N76-16765

EVALUATION OF RIDE QUALITY MEASUREMENT PROCEDURES BY SUBJECTIVE EXPERIMENTS USING SIMULATORS*

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Summary

For the purposes of vehicle design and procurement, well-defined procedures are needed for measuring ride quality. A number of more or less different Ride Quality Measurement Procedures (RQMP's) have been proposed and/or used in the past, e.g., ISO, ISO alternate, or Shaevitz exceedance counts.

Since ride quality is, by definition, a matter of passenger response, there is need for a Qualification Procedure (QP) for establishing the degree to which any particular RQMP does correlate with passenger responses. Once established, such a QP will provide very useful guidance for optimal adjustment of the various parameters which any given RQMP contains.

The present paper proposes a QP based on use of a ride motion simulator and on test subject responses to recordings of actual vehicle motions. Test subject responses are used to determine simulator gain settings for the individual recordings such as to make all of the simulated rides equally uncomfortable to the test subjects. Simulator platform accelerations vs. time are recorded with each ride at its equal discomfort gain setting. The equal discomfort platform acceleration recordings are then digitized. A computer is used to apply a prospective RQMP to each of the equally uncomfortable simulator motions and to determine the scatter among the ride index values which the RQMP assigns to these motions. The best RQMP will be taken to the one for which the scatter is smallest.

*This work was supported in part by AMTRAK and by NASA Langley Research Center.

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This program has been carried out on a pilot basis using the Passenger Ride Quality Apparatus at NASA Langley Research Center, using recordings of 19 passenger railcar ride motions (vertical and lateral), and working with subjective responses from a panel of four subjects.

The present paper includes a discussion of various RQMP's which are available, a description of the experimental procedure, and preliminary results illustrating the extent to which several particular RQMP's deviate from ideal correlation with passenger response.

1. The Role of Ride Motion Measurement in Vehicle Specifications

This article is motivated to a large extent by the needs of the engineer who is responsible for drawing up specifications for railroad or rail-transit cars and who seeks to insure that the cars will "ride" well.

The engineer can use either or both of the following two basic approaches:

1) he can set forth a prescription for measuring the ride motion of the new cars at stated speeds on stated trackage and require that the measured motion not exceed stated limits, or

2) he can rely on analysis and/or experience as a basis for requiring that the new car suspension incorporate specific ride quality related features he believes will help to secure a satisfactory ride.

One weakness of the second approach is that it limits the manufacturer's control over running gear design and may reduce the likelihood that the manufacturer can be held responsible for the ride quality consequences of the many other features which he himself must contribute to the suspension. Thus, for specifications on which there is to be competitive bidding, the engineer is likely to be more interested in specifying upper limits for measured motion of the resulting ride than in specifying details of suspension design.

A satisfactory specification of the manner in which the ride motion of a new car is to be tested must include a prescription for converting the vehicle's actual ride motion (e.g. vertical, lateral, and longitudinal acceleration as functions of time) into a number (or set of numbers) which can serve as a "measure" of the amount of motion as far as ride quality is concerned. A quantitative prescription of this type will be referred to as a measure of ride motion, or simply as a <u>ride measure</u>.

Section 2 below reviews the nature of the empirical data on human sensitivity to some particular motions. Section 3 reviews some ride measures which are available. Section 4 proposes a method for characterizing the extent to which any given ride measure represents discomfort as it is actually perceived by passengers. Section 5 describes an experimental procedure for obtaining the necessary empirical data. Section 6 describes some recent experimental work using the ride motion simulator at NASA Langley Research Center. Section 7 presents results of a preliminary evaluation of several ride measures.

2. Data on Passenger Sensitivity to Specific Motions

A number of investigators have published results of empirical studies on human sensitivity to sinusoidal motion and a few workers have reported on sensitivity to vibratory motion composed of randomly varying contributions having frequencies within a narrow band about a nominal central frequency. (See for example ref. 1.) The results of these studies are normally expressed via contours of vibration amplitude as a function of frequency with the contours being drawn so that the discomfort experienced by the average test subject is constant along any one contour. The contours are sometimes approximated via straight lin segments for ease of representation.

It will be convenient to have a name for referring to these contours. While the term isocomfort has been used, we will refer to each contour of equal discomfort as an isobother. Empiri research will presumably reveal that isobothers which differ in discomfort also show some variation in shape, analogous to that of the Fletcher-Munson curves for aural sensitivity. However, we will ignore such dependence and will denote the r.m.s. amplitude of the acceleration as a function of frequency along an isobother simply as $I(f)$.

The main appeal of sinusoidal motions is that the number of distinct sinusoidal motions (e.g., distinct combinations of frequency and amplitude) which are likely to be important in a given passenger environment is only about 300 (20 one-third octaves from 0.5 to 50 Hz , 5 amplitude levels for each one-third octave, and three directions of motion). This makes it practical to gather empirical data which will cover any sinusoidal motion which might be encountered.

When attention is turned to motions of a more general character, it becomes difficult even to find a way to ennumerating a set of distinct representative motions, and if a comprehensive ennumeration could be devised, testing of all of the representative would be a staggering task. On account of the foregoing, more general motions are not approached with the assumption that all possible types can be ennumerated. Instead, they are approached with the assumption that it will be possible to devise quantitative prescriptions (ride measures) for converting recorded acceleration histories directly into numerical measures of discomfort.

3. Examples of Ride Quality Measurement Procedures

The term ride measure was introduced at the end of Section 1 as a means of referring to a prescription for converting a record of acceleration as a function of time into a number which is intended to be a measure of the discomfort produced by the corresponding motion. The present section discusses a few examples of specific ride measures which have been formulated in the past and a few ways in which they can be generalized.

A. Exceedance Count Measures

These measures are based on counting the number of times that the acceleration crosses each of several acceleration thresholds. Prior to the development of modern electronic equipment, it was a standard railroad practice to have the acceleration recorded in an approximate manner on a strip chart by pens actuated mechanically by suspended masses. The thresholds were represented by grid lines printed on the charts, and the number of times that the signal crossed a grid line was counted by hand. With modern instrumentation, these functions can be accomplished electronically, and at least one firm (Schaevitz Engineering Co.) has marketed a ride recording instrument package set up on this basis.

If there is a need to determine which of two given rides is to be considered the more comfortable, and if the exceedance counts are selected as the basic measured data, then a formula must be chosen for converting each set of recorded exceedance counts into a single number which is to be the measure of the corresponding ride motion.

A formula used by the Pennsylvania Railroad to reduce exceedance counts from mechanical recorders was as follows: give each count a weight proportional to the square of the associated acceleration level and form the weighted average number of counts per unit time. Or, expressed in symbols,

$$\text{RMEC3} \propto \frac{\Delta a}{D} \sum_{\ell} \frac{3}{2} a_{\ell}^{2} C_{\ell}$$

where RMEC stands for "Ride Measure- Exceedance Count", the suffix 3 is included in preference to a suffix 2 (the exponent) for reasons which will appear later, D is the duration of the time of counting, a_{ℓ} is the acceleration at the ℓ th threshold, C_{ℓ} is the count for that threshold, the summation is over all of the thresholds, and the factor Δa, which is the spacing between adjacent thresholds, is included so that the whole expression will approach a finite limit if the spacing between thresholds approaches zero. The factor of 3/2 is included for later convenience. The symbol $\propto$ denoting "is proportional to" will be used for the time being, and a specific normalization will be suggested at the end of this section.

Having introduced this measure, we will now explore some of its features.

For conceptual purposes it is convenient to work with the limit in which the spacing between adjacent acceleration thresholds does approach zero. Thus we will use

$$\text{RMEC3} \propto \lim_{\Delta a \to 0} \left[\frac{\Delta a}{D} \sum_{\ell} \frac{3}{2} a_{\ell}^{2} \, C_{\ell} \right]$$

$$\propto \frac{1}{D} \int_{-\infty}^{\infty} da \, \frac{3}{2} a^{2} \, C(a)$$

When it is helpful to be more explicit, we can express the value obtained when this ride measure is applied to the acceleration signal $a(t)$ as

$$\text{RMEC3}\left[a(t)\right] \propto \frac{1}{D} \int_{-\infty}^{\infty} dx \, \frac{3}{2} x^{2} \, C_{\left[a(t)\right]}(x)$$

where $C_{[a(t)]}(x)$ is the number of times that the signal a(t) passes the threshold x during the interval D .

The general properties of C are $C(x) \geq 0$ for all x and $C(-\infty) = C(\infty) = 0$.

Applying the foregoing ride measure to a sinusoidal motion,

$$a(t) = A \sin(2\pi ft),$$

one has

$$\text{RMEC3}\left[A \sin(2\pi ft)\right] \propto \frac{1}{D} \int_{-A}^{A} da \, \frac{3}{2} a^{2} \quad f D$$

$$\propto \quad f A^{3}$$

where f and A are respectively the frequency and amplitude of the sinusoidal motion. The fact that the result is proportional to the third power of the amplitude provides the motive for use of the suffix 3 .

As there is likely to be interest in a ride measure which, when applied to a sinusoidal motion, will give a value proportional to $f A^2$, we may note two ways of arriving at such a measure.

From the preceding exercise with RMEC3 it is easy to see that the measure defined by

$$\text{RMEC2} \propto \frac{1}{D} \int_{-\infty}^{\infty} da\, |a|\, C(a)$$

will vary as the square of the amplitude when it is applied to a sinusoid.

Another definition with this feature may be obtained in a somewhat more intuitive manner as follows. Thinking in terms of the sum over discrete levels and using the a^2 values as weights, we want to increment the count for a given level only when that level is the highest (or lowest) one reached by a local peak (or valley) of the wave form. As that idea can be expressed in terms of differences between the counts which have been defined already we can write

$$\frac{1}{D} \left[\sum_{\ell=-\infty}^{-1} \frac{a_\ell^2}{2} (C_\ell - C_{\ell-1}) + \sum_{\ell=1}^{\infty} \frac{a_\ell^2}{2} (C_\ell - C_{\ell+1}) \right]$$

Putting $C_\ell - C_{\ell-1} = \Delta C_\ell$ and going to the limit of zero spacing between levels, this becomes an integral over acceleration, namely

$$\propto \frac{1}{D} \left[\int_{-\infty}^{0} \frac{a^2}{2}\, dC(x) - \int_{0}^{\infty} \frac{a^2}{2}\, dC(x) \right]$$

$$\propto \frac{1}{D} \left[- \int_{-\infty}^{0} da\, a\, C(a) + \int_{0}^{\infty} da\, a\, C(a) \right]$$

$$\propto \frac{1}{D} \int_{-\infty}^{\infty} da\, |a|\, C(a)$$

Thus we find that the two approaches give the same result.

We observe next that various forms of weight function can be tried in order to see which weight functions lead to ride measures which correlate best with passenger judgements. In this vein, let $w(a)$ represent an arbitrary weight function, and denote the corresponding exceedance count ride measure by

$$\text{RMECw} \propto \frac{1}{D} \left[\int_{-\infty}^{0} \frac{w(x)}{2}\, dC - \int_{0}^{\infty} \frac{w(x)}{2}\, dC \right]$$

In the interests of a simple notation, we take it as an axiom that the zero point on the axis of acceleration values is located at the point of minimum discomfort and that we will always have $w(0) = 0$ so that all measures will give the value zero when $a(t) = 0$ for all time t.

Then, integrating by parts, we have

$$\text{RMECw} \propto \frac{1}{D}\left[-\int_{-\infty}^{0} C\,\frac{w'}{2}\,dx + \int_{0}^{\infty} C\,\frac{w'}{2}\,dx\right]$$

If from physical symmetry it can be assured that $w(-x) = w(x)$, then the foregoing becomes

$$\propto \frac{1}{D}\int_{0}^{\infty}\left[C(x) + C(-x)\right]\frac{w'(x)}{2}\,dx$$

However, the original expression in terms of dC will usually be the more convenient one.

As an example of the application of the general definiti the value obtained when it is applied to a sinusoid is

$$\text{RMECw}\,[A_0 + A_1 \sin(2\pi ft)]$$

$$\propto \begin{cases} (f/2)[w(A_0 + A_1) + w(A_0 - A_1)], & \text{if } A_0 < A_1 \\ (f/2)[w(A_0 + A_1) - w(A_0 - A_1)], & \text{if } A_0 > A_1 \end{cases}$$

Whereas the above definitions assumed counts based on preset absolute acceleration values, one can also define counts based on thresholds whose locations are dependent on the recent past behavior of the acceleration.

The following is one simple way of obtaining counts based on moving thresholds. Namely, look at the local peaks and local valleys of the acceleration waveform and treat the wave form as a sequence: $a_1, a_2, \ldots, a_n$ where all the odd members are local peaks and the even members are local valleys (or vice versa). Then apply one of the previously described exceedance count ride measures as though the ride consisted of a sequence of unconnected segments:

from $-|a_1-a_2|/2$ to $+|a_1-a_2|/2$

then

from $-|a_2-a_3|/2$ to $+|a_2-a_3|/2$

etc.

An indication of the magnitude of the change in results which will follow from use of moving thresholds may be obtained by applying the formulae given above for:

$$a(t) = A_0 + A_1 \sin(2\pi ft)$$

to the case that $w(x)$ is x^2 or x^3 .

With static thresholds we have

$$\text{RMEC2}\left[A_0 + A_1 \sin(2\pi ft)\right] \propto \begin{cases} f(A_0^2 + A_1^2), & \text{if } A_0 < A_1 \\ f(2A_0A_1), & \text{if } A_0 > A_1 \end{cases}$$

and

$$\text{RMEC3}\left[A_0 + A_1 \sin(2\pi ft)\right] \propto f(A_1^3 + 3A_1A_0^2)$$

The results which apply if the static thresholds are replaced by moving ones are $f A_1^2$ and $f A_1^3$ respectively. Comparison with the preceding results indicates that the choice of the type of threshold can have a pronounced effect on the results.

While general discussion of the criteria of ride measure validity is reserved for Section 4, one criterion will be introduced here. Namely, if it were to be completely satisfactory, a ride measure ought, among other things, to yield the same value for all points on any one isobother (isobother being the term used in Section 2 to refer to a sinusoidal motion amplitude vs. frequency contour along which the average person judges annoyance to be constant).

In the limit that the acceleration discrimination level spacing tends to zero, any reasonable exceedance count ride measure can be made to satisfy this particular criterion exactly. All that is required is that the acceleration signal pass through a suitably chosen filter prior to counting of the exceedances.

Let $I(f)$ denote the isobother's amplitude as a function of frequency, and let $K(f)$ denote the magnitude of the transfer function of the filter. Then referring to the earlier expression for the value obtained when RMECw is applied to a sinusoid and denoting the even part of w by w_e , we have

$$\text{RMECw}\left[K_w(f)\ I(f) \sin(2\pi f t)\right] \propto f\, w_e\left[K_w(f)\ I(f)\right]$$

and requiring that this expression have a constant value, B, independent of f , we find

$$w_e\left[K_w(f)\ I(f)\right] = B/f$$

Thus the desired filter characteristic is determined to be

$$K_w(f) = w_e^{-1} (B/f) / I(f)$$

where w_e^{-1} denotes the function inverse to w_e. The inverse will exist because $w_e(x)$ must be monotonically increasing function of x if it is to be physically reasonable.

If the criterion of constant value at all points on any one isobother were assumed to be a sufficient test of ride measur validity, then the preceding consideration would settle the question of the relative manner in which any exceedance count ride measure should treat different frequency components in the signal it receives. However, the foregoing consideration will be regarded here as a motivation for introducing the filter rather than as a basis for deciding what characteristic the filter shoul have.

