \frac{\partial \bar{A}}{\partial t} - \mu \bar{J}_e \tag{8}$$

Since we are dealing with harmonic systems it is favourable to replace $\bar{A}$ by $\bar{A}e^{j\omega t}$ and $\bar{J}_e$ by $\bar{J}_e e^{j\omega t}$, with j standing for operator $\sqrt{-1}$. This will make Eq. (8) read:

$$\nabla^2 \bar{A} - j\omega\mu\sigma\bar{A} + \mu\bar{J}_e = 0 \tag{9}$$

which describes a stationary rather than a transient state.
However we have to pay a penalty since we are now dealing with complex quantity $\bar{A}$.
The boundary conditions valid in most cases are reflective ($\frac{\partial \bar{A}}{\partial n} = 0$) or consist of prescribed values of $\bar{A}$. For certain types of problems the location of the boundary may not be clearly established and is in fact situated at infinite distance from the area of interest. Moreover we have to meet the requirement that eddy currents should not give any contribution to the total current. This means the eddy current density integrated over the volume V vanishes, i.e.

$$\int_V \bar{A} \, dV = 0 \tag{10}$$

The solution of Eq. (9) is obtained by solving the equivalent variational problem, i.e. by imposing the requirement of the relevant energy functional P to be stationary.

$$P = \iiint_V \left(\tfrac{1}{2} \frac{\partial \bar{A}}{\partial x_i} \frac{\partial \bar{A}}{\partial x_i} - \frac{j\omega\mu\sigma}{2} \bar{A}^2\right) dV + \iiint_V \mu \bar{J}_e \bar{A} \, dV = \text{Stationary} \tag{11}$$

When contrasting the first term of this functional with the thermal potential function it shows out to the neglect of a constant factor this term contains

the thermal potential function. Moreover when omitting a constant factor the capacitive energy function is implied in it.
This indicates electromagnetic field problems may be computed using finite element heat conduction computer programs at least modified in such a sense th the heat capacity matrix is added to the heat conduction matrix. Alternatively finite element structural mechanics codes with provisions to account for anisotropic material properties may be used. Specification of particular material properties and addition of the mass (or damping) matrix to the stiffness matri is required in enabling the application of this alternative approach.

THREE PHASE BUS BAR SYSTEM

As a first example on how NASTRAN can be applied to deal with these problems we will discuss the interaction of electromagnetic fields in a system of three parallel conductors carrying alternating currents with mutual phase lags equal to $\frac{2\pi}{3}$.
For this type of problem the following assumptions are valid:

1) The magnetic vector potential and the source current densities in the bars have only components in the longitudinal direction of the system. They are invariant with this direction and vary sinusoidally with time, so the problem is essentially two-dimensional.

2) The fields are assumed quasi-stationary, so that displacement currents may be neglected.

3) Hysteresis, magnetic saturation and temperature effects of resistivity are negligible.

Because of the first of these assumptions $\bar{A}$ and $\bar{J}$ may be considered to be scalars. At infinite distance around the system this quantity vanishes and in order to apply this boundary condition properly it seems desirable to make use of something like the apparently contradictory concept of infinite finite elements recently introduced by P. Bettes, University of Wales. The lack of this type of elements can be overcome by restricting ourselves to a usual finite mes (Fig. |1|) and diminishing of the results with a certain quantity, resulting fro Eq. (10).
The phase lagged volumetric quantity μJ_e present in Eq. (9) is composed of thre sets; one for each of the bars. Since the unknown is a scalar quantity and sinc in the displacement approach volumetric loading (GRAV bulk data cards) can neither be computed to obtain complex quantities nor be varied over subcases it is appropriate to use the NASTRAN heat conduction capability where we can use QVOL cards. Complex values of volumetric heat generation rate cannot be specified. This deficiency can be overcome by stepwise generation of the electromagnetic load in three different subcases and subsequent phase lagged superposition of the load vectors, making use of complex multiplication factors specified in the parameter section of DMAP module ADD.
The load vectors have to be computed separately instead of being appended. This can be accomplished by specification of different single point constraints

