GROUND VIBRATION TESTING OF COMPLEX STRUCTURES

C. V. Stahle & W. R. Forlifer — The Martin Co., Baltimore, Md.

Abstract

Planning of flight flutter testing and interpretation of results require reliable information about the ground vibration behavior of the aircraft. Conventional GVS techniques are unsatisfactory in that internal damping and closeness of frequencies lead to sensitivity of the measured frequencies and modes to the specific excitation points used.

In fact, there is no unique definition of resonance of a multi-degree-of-freedom structure having internal damping. Following suggestions by DeVries, a method of measuring separately the in-phase and quadrature components of the vibration response, designed by APL, has been developed and applied. Both analysis and test results show immediately a much improved definition of mode shapes and frequencies.

The approach has been further developed. It allows to measure damping in the different natural modes, and to determine the exact shape of the normal modes, i.e., to eliminate the coupling effect due to structural damping. It is expected to be used in flight flutter testing also.

INTRODUCTION

The present paper presents a method for accurately determining the vibration characteristics of complex structures from test data obtained during a Ground Vibration Survey using only simple excitation techniques. The accurate measurement of the vibration characteristics of an aircraft during a Ground Vibration Survey is necessary for the planning of flight flutter tests. These data provide the first check of the predicted flutter behavior by establishing the accuracy of the calculated vibration modes and resonant frequencies of the aircraft and are used for the interpretation of the flight flutter test results.

In the past, two methods have been used to determine the vibration behavior of complex structures but neither is satisfactory. The first method measures the response to excitation at several points on the structure. Since the response of the Structure using this method is a combination of all the structural modes, it is unsatisfactory in that the mode shapes and resonant frequencies depend on the excitation points selected. The other method uses a multiple Shaker system to separate the structural modes with excitation techniques. Because a large number of exciting points are required and the individual exciting forces must be adjusted for each mode, this method is undesirable since it is extremely time consuming. When the structure being tested has resonant frequencies close together, the difficulties are magnified and a mode may be obscured and lost. The need for a simple technique which permits the accurate determination of the resonant frequencies, mode shapes, and modal damping coefficients without utilizing complicated methods of excitation has been evident.

Our approach is to measure quantities which decrease the effects of modal interaction and to analytically separate the modes of vibration from the measured data. Hence, the vibration characteristics can be accurately determined with simple methods of excitation.

The components of response in-phase and 90° out-of-phase with the exciting force are used to determine the resonant frequencies and damping coefficients. Our analytical method separates the structural modes of vibration from the component of response 90° out-of-phase with the exciting force when the structural damping is small. Figure 1 illustrates the components of response and their relation to the exciting force. The total response is defined as the structural displacement per unit force. The total response can be resolved into a vector component in-phase with the force, the in-phase response, and the vector component 90° out-of-phase with the force, the quadrature response. The representation of the vibration response in this manner was suggested by DeVries. (1) Kennedy and Pancu, (2) in a later paper utilized vector response to determine modal properties from polar plots. Theoretical calculations of Veubeke (3) indicated that the quadrature response determined more accurately the modes of vibration of a uniform beam excited at a single point. A device which enables us to measure separately the in-phase and quadrature response, a Component Analyzer, was developed by Kearns of John Hopkins Applied Physics Laboratory. (4) The results of their investigations are used as the basis of our method and its application to actual structures.

The problem that concerns us and our method of solution are illustrated in Figure 2. This graph presents the frequency response at a particular point of a two degree of freedom system with resonant frequencies close together. This example has been selected to illustrate the problem which occurs often in complex structures. The solid line represents the total response which is the quantity generally measured. The quadrature response of the system is shown by the dashed-dot line and can be measured with the Component Analyzer. The dashed lines represent the quadrature response in each of the modes and the peak values when taken at a number of locations define the mode shapes of the system. The negative quadrature response in the second mode is caused by a mode between the point of excitation and the point we are considering. If we compare the total response with the quadrature response, it can be seen that the quadrature response determines more accurately the resonant frequencies and mode shapes of the system. In fact, only one resonant frequency is apparent from the total response. Finally, we can analytically separate the quadrature response at each resonant frequency into the response of the resonant mode and the response of the non-resonant mode. Therefore, we can accurately determine the mode shapes.

In the body of this paper we will:

1) Review the significance of the in-phase and quadrature responses.

2) Present our method for analytically separating the modes of vibration from the quadrature response.

3) Describe the Component Analyzer which we used and the results of some of our laboratory tests.