Stephens (reference 2) has given interesting data characterizing vehicle motions in terms of the maximum value of $a(t)$ in each motion recording. That ride measure can be regarded as a representative of a group of measures which can be written in terms of the function inverse to $C_{[a(t)]}(x)$. Namely, letting $A(c)$ be the acceleration at the largest threshold which is crossed c times by the signal $|a(t)|$, one can write a measure in the form

$$\sum_{c=1}^{\infty} w[A(c)]\ V(c)$$

The specific example used by Stephens has $w[A] = A$, $V(1) = 1$, and $V(i) = 0$ for $i>1$.

B. Exceedance Time Measures

When reliance had to be placed on mechanical means, exceedance counts were used because it was easier to count the number of times that the acceleration crossed each of several thresholds than it was to determine the cumulative time spent above each one of them. However, as the development of electroni has made it easy to determine exceedance times, exceedance time measures have become of interest. United Aircraft Corp. was an early user of this approach.

Let $T_{[a(t)]}(x)$ be the cumulative time during which the acceleration

$$a(t) > x \quad \text{if} \quad x > 0$$

and during which

$$a(t) < x \text{ if } x < 0$$

Then the common exceedance time ride measure may be defined as

$$RMETw \propto \frac{1}{D} \left[\int_{-\infty}^{0} w(x)\ dT(x) - \int_{0}^{\infty} w(x)\ dT(x) \right]$$

This measure may be more familiar in the guise,

$$RMETw \propto \frac{1}{D} \int_{0}^{D} dt\ w\left[a(t)\right]$$

The latter form calls attention to the fact that this exceedance time measure is the same as the time average of the corresponding function of the acceleration. It is usually also the more convenient form when $a(t)$ is a mathematical function, such as a sinusoid. (To show the equivalence of the two forms, one may express the second form in terms of a series based on division of the acceleration range into a number of equal sized small segments and then let the segment size tend to zero so that the series becomes an integral over the acceleration range.)

The weighting function which has generally been used in past work is $w(a) = a^2$, in which case the exceedance time measure is the mean square value of the acceleration (for example, see ref. 3).

Taking $w(x) = x^2$, and applying the measure to a sinusoid, one obtains

$$RMET2\left[A_0 + A_1 \sin(2\pi f t)\right] \propto A_0^2 + \frac{1}{2} A_1^2$$

Thus, with the static thresholds which have been assumed, a constant term in the acceleration appears to affect exceedance time measures more strongly than it affects the corresponding exceedance count measures.

One other specific form of exceedance time measure which has occasionally been used in procurement specifications is that based on the weighting function

$$w(x) = \text{stepA}(x) \equiv \begin{cases} 0 & \text{if } |x| \leq A \\ 1 & \text{if } |x| > A \end{cases}$$

In this case it is convenient to integrate the first definition by parts to obtain

$$\text{RMETw} \propto \frac{1}{D}\left[- \int_{-\infty}^{0} T \, dw + \int_{0}^{\infty} T \, dw \right]$$

from which we have

$$\text{RMETstepA} = [T(-A) + T(A)]/D$$

Thus this measure is seen to give the fraction of the time that the magnitude of the acceleration exceeds the value A. The only virtues this measure possesses are that it is easy to understand and easy to implement.

Since the values obtained when exceedance time measures are applied to a sinusoid are independent of the frequency of the sinusoid, every exceedance time measure will be consistant with the isobother data if the acceleration signal is passed through a filter with transfer function magnitude proportional to $1/I(t)$ prior to determination of the exceedance times.

Another measure used by Stephens (reference 2) is defined as the value A such that $|a(t)| > A$ for 10 percent of the duration of the ride. This measure may be treated as being of the form

$$- \frac{1}{D} \int_{0}^{\infty} w(x) \; v(T(x)) \; dT(x)$$

with $T(x)$ defined as $T\left[|a(t)|\right](x)$, with $w(x) = x$, and with $v(T) = \delta(T - .9D)$ (where $\delta(x)$ is the symbol commonly used for the derivative of the unit step function with step at $x = 0$). The additional freedom which can be introduced by varying the weighting function $v(T)$ may turn out to be useful.

C. Spectral Measures

Whereas the measures discussed above deal directly with the acceleration as a function of time it is also possible to deal with the Fourier transform of the acceleration. To simplify the discussion, we will assume that suitable weighting of the various spectral components (such as might be needed for consistency with isobother data) has already been accomplished via filtering prior to the Fourier transformation or via numerical scaling of each of the spectral components after the transformation.

There are two forms of spectral measure which have been discussed extensively in the past. One is something like an exceedance count measure and the other is analogous to an exceedance time measure.

The former is the prescription recommended in International Standard 2631 (ref. 4). The prior scaling of the various frequency components is specified based on isobother data.

The prescription requires ascertaining the r.m.s. value of each standard 1/3 octave band contribution in the spectrum. The value assigned by this measure is the largest of the r.m.s. values obtained in that manner. While this measure is quite adequate for dealing with sinusoidal motions, it is not a plausible approach to more general motions. (For example, if two sinusoidal motions which are separated in frequency by an octave or so are valued equally by this measure, the motion obtained by superposing them will receive the same value as either one alone.)

The other spectral measure which has been discussed frequently in the past (refs. 3, 4, 5) is that obtained by integrating the square of the magnitude of the transform with respect to frequency. By Parcival's theorem, this particular measure is equivalent to the corresponding exceedance time measure, namely the mean square value of the acceleration. However, integration of functions of the magnitude of Fourier transform other than the square will lead to measures which do not have simple exceedance time measure equivalents.

Mention may also be made of the interesting hybrid measure introduced by Brickman, Wambold, and Zimmermann (refs. 6 and 7). This measure is based on obtaining the spectra of a succession of short samples of motion, tabulating transform amplitude threshold exceedance counts, and forming an average weighted both with respect to amplitude and frequency.

D. Scaling and Normalization

The specific sample ride measures discussed above incorporate weighting functions which are proportional to a power of the acceleration. Thus, they are homogeneous in the sense that

$$RMn[b\ a(t)] = b^n\ RMn[a(t)]$$

where RM denotes the measure, b is an overall factor by which the acceleration function is multiplied, and n is the exponent of acceleration in the weighting function. Taking the case of power law exceedance count measure as an example we have

$$\mathrm{RMECn}\left[b\ a(t)\right] \propto \frac{1}{2D}\left\{\int_{-\infty}^{0} |x|^n \, dC_{\left[b\ a(t)\right]}(x) - \int_{0}^{\infty} |x|^n \, dC_{\left[b\ a(t)\right]}\right\}$$

where it will be recalled that $C_{[b\ a(t)]}(x)$ is the number of times that the signal $b\ a(t)$ crosses the threshold x during the interval D.

It follows from the definition of the exceedance count function that

$$C_{\left[b\ a(t)\right]}(bx) = C_{\left[a(t)\right]}(x)$$

so that

$$C_{[b\ a(t)]}(x) = C_{[a(t)]}(x/b)$$

Thus,

$$\mathrm{RMECn}[b\ a(t)]$$

$$\propto \frac{1}{2\ D}\left\{\int_{-\infty}^{0} |x|^n \, dC_{\left[a(t)\right]}(x/b) - \int_{0}^{\infty} |x|^n \, dC_{\left[a(t)\right]}(x/b)\right\}$$

$$\frac{b^n}{2\ D}\left\{\int_{-\infty}^{0} |y|^n \, dC_{\left[a(t)\right]}(y) - \int_{0}^{\infty} |y|^n \, dC_{\left[a(t)\right]}(y)\right\}$$

$$= b^n \ \mathrm{RMECn}[a(t)]$$

Any measure which is homogeneous may be rescaled so as to be linear. That is, defining the rescaled measure as the nth root of the original measure, we have

$$\mathrm{LRMn}[b\ a(t)] \equiv \left[\mathrm{RMn}[b\ a(t)]\right]^{1/n}$$

$$= b\ \mathrm{LRMn}[a(t)]$$

A linear measure can be normalized so as to assign the r.m.s. value to sinusoidal motion at some reference frequency. Then to the extent that the measure correlates well with comfort, the value which it assigns to any other motion will be the r.m.s. amplitude of an equally uncomfortable sinusoidal motion with frequency equal to the reference frequency.

Homogeneity is convenient because it permits a measure to be interpreted in the simple manner indicated above. However, it may be found that the ride measures which correlate best with subjective judgements of ride quality are not homogeneous.

The nonhomogeneous examples which come most easily to mind are those obtained when the simple power of acceleration which occurs in one of the homogeneous measures is replaced by some more general function of the acceleration such as a polynomial or a combination of exponentical functions.

One example using the hyperbolic cosine is

$$\text{RMET } \cosh\left[a(t)\right] \propto \frac{1}{D}\int_0^D dt \cosh\left[k\, a(t)\right]$$

where k is an adjustable parameter.

Looking at the example

$$a(t) = A\cos(2\pi f t)$$

one has

$$\begin{aligned}\text{RMET } \cosh\left[A\cos(2\pi f t)\right] &\propto \frac{1}{D}\int_0^D dt \cosh\left[k A \cos(2\pi f t)\right] \\ &\propto \frac{1}{\pi}\int_0^\pi d\theta \cosh\left[k A \cos\theta\right] \\ &\propto I_0(k A)\end{aligned}$$

where $I_0(x)$ is a modified Bessel function (reference 8) whose behavior is somewhat like that of the exponential function.

This measure may be rescaled so that (ignoring the effect of preliminary filtering) it assigns the r.m.s. value to any sinusoidal motion. Namely, writing $I_0^{-1}(x)$ for the function inverse to $I_0(x)$, the rescaled measure is

$$\text{LRMET} \quad \cosh\,[a(t)] \;\propto\; \frac{1}{k}\, I_0^{-1} \left\{ \frac{1}{D}\int_0^D dt\, \cosh\left[k\; a(t)\right] \right\}$$

Rescaling of the type just illustrated may be applied to any measure of this general sort, but whereas with a homogeneous measure the result would be a linear measure, here the result is a measure which is linear only so long as the motion being measured is sinusoidal.

In order to facilitate exchange of information, it might be desirable for all ride measures to have their outputs scaled so as to assign the r.m.s. value to the motion consisting of sinusoidal vertical oscillation at a chosen frequency such as 1 or 6.0 Hz.

4. A Method for Testing and Development of Ride Measures

The need which engineers have for a means of specifying ride comfort was discussed in Section 1. Section 3 has indicated that there are many different measures available for this purpose. Supposing that two such measures are under consideration, we come now to the question of how to decide which one is better. We will argue that this question has a reasonably definite answer and that that answer suggests a practical program for ride measure development and validation.

We take it as a postulate that a ride measure will be completely satisfactory only if it correlates fully with discomfort as perceived by the average passenger. (Here, as elsewhere, we assume that it is meaningful to talk about an "average passenger" and that the average passenger perceives discomfort due to ride motion as a scalar quantity. Naturally, the average passenger's response can be expected to vary depending on duration of exposure, type of seat, activity during travel, etc.) Expressed symbolically our postulate is that a ride measure, RM , will not be completely satisfactory unless it has the property that $RM(R_1) = RM(R_2)$ for every pair of ride motions R_1 and R_2 such that R_1 and R_2 are equally annoying to the average passenger.

This postulate suggests two different ways of determining how satisfactory a given ride measure is. The first way is to look at the scatter in the values assigned by the ride measure to a number of rides which are equally uncomfortable to the average passenger. That is the method which we propose. The other way is to look at the variation in perceived discomfort for a number of rides all of which are assigned the same value by the ride measure. Since difference in discomfort is somewhat ambiguous from an experimental point of view, we regard the proposed approach as the proper one in principle.

The first step in conducting either kind of correlation deficiency test is to make or select recordings of the ride motions on which the test is to be based. One might seek to develop a ride measure which could be applied to any motion environment. However, the specific ride measure which correlates best with comfort for one mode of travel and range of speeds may not be the same as the specific ride measure which correlates best with comfort for a larger group of modes and speeds. To the extent that this is so, development and testing of a ride measure should be based on ride recordings exhibiting the kinds of motion that might actually be produced by the equipment in whose specification the ride measure is to be used.

The proposed approach (i.e. determine the scatter of the values which the measure assigns to the members of a group of equally uncomfortable rides) may be carried out by: 1) using a dynamic ride simulator to reproduce each of the chosen ride motions, 2) adjusting the overall motion amplitude of each ride until the test subjects sitting in the simulator judge its discomfort to be equal to that of each of the other rides, and 3) determining the value assigned to each of the equal discomfort motions by the ride measure under test.

This method of testing has a feature which makes it very convenient for the purpose of ride measure development and optimization. Namely, since the necessary empirical data consists just of recordings of ride motions which have all been normalized to a common level of perceived discomfort, the data may be gathered without reference to any particular ride measure. Once the normalized ride motions have been recorded in digital form, the task of testing and optomizing a prospective measure (with respect to that library of normalized rides) becomes one of computation alone.

The other method of testing would require that the ride measure under test be known and in operation for the gathering of the emperical data and would make the data specific to the ride measure used. Thus it is not only inferior in principle but would be very inconvenient in practice as well.

The indicated advantage of the proposed method of testing is a reflection of the fact that it treats discomfort as the independent variable and the corresponding ride measure values measured as dependent variables. Thus, results obtained using the proposed method are convenient from the point of view of the engineer who begins with a design goal for comfort and who wishes to know what limit he must place on the measured value of the motion.

Jacobson and Kuhlthau (ref. 3) have described an alternative approach to testing and development of ride measures which has an advantage of greater realism of motion environment due to gathering of test subject responses in actual vehicle travel but in which the bases for test subject judgements cannot be as clearly defined.

5. A Symmetrized Experimental Procedure

The authors' thinking in the area of experimental method was stimulated by a paper by C. Ashley (ref. 9). Ashley determined isobother curve amplitudes at various frequency points by adjusting the amplitude until the test subject judged the sinusoid to be equal in discomfort to a quasi-constant random reference signal to which the test subject was alternately exposed. Ashley's procedure constitutes a significant improvement over procedures which seek to have subjects compare ride motions which differ in discomfort, and it could be used for the program outlined in Section 4 above. However, it may be feared that singling any one motion out as the standard of reference for all of the others could cause some undetectable bias. (For example, repeated exposure to the reference motion could cause test subjects to become unduly sensitive to other motions which were similar to it.)

Partly from fear of bias, and partly because of aesthetic dissatisfaction with the lack of symmetry if one motion is singled out as a standard, the authors have employed a symmetrical procedure as follows:

Let the number of ride motion samples to be used be n. Imagine that ride i is fed to the simulator with variable gain and that it is compared to ride j which is fed to the simulator with the gain at which it is recorded. Let g_{ij} denote the gain value which makes ride i's discomfort equal to that of ride j. Note that g_{ij} is defined in terms of a "true" equality and is not meant to be effected by inconsistancies in test subject responses. While there are n(n-1)/2 different (i j) combinations, the set of g_{ij}'s possesses only (n-1) degrees of freedom; namely they may all be determined from the values g_{n1}, g_{n2}, $g_{n,n-1}$ via the relations,

$$g_{ij} = g_{in}\, g_{nj}$$
$$= g_{nj}/g_{ni}$$

On the other hand, let r_{ij} denote the corresponding gain settings as determined from test subject responses during a particular set of comparisons using a ride motion simulator. Because of experimental error the r_{ij} values will not be transitive (i.e. $r_{ij} r_{jk}$ will not equal r_{ik}).

We, therefore, seek the set of g_{ij} values which provides the best fit to the experimental r_{ij} values.

