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VOLUME I

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George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama

APPROVAL

ASTRONAUTIC STRUCTURES MANUAL
VOLUME I

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

A. A. McCOOL
Director, Structures and Propulsion Laboratory

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16. ABSTRACT

This document (Volumes I, II, and III) presents a compilation of industry-wide methods in aerospace strength analysis that can be carried out by hand, that are general enough in scope to cover most structures encountered, and that are sophisticated enough to give accurate estimates of the actual strength expected. It provides analysis techniques for the elastic and inelastic stress ranges. It serves not only as a catalog of methods not usually available, but also as a reference source for the background of the methods themselves.

An overview of the manual is as follows: Section A is a general introduction of methods used and includes sections on loads, combined stresses, and interaction curves; Section B is devoted to methods of strength analysis; Section C is devoted to the topic of structural stability; Section D is on thermal stresses; Section E is on fatigue and fracture mechanics; Section F is on composites; Section G is on rotating machinery; and Section H is on statistics.

These three volumes supersede NASA TM X-60041 and NASA TM X-60042.

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STRUCTURES MANUAL

FOREWORD

This manual is issued to the personnel of the Strength Analysis Branch to provide uniform methods of structural analysis and to provide a ready reference for data. Generally, the information contained in this manual is a condensation of material published by universities, scientific journals, missile and aircraft industries, text book publishers, and government agencies.

Illustrative problems to clarify either the method of analysis or the use of the curves and tables are included wherever they are considered necessary. Limitations of the procedures and the range of applicability of the data are indicated wherever possible.

It is recognized that all subjects in the Table of Contents are not present in the body of the manual; some sections remain to be developed in the future. However, an alphabetical index of content material is provided and is updated as new material is added. New topics not listed in the Table of Contents will be treated as the demand arises. This arrangement has been utilized to make a completed section available as soon as possible. In addition, revisions and supplements are to be incorporated as they become necessary.

Many of the methods included have been adapted for computerized utilization. These programs are written in Fortran Language for utilization on the MSFC Executive VIII, Univac 1108, or IBM 7094 and are cataloged with example problems in the Structural Analysis Computer Utilization Manual.

It is requested that any comments concerning this manual be directed to:

Chief, Structural Requirements Section
Strength Analysis Branch
Analytical Mechanics Division
Astronautics Laboratory
National Aeronautics and Space Administration
Marshall Space Flight Center, Alabama 35812

SECTION A1

STRESS AND STRAIN

TABLE OF CONTENTS

A1.0.0 Stress and Strain

The relationship between stress and strain and other material properties, which are used throughout this manual, are presented in this section. A brief introduction to the theory of elasticity for elementary applications is also presented in this section.

A1.1.0 Mechanical Properties of Materials

A brief account of the important mechanical properties of materials is given in this subsection; a more detailed discussion may be found in any one of a number of well known texts on the subject. The numerical values of the various mechanical properties of most aerospace materials are given in MIL-HDBK-5 (reference 1). Many of these values are obtained from a plotted set of test results of one type or another. One of the most common sets of these plotted sets is the stress-strain diagram. A typical stress-strain diagram is discussed in the next subsection.

A1.1.1 Stress-Strain Diagram

Some of the more useful properties of materials are obtained from a stress-strain diagram. A typical stress-strain curve for aerospace metals is shown in Figure A1.1.1-1.

The curve in Figure A1.1.1-1 is composed of two regions; the straight line portion up to the proportional limit where the stress varies linearly with strain, and the remaining part where the stress is not proportional to strain. In this manual, stresses below the ultimate tensile stress (F_{tu}) are considered to be elastic. However, a correction (or plasticity reduction) factor is sometimes employed in certain types of analysis for stresses above the proportional limit stress.

Commonly used properties shown on a stress-strain curve are described briefly in the following paragraphs:

E	Modulus of elasticity; average ratio of stress to strain for stresses below the proportional limit. In Figure A1.1.1-1 $E = \tan\theta$

A1.1.1 Stress-Strain Diagram (Cont'd)

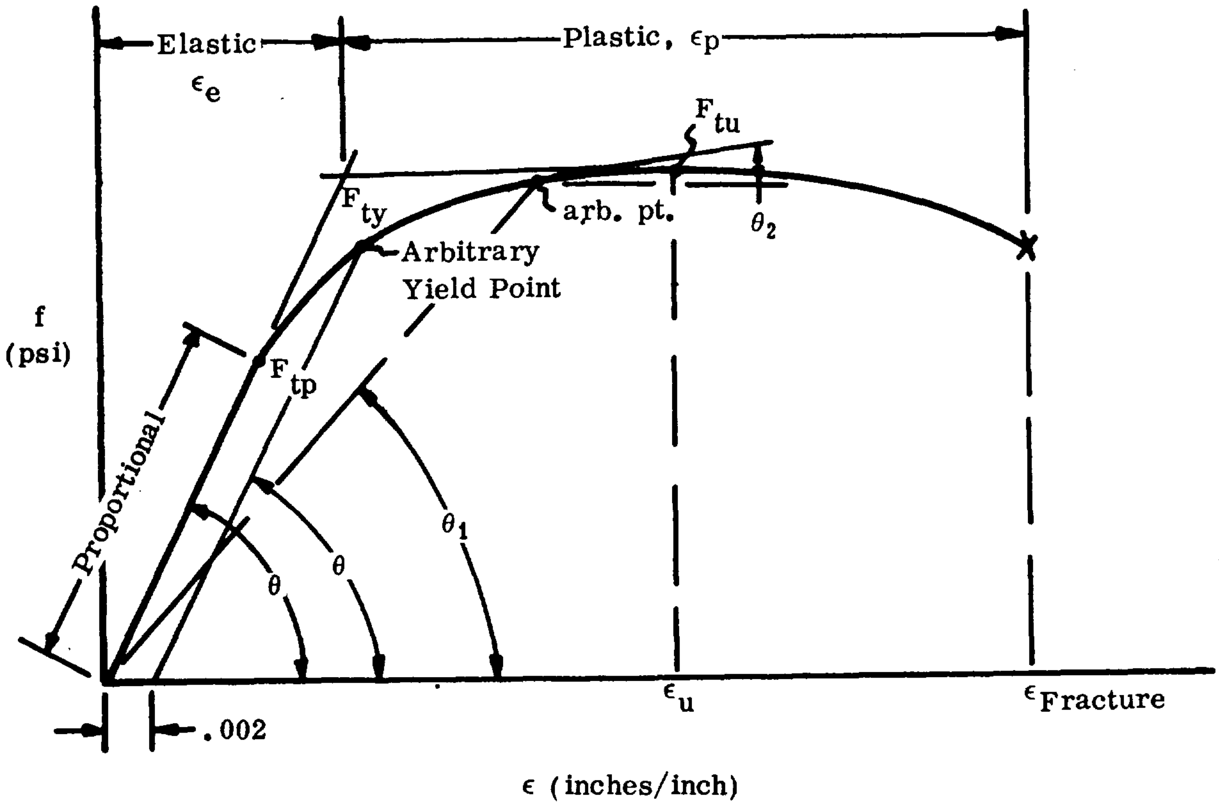


Figure A1.1.1-1 A Typical Stress-Strain Diagram

E_s — Secant modulus; ratio of stress to strain above the proportional limit; reduces to E in the proportional range. In Figure A1.1.1-1 $E_s = \tan \theta_1$

E_t — Tangent modulus; slope of the stress-strain curve at any point; reduces to E in the proportional range. In Figure A1.1.1-1 $E_t = \frac{df}{d\epsilon} = \tan \theta_2$.

A1.1.1 Stress-Strain Diagram (Cont'd)

F_{ty} or F_{cy}	Tensile or compressive yield stress; since many materials do not exhibit a definite yield point, the yield stress is determined by the .2% offset method. This entails the construction of a straight line with a slope E passing through a point of zero stress and a strain of .002 in./in. The intersection of the stress-strain curve and the constructed straight line defines the magnitude of the yield stress.
F_{tp} or F_{cp}	Proportional limit stress in tension or compression; the stress at which the stress ceases to vary linearly with strain.
F_{tu}	Ultimate tensile stress; the maximum stress reached in tensile tests of standard speçimens.
F_{cu}	Ultimate compressive stress; taken as F_{tu} unless governed by instability.
ϵ_u	The strain corresponding to F_{tu}.
ϵ_e	Elastic strain; see Figure A1.1.1-1.
ϵ_p	Plastic strain; see Figure A1.1.1-1.
$\epsilon_{fracture}$ (% elongation)	Fracture strain; percent elongation in a predetermined gage length associated with tensile failures, and is a relative indication of ductility of the material.

A1.1.2 Other Material Properties

The definition of various other material properties and terminology used in stress analysis work is given in this subsection.

A1.1.2 Other Material Properties (Cont'd)

F_{bry}, F_{bru}	Yield and ultimate bearing stress; determined in a manner similar to those for tension and compression. A load-deformation curve is plotted where the deformation is the change in the hole diameter. Bearing yield (F_{bry}) is defined by an offset of 2% of the hole diameter; bearing ultimate (F_{bru}) is the actual failing stress divided by 1.15.
F_{su}	Ultimate shear stress.
F_{sp}	Proportional limit in shear; usually taken equal to 0.577 times the proportional limit in tension for ductile materials.
ν	Poisson's ratio; the ratio of transverse strain to axial strain in a tension or compression test. For materials stressed in the elastic range, ν may be taken as a constant but for inelastic strains ν becomes a function of axial strain.
ν_p	Plastic Poisson's ratio; unless otherwise stated, ν_p may be taken as 0.5.
$G = \frac{E}{2(1 + \nu)}$	Modulus of rigidity or shearing modulus of elasticity for pure shear in isotropic materials.
Isotropic	Elastic properties are the same in all directions.
Anisotropic	Elastic properties differ in different directions.
Orthotropic	Distinct material properties in mutually perpendicular planes.

A1.1.3 Strain-Time Diagram

The behavior of a structural material is dependent on the duration of loading. This behavior is exhibited with the aid of a strain-time diagram such as that shown in Figure A1.1.3-1. This diagram consists of regions that are dependent

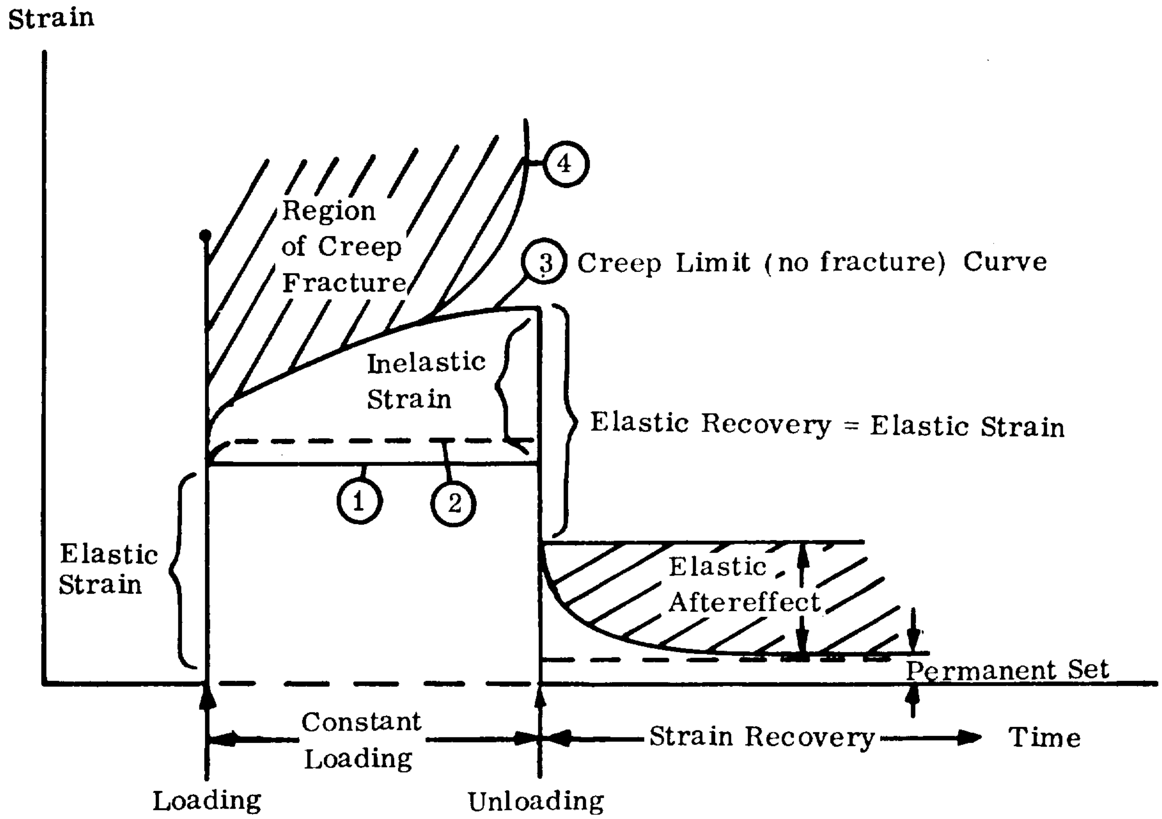


Figure A1.1.3-1 Strain-Time Diagram

upon the four loading conditions as indicated on the time coordinate. These loading conditions are as follows:

1. Loading

2. Constant loading

A1.1.3 Strain-Time Diagram (Cont'd)

3. Unloading

4. Recovery (no load)

The interval of time when the load is held constant is usually measured in weeks or months. Whereas the time involved in loading and unloading is relatively short (usually seconds or minutes) such that the corresponding strain-time curve can be represented by a straight vertical line.

The following discussion of the diagram will be confined to generalities due to the complexity of the phenomena of creep and fracture. A more detailed discussion on this subject is presented in reference 5.

The condition referred to as "loading" represents the strain due to a load which is applied over a short interval of time. This strain may vary from zero to the strain at fracture ($\epsilon_{fracture}$ - See Figure A1.1.1-1) depending upon the material and loading.

During the second loading condition, where the load is held constant, the strain-time curve depends on the initial strain for a particular material. The possible strain-time curves (Figure A1.1.3-1) that could result are discussed below.

a. In curve 1, the initial strain is elastic and no additional strain is experienced for the entire time interval. This curve typifies elastic action.

b. In curve 2, the initial strain increases for a short period after the load becomes constant and then remains constant for the remainder of the period. This action is indicative of slip which is characterized by a permanent set resulting from the shifting (slip) of adjacent crystalline structures along planes most favorably oriented with respect to the direction of the principal shearing stress.

c. In curve 3, there is a continuous increase in strain after the initial slip until a steady state condition is attained. This curve is indicative of creep which is generally the result of a combined effect of the predominantly viscous inelastic deformation within the unordered intercrystalline boundaries and the complex deformations by slip and fragmentation of the ordered crystalline domains.

A1.1.3 Strain-Time Diagram (Cont'd)

d. Curve 4 is also a combination of slip and creep. The only difference from curve 3 is that the creep action continues until the material fails in fracture. This fracture may take place at any time during the constant load period and is indicated by the upper shaded area in Figure A1.1.3-1.

During unloading, the reduction in strain of curves 1, 2 and 3 is equal to the elastic strain incurred during loading. This reduction is referred to as the "elastic recovery." It can be seen in Figure A1.1.3-1 that in the case of curve 1 the structural member will return to its initial configuration immediately after unloading. This is not the case for curves 2 and 3 as there will be some residual strain.

The last condition to be discussed on the strain-time diagram concerns the recovery period. In this period, some of the strain indicated as inelastic strain is recoverable. This is true particularly for many viscoelastic materials (such as flexible plastics) that do not show real creep, only delayed recoverable strains.

The height of the lower shaded area in Figure A1.1.3-1 is called the elastic after effect. The upper bound is the maximum possible permanent set and is indicated by the solid horizontal line. The lower bound could be any one of the family of possible strain-time curves confined within the lower shaded area. The limiting curve of the lower bound would approach the permanent set curve due to slip as indicated by the horizontal dashed line. If slip action is negligible, this limiting curve would be represented by a line that approaches zero asymptotically with increasing time.

A1.1.4 Temperature Effects

The mechanical properties of a material are usually affected by its temperature. This effect will be discussed in general terms in this section. For specific information, see the applicable chapter in reference 1.

In general, temperatures below room temperature increase the strength properties of metals. Ductility is usually decreased and the notch sensitivity of the metal may become of primary importance. The opposite is generally true for temperatures above room temperature.

A representative example for the effect of temperature on the mechanical properties of aluminum alloys is given in Figures A1.1.4-1 through 4. Most steels behave in a similar manner but generally are less sensitive to temperature magnitudes.

A1.1.4 Temperature Effects (Cont'd)

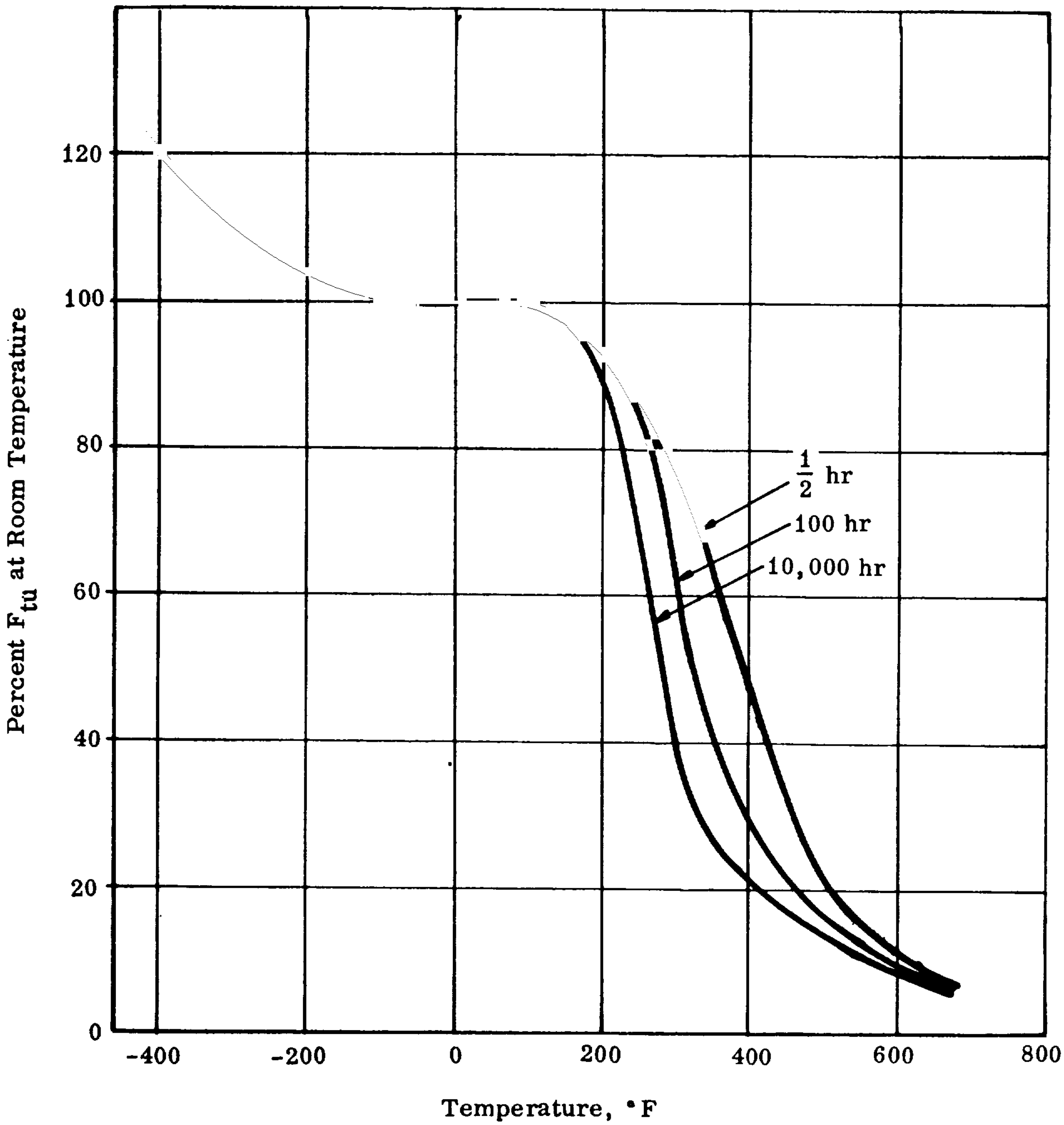


Figure A1.1.4-1 Effects of Temperature on the Ultimate Tensile Strength (F_{tu}) of 7079 Aluminum Alloy (from Ref. 1)

A1.1.4 Temperature Effects (Cont'd)

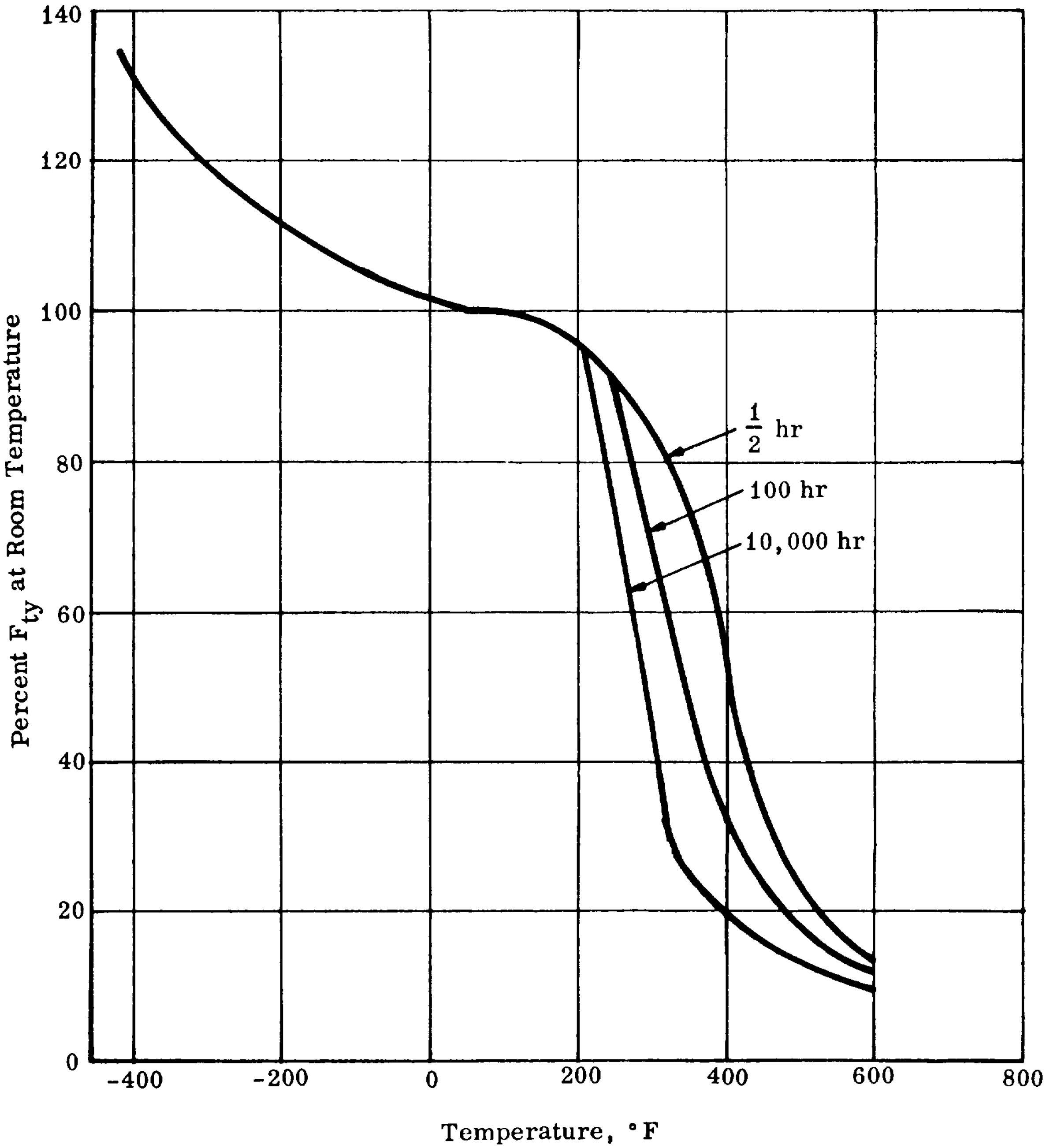


Figure A1.1.4-2 Effects of Temperature on the Tensile Yield Strength (F_{ty}) of 7079 Aluminum Alloy (from Ref. 1)

A1.1.4 Temperature Effects (Cont'd)

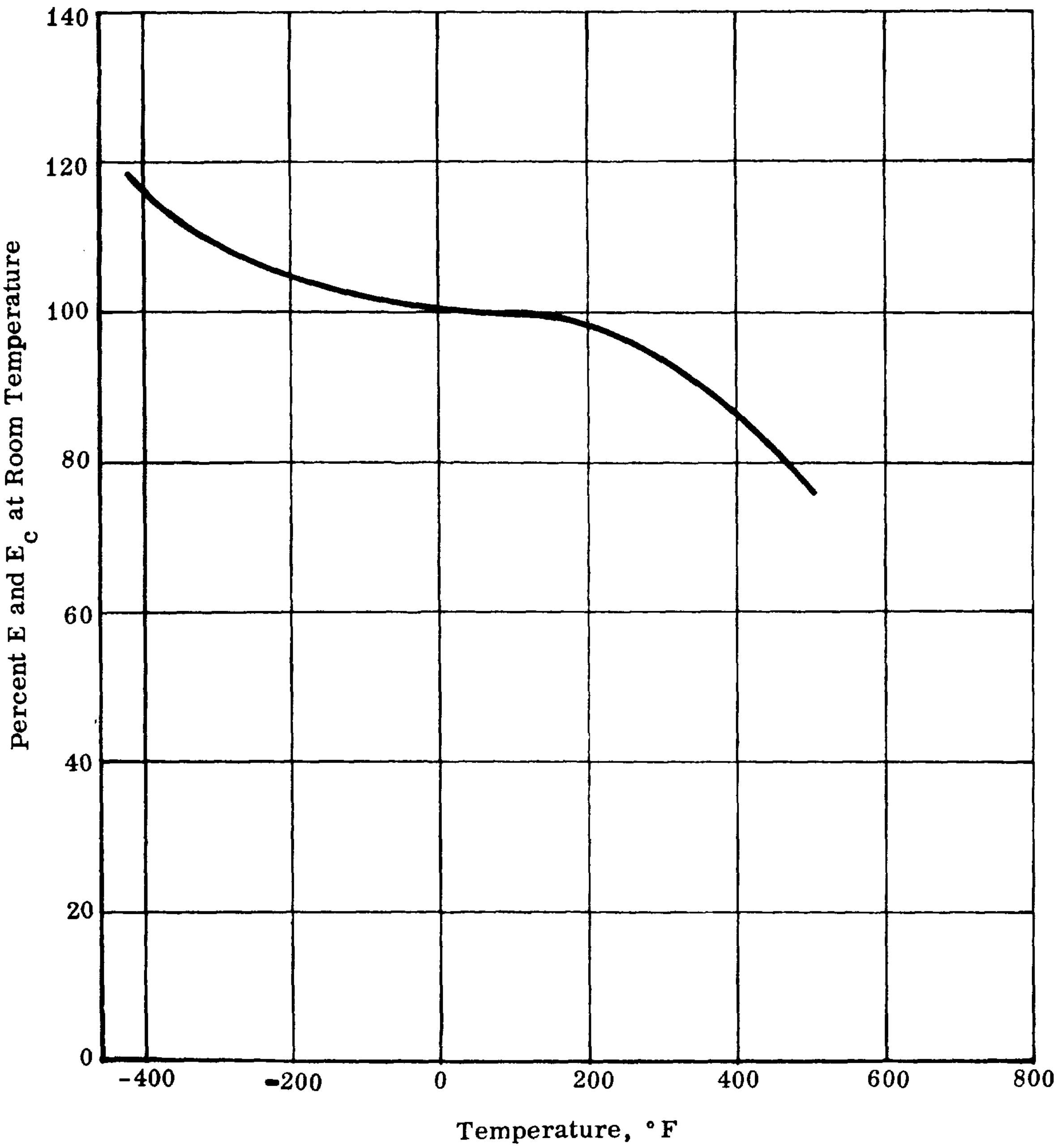


Figure A1.1.4-3 Effect of Temperature on the Tensile and Compressive Modulus (E and E_c) of 7079 Aluminum Alloy (from Ref. 1)

A1.1.4 <u>Temperature Effects (Cont'd)</u>

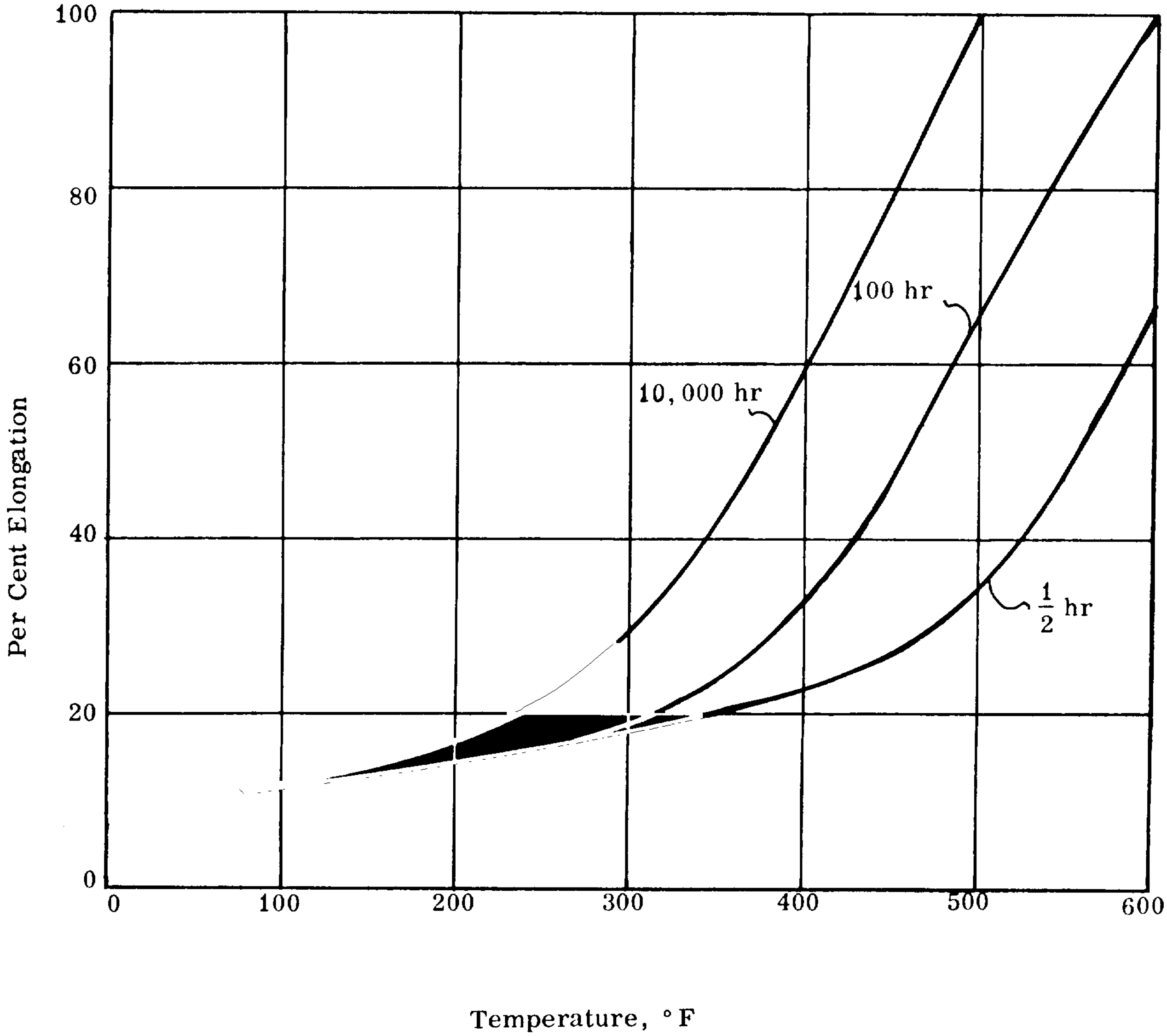


Figure A1.1.4-4 Effect of Temperature on the Elongation of 7079-T6 Aluminum Alloy (from Ref. 1)

A1.1.5 Hardness Conversion Table

A table for converting hardness numbers to ultimate tensile strength values is presented in this section. In this table, the ultimate strength values are in the range, 50 to 304 ksi. The corresponding hardness number is given for each of three hardness machines; namely, the Vickers, Brinell and the applicable scale(s) of the Rockwell machine.

This table is given in the remainder of this section. The appropriate materials-property handbook should be consulted for additional information whenever necessary.

Tensile Strength	Vickers-Firth Diamond	Brinell 3000 kg 10mm Stl Ball	Rockwell		
			A Scale	B Scale	C Scale
ksi	Hardness Number	Hardness Number	60 kg 120 deg Diamond Cone	100 kg 1/16 in. Dia Stl Ball	150 kg 120 deg Diamond Cone
50	104	92	--	58	--
52	108	96	--	61	--
54	112	100	--	64	--
56	116	104	--	66	--
58	120	108	--	68	--
60	125	113	--	70	--
62	129	117	--	72	--
64	135	122	--	74	--

Table A1.1.5-1 Hardness Conversion Table

A1.1.5 Hardness Conversion Table (Cont'd)

Tensile Strength	Vickers-Firth Diamond	Brinell 3000 kg 10mm Stl Ball	Rockwell		
			A Scale	B Scale	C Scale
ksi	Hardness Number	Hardness Number	60 kg 120 deg Diamond Cone	100 kg 1/16 in. Dia Stl Ball	150 kg 120 deg Diamond Cone
66	139	127	--	76	--
68	143	131	--	77.5	--
70	149	136	--	79	--
72	153	140	--	80.5	--
74	157	145	--	82	--
76	162	150	--	83	--
78	167	154	51	84.5	--
80	171	158	52	85.5	--
82	177	162	53	87	--
83	179	165	53.5	87.5	--
85	186	171	54	89	--
87	189	174	55	90	--
89	196	180	56	91	--

Table A1.1.5-1 Hardness Conversion Table (Cont'd)

A1.1.5 Hardness Conversion Table (Cont'd)

Tensile Strength	Vickers-Firth Diamond	Brinell 3000 kg 10mm Stl Ball	Rockwell		
			A Scale	B Scale	C Scale
ksi	Hardness Number	Hardness Number	60 kg 120 deg Diamond Cone	100 kg 1/16 in. Dia Stl Ball	150 kg 120 deg Diamond Cone
91	203	186	56.5	92.5	--
93	207	190	57	93.5	--
95	211	193	57	94	--
97	215	197	57.5	95	--
99	219	201	57.5	95.5	--
102	227	210	59	97	--
104	235	220	60	98	19
107	240	225	60.5	99	20
110	245	230	61	99.5	21
112	250	235	61.5	100	22
115	255	241	62	101	23
118	261	247	62.5	101.5	24
120	267	253	63	102	25

Table A1.1.5-1 Hardness Conversion Table (Cont'd)

A1.1.5 Hardness Conversion Table (Cont'd)

Tensile Strength	Vickers-Firth Diamond	Brinell 3000 kg 10mm Stl Ball	Rockwell		
			A Scale	B Scale	C Scale
ksi	Hardness Number	Hardness Number	60 kg 120 deg Diamond Cone	100 kg 1/16 in. Dia Stl Ball	150 kg 120 deg Diamond Cone
123	274	259	63.5	103	26
126	281	265	64	--	27
129	288	272	64.5	--	28
132	296	279	65	--	29
136	304	286	65.5	--	30
139	312	294	66	--	31
142	321	301	66.5	--	32
147	330	309	67	--	33
150	339	318	67.5	--	34
155	348	327	68	--	35
160	357	337	68.5	--	36
165	367	347	69	--	37
170	376	357	69.5	--	38
176	386	367	70	--	39

Table A1.1.5-1 Hardness Conversion Table (Cont'd)

A1.1.5 Hardness Conversion Table (Cont'd)

Tensile Strength	Vickers-Firth Diamond	Brinell 3000 kg 10mm Stl Ball	Rockwell		
			A Scale	B Scale	C Scale
ksi	Hardness Number	Hardness Number	60 kg 120 deg Diamond Cone	100 kg 1/16 in. Dia Stl Ball	150 kg 120 deg Diamond Cone
181	396	377	70.5	--	40
188	406	387	71	--	41
194	417	398	71.5	--	42
201	428	408	72	--	43
208	440	419	72.5	--	44
215	452	430	73	--	45
221	465	442	73.5	--	46
231	479	453	74	--	47
237	493	464	75	--	48
246	508	476	75.5	--	49
256	523	488	76	--	50
264	539	500	76.5	--	51
273	556	512	77	--	52
283	573	524	77.5	--	53

Table A1.1.5-1 Hardness Conversion Table (Cont'd)

A1.1.5 Hardness Conversion Table (Cont'd)

Tensile Strength	Vickers-Firth Diamond	Brinell 3000 kg 10mm Stl Ball	Rockwell		
			A Scale	B Scale	C Scale
ksi	Hardness Number	Hardness Number	60 kg 120 deg Diamond Cone	100 kg 1/16 in. Dia Stl Ball	150 kg 120 deg Diamond Cone
294	592	536	78	--	54
304	611	548	78.5	--	55

Table A1.1.5-1 Hardness Conversion Table (Concluded)

A1.2.0 Elementary Theory of the Mechanics of Materials

In the elementary theory of mechanics of materials, a uni-axial state of strain is generally assumed. This state of strain is characterized by the simplified form of Hooke's law; namely $f = E\epsilon$, where ϵ is the unit strain in the direction of the unit stress f, and E is the Modulus of Elasticity. The strains in the perpendicular directions (Poisson's ratio effect) are neglected. This is generally justified in most elementary and practical applications considered in the theory of mechanics of materials. In these applications, the structural members are generally subjected to a uni-axial state of stress and/or the strains and displacements are of secondary importance. Also, in these applications, the magnitude of each of a set of bi-axial stresses (when this occurs) is generally independent of the Poisson's ratio effect.

Frequently in design, there are applications in which the magnitude of each of a set of bi-axial (or tri-axial) stresses are dependent upon the Poisson's ratio effect; and/or the magnitude of the strains and displacements are of primary importance. This type of application must be generally analyzed by the theory of elasticity. A brief account on the use of the theory of elasticity for elementary applications is given in the next subsection.

A1.3.0 Elementary Applications of the Theory of Elasticity

The difference between the method of ordinary mechanics and the theory of elasticity is that no simplifying assumption is made concerning the strains in the latter. Because of this, it becomes necessary to take into account the complete distribution of the strains in the body and to assume a more general statement of Hooke's law in expressing the relation between stresses and strains. It is noted that the stresses calculated by both methods are only approximate since the material in the physical body deviates from the ideal material assumed by both methods.

Some of the following subsections are written for a three dimensional stress field but are applicable to problems in two dimension simply by neglecting all terms containing the third dimension.

A1.3.1 Notation for Forces and Stresses

The stresses acting on the side of a cubic element can be described by six components of stress, namely the three normal stresses f_{11}, f_{22}, f_{33}, and the three shearing stresses $f_{12} = f_{21}$, $f_{13} = f_{31}$, $f_{23} = f_{32}$.

In Figure A1.3.1-1 shearing stresses are resolved into two components parallel to the coordinate axis. Two subscript numbers are used, the first indicating the direction normal to the plane under consideration and the second indicating the direction of the component of the stress. Normal stresses have like subscripts and positive directions are as shown in the figure. An analogous notation for the x-y coordinate system is:

$f_{11} = f_x$

$f_{22} = f_y$

$f_{12} = f_s$

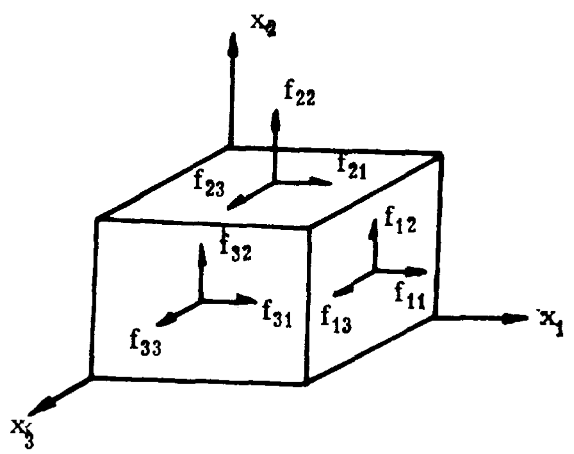


Figure A1.3.1-1 Representation of Stresses on an Element of a Body

A1.3.1 Notation for Forces and Stresses (Cont'd)

Surface forces

Forces distributed over the surface of the body, such as pressure of one body on another, or hydrostatic pressure, are called surface forces.

Body forces

Body forces are forces that are distributed over the volume of a body, such as gravitational forces, magnetic forces, or inertia forces in the case of a body in motion.

A1.3.2 Specification of Stress at a Point

If the components of stress in Figure A1.3.1-2 are known for any given point, the stress acting on any inclined plane through this point can be calculated from the equations of statics. Body forces, such as weight of the element, can generally be neglected since they are of higher order than surface forces.

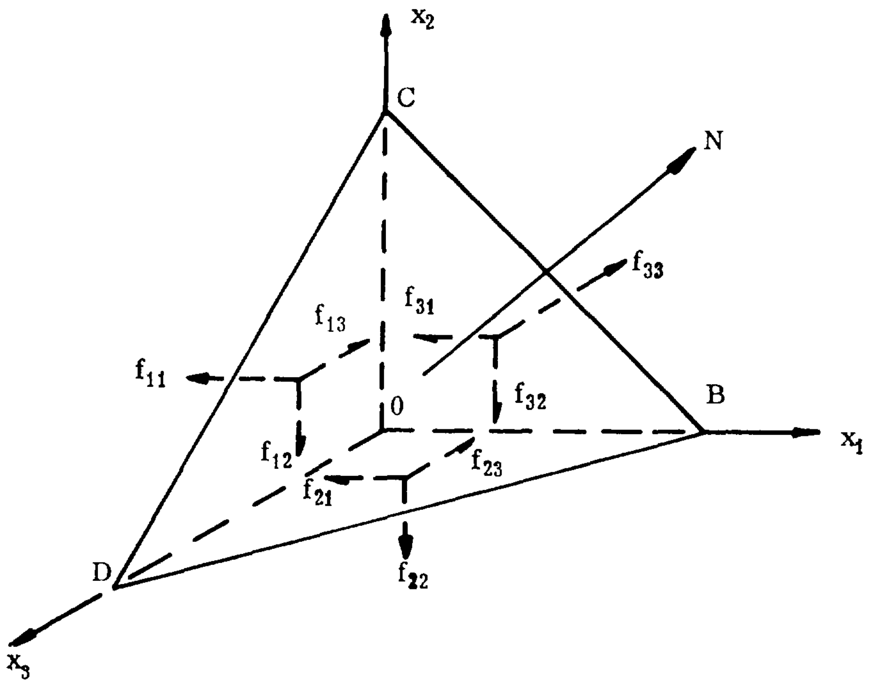


Figure A1.3.1-2 An Element Used in Specifying Stress at a Point

A1.3.2 Specification of Stress at a Point (Cont'd)

If A denotes the area of the inclined face BCD of the tetrahedron in Figure A1.3.1-2, then the areas of the three faces are obtained by projecting A on the three coordinate planes. Letting N be the stress normal to the plane BCD, the three components of stress acting parallel to the coordinate axes, are denoted by N_1, N_2, and N_3. The components of force acting in the direction of the coordinates X_1, X_2, X_3 are AN_1, AN_2, and AN_3 respectively. Another useful relationship can be written as:

$$\cos(N_1) = k, \qquad \cos(N_2) = m, \qquad \cos(N_3) = n \tag{1}$$

and the areas of the other faces are Ak, Am, An.

The equations of equilibrium of the tetrahedron can then be written as:

$$\begin{aligned} N_1 &= f_{11}\,k + f_{12}\,m + f_{13}\,n \\ N_2 &= f_{12}\,k + f_{22}\,m + f_{32}\,n \\ N_3 &= f_{13}\,k + f_{23}\,m + f_{33}\,n \end{aligned} \tag{2}$$

The principal stresses for a given set of stress components can be determined by the solution of the following cubic equation:

$$\begin{aligned} &f_p^3 - (f_{11} + f_{22} + f_{33})\,f_p^2 + (f_{11}\,f_{22} + f_{22}\,f_{33} + f_{11}\,f_{33} - f_{23}^2 \\ &\quad - f_{13}^2 - f_{12}^2)\,f_p - (f_{11}\,f_{22}\,f_{33} + 2f_{23}\,f_{13}\,f_{12} - f_{11}\,f_{23}^2 - f_{22}\,f_{13}^2 - f_{33}\,f_{12}^2) = 0 \end{aligned} \tag{3}$$

The three roots of this equation give the values of the three principal stresses. The three corresponding sets of direction cosines for the three principal planes can be obtained by substituting each of these stresses (one set for each principal stress) into Equations 3 and using the relation $k^2 + m^2 + n^2 = 1$.

A1.3.2 Specification of Stress at a Point (Cont'd)

$$
\begin{aligned}
&(f_p - f_{11})\, k - f_{12}\, m - f_{13}\, n = 0 \\
&f_{12}\, k + (f_p - f_{22})\, m - f_{23}\, n = 0 \qquad (4) \\
&f_{13}\, k - f_{23}\, m + (f_p - f_{33})\, n = 0
\end{aligned}
$$

The shearing stresses associated with the three principal stresses can be obtained by:

$$
\begin{aligned}
&f^{12} = \pm \frac{1}{2}(f_{p1} - f_{p2}),\; f^{13} = \pm \frac{1}{2}(f_{p1} - f_{p3}), \\
&f^{23} = \pm \frac{1}{2}(f_{p2} - f_{p3}) \qquad (5)
\end{aligned}
$$

where the superscript notation is used to distinguish between the applied shearing stresses and the stresses associated with the principal normal stresses f_{p1}, f_{p2}, and f_{p3}.

The maximum shearing stress acts on the plane bisecting the angle between the largest and the smallest principal stresses and is equal to half the difference between these two principal stresses.

A1.3.3 Equations of Equilibrium

Since no simplifying assumption is permitted as to the distribution of strain in the theory of elasticity, the equilibrium and the continuity of each element within the body must be considered. These considerations are discussed in this and the subsequent subsections.

Let the components of the specific body force be denoted by X_1, X_2, X_3, then the equation of equilibrium in a given direction is obtained by summing all the forces in that direction and proceeding to the limit. The resulting differential equations of equilibrium for three dimensions are:

A1.3.3 Equations of Equilibrium (Cont'd)

$$\frac{\partial f_{11}}{\partial x_1} + \frac{\partial f_{12}}{\partial x_2} + \frac{\partial f_{13}}{\partial x_3} + X_1 = 0$$

$$\frac{\partial f_{22}}{\partial x_2} + \frac{\partial f_{12}}{\partial x_1} + \frac{\partial f_{23}}{\partial x_3} + X_2 = 0 \qquad (6)$$

$$\frac{\partial f_{33}}{\partial x_3} + \frac{\partial f_{13}}{\partial x_1} + \frac{\partial f_{23}}{\partial x_2} + X_3 = 0$$

These equations must be satisfied at all points throughout the body. The internal stresses must be in equilibrium with the external forces on the surface of the body. These conditions of equilibrium at the boundary are obtained by considering the stresses acting on Figure A1.3.3-1.

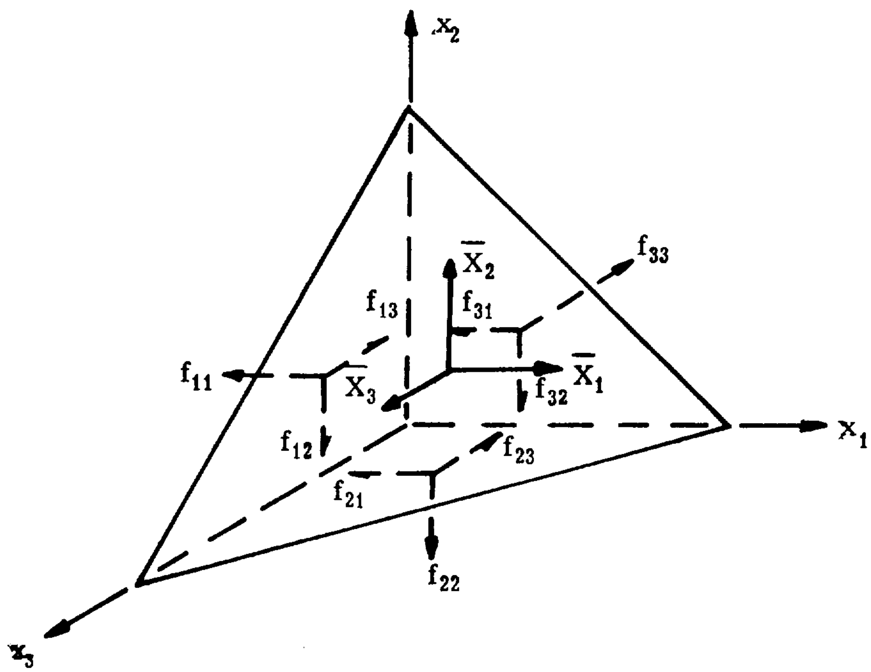


Figure A1.3.3-1 An Element Used in Deriving the Equations of Equilibrium

A1.3.3 Equations of Equilibrium (Cont'd)

By use of Equations 1 and summing forces the boundary equations are:

$$\overline{X}_1 = f_{11} k + f_{12} m + f_{13} n$$

$$\overline{X}_2 = f_{22} m + f_{23} n + f_{12} k \qquad (7)$$

$$\overline{X}_3 = f_{33} n + f_{13} k + f_{23} m$$

in which k, m, n are the direction cosines of the external normal to the surface of the body at the point under consideration and $\overline{X}_1$, $\overline{X}_2$, $\overline{X}_3$ are the components of the surface forces per unit area.

The Equations 6 and 7 in terms of the six components of stress, f_{11}, f_{22}, f_{33}, f_{12}, f_{13}, f_{23} are statically indeterminate. Consideration of the elastic deformations is necessary to complete the description of the stressed body. This is done by considering the elastic deformations of the body.

A1.3.4 Distribution of Strains in a Body

The relations between the components of stress and the components of strain have been established experimentally and are known as Hooke's law. For small deformations where superposition applies, Hooke's law in three dimensions for normal strain is written as:

$$\epsilon_1 = \frac{1}{E} [f_{11} - \nu (f_{22} + f_{33})]$$

$$\epsilon_2 = \frac{1}{E} [f_{22} - \nu (f_{11} + f_{33})] \qquad (8)$$

$$\epsilon_3 = \frac{1}{E} [f_{33} - \nu (f_{11} + f_{22})]$$

A1.3.4 Distribution of Strains in a Body (Cont'd)

and for shearing strain

$$\gamma_{12} = \frac{2(1+\nu)}{E} f_{12} = \frac{f_{12}}{G}$$

$$\gamma_{13} = \frac{2(1+\nu)}{E} f_{13} = \frac{f_{13}}{G} \qquad (9)$$

$$\gamma_{23} = \frac{2(1+\nu)}{E} f_{23} = \frac{f_{23}}{G}$$

These six components of strains can be expressed in terms of the three components of displacements. By considering the deformation of a small element dx_1, dx_2, dx_3 of an elastic body with u, v, w as the components of the displacement of the point 0. The displacement in the x_1 - direction of an adjacent point A on the x_1 axis is

$$u + \frac{\partial u}{\partial x_1} dx_1$$

due to the increase $(\partial u/\partial x_1)\,dx_1$ of the function u with increase of the coordinate x_1. It follows that the unit elongation at point 0 in the x_1 direction is $\partial u/\partial x_1$. In the same manner it can be shown that the unit elongations in the x_2 - and x_3 - directions are given by $\partial v/\partial x_2$ and $\partial w/\partial x_3$ respectively.

The distortion of the angle from AOB to A'O'B' can be seen from Figure A1.3.4-1 to be $\partial v/\partial x_1 + \partial u/\partial x_2$. This is the shearing strain between the planes $x_1\,x_3$ and $x_2\,x_3$. The shearing strains between the other two planes are obtained similarly.

The six components of strains in terms of the three displacements are:

$$\epsilon_1 = \frac{\partial u}{\partial x_1}, \quad \epsilon_2 = \frac{\partial v}{\partial x_2}, \quad \epsilon_3 = \frac{\partial w}{\partial x_3}$$

$$\gamma_{12} = \frac{\partial u}{\partial x_2} + \frac{\partial v}{\partial x_1}, \quad \gamma_{13} = \frac{\partial u}{\partial x_3} + \frac{\partial w}{\partial x_1}, \quad \gamma_{23} = \frac{\partial v}{\partial x_3} + \frac{\partial w}{\partial x_2} \qquad (10)$$

A1.3.4 Distribution of Strains in a Body (Cont'd)

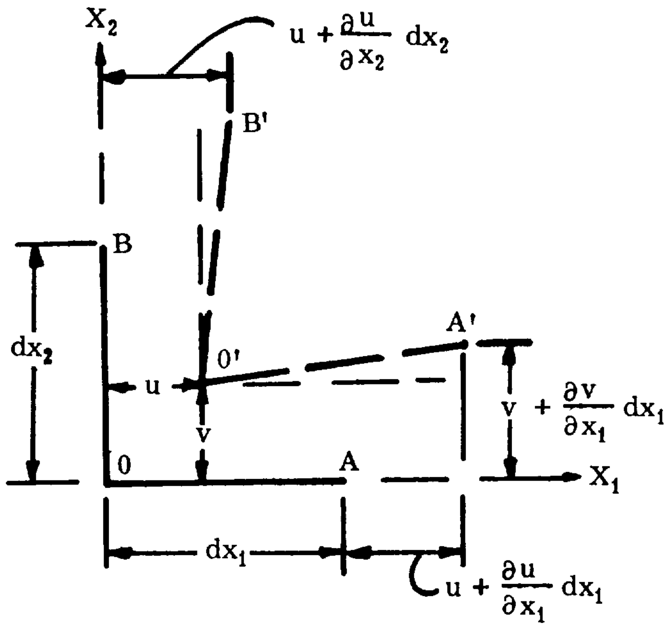


Figure A1.3.4-1 Distortions Due to Normal and Shearing Stresses Used to Define Strains in Terms of Displacements

A1.3.5 Conditions of Compatibility

The conditions of compatibility, that assure continuity of the structure, can be satisfied by obtaining the relationship between the strains in Equations 10. The relationship can be obtained by purely mathematical manipulation as follows:

Differentiating ϵ_1 twice with respect to x_2; ϵ_2 twice with respect to x_1; and γ_{12} once with respect to x_1 and once with respect x_2. The sum of the derivatives of ϵ_1 and ϵ_2 is found to be identical to the derivative of γ_{12}. Therefore,

$$\frac{\partial^2 \epsilon_1}{\partial x_2^2} + \frac{\partial^2 \epsilon_2}{\partial x_1^2} = \frac{\partial^2 \gamma_{12}}{\partial x_1 \partial x_2}$$

A1.3.5 Conditions of Compatibility (Cont'd)

Two more relationships of the same kind can be obtained by cyclic interchange of the subscripts 1, 2, 3.

Another set of equations can be found by further mathematical manipulation as follows:

Differentiate ϵ_1 once with respect to x_1 and once with respect to x_3; γ_{12} once with respect to x_1 and once with respect to x_3; γ_{13} once with respect to x_1 and once with respect to x_2; and γ_{23} twice with respect to x_1. It then follows that

$$2\,\frac{\partial^2 \epsilon_1}{\partial x_2 \partial x_3} = \frac{\partial^2 \gamma_{12}}{\partial x_1 \partial x_3} + \frac{\partial^2 \gamma_{13}}{\partial x_1 \partial x_2} - \frac{\partial^2 \gamma_{12}}{\partial x_1^2}$$

Two additional relationships can be found by the cyclic interchange of subscripts as before.

The six differential relations between the components of strain are called the equations of compatibility and are given below.

$$\frac{\partial^2 \epsilon_1}{\partial x_2^2} + \frac{\partial^2 \epsilon_2}{\partial x_1^2} = \frac{\partial^2 \gamma_{12}}{\partial x_1 \partial x_2}, \quad \frac{2\partial^2 \epsilon_1}{\partial x_2 \partial x_3} = \frac{\partial}{\partial x_1}\left(\frac{\partial \gamma_{12}}{\partial x_3} + \frac{\partial \gamma_{13}}{\partial x_2} - \frac{\partial \gamma_{22}}{\partial x_1} \right),$$

$$\frac{\partial^2 \epsilon_2}{\partial x_3^2} + \frac{\partial^2 \epsilon_3}{\partial x_2^2} = \frac{\partial^2 \gamma_{33}}{\partial x_2 \partial x_3}, \quad \frac{2\partial^2 \epsilon_2}{\partial x_1 \partial x_3} = \frac{\partial}{\partial x_2}\left(\frac{\partial \gamma_{12}}{\partial x_3} - \frac{\partial \gamma_{13}}{\partial x_2} + \frac{\partial \gamma_{23}}{\partial x_1} \right), \qquad (11)$$

$$\frac{\partial^2 \epsilon_3}{\partial x_1^2} + \frac{\partial^2 \epsilon_1}{\partial x_2^2} = \frac{\partial^2 \gamma_{13}}{\partial x_1 \partial x_3}, \quad \frac{2\partial^2 \epsilon_3}{\partial x_1 \partial x_2} = \frac{\partial}{\partial x_3}\left(-\frac{\partial \gamma_{12}}{\partial x_3} + \frac{\partial \gamma_{13}}{\partial x_2} + \frac{\partial \gamma_{23}}{\partial x_1} \right)$$

These equations of compatibility may be stated in terms of the stresses if the strains in Equations 11 are expressed in terms of the stresses by Hooke's law (Equations 8 and 9). Differentiating each of Equations 8 and 9 as required for substitution, we have

A1.3.5 Conditions of Compatibility (Cont'd)

$$(1+\nu)\ \nabla^2 f_{11} + \frac{\partial^2\theta}{\partial x_1^2} = 0 \quad , \quad (1+\nu)\ \nabla^2 f_{23} + \frac{\partial^2\theta}{\partial x_2 \partial x_3} = 0$$

$$(1+\nu)\ \nabla^2 f_{22} + \frac{\partial^2\theta}{\partial x_2^2} = 0 \quad , \quad (1+\nu)\ \nabla^2 f_{13} + \frac{\partial^2\theta}{\partial x_1 \partial x_3} = 0 \qquad (12)$$

$$(1+\nu)\ \nabla^2 f_{33} + \frac{\partial^2\theta}{\partial x_3^2} = 0 \quad , \quad (1+\nu)\ \nabla^2 f_{12} + \frac{\partial^2\theta}{\partial x_1 \partial x_2} = 0$$

where:

$$\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}$$

and

$$\theta = f_{11} + f_{22} + f_{33}$$

For most cases where strains are linear and superposition applies, the system of Equations 6, 7, and 11 or 12 are sufficient to determine the stress components without ambiguity. The use of stress functions to aid in the solution of these equations are discussed below.

A1.3.6 Stress Functions

It has been shown in the previous sections that the differential equations of equilibrium (Equations 6) ensure a distribution of stress in a body that preserves the equilibrium of every element in the body. The fact that these are satisfied does not necessarily mean that the distribution of stresses are correct since the boundary stresses must also be satisfied. The compatibility equations (Equations 11) must also be satisfied to ensure the proper strain distribution throughout the body. The problem is then to find an expression that satisfies all

A1.3.6 Stress Functions (Cont'd)

these conditions. The usual procedure is to introduce a function called a stress function that meets this requirement. For the sake of simplicity, this section will deal only with problems in two dimensions. The stresses due to the weight of the body will also be neglected.

In 1862, G. B. Airy introduced a stress function ($\phi(x_1, x_2)$) which is an expression that satisfies both Equations 6 and 11 (in two dimension) when the stresses are described by:

$$f_{11} = \frac{\partial^2\phi}{\partial x_2^2} \quad , \quad f_{22} = \frac{\partial^2\phi}{\partial x_1^2} \quad , \quad f_{12} = -\frac{\partial^2\phi}{\partial x_1 \partial x_2} \tag{13}$$

By operating on Equations 13 and substituting into Equations 11, we find that the stress function ϕ must satisfy the equation

$$\frac{\partial^4\phi}{\partial x_1^4} + 2\frac{\partial^4\phi}{\partial x_1^2 \partial x_2^2} + \frac{\partial^4\phi}{\partial x_2^4} = \nabla^4\phi = 0 \tag{14}$$

Thus the solution of a two-dimensional problem reduces to finding a solution of the biharmonic equation (Equation 14) which satisfies the boundary conditions (7) of the problem.

A1.3.7 Use of Equations from the Theory of Elasticity

Proficiency in the use of stress functions is gained mainly by experience. It is not unusual to find an expression that satisfies Equation 14 first and then try to determine what problem it solves.

The following problem is presented to illustrate the basic procedure in the use of stress functions.

A1.3.7 Use of Equations from the Theory of Elasticity (Cont'd)

Statement of the problem:

Determine the stress function that corresponds to the boundary conditions for a cantilever beam of rectangular cross section of unit width and loaded as shown in Figure A1.3.7-1. From this stress function determine the stresses and compare with the maximum flexure stresses as obtained by the method of mechanics.

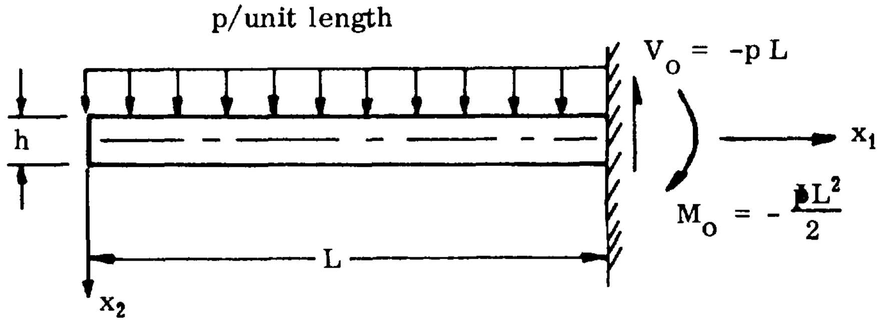


Figure A1.3.7-1 Sample Problem

Solution:

Assume that the stress function is

$$\phi = ax_2^5 + bx_2^3x_1^2 + cx_2^3 + dx_2x_1^2 + ex_1^2$$

Operate on ϕ to satisfy Equation 14

$$\nabla^4\phi = (5 \cdot 4 \cdot 3 \cdot 2)\, ax_2 + 2(3 \cdot 2 \cdot 2\, bx_2) = 0$$

$$24\, x_2\, (5a + b) = 0$$

from which $a = -b/5$ (a)

A1.3.7 Use of Equations from the Theory of Elasticity (Cont'd)

Since Equation 14 can now be satisfied by letting $a = -b/5$, the only other condition to satisfy is the boundary conditions.

From Figure A1.3.7-1 the boundary conditions are as follows:

1. $f_{22} = -p \quad \text{at} \quad x_2 = -h/2$

2. $f_{22} = 0 \quad \text{at} \quad x_2 = h/2$

3. $\int_{-h/2}^{h/2} f_{12}\,dx_2 = -pL \quad \text{at} \quad x_1 = L \quad \text{from} \quad \Sigma F = 0$

4. $\int_{-h/2}^{h/2} f_{11} x_2\,dx_2 = -pL^2/2 \quad \text{at} \quad x_1 = L \quad \text{from} \quad \Sigma M = 0$

5. $f_{12} = 0 \quad \text{at} \quad x_2 = h/2$

From Equation 13

$$f_{11} = \frac{\partial^2 \phi}{\partial x^2} = 20ax_2^3 + 6bx_1^2x_2 + 6cx_2$$

$$f_{22} = \frac{\partial^2 \phi}{\partial x_1^2} = 2bx_2^3 + 2hx_2 + 2e \tag{b}$$

$$f_{12} = -\frac{\partial^2 \phi}{\partial x_1 \partial x_2} = -6bx_2^2x_1 - 2hx_1$$

Using boundary condition 1

$$f_{22} = -p = -\frac{2bh^3}{8} - \frac{2dh}{2} + 2e \tag{c}$$

A1.3.7 Use of Equations from the Theory of Elasticity (Cont'd)

from boundary condition 2

$$f_{22} = 0 = \frac{2bh^3}{8} + \frac{2dh}{2} + 2e \qquad \text{(d)}$$

adding (c) and (d)

$$4e = -p \quad \text{or} \quad \underline{\underline{e = -p/4}} \qquad \text{(e)}$$

from boundary condition 3

$$\int_{-h/2}^{h/2} f_{12}\,dx_2 = \int_{-h/2}^{h/2} [-6bx_2^2x_1 - 2hx_1]\,dx_2$$

$$= 2\left[-\frac{6}{3}\,bLx_2^3 - 2hLx_2\right]_0^{h/2} = -pL \qquad \text{(f)}$$

or $$\underline{\underline{\frac{bh^3}{2} + 2dh = p}}$$

from boundary condition 4

$$\int_{-h/2}^{h/2} f_{11}\,x_2\,dx_2 = \int_{-h/2}^{h/2} [20ax_2^4 + 6bx_1^2x_2^2 + 6cx_2^2]\,dx_2$$

$$= 2\left[\frac{20a}{5}\,x_2^5 + \frac{6b}{3}\,x_1^2x_2^3 + \frac{6}{3}\,cx_2^3\right]_0^{h/2}$$

$$= \frac{ah^5}{4} + \frac{bL^2h^3}{2} + \frac{ch^3}{2} = -pL^2/2$$

A1.3.7 Use of Equations from the Theory of Elasticity (Cont'd)

substituting Equation a and solving for c

$$c = \frac{-pL^2 - b(L^2h^3 - h^5/10)}{h^3} \qquad (g)$$

from boundary condition 5

$$f_{12} = \frac{-6}{4} bh^2x_1 - 2dx_1 = 0$$

$$= -x_1\left(\frac{3}{2} bh^2 + 2d\right)$$

or

$$b = \frac{-4d}{3h^2} \qquad (h)$$

Solving Equations f and h simultaneously we get

$$d = \frac{3p}{4h} \quad \text{and} \quad b = -p/h^3 \qquad (i)$$

Substituting $b = -p/h^3$ into Equation g

$$c = -\frac{p}{10L} \qquad (j)$$

A1.3.7 Use of Equations from the Theory of Elasticity (Cont'd)

The stress function can now be written as

$$\phi = -px_1^2 \left(x_2^3/h^3 - 3x_2/4h + 1/4\right) + (ph^2/5)\left(x_2^5/h^5 - x_2^3/2h^3\right) \tag{k}$$

and the stresses as (see Equations b)

$$f_{11} = -\frac{p}{2I}\left(x_1^2 x_2 + h^2 x_2/10 - 2x_2^3/3\right) \tag{l}$$

$$f_{22} = -\frac{p}{2I}\left(x_2^3/3 - h^2 x_2/4 + h^3/12\right) \tag{m}$$

$$f_{12} = \frac{p}{2I}\left(x_2^2 x_1 - h^2 x_1/4\right) \tag{n}$$

where $I = h^3/12$

Comparison of maximum flexure stresses from Equation l with $x_1 = L$, $x_2 = -h/2$

$$f_{11}^{\text{elasticity}} = \frac{ph}{4I}\left(L^2 - \frac{h^2}{15}\right) \tag{o}$$

from elementary mechanics

$$f_{11}^{\text{mechanics}} = \frac{Mc}{I} = \frac{pL^2}{4I}\, h \tag{p}$$

The difference is then

$$f_{11}^{\text{elasticity}} - f_{11}^{\text{mechanics}} = -\frac{ph^3}{60I} = -\frac{p}{5} \tag{q}$$

A1.4.0 Theories of Failure

Several theories have been advanced to aid in the prediction of the critical load combination on a structural member. Each theory is based on the assumption that a specific combination of stresses or strains constitutes the limiting condition. The margin of safety of a member is then predicted by comparing the stress, the strain, or combination of stress and strain with the corresponding factors as determined from tests on the material.

Three of the more useful theories are stated in this subsection. A more detailed discussion on these and other theories of failure can be found in most elementary strength analysis text books such as references 2 and 3.

The Maximum Normal Stress Theory

The maximum normal stress theory of failure states that inelastic action at any point in a material begins only when the maximum principal stress at the point reaches a value equal to the tensile (or compressive) yield strength of the material as found in a simple tension (or compression) test. The normal or shearing stresses that occur on other planes through the point are neglected.

The Maximum Shearing Stress Theory

The maximum shearing stress theory is based on the assumption that yielding begins when the maximum shear stress in the material becomes equal to the maximum shear stress at the yield point in a simple tension specimen. To apply it, the principal stresses are first determined, then, according to Equation 5,

$$f_{max}^{ij} = \frac{1}{2}(f_{pi} - f_{pj})$$

where i and j are associated with the maximum and minimum principal stresses respectively.

The Maximum Energy of Distortion Theory

The maximum energy of distortion theory states that inelastic action at any point in a body under any combination of stresses begins only when the strain energy of distortion per unit volume absorbed at the point is equal to the strain

A1.4.0 Theories of Failure (Cont'd)

energy of distortion absorbed per unit volume at any point in a bar stressed to the elastic limit under a state of uniaxial stress as occurs in a simple tension (or compression) test. The value of this maximum strain energy of distortion as determined from the uniaxial test is

$$w_1 = \frac{1 + \nu}{3E} F_{yp}^2$$

and the strain energy of distortion in the general case is

$$w = \frac{1 + \nu}{6E} [(f_{p1} - f_{p2})^2 + (f_{p2} - f_{p3})^2 + (f_{p1} - f_{p3})^2]$$

where f_{p1}, f_{p2}, f_{p3} are the principal stresses and F_{yp} is the yield point stress. (For the case of a biaxial state of stress, $f_{p3} = 0.$)

The condition for yielding is then, $w = w_1$ or

$$(f_{p1} - f_{p2})^2 + (f_{p2} - f_{p3})^2 + (f_{p1} - f_{p3})^2 = 2 F_{yp}^2$$

A1.4.1 Elastic Failure

The choice of the proper theory of failure is dependent on the behavior of the material. It is suggested that the maximum principal stress theory be used for brittle materials and either the maximum energy of distortion theory or the maximum-shearing-stress theory for ductile materials.

The choice between the two methods for ductile materials may be made by considering the particular application. When failure of the component leads to catastrophic results, the maximum-shearing-stress theory should be used since the results are on the safe side.

A1.4.2 Interaction Curves

No general theory exists which applies in all cases for combined loading conditions in which failure is caused by instability. Interaction curves for the instability case or other critical load conditions are usually determined from or substantiated by structural tests. The analysis of various loading combinations are discussed in Section A3.

A1.0.0 Stress and Strain

REFERENCES

1. MIL-HDBK-5, "Metallic Materials and Elements for Flight Vehicle Structures," Department of Defense, Washington, D. C., August, 1962.

2. Murphy, Glenn, Advanced Mechanics of Materials, McGraw-Hill Book Company, Inc., New York, 1946.

3. Seely, Fred B. and James O. Smith, Advanced Mechanics of Materials, Second Edition, John Wiley and Sons, Inc., New York, 1957.

4. Timoshenko, S. and J. N. Goodier, Theory of Elasticity, Second Edition, McGraw-Hill Book Company, Inc., New York, 1951.

5. Freudenthal, Alfred M., The Inelastic Behavior of Engineering Materials and Structures, John Wiley and Sons, Inc., New York, 1950.

SECTION A2

LOADS

TABLE OF CONTENTS

A2 SPACE VEHICLE LOADS.

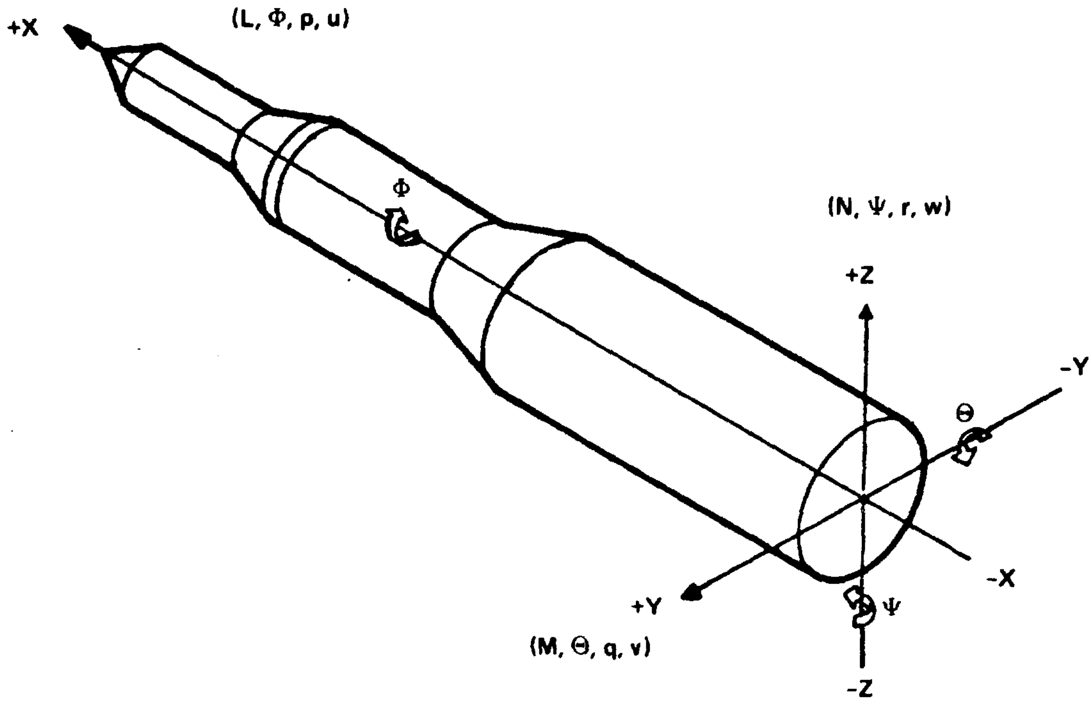
A. 2. 1 COORDINATE SYSTEMS.

The standard coordinate axes which have been used for rockets, missiles, and launch vehicles are shown in Figure A2. 1-1. The longitudinal X axis is taken as positive in the flight direction. The Y and Z axes are taken in appropriate directions to form a right-handed system. Moments are positive as determined by the right-hand rule.

For aircraft analysis the sign conventions used are shown in Figure A2. 1-2. In this figure externally applied loads acting at the airplane center of gravity are defined as positive when directed aft in the X direction, outboard to the left in the Y direction, and upward in the Z direction. Externally applied moments about the airplane center of gravity are defined as positive when acting as shown in Figure A2. 1-2 (left-hand rule). At any section under positive shear the rear, left outboard, or upper part tends to move aft, left, or up; the right outboard, or upper part, tends to move aft, right, or up. Any section under positive torsion tends to rotate clockwise when viewed from the rear, left, or above. Positive bending moments produce compression in the rear, left, and upper fibers. Positive axial load produces tension across any section.

The external loads which may act on a space vehicle are categorized as follows:

1. Flight Loads
2. Launch Pad Loads



AXIS	FORCE SYMBOL	MOMENT SYMBOL	LINEAR VELOCITY
LONGITUDINAL	X	L	u
LATERAL	Y	M	v
YAW	Z	N	w

ANGLE	SYMBOL	POSITIVE DIRECTION	ANGULAR VELOCITY
ROLL	Φ	Y to Z	p
PITCH	Θ	Z to X	q
YAW	Ψ	X to Y	r

NOTE: Sign convention follows right-hand rule.

Figure A2.1-1. Coordinate axes and symbols for a space vehicle.

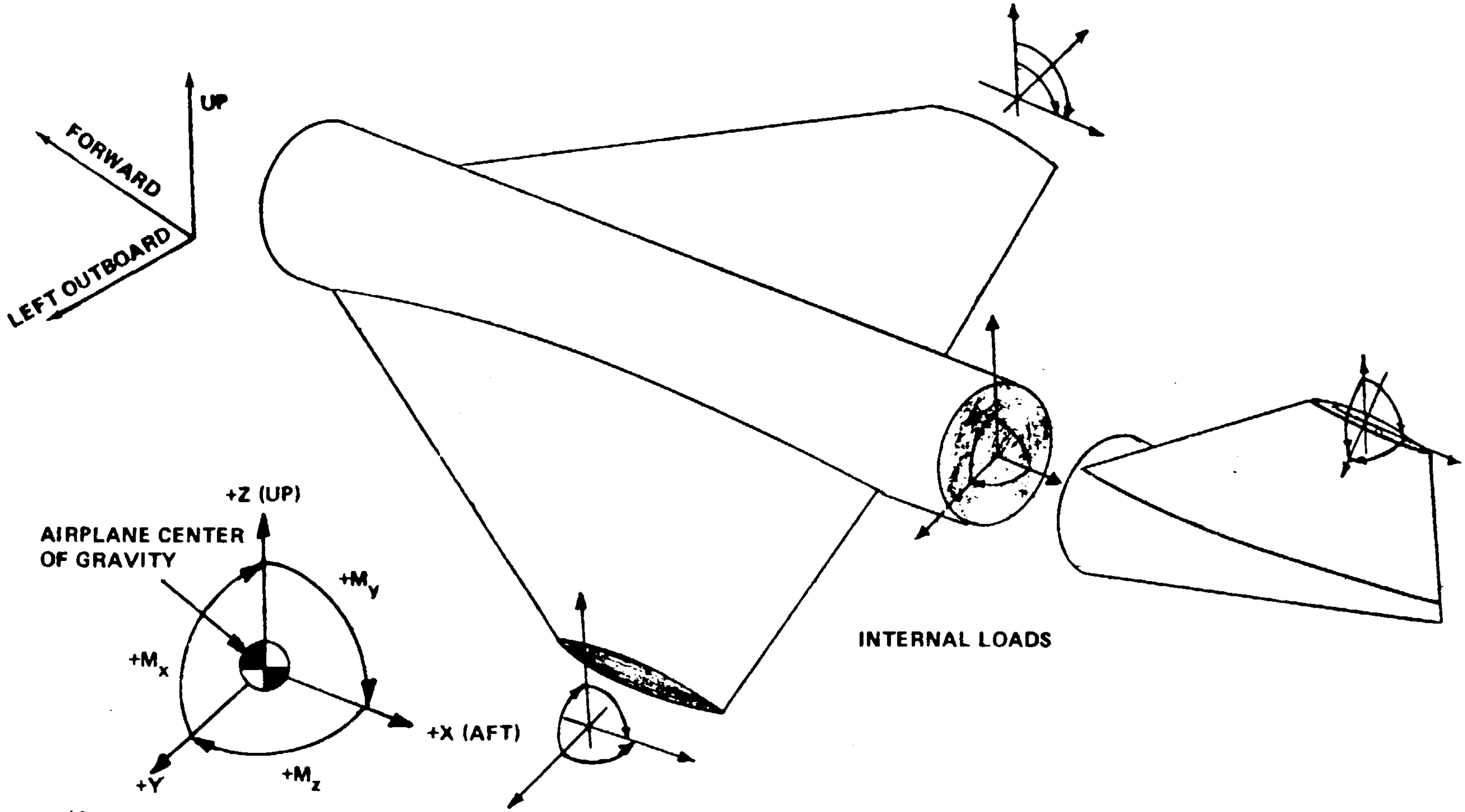


Figure A2.1-2. Positive sense of internal shear, torsion and bending moment and positive designation and sign convention of external applied loads and moments for an airplane.

3. Transportation and Handling Loads

4. Static Test Loads

5. Recovery Loads.

Since it is universal practice in the airframe industry for the stress analyst to obtain the magnitudes of external loads for the space vehicle from the cognizant "Loads Group" in his organization, the methods of calculating these quantities will not be presented in this manual. Rather, it will be assumed that these loads are furnished to the stress analyst so that only their qualitative description is required. These loads are generally resolved along the coordinate axes for stress and aerodynamic analysis.

A2.2.0 Loading Curves

The loads are usually presented in the form of load versus vehicle station curves, where locations along the longitudinal coordinate are referred to as vehicle stations. These curves are plotted for various times during the flight of the vehicle. At each of these times, the longitudinal force, the shear and the bending moment are plotted as a function of the vehicle station. Typical curves showing the bending moment and longitudinal force distribution along a vehicle can be seen in Figure A2.2.0-1.