4) Discuss the application of the Component Analyzer to flight flutter testing.

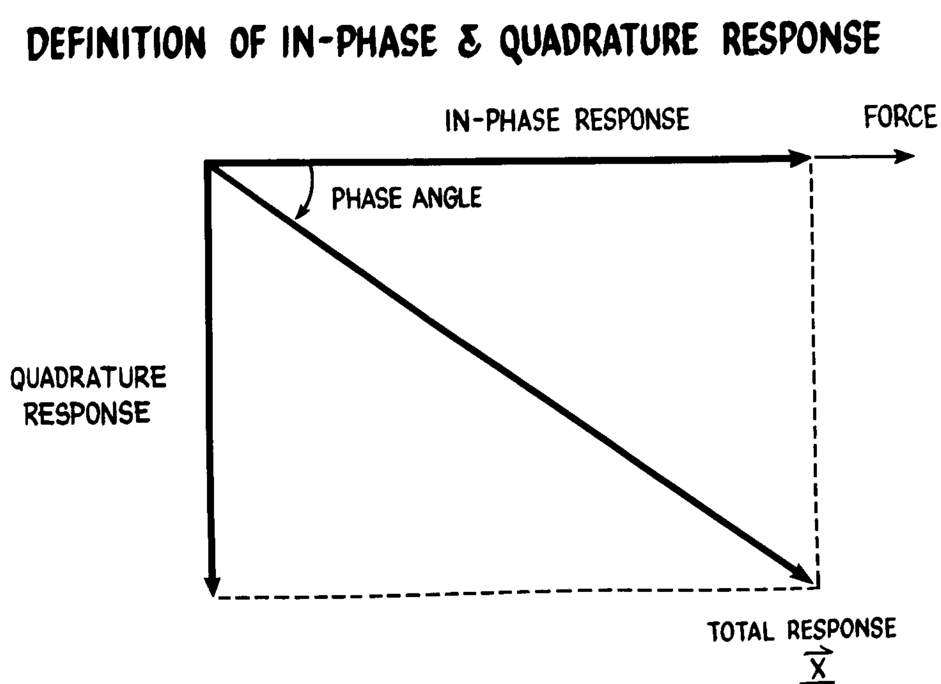


Figure 1. Definition of In-Phase and Quadrature Response

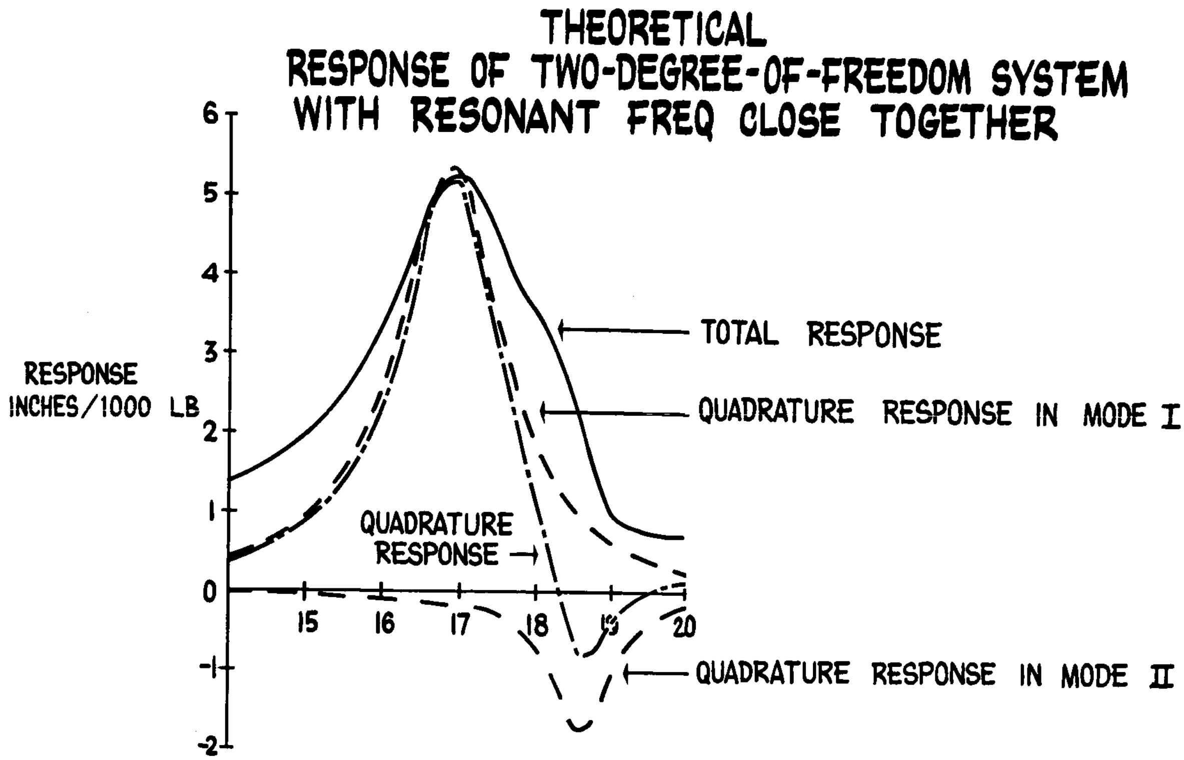


Figure 2. Theoretical Response of Two-Degree-of-Freedom System with Resonant Frequencies Close Together

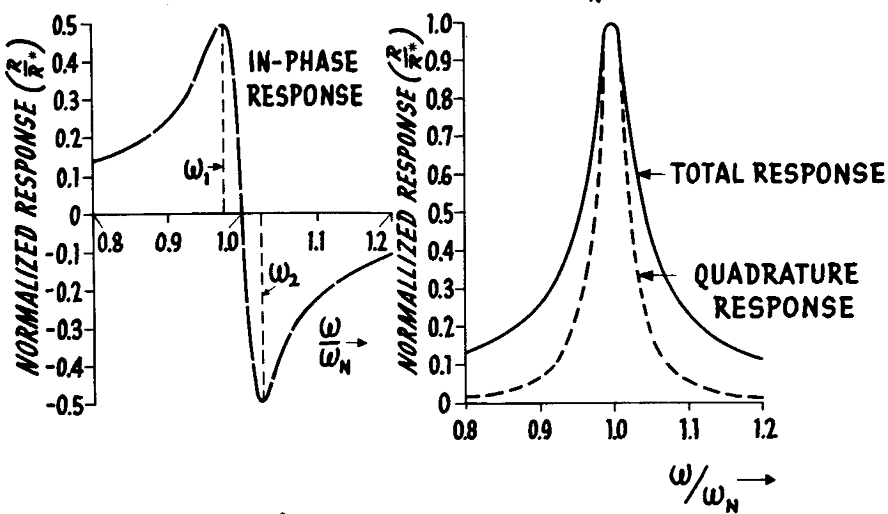
SIGNIFICANCE OF THE IN-PHASE AND QUADRATURE RESPONSE

First we will review the significance of in-phase and quadrature response for a single degree of freedom system. Then we will consider more degrees of freedom. When the structural damping is small, we can represent the forced response of a single degree of freedom system by equation (1) of Figure 3 where R is the total response at any frequency, R* is the total response at resonance, g is the structural damping coefficient, ω is the exciting frequency, and ω_N is the resonant frequency. The real and imaginary terms are the in-phase and quadrature response, respectively. The frequency variations of the total, in-phase, and quadrature responses are also shown in Figure 3. Note that the shape of each curve is completely determined by the resonant frequency and damping coefficient. We can use the frequencies at which the in-phase response peaks ω_1, and ω_2, to determine the structural damping coefficient from equation (2). For this single degree of freedom system, resonance is defined by maximum total response and a 90° phase relationship between the total response and exciting force. This is indicated by equal peak values of the total and quadrature responses with zero in-phase response.

If there is more than one degree of freedom, the response of the structure will be the sum of the responses in each of the modes. This implies that each mode will retain the response characteristics of a single degree of freedom. However, we can no longer define resonance of the system by a 90° phase relationship between the total response and the exciting force or by the maximum total response. The peaks of the quadrature response will determine the resonant frequency and response of each mode more accurately than the total response since the quadrature response of each mode peaks more sharply and the quadrature response contributions of non-resonant modes are smaller. Although the non-resonant modes effect the in-phase response more than the quadrature response, we can still use equation (2) to determine the damping if we select a point such that the response is predominantly that of the mode of interest. Damping can be determined by this method under conditions where the decay of the total response fails to give valid results. Although the mode shape which we obtain from the quadrature response will be more accurate than that of the total response, it will be necessary to separate the quadrature response into the response of each mode when the structure has resonant frequencies close together.