The variables to be determined are $g_{n1}, g_{n2}, \ldots g_{n,n-1}$, which we will abbreviate as $g_1, g_2, \ldots g_{n-1}$. For the error function which is to be minimized we take

$$E = \frac{1}{2} \sum_{i \neq j}' \left[r_{ij}/g_{ij} - 1 \right]^2 = \frac{1}{2} \sum_{i \neq j}' \left[g_i r_{ij}/g_j - 1 \right]^2$$

where the prime over the summation symbol is to indicate that a given (ij) pair is not to be included in the sum if the corresponding r_{ij} was not measured. We presume that the error function given above is the best choice. However, we are not aware of any theorem to that effect, and there are other simple positive definite functions which could be used.

The g_i values which minimize E are found with the help of a simple computer code which uses Newton's method and iterates until the partial derivatives, $\partial E/\partial g_i$, are all close to zero.

The level of discomfort to which all of the rides are to be adjusted is chosen to be that of ride n when its gain is multiplied by

$$g_{n,mean} = \left[g_1 \, g_2 \cdots g_{n-1} \right]^{1/n}$$

The comfort of ride i is brought to that level by multiplying its gain by the factor

$$S_i = g_{n,mean} / g_{ni}$$

This choice of settings has the desirable property that the product

$$S_1 \, S_2 \ldots S_n = 1$$

and thus that the passenger reponses can not cause any rise or fall in the geometric mean of all of the settings.

Determination of the S_i's should be done in two or three stages with the first one serving to bring all of the ride samples close to a common level of discomfort so that adjustments in subsequent stages will be small. The motive here is to minimize errors which would arise from nonlinearity in simulator and test subject responses.

As a further detail of procedure, the ride i - ride j pairs are presented to the test subjects in a random order.

6. Production of a Library of 19 Equal Discomfort Rides

The authors have carried out the steps set forth above on a pilot basis as follows:

A) Selection of Sample Motions

Seventeen samples of passenger rail car ride motion were selected so as to include a number of distinctly different types of disturbing motion as well as several "good" rides. Each rail car sample included vertical acceleration and lateral acceleration as sensed by accelerometers located on the floor of the car over one of the trucks. Two sinusoidal samples were added to the collection so as to facilitate comparison with work by others.

The numbers of segments from the various sources were:

Car Type	Truck Type	# of segments
Metroliner	G70	5
St. Louis Silver Liner	G70	4
Penn Central E5	Commonwealth inside S.H.	2
DOT Test Car	Pioneer	2
Santa Fe High Level	Commonwealth outside S.H.	2
Budd Silverliner	Pioneer	1
GE Silverliner	G70	1
Sine Wave, 6 Hz.	1 lateral, 1 vertical	2
	TOTAL	19

The disturbing motions which are represented were described when they were recorded by terms such as, brake shudder, chafing, grinding, resonance, bounding, growling, lurching, and bottoming.

B) Presentation of Pairs of Rides to the Test Subjects

The ride motion simulator used in this work was the Passenger Ride Quality Apparatus (PRQA) at NASA Langley Research Center. Data were gathered on the basis of responses from three men and one woman seated in aircraft "tourist class" type seats.

Let A and B denote two ride motions being compared. The two rides were fed to the PRQA in accordance with the following protocol:

ride A , 10 sec
pause , 2 sec
ride B , 10 sec
pause , 2 sec
ride A , 10 sec

stop tape drive

have subjects say which ride was more annoying

manually adjust the separate gain controls provided for rides A and B so as to reduce the difference in annoyance

ride B
ride A
ride B

sample and pause durations as before

stop tape drive.

have subjects say which ride was more annoying

manually adjust gain settings so as to further reduce the difference in annoyance.

The above sequence was repeated until the test subjects indicated that the two rides were equally annoying. At that point the gain settings for both rides were recorded and the test tape was run forward to the next pair of rides.

The 10 sec and 2 sec durations appeared to be satisfactory. The ordering of pairs on the test tapes was randomized. Independent control of the gains for rides A and B was accomplished by means of an electronic control module located between the tape drive and the PRQA and controlled by timing and switching signals on tape channels 7 and 8. The person conducting the test was kept informed of the identities of the individual rides via a digital read out operated by coding on tape channels 9 through 14. That module also operated a pair of lights for keeping the test subjects informed as to whether the ride in progress was A or B.

C) Determination of Gain Settings for Equal Discomfort

The testing accomplished to date has consisted of only one cycle. Thus the gain setting ratios, r_{ij}, which have been measured are fairly large. However, the procedure set forth in Section 5 has been carried out and a recording of simulator platform motion has been made for each ride with gain setting equal to 0.496 times the S_i value defined in Section 5. (The extra factor of 0.496 was introduced to assure that the PRQA would not be over driven.) The final gain settings based on 76 pair comparisons were:

RIDE NO.	GAIN SETTING
1	0.621
2	0.525
3	0.504
4	0.362
5	0.306
6	0.509
7	0.429
8	0.482
9	0.572
10	0.362
11	0.531
12	0.384
13	0.302
14	0.800
15	0.609
16	1.500
17	0.860
18	0.360
19	0.300

The characteristics of the signals fed to the gain control module and of the accelerations of the PRQA platform pursuant to the final gain settings are both illustrated by the computer generated oscillograms reproduced in figures 1 through 19. Figures 20 and 21 show the r.m.s. values of the vertical and horizontal components of each of the rides both by half octave band and overall.

For the recording of the PRQA motions in response to the rides at their final settings, the PRQA was ballasted with 3 passengers and 68 kg (150 lbs) of bagged sand. Rides 1 through 19 were played in sequence with brief pauses between rides. As a matter of curiosity, each passenger was asked to rate each ride on a numerical scale from 0 (no discomfort) to 8 (maximum discomfort). No further verbal instruction was given. The results were as follows:

DISCOMFORT RATINGS BY "BALLAST" PASSENGERS

(ratings shown for each subject have been divided by the mean value of the ratings which that subject assigned)

RIDE	SUBJECT 1	SUBJECT 2	SUBJECT 3	MEAN X	SAMPLE STANDARD DEVIATION S
1	1.28	.57	1.02	.96	.36
2	1.06	.75	1.02	.94	.17
3	1.17	.94	1.02	1.02	.12
4	1.40	1.64	1.10	1.38	.27
5	1.01	.50	1.02	.84	.30
6	.84	1.01	1.00	.95	.10
7	1.01	1.13	1.02	1.05	.07
8	.67	1.07	1.02	.92	.22
9	1.01	1.38	1.05	1.15	.20
10	1.51	1.57	1.30	1.46	.14
11	1.89	1.19	1.02	1.03	.15
12	.61	.88	.97	.82	.19
13	1.17	1.32	1.02	1.17	.15
14	.50	157	.90	.66	.21
15	1.06	1.19	1.10	1.12	.07
16	.84	.75	1.02	.87	.14
17	1.01	.31	.27	.53	.42
18	1.56	1.13	1.05	1.25	.27
19	.45	1.31	1.05	.88	.37
S	.31	.36	.19	.23	

The "ballast" subjects appear to find some significant differences in discomfort among the final rides. The following may be noted as possible sources of difference:

- the "ballast" subjects rode in the simulator for a much shorter time than the original subjects
- the empirical gain ratios were larger than one would wish because circumstances have not yet allowed for a second stage of comparisons with the starting gains equal to the final gains from the first cycle.
- the judgements of the ballast subjects may have included some extra randomness due to ambiguity as to frame of reference.

7. Preliminary Evaluation of Several Ride Measures

We have begun to carry out the program of section four using the library of 19 rides described in section six. Work to date has been limited to an initial scrutiny of the family of ride measures given by the formula

$$RM = \sum_{k=1}^{4} \sum_{i}^{\text{band } k} \left[A_k + B_k \log(f_i) \right] V_i^n + \sum_{k=6}^{7} \sum_{i}^{\text{band } k} \left[A_k + B_k \log(f_i) \right] H_i^n$$

where V_i and H_i are the magnitudes of the vertical and lateral acceleration Fourier components at frequency f_i, where i is summed over the frequency points in each of the bands into which the frequency range is divided, and where the disposable parameters of the measure are the exponent n, the constants A_k and B_k which define the semi-log straight line weighting function in frequency band k, and the locations of the boundaries of the bands in the frequency range. The A's and B's are constrained so as to make the weighting function continuous at the band boundaries. Thus, for any fixed choice of frequency band boundaries, the weighting curves offer eight disposable parameters. Overall normalization effectively reduces that number to seven. This ride measure is convenient for purposes of exploration because it depends linearly on the weighting function height parameters.

A least squares fitting routine was used to find the weighting curve height parameters which minimize the error function

$$\text{Error} = \sum_{i=1}^{19} \left(RM_i - 1 \right)^2$$

where RM_i is the value assigned to the i_{th} ride.

This fitting was done with the exponent, n, and the frequency band boundary points fixed and was repeated for several combinations of exponent and frequency band boundaries.

For the purpose of comparing measures with different exponent values we use the sample standard deviation of the linearized form of each measure, namely

$$\text{Deviation} = \left\{ \frac{1}{18} \sum_{i=1}^{19} \left[RM_i^{1/n} - 1 \right]^2 \right\}^{1/2}$$

Table 1 shows some sample results with the weighting curve heights scaled so that each curve has height unity at the beginning of the fourth band of the vertical spectrum. An exponent value of four was also tried but was found to give residual errors larger than those obtained with the exponent value three.

One may note that some of the weighting curve heights are negative. While it is clear that the occurrance of negative weighting values can be legitimate relative to a fixed set of ride motions, it is also clear that a ride measure with some negative spectral weights will fail badly if it is applied to a sinusoidal motion with a frequency such that the corresponding weighting is negative. Thus for results which are to be used in practice, the weighting would need to be made everywhere positive, either by constraint, or by augmenting the library of equal discomfort rides with rides having appreciable energy at frequencies where negative weights had been obtained.

While the specific results obtained to date must be considered tentative because of the limitations of the equal discomfort ride data base both as to number of rides and as to likelihood of scatter in actual discomfort, they suggest the following three conclusions.

First, to obtain parameter optimization results which are not unduly sensitive to minor variations in the structure of the model, the empirical data base of equal discomfort ride motions will need to be a good deal larger than the one discussed here.

Second, for rail car comfort the upper portion of the frequency range appears to be more important than the isobother type data would suggest.

Third, the square of the acceleration appears to provide a better measure than does either the first or the third power.

8. Acknowledgements

The ride recordings and test tapes referred to in Section 6 were prepared by R. Arendt of Calspan Corporation with assistance from C. A. Woodbury of Louis T. Klauder and Associates, and constituted an essential part of the experimental work reported here.

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TABLE 1 – VALUES FOR WEIGHTING CURVE HEIGHTS AS DETERMINED BY LEAST SQUARES FIT AND CORRESPONDING RESIDUAL DEVIATION, AS DEFINED IN SECTION 7

FREQUENCY BAND BOUNDARIES VERTICAL	FREQUENCY BAND BOUNDARIES HORIZONTAL	EXPONENT VALUE	DEVIATION	RELATIVE HEIGHTS OF RESULTING WEIGHTING CURVE AT FREQUENCY BAND BOUNDARIES – VERTICAL					HORIZONTAL		
.5 1.5 4 12 30	.5 6 30	1	0.21	−1.49	.59	−.19	1.0	−1.30	−1.81	1.27	5.82
		2	0.17	− .16	.02	−.77	−1.0	−18.05	− .41	2.80	9.19
		3	0.21	− .01	−.01	−.19	−1.0	−4.52	− .19	1.85	6.28
.5 1 2 6 30	.5 6 30	1	0.21	−3.68	.10	4.09	1.0	12.15	−4.26	3.59	5.16
		2	0.163	− .11	.32	.89	1.0	5.61	− .36	.78	2.08
		3	0.19	.57	.34	.87	1.0	4.13	− .38	3.64	12.30
.5 1 2 6 30	.5 4 30	1	0.21	−4.80	.34	4.61	1.0	17.15	−4.37	3.83	−3.61
		2	0.165	− .10	.31	.87	1.0	5.68	− .34	.44	1.74
		3	0.19	.82	.41	1.06	1.0	4.09	− .50	3.20	25.27