or each of the individual subcases. Therefore an SPOINT has been introduced ith three differing prescribed values.
n order to determine the solution at the right level the subsidiary condition Eq. 10) has to be applied. This means all elements of the solution vector $\{u_g\}$ ave to be subtracted by its mean value:

$$u_m = \frac{1}{S} \int \{u_g\} dS \tag{12}$$

here S stands for the cross-sectional surface of the conductors. This multipliation factor is computed from:

$$\frac{1}{\rho S} \{u_g\}^T |M_{gg}| \{I_g\} \tag{13}$$

ith $\{I_g\}$ denoting a vector of order g with all elements equal to 1.
ince SPR's 458 and 483 have not yet been corrected in Level 15.5 of NASTRAN odule ADD is unable to add complex input matrices. Consequently, at present the ubtraction mentioned has to be performed either by hand or in a stand-alone rogram. For the same reason the computation of the current density can not be one in NASTRAN.
pecial care has to be taken that complex quantities are output from module DR2. This requires reformatting of some of its input data blocks. In order to ccomplish this a dummy table of eigenvectors CLAMA has been specified on DTI ulk data cards. A dummy EIGC card was used in order to have IFP create a YNAMICS data block. Moreover modules CASE and DPD were introduced to provide ASEXX, EQDYN and SILD required to make SDR2 print complex eigenvectors. For the urpose of this application this means the printing of complex nodal vector potential values. The rigid format Alter resulting from these modifications is iven in Appendix I.

The range of parameters used for the computation of $\overline{A}$ reads as follows:

specific conductivity	$\sigma = 3.7_{10}{}^{7}$ Ohm^{-1} m^{-1}
permeability	$\mu = 4\pi_{10}{}^{-7}$ Henry m^{-1}
electrical current density	$J_e = 8.711$ A m^{-2}
angular frequency	$\omega = 314.16$ rad. sec^{-1}
RMS of total current through each conductor	$J = 1500$ A
cross-sectional surface of one conductor	$S = 1722_{10}{}^{-6}$ m^{2}

The distribution of the current density $\overline{J}$ derived from $\overline{A}$ demonstrates the large impact on the uniform distribution due to the presence of the magnetic field. This phenomenon is called the proximity effect.
For cross-sectional surface S and specific electric conductivity σ the heat production per unit length is computed from

$$Q = \iint_S \frac{1}{\sigma} |\overline{J}^2| \, dS \tag{14}$$

Obviously, for any fixed net current through the system, the minimum value of Q ($=Q_{min}$) is obtained when $\overline{J}$ is uniformly distributed.
The economy of the conducting system is determined by the ratio of Q over Q_{min}, which is called the resistance ratio. Q cannot be computed in the DMAP sequence since SPR's 458 and 483 prohibit the derivation of $\overline{J}$ (Eq. 7). A stand-alone calculation of Q is simplified when making use of

$$ADJ2 = \{u_g\}^T |M_{gg}| \{u_g\}, \tag{15}$$

referred to in the ALTER packet.
To the neglect of J_e the heat production Q^* can be computed from

$$Q^* = \{u_g^*\}^T |M_{gg}| \{u_g\} \tag{16}$$

with $\{u_g^*\}^T$ standing for the transposed complex conjugate of $\{u_g\}$.

$$Q^* = \sqrt{\{Re(ADJ2)\}^2 + \{Im(ADJ2)\}^2} \tag{17}$$

For the present configuration the resistance ratio was found to be 2.4.

MAGNETIC FIELDS IN CIRCUIT BREAKERS

One of the aspects to be taken into account in the design of circuit breakers is the phenomenon of electrical discharge.
The occurrence of this phenomenon depends on the amplitude and phase angle of the magnetic field present in the neighbourhood of the contact. The magnetic field is studied in an arrangement of two parallel circular disks situated at the center of a Helmholtz configuration (Fig. |2|). This assembly of two identical coils with mutual distance equal to their diameter is known for the uniformity of the magnetic field that can be generated in it.