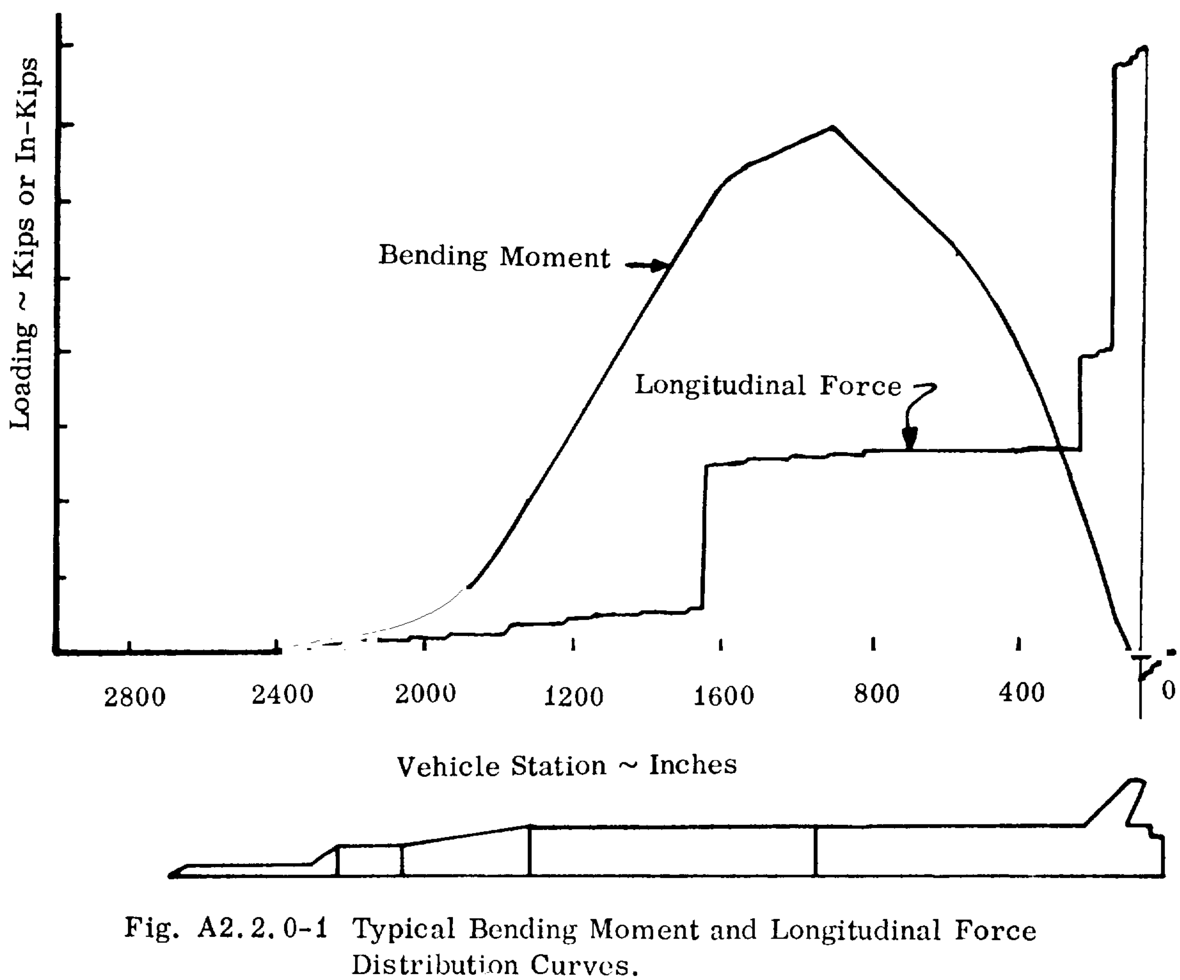


Fig. A2.2.0-1 Typical Bending Moment and Longitudinal Force Distribution Curves.

A2.2.0 Loading Curves (Cont'd)

It is necessary to know the circumferential pressure distribution along the vehicle at times of critical loading. This circumferential pressure is applied to the structure along with the critical loads during strength analysis of the vehicle. Typical distribution of this circumferential pressure at a particular vehicle station may appear as in Figure A2.2.0-2.

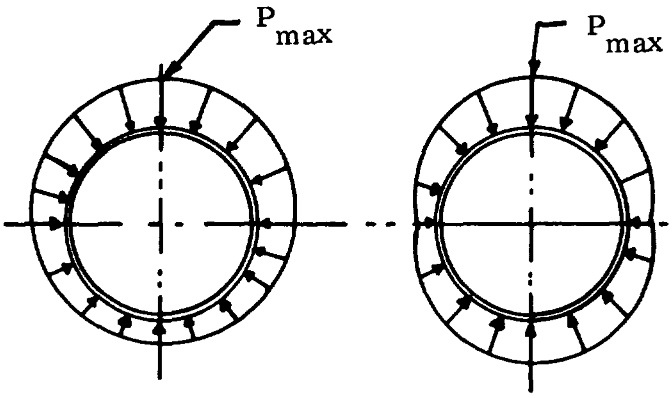


Figure A2.2.0-2 Typical Circumferential Pressure Distribution Curves at a Vehicle Station

A2.3.0 Flight Loads

A2.3.1 General

A space vehicle is subjected to flight loads of varying magnitudes during its flight. These flight loads must be investigated to determine the critical loads on the vehicle. Although it is not possible to know when these critical loads will occur without considering the entire flight history, there are certain times during the flight where conditions exist which are favorable for the build-up of critical loads. These times and the loads which occur may be summarized as follows:

1. Liftoff - As the vehicle lifts off the launch pad there is a sudden application and redistribution of loads on the vehicle. This causes dynamic loads which may be critical.

2. Maximum Dynamic Pressure (Maximum q) - At this time the combination of vehicle velocity and air density is such that the maximum airloads result.

3. Maximum $q\alpha$ - At this time the combination of vehicle velocity, air density and vehicle angle of attack is such that high bending moments due to airloads and vehicle acceleration result.

4. Engine Cutoff - Engine thrust and longitudinal inertia loads are maximum just before cutoff. During cutoff, high dynamic loads may result because of the redistribution of these loads.

A2.3.2 Dynamic and Acoustic Loads

Dynamic loads are loads which are characterized by an intensity that varies with time. These loads may be analyzed by one of two methods. One method is to replace the dynamic load by an equivalent static load, and it is the preferred method for most cases. The other method is a fatigue analysis and it is justified only in those cases where the confidence in the load time-history is good and the design is felt to be marginal.

Acoustic loads are loads induced by pressure fluctuations resulting from extraneous disturbances such as engine noise. The effects of these loads are determined by using an equivalent static pressure load. This equivalent static pressure acts in both the positive and negative directions, since the pressure fluctuates about a zero mean value. This pressure should be combined with the design inflight pressure to obtain the total pressure, and should be considered only in shell or panel stress analysis, not in the analysis of primary or supporting structure.

A2.3.3 Other Flight Loads

Other flight loads, which are caused by pressure and temperature differentials, must be considered in the stress analysis. In addition to the

A2. 3. 3 Other Flight Loads (Cont'd)

longitudinal loads presented in the loading diagrams in Section A2. 1. 1, there is a longitudinal load resulting from the difference between the ambient external pressure and the vehicle internal pressure at any time during flight. The ambient external pressure is a function of the vehicle's altitude only, while the vehicle internal pressure depends on vehicle trajectory and venting effects. These pressures in combination usually produce positive net internal pressures which either increases the tensile or decreases the compressive longitudinal load in the vehicle.

In order to determine the hoop loads at a particular vehicle station, the difference between the local external pressure and the vehicle internal pressure must be known at the desired time. The local external pressure is a function of the angle of attack, dynamic pressure and ambient pressure. The pressure difference may be positive or negative depending on the circumferential and longitudinal location of the point in question and on the range of values used in the aerodynamic analysis. This range of values results in a maximum and a minimum design curve.

Temperature magnitudes and temperature differentials caused by aerodynamic heating, retro or ullage rocket heating and cryogenic propellants result in additional vehicle loads which must be considered. The effects of these temperatures on material properties must also be investigated.

A2. 4. 0 Launch Pad Loads

The vehicle may be subjected to various loads while it is on the launch pad. These loads are referred to as launch pad loads and are generally categorized as follows:

1. Holddown Loads - The vehicle is usually held onto the launch pad by a holddown mechanism during engine ignition. The loads on the vehicle during this time are referred to as the holddown loads.

2. Rebound Loads - During engine ignition it may be necessary to shut down the engines due to some malfunction. The loads on the vehicle as it settles back onto the launch pad are referred to as rebound loads.

A2.4.0 Launch Pad Loads (Cont'd)

3. Surface Wind Loads - While the vehicle is freestanding on the launch pad, i.e., unsupported except for the holddown mechanism, it is exposed to surface wind loads. The magnitude of these loads will depend on the geographical location and should be specified in the design specifications.

4. Air-blast Loads - The vehicle may be subjected to an air-blast load from an accidental explosion at an adjacent vehicle launch site. The potential effect of this air-blast on the vehicle must be determined.

A2.5.0 Static Test Loads

The static test loads are the loads on the vehicle during static testing of the vehicle. These loads are summarized as follows:

1. Engine gimbaling loads

2. Longitudinal loads due to various propellant loadings during the holddown and rebound conditions

3. Wind loads

The dynamic and acoustic loads for static firing tests should also be investigated since they are higher during static test than in flight, in many cases.

A2.6.0 Transportation and Handling Loads

The transportation and handling loads are the loads which occur during transportation and handling of the space vehicle. In the design of the vehicle, these loads are required primarily for the design of tiedown and handling attachments.

A2.7.0 Recovery Loads

The recovery loads are the loads which occur during the recovery of a particular structural component or stage of the vehicle. These recovery loads also include the loads which may occur during descent and impact.

SECTION A

GENERAL

ASTRONAUTICS STRUCTURES MANUAL

SECTION SUBJECT INDEX

SECTION SUBJECT INDEX

(CONTINUED)

SECTION A3

COMBINED STRESSES

TABLE OF CONTENTS

A 3.0.0 Combined Stresses and Stress Ratio

A 3.1.0 Combined Stresses

When an element of structure is subjected to combined stresses such as tension, compression and shear, it is oftentimes necessary to determine resultant maximum stress values and their respective principal axes.

The solution may be attained through the use of equations or the graphical construction of Mohr's circle.

Relative Orientation and Equations of Combined Stresses

f_x and f_y are applied normal stresses.

f_s is applied shear stress.

f_{max} and f_{min} are the resulting principal normal stresses.

$f_{s_{max}}$ is the resulting principal shear stress.

θ is the angle of principal axes.

Sign Convention:

Tensile stress is positive.
Compressive stress is negative.
Shear stress is positive as shown.
Positive θ is counter-clockwise as shown.

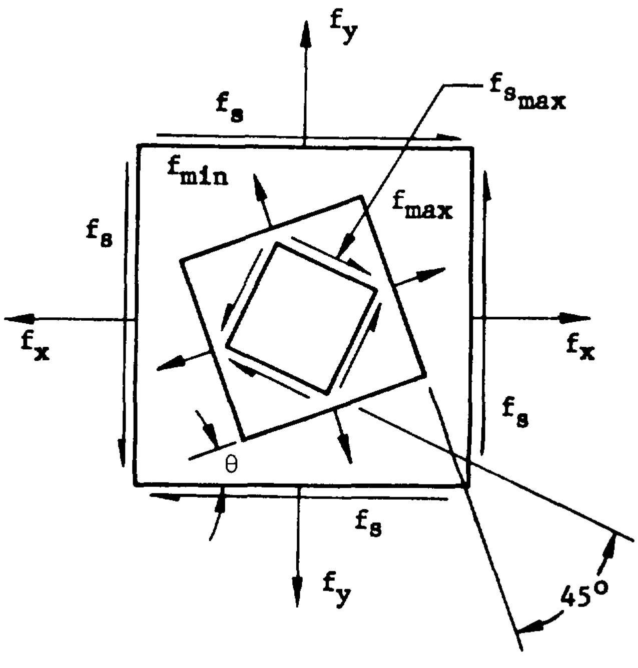


Fig. A 3.1.0-1

Note:
This convention of signs for shearing stress is adopted for this work only.

A 3.1.0 Combined Stresses (Cont'd)

Distributed Stresses on a 45^{o} Element

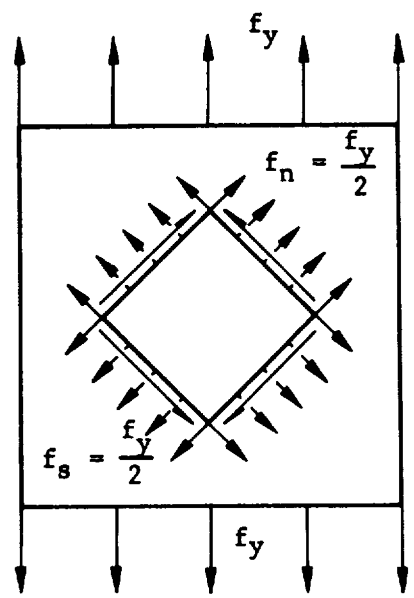


Fig. A 3.1.0-2
Pure Tension

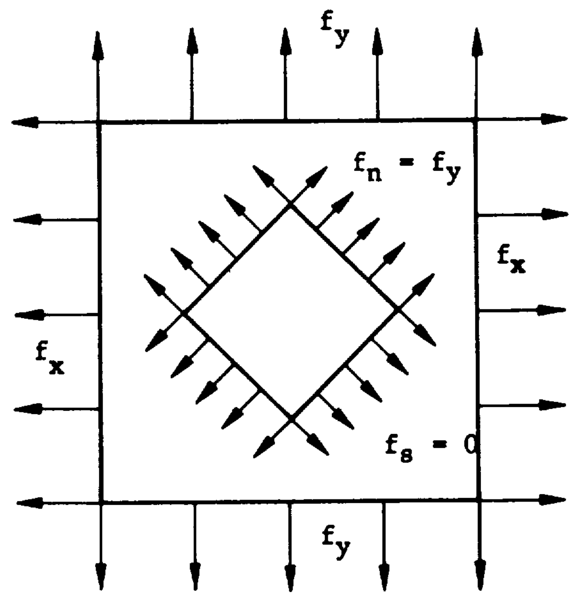


Fig. A 3.1.0-3
Equal Biaxial Tension

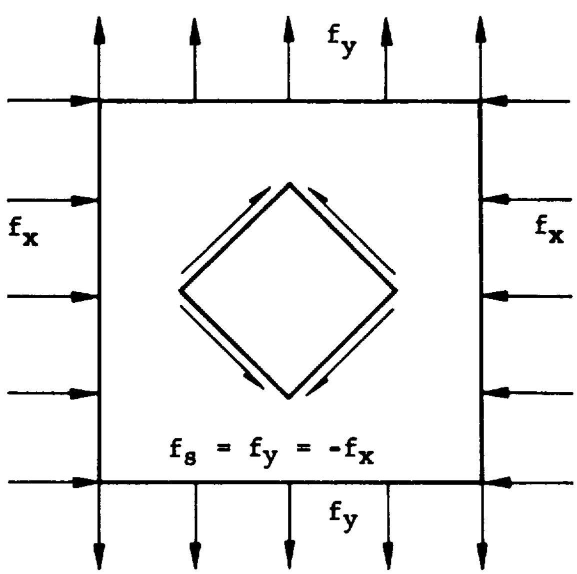


Fig. A 3.1.0-4
Equal Tension &
Compression

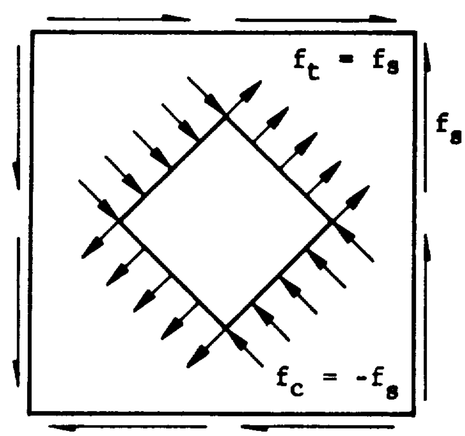


Fig. A 3.1.0-5
Pure Shear

A 3.1.0 Combined Stresses (Cont'd)

$$f_{max} = \frac{f_x + f_y}{2} + \sqrt{\left(\frac{f_x - f_y}{2}\right)^2 + f_s^2} \qquad \text{............ (1)}$$

$$f_{min} = \frac{f_x + f_y}{2} - \sqrt{\left(\frac{f_x - f_y}{2}\right)^2 + f_s^2} \qquad \text{............ (2)}$$

$$\text{TAN } 2\,\theta = \frac{2f_s}{f_x - f_y} \qquad \left[\begin{array}{l}\text{The solution results in} \\ \text{two angles representing} \\ \text{the principal axes of} \\ f_{max} \text{ and } f_{min}.\end{array}\right] \text{....... (3)}$$

$$f_{s_{max}} = \sqrt{\left(\frac{f_x - f_y}{2}\right)^2 + f_s^2} \quad \text{(Disregard Sign) .... (4)}$$

Constructing Mohr's Circle (for the stress condition shown in Fig. A 3.1.0-6a)

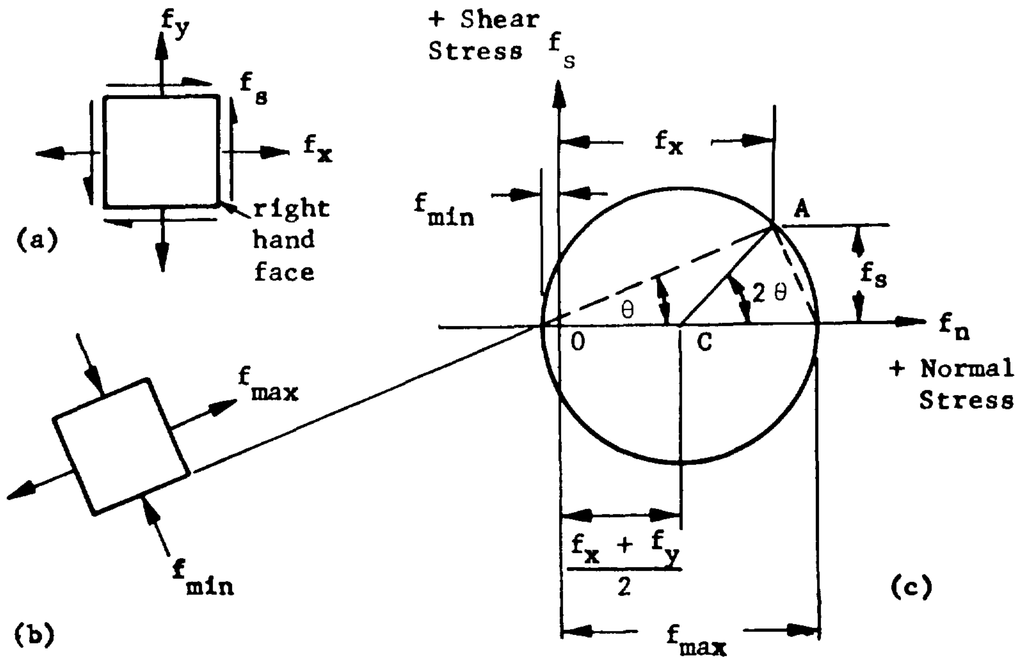


Fig. A 3.1.0-6

A 3.1.0 Combined Stresses (Cont'd)

1. Make a sketch of an element for which the normal and shearing stresses are known and indicate on it the proper sense of these stresses.

2. Set up a rectangular co-ordinate system of axes where the horizontal axis is the normal stress axis, and the vertical axis is the shearing stress axis. Directions of positive axes are taken as usual, upward and to the right.

3. Locate the center of the circle, which is on the horizontal axis at a distance of $(f_x + f_y)/2$ from the origin. Tensile stresses are positive, compressive stresses are negative.

4. From the right-hand face of the element prepared in step (1), read off the values for f_x and f_s and plot the controlling point "A". The co-ordinate distances to this point are measured from the origin. The sign of f_x is positive if tensile, negative if compressive; that of f_s is positive if upward, negative if downward.

5. Draw the circle with center found in step (3) through controlling point "A" found in step (4). The two points of intersection of the circle with the normal-stress axis give the magnitudes and sign of the two principal stresses. If an intercept is found to be positive, the principal stress is tensile, and conversely.

6. To find the direction of the principal stresses, connect point "A" located in step (4) with the intercepts found in step (5). The principal stress given by the particular intercept found in step (5) acts normal to the line connecting this intercept point with the point "A" found in step (4).

7. The solution of the problem may then be reached by orienting an element with the sides parallel to the lines found in step (6) and by indicating the principal stresses on this element.

To determine the maximum or the principal shearing stress and the associated normal stress:

1. Determine the principal stresses and the planes on which they act per previous procedure.

2. Prepare a sketch of an element with its corners located on the principal axes. The diagonals of this element will thus coincide with the directions of the principal stresses. (See Fig. A 3.1.0-7).

3. The magnitude of the maximum (principal) shearing stresses acting on mutually perpendicular planes is equal to the radius of the circle. These shearing stresses act along the faces of the element prepared in step (2) toward the diagonal, which coincides with the direction of the algebraically greater normal stress.

A 3.1.0 Combined Stresses (Cont'd)

4. The normal stresses acting on all faces of the element are equal to the average of the principal stresses, considered algebraically. The magnitude and sign of these stresses are also given by the distance from the origin of the co-ordinate system to the center of Mohr's circle.

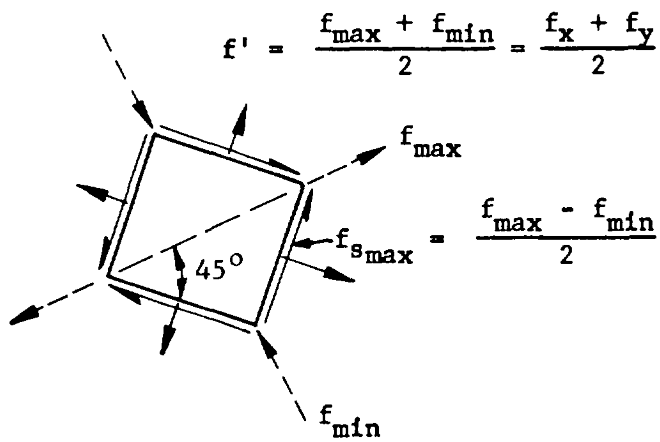


Fig. A 3.1.0-7

A 3.1.0 Combined Stresses (Cont'd)

Mohr's Circle for Various Loading Conditions

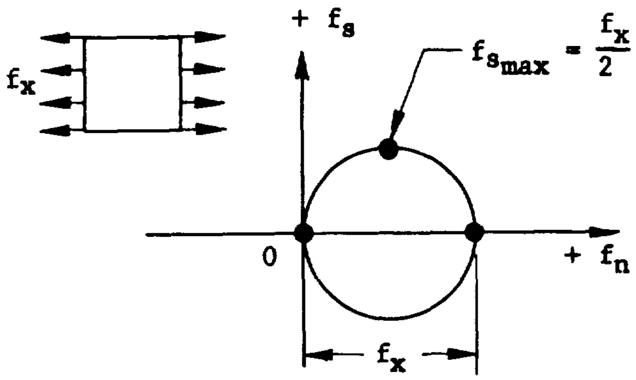


Fig. A 3.1.0-8 Simple Tension

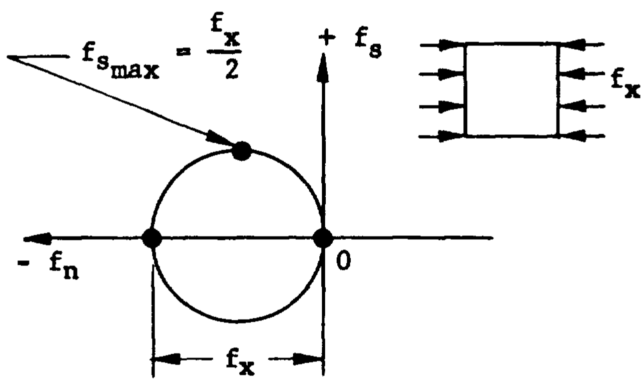


Fig. A 3.1.0-9 Simple Compression

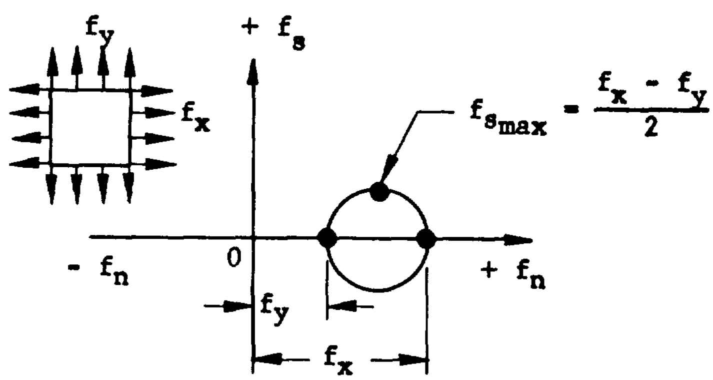


Fig. A 3.1.0-10 Biaxial Tension

A 3.1.0 Combined Stresses (Cont'd)

Mohr's Circle for Various Loading Conditions

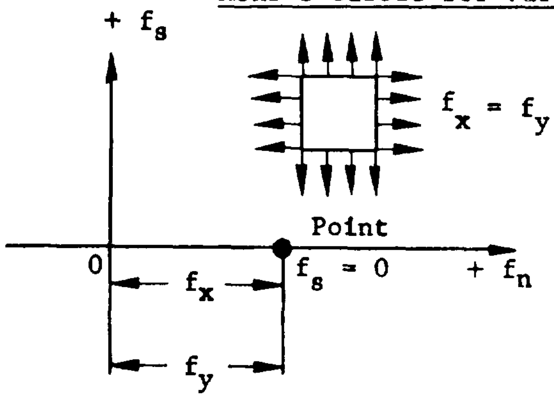


Fig. A 3.1.0-11 Equal Biaxial Tension

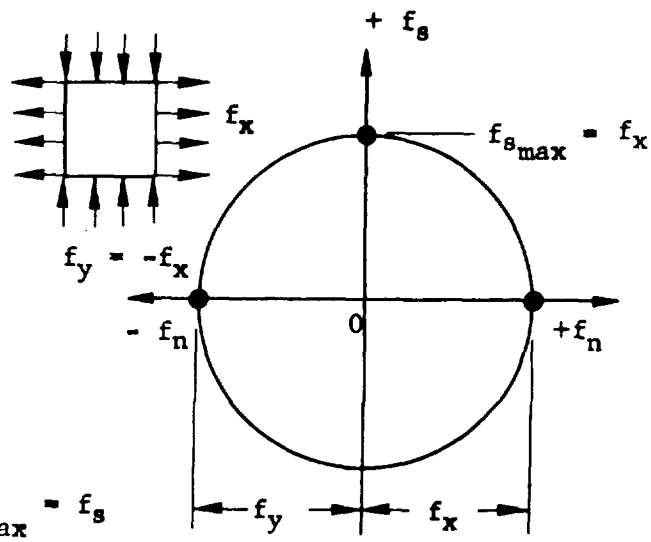


Fig. A 3.1.0-12
Equal Tension and Compression

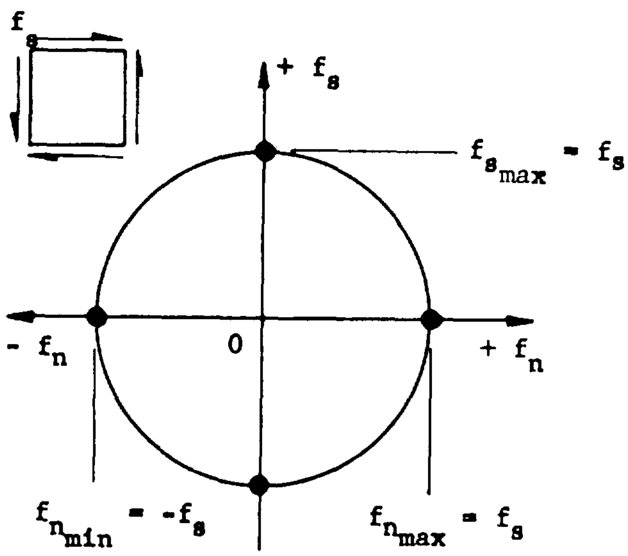


Fig. A 3.1.0-13
Pure Shear

A 3.2.0 Stress Ratios, Interaction Curves, and Factor of Safety

A means of predicting structural failure under combined loading without determining principal stresses is known as the interaction method.

The basis for this method is as follows:

1. The strength under each simple loading condition (tension, shear, bending, buckling, etc.) is determined by test or theory.
2. The combined loading condition is represented by either load or stress ratios, "R" where

$$R = \frac{\text{APPLIED LOAD OR STRESS}}{\text{FAILING LOAD OR STRESS}}$$

Failing can mean yield, rupture, buckling, etc.

The effect of one loading R_1 on another simultaneous loading R_2 is represented by an equation or interaction curve involving R_1 and R_2. The equation or curve may have been determined by theory, by test, or by a combination of both.

A schematic interaction curve is shown in Fig. A 3.2.0-1. Type of material or size effects will not influence it. This curve represents all the possible combinations of R_1 and R_2 that will cause failure.

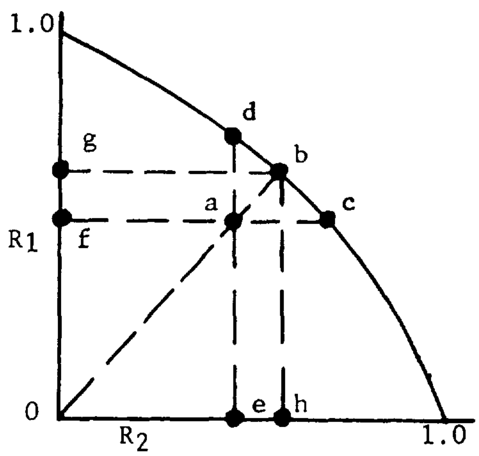


Fig. A 3.2.0-1

Using the curve:

1. Let the value of R_1 and R_2 locate point a.
2. R_1 and R_2 can increase proportionately until failure occurs at point b.
3. If R_1 remains constant, R_2 can increase until failure occurs at point c.
4. If R_2 remains constant, R_1 can increase until failure occurs at point d.
5. The factor of safety for (2) is F. S. = (ob÷oa), or (oh÷oe), (or og÷of) and the factor of safety for (3) is F. S. = (fc÷ fa).

A 3.2.0 Stress Ratios, Interaction Curves, and Factor of Safety (Cont'd)

In general, the formula for the factor of safety stated analytically for interaction equations where the exponents are only 1 or 2 (one term may be missing) is as follows:

$$F.S. = \frac{2}{\left[R' + \sqrt{(R')^2 + (R'')^2}\right]} \qquad \text{.................... (1)}$$

where

R' designates the sum of all first-power ratios.

R'' designates the sum of all second-power ratios.

A 3.2.1 A Theoretical Approach to Interaction

For combining normal and shear stresses, the principal stress equations are convenient to use.

Let F and F_s be defined as the failing stress, such as yielding or rupture.

Let $k=F_s/F$; tests of most materials will show this ratio to vary from 0.50 to 0.75.

$$R_f = f/F; \quad R_s = f_s/F_s$$

Maximum Normal Stress Theory

$$f_{max} = \frac{f}{2} + \sqrt{\left(\frac{f}{2}\right)^2 + f_s^2} \qquad \text{Ref Eq. (1) Sec. A 3.1.0}$$

Divide by F; replace f_s by R_sF_s, f/F by R_f and F_s/F by k.

The resulting equation when $f_{max} = F$ is

$$1 = \frac{R_f}{2} + \sqrt{\left(\frac{R_f}{2}\right)^2 + (kR_s)^2} \qquad \dots\dots\dots\dots\dots\dots \text{(1)}$$

A plot of this equation for k = 0.50 and k = 0.70 is shown in Fig. A 3.2.1-1.

Maximum Shear Stress Theory

$$f_{s_{max}} = \sqrt{\left(\frac{f}{2}\right)^2 + f_s^2} \qquad \text{Ref Eq. (4) Sec. A 3.1.0}$$

Divide by F_s; replace f by R_fF, f_s/F_s by R_s and F/F_s by 1/k.

The resulting equation when $f_{s_{max}} = F_s$ is

$$1 = \sqrt{\left(\frac{R_f}{2k}\right)^2 + R_s^2} \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(2)}$$

A plot of this equation for k = 0.50 and k = 0.70 is shown in Fig. A 3.2.1-1.

A 3.2.1 <u>A Theoretical Approach to Interaction (Cont'd)</u>

Conclusion

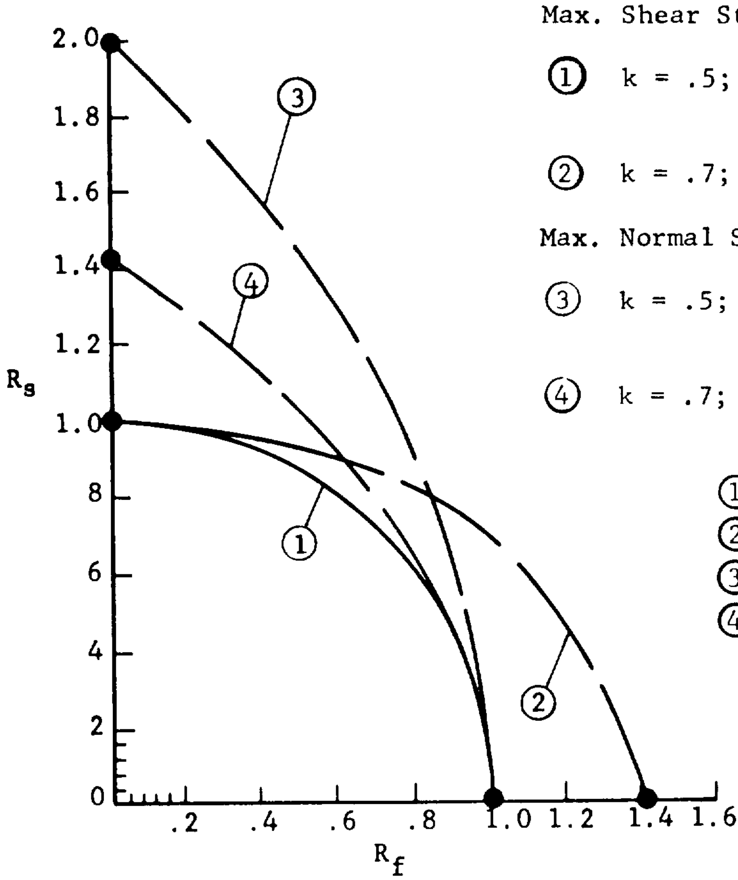
From the foregoing analysis, only Equation (2) with k = 0.5 is valid for all values of R_f and R_s. It is conservatively safe to use the resulting Equation (3) for values of k ranging from 0.5 to 0.7, since all values within curve ① must also be within the other curves. The use of other curves of Fig. A 3.2.1-1 may lead to unconservative results.

$$R_f^2 + R_s^2 = 1 \quad \ldots\ldots \quad (3)$$

and the Factor of Safety

$$F.S. = \frac{1}{\sqrt{R_f^2 + R_s^2}} \quad \ldots\ldots \quad (4)$$

For the graphical solution for Factor of Safety, the curve $R_1^2 + R_2^2 = 1$ of Fig. A 3.4.0-1 may be used.



Max. Shear Stress Theory

① $k = .5;\ R_f^2 + R_s^2 = 1$

② $k = .7;\ .5R_f^2 + R_s^2 = 1$

Max. Normal Stress Theory

③ $k = .5;\ R_f + \sqrt{R_f^2 + R_s^2} = 2$

④ $k = .7;\ R_f + \sqrt{R_f^2 + (1.4\, R_s)^2} = 2$

① Valid
② Partly Valid
③ Invalid
④ Partly Valid

Fig. A 3.2.1-1

A 3.3.0 Interaction for Beam-Columns

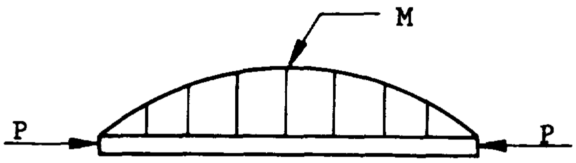


Fig. A 3.3.0-1
Sinusoidal Moment Curve

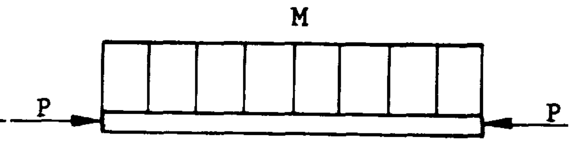


Fig. A 3.3.0-2
Constant Moment Curve

P = applied load.

$P_e = \dfrac{\pi^2 EI}{L^2}$ (Euler load). (Reference Section C 1.0.0).. (1)

P_o = buckling load $= \dfrac{\pi^2 E_t I}{L^2}$ .... (2)

or applicable short column formula. (Reference Section C 1.0.0)

M = maximum applied bending moment as a beam only.

M_o = ultimate bending moment as a beam only. (Reference Section B 4.0.0)

$R_a = \dfrac{P}{P_o}$ (column stress ratio) .... (3)

$R_b = \dfrac{M}{M_o}$ (beam stress ratio) .... (4)

$f = \dfrac{P}{A} + k\,\dfrac{Mc}{I}$

from which the interaction equation is:

$R_a + kR_b = 1$ .... (5)

Let $\eta = \dfrac{P_o}{P_e} = \dfrac{E_t}{E}$ (plasticity coefficient).............. (6)

For sinusoidal bending moment curves

$$k = \frac{1}{1 - P/P_e}$$

$R_b = (1 - R_a)\,(1 - \eta\, R_a)$ .... (7)

A 3.3.0 Interaction for Beam-Columns (Cont'd)

Interaction curves for various values of η are shown in Fig. A 3.3.1-5.

For constant bending moment curves

$$k = \frac{1}{\cos\left(\frac{\pi}{2}\sqrt{P/P_e}\right)}$$

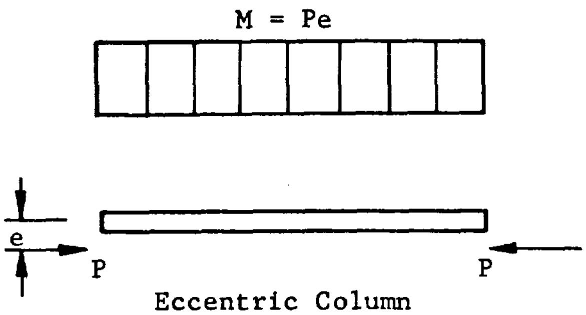
$$R_b = (1 - R_a)\cos\left(\frac{\pi}{2}\sqrt{\eta\, R_a}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

Interaction curves for various values of η are shown in Fig. A 3.3.1-6.

Conclusion

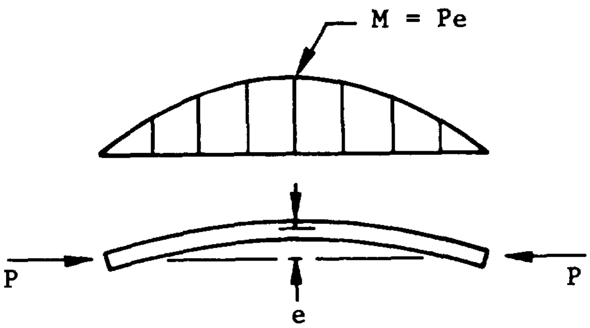
Comparison of Figs. A 3.3.1-5 and A 3.3.1-6 show that significant changes in shape of the primary bending moment diagram do not greatly influence the interaction curves. Therefore, Figs. A 3.3.1-5 and A 3.3.1-6 should be adequate for many types of simple beam columns.

A 3.3.1 Interaction for Eccentrically Loaded and Crooked Columns



Eccentric Column

Fig. A 3.3.1-1



Crooked Column

Fig. A 3.3.1-2

Reference Section A 3.3.0 for beam-column terms

$$R_e = \frac{e}{e_o} \text{ (eccentricity ratio) } \quad \ldots\ldots \quad (1)$$

$$e_o = \frac{M_o}{P_o} \text{ (base eccentricity, which is that required for } P_o \text{ to induce a moment } M_o) \quad \ldots \quad (2)$$

For a particular e, M would be a linear function of P as shown in Fig. A 3.3.1-3. A family of such lines could be drawn which would represent all eccentric columns.

To obtain Fig. A 3.3.1-4 (a nondimensional one-one diagram of the same form as the interaction curves of Figs. A 3.3.1-5 and A 3.3.1-6), P, M, and e of Fig. A 3.3.1-3 may be divided by P_o, M_o and e_o respectively.

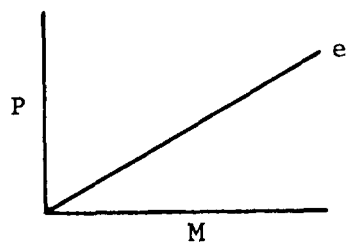


Fig. A 3.3.1-3

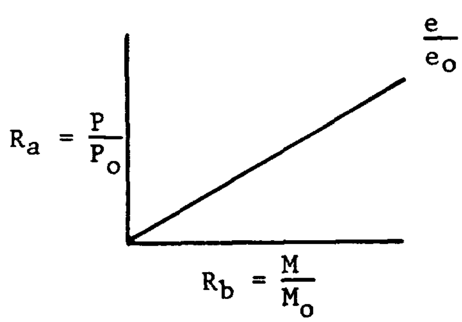


Fig. A 3.3.1-4

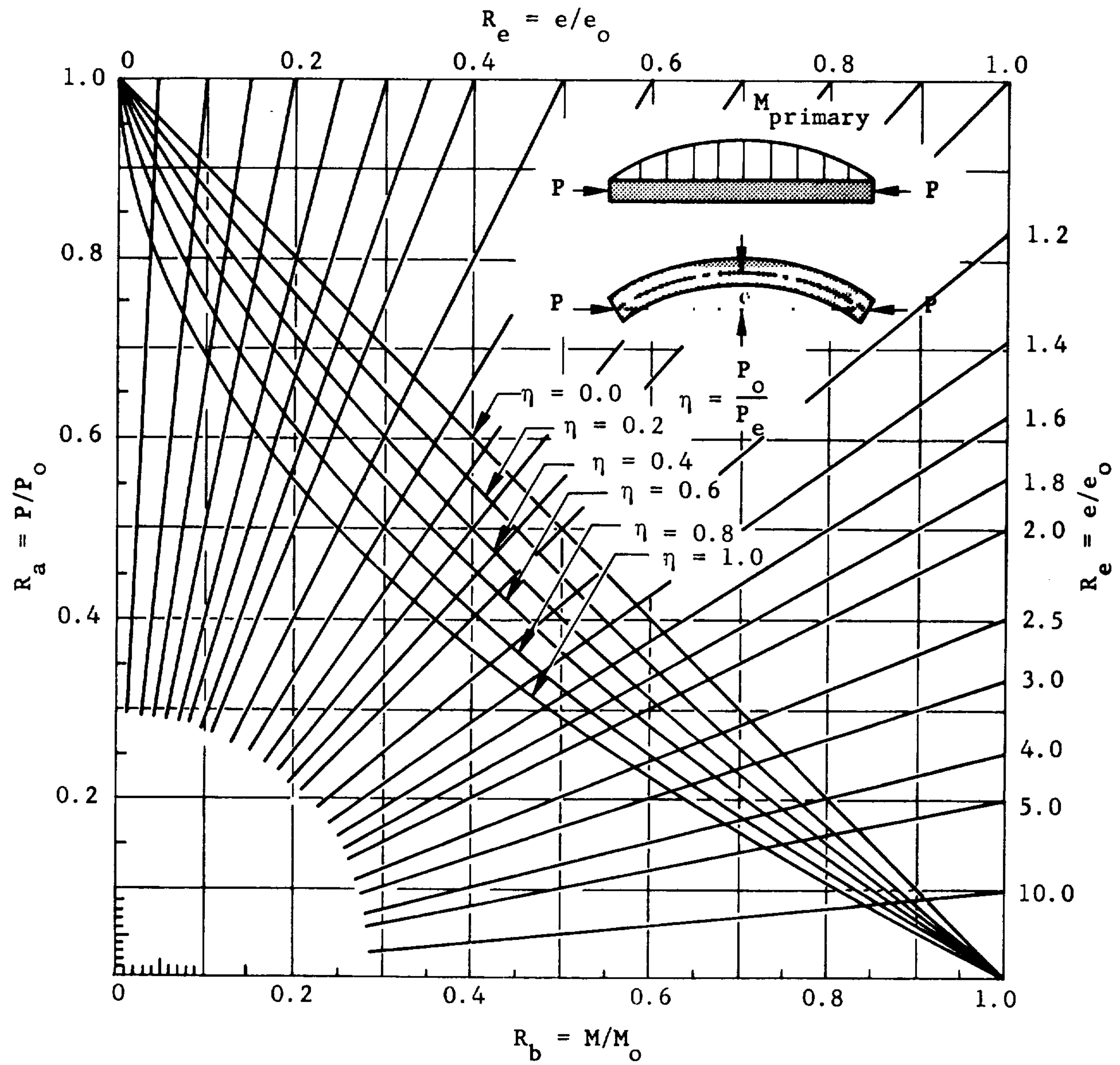
A 3.3.1 Interaction for Eccentrically Loaded and Crooked Columns (Cont'd)

In using Fig. A 3.3.1-6 for eccentric columns and Fig. A 3.3.1-5 for crooked columns the following steps are taken:

1. Determine P_o, the buckling load by $\pi^2 E_t I/L^2$ or applicable short column formula.
2. Calculate $P_e = \pi^2 EI/L^2$, the Euler load.
3. Determine M_o, the ultimate bending moment as a beam only using Section B 4.0.0.
4. Calculate $e_o = M_o/P_o$, the base eccentricity.
5. Calculate $R_e = e/e_o$.
6. Calculate $\eta = P_o/P_e$, the plasticity coefficient.
7. Knowing R_e and η, $R_a = P/P_o$ may be determined from the appropriate curve. This value of R_a corresponds to a Factor of Safety of 1.0.
8. The ultimate load is $P_u = P_o \times R_a$.
9. The Factor of Safety for an applied load P is

$$F.S. = \frac{P_u}{P}$$

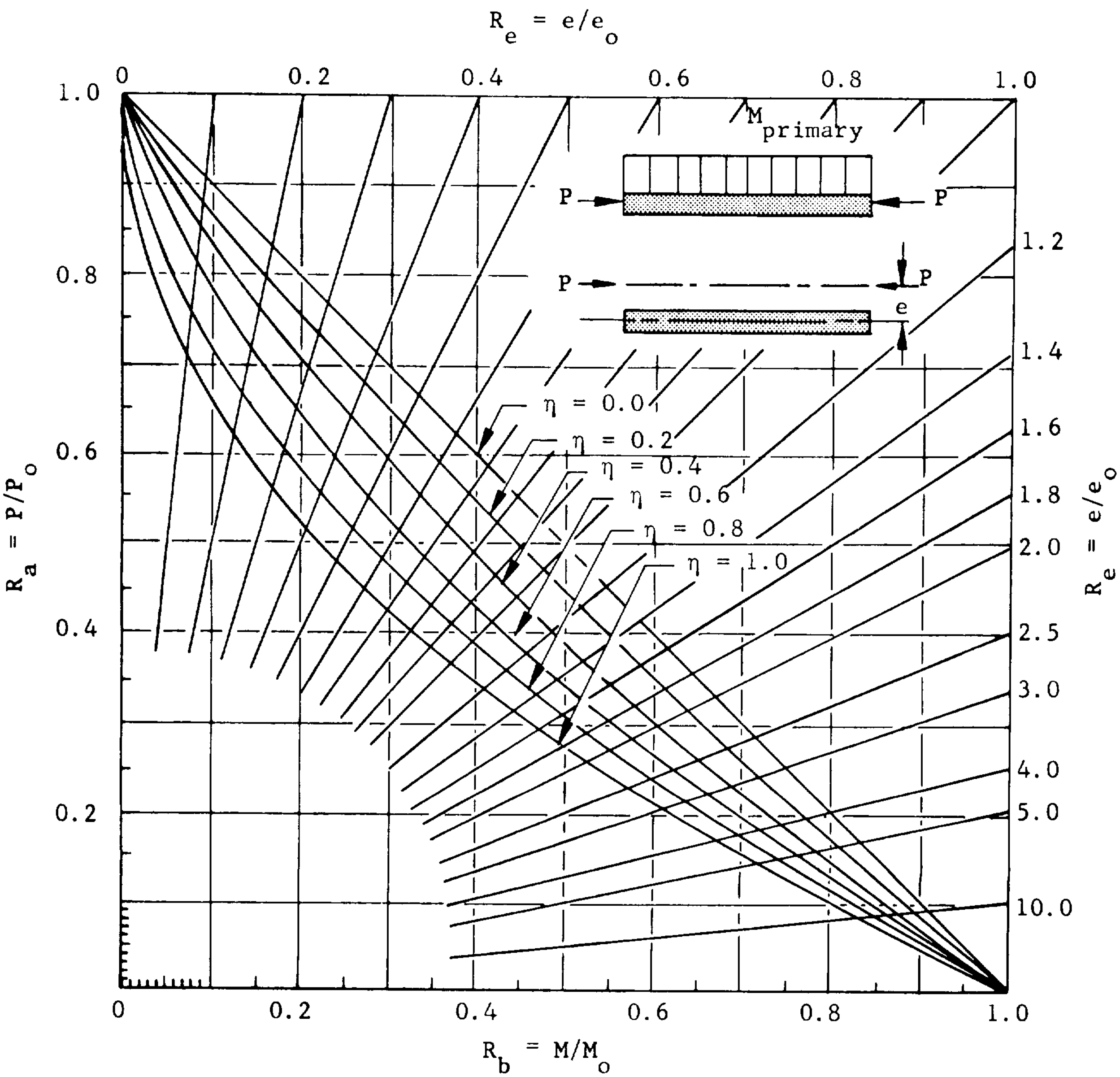
A 3.3.1 Interaction for Eccentrically Loaded and Crooked Columns (Cont'd)



Interaction Curves for Straight or Crooked Columns
with Sinusoidal Primary Bending Moment and Compression

Fig. A 3.3.1-5

A 3.3.1 Interaction for Eccentrically Loaded and Crooked Columns (Cont'd)



Interaction Curves for Columns with Constant Primary Bending Moment and Axial or Eccentric Compression

Fig. A 3.3.1-6

A 3.4.0 General Interaction Relationships

Table A 3.4.0-1

STRUCTURE	LOADING COMBINATION	FIGURE	INTERACTION EQUATION	EQ. FOR FACTOR OF SAFETY	REMARKS
Compact	Biaxial Tension or Biaxial Compression		$R_x = \frac{f_x}{F}$; $R = \frac{f_y}{F}$	$\frac{1}{R_{max}}$	Use R_x or R_y, whichever is greater
Compact and Round Tubes (a)	Axial and Bending Stresses	A 3.4.0-1	$R_s + R_b = 1$	$\frac{1}{R_a + R_b}$	For R_b, see Sect. B 4.0.0 on Plastic Bending
	Normal and Shear Stresses	A 3.4.0-1	$R_f^2 + R_s^2 = 1$ $R_f = R_a + R_b$	$\frac{1}{\sqrt{R_f^2 + R_s^2}}$	(b) for $.5 < \frac{F_s}{F} < .75$; For all other values use max. stress equations or Mohr's circle
Round Tubes	Bending, Torsion and Compression		$R_b^2 + R_{st}^2 = (1 - R_c)^2$	$\frac{1}{R_c + \sqrt{R_b^2 + R_{st}^2}}$	
Stream-Line Tubes	Bending and Torsion	A 3.4.0-1	$R_b + R_{st} = 1$	$\frac{1}{R_b + R_{st}}$	
Bolts	Tension and Shear	A 3.4.0-1 A 3.4.0-2	$R_t^2 + R_s^3 = 1$		

A 3.4.0 General Interaction Relationships (Cont'd)

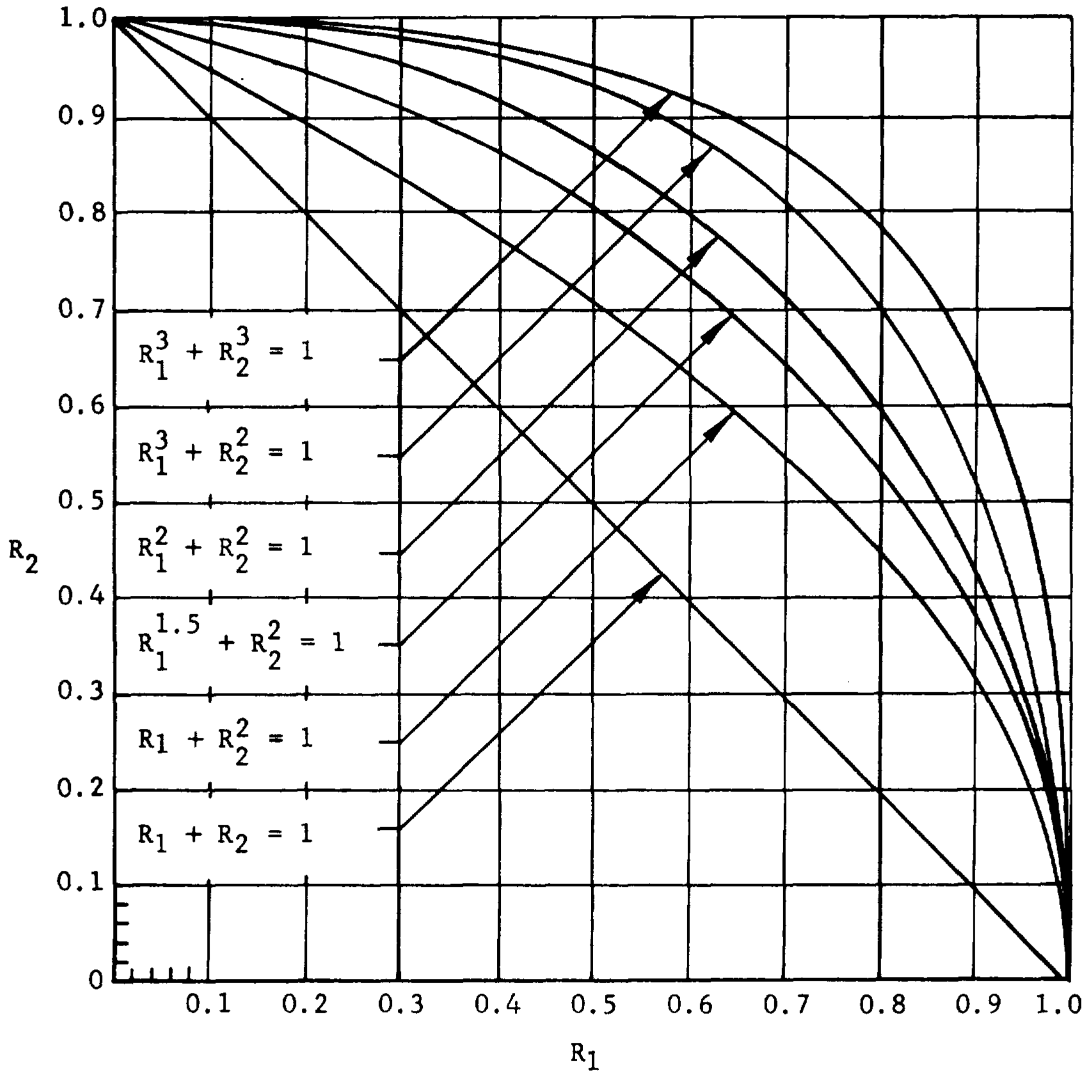
Table A 3.4.0-1 (Cont'd)

NOTE: Care must be exercised in determining whether to check Factor of Safety for limit or ultimate loads.

(a) For round tubes in compression see Section C.3.0.0.

(b) See Section A 3.2.1 for discussion of range.

A 3.4.0 General Interaction Relationships (Cont'd)



Interaction Curves

Fig. A 3.4.0-1

A 3.4.0 General Interaction Relationships (Cont'd)

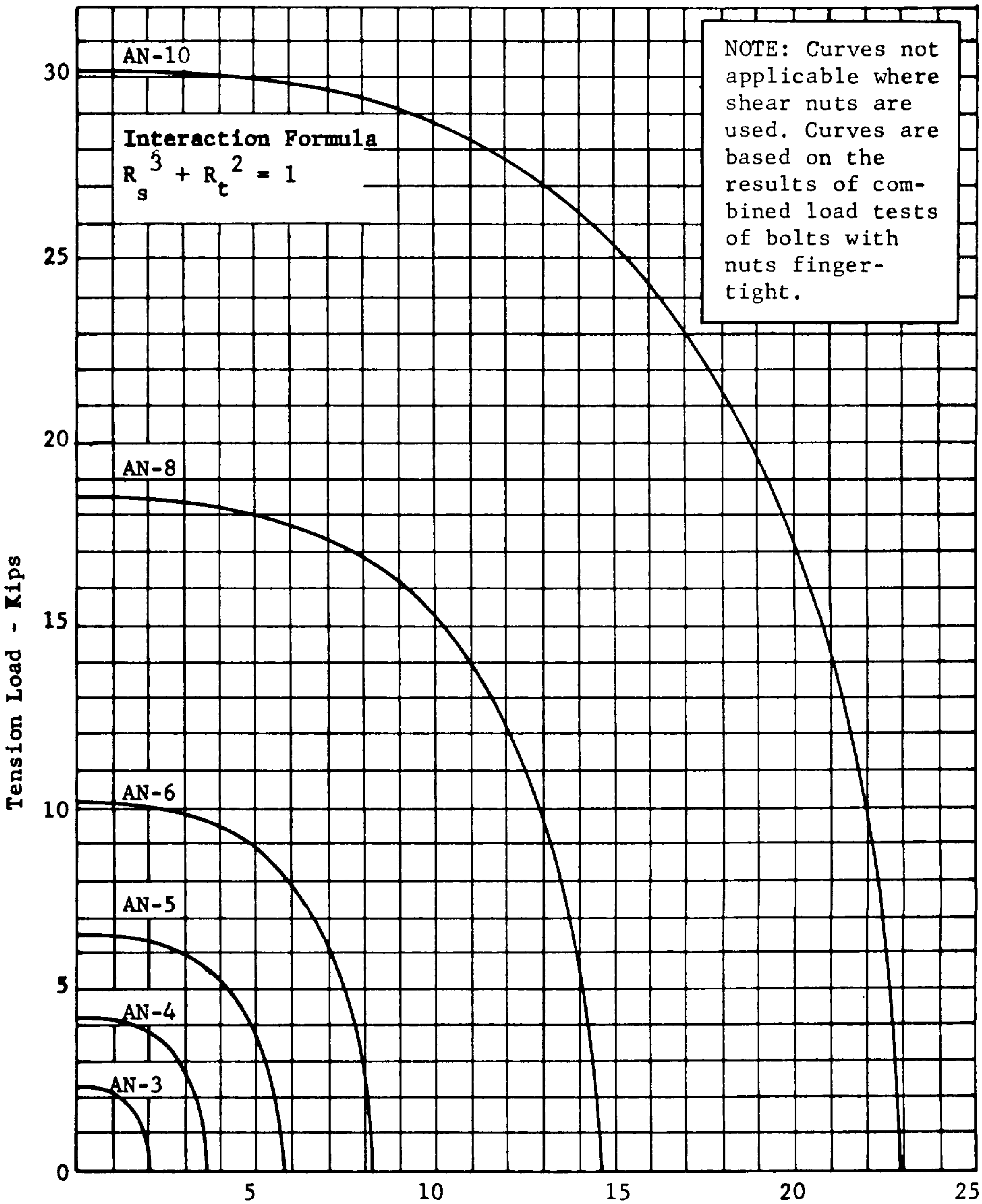
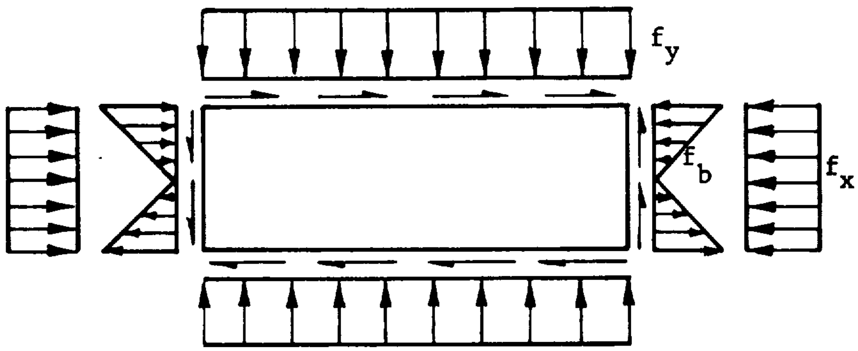


Fig. A 3.4.0-2

AN Steel Bolts (125,000 H.T.) Interaction Curves*

A 3.5.0 Buckling of Rectangular Flat Plates Under Combined Loading

NOTE: See discussion in Sec. C 2.1.4



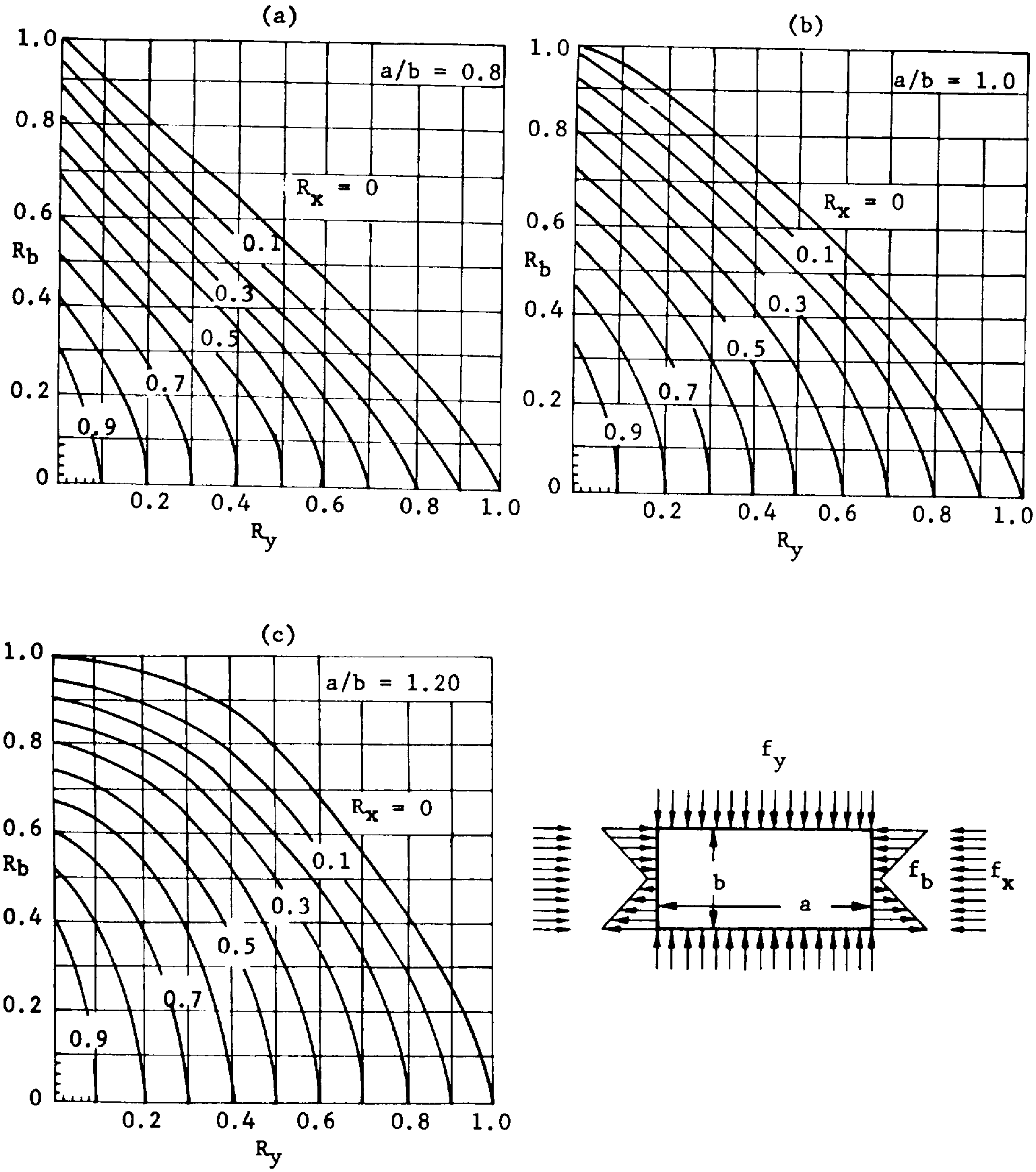
Combined Loading

Fig. A 3.5.0-1

Table A 3.5.0-1

THEORY	LOADING COMBINATION	FIGURE	INTERACTION EQUATION	EQ FOR FACTOR OF SAFETY
Elastic	Biaxial Compression	A 3.5.0-2	For plates that buckle in sq. waves, $R_x + R_y = 1$	$\frac{1}{R_x + R_y}$
	Longitudinal Compression and Shear		For Long Plates, $R_c + R_s^2 = 1$	$\frac{2}{R_c + \sqrt{R_c^2 + 4R_s^2}}$
	Longitudinal Compression and Bending	A 3.5.0-2		
	Bending and Shear	A 3.4.0-1	$R_b^2 + R_s^2 = 1$	$\frac{1}{\sqrt{R_b^2 + R_s^2}}$
	Bending, Shear, & Transverse Compression	A 3.5.0-3		
	Longitudinal Compression, Bending and Transverse Compression	A 3.5.0-2		
Inelastic	Longitudinal Compression and Shear	A 3.4.0-1	$R_c^2 + R_s^2 = 1$	$\frac{1}{\sqrt{R_c^2 + R_s^2}}$

A 3.5.0 Buckling of Rectangular Flat Plates Under Combined Loading (Cont'd)



Interaction Curves for Simply Supported Flat Rectangular Plates Under Combined Biaxial-Compression and Longitudinal Bending Loadings

Fig. A 3.5.0-2

A 3.5.0 Buckling of Rectangular Flat Plates under Combined Loading (Cont'd)

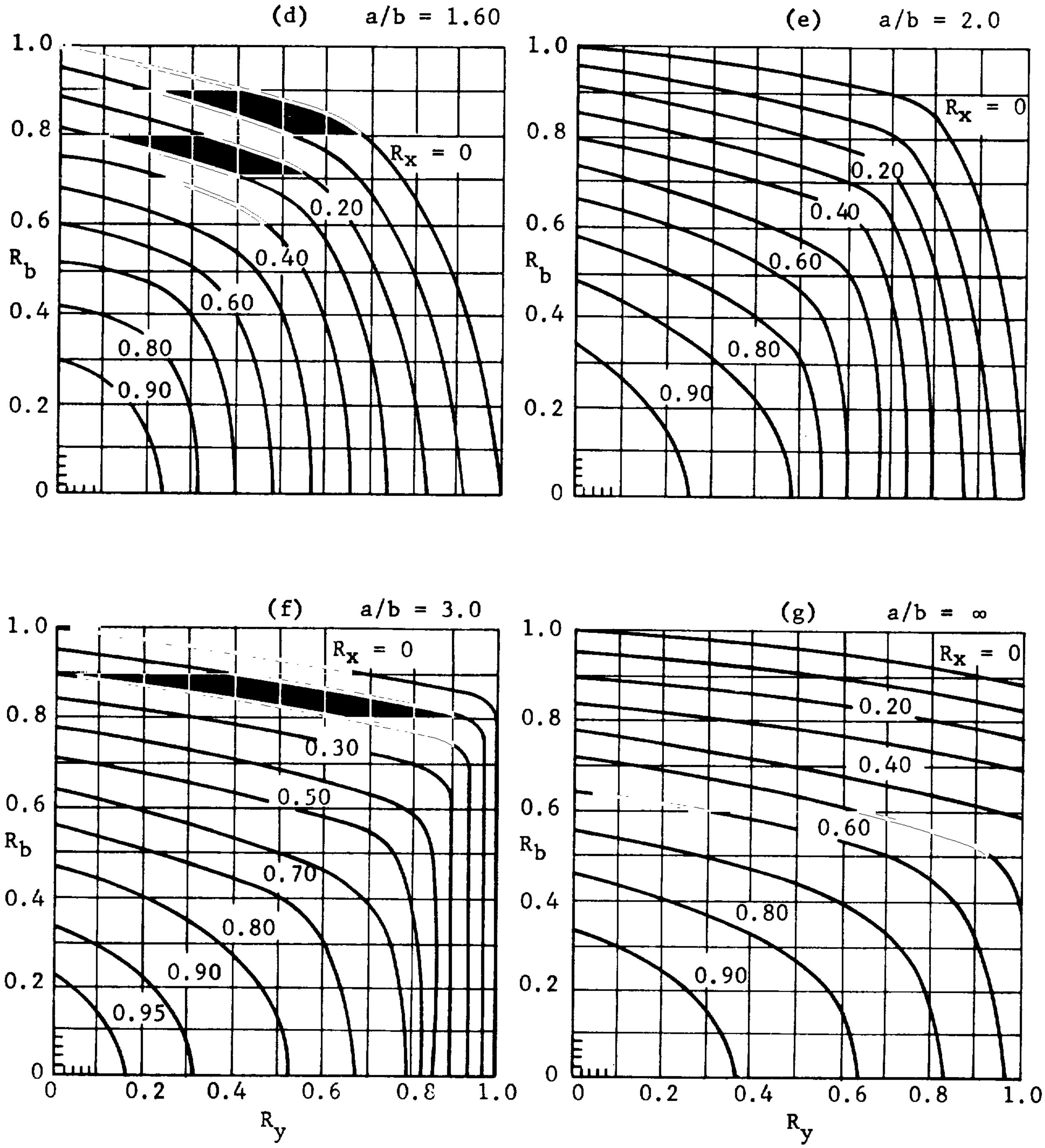
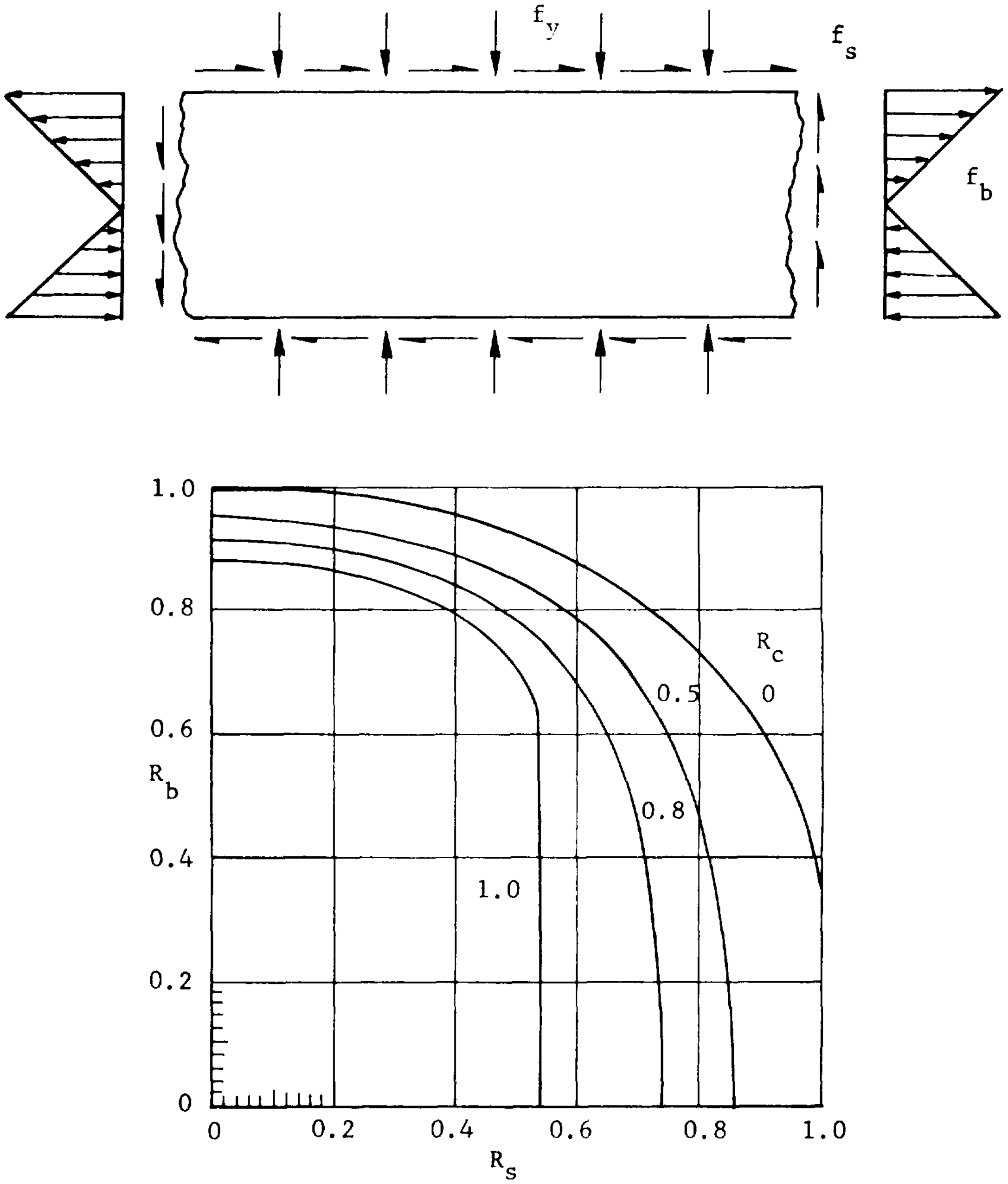


Fig. A 3.5.0-2 (Cont'd)

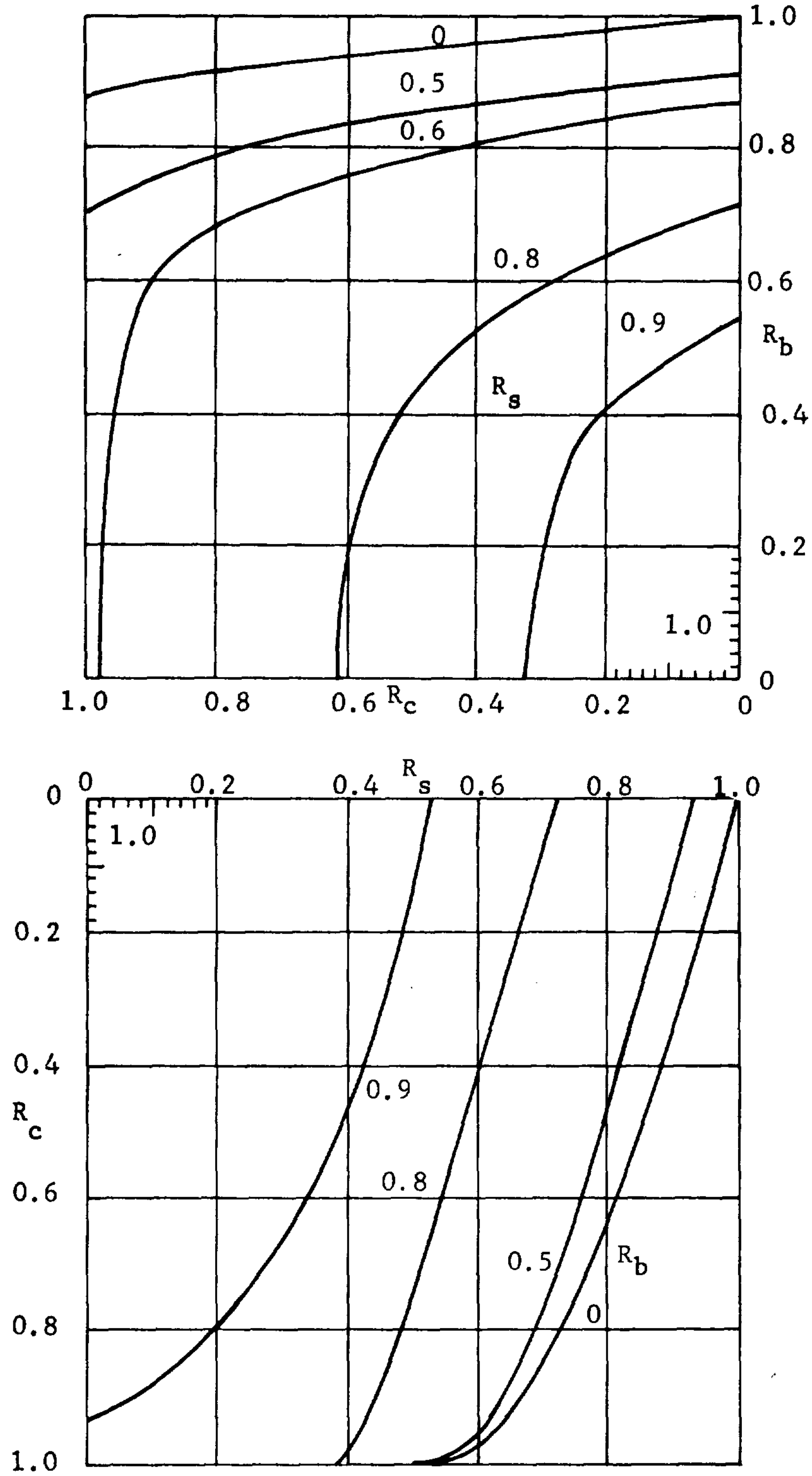
A 3.5.0 Buckling of Rectangular Flat Plates under Combined Loading (Cont'd)



Interaction Curves for Simply Supported Long Flat Plates
Under Various Combinations of Shear, Bending, and Transverse Compression

Fig. A 3.5.0-3

A 3.5.0 <u>Buckling of Rectangular Flat Plates under Combined Loading</u> <u>(Cont'd)</u>



Interaction Curves (Cont'd)

Fig. A 3.5.0-3

A3.6.0 Buckling of Circular Cylinders, Elliptical Cylinders, and Curved Plates Under Combined Loading

Table A 3.6.0-1

STRUCTURE	LOADING COMBINATION	FIGURE	INTERACTION EQUATION	EQ. FOR FACTOR OF SAFETY	NOTES
Curved Plates	Longitudinal Compression and Shear	A 3.4.0-1	$R_s^2 + R_c = 1$	$\frac{2}{R_c + \sqrt{R_c^2 + 4R_s^2}}$	
	Longitudinal Compression and Internal Pressure	A 3.6.0-1	$R^2 - \lvert R_p \rvert = 1$		
	Shear and Internal Pressure		where $R = R_c$ or R_s		1
Circular Cylinders	Longitudinal Compression and Pure Bending	A 3.4.0-1	$R_c + R_b = 1$	$\frac{1}{R_c + R_b}$	
	Longitudinal Compression and Torsion	A 3.4.0-1	$R_c + R_{st}^2 = 1$	$\frac{2}{R_c + \sqrt{R_c^2 + 4R_{st}^2}}$	
	Torsion and Longitudinal Tension	A 3.6.0-1	$R_{st}^3 - \lvert R_t \rvert = 1$		2
	Pure Bending and Tension	A 3.4.0-1	$R_b^{1.5} + R_t = 1$		
	Pure Bending and Transverse Shear	A 3.4.0-1	$R_b^3 + R_s^3 = 1$		3

A 3.6.0 Buckling of Circular Cylinders, Elliptical Cylinders, and Curved Plates under Combined Loading (Cont'd)

Table A 3.6.0-1 (Cont'd)

STRUCTURE	LOADING COMBINATION	FIGURE	INTERACTION EQUATION	EQ. FOR FACTOR OF SAFETY	NOTES
Circular Cylinders (Cont'd)	Longitudinal Compression, Pure Bending and Transverse Shear		$R_c + \sqrt[3]{R_s^3 + R_b^3} = 1$		3
	Pure Bending Torsion, and Transverse Shear		$R_b^p + (R_s + R_{st})^q = 1$		4
	Longitudinal Compression, Pure Bending Transverse Shear, and Torsion		$R_c + R_{st}^2 + \sqrt[3]{R_s^3 + R_b^3} = 1$		3
	Longitudinal Compression, Pure Bending and Torsion		$R_c + R_b + R_{st}^2 = 1$	$R_c + R_b + \sqrt{(R_c + R_b)^2 + 4R_{st}^2}$	
Elliptical Cylinders	Bending and Transverse Shear	A 3.4.0-1	$R_b^2 + R_s^2 = 1$	$\frac{1}{\sqrt{R_b^2 + R_s^2}}$	3
	Bending and Torsion	A 3.4.0-1	$R_b^2 + R_{st}^2 = 1$	$\frac{1}{\sqrt{R_b^2 + R_{st}^2}}$	

A 3.6.0 Buckling of Circular Cylinders, Elliptical Cylinders, and Curved Plates under Combined Loading (Cont'd)

Table A 3.6.0-1 (Cont'd)

NOTES:

1. $R_p = \dfrac{\text{internal pressure}}{\text{external collapsing pressure}}$

2. $R_t = \dfrac{\text{applied tensile stress}}{\text{compression buckling stress}}$; $R_t < 0.8$

3. Use f_s and f_b each as maximum calculated values even though the locations of the two maxima do not coincide. Use F_s as smaller of F_{su} and 1.25 F_{st}. Use buckling stress in bending for F_b.