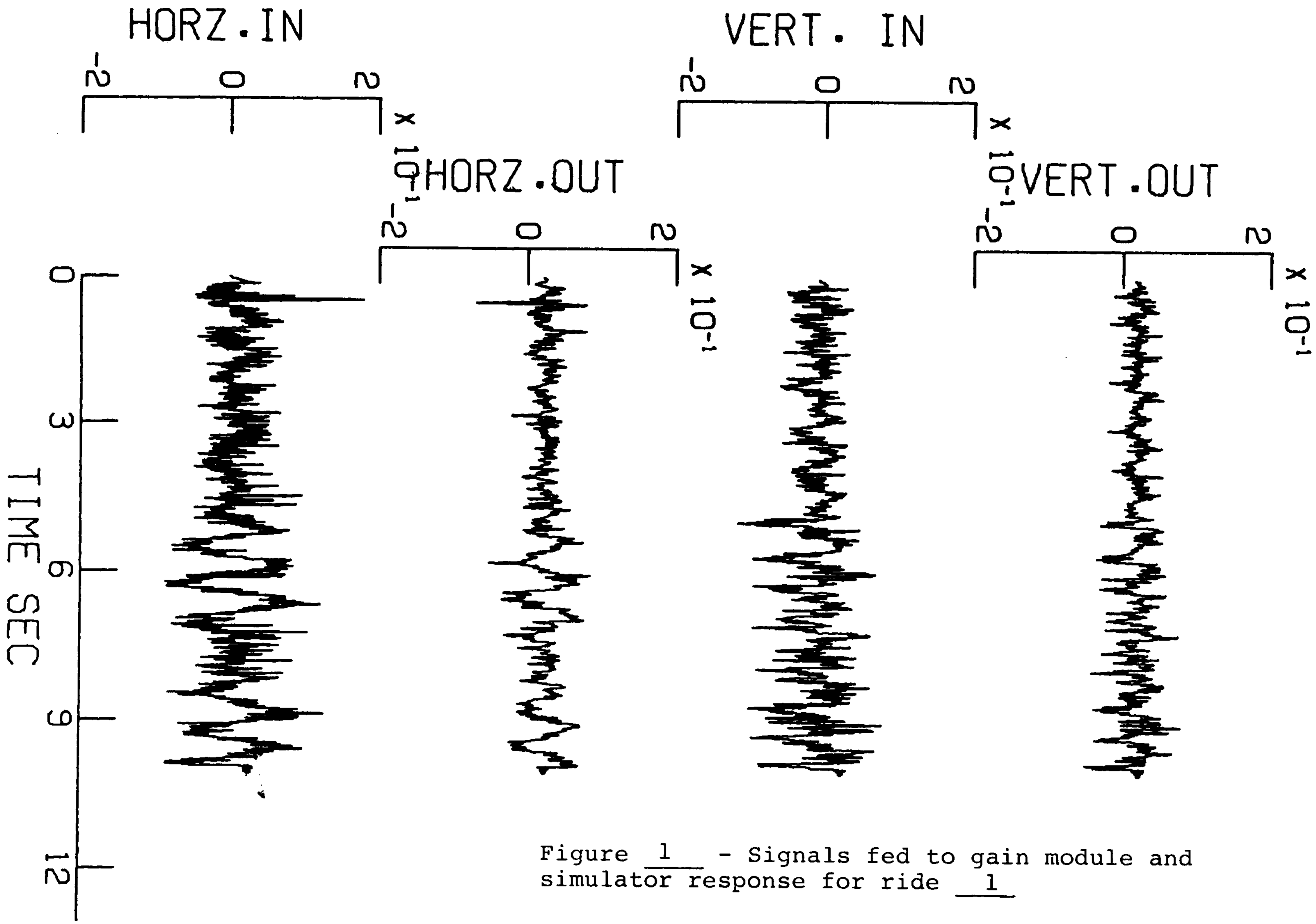

Figure 1 - Signals fed to gain module and simulator response for ride 1

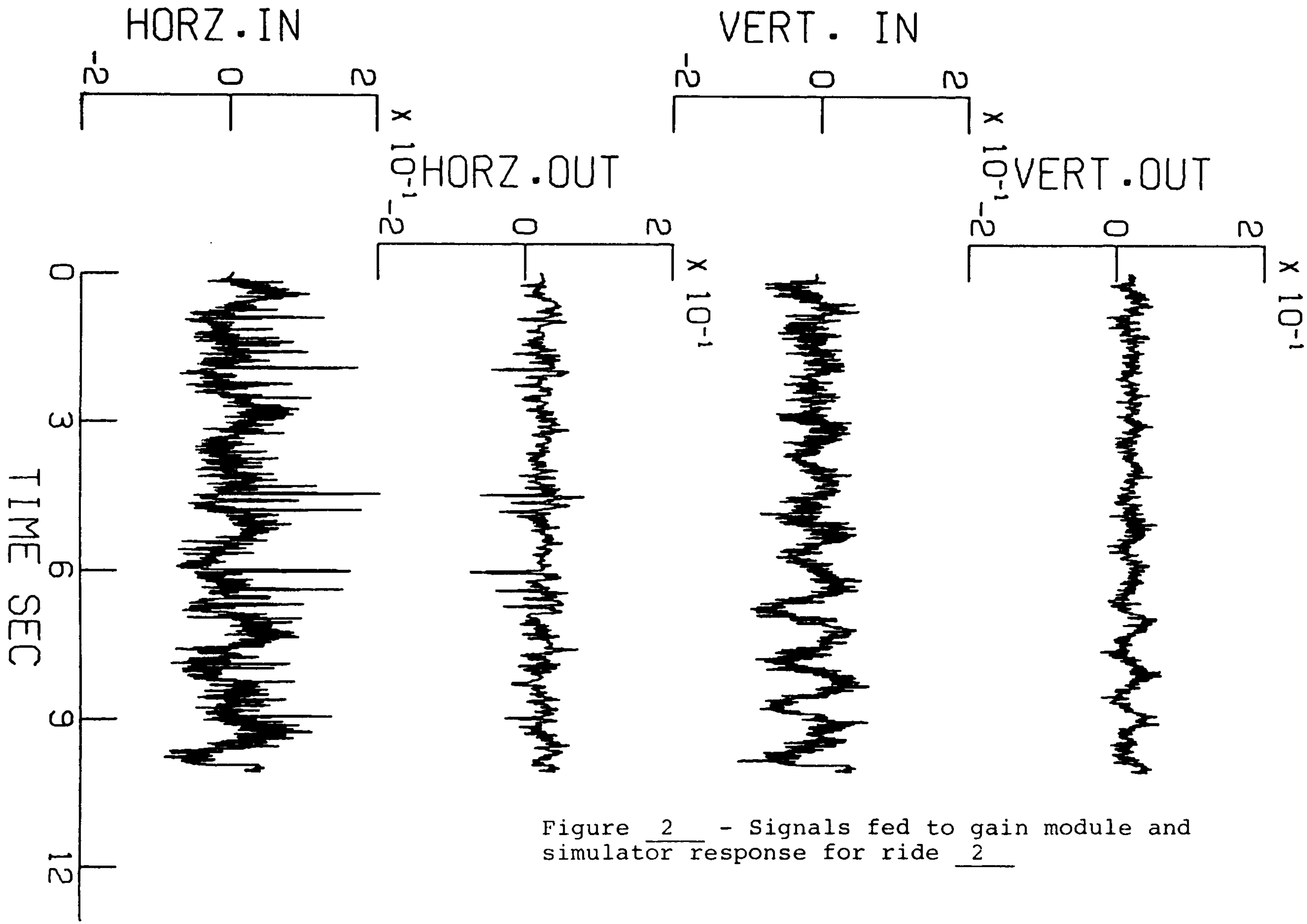

Figure 2 - Signals fed to gain module and simulator response for ride 2

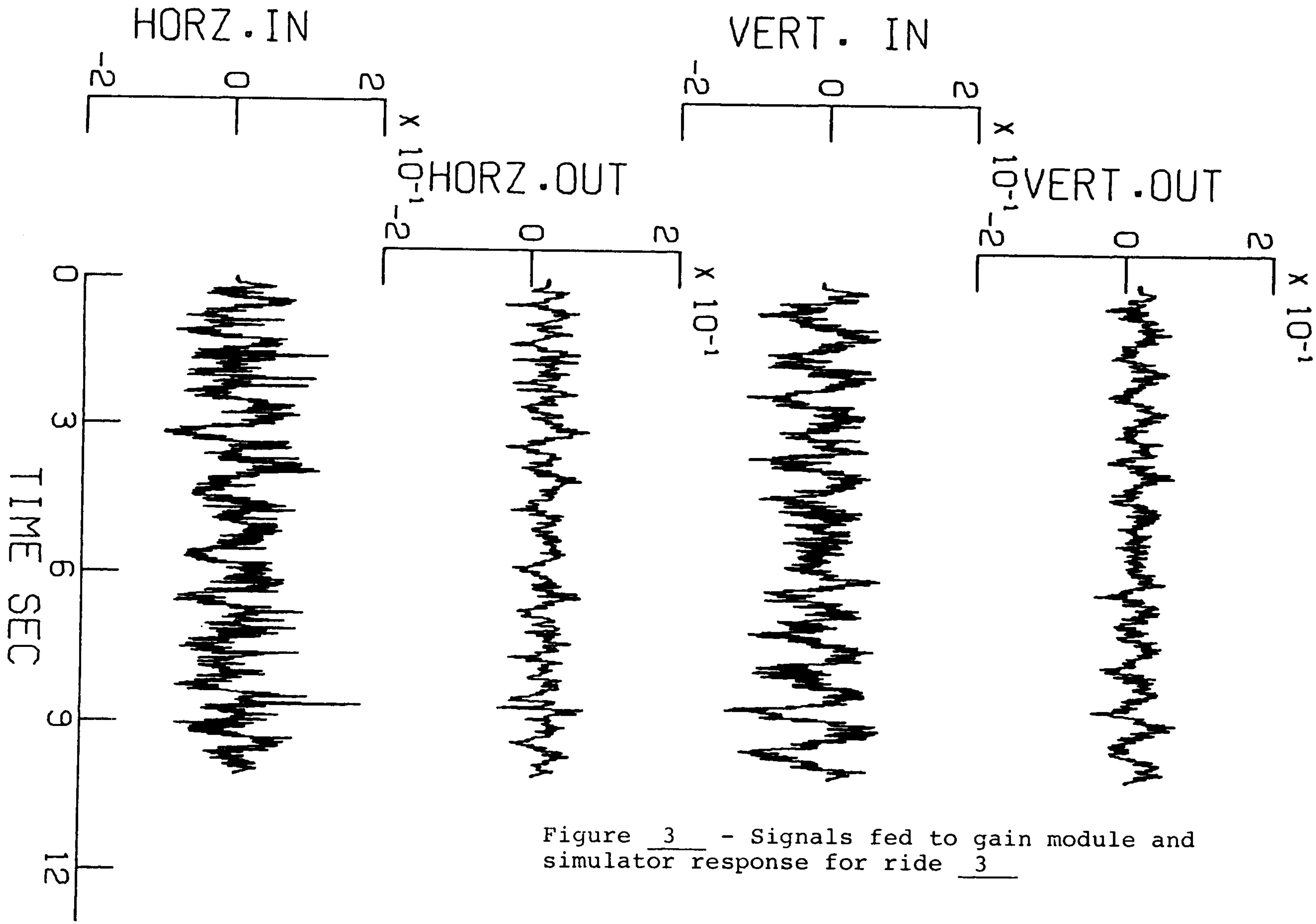

Figure 3 - Signals fed to gain module and simulator response for ride 3

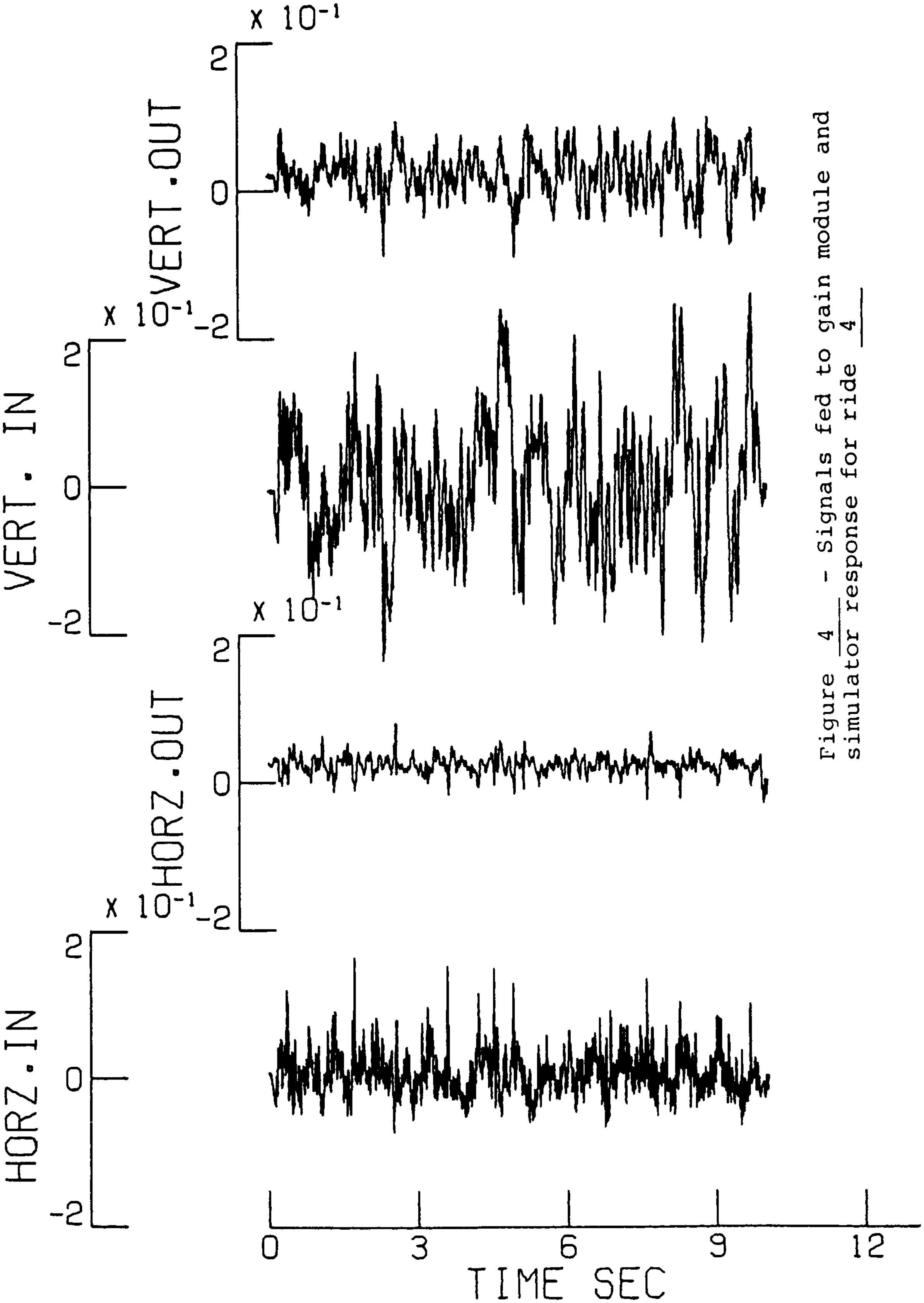

Figure 4 - Signals fed to gain module and simulator response for ride 4

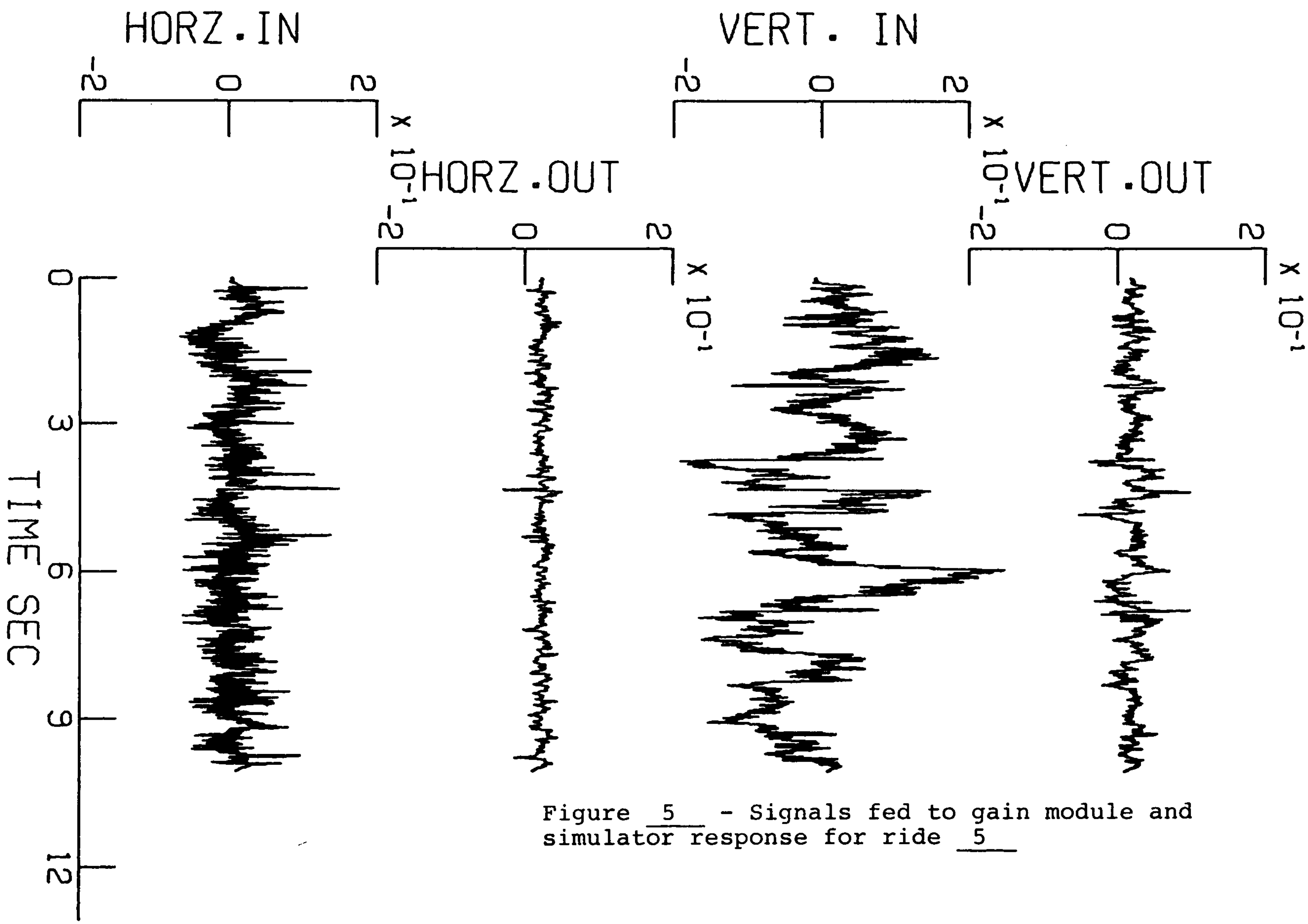

Figure 5 - Signals fed to gain module and simulator response for ride 5

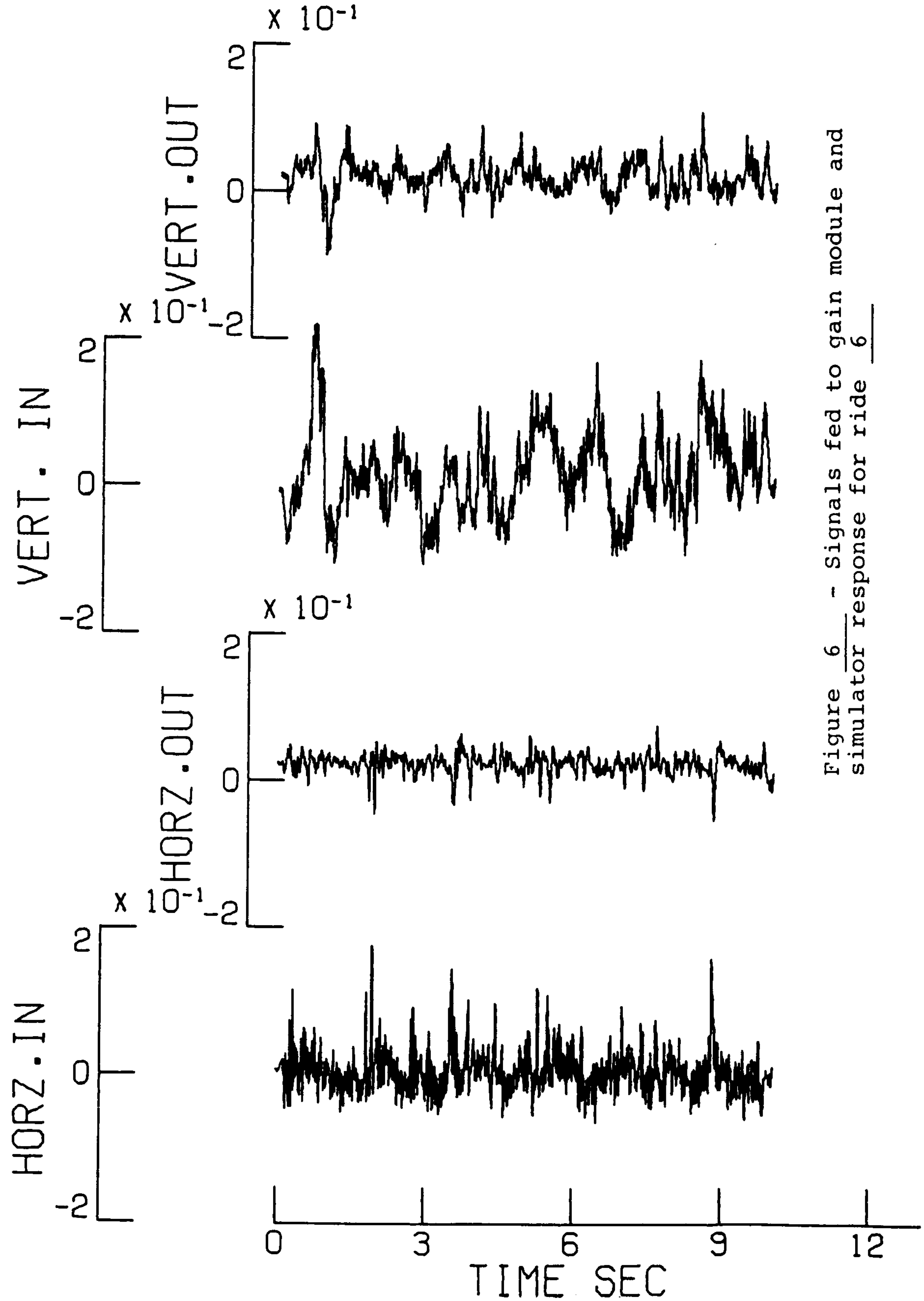

Figure 6 - Signals fed to gain module and simulator response for ride 6

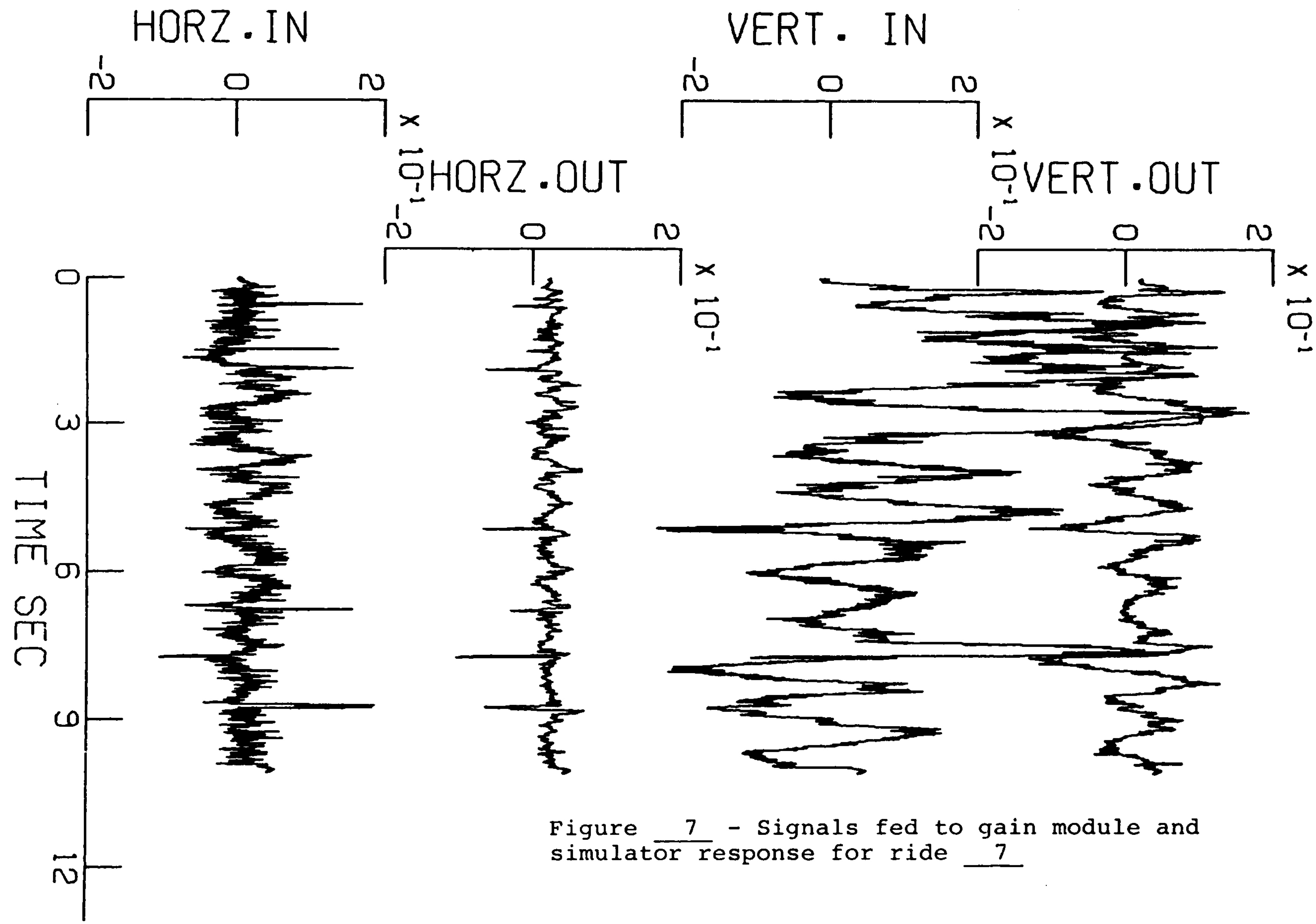

Figure 7 - Signals fed to gain module and simulator response for ride 7

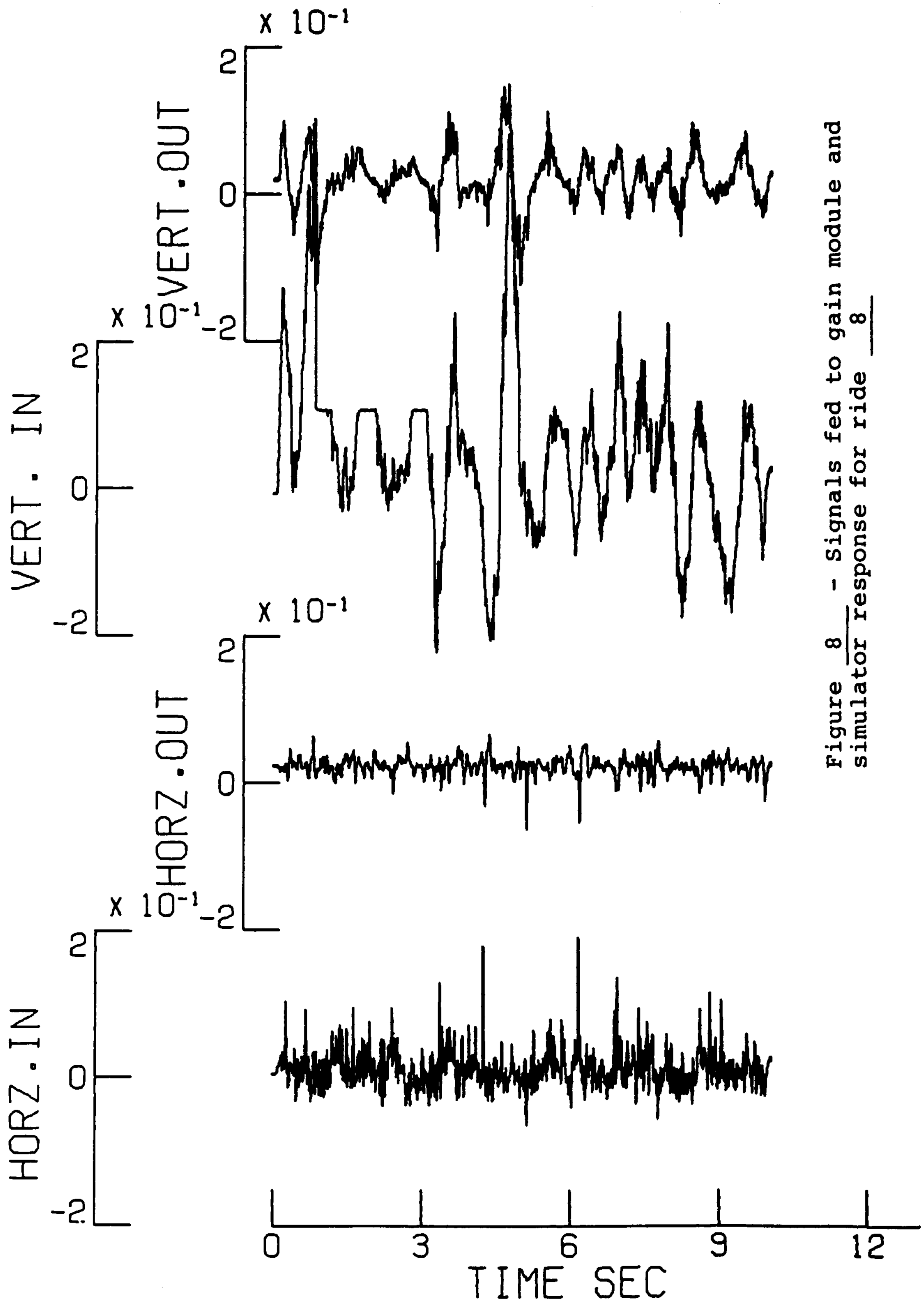

Figure 8 - Signals fed to gain module and simulator response for ride 8

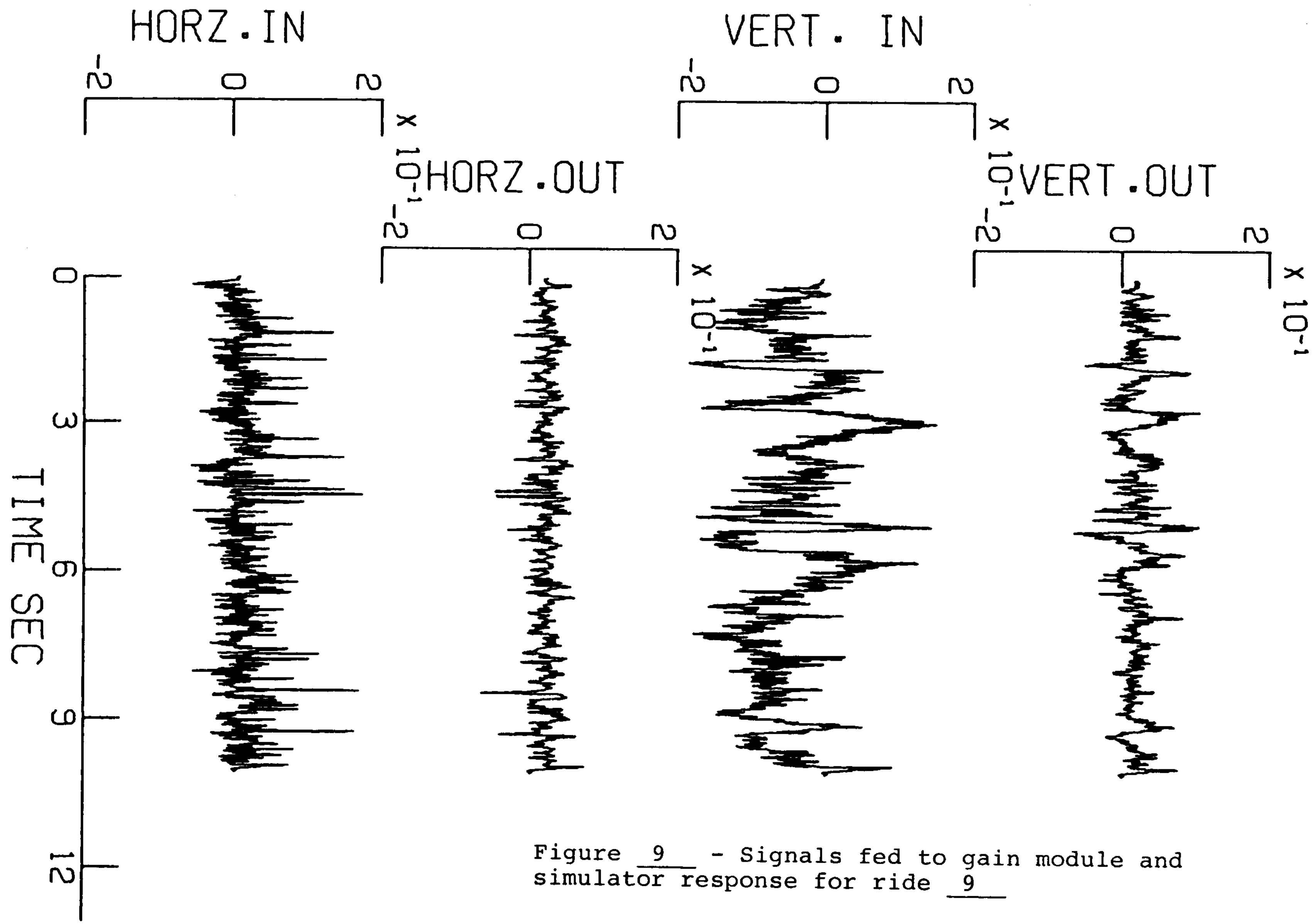

Figure 9 - Signals fed to gain module and simulator response for ride 9

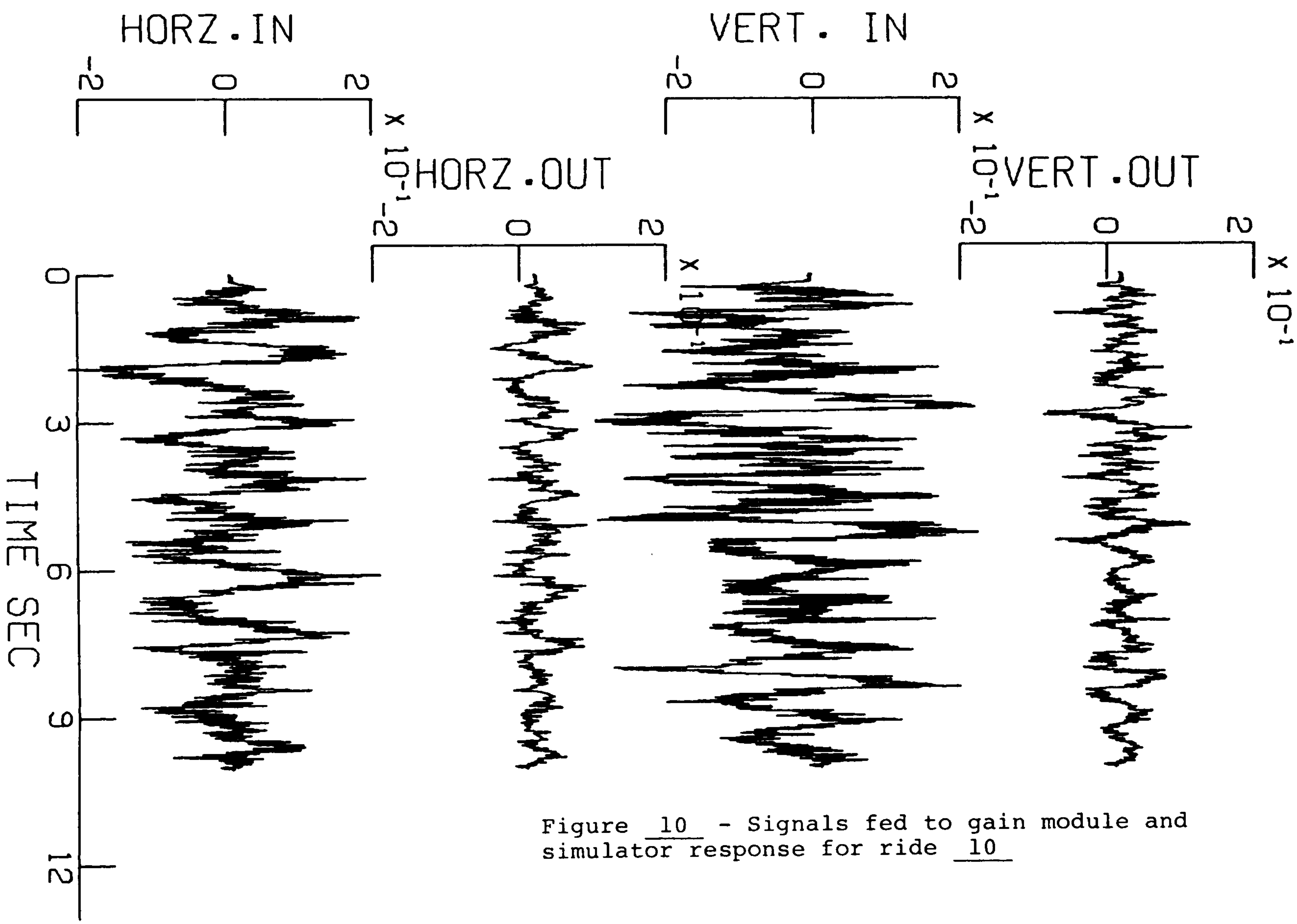

Figure 10 - Signals fed to gain module and simulator response for ride 10

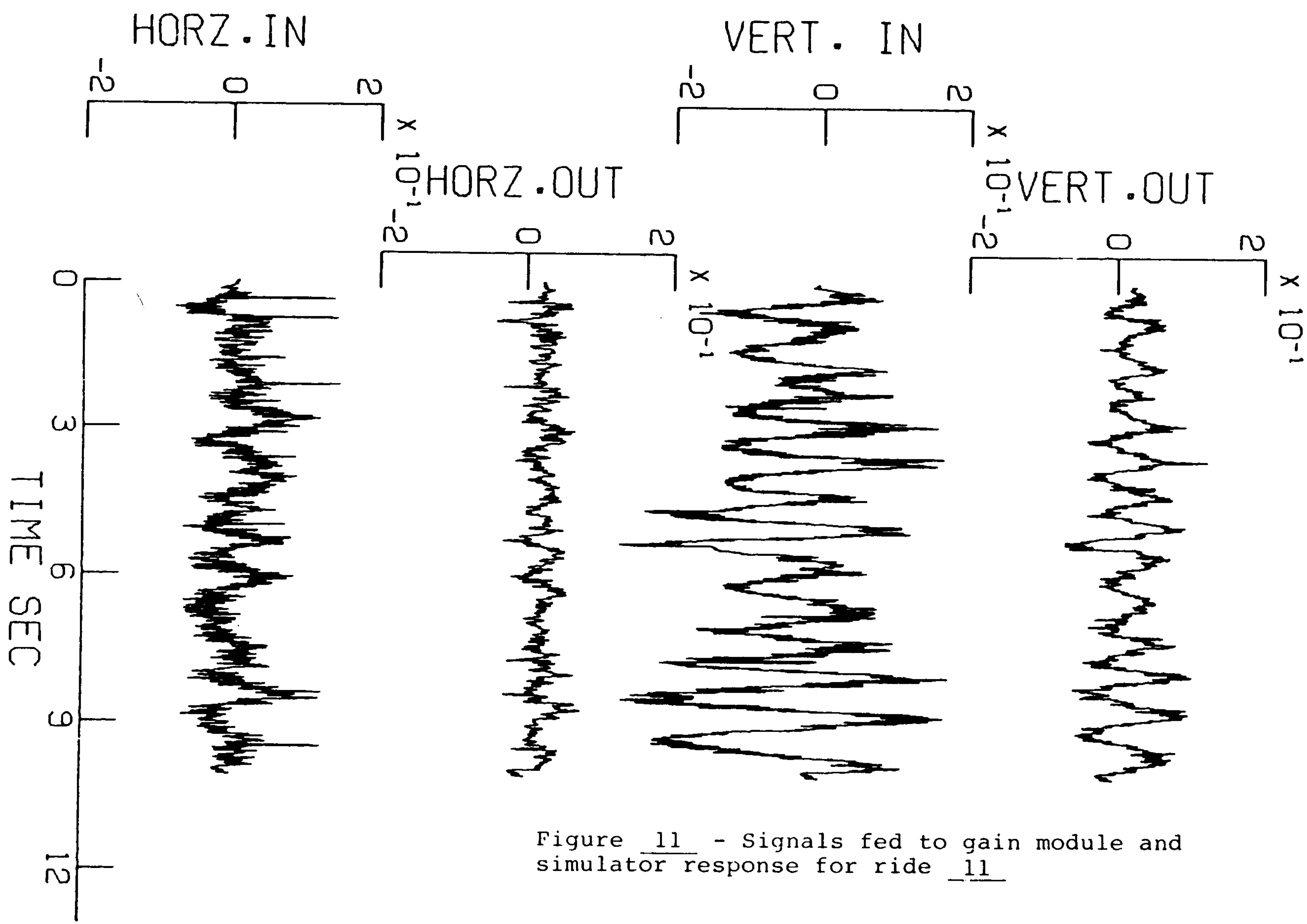

Figure 11 - Signals fed to gain module and simulator response for ride 11

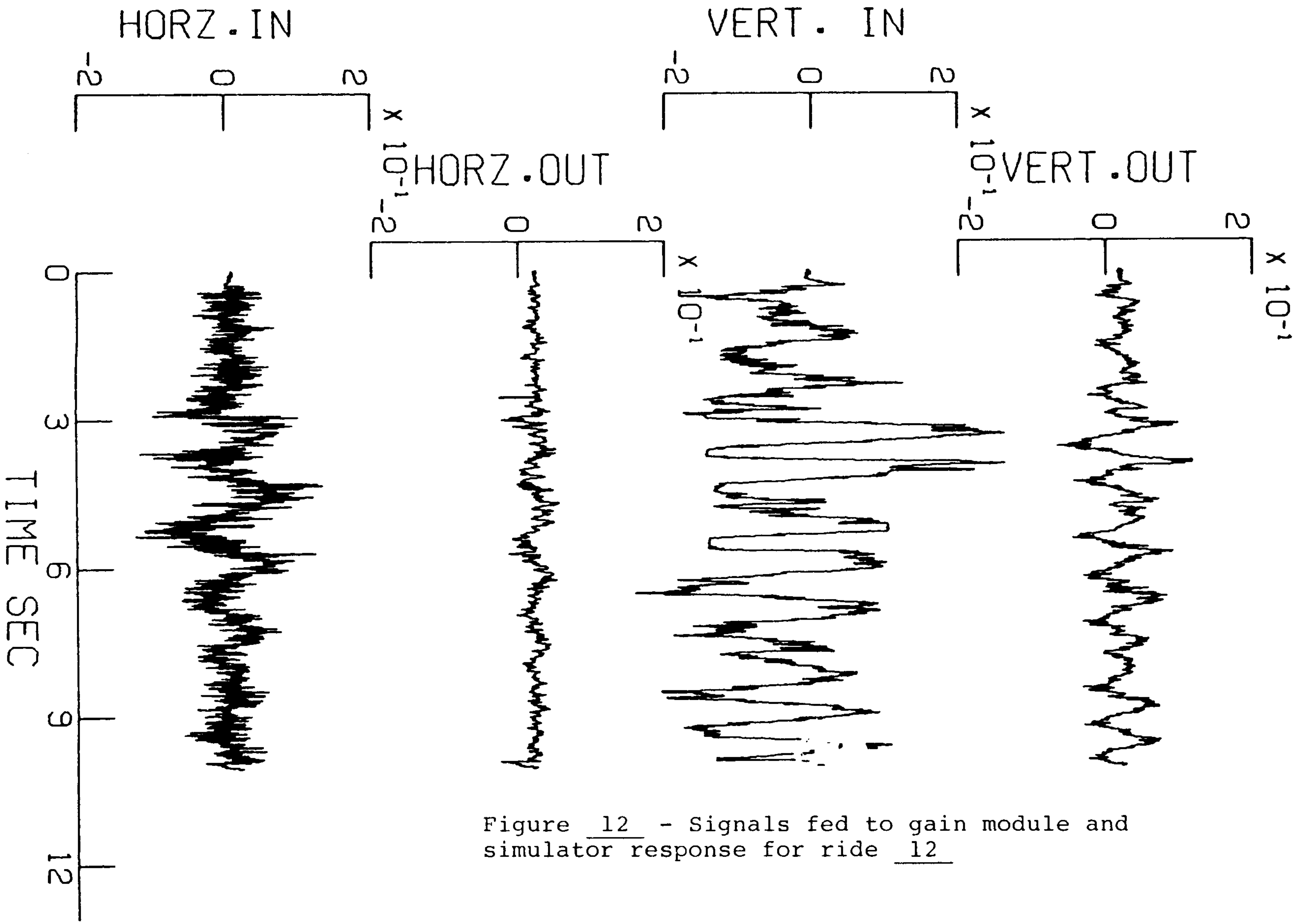

Figure 12 - Signals fed to gain module and simulator response for ride 12

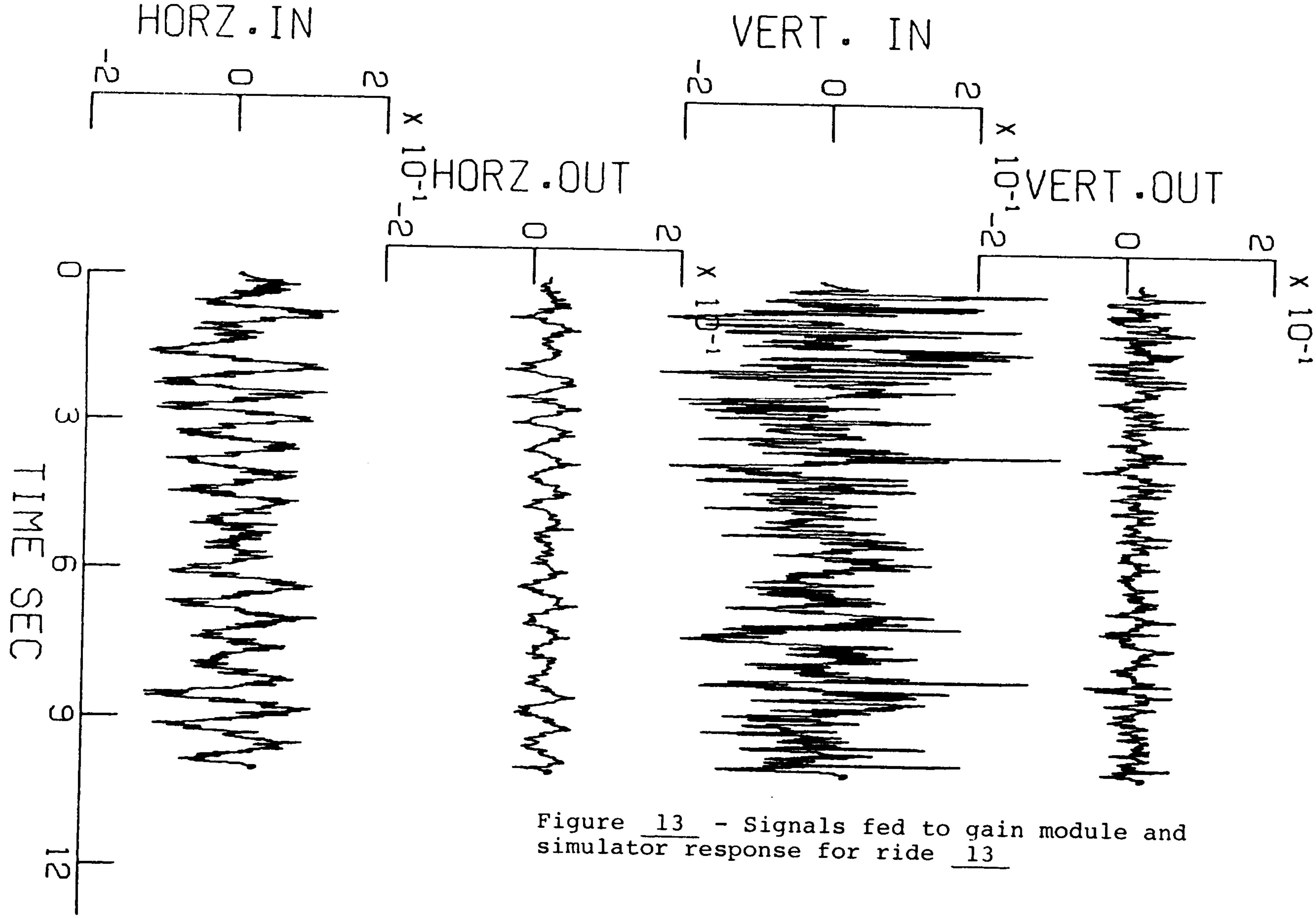

Figure 13 - Signals fed to gain module and simulator response for ride 13

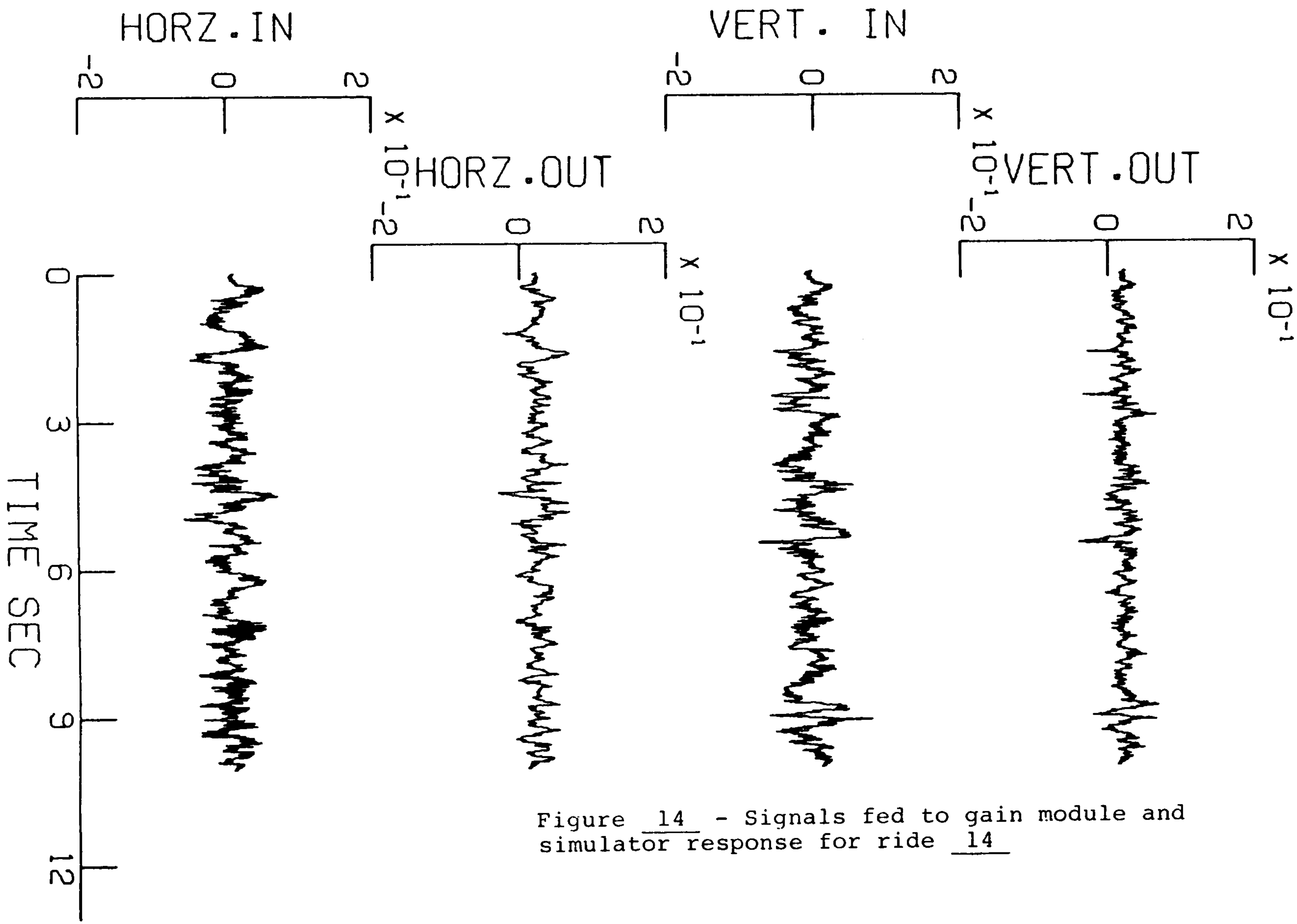