In contrast to the first application where we determined $\overline{A}$, this time the interest is in curl $\overline{A}$ (Eq. 4) which is obtained from linear combinations of it derivatives.
There is an analogy with the derivation of stresses from displacements and sin the problem is essentially two-dimensional the choice for solid ring elements obvious. However according to SPR 933 complex stresses cannot be derived for these types of elements. That is why the use of three-dimensional elements may be considered.
$\overline{A}$ is no longer unidirectional and Eq. (9) in fact consists of two independent equations, one for each of the components of $\overline{A}$. Since no relation between thes components exists the equations can be solved in two subcases using the dis-

lacement approach. This requires that E = G and $\nu = 0$ such that the stress-train matrix transforms into the identity matrix multiplied by a constant. his stress-strain relation implies the material is anisotropic. It can be pecified on MAT1 bulk data cards while for solid elements the material axes o which it refers are the axes of the basic coordinate system.

For the first subcase degrees of freedom 2 thru 6 of all grid points are con-trained. This means the σ_x data of subcase 1 will denote $E\,\frac{\partial A_1}{\partial x}$, $\tau_{zx} = E\,\frac{\partial A_1}{\partial z}$ and $_{xy} = E\,\frac{\partial A_1}{\partial y}$. When in the second subcase constraining all degrees of freedom but he third at all grid points stress data σ_z contain $E\,\frac{\partial A_3}{\partial z}$, $\tau_{yz} = E\,\frac{\partial A_3}{\partial y}$ and $_{zx} = E\,\frac{\partial A_3}{\partial x}$. To the neglect of constant E the components of $\overline{B}$ (Eq. 4) can now e obtained. The first component of rot $\overline{A}$ is found to be τ_{yz} given in the second ubcase. The second component is represented by the τ_{zx} data obtained from a UBCOM in which SUBCASE 1 data are subtracted from SUBCASE 2. The third compo-ent is the negative value of τ_{xy} output for the first subcase.

nfortunately the SUBCOM results were erroneous which is presumably due to any f the SPR's mentioned before. As a consequence the subtraction of results has o be performed separately. Attempts to output complex stresses via OFP lead to 'AKUNPK errors when more than one subcase is involved. Consequently τ_{yz} and $_{xy}$ are obtained from a printout of table OES1.

The ALTER packet required for this class of applications is a simplification f the one given in Appendix I. The modifications made to it appear from the ollowing:

1) The FILE instruction is removed.
2) Since the displacement approach is used the SMA2 instruction is the one standard in rigid format 1.
3) The part related to multiplication and addition of load vectors is deleted.
4) The instructions dealing with the computation of quantities required for heat production calculation are no longer appropriate.
5) The standard rigid format 1 call for SDR2 may be used but OES1 has to be printed out.

The resulting packet is given in Appendix II.

CONCLUDING REMARKS

Electromagnetic field calculations mean an area of relatively new finite element applications. Since the development of a production program based on finite element techniques has shown out to be an extensive task, those inter-ested in these calculations are advised to verify to which extent they can reap the fruits of what has already been accomplished in structural analysis. The convenient way of programming offered in the DMAP language and the large number of available features such as checkpointing, direct matrix and table input, parameter operations and plotting facilities makes consideration of NASTRAN application worthwhile. Since many of the problems dealt with in electromagnet: field analysis are axisymmetric it is felt desirable to increase the 1.0 prior- ity status of SPR 933.

Moreover fixes of SPR's 458 and 483 will allow for proper execution of modules ADD, ADD5 and MPYAD when processing complex input matrices. Proper accounting the boundary conditions and the capability of determination of quantities deri from the magnetic vector potential will be enabled by these corrections.