4. When $\dfrac{R_{st}}{R_s} \leq 1$:

   $$p = 3 - \frac{3}{4}\left(\frac{R_{st}}{R_s}\right)$$

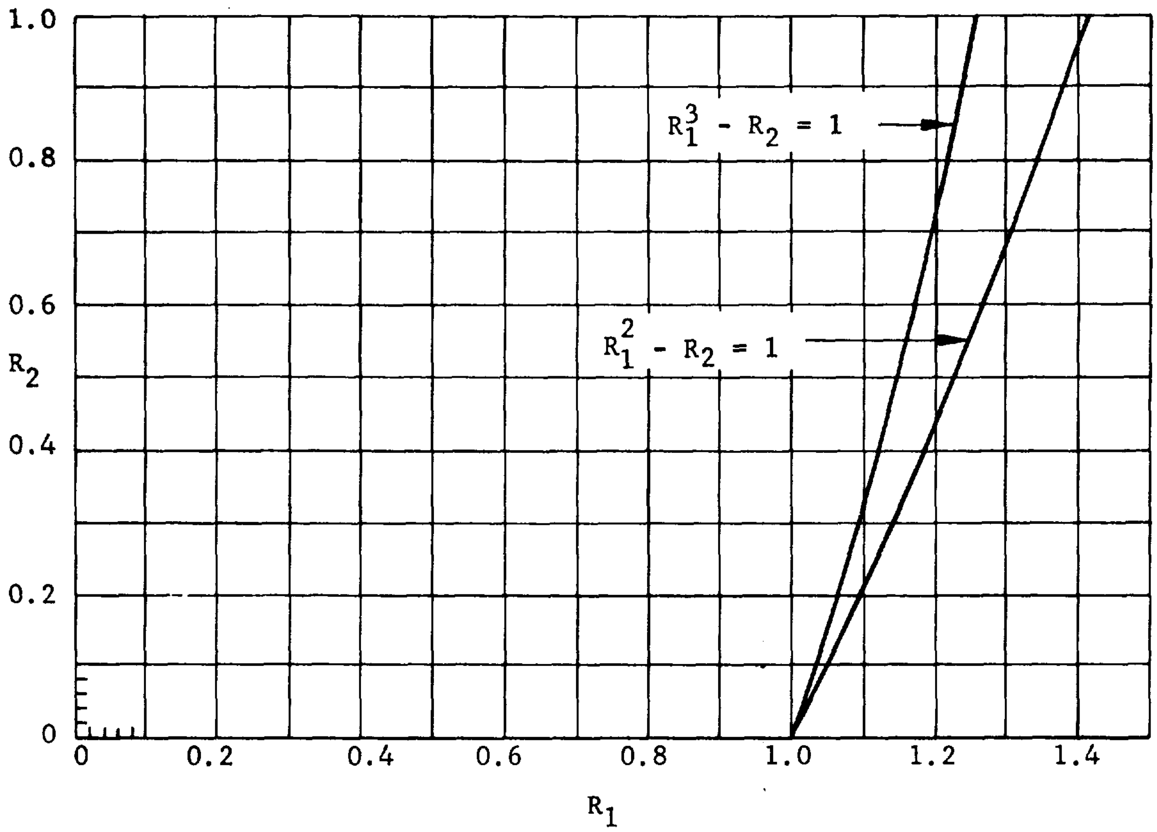
   $$q = 3 - \frac{1}{2}\left(\frac{R_{st}}{R_s}\right)$$

   When $\dfrac{R_s}{R_{st}} \leq 1$:

   $$p = 1.5 + \frac{3}{4}\left(\frac{R_{st}}{R_s}\right)$$

   $$q = 2 + \frac{1}{2}\left(\frac{R_{st}}{R_s}\right)$$

A 3.6.0 Buckling of Circular Cylinders, Elliptical Cylinders, and Curved Plates under Combined Loading (Cont'd)



Interaction Curves

Fig. A 3.6.0-1

A3.7.0 Modified Stress-Strain Curves Due to Combined Loading Effects

An analysis that uses a uniaxial stress-strain curve or material properties derived from such a curve (analysis of beams, columns, thermal effects, plastic bending, elastic and plastic buckling, Elastic-Plastic Energy Theory of Section B4. 5.7 , etc) may require a modified stress-strain curve or properties derived from a modified curve when combined loading is involved. Loads or stresses in one plane affect the loads and stresses in other planes due to the Poisson effect. For example, a tension member fails when the average stress, (P/A), reaches the ultimate tensile stress F_{tu} of the material, but a member resisting combined loading may fail before the maximum principal stress reaches F_{tu} (Reference Section A1). When buckling or other empirical parameters include combined loading effects, modified stress-strain curves are not required.

Several methods of modifying uniaxial stress-strain curves have been developed; the method presented here is derived from the Octahedral Shear Stress Theory.

Assumptions & Conditions:

1. f_1, f_2 and f_3, the three principal stresses, are in proportion; i. e. ,

$$f_2 = K_1 f_1 \quad (1)$$

$$f_3 = K_2 f_1 \quad (2)$$

$$K_1 \neq K_2$$

See Fig. A3.7.0-1 for direction of principal stresses.

A3.7.0 Modified Stress-Strain Curves Due to Combined Loading Effect (Cont'd)

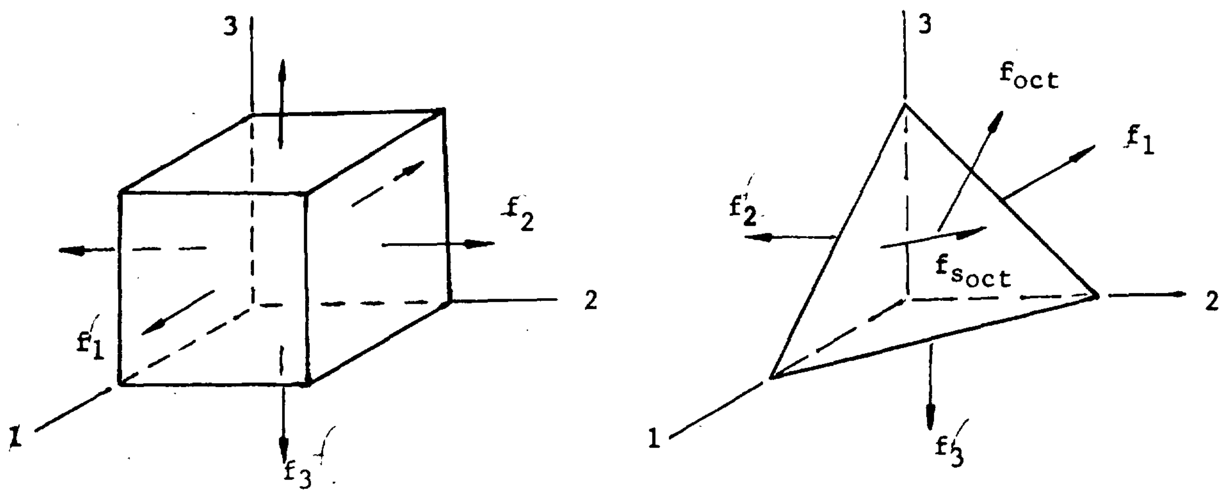


Figure A3.7.0-1

Directions of Principal Stresses

2. Prime (') denotes a modified value:

   ϵ' = modified strain

   E' = modified modulus of elasticity.

3. In this method, for any principal stress f_i, the total strains and modulus of elasticity are modified to include the effects of the other principal stresses.

Procedure:

1. Calculate the principal stresses for a given load condition (Reference Section A3.1.0).

2. Determine the effective uniaxial stress:

$$\bar{f}_1 = \frac{1}{\sqrt{2}}\sqrt{(f_1 - f_2)^2 - (f_2 - f_3)^2 + (f_3 - f_1)^2} \qquad (3)$$

A3.7.0 Modified Stress-Strain Curves Due to Combined Loading Effect (Cont'd)

and calculate an effective modulus of elasticity, E'_1, by:

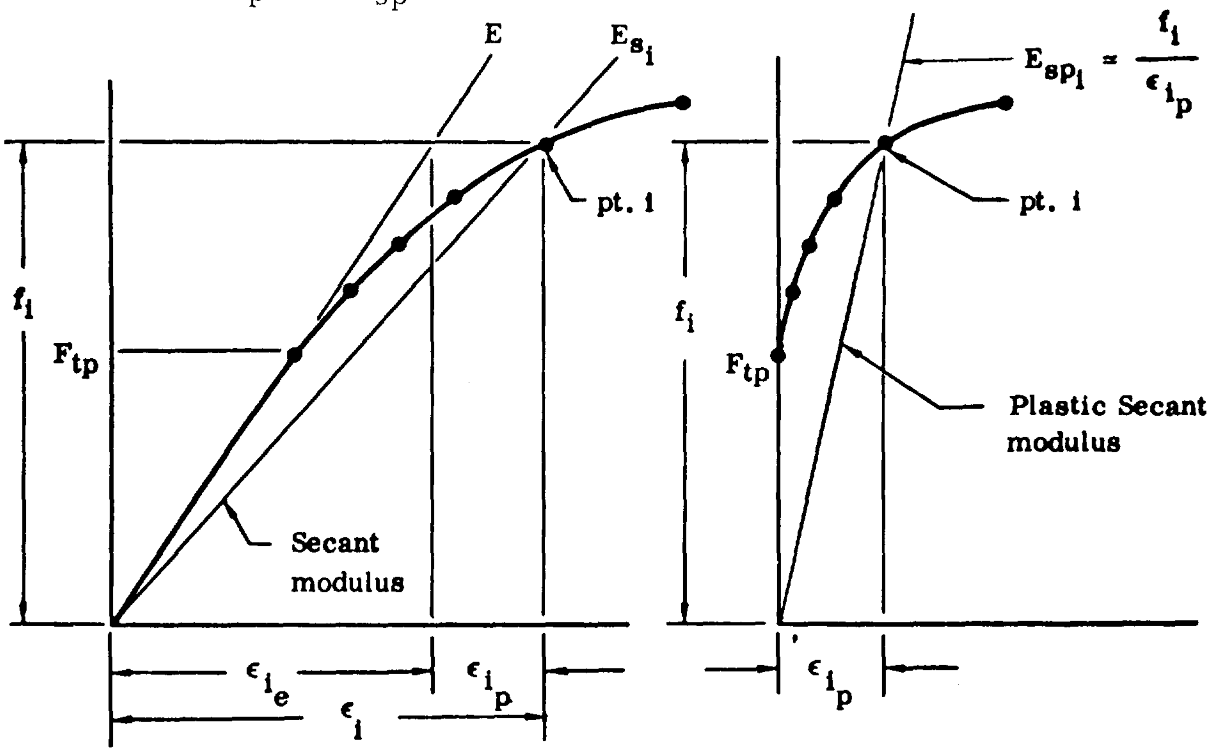
$$E'_1 = \left(\frac{f_1}{\bar{f}_1}\right) E_1 \qquad (4)$$

3. Enter the plastic stress-strain diagram for simple tension of the material, if available, at the value of $\bar{f}_1$ and determine E'_{sp}. (See Figure (A3.7.0-2b) Otherwise, enter the simple tension stress-strain curve at $\bar{f}_1$ and determine E'_{sp} (see Figure A3.7.0-2a) by:

$$E'_{sp} = \frac{\bar{f}_1}{\epsilon_1 - \epsilon_{1_e}} \qquad (5)$$

4. Use this value of E'_{sp} and a value of $\mu_p = 0.5$, if not accurately known, find e'_{1_p} from

$$\epsilon'_{1_p} = \frac{1}{E'_{sp}} (f_1 - \mu_p f_2 - \mu_p f_3) \qquad (6)$$



(a) Engineering stress-strain curve (b) Plastic stress-strain curve

Figure A3.7.0-2

A3.7.0 Modified Stress-Strain Curves Due to Combined Loading Effect (Cont'd)

5. Once E_1' has been found, ϵ_{1_e}' can be determined for any value of f_1 by:

$$\epsilon_{1_e}' = \frac{f_1}{E_1'} \qquad (7)$$

6. Determine the total effect strain, ϵ_1', for each value of f_1 by:

$$\epsilon_1' = \epsilon_{1_e}' + \epsilon_{1_p}' \qquad (8)$$

7. Repeat all steps until sufficient points are obtained to construct a plot of f_1 vs ϵ_1' (see Figure A3.7.0-3) which is the modified stress-strain curve.

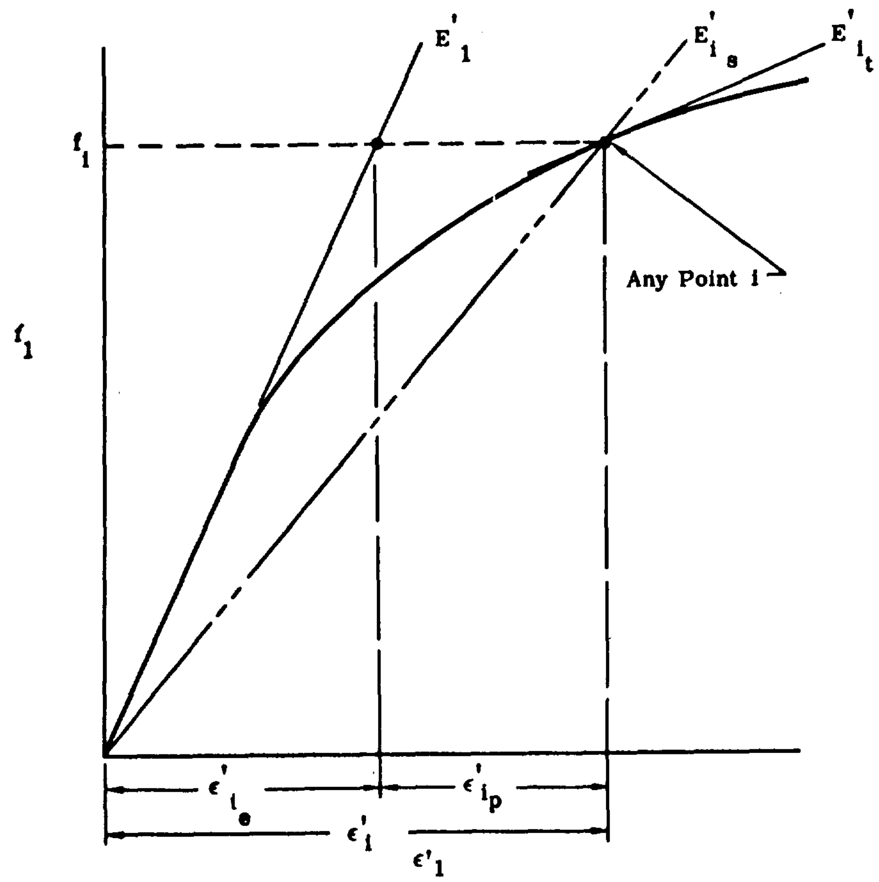


Figure A3.7.0-3 Modified Stress Strain Diagram Due to Combined Loading

References:

Popov, E. P., Mechanics of Materials, Prentice-Hall, Inc., New York, 1954.

Structures Manual, Convair Division of General Dynamics Corporation, Fort Worth.

SECTION A4

METRIC SYSTEM

TABLE OF CONTENTS

TABLE OF CONTENTS (Continued)

A4.0.0 METRIC SYSTEM

A4.1.0 Introduction

The purpose of this section is to acquaint the reader with the metric system and its advantages over the English system. This section also presents definitions, symbols, and conversion tables.

Units of length, mass, and time are basic to both the English System and to the Metric System. In the English System these are: length, the foot; mass, the pound; and time, the second. Note that the second, based on the sexagesimal system, is common to both the English System and the Metric System.

A4.2.0 The International System of Units (SI)

The International System of Units, or Systeme Internationale (SI), is sometimes referred to, in less precise terms, as the Meter-Kilogram-Second-Ampere (MKSA) system. The SI, therefore, should be considered as the definitive metric system, although it is much broader in scope and purpose than any previous system.

A4.2.1 Advantages of SI

The use of SI has significant advantages in all phases of research and development work relating to space technology. For instance, the use of SI will tend to eliminate wasted time and costly errors in computations now involving varied terms derived from a multiplicity of sources. The utilization of a uniform system of measurement such as the SI thus simplifies the exchange of in-house data among NASA centers and installations and will do so, eventually, among associated contractors and space-oriented organizations throughout the world.

A4.3.0 Basic SI Units

The name, International System of Units, has been recommended by the Conférence Générale des Poids et Mesures in 1960 for the following basic units:

meter	m	ampere	A
kilogram	kg	degree Kelvin	^{o}K
second	s	candela	cd

In addition, it was also determined that the amount of substance would be treated as a basic quantity. The recommended basic unit is the mole, symbol: mol. The mole (mol), a unit of quantity in chemistry, is defined as the amount of a substance in grams (gram mole; gram molecular weight; or pound mole, or pound molecular weight) which corresponds to the sum of the atomic weights of all the atoms constituting the molecule. These atomic weights are based upon Carbon 12.

A4.4.0 International Symbols for Units, SI

In order that SI may be in fact an international system, it was necessary to reach agreement on the symbols, names, and abbreviations.

A4.4.1 CGS System

In the field of mechanics, the following units of this system have special names and symbols which have been approved by the General Conference on Weights and Measures:

l, b, h	centimeter	cm
t	second	s
m	gram	g
f, ν	hertz (= s^{-1})	Hz
F	dyne (= g·cm/s^2)	dyn
E, U, W, A	erg (= g·cm^2/s^2)	erg
p	microbar (= dyn/cm^2)	μ bar
μ	poise (= dyn·s/cm^2)	p

A4.4.2 Giorgi System

The MKSA system or m-kg-s-A system is a coherent system of units for mechanics, electricity, and magnetism, based on four basic quantities: length, mass, time, and electric current intensity.

meter	m
kilogram	kg
second	s
ampere	A

The system based on these four units was given the name "Giorgi system" by the International Electrotechnical Committee in 1958. The mechanical system, which is based on the first three units only, has the name MKS system.

The MKSA system of units forms a coherent system of units in the four-dimensional system of equations previously mentioned, and is most commonly used together with these equations.

A4. 4. 3 Incoherent Units

l	ångström	Å
σ	barn (= 10^{-24} cm^2)	b
V	liter (= 1 dm^3)	l
t, τ	minute	min
t, τ	hour	h
t, τ	day	d
t, τ	year	a
p	atmosphere	atm
p	kilowatt-hour	kWh
Q	calorie	cal
Q	kilocalorie	kcal
E, Q	electronvolt	eV
m	ton (= 1000kg)	t
M_a, m	(unified) atomic mass unit	u
p	bar (= 10^6 dyn/cm^2) (= 10^5 N/m^2)	bar

A4. 5. 0 Physical Quantities

The symbol for a physical quantity (French: 'grandeur physique'; German: 'physikalische Grosse'; English, sometimes: 'physical magnitude') is equivalent to the product of the numerical value (or the measure, a pure number) and a unit, i. e. , physical quantity = numerical value x unit.

A4. 5. 1 Dimensionless Physical Quantities

For dimensionless physical quantities the unit often has no name or symbol and is not explicitly indicated.

Examples: E = 200 erg n_{qu} = 1. 55

F = 27 N ν = 3 x 10^8 s^{-1}

A4.6.0 Other SI Symbols

The following units of the MKSA system have special names and symbols which have been approved by the General Conference on Weights and Measures:

I	ampere	A
Q	coulomb (= A· s)	C
C	farad (= C/V)	F
L	henry (= Vs/A)	H
E	joule (= kg· m^2/s^2)	J
m	kilogram	kg
l, b, h	meter	m
F	newton (= kg· m/s^2)	N
R	ohm (= V/A)	Ω
B	tesla (= Wb/m^2)	T
V	volt (= W/A)	V
P	watt (= J/s)	W
Φ	weber (= V· s)	Wb

A4.6.1 Photometric Units

In the field of photometry an additional basic unit is introduced corresponding to the basic quantity, luminous intensity. This unit is the candela, symbol: cd. Special names for units in this field are:

I	candela (candle)	cd
Φ	lumen	lm
E	lux (= lm/m^2)	lx

A4.6.2 Rules for Notation

a. Symbols for units of physical quantities shall be printed in Roman upright type.

b. Symbols for units shall not contain a final full stop (a period), and shall remain unaltered in the plural, e.g.: 7cm, not 7 cms.

c. Symbols for units shall be printed in lower case Roman upright type. However, the symbol for a unit derived from a proper name shall start with a capital Roman letter, e.g.: m (meter); A (ampere); Wb (weber); Hz (hertz).

d. The following prefixes shall be used to indicate decimal fractions or multiples of a unit.

Prefix	Equiv	Symbol
deci	(10^{-1})	d
centi	(10^{-2})	c
milli	(10^{-3})	m
micro	(10^{-6})	µ
nano	(10^{-9})	n
pico	(10^{-12})	p
femto	(10^{-15})	f
atto	(10^{-18})	a
deka	(10^{1})	da
hecto	(10^{2})	h
kilo	(10^{3})	k
mega	(10^{6})	M
giga	(10^{9})	G
tera	(10^{12})	T

e. The use of double prefixes shall be avoided when single prefixes are available.

Not: mμs,	but: ns (nanosecond)
Not: kMW,	but: GW (gigawatt)
Not: μμF,	but: pF (picofarad)

f. When a prefix symbol is placed before a unit symbol, the combination shall be considered as a new symbol, which can be squared or cubed without using brackets.

Examples: cm^2, mA^2, μs^2

A numerical prefix shall never be used before a unit symbol which is squared, thus, cm^2 is never written, and never means, $0.01\ (m^2)$ but always means $(0.01m)^2$.

g. No periods or hyphens shall be used with SI abbreviations, symbols, or prefixes. Prefixes are joined directly to units, as in the following examples:

MN	mN
kV	kHz
MV	mA
GHz	cm

A4.7.0 SI Units on Drawings and in Analyses

The following paragraphs describe general techniques for using SI units on drawings and in analyses.

A4.7.1 Dual Units

When SI units are specified for use on a drawing or in an analysis, the non-SI units of measure shall be used parenthetically to facilitate comprehension of the drawing or analysis. Non-SI units shall never be omitted on the assumption that users are familiar with the SI units.

A4.7.2 Identification of Units

Basic units of measure used frequently on a drawing shall be identified by a note on the drawing to avoid repetition of unit names throughout the drawing. For example,

NOTE: ALL DIMENSIONS ARE IN mm (in.).

A4.7.3 Tabular Data

To provide maximum clarity of presentation, SI and non-SI units shall be placed in separate columns or in separate tables if the need is indicated.

A4.7.4 Collateral Use, SI and Non-SI Units

Place the metric units first, followed immediately by the equivalents in parentheses. In tables, other formats may be desirable, such as one unit in a row or column, followed by the other unit in another row or column. In some complex tables and drawings it may be desirable to present the equivalent units in separate tables and drawings. Figure A4-1 shows a drawing with both units given.

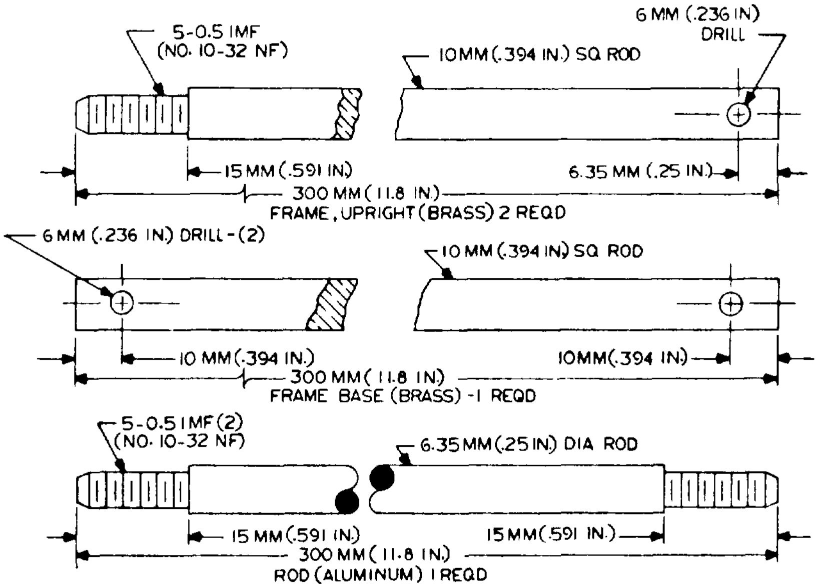


Figure A4-1. Collateral Use of Units

A4.7.5 Temperature Scales

Either the Kelvin or the Celsius temperature scale may be used as the SI unit, with the Fahrenheit scale being optional as a parenthetical non-SI unit.

A4.8.0 Transitional Indices

The following explanations indicate nomenclatures, methods, and preferred styles which are to be used during the transition from non-SI systems to SI.

A4.8.1 Mass vs Force

The term "mass" (and not weight) shall be used to specify the quantity of matter contained in material objects.

The term "weight" shall be defined as the gravitational force acting on a material object at a specified location. Accordingly, the statement of the weight of an object should be accompanied by a statement of the corresponding location of the object or by a statement of the gravitational acceleration in m/s^2 at the location where the object was weighed or is assumed to be located.

The pound mass (lbm), defined as being exactly 0.453 592 37 kilogram by the U. S. National Bureau of Standards; the pound force (lbf), defined as being exactly 4.448 221 615 260 5 newtons by the NBS; and the pound thrust, defined as being exactly 4.448 221 615 260 5 newtons, shall be abandoned at the earliest practicable date.

During the transition period to SI units, the pound mass shall be abbreviated lbm, the pound force shall be abbreviated lbf, and the pound thrust shall be abbreviated to lbf.

The kilogram (kg), the SI unit of mass, shall not be used as a unit of force, weight, or thrust.

A4.8.2 Examples of Nomenclature

The dry mass of the S-I (first) stage of the Saturn I launch vehicle is 48 600 kg (107 139 lbm).

The weight of a man of 70.0 kg (154 lbm) mass, standing on the surface of the moon where the gravitational acceleration is 1.62 m/s^2, is 113 newtons (25.4 lbf).

The thrust of the S-I (first) stage of the Saturn I launch vehicle is 6.689 MN (1 504 000 lbf).

The preferred unit of force, weight, and thrust is the newton.

A4.9.0 Measurement of Angles

A circle cannot be divided into a rational number of radians (rad), there being 2π radians (approximately 6.283 rad) in a circle. However, the radian, arc degree, arc minute, and arc second may all be used for the measurement of plane angles. Decimal multiples of the degree or radian are preferred.

The "grad" is a unit of angular measure wherein 100 grads constitute a right angle. This is not an SI unit, but, since it is based on decades it will be found useful for many purposes.

A4.10.0 Preferred Style

In order to ensure maximum accuracy, the following style shall be adhered to wherever practicable in engineering analysis documentation.

a. Spell out a term in full when first used, followed by the related symbol in parentheses. Thereafter, use the related symbol for measurement applications.

b. In general, state the measurement in terms of the system of units used, followed by the applicable translated value in parentheses: for example, 48 000 kg (107 000 lbm), and 25.4 lbf (113 newtons).

c. In using numerical values involving more than three digits, place a space between each group of three digits. Such spaces shall be used to the right and left of decimal points. Commas are not used: 126 306.204 359 60.

A4.10.1 Volume

The cubic meter (m^3) should be used in preference to the liter. The liter is now defined as exactly 1 dm^3.

A4.10.2 Time

The preferred unit of time associated with time rates is the second.

A4.10.3 Energy

The preferred unit of energy (mechanical, electrical, thermal, and all other forms) is the joule (J). The Btu, calorie, and kilocalorie, although listed in this document for information, are poorly defined and should be avoided.

A4.10.4 Temperature

Either the Thermodynamic Kelvin Temperature Scale, the International Practical Kelvin Temperature Scale, or the International Practical Celsius Temperature Scale may be used. Equivalent temperatures in degrees Rankine, Fahrenheit, etc., may be included in parentheses. Note that temperature differences expressed in Kelvin degrees (°K) and in Celsius degrees (°Cels) are numerically equal and that degrees Celsius and degrees Centigrade are identical. See Temperature Nomograph, Figure A4-2, and Table A4-15.

The International Practical Kelvin Temperature Scale of 1960, and the International Practical Celsius Temperature Scale of 1960 are defined by a set of interpolation equations based on the reference temperatures in Table A4-1.

Table A4-1. Reference Temperatures, International Practical Temperature Scale

Temperature of	°K	°C
Oxygen: liquid-gas equilibrium	90.18	-182.97
Water: solid-liquid equilibrium	273.15	0.00
Water: solid-liquid-gas equilibrium	273.16	0.01
Water: liquid-gas equilibrium	373.15	100.00
Zinc: solid-liquid equilibrium	692.655	419.505
Sulphur: liquid-gas equilibrium	717.75	444.6
Silver: solid-liquid equilibrium	1233.95	960.8
Gold: solid-liquid equilibrium	1336.15	1063.0

A4.10.5 Prefixes

"Coherent units" are units that can be used directly in equations without the application of numerical coefficients. The exclusive use of coherent units over the entire range of numerical values of physical quantities is highly desirable. As previously stated, the SI is the only complete system of coherent units available to meet the needs of all branches of science and technology. The full range of numerical values of physical quantities can be represented rationally and conveniently by utilizing SI units.

To facilitate this, either a power of ten is employed, or an approved prefix representing a power of ten is placed before an SI unit (or before any combination of SI units).

Only previously listed prefixes shall be used to indicate decimal fractions or multiples of an SI unit.

A4.11.0 Conversion Factors

Measurements in units other than those of the SI are preferably converted by the application of approved numerical conversion factors.

The number of decimal places should be governed by the purpose to which the information is to be put and by the degree of accuracy attainable with applicable measuring instruments and methods. These conversion factors have been tabulated according to physical quantity.

A4. 11. 1 Basic Linear Unit

The basic unit of measurement is the meter. Prefixes shall be used in accordance with A4. 6. 2(d). Once a prefix is chosen, no other prefix shall be used on a drawing or in an analysis. The inch (in.) is defined as exactly 2. 54 cm.

A4. 11. 2 Noncritical Conversion

If dimensions are not critical, non-SI data converted to mm shall be rounded to convenient numbers and the word "nominal" appended in parentheses.

A4. 11. 3 Conversion to Other SI Units

Conversion to SI units other than mm shall follow the rules as herein set forth.

A4. 12. 0 Conversion Tables

The conversion factors given in the following tables will facilitate the conversion of most commonly used units of the English system to units of the Metric System (or conversion of non-SI units to SI units).

Table A4-2. Acceleration

To Convert	To	Symbol	Multiply by
foot/second squared	meter/second squared	m/s^2	*3.048×10^{-1}
galileo (gal)	meter/second squared	m/s^2	*1.000×10^{-2}
inch/second squared	meter/second squared	m/s^2	*2.54×10^{-2}

Table A4-3. Area

To Convert	To	Symbol	Multiply by
sq foot	sq meter	m^2	*$9.290\,304 \times 10^{-2}$
sq inch	sq meter	m^2	*$6.451\,6 \times 10^{-4}$
circular mil	sq meter	m^2	$5.067\,074\,8 \times 10^{-10}$

Table A4-4. Density

To Convert	To	Symbol	Multiply by
gram/cu centimeter	kilogram/cu meter	kg/m^3	*1.00×10^3
pound mass/cu inch	kilogram/cu meter	kg/m^3	$2.767\,990\,5 \times 10^4$
pound mass/cu foot	kilogram/cu meter	kg/m^3	$1.601\,846\,3 \times 10^1$
slug/cu foot	kilogram/cu meter	kg/m^3	$5.153\,79 \times 10^2$

Table A4-5. Electrical

To Convert	To	Symbol	Multiply by
ampere (Int of 1948)	ampere	A	$9.998\,35 \times 10^{-1}$
ampere hour	coulomb	$C = A \cdot s$	*3.60×10^3
coulomb (Int of 1948)	coulomb	$C = A \cdot s$	$9.998\,35 \times 10^{-1}$
faraday (physical)	coulomb	$C = A \cdot s$	$9.652\,19 \times 10^4$
farad (Int of 1948)	farad	$F = A \cdot s/V$	$9.995\,05 \times 10^{-1}$
henry (Int of 1948)	henry	$H = V \cdot s/A$	1.000 495
ohm (Int of 1948)	ohm	$\Omega = V/A$	1.000 495
gamma	tesla	$T = Wb/m^2$	*1.00×10^{-9}

*Exact, as defined by the National Bureau of Standards.

Table A4-5. Electrical (Cont'd)

To Convert	To	Symbol	Multiply by
gauss	tesla	$T = Wb/m^2$	$*1.00 \times 10^{-4}$
volt (Int of 1948)	volt	$V = W/A$	1.000 330
maxwell	weber	$Wb = V \cdot s$	$*1.00 \times 10^{-8}$

Table A4-6. Energy

To Convert	To	Symbol	Multiply by
Btu (mean)	joule	$J = N \cdot m$	$1.055\ 87 \times 10^{3}$
calorie (mean)	joule	$J = N \cdot m$	4.190 02
calorie (thermochemical)	joule	$J = N \cdot m$	*4.184
electron volt	joule	$J = N \cdot m$	$1.602\ 10 \times 10^{-19}$
erg	joule	$J = N \cdot m$	$*1.00 \times 10^{-7}$
foot pound force	joule	$J = N \cdot m$	1.355 817 9
foot poundal	joule	$J = N \cdot m$	$4.214\ 011\ 0 \times 10^{-2}$
joule (Int of 1948)	joule	$J = N \cdot m$	1.000 165
kilowatt hour (Int of 1948)	joule	$J = N \cdot m$	$3.600\ 59 \times 10^{6}$
ton (nuclear equiv of TNT)	joule	$J = N \cdot m$	4.20×10^{9}
watt hour	joule	$J = N \cdot m$	$*3.60 \times 10^{3}$

Table A4-7. Energy/Area: Time

To Convert	To	Symbol	Multiply by
**Btu/sq foot·sec	watt/sq meter	W/m^2	$1.134\ 893\ 1 \times 10^{4}$
**Btu/sq foot·min	watt/sq meter	W/m^2	$1.891\ 488\ 5 \times 10^{2}$
**Btu/sq inch·sec	watt/sq meter	W/m^2	$1.634\ 246\ 2 \times 10^{6}$
erg/sq centimeter·sec	watt/sq meter	W/m^2	$*1.00 \times 10^{-3}$
watt/sq centimeter	watt/sq meter	W/m^2	$*1.00 \times 10^{4}$

*Exact, as defined by the National Bureau of Standards.

**(thermochemical)

Table A4-8. Force

To Convert	To	Symbol	Multiply by
dyne	newton	$N=kg\cdot m/s^2$	*1.00×10^{-5}
kilogram force (kgf)	newton	$N=kg\cdot m/s^2$	*9.806 65
pound force (avoirdupois)	newton	$N=kg\cdot m/s^2$	*4.448 221 615 260 5
ounce force (avoirdupois)	newton	$N=kg\cdot m/s^2$	$2.780\ 138\ 5 \times 10^{-1}$

Table A4-9. Length

To Convert	To	Symbol	Multiply by
angstrom	meter	m	*1.00×10^{-10}
astronomical unit	meter	m	1.495×10^{11}
foot	meter	m	*3.048×10^{-1}
foot (U. S. survey)	meter	m	*1200/3937
foot (U. S. survey)	meter	m	$3.048\ 006\ 096 \times 10^{-1}$
inch	meter	m	*2.54×10^{-2}
light year	meter	m	$9.460\ 55 \times 10^{15}$
meter	wavelengths Kr^{86}	m	*$1.650\ 763\ 73 \times 10^{6}$
micron	meter	m	*1.00×10^{-6}
mil	meter	m	*2.54×10^{-5}
mile (U. S. statute)	meter	m	*$1.609\ 344 \times 10^{3}$
yard	meter	m	*9.144×10^{-1}

Table A4-10. Mass

To Convert	To	Symbol	Multiply by
gram	kilogram	kg	*1.00×10^{-3}
kilogram force$\cdot$ sec^2/meter (mass)	kilogram	kg	*9.806 65
kilogram mass	kilogram	kg	*1.00
pound mass (avoirdupois)	kilogram	kg	*$4.535\ 923\ 7 \times 10^{-1}$

*Exact, as defined by the National Bureau of Standards.

Table **A4-10.** Mass (Cont'd)

To Convert	To	Symbol	Multiply by
ounce mass (avoirdupois)	kilogram	kg	*2.834 952 312 5 x 10^{-2}
ounce mass (troy or apothecary)	kilogram	kg	*3.110 347 68 x 10^{-2}
pound mass (troy or apothecary)	kilogram	kg	*3.732 417 216 x 10^{-1}
slug	kilogram	kg	1.459 390 29 x 10^{1}
ton (short, 2000 pound)	kilogram	kg	*9.071 847 4 x 10^{2}

Table **A4-11.** Miscellaneous

To Convert	To	Symbol	Multiply by
degree (angle)	radian	rad	1.745 329 251 994 3 x 10^{-2}
minute (angle)	radian	rad	2.908 882 086 66 x 10^{-4}
second (angle)	radian	rad	4.848 136 811 x 10^{-6}
cu foot/second	cu meter/second	m^3/s	*2.831 684 659 2 x 10^{-2}
cu foot/minute	cu meter/second	m^3/s	4.719 474 4 x 10^{-4}
**Btu/pound mass °F	joule/kilogram °C	J/kg°C	*4.184 x 10^{3}
**Kilocalorie/kg °C	joule/kilogram °C	J/kg°C	*4.184 x 10^{3}
**Btu/pound mass	joule/kilogram	J/kg	2.324 444 4 x 10^{3}
Rad (radiation dose absorbed)	joule/kilogram	J/kg	*1.00 x 10^{-2}
roentgen	coulomb/kilogram	A·s/kg	*2.579 76 x 10^{-4}
curie	disintegration/second	1/s	*3.70 x 10^{10}

Table **A4-12.** Power

To Convert	To	Symbol	Multiply by
**Btu/second	watt	W= J/s	1.054 350 264 488 888 x 10^{3}
**Btu/minute	watt	W= J/s	1.757 250 4 x 10^{1}
**calorie/second	watt	W= J/s	*4.184
**calorie/minute	watt	W= J/s	6.973 333 3 x 10^{-2}
foot pound force/second	watt	W= J/s	1.355 817 9

* Exact, as defined by the National Bureau of Standards
** (thermochemical)

Table A4-12. Power (Cont'd)

To Convert	To	Symbol	Multiply by
foot pound force/minute	watt	W= J/s	$2.259\ 696\ 6 \times 10^{-2}$
foot pound force/hour	watt	W= J/s	$3.766\ 161\ 0 \times 10^{-4}$
horsepower (550 ft lb force/sec)	watt	W= J/s	$7.456\ 998\ 7 \times 10^{2}$
horsepower (electric)	watt	W= J/s	*7.46×10^{2}
**kilocalorie/sec	watt	W= J/s	*4.184×10^{3}
**kilocalorie/min	watt	W= J/s	$6.973\ 333\ 3 \times 10^{1}$
watt (Int of 1948)	watt	W= J/s	1.000 165

Table A4-13. Pressure

To Convert	To	Symbol	Multiply by
atmosphere	newton/sq meter	N/m^2	*$1.013\ 25 \times 10^{5}$
centimeter of mercury (0°C)	newton/sq meter	N/m^2	$1.333\ 22 \times 10^{3}$
centimeter of water (4°C)	newton/sq meter	N/m^2	$9.806\ 38 \times 10^{1}$
dyne/sq centimeter	newton/sq meter	N/m^2	*1.00×10^{-1}
foot of water (39.2°F)	newton/sq meter	N/m^2	$2.988\ 98 \times 10^{3}$
inch of mercury (60°F)	newton/sq meter	N/m^2	$3.376\ 85 \times 10^{3}$
inch of water (60°F)	newton/sq meter	N/m^2	$2.488\ 4 \times 10^{2}$
kilogram force/sq centimeter	newton/sq meter	N/m^2	*$9.806\ 65 \times 10^{4}$
kilogram force/sq meter	newton/sq meter	N/m^2	*9.806 65
pound force/sq inch (psi)	newton/sq meter	N/m^2	$6.894\ 757\ 2 \times 10^{3}$
pound force/sq foot	newton/sq meter	N/m^2	$4.788\ 025\ 8 \times 10^{1}$
millimeter of mercury (0°C)	newton/sq meter	N/m^2	$1.333\ 224 \times 10^{2}$
torr (0°C)	newton/sq meter	N/m^2	$1.333\ 22 \times 10^{2}$

*Exact, as defined by the National Bureau of Standards

**(thermochemical)

Table A4-14. Speed

To Convert	To	Symbol	Multiply by
foot/second	meter/second	m/s	*3.048 x 10^{-1}
foot/minute	meter/second	m/s	*5.08 x 10^{-3}
foot/hour	meter/second	m/s	8.466 666 6 x 10^{-5}
inch/second	meter/second	m/s	*2.54 x 10^{-2}
kilometer/hour	meter/second	m/s	2.777 777 8 x 10^{-1}
mile/second (U.S. statute)	meter/second	m/s	*1.609 344 x 10^{3}
mile/minute (U.S. statute)	meter/second	m/s	*2.682 24 x 10^{1}
mile/hour (U.S. statute)	meter/second	m/s	*4.470 4 x 10^{-1}

Table A4-15. Temperature

To Convert	Symbol	To	Symbol	Computation
°Celsius	°Cels.	°Centigrade	°C	°Cels. = °C
°Fahrenheit	°F	°Centigrade	°C	°C = 5/9 (°F-32)
°Rankine	°R	°Centigrade	°C	°C = 5/9 (°R-491.69)
°Reaumur	°Re	°Centigrade	°C	°C = 5/4 °Re
°Fahrenheit	°F	°Celsius	°Cels.	°Cels. = 5/9 (°F-32)
°Fahrenheit	°F	°Reaumur	°Re	°Re = 4/9 (°F-32)
°Fahrenheit	°F	°Rankine	°R	°R = °F + 459.69
°Rankine	°R	°Celsius	°Cels.	°Cels.= 5/9 (°R-491.69)
°Rankine	°R	°Reaumur	°Re	°Re = 4/9 (°R-491.69)
°Reaumur	°Re	°Celsius	°Cels.	°Cels. = 5/4 °Re
°Centigrade	°C	°Kelvin	°K	°K = °C + 273.16

Table A4-16. Thermal Conductivity

To Convert	To	Symbol	Multiply by
Btu·inch/sq foot·second·°F	joule/meter·second·°Kelvin	J/m·s·°K	5.188 731 5 x 10^{2}

*Exact, as defined by the National Bureau of Standards.

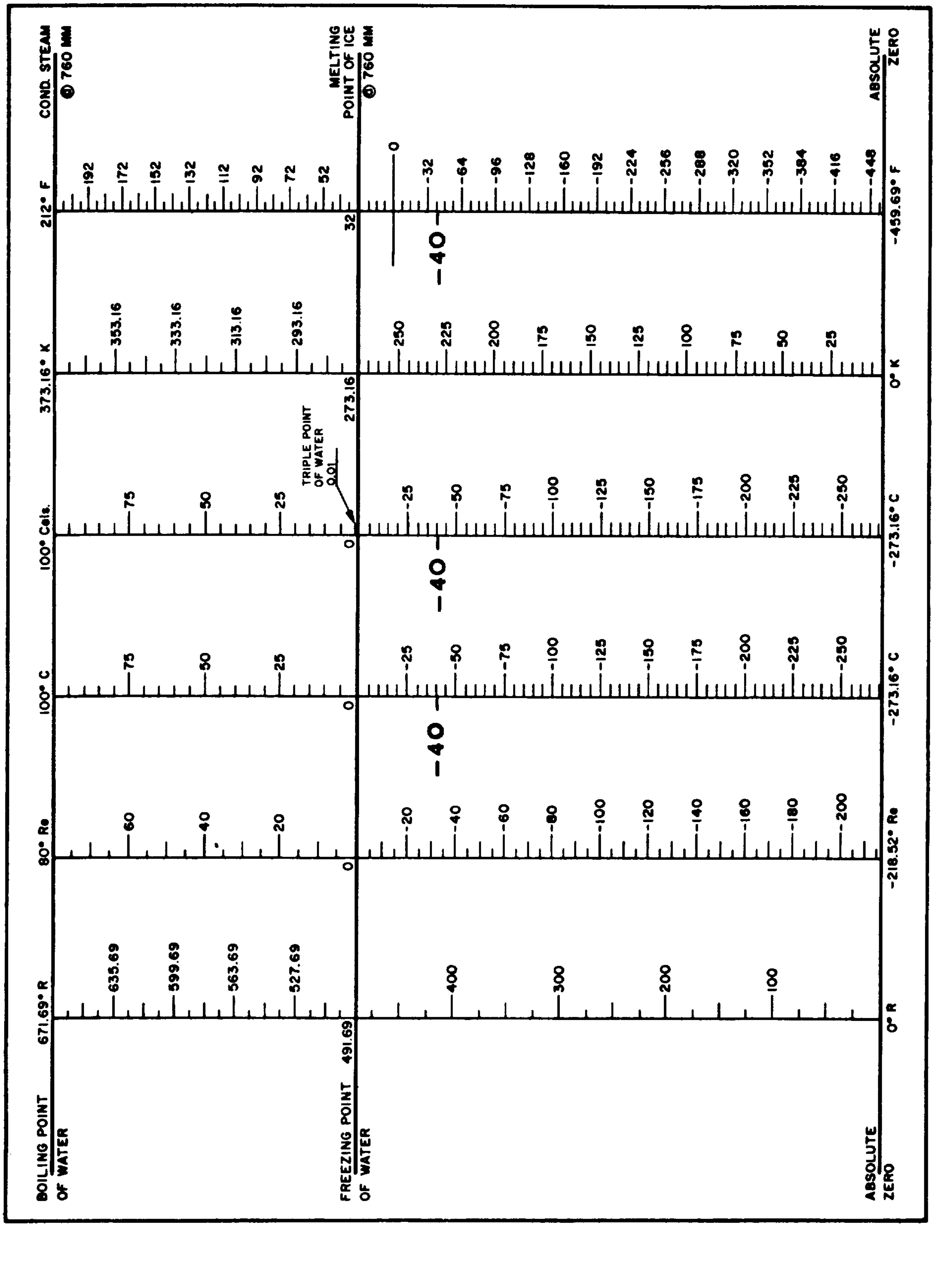


Figure A4-2. Temperature Nomograph

Table A4-17. Time

To Convert	To	Multiply by
day (mean solar)	second (mean solar)	*8.64 x 10^{4}
day (sidereal)	second (mean solar)	8.616 409 0 x 10^{4}
hour (mean solar)	second (mean solar)	*3.60 x 10^{3}
hour (sidereal)	second (mean solar)	3.590 170 4 x 10^{3}
minute (mean solar)	second (mean solar)	*6.00 x 10^{1}
minute (sidereal)	second (mean solar)	5.983 617 4 x 10^{1}
month (mean calendar)	second (mean solar)	*2.628 x 10^{6}
second (mean solar)	second (ephemeris)	Use equation of time
second (sidereal)	second (mean solar)	9.972 695 7 x 10^{-1}
tropical year 1900, Jan, day 0, hour 12	second (ephemeris)	*3.155 692 597 47 x 10^{7}
year (calendar)	second (mean solar)	*3.153 6 x 10^{7}
year (sidereal)	second (mean solar)	3.155 815 0 x 10^{7}
year (tropical)	second (mean solar)	3.155 692 6 x 10^{7}

Table A4-18. Viscosity

To Convert	To	Symbol	Multiply by
sq foot/second	sq meter/second	m^2/s	*9.290 304 x 10^{-2}
centipoise	newton· second/sq meter	$N \cdot s/m^2$	*1.00 x 10^{-3}
pound mass/foot· second	newton· second/sq meter	$N \cdot s/m^2$	1.488 163 9
pound force· second/sq foot	newton· second/sq meter	$N \cdot s/m^2$	4.788 025 8 x 10^{1}
poise	newton· second/sq meter	$N \cdot s/m^2$	*1.00 x 10^{-1}
poundal· second/sq foot	newton· second/sq meter	$N \cdot s/m^2$	1.488 163 9
slug/foot· second	newton· second/sq meter	$N \cdot s/m^2$	4.788 025 8 x 10^{1}

* Exact, as defined by the National Bureau of Standards.

Table A4-19. Volume

To Convert	To	Symbol	Multiply by
fluid ounce (U.S.)	cu meter	m^3	*2.957 352 956 25 x 10^{-5}
cu foot	cu meter	m^3	*2.831 684 659 2 x 10^{-2}
gallon (U.S. liquid)	cu meter	m^3	*3.785 411 784 x 10^{-3}
cu inch	cu meter	m^3	*1.638 706 4 x 10^{-5}
liter	cu meter	m^3	1.000 000 x 10^{-3}
pint (U.S. liquid)	cu meter	m^3	*4.731 764 73 x 10^{-4}
quart (U.S. liquid)	cu meter	m^3	9.463 529 5 x 10^{-4}
ton (register)	cu meter	m^3	*2.831 684 659 2

Table A4-20. Alphabetical Listing of Conversion Factors

To Convert	To	Symbol	Multiply by
abampere	ampere	A	*1.00 x 10^{1}
abcoulomb	coulomb	C= A·s	*1.00 x 10^{1}
abfarad	farad	F= A·s/V	*1.00 x 10^{9}
abhenry	henry	H= V·s/A	*1.00 x 10^{-9}
abmho	mho		*1.00 x 10^{9}
abohm	ohm	Ω = V/A	*1.00 x 10^{-9}
abvolt	volt	V= W/A	*1.00 x 10^{-8}
acre	sq meter	m^2	*4.046 856 422 4 x 10^{3}
ampere (Int of 1948)	ampere	A	9.998 35 x 10^{-1}
angstrom	meter	m	*1.00 x 10^{-10}
are	sq meter	m^2	*1.00 x 10^{2}
astronomical unit	meter	m	1.495 98 x 10^{11}
atmosphere	newton/sq meter	N/m^2	*1.013 25 x 10^{5}
bar	newton/sq meter	N/m^2	*1.00 x 10^{5}
barn	sq meter	m^2	*1.00 x 10^{-28}

* Exact, as defined by the National Bureau of Standards.

Table A4-20. Alphabetical Listing of Conversion Factors (Cont'd)

To Convert	To	Symbol	Multiply by
barye	newton/sq meter	N/m^2	*1.00×10^{-1}
Btu (Int Steam Table)	joule	$J = N \cdot m$	$1.055\,04 \times 10^3$
Btu (mean)	joule	$J = N \cdot m$	$1.055\,87 \times 10^3$
Btu (thermochemical)	joule	$J = N \cdot m$	$1.054\,350\,264\,488\,888 \times 10^3$
Btu (39°F)	joule	$J = N \cdot m$	$1.059\,67 \times 10^3$
Btu (60°F)	joule	$J = N \cdot m$	$1.054\,68 \times 10^3$
bushel (U.S.)	cu meter	m^3	*$3.523\,907\,016\,688 \times 10^{-2}$
cable	meter	m	*$2.194\,56 \times 10^2$
caliber	meter	m	*2.54×10^{-4}
calorie (Int Steam Table)	joule	$J = N \cdot m$	4.186 8
calorie (mean)	joule	$J = N \cdot m$	4.190 02
calorie (thermochemical)	joule	$J = N \cdot m$	*4.184
calorie (15°C)	joule	$J = N \cdot m$	4.185 80
calorie (20°C)	joule	$J = N \cdot m$	4.181 90
calorie (kilogram, Int Steam Table)	joule	$J = N \cdot m$	$4.186\,8 \times 10^3$
calorie (kilogram, mean)	joule	$J = N \cdot m$	$4.190\,02 \times 10^3$
calorie (kilogram, thermochemical)	joule	$J = N \cdot m$	*4.184×10^3
carat (metric)	kilogram	kg	*2.00×10^{-4}
°Celsius (temperature)	°Kelvin	°K	°K = °C + 273.16
centimeter of mercury (0°C)	newton/sq meter	N/m^2	$1.333\,22 \times 10^3$
centimeter of water (4°C)	newton/sq meter	N/m^2	$9.806\,38 \times 10^1$
chain (surveyor or gunter)	meter	m	*$2.011\,68 \times 10^1$
chain (engineer or ramden)	meter	m	*3.048×10^1
circular mil	sq meter	m^2	$5.067\,074\,8 \times 10^{-10}$
cord	cu meter	m^3	3.624 556 3

*Exact, as defined by the National Bureau of Standards.

Table A4-20. Alphabetical Listing of Conversion Factors (Cont'd)

To Convert	To	Symbol	Multiply by
coulomb (Int of 1948)	coulomb	C= A· s	$9.998\ 35 \times 10^{-1}$
cubit	meter	m	*4.572×10^{-1}
cup	cu meter	m^3	*$2.365\ 882\ 365 \times 10^{-4}$
curie	disintegration/second	1/s	*3.70×10^{10}
day (mean solar)	second (mean solar)		*8.64×10^{4}
day (sidereal)	second (mean solar)		$8.616\ 409\ 0 \times 10^{4}$
degree (angle)	radian	rad	$1.745\ 329\ 251\ 994\ 3 \times 10^{-2}$
denier (International)	kilogram/meter	kg/m	*1.00×10^{-7}
dram (avoirdupois)	kilogram	kg	*$1.771\ 845\ 195\ 312\ 5 \times 10^{-3}$
dram (troy or apothecary)	kilogram	kg	*$3.887\ 934\ 6 \times 10^{-3}$
dram (U.S. fluid)	cu meter	m^3	*$3.696\ 691\ 195\ 312\ 5 \times 10^{-6}$
dyne	newton	$N= kg\cdot m/s^2$	*1.00×10^{-5}
electron volt	joule	J= N· m	$1.602\ 10 \times 10^{-19}$
erg	joule	J= N· m	*1.00×10^{-7}
°Fahrenheit (temperature)	°Celsius	°C	°C = 5/9 (°F - 32)
°Fahrenheit (temperature)	°Kelvin	°K	°K = 5/9 (°F + 459.69)
farad (Int of 1948)	farad	F= A· s/V	$9.995\ 05 \times 10^{-1}$
faraday (based on carbon 12)	coulomb	C= A· s	$9.648\ 70 \times 10^{4}$
faraday (chemical)	coulomb	C= A· s	$9.649\ 57 \times 10^{4}$
faraday (physical)	coulomb	C= A· s	$9.652\ 19 \times 10^{4}$
fathom	meter	m	*1.828 8
fermi	meter	m	*1.00×10^{-15}
fluid ounce (U.S.)	cu meter	m^3	*$2.957\ 352\ 956\ 25 \times 10^{-5}$
foot	meter	m	*3.048×10^{-1}
foot (U.S. survey)	meter	m	*1200/3937

*Exact, as defined by the National Bureau of Standards.

Table A4-20. Alphabetical Listing of Conversion Factors (Cont'd)

To Convert	To	Symbol	Multiply by
foot (U.S. survey)	meter	m	$3.048\ 006\ 096 \times 10^{-1}$
foot of water (39.2°F)	newton/sq meter	N/m^2	$2.988\ 98 \times 10^{3}$
foot-candle	lumen/sq meter	lm/m^2	$1.076\ 391\ 0 \times 10^{1}$
furlong	meter	m	*$2.011\ 68 \times 10^{2}$
gal	meter/second squared	m/s^2	*1.00×10^{-2}
gallon (British)	cu meter	m^3	$4.546\ 087 \times 10^{-3}$
gallon (U.S. dry)	cu meter	m^3	*$4.404\ 883\ 770\ 86 \times 10^{-3}$
gallon (U.S. liquid)	cu meter	m^3	*$3.785\ 411\ 784 \times 10^{-3}$
gamma	tesla	$T = Wb/m^2$	*1.00×10^{-9}
gauss	tesla	$T = Wb/m^2$	*1.00×10^{-4}
gilbert	ampere turn		$7.957\ 747\ 2 \times 10^{-1}$
gill (British)	cu meter	m^3	$1.420\ 652 \times 10^{-4}$
gill (U.S.)	cu meter	m^3	$1.182\ 941\ 2 \times 10^{-4}$
grad	degree (angular)	1°	*9.00×10^{-1}
grad	radian	rad	$1.570\ 796\ 3 \times 10^{-2}$
grain	kilogram	kg	*$6.479\ 891 \times 10^{-5}$
gram	kilogram	kg	*1.00×10^{-3}
hand	meter	m	*1.016×10^{-1}
hectare	sq meter	m^2	*1.00×10^{4}
henry (Int of 1948)	henry	$H = V \cdot s/A$	1.000 495
hogshead (U.S.)	cu meter	m^3	*$2.384\ 809\ 423\ 92 \times 10^{1}$
horsepower (550 foot lbf/second)	watt	$W = J/s$	$7.456\ 998\ 7 \times 10^{2}$
horsepower (boiler)	watt	$W = J/s$	$9.809\ 50 \times 10^{3}$
horsepower (electric)	watt	$W = J/s$	*7.46×10^{2}
horsepower (metric)	watt	$W = J/s$	$7.354\ 99 \times 10^{2}$

*Exact, as defined by the National Bureau of Standards.

Table A4-20. Alphabetical Listing of Conversion Factors (Cont'd)

To Convert	To	Symbol	Multiply by
horsepower (water)	watt	W= J/s	7.460 43 x 10^2
hour (mean solar)	second (mean solar)		*3.60 x 10^2
hour (sidereal)	second (mean solar)		3.590 170 4 x 10^3
hundredweight (long)	kilogram	kg	*5.080 234 544 x 10^1
hundredweight (short)	kilogram	kg	*4.535 923 7 x 10^1
inch	meter	m	*2.54 x 10^{-2}
inch of mercury (32°F)	newton/sq meter	N/m^2	3.386 389 x 10^3
inch of mercury (60°F)	newton/sq meter	N/m^2	3.376 85 x 10^3
inch of water (39.2°F)	newton/sq meter	N/m^2	2.490 82 x 10^2
inch of water (60°F)	newton/sq meter	N/m^2	2.488 4 x 10^2
joule (Int of 1948)	joule	J= N·m	1.000 165
kayser	1/meter	1/m	*1.00 x 10^2
°Kelvin (temperature)	°Celsius	°C	°C= °K - 273.16
kilocalorie (Int Steam Table)	joule	J= N·m	4.186 74 x 10^3
kilocalorie (mean)	joule	J= N·m	4.190 02 x 10^3
kilocalorie (thermochemical)	joule	J= N·m	*4.184 x 10^3
kilogram mass	kilogram	kg	*1.00
kilogram force	newton	N= kg·m/s^2	*9.806 65
kilopond force	newton	N= kg·m/s^2	*9.806 65
kip	newton	N= kg·m/s^2	*4.448 221 615 260 5 x 10^3
knot (International)	meter/second	m/s	5.144 444 444 x 10^{-1}
lambert	candela/sq meter	cd/m^2	*1/pi x 10^4
lambert	candela/sq meter	cd/m^2	3.183 098 8 x 10^3
langley	joule/sq meter	J/m^2	*4.184 x 10^4
lbf (pound force, avoirdupois)	newton	N= kg·m/s^2	*4.448 221 615 260 5

* Exact, as defined by the National Bureau of Standards.

Table A4-20. Alphabetical Listing of Conversion Factors (Cont'd)

To Convert	To	Symbol	Multiply by
lbm (pound mass, avoirdupois)	kilogram	kg	*4.535 923 7 x 10^{-1}
league (British nautical)	meter	m	*5.559 552 x 10^{3}
league (Int nautical)	meter	m	*5.556 x 10^{3}
league (statute)	meter	m	*4.828 032 x 10^{3}
light-year	meter	m	9.460 55 x 10^{15}
link (surveyor or gunter)	meter	m	*2.011 68 x 10^{-1}
link (engineer or ramden)	meter	m	*3.048 x 10^{-1}
liter	cu meter	m^3	1.000 000 x 10^{-3}
lux	lumen/sq meter	lm/m^2	1.00
maxwell	weber	Wb= V·s	*1.00 x 10^{-8}
meter	wavelengths Kr^{86}		*1.650 763 73 x 10^{6}
micron	meter	m	*1.00 x 10^{-6}
mil	meter	m	*2.54 x 10^{-5}
mile (U.S. statute)	meter	m	*1.609 344 x 10^{3}
mile (British nautical)	meter	m	*1.853 184 x 10^{3}
mile (Int nautical)	meter	m	*1.852 x 10^{3}
mile (U.S. nautical)	meter	m	*1.852 x 10^{3}
millimeter of mercury (0°C)	newton/sq meter	N/m^2	1.333 224 x 10^{2}
millibar	newton/sq meter	N/m^2	*1.00 x 10^{2}
minute (angle)	radian	rad	2.908 882 086 66 x 10^{-4}
minute (mean solar)	second (mean solar)		*6.00 x 10^{1}
minute (sidereal)	second (mean solar)		5.983 617 4 x 10^{1}
month (mean calendar)	second (mean solar)		*2.628 x 10^{6}
oersted	ampere/meter	A/m	7.957 747 2 x 10^{1}
ohm (Int of 1948)	ohm	Ω = V/A	1.000 495

*Exact, as defined by the National Bureau of Standards.

Table A4-20. Alphabetical Listing of Conversion Factors (Cont'd)

To Convert	To	Symbol	Multiply by
ounce mass (avoirdupois)	kilogram	kg	*2.834 952 312 5 x 10^{-2}
ounce force (avoirdupois)	newton	N= kg· m/s²	2.780 138 5 x 10^{-1}
ounce mass (troy or apothecary)	kilogram	kg	*3.110 347 68 x 10^{-2}
ounce (U.S. fluid)	cu meter	m^3	*2.957 352 956 25 x 10^{-5}
pace	meter	m	*7.62 x 10^{-1}
parsec	meter	m	3.083 74 x 10^{16}
pascal	newton/sq meter	N/m^2	*1.00
peck (U.S.)	cu meter	m^3	*8.809 767 541 72 x 10^{-3}
pennyweight	kilogram	kg	*1.555 173 84 x 10^{-3}
perch	meter	m	*5.029 2
phot	lumen/sq meter	lm/m^2	1.00 x 10^4
pica (printers')	meter	m	*4.217 517 6 x 10^{-3}
pint (U.S. dry)	cu meter	m^3	*5.506 104 713 575 x 10^{-4}
pint (U.S. liquid)	cu meter	m^3	*4.731 764 73 x 10^{-4}
point (printers')	meter	m	*3.514 598 x 10^{-4}
poise	newton·second/sq meter	$N \cdot s/m^2$	*1.00 x 10^{-1}
pole	meter	m	*5.029 2
pound mass (lbm, avoirdupois)	kilogram	kg	*4.535 923 7 x 10^{-1}
pound force (lbf, avoirdupois)	newton	$N = kg \cdot m/s^2$	*4.448 221 615 260 5
pound mass (troy or apothecary)	kilogram	kg	*3.732 417 216 x 10^{-1}
poundal	newton	$N = kg \cdot m/s^2$	*1.382 549 543 76 x 10^{-1}
quart (U.S. dry)	cu meter	m^3	*1.101 220 942 715 x 10^{-3}
quart (U.S. liquid)	cu meter	m^3	9.463 529 5 x 10^{-4}
Rad (radiation dose absorbed)	joule/kilogram	J/kg	*1.00 x 10^{-2}
°Reaumur (temperature)	°Centigrade	°C	°C = 5/4 °Re

*Exact, as defined by the National Bureau of Standards.

Table A4-20. Alphabetical Listing of Conversion Factors (Cont'd)

To Convert	To	Symbol	Multiply by
rhe	sq meter/newton·second	$m^2/N\cdot s$	*1.00×10^{1}
rod	meter	m	*5.029 2
roentgen	coulomb/kilogram	C/kg	*$2.579\ 76 \times 10^{-4}$
second (angle)	radian	rad	$4.848\ 136\ 811 \times 10^{-6}$
second (mean solar)	second (ephemeris)		Use equation of time.
second (sidereal)	second (mean solar)		$9.972\ 695\ 7 \times 10^{-1}$
section	sq meter	m^2	*$2.589\ 988\ 110\ 336 \times 10^{6}$
scruple (apothecary)	kilogram	kg	*$1.295\ 978\ 2 \times 10^{-3}$
shake	second	s	1.00×10^{-8}
skein	meter	m	*$1.097\ 28 \times 10^{2}$
slug	kilogram	kg	$1.459\ 390\ 29 \times 10^{1}$
span	meter	m	*2.286×10^{-1}
statampere	ampere	A	$3.335\ 640 \times 10^{-10}$
statcoulomb	coulomb	$C = A\cdot s$	$3.335\ 640 \times 10^{-10}$
statfarad	farad	$F = A\cdot s/V$	$1.112\ 650 \times 10^{-12}$
stathenry	henry	$H = V\cdot s/A$	$8.987\ 554 \times 10^{11}$
statmho	mho		$1.112\ 650 \times 10^{-12}$
statohm	ohm	$\Omega = V/A$	$8.987\ 554 \times 10^{11}$
statvolt	volt	$V = W/A$	$2.997\ 925 \times 10^{2}$
stere	cu meter	m^3	*1.00
stilb	candela/sq meter	cd/m^2	1.00×10^{4}
stoke	sq meter/second	m^2/s	*1.00×10^{-4}
tablespoon	cu meter	m^3	*$1.478\ 676\ 478\ 125 \times 10^{-5}$
teaspoon	cu meter	m^3	*$4.928\ 921\ 593\ 75 \times 10^{-6}$

*Exact, as defined by the National Bureau of Standards.

Table A4-20. Alphabetical Listing of Conversion Factors (Cont'd)

To Convert	To	Symbol	Multiply by
ton (assay)	kilogram	kg	$2.916\,666\,6 \times 10^{-2}$
ton (short, 2000 pound)	kilogram	kg	*$9.071\,847\,4 \times 10^{2}$
ton (long)	kilogram	kg	*$1.016\,046\,908\,8 \times 10^{3}$
ton (metric)	kilogram	kg	*1.00×10^{3}
ton (nuclear equiv. of TNT)	joule	$J = N \cdot m$	4.20×10^{9}
ton (register)	cu meter	m^3	*2.831 684 659 2
torr (0°C)	newton/sq meter	N/m^2	$1.333\,22 \times 10^{2}$
township	sq meter	m^2	$9.323\,957\,2 \times 10^{7}$
unit pole	weber	$Wb = V \cdot s$	$1.256\,637 \times 10^{-7}$
volt (Int of 1948)	volt	$V = W/A$	1.000 330
watt (Int of 1948)	watt	$W = J/s$	1.000 165
yard	meter	m	*9.144×10^{-1}
year (calendar)	second (mean solar)		*$3.153\,6 \times 10^{7}$
year (sidereal)	second (mean solar)		$3.155\,815\,0 \times 10^{7}$
year (tropical)	second (mean solar)		$3.155\,692\,6 \times 10^{7}$
year 1900, tropical, Jan, day 0, hour 12	second (ephemeris)	s	*$3.155\,692\,597\,47 \times 10^{7}$

*Exact, as defined by the National Bureau of Standards.

Table A4-21. Decimal and Metric Equivalents of Fractions of an Inch

Inch Decimal	Inch Fraction	Millimeter (mm)	Centimeter (cm)	Meter (m)
0.015 625	1/64	0.396 87	0.039 687	0.000 396 87
0.031 25	1/32	0.793 74	0.079 374	0.000 793 74
0.046 875	3/64	1.190 61	0.119 061	0.001 190 61
0.062 5	1/16	1.587 48	0.158 748	0.001 587 48
0.078 125	5/64	1.984 35	0.198 435	0.001 984 35
0.093 75	3/32	2.381 23	0.238 123	0.002 381 23
0.109 375	7/64	2.778 09	0.277 809	0.002 778 09
0.125	1/8	3.174 97	0.317 497	0.003 174 97
0.140 625	9/64	3.571 83	0.357 183	0.003 571 83
0.156 25	5/32	3.968 71	0.396 871	0.003 968 71
0.171 875	11/64	4.365 57	0.436 557	0.004 365 57
0.187 5	3/16	4.762 45	0.476 245	0.004 762 45
0.203 125	13/64	5.159 31	0.515 931	0.005 159 31
0.218 75	7/32	5.556 20	0.555 620	0.005 556 20
0.234 375	15/64	5.953 05	0.595 305	0.005 953 05
0.25	1/4	6.349 94	0.634 994	0.006 349 94
0.265 625	17/64	6.746 79	0.674 679	0.006 746 79
0.281 25	9/32	7.143 68	0.714 368	0.007 143 68
0.296 875	19/64	7.540 53	0.754 053	0.007 540 53
0.312 5	5/16	7.937 43	0.793 743	0.007 937 43
0.328 125	21/64	8.334 27	0.833 427	0.008 334 27
0.343 75	11/32	8.731 17	0.873 117	0.008 731 17
0.359 375	23/64	9.128 01	0.912 801	0.009 128 01
0.375	3/8	9.524 91	0.952 491	0.009 524 91
0.390 625	25/64	9.921 75	0.992 175	0.009 921 75

Table A4-21. Decimal and Metric Equivalents of Fractions of an Inch (Cont'd)

Inch Decimal	Inch Fraction	Millimeter (mm)	Centimeter (cm)	Meter (m)
0.406 25	13/32	10.318 65	1.031 865	0.010 318 65
0.421 875	27/64	10.715 49	1.071 549	0.010 715 49
0.437 5	7/16	11.112 40	1.111 240	0.011 112 40
0.453 125	29/64	11.509 23	1.150 923	0.011 509 23
0.468 75	15/32	11.906 14	1.190 614	0.011 906 14
0.484 375	31/64	12.302 97	1.230 297	0.012 302 97
0.5	1/2	12.699 88	1.269 988	0.012 699 88
0.515 625	33/64	13.096 71	1.309 671	0.013 096 71
0.531 25	17/32	13.493 62	1.349 362	0.013 493 62
0.546 875	35/64	13.890 45	1.389 045	0.013 890 45
0.562 5	9/16	14.287 37	1.428 737	0.014 287 37
0.578 125	37/64	14.684 19	1.468 419	0.014 684 19
0.593 75	19/32	15.081 11	1.508 111	0.015 081 11
0.609 375	39/64	15.477 93	1.547 793	0.015 477 93
0.625	5/8	15.874 85	1.587 485	0.015 874 85
0.640 625	41/64	16.271 67	1.627 167	0.016 271 67
0.656 25	21/32	16.668 59	1.666 859	0.016 668 59
0.671 875	43/64	17.065 41	1.706 541	0.017 065 41
0.687 5	11/16	17.462 34	1.746 234	0.017 462 34
0.703 125	45/64	17.859 15	1.785 915	0.017 859 15
0.718 75	23/32	18.256 08	1.825 608	0.018 256 08
0.734 375	47/64	18.652 89	1.865 289	0.018 652 89
0.75	3/4	19.049 82	1.904 982	0.019 049 82
0.765 625	49/64	19.446 63	1.944 663	0.019 446 63
0.781 25	25/32	19.843 56	1.984 356	0.019 843 56

SECTION B

STRENGTH ANALYSIS

SECTION B1

JOINTS AND FASTENERS

TABLE OF CONTENTS

Page

B1.0.0 Joints and Fasteners

B 1.0.0 Joints and Fasteners

B 1.1.0 Mechanical Joints and Fasteners

B 1.1.1 Riveted Joints

Although the actual state of stress in a riveted joint is complex, it is customary to ignore such considerations as stress concentration at the edge of rivet holes, unequal division of load among fasteners, and nonuniform distribution of shear stress across the section of the rivet and of the bearing stress between rivet and plate. Simplifying assumptions are made, which are summarized as follows:

(1) The applied load is assumed to be transmitted entirely by the rivets, friction between the connected plates being ignored.

(2) When the center of cross-sectional area of each of the rivets is on the line of action of the load, or when the centroid of the total rivet area is on this line, the rivets of the joint are assumed to carry equal parts of the load if of the same size; and to be loaded proportionally to their section areas otherwise.

(3) The shear stress is assumed to be uniformly distributed across the rivet section.

(4) The bearing stress between plate and rivet is assumed to be uniformly distributed over an area equal to the rivet diameter times the plate thickness.

(5) The stress in a tension member is assumed to be uniformly distributed over the net area.

(6) The stress in a compression member is assumed to be uniformly distributed over the gross area.

The design of riveted joints on the basis of these assumptions is the accepted practice, although none of them is strictly correct.

The possibility of secondary failure due to secondary causes, such as the shearing or tearing out of a plate between rivet and edge of plate or between adjacent rivets, the bending or insufficient upsetting of long rivets, or tensile failure along a zigzag line when rivets are staggered, are guarded against in standard specifications by provisions summarized as follows:

(1) The distance from a rivet to a sheared edge shall not be less than 1 3/4 diameters, or to a planed or rolled edge, 1 1/2 diameters.

(2) The minimum rivet spacing shall be 3 diameters.

B 1.1.1 Riveted Joints (Cont'd)

(3) The maximum rivet pitch in the direction of stress shall be 7 diameters, and at the ends of a compression member it shall be 4 diameters for a distance equal to 1 1/2 times the width of the member.

(4) In the case of a diagonal or zigzag chain of holes extending across a part, the net width of the part shall be obtained by deducting from the gross width the sum of the diameters of all the holes in the chain, and adding, for each gauge space in the chain, the quantity $S^2/4g$, where S = longitudinal spacing of any two successive holes in the chain and g = the spacing transverse to the direction of stress of the same two holes. The critical net section of the part is obtained from that chain which gives the least net width.

(5) The shear and bearing stresses shall be calculated on the basis of the nominal rivet diameter, the tensile stresses on the hole diameter.

If the rivets of a joint are so arranged that the line of action of the load does not pass through the centroid of the rivet areas then the effect of eccentricity must be taken into account.

B 1.1.2 Bolted Joints

Bolted joints that are designed on the basis of shear and bearing are analyzed in the same way as riveted joints. The simplifying assumptions listed in Section B 1.1.1 are valid for short bolts where bending of the shank is negligible.

In general when bolts are designed by tension, the Factor of Safety should be at least 1.5 based on design load to take care of eccentricities which are impossible to eliminate in practical design. Avoid the use of aluminum bolts in tension.

Hole-filling fasteners (such as conventional solid rivets) should not be combined with non-hole-filling fasteners (such as conventional bolt or screw installation).

B 1.1.3 Protruding-Head Rivets and Bolts

The load per rivet or bolt, at which the shear or bearing type of failure occurs, is separately calculated and the lower of the two governs the design. The ultimate shear and tension stress, and the ultimate loads for steel AN bolts and pins are given in Table B 1.1.3.1 and B 1.1.3.2. Interaction curves for combined shear and tension loading on AN bolts are given in Fig. B 1.1.3-1. Shear and tension ultimate loads for MS internal wrenching bolts are specified in Table B 1.1.3.3.

B 1.1.3 Protruding-Head Rivets and Bolts (Cont'd)

In computing aluminum rivet shear strength, the correction factors given in Table B 1.1.3.5 should be used to compensate for the reductions in rivet shear strength resulting from high bearing stresses on the rivet at D/t ratios in excess of 3.0 for single-shear joints, and 1.5 for double-shear joints. The basic shear strength for protruding-head aluminum-alloy rivets is given in Table B 1.1.3.6.

The yield and ultimate bearing stresses for various materials at room and elevated temperatures are given in the strength properties stated for each alloy or group of alloys, and are applicable to riveted or bolted joints where cylindrical holes are used and where $D/t < 5.5$. Where $D/t > 5.5$, tests to substantiate yield and ultimate bearing strengths must be performed. These bearing stresses are applicable only for the design of rigid joints where there is no possibility of relative motion of the parts joined without deformation of the parts. Yield and ultimate stresses at low temperatures will be higher than those specified for room temperature; however, no quantitative data are available.

For convenience, "unit" sheet bearing strength for rivets, based on a stress of 100 ksi and nominal hole diameters, is given in Table B 1.1.3.7. Factors representing the ratio of actual sheet bearing strength to 100 ksi are given in Table B 1.1.3.8. Table B 1.1.3.9 contains unit bearing strength of sheets on bolts. For magnesium-alloy riveting, it is unnecessary to use the correction factors of Table B 1.1.3.5, which account for high bearing stresses on the rivet.

B 1.1.3 Protruding-Head Rivets and Bolts (Cont'd)

Table B 1.1.3.1 Ultimate Strength of Bolts

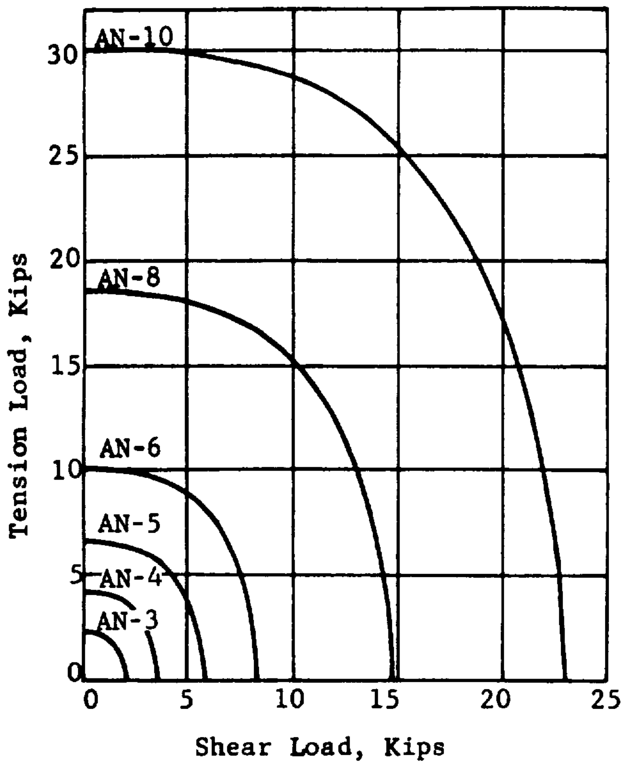
Bolt Size	Nominal Shank Area	Basic Minor Dia	Basic Area at Minor Dia	Steel Bolts Heat Treat 125,000 psi Min			Steel Bolts Heat Treat 160,000 psi Min		Al Alloy Bolts Heat Treat 62,000 psi Min	
				Tension (lb)	Single Shear (lb)	Bending (in.lb)	Tension (lb)	Single Shear (lb)	Tension (lb)	Single Shear (lb)
10-32	.0283	.1494	.0175	2,210	2,125		2,800	2,620		
1/4-28	.0494	.2036	.0326	4,080	3,680	192	5,000	4,650	1,310	1,715
5/16-24	.0767	.2584	.0524	6,500	5,750	375	8,200	7,300	2,110	2,685
3/8-24	.1105	.3209	.0809	10,100	8,290	647	12,700	10,500	3,260	3,870
7/16-20	.1503	.3725	.1090	13,600	11,250	1,028	17,100	14,300	4,400	5,250
1/2-20	.1964	.4350	.1486	18,500	14,700	1,534	23,400	18,650	6,000	6,850
9/16-18	.2485	.4903	.1888	23,600	18,700	2,184	29,800	23,600	....	8,700
5/8-18	.3068	.5528	.2400	30,100	23,000	2,996	38,000	29,150	....	10,750
3/4-16	.4418	.6688	.3513	44,000	33,150	5,177	55,600	41,950	....	15,500
7/8-14	.6013	.7822	.4805	60,000	45,050	8,221	76,200	57,100	....	21,050
1-14	.7854	.9072	.6464	80,700	58,900	12,272	102,500	74,600	....	27,500
1 1/8-12	.9940	1.0167	.8118	101,800	73,750	17,470	128,800	94,450	....	34,500
1 1/4-12	1.2272	1.1417	1.0237	130,200	91,050	23,970	162,600	116,600	....	42,500
1 3/8-12	1.4849	1.2667	1.2602	....	....	....	200,300	141,050		
1 1/2-12	1.7671	1.3917	1.5212	....	....	....	241,200	167,900		

B 1.1.3 Protruding-Head Rivets and Bolts (Cont'd)

Table B 1.1.3.2 Shear and Tensile Strengths, Areas, and Moments of Inertia of Steel Bolts and Pins

Material				Low Carbon Steel	Heat-treated steel			AN standard bolt designation specification MIL-B-1812
Ultimate tensile strength, ksi				55	100	125	125	
Ultimate shear strength, ksi				35	65	75	75	
Size of pin or bolt	Machine screw size, No.	Area of solid section, in.^{2}	Moment of inertia of solid, in^4	Ultimate single shear strength at full diameter, lb			Ultimate tensile strength (in thread), lb	
1/16		0.003068	0.00000075	107	199	230		
3/32		.006902	.00000379	242	449	518		
0.112	4	.009852	.00000772	345	640	739		
1/8		.012272	.00001198	430	798	920		
0.138	6	.014957	.00001781	523	972	1,122		
5/32		.01918	.00002926	671	1,247	1,438		
0.164	8	.02112	.00003549	739	1,372	1,584		
3/16		.02761	.00006066	966	1,794	2,070		
0.190	10	.02835	.00006399	992	1,842	2,126	2,210	AN-3
0.216	12	.03664	.0001069	1,282	2,381	2,748		
7/32		.03758	.0001125	1,315	2,442	2,818		
1/4		.04908	.0001918	1,717	3,190	3,680	4,080	AN-4
5/16		.07669	.0004682	2,684	4,984	5,750	6,500	AN-5
3/8		.1105	.9009710	3,868	7,183	8,280	10,100	AN-6
7/16		.1503	.001797	5,261	9,770	11,250	13,600	AN-7
1/2		.1963	.003069	6,871	12,760	14,700	18,500	AN-8
9/16		.2485	.004914	8,697	16,152	18,700	23,600	AN-9
5/8		.3068	.007492	10,738	19,942	23,000	30,100	AN-10
3/4		.4418	.01553	15,463	28,717	33,150	44,000	AN-12
7/8		.6013	.02878	21,046	39,085	45,050	60,000	AN-14
1		.7854	.04908	27,489	51,651	58,900	80,700	AN-16
1 1/8						73,750	101,800	AN-18
1 1/4						91,050	130,200	AN-20

B 1.1.3 Protruding-Head Rivets and Bolts (Cont'd)



Interaction Formula

$$\frac{x^3}{a^3} + \frac{y^2}{b^2} = 1$$

Where:

x = shear load
y = tension load
a = shear allowable
b = tension allowable

Note: Curves not applicable where shear nuts are used. Curves are based on the results of combined load tests of bolts with nuts fingertight.