THEORETICAL RESPONSE OF A SINGLE DEGREE OF FREEDOM SYSTEM WITH STRUCTURAL DAMPING

$$R = R^{*}\left\{\left[\frac{g\left(1-\frac{\omega^2}{\omega_N^2}\right)}{\left(1-\frac{\omega^2}{\omega_N^2}\right)^2+g^2}\right]+i\left[\frac{g^2}{\left(1-\frac{\omega^2}{\omega_N^2}\right)^2+g^2}\right]\right\} \qquad (1)$$



$$g = \frac{\left(\frac{\omega_2}{\omega_1}\right)^2 - 1}{\left(\frac{\omega_2}{\omega_1}\right)^2 + 1} \qquad (2)$$

Figure 3. Theoretical Response of a Single Degree of Freedom System with Structural Damping

ANALYTICAL SEPARATION OF MODES

Our method of analytically separating the modes of vibration from the quadrature response, is presented below. We will refer again to Figure 2 to explain our method of analytical separation as applied to the two degrees of freedom system. The modal responses are indicated by the dashed lines and define the mode shapes of the structure. First, we will obtain the resonant frequencies of the modes from the peaks of the quadrature response. The damping coefficient for each mode will be obtained from the peaks of the in-phase response. Having determined these parameters, the shape of the quadrature response curves for each of the modes will be completely determined as we have pointed out in equation (1). The problem now is to find the modal amplitudes which when added together will give the measured quadrature response. At each resonant frequency, we will equate the sum of the quadrature responses in each of the modes to the total quadrature response. Two simultaneous equations will be obtained which can be solved for the peak amplitude of each of the modal responses. Figure 4 shows the equations which we obtain for this two degree of freedom system. R_{Q_1} is the measured quadrature response at the resonant frequency of the first mode, R_{Q_2} is the measured

EQUATIONS FOR MODAL RESPONSES FOR TWO-DEGREE-OF-FREEDOM SYSTEM

$$\underset{\omega=\omega_{N1}}{R_{Q1}} = R_1^{*} + R_2^{*}\left\{\frac{g_2^2}{\left(1-\frac{\omega_{N1}^2}{\omega_{N2}^2}\right)^2+g_2^2}\right\}$$

$$\underset{\omega=\omega_{N2}}{R_{Q2}} = R_1^{*}\left\{\frac{g_1^2}{\left(1-\frac{\omega_{N2}^2}{\omega_{N1}^2}\right)^2+g_1^2}\right\} + R_2^{*}$$

Figure 4. Equations for Modal Responses for Two-Degree-of-Freedom System

quadrature response at the resonant frequency of the second mode. R_1^* and R_2^* are the maximum modal responses in each of the modes which determine the mode shapes of the system. These are found from the solution of the equations. Effectively, the equations eliminate the response of the non-resonant modes caused by the structural damping.

The extension of our method of separation to more degrees of freedom can easily be seen. For n degrees of freedom, there will be n simultaneous equations which can be solvent for the n modal responses. The ease of application of our analytical method can be seen by expressing the equations in matrix form.

$$\{R_Q\} = [A]\ \{R^*\}$$

where $\{R_Q\}$ is the column matrix of the measured quadrature response at each resonant frequency.

$[A]$ is a square matrix determined by the modal damping coefficients and the resonant frequencies.

$\{R^*\}$ is a column matrix of modal responses.

Transposing, we write the equation in the desired form:

$$\{R^*\} = [A]^{-1}\{R_Q\}$$

Our analytical method is easily applied since the inverted matrix, $[A]^{-1}$, is the same for all locations. Hence, the modal responses at all locations can be found by a simple matrix multiplication once the $[A]^{-1}$ matrix has been determined. It will be noted that only the test data normally required is used for the application of our analytical method; that is, the resonant frequencies, the mode shapes at each resonant frequency and the modal damping coefficients.

COMPONENT ANALYZER

We now proceed with the third point of the discussion, the description of the Component Analyzer and the results of some of our laboratory tests. To apply our method to the tests which we performed, we used a Component Analyzer which measures separately the in-phase and quadrature response. Figure 5 is block diagram of our Component Analyzer. It consists of an undamped strain gage accelerometer powered by the exciting force signal. Our Component Analyzer differs from that of Kearns in that the actual exciting force signal is used where Kearns used the current of an electro-magnetic shaker. This modification was necessary since the inertia and spring force of the shaker armature can cause large phase shifts between the armature current and the force applied to the structure particularly at resonance. The accelerometer was undamped to eliminate phase shifts in the transducer. When the in-phase response