It is hoped the present work demonstrates the usefulness of NASTRAN in this are of engineering and will lead to the increasing number of non-structural applica tions it deserves.

APPENDIX I

```
$         ************************************************************
$         *                                                          *
$         *     ALTER PACKET FOR THE COMPUTATION OF THE MAGNETIC     *
$         *     VECTOR POTENTIAL IN THREE-PHASE CONDUCTING SYSTEMS   *
$         *                                                          *
$         ************************************************************
$
ALTER 3
FILE        PG1=SAVE/PG2=SAVE $
ALTER 22,22
PARAM       //C,N,NOP/V,N,SKPMGG=1 $ GENERATE HEAT CAPACITY MATRIX
ALTER 35,35
SMA2        CSTM,MPT,ECPT,GPCT,DIT/,MGG/V,Y,WTMASS=1.0/V,N,NOMGG/
            V,N,NOBGG/C,N,-1 $
ALTER 44,44
ADD         MGG,KGGX/KGG1/C,Y,OMEGA=(0.0,-314.1592654) $
CHKPNT      KGG1 $ NOW MODIFY TO ENABLE PROCESSING OF KGG1
EQUIV       KGG1,KGG/NOGENL $ EXCLUDES USE OF GENERAL ELEMENTS
ALTER 47,47
SMA3        GEI,KGG1/KGG/V,N,LUSET/V,N,NOGENL/V,N,NOSIMP $
ALTER 89,89
PARAM       //C,N,NOP/V,N,COUNT=3 $ SET LOOP CONTROL COUNTER
DECOMP      KLL/LLL,ULL/C,N,1/C,N,0 /V,N,MINDIAG/V,N,DET/V,N,POWER/
            V,N,SING $          DECOMPOSITION BY RBMG2 REPLACED
SAVE        MINDIAG,DET,POWER,SING $
COND        HERROR3,SING $ TERMINATE COMPUTATION IF KLL IS SINGULAR
ALTER 97,97
$           BEGIN OF ALTER TO MPYADD LOAD VECTORS OBTAINED
$           FROM SUBCASES 1, 2 AND 3
PARAM       //C,N,ADD/V,N,NSKIP/V,N,NSKIP $
PARAM       //C,N,SUB/V,N,COUNT/V,N,COUNT $
PARAM       //C,N,SUB/V,N,CASE3/V,N,COUNT $
PARAM       //C,N,SUB/V,N,CASE2/V,N,CASE3 $
COND        LOAD3,CASE3 $
COND        LOAD2,CASE2 $
ADD         PG,/PG1 $
CHKPNT      PG1 $
JUMP        END3 $
LABEL       LOAD2 $
ADD         PG,/PG2 $
CHKPNT      PG2 $
JUMP        END3 $
```

```
LABEL          LOAD3 $
ADD5           PG,PG2,PG1,,/PSUM/C,Y,PHASE3=(-.5,-.8660254)/
                                   C,Y,PHASE2=(-.5, .8660254) $
CHKPNT         PSUM $
SETVAL         //V,N,REPEAT/C,N,-1 $
SAVE           REPEAT $
LABEL          END3 $
REPT           LBL7,2 $
$              END OF ALTER TO MPYADD LOAD VECTORS OBTAINED FROM
$              SUBCASES 1, 2 AND 3
EQUIV          PSUM,PL/NOSET $
ALTER 100,100
SSG2           USET,GM,YS,KFS,GO,DM,PSUM/QR,PO,PS,PL $
ALTER 103,111
$              SSG3 REPLACED BY FBS. LAST PARAMETER FOR CSP OUTPUT
FBS            LLL,ULL,PL/ULV/C,N,1/C,N,1/C,N,2/C,N,3 $
CHKPNT         ULV $
SDR1           USET,PG,ULV,,YS,GO,GM,PS,KFS,KSS,QR/UGV,PGG,QG/C,N,1/
               C,N,STATICS $
CHKPNT         PGG,UGV $
$              BUILD VECTOR WITH ALL COMPONENTS EQUAL TO 1
VEC            USET/FSET/C,N,G/C,N,COMP/C,N,L $
VEC            USET/SSET/C,N,G/C,N,L/C,N,COMP $
CHKPNT         SSET,FSET $
ADD            SSET,FSET/UNIT $
MPYAD          MGG,UGV,/TST/C,N,0 $
CHKPNT         UNIT,TST $
MPYAD          UNIT,TST,/ADJ1/C,N,1 $ QUANTITIES ADJ1 AND ADJ2 ENABLE
MPYAD          UGV ,TST,/ADJ2/C,N,1 $ COMPUTATION OF HEAT PRODUCTION
CHKPNT         ADJ1,ADJ2 $
MATPRN         ADJ1,ADJ2,,,// $
ALTER 119,119
$              ADJUST SDR2 INPUT TO ALLOW FOR COMPLEX UGV OUTPUT
CASE           CASECC,/CASEXX/C,N,CEIGEN/V,N,REPEAT $
DPD            DYNAMICS,GPL,SIL,USET/GPLD,SILD,USETD,,,,,,,,EQDYN/V,N,
               LUSET/V,N,LUSETD/V,N,NOTFL/V,N,NODLT/V,N,NOPSDL/V,N,
               NOFRL/V,N,NONFLT/V,N,NOTRL/V,N,NOEED/C,N,123/V,N,NOUE $
SDR2           CASEXX,CSTM,MPT,DIT,EQDYN,SILD,GPTT,EDT,BGPDT,CLAMA,QG,
               UGV ,EST,/OPG1,OQG1,OUGV1,OES1,OEF1,PUGV1/C,N,CEIG $
ALTER 137
LABEL          HERROR3 $
PRTPARM        //C,N,-3/C,N,HSTATICS $
ENDALTER
```