Fig. B 1.1.3-1 Combined Shear and Tension on AN Steel Bolts.

B 1.1.3 Protruding-Head Rivets and Bolts (Cont'd)

Table B 1.1.3.3 Shear and Tensile Strengths of Internal Wrenching Bolts

Material		Heat-treated alloy steel (160-180 ksi)		Material		Heat-treated alloy steel (160-180 ksi)	
Specification		MIL-S-8503 and MIL-S-6049	MIL-S-5000	Specification		MIL-S-8503 and MIL-S-6049	MIL-S-5000
Size	Standard	Ultimate tensile strength (minimum), lb	Double shear strength (minimum), lb	Size	Standard	Ultimate tensile strength (minimum), lb	Double shear strength (minimum), lb
1/4	MS20004	5,000	9,300	3/4	MS20012	55,600	83,900
5/16	MS20005	8,200	14,600	7/8	MS20014	76,200	114,200
3/8	MS20006	12,700	21,000	1	MS20016	102,500	149,200
7/16	MS20007	17,100	28,600	1 1/8	MS20018	128,800	188,900
1/2	MS20008	23,400	37,300	1 1/4	MS20020	162,600	233,200
9/16	MS20009	29,800	47,200	1 3/8	MS20022	200,300	282,100
5/8	MS20010	38,000	58,300	1 1/2	MS20024	241,200	335,800

Note: Nuts designed to develop the ultimate tensile strength of the bolts are required in applications depended upon to develop the tabulated bolt loads.

B 1.1.3 Protruding-Head Rivets and Bolts (Cont'd)

Table B 1.1.3.4 Ultimate Tension Loads For Solid Rivets
2024-T3 and 2024-T36 Alclad Sheet

	Universal Head						All Flush Riveting						
DIA.	3/32	1/8	5/32	3/16	1/4	5/16	3/32	1/8	5/32	3/16	1/4	5/16	DIA.
Head Tension	296	521	810	1506	2695	4200	261	445	729	1445	2610	3642	Head Tension
.016	130	160											.016
.020	145	175											.020
.025	170	210	220				75						.025
.032	199	248	290	390			104	146					.032
.040	233	290	380	490			138	200	235				.040
.045	254	317	435	550	970		160	232	275				.045
.051		348	490	622	1020		185	272	318	415			.051
.064		414	640	782	1130	1175	242	362	430	589	725		.064
.072		456	725	882	1195	1260		415	490	695	825		.072
.080		497		980	1265	1495			560	807	925	1092	.080
.091				1110	1355	1800			645	952	1062	1270	.091
.100				1220	1430	2065			720	1075	1175	1420	.100
.102				1240	1446	2120				1105	1200	1450	.102
.110				1340	1512	2350				1218	1300	1585	.110
.120				1460	1598	2645				1348	1427	1752	.120
.128					1665	2877					1527	1885	.128
.130					1680	2935					1552	1920	.130
.140					1762	3220					1675	2087	.140
.150					1845	3505					1802	2252	.150
5/32					1898	3700					1880	2362	5/32
.160					1930	3790					1925	2420	.160
.170					2015						2015	2585	.170
.180					2095						2095	2750	.180
3/16					2160						2160	2875	3/16
.20					2265						2265	3080	.20
.25					2680								.25

B 1.1.3 Protruding-Head Rivets and Bolts (Cont'd)

Table B 1.1.3.5 Shear Strengths of Protruding and Flush-Head Aluminum-Alloy Rivets

Diameter of rivet, (in.)	Mat. Code	1/16	3/32	1/8	5/32	3/16	1/4	5/16	3/8
Shear strength, lb:									
5056, F_{su}=28 ksi	B	99	203	363	556	802	1,450	2,290	3,280
2117-T3, F_{su}=30 ksi	AD	106	217	388	596	862	1,550	2,460	3,510
2017-T31, F_{su}=34 ksi	D	120	247	442	675	977	1,760	2,790	3,970
2017-T3, F_{su}=38 ksi	D	135	275	494	755	1,090	1,970	3,110	4,450
2024-T31, F_{su}=41 ksi	DD	145	296	531	815	1,180	2.120	3,360	4,800
Single-shear rivet strength factors									
Sheet thickness, in.:									
0.016		0.964							
0.018		.984							
0.020		.996							
0.025		1.000	0.972						
0.032			1.000	0.964					
0.036				.980					
0.040				.996	0.964				
0.045				1.000	.980				
0.050					.996	0.972			
0.063					1.000	1.000	0.964		
0.071							.980	0.964	
0.080							.996	.974	
0.090							1.000	.984	
0.100								.996	0.972
0.125								1.000	1.000
0.160									
0.190									
0.250									

B 1.1.3 Protruding-Head Rivets and Bolts (Cont'd)

Table B 1.1.3.5 Shear Strengths of Protruding and Flush-Head Aluminum-Alloy Rivets **(Cont'd)**

Diameter of rivet, (in.)	1/16	3/32	1/8	5/32	3/16	1/4	5/16	3/8
Double-shear rivet strength factors								
Sheet thickness, in.:								
0.016	0.688							
0.018	.753							
0.020	.792							
0.025	.870	0.714						
0.032	.935	.818	0.688					
0.036	.974	.857	.740					
0.040	.987	.896	.792	0.688				
0.045	1.000	.922	.831	.740				
0.050		.961	.870	.792	0.714			
0.063		1.000	.935	.883	.818	0.688		
0.071			.974	.919	.857	.740		
0.080			1.000	.948	.896	.792	0.688	
0.090				.974	.922	.831	.753	
0.100				1.000	.961	.870	.792	0.714
0.125					1.000	.935	.883	.818
0.160						.987	.935	.883
0.190						1.000	.974	.935
0.250							1.000	1.000

Note: Values of shear strength should be multiplied by the factors given herein whenever the D/t ratio is large enough to require such a correction.

Shear values are based on areas corresponding to the nominal hole diameters specified in table 4.12.0.9, note e.

Shear stresses in table 4.12.0.9 corresponding to 90 percent probability data are used wherever available.

Sheet thickness is that of the thinnest sheet in single-shear joints and the middle sheet in double-shear joints.

B 1.1.3 Protruding-Head Rivets and Bolts (Cont'd)

Table B 1.1.3.6 Ultimate Shear Strength for Aluminum Alloy Rivets

Type	Protruding-head rivets[a]									
Alloy	2117		2017				2024		5056	
Specification	MIL-R-5674									
Condition	-T3		-T31[b]		-T3[c]		-T31		-H321	
Basis[d]	A	B	A	B	A	B	A	B	A	B
Ultimate Shear Strength, ksi:										
F_{su}[e] (for driven rivets)	28	30	33	34	35	38	37	41	27	28
F_{su} (for undriven rivets and rivet wire)	26	29			33	37	37	38	24	27

a The driven head diameter shall be at least 1.3 times the nominal shank diameter of the rivet.

b The 2017-T31 designation refers to rivets that have been heat treated and then maintained in the heat-treated condition until driving.

c The 2017-T3 designation refers to 2017 rivets which are fully aged at room temperature for at least 4 days after quenching, and then driven. (The higher strength properties of the 2017-T3 rivets result from the cold-working effects obtained when the rivets are driven in the aged condition.)

d A is the mechanical-property column based upon the minimum guaranteed tensile properties; B is the mechanical-property column based upon probability data.

e Shear and bearing strength values for driven rivets may be based on areas corresponding to the nominal hole diameter, provided that the nominal hole diameter is not larger than the values listed below. If the nominal hole diameter is larger than the listed values, the listed value shall be used.

Standard Rivet-Hole Drill Sizes and Nominal Hole Diameters

Rivet size, in.	1/16	3/32	1/8	5/32	3/16	1/4	5/16	3/8
Drill No.	51	41	30	21	11	F	P	W
Nominal hole diameter, (in.)	0.067	0.096	0.1285	0.159	0.191	0.257	0.323	0.386

B 1.1.3 Protruding-Head Rivets and Bolts (Cont'd)

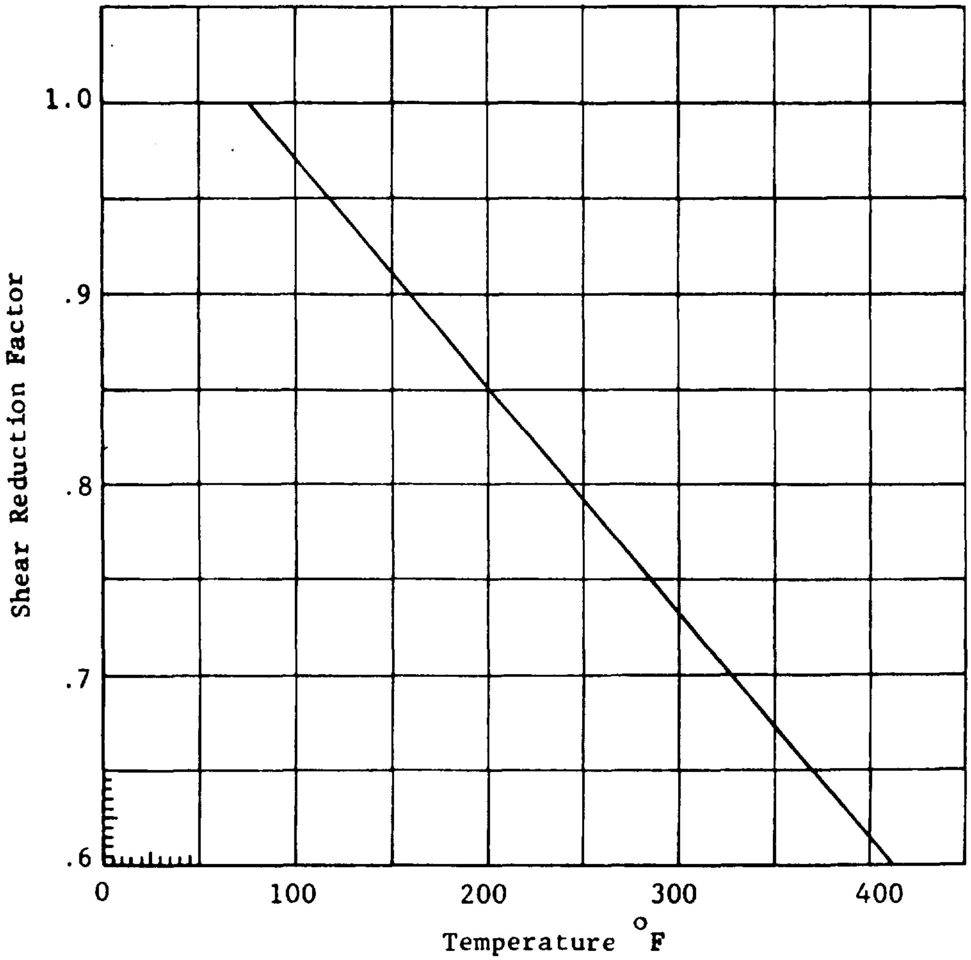


Fig. B 1 1.3-2 Reduction Factor for Allowables of Protruding Head, AN470-AD (2117-T3), Rivets at Elevated Temperature for Five Minutes

B 1.1.3 Protruding-Head Rivets and Bolts (Cont'd)

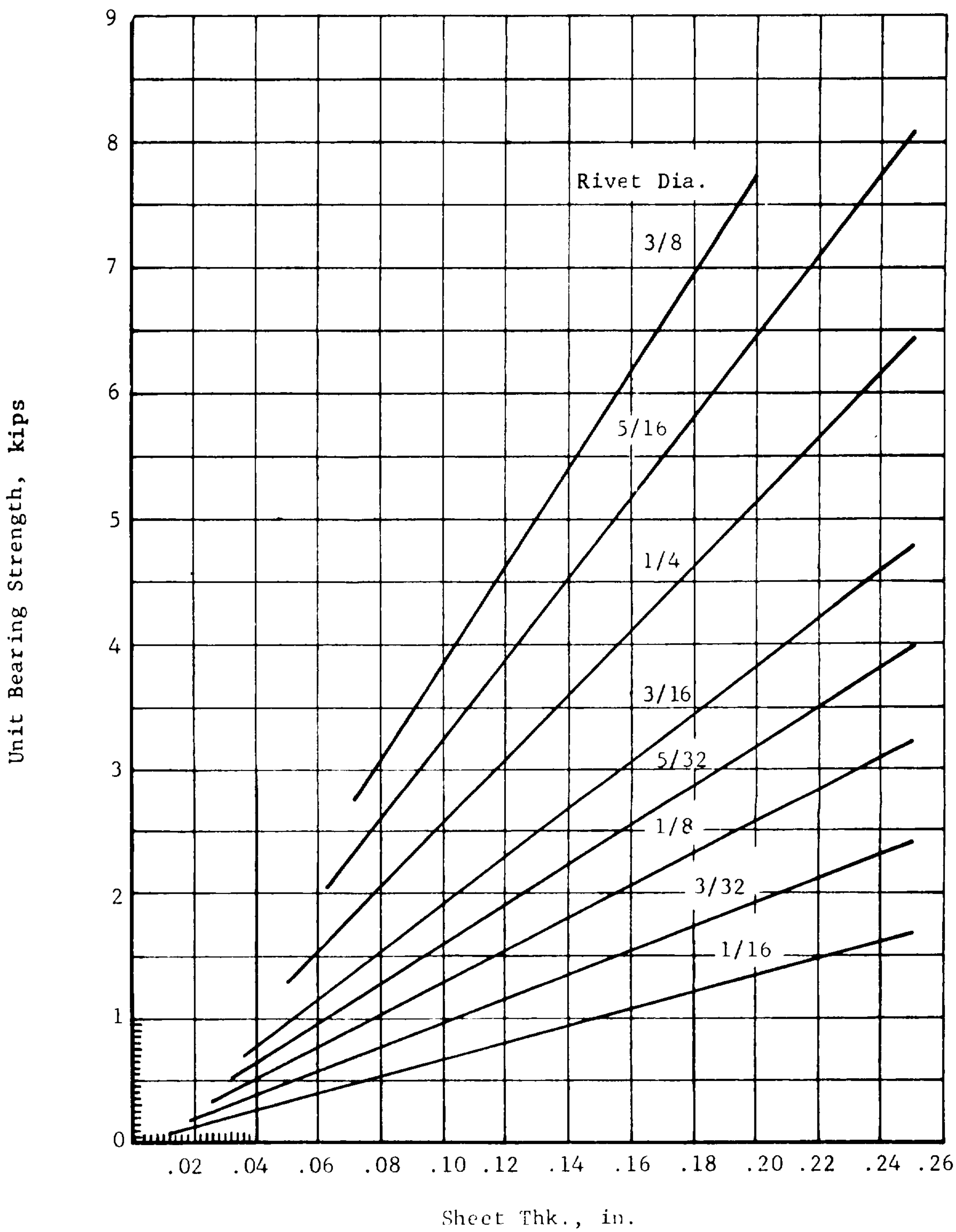


Fig. B 1.1.3-3 Unit Bearing Strengths of Sheets on Rivets

F_{br} = 100 ksi

B 1.1.3 Protruding-Head Rivets and Bolts (Cont'd)

Table B 1.1.3.7 Unit Bearing Strength of Sheet on Rivets, F_{br} = 100 ksi

Sheet thickness (in.)	Unit bearing strength for rivet diameter indicated, lb[a]							
	1/16 in.	3/32 in.	1/8 in.	5/32 in.	3/16 in.	1/4 in.	5/16 in.	3/8 in.
0.012	80							
0.016	107							
0.018	121	173						
0.020	134	192						
0.025	168	240	321					
0.032	214	307	411	509				
0.036	241	346	463	572	688			
0.040	268	384	514	636	764			
0.045	302	432	578	716	860			
0.050	335	480	643	795	955	1285		
0.063	422	605	810	1002	1203	1619	2035	
0.071	476	682	912	1129	1356	1825	2293	2741
0.080	536	768	1028	1272	1528	2056	2584	3088
0.090	603	864	1157	1431	1719	2313	2907	3474
0.100	670	960	1285	1590	1910	2570	3230	3860
0.125	838	1200	1606	1988	2388	3213	4038	4825
0.160	1072	1536	2056	2544	3056	4112	5168	6176
0.200	1340	1920	2570	3180	3820	5140	6460	7720
0.250	1670	2400	3210	3970	4770	6420	8070	......

a Bearing values are based on areas computed using the nominal hole diameters specified in table B 1.1.3.6

B 1.1.3 Protruding-Head Rivets and Bolts (Cont'd)

Table B 1.1.3.8 Aluminum-Alloy Sheet and Plate Bearing Factors[a]
(K = ratio of actual bearing strength to 100 ksi)

Material	Thickness, in.	A values K (Ultimate) e/D=2.0	A values K (Ultimate) e/D=1.5	A values K (Yield) e/D=2.0	A values K (Yield) e/D=1.5	B values K (Ultimate) e/D=2.0	B values K (Ultimate) e/D=1.5	B values K (Yield) e/D=2.0	B values K (Yield) e/D=1.5
2024-T42 (heat treated by user)	<.250	1.18	0.93	0.64	0.56				
	.250- .500	1.22	.96	.61	.53				
	.501-1.000	1.18	.93	.61	.53				
2024-T3	<.250	1.24	.98	.79	.69	1.29	1.02	0.82	0.71
	.250- .500	1.24	.98	.74	.64	1.27	1.01	.78	.69
2024-T4	.501-1.000	1.20	.95	.70	.62	1.29	1.02	.77	.67
2024-T36	≤.500	1.33	1.05	.96	.84	1.37	1.08	1.00	.88
2024-T4 (coiled)	<.250	1.18	.93	.64	.56	1.26	.99	.66	.57
Clad 2024-T42 (heat treated by user)	<.063	1.06	.84	.54	.48	1.10	.87	.56	.49
	.064- .249	1.12	.89	.58	.50	1.16	.92	.68	.53
	.250- .499	1.18	.93	.61	.53				
	.500-1.000	1.14	.90	.58	.50				
Clad 2024-T3	.010- .063	1.14	.90	.73	.64	1.18	.93	.76	.67
	.064- .249	1.20	.95	.74	.64	1.24	.98	.78	.69
Clad 2024-T4	.250- .499	1.20	.95	.74	.64	1.24	.98	.78	.69
	.500-1.000	1.16	.92	.67	.59	1.24	.98	.74	.64
Clad 2024-T36	.019- .063	1.20	.95	.88	.77	1.25	.99	.93	.81
	.064- .500	1.27	1.01	.93	.81	1.31	1.04	.96	.84
Clad 2024-T4 (coiled)	.012- .062	1.10	.87	.59	.52	1.16	.92	.61	.53
	.063	1.16	.92	.61	.53	1.20	.95	.64	.56
Clad 2024-T6	<.063	1.14	.90	.75	.66				
	≥.063	1.18	.93	.78	.69				
Clad 2024-T81	<.063	1.22	.96	.90	.78				
	≥.063	1.27	1.00	.94	.83				
Clad 2024-T86	<.063	1.33	1.05	1.04	.91				
	≥.063	1.35	1.06	1.09	.95				
7075-T6	.016- .039	1.44	1.14	1.06	.92	1.48	1.17	1.10	.97
	.040- .249	1.49	1.16	1.07	.94	1.50	1.19	1.12	.98
	.250- .500	1.39	1.08	1.00	.87	1.42	1.10	1.04	.90
	.501-1.000	1.42	1.10	1.04	.90	1.47	1.15	1.08	.94

B 1.1.3 Protruding-Head Rivets and Bolts (Cont'd)

Table B 1.1.3.8 Aluminum-Alloy Sheet and Plate Bearing Factors[a]
(K = ratio of actual bearing strength to 100 ksi) Cont'd

Material	Thickness, in.	A values				B values			
		K (Ultimate)		K (Yield)		K (Ulitmate)		K (Yield)	
		e/d=2.0	e/D=1.5	e/D-2.0	e/D=1.5	e/D=2.0	e/D=1.5	e/D=2.0	e/D=1.5
Clad 7075-T6	.016- .039	1.33	1.05	.98	.85	1.39	1.10	1.02	.90
	.040- .249	1.37	1.08	1.01	.88	1.41	1.11	1.04	.91
	.250- .499	1.30	1.01	.94	.84	1.33	1.04	.98	.82
	.500-1.000	1.33	1.04	.96	.83	1.37	1.06	1.00	.87
Clad 2014-T6	≤.039	1.22	.96	.90	.78	1.22	.96	.90	.78
	.040-1.000	1.24	.98	.93	.81	1.27	1.01	.96	.84
2014-T6	.040-1.000	1.29	1.02	.96	.84	1.33	1.05	.99	.87
5052-H32 (1/4H)		.65	.50	.34	.29				
5052-H32 (1/2H)		.71	.54	.38	.34				
5052-H36 (3/4H)		.78	.59	.46	.41				
5052-H38 (H)		.82	.62	.53	.46				
6061-T4		.63	.48	.26	.22				
6061-T6		.88	.67	.58	.50				

[a]For e/D values between 1.5 and 2.0 bearing factors may be obtained by linear interpolation. (e = edge distance, D = hole diameter).

B 1.1.3 Protruding-Head Rivets and Bolts (Cont'd)

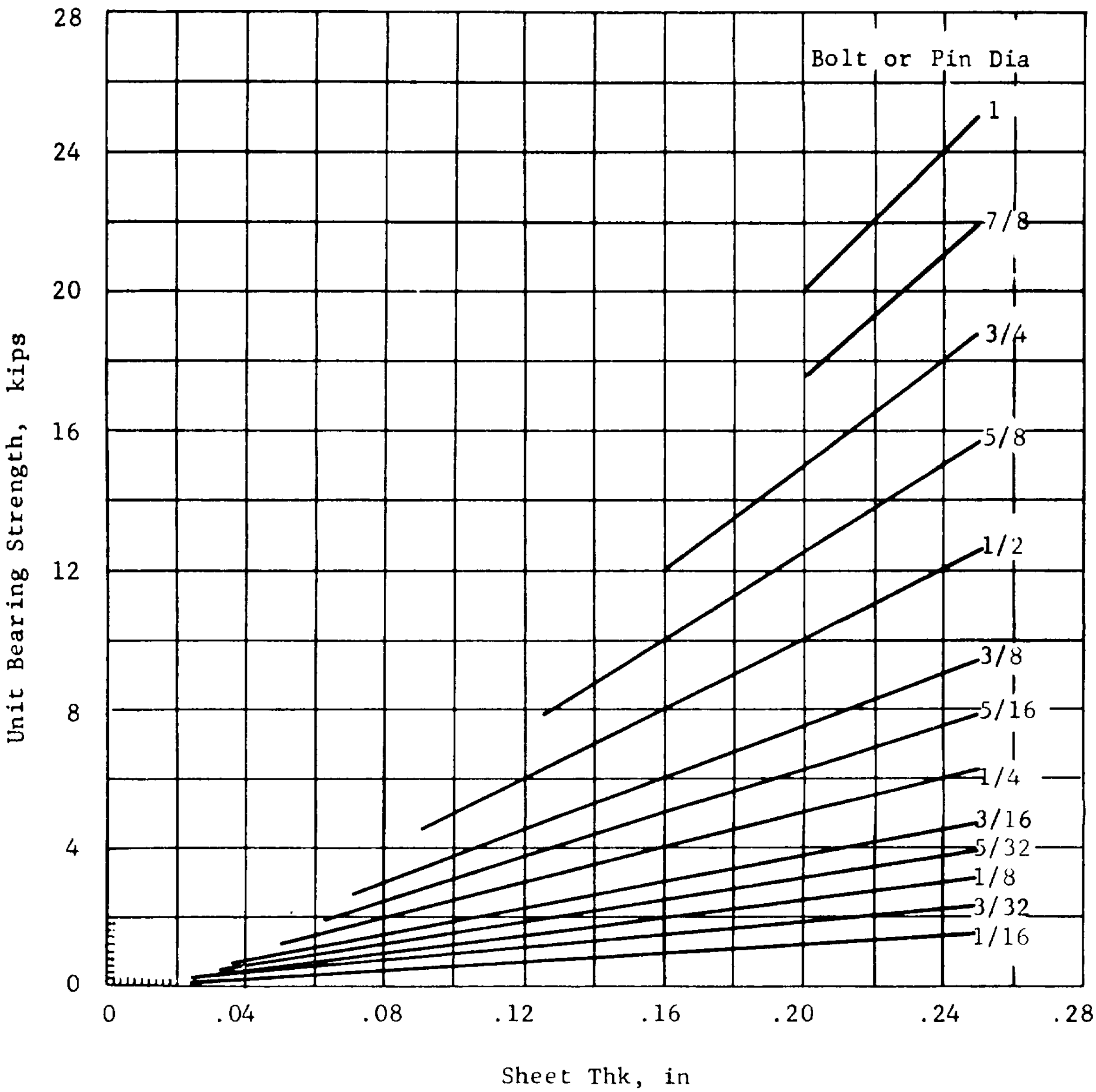


Fig. B 1.1.3-4 Unit Bearing Strengths of Sheets on Bolts and Pins
F_{br} = 100 ksi

B 1.1.3 Protruding-Head Rivets and Bolts (Cont'd)

Table B 1.1.3.9 Unit Bearing Strengths of Sheets on Bolts and Pins; F_{br} = 100 ksi

Plate sizes, in.	Bearing strength of plate for rivet size indicated, lb												
	1/16 in.	3/32 in.	1/8 in.	5/32 in.	3/16 in.	1/4 in.	5/16 in.	3/8 in.	1/2 in.	5/8 in.	3/4 in.	7/8 in.	1 in.
0.025	156	234	313										
0.032	200	300	400	500									
0.036	225	338	450	563	675								
0.040	250	375	500	625	750								
0.045	281	422	563	704	845								
0.050	313	469	625	781	940	1,250							
0.063	394	590	788	985	1,180	1,575	1,969						
.0.071	444	665	888	1,110	1,330	1,775	2,219	2,663					
0.080	500	750	1,000	1,250	1,500	2,000	2,500	3,000					
0.090	563	845	1,125	1,407	1,690	2,250	2,813	3,375	4,500				
0.100	625	938	1,250	1,562	1,875	2,500	3,125	3,750	5,000				
0.125	781	1,170	1,563	1,953	2,340	3,125	3,906	4,688	6,250	7,812			
0.160	1,000	1,500	2,000	2,500	3,000	4,000	5,000	6,000	8,000	10,000	12,000		
0.200	1,250	1,875	2,500	3,125	3,750	5,000	6,250	7,500	10,000	12,500	15,000	17,500	20,000
0.250	1,563	2,344	3,125	3,916	4,688	6,250	7,813	9,375	12,500	15,625	18,750	21,875	25,000

Note: For intermediate values see Fig. B 1.1.3-4

B 1.1.4 Flush Rivets

Table B 1.1.4.1 through B 1.1.4.3 contain ultimate and yield allowable single-shear strength values for both machine-countersunk and dimpled flush riveted joints employing solid rivets with a head angle of 100°. These strength values are applicable when the edge distance is equal to or greater than two times the nominal rivet diameter. Other strength values and edge distances may be used if substantiated by tests.

The allowable ultimate loads were established from test data using the average failing load divided by a factor of 1.15. The yield loads were established from test data wherein the yield load was defined as the average test load at which the following permanent set across the joint is developed:

(1) 0.005 inch, up to and including 3/16 inch diameter rivets.

(2) 2.5 percent of the rivet diameter for rivet sizes larger than 3/16 inch diameter.

B 1.1.4 Flush Rivets (Cont'd)

Table B 1.1.4.1 Ultimate and Yield Strengths of Solid 100° Machine-Countersunk Rivets

	Strength, lb								
Rivet material	2117-T3				2017-T3			2024-T31	
Clad sheet material	2024-T3, 2024-T4, 2024-T6, 2024-T81, 2024-T86, and 7075-T6								
Rivet diameter, (in.)	3/32	1/8	5/32	3/16	5/32	3/16	1/4	3/16	1/4
Ultimate strength									
Sheet thickness (in.):[ab]									
0.020	132	163							
0.025	156	221	[c]250						
0.032	178	272	[c]348	......	......	......	......	[c]324	
0.040	193	309	[c]418	[c]525	[c]476	......	......	[c]555	
0.050	206	340	[c]479	[c]628	[c]580	[c]726	......	758	[c]975
0.063	216	363	523	705	[c]657	[c]859	[c]1,200	886	[c]1,290
0.071	......	373	542	739	690	[c]917	[c]1,338	942	[c]1,424
0.080	......	......	560	769	720	[c]969	[c]1,452	992	[c]1,543
0.090	......	......	575	795	746	1,015	[c]1,552	1,035	[c]1,647
0.100	......	......	......	818	......	1,054	[c]1,640	1,073	[c]1,738
0.125	......	......	......	853	......	1,090	1,773	1,131	1,877
0.160	......	......	......	......	......	......	1,891	......	2,000
0.190	......	......	......	......	......	......	1,970	......	2,084
Shear	217	388	596	862	755	1,090	1,970	1,180	2,120

B 1.1.4 Flush Rivets (Cont'd)

Table B 1.1.4.1 Ultimate and Yield Strengths of Solid 100° Machine-Countersunk Rivets (Cont'd)

	Strength, lb								
Rivet material	2117-T3				2017-T3			2024-T31	
Clad sheet material	2024-T3, 2024-T4, 2024-T6, 2024-T81, 2024-T86, and 7075-T6								
Rivet diameter, (in.):[a][b]	3/32	1/8	5/32	3/16	5/32	3/16	1/4	3/16	1/4
	Yield strength								
0.020	91	98							
0.025	113	150	110						
0.032	132	198	200					204	
0.040	153	231	265	273	270			362	
0.050	188	261	321	389	345	419		538	594
0.063	213	321	402	471	401	515	610	614	811
0.071		348	453	538	481	557	706	669	902
0.080			498	616	562	623	788	761	982
0.090			537	685	633	746	861	842	1,053
0.100				745		854	1,017	913	1,115
0.125				836		1,018	1,313	1,021	1,357
0.160							1,574		1,694
0.190							1,753		1,925

Note: The values in this table are based on "good" manufacturing practice, and any deviation from this will produce significantly reduced values.

[a] Sheet gage is that of the countersunk sheet. In cases where the lower sheet is thinner than the upper, the shear-bearing allowable for the lower sheet-rivet combination should be computed.

[b] Increased attention should be paid to detail design in cases where D/t >4.0 because of possibly greater incidence of difficulty in service.

[c] Yield values of the sheet-rivet combinations are less than 2/3 of the indicated ultimate values.

B 1.1.4 Flush Rivets (Cont'd)

Table B 1.1.4.2 Ultimate Strength of Solid 100° Dimpled Rivets

Rivet Material	Ultimate strength, lb																
	2117-T3							2017-T3						2024-T31			
Clad sheet material	2024-T3, 2024-T4, 2024-T6, and 2024-T81		2024-T86	2024-T3, and 2024-T4		2024-T6, 2024-T81, 2024-T86, 7075-T6		2024-T3, 2024-T4, 2024-T6, and 2024-T81		2024-T86 and 7075-T6		2024-T3 and 2024-T4	2024-T6 2024-T81, 2024-T86, and 7075-T6	2024-T3, and 2024-T4		2024-T6, 2024-T81, 2024-T86, and 7075-T6	
Rivet diameter (in.)	3/32	1/8	1/8	5/32	3/16	5/32	3/16	5/32	3/16	5/32	3/16	1/4	1/4	3/16	1/4	3/16	1/4
Sheet thickness, (in.):[a]																	
0.016	177																
0.020	209	299	302														
0.025	235	360	383	474	......	462	......	419	......	530							
0.032	257	413	454	568	722	599	725	600	681	672	822	......	......	744	......	786	
0.040	273	451	505	635	839	695	891	728	905	775	1,000	845	1,108	941	879	982	1,300
0.050	......	484	548	693	940	778	1,036	840	1,097	864	1,153	1,332	1,508	1,110	1,359	1,152	1,705
0.063	......	......	......	736	1,012	840	1,142	922	1,240	930	1,267	1,695	1,803	1,236	1,727	1,277	2,010
0.071	......	......	......	755	1,045	867	1,190	958	1,301	957	1,315	1,853	1,930	1,291	1,883	1,332	2,150
0.081	......	......	......	......	1,074	......	1,230	......	1,357	......	1,358	1,995	2,044	1,340	2,025	1,380	2,260
0.090	......	......	......	......	1,098	......	1,267	......	1,405	......	1,398	2,115	2,145	1,382	2,150	1,424	2,365
0.100	......	......	......	......	......	......	......	......	......	......	......	2,220	2,232	......	2,255	......	2,455

Note: The values in this table are based on "good" manufacturing practice and any deviation from this will produce significantly reduced values.

[a]The values apply to double dimpled sheets and to the upper sheet dimpled into a machine-countersunk lower sheet. Sheet gage is that of the thinnest sheet for double dimpled joints and of the upper dimple sheet for dimpled, machine-countersunk joints. The thickness of the machine-countersunk sheet must be at least 1 tabulated gage thicker than the upper sheet. In no case shall allowables be obtained by extrapolation for skin gages other than those shown.

B 1.1.4 Flush Rivets (Cont'd)

Table B 1.1.4.3 Yield Strength of Solid 100° Dimpled Rivets

Rivet material	Yield strength, lb																	
	2117-T3						2017-T3						2024-T31					
Clad sheet material	2024-T3, 2024-T4, 2024-T6, and 2024-T81	2024-T3, 2024-T4, 2024-T6, 2024-T81, and 2024-T86	2024-T3, 2024-T4, 2024-T6, and 2024-T81		2024-T86 and 7075-T6		2024-T3, 2024-T4, 2024-T6, and 2024-T81			2024-T86 and 7075-T6			2024-T3, and 2024-T4		2024-T6 and 2024-T81		2024-T86 and 7075-T6	
Rivet diameter (in.)	3/32	1/8	5/32	3/16	5/32	3/16	5/32	3/16	1/4	5/32	3/16	1/4	3/16	1/4	3/16	1/4	3/16	1/4
Sheet thickness, in.:[a]																		
0.016...	154																	
0.020...	184	257																
0.025...	209	315	324	...	410	...	336	...	...	450								
0.032...	231	367	430	512	525	640	483	546	...	581	705	...	582	...	649	...	786	
0.040...	246	404	506	644	606	782	589	730	845	675	867	978	666	879	816	962	982	978
0.050...	...	436	571	757	677	905	681	888	1,187	756	1,007	1,508	738	1,308	961	1,308	1,152	1,543
0.063...	...	...	619	841	729	995	748	1,006	1,415	816	1,111	1,803	925	1,564	1,068	1,564	1,277	1,958
0.071...	...	...	641	878	752	1,034	778	1,056	1,656	842	1,156	1,930	1,045	1,711	1,115	1,711	1,332	2,140
0.080...	...	...	...	910	...	1,070	...	1,102	1,870	...	1,196	2,044	1,152	1,928	1,177	1,928	1,380	2,260
0.090...	...	...	...	939	...	1,100	...	1,142	2,057	...	1,231	2,145	1,246	2,121	1,324	2,121	1,424	2,365
0.100...	...	...	...	...	...	...	...	...	2,220	...	...	2,232	...	2,255	...	2,268	...	2,455

Note: The values in this table are based on "good" manufacturing practice and any deviation from this will produce significantly reduced values.

[a] These values apply to double dimpled sheets and to the upper sheet dimpled into a machine-countersunk lower sheet. Sheet gage is that of the thinnest sheet for double dimpled joints and of the upper dimple sheet for dimpled, machine-countersunk joints. The thickness of the machine-countersunk sheet must be at least 1 tabulated gage thicker than the upper sheet. In no case shall values be obtained by extrapolation for skin gages other than those shown.

B 1.1.5 Flush Screws

Table B 1.1.5.1 contains ultimate and yield allowable strength values for 100° flush-head screws with recessed heads installed in machine-countersunk clad 2024 and 7075 sheet. These strength values are applicable when the edge distance is equal to or greater than two times the nominal screw diameter. Other strength values and edge distances may be used if substantiated by tests.

These strength values may be used for the design of dimpled joints. Higher values may be used for dimpled joints if based on test results.

The allowable ultimate loads were established from test data using the average failing load divided by a factor of 1.15. The yield loads were established from test data, wherein the yield load was defined as the average test load at which the following permanent set across the joint is developed:

(1) 0.012 inch, up to and including 1/4 inch diameter screws.

(2) 4.0 percent of the screw diameter for screw sizes larger than 1/4 inch diameter.

The test specimens used were made up of two equal-gage sheets lap jointed and machine countersunk with washers to build up thickness to minimum grip. All joints had 2D nominal edge distance in the direction of the load and were either of the three-screws-across or the two-screws-in-tandem type. For the latter type, the flush heads were placed on opposite sides of the joint to assure 2D edge distances.

B 1.1.5 Flush Screws (Cont'd)

Table B 1.1.5.1 Ultimate and Yield Strengths of 100° Machine-Countersunk Screw Joints[ab]

	Strength, lb									
Type fastener	AN 509 steel screw with MS20365 steel nut									
Sheet material	Clad 2024-T3					Clad 7075-T6				
Screw diameter (in.)	3/16	1/4	5/16	3/8	1/2	3/16	1/4	5/16	3/8	1/2
	Ultimate strength									
Sheet thickness, in.[c]										
0.032	493					569				
0.040	657	761				791	905			
0.050	903	1,074	1,244			[d]1,080	1,277	1,454		
0.063	[d]1,211	1,439	1,690	1,887		[d]1,365	1,748	1,995	2,211	
0.071	[d]1,392	[d]1,693	1,955	2,235		[d]1,501	[d]2,006	[d]2,386	2,608	
0.080	[d]1,567	[d]1,965	[d]2,288	2,600		[d]1,632	[d]2,252	[d]2,777	[d]3,105	
0.090	[d]1,726	[d]2,263	[d]2,679	3,022	3,690	[d]1,762	[d]2,488	[d]3,162	[d]3,693	4.263
0.100	[d]1,877	[d]2,576	[d]3,105	[d]3,519	4,292	[d]1,892	[d]2,723	[d]3,536	[d]4,222	5,100
0.125	[d]2,126	[d]3,054	[d]3,922	[d]4,579	5,586	2,126	[d]3,109	[d]4,180	[d]5,216	6,791
0.160	2,126	[d]3,536	[d]4,772	[d]5,878	7,482	2,126	[d]3,551	[d]4,858	[d]6,193	8,673
0.190		3,680	[d]5,405	[d]6,872	[d]9,408		3,680	[d]5,433	[d]6,996	10,202
0.250			[d]5,750	[d]8,280	[d]12,201			5,750	[d]8,280	[d]12,421
0.312				[d]8,280	[d]14,141				8,280	14,185
0.375					14,700					14,700

B 1.1.5 Flush Screws (Cont'd)

Table B 1.1.5.1 Ultimate and Yield Strengths of 100° Machine-Countersunk Screw Joints[ab] (Cont'd)

Type fastener	Strength, lb									
	AN 509 steel screw with MS20365 steel nut									
Sheet material	Clad 2024-T3					Clad 7075-T6				
Screw diameter (in.)	3/16	1/4	5/16	3/8	1/2	3/16	1/4	5/16	3/8	1/2
	Yield strength									
0.032	436	...	...	...	...	559				
0.040	508	732	...	...	...	616	931			
0.050	617	854	1,035	...	...	710	1,041	1,156		
0.063	744	1,012	1,248	1,531	...	819	1,181	1,374	1,722	
0.071	818	1,122	1,380	1,697	...	884	1,269	1,495	1,887	
0.080	903	1,232	1,512	1,871	...	965	1,369	1,610	2,045	
0.090	989	1,354	1,633	2,070	3,395	1,063	1,479	1,731	2,219	3,925
0.100	1,084	1,490	1,765	2,244	3,719	1,179	1,600	1,857	2,401	4,292
0.125	1,296	1,748	2,001	2,559	4,336	1,462	1,895	2,098	2,699	5,145
0.160	1,615	2,116	2,334	2,939	5,189	1,913	2,363	2,501	3,088	6,085
0.190	...	2,484	2,702	3,361	6,012	...	2,926	3,018	3,601	6,835
0.250	...	...	3,404	4,197	7,306	...	...	4,312	4,868	8,041
0.312	...	...	...	5,092	8,452	...	...	...	6,624	9,437
0.375	...	...	...	...	9,996	...	...	...	...	11,686

NOTE: The values in this table are based on "good" manufacturing practice, and any deviation from this will produce significantly reduced values.

[a] This table refers to recessed-head screws only. Values for sheet thicknesses above or below the range for which values are indicated shall not be determined by extrapolation.

[b] Values for alloys in other physical conditions, for joint configurations other than that indicated in section B 1.1.5 or for section thicknesses outside the range for which values are indicated shall be substantiated by test data.

[c] Sheet thickness of countersunk sheet.

[d] The yield values of the sheet screw combinations are less than 2/3 of the indicated ultimate values.

B 1.1.6 Blind Rivets

Tables B 1.1.6.1 through B 1.1.6.6 contain ultimate and yield allowable single-shear strengths for protruding and flush-head blind rivets. These strengths are applicable only when the grip lengths and rivet-hole tolerances are as recommended by the respective manufacturers, and may be substantially reduced if oversize holes or improper grip lengths are used.

The strength values were established from test data obtained from tests of specimens having values of e/D equal to or greater than 2.0. Where e/D values less than 2.0 are used, tests to substantiate yield and ultimate strengths must be made. Ultimate strength values of protruding and flush blind rivets were obtained from the average failing load of test specimens divided by 1.15. Yield strength values were obtained from average yield load test data wherein the yield load is defined as the load at which the following permanent set across the joint is developed:

(1) 0.005 inch, up to and including 3/16 inch diameter rivets.

(2) 2.5 percent of the rivet diameter for rivet sizes larger than 3/16 inch diameter.

For tables B 1.1.6.2 and B 1.1.6.3 the ultimate rivet shear strength was based on the comparable rivet shear strength of 2117 solid rivets, as noted in table B 1.1.6.3. Test data on which the strength values of these tables were based were obtained using standard degreased clad 2024-T4 specimens.

In view of the wide variance in dimpling methods and tolerances for aluminum and magnesium alloys, no standard or uniform load allowables are recommended. Allowables for ultimate and shear strengths of blind rivets in double-dimpled or dimpled, machine-countersunk application should be established on the basis of specific tests. In the absence of such data, allowables for blind rivets in machine-countersunk sheet may be used.

Since blind rivets are primarily shear-type fasteners, they should not be used in applications where appreciable tensile loads on the rivets will exist. Reference should be made to the requirements of the applicable use of blind rivets, such as the limitations of usage on Drawing MS33522.

B 1.1.6 Blind Rivets (Cont'd)

Table B 1.1.6.1 Ultimate and Yield Strengths for Blind Monel Cherry Rivets in Corrosion-Resistant Sheet

	Strength, lb[a]											
Installation	Protruding heat				100° Doubled dimpled[b]				100° Machine countersunk[c]			
Rivet type	CR 563				CR 562							
Sheet material	18-8 (1/2 hard)											
Diameter of rivet (in.)	1/8	5/32	3/16	1/4	1/8	5/32	3/16	1/4	1/8	5/32	3/16	1/4
	Ultimate strength											
Sheet thickness, in.[d]												
0.008	150	178			252	327						
0.012	242	286	335		302	428	560					
0.020	402	530	620		415	542	705	1135				
0.025	456	621	785		482	642	808	1230				
0.032	522	712	937	1362	543	750	963	1400				
0.040	580	810	1050	1615	585	833	1110	1660	387			
0.050	635	903	1200	1845	628	910	1240	1940	480	607		
0.063	678	980	1325	2090		964	1330	2175	554	744	912	
0.071	701	1013	1385	2220		993	1375	2275	585	793	1013	
0.080	717	1050	1438	2340			1420	2360	612	882	1109	1570
0.090	735	1081	1486	2450				2440	637	910	1196	1743
0.100	747	1100	1540	2540					662	952	1270	1915
0.125	772	1147	1605	2710					697	1012	1380	2180
Rivet shear strength[e]	775	1190	1720	3110					775	1190	1720	3110

B 1.1.6 Blind Rivets (Cont'd)

Table B 1.1.6.1 Ultimate and Yield Strengths for Blind Monel Cherry Rivets in Corrosion-Resistant Sheet (Cont'd)

Installation	Strength, lb[a]											
	Protruding head				100° Double dimpled[b]				100° Machine countersunk[c]			
Rivet type	CR 563				CR 562							
Sheet material	18-8 (1/2 hard)											
Diameter of rivet (in.)	1/8	5/32	3/16	1/4	1/8	5/32	3/16	1/4	1/8	5/32	3/16	1/4
	Yield strength											
0.008	150	178	...	...	151	185						
0.012	242	286	335	...	252	291	344					
0.020	402	530	620	...	412	535	653	793				
0.025	456	621	785	...	473	637	800	1072				
0.032	522	712	937	1338	497	743	963	1395				
0.040	580	810	1050	1615	582	827	1090	1650	236			
0.050	635	903	1200	1845	620	900	1220	1930	364	393		
0.063	678	980	1325	2090	...	958	1315	2145	457	572	628	
0.071	701	1013	1385	2220		977	1360	2250	500	643	757	
0.080	717	1050	1438	2340	...	...	1395	2350	534	720	877	995
0.090	735	1081	1486	2450	...	...	...	2425	565	774	972	1242
0.100	747	1100	1540	2540	...	...	...	...	597	834	1075	1460
0.125	772	1147	1605	2710	...	...	...	...	635	917	1212	1800

[a] The strength values listed are based on the results of laboratory tests conducted under optimum conditions and should be used with caution.

[b] In dimpled installations, values shall not be obtained by extrapolation for skin gages other than those shown.

[c] In the case of machine-countersunk joints where the lower sheet is thinner than the upper, the bearing allowable for the lower sheet-rivet combination should be computed.

[d] Sheet gage is that of the thinnest sheet for protruding-head and double-dimpled installations. For machine-countersunk installations, sheet gage is that of the upper sheet.

[e] Rivet shear strength computed using nominal hole size and the following values for rivet and pin materials: Rivet - R monel, annealed - F_{su} = 55 ksi, Pin - R monel, cold worked - F_{su} = 65 ksi.

B 1.1.6 Blind Rivets (Cont'd)

Table B 1.1.6.2 Ultimate and Yield Strengths for Protruding-Head (MS-20600 and MS-20602) Aluminum-Alloy Blind Rivets[ab]

Installation	Strength, lb			
	Universal		Brazier head	
Rivet type	MS-20600 AD (2117)	and/or	MS-20602 AD (2017)	
Sheet material	For clad 2024-T4 and higher strength aluminum sheet materials			
Rivet diameter (in.)	1/8	5/32	3/16	1/4
Ultimate strength				
Sheet thickness, in.:[c]				
0.020	186			
0.025	233	286		
0.032	277	368	445	601
0.040	321	425	544	750
0.050	386	506	643	961
0.063	388	596	753	1,110
0.071	388	596	823	1,200
0.080	...	...	862	1,305
0.090	...	...	862	1,415
0.100	...	...	...	1,548
0.125	...	...	...	1,550

B 1.1.6 Blind Rivets (Cont'd)

Table B 1.1.6.2 Ultimate and Yield Strengths for Protruding-Head (MS-20600 and MS-20602) Aluminum-Alloy Blind Rivets[ab] (Cont'd)

	Strength, lb			
Installation	Universal		Brazier head	
Rivet type	MS-20600 AD (2117)	and/or	MS-20602 AD (2017)	
Sheet material	For clad 2024-T4 and higher strength aluminum sheet materials			
Rivet diameter (in.)	1/8	5/32	3/16	1/4
Yield strength				
0.020	180			
0.025	226	271		
0.032	264	356	431	572
0.040	304	406	523	720
0.050	362	475	610	925
0.063	388	560	709	1,058
0.071	388	596	771	1,135
0.080	...	...	862	1,230
0.090	...	...	862	1,330
0.100	...	...	...	1,450
0.125	...	...	...	1,550

[a]Protruding-head blind-rivet yield values are included for information purposes.

[b]Values for alloys in other physical conditions for joint configurations other than that indicated in section B 1.1.6 or for section thickness outside the range for which values are indicated shall be substantiated by test data.

[c]Thickness of thinnest sheet.

B 1.1.6 Blind Rivets (Cont'd)

Table B 1.1.6.3 Ultimate and Yield Strengths for 100° Countersunk-Head (MS-20601 and MS-20603) Aluminum-Alloy Blind Rivets[ab]

	Strength, lb			
Installation	100° Countersunk head			
Rivet type	MS-20601 AD (2117)		MS-20603 AD (2017)	
Sheet material	For clad 2024-T4 and higher strength aluminum sheet materials			
Rivet diameter (in.)	1/8	5/32	3/16	1/4
Ultimate strength				
Sheet thickness, in.[c]				
0.040	159			
0.050	236	258		
0.063	327	369	398	
0.071	360	439	485	
0.080	388	511	577	654
0.090	388	561	684	795
0.100	....	596	768	945
0.125	....	596	862	1,270

B 1.1.6 Blind Rivets (Cont'd)

Table B 1.1.6.3 Ultimate and Yield Strengths for 100° Countersunk-Head (MS-20601 and M -20603) Aluminum-Alloy Blind Rivets[ab] (Cont'd)

	Strength, lb			
Installation	100° Countersunk head			
Rivet type	MS-20601 AD (2117)		MS-20603 AD (2017)	
Sheet material	For clad 2024-T4 and higher strength aluminum sheet materials			
Rivet diameter (in.)	1/8	5/32	3/16	1/4
Yield strength				
0.040	[d]110			
0.050	198	[d]185		
0.063	300	308	[d]296	
0.071	336	384	391	
0.080	377	468	497	[d]456
0.090	338	524	614	621
0.100	....	592	709	793
0.125	....	596	862	1,150

a Values for sheet thicknesses above or below the range for which values are indicated shall not be determined by extrapolation.

b Values for alloys in other physical conditions, for joint configurations other than that indicated in section B 1.1.6 or for section thicknesses outside the range for which values are indicated, shall be substantiated by test data.

c Sheet thicknesses of countersunk sheet.

d These yield values of the sheet-rivet combinations are less than 77 percent (i.e., $\frac{\text{Average yield}}{1.5} \times 1.15$) of the indicated ultimate values.

The remaining countersunk-head blind-rivet values are included for information purposes.

B 1.1.6 <u>Blind Rivets (Cont'd)</u>

Table B 1.1.6.4 Ultimate and Yield Strengths for Blind 5056 Aluminum Rivets in Magnesium Sheet

Installation	Strength, lb													
	Protruding head							Machine countersunk						
Rivet type	MS-20600				MS-20602			MS-20601				MS-20603		
Sheet material	AZ31A-0													
Rivet diameter (in.)	1/8	5/32	3/16	1/4	1/8	5/32	3/16	1/8	5/32	3/16	1/4	1/8	5/32	3/16
	Ultimate strength													
Sheet thickness, in.[a]														
0.020	134													
0.025	165	210	...	...	161	186								
0.032	210	260	318	435	220	261	334							
0.040	268	324	391	535	272	354	391	197						
0.050	311	410	494	667	309	440	490	248	310	...	...	290	465	
0.063	363	481	607	837	322	497	633	307	385	470	...	336	503	652
0.071	...	524	665	930	...	521	670	344	427	523	...	...	545	750
0.080	...	556	720	1040	...	...	720	385	482	584	796	...	...	756
0.090	...	...	785	1140	...	...	725	417	537	654	895			
0.100	...	...	802	1240	...	...	...	...	596	723	992			
0.125	...	...	...	1440	...	...	...	...	637	895	1130			

B 1.1.6 Blind Rivets (Cont'd)

Table B 1.1.6.4 Ultimate and Yield Strength for Blind 5056 Aluminum Rivets in Magnesium Sheet (Cont'd)

	Strength, lb													
Installation	Protruding head							Machine countersunk						
Rivet type	MS-20600				MS-20602			MS-20601				MS-20603		
Sheet material	AZ31A-0													
Rivet diameter (in.)	1/8	5/32	3/16	1/4	1/8	5/32	3/16	1/8	5/32	3/16	1/4	1/8	5/32	3/16
	Yield strength													
0.020	100													
0.025	120	157	......	......	116	162								
0.032	150	192	238	336	137	188	[b]237							
0.040	183	233	286	396	161	218	278	[b]76						
0.050	213	290	354	495	194	260	325	[b]117	[b]124	......	......	[b]148		
0.063	241	326	431	600	226	312	382	[b]161	[b]184	[b]197	......	[b]170	[b]236	
0.071	[b]259	350	456	677	229	342	416	[b]197	[b]217	[b]240	......	[b]185	[b]254	[b]320
0.080	[b]259	[b]375	485	746	235	362	457	[b]228	[b]263	[b]288	[b]317	[b]200	[b]275	[b]345
0.090	300	[b]402	519	800	240	372	502	[b]266	[b]306	[b]343	[b]390	[b]216	[b]296	[b]370
0.100	300	432	[b]559	852	[b]248	381	511	298	[b]356	[b]405	[b]470	235	[b]321	[b]400
0.125		460	642	950	[b]263	399	533	340	448	[b]540	[b]640	275	[b]372	[b]457

[a] Sheet gage is that of the thinnest sheet for protruding-head applications, and that of the upper sheet for machine countersunk applications. In the case of machine countersunk joints where the lower sheet is thinnest, bearing allowable for the lower sheet-rivet combinations should be computed.

[b] Yield values of the sheet-rivet combinations are less than 2/3 of the indicated ultimate values.

B 1.1.6 Blind Rivets (Cont'd)

Table B 1.1.6.5 Ultimate and Yield Strengths for Protruding and Flush Head Blind A-286 Rivets

	Strength, lbs													
Installation	Protruding head				Flush head				Protruding head			Flush head		
Manufacturer	Cherry								Dupont					
Rivet type	CR-6636, soft stem				CR-6626, soft stem									
Sheet material	4130 Steel 41-43Rc								A-286 Age hardened					
Temperature	RT													
Rivet diameter (in.)	1/8	5/32	3/16	1/4	1/8	5/32	3/16	1/4	1/8	5/32	3/16	1/8	5/32	3/16
	Ultimate strength													
Sheet thickness:														
0.020	530								440	617	791			
0.025	675	825							512	712	895			
0.032	800	1060	1285						562	829	1040			
0.040	890	1220	1570	2125	475					910	1203	435		
0.050	960	1360	1820	2700	640	760					1265	477	703	
0.063	970	1465	2000	3160	860	1015	1120					529	766	999
0.071		1490	2080	3340	960	1165	1330	1650				560	808	1049
0.080			2150	3510	975	1320	1540	1950				562	853	1103
0.090				3650		1490	1750	2240					904	1157
0.100				3800			2010	2540					910	1224
0.125				3900			2160	3270						1265
0.156								3900						

B 1.1.6 Blind Rivets (Cont'd)

Table B 1.1.6.5 Ultimate and Yield Strengths for Protruding and Flush Head Blind A-286 Rivets (Cont'd)

	Strength, lbs													
Installation	Protruding head				Flush head				Protruding head			Flush head		
Manufacturer	Cherry								Dupont					
Rivet type	CR-6636, soft stem				Cr-6626, soft stem									
Sheet material	4130 Steel 41-43Rc								A-286 Age hardened					
Temperature	RT													
Rivet diameter (in.)	1/8	5/32	3/16	1/4	1/8	5/32	3/16	1/4	1/8	5/32	3/16	1/8	5/32	3/16
	Yield strength													
0.020									[a]375	[a]537	[a]704			
0.025									[a]430	[a]606	[a]775			
0.032									[a]474	[a]696	[a]881			
0.040					[b]275					[a]767	[a]1005	308		
0.050					430	[b]445					[a]1067	358	499	
0.063					595	[b]670	[b]660					417	575	722
0.071					700	800	[b]860	[b]925				451	620	775
0.080					825	950	1040	[b]1120				474	672	837
0.090					950	1120	1240	[b]1410					727	899
0.100						1300	1460	[b]1680					767	976
0.125						1460	1935	2310						1667
0.156								3140						

[a]Yield strength is 80% ultimate or higher.

[b]Yield values of the sheet-rivet combinations are less than 2/3 of the indicated ultimate values.

B 1.1.6 Blind Rivets (Cont'd)

Table B 1.1.6.6 Explosive Rivets, DuPont Extended Cavity

Ultimate Rivet Load, Lb/Rivet							
Rivet Size	Sheet Gauge						
	.025	.032	.040	.051	.064	.072	.081
5/32	320	410	513	610	610	610	--
3/16	--	495	620	796	880	880	880

B 1.1.7 Hollow-End Rivets

If hollow-end rivets with solid cross sections for a portion of the length (AN 450) are used, the strength of these rivets may be taken equal to the strength of solid rivets of the same material, provided that the bottom of the cavity is at least 25 percent of the rivet diameter from the plane of shear, as measured toward the hollow end, and further provided that they are used in locations where they will not be subjected to appreciable tensile stresses.

B 1.1.8 Hi-Shear Rivets

The allowable shear load for "Hi-Shear" rivets is the same as that specified for the standard aircraft bolts heat treated to 125 ksi and given in table B 1.1.3.2.

B 1.1.9 Lockbolts

Lockbolts and lockbolt stumps shall be installed in conformance with the lockbolt manufacturer's recommended practices, and shall be inspected in accordance with procedures recommended by the manufacturer or by an equivalent method. The ultimate allowable shear and tensile strengths for protruding and flush-head Huck lockbolts and lockbolt stumps are contained in table B 1.1.9.1. These strength values were established from test data and are minimum values guaranteed by the manufacturer. Shear and tensile yield strengths and ultimate and yield bearing strengths will be added when available.

For all lockbolts but the BL type (blind) under combined loading of shear and tension installed in material having a thickness large enough to make the shear cutoff strength critical for the shear loading, the following interaction equations are applicable:

Steel lockbolts - $R_t + R_s^{10} = 1.0$, 7075-T6 lockbolts - $R_t + R_s^{5} = 1.0$, where R_t and R_s, are the ratios of applied load to allowable load in tension and shear, respectively.

B 1.1.9 Lockbolts (Cont'd)

Table B 1.1.9.1 Ultimate Single-Shear and Tensile Strengths of Protruding and Flush-Head Lockbolts[a] and Lockbolt Stumps

Lockbolt diameter, in.	Heat treated alloy steel				7075-T6 Aluminum alloy	
	Single-shear strength, lb	Tensile strength, lb			Single-shear strength, lb	Tensile strength, lb
		Standard-type		Shear-type	Stumps and pull gun pins[b]	
		Stumps	Pull-gun pins	Stumps and pull gun pins		
5/32	...	...	...	...	995	850
3/16	2,620	2,210	2,210	1,105	1,330	1,375
1/4	4,650	4,080	4,080	2,040	2,280	2,535
5/16	7,300	6,500	[c]4,225	3,750	3,620	4,025
3/8	10,500	10,100	[d]5,050	5,050	5,270	6,275

[a]Lockbolts are pull-gun driven: lockbolt stumps are hammer or squeeze driven.

[b]Collars are heat-treated 6061S.

[c]Huck designation is R-1028 or R-1029.

[d]Collar material is heat-treated 6061S. All other collars are heat-treated 2024S.

B 1.1.10 Jo-Bolts

The ultimate and yield allowable shear strengths for flush-head steel and aluminum Jo-Bolts in clad aluminum-alloy sheet are given in Tables B 1.1.10.1 and B 1.1.10.2.

B 1.1.10 Jo-Bolts (Cont'd)

Table B 1.1.10.1 Allowable Ultimate and Yield Shear Strengths of Steel Jo-Bolts in Machine-Countersunk Joints in Clad 2024-T3 and Clad 7075-T6 Aluminum Alloys

	Shear strength, lb					
Type	F 200		F 260		F 312	
Sheet	Clad	Clad	Clad	Clad	Clad	Clad
Material	2024-T3	7075-T6	2024-T3	7075-T6	2024-T3	7075-T6
Ultimate strength						
Sheet thickness, in.:						
0.032	420	520				
0.040	445	545	[a]520	580		
0.050	[a]580	[a]700	[a]620	[a]700		
0.063	[a]1000	[a]1200	[a]1040	[a]1230	[a]1540	[a]1720
0.071	[a]1220	[a]1360	[a]1300	[a]1540	[a]1580	[a]1760
0.080	[a]1380	[a]1500	[a]1580	[a]1870	[a]1660	[a]1860
0.090	[a]1520	[a]1620	[a]1900	[a]2260	[a]1780	[a]2130
0.100	[a]1650	[a]1740	[a]2250	[a]2700	[a]2060	[a]2440
0.125	[a]1890	1960	[a]2940	[a]3220	[a]2720	[a]3080
0.160	2160	2200	[a]3390	[a]3570	[a]3600	[a]3940
0.190	2400	2420	[a]3730	[a]3860	[a]4490	[a]4810
0.250	2620	2620	[a]4260	4320	[a]5550	[a]6110
0.312			4650	4650	[a]6020	6500
0.375					6500	6790

B 1.1.10 Jo-Bolts (Cont'd)

Table B 1.1.10.1 Allowable Ultimate and Yield Shear Strengths of Steel Jo-Bolts in Machine-Countersunk Joints in Clad 2024-T3 and Clad 7075-T6 Aluminum Alloys (Cont'd)

	Shear strength, lb					
Type	F 200		F 260		F 312	
Sheet	Clad	Clad	Clad	Clad	Clad	Clad
Material	2024-T3	7075-T6	2024-T3	7075-T6	2024-T3	7075-T6
Yield strength						
Sheet thickness, in.:						
0.032	310	390				
0.040	320	400	320	400		
0.050	340	430	340	430		
0.063	610	770	690	790	870	950
0.071	685	850	780	870	930	1000
0.080	770	930	880	980	1000	1070
0.090	870	1025	990	1110	1090	1160
0.100	980	1130	1120	1280	1200	1280
0.125	1200	1350	1380	1600	1440	1540
0.160	1500	1640	1700	2050	1820	1980
0.190	1800	1960	2010	2470	2200	2520
0.250	2400	2550	2600	3190	2950	3710
0.312			3200	3880	3690	4830
0.375					4450	5790

[a] Yield values are less than 2/3 of the indicated ultimate values.

B 1.1.10 Jo Bolts (Cont'd)

Table B 1.1.10.2 Allowable Ultimate and Yield Shear Strengths of Aluminum Jo-Bolts in Machine-Countersunk Joints in Clad 2024-T3 and Clad 7075-T6 Aluminum Alloys

	Shear strength, lb			
Type	FA-200		FA-260	
Sheet	Clad	Clad	Clad	Clad
Material	2024-T3	7075-T6	2024-T3	7075-T6
Ultimate strength				
Sheet thickness, in.:				
0.032	390	450		
0.040	420	500	620	740
0.050	500	590	790	940
0.063	640	750	1010	1170
0.071	790	880	1150	1310
0.080	1040	1060	1310	1480
0.090	1270	1270	1480	1650
0.100	1450	1450	1680	1850
0.125	1595	1595	2010	2250
0.160	1595	1595	2300	2650
0.190			2520	2790
0.250			[a]2790	

Table B 1.1.10.2 Allowable Ultimate and Yield Shear Strengths of Aluminum Jo-Bolts in Machine-Countersunk Joints in Clad 2024-T3 and Clad 7075-T6 Aluminum Alloys (Cont'd)

	Shear strength, lb			
Type	FA-200		FA-260	
Sheet	Clad	Clad	Clad	Clad
Material	2024-T3	7075-T6	2024-T3	7075-T6
	Yield strength			
Sheet thickness, in.:				
0.032	380	390		
0.040	420	430	450	590
0.050	500	520	520	720
0.063	630	700	705	910
0.071	740	800	820	1020
0.080	860	915	940	1160
0.090	990	1040	1080	1300
0.100	1130	1180	1230	1460
0.125	1340	1420	1550	1790
0.160	1540	1590	1980	2240
0.190			2420	2700
0.250				

[a] Extrapolated value.

B 1.2.0 Welded Joints

Whenever possible, joints to be welded should be so designed that the welds will be loaded in shear.

B 1.2.1 Fusion Welding - Arc and Gas

In the design of welded joints, the strength of both the weld metal and the adjacent parent metal must be considered. The allowable strength for the adjacent parent metal is given in section B 1.2.2 and the allowable strength for the weld metal is given in section B 1.2.3. The weld-metal section will be analyzed on the basis of its loading, allowables, dimensions, and geometry.

B 1.2.2 Effect on Adjacent Parent Metal Due to Fusion Welding

For joints welded after heat treatment, the allowable stresses near the weld are given in Tables B 1.2.2.1 and B 1.2.2.2.

For materials heat treated after welding, the allowable stresses in the parent metal near a welded joint may equal the allowable stress for the material in the heat-treated condition as given in tables of design mechanical properties of the specific alloys.

Table B 1.2.2.1 Allowable Ultimate Tensile Stresses Near Fusion Welds in 4130, 4140, 4340, or 8630 Steels[a]

(Section thickness 1/4 inch or less)	
Type of joint	Ultimate tensile stress, ksi
Tapered joints of 30° or less[b]	90
All others	80

[a] Welded after heat treatment or normalized after weld.
[b] Gussets or plate inserts considered 0° taper with center line.

Table B 1.2.2.2 Allowable Bending Modulus of Rupture Near Fusion Welds in 4130, 4140, 4340, or 8630 Steels[a]

Type of joint	Bending modulus of rupture, ksi
Tapered joints of 30° or less[b]	F_b, figure B 1.2.2-1 for F_{tu} = 90 ksi
All others	0.9 of the values of F_b from figure B 1.2.2-1 for F_{tu} = 90 ksi

[a] Welded after heat treatment or normalized after weld.
[b] Gussets or plate inserts considered 0° taper with center line.

B 1.2.2 Effect on Adjacent Parent Metal Due to Fusion Welding (Cont'd)

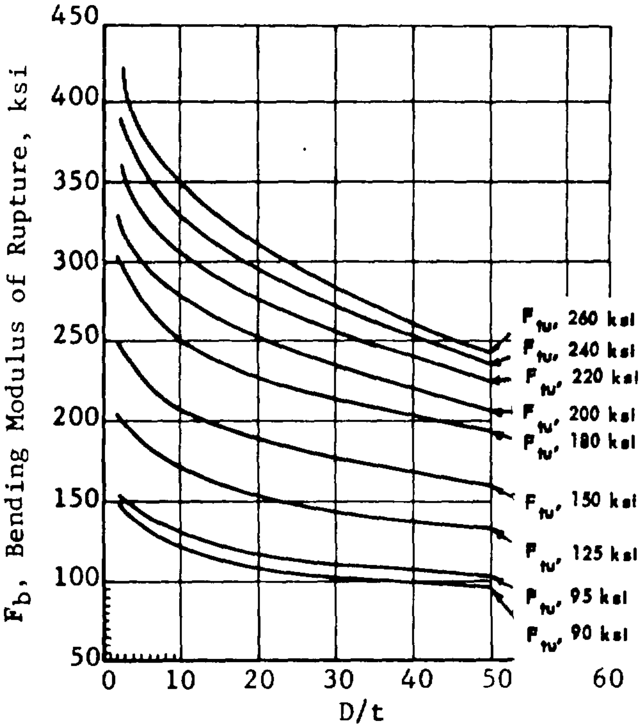


Fig. B 1.2.2-1 Bending Modulus of Rupture for Round Alloy-Steel Tubing.

B 1.2.3 Weld-Metal Allowable Strength

Allowable weld-metal strengths are shown in Table B 1.2.3.1. Design allowable stresses for the weld metal are based on 85 percent of the respective minimum tensile ultimate test values.

B 1.2.3 Weld-Metal Allowable Strength (Cont'd)

Table B 1.2.3.1 Strengths of Welded Joints

Material	Heat treatment subsequent to welding	Welding rod or electrode	F_{su}, ksi	F_{tu}, ksi
Carbon and alloy steels..	None	MIL-R-5632, class 1 MIL-E-15599, classes E-6010 and E-6013	32 32	51 51
Alloy steels	None	MIL-R-5632, class 2	43	72
Alloy steels	Stress relieved	MIL-E-6843, class 10013 MIL-E-18038, classes E-10015 and E-10016	50	85
Alloy steels	Stress relieved	MIL-E-18038, classes E-12015 and E-12016	60	100
Steels	Quench and temper ...			
4130 .... 4140 .... 4340	125 ksi .... 150 ksi .... 180 ksi	MIL-E-8697, classes HT-4130, HT-4140, and HT-4340	63 75 90	105 125 150

B 1.2.4 Welded Cluster

In a welded structure where seven or more members converge, the allowable stress shall be determined by dividing the normal allowable stress by a material factor of 1.5, unless the joint is reinforced. A tube that is continuous through a joint should be assumed as two members.

B 1.2.5 Flash Welding

The tensile ultimate allowable stresses and bending allowable modulus of rupture for flash welds are given in Tables B 1.2.5.1 and B 1.2.5.2.

Table B 1.2.5.1 Allowable Ultimate Tensile Stress for Flash Welds in Steel Tubing

Tubing	Allowable ultimate tensile stress of welds
Normalized tubing - not heat treated (including normalizing) after welding.	1.0 F_{tu} (based on F_{tu} of normalized tubing)
Heat-treated tubing welded after heat treatment.	1.0 F_{tu} (based on F_{tu} of normalized tubing)
Tubing heat treated (including normalizing) after welding. F_{tu} of unwelded material in heat-treated condition:	
$<$100 ksi	0.9 F_{tu}
100 to 150 ksi	0.6 F_{tu} + 30
$>$150 ksi	0.8 F_{tu}

B 1.2.5 Flash Welding (Cont'd)

Table B 1.2.5.2 Allowable Bending Modulus of Rupture for Flash Welds in Steel Tubing

Tubing	Allowable bending modulus of rupture of welds (F_b from Fig. B 1.2.2-1 using values of F_{tu} listed)
Normalized tubing-not heat treated (including normalizing) after welding.	1.0 F_{tu} for normalized tubing
Heat-treated tubing welded after heat treatment.	1.0 F_{tu} for normalized tubing
Tubing heat treated (including normalizing) after welding. F_{tu} of unwelded material in heat-treated condition:	
$<$100 ksi	0.9 F_{tu}
100 to 150 ksi	0.6 F_{tu} + 30
$>$150 ksi	0.8 F_{tu}

B 1.2.6 Spot Welding

Design shear strength allowables for spot welds in various alloys are given in Tables B 1.2.6.1, B 1.2.6.2, and B 1.2.6.3; the thickness ratio of the thickest sheet to the thinnest outer sheet in the combination should not exceed 4:1. Table B 1.2.6.4 gives the minimum allowable edge distance for spot welds, these values may be reduced for non-structural applications, or for applications not depended upon to develop full tabulated weld strength. Combinations of aluminum alloys suitable for spot welding are given in Table B 1.2.6.5.

B 1.2.6 Spot Welding (Cont'd)

Table B 1.2.6.1 Spot-Weld Maximum Design Shear Strengths for Uncoated Steels[a] and Nickel Alloys

Nominal thickness of thinner sheet, in.	Material ultimate tensile strength, lb		
	150 ksi and above	70 ksi to 150 ksi	Below 70 ksi
0.006	70	57	....
0.008	120	85	70
0.010	165	127	92
0.012	220	155	120
0.014	270	198	142
0.016	320	235	170
0.018	390	270	198
0.020	425	310	225
0.025	580	425	320
0.030	750	565	403
0.032	835	623	453
0.040	1,168	850	650
0.042	1,275	920	712
0.050	1,700	1,205	955
0.056	2,039	1,358	1,166
0.060	2,265	1,558	1,310
0.063	2,479	1,685	1,405
0.071	3,012	2,024	1,656
0.080	3,540	2,405	1,960
0.090	4,100	2,810	2,290
0.095	4,336	3,012	2,476
0.100	4,575	3,200	2,645
0.112	5,088	3,633	3,026
0.125	5,665	4,052	3,440

[a]Refers to plain carbon steels containing not more than 0.20 percent carbon and to austenitic steels. The reduction in strength of spot-welds due to the cumulative effects of time-temperature-stress factors is not greater than the reduction in strength of the parent metal.

B 1.2.6 Spot Welding (Cont'd)

Table B 1.2.6.2 Spot-Weld Maximum Design Shear Strength Standards for Bare and Clad Aluminum Alloys[a]

Nominal thickness of thinner sheet, in.	Material ultimate tensile strength, lb			
	Above 56 ksi	28 ksi to 56 ksi	20 ksi to 27.5 ksi	19.5 ksi and below
0.012	60	52	24	16
0.016	86	78	56	40
0.020	112	106	80	62
0.025	148	140	116	88
0.032	208	188	168	132
0.040	276	248	240	180
0.050	374	344	321	234
0.063	539	489	442	314
0.071	662	578	515	358
0.080	824	680	609	417
0.090	1,002	798	695	478
0.100	1,192	933	750	536
0.112	1,426	1,064	796	584
0.125	1,698	1,300	840	629
0.160	2,490			

[a]Spot welding of aluminum-alloy combinations conforming to QQ-A-277, QQ-A-355, and QQ-A-255 may be accomplished. The reduction in strength of spotwelds due to cumulative effects of time-temperature-stress factors is not greater than the reduction in strength of the parent metal.

B 1.2.6 Spot Welding (Cont'd)

Table B 1.2.6.3 Spot-Weld Maximum Design Shear Strength Standards for Magnesium Alloys[a]
Welding Specification MIL-W-6858

Nominal thickness of thinner sheet, in.	Design shear strength, lb
0.020	72
0.022	84
0.025	100
0.028	120
0.032	140
0.036	164
0.040	188
0.045	220
0.050	248
0.056	284
0.063	324
0.071	376
0.080	428
0.090	496
0.100	572
0.112	648
0.125	720

[a]Magnesium alloys AZ31B and HK31A may be spot welded in any combination.

B 1.2.6 Spot Welding (Cont'd)

Table B 1.2.6.4 Minimum Edge Distances for Spot-Welded Joints[ab]

Nominal thickness of thinner sheet, in.	Edge distance, E, in.
0.016	3/16
0.020	3/16
0.025	7/32
0.032	1/4
0.036	1/4
0.040	9/32
0.045	5/16
0.050	5/16
0.063	3/8
0.071	3/8
0.080	13/32
0.090	7/16
0.100	7/16
0.125	9/16
0.160	5/8

[a]Intermediate gages will conform to the requirement for the next thinner gage shown.

[b]For edge distances less than those specified above, appropriate reductions in the spot-weld allowable loads shall be made.

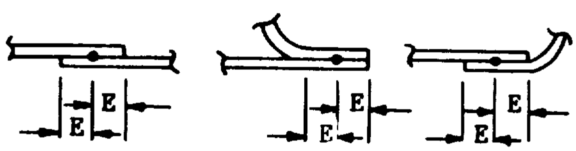


Fig. B 1.2.6-1 Edge Distances for Spot-Welded Joints.

B 1.2.6 Spot Welding (Cont'd)

Table B 1.2.6.5 Acceptable Aluminum and Aluminum-Alloy Combinations[a] for Spot Welding

Specification No.	Material	QQ-A-315 (5052)	QQ-A-318 (5052)	QQ-A-327 (6061)	QQ-A-354 (Bare 2024)	QQ-A-355 (Bare 2024)	QQ-A-359 (3003)	QQ-A-362 (Clad 2024)	QQ-A-411 (1100)	QQ-A-561 (1100)	QQ-A-287 (Clad 7075)[b]	QQ-A-277 (Bare 7075)	QQ-A-355 (Bare 2024)	QQ-A-362a (Clad 2024)	QQ-A-255 (Clad 2014)	QQ-A-261 and QQ-A-266 (Bare 2014)
QQ-A-315	(5052)															
QQ-A-318	(52S)(5052)															
QQ-A-327	(61S)(6061)															
QQ-A-354	(Bare 2024)				(*)	(*)						(*)	(*)		(*)	(*)
QQ-A-355	(Bare 2024)				(*)	(*)						(*)	(*)		(*)	(*)
QQ-A-359	(3003)															
QQ-A-362	(Clad 2024)															
QQ-A-411	(1100)															
QQ-A-561	(1100)															
QQ-A-287	(Clad 7075)[b]															
QQ-A-277	(Bare 7075)				(*)	(*)						(*)	(*)		(*)	(*)
QQ-A-355	(Bare 2024)				(*)	(*)						(*)	(*)		(*)	(*)
QQ-A-362a............	(Clad 2024)															
QQ-A-255	(Clad 2014)				(*)	(*)						(*)	(*)			(*)
QQ-A-261 and QQ-A-266	(Bare 2014)				(*)	(*)						(*)	(*)		(*)	(*)

[a]The various aluminum and aluminum-alloy materials referred to in this table may be spot welded in any combinations except the combinations indicated by the asterisk (*) in the table.
[b]Clad heat-treated and aged 7075 material in thicknesses less than 0.020 inch shall not be welded.

B 1.2.7 Reduction in Tensile Strength of Parent Metal Due to Spot Welding

In applications of spot welding where ribs, intercostals, or doublers are attached to sheet, either at splices or at other points on the sheet panels, the allowable ultimate strength of the spot-welded sheet shall be determined by multiplying the ultimate tensile sheet strength ("A" values where available) by the appropriate efficiency factor shown on Figures B 1.2.7-1 through B 1.2.7-4. The minimum values of the basic sheet efficiency in tension should not be considered applicable to cases of seam welds. Allowable ultimate tensile strengths for spot-welded sheet gages of less than 0.012 inch for steel and 0.020 inch for aluminum shall be established on the basis of tests.

