ON THE PREDICTION OF CRITICAL FLUTTER CONDITIONS FROM SUBCRITICAL RESPONSE DATA AND SOME RELATED WIND-TUNNEL EXPERIENCE

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Abstract

Methods of interpreting response measurements which could be amenable to flight flutter testing procedures are being studied analytically and in the wind tunnel. One suggested scheme, which requires evaluation, is an iterative technique in which derivatives obtained from subcritical response data are used to indicate the approach to flutter. This paper considers a simplification of this procedure by examining the manner in which a single characteristic of the subcritical response behaves in relation to variations of the density or dynamic pressure in the approach to flutter. The use of this single parameter scheme is examined for random excitation as well as for sinusoidal forcing. The feasibility of the method is illustrated by several examples and the relative merits of random and sinusoidal excitation are discussed.

INTRODUCTION

In this paper certain new slants are given on the prediction of critical flutter condition from subcritical response data. Specifically, the technique considered herein deals with the manner in which the forced response behavior of an aeroelastic system varies with changes in air density, while velocity is being held essentially fixed. The impression is not to be given that density considerations are necessarily new, but rather the point of view is held that a further examination of density effects may lead to a simple index which may be useful in the prediction of flutter. The motivation stems from the fact that density appears in a rather clean-cut fashion in the equations for flutter, in contrast to the complex way in which velocity enters. Actually, the work started when we were considering the application of ideas suggested by Professor Mollø-Christensen. The present work evolved as a special consideration, and we thought it to be of enough interest to merit separate attention.

In the first part of the paper an elementary but rational analysis is given to show how the response of a wing system might be expected to depend on air density, for both the cases of sinusoidal and random force input. A theoretical model illustrating the technique of extrapolation to the flutter condition is then considered. Then, in the second part of the paper, attention is focused on the experimental testing of the approach by application to some wind-tunnel studies.

ANALYTICAL TREATMENT

Derivation of Extrapolation Equations

Let us consider an aeroelastic system which is being excited into motion by either a sinusoidal shaker or a sinusoidal gust, and then proceed to investigate how the amplitude of the response, such as deflection, is dependent on the density of the air flow. To do this, introduce the equation governing the motion of the system as follows

$$Dw = p = \omega^2 mw + \rho v^2 D_L w + F_s + \rho F_g \quad (1)$$

where the equation may be interpreted either in differential operator form or in matrix notation. The operator D on the left hand side converts the surface deflection w into the total surface loading composed of the inertia, aerodynamic, and applied loadings on the right hand side. The operator D_L is complex and is a function of Mach number and reduced frequency, and when operating on the deflection, leads to the aerodynamic loading; the shaker force F_s (considered to be distributed over a small area to give an intensity) and the gust loading ρF_g are treated together for convenience, and will be separated later. It is remarked that the sinusoidal gust condition is introduced because this condition yields a necessary part — the transfer function — of the solution for response when random inputs are involved; the density ρ is shown specifically as an ingredient of the gust loading so as to keep the density in an explicit sense throughout the analysis.

We now choose to make an approximate solution of equation (1), since our essential result is arrived at rather quickly, and will leave a more rigorous, but lengthier, treatment which leads to the same result to an appendix. The approximate solution is of the Galerkin type and is made by assuming that the deflection is expressed in terms of the modal shape which occurs at flutter, thus

$$w = a_1 w_f \tag{2}$$

where a_1 is a coefficient to be determined and w_f is the flutter deflection shape which satisfies the equation

$$Dw_f = \omega_f^2 \, mw_f + \rho_f v_f^2 D_{L_f} w_f \tag{3}$$

which is simply equation (1) with the forcing terms suppressed. Substitute equation (2) into (1), use equation (3), multiply by w_f and integrate over the wing surface; the result leads to the following solution for a_1

$$a_1 = \frac{Q_s + \rho Q_g}{\omega_f^2 - \omega^2 M + \rho_f v_f^2 A_f - \rho v^2 A} \tag{4}$$

where Q_s and Q_g are in the nature of generalized forces

$$Q_s = \int w_f F_s dS \, , \; Q_g = \int w_f F_g dS$$

and

$$M = \int m w_f^2 dS, \; A_f = \int w_f D_{L_f} w_f dS, \; A = \int w_f D_L w_f dS \tag{5}$$

