IN-FLIGHT DAMPING MEASUREMENT

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Abstract

This paper describes a new testing technique which can be applied in determining the damping coefficient of the critical vibration modes of an airplane in flight. The damping coefficient can be determined in several different ways from the same data using different features of a modified response curve which implies the possibility of checking one value against the other.

The method introduces the effect of sweep rate in the driving system. This effect on the frequency response curve of the critical vibration mode and its various characteristics are used in the determination of damping coefficient. A theoretical examination is made of these characteristics for single degree of freedom systems.

INTRODUCTION

The main objective of flight flutter tests is to demonstrate that an airplane is flutter safe in its designed range of speed and altitude. An airplane can be considered as flutter safe if all structural vibration modes exceed a minimum requirement in damping. The minimum requirement is a matter of experience and may be agreed upon between airframe manufacturer and customer. A certain safety margin from the critical speed must be observed. The airplane cannot be flown and tested at the critical speed unless artificial damping of predictable magnitude can be applied. This is one reason why flight test data cannot be immediately compared with data from flutter analysis which mainly deals with the critical speed or zero damping condition. A comparison is only possible with derived data. But even an indirect comparison is very useful in order to insure that the data from analysis are reliable. Before flight test, the various structural modes of an airplane are determined in a ground shake test where only structural damping is present. During flight, additional aerodynamic forces are present which vary with speed and altitude. They affect the frequency and damping of the modes.

In flight vibration tests, the various modes of vibration have to be excited by means of some controllable source of energy and the variation of the response with speed and altitude has to be measured.

The method of excitation and the method of evaluation of the response curves are closely related. There are different types of exciters:

Mechanical exciter with a rotating single out-of-balance weight or with a pair of out-of-balance weights coupled with each other in this way that one component of the force is cancelled. The balance weight can be preloaded by a spring in order to obtain a desired function of the exciting force versus frequency.

Aerodynamic exciter can be any flap in the free airstream placed in the proper position, e.g. any control surface or additional flaps. The real force or moment of excitation cannot be determined due to the interaction between exciter and airplane. This type of exciter may be mandatory if no place for a mechanical exciter is available.

By using a small explosive charge suitably located it is possible to excite transient response in all the various modes of vibration.

The mechanical and the aerodynamic exciters allow the application of sinusoidal input function with step by step variable frequency. The response function is the so-called "frequency response curve". The test procedure is to excite the system at a fixed and constant frequency until a steady-state amplitude is achieved. This procedure has to be repeated for each frequency and each flight condition. It is extremely time-consuming especially when the frequency interval has to be chosen very small in case of a response function with a high maximum response and a steep slope of the response function.

Both exciters can also be used for application of a variable input frequency. The input frequency function versus time may be described by a polynomial. The simplest polynomial is the straight line. It implies a new variable, the slope of the straight line or the "sweep rate" of the frequency variation. The sweep rate can be made proportional to the frequency, but this method does not give more information (Applied by H. G. S. Peacock, Gloster Aircraft Co., Reference 1).

Any variation of the input frequency makes the response function dependent on the time. We may call it a "time response curve" in order to distinguish it from the "frequency response curve" obtained by applying a constant input frequency.

The method with variable frequency excitation requires considerably less time than the method with constant driving frequency. The entire frequency range of interest can be covered in one sweep up and down for each flight condition.

The excitation with a short sharp impulse gives a transient response function followed by a decay. It is theoretically possible to excite transient response in all the various modes of vibration.

Common to all response functions obtained in flight test is the superimposition of the response to random input which tends to mask the response curve. It is impossible in flight test to avoid the random input. The different response functions are more or less sensitive with respect to random input. Especially sensitive is the transient response to a sharp impulse. The frequency spectrum of a sharp impulse covers theoretically a wide range of input frequencies which can be viewed as the sum of sinusoidal waves. Therefore, the response of a linear system to a transient input can be viewed as its response to the sum of sinusoidal waves contained in the transient input. The procedure for converting transient data from the time to the frequency domain is based on the use of the Fourier integral. It has to be taken separately for the input and output function. This method requires steady state condition in some finite time which is quite difficult to obtain in flight test.

The frequency spectrum of the random input which is not contained in the integral of the input function may have a pretty high magnitude at certain frequencies compared with the magnitude of the input which is contained in the integral. In this case the frequency response curve will be in error at these frequencies.

The determination of damping coefficient from transient response data must be approached with care. It is difficult to determine that no other input forcing function has been applied during the time the determination is being made. Further confusion can arise if the energy put into one mode is transferred slowly to some more complex mode. This can give rise to apparent rapid decays and high damping simply due to unfortunate choice of either the location or direction of forcing function.

The decay of the free oscillation is also very sensitive to random input. If the damping of the system is low, a very small impulse is necessary to excite the system and vary the amplitude of the response. Also the presence of other structural modes and even the motion of the rigid airplane make the evaluation of the decay quite questionable.

While, as stated earlier, the purpose of inflight vibration testing was to gain information about the damping characteristics of the various modes of interest, several other ground rules were used to arrive at the procedure to be described more fully.