B 1.2.7 Reduction in Tensile Strength of Parent Metal Due to Spot Welding (Cont'd)

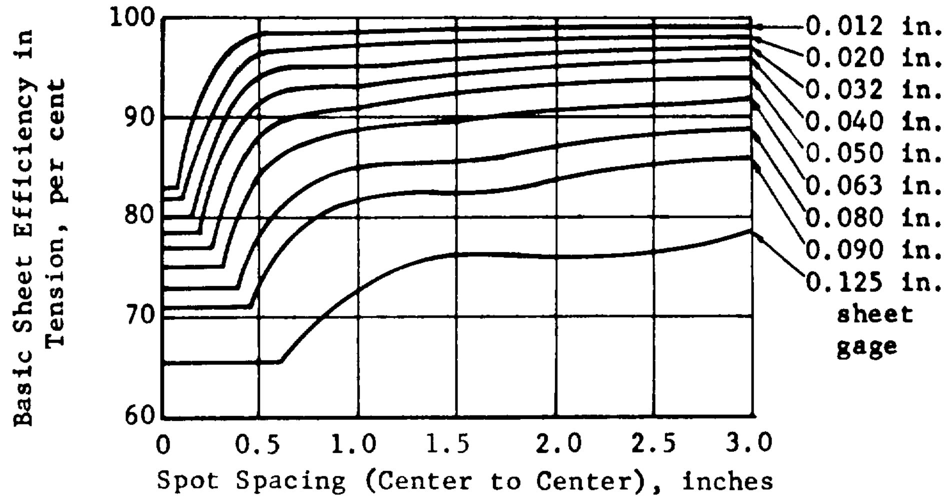


Fig. B 1.2.7-1 Efficiency of the Parent Metal in Tension for Spot-Welded 301-1/2 H Corrosion-Resistant Steel

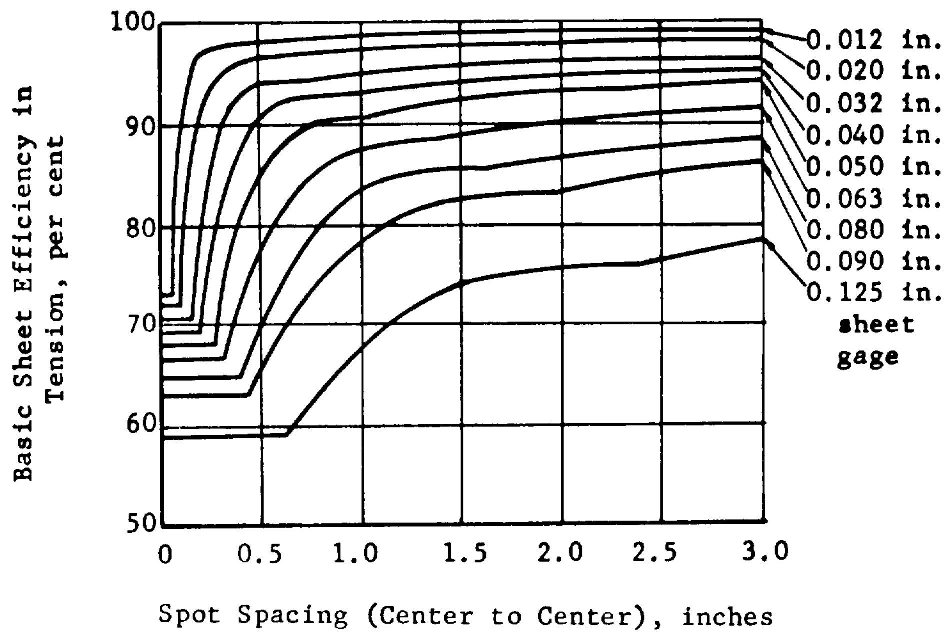


Fig. B 1.2.7-2 Efficiency of the Parent Metal in Tension for Spot-Welded 301-H Corrosion-Resistant Steel

B 1.2.7 Reduction in Tensile Strength of Parent Metal Due to Spot Welding (Cont'd)

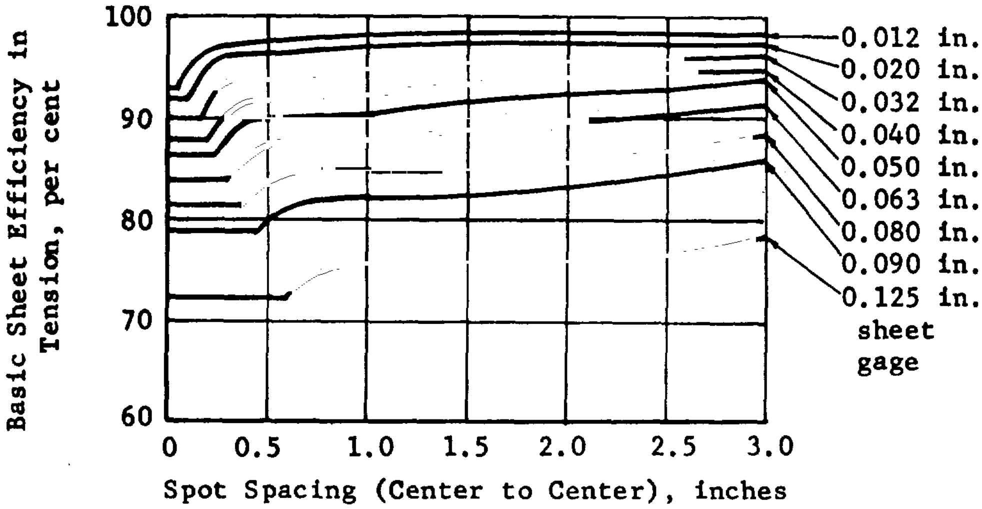


Fig. B 1.2.7-3 Efficiency of the Parent Metal in Tension for Spot-Welded 301-A, 347-A, and 301-1/4 H Corrosion-Resistant Steel

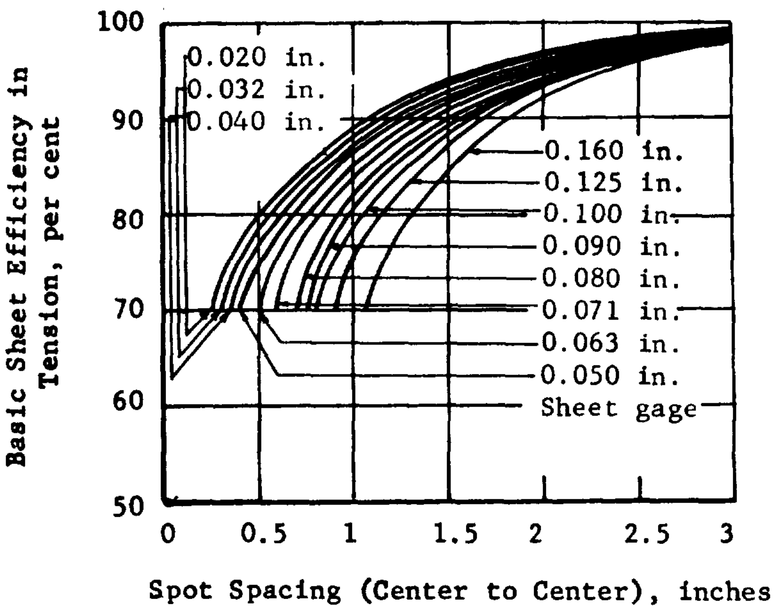


Fig. B 1.2.7-4 Efficiency of the Parent Metal in Tension for Spot-Welded Aluminum Alloys

B 1.3.0 Brazing

Insofar as discussed herein, brazing is applicable only to steel. Brazing is defined as a weld wherein coalescence is produced by heating to suitable temperatures above 800° F and by using a nonferrous filler metal having a melting point below that of the base metal. The filler metal is distributed through the joint by capillarity.

The effect of the brazing process upon the strength of the parent or base metal shall be considered in the structural design. Where copper furnace brazing or silver brazing is employed the calculated allowable strength of the base metal which is subjected to the temperatures of the brazing process shall be in accordance with the following:

Material	Allowable Strength
Heat-treated material (including normalized) used in "as-brazed" condition	Mechanical properties of normalized material
Heat-treated material (including normalized) reheat-treated during or after brazing	Mechanical properties corresponding to heat treatment performed

B 1.3.1 Copper Brazing

The allowable shear stress for design shall be 15 ksi, for all conditions of heat treatment.

B 1.3.2 Silver Brazing

The allowable shear stress for design shall be 15 ksi, provided that clearances or gaps between parts to be brazed do not exceed 0.010 inch. Silver-brazed areas should not be subjected to temperatures exceeding 900° F. Acceptable brazing alloys, with the exception of Class 3, are listed in Federal Specification QQ-S-561d.

Reference:

(1) MIL-HDBK-5, Strength of Metal Aircraft Elements, Armed Forces Supply Support Center, Washington 25, D.C., March 1959.

SECTION B2

LUGS AND SHEAR PINS

TABLE OF CONTENTS

B 2.0.0 Analysis of Lugs and Shear Pins

The method described in this section is semi-empirical and is applicable to aluminum or steel alloy lugs. The analysis considers loads in the axial, transverse and oblique directions. Each of these loads are treated in Sections B 2.1.0, B 2.2.0, and B 2.3.0 respectively. See Fig. B 2.0.0-1 for description of directions.

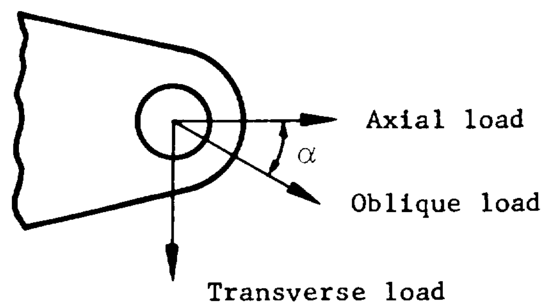


Fig. B 2.0.0-1

B 2.1.0 Analysis of Lugs with Axial Loading

A lug-pin combination under tension load can fail in any of the following ways, each of which must be investigated by the methods presented in this section:

1. Tension across the net section. Stress concentration must be considered.

2. Shear tear-out or bearing. These two are closely related and are covered by a single calculation based on empirical curves.

3. Shear of the pin. This is analyzed in the usual manner.

4. Bending of the pin. The ultimate strength of the pin is based on the modulus of rupture.

5. Excessive yielding of bushing (if used).

6. Yielding of the lug is considered to be excessive at a permanent set equal to .02 times pin diameter. This condition must always be checked as it is frequently reached at a lower load than would be anticipated from the ratio of the yield stress, F_{ty} to the ultimate stress, F_{tu} for the material.

Notes:

a. Hoop tension at tip of lug is not a critical condition, as the shear-bearing condition precludes a hoop tension failure.

b. The lug should be checked for side loads (due to mis-alignment, etc.) by conventional beam formulas (Fig. B 2.1.0-1).

Analysis procedure to obtain ultimate axial load.

1. Compute (See Fig. B 2.1.0-1 for nomenclature)

   e/D, W/D, D/t; $A_{br} = Dt$, $A_t = (W-D)t$

2. Ultimate load for shear-bearing failure:
   Note: In addition to the limitations provided by curves "A" and "B" of Fig. B 2.1.0-3, K_{br} greater than 2.00 shall not be used for lugs made from .5 inch thick or thicker aluminum alloy plate, bar or hand forged billet.

B 2.1.0 Analysis of Lugs with Axial Loading (Cont'd)

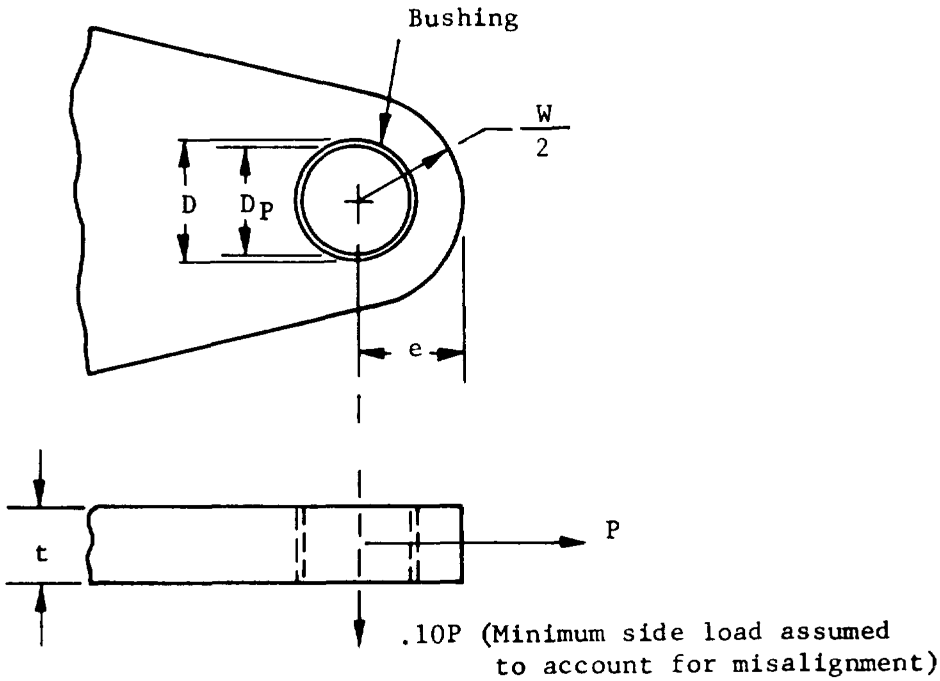


Fig. B 2.1.0-1

(a) Enter Fig. B 2.1.0-3 with e/D and D/t to obtain K_{br}

(b) The ultimate load for shear bearing failure, P'_{bru} is

$$P'_{bru} = K_{br} F_{tux} A_{br} \quad \text{.......................} \quad (1)$$

where

F_{tux} = Ultimate tensile strength of lug material in transverse direction.

3. Ultimate load for tension failure:

(a) Enter Fig. B 2.1.0-4 with W/D to obtain K_t for proper material

(b) The ultimate load for tension failure P'_{tu} is

$$P'_{tu} = K_t F_{tu} A_t \quad \text{.........................} \quad (2)$$

B 2.1.0 Analysis of Lugs with Axial Loading (Cont'd)

where

F_{tu} = ultimate tensile strength of lug material

4. Load for yielding of the lug

(a) Enter Fig. B 2.1.0-5 with e/D to obtain K_{bry}.

(b) The yield load, P'_y is

$$P'_y = K_{bry} A_{br} F_{ty} \quad (3)$$

where

F_{ty} = Tensile yield stress of the lug material.

5. Load for yielding of the bushing in bearing (if used):

$$P'_{bry} = 1.85 F_{ty} A_{brb} \quad (4)$$

where

F_{ty} = Compressive yield stress of bushing material

$A_{brb} = D_p t$ (Fig. B 2.1.0-1)

6. Pin bending stress.

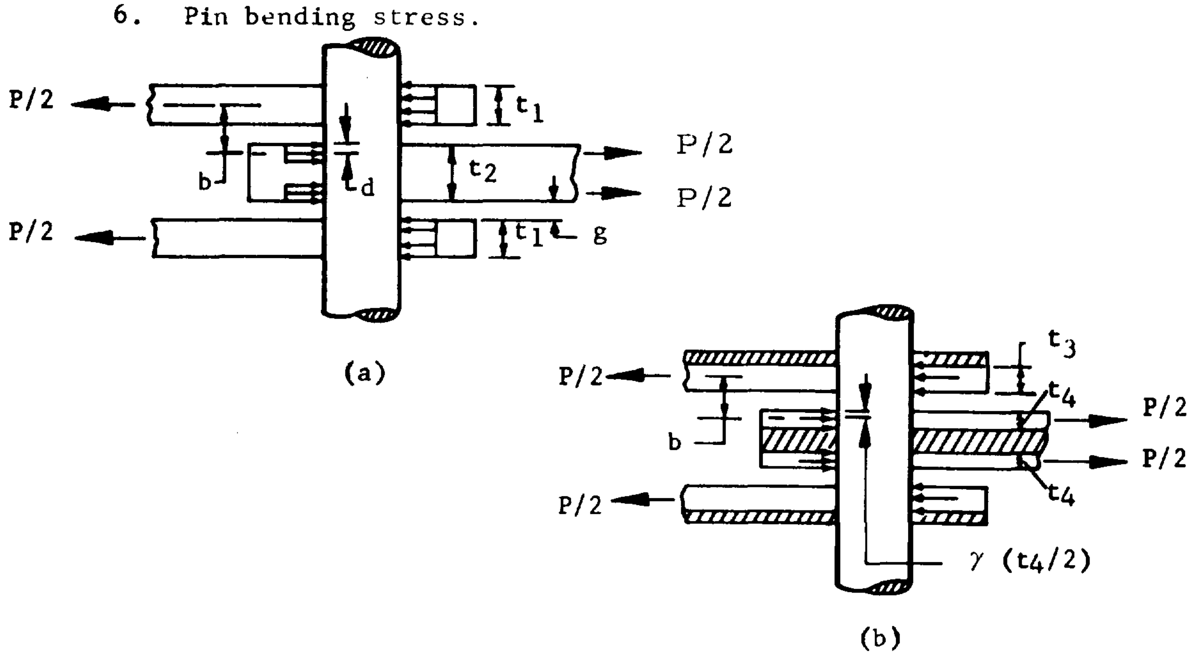


Fig. B 2.1.0-2

B 2.1.0 Analysis of Lugs with Axial Loading (Cont'd)

(a) Obtain moment arm "b" as follows:
(See Fig. B 2.1.0-2a) compute for the inner lug

$$r = \left[\left(\frac{e}{D}\right) - \frac{1}{2}\right] \frac{D}{t_2} \quad \text{.....................} \quad (5)$$

Take the smaller of P'_{bru} and P'_{tu} for the inner lug as $(P'_u)_{min}$ and compute $(P'_u)_{min}/A_{br}F_{tux}$.
Enter Fig. B 2.1.0-6 with $(P'_u)_{min}/A_{br}F_{tux}$ and "r" to obtain the reduction factor, "γ" which compensates for the "peaking" of the distributed pin bearing load near the shear plane. Calculate the moment arm "b" from

$$b = \left(\frac{t_1}{2}\right) + g + \gamma\left(\frac{t_2}{4}\right) \quad \text{.....................} \quad (6)$$

Where "g" is the gap between lugs as in Fig. B 2.1.0-2a and may be zero. Note that the peaking reduction factor applies only to the inner lugs.

(b) Calculate maximum pin bending moment M, from the equation

$$M = P\left(\frac{b}{2}\right) \quad \text{.................................} \quad (7)$$

(c) Calculate bending stress resulting from "M", assuming an My/I distribution.

(d) Obtain ultimate strength of the pin in bending by use of Section B 4.5.2. If the analysis should show inadequate pin bending strength, it may be possible to take advantage of any excess lug strength to show adequate strength for the pin by continuing the analysis as follows:

(e) Consider a portion of the lugs to be inactive as indicated by the shaded area of Fig. B 2.1.0-2b. The portion of the thickness to be considered active may have any desired value sufficient to carry the load and should be chosen by trial and error to give approximately equal Factors of Safety for the lugs and pin.

(f) Recalculate all lug Factors of Safety, with ultimate loads reduced in the ratio of active thickness to actual thickness.

(g) Recalculate pin bending moment, M = P (b/2), and Factor of Safety using a reduced value of "b" which is obtained as follows:

B 2.1.0 Analysis of Lugs with Axial Loading (Cont'd)

Compute for the inner lug, Fig. B 2.1.0-2b

$$r = \left[\left(\frac{e}{D} \right) - \frac{1}{2} \right] \frac{D}{2t_4} \quad \text{.................. (8)}$$

Take the smaller of P'_{bru} and P'_{tu} for the inner lug, based upon the active thickness, as $(P'_u)_{min}$ and compute $(P'_u)_{min}/A_{br} F_{tux}$, where $A_{br} = 2t_4 D$. Enter Fig. B 2.1.0-6 with $(P'_u)_{min}/A_{br} F_{tux}$ and "r " to obtain the reduction factor " γ " for peaking. Then the moment arm is

$$b = \frac{t_3}{2} + g + \gamma \left(\frac{t_4}{2} \right) \quad \text{.................... (9)}$$

This reduced value of "b" should not be used if the resulting eccentricity of load on the outer lugs introduce excessive bending stresses in the adjacent structure. In such cases the pin must be strong enough to distribute the load uniformly across the entire lug.

7. Factor of Safety, F.S.
Compute the following Factors of Safety:

(a) Lug

$$\text{Ultimate F.S. in shear-bearing} = \frac{P'_{bru}}{P} \quad \text{...... (10)}$$

$$\text{Ultimate F.S. in tension} = \frac{P'_{tu}}{P} \quad \text{............. (11)}$$

$$\text{Yield F.S.} = \frac{P'_y}{P} \quad \text{......................... (12)}$$

(b) Pin

$$\text{Ultimate F.S. in shear} = \frac{F_{su}}{f_s} \quad \text{............... (13)}$$

$$\text{Ultimate F.S. in bending} = \frac{F_b}{f_b} \quad \text{............... (14)}$$

(c) Bushing (if used)

$$\text{Yield F.S. in bearing} = \frac{P'_{bry}}{P} \quad \text{.............. (15)}$$

An analysis for yielding of the pin and ultimate bearing failure of the bushing is not required.

B 2.1.0 Analysis of Lugs with Axial Loading (Cont'd)

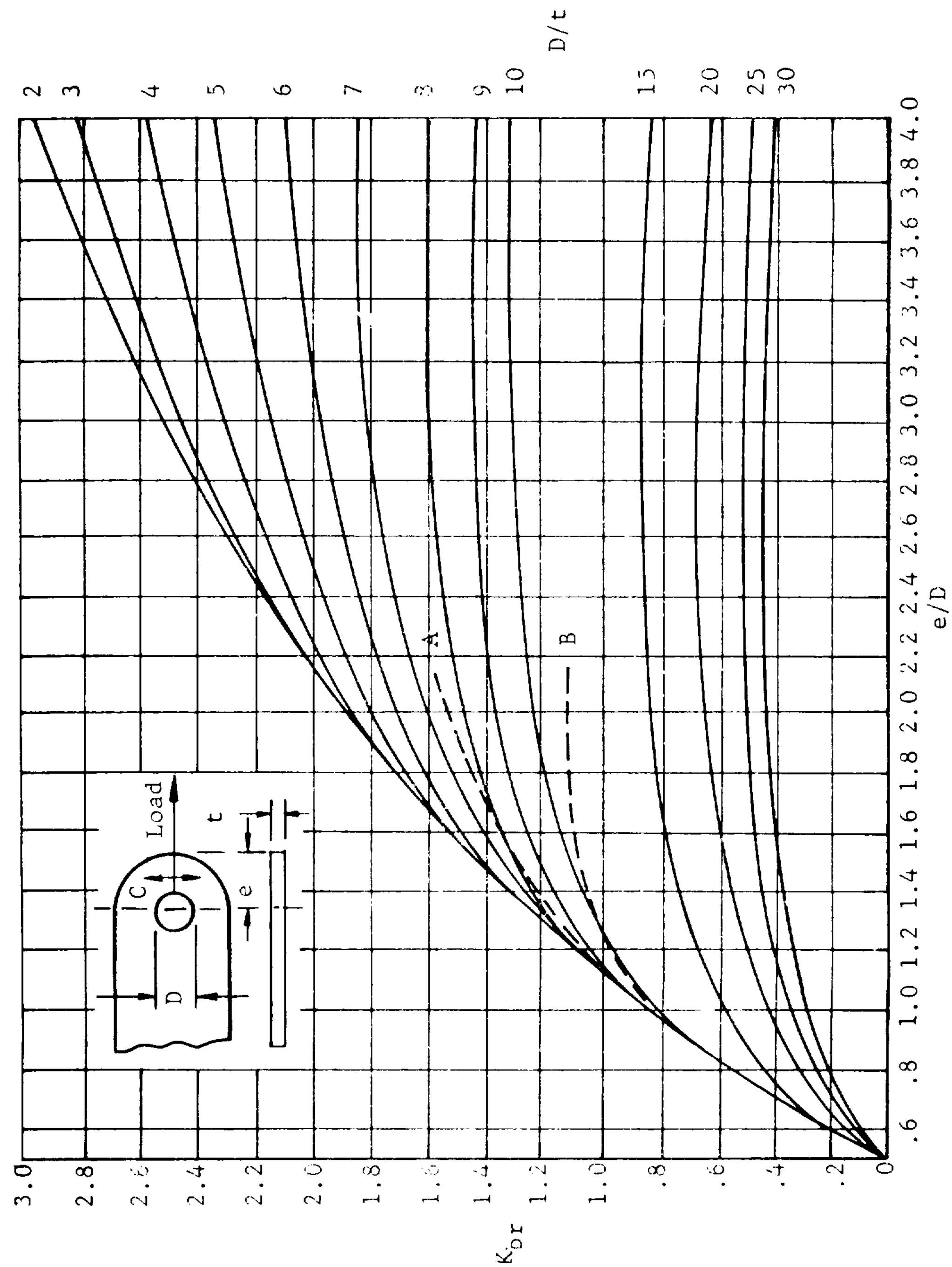


Fig. B 2.1.0-3

See notes on following page.

B 2.1.0 Analysis of Lugs with Axial Loading (Cont'd)

Curve A is a cutoff to be used for all aluminum alloy hand forged billet when the long transverse grain direction has the general direction C in the sketch.

Curve B is a cutoff to be used for all aluminum alloy plate, bar and hand forged billet when the short transverse grain direction has the general direction C in the sketch, and for die forging when the lug contains the parting plane in a direction approximately normal to the direction C.

B 2.1.0 Analysis of Lugs with Axial Loading (Cont'd)

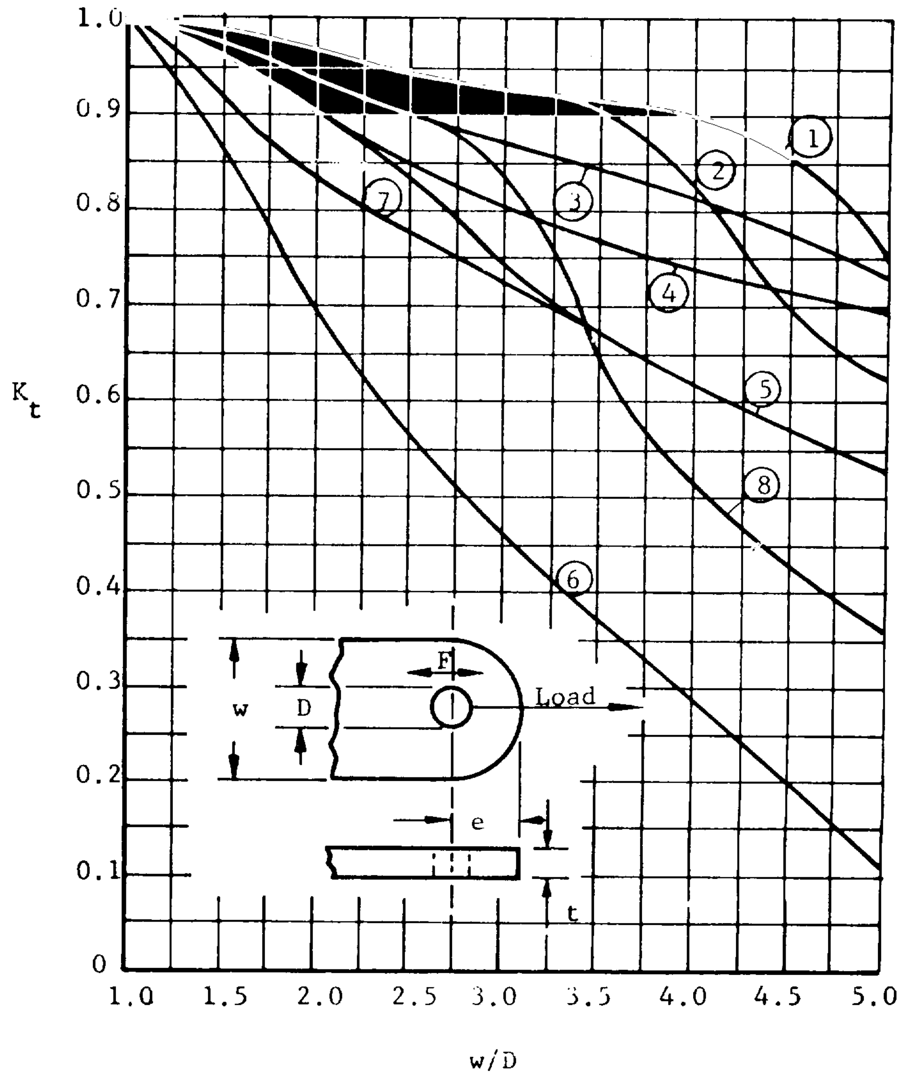


Fig. B 2.1.0-4

Legend on following pages.

B 2.1.0 Analysis of Lugs with Axial Loading (Cont'd)

Legend - Figure B 2.1.0-4 - L, T, N, indicate grain in direction F in sketch

L = longitudinal

T = long transverse

N = short transverse (normal)

Curve 1

4130, 4140, 4340 and 8630 steel
2014-T6 and 7075-T6 plate ≤ 0.5 in (L,T)
7075-T6 bar and extrusion (L)
2014-T6 hand forged billet ≤ 144 sq. in. (L)
2014-T6 and 7075-T6 die forgings (L)

Curve 2

2014-T6 and 7075-T6 plate > 0.5 in., ≤ 1 in.
7075-T6 extrusion (T,N)
7075-T6 hand forged billet ≤ 36 sq.in. (L)
2014-T6 hand forged billet > 144 sq.in. (L)
2014-T6 hand forged billet ≤ 36 sq.in. (T)
2014-T6 and 7075-T6 die forgings (T)
17-4 PH
17-7 PH-THD

Curve 3

2024-T6 plate (L,T)
2024-T4 and 2024-T42 extrusion (L,T,N)

Curve 4

2024-T4 plate (L,T)
2024-T3 plate (L,T)
2014-T6 and 7075-T6 plate > 1 in.(L,T)
2024-T4 bar (L,T)
7075-T6 hand forged billet > 36 sq.in. (L)
7075-T6 hand forged billet ≤ 16 sq.in. (T)

Curve 5

195T6, 220T4, and 356T6 aluminum alloy casting
7075-T6 hand forged billet > 16 sq.in. (T)
2014-T6 hand forged billet > 36 sq.in. (T)

B 2.1.0 Analysis of Lugs with Axial Loading (Cont'd)

Curve 6

Aluminum alloy plate, bar, hand forged billet, and die forging (N). Note: for die forgings, N direction exists only at the parting plane. 7075-T6 bar (T)

Curve 7

18-8 stainless steel, annealed

Curve 8

18-8 stainless steel, full hard, Note: for 1/4, 1/2 and 3/4 hard, interpolate between Curves 7 and 8.

B 2.1.0 Analysis of Lugs with Axial Loading (Cont'd)

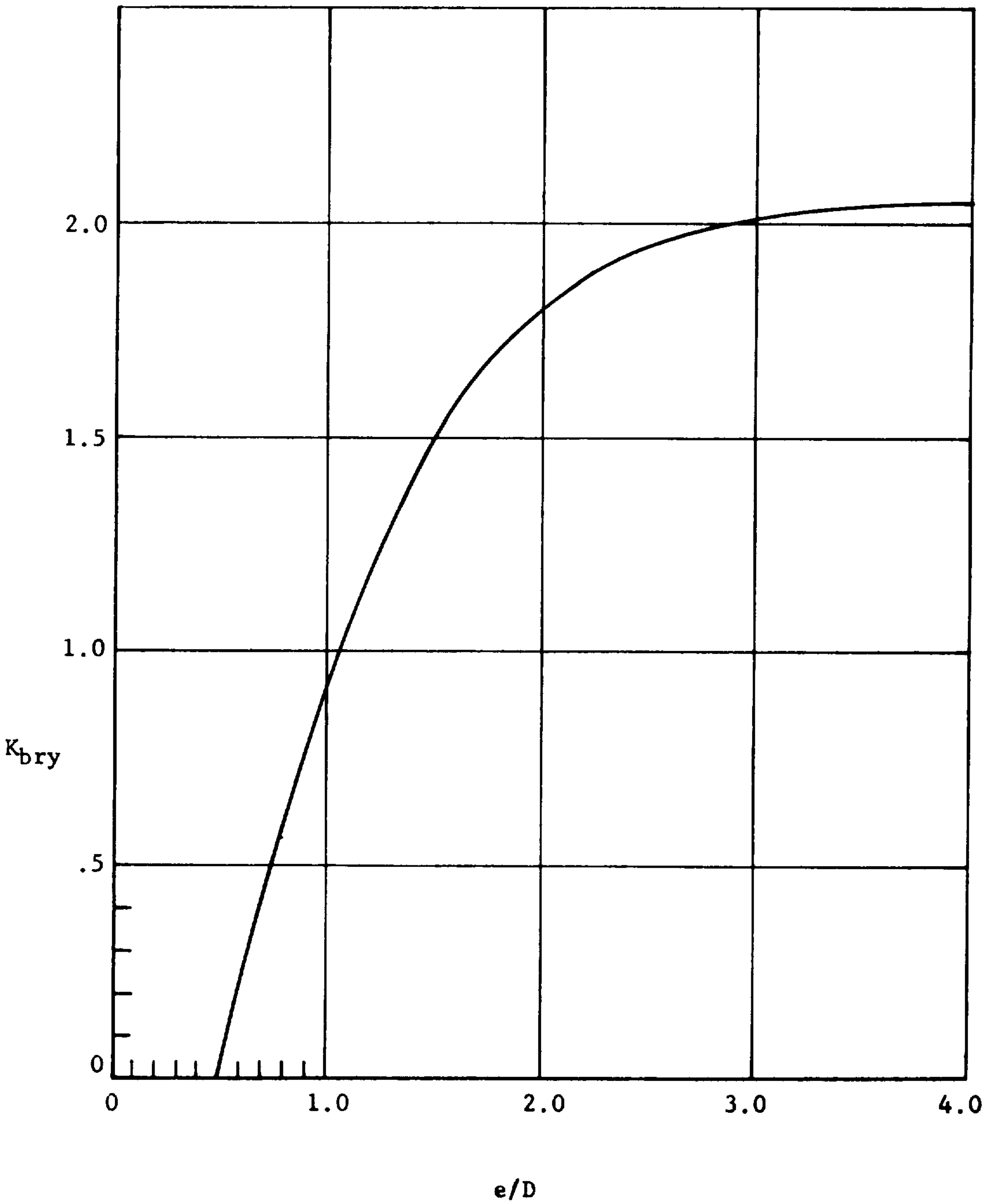


Fig. B 2.1.0-5

B 2.1.0 Analysis of Lugs with Axial Loading (Cont'd)

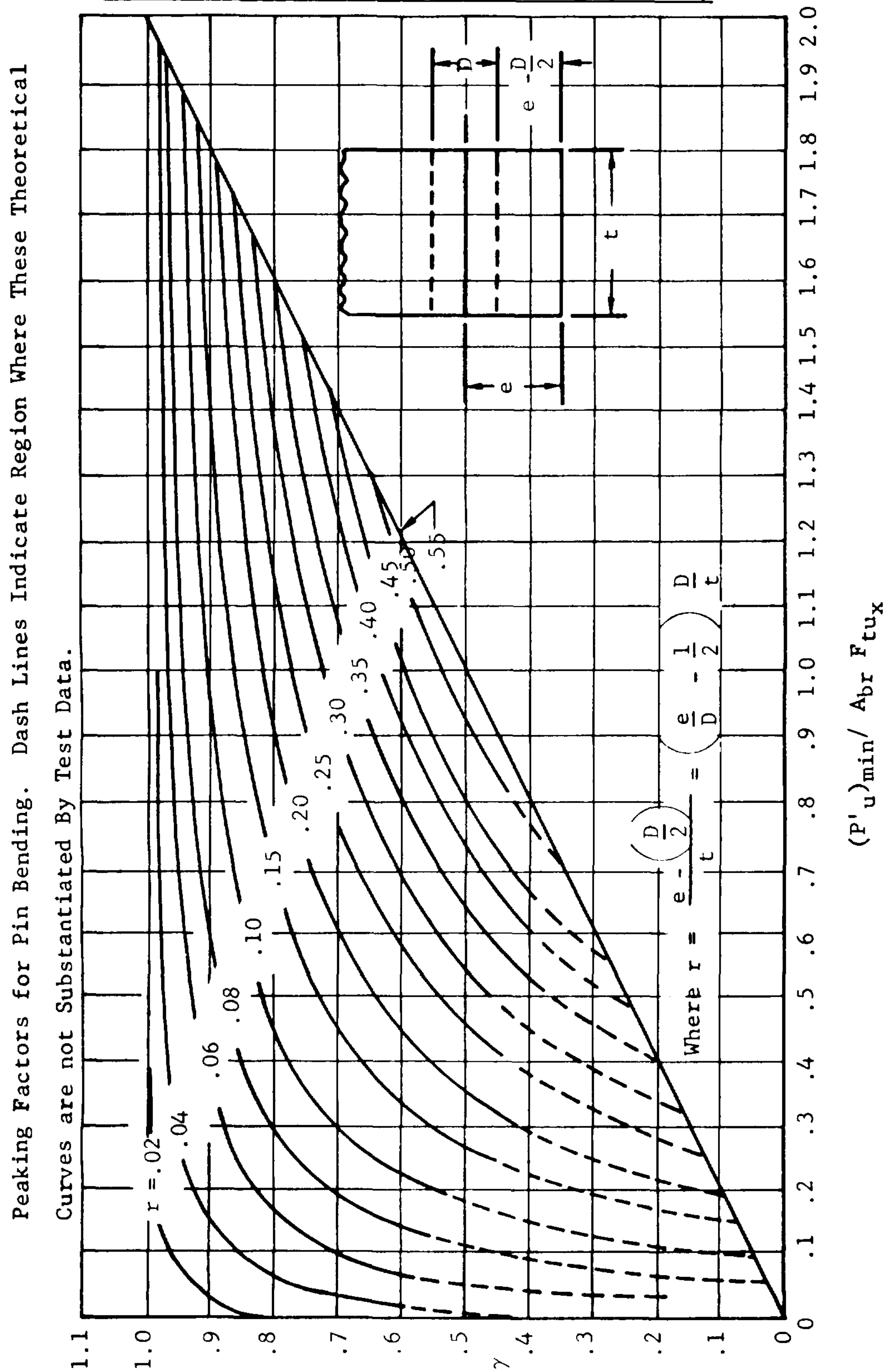


Fig. B 2.1.0-6

B 2.1.0 Analysis of Lugs with Axial Loading (Cont'd)

Special Applications

1. Irregular lug section - bearing load distributed over entire thickness.

For lugs of irregular section having bearing stress distributed over the entire thickness, an analysis is made based on an equivalent lug with rectangular sections having an area equal to the original section.

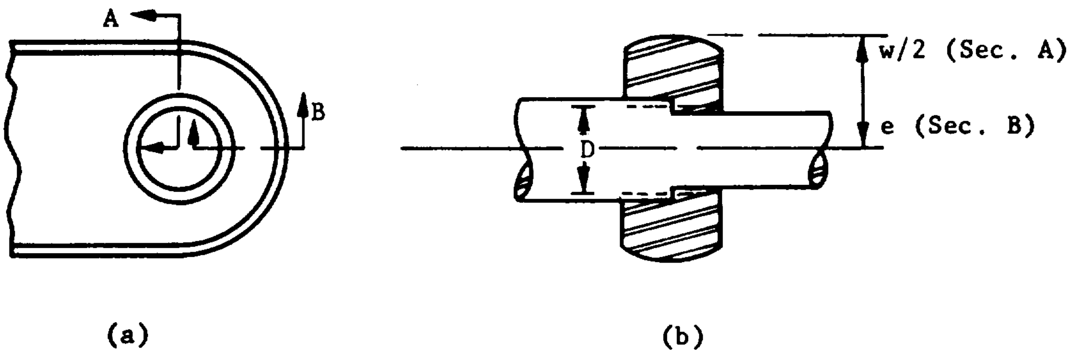


Fig. B 2.1.0-7

Dashed lines show equivalent lug

2. Critical bearing stress

NASA Design Manual Section 3.0.0 lists the values of the ultimate and yield bearing stress of materials for e/D values of 2.0 and 1.5, these are valid for values of D/t to 5.5. The ultimate and yield bearing stress for geometrical conditions outside of the above range may be determined in the following manner:

(a) Ultimate bearing stress: For the particular D/t and e/D, obtain K_{br} from Fig. B 2.1.0-3 then

$$F_{bru} = K_{br} F_{tux}$$

where

F_{tux} = Ultimate strength of lug material in transverse direction.

B 2.1.0 Analysis of Lugs with Axial Loading (Cont'd)

(b) Yield bearing stress: With the particular e/D obtain K_{bry} from Fig. B 2.1.0-5. Then

$$F_{bry} = K_{bry} F_{tyx}$$

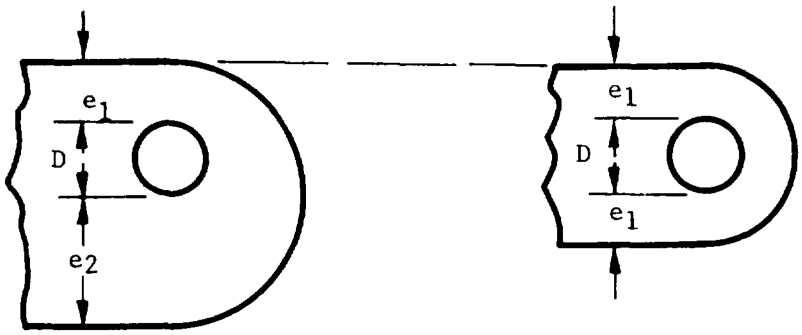
where

F_{tyx} = Tensile yield stress of lug material in transverse direction.

3. Eccentrically located hole

If the hole is located as in Fig. B 2.1.0-8 (e_1 less than e_2), the ultimate and yield lug loads are determined by obtaining P'_{bru}, P'_{tu} and P'_y for the equivalent lug shown and multiplying by the factor

$$\text{factor} = \frac{e_1 + e_2 + 2D}{2e_2 + 2D}$$



Actual lug — Equivalent lug

Fig. B 2.1.0-8

4. Multiple shear connections

Lug-pin combinations having the geometry shown in Fig. B 2.1.0-9 are analyzed according to the following criteria:

(a) The load carried by each lug is determined by distributing the total applied load "P" among the lugs as shown on Fig. B 2.1.0-9 and the value of "C" is obtained from Table B 2.1.0.1.

B 2.1.0 Analysis of Lugs with Axial Loading (Cont'd)

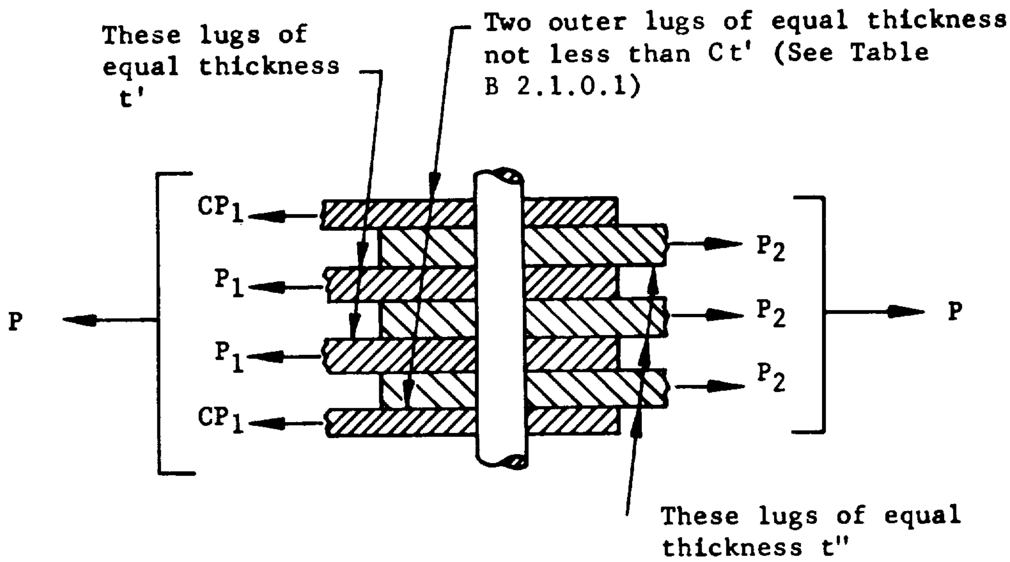


Fig. B 2.1.0-9

(b) The maximum shear load on the pin is given in Table B 2.1.0.1.

(c) The maximum bending moment in the pin is given by the formula, $M = \frac{P_1 b}{2}$ where "b" is given in Table B 2.1.0.1.

Table B 2.1.0.1

Total number of lugs including both sides	C	Pin Shear	b
5	.35	.50 P_1	$.28 \frac{t' + t''}{2}$
7	.40	.53 P_1	$.33 \frac{t' + t''}{2}$
9	.43	.54 P_1	$.37 \frac{t' + t''}{2}$
11	.44	.54 P_1	$.39 \frac{t' + t''}{2}$
∞	.50	.50 P_1	$.50 \frac{t' + t''}{2}$

B 2.2.0 Analysis of Lugs with Transverse Loading

Shape Parameter

In order to determine the ultimate and yield loads for lugs with transverse loading, the shape of the lug must be taken into account. This is accomplished by use of a shape parameter given by

$$\text{Shape parameter} = \frac{A_{av}}{A_{br}}$$

where

A_{br} is the bearing area = Dt

A_{av} is the weighted average area given by

$$A_{av} = \frac{6}{\left(\frac{3}{A_1}\right) + \left(\frac{1}{A_2}\right) + \left(\frac{1}{A_3}\right) + \left(\frac{1}{A_4}\right)}$$

A_1, A_2, A_3 and A_4 are areas of the lug sections indicated in Fig. B 2.2.0-1.

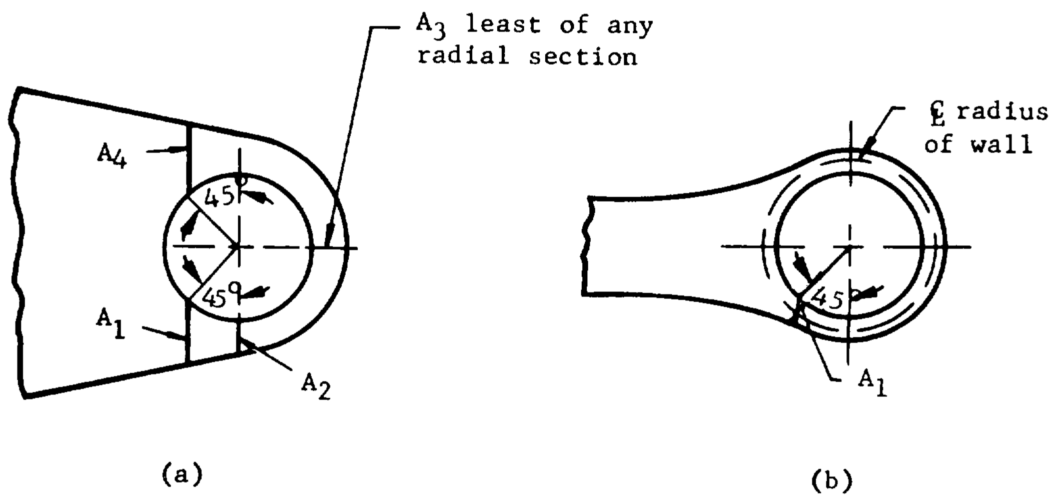


Fig. B 2.2.0-1

(1) Obtain the areas A_1, A_2, A_3, and A_4 as follows:

(1a) A_1, A_2, and A_4 are measured on the planes indicated in Fig. B 2.2.0-1a (perpendicular to the axial center line), except that in a necked lug, as shown in Fig. B 2.2.0-1b, A_1 and A_4 should be measured perpendicular to the local line.

B 2.2.0 Analysis of Lugs with Transverse Loading (Cont'd)

(1b) A_3 is the least area on any radial section around the hole.

(1c) Thought should always be given to assure that the areas A_1, A_2, A_3, and A_4 adequately reflect the strength of the lug. For lugs of unusual shape (e.g. with sudden changes of cross section), an equivalent lug should be sketched as shown in Fig. B 2.2.0-2 and used in the analysis.

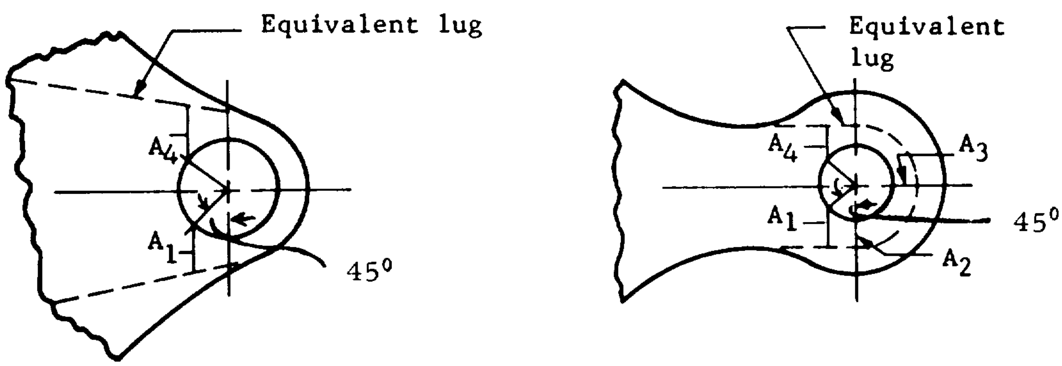


Fig. B 2.2.0-2

(2) Obtain the weighted average

$$A_{av} = \frac{6}{(3/A_1) + (1/A_2) + (1/A_3) + (1/A_4)}$$

(3) Compute $A_{br} = Dt$ and A_{av}/A_{br}

(4) Ultimate load P'_{tru} for lug failure:

(a) Obtain K_{tru} from Fig. B 2.2.0-4

(b) $P'_{tru} = K_{tr_u} A_{br} F_{tu_x}$

(5) Yield load P'_y of the lug:

(a) Obtain K_{tr_y} from Fig. B 2.2.0-4

(b) $P'_y = K_{tr_y} A_{br} F_{ty_x}$

(6) Check bushing yield and pin shear as outlined previously.

B 2.2.0 Analysis of Lugs with Transverse Loading (Cont'd)

(7) Investigate pin bending as for axial load with following modifications: Take $(P'_u)min = P'_{tru}$. In the equation $r = [e - (D/2)] /t$ use for the $[e - (D/2)]$ term the edge distance at $\alpha = 90^o$.

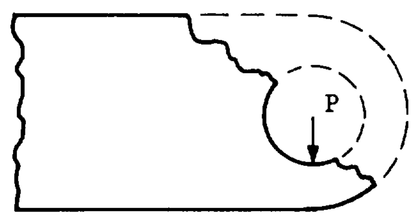


Fig. B 2.2.0-3

B 2.2.0 Analysis of Lugs with Transverse Loading (Cont'd)

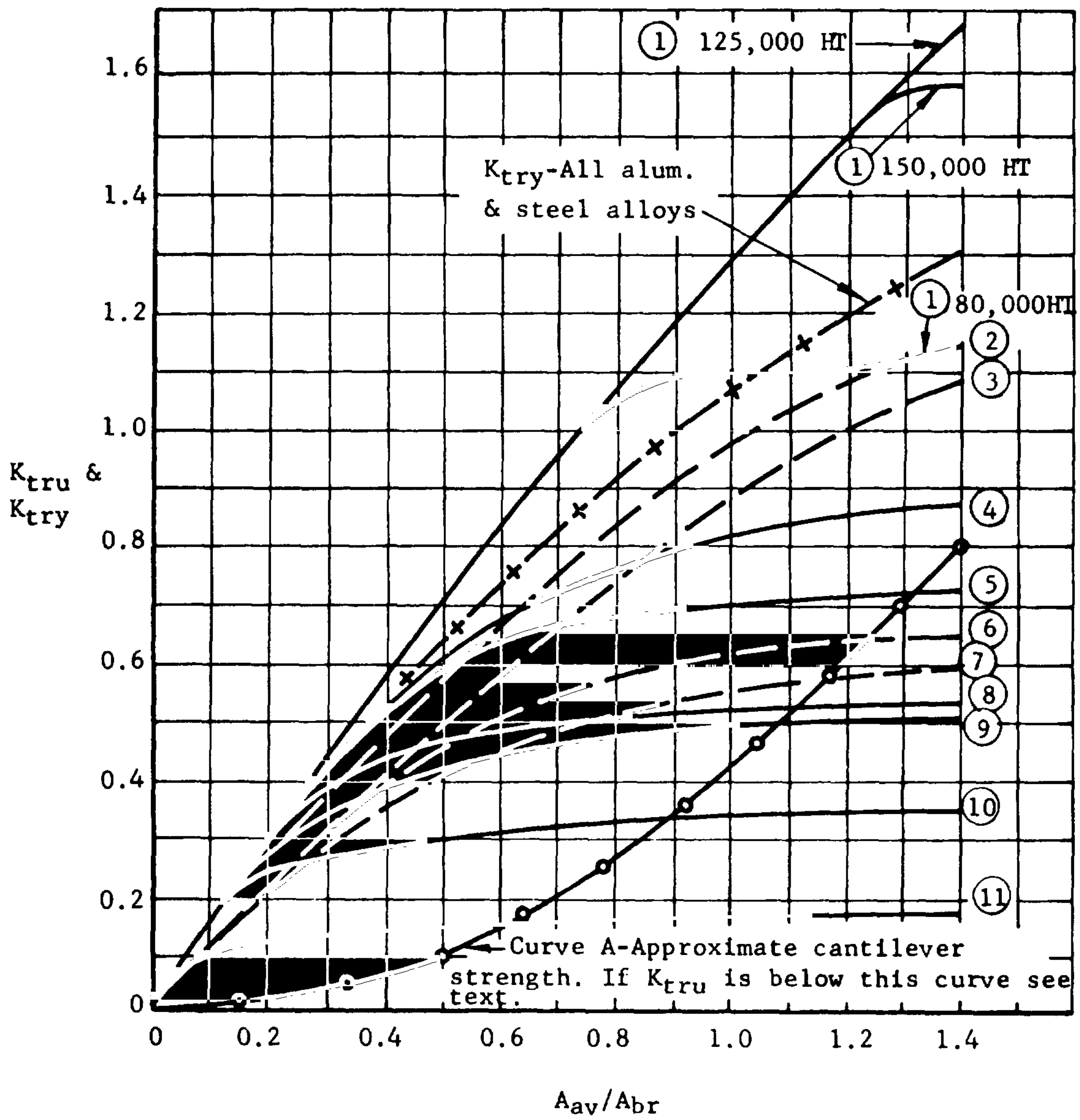


Fig. B 2.2.0-4

Legend on following pages

B 2.2.0 Analysis of Lugs with Transverse Loading (Cont'd)

Legend - Fig. B 2.2.0-4

Curve 1:

4130, 4140, 4340, and 8630 steels, heat treatment as noted.

Curve 2:

2024-T4 and 2024-T3 plate $\leq$0.5 in.

Curve 3:

220-T4 aluminum alloy casting

Curve 4:

17-7 PH (THD)

Curve 5:

2014-T6 and 7075-T6 plate $\leq$0.5 in.

Curve 6:

2024-T3 and 2024-T4 plate $>$0.5 in., 2024-T4 bar

Curve 7:

195-T6 and 356-T6 aluminum alloy casting

Curve 8:

2014-T6 and 7075-T6 plate $>$0.5 in., $\leq$1 in.
7075-T6 extrusion
2014-T6 hand forged billet $\leq$ 36 sq. in.
2014-T6 and 7075-T6 die forgings

Curve 9:

2024-T6 plate
2024-T4 and 2024-T42 extrusion

Curve 10:

2014-T6 and 7075-T6 plate $>$ 1 in.
7075-T6 hand forged billet $\leq$16 sq. in.

B 2.2.0 Analysis of Lugs with Transverse Loading (Cont'd)

Legend - Fig. B 2.2.0-4 Cont'd

Curve 11:

7075-T6 hand forged billet > 16 sq. in.
2014-T6 hand forged billet > 36 sq. in.

All curves are for K_{tru} except the one noted as K_{try}
Note: The curve for 125,000 HT steel in Fig. B 2.2.0-4 agrees closely with test data. Curves for all other materials have been obtained by the best available means of correcting for material properties and may possibly be very conservative to some places.

In no case should the ultimate transverse load be taken as less than that which could be carried by cantilever beam action of the portion of the lug under the load (Fig. B 2.2.0-3). The load that can be carried by cantilever beam action is indicated very approximately by curve (A) in Fig. B 2.2.0-4, should K_{tru} be below curve (A), separate calculation as a cantilever beam is warranted.

B 2.3.0 Analysis of Lugs with Oblique Loading

Interaction Relation

In analyzing lugs subject to oblique loading it is convenient to resolve the loading into axial and transverse components (denoted by subscripts "a" and "tr" respectively), analyze the two cases separately and utilize the results by means of an interaction equation. The interaction equation $R_a^{1.6} + R_{tr}^{1.6} = 1$, where R_a and R_{tr} are ratios of applied to critical loads in the indicated directions, is to be used for both ultimate and yield loads for both aluminum and steel alloys.
where, for ultimate loads

R_a = (Axial component of applied load) divided by (smaller of P'_{bru} and P'_{tu} from Eq. 1 and Eq. 2.)

R_{tr} = (Transverse component of applied load) divided by (P'_{tru} from analysis procedure for α = 90 deg.)

and for yield load:

R_a = (Axial component of applied load) divided by (P'_y from Eq. 3.)

R_{tr} = (Transverse component of applied load) divided by (P'_{try} from Analysis Procedure for α = 90 deg.)

Analysis Procedure

(1) Resolve the applied load into axial and transverse components and obtain the lug ultimate and yield Factor of Safety from the interaction equation:

$$F.S. = \frac{1}{\left[R_a^{1.6} + R_{tr}^{1.6}\right]^{0.625}}$$

(2) Check pin shear and bushing yield as in Section B 2.1.0.

(3) Investigate pin bending using the procedure for axial load modified as follows:

$$\text{Take } (P'_u)_{min} = \frac{P}{\left(R_a^{1.6} + R_{tr}^{1.6}\right)^{0.625}}$$

In the equation $r = [e - (D/2)] / t$ use for the $[e-(D/2)]$ term the edge distance at the value of "α" corresponding to the direction of load on the lug.

B 2.3.0 Analysis of Lugs with Oblique Loading (Cont'd)

Reference

Melcone, M. A. and F. M. Hoblit, Developments in the Analysis of Method for Determining the Strength of Lugs Loaded Obliquely or Transversely, Product Engineering, June, 1953.

SECTION B3

SPRINGS

TABLE OF CONTENTS

B 3.0.0 SPRINGS

B 3.1.0 Helical Springs

B 3.1.1 Helical Compression Springs

Most compression springs are open-coil, helical springs which offer resistance to loads acting to reduce the length of the spring. The longitudinal deflection of springs produces shearing stresses in the spring wire. Where particular load-deflection characteristics are desired, springs with varying pitch diameters may be used. These springs may have any number of configurations, including cone, barrel, and hourglass.

Round-Wire Springs

The relation between the applied load and the shearing stress for helical springs formed from round wire is

$$f_s = \frac{8PD}{\pi d^3} \quad \ldots\ldots \quad (1)$$

where

f_s = shearing stress in pounds per sq. inch. (not corrected for curvature)
P = axial load in pounds
D = mean diameter of the spring coil (Outside diameter minus wire diameter or inside diameter plus wire diameter)
d = diameter of the wire in inches.

Equation (1) is based on the assumption that the magnitude of the stress varies directly with the distance from the center of the wire; but, actually, the stress is greater on the inside of the cross section due to the curvature. The stress correction factor (k) used to determine the maximum shearing stress for static loads is found in Fig. B 3.1.1-1. This correction factor gives the effect of both torsion and direct shear. The equation for the maximum stress is

$$f_{max} = k\, f_s = k\, \frac{8PD}{\pi d^3} \quad \ldots\ldots \quad (2)$$

B 3.1.1 Helical Compression Springs (Cont'd)

where

$k = k_c + \frac{0.615}{C_1}$	Stress concentration factor plus the effect of direct shear
$k_c = \frac{4C_1 - 1}{4C_1 - 4}$	Stress concentration factor due to curvature
$C_1 = \frac{D}{d}$	Ratio of mean diameter of helix to the diameter of the bar or wire

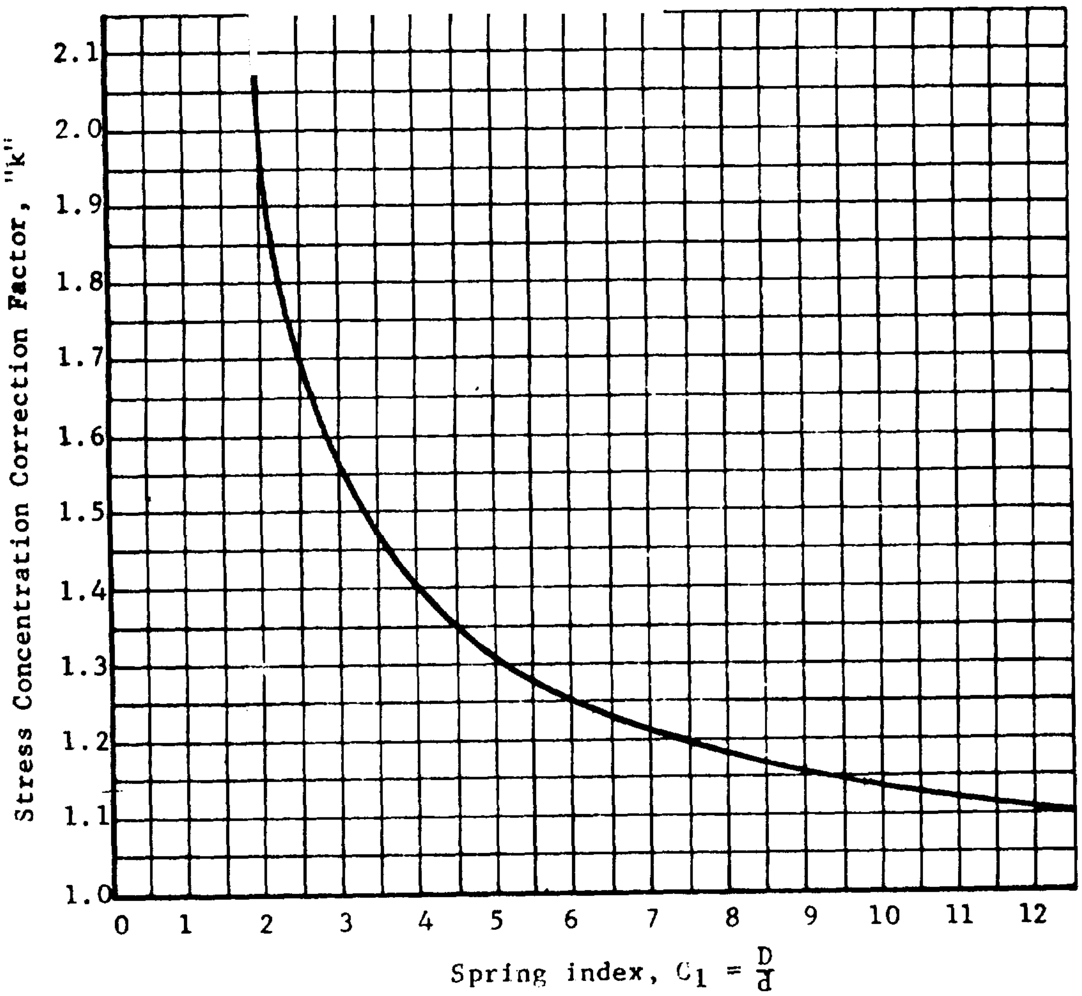


Fig. B 3.1.1-1

B 3.1.1 Helical Compression Springs (Cont'd)

Stress correction for temperature.

Corrections must be made to account for changes in strength and in elastic properties of spring materials at elevated temperatures. This correction is made to the allowable stress of the spring material. Values for various materials at various temperatures may be obtained from Section 3.0.0 of the Design Manual.

Deflection

The formula for the relation between deflection and load when using round wire in helical springs is

$$\delta = \frac{8NPD^3}{Gd^4} \qquad (3)$$

where

δ = total deflection
N = number of coils
G = modulus of rigidity

The deflection may also be given in terms of the shearing stress by combining Eqs. (1) and (3). Stress concentration usually does not affect deflection to an appreciable degree, and no adjustment is needed in Eq. (1). The expression for the deflection is

$$\delta = \frac{N f_s \pi D^2}{Gd} \qquad (4)$$

Spring rate

The spring rate (K) is defined as the amount of force required to deflect the spring a unit length. By proper substitution of the previous equations, the spring rate may be shown to be

$$K = \frac{d^4 G}{8ND^3} \qquad (5)$$

Buckling of Compression Springs

A compression spring which is long compared to its diameter will buckle under relatively low loads in the same manner as a column. However, the problem of buckling is of little consequence if the compression spring operates inside a cylinder or over a rod.

B 3.1.1 Helical Compression Springs (Cont'd)

As the critical buckling load of a column is dependent upon the end fixity at the supports, so is the critical buckling load of a spring dependent upon the fixity of the ends. In general, a compression spring with ends squared, ground, and compressed between two parallel surfaces can be considered a fixed-end spring. The following formula gives the critical buckling load.

$$P_c = JKL \qquad (6)$$

where

J = factor from Fig. B 3.1.1-2 = $\frac{\text{Deflection}}{\text{Free Length}}$
L = free length of spring
K = spring rate (See Eq. 5)

Fig. B 3.1.1-2 (curve 1) is for squared and ground springs with one end on a flat surface and the other on a ball. Curve 2 indicates buckling for a squared and ground spring both ends of which are compressed against parallel plates. This is the most common condition with which the user must contend.

Helical Springs of Rectangular Wire.

When rectangular wire is used for helical springs, the value of the shearing stress can be found by use of the equations for rectangular shafts. A stress concentration factor is applied in the usual way to compensate for the effect of curvature and direct shear. For the springs in Fig. B 3.1.1-3 (a) and (b) the stresses at points A_1 and A_2 are as follows:

$$f_s = \frac{kPR}{\alpha_1 bc^2} \quad \text{for point } A_1 \qquad (7)$$

$$f_s = \frac{kPR}{\alpha_2 bc^2} \quad \text{for point } A_2 \qquad (8)$$

values for α_1 and α_2 for various b/c ratios are found in Table B 3.1.1-1.

The stress concentration factor should be applied for point A_1 in Fig. B 3.1.1-3 (a) and to point A_2 in Fig. B 3.1.1-3 (b). The stress concentration factors of Fig. B 3.1.1-1 may be used as an approximate value for rectangular wire.

B 3.1.1 Helical Compression Springs (Cont'd)

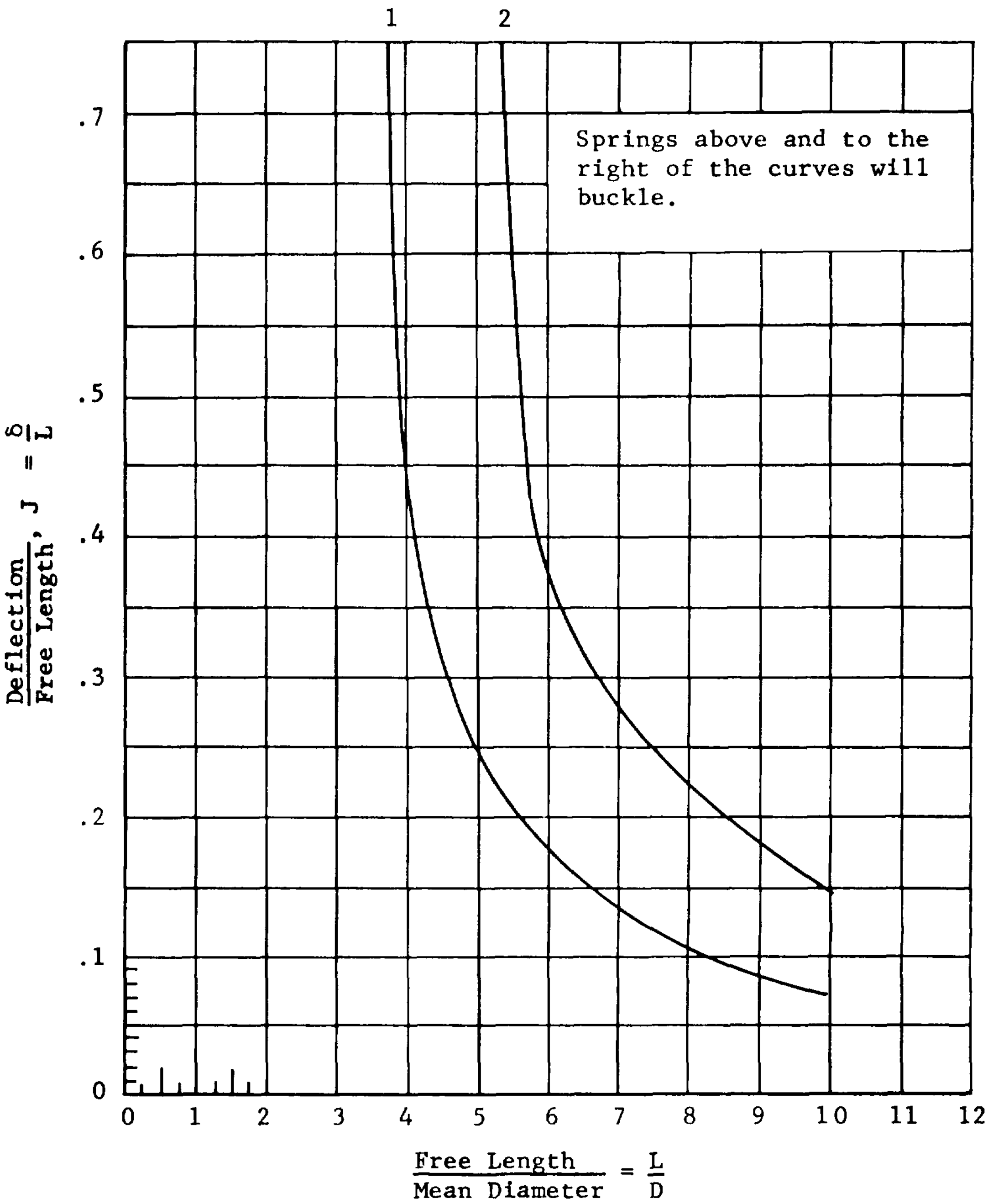


Fig. B 3.1.1-2

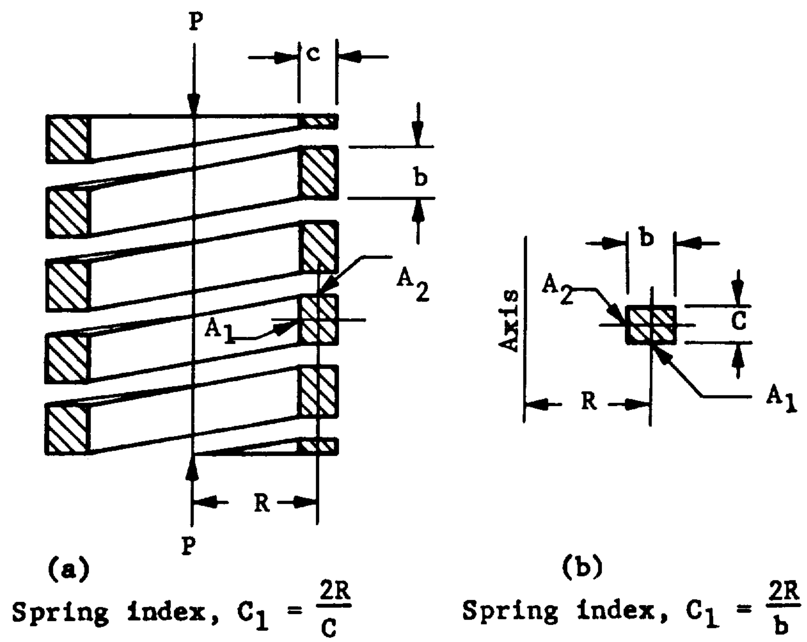


Fig. B 3.1.1-3

b/c	1.00	1.20	1.50	1.75	2.00	2.50	3.00	4.00	5.00	6.00	8.00	10.00	∞
α_1	.208	.219	.231	.239	.246	.258	.267	.282	.291	.299	.307	.312	.333
α_2	.208	.235	.269	.291	.309	.336	.355	.378	.392	.402	.414	.421	--
β	.1406	.166	.196	.214	.229	.249	.263	.281	.291	.299	.307	.312	.333

Table B 3.1.1-1

The equation for the relation between the load (P) and the deflection (δ) is

$$\delta = \frac{2\pi P R^3 N}{\beta G b c^3} \qquad (9)$$

where:

β is obtained from Table B 3.1.1-1
b and c are as shown in Fig. B 3.1.1-3
R is the mean radius of the spring
N is the number of coils
G is the modulus of rigidity
P is the axial load

B 3.1.2 Helical Extension Springs

Helical extension springs differ from helical compression springs only in that they are usually closely coiled helices with ends formed to permit their use in applications requiring resistance to tensile forces. It is also possible for the spring to be wound so that it is preloaded, that is, the spring is capable of resisting an initial tensile load before the coils separate. This initial tensile load does not affect the spring rate. See Figure B 3.1.2-1 for the load-deflection relationship of a preloaded helical extension spring.

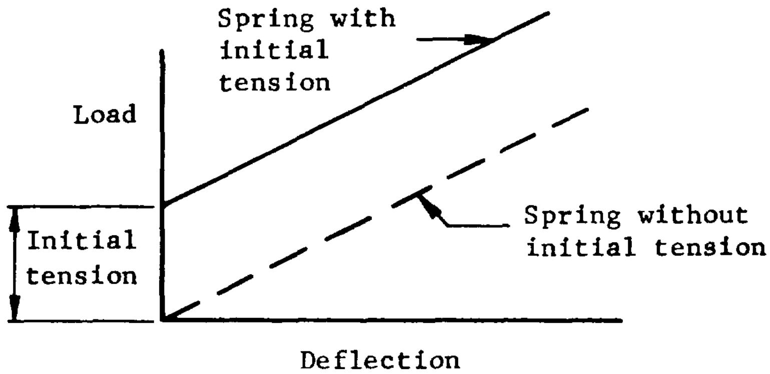


Fig. B 3.1.2-1

Stresses and Deflection

In helical extension springs, the shape of the hook or end turns for applying the load must be designed so that the stress concentration effects caused by the presence of sharp bends are decreased as much as possible. This problem is covered in the next article.

If the extension spring is designed with initial tension, formulas (1) through (9) from Section B 3.1.1 are valid, but must be applied with some understanding of the nature of the forces involved.

B 3.1.2 Helical Extension Springs (Cont'd)

Stress concentration in hooks on extension springs.

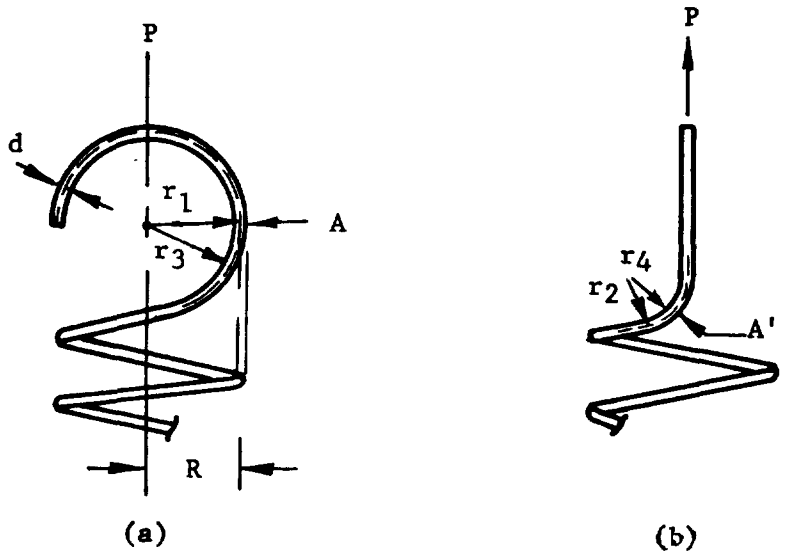


Fig. B 3.1.2-2

a. Bending Stress

Fig. B 3.1.2-2(a) illustrates the bending moment (PR) due to the load (P). The bending stress at point A is:

$$f_b = \frac{32PR}{\pi d^3} k + \frac{4P}{\pi d^2} \quad \text{.................................} \quad (10)$$

where k is the correction factor obtained from Fig. B 3.1.2-3, using the ratio $2r_1/d$.

r_1 = radius of center line of maximum curvature.

A simplified equation is:

$$f_b = \frac{32PR}{\pi d^3} \left(\frac{r_1}{r_3}\right) \quad \text{.................................} \quad (11)$$

r_3 = inside radius of bend.

The maximum bending stress obtained by this simplified form will always be on the safe side and, under normal conditions, only slightly higher than the true stress.

B 3.1.2 Helical Extension Springs (Cont'd)

b. Torsional Stress

At point A', Fig. B 3.1.2-2(b), where the bend joins the helical portion of the spring, the stress condition is primarily torsion. The maximum torsional shear stress due to the moment (PR) is

$$f_s = \frac{16PR}{\pi d^3}\left(\frac{4C_1 - 1}{4C_1 - 4}\right) \quad \ldots\ldots\ldots\ldots \quad (12)$$

$$C_1 = \frac{2r_2}{d}$$

A simplified form similar to the one for bending is

$$f_s = \frac{16PR}{\pi d^3}\left(\frac{r_2}{r_4}\right) \quad \ldots\ldots\ldots\ldots \quad (13)$$

This will also give safe results.

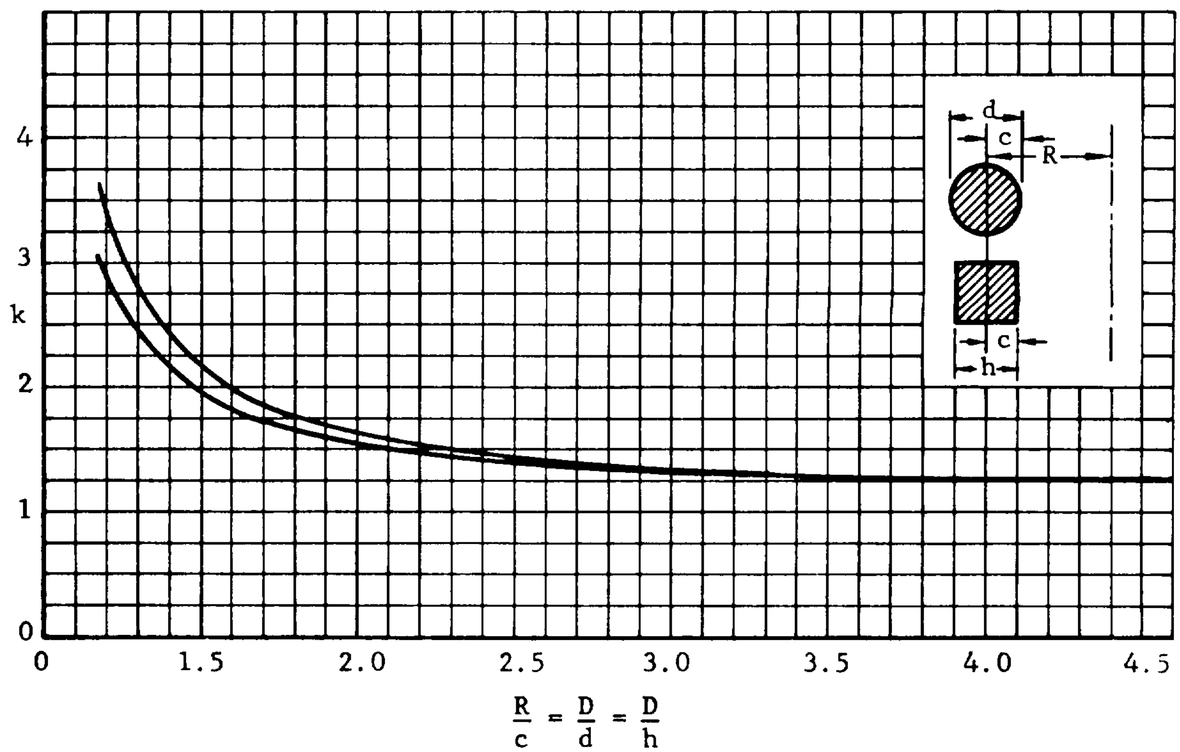


Fig. B 3.1.2-3

B 3.1.3 Helical Springs with Torsional Loading

A helical spring can be loaded by a torque about the axis of the helix. Such loading, as shown in Fig. B 3.1.3-1(a), is similar to the torsional loading of a shaft. The torque about the axis of the helix acts as a bending moment on each section of the wire as shown in Fig. B 3.1.3-1(b). The stress is then

$$f_b = k_n \frac{Mc}{I} \quad \text{.......................................} \quad (14)$$

where the stress concentration factor, K_n, is given as

$$\left.\begin{array}{ll} k_1 = \dfrac{3C_1^2 - C_1 - 0.8}{3C_1(C_1 - 1)} & \text{inner edge} \\[2ex] k_2 = \dfrac{3C_1^2 + C_1 - 0.8}{3C_1(C_1 - 1)} & \text{outer edge} \end{array}\right\} \text{Rectangular cross section wire}$$

$C_1 = \frac{2R}{h}$; "h" is the depth of section perpendicular to the axis.

$$\left.\begin{array}{ll} k_3 = \dfrac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)} & \text{inner edge} \\[2ex] k_4 = \dfrac{4C_1^2 + C_1 - 1}{4C_1(C_1 + 1)} & \text{outer edge} \end{array}\right\} \text{Round cross section wire}$$

$$C_1 = \frac{2R}{d}$$

Angular deformation

The deformation of the wire in the spring is the same as for a straight bar of the same length "S". The total angular deformation θ between tangents drawn at the ends of the bar is:

$$\theta = \frac{MS}{EI} \quad \text{.......................................} \quad (15)$$

Angle θ in some cases may amount to a number of revolutions.