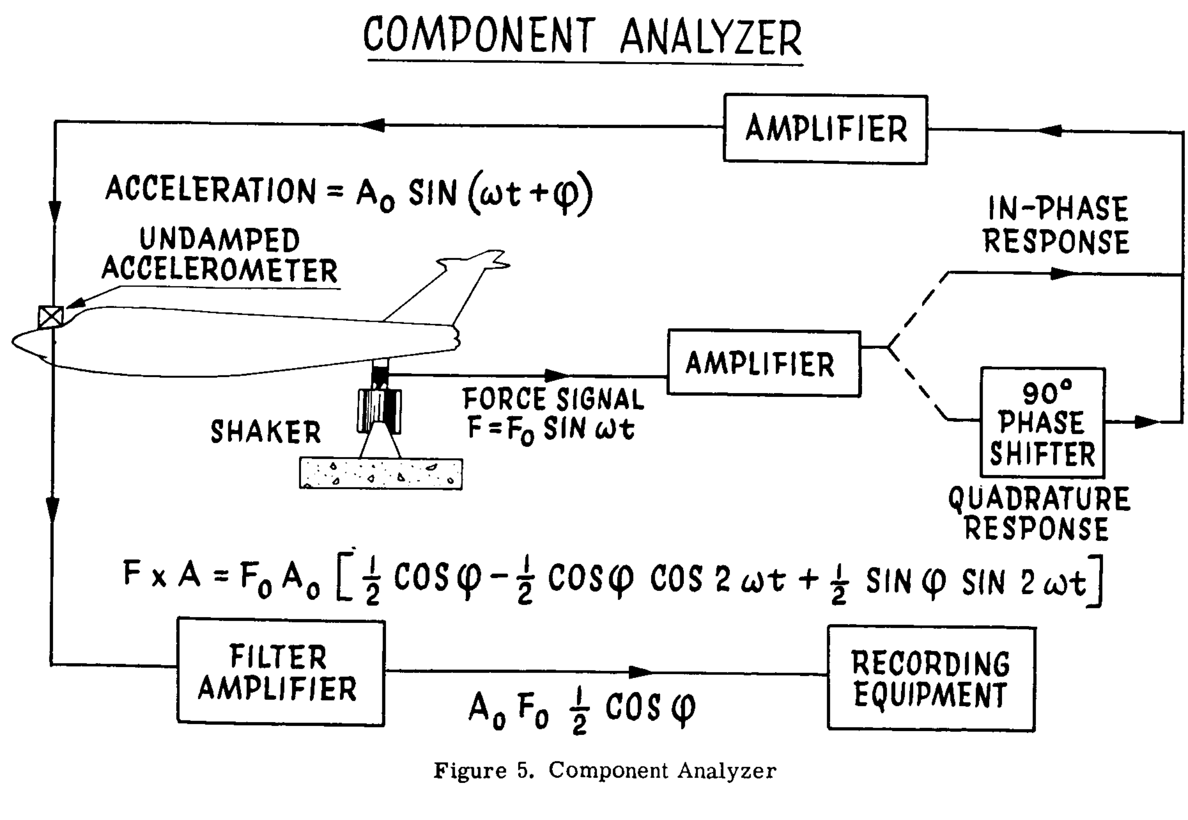


Figure 5. Component Analyzer

is measured, the force signal, $F_0 \ SIN \ \omega t$ is amplified and applied to the accelerometer. The accelerometer multiplies the force signal and the acceleration, $A \ SIN \ (\omega t + \varphi)$ with the steady component of the output, $1/2 \ A_0 F_0 \ COS \ \varphi$ being proportional to the in-phase response. The oscillatory parts of the signal are filtered out and the steady signal recorded. In the same manner, we obtain the quadrature response by shifting the phase of the force signal 90° before applying it to the accelerometer. Since the electrical signal from the Component Analyzer is not oscillatory, it can be applied to recording equipment such as an x-y plotter or an array of vertical deflecting galvanometers providing immediate records of frequency response and mode shape. The use of the accelerometer in this manner provides a measurement of the response only at the frequency of excitation. (5) The block diagram indicates the simplicity of the Component Analyzer.