These ground rules were:

(1) That the method requires as small a time as possible to gather the data. This is to relieve the problems of very high speed low altitude testing.

(2) The method requires an absolute minimum of rework to the airplane. The surfaces in question in one case were all blind structures, very thin and were not amenable to additional weight without danger of adding a new unknown problem.

(3) If possible, the method should not require an absolute value of input force since this would nearly always present a more difficult problem.

(4) The method did not necessarily require a firm theoretical foundation, preferably it should have.

(5) The method should be fairly simple to apply so that the flight program would not be unduly impeded by lack of information.

(6) The method should arrive at least a reasonable prediction as to the safety for the next several steps in approaching a flutter boundary.

Response to Variable Frequency Input

Before discussing the testing technique with a variable frequency input function, we need some information about the effect of the sweep rate on the response.

Existing references indicate neglect of the effect of the sweep rate or assume constant correction. It can be shown that this assumption is misleading in cases of low damping which we are mostly concerned with.

Some information we get from Frank M. Lewis' report about "Vibration During Acceleration Through a Critical Speed" (Reference 2). We extended this work to the method covered in the paper. We will now discuss the response of a linear single degree of freedom system to a forcing function of variable frequency with constant sweep rate. The case of constant driving frequency is included as boundary case with zero sweep rate.

For better understanding of the curves the symbols used may be explained. The differential equation for a single degree of freedom system with variable frequency excitation and with unit input can be expressed as:

$$y + 2\,ny + p^2 y = \sin\,(m_o t + m_1 t^2)$$

where:

y	= response for unit input
$p = 2\pi f_o$	= system frequency in radians per second
f_o	= system frequency in cycles per second
m_o	= input frequency at $t = 0$ in radians per second
$2m_1 = 2\pi f'$	= rate of change of input frequency in radians per second squared
f'	= rate of change of input frequency in cycles per second squared
$\overline{m}_1 = \frac{m_1}{p^2} = \frac{f'}{4\pi f_o^2}$	= dimensionless rate of change of input frequency, called "sweep rate"
$\gamma = \frac{2n}{p}$	= damping coefficient
f_i	= variable input frequency in cycles per second
f_m	= input frequency at maximum response in cycles per second

The argument of the forcing function on the right side is a quadratic function of time. The first derivative of the argument with respect to time is the input frequency.

$$2\pi f_i = m_o + 2m_1 t$$

where m_o = input frequency at $t = 0$ in radians per second

and m_1 = rate of change of input frequency, called "sweep rate", in radians per second squared

Setting $m_1 = 0$, we get the classical case of constant input frequency. In all cases $m_1 > 0$ we may set the initial frequency $m_o = 0$ and in cases $m_1 < 0$ we may set $m_o = 2p$.

Figure 1 shows the frequency response curve obtained by applying a constant frequency forcing function ($m_1 = 0$) compared with two response curves to variable frequency excitation. The damping coefficient in all three cases is $\gamma = 0.1$. The response curves for $m_1 \neq 0$ are "pseudo frequency response curves", because the frequency depends on the time.

The first curve ($\overline{m}_1 = 0$) depends only on the damping γ and the input frequency. Some features of the curve depend only on γ. The maximum response — the amplitude ratio R — is proportional $1/\gamma$ for small damping. The proportionality factor is the ratio of the maximum response to the response at zero input frequency (static condition). The static response is difficult to measure in flight test. Another feature of the response curve is the width of the response peak at 0.707R. It is well known that the width at this response (3 db down point) is equal to the damping γ. We know that the maximum response occurs at the frequency ratio "one", if the damping is small, and that the maximum response shifts to lower frequency ratios if the damping is high.

RESPONSE AMPLITUDE OF A SINGLE DEGREE OF FREEDOM SYSTEM VS. FREQUENCY

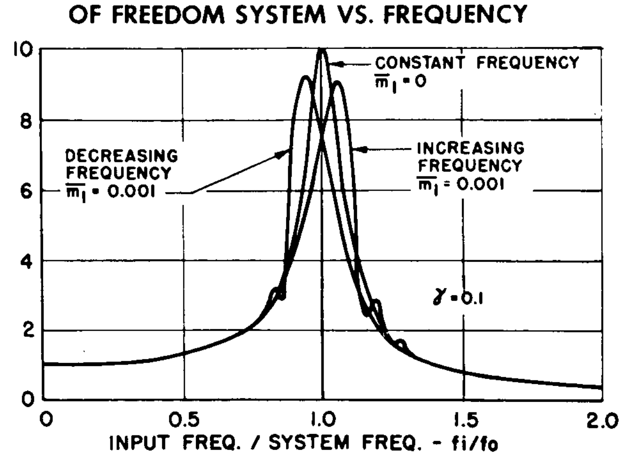

Figure 1. Response Amplitude of a Single Degree of Freedom System Versus Frequency

In case of variable frequency excitation we have one additional new variable in the input frequency function, the slope of the frequency function, called the "sweep rate" f' or $\overline{m}_1$ (dimensionless). The sweep rate causes a delay in the response. In case of increasing frequency the maximum response occurs at higher frequency and in case of decreasing frequency at lower frequency. The maximum response is in both cases lower than in the case of zero sweep rate, because the excited system has not enough time to build up higher amplitudes.