B 3.1.3 Helical Springs with Torsional Loading (Cont'd)

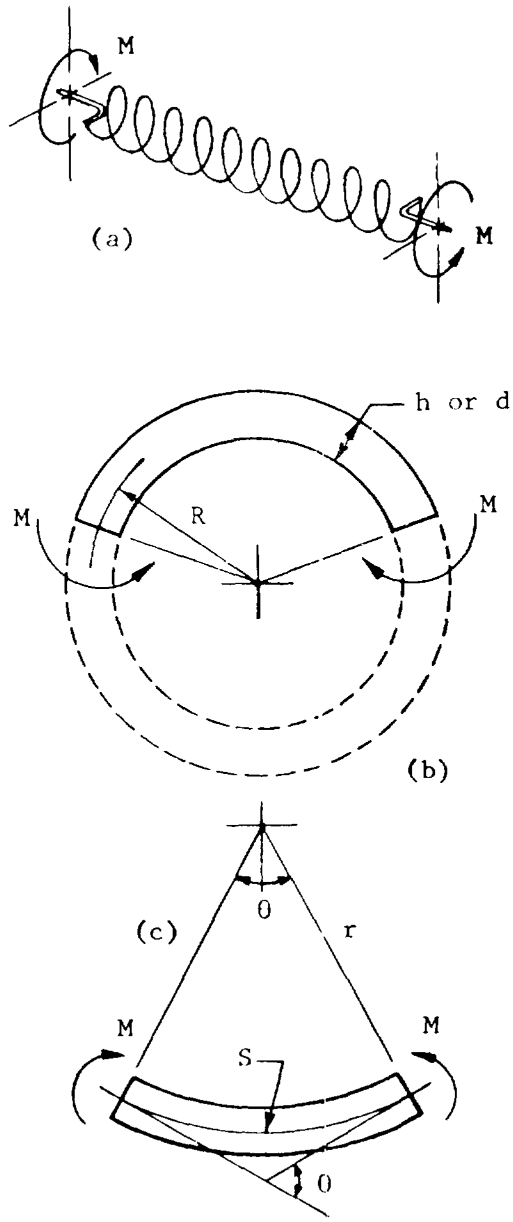


Fig. B 3.1.3-1

B 3.1.4 Analysis of Helical Springs by Use of Nomograph

The procedure for using the nomographs (Fig. B 3.1.4-1 and Fig. B 3.1.4-2) for helical springs are as follows:

1. Set the appropriate wire diam. on the "d" scale.

2. Set the appropriate mean diam. on the "D" scale.

3. Connect the two points and read the curvature correction factor:

   a. For tension and compression springs read the "y" scale on Fig. B 3.1.4-1.

   b. For torsion springs read the "k" scale on Fig. 3.1.4-2.

4. Set the correction factor, obtained in step 3, on the appropriate "y" or "k" scale to the right of Fig. B 3.1.4-1 and Fig. B 3.1.4-2.

5. Set the "calculated" (Eq. 1 or Eq. 14 with $k_n = 1$) stress on the "f" scale.

6. Connect the two points, from steps 4 and 5, and read the corrected stress on the (f') scale.

B 3.1.4 Analysis of Helical Springs by Use of Nomograph (Cont'd)

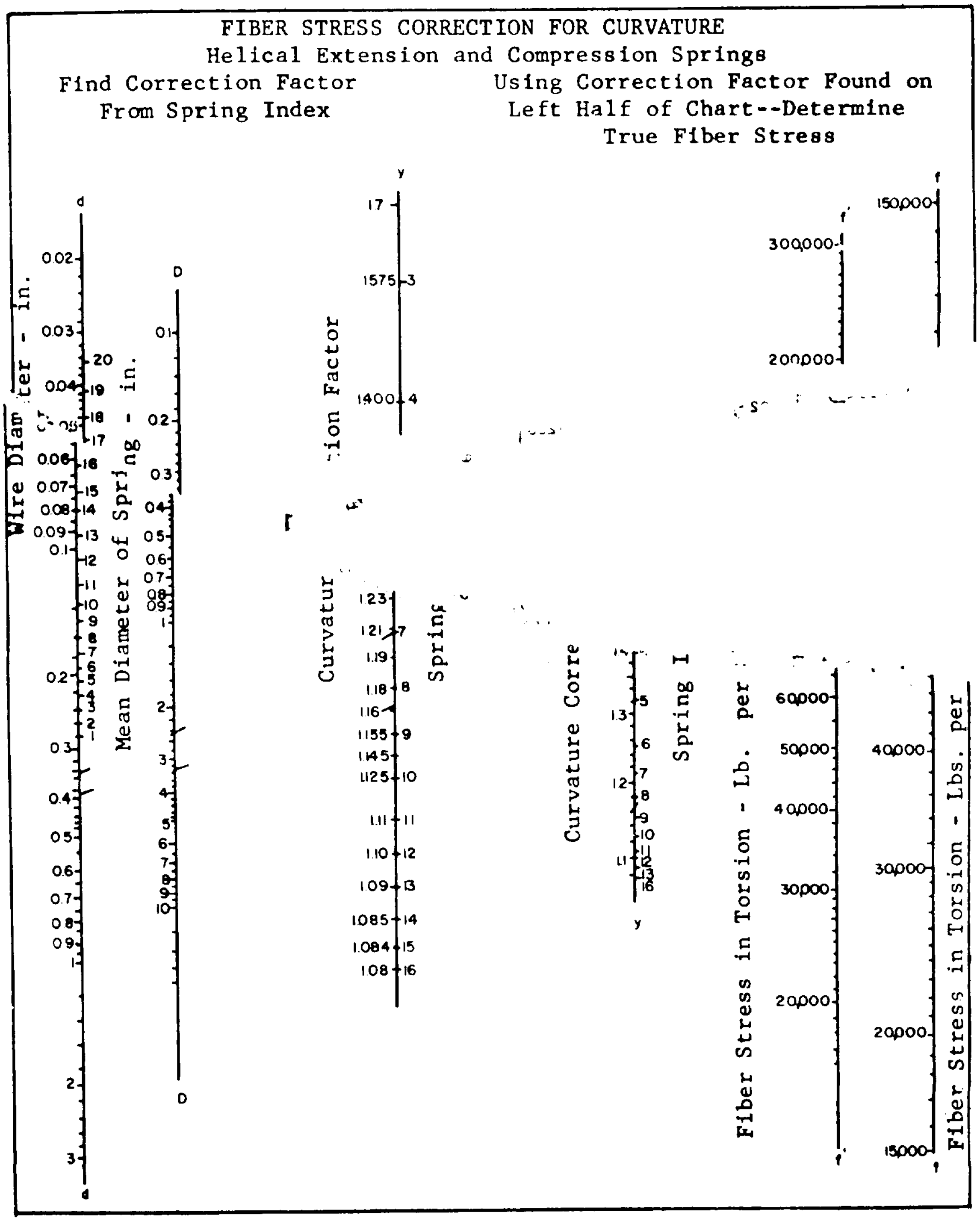


Fig. B 3.1.4-1

B 3.1.4 Analysis of Helical Springs by Use of Nomograph (Cont'd)

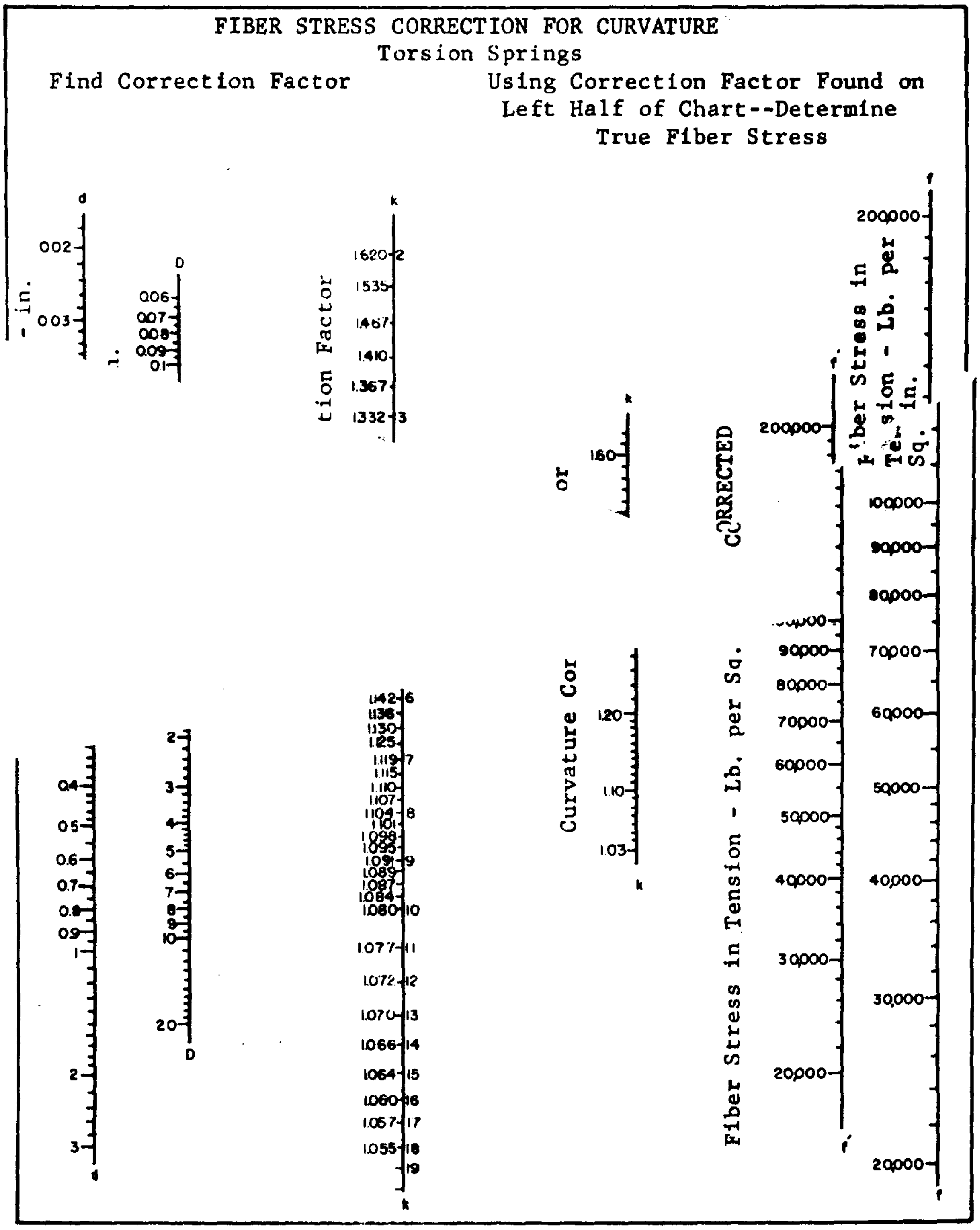


Fig. B 3.1.4-2

B 3.1.5 Maximum Design Stress for Various Spring Materials

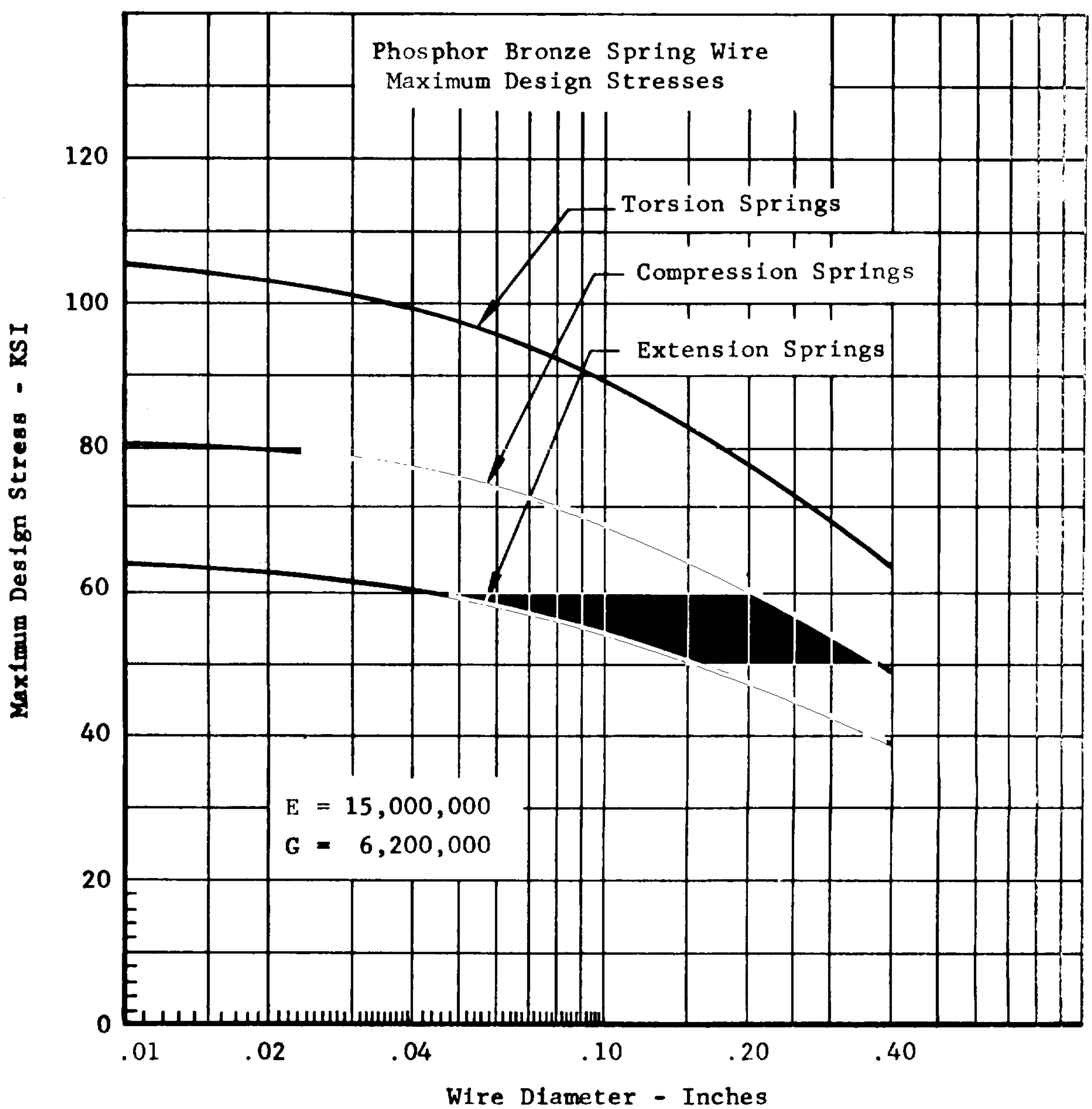


Fig. B 3.1.5-1

B 3.1.5 Maximum Design Stresses for Various Spring Materials (Cont'd)

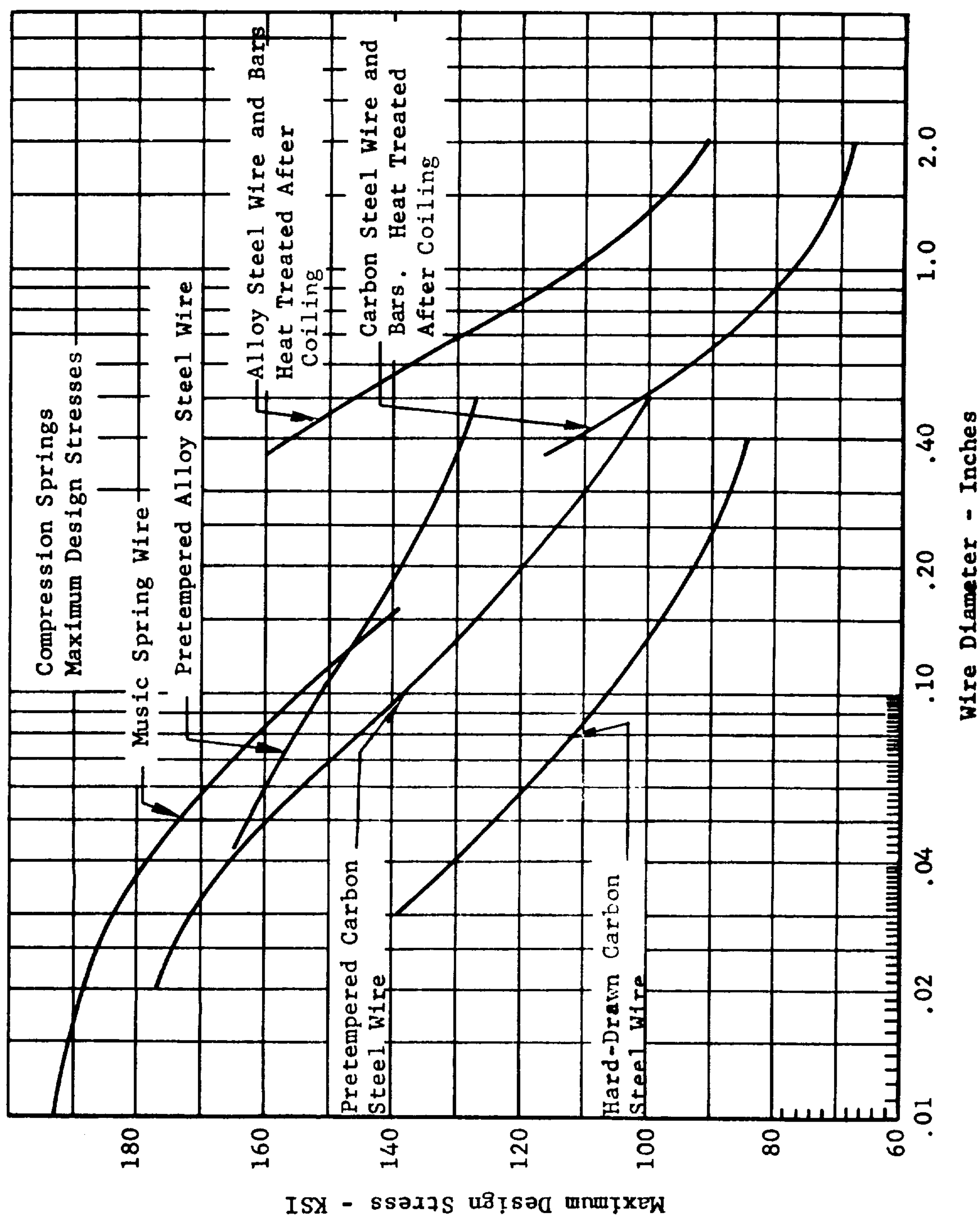


Fig. B 3.1.5-2

B 3.1.5 Maximum Design Stresses for Various Spring Materials (Cont'd)

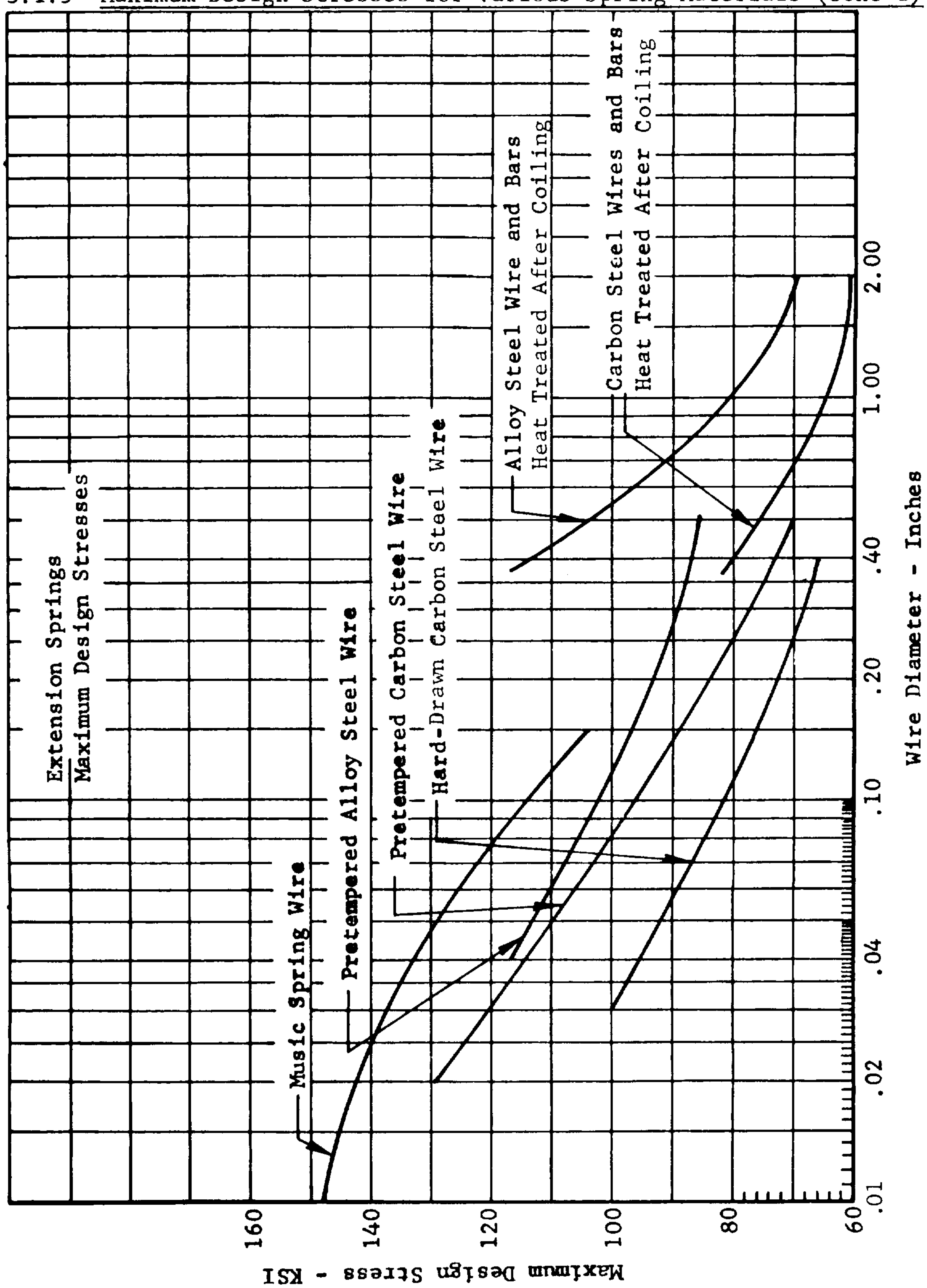


Fig. B 3.1.5-3

B 3.1.5 Maximum Design Stresses for Various Spring Materials (Cont'd)

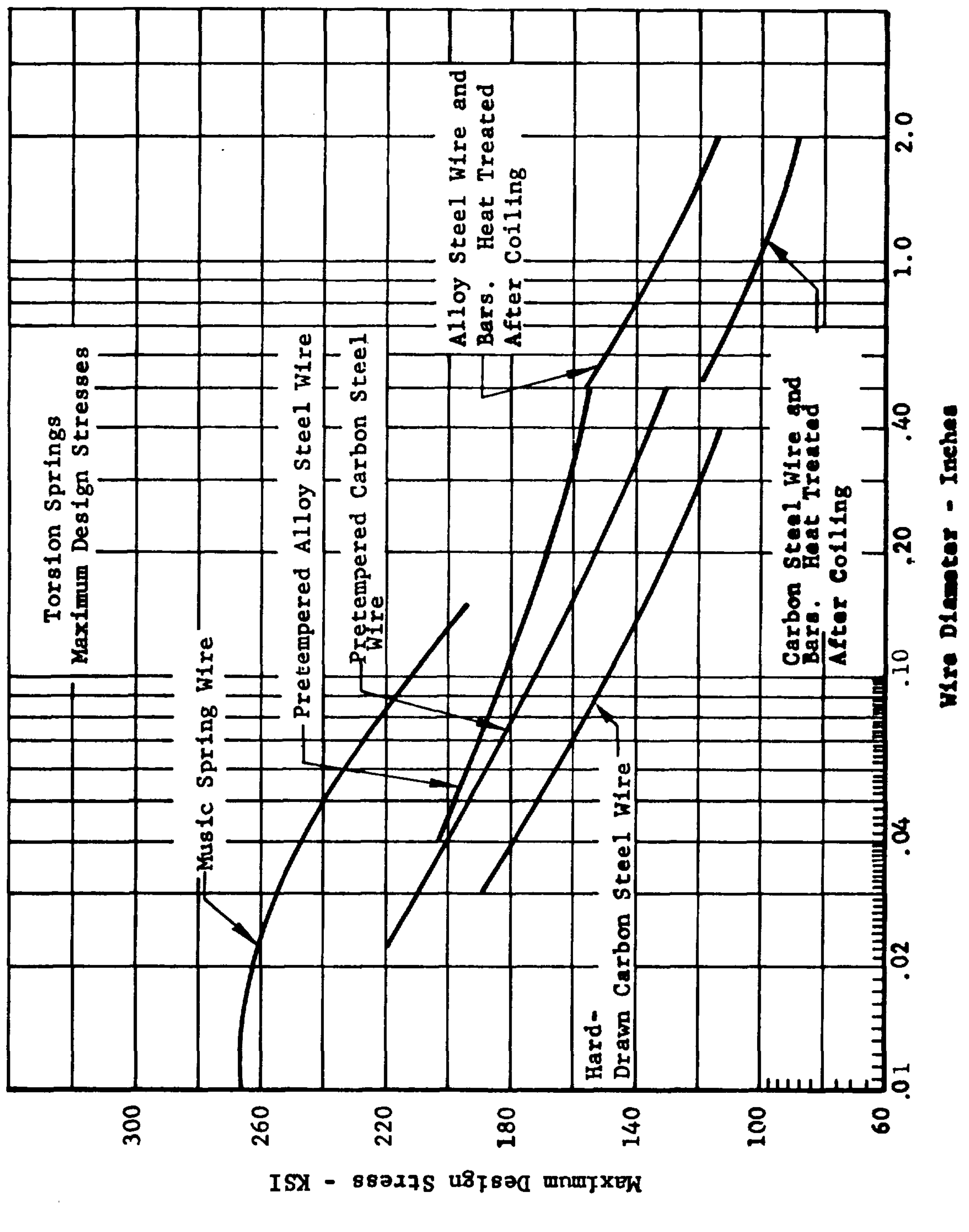


Fig. B 3.1.5-4

B 3.1.6 Dynamic or Suddenly Applied Spring Loading

A freely falling weight, or moving body, that strikes a structure delivers a dynamic or impact load, or force. Problems involving such forces may be analyzed on the basis of the following idealizing assumptions:

1. Materials behave elastically, and no dissipation of energy takes place at the point of impact or at the supports owing to local inelastic deformation of materials.

2. The deflection of a system is directly proportional to the magnitude of the dynamically or statically applied force.

Then, on the basis of the principle of conservation of energy, if it may be further assumed that at the instant a moving body is stopped, its kinetic energy is completely transformed into the internal strain energy of the resisting system, the following formulas will apply:

(a) For very slowly applied loads

$$\delta = \frac{P}{K} \qquad (16)$$

(b) For loads suddenly applied

$$\delta = \frac{2P}{K} \qquad (17)$$

(c) For loads dropped from a given height

$$\delta^2 = \frac{2P(S + \delta)}{K} \qquad (18)$$

where:

δ = Total deflection
K = Spring rate
P = Load on spring
S = Height load is dropped.

The following problems will illustrate such conditions and their solutions:

Problem 1.

Given a spring which compresses one inch for each pound of load, determine the maximum load and deflection resulting from a 4 lb. weight.

B 3.1.6 Dynamic or Suddenly Applied Spring Loading (Cont'd)

(a) Case 1:

The weight is laid gently on the spring.

$$\delta = \frac{P}{K} = \frac{4}{1} = 4 \text{ in. from Eq. 16}$$

since K = 1 maximum load = 4 lb.

(b) Case 2:

The weight is suddenly dropped on the spring from zero height.

$$\delta = \frac{2P}{K} = \frac{2(4)}{1} = 8 \text{ in. from Eq. 17}$$

maximum load = 8 lb.

(c) Case 3:

The weight is dropped on the spring from a height of 12 inches.

$$\delta^2 = \frac{2P(S + \delta)}{K} = \frac{2(4)}{1}(12 + \delta) \quad \text{from Eq. 18}$$

or

$$\delta^2 - 8\delta - 96 = 0$$

$$\delta = \frac{8 + \sqrt{8^2 + 4(96)}}{2} = 14.6 \text{ in.}$$

maximum load 14.6 lb.

From the maximum load produced, Eq. 2 section B 3.1.1 may be used to calculate the stress produced. This should be within the limits indicated on Fig. B 3.1.5-1, B 3.1.5-2, B 3.1.5-3 and B 3.1.5-4.

Problem 2.

For many uses it is necessary to know the return speed of a spring or the speed with which it will return a given weight. A typical example of this problem could be stated as follows: a spring made of 5/16 in. by 3/16 in. rectangular steel contains 4 3/8 total coils, 2 3/8 active coils, on a mean diameter of 1 5/16 in. The spring compresses 5/32 in. for 200 lb. load. If the spring is compressed and then instantaneously released, how fast will it be moving at its original free length position of 1 21/32 in.? The solution is as follows:

B 3.1.6 Dynamic or Suddenly Applied Spring Loading (Cont'd)

Weight per turn = $\pi(1\ 5/16)(3/16)(5/16)(.283)$ lb. per cu. in.
= .0685 lb. of steel in each coil

.0685 (2 3/8) active coils (1/3) = .0542 lb (one-third of the weight of active spring material involved.)

To this .0542 lb. we add the dead coil at the end plus the moving weight, if any.

The equivalent total weight (.0542 + .0685) is 0.1227 lb.

The potential energy of a spring is equal to 1/2 the total load times the distance moved, or 1/2(200)(5/32) = 15.6 in. lb. The kinetic energy equals 1/2 Mv^2 wherein (M) is the mass and (v) is the velocity.

$$\text{Mass} = \frac{\text{Weight}}{g}$$ where g is the gravitational acceleration,

or 32.16 ft/sec./sec.

$$\text{Therefore } 15.6 = \frac{.1227v^2}{2(32.16)(12)} \quad \text{or } v = 314 \text{ in/sec}$$

Often springs are used to absorb energy of impact. In most such instances springs must be designed so that they will absorb the entire energy. In a few cases partial absorption is tolerated. A typical problem of this type follows.

Problem 3.

A 30 lb. weight has a velocity of 4 ft. per second. How far will a spring that has a spring rate of 10 lb/in. be compressed?

KINETIC ENERGY

$$K.E. = \frac{1}{2} Mv^2 = \frac{30(4)(4)}{2(32.16)} = 7.46 \text{ ft. lb.}$$

or

7.46(12) = 89.52 in.lb.

Spring energy = $\frac{1}{2}$ load times deflection

Load = rate per inch times deflection

Spring energy = $\frac{1}{2} K\delta^2$ = 89.52 in. lb.

B 3.1.6 Dynamic or Suddenly Applied Spring Loading (Cont'd)

$$\frac{1}{2}\,10\delta^2 = 89.52$$

$$\delta^2 = 17.90$$

deflection, δ = 4.23 in.

If springs are used to propel a mass, a parallel attack using velocity and acceleration applies.

Problem 4.

Let it be required to find the spring load that will propel a 1-lb. ball 15 ft. vertically upward in 1/2 second. It is assumed that the spring can be compressed a distance of 1 ft.

In order to travel 15 ft. in 1/2 second the 1-lb. load must have a certain initial velocity. This can be found as follows:

$$v = \frac{h}{t} + \frac{gt}{2}$$

wherein: h = height
g = 32.16 ft. per sec^2
t = time

$$v = \frac{15}{\frac{1}{2}} + \frac{32.16\left(\frac{1}{2}\right)}{2} = 38.04 \text{ ft. per sec.}$$ Spring velocity at free height.

$$\text{Spring acceleration} = \frac{v^2}{2s} = \frac{(38.04)^2}{2(1)} = 723 \text{ ft/sec/sec}$$

Force equals mass times acceleration so

$$F = \frac{723(1)}{32.16} = 22.5 \text{ lb. avg.}$$

The average spring pressure is 1/2 the total load. Hence the spring will compress 1 ft. with 2(22.5) or 45-lb. of load. Often, it is desired to know how high the weight would be propelled. This can be determined by equating the work performed by the spring to the work of the falling weight; thus work equals force times distance.

In the spring we have $\frac{45}{2}$ (1) ft.

In the weight we have 1-lb. (h)

Hence 1(h) = $\frac{45}{2}$ (1) = 22.5 ft., the height to which the weight would be thrown.

B 3.1.6 Dynamic or Suddenly Applied Spring Loading (Cont'd)

If we were to apply the previous formula

$$\delta^2 = \frac{2P(S + \delta)}{K}$$ to the springs,

we must remember (S) is the height the load is dropped. The total distance traveled by the weight is $(S + \delta)$.

Therefore, $h = S + \delta$ in this case

Substituting $1 = \frac{2(1)h}{45}$

$$h = 22.5 \text{ ft.}$$

B 3.1.7 Working Stress for Springs

If the loading on the spring is continuously fluctuating, due allowance must be made in the design for fatigue and stress concentration. A method of determining the allowable or working stress for a particular spring is dependent on the application as well as the physical properties of the material.

B 3.2.0 Curved Springs

The analytical expression for determining stresses for curved springs is

$$f = \pm \frac{P}{A} \pm \frac{M}{AR}\left(1 + \frac{1}{Z}\frac{y}{R+y}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (19)$$

in which the quantities have the same meaning defined in section B 4.3.1.

Displacement of curved springs is determined by use of Castigliano's theorem.

$$\delta_p = \frac{\partial v}{\partial P} = \int \frac{N}{EA}\frac{\partial N}{\partial P}\,ds + \int \frac{V}{GA}\frac{\partial V}{\partial P}\,ds + \int \frac{M}{EAy_0R}\frac{\partial M}{\partial P}\,ds$$

$$+ \int \frac{M}{EAR}\frac{\partial N}{\partial P}\,ds + \int \frac{N}{EAR}\frac{\partial M}{\partial P}\,ds \quad \ldots\ldots\ldots\ldots \quad (20)$$

in which

N = normal force
E = Modulus of Elasticity
G = Modulus of Rigidity
A = Cross-sectional area
R = Radius to centroid
ds = Incremental length

$y_0 = \frac{ZR}{Z+1}$

$Z = -\frac{1}{A}\int_A \frac{y}{R+y}\,dA$

y = is measured from the centroid

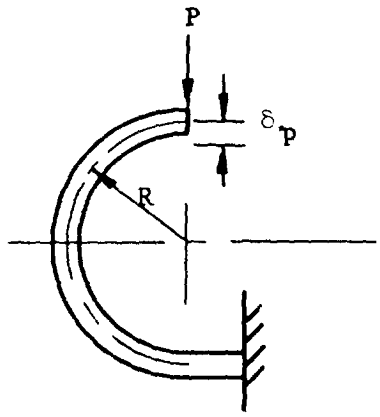


Fig. B 3.2.0-1

These expressions for stresses and displacements are quite cumbersome; therefore, correction factors are used to simplify the analysis. The correction factors (K) used to determine the stresses are given in section B 4.3.1. The expression for the stress is

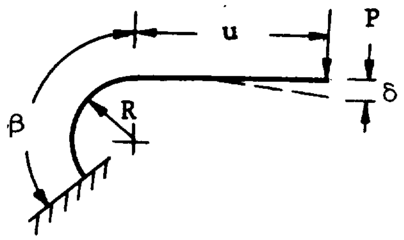
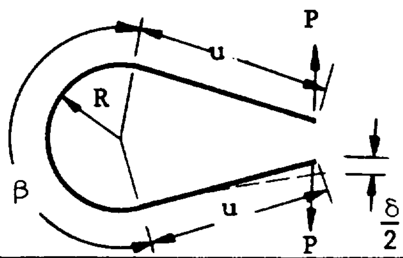
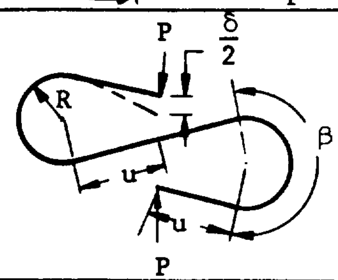
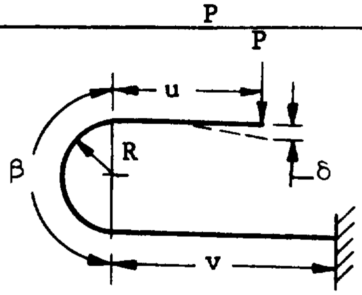
$$f = K\frac{Mc}{I} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (21)$$

See Table B 4.3.1-1 for values of correction factor K.

Deflection formulas for some basic types of curved springs are given in Table B 3.2.0-1. Complex spring shapes may be analyzed by combination of two or more basic types.

B 3.2.0 Curved Springs (Cont'd)

Table B 3.2.0-1

Spring types	Deflection
A 	$\delta = \frac{KPR^3}{3EI}(m+\beta)^3$ * where $\alpha = \beta$ for finding K
B 	$\delta = \frac{2KPR^3}{3EI}\left(m+\frac{\beta}{2}\right)^3$ where $\alpha = \frac{\beta}{2}$ for finding K
C 	$\delta = \frac{4KPR^3}{3EI}\left(m+\frac{\beta}{2}\right)^3$ where $\alpha = \frac{\beta}{2}$ for finding K
D 	$\delta = \frac{P}{3EI}\left[2KR^3\left(m+\frac{\beta}{2}\right)^3\left(v-u\right)^3\right]$ where $\alpha = \frac{\beta}{2}$ for finding K

* $m = \frac{u}{R}$

B 3.2.0 Curved Springs (Cont'd)

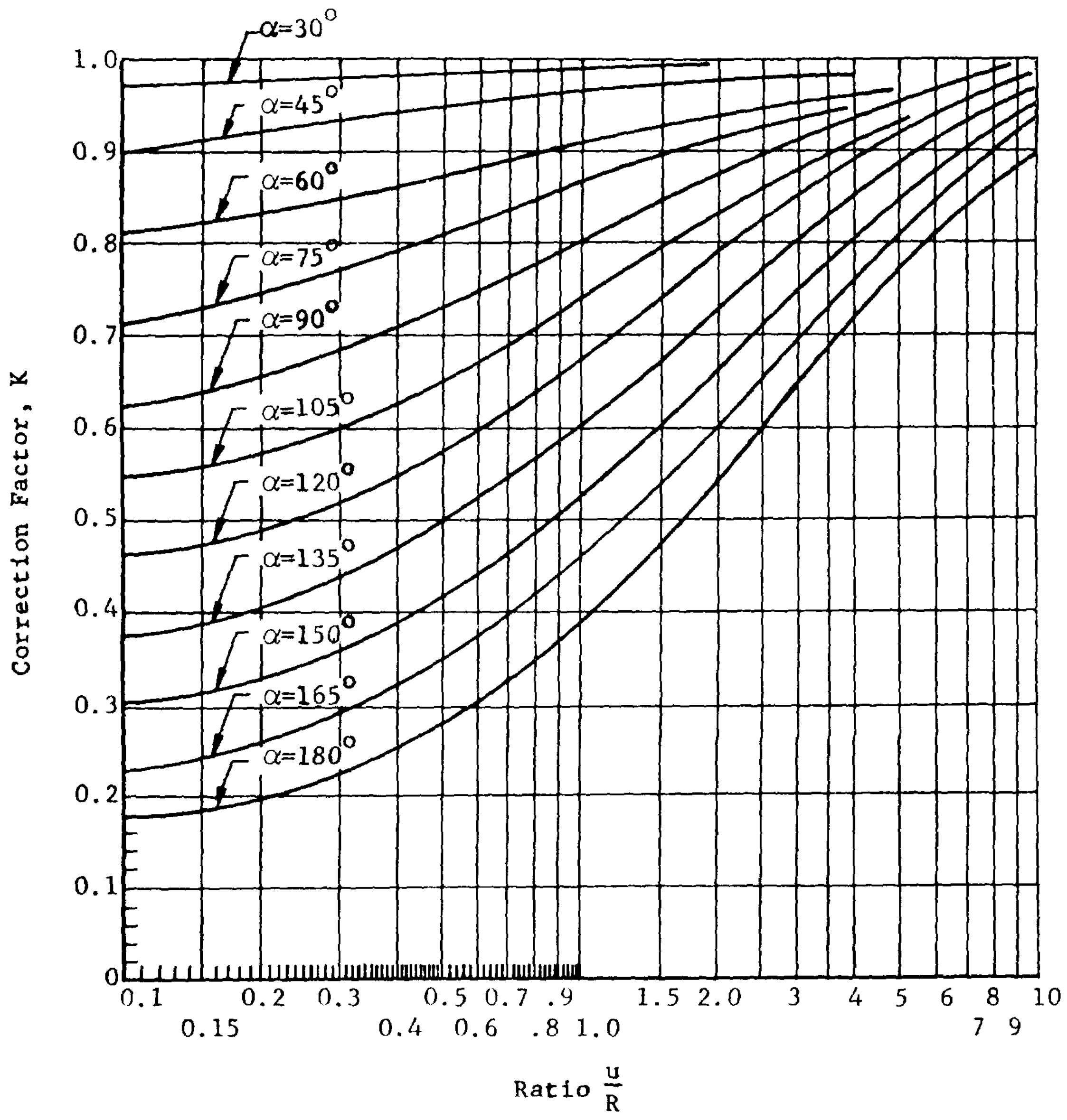


Fig. B 3.2.0-2

B 3.2.0 Curved Springs (Cont'd)

For close approximations, the following conditions should be met:

flat springs

$$\frac{\text{spring thickness}}{\text{radius of curvature}} = \frac{h}{R} < 0.6$$

round wire

$$\frac{\text{wire diameter}}{\text{radius of curvature}} = \frac{d}{R} < 0.6$$

Figure B 3.2.0-3 is a typical curved spring. The deflection of the spring at point A is calculated as follows:

Spring Characteristics

$\frac{h}{R} < 0.6$ $u_2 = 2.5"$

$\beta_1 = \pi$ $R_1 = R_2 = 1"$

$\beta_2 = \frac{\pi}{2}$

$u_1 = 1"$

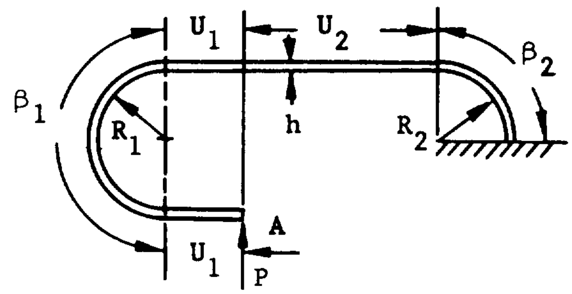


Fig. B 3.2.0-3

Solution -

The solution involves two basic types (type B and A of Table B 3.2.0-1). Type A solution is used for that portion of the spring denoted by subscript (2), and type B solution is used for that portion of the spring denoted by subscript (1).

Correction factor, K (from Fig. B 3.2.0-2)

$\frac{u_1}{r_1} = \frac{1}{1} = 1,\ \beta_1 = 180^o,\ K_1 = .80$

$\alpha_1 = 90^o$

$\frac{u_2}{r_2} = \frac{2.5}{1} = 2.5,$

$\beta_2 = 90^o,\ K_2 = .86$

Deflection at point A

$$\delta_A = \frac{2K_1PR_1^3}{3EI}\left(m + \frac{\beta_1}{2}\right)^3 + \frac{K_2PR_2^3}{3EI}\left(m + \beta_2\right)^3$$

$$\delta_A = \frac{2(.8)\ P(1)^3}{3EI}\left(1 + \frac{\pi}{2}\right)^3 + \frac{.86P(1)^3}{3EI}\left(2.5 + \frac{\pi}{2}\right)^3$$

$$\delta_A = \frac{28.4P}{EI}$$

B 3.3.0 Belleville Springs or Washers

Belleville type springs are used where space requirements necessitate high stresses and short range of motion. A complete derivation of data that is presented in this section will be found in "Transactions of Amer. Soc. of Mech. Engineers", May 1936, Volume 58, No. 4, by Almen and Laszlo.

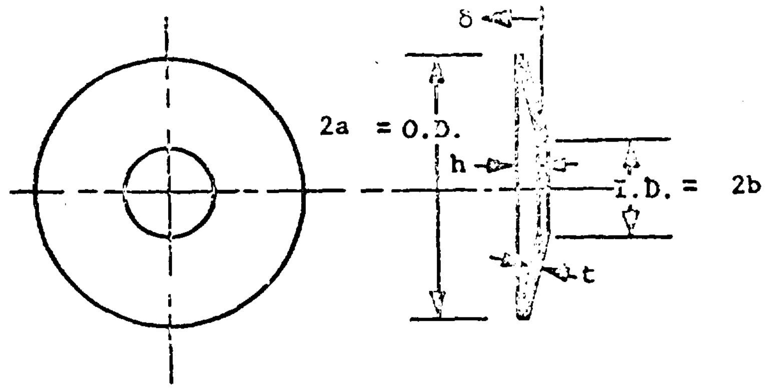


Fig. B 3.3.0-1

Symbols

- P = Load in pounds
- δ = Deflection in inches
- t = Thickness of material in inches
- h = Free height minus thickness in inches
- a = One-half outside diameter in inches
- E = Young's modulus
- f = Stress at inside circumference
- k = ratio of $\frac{O.D.}{I.D.} = \frac{a}{b}$
- ν = Poisson's ratio

M, C_1 and C_2 are constants which can be taken from the chart, Fig. B 3.3.0-4, or calculated from the formulas given.

The formulas are:

$$M = \frac{6}{\pi \log_e k} \frac{(k-1)^2}{k^2} \qquad (22)$$

$$C_1 = \frac{6}{\pi \log_e k} \left[\frac{(k-1)}{\log_e k} - 1 \right] \qquad (23)$$

$$C_2 = \frac{3}{\pi \log_e k} \frac{(k-1)}{} \qquad (24)$$

B 3.3.0 Belleville Springs or Washers (Cont'd)

The deflection-load formula, using these constants is

$$P = \frac{E\delta}{(1-\nu^2)Mk^2b^2}\left[\left(h-\frac{\delta}{2}\right)\left(h-\delta\right)t+t^3\right] \quad \text{.........} \quad (25)$$

The stress formula is as follows:

$$f = \frac{E\delta}{(1-\nu^2)Mk^2b^2}\left[C_1\left(h-\frac{\delta}{2}\right)+C_2t\right] \quad \text{................} \quad (26)$$

Before using these formulas to calculate a sample problem, there are some facts which should be considered. In the stress formula it is possible for the term $(h - \delta/2)$ to become negative if (δ) is large. When this occurs, the term inside the brackets should be changed to read $C_1(h - \delta/2) - C_2t$. Such an occurrence means that the maximum stress is tensile.

For a spring life of less than one-half million stress cycles, a fiber stress of 200,000 p.s.i. can be substituted for f, even though this might be slightly beyond the elastic limit of the steel. This is because the stress is calculated at the point of greatest intensity, which is on an extremely small part of the disc. Immediately surrounding this area is a much lower-stressed portion which so supports the higher-loaded corner that very little setting results at atmospheric temperatures. For higher operating temperatures and longer spring life lower stresses must be employed.

Fig B 3.3.0-2 displays the load-deflection characteristics of a .040 in. thick washer for various h/t ratios.

It is noted (from Fig. B 3.3.0-2) that for ratios of (h/t) under 1.41 the load-deflection curve is somewhat similar to that of other conventional springs. As this ratio approaches the value of 1.41, the spring rate approaches zero (practically horizontal load-deflection curve) at the flat position. When the (h/t) ratio is 2.83 or over there is a portion of the curve where further deflection produces a lower load. This is illustrated in the curves for the washers having (h/t) ratios of 2.83 and 3.50. Such a spring, when deflected to a certain point, will snap through center and require a negative loading to return it to its original position.

The washers may sometimes be stacked so as to obtain the load-deflection characteristics desired. The accepted methods are illustrated in Fig. B 3.3.0-3.

B 3.3.0 Belleville Springs or Washers (Cont'd)

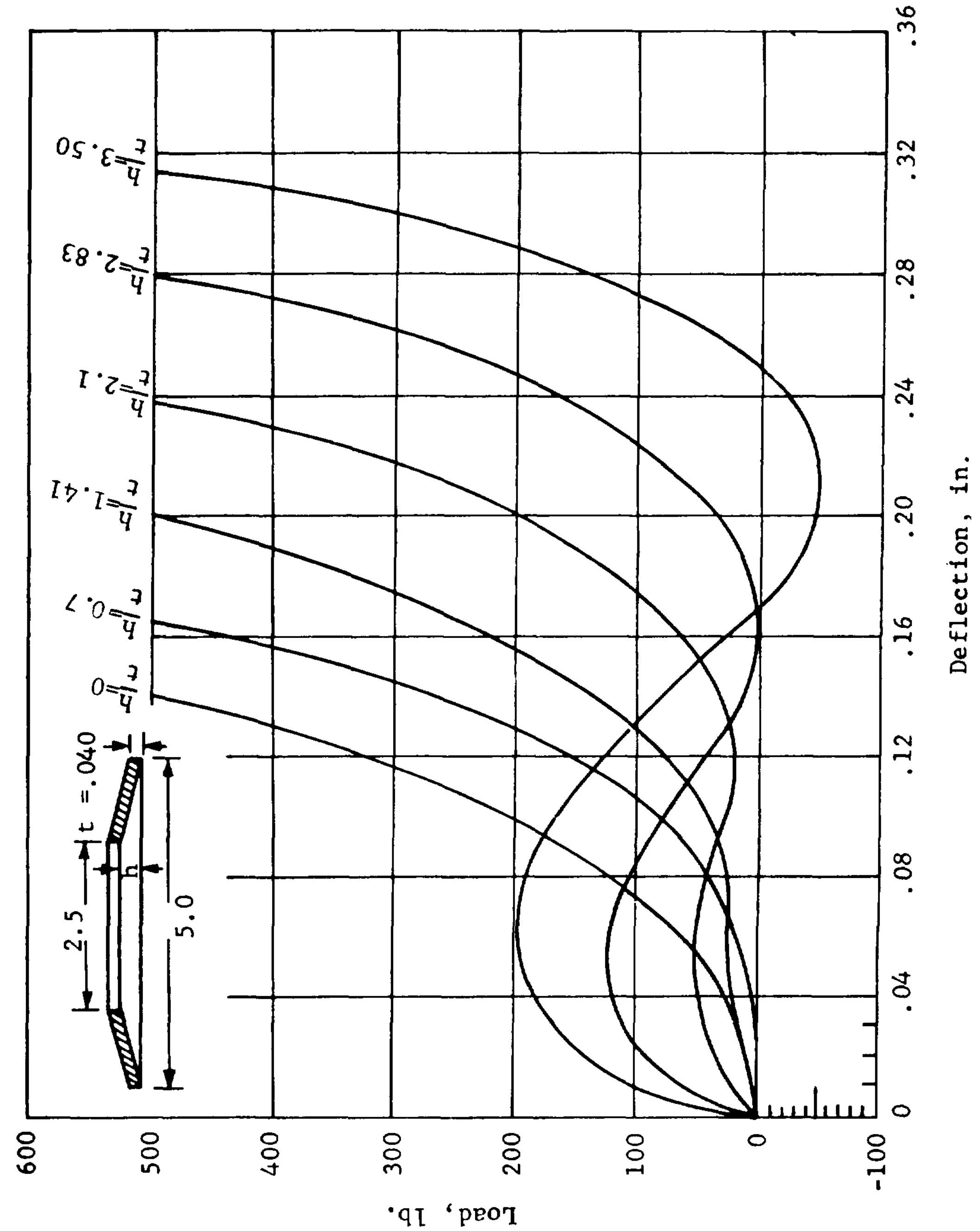


Fig. B 3.3.0-2

B 3.3.0 Belleville Springs or Washers (Cont'd)

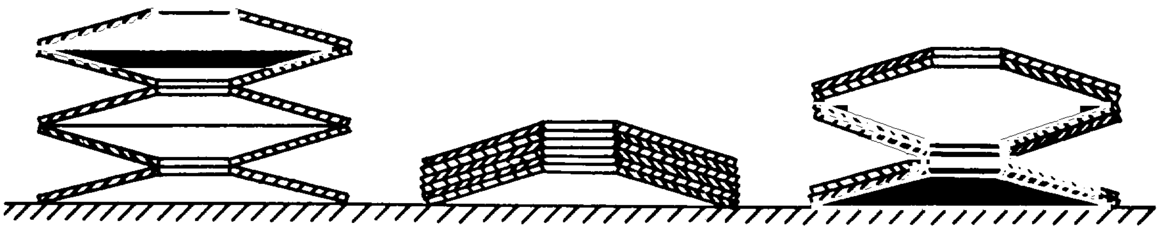


Fig. B 3.3.0-3

As the number of washers used increases, so does the friction in the stacks. This is not uniform and could result in spring units which are very erratic in their load-carrying capacities. Belleville springs, as a class, are one of the most difficult to hold to small load-limit tolerances.

B 3.3.0 Belleville Springs or Washer (Cont'd)

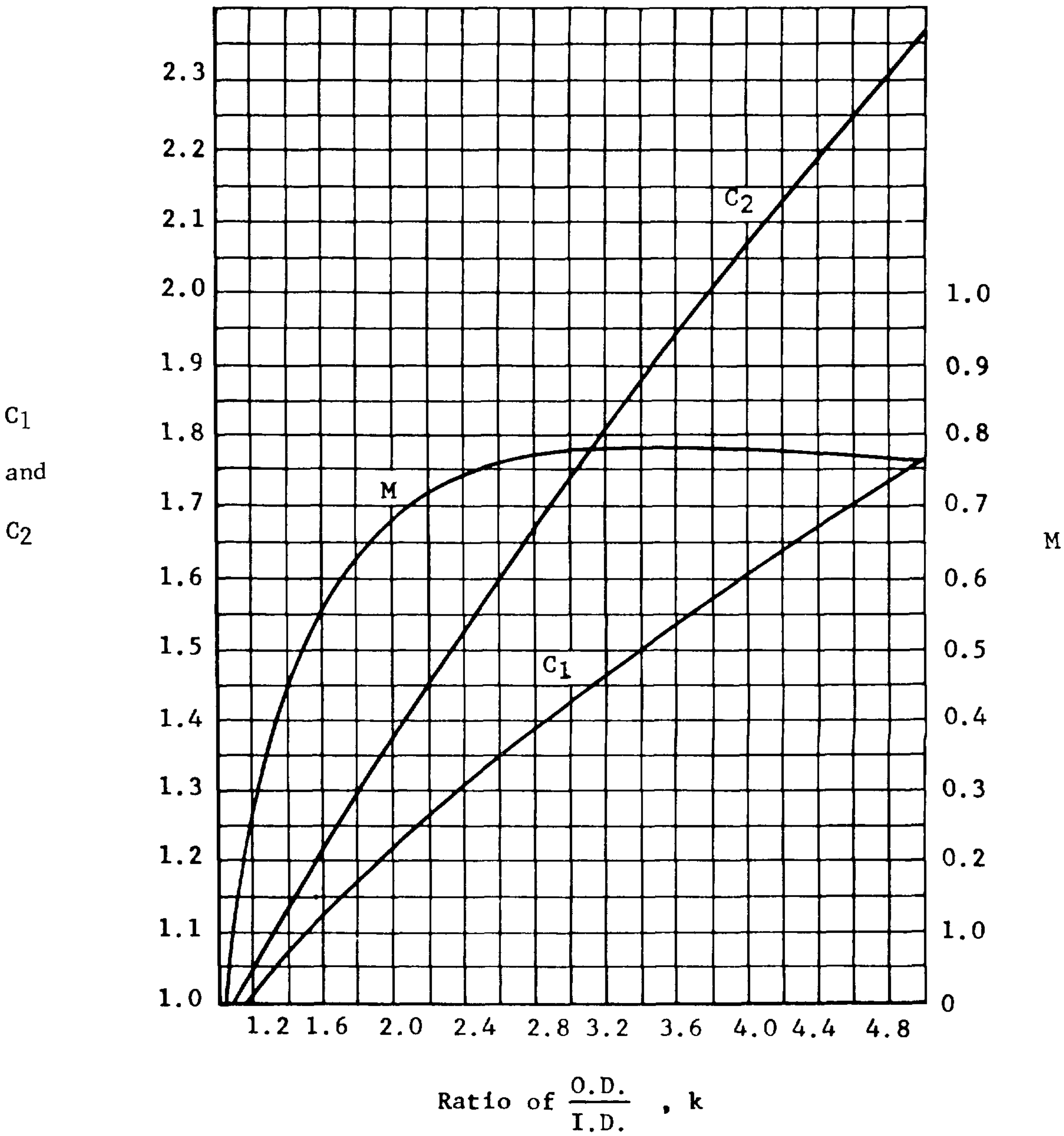


Fig.B 3.3.0-4 Belleville Spring Constants: M, C_1 and C_2

B 3.3.0 Belleville Springs or Washers (Cont'd)

Example Problem

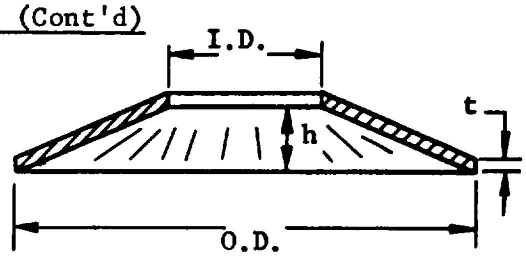


Fig. B 3.3.0-5

Given:

O.D. = 2"
I.D. = 1.25"
Load to deflect .02" = 675 lb.

Required:

Determine required thickness, t and dimension h.

Solution:

$$k = \frac{O.D.}{I.D.} = \frac{2.00}{1.25} = 1.6$$

The constants M, C_1 and C_2 may be taken from the curves in Fig. B 3.3.0-4 or may be calculated as follows:

$$M = \frac{6}{\pi \log_e k} \frac{(k-1)^2}{k^2} = \frac{6}{3.14(0.47)} \frac{(1.6 - 1.0)^2}{(1.6)^2} = 0.57$$

$$C_1 = \frac{6}{\pi \log_e k}\left[\frac{k-1}{\log_e k} - 1\right] = \frac{6}{3.14(0.47)}\left[\frac{1.6 - 1.0}{0.47} - 1\right] = 1.123$$

$$C_2 = \frac{6}{\pi \log_e k}\left[\frac{k-1}{2}\right] = \frac{6}{3.14(0.47)}\left[\frac{1.6 - 1.0}{2}\right] = 1.220$$

The Deflection-Stress Formula

$$f = \frac{E\delta}{(1-\nu^2)Ma^2}\left[C_1\left(h - \frac{\delta}{2}\right) + C_2 t\right]$$

may be written in the form

$$h = \frac{fMa^2(1-\nu^2)}{C_1 E\delta} + \frac{\delta}{2} - \frac{C_2}{C_1}t$$

B 3.3.0 Belleville Springs or Washers (Cont'd)

Assume that the washer shown in Fig. B 3.3.0-5 is steel

f_{max} = 200,000 psi

E = 30,000,000 psi

ν = .3

δ = 0.02 in.

M = 0.57

C_1 = 1.123

C_2 = 1.220

a = 1.00 (half outside dia., inches)

try t = .04 and solve for dimension h

$$h = \frac{200{,}000\ (1)^2(.57)(1-.3^2)}{(1.123)(.02)(30)10^6} + \frac{.02}{2} - \frac{1.220}{1.123}(.04) = .120 \text{ in.}$$

This value for (h) is then substituted in the deflection-load formula to obtain the load.

$$P = \frac{E\,\delta}{(1-\nu^2)M\,a^2}\left[\left(h - \frac{\delta}{2}\right)\left(h - \delta\right)t + t^3\right]$$

$$= \frac{30(10^6)(.02)}{(1-.3^2)(.57)(1)^2}\left[(.121-.01)(.121-.02)(.04)+(.04)^3\right] = 600 \text{ lb.}$$

Since this load is too low, the calculation is repeated using a stock thickness (t) of .05 in.

Then, solving again for h, the result is

h = .110 in.

Therefore, substituting this value of h into the formula used to calculate the previous value of load (P), the new value of (P) is

P = 665 lb.

This is as close as calculation need be carried. It is not expected that this or a similarly calculated spring will be deflected beyond the amount used in the calculation.

REFERENCES

Manuals

1. Chrysler Missile Operation, Design Practices, Sec. 108, Dtd. 1 November 1957.

2. Convair (Fort Worth), Structures Manual, Sec. 10.4.0, Vol. 1.

Handbook

1. Associated Spring Corporation, Mechanical Spring Design, Bristol, Conn.

Periodicals

1. Klaus, Thomas and Joachim Palm, Product Engineering, Design Digest Issue, Mid-September 1960.

Text Books

1. Seely, Fred, B. and Smith, J. O., Advanced Mechanics of Materials, Second Edition, John Wiley & Sons, Inc., New York, 1957.

2. Spotts, M. F., Design of Machine Elements, Second Edition, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1955.

SECTION B4

BEAMS

TABLE OF CONTENTS

B 4.0.0 BEAMS

B 4.1.0 Simple Beams

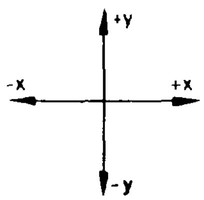
B 4.1.1 Shear, Moment, and Deflection

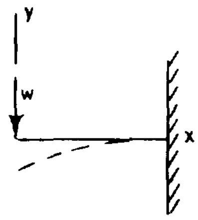
The general equations relating load, shear, bending moment, and deflection are given in Table B 4.1.1.1. These equations are given in terms of deflection and bending moments.

Title	Y	M
Deflection	$\Delta = y$	$\Delta = \iint \frac{M}{EI} dx\, dx$
Slope	$\theta = dy/dx$	$0 = \int \frac{M}{EI} dx$
Bending Moment	$M = EI\ d^2y/dx^2$	M
Shear	$V = EI\ d^3y/dx^3$	$V = dM/dx$
Load	$W = EI\ d^4y/dx^4$	$W = dv/dx = d^2M/dx^2$

Table B 4.1.1.1

Sign Convention





a) x is positive to the right.
b) y is positive upward.
c) M is positive when the compressed fibers are at the top.
d) W is positive in the direction of negative y.
e) V is positive when the part of the beam to the left of the section tends to move upward under the action of the resultant of the vertical forces.

The limiting assumptions are:

a) The material follows Hooke's Law.
b) Plane cross sections remain plane.
c) Shear deflections are negligible.
d) The deflections are small.

B 4.1.1 Shear, Moment and Deflection (Cont'd)

The deflection of short, deep beams due to vertical shear may need to be considered. The differential equation of the deflection curve including the effects of shearing deformation is:

$$y = \int\int \frac{M\, dxdx}{EI} + \int \frac{KV}{AG}\, dx$$

(K) is the ratio of the maximum shearing stress on the cross section to the average shearing stress. The value of (K) is given by the equation:

$$K = \frac{A}{I\ b} \int_{o}^{a} b' y dy$$

(I) is the moment of inertia of the cross-section with respect to the centrodial axis and (a), (b), (b'), and (y) are the dimensions shown in Fig. B 4.1.1-1.
(A) is the area of the cross-section

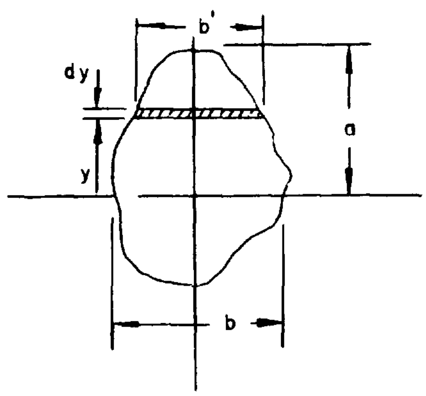
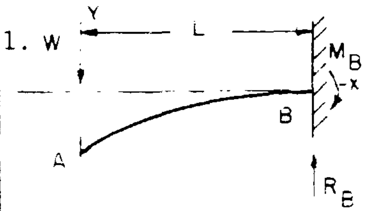
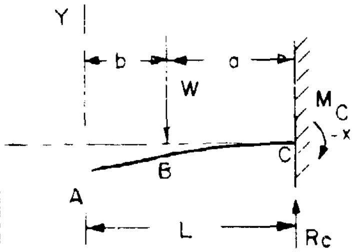


Fig. B 4.1.1-1

B4.1.1 Shear, Moment and Deflection (Cont'd)

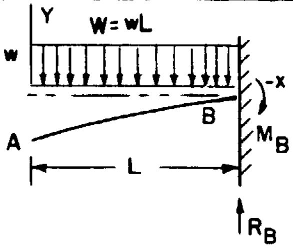
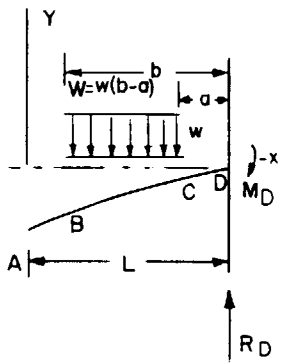
Table B 4.1.1-2 Beam Formulas

Notation: W = load (lb); w = unit load (lb. per linear in.); M is positive when clockwise; V is positive when acting upward; y is positive when upward. Constraining moments, applied couples, loads, and reactions are positive when acting as shown. All forces are in pounds, all moments in inch-pounds; all deflections and dimensions in inches. θ is in radians and $\tan \theta = \theta$.

Cantilever Beams	
Type of loading and Case number	Reactions, Vertical Shear, Bending Moments, Deflection y, and Slope
1. 	$R_B = +W;\ V = -W$ $M_x = -Wx$; Max $M = -WL$ at B $y = -\frac{1}{6}\frac{W}{EI}(x^3 - 3L^2x + 2L^3)$; Max $y = -\frac{1}{3}\frac{WL^3}{EI}$ at A; $\theta = \frac{1}{2}\frac{WL^2}{EI}$ at A
2. 	$R_C = +W$; (A to B) $V = 0$; (B to C) $V = -W$ (A to B) $M = 0$; (B to C) $M = -W(x-b)$; Max $M = -Wa$ at C (A to B) $y = -\frac{1}{6}\frac{W}{EI}(-a^3 + 3a^2L - 3a^2x)$; (B to C) $y = -\frac{1}{6}\frac{W}{EI}\left[(x-b)^3 - 3a^2(x-b) + 2a^3\right]$; Max $y = -\frac{1}{6}\frac{W}{EI}(3a^2L - a^3)$; $\theta = \frac{1}{2}\frac{Wa^2}{EI}$ (A to B)

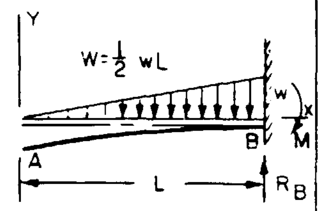
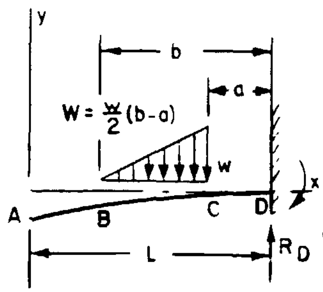
B 4.1.1 Shear, Moment and Deflection (Cont'd)

Table 4.1.1-2 Beam Formulas (Cont'd)

Cantilever Beams	
Type of loading and case number	Reactions, Vertical Shear, Bending Moments, Deflection Y, and Slope
3. 	$R_B = +W;\ V = -\frac{W}{L}x$ $M = -\frac{1}{2}\frac{W}{L}x^2;$ Max $M = -\frac{1}{2}WL$ at B $y = -\frac{1}{24}\frac{W}{EIL}(x^4-4L^3x+3L^4);$ Max $y = -\frac{1}{8}\frac{WL^3}{EI}$ $\theta = +\frac{1}{6}\frac{WL^2}{EI}$ at A
4. 	$R_D = +W;$ (A to B)$V = 0$: (B to C)$V = -\frac{W}{b-a}(x-L+b);$ (C to D)$V = -W$ (A to B)$M = 0$; (B to C)$M = -\frac{1}{2}\frac{W}{b-a}(x-L+b)^2;$ (C to D)$M = -\frac{1}{2}W(2x-2L+a+b);$ Max $M = -\frac{1}{2}W(a+b)$ at D (A to B)$y = -\frac{1}{24}\frac{W}{EI}\left[4(a^2+ab+b^2)(L-x)-a^3-ab^2-a^2b-b^3\right]$ (B to C)$y = -\frac{1}{24}\frac{W}{EI}\left[6(a+b)(L-x)^2-4(L-x)^3+\frac{(L-x-a)^4}{b-a}\right];$ (C to D)$y = -\frac{1}{12}\frac{W}{EI}\left[3(a+b)(L-x)^2-2(L-x)^3\right]$ Max $y = -\frac{1}{24}\frac{W}{EI}\left[4(a^2+ab+b^2)L-a^3-ab^2-a^2b-b^3\right]$ at A; $\theta = +\frac{1}{6}\frac{W}{EI}\left(a^2+ab+b^2\right)$ (A to B)

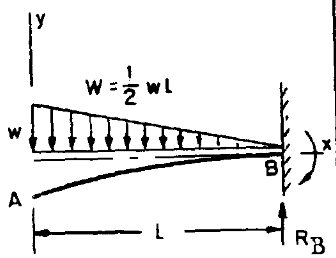
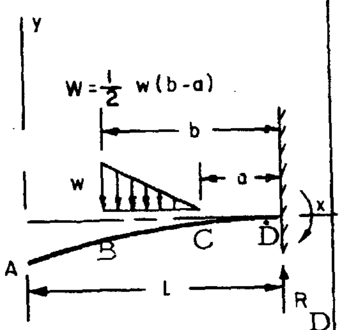
B 4.1.1 Shear, Moment and Deflection (Cont'd)

Table 4.1.1-2 Beam Formulas (Cont'd)

Cantilever Beams	
Type of loading and case number	Reactions, Vertical Shear, Bending Moments, Deflection Y, and Slope
5. 	$R_B = +W$; $V = -\frac{W}{L^2} x^2$ $M = -\frac{1}{3}\frac{W}{L^2} x^3$; Max $M = -\frac{1}{3} WL$ at B $y = -\frac{1}{60}\frac{W}{EIL^2}(x^5-5L^4x+4L^5)$; Max $y = -\frac{1}{15}\frac{WL^3}{EI}$ at A; $\theta = +\frac{1}{12}\frac{WL^2}{EI}$ at A
6. 	$R_D = +W$; (A to B)$V = 0$; (B to C)$V = -\frac{W(x-L+b)^2}{(b-a)^2}$; (C to D)$V = -W$ (A to B) $M = 0$; (B to C) $M = -\frac{1}{3}\frac{W(x-L+b)^3}{(b-a)^2}$; (C to D) $M = -\frac{1}{3}W(3x-3L+b+2a)$; Max $M = -\frac{1}{3}W(b+2a)$ at D (A to B) $y = -\frac{1}{60}\frac{W}{EI}\left[(5b^2+10ba+15a^2)(L-x) -4a^3 -2ab^2-3a^2b-b^3\right]$ (B to C) $y = -\frac{1}{60}\frac{W}{EI}\left[(20a+10b)(L-x)^2 -10(L-x)^3+5\frac{(L-x-a)^4}{b-a} - \frac{(L-x-a)^5}{(b-a)^2}\right]$; (C to D) $y = -\frac{1}{6}\frac{W}{EI}\left[(2a+b)(L-x)^2 -(L-x)^3\right]$; Max $Y = -\frac{1}{60}\frac{W}{EI}\left[(5b^2+10ba+15a^2)L-4a^3-2ab^2-3a^2b-b^3\right]$ at A $\theta = +\frac{1}{12}\frac{W}{EI}(3a^2+2ab+b^2)$ (A to B)

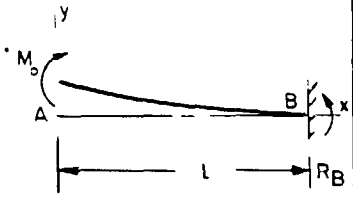
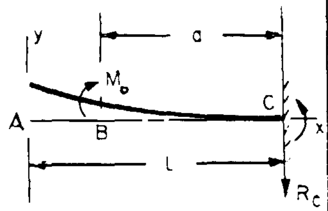
B4.1.1 Shear, Moment and Deflection (Cont'd)

Table 4.1.1-2 Beam Formulas (Cont'd)

Cantilever Beams	
Type of loading and case number	Reactions, Vertical Shear, Bending Moments, Deflection Y, and Slope
7. 	$R_B = +W;\ V = -W\left(\frac{2Lx-x^2}{L^2}\right)$ $M = -\frac{1}{3}\frac{W}{L^2}(3Lx^2-x^3)$; Max $M = -\frac{2}{3}WL$ at B $y = -\frac{1}{60}\frac{W}{EIL^2}(-x^5-15L^4x+5LX^4+11L^5)$; Max $Y = -\frac{11}{60}\frac{WL^3}{EI}$ at A $\theta = +\frac{1}{4}\frac{W}{EI}L^2$ at A
8. 	$R_D = +W$; (A to B) $V = 0$; (B to C) $V = -W\left[1-\frac{(L-a-x)^2}{(b-a)^2}\right]$; (C to D) $V = -W$ (A to B) $M = 0$; (B to C) $M = -\frac{1}{3}W\left[\frac{3(x-L+b)^2}{b-a}-\frac{(x-L+b)^3}{(b-a)^2}\right]$ (C to D) $M = -\frac{1}{3}W(-3L+3x+2b+a)$; Max $M = -\frac{1}{3}W(2b+a)$ at D (A to B) $y = -\frac{1}{60}\frac{W}{EI}\left[(5a^2+10ab+15b^2)(L-x)-a^3-2a^2b-3ab^2-4b^3\right]$; (B to C) $y = -\frac{1}{60}\frac{W}{EI}\left[\frac{(L-x-a)^5}{(b-a)^2}-10(L-x)^3+(10a+20b)(L-x)^2\right]$ (C to D) $y = -\frac{1}{6}\frac{W}{EI}\left[(a+2b)(L-x)^2-(L-x)^3\right]$ Max $Y = -\frac{1}{60}\frac{W}{EI}\left[(5a^2+10ab+15b^2)L-a^3-2a^2b-3ab^2-4b^3\right]$ at A $\theta = +\frac{1}{12}\frac{W}{EI}(a^2+2ab+3b^2)$ (A to B)

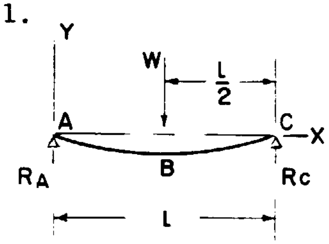
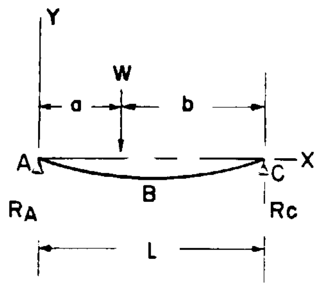
B 4.1.1 Shear, Moment and Deflection (Cont'd)

Table 4.1.1-2 Beam Formulas (Cont'd)

Cantilever Beams	
Type of loading and case number	Reactions, Vertical Shear, Bending Moments, Deflection Y, and Slope
9. 	$R_B = 0; V = o$ $M = M_o$; Max $M = M_o$ (A to B) $y = \frac{1}{2}\frac{M_o}{EI}(L^2-2Lx+x^2)$; Max $y = \frac{1}{2}\frac{M_oL^2}{EI}$ at A; $\theta = -\frac{M_oL}{EI}$ at A
10. 	$R_C = 0; V = 0$ (A to B) $M = 0$; (B to C) $M = M_o$; Max $M = M_o$ (B to C) (A to B) $y = \frac{M_oa}{EI}\left(L - \frac{1}{2}a - x\right)$; (B to C) $y = \frac{1}{2}\frac{M_o}{EI}\left[(x-L+a)^2 - 2a(x-L+a) + a^2\right]$ Max $y = \frac{M_oa}{EI}\left(L - \frac{1}{2}a\right)$ at A; $\theta = -\frac{M_oa}{EI}$ (A to B)

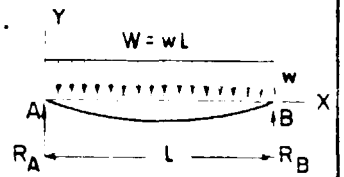
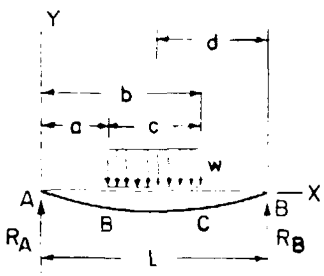
B 4.1.1 <u>Shear, Moment and Deflection (Cont'd)</u>

Table 4.1.1-2 Beam Formulas (Cont'd)

Simply Supported Beams	
Type of loading and case number	Reactions, Vertical Shear, Bending Moments, Deflection Y, and Slope
11. 	$R_A = \frac{1}{2} W$; $R_C = \frac{1}{2} W$; (A to B) $V = \frac{1}{2} W$; (B to C) $V = -\frac{1}{2} W$ (A to B) $M = \frac{1}{2} Wx$; (B to C) $M = \frac{1}{2} W (L-x)$; Max $M = \frac{1}{4} WL$ at B (A to B) $y = -\frac{1}{48} \frac{W}{EI} (3L^2x-4x^3)$; Max $y = -\frac{1}{48} \frac{WL^3}{EI}$ at B; $\theta = -\frac{1}{16} \frac{WL^2}{EI}$ at A; $\theta = +\frac{1}{16} \frac{WL^2}{EI}$ at C
12. 	$R_A = +\frac{Wb}{L}$; $R_C = +\frac{Wa}{L}$; (A to B) $V = +\frac{Wb}{L}$; (B to C) $V = -\frac{Wa}{L}$ (A to B) $M = +\frac{Wb}{L} x$; (B to C) $M = +\frac{Wa}{L} (L-x)$; Max $M = +\frac{Wab}{L}$ at B (A to B) $y = -\frac{Wbx}{6EIL} \left[2L(L-x) - b^2 - (L-x)^2\right]$; (B to C) $y = -\frac{Wa(L-x)}{6\ EIL} \left[2\ Lb - b^2 - (L-x)^2\right]$; Max $y = -\frac{Wab}{27EIL} (a+2b) \sqrt{3a(a+2b)}$ at $x = \sqrt{\frac{a}{3}(a+2b)}$ When $a > b$; $\theta = -\frac{1}{6} \frac{W}{EI} \left(bL - \frac{b^3}{L}\right)$ at A; $\theta = +\frac{1}{6} \frac{W}{EI} \left(2bL + \frac{b^3}{L} - 3b^2\right)$ at C

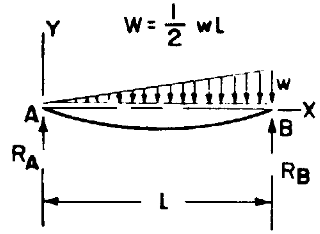
B4.1.1 Shear, Moment and Deflection (Cont'd)

Table 4.1.1-2 Beam Formulas (Cont'd)

Simply Supported Beams	
Type of loading and case number	Reactions, Vertical Shear, Bending Moments, Deflection Y, and Slope
13. 	$R_A = \frac{W}{2}$; $R_B = \frac{W}{2}$; $V = +\frac{W}{2}\left(1 - \frac{2x}{L}\right)$; $M = +\frac{W}{2}\left(x - \frac{x^2}{L}\right)$ Max. $M = +\frac{WL}{8}$ at $x = \frac{L}{2}$; $y = -\frac{Wx}{24EIL}(L^3 - 2Lx^2 + x^3)$; Max. $y = -\frac{5WL^3}{384EI}$ at $x = \frac{L}{2}$; $\theta = -\frac{WL^2}{24EI}$ at A; $\theta = +\frac{WL^2}{24EI}$ at B
14. W = wc $d = L - \frac{1}{2}b - \frac{1}{2}a$	$R_A = \frac{Wd}{L}$; $R_D = \frac{W}{L}\left(a + \frac{c}{2}\right)$; (A to B) $V = R_A$; (B to C) $V = R_A - \frac{W(x-a)}{c}$ (C to D) $V = R_A - W$; (A to B) $M = R_A x$; (B to C) $M = R_A x - \frac{W(x-a)^2}{2c}$ (C to D) $M = R_A x - W\left(x - \frac{a}{2} - \frac{b}{2}\right)$; Max. $M = \frac{Wd}{L}\left(a + \frac{cd}{2L}\right)$ at $x = a + \frac{cd}{L}$ (A to B) $y = \frac{1}{48EI}\left\{8R_A(x^3 - L^2x) + Wx\left[\frac{8d^3}{L} - \frac{2bc^2}{L} + \frac{c^3}{L} + 2c^2\right]\right\}$ (B to C) $y = \frac{1}{48EI}\left\{8R_A(x^3 - L^2x) + Wx\left[\frac{8d^3}{L} - \frac{2bc^2}{L} + \frac{c^3}{L} + 2c^2\right] - \frac{2W(x-a)^4}{c}\right\}$ (C to D) $y = \frac{1}{48EI}\left\{8R_A(x^3 - L^2x) + Wx\left[\frac{8d^3}{L} - \frac{2bc^2}{L} + \frac{c^3}{L}\right] - 8W\left(x - \frac{a}{2} - \frac{b}{2}\right)^3 + W(2bc^2 - c^3)\right\}$