APPENDIX II

```
$         ***************************************************************
$         *                                                             *
$         *     ALTER PACKET FOR THE COMPUTATION OF MAGNETIC FIELDS     *
$         *                                                             *
$         ***************************************************************
$
ALTER 22,22
PARAM          // C,N,NOP/V,N,SKPMGG=1 $ GENERATE MASS MATRIX
ALTER 44,44
ADD            MGG,KGGX/KGG1/C,Y,OMEGA=(0.0,-314.1592654) $
CHKPNT         KGG1 $ NOW RENAME TO ENABLE PROCESSING OF KGG1
EQUIV          KGG1,KGG/NOGENL $ USAGE OF GENERAL ELEMENTS IS EXCLUDED
ALTER 47,47
SMA3           GEI,KGG1/KGG/V,N,LUSET/V,N,NOGENL/V,N,NOSIMP $
ALTER 89,89
DECOMP         KLL/LLL,ULL/C,N,1/C,N,0/V,N,MINDIAG/V,N,DET/V,N,POWER/
               V,N,SING $ REPLACE RBMG2 DECOMPOSITION, KLL IS COMPLEX
SAVE           MINDIAG,DET,POWER,SING$
COND           HERROR3,SING $ TERMINATE COMPUTATION IF KLL IS SINGULAR
ALTER 103,110
$              SSG3 REPLACED BY FBS, LAST PARAMETER FOR CSP OUTPUT
FBS            LLL,ULL,PL/ULV/C,N,1/C,N,1/C,N,2/C,N,3 $
CHKPNT         ULV $
SDR1           USET,PG,ULV,,YS,GO,GM,PS,KFS,KSS,QR/UGV,PGG,QG/V,N,
               NSKIP/C,N,STATICS $
ALTER 121,121
CHKPNT         OES1 $         COMPLEX STRESSES ARE IN OES1 BUT FORMAT
TABPT          OES1,,,,// $   IS IMPROPER FOR CORRECT OFP PROCESSING
ALTER 137
LABEL          HERROR3 $
PRTPARM        //C,N,-3/C,N,HSTATICS $
ENDALTER
```