Figure 14 - Signals fed to gain module and simulator response for ride 14

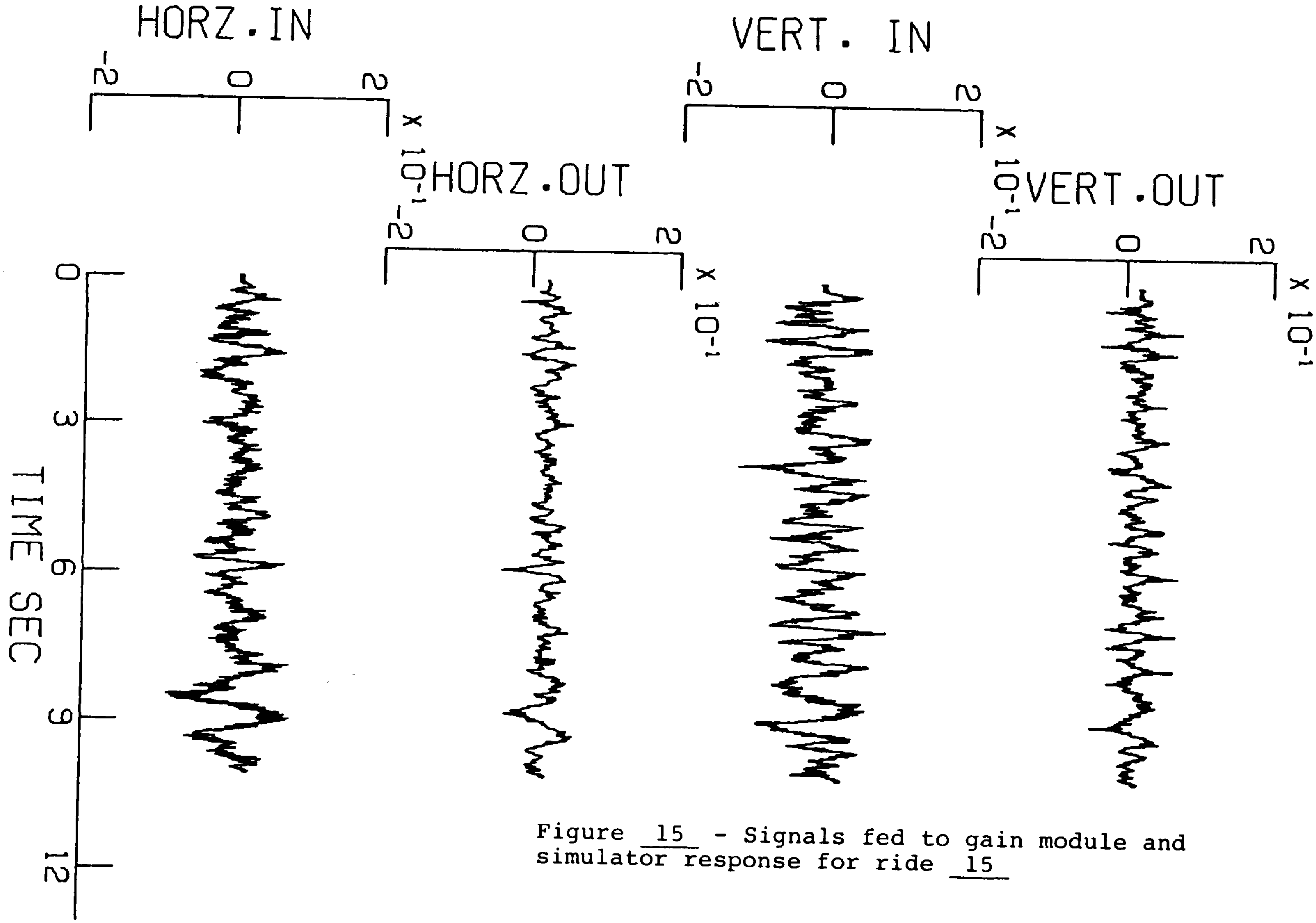

Figure 15 - Signals fed to gain module and simulator response for ride 15

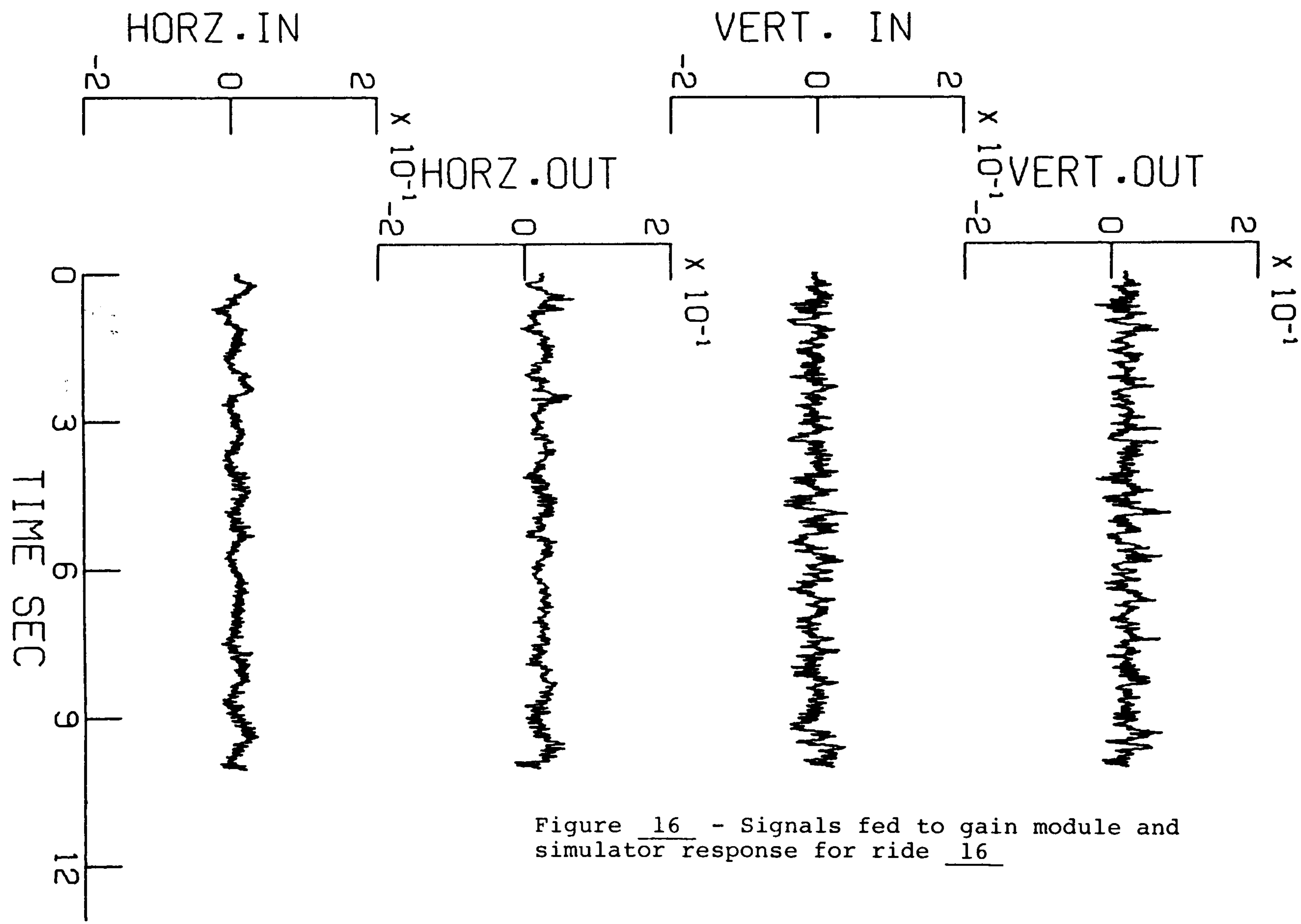

Figure 16 - Signals fed to gain module and simulator response for ride 16

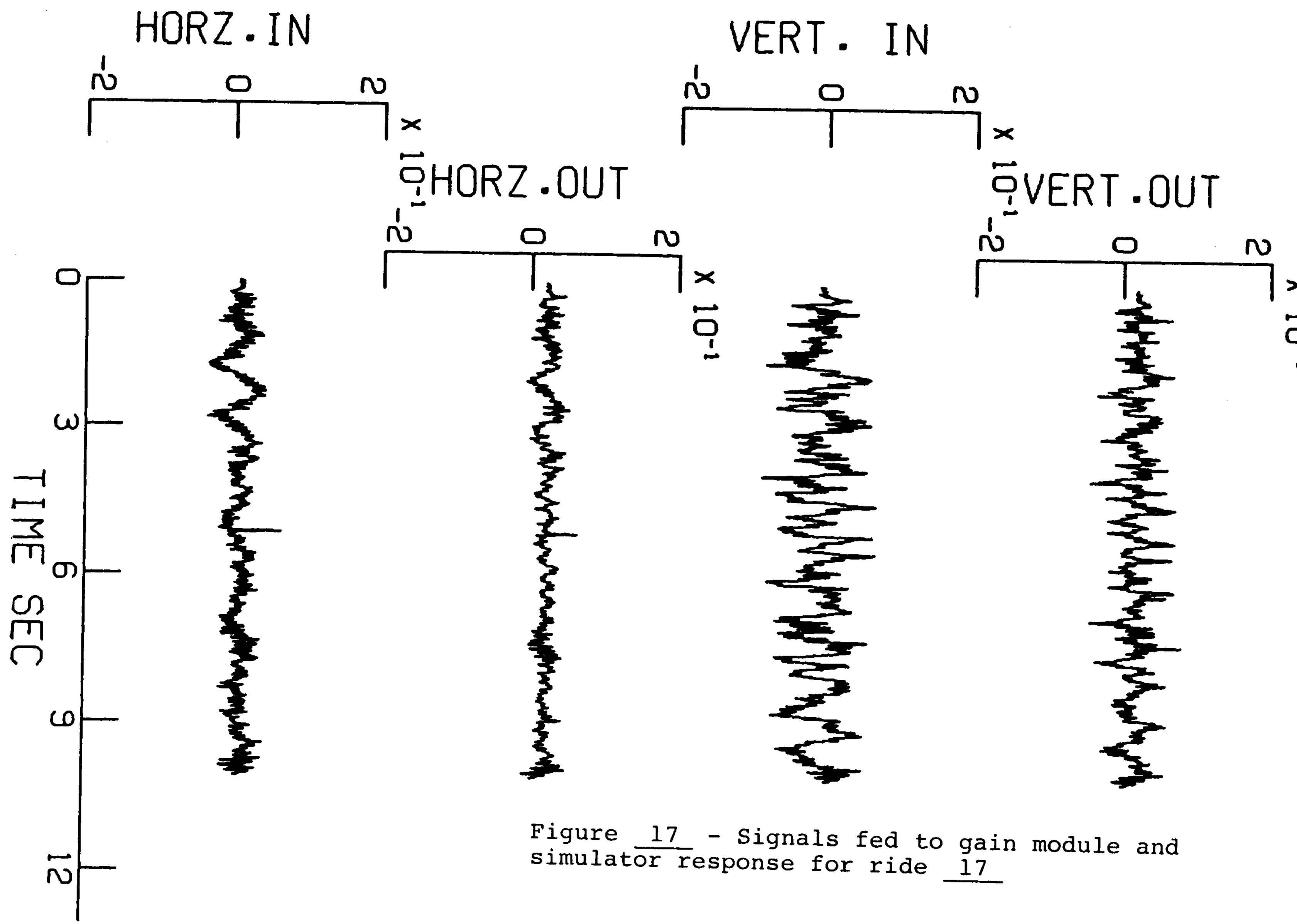

Figure 17 - Signals fed to gain module and simulator response for ride 17

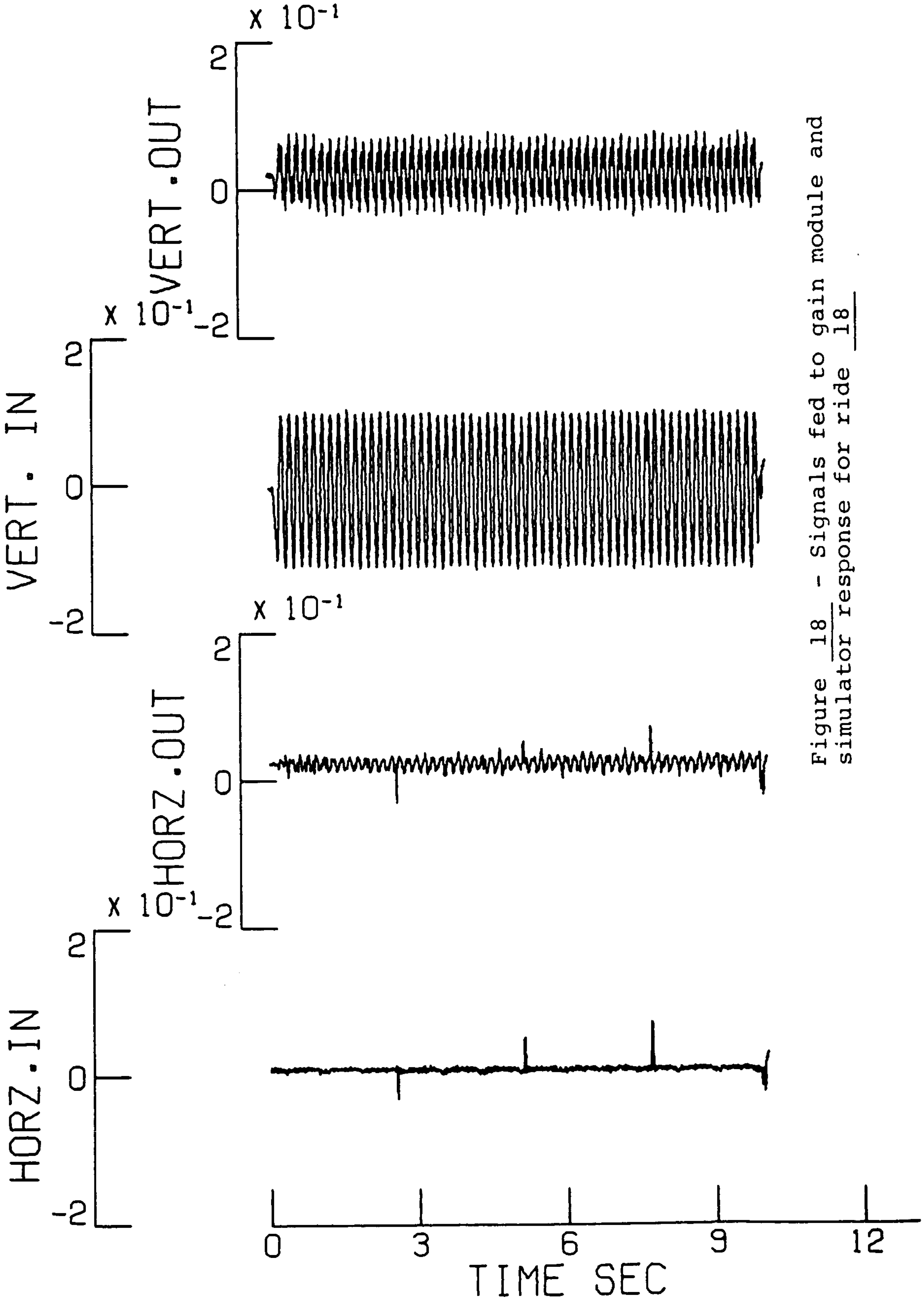

Figure 18 - Signals fed to gain module and simulator response for ride 18

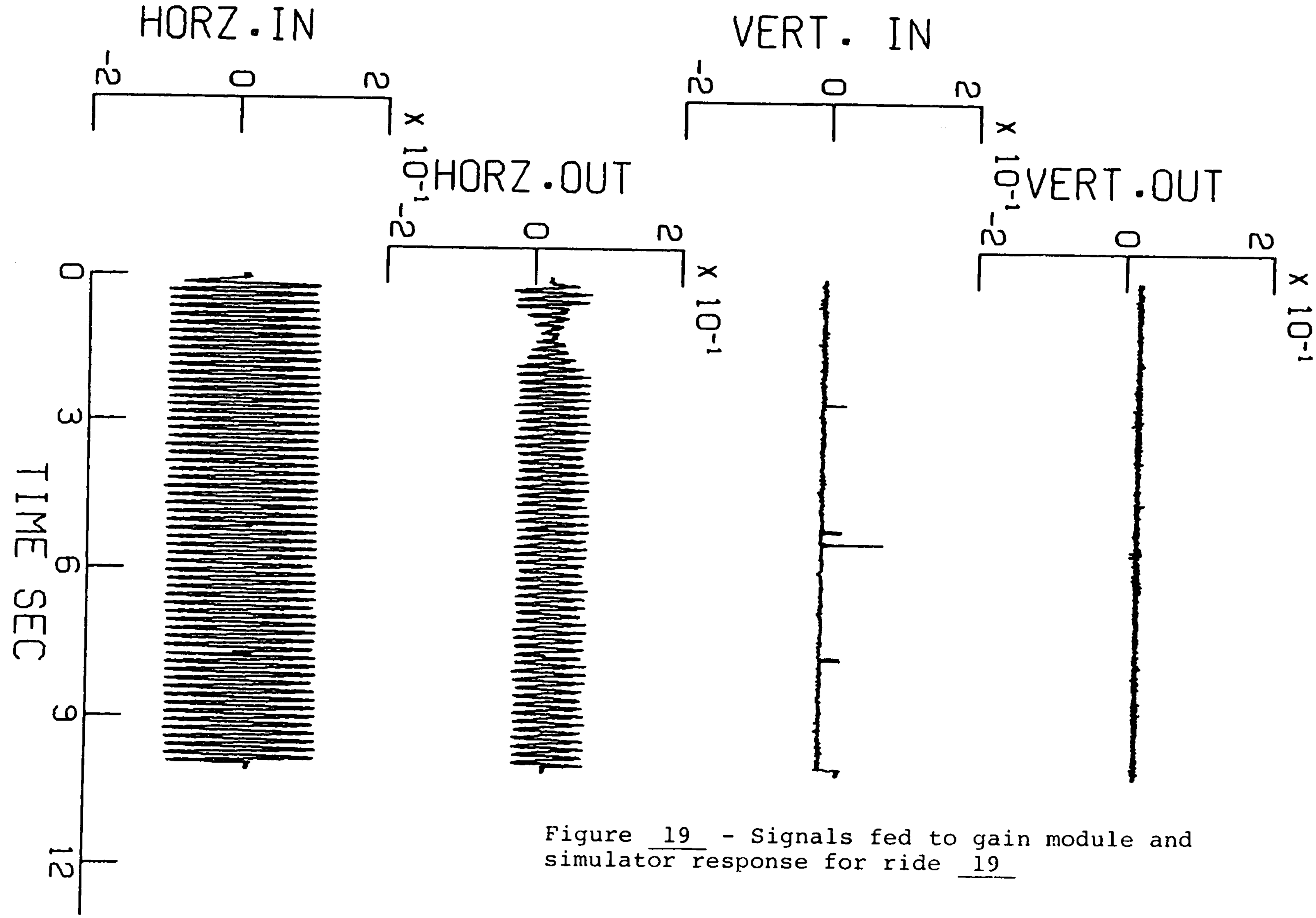

Figure 19 - Signals fed to gain module and simulator response for ride 19

RMS VERTICAL CONTRIBUTIONS BY HALF OCTAVE

RIDE	0.25–0.35	0.35–0.50	0.50–0.71	0.71–1.00	1.00–1.41	1.41–2.00	2.00–2.83	2.83–4.00	4.00–5.66	5.66–8.00	8.00–11.31	11.31–16.00	16.00–22.63	22.63–32.00	32.00–45.25	45.25–64.00
	HALF OCTAVE BOUNDARY FREQUENCIES															
1	.00016	.00020	.00047	.00132	.00393	.00817	.00449	.00530	.00670	.00402	.00663	.00391	.00477	.00398	.00129	.00146
2	.00008	.00026	.00054	.00177	.007[illegible]8	.00295	.00234	.00216	.00167	.00299	.00275	.00344	.00363	.00419	.00111	.00156
3	.00037	.00050	.0004[illegible]	.00191	.00772	.00999	.00418	.00299	.00427	.00418	.00346	.00429	.00471	.00408	.00116	.00158
4	.00024	.00083	.00232	.00539	.00831	.01492	.00912	.00702	.00642	.01645	.00719	.00713	.00675	.00433	.00201	.00174
5	.00010	.00059	.00241	.00626	.00568	.00506	.00527	.00489	.00473	.00576	.00397	.00455	.00402	.00302	.00155	.00165