Figure 2 shows how the maximum response and the frequency at the maximum response depend on the damping γ of the excited system and on the sweep rate of the input function. The up or down going lines are lines of constant sweep rate. In the middle is the line for zero sweep rate (classical case), on the right for positive, and on the left for negative sweep rates. The lines going from the left to the right are lines of constant damping γ. The higher the sweep rate is, the higher is the effect on the maximum response and the frequency shift at maximum response. This dependency allows us to pick up more information from the response curves to variable input frequency then from the classical response curve. Applying a positive and a negative sweep rate of same magnitude in two test runs under same conditions, we can measure a total frequency shift which depends on the damping γ and the sweep rate $\overline{m}_1$.

Before we discuss the crossplottings along the lines of constant damping and constant sweep rate, let's look at the phase angle of the response for the same three cases. Figure 3 shows the phase angle vs. frequency. From the classical case ($\overline{m}_1 = 0$) we know that the phase angle starts with zero degree at frequency ratio "one" and approaches 180° for very high frequencies. The slope of the phase angle at the maximum response is proportional $1/\gamma$ for small damping. The phase angle of the response to variable frequency input is also affected by the sweep rate. The phase angle at the maximum response shifts to higher values for increasing frequency and to lower values for decreasing frequency.

The slope of the phase angle curve at the maximum response is lower than that for zero sweep rate. The maximum slope which occurs somewhat later is nearly the same as that for zero sweep rate. Figure 4 shows the phase angle at the maximum response vs. frequency for different damping values γ, and different sweep rates m_1. Also here we can state that the effect of the sweep rate is increasing with decreasing γ and that the shift of the phase angle is opposite for positive and negative sweep rates. The magnitude of the total phase angle shift can again be utilized in determining the damping.

The following figures are crossplottings of the different features vs. sweep rate $\overline{m}_1$ and vs. damping γ.

In Figure 5, we see the maximum reponse R vs. sweep rate $\overline{m}_1$ for different γ. The effect of the

PHASE ANGLE OF A SINGLE DEGREE OF FREEDOM SYSTEM VS. FREQUENCY

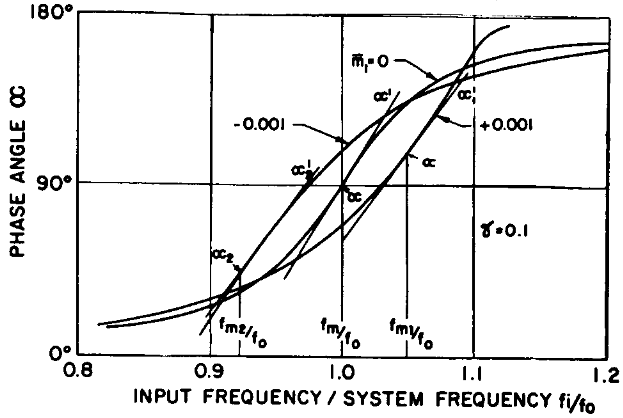

Figure 3. Phase Angle of a Single Degree of Freedom System Versus Frequency

MAXIMUM RESPONSE R VS. FREQUENCY

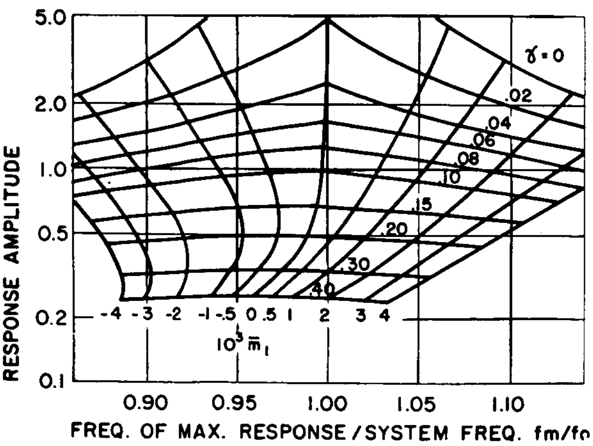

Figure 2. Maximum Response Versus Frequency

PHASE ANGLE α AT MAXIMUM RESPONSE VS. FREQUENCY

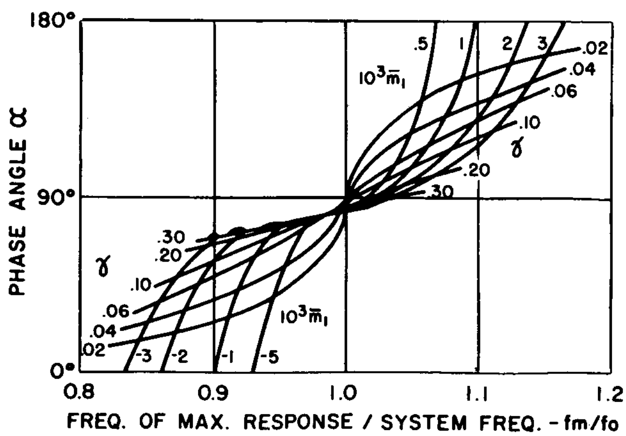

Figure 4. Phase Angle at Maximum Response Versus Frequency

sweep rate is very little in case of high damping γ, but remarkable in case of low damping. In all cases but zero sweep rate we get a finite maximum response, even for $\gamma = 0$.