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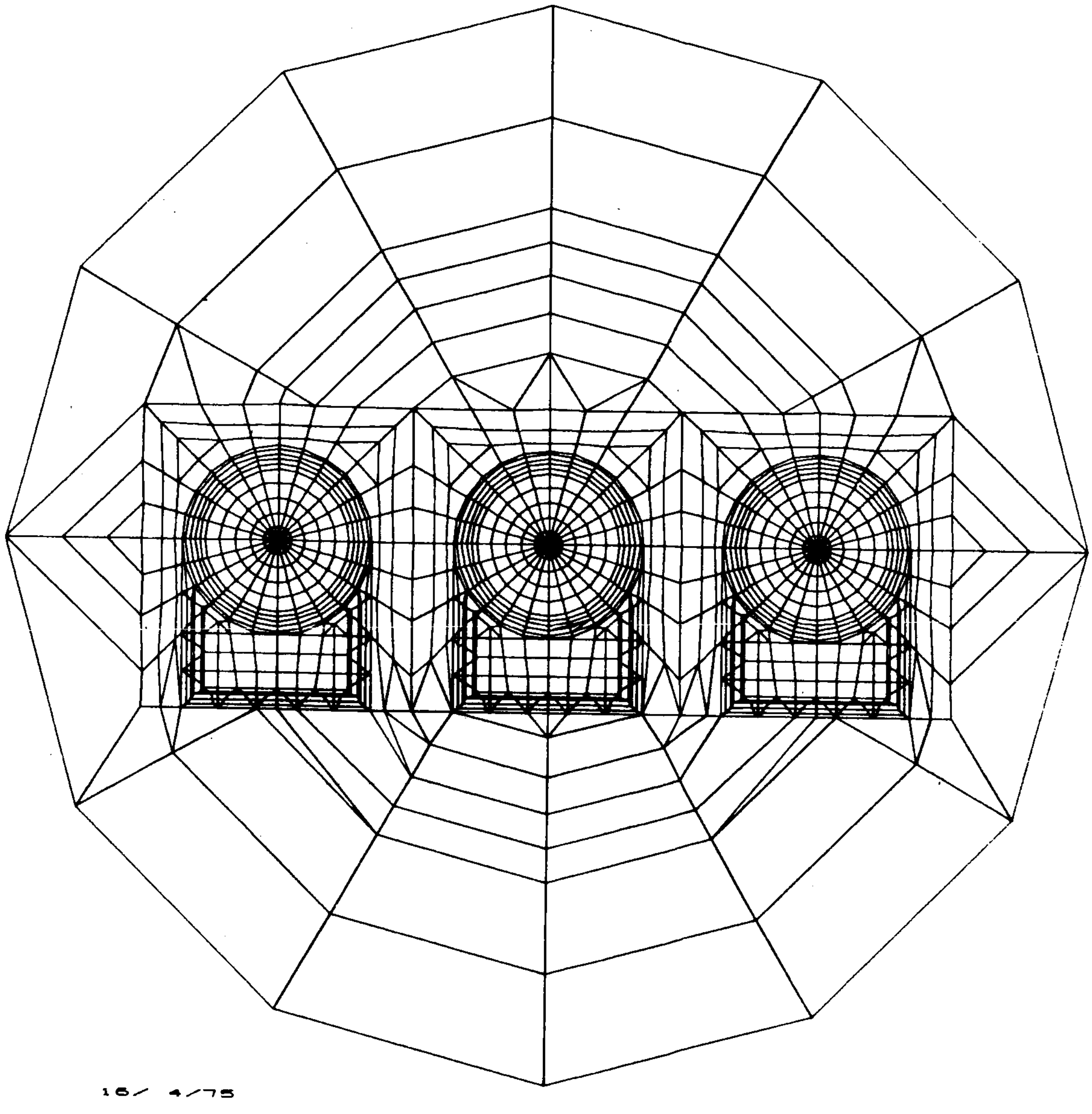

Fig. 1. Finite element model of bus bar geometry.