One parameter which can cause considerable error in the Component Analyzer measurements is the rate of change of excitation frequency, the sweep rate. The effect of sweep on the in-phase and quadrature response is much greater than the effect on total response. Figure 6 shows the effect of sweep rate on the quadrature and in-phase velocity responses of a typical single degree of freedom system. The dashed lines represent the steady state response and the solid lines represent the swept responses with increasing frequency. Sweep causes a shift in the frequencies of the peak responses, variation in amplitudes, and causes oscillations in the response. These curves are based on the theoretical results of Hok (6) which were obtained for electrical circuits and agree qualitatively with those observed during tests.

EXPERIMENTAL RESULTS FROM TESTS

Although we have used this technique for Ground Vibration Tests of the YP6M, we will confine our discussion to results obtained from laboratory tests. We will describe the results we obtained from tests on a two degree of freedom system with resonant frequencies close together. The system on which our tests were conducted is shown in Figure 7, a rigid beam mounted on rubber vibration isolators at the approximate radius of gyration. The resonant frequencies of the beam on the isolators, rigid translation and pitch about the center, were close together. We applied excitation at a single point slightly off the center of the beam. The total, in-phase, and quadrature response at each end and the center of the beam were measured. The first mode at 16.9 cps is the translation mode. The mode shape determined from the total response is indicated by the solid line. The mode shape obtained from the quadrature response is shown by the dashed-dot line and the mode shape obtained from the analytical separation of the modes by the dashed line. The translation mode shapes obtained