In general, all of these generalized coefficients are complex. At a velocity and frequency equal to the values at flutter but at a subcritical value for density, the value of a_1 is particularly significant and is

$$a_1 = \frac{Q_s + \rho \, Q_g}{v_f^2 A_f \; (\rho_f - \rho)} \tag{6}$$

By inverting this equation and at the same time separating the effects of the shaker and gust terms, we arrive at the final two equations which indicate how the amplitude of wing deflection varies with density

$$\frac{1}{|a_1|} = \frac{v_f^2 |A_f|}{|Q_s|} (\rho_f - \rho) \qquad \text{shaker only} \tag{7a}$$

$$\frac{1}{|a_1|} = \frac{\rho_f v_f^2 |A_f|}{|Q_g|} \left(\frac{1}{\rho} - \frac{1}{\rho_f}\right) \qquad \text{gust only} \tag{7b}$$

These two equations suggest the basic linear extrapolation procedure of this paper. Thus, assume that in-flight measurements of response are made according to the following plan: we fly at a velocity near the expected flutter speed (or at a velocity for which we want to prove the aircraft safe), but take care to first fly at a high altitude where the density is low. Then, repeat the tests at successively lower altitudes. Then, for tests utilizing a sinusoidal shaker input, we might expect a plot of the reciprocal of the amplitude versus density to form a straight line, which when extrapolated to $\frac{1}{|a_1|} = 0$ yields the density that ought to produce flutter. For the case of a gust input, $\frac{1}{|a_1|}$ is plotted against $\frac{1}{\rho}$ for an expected linear relationship. In the actual testing in a random force input environment, the output spectrum of response will be found. But since this spectrum is proportional to the square of the frequency response function for sinusoidal gust input, we see that the reciprocal of the square root of the output spectrum should be plotted against $\frac{1}{\rho}$, to arrive at a condition consistent with that indicated by equation (7b).

In applying equations (7a) and (7b), it is implied that the frequency of flutter is known. This is, of course, not so; therefore the procedure to follow is to observe the amplitude-density behavior at several frequencies until it becomes clear from the frequency response plots what frequency is emerging as the flutter frequency.

Example of Calculated Results

As a test of the possible range of applicability of equations (7a) and (7b), response calculations were made for a rectangular cantilever wing, and interpreted in accordance with these equations. The response analysis was limited to two degrees of freedom, one bending and one torsion, and employed the aerodynamic coefficients for M = 0.8 in a strip fashion. The frequency response functions obtained for amplitude of torsional displacement at the wing tip are shown in Figure 1, where the curves at the left are for a sinusoidal gust input, whereas the curves at the right are for a sinusoidal shaker input located at the tip and at 10 percent chord position. The parameter μ is a ratio of structural mass to air mass, and