MAXIMUM RESPONSE R VS. SWEEP RATE $10^3\bar{m}_1$

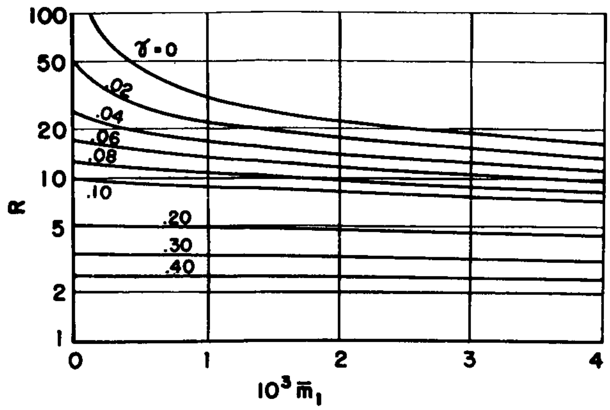

Figure 5. Maximum Response Versus Sweep Rate

This finding is very important for practical flight flutter tests. The method with variable frequency excitation applied with caution is not more dangerous than a straight flight with always present random excitation.

The next plotting (Figure 6) is more suitable for practical application. It shows the maximum response vs. damping for different sweep rates. Using the maximum response for determining the damping coefficient γ a preliminary study of the proportionality or magnification factor is necessary. It can be assumed as a first approximation that this factor is constant in a certain speed and altitude range.

MAXIMUM RESPONSE R VS. DAMPING γ

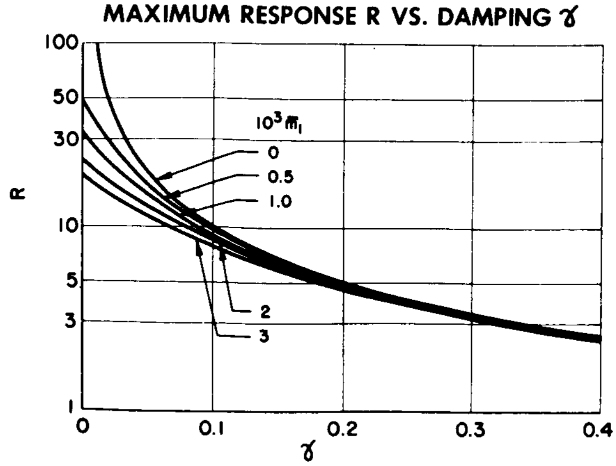

Figure 6. Maximum Response Versus Damping

In the following Figure 7 the frequency shift of the maximum response is plotted vs. sweep rate. The maximum response shifts to higher frequencies in case of increasing frequency and to lower frequencies for decreasing frequency. The frequency shift is remarkable and well measurable in case of low damping. This plotting is very useful in determining the frequency and the damping of the excited system. In order to get a well measurable frequency shift it is advisable to apply a positive and a negative sweep rate of same magnitude under the same flight condition. The frequency shift is independent on the magnitude of the input function; it depends only on the damping and the sweep rate. Therefore, the damping can be determined directly without knowledge of the real input function and the magnification factor.

FREQUENCY SHIFT OF MAXIMUM RESPONSE $\frac{f_m}{f_o}$ VS. SWEEP RATE $10^3\bar{m}_1$

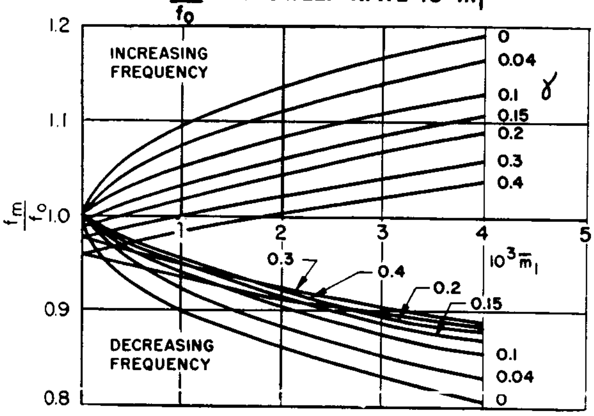

Figure 7. Frequency Shift of Maximum Response Versus Sweep Rate

Crossplottings of the frequency shift vs damping γ for different sweep rates are presented in Figure 8. It shows the effect of the sweep rate and the damping on the frequency shift.