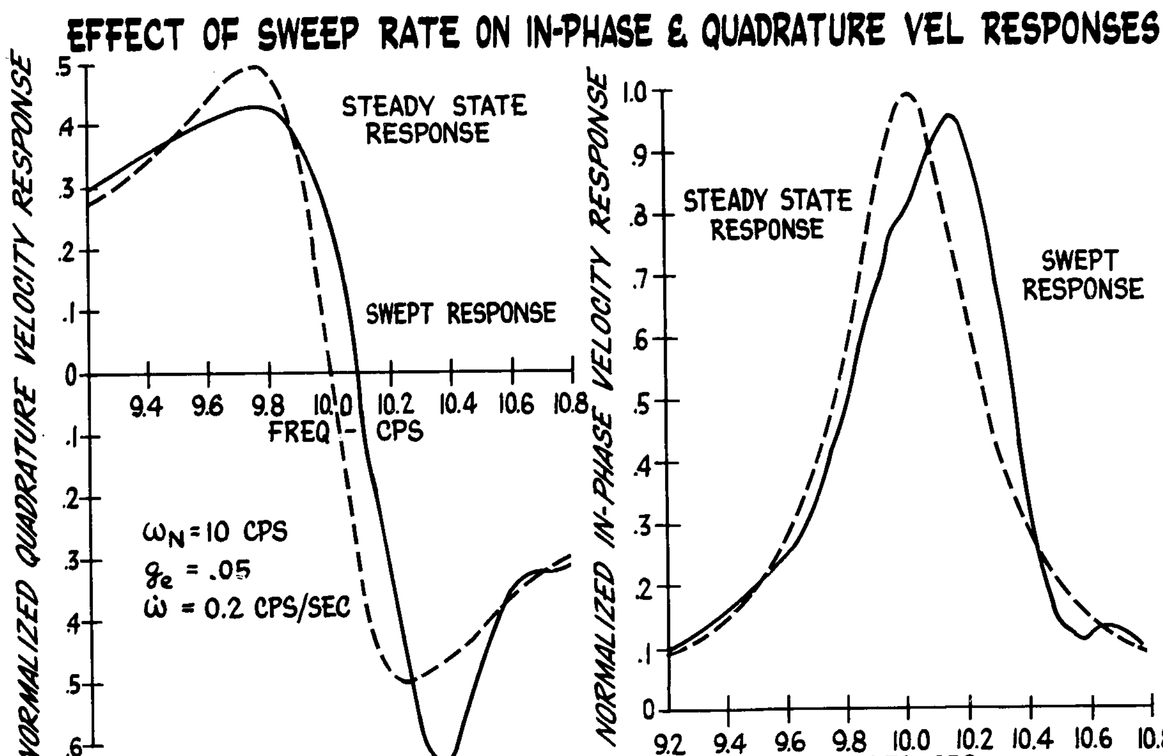


Figure 6. Effect of Sweep Rate on In-Phase and Quadrature Velocity Responses

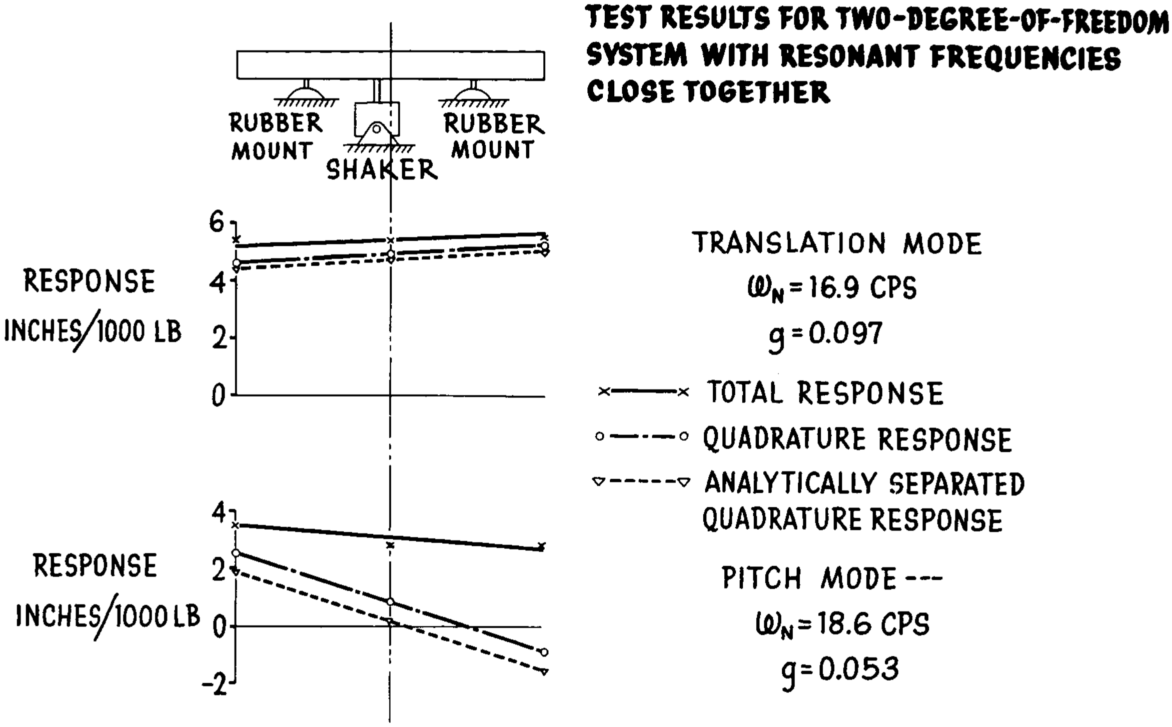


Figure 7. Test Results for Two-Degree-of-Freedom System with Resonant Frequencies Close Together

with the various techniques are practically the same. The slight pitch in the mode shapes was caused by a lack of symmetry in the beam and mounts, that is, the beam was not uniform and the mounts were not equally stiff. However, the mode shapes obtained for the pitch mode at 18.6 cps are substantially different. Since the excitation was applied near the mode of the pitch mode, the response in the translation mode was sufficiently large to distort the pitch mode. It is barely recognizable from the total response but begins to take form when determined from the quadrature response. The mode shape which we obtain from our method of analytical separation agrees almost exactly with the predicted mode shape (a straight line through the center of the beam).

If we compare the quadrature and total responses in the pitch mode at the end of the beam farthest away from the shaker, we see that the total response does not indicate the node line but the quadrature response does. Figure 8 shows the measured quadrature and total responses at this point. We have already demonstrated the improvement in mode shape. Now let us consider the resonant frequencies indicated by the measurements. The frequency of the translation mode indicated by the quadrature response differs only slightly from the actual frequency of 16.9 cps. The total response, however, indicates a resonant frequency of 17.15 cps, a shift of 1/4 cps. The resonant frequency of the pitch mode indicated by the quadrature response is 18.65 cps a shift of only .05 of a cps. The resonant frequency of the pitch mode is not apparent from the total response. Figure 2 which was discussed before shows the theoretical response at this point. The measured and theoretical responses are in very close agreement. In addition, the values of the damping coefficients obtained from the in-phase response agree within 5 percent with those measured from the decay of the total response.