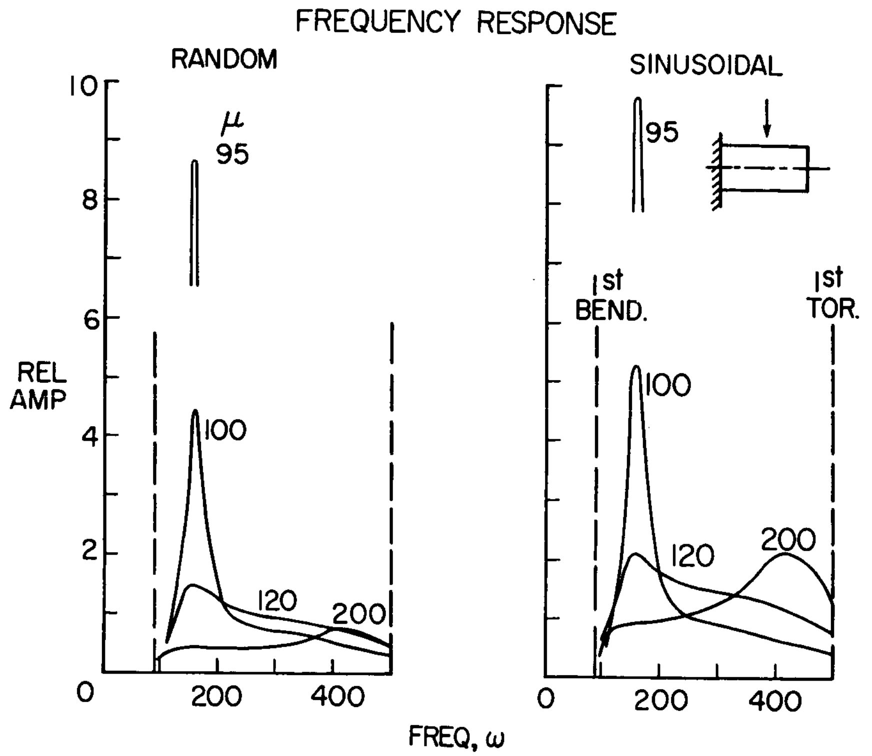

Figure 1. Frequency Response

therefore may be regarded as inversely proportional to air density. It is seen that as the air density increases (μ decreasing) an ever growing and sharper peak develops at a frequency of 158 cps, thus suggesting a frequency of flutter.

Application of equations (7) to the amplitude values at this frequency gives the curves shown in Figure 2. Extrapolation of the curves to $\frac{1}{|\phi|} = 0$ indicates a flutter density (μ = 89) which agrees identically with that given by a conventional flutter analysis. The very pronounced range of linearity is also to be noted; in fact, using only the data at densities of 45 and 75 percent of the flutter density would give a flutter prediction erring by only a few percent. It is

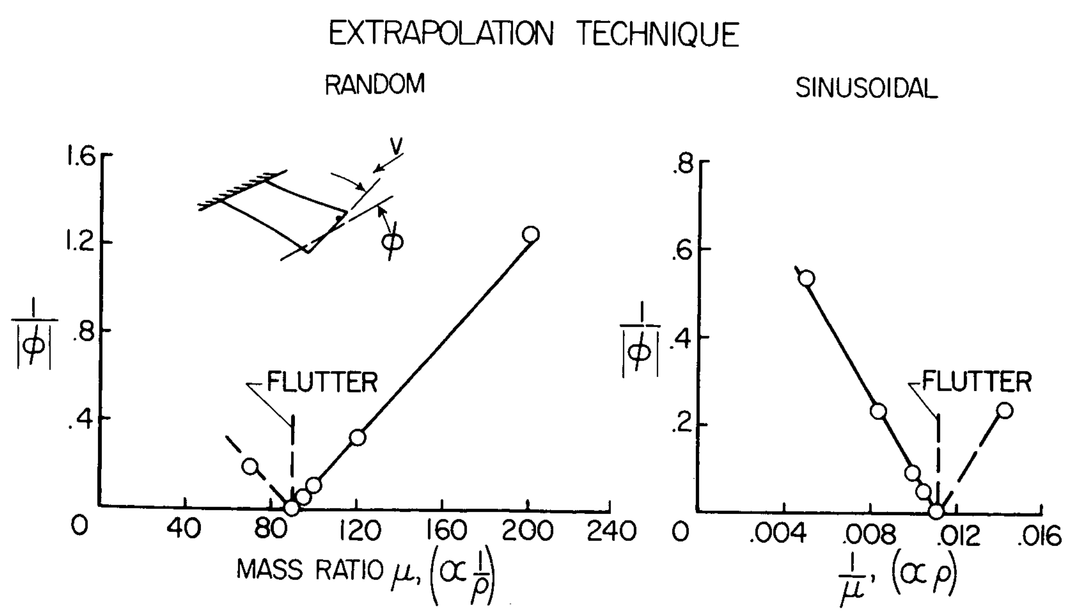

Figure 2. Extrapolation Technique

significant to note also that the data point corresponding to the 45 percent of critical density condition is not a major peak in the frequency response curve for this density. Thus, subcritical response data which have not yet indicated peaks may still be useful.

The single data point and dashed curve shown for densities above the critical value are shown simply as a matter of interest to indicate that the theoretical response calculations based on sinusoidal conditions show a branch above the flutter condition as well as below.