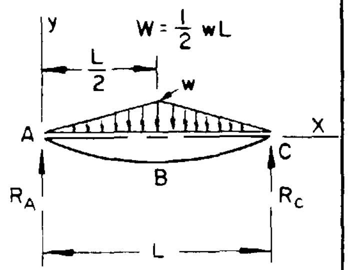
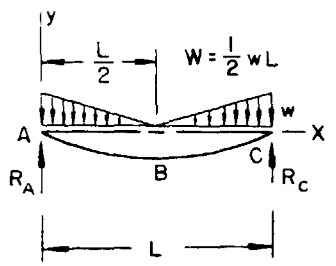
B 4.1.1 Shear, Moment and Deflection (Cont'd)

Table 4.1.1-2 Beam Formulas (Cont'd)

Simply Supported Beams	
Type of loading and case number	Reactions, Vertical Shear, Bending Moments, Deflection Y, and Slope
14. (Cont'd)	$\theta = \frac{1}{48EI}\left[-8R_AL^2 + W\left(\frac{8d^3}{L} - \frac{2bc^2}{L} + \frac{c^3}{L} + 2c^2\right)\right]$ at A $\theta = \frac{1}{48EI}\left[16R_AL^2 - W\left(24d^2 - \frac{8d^3}{L} + \frac{2bc^2}{L} - \frac{c^3}{L}\right)\right]$ at B
15. 	$R_A = \frac{W}{3}$; $R_B = \frac{2W}{3}$; $V = W\left(\frac{1}{3} - \frac{x^2}{L^2}\right)$; $M = \frac{W}{3}\left(x - \frac{x^3}{L^2}\right)$; Max. $M = 0.128\ WL$ at $x = \frac{L\sqrt{3}}{3}$; $y = \frac{-Wx(3x^4 - 10L^2x^2 + 7L^4)}{180EIL^2}$ Max. $y = -\frac{0.01304\ WL^3}{EI}$ at $x = 0.519\ L$ $\theta = -\frac{7WL^2}{180EI}$ at A ; $\theta = \frac{8WL^2}{180EI}$ at B

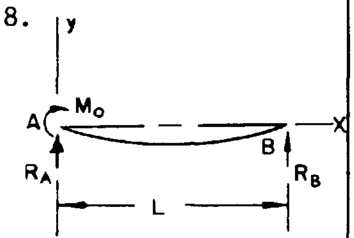
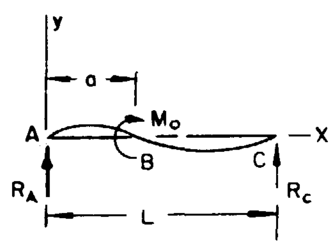
B 4.1.1 Shear, Moment and Deflection (Cont'd)

Table 4.1.1-2 Beam Formulas (Cont'd)

Simply Supported Beams	
Type of loading and case number	Reactions, Vertical Shear, Bending Moments, Deflection Y, and Slope
16. 	$R_A = \frac{W}{2}$; $R_C = \frac{W}{2}$; (A to B) $V = \frac{W}{2}\left(1 - \frac{4x^2}{L^2}\right)$; (B to C) $V = -\frac{W}{2}\left[1 - \frac{4(L-x)^2}{L^2}\right]$; (A to B) $M = \frac{W}{6}\left(3x - \frac{4x^3}{L^2}\right)$ (B to C) $M = \frac{W}{6}\left[3(L-x) - \frac{4(L-x)^3}{L^2}\right]$; Max. $M = \frac{WL}{6}$ at B (A to B) $y = \frac{Wx}{6EIL^2}\left(\frac{L^2x^2}{2} - \frac{x^4}{5} - \frac{5L^4}{16}\right)$; Max. $y = -\frac{WL^3}{60EI}$ at B $\theta = -\frac{5WL^2}{96EI}$ at A ; $\theta = +\frac{5WL^2}{96EI}$ at C
17. 	$R_A = \frac{W}{2}$; $R_B = \frac{W}{2}$; (A to B) $V = \frac{W}{2}\left(\frac{L-2x}{L}\right)^2$ (B to C) $V = -\frac{W}{2}\left(\frac{2x-L}{L}\right)^2$; (A to B) $M = \frac{W}{2}\left(x - \frac{2x^2}{L} + \frac{4x^3}{3L^2}\right)$ (B to C) $M = \frac{W}{2}\left[(L-x) - \frac{2(L-x)^2}{L} + \frac{4(L-x)^3}{3L^2}\right]$; Max. $M = \frac{WL}{12}$ at B (A to B) $y = \frac{W}{12EI}\left(x^3 - \frac{x^4}{L} + \frac{2x^5}{5L^2} - \frac{3L^2x}{8}\right)$; Max. $y = -\frac{3WL^3}{320EI}$ at B $\theta = -\frac{WL^2}{32EI}$ at A; $\theta = \frac{WL^2}{32EI}$ at C

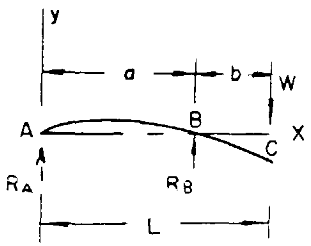
B 4.1.1 Shear, Moment and Deflection (Cont'd)

Table 4.1.1-2 Beam Formulas (Cont'd)

Simply Supported Beams	
Type of loading and case number	Reactions, Vertical Shear, Bending Moments, Deflection Y, and Slope
18. 	$R_A = -\frac{M_o}{L}$; $R_B = \frac{M_o}{L}$; $V = R_A$; $M = M_o + R_A x$; Max. $M = M_o$ at A $y = \frac{M_o}{6EI}\left(3x^2 - \frac{x^3}{L} - 2Lx\right)$; Max. $y = -0.0642\frac{M_o L^2}{EI}$ at $x = 0.422L$ $\theta = -\frac{M_o L}{3EI}$ at A; $\theta = \frac{M_o L}{6EI}$ at B
19. 	$R_A = -\frac{M_o}{L}$; $R_C = \frac{M_o}{L}$; (A to C) $V = +R_A$; (A to B) $M = +R_A x$ (B to C) $M = +R_A x + M_o$; Max. $(-M) = +R_A a$ just left of B Max. $(+M) = +R_A a + M_o$ just right of B (A to B) $y = \frac{M_o}{6EI}\left[\left(6a - \frac{3a^2}{L} - 2L\right)x - \frac{x^3}{L}\right]$ (B to C) $y = \frac{M_o}{6EI}\left[3a^2 + 3x^2 - \frac{x^3}{L} - \left(2L + \frac{3a^2}{L}\right)x\right]$ $\theta = -\frac{M_o}{6EI}\left(2L - 6a + \frac{3a^2}{L}\right)$ at A; $\theta = \frac{M_o}{6EI}\left(L - \frac{3a^2}{L}\right)$ at C $\theta = \frac{M_o}{EI}\left(a - \frac{a^2}{L} - \frac{L}{3}\right)$ at B

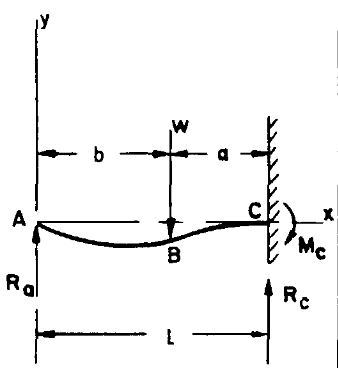
B4.1.1 Shear, Moment and Deflection (Cont'd)

Table 4.1.1-2 Beam Formulas (Cont'd)

Simply Supported Beams	
Type of loading and case number	Reactions, Vertical Shear, Bending Moments, Deflection Y, and Slope
20. 	$R_A = \frac{-Wb}{a}$; $R_B = \frac{WL}{a}$; (A to B) $V = +R_A$; (B to C) $V = +W$; (A to B) $M = +R_A x$; (B to C) $M = +R_A a + W(x-a)$; Max. $M = +R_A a$ at B (A to B) $y = \frac{-Wbx}{6aEI}(x^2-a^2)$; (B to C) $y = \frac{-W}{6EI}\left[(L-x)^3 - b(L-x)(2L-b) + 2b^2L\right]$ Max. $y = \frac{-Wb^2L}{3EI}$ at C; $\theta = \frac{Wab}{6EI}$ at A; $\theta = -\frac{Wab}{3EI}$ at B $\theta = -\frac{Wb}{6EI}(2L+b)$ at C

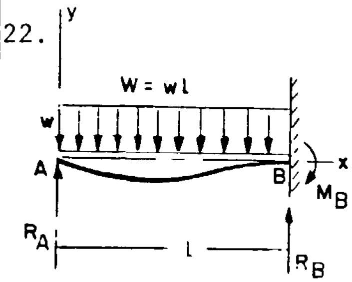
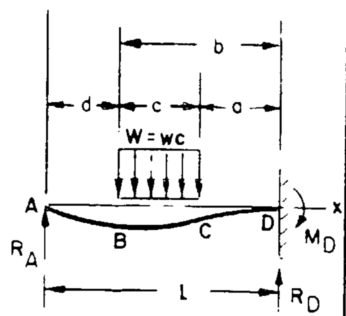
B 4.1.1 Shear, Moment and Deflection (Cont'd)

Table 4.1.1-2 Beam Formulas (Cont'd)

Statically Indeterminate Cases	
Type of loading and case number	Reactions, Vertical Shear, Bending Moments, Deflection Y, and Slope
21.  	$R_A = \frac{W}{2}\left(\frac{3a^2L-a^3}{L^3}\right)$; $R_C = W - R_A$; $M_c = \frac{W}{2}\left(\frac{a^3+2aL^2-3a^2L}{L^2}\right)$ (A to B) $V = R_A$; (B to C) $V = R_A - W$; (A to B) $M = R_Ax$ (B to C) $M = R_Ax - W(x-L+a)$; Max. $(+M) = R_A(L-a)$ at B max. possible value = 0.174 WL when a = 0.634 L Max. $(-M) = -M_c$ at C max. possible value = -0.1927 WL when a = 0.4227 L (A to B) $y = \frac{1}{6EI}\left[R_A(x^3-3L^2x) + 3Wa^2x\right]$ (B to C) $y = \frac{1}{6EI}\left\{R_A(x^3-3L^2x) + W\left[3a^2x - (x-b)^3\right]\right\}$ If $a < 0.586$ L, max. y is between A and B at $x = L\sqrt{1 - \frac{2L}{3L-a}}$ If $a > 0.586$ L, max. y is at $x = \frac{L(L^2+b^2)}{3L^2-b^2}$ If a = 0.586 L, max. y is at B and = $-0.0098 \frac{WL^3}{EI}$, max. possible deflection $\theta = \frac{W}{4EI}\left(\frac{a^3}{L} - a^2\right)$ at A

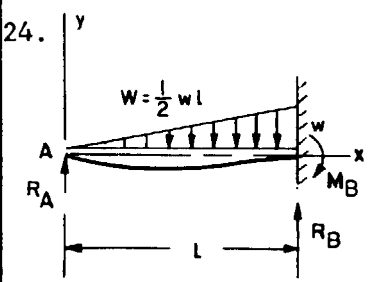
B4.1.1 Shear, Moment and Deflection (Cont'd)

Table 4.1.1-2 Beam Formulas (Cont'd)

Statically Indeterminate Cases	
Type of loading and case number	Reactions, Vertical Shear, Bending Moments, Deflection Y, and Slope
22. 	$R_A = \frac{3W}{8}$; $R_B = \frac{5W}{8}$; $M_B = \frac{WL}{8}$; $V = W\left(\frac{3}{8} - \frac{x}{L}\right)$; $M = W\left(\frac{3x}{8} - \frac{x^2}{2L}\right)$ Max. $(+M) = \frac{9WL}{128}$ at $x = \frac{3L}{8}$; Max. $(-M) = -\frac{WL}{8}$ at B $y = \frac{W}{48EIL}(3Lx^3 - 2x^4 - L^3x)$; Max. $y = -0.0054\frac{WL^3}{EI}$ at $x = 0.4215\ L$ $\theta = -\frac{WL^2}{48EI}$ at A
23. 	$R_A = \frac{W}{8L^3}\left[4L(a^2+ab+b^2) - a^3 - ab^2 - a^2b - b^3\right]$; $R_D = W - R_A$; $M_D = -R_AL + \frac{W}{2}(a+b)$; (A to B) $V = R_A$; (B to C) $V = R_A - W\left(\frac{x-d}{c}\right)$ (C to D) $V = R_A - W$; (A to B) $M = R_Ax$; (B to C) $M = R_Ax - \frac{W(x-d)^2}{2c}$ (C to D) $M = R_Ax - W\left(x - d - \frac{c}{2}\right)$; Max. $(+M) = R_A\left(d + \frac{R_Ac}{2W}\right)$ at $x = \left(d + \frac{R_Ac}{W}\right)$; Max. $(-M) = -M_D$ (A to B) $y = \frac{1}{EI}\left[R_A\left(\frac{x^3}{6} - \frac{L^2x}{2}\right) + Wx\left(\frac{a^2}{2} + \frac{ac}{2} + \frac{c^2}{6}\right)\right]$ (B to C) $y = \frac{1}{EI}\left[R_A\left(\frac{x^3}{6} - \frac{L^2x}{2}\right) + Wx\left(\frac{a^2}{2} + \frac{ac}{2} + \frac{c^2}{6}\right) - \frac{W(x-d)^4}{24c}\right]$

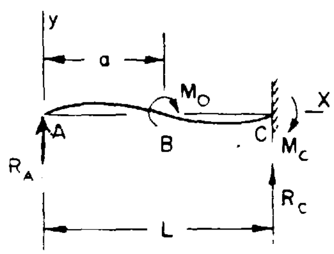
B 4.1.1 Shear, Moment and Deflection (Cont'd)

Table 4.1.1-2 Beam Formulas (Cont'd)

Statically Indeterminate Cases	
Type of loading and case number	Reactions, Vertical Shear, Bending Moments, Deflection Y, and Slope
23. (Cont'd)	(C to D) $y = \frac{1}{EI}\left\{R_A\left(\frac{x^3}{6} - \frac{L^2x}{2} + \frac{L^3}{3}\right) + W\left[\frac{1}{6}\left(a + \frac{c}{2}\right)^3 - \frac{1}{2}\left(a + \frac{c}{2}\right)^2 L - \frac{1}{6}\left(x - d - \frac{c}{2}\right)^3 + \frac{1}{2}\left(a \div \frac{c}{2}\right)^2 x\right]\right\}$ $\theta = -\frac{1}{EI}\left[\frac{R_AL^2}{2} - W\left(\frac{a^2}{2} + \frac{ac}{2} + \frac{c^2}{6}\right)\right]$ at A
24. 	$R_A = \frac{W}{5}$; $R_B = \frac{4W}{5}$; $M_B = \frac{2WL}{15}$; $V = W\left(\frac{1}{5} - \frac{x^2}{L^2}\right)$; $M = W\left(\frac{x}{5} - \frac{x^3}{3L^2}\right)$ Max. (+M) = 0.06 WL at x = 0.4474 L; Max. (-M) = $-M_B$ $y = \frac{W}{60EIL}\left(2Lx^3 - L^3x - \frac{x^5}{L}\right)$; Max. $y = -0.00477\frac{WL^3}{EI}$ at $x = L\sqrt{\frac{1}{5}}$ $\theta = -\frac{WL^2}{60EI}$ at A

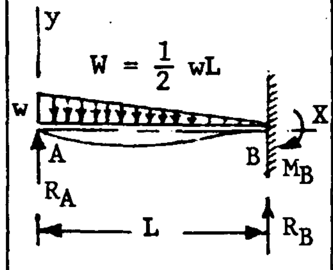
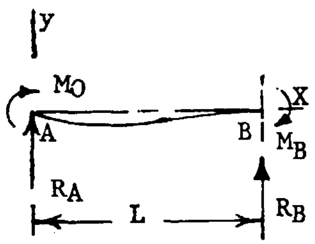
B4.1.1 Shear, Moment and Deflection (Cont'd)

Table 4.1.1-2 Beam Formulas (Cont'd)

Statically Indeterminate Cases	
Type of loading and case number	Reactions, Vertical Shear, Bending Moments, Deflection Y, and Slope
25. 	$R_A = -\frac{3M_o}{2L}\left(\frac{L^2-a^2}{L^2}\right)$; $R_C = \frac{3M_o}{2L}\left(\frac{L^2-a^2}{L^2}\right)$; $M_c = \frac{M_o}{2}\left(1 - \frac{3a^2}{L^2}\right)$ (A to B) $V = +R_A$; (B to C) $V = +R_A$; (A to B) $M = +R_A x$ (B to C) $M = +R_A x + M_o$; Max. $(+M) = M_o\left[1 - \frac{3a(L^2-a^2)}{2L^3}\right]$ just to the right of B Max. $(-M) = -M_c$ at C when $a < 0.275\,L$ Max. $(-M) = R_A a$ to the left of B when $a > 0.275\,L$ (A to B) $y = \frac{M_o}{EI}\left[\frac{L^2-a^2}{4L^3}(3L^2x-x^3) - (L-a)x\right]$ (B to C) $y = \frac{M_o}{EI}\left[\frac{L^2-a^2}{4L^3}(3L^2x-x^3) - Lx + \frac{(x^2+a^2)}{2}\right]$ $\theta = \frac{M_o}{EI}\left(a - \frac{L}{4} - \frac{3a^2}{4L}\right)$ at A

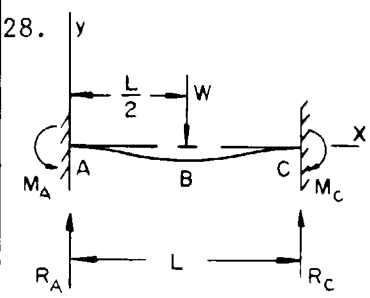
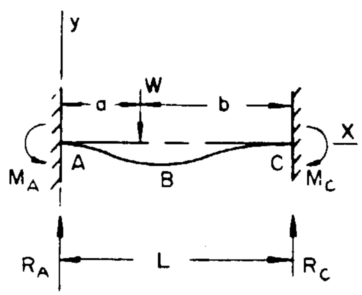
B4.1.1 Shear, Moment and Deflection (Cont'd)

Table 4.1.1-2 Beam Formulas (Cont'd)

Type of loading and case number	Statically Indeterminate Cases Reactions, Vertical Shear, Bending Moments, Deflection Y, and Slope
26.  	$R_A = \frac{11W}{20}$; $R_B = \frac{9W}{20}$; $M_B = \frac{7WL}{60}$; $V = W\left(\frac{11}{20} - \frac{2x}{L} + \frac{x^2}{L^2}\right)$; $M = W\left(\frac{11x}{20} - \frac{x^2}{L} + \frac{x^3}{3L^2}\right)$ Max. (+M) = 0.0846 WL at x = 0.329 L; Max. (-M) = $-\frac{7WL}{60}$ at B $y = \frac{W}{120EIL}\left(11Lx^3 - 3L^3x - 10x^4 + \frac{2x^5}{L}\right)$ Max. $y = -0.00609 \frac{WL^3}{EI}$ at x = 0.402 L; $= \frac{WL^2}{40EI}$ at A
27.  	$R_A = -\frac{3M_O}{2L}$; $R_B = \frac{3M_O}{2L}$; $M_B = \frac{M_O}{2}$; $V = -\frac{3M_O}{2L}$; $M = \frac{M_O}{2}\left(2 - \frac{3x}{L}\right)$ Max. (+M) = M_o at A; Max. (-M) = $-\frac{M_O}{2}$ at B $y = \frac{M_O}{4EI}\left(2x^2 - \frac{x^3}{L} - Lx\right)$; Max. $y = -\frac{M_OL^2}{27EI}$ at $x = \frac{L}{3}$ $\theta = -\frac{M_OL}{4EI}$ at A

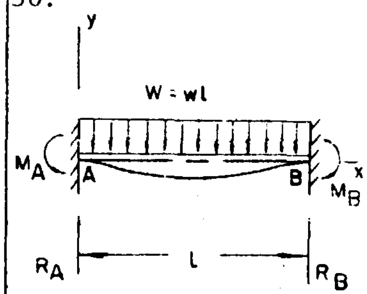
B 4.1.1 Shear, Moment and Deflection (Cont'd)

Table 4.1.1-2 Beam Formulas (Cont'd)

Statically Indeterminate Cases	
Type of loading and case number	Reactions, Vertical Shear, Bending Moments, Deflection Y, and Slope
28. 	$R_A = \frac{W}{2}$; $R_C = \frac{W}{2}$; $M_A = \frac{WL}{8}$; $M_c = \frac{WL}{8}$; (A to B) $V = \frac{W}{2}$; (B to C) $V = -\frac{W}{2}$ (A to B) $M = \frac{W(4x-L)}{8}$; (B to C) $M = \frac{W(3L-4x)}{8}$; Max. $(+M) = \frac{WL}{8}$ at B Max. $(-M) = -\frac{WL}{8}$ at A and C; (A to B) $y = -\frac{W}{48EI}(3Lx^2-4x^3)$ Max. $y = -\frac{WL^3}{192EI}$ at B
29. 	$R_A = \frac{Wb^2}{L^3}(3a+b)$; $R_C = \frac{Wa^2}{L^3}(3b+a)$; $M_a = \frac{Wab^2}{L^2}$; $M_c = \frac{Wba^2}{L^2}$ (A to B) $V = R_A$; (B to C) $V = R_A - W$; (A to B) $M = -\frac{Wab^2}{L^2} + R_A x$ (B to C) $M = -\frac{Wab^2}{L^2} + R_A x - W(x-a)$; Max. $(+M) = -\frac{Wab^2}{L^2} + R_A a$ at B, max. value $= \frac{WL}{8}$ when $a = \frac{L}{2}$ Max. $(-M) = -M_A$ when $a < b$, max. possible value = -0.1481 WL when $a = \frac{L}{3}$ Max. $(-M) = -M_c$ when $a > b$, max. possible value = -0.1481 WL when $a = \frac{2L}{3}$

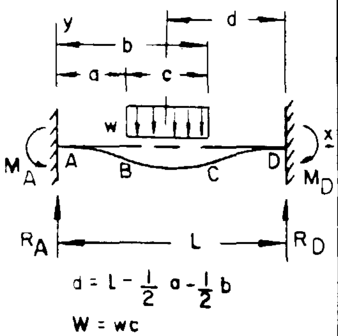
B4.1.1 Shear, Moment and Deflection (Cont'd)

Table 4.1.1-2 Beam Formulas (Cont'd)

Statically Indeterminate Cases	
Type of loading and case number	Reactions, Vertical Shear, Bending Moments, Deflection Y, and Slope
29. (Cont'd)	(A to B) $y = \frac{Wb^2x^2}{6EIL^3}(3ax + bx - 3aL)$ (B to C) $y = \frac{Wa^2(L-x)^2}{6EIL^3}\left[(3b+a)(L-x) - 3bL\right]$ Max. $y = -\frac{2Wa^3b^2}{3EI(3a+b)^2}$ at $x = \frac{2aL}{3a+b}$ if $a > b$ Max. $y = -\frac{2Wa^2b^3}{3EI(3b+a)^2}$ at $x = L - \frac{2bL}{3b+a}$ if $a < b$
30. 	$R_A = \frac{W}{2}$; $R_B = \frac{W}{2}$; $M_A = \frac{WL}{12}$; $M_B = \frac{WL}{12}$; $V = \frac{W}{2}(1 - \frac{2x}{L})$ $M = \frac{W}{2}\left(x - \frac{x^2}{L} - \frac{L}{6}\right)$; Max. $(+M) = \frac{WL}{24}$ at $x = \frac{L}{2}$ Max. $(-M) = -\frac{WL}{12}$ at A and B; $y = \frac{Wx^2}{24EIL}(2Lx - L^2 - x^2)$ Max. $y = \frac{-WL^3}{384EI}$ at $x = \frac{L}{2}$

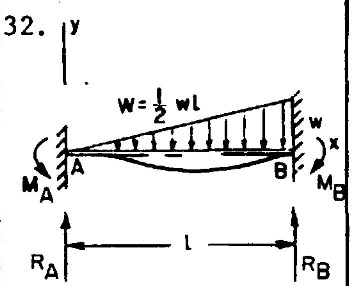
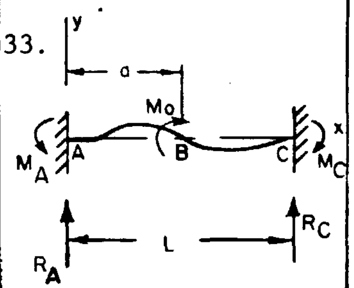
B 4.1.1 Shear, Moment and Deflection (Cont'd)

Table 4.1.1-2 Beam Formulas (Cont'd)

Statically Indeterminate Cases	
Type of loading and case number	Reactions, Vertical Shear, Bending Moments, Deflection Y, and Slope
31. 	$R_A = \frac{W}{4L^2}\left(12d^2 - \frac{8d^3}{L} + \frac{2bc^2}{L} - \frac{c^3}{L} - c^2\right)$; $R_D = W - R_A$ $M_A = -\frac{W}{24L}\left(\frac{24d^3}{L} - \frac{6bc^2}{L} + \frac{3c^3}{L} + 4c^2 - 24d^2\right)$ $M_D = \frac{W}{24L}\left(\frac{24d^3}{L} - \frac{6bc^2}{L} + \frac{3c^3}{L} + 2c^2 - 48d^2 + 24dL\right)$; (A to B) $V = R_A$ (B to C) $V = R_A - W\left(\frac{x-a}{c}\right)$; (C to D) $V = R_A - W$; (A to B) $M = -M_A + R_A x$ (B to C) $M = -M_A + R_A x - W\frac{(x-a)^2}{2c}$; (C to D) $M = -M_A + R_A x - W(x-L+d)$ Max. (+M) is between B and C at $x = a + \frac{R_A c}{W}$; Max. $(-M) = -M_A$ when $a < (L-b)$; Max. $(-M) = -M_D$ when $a > (L-b)$ (A to B) $y = \frac{1}{6EI}(R_A x^3 - 3M_A x^2)$ (B to C) $y = \frac{1}{6EI}\left(R_A x^3 - 3M_A x^2 - \frac{W(x-a)^4}{4c}\right)$ (C to D) $y = \frac{1}{6EI}\left[R_D(L-x)^3 - 3M_D(L-x)^2\right]$

B 4.1.1 Shear, Moment and Deflection (Cont'd)

Table 4.1.1-2 Beam Formulas (Cont'd)

Statically Indeterminate Cases	
Type of loading and case number	Reactions, Vertical Shear, Bending Moments, Deflection Y, and Slope
32. 	$R_A = \frac{3W}{10}$; $R_B = \frac{7W}{10}$; $M_A = \frac{WL}{15}$; $M_B = \frac{WL}{10}$; $V = W\left(\frac{3}{10} - \frac{x^2}{L^2}\right)$ $M = W\left(\frac{3x}{10} - \frac{L}{15} - \frac{x^3}{3L^2}\right)$ Max. (+M) = 0.043 WL at x = 0.548 L; Max. (-M) = $-\frac{WL}{10}$ at B $y = \frac{W}{60EI}\left(3x^3 - 2Lx^2 - \frac{x^5}{L^2}\right)$; Max. y = -0.002617 $\frac{WL^3}{EI}$ at x = 0.525 L
33. 	$R_A = -\frac{6M_o}{L^3}(aL-a^2)$; $R_C = \frac{6M_o}{L^3}(aL-a^2)$; $M_A = -\frac{M_o}{L^2}(4La-3a^2-L^2)$ $M_C = \frac{M_o}{L^2}(2La-3a^2)$; $V = +R_A$; (A to B) $M = -M_A + R_Ax$ (B to C) $M = -M_A + R_Ax + M_o$ Max. (+M) $= M_o\left(\frac{4a}{L} - \frac{9a^2}{L^2} + \frac{6a^3}{L^3}\right)$ just to the right of B Max. (-M) $= M_o\left(\frac{4a}{L} - \frac{9a^2}{L^2} + \frac{6a^3}{L^3} - 1\right)$ just to the left of B (A to B) $y = \frac{-1}{6EI}(3M_Ax^2 - R_Ax^3)$

B4.1.1 Shear, Moment and Deflection (Cont'd)

Table 4.1.1-2 Beam Formulas (Cont'd)

Type of loading and case number	Statically Indeterminate Cases: Reactions, Vertical Shear, Bending Moments, Deflection Y, and Slope
33. (Cont'd)	(B to C) $y = \frac{1}{6EI} \left[(M_o - M_A)(3x^2 - 6Lx + 3L^2) - R_A (3L^2 x - x^3 - 2L^3) \right]$ Max (-y) at $x = L - \frac{2M_C}{R_C}$ if $a < \frac{2L}{3}$ Max (+y) at $x = \frac{2M_A}{R_A}$ if $a > \frac{L}{3}$

B4.1.2 Stress Analysis

The maximum bending stress is:

$$f = \frac{Mc}{I} \qquad (1)$$

The limitations are:

a) The loads on the beam must be static loads.
b) The value of f is the result of external forces only.
c) The beam acts as a unit with bending as the dominant action.
d) The initial curvature of the member must be relatively small. (Radius of curvature at least ten times the depth.)
e) Plane cross sections remain plane.
f) The material follows Hooke's law.

If the calculated stress does exceed the proportional limit, a suitable reduced modulus must be used.

The maximum shearing stress in a beam in combined bending and shear is:

$$f_s = K \frac{V}{A} \qquad (2)$$

where (K) is the ratio of the maximum shearing stress on the cross section to the average shearing stress. The maximum shearing stress is often expressed as:

$$f_s = \frac{VQ}{It} \qquad (3)$$

Where $Q = \int_{Area} y\,dA$ (First moment about the neutral axis of the area between the neutral axis and the extreme outer fiber.)

B 4.1.3 Variable Cross Section

The following formulas and figures present a method of analyzing beams with uniformly tapering cross sections. Figure B 4.1.3-1 shows a tapered cantilever beam consisting of two concentrated flange areas joined by a vertical web which resists no bending. The vertical components of the loads in the flanges, P tan α_1, and P tan α_2, resist some of the external force V. Letting V_f equal the force resisted by the flanges and V_w the force resisted by the webs, then:

$$V = V_f + V_w \quad (1)$$

$$V_f = P\,(\tan \alpha_1, + \tan \alpha_2) \quad (2)$$

From Fig. B 4.1.3-1, $\tan \alpha_1 = \frac{h_1}{c}$, $\tan \alpha_2 = \frac{h_2}{c}$, and $\tan \alpha_1 + \tan \alpha_2 = \frac{h_1 + h_2}{c} = \frac{h}{c}$. From this $V_f = P\,\frac{h}{c}$, and since $P = V\,\frac{b}{h}$,

then

$$V_f = V\,\frac{b}{c} \quad (3)$$

The load in the web is $V\,\frac{a}{c}$, so by writing a, b, and c, in terms of h_o and h, we have

$$V_w = V\,\frac{h_o}{h} \quad (4)$$

$$V_f = V\,\frac{(h - h_o)}{h} \quad (5)$$

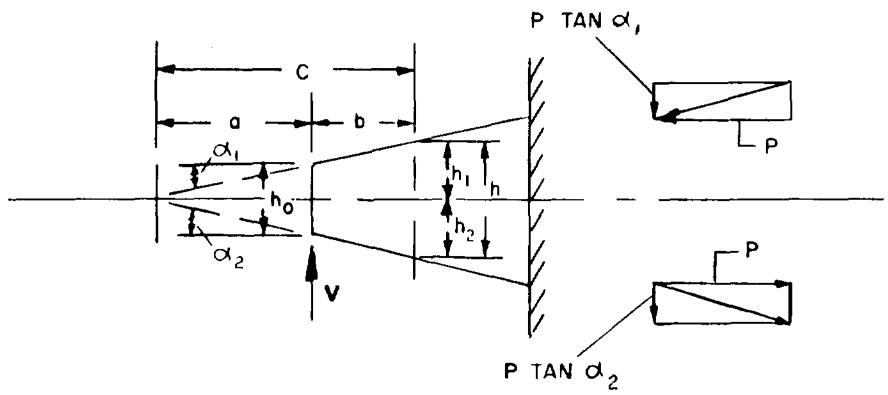


Fig. B 4.1.3-1

B 4.1.4 Symmetrical Beams of Two Different Materials

The analysis of symmetrical beams of two or more materials within the elastic range may be analyzed by transforming the section into an equivalent beam of one material. The usual elastic flexure formula then applies.

The transformation is accomplished by changing the dimension perpendicular to the axis of symmetry of the various materials in the ratio of their elastic moduli. Examples to illustrate the method for various conditions follows.

Example 1. Consider a beam made of two materials whose cross section is shown in Fig. B 4.1.4-1a. Assuming n = Ea/Es, the transformation in terms of material (S) (Fig. B 4.1.4-1b) is then $b_1 = nb$. The maximum stress in member (S) is then $f_{(s)} \max = \frac{Mh_1}{I}$ and the maximum stress in member (A) is $f_{(a)} \max = \frac{-Mh_2}{nI}$. It is noted that the section could have been transformed in terms of member (A) (Fig. B 4.1.4-1c) giving the same results for the maximum fiber stresses.

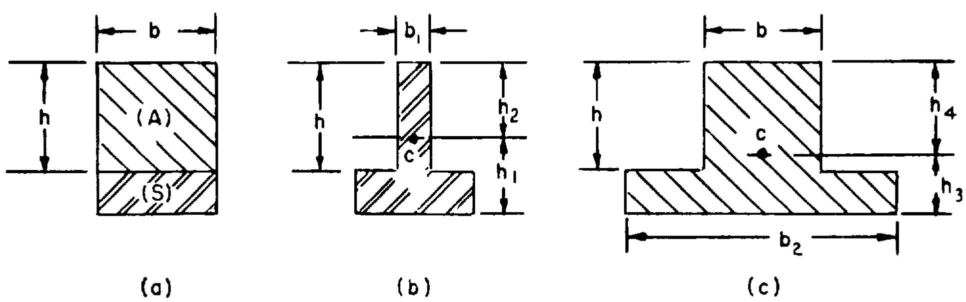


Fig. B 4.1.4-1

Example 2. Reinforced-Concrete Beams. It is the established practice in calculating bending stresses in reinforced-concrete beams to assume that concrete can withstand only compressive stress. The steel or other reinforcing member then is transformed into an equivalent area as shown in Fig. B 4.1.4-2b. The distribution of internal forces for a beam (Fig. B 4.1.4-2a) over any cross section ab is shown in Fig. B 4.1.4-2c.

B 4.1.4 Symmetrical Beams of Two Different Materials (Cont'd)

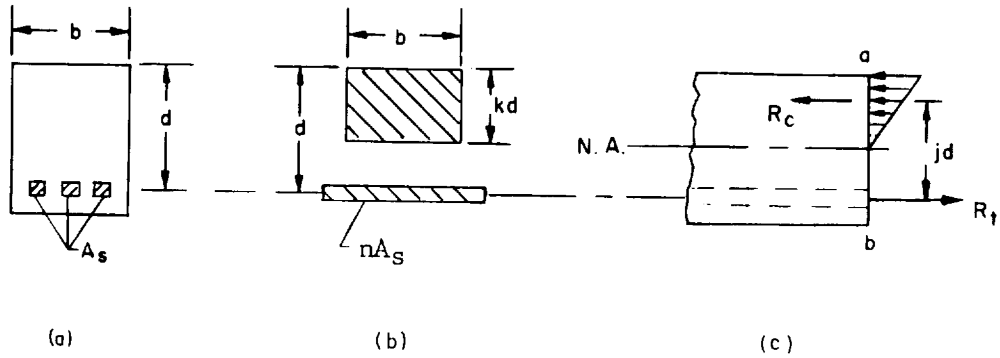


Fig. B 4.1.4-2

To satisfy the equation of equilibrium, the internal moment must equal the external moment. The mathematical statement is:

$$\underbrace{b\ (kd)}_{\text{Concrete area}}\ \underbrace{\left(\frac{kd}{2}\right)}_{\text{arm}} = \underbrace{nAs}_{\text{Transformed steel area}}\ \underbrace{(d-kd)}_{\text{arm}} \quad \cdots\cdots\cdots\cdots \quad (1)$$

From which

$$kd = \frac{nAs}{b}\left(\sqrt{1 + \frac{2bd}{nAs}} - 1\right) \quad \cdots\cdots\cdots\cdots \quad (2)$$

$$\text{where } n = \frac{E\ \text{steel}}{E\ \text{concrete}}$$

The stress in the concrete f_c and the stress in the steel f_{st} is

$$f_c = \frac{M\ (kd)}{I} \quad \cdots\cdots\cdots\cdots \quad (3)$$

$$f_{st} = \frac{nM(d-kd)}{I} \quad \cdots\cdots\cdots\cdots \quad (4)$$

where

I = Moment of inertia of the transformed section.

B 4.1.4 Symmetrical Beams of Two Different Materials (Cont'd)

Alternate solution: After kd is determined, instead of computing I, a procedure evident from Fig. B 4.1.4-2c may be used. The resultant force developed by the stresses acting in a "hydrostatic" manner on the compression side of the beam must be located $\frac{kd}{3}$ below the top of the beam. Moveover, if b is the width of the beam, this resultant force $R_c = \frac{b}{2}(kd)\, f_c$ max, (average stress times area). The resultant tensile force R_t acts at the center of the steel and is equal to $A_s f_{st}$, where A_s is the cross-sectional area of the steel. Then if jd is the distance between R_c and R_t, and since $R_c = R_t$, the applied moment M is resisted by a couple equal to $R_c jd$ or $R_t jd$.

$$jd = d - \frac{1}{3} kd \qquad (5)$$

The stress in the steel and concrete is

$$f_s = \frac{M}{A_s\, jd} \qquad (6)$$

$$f_c = - \frac{2M}{b(kd)\,(jd)} \qquad (7)$$

B 4.2.0 Continuous Beams

B 4.2.1 Castigliano's Theorem

Castigliano's Theorem is useful in the solution of problems involving continuous beams with only one or two redundant supports. The theorem can be written as

$$\delta_Q = \frac{\partial U}{\partial Q} = \frac{1}{EI}\int_o^L M \frac{\partial M}{\partial Q}\, ds \quad \ldots\ldots (1)$$

$$\delta_Q = \frac{\partial Us}{\partial Q} = \frac{1}{GA}\int_o^L V \frac{\partial V}{\partial Q}\, ds \quad \ldots\ldots (2)$$

$$\theta_a = \frac{\partial U}{\partial M_a} = \frac{1}{EI}\int_o^L M \frac{\partial M}{\partial M_a}\, ds \quad \ldots\ldots (3)$$

where

δ_Q is the deflection at the load Q in the direction of Q.

Q may be a real or fictitious load.

Q_a is the slope at the moment M_a in the direction of M_a.

M_a may be a real or fictitious moment.

U is the strain energy of the beam.

M is the bending moment due to all loads.

V is the vertical shearing forces due to all loads.

A is the cross-sectional area of the beam.

E is the modulus of elasticity.

I is the moment of inertia.

B 4.2.2 Unit Load or Dummy Load Method

The unit load or dummy load method may be used to determine deflection at elastic or inelastic members. Deflection of inelastic members by this method is given in section B 4.5.0. The theorem as applied to elastic beams is written in integral form as

$$\delta = \int_{o}^{L} \frac{Mm}{EI}\, dx \quad \text{...} \quad (1)$$

$$\theta = \int_{o}^{L} \frac{Mm'}{EI}\, dx \quad \text{...} \quad (2)$$

Where (δ) is the deflection at the unit load and (θ) is the rotation at the unit moment. The Moment (M) is the bending moment at any section caused by the actual loads. (m) is the bending moment at any section of the beam caused by a dummy load of unity acting at the point whose deflection is to be found and in the direction of the desired deflection. The bending moment (m') is the bending moment at any section of the beam caused by a dummy couple of unity applied at the section where the change in slope is desired. It is noted that although (m') may be thought of as a bending moment, it is evident from the expression $m' = \frac{\partial M}{\partial M_a}$ that it is actually dimensionless.

B 4.2.2 Unit Load or Dummy Load Method (Cont'd)

Illustrative Problem: Find the elastic vertical deflection of the point A (Fig. B 4.2.2-1a) of the simply supported beam subjected to two concentrated loads.

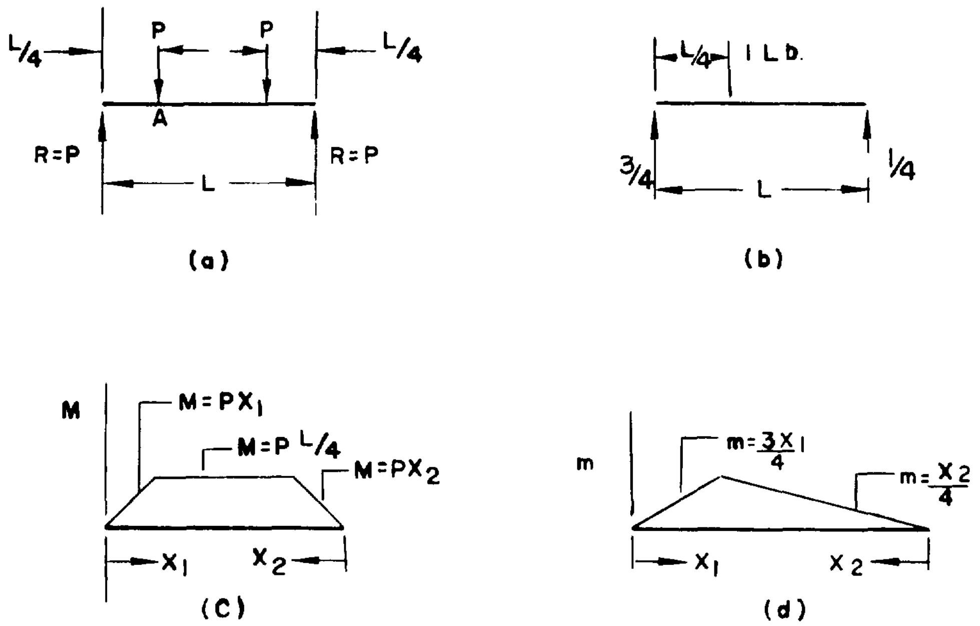


Fig. B 4.2.2-1

Solution: The actual loading is shown in Fig. B 4.2.2-1a and the dummy loading is shown on Fig. B 4.2.2-1b. The moment for the actual loading is shown on Fig. B 4.2.2-1c and the corresponding moment diagram for the dummy loading is shown on Fig. B 4.2.2-1d.

The deflection by use of equation (1) noting that x_1 starts at the left and x_2 starts at the right is

$$\delta = \int_0^L \frac{Mm}{EI}\,dx = \int_0^{\frac{L}{4}} \frac{Px_1}{EI}\left(\frac{3x_1}{4}\right) dx_1 + \int_0^{\frac{L}{4}} \frac{Px_2}{EI}\left(\frac{x_2}{4}\right) dx_2$$

$$+ \int_{\frac{L}{4}}^{\frac{3L}{4}} \frac{PL}{4EI}\left(\frac{x_2}{4}\right) dx_2 = \frac{PL^3}{48EI}$$

B 4.2.3 The Two-Moment Equation

The two-moment equation may be used to determine the bending moment at one section of the beam when the shear and bending moment at another section and the loads applied to the beam between the two sections are known. The expressions for the moment and shear corresponding to Fig. B 4.2.3-1 is

$$M_2 = M_1 + V_1 d + Fx \quad (1)$$

$$V_2 = V_1 + F \quad (2)$$

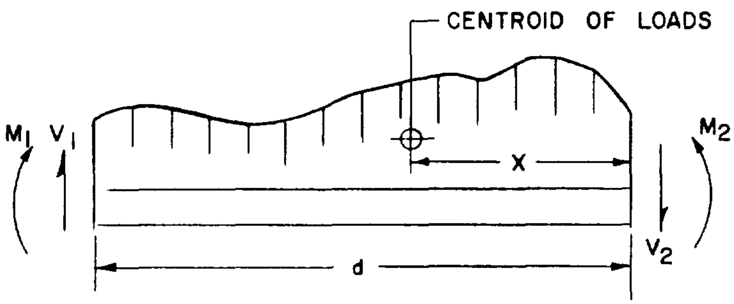


Fig. B 4.2.3-1

The two-moment equation is particularly useful in determining the curve of bending moments and shears in the case of a cantilever beam subjected to distributed loads, such as shown in Fig. B 4.2.3-2. The calculations may be done in tabular form as in Table B 4.2.3-1.

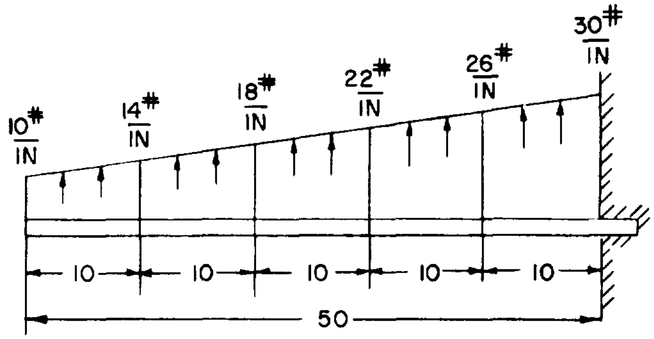


Fig. B 4.2.3-2

B 4.2.3 The Two-Moment Equation (Cont'd)

Station (inches from end)	W = Load lb/in	Shear $V=(W_1+W_2)\left(\frac{\Delta x}{2}\right)$		* Bending Moment $M=(2W_1+W_2)\left(\frac{\Delta x^2}{6}\right)+V_1(\Delta x)$	
0	10		0		0
		(10+14)10/2=	120	$(2\cdot10+14)10^2/6$ =	567
10	14		120		567
		(14+18)10/2=	160	$(2\cdot14+18)10^2/6$ = $120\cdot10$ =	767 1200
20	18		280		2534
		(18+22)10/2=	200	$(2\cdot18+22)10^2/6$ = $280\cdot10$ =	976 2800
30	22		480		6301
		(22+26)10/2=	240	$(2\cdot22+26)10^2/6$ = $480\cdot10$ =	1167 4800
40	26		720		12,268
		(26+30)10/2=	280	$(2\cdot26+30)10^2/6$ $720\cdot10$ =	1367 7200
50	30		1000		20,835

Table B 4.2.3-1

*The increment of bending moment between stations may be calculated from the relation:

$$M = (\Delta V)\ (\text{Dist. from centroid of trapezoid to inboard station})$$

$$+ V_1\ (\Delta x)$$

B 4.2.4 <u>The Three-Moment Equation</u>

The three-moment equation is useful in the solution of problems involving continuous beams with relatively few redundant supports. The equation is:

$$\frac{M_a L_1}{I_1} + \frac{2M_b L_1}{I_1} + \frac{2M_b L_2}{I_2} + \frac{M_c L_2}{I_2} =$$

$$K_1 + K_2 + \frac{6E}{L_1}(Y_a - Y_b) + \frac{6E}{L_2}(Y_c - Y_b) \quad \cdots\cdots\cdots\cdots \quad (1)$$

Where (K_1) and (K_2) are functions of loading on span (L_1) and span (L_2), respectively.

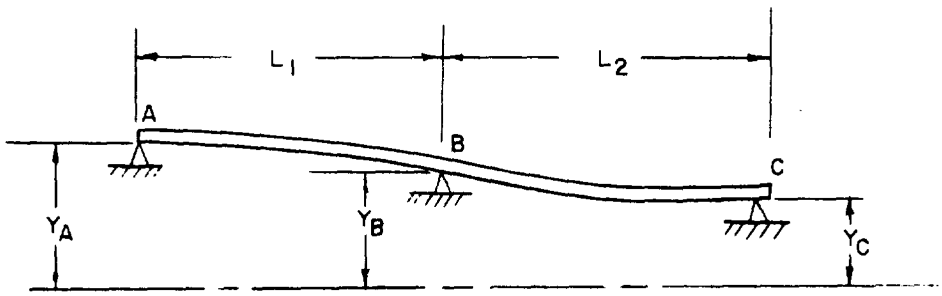


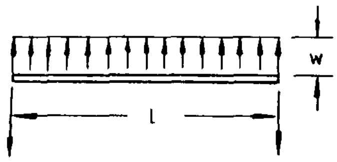
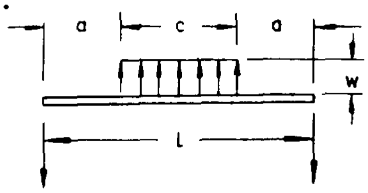
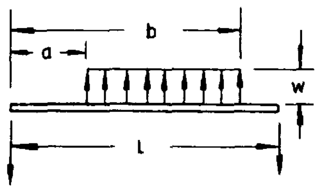
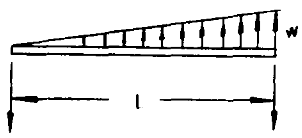
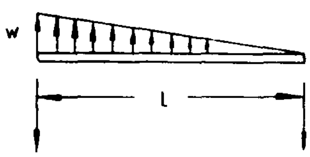
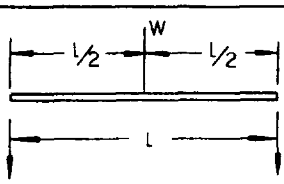
Fig. B 4.2.4-1

One equation must be written for each intermediate support. The system of simultaneous equations is then solved for the moments at the intermediate supports.

Values at (K_1) and (K_2) for various types of loading are tabulated in Table B 4.2.4-1.

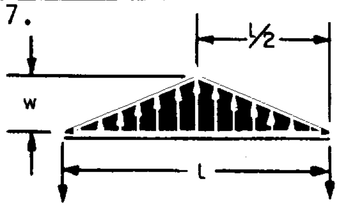
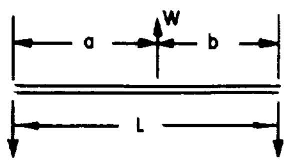
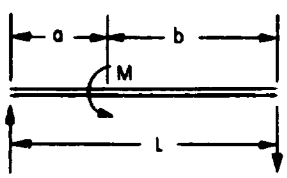
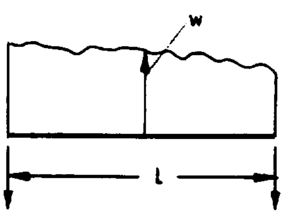
B 4.2.4 <u>The Three-Moment Equation (Cont'd)</u>

Table B 4.2.4-1 K_1 and K_2 Values for Various Load Conditions

Type of Loading	Left Bay - K_1	Right Bay - K_2
1. 	$\dfrac{+w_1 L_1^3}{4I_1}$	$\dfrac{+w_2 L_2^3}{4I_2}$
2. 	$\dfrac{+w_1 c_1 (3L_1^2 - C_1^2)}{8I_1}$	$\dfrac{+w_2 c_2 (3L_2^2 - c_2^2)}{8I_2}$
3. 	$K_1 = + \dfrac{w_1 \left[b_1^2 (2L_1^2 - b_1^2) - a_1^2 (2L_1^2 - a_1^2) \right]}{4I_1 L_1}$ $K_2 = + \dfrac{w_2 \left[b_2^2 (2L_2^2 - b_2^2) - a_2^2 (2L_2^2 - a_2^2) \right]}{4I_2 L_2}$	
4. 	$\dfrac{+2w_1 L_1^3}{15I_1}$	$\dfrac{+7w_2 L_2^3}{60\ I_2}$
5. 	$\dfrac{+7w_1 L_1^3}{60I_1}$	$\dfrac{+2w_2 L_2^3}{15I_2}$
6. 	$\dfrac{+3L_1^2 w_1}{8I_1}$	$\dfrac{+3L_2^2 w_2}{8I_2}$

B 4.2.4 <u>The Three-Moment Equation (Cont'd)</u>

Table 4.2.4-1 K_1 and K_2 Values for Various Load Conditions

Type of Loading	Left Bay - K_1	Right Bay - K_2
7. 	$+\frac{5w_1L_1^3}{32I_1}$	$+\frac{5w_2L_2^2}{32I_2}$
8. 	$+\frac{w_1a_1\left(L_1^2-a_1^2\right)}{I_1L_1}$	$+\frac{w_2b_2\left(L_2^2-b_2^2\right)}{I_2L_2}$
9. 	$+\frac{M_1}{I_1}\left(L_1-\frac{3a_1^2}{L_1}\right)$	$+\frac{M_2}{I_2}\left(\frac{3b_2^2}{L_2}-L_2\right)$
10. 	$K_1 = \frac{6}{I_2L_2}\int_0^L M_1\, x_1\, dx_1$ $K_2 = \frac{6}{I_2L_2}\int_0^L M_2\, x_2\, dx_2$ Where M is the bending moment.	

B 4.2.5 Moment Distribution Method

The moment distribution method is suggested for the solution of problems involving continuous beams with many redundant supports. The discussion and application of the method is given in section B 5.0.0 (Frames).

B 4.3.0 Curved Beams

B 4.3.1 Correction Factors for Use in Straight-Beam Formula

When a curved beam is bent in the plane of initial curvature, plane sections remain plane, but the strains of the fibers are not proportional to the distance from the neutral axis because the fibers are not at equal length. If (K) denotes a correction factor, the stress at the extreme fiber of a curved beam is given by

$$f = K \frac{Mc}{I}$$

in which

$$K = \frac{\frac{M}{AR}\left[1 + \frac{c}{Z(R + c)}\right]}{\frac{Mc}{I}}$$

where

M is the bending moment
A is the cross sectional area
R is the radius of curvature to the centroidal axis
c is the distance from the centroidal axis to the extreme outer fiber
I is the moment of inertia

$$Z = -\frac{1}{A}\int \frac{y}{R+y}\, dA$$

Values of K for different sections are given in Table B 4.3.1.1.

Table B 4.3.1-1

Values of K for Different Sections and Different Radii of Curvature.

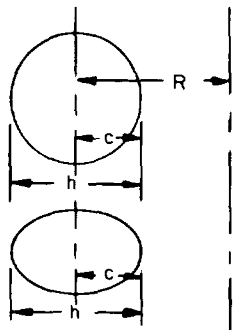
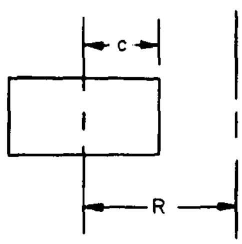
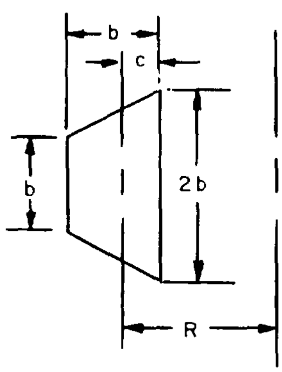
Section	$\frac{R}{c}$	Factor K Inside Fiber	Factor K Outside Fiber
1. 	1.2	3.41	0.54
	1.4	2.40	0.60
	1.6	1.96	0.65
	1.8	1.75	0.68
	2.0	1.62	0.71
	3.0	1.33	0.79
	4.0	1.23	0.84
	6.0	1.14	0.89
	8.0	1.10	0.91
	10.0	1.08	0.93
K the same for circle and ellipse and independent of dimensions.			
2. 	1.2	2.89	0.57
	1.4	2.13	0.63
	1.6	1.79	0.67
	1.8	1.63	0.70
	2.0	1.52	0.73
	3.0	1.30	0.81
	4.0	1.20	0.85
	6.0	1.12	0.90
	8.0	1.09	0.92
	10.0	1.07	0.94
K independent of section dimensions			
3. 	1.2	3.01	0.54
	1.4	2.18	0.60
	1.6	1.87	0.65
	1.8	1.69	0.68
	2.0	1.58	0.71
	3.0	1.33	0.80
	4.0	1.23	0.84
	6.0	1.13	0.88
	8.0	1.10	0.91
	10.0	1.08	0.93

Table B 4.3.1-1 (Cont'd)

Values of K for Different Sections and Different Radii of Curvature

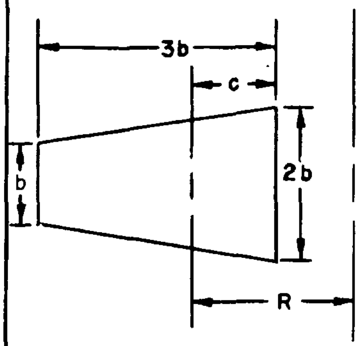
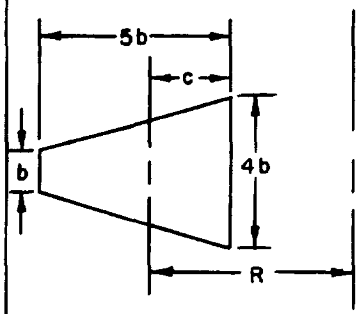
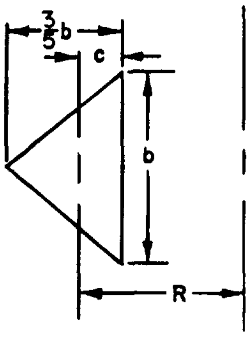
Section	$\frac{R}{c}$	Factor K	
		Inside Fiber	Outside Fiber
4. 	1.2	3.09	0.56
	1.4	2.25	0.62
	1.6	1.91	0.66
	1.8	1.73	0.70
	2.0	1.61	0.73
	3.0	1.37	0.81
	4.0	1.26	0.86
	6.0	1.17	0.91
	8.0	1.13	0.94
	10.0	1.11	0.95
5. 	1.2	3.14	0.52
	1.4	2.29	0.54
	1.6	1.93	0.62
	1.8	1.74	0.65
	2.0	1.61	0.68
	3.0	1.34	0.76
	4.0	1.24	0.82
	6.0	1.15	0.87
	8.0	1.12	0.91
	10.0	1.10	0.93
6. 	1.2	3.26	0.44
	1.4	2.39	0.50
	1.6	1.99	0.54
	1.8	1.78	0.57
	2.0	1.66	0.60
	3.0	1.37	0.70
	4.0	1.27	0.75
	6.0	1.16	0.82
	8.0	1.12	0.86
	10.0	1.09	0.88

Table B 4.3.1-1 (Cont'd)

Values of K for Different Sections and Different Radii of Curvature

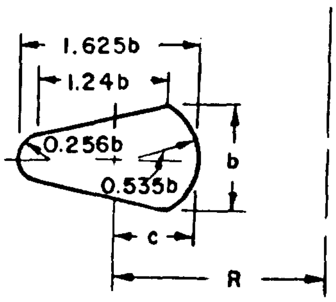
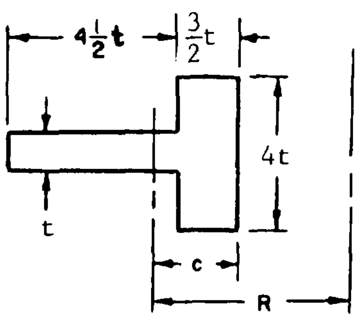
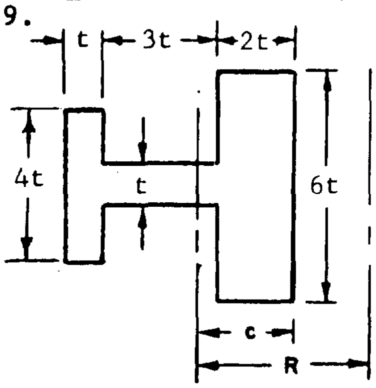
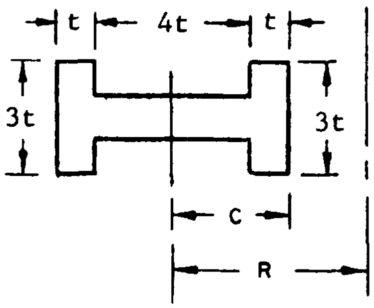
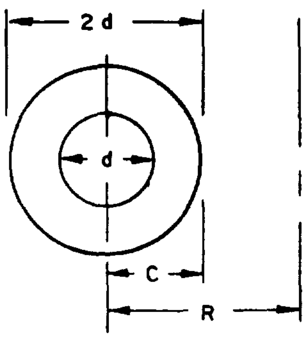
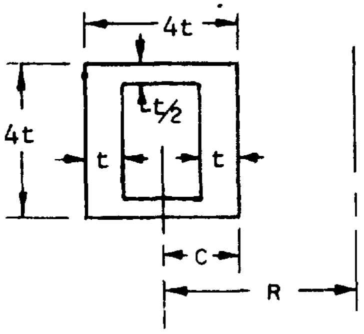
Section	R/c	Factor K	
		Inside Fiber	Outside Fiber
7.  $A = 1.05b^2$ $I = 0.18b^4$ $C = 0.70b$	1.2	3.65	0.53
	1.4	2.50	0.59
	1.6	2.08	0.63
	1.8	1.85	0.66
	2.0	1.69	0.69
	2.5	1.49	0.74
	3.0	1.38	0.78
	4.0	1.27	0.83
	6.0	1.19	0.90
	8.0	1.14	0.93
	10.0	1.12	0.96
8. 	1.2	3.63	0.58
	1.4	2.54	0.63
	1.6	2.14	0.67
	1.8	1.89	0.70
	2.0	1.73	0.72
	3.0	1.41	0.79
	4.0	1.29	0.83
	6.0	1.18	0.88
	8.0	1.13	0.91
	10.0	1.10	0.92
9. 	1.2	3.55	0.67
	1.4	2.48	0.72
	1.6	2.07	0.76
	1.8	1.83	0.78
	2.0	1.69	0.80
	3.0	1.38	0.86
	4.0	1.26	0.89
	6.0	1.15	0.92
	8.0	1.10	0.94
	10.0	1.08	0.95

Table B 4.3.1-1 (Cont'd)

Values of K for Different Sections and Different Radii of Curvature

Section	$\frac{R}{c}$	Factor K	
		Inside Fiber	Outside Fiber
10. 	1.2	2.52	0.67
	1.4	1.90	0.71
	1.6	1.63	0.75
	1.8	1.50	0.77
	2.0	1.41	0.79
	3.0	1.23	0.86
	4.0	1.16	0.89
	6.0	1.10	0.92
	8.0	1.07	0.94
	10.0	1.05	0.95
11. 	1.2	3.28	0.58
	1.4	2.31	0.64
	1.6	1.89	0.68
	1.8	1.70	0.71
	2.0	1.57	0.73
	3.0	1.31	0.81
	4.0	1.21	0.85
	6.0	1.13	0.90
	8.0	1.10	0.92
	10.0	1.07	0.93
12. 	1.2	2.63	0.68
	1.4	1.97	0.73
	1.6	1.66	0.76
	1.8	1.51	0.78
	2.0	1.43	0.80
	3.0	1.23	0.86
	4.0	1.15	0.89
	6.0	1.09	0.92
	8.0	1.07	0.94
	10.0	1.06	0.95

B 4.4.0 Bending-Crippling Failure of Formed Beams

This section contains methods of analysis applicable to formed or built-up sections which are critical in the bending-crippling mode of failure. This method is to be used when plastic bending curves are not available, otherwise use Section B4.5.

It is noted that a positive margin of safety derived from this analysis does not preclude failure in another mode.

The analysis procedure is divided into two sections according to the type of applied loading as follows:

Section

B 4.4.1 Bending moment only.
B 4.4.2 Combined bending moment and axial load.

Examples are given to illustrate the procedure for each type of loading.

B 4.4.1 Analysis Procedure for Bending Moment Only

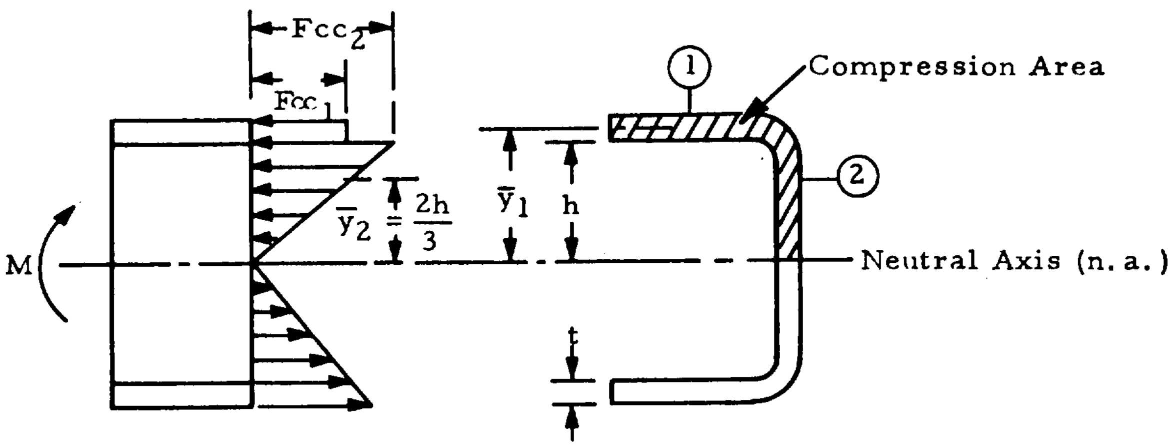


Figure 1. Bending-Crippling of Formed Shapes

B4.4.1 Analysis Procedure for Bending Moment Only (Cont'd)

(a) Locate the neutral axis (line of zero fiber stress) of the cross-section assuming a linear stress distribution.
(b) Divide the compression area into elements according to Section C 1, pages 11-16.
(c) Calculate $Fcc_{n,m}$ according to Figure C 1.3.1-13 for each element.
(d) Calculate the allowable bending-crippling moment by summing moments about the neutral axis for the compression area and doubling the result.

$$\overline{M} = 2\,[\underbrace{\Sigma\, Fcc_n\, b_n\, t_n\, \overline{y}_n}_{\text{For flange members}} + \underbrace{\Sigma\, Fcc_m\, b_m\, t_m\, \overline{y}_m/2}_{\text{For web members}}] \quad \ldots\ldots\ldots\ldots (1)$$

where:

$\overline{y}_{n,m}$ = distance from neutral axis to the resultant force of each element.

This equation is applicable for all shapes although only a formed channel is shown in Figure 1.

(e) The margin of safety is given by:

$$(\text{ult})\ M.S. = \frac{\overline{M}}{(FS)_{ult}\, M} - 1 \quad \ldots\ldots\ldots\ldots (2)$$

where:

M = applied bending moment at the cross-section in question.

$\overline{M}$ = allowable bending-crippling moment from Eq. (1).

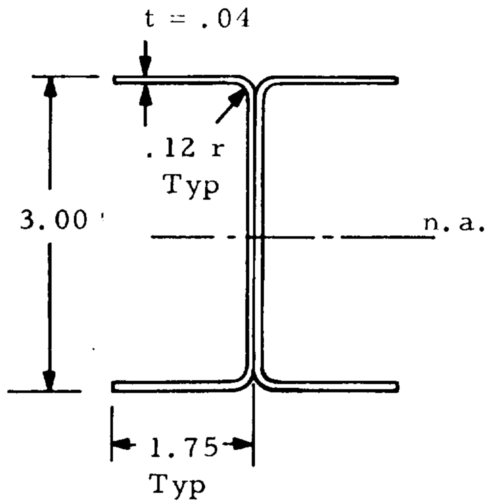
$(FS)_{ult}$ = ultimate factor of safety.

Note: if the section is unsymmetrical, the tension flange should be analyzed in the conventional manner.

B4.4.1 Analysis Procedure for Bending Moment Only (Cont'd)

Example 1

Determine the margin of safety in bending-crippling for the cross-section shown below if the bending moment is 4000 in.-lb and a factor of safety of 1.4 is desired.



Given:

Mat'l = 6061-T6 Bare Sht.

Mech. Prop.

F_{tu} = 42 ksi

F_{cy} = 35 ksi

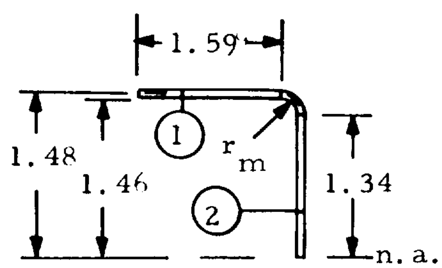
$E = 9.9 \times 10^3$ ksi

Channels are intermittently riveted together.

Figure 2. Back-to-Back Formed Channels

Analysis

(a) The neutral (centroidal) axis for this case is determined by inspection.



$r_m = .12 + .02 = .14$ in.

$b_1 = 1.59 + .535(.14) = 1.665$ in.

$b_2 = 1.34 + .535(.14) = 1.415$ in.

B4.4.1 Analysis Procedure for Bending Moment Only (Cont'd)

Example 1 (Cont'd)

(c) $$\sqrt{\frac{F_{cy}}{E}}\ \frac{b_1}{t} = \sqrt{\frac{35}{9.9 \times 10^3}}\ \frac{1.665}{.040} = 2.475, \quad Fcc_1 = .275(35)$$
$$= 9.62 \text{ ksi}$$

$$\sqrt{\frac{F_{cy}}{E}}\ \frac{b_2}{t} = \sqrt{\frac{35}{9.9 \times 10^3}}\ \frac{1.415}{.040} = 2.103, \quad Fcc_2 = .76(35)$$
$$= 26.6 \text{ ksi}$$

(d) $\overline{M} = 2[M_{flange} + M_{web}]$ Ref. Eq. (1)

$$= 2\,[(9.62)(1.665)(.04)(1.48) + (26.6)(1.415)(.04)(1.46)/3\,]$$

= 3.36 in-kips (for each channel) See page 278 for Fcc_n, b_n, t_n, and $\bar{y}_n$

(e) Margin of safety

$$\text{(ult) M.S.} = \frac{\overline{M}}{(FS)_{ult}\ M/2} - 1 = \frac{3360}{1.4(2000)} - 1 = \underline{\underline{+0.20}}$$

where:

$M/2$ = 2000 in-lb (per channel)

$(FS)_{ult} = 1.4$

Ref. page 278

B4.4.2 Analysis Procedure for Combined Bending Moment and Axial Load

1. Calculate steps (a) through (d) according to the procedure of Section B 4.6.1. If the neutral axis falls outside the cross-section, consider the section to be stressed as a column and compare with the maximum applied fiber stress.

B4.4.2 Analysis Procedure for Combined Bending Moment and Axial Load (Cont'd)

2. Calculate the section modulus of the compression area about the neutral axis assuming a mirror image.

$$Z_{c_{n.a.}} = \frac{I_{n.a.}}{C_c} \text{ (see Example 2) } \quad (3)$$

3. Compute the equivalent allowable stress

$$\overline{F} = \frac{\overline{M}}{Z_{c_{n.a.}}} \quad (4)$$

4. The margin of safety is given by

$$\text{(ult) MS} = \frac{\overline{F}}{(FS)_{ult}\, f_c} - 1 \quad (5)$$

where:

$$f_c = \frac{P}{A} \pm \frac{Mc}{I_{c.g.}} \text{ (maximum applied compressive stress)}$$

Note: If the normal load (P) is tensile, the tension flange should be analyzed in the conventional manner.

Example 2

Determine the margin of safety for bending-crippling failure for the beam column shown in Figure 3.

B4.4.2 Analysis Procedure for Combined Bending Moment and Axial Load (Cont'd)

Example 2 (Cont'd)

Given:

Mat'l = 6061-T6 Bare Sht

Mech. Prop.

F_{tu} = 42 ksi

F_{cy} = 35 ksi

E = 9.9 x 10^3 ksi

P = 5000 lb

M = 6400 in-lb

$(FS)_{ult}$ = 1.4

Angles are intermittently riveted together.

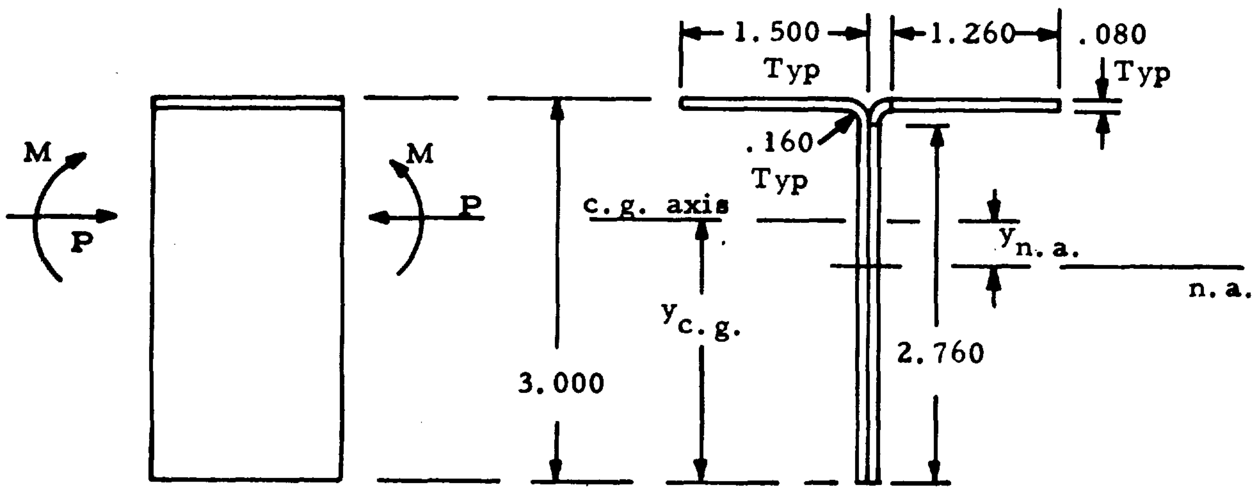
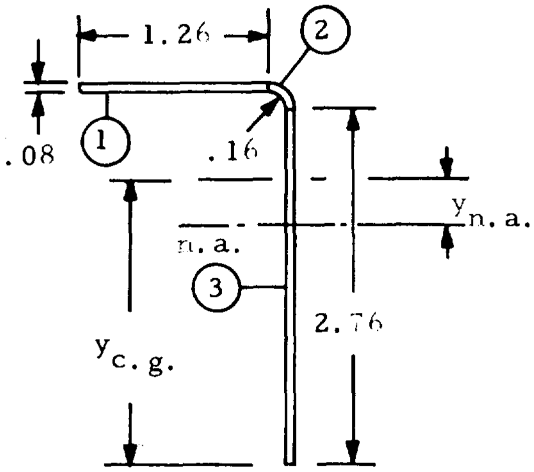


Figure 3. Back-to-Back Formed Angle

B4.4.2 Analysis Procedure for Combined Bending Moment and Axial Load (Cont'd)

Example 2 (Cont'd)

1.



ELE	A	y	Ay	Ay^2	I_o
①	.1008	2.96	.298	.883	-
②	.0250	2.89	.072	.209	-
③	.2208	1.38	.305	.420	.140
TOT.	.3466		.675	1.512	.140

$$y_{c.g.} = \frac{\Sigma Ay}{\Sigma A} = \frac{.675}{.3466} = 1.947 \text{ in.}$$

$$I_{c.g.} = \Sigma Ay^2 + \Sigma I_o - y_{c.g.}^2 \ \Sigma A = 1.512 + .140 - (1.947)^2 \ (.3466)$$

$$= .338 \text{ (for each angle)}$$

$$0 = \frac{P}{A} \pm \frac{My_{n.a.}}{I_{c.g.}}$$

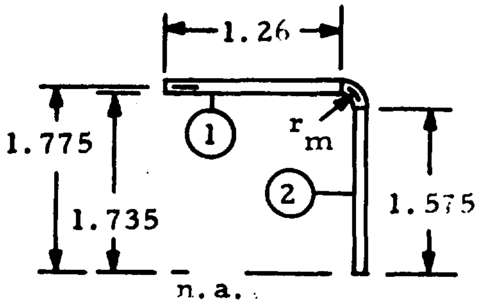
(a)

$$y_{n.a.} = -\frac{P}{A} \ \frac{I_{c.g.}}{M} = -\frac{5000\ (2)(.338)}{2(.3466)\ 6400} = -.762 \text{ in.}$$

B4.4.2 Analysis Procedure for Combined Bending Moment and Axial Load (Cont'd)

Example 2 (Cont'd)

(b)



$r_m = .16 + .04 = .20$ in.

$b_1 = 1.26 + .535(.20) = 1.367$ in.

$b_2 = 1.575 + .535(.20) = 1.682$ in.

(c)

$$\sqrt{\frac{F_{cy}}{E}}\,\frac{b_1}{t} = \sqrt{\frac{35}{9.9 \times 10^3}}\,\frac{1.367}{.08} = 1.016, \quad Fcc_1 = .56(35) = 19.60 \text{ ksi}$$

$$\sqrt{\frac{F_{cy}}{E}}\,\frac{b_2}{t} = \sqrt{\frac{35}{9.9 \times 10^3}}\,\frac{1.682}{.08} = 1.250, \quad Fcc_2 = 1.1(35) = 38.5 \text{ ksi}$$

(d)

$$\overline{M} = 2[M_{flange} + M_{web}] \quad \text{Ref. Eq. (1)}$$

$$= 2[19.60(1.367)(.08)(1.775) + 38.5(1.682)(.08)(1.735)/3]$$

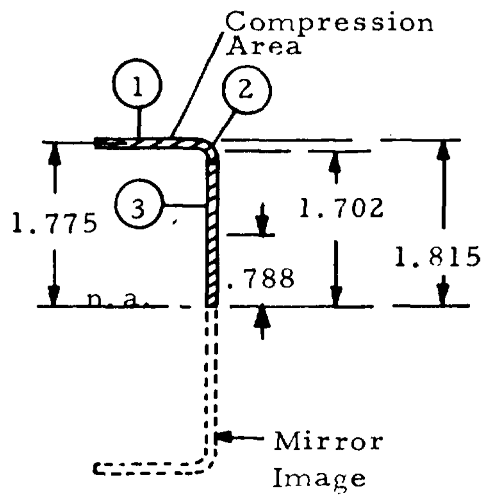
$$= 13.60 \text{ in-kip (each angle)}$$

B4.4.2 Analysis Procedure for Combined Bending Moment and Axial Load (Cont'd)

Example 2 (Cont'd)

2. Section modulus about n. a.

ELE	A	y	Ay	Ay^2	I_o
(1)	.1008	1.775	.179	.318	-
(2)	.0250	1.702	.042	.072	-
(3)	.1260	.788	.099	.078	.026
TOT.				.468	.026



$$I_{n.a.} = 2[\Sigma Ay^2 + \Sigma I_o]$$

$$= 2[.468 + .026] = .988 \text{ in}^4 \text{ (for each angle)}$$

$$Z_{c_{n.a.}} = \frac{2(.988)}{1.815} = 1.089 \text{ in}^3$$

3. Equivalent allowable stress

$$\overline{F} = \frac{2\,(\overline{M})}{Z_{c_{n.a.}}} = \frac{2(13.60)}{1.089} = 25.0 \text{ ksi}$$

B4.4.2 Analysis Procedure for Combined Bending Moment and Axial Load (Cont'd)

Example 2 (Cont'd)

Applied compressive stress

$$f_c = \frac{P}{A} + \frac{Mc}{I_{c.g.}}$$

$$= \frac{5}{2(.3466)} + \frac{6.4(1.053)}{2(.338)} = 17.2 \text{ ksi}$$

4. The margin of safety is

$$(\text{ult})\ MS = \frac{\overline{F}}{(FS)_{ult}\, f_c} - 1 = \frac{25.0}{1.4(17.2)} - 1 = +\underline{\underline{0.04}}$$

where:

$(FS)_{ult} = 1.4$ Ref. page 281

REFERENCE

B4.0.0 Beams

4.1.0 Perry, D. J., PhD. Aircraft Structures, McGraw-Hill Book Co., 1950.

Popov, E. P., Mechanics of Materials, Prentice-Hall Inc., New York, 1954.

Roark, Raymond J., Formulas for Stress and Strain, Third Edition, McGraw-Hill Book Company, Inc., New York, 1954.

Seely, Fred B. and Smith, J.O., Advanced Mechanics of Materials, Second Edition, John Wiley & Sons, Inc., New York, 1957.

Timoshenko, S., Strength of Materials, Part I, Third Edition, D. Van Nostrand Company, Inc., New York, 1955.

4.2.0 Deyarmond, Albert and Arslan, A., Fundamentals of Stress Analyses, Aero Publishers, Los Angeles, 1960.

Seely, Fred B. and Smith, J. O., Advanced Mechanics of Materials, Second Edition, John Wiley & Sons, Inc., New York, 1957.

Wilber, John B. and Norris, C. H., Elementary Structural Analyses, First Edition, McGraw-Hill Book Company, Inc.,

4.3.0 Roark, Raymond J., Formulas for Stress and Strain, Third Edition, McGraw-Hill Book Company, Inc., New York, 1954.

Seely, Fred B. and Smith, J. O., Advanced Mechanics of Materials, Second Edition, John Wiley & Sons, Inc., New York, 1957.