6	.00016	.00099	.00137	.00537	.01194	.00873	.00706	.00602	.00775	.00677	.00562	.00497	.00464	.00318	.00138	.00157
7	.00023	.00163	.00520	.01172	.03135	.02142	.01183	.00834	.00740	.00821	.00662	.00627	.00596	.00487	.00239	.00192
8	.00029	.00178	.00326	.00687	.01940	.02027	.01198	.00584	.00473	.00635	.00578	.00489	.00520	.00383	.00168	.00163
9	.00023	.00103	.00207	.00360	.00912	.00921	.00803	.00682	.00707	.00595	.00584	.00420	.00466	.00366	.00171	.00161
10	.00003	.00037	.00074	.00233	.00829	.01715	.00527	.00725	.00786	.00562	.00349	.00566	.00748	.00479	.00247	.00164
11	.00031	.00062	.00063	.00393	.01129	.02527	.00719	.00458	.00291	.00499	.00345	.00450	.00517	.00400	.00218	.00168
12	.00046	.00086	.00227	.00314	.01550	.01505	.00542	.00506	.00415	.00437	.00327	.00340	.00406	.00313	.00154	.00160
13	.00005	.00034	.00111	.00142	.00510	.00742	.00527	.00811	.00458	.01008	.00697	.00669	.00579	.00351	.00161	.00157
14	.00007	.00007	.00035	.00161	.00489	.00336	.00311	.00581	.00279	.00398	.00275	.00298	.00267	.00210	.00100	.00147
15	.00017	.00041	.0004[illegible]	.00175	.00532	.00581	.00669	.00784	.00303	.00398	.00245	.00319	.00320	.00251	.00120	.00150
16	.00016	.00027	.00092	.00161	.00400	.00365	.00533	.00576	.00432	.00410	.00408	.00565	.00307	.00246	.00123	.00148
17	.00008	.00017	.00047	.00180	.00789	.00531	.00468	.00908	.00369	.00503	.00409	.00624	.00344	.00270	.00131	.00154
18	.00009	.00013	.00016	.00033	.00058	.00047	.00046	.00059	.00330	.02330	.00080	.00160	.01476	.00614	.00128	.00162
19	.00003	.00002	.00003	.00005	.00007	.00013	.00025	.00012	.00017	.00018	.00049	.00067	.00097	.00042	.00055	.00137