APPLICATION TO FLIGHT FLUTTER TESTING

The fourth and last point of the paper, the application of the Component Analyzer to flight flutter testing, will now be presented. The Component Analyzer has one intrinsic property which makes it particularly suited to flight flutter testing with sinusoidal excitation. The Component Analyzer measures only the component of response at the excitation frequency either in-phase or 90° out-of-phase with the exciting force. Therefore, the response caused by atmospheric turbulence, which has been a problem in the past, will have a substantially decreased effect on the response measured with the Component Analyzer. A procedure which might be suggested is to use the Component Analyzer with the usual sweep technique of excitation. This procedure, however, has an undesirable feature since a very slow sweep rate would be required to obtain accurate measurements, as we have pointed out earlier. A technique which would eliminate the undesirable feature would be to slave the exciter frequency to the resonant frequency of the

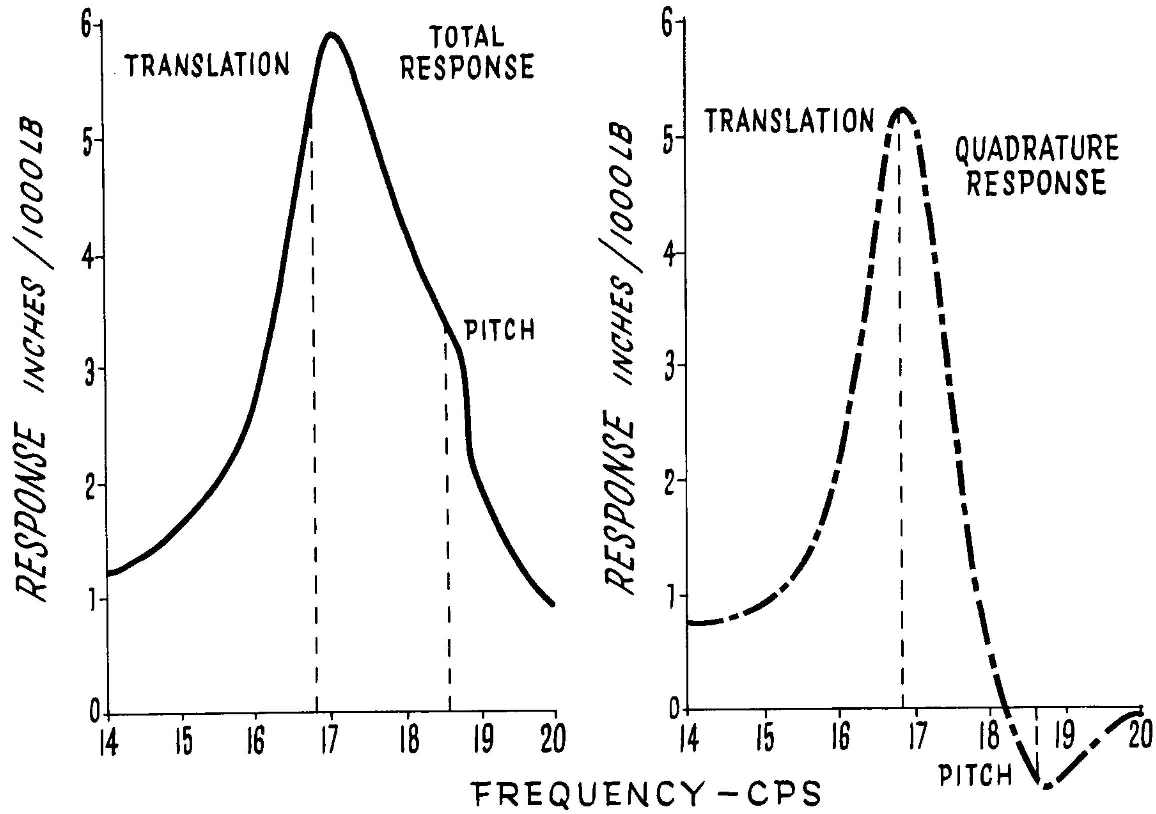


Figure 8. Measured Response at Far End of Beam

mode which we want to investigate. The variation of the quadrature response with aircraft velocity will be observed; an increase in this response amplitude with increasing flight speed will indicate approach to flutter.

In the procedure just discussed, each mode has to be investigated separately. A possible extension of this technique is to investigate several modes simultaneously. One exciter and one Component Analyzer for each mode has to be provided. Since each Component Analyzer responds only to its specific excitation, several modes may be investigated simultaneously.

CONCLUSION

The results of our tests indicate an immediate marked improvement in the determination of the vibration characteristics without the use of complicated methods of excitation. Analytical separation of the quadrature responses of the several structural modes yields a further improvement when the structure has resonant frequencies close together.

A procedure for extending the present technique to flight flutter testing has also been suggested. This procedure would decrease the effects of atmospheric turbulence; it might also be possible to eliminate the errors caused by a finite sweep rate.

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