4.4.0 Roark, Raymond J., Formulas for Stress and Strain, Third Edition, McGraw-Hill Book Company, Inc., New York, 1954.

SECTION B4.5

PLASTIC BENDING

TABLE OF CONTENTS (Continued)

TABLE OF CONTENTS (Concluded)

B4.5.5.6 Magnesium-Minimum Properties

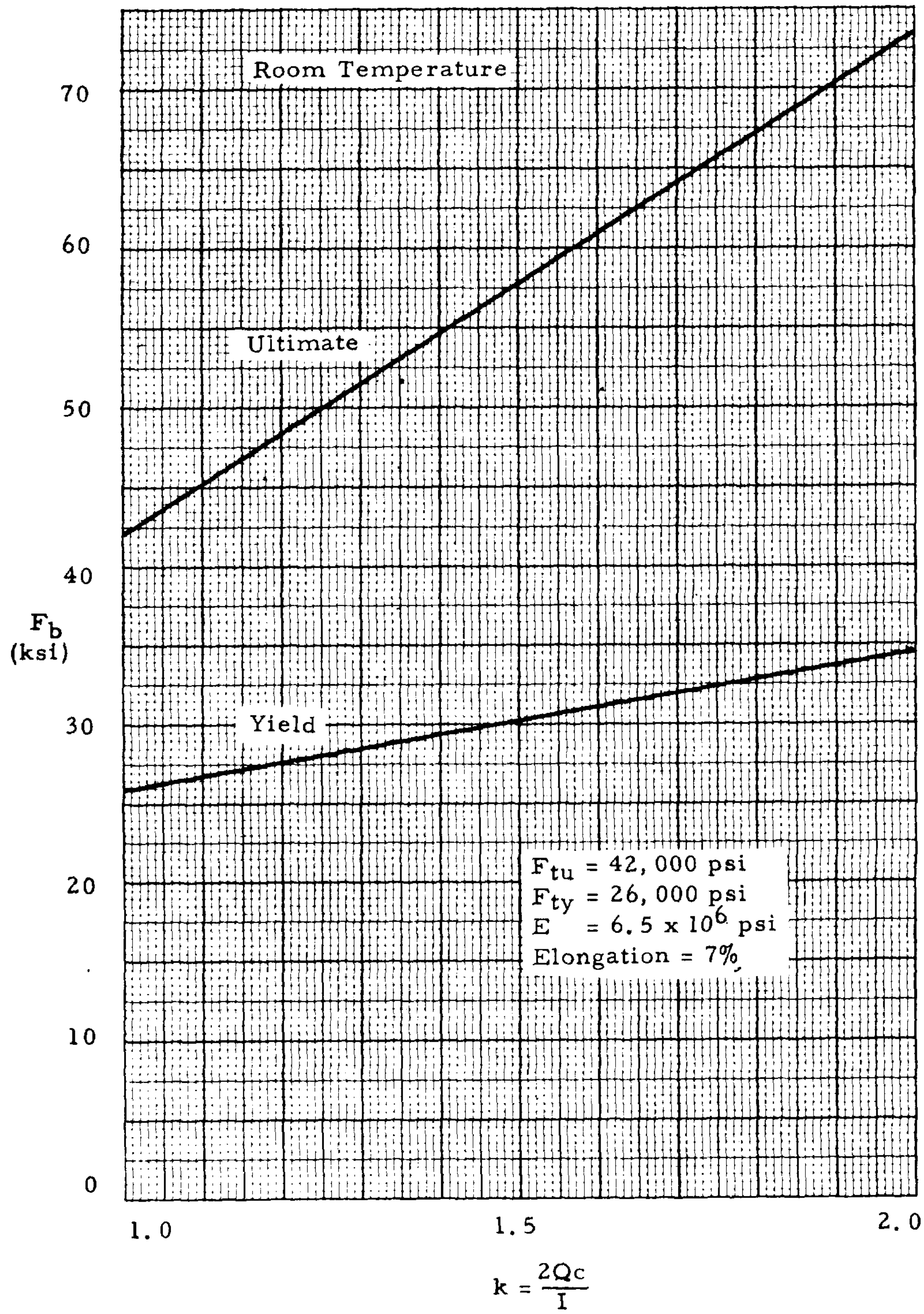


Fig. B4.5.5.6-5 Minimum Bending Modulus of Rupture for Symmetrical Sections ZK60A Magnesium Alloy Forgings (Longitudinal)

B4.5.6.6 Magnesium-Minimum Properties

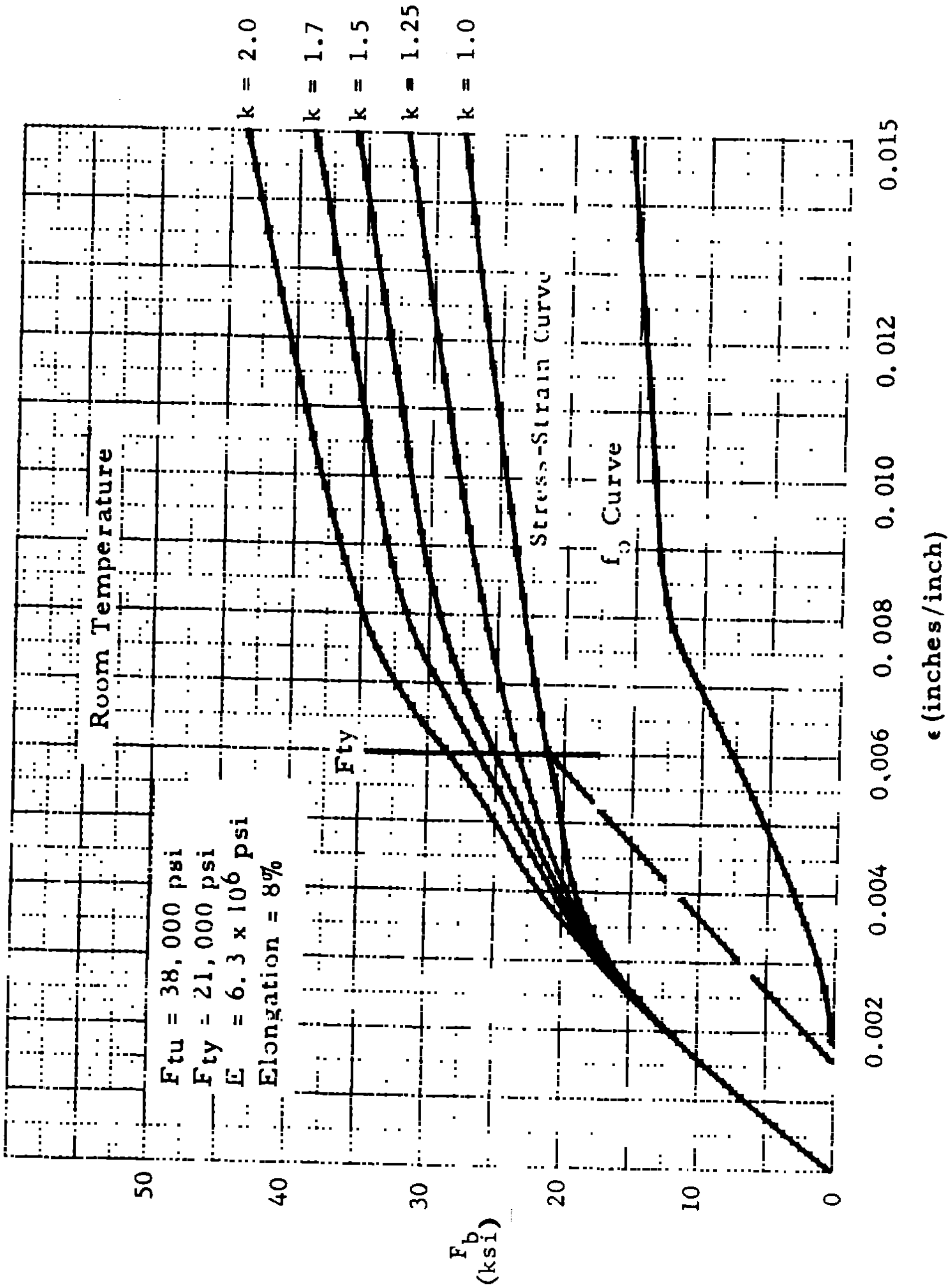


Fig. B4.5.6.6-1 Minimum Plastic Bending Curves A-Z61A Magnesium Alloy Extrusions (Longitudinal) ≤ 0.249 in.

Table A4-21. Decimal and Metric Equivalents of Fractions of an Inch (Cont'd)

Inch Decimal	Inch Fraction	Millimeter (mm)	Centimeter (cm)	Meter (m)
0.796 875	51/64	20.240 37	2.024 037	0.020 240 37
0.812 5	13/16	20.637 31	2.063 731	0.020 637 31
0.828 125	53/64	21.034 11	2.103 411	0.021 034 11
0.843 75	27/32	21.431 05	2.143 105	0.021 431 05
0.859 375	55/64	21.827 85	2.182 785	0.021 827 85
0.875	7/8	22.224 79	2.222 479	0.022 224 79
0.890 625	57/64	22.621 59	2.262 159	0.022 621 59
0.906 25	29/32	23.018 53	2.301 853	0.023 018 53
0.921 875	59/64	23.415 33	2.341 533	0.023 415 33
0.937 5	15/16	23.812 28	2.381 228	0.023 812 28
0.953 125	61/64	24.209 07	2.420 907	0.024 209 07
0.968 75	31/32	24.606 02	2.460 602	0.024 606 02
0.984 375	63/64	25.002 81	2.500 281	0.025 002 81
1.0	1	25.4	2.54	0.025 4

B4.5.0 Plastic Analysis of Beams

Introduction

The conventional flexure formula f=Mc/I is correct only if the maximum fiber stress is within the proportional limit. In the plastic range, the assumption that plane sections remain plane is valid while the stress corresponds with the stress-strain relationship of the material.

This section provides a method of approximating the true stress which depends upon the shape and the material properties. It is noted that deflection requirements are investigated when using this method since large deflections are possible while showing adequate structural strength.

The method outlined in this section is not applicable if the member is subjected to high fluctuating loads.

The following glossary is given for convenience:

Simple bending: This condition occurs when the resultant applied moment vector is parallel to a principal axis.

Complex bending: This condition occurs when the resultant applied moment vector is not parallel to a principal axis.

Development of the Theory

A rectangular cross section will be used in this development; any other symmetrical cross section would yield the same results.

B 4.5.0 Plastic Analysis of Beams (Cont'd)

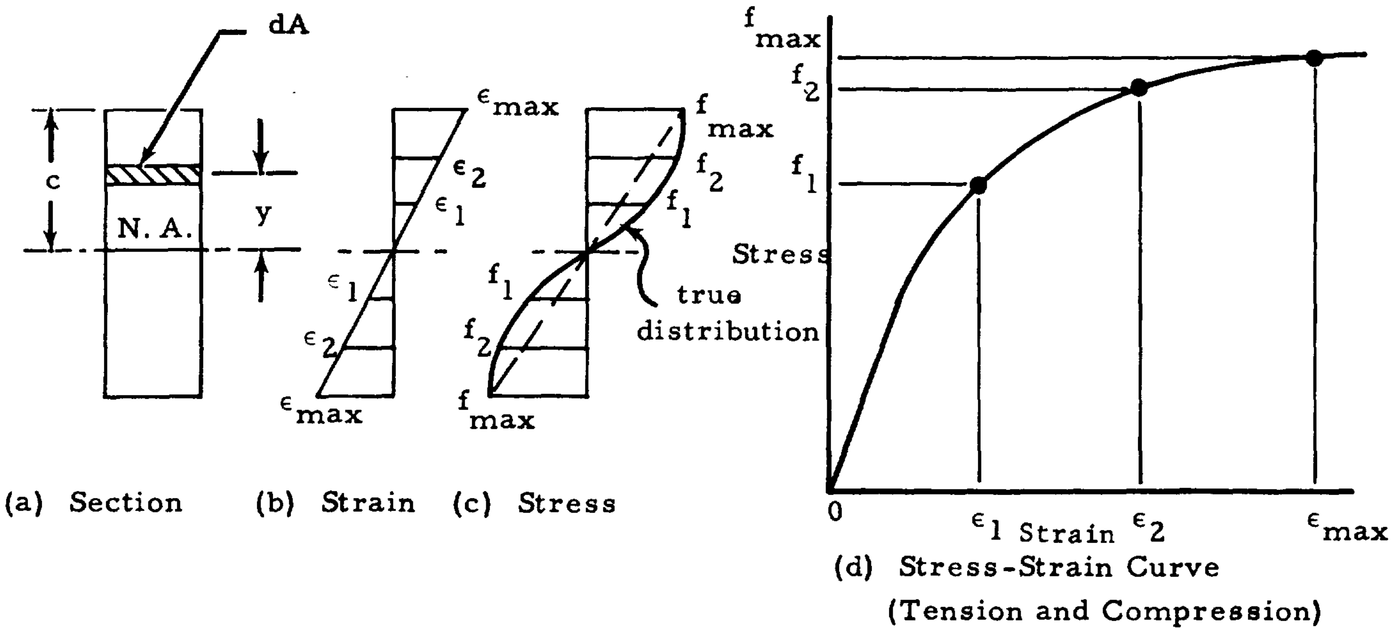
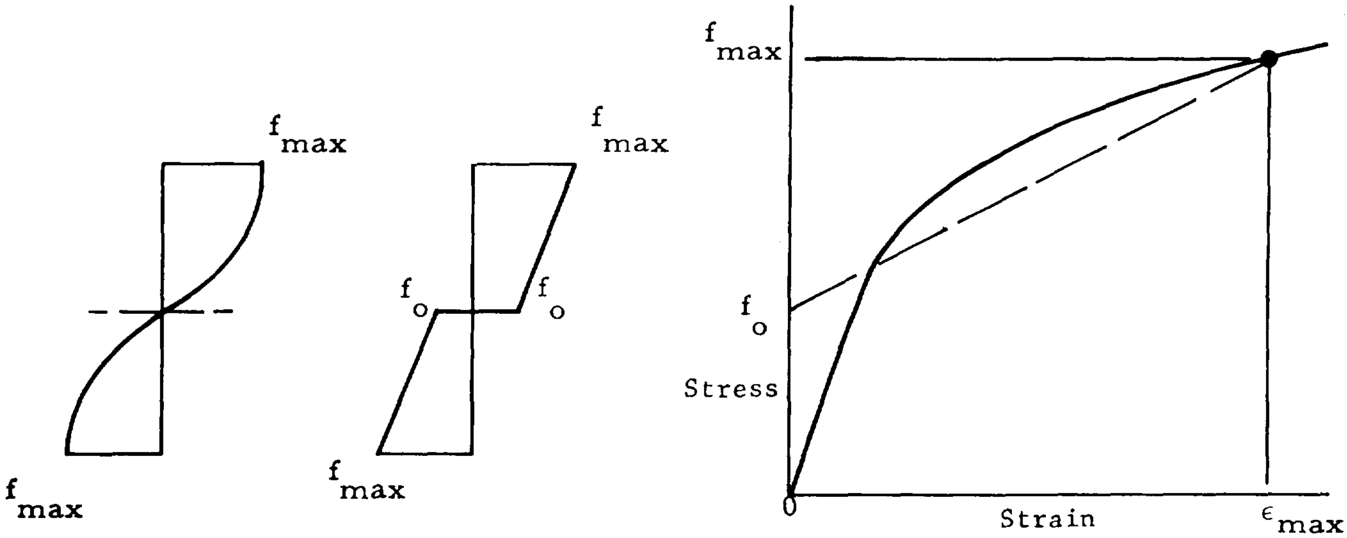


Figure B4.5.0-1

Since the bending moment of the true stress distribution about the neutral axis is greater than that of a linear Mc/I distribution as used in the elastic range, a trapezoidal stress distribution is used to approximate the true stress distribution. f_{max} may be defined as a yield stress, a buckling stress, an ultimate stress or any other stress level above the proportional limit.

Let f_o be a fictitious stress at zero strain in the trapezoidal stress distribution as shown in Figure B4.5.0-2. The value of f_o may be determined by integrating graphically the moment of (not the area of) the true stress distribution and equating this to the bending moment of the trapezoidal stress distribution.

B 4.5.0 Plastic Analysis of Beams (Cont'd)



(a) True Stress (b) Trapezoidal Stress (c) Stress Strain Curve

Figure B4.5.0-2

M_{b_a} = Allowable bending moment of the true stress distribution for a particular cross-section at a prescribed maximum stress level.

$F_b = \frac{M_{b_a} c}{I}$ = Fictitious allowable $\frac{Mc}{I}$ stress or the bending modulus of rupture for a particular cross-section at a prescribed maximum stress level.

$M_{b_t} = \frac{I}{c} f_{max} + (2Q - \frac{I}{c}) f_o$ = The bending moment of a trapezoidal stress distribution that is equivalent to M_{b_a}

Where for symmetrical sections,

$I = \int_{-c}^{c} y^2 \, dA$ (moment of inertia)

$Q = \int_{0}^{c} y \, dA$ (static moment of cross-section)

c = Distance from centroidal axis to the extreme fiber

B 4.5.0 Plastic Analysis of Beams (Cont'd)

Therefore, from $F_b = \frac{M_b c_t}{I}$ we obtain:

$$F_b = f_{max} + (k-1)f_o \qquad (4.5.0\text{-}1)$$

Where,

$$k = \frac{2Qc}{I} \qquad (4.5.0\text{-}2)$$

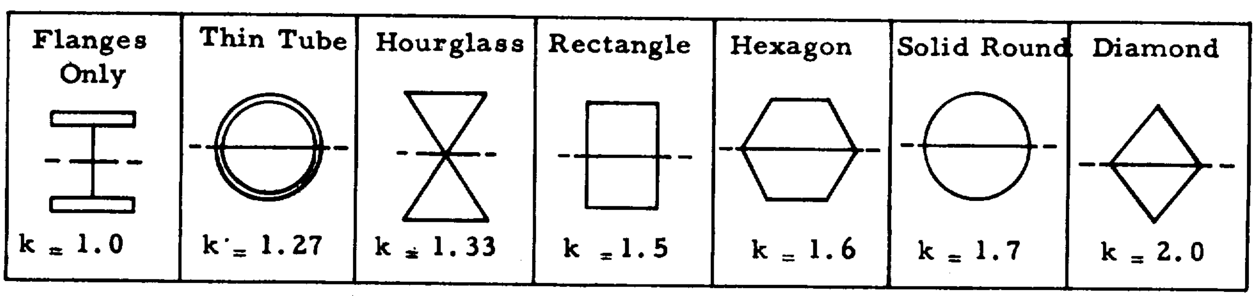
Two types of figures are presented for plastic bending analysis. One type presents Bending Modulus of Rupture Curves for Symmetrical Sections at yield and ultimate (Reference Section B4.5.4). The other type presents Plastic Bending Curves (Reference Section B4.5.5) which are necessary for the following:

1) Limiting stress other than yield or ultimate.

2) Tension and compression stress strain curves which are significantly different.

3) Unsymmetrical cross-section.

Figure B4.5.0-3 shows values of k for various symmetrical cross-sections. Generally, validity of the approach has been shown only for typical aircraft sections which are geometric thin sections.



SHAPE FACTOR

Figure B4.5.0-3

B4.5.1 Analysis Procedure When Tension and Compression Stress-Strain Curves Coincide.

B4.5.1.1 Simple Bending about a Principal Axis---Symmetrical Sections.

The procedure is as follows:

1. Determine k by Equation (4.5.0-2) or by the use of Figure B4.5.0-3.

2. For yield or ultimate limiting stress, use the Bending Modulus of Rupture Curves (F_b vs. k - see section B4.5.5 for index) to determine F_b.

3. For a limiting stress other than yield or ultimate, use the Plastic Bending Curves (See section B4.5.6 for index). Locate the limiting stress on the stress-strain (or k=1) curve and move directly up to the appropriate k curve to read F_b for the same strain.

4. F_b from Step 3 and f_b, the calculated Mc/I stress, may be used in determining the bending stress ratio for combined stresses and the margin of safety for pure bending as follows:

$$R_b = \frac{f_b}{F_b} \qquad (4.5.1.1\text{-}1)$$

$$M.S. = \frac{1}{R_b(S.F.)} - 1 \qquad (4.5.1.1\text{-}2)$$

Where: S.F. is the appropriate (yield or ult) safety factor

B4.5.1.2 Simple Bending about a Principal Axis---Unsymmetrical Sections with an Axis of Symmetry Perpendicular to the Axis of Bending

This procedure also applies to any section with bending about one of its principal axes.

B4.5.1.2 Simple Bending about a Principal Axis---Unsymmetrical Sections with an Axis of Symmetry Perpendicular to the Axis of Bending. (Cont'd)

The procedure is as follows:

1. Divide the section into two parts, (1) and (2), on either side of the principal axis (not the axis of symmetry) similar to that shown in Figure B4.5.1.2-1.

2. Calculate:

$$k_1 = \frac{Q_1 c_1}{I_1} \qquad (4.5.1.2\text{-}1)$$

where I_1 is the moment of inertia of part (1) only about the principal axis of the entire cross-section. This would be identical to k of a section made up of part (1) and its mirror image. Figure B4.5.0-3 may be used where part (1) and its mirror image form one of the sections shown.

3. Calculate similarly:

$$k_2 = \frac{Q_2 c_2}{I_2} \qquad (4.5.1.2\text{-}2)$$

4. Assuming part (1) is critical in yield (or crippling, ultimate, etc.) use the Plastic Bending Curves and locate this stress on the stress-strain (or k=1) curve. Move directly up to the appropriate k curve and read F_{b_1} for the same strain.

5. Read the strain ϵ_1.

6. Calculate:

$$\epsilon_2 = \frac{\epsilon_1 c_2}{c_1} \qquad (4.5.1.2\text{-}3)$$

7. Locate ϵ_2 on the strain scale and move directly up to the appropriate k curve to read F_{b_2}

8. Calculate M_{b_a} by

$$M_{b_a} = \frac{F_{b1} I_1}{c_1} + \frac{F_{b2} I_2}{c_2} \qquad (4.5.1.2\text{-}4)$$

B 4.5.1.2 Simple bending about a principal axis--unsymmetrical sections with an axis of symmetry perpendicular to the axis of bending. (Cont'd)

9. M_{b_a} must be used in determining the moment ratio for bending and and the margin of safety for pure bending as follows:

$$R_b = \frac{M}{M_{b_a}} \tag{4.5.1.2-5}$$

$$M.S. = \frac{1}{R_b \quad (S.F.)} - 1 \tag{4.5.1.2-6}$$

Where: S. F. is the appropriate (yield or ult) safety factor

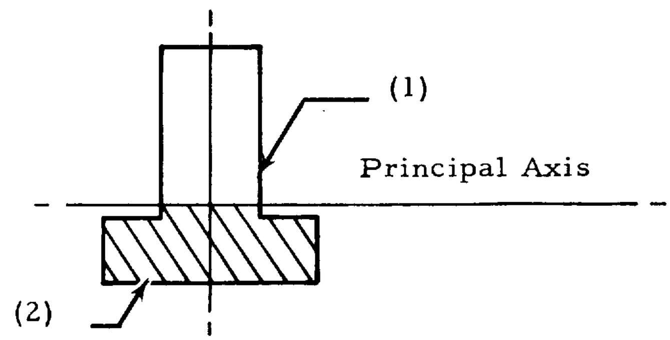
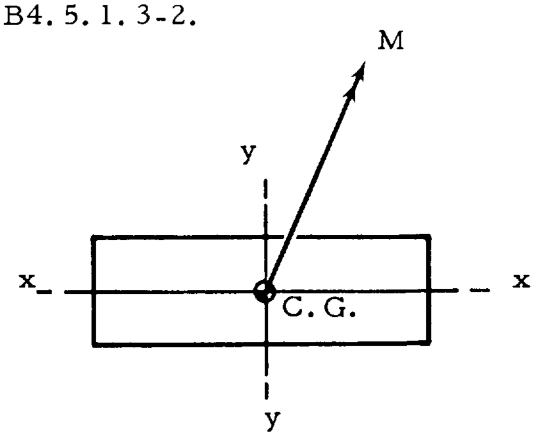


Figure B4.5.1.2-1

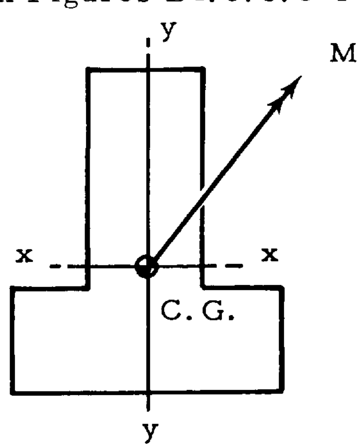
B4.5.1.3 Complex Bending--Symmetrical Sections; Also Unsymmetrical Sections with One Axis of Symmetry.

This condition occurs when the resultant applied moment vector is not parallel to a principal axis as shown in Figures B4.5.1.3-1 or B4.5.1.3-2.



Symmetrical Section

Figure B4.5.1.3-1



Unsymmetrical Section
with one axis of symmetry
Figure B4.5.1.3-2

B 4.5.1.3 Complex Bending-- Symmetrical Sections; also Unsymmetrical Sections with One Axis of Symmetry.

The procedure is as follows (this procedure is always conservative and may be very conservative for some cross-sectional shapes):

1. Obtain M_x and M_y, the components of M with respect to the principal axes.

2. Follow the procedure outlined in Section B4.5.1.1 or B4.5.1.2 to determine R_{b_x} and R_{b_y}

3. $R_b = R_{b_x} + R_{b_y}$ (4.5.1.3-1)

4. For pure bending,

$$M.S. = \frac{1}{R_b\,(S.F.)} - 1 \qquad (4.5.1.3\text{-}2)$$

Where:

S.F. is the appropriate (yield or ult) safety factor.

B4.5.1.4 Complex Bending--Unsymmetrical Sections with No Axis of Symmetry

This condition occurs when the resultant applied moment vector is not parallel to a principal axis as shown in Figure B4.5.1.4-1.

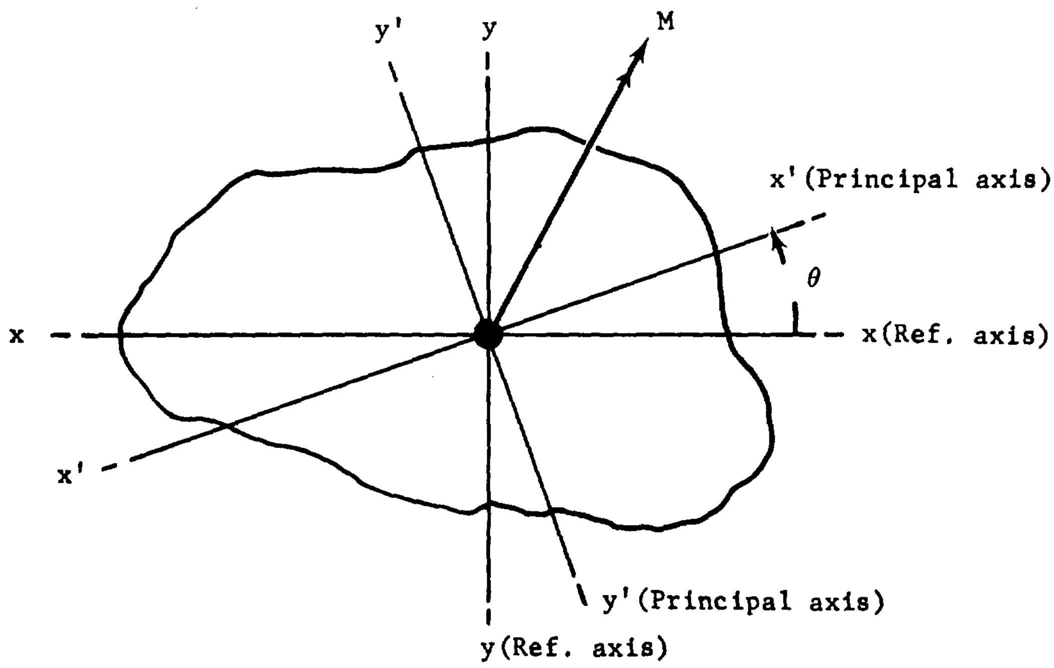


Figure B4.5.1.4-1

B 4.5.1.4 Complex Bending-- Unsymmetrical Sections with no Axis of Symmetry.

The procedure is as follows:

1. Determine the principal axes by the equation

$$\tan 2\theta = \frac{2 I_{xy}}{I_y - I_x} \qquad (4.5.1.4\text{-}1)$$

where x and y are centroidal axes and

$$I_x = \int y^2 \, dA \qquad \text{(Moment of inertia)}$$

$$I_y = \int x^2 \, dA \qquad \text{(Moment of inertia)}$$

$$I_{xy} = \int xy \, dA \qquad \text{(Product of inertia)}$$

2. Obtain $M_{x'}$ and $M_{y'}$, the components of M with respect to the principal axes.

3. Follow the procedure outlined in Section B4.5.1.2 to determine $R_{b_{x'}}$ and $R_{b_{y'}}$

4. The stress ratio for complex bending is

$$R_b = R_{b_{x'}} + R_{b_{y'}} \qquad (4.5.1.4\text{-}2)$$

5. For pure complex bending, the margin of safety is

$$M.S. = \frac{1}{R_b(S.F.)} - 1 \qquad (4.5.1.4\text{-}3)$$

where:

S.F. is the appropriate (yield or ult) safety factor.

B4.5.1.5 Shear Flow for Simple Bending About a Principal Axis-- Symmetrical Sections

When plastic bending occurs, the classical formula SQ/I is correct only for cross-sections with k=1.0. For k greater than 1.0, SQ/I is unconservative. The derivation of a correction factor β SQ/I is as follows for an ultimate strength assessment:

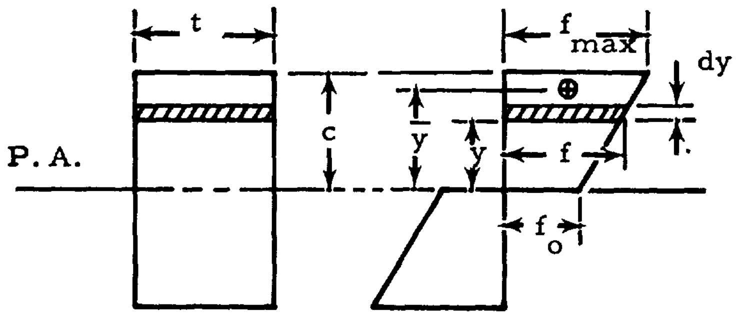


Figure B4.5.1.5-1

Let P = load on cross-section between y and c

$$P = \int_y^c tf\,dy$$

From the geometry of Figure B4.5.1.5-1

$$f = f_o + (f_{max} - f_o)\frac{y}{c}$$

$$\therefore P = \int_y^c f_o t\,dy + \int_y^c (f_{max} - f_o)\frac{ty}{c}\,dy$$

Let A= area of cross-section between y and c

$$A = \int_y^c t\,dy \quad \text{then,}$$

B 4.5.1.5 Shear Flow for Simple Bending about a Principal Axis-- Symmetrical Section. (Cont'd)

$$P = f_o A + (f_{max} - f_o)\frac{A\bar{y}}{c} \text{ and since } Q = A\bar{y},$$

$$P = f_{max}\left(\frac{Q}{c}\right) + f_o\left(A - \frac{Q}{c}\right) \qquad \text{(a)}$$

From Equation (4.5.0-1)

$$F_b = f_{max} + f_o(k-1) = \frac{Mc}{I} \text{ then,}$$

$$\frac{c}{I}\frac{dM}{dx} = \frac{df_{max}}{dx} + \frac{df_o}{dx}(k-1)$$

Since, by definition, $S = dM/dx$ and $q = dP/dx$,

$$\frac{c}{I}(S) = \frac{df_{max}}{dx}\left[1 + \frac{df_o}{df_{max}}(k-1)\right] \text{ and from (a),} \qquad \text{(b)}$$

$$q = \frac{df_{max}}{dx}\frac{Q}{c} + \frac{df_o}{df_{max}}\frac{df_{max}}{dx}\left(A - \frac{Q}{c}\right)$$

Let $\lambda = df_o/df_{max}$ and $\beta = \dfrac{1 + \lambda(c/\bar{y} - 1)}{1 + \lambda(k-1)}$ then,

$$q = \frac{df_{max}}{dx}\left[\frac{Q}{c} + \lambda\left(A - \frac{Q}{c}\right)\right] \qquad \text{(c)}$$

But since from (b): $df_{max}/dx = \dfrac{Sc}{I} \Big/ \left|1 + \lambda(k-1)\right|$ (d)

and substituting (d) into (c) we have

$$q = \left(\frac{Sc}{I}\right)\left(\frac{Q}{c}\right)\frac{[1 + \lambda(Ac/Q - 1)]}{[1 + \lambda(k-1)]}$$

Since $Q = A\bar{y}$,

$$q = \beta\frac{SQ}{I}$$

B 4.5.1.5 Shear Flow for Simple Bending about a Principal Axis-- Symmetrical Section. (Cont'd)

The method outlined below shows how to correct SQ/I and calculate the margin of safety for simple plastic bending about a principal axis of a symmetrical section.

The procedure is as follows:

1. Determine Mc/I

2. Determine k by Equation (4.5.0-2) or by the use of Figure B4.5.0-3.

3. Refer to the applicable Plastic Bending Curves and locate Mc/I on the F_b scale. Move across to the appropriate k curve to read the true strain, ϵ.

4. By use of the stress-strain (or k = 1) curve and the f_o curve, determine the rate of change of f_o with respect to f at the true strain ϵ, which would be expressed as

$$\lambda = \frac{df_o}{df} = \frac{df_o/d\epsilon}{df/d\epsilon} \qquad (4.5.1.5\text{-}1)$$

5. To determine the shear flow at any point on a cross-section, determine the distance from the principal axis to the centroid of the area outside of the point in question as shown in Figure B4.5.1.5-1. This is defined as $\bar{y}$.

6. Determine the shear flow at distance "a" from the principal axis by

$$q_a = \beta \frac{SQ_a}{I} \qquad (4.5.1.5\text{-}2)$$

where,

$$\beta = \frac{1 + \lambda\ (c/\bar{y} - 1)}{1 + \lambda\ (k - 1)} \qquad (4.5.1.5\text{-}3)$$

$$Q_a = \int_a^c y\, dA \qquad (4.5.1.5\text{-}4)$$

B 4.5.1.5. Shear Flow for Simple Bending about a Principal Axis-- Symmetrical Section. (Cont'd)

7. Calculate $f_{s_{max}} = (f_s)_{a=0} = (q/t)_{a=0}$ and the stress ratio is

$$R_s = \frac{f_{s_{max}}}{F_s} \qquad (4.5.1.5\text{-}5)$$

8. For the margin of safety with pure bending and shear use

$$M.S. = \frac{1}{(S.F.)\sqrt{R_b^2 + R_s^2}} - 1 \qquad (4.5.1.5\text{-}6)$$

9. For the margin of safety with axial load, bending and shear use

$$M.S. = \frac{1}{(S.F.)\sqrt{(R_b + R_a)^2 + R_s^2}} - 1 \qquad (4.5.1.5\text{-}7)$$

where,

$$R_a = \frac{fa}{Fa}$$

S.F. is the appropriate (yield or ult) safety factor.

B4.5.1.6 Shear Flow for Simple Bending About a Principal Axis-- Unsymmetrical Section with an Axis of Symmetry Perpendicular to the Axis of Bending

A procedure similar to that shown in Section B4.5.1.5 for Figure B4.5.1.6-1 is as follows:

1. Divide the section into two parts, (1) and (2), on either side of the principal axis (not the axis of symmetry) similar to that shown in Figure B4.5.1.2-1.

2. Calculate:

$$k_1 = \frac{Q_1 c_1}{I_1} \qquad (4.5.1.2\text{-}1)$$

where I_1 is the moment of inertia of part (1) only about the principal axis of the entire cross-section. This would be identical to k of a section made up of part (1) and its mirror image. Figure B4.5.0-3 may be used where part (1) and its mirror image form one of the sections shown.

B 4.5.1.6 Shear Flow for Simple Bending about a Principal Axis-- Unsymmetrical Section with an Axis of Symmetry Perpendicular to the Axis of Bending.

3. Calculate similarly:

$$k_2 = \frac{Q_2 c_2}{I_2} \qquad (4.5.1.2\text{-}2)$$

4. Calculate $\frac{Mc_1}{I}$ and $\frac{Mc_2}{I}$

5. Refer to the applicable Plastic Bending Curves and locate $\frac{Mc_1}{I}$ on the F_b scale. Move across to the appropriate k (use k_1) curve to read the true strain ϵ_1.

6. Find ϵ_2, similarly, or by $\epsilon_2 = \frac{\epsilon_1 c_2}{c_1}$ (4.5.1.2-3)

7. By use of the stress-strain (or k = 1) curve and the f_o curve, determine the rate of change of f_o with respect to f for the true strain ϵ_1 which would be expressed as

$$\lambda_1 = \left(\frac{df_o}{df}\right)_1 = \left(\frac{df_o/d\epsilon}{df/d\epsilon}\right)_1 \qquad (4.5.1.6\text{-}1)$$

8. Calculate similarly:

$$\lambda_2 = \left(\frac{df_o}{df}\right)_2 = \left(\frac{df_o/d\epsilon}{df/d\epsilon}\right)_2 \qquad (4.5.1.6\text{-}2)$$

9. To determine the shear flow at any point on a cross-section, determine the distance from the principal axis to the centroid of the area above or below the point in question as shown in Figure B4.5.1.6-1. This is defined as $\bar{y}_a$ or $\bar{y}_b$.

10. Determine the shear flow in part (1) at distance "a" from the neutral axis by

$$q_a = \beta_a \left(\frac{SQ_a}{I}\right) \qquad (4.5.1.6\text{-}3)$$

B 4.5.1.6 Shear Flow for Simple Bending about a Principal Axis-- Unsymmetrical Section with an Axis of Symmetry Perpendicular to the Axis of Bending.

where,

$$\beta_a = \frac{1 + \lambda_1 \left(\frac{c_1}{\overline{y}_a} - 1 \right)}{1 + \lambda_1 (k_1 - 1)} \tag{4.5.1.6-4}$$

$$Q_a = \int_a^{c_1} y dA \tag{4.5.1.6-5}$$

11. Determine the shear flow in part (2) at distance "b" from the neutral axis by

$$q_b = \beta_b \left(\frac{SQ_b}{I} \right) \tag{4.5.1.6-6}$$

where,

$$\beta_b = \frac{1 + \lambda_2 \left(\frac{c_2}{\overline{y}_b} - 1 \right)}{1 + \lambda_2 (k_2 - 1)} \tag{4.5.1.6-7}$$

$$Q_b = \int_b^{c_2} y dA \tag{4.5.1.6-8}$$

12. For the shear flow at the principal axis, calculate q using both parts of the cross-section and use the larger. A rigorous analysis could be made so that shear flow calculations at the neutral axis would result in the same value regardless of which side was used in the calculations. This would involve determining the amount of transverse shear distributed to each side of the neutral axis. If the bending moment is developed entirely from external shear, then the shear distributed on each side is proportional to the corresponding moments and Equations (4.5.1.6-3) and (4.5.1.6-6) become as follows:

$$q_a = \beta_a \frac{S_1 Q_a}{I_1} \quad \text{where } S_1 = \frac{M_1}{M_1 + M_2} S, \; M_1 = \frac{F_{b1} I_1}{C_1}$$

(See B4.5.1.2)

B 4.5.1.6 **Shear Flow for Simple Bending about a Principal Axis-- Unsymmetrical Section with an Axis of Symmetry Perpendicular to the Axis of Bending.**

$$q_b = \beta_b \frac{S_2 Q_b}{I_2} \quad \text{where } S_2 = \frac{M_2}{M_1 + M_2} S, \; M_2 = F_{b_2} \frac{I_2}{C_2}$$

(See B4.5.1.2)

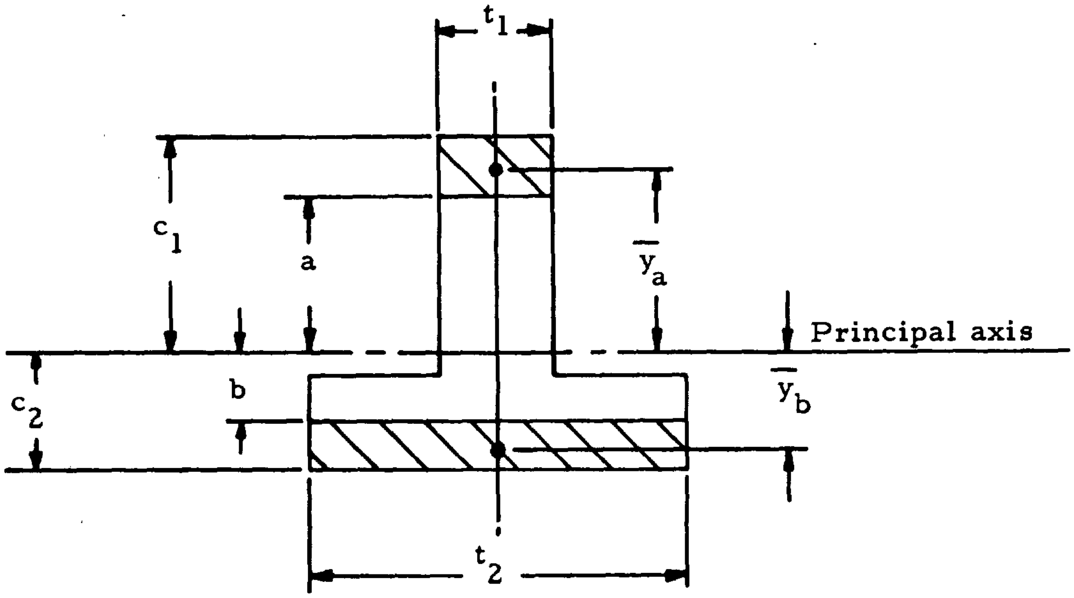


Figure B4.5.1.6-1

13. Calculate $f_{s_a} = \frac{q_a}{t_1}$ and $f_{s_b} = \frac{q_b}{t_2}$ to obtain the shear ratios:

$$R_{s_a} = \frac{f_{s_a}}{F_s} \quad (4.5.1.6\text{-}9)$$

$$R_{s_b} = \frac{f_{s_b}}{F_s} \quad (4.5.1.6\text{-}10)$$

14. For the margin of safety with pure bending and shear use

$$M.S. = \frac{1}{(S.F.)\sqrt{R_b^2 + R_s^2}} - 1 \quad (4.5.1.6\text{-}11)$$

where:

S.F. is the appropriate (yield or ult) safety factor.

B 4.5.1.6 Shear Flow for Simple Bending about a Principal Axis-- Unsymmetrical Section with an Axis of Symmetry Perpendicular to the Axis of Bending. (Cont'd)

15. For the margin of safety with axial load, bending and shear use

$$M.S. = \frac{1}{(S.F.)\sqrt{(R_b + R_a)^2 + R_s^2}} - 1 \qquad (4.5.1.6\text{-}12)$$

where:

$$R_a = \frac{f_a}{F_a}$$

S. F. is the appropriate (yield or ult) safety factor.

B4.5.1.7 Shear Flow for Complex Bending--Any Cross-Section

The procedure is as follows:

1. Determine the principal axes by inspection or, if necessary, by Equation (4.5.1.4-1).

2. Obtain $S_{x'}$ and $S_{y'}$, the components of S with respect to the principal axes. The principal axes are denoted by x' and y' as indicated in Figure B4.5.1.4-1.

3. Follow the same procedure of Section B4.5.1.6 about both principal axes to obtain the shear stresses $f_{s_{x'}}$ and $f_{s_{y'}}$ at a prescribed point.

4. The shear stress ratios at this point are

$$R_{s_{x'}} = \frac{f_{s_{x'}}}{F_s} \qquad (4.5.1.7\text{-}1)$$

$$R_{s_{y'}} = \frac{f_{s_{y'}}}{F_s} \qquad (4.5.1.7\text{-}2)$$

5. For the margin of safety with complex bending and shear use

$$M.S. = \frac{1}{S.F.\sqrt{R_b^2 + R_s^2}} - 1 \qquad (4.1.5.7\text{-}3)$$

B 4.5.1.7. Shear Flow for Complex Bending-- Any Cross-Section. (Cont'd)

where,

$$R_s = \sqrt{R_{s_{x'}}^2 + R_{s_{y'}}^2} \qquad (4.1.5.7\text{-}4)$$

S. F. is the appropriate (yield or ult.) safety factor.

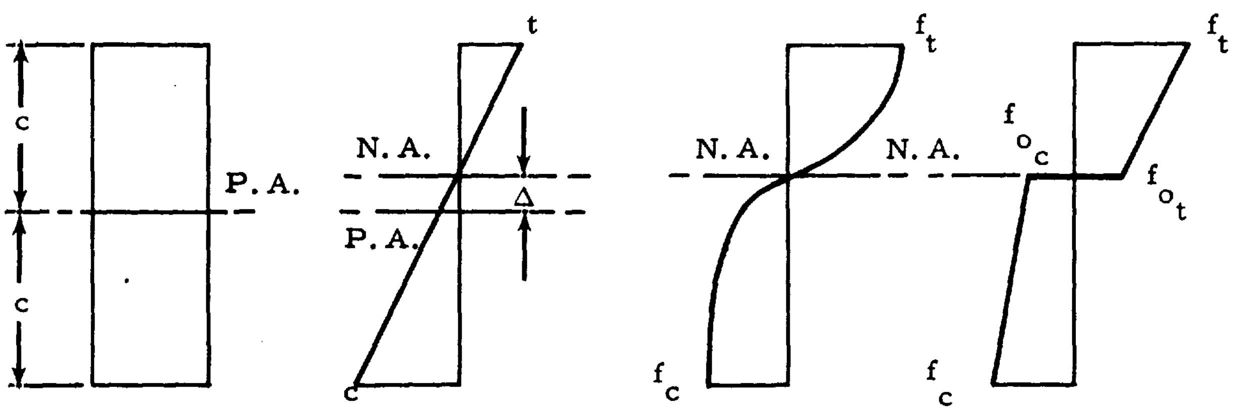
6. For the margin of safety with axial load, complex bending and shear use

$$M.S. = \frac{1}{S.F.\sqrt{(R_b + R_a)^2 + R_s^2}} - 1 \qquad (4.1.5.7\text{-}5)$$

B4.5.2 Analysis Procedure When Tension and Compression Stress-Strain Curves Differ Significantly

This may occur in materials such as the AISI 301 strainless steels in the longitudinal grain direction where the tensiốn stress-strain curve is higher than the compression curve.

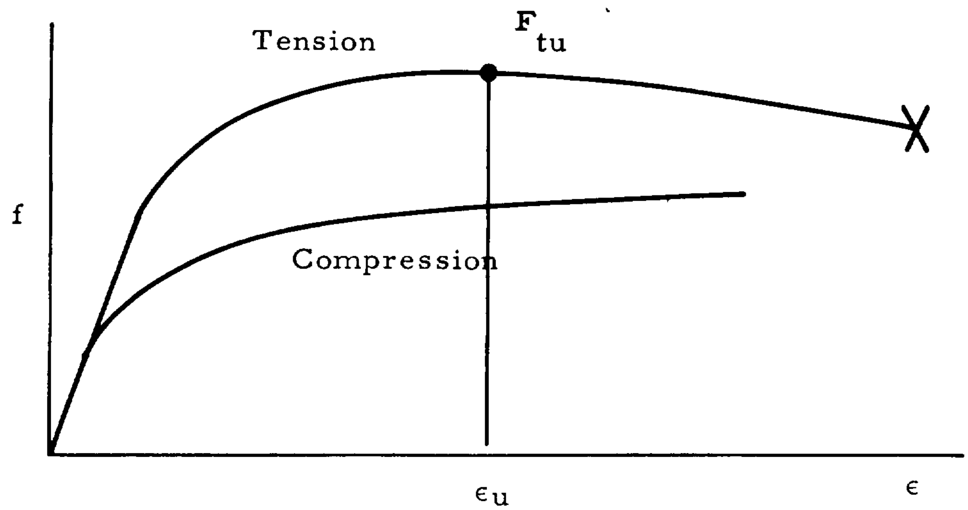
B4.5.2.1 Simple Bending about a Principal Axis--Symmetrical Sections



(a) Section (b) Strain (c) True Stress (d) Trapezoidal Stress

Figure B4.5.2.1-1

B 4.5.2.1 Simple Bending about a Principal Axis-- Symmetrical Sections. (Cont'd)



(e) Stress-Strain Curves
Figure B4.5.2.1-1 (cont.)

The procedure for a symmetrical section such as that in Figure B4.5.2.1-1 is as follows:

1. Determine k by Equation (4.5.0-2) or by the use of Figure B4.5.0-3.

2. For yield or ultimate limiting stress, use the Bending Modulus of Rupture Curves to determine F_b. The correction for shifting the neutral axis away from the principal axis by Δ has been taken into account in the development of the curves. Note Step 8.

3. For a limiting stress (or strain) other than yield or ultimate, use the Plastic Bending Curves. Locate the limiting stress on the appropriate (tension or compression) stress-strain (or k=1) curve and call this f_1 at ϵ_1 with its trapezoidal intercept f_{o1}. Note Step 9.

B 4.5.2.1 Simple Bending about a Principal Axis-- Symmetrical Sections. (Cont'd)

4. Locate the neutral axis which would be some distance Δ from the principal axis toward the tension side. This may be difficult to determine, but trial and error type formulae are provided in Figure B4.5.2.1-2 for several symmetrical cross-sections. See Example B4.5.4.1 for typical procedure in determining Δ, ϵ_2, f_2, f_{o2}, k_1, and k_2. Note that k_1 and k_2 are with respect to the neutral axis.

5. Find F_{b1} and F_{b2} for ϵ_1 and ϵ_2, using the correct values of k_1 and k_2 which may or may not be equal.

6. Calculate I_1 and I_2, the moments of inertia of the elements with respect to the neutral axis of the entire cross-section.

7. Calculate M_{b_a} by

$$M_{b_a} = \frac{F_{b_1} I_1}{c_1} + \frac{F_{b_2} I_2}{c_2} \qquad (4.5.2.1\text{-}1)$$

8. For cases as in Step 2, F_b may be used with f_b, the calculated Mc/I stress, in determining the stress ratio for bending and the margin of safety for pure bending as follows:

$$R_b = \frac{f_b}{F_b} \qquad (4.5.2.1\text{-}2)$$

$$M.S. = \frac{1}{(S.F.)\, R_b} - 1 \qquad (4.5.2.1\text{-}3)$$

where:

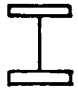
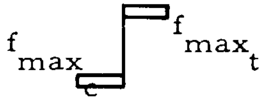
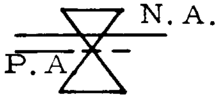
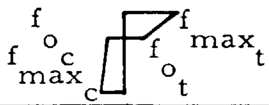
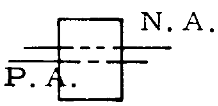
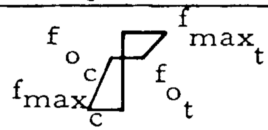
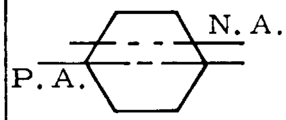
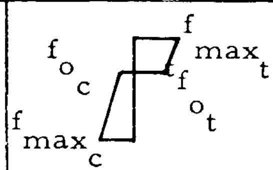
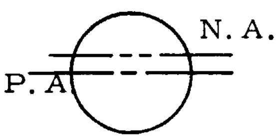
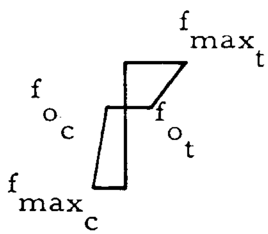
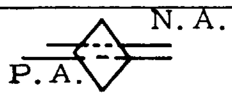
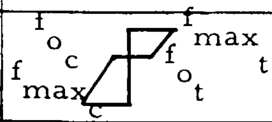
S.F. is the appropriate (yield or ult) safety factor.

9. For cases as in Step 3, M_{b_a} must be used in determining the moment ratio for bending and the margin of safety for pure bending as follows:

$$R_b = \frac{M}{M_{b_a}} \qquad (4.5.2.1\text{-}4)$$

$$M.S. = \frac{1}{(S.F.)\, R_b} - 1 \qquad (4.5.2.1\text{-}5)$$

B 4.5.2.1 Simple Bending about a Principal Axis-- Symmetrical Sections. (Cont'd)

k	CROSS-SECTION	STRESS DISTRIBUTION	FORMULA (Correct Assumption of Δ Required to Satisfy the Equality of the Equation)
1.0			Calculation of Δ not required. Modulus of rupture is equal to the limiting value of stress.
1.33			$(1-\Delta)\left[\frac{2\Delta+1}{6} f_{o_t} + \frac{\Delta+2}{6} f_{max_t}\right] = \frac{3\Delta^2+1}{6} f_{o_c} + \frac{1}{3} f_{max_c}$
1.5			$(f_{max_t} + f_{o_t})(1-\Delta) = (f_{max_c} + f_{o_c})(1+\Delta)$
1.6			$(1-\Delta)\left[f_{o_t}\left(0.9623 - \frac{\Delta}{3}\right) + f_{max_t}\left(0.7698 - \frac{\Delta}{6}\right)\right] =$ $f_{o_c}\left[\Delta\left(2.3094 - \frac{\Delta}{2}\right) + 0.9623\right] + 0.7698\, f_{max_c}$
1.7			$\left[f_{o_t} - \frac{\Delta}{1-\Delta}\left(f_{max_t} - f_{o_t}\right)\right]\left[\frac{\pi}{2} - \Delta\left(1-\Delta^2\right)^{1/2} - \sin^{-1}\Delta\right]$ $+ \frac{2}{3}\left[\frac{f_{max_t} - f_{o_t}}{1-\Delta}\right]\left(1-\Delta^2\right)^{3/2} = \left[f_{o_c} + \frac{\Delta}{1+\Delta}\left(f_{max_c} - f_{o_c}\right)\right] X$ $\left[\frac{\pi}{2} + \Delta\left(1-\Delta^2\right)^{1/2} + \sin^{-1}\Delta\right] + \frac{2}{3}\left[\frac{f_{max_c} - f_{o_c}}{1+\Delta}\right]\left(1-\Delta^2\right)^{3/2}$
2.0			$f_t + 2f_{o_t} = \frac{1}{\left(1-\Delta^2\right)}\left[f_c + f_{o_c}\left(2 + 6\Delta - 3\Delta^2\right)\right]$

TRIAL AND ERROR FORMULAE FOR SHIFT OF NEUTRAL AXIS IN MATERIAL WHOSE TENSION AND COMPRESSION STRESS-STRAIN CURVES DIFFER SIGNIFICANTLY

FIGURE B4.5.2.1-2

B4.5.2.2 Simple Bending about a Principal Axis--Unsymmetrical Sections with an Axis of Symmetry Perpendicular to the Axis of Bending

The procedure is as follows:

1. Locate the neutral axis which would be some distance Δ from the principal axis toward the tension side by a method similar to that outlined in Example B4.5.4.1 for symmetrical sections. This will require the derivation of an expression relating Δ, f_{max_t}, f_{o_t}, f_{max_c}, and f_{o_c} by use of the equilibrium of axial loads due to the bending stress distribution. This expression is likely to contain higher powers of Δ and the solution for Δ can best be obtained by trial and error using the stress-strain (or k = 1) and f_o curves. Refer to Example B4.5.4.2 for typical procedure.

2. Now, follow the identical procedure to that of Section B4.5.1.2 with the exception of using the neutral axis instead of the principal axis. Remember that F_b on the compression side is obtained from the compression Plastic Bending Curves.

B4.5.2.3 Complex Bending--Symmetrical Sections; also Unsymmetrical Sections with One Axis of Symmetry

Refer to Section B4.5.1.3 and follow the identical procedure except for Step 2 which is expressed as follows:

2. Follow the applicable procedure outlined in Section B4.5.2.1 and/or B4.5.2.2 to determine R_{b_x} and R_{b_y}.

B4.5.2.4 Complex Bending--Unsymmetrical Sections with No Axis of Symmetry

Refer to Section B4.5.1.4 and follow the identical procedure except for Step 3 which is expressed as follows:

3. Follow the procedure outlined in Section B4.5.2.2 to determine R_{b_x} and R_{b_y}

B4.5.2.5 Shear Flow for Simple Bending about a Principal Axis--Symmetrical Sections

Refer to Section B4.5.1.5 and follow the identical procedure with the following exception:

If shear flow is being determined on the tension side, use the tension Plastic Bending Curves in the evaluation of λ for Equation (4.5.1.5-3). The compression side is considered similarly using the compression Plastic Bending Curves.

B4.5.2.6 Shear Flow for Simple Bending about a Principal Axis--Unsymmetrical Sections with an Axis of Symmetry Perpendicular to the Axis of Bending

Refer to Section B4.5.1.6 and follow the identical procedure with the following exceptions:

If shear flow is being determined on the tension side, use the tension Plastic Bending Curves in the evaluation of λ for Equations (4.5.1.6-1) or (4.5.1.6-2). The compression side is considered similarly using the compression Plastic Bending Curves.

B4.5.2.7 Shear Flow for Complex Bending--Any Cross-Section

Refer to Section B4.5.1.7 and follow the identical procedure except for Step 3 which is expressed as follows:

3. Follow the same procedure of Section B4.5.2.6 about both principal axes and obtain the shear stresses $f_{s_{x'}}$ and $f_{s_{y'}}$ at a prescribed point.

B4.5.3 The Effects of Transverse Stresses on Plastic Bending

The elastic and plastic portions of stress-strain curves for combined loading do not correspond to those of the uniaxial curves due to the effects of Poisson's ratio. The plastic bending curves depend on the magnitude and shape of the stesss-strain curve of a given material. Therefore, under combined loading the plastic bending curves may have to be modified if the uniaxial stress-strain curve is significantly affected.

B 4.5.3 The Effects of Transverse Stresses on Plastic Bending.

When modification of the plastic bending curves is necessary, the procedure is as follows:

1. Modify the uniaxial stress-strain curve by Section A3.7.0.

2. Determine and construct the modified plastic bending curves, F_b vs ϵ. Refer to Section B4.5.0 for the theoretical background; the f_o curve must first be plotted and then k curves constructed by the use of Equation (4.5.0-1).

B4.5.4 Example Problems

B4.5.4.1 A Diamond Cross-Section of AISI 301 Extra Hard (2% Elongation) Stainless Steel Sheet Is Subjected to Pure Bending in the Longitudinal Direction at Room Temperature. Find the Shift in Neutral Axis (Δ) from the Principal Axis When the Limiting Stress Is (a) F_{tu} and (b) F_{cy}. (F_{ty} is Is Not Used Since $F_{ty} > F_{cy}$.)

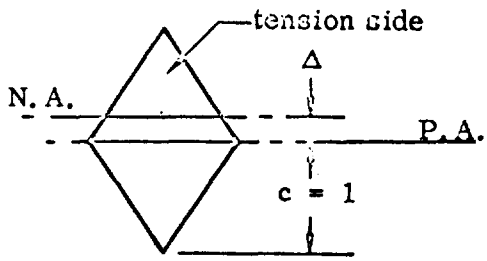
Reference the Minimum Plastic Bending Curves on page 133 and 134.

(a)

$F_{tu} = 200$ ksi

$\epsilon_t = .020$ in./in.

$f_{o_t} = 146$ ksi



From Figure B4.5.2.1-2 use the equation for determining Δ when k = 2 (for a diamond).

$$f_t + 2f_{o_t} = \frac{1}{(1 - \Delta^2)} [f_c + f_{o_c} (2 + 6\Delta - 3\Delta^2)]$$

By trial and error, equality is reached within 0.8% when Δ is assumed equal to 0.187c as shown below:

From Equation (4.5.1.2-3)

$$\epsilon_2 = \frac{\epsilon_1 c_2}{c_1} \quad \text{or} \quad \epsilon_c = \frac{\epsilon_t c_c}{c_t}$$

$$\epsilon_c = \frac{0.020 \times 1.187c}{.813c} = 0.0292 \text{ in./in.}$$

B 4.5.4.1 Example Problem B 4.5.4.1 (Cont'd)

From the stress-strain (k=1) curve page 135 at ϵ_c.

$f_c = 149$ ksi

$f_{o_c} = 107$ ksi

Therefore,

$$200 + 2 \times 146 \approx \frac{1}{1 - \overline{0.187}^2}[149 + 107(2 + 6 \times 0.187 - 3 \times \overline{0.187}^2]$$

$$492 \approx 488$$

(b) $f_{cy} = 97$ ksi

$\epsilon_c = 0.00575$ in./in.

$f_{o_c} = 27$ ksi

From Figure B4.5.2.1-2, use the equation for determining Δ when k = 2 (for a diamond).

$$f_t + 2 f_{o_t} = \frac{1}{(1 - \Delta^2)} [f_c + f_{o_c} (2 + 6\Delta - 3\Delta^2)]$$

By trial and error, equality is reached within 0.60% when Δ is assumed equal to 0.024c as shown below:

From Equation (4.5.1.2-3)

$$\epsilon_2 = \frac{\epsilon_1 c_2}{c_1} \quad \text{or} \quad = \epsilon_t = \frac{\epsilon_c c_t}{c_c}$$

$$\epsilon_t = \frac{0.00575 \times 0.976c}{1.024c} = 0.00548 \text{ in./in.}$$

From the stress-strain (k=1) curve page 132 at ϵ_t

$f_t = 121$ ksi

$f_{o_t} = 16.5$ ksi

B 4.5.4.1 Example Problem B 4.5.4.1 (Cont'd)

Therefore,

$$121 + 2 \times 16.5 \approx \frac{1}{1-0.024^2}\left[97 + 27\,(2 + 6\times 0.024 - 3\times \overline{0.024}^2)\right]$$

$$154 \approx 154.93$$

B4.5.4.2 Calculate the Ultimate Allowable Bending Moment About the Principal Axis for the Built-Up Tee Section Shown. The Flange Is in Compression and the Crippling Stress Is 60 ksi. The Material Is AISI 301 $\frac{1}{2}$ Hard Stainless Steel Sheet at Room Temperature with Bending in the Longitudinal Grain Direction.

Since the longitudinal tension and compression stress-strain curves are significantly different, the procedure of Section B4.5.2.2 will be followed.

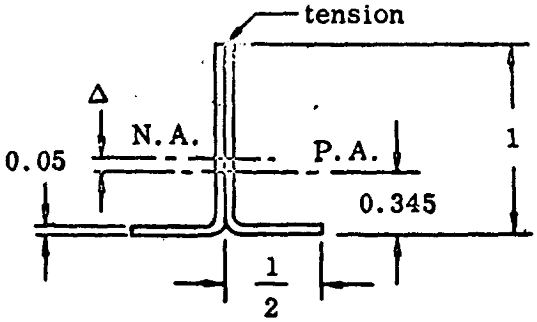
For ultimate design, crippling is the limiting stress rather than F_{tu}.

$F_{cc} = 60$ ksi

From page 114

$\epsilon_c = 0.0046$ in./in.

$f_{o_c} = 27$ ksi



A trial and error equation for shifting the neutral axis from the principal axis toward the tension side is determined by equating the load on the tension side to the load on the compression side. (See Section B4.5.2.2):

$$c_t\left(\frac{f_t + f_{o_t}}{2}\right) = c_c\left(\frac{F_{cc} + f_{o_c}}{2}\right) + \left(\frac{1}{2} - .05\right) F_{cc}$$

B 4.5.4.2 Example Problem B 4.5.4.2 (Cont'd)

Equality is reached within 0.70% when Δ is assumed equal to 0.065:

$$c_c = 0.345 + \Delta = 0.410 \text{ in.}$$

$$c_t = 1 - c_c = 0.590 \text{ in.}$$

Using Equation (4.5.1.2-3)

$$\epsilon_t = \frac{\epsilon_c c_t}{c_c} = \frac{0.0046 \times 0.590}{0.410} = 0.00662 \text{ in./in.}$$

From page 112 at ϵ_t

$$f_t = 113 \text{ ksi}$$

$$f_{o_t} = 38 \text{ ksi}$$

Substitution of these values into the above trial and error equation results in equality within 0.70%.

Section properties about the neutral axis:

$$I_t = \frac{2\,t\,c_t^3}{3} = \frac{2 \times 0.05 \times \overline{0.590}^3}{3} = 0.006845 \text{ in.}^4$$

$$Q_t = t\,c_t^2 = 0.05 \times \overline{0.590}^2 = 0.01740 \text{ in.}^3$$

$$k_t = \frac{Q_t c_t}{I_t} = \frac{0.01740 \times 0.590}{0.006845} = 1.5$$

(which checks with Figure B4.5.0-3 for a rectangle)

$$I_c = \frac{2\,t\,c_c^3}{3} + 2 \times 0.450t\left(c_c - \frac{t}{2}\right)^2 = \frac{2\text{x}0.05\text{x}0.410^3}{3}$$

$$+ 2\text{x}0.450\text{x}0.05\text{x}\overline{0.385}^2 = 0.008967 \text{ in.}^4$$

B 4.5.4.2 Example Problem B 4.5.4.2 (Cont'd)

$$Q_c = t c_c^2 + 2 \times 0.450t \left(c_c - \frac{t}{2}\right) = 0.05 \times \overline{0.410}^2 + 2 \times 0.450$$

$$\times\ 0.05 \times 0.385 = 0.02572 \text{ in.}^3$$

$$k_c = \frac{Q_c c_c}{I_c} = \frac{0.02572 \times 0.410}{0.008967} = 1.17$$

From page 112 using ϵ_t and k_t

$$f_{b_t} = 132 \text{ ksi}$$

Let's check this value by numerically evaluating Equation (4.5.0-1).

$$F_{b_t} = f_t + (k - 1) f_{o_t} = 113 + (1.5-1)38 = 132 \text{ ksi (check)}$$

From page 114 using ϵ_c and k_c

$$F_{b_c} = 64.5 \text{ ksi}$$

Calculate the ultimate allowable moment by Equation (4.5.1.2-4)

$$M_{b_{ult}} = \frac{F_{b_t} I_t}{c_t} + \frac{F_{b_c} I_c}{c_c} = \frac{132 \times 0.006845}{0.590}$$

$$+ \frac{64.5 \times 0.008967}{0.410} = \underline{\underline{2.94 \text{ in. - kips}}} \text{ . (answer)}$$

B4.5.4.3 Find the Shear Flow at the Neutral Axis of the Example Problem B4.5.4.2 If the Transverse Shear, (s), Is Equal to 5 kips. and the Bending Moment Is Equal to the Ultimate (2.94 in-kips.)

Shear flow at the neutral axis should be determined by using the section properties from each side of the neutral axis. The larger of the two shear flows should be (conservatively) used.

The procedure of Section B4.5.2.6 and consequently Section B4.5.1.6 will be used.

B4.5.4.3 Example Problem B4.5.4.3 (Cont'd)

Shear Flow From Tension Properties

The required section properties already known from example B4.5.4.2 are:

$I_t = 0.006845 \text{ in.}^4$ (tension side only)

$I_c = 0.008967 \text{ in.}^4$ (compression side only)

$I = I_t + I_c = 0.015812 \text{ in.}^4$

$c_t = 0.590 \text{ in.}$

$Q_t = 0.01740 \text{ in.}^3$

$k_t = 1.5$ (rectangle)

$\epsilon_t = 0.00662 \text{ in./in.}$

From the stress-strain (k= 1) curve on page 112 at ϵ_t

$$\left(\frac{df}{d\epsilon}\right)_1 = 7 \text{ ksi}/.001$$

$$\left(\frac{df_o}{d\epsilon}\right)_1 = 8 \text{ ksi}/.001$$

Using Equation (4.5.1.6-1)

$$\lambda_1 = \left(\frac{df_o}{df}\right)_1 = \left(\frac{df_o/d\epsilon}{df/d\epsilon}\right)_1$$

$$\lambda_1 = \frac{8}{7} = 1.14$$

Using Equation (4.5.1.6-4)

$$\beta_a = \frac{1 + \lambda_1\left(\frac{c_1}{\bar{y}_a} - 1\right)}{1 + \lambda_1 (k_1 - 1)}$$

$$\bar{y}_a = \frac{c_t}{2} = 0.295 \text{ in.}$$

B 4.5.4.3 Example Problem B 4.5.4.3 (Cont'd)

$$\beta_a = \frac{1 + 1.14\left(\frac{0.590}{0.295} - 1\right)}{1 + 1.14\,(1.5 - 1)} = 1.36$$

Shear flow at the NA from Equation (4.5.1.5-2)

$$q_a = \frac{\beta_a \, S Q_a}{I}$$

$$q_a = 1.36 \; \frac{5 \times 0.01740}{0.015812} = 7.48 \text{ kips/in.}$$

[Note that the conventional $\frac{SQ}{I}$ formula is 36% unconservative here.]

Shear Flow From Compression Properties

The required section properties already known from example B4.5.4.2 are:

$I = 0.015812 \text{ in.}^4$ (Reference Page 82)

$c_c = 0.410 \text{ in.}$

$Q_c = 0.02572 \text{ in.}^3$

$k_c = 1.17$

$\epsilon_c = 0.0046 \text{ in./in.}$

From the stress-strain (k = 1) curve on page 114 at ϵ_c

$$\left(\frac{df}{d\epsilon}\right)_2 = 4.6 \text{ ksi/.001}$$

$$\left(\frac{df_o}{d\epsilon}\right)_2 = 5.0 \text{ ksi/.001}$$

B 4.5.4.3 Example Problem B 4.5.4.3 (Cont'd)

Using Equation (4.5.1.6-2)

$$\lambda_2 = \left(\frac{df_o}{df}\right)_2 = \left(\frac{df_o/d\epsilon}{df/d\epsilon}\right)_2$$

$$\lambda_2 = \frac{5.0}{4.6} = 1.09$$

Using Equation (4.5.1.6-7)

$$\beta_b = \frac{1 + \lambda_2\left(\frac{c_2}{\bar{y}_b} - 1\right)}{1 + \lambda_2 (k_2 - 1)}$$

$$\bar{y}_b = \frac{Q_b}{A_b} = \frac{Q_c}{A_c}$$

$$A_c = 2\,[0.05\,(0.410 + 0.450)] = 0.086 \text{ in.}^2$$

$$\bar{y}_b = \frac{0.02572}{0.086} = 0.299 \text{ in.}$$

$$\beta_b = \frac{1 + 1.09\left(\frac{0.410}{0.299} - 1\right)}{1 + 1.09(1.17 - 1)} = 1.18$$

Shear flow at the NA from Equation (4.5.1.6-6).

$$q_b = \beta_b \frac{SQ_b}{I}$$

$$q_b = 1.18\,\frac{5 \times 0.02572}{0.015812} = 9.6 \text{ kips/in.}$$

[Note that the conventional $\frac{SQ}{I}$ formula is 18% unconservative here.]

The larger shear flow should be used which is

$$q = 9.6 \text{ kips/in.}$$

B4.5.5 Index for Bending Modulus of Rupture Curves for Symmetrical Sections

These curves provide yield and ultimate modulus of rupture values for symmetrical sections only. For materials with significantly different tension and compression stress-strain curves, the necessary corrections for shifting of the neutral axis are already included. In the case of work hardened stainless steels in longitudinal bending with all fibers in tension (as in pressurized cylinders), the transverse Modulus of Rupture Curves are applicable.

It is recommended that MIL-HDBK-5 or other official sources be used for allowable material properties. Where these values correspond directly to the values called out on the graphs of this section, the modulus of rupture values are applicable as shown.

Where material allowables vary with thickness, cross-sectional area, etc., only one or two Modulus of Rupture Curves are presented. Therefore, for material properties slightly higher or lower than those used in the given curve, the modulus values may be ratioed up or down (provided the % elongations are practically the same).

B4.5.5.1 Stainless Steels - Minimum Properties

*(RT) - Room Temperature.

** Not presently available

*Alloy Steels include AISI 4130, 4140, 4340, 8630, 8735, 8740, and 9840.

B4.5.6 Index for Plastic Bending Curves

These curves represent modulus of rupture values relative to the stress-strain curve for a section on either side of the neutral axis. These curves are particularly useful for unsymmetrical sections and when the allowable stress is other than yield or ultimate for symmetrical or unsymmetrical sections.

For materials with significantly different tension and compression stress-strain curves, Plastic Bending Curves for both are presented. When the difference is slight, the Plastic Bending Curves for the lower stress-strain curve only are presented.

It is recommended that MIL-HDBK-5 or other official sources be used for allowable material properties. Where these values correspond directly to the values called out on the graphs of this section, the modulus of rupture values are applicable as shown.

Where material allowables vary with thickness, cross-sectional area, etc., only one or two Plastic Bending Curves are presented. Therefore, for material properties slightly higher or lower than those used in the given curve, the modulus values may be ratioed up or down (provided the % elongations are practically the same).

Stress directions called out in the headings of the Plastic Bending Curves apply only to stress caused by pure bending. The bending modulus for compression is higher in the transverse grain direction (in fact, slightly higher than the tension modulus) than in the longitudinal grain direction, for the AISI 301 stainless steels. Separate curves are therefore presented for AISI stainless steels in the longitudinal compression.

B4.5.6.1 Stainless Steels-Minimum Properties

B4.5.6.2 Low Carbon and Alloy Steels**-Minimum Properties

* (RT) - Room Temperature.
** Alloy Steels include AISI 4130, 4140, 4340, 8630, 8735, 8740 and 9840.

B4.5.6.2 Low Carbon and Alloy Steels - Minimum Properties (Cont'd)

B4.5.6.5 Aluminum - Minimum Properties (Cont'd)

B4.5.6.6 Magnesium-Minimum Properties

B4.5.5.1 Stainless Steels-Minimum Properties

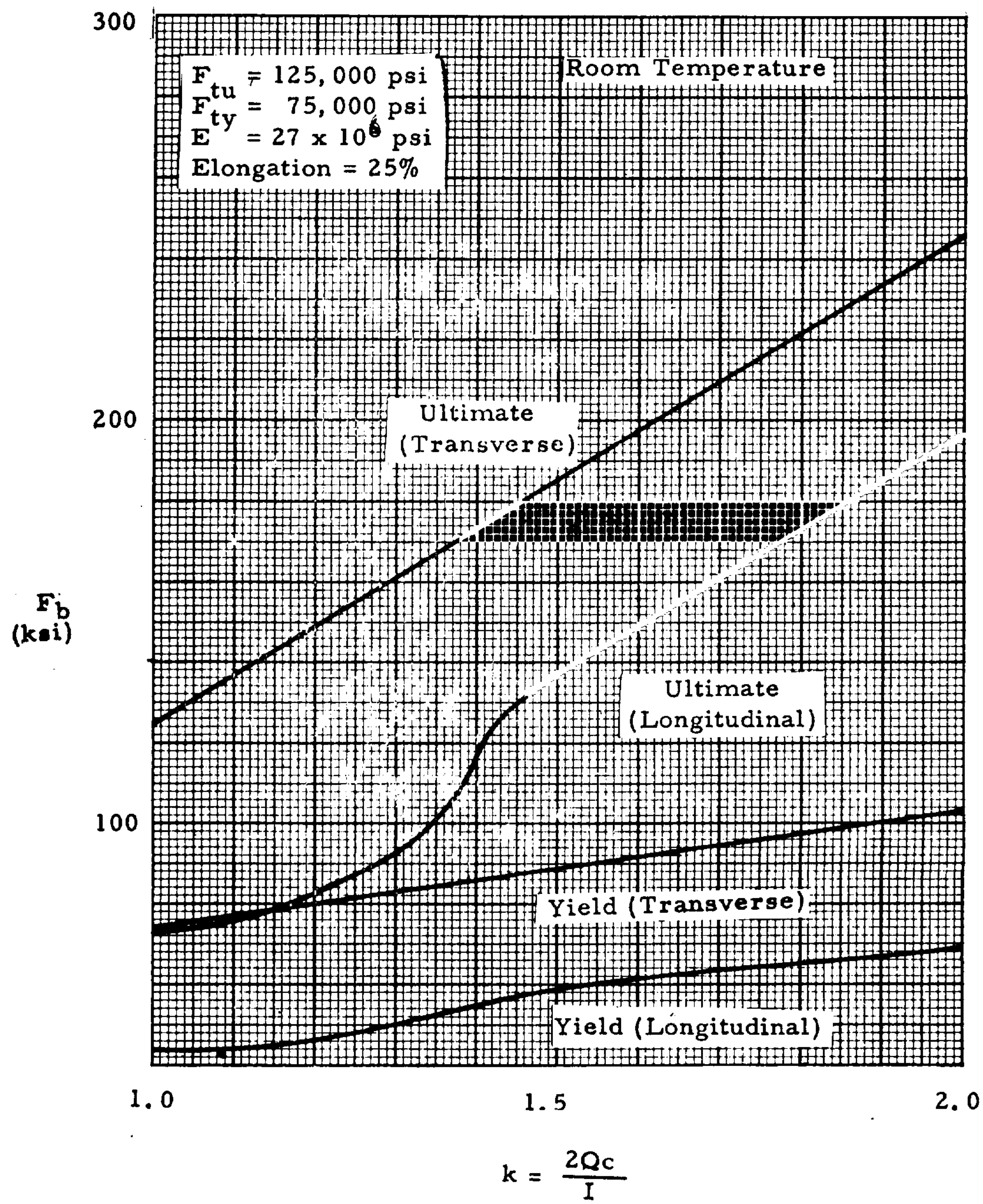


Fig. B4.5.5.1-1 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections 1/4 Hard AISI 301 Stainless Steel Sheet

B4.5.5.1 Stainless Steels-Minimum Properties

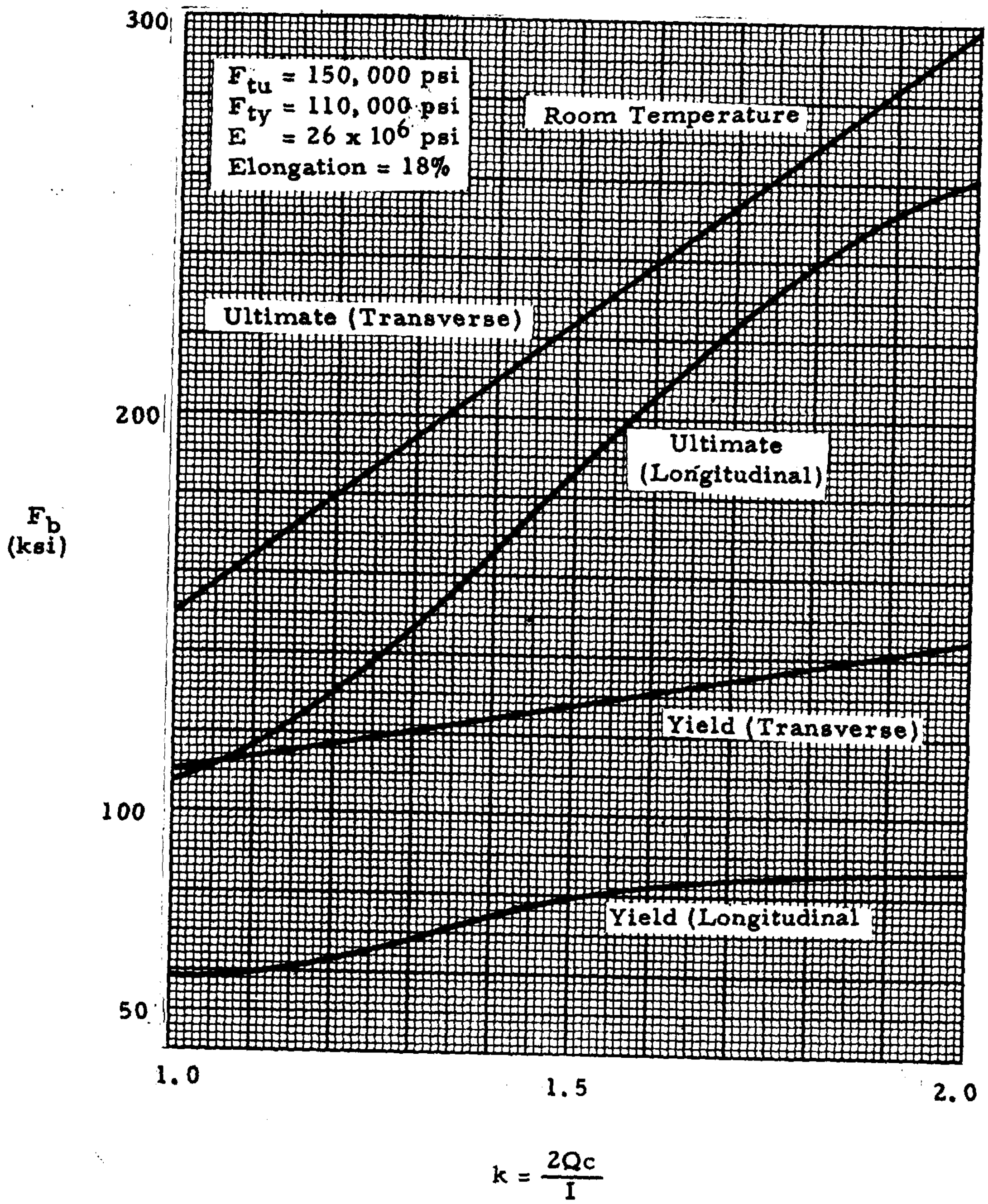


Fig. B4.5.5.1-2 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections 1/2 Hard AISI 301 Stainless Steel Sheet

B4.5.5.1 Stainless Steels-Minimum Properties