RIDE	OVERALL RMS VALUE
1	0.01718
2	0.01206
3	0.01729
4	0.03074
5	0.01666
6	0.02312
7	0.04574
8	0.03454
9	0.02180
10	0.02597
11	0.03116
12	0.02505
13	0.02114
14	0.01167
15	0.01527
16	0.01406
17	0.01778
18	0.02860
19	0.00203

Figure 20 - RMS value of vertical component of each ride by half octave band and overall

RMS LATERAL CONTRIBUTIONS BY HALF OCTAVE

HALF OCTAVE BOUNDARY FREQUENCIES

RIDE	0.25–0.35	0.35–0.50	0.50–0.71	0.71–1.00	1.00–1.41	1.41–2.00	2.00–2.83	2.83–4.00	4.00–5.66	5.66–8.00	8.00–11.31	11.31–16.00	16.00–22.63	22.63–32.00	32.00–45.25	45.25–64.00
1	.00035	.00053	.00076	.00155	.00660	.01322	.00438	.00449	.00474	.00508	.00559	.00605	.00401	.00228	.00093	.00072
2	.00011	.00041	.00243	.00293	.00529	.00297	.00271	.00407	.00512	.00802	.00589	.00753	.00400	.00182	.00086	.00074
3	.00007	.00032	.00042	.00268	.00746	.00643	.00880	.00362	.00414	.00594	.00418	.00627	.00412	.00187	.00089	.00075
4	.00007	.00019	.00051	.00154	.00263	.00382	.00418	.00333	.00425	.00545	.00360	.00445	.00284	.00160	.00090	.00074
5	.00004	.00013	.00082	.00193	.00174	.00268	.00111	.00203	.00318	.00346	.00289	.00339	.00239	.00128	.00066	.00070
6	.00015	.00017	.00046	.00218	.00238	.00349	.00381	.00438	.00469	.00605	.00509	.00495	.00412	.00169	.00084	.00069
7	.00003	.00024	.00151	.00221	.00153	.00703	.00289	.00459	.00524	.00532	.00424	.00484	.00329	.00173	.00089	.00074
8	.00006	.00014	.00069	.00176	.00169	.00336	.00223	.00419	.00476	.00515	.00408	.00439	.00325	.00165	.00091	.00075