FREQUENCY SHIFT OF MAXIMUM RESPONSE $\left(\frac{f_m}{f_o}\right)$ VS. DAMPING γ

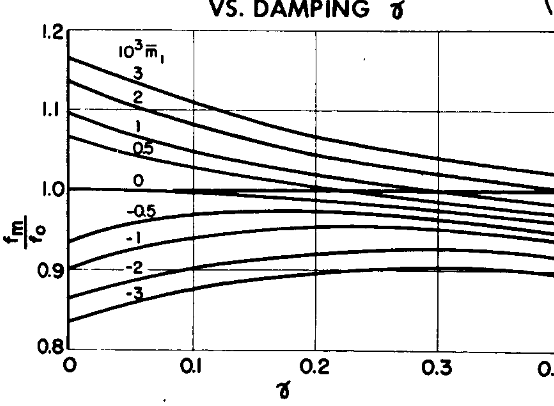

Figure 8. Frequency Shift of Maximum Response Versus Damping

The next plotting (Figure 9) is very convenient for a quick estimation of the damping from the total frequency shift between the positive and negative sweep rate of the same magnitude. All three plottings of the frequency shift indicate that the accuracy of reading is better in case of low damping than of high damping.

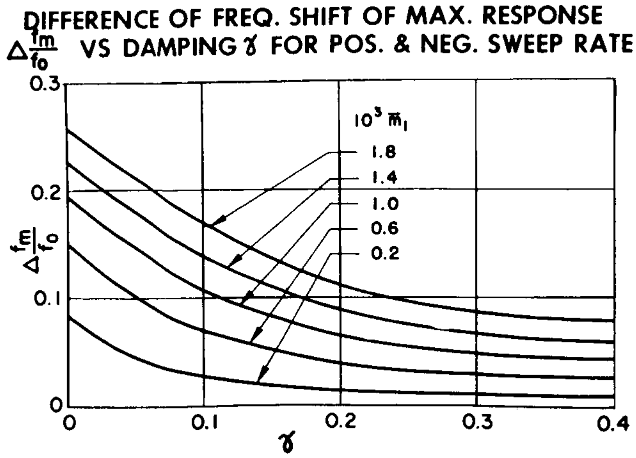

Figure 9. Difference of Frequency Shift of Maximum Response Versus Damping for Positive and Negative Sweep Rate

Another feature of the response function which can be used for direct reading of the damping coefficient without knowledge of the input function is the width of the response curve at 0.707R (Figures 10 and 11). The width $w = \gamma$ for the classical case of zero sweep rate $\bar{m}_1 = 0$ and small damping. The effect of the sweep rate on the width w is quite remarkable at low damping. Neglecting the effect of the sweep rate can be dangerous.

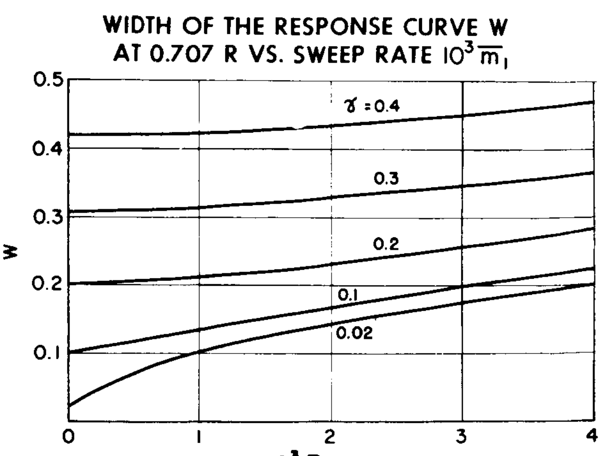

Figure 10. Width of the Response Curve at 0.707R Versus Sweep Rate

Figure 12 represents the crossplotting of the phase angle at maximum response α vs. sweep rate. The phase angle is more sensitive with respect to variation of the input frequency than the frequency at

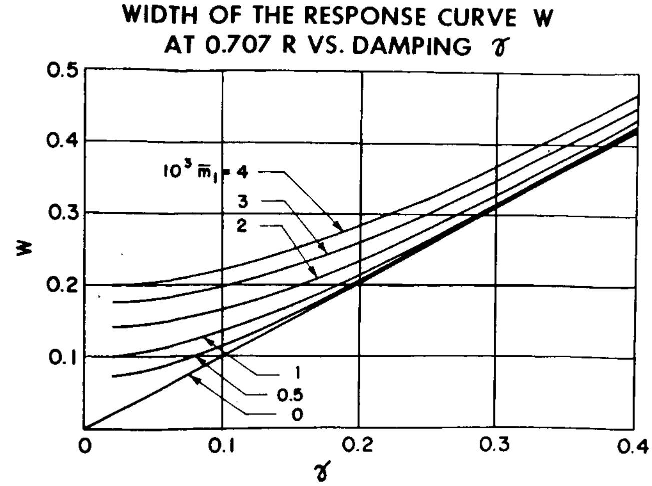

Figure 11. Width of the Response Curve at 0.707R Versus Damping

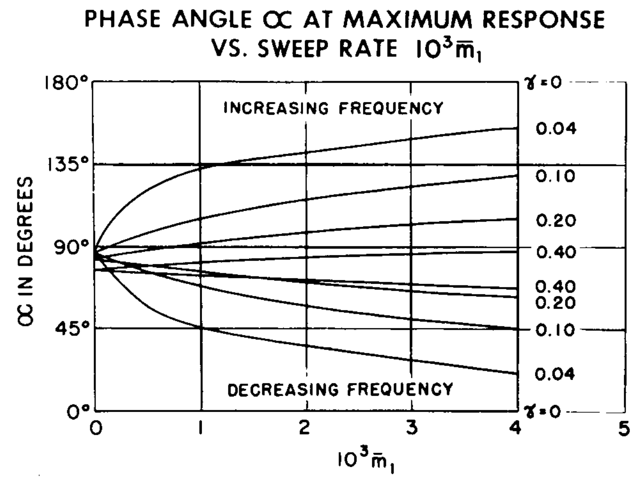

Figure 12. Phase Angle at Maximum Response Versus Sweep Rate

maximum response, but the character of the curves is quite similar to those in Figure 7.

The crossplotting of the phase angle vs. damping (Figure 13) can be compared with the plotting (Figure 8): frequency shift vs. damping. The phase angle shift in case of low damping is remarkable.

The difference of the phase angle $\Delta\alpha$ at maximum response for positive and negative sweep rate is shown in the next Figure 14. This plotting is useful for a quick estimation of the damping.

Finally, lets take a look at the increment of the phase angle at maximum response. In Figure 15 the slope of the phase angle α' is plotted vs. sweep rate. These curves look quite similar to those in Figure 5, maximum response vs. sweep rate. The plotting of the slope α' vs. damping (Figre 16) is similar to Figure 6.

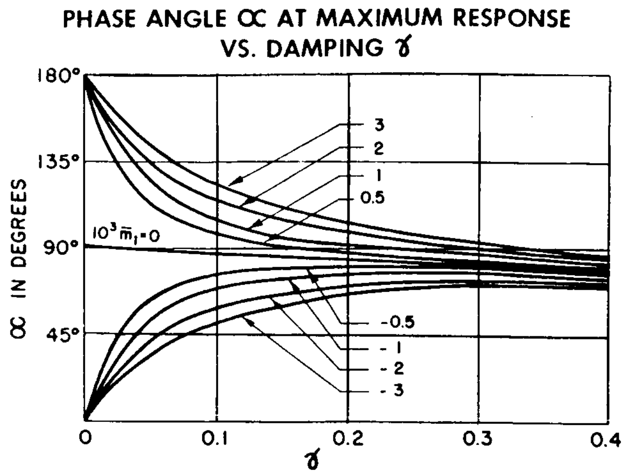

Figure 13. Phase Angle at Maximum Response Versus Damping

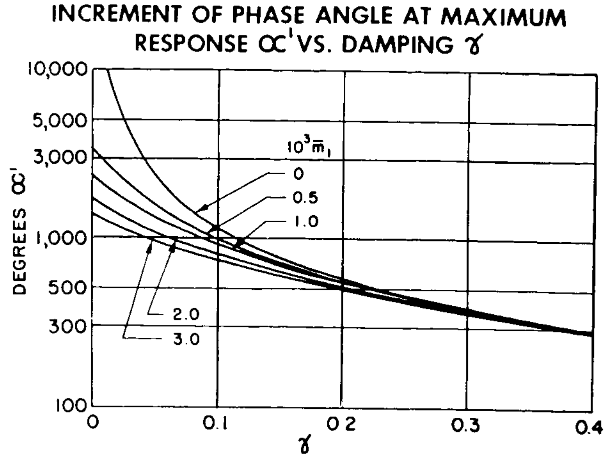

Figure 16. Increment of Phase Angle at Maximum Response Versus Damping

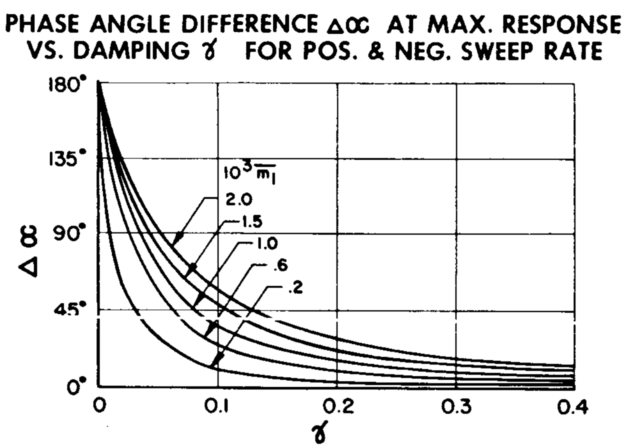

Figure 14. Phase Angle Difference at Maximum Response Versus Damping for Positive and Negative Sweep Rate

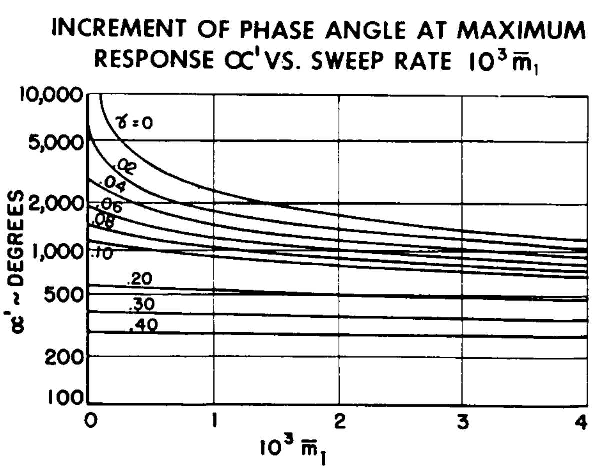

Figure 15. Increment of Phase Angle at Maximum Response Versus Sweep Rate

The phase angle and the slope of the phase angle are pretty sensitive with respect to any random input. Therefore, the data obtained from the phase angle curve are less reliable than those obtained from the response curve. Some experience is required in judging how to weigh each of the features. The possibility to use quite a number of the features of the response curve for determining the damping coefficient provides the opportunity of checking.