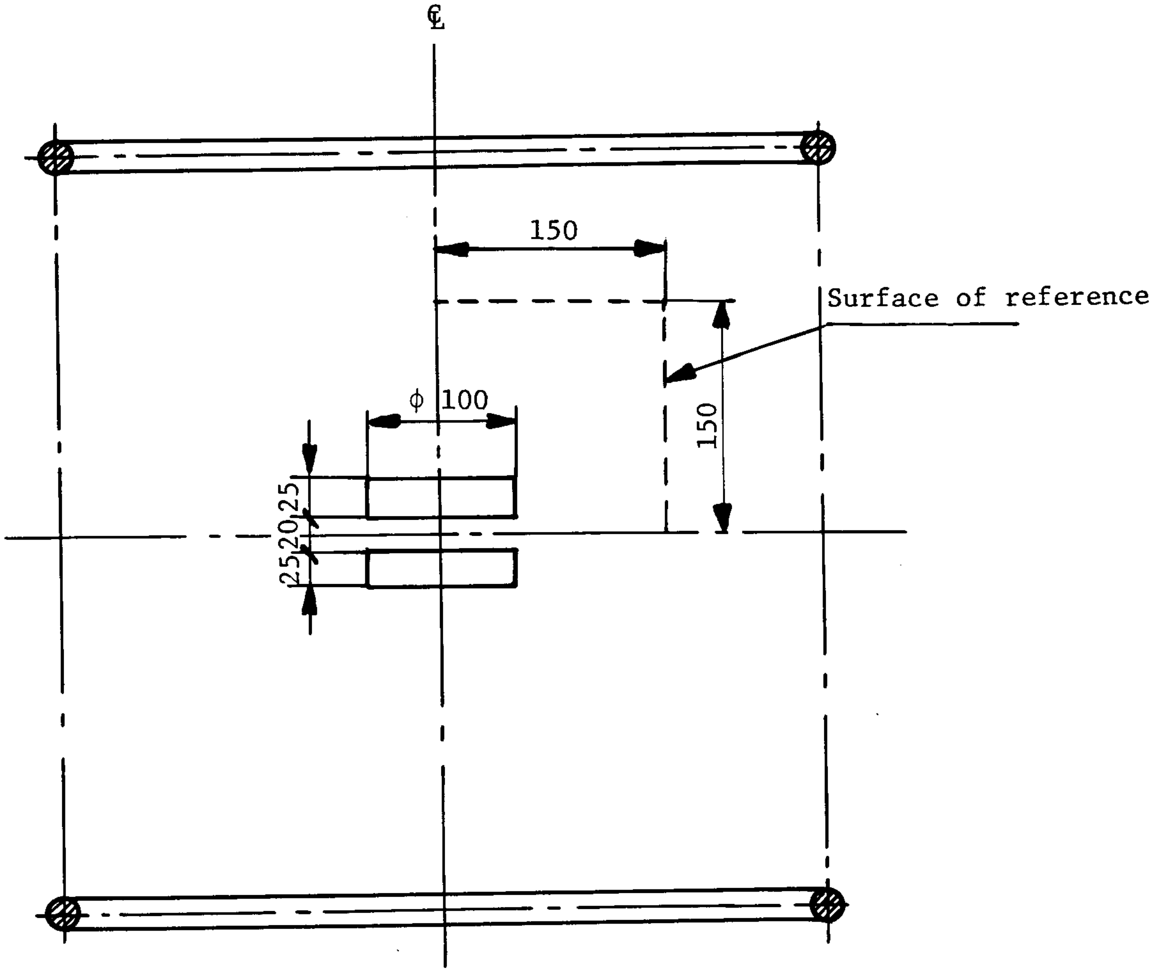

Fig. 2. Helmholtz configuration with two circular disks.