9	.00010	.00012	.00053	.00235	.00353	.00226	.00422	.00453	.00602	.00565	.00612	.00622	.00445	.00174	.00091	.00078
10	.00015	.00075	.00061	.00430	.01184	.01218	.00532	.00518	.00452	.00475	.00605	.00777	.00466	.00156	.00088	.00073
11	.00008	.00031	.00140	.00290	.00668	.00702	.00430	.00361	.00359	.00513	.00443	.00552	.00427	.00173	.00082	.00073
12	.00005	.00024	.00068	.00564	.00286	.00190	.00252	.00240	.00316	.00339	.00275	.00265	.00284	.00182	.00068	.00071
13	.00013	.00046	.00085	.00135	.00267	.01313	.00620	.00266	.00236	.00314	.00360	.00283	.00238	.00149	.00077	.00072
14	.00003	.00023	.00054	.00152	.00770	.00376	.00427	.00692	.00296	.00355	.00304	.00330	.00237	.00139	.00067	.00065
15	.00023	.00050	.00051	.00343	.00738	.00414	.00391	.00738	.00297	.00230	.00260	.00247	.00227	.00161	.00071	.00063
16	.00017	.00042	.00114	.00216	.00635	.00154	.00224	.00340	.00164	.00166	.00329	.00379	.00216	.00119	.00069	.00065
17	.00016	.00042	.00106	.00304	.00444	.00135	.00181	.00332	.00217	.00239	.00259	.00416	.00173	.00117	.00056	.00063
18	.00007	.00005	.00004	.00007	.00017	.00039	.00035	.00041	.00074	.00656	.00176	.00208	.00148	.00156	.00105	.00074
19	.00006	.00018	.00022	.00019	.00041	.00043	.00029	.00090	.00401	.02657	.00232	.00400	.00238	.00518	.00225	.00097

RIDE	OVERALL RMS VALUE
1	0.02000
2	0.01667
3	0.01805
4	0.01194
5	0.00838
6	0.01366
7	0.01417
8	0.01183
9	0.01517
10	0.02300
11	0.01574
12	0.01004
13	0.01657
14	0.01386
15	0.01376
16	0.01015
17	0.00927
18	0.00760
19	0.02798

Figure 21 - RMS value of lateral component of each ride by half octave band and overall