The main conclusion to be drawn from this example is that the present technique for predicting flutter appears quite promising. In the second part of the paper we shall see how well it works when applied to wind-tunnel studies.

Before looking at the experimental results, we might make a few comments on the general applicability of the density extrapolation technique. As with other flutter extrapolation techniques, there will undoubtedly be cases where this scheme breaks down. One possible example is that associated with wing systems which are capable of a single degree of freedom type flutter. Interestingly enough, equation (7) can be used to demonstrate why. Up to now we have tacitly assumed that unbounded response ($a_1 \longrightarrow \infty$) occurs when $\rho_f - \rho$ becomes zero. It, of course, also is possible for the response to become infinite when A vanishes, and this may occur either in a classical way for attached flow, or what is more likely, when the flow becomes separated, such as in stall flutter. The equation indicates that density is unimportant in these instances, and this is actually what the experiment shows. Thus, any flight investigation should keep this possibility in mind.

EXPERIMENTAL RESULTS

The previous section concerned the analytical background which has formed a guide to some wind-tunnel experiments discussed in this section.

The linear extrapolation technique has been examined experimentally for six cases involving random excitation and for one case of sinusoidal excitation. These various cases are illustrated in Figure 3, where a typical flutter boundary is used to illustrate the manner in which the flutter condition was approached. Geometric properties of the four semi-span, cantilever mounted models are listed in Table I. Model A was used to obtain three sets of subcritical response data — Case I and Case II at two different stagnation pressures, but increasing velocity, and Case III at constant velocity but increasing density. Models B and C were tested at constant stagnation pressure and increasing velocity. Model D was equipped with an electro-hydraulic shaker housed in a tip tank. This model was examined for two cases — Case I, random excitation at constant stagnation pressure, and Case II, sinusoidal excitation at constant velocity. In all of the cases examined the type of flutter encountered was classical bending torsion involving the coupling of well separated modes.

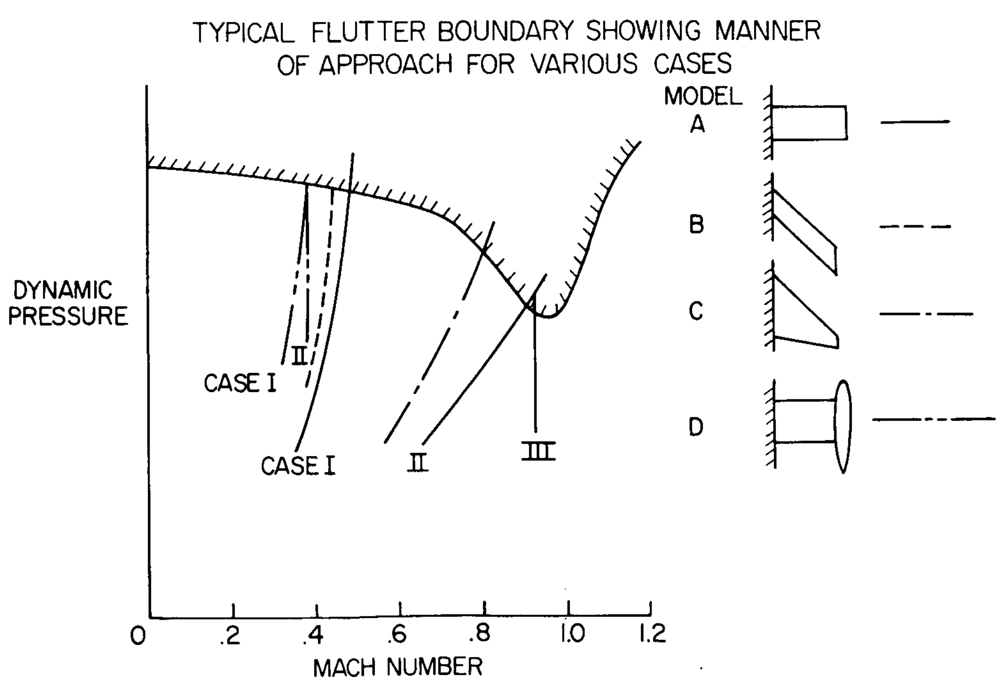

Figure 3. Typical Flutter Boundary Showing Manner of Approach for Various Cases

TABLE I

GEOMETRIC PROPERTIES OF MODELS TESTED

Model	Aspect Ratio	Taper Ratio	Sweep at 1/4 C	Airfoil Section
A	5	1.0	0°	6 percent Circular Arc
B	6	1.0	45°	Flat Plate
C	3	1/7	45°	NACA 65A004
D	3	1.0	0°	NACA 65A010