Summarizing, we can say that the new variable, the sweep rate, causes more variation in the response curve. The evaluation seems to be more difficult at first sight, but with the knowledge of the dependence of the different features on damping and sweep rate we can determine the damping in different ways. We can pick up more information from the response to variable input frequency than from the frequency response curve for zero sweep rate ($\bar{m}_1 = 0$).

DISCUSSION

A theoretical study on a single degree of freedom system showed that the response to a forcing function of variable frequency with constant rate of frequency change depends on the sweep rate and the damping of the system. The sweep rate causes a diminution of the maximum response and a frequency shift of the maximum response to higher or lower input frequencies. Also, the phase angle between output and input function and the slope of the phase angle function at the maximum response vary with the sweep rate. The width of the response curve is another feature which varies with the sweep rate. The variation of all the features just mentioned is of such a magnitude, especially in case of small system damping, that it cannot be neglected. It can rather be an aid in determining the damping coefficient of the system if the sweep rate is properly chosen and kept constant in the frequency range of interest.

A new flight testing technique can be based on the comparison of the measured response curve with the response curve of a system with one degree of freedom. The different features of the response function which depend on the sweep rate of the input function and the damping of the system allow the determination of the damping coefficient. A practically convenient sweep rate $\bar{m}_1 = \frac{f'}{4\pi f_0^2}$ lies in the range of 0.0005 to 0.0015. The sweep rate has to be constant in order to avoid additional response to variation of the sweep rate. The determination of the damping coefficient from the different features provides the possibility of checking one value against the other.

A BEAC study was made on a three degree of freedom system with one predominant mode of small damping. The amplitude of the input force was kept constant and the varying frequency was controlled by hand. Figure 17 shows a comparison of the frequency shift of the maximum response of the three degree of freedom system with that of a single degree of freedom system. The frequency shift curves plotted versus rate of change of input frequency show fairly good agreement.

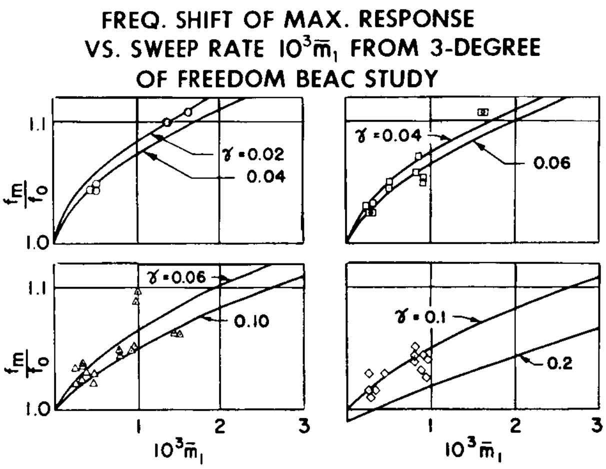

Figure 17. Frequency Shift of Maximum Responses Versus Sweep Rate from 3-Degree of Freedom BEAC Study

Flight Test Results

We applied the new testing technique successfully on the F-104A and other airplanes. Here are a few results. The tests indicated that there were no satisfactory means of determining the exact input forcing function. Only an indirect input function could be applied through the yaw damper. So the yaw damper deflection was used as an indication of the input function. The bending and torsion moment at the fin root was used as output. Any other measured and recorded quantity which is closely related to the structural mode of interest can be considered as an output.

The time response function, output amplitude divided by the input amplitude, can be replotted versus input frequency, as shown in Figure 18, for increasing and decreasing frequency. Most information used in determining the damping coefficient can be picked up from these response functions: the maximum response, the frequency shift of the maximum response, and the width of the response curve. The sweep rate is taken from the frequency function versus time. The reciprocal of the maximum response 1/R is a good indication of the damping, it increases with increasing damping and decreases with decreasing damping. The damping coefficient determined by comparison of the measured response curve with the response curve of a system with one degree of freedom is plotted in Figure 19 versus Mach number for constant altitude.

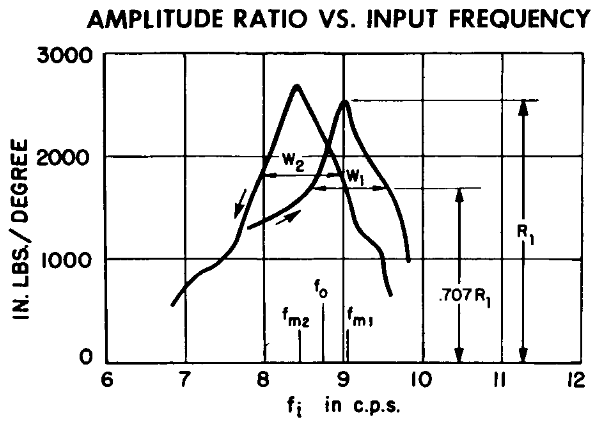

Figure 18. Amplitude Ratio: Fin Root Torsion Moment per Degree Yaw Damper Deflection Versus Yaw Damper Frequency

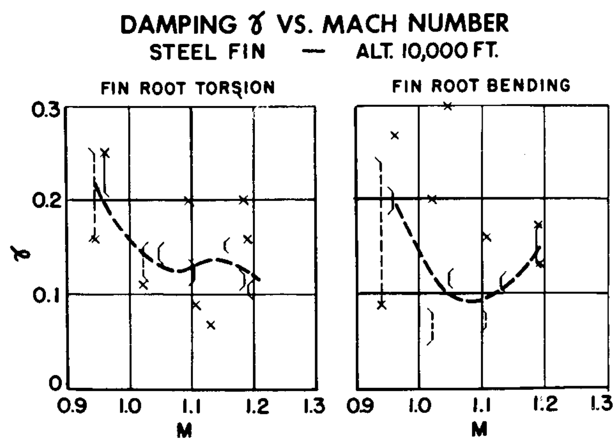

Figure 19. Damping Coefficient Versus Mach Number

The tests were repeated at different altitudes. The minimum damping picked up from these plottings is now plotted versus altitude. Figures 20 and 21 show the minimum damping versus altitude for the F-104A fin with aluminum and steel skin respectively. The altitude for zero damping can be found by extrapolation. Figure 22 shows a comparison of the flight test results with the analytical and wind tunnel results. A fairly good agreement can be stated.

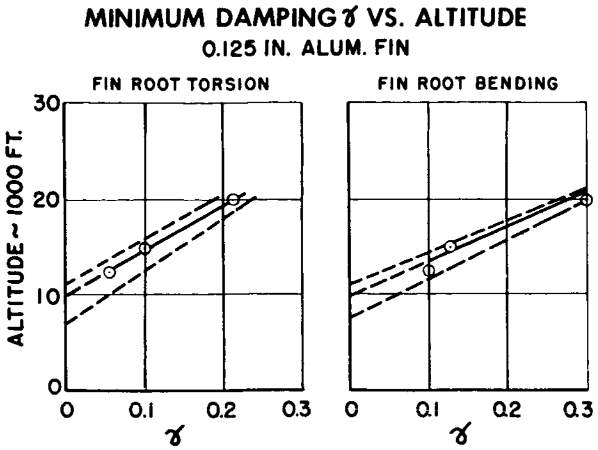

Figure 20. Minimum Damping Versus Altitude for Aluminum Fin

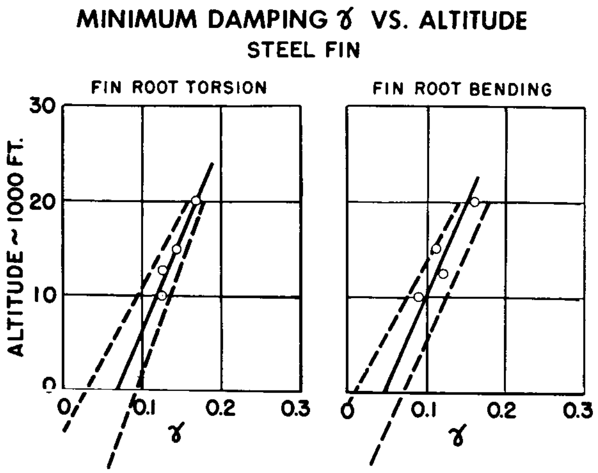

Figure 21. Minimum Damping Versus Altitude for Steel Fin

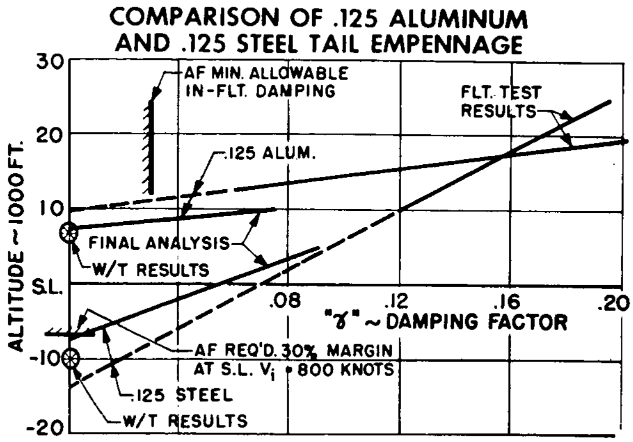

Figure 22. Comparison of Flight Test Results with the Analytical and Wind Tunnel Results

This was a brief survey about the application of the testing technique with variable input frequency in flight flutter tests because of the limited time available.

REFERENCES

1. H. G. S. Peasock, "Flight Flutter Tests on the Gloster Javelin".
2. Frank M. Lewis, "Vibration During Acceleration Through a Critical Speed".