Random Excitation

The subcritical response data for Models A, B, and C were obtained by recording the output of resistance wire strain gage bridges mounted near the root of the model, while the model was responding to the normal turbulence in the wind-tunnel airstream. The response data were recorded on magnetic tape using frequency modulation amplifiers (ref. 1). After completing the tunnel runs, thirty-second samples of the tape records were analyzed using analog data reduction equipment described in reference 1. The peak values in the power spectra of strain response were operated on to yield numbers proportional to the reciprocal of the absolute magnitude of the strain response. These results are illustrated in Figure 4 where the response magnitudes are shown as functions of the ratio of the dynamic pressure at flutter to the dynamic pressure associated with each point.

It should be pointed out that this form of presentation is not identical to that suggested by the analysis. Some of the experiments were completed before the analysis was available, and the form of presentation chosen was such that all of the experiments would be consistent within themselves. For example, the velocity squared term has been combined with the density to form the dynamic pressure. This is a necessary step in that some of the experiments involved an approach to the flutter condition primarily

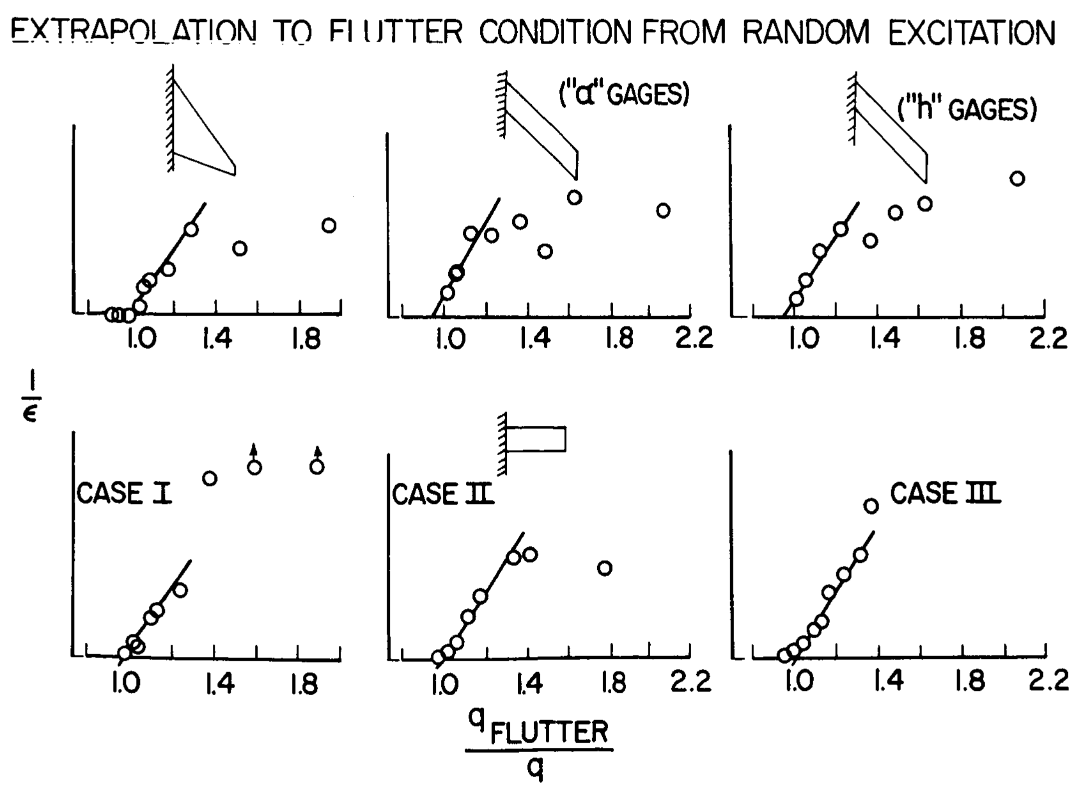

Figure 4. Extrapolation to Flutter Condition from Random Excitation

through increases in velocity. These variations in velocity require the statement of additional qualifications to those already mentioned if one is to expect a linear extrapolation of the response data. Perhaps the most important of these additional assumptions is that near the flutter condition, the air forces associated with flutter do not vary rapidly with the reduced frequency and Mach number.

An idea of the usefulness of these extrapolation methods can be gained by examining Figure 4. A reasonable degree of linearity of the response data is indicated for all of the cases, when the dynamic pressure is within about 20 percent of the critical value and the extrapolation gives a good indication of the flutter condition. The least encouraging results were obtained for Model B which was poorly instrumented. The strain gage bridges were mounted very near the root and were about equally sensitive to bending or torsional motions. The response data for the other cases were taken from strain gages arranged such that they were sensitive primarily to torsional strains. It might be mentioned that the results shown for the third case of Model A indicate a linear relation to lower values of dynamic pressure than most of the other cases. This result may be associated with the constant velocity method of obtaining the response data in this case.

Sinusoidal Excitation

In order to gain some insight regarding the relative merits of sinusoidal excitation as opposed to random excitation, two cases have been examined for a model equipped with an electro-hydraulic shaker contained in a tip tank (Model D). These results are shown in Figure 5. The data in the left hand part of the figure were obtained in the same manner as the data of the previous figure except that the angular motion of the tip of the model was deduced from the combined output of two linear accelerometers mounted in the tip tank.

The data shown in the right hand part of Figure 5 were obtained by measuring the amplitude of response at the two accelerometer stations due to a sinusoidal applied force. The amplitudes were measured after the shaker had been tuned to the frequency of maximum response which, in this case, appeared to be associated with the torsional mode. Although some response due to turbulence was present during the shaker tests, the phase sensitive instrumentation used effectively eliminated its effects.

It is noted that both sets of response data indicate an equally good extrapolation to the flutter condition. If it is assumed that random excitation and sinusoidal excitation will yield equally adequate extrapolation results, the question of relative cost or difficulty of the two methods is of interest. It was mentioned earlier that six cases of random excitation as opposed to one case of sinusoidal excitation have been examined. In the wind tunnel, at least, it is believed that this six-to-one ratio is a fair estimate of the relative difficulty of the two methods. This is due, primarily, to the fact that the turbulence is always available while the shaker must be constructed and installed. Although turbulence also exists in the atmosphere, the problem of finding it during a flight test and determining enough of its properties to permit its use might improve the relative attractiveness of a sinusoidal shaker as a source of excitation.

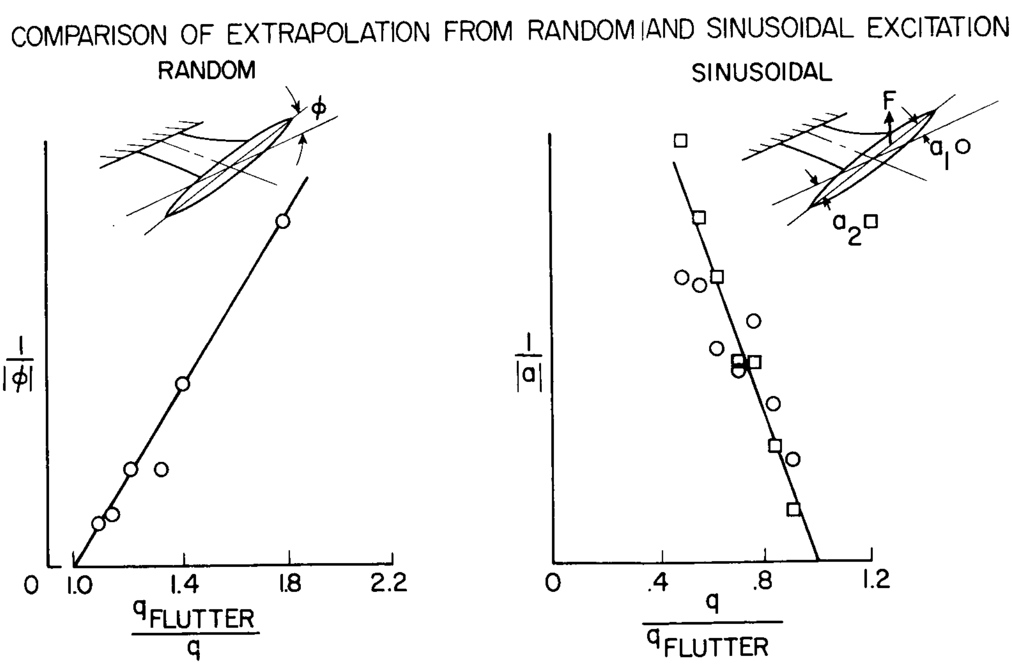

Figure 5. Comparison of Extrapolation from Random and Sinusoidal Excitation

APPENDIX

The Response-Density Relationship

A more rigorous development of equation (7) can be made along the following lines. Introduce the two equations

$$(D - \omega^2 m)w = \rho v^2 D_L w \qquad (A1a)$$

$$(D - \omega^2 m)z = \rho v^2 D_L' z \qquad (A1b)$$

where the first is simply the statement of flutter, i.e., equation (1) with forcing terms suppressed, and the second is what we shall term the transposed mate of equation (A1a). For fixed v and ω, these equations may be regarded as eigenvalue statements of ρ; they may be shown to have the same eigenvalues ρ_n (which in general may be complex), and hence may be written

$$Bw_n = \rho_n v^2 D_L w_n \qquad (A2a)$$

$$Bz_m = \rho_m v^2 D_L' z_m \qquad (A2b)$$

where $B = D - \omega^2 m$. Considered jointly, some significant relations between w_n and z_m may be found. Thus, multiply equation (A2a) by z_m, equation (A2b) by w_n, integrate both over the wing surface, then subtract the resulting expressions and make use of the fact that $\int z_m B w_n dS = \int w_n B z_m dS$ and $\int z_m D_L w_n dS = \int w_n D_L' z_m dS$; there results the relation

$$(\rho_m - \rho_n) \int z_m D_L w_n dS \qquad (A3)$$

From this equation we arrive at the basic orthogonality properties of w_n and z_m as given by the following equation

$$\int z_m D_L w_n dS = O \qquad m \neq n \qquad (A4a)$$

$$= A_n \qquad m = n \qquad (A4b)$$

We may now proceed to solve equation (1) by expressing the deflection by the following series expansion involving w_n

$$w = a_1 w_1 + a_2 w_2 + a_3 w_3 + \ldots \qquad (A5)$$

where the a_n's are unknown coefficients to be determined. Substitute into equation (1), use equation (A2a), multiply by z_m, integrate over the surface and then apply equation (A4); the result is an independent solution for a_n as follows

$$a_n = \frac{\int z_m F_s dS + \rho \int z_m F_g dS}{(\rho_n - \rho) v^2 A_n} \qquad (A6)$$

Now, if ω, v, and ρ_1 are chosen to represent an actual flutter condition ($\omega = \omega_f$, $v = v_f$, $\rho_1 = \rho_f$), then w_1 will represent the associated flutter mode shape, and the solution for a_1 becomes

$$a_1 = \frac{\int z_1 F_s dS + \rho \int z_1 F_g dS}{(\rho_f - \rho) v_f^2 A_1} \qquad (A7)$$

This solution thus confirms the validity of equation (7) presented in the body of the paper. The form of the equations is the same, but it is of interest to note that the more rational analysis presented here indicates that the generalized forces are associated with the work done by the applied forces in moving through the modal displacements of the transposed system.

REFERENCE

1. Smith, Francis B.: Analog Equipment For Processing Randomly Fluctuating Data. Aero. Egnr. Review, Vol. 14, No. 5, pp. 112-119, May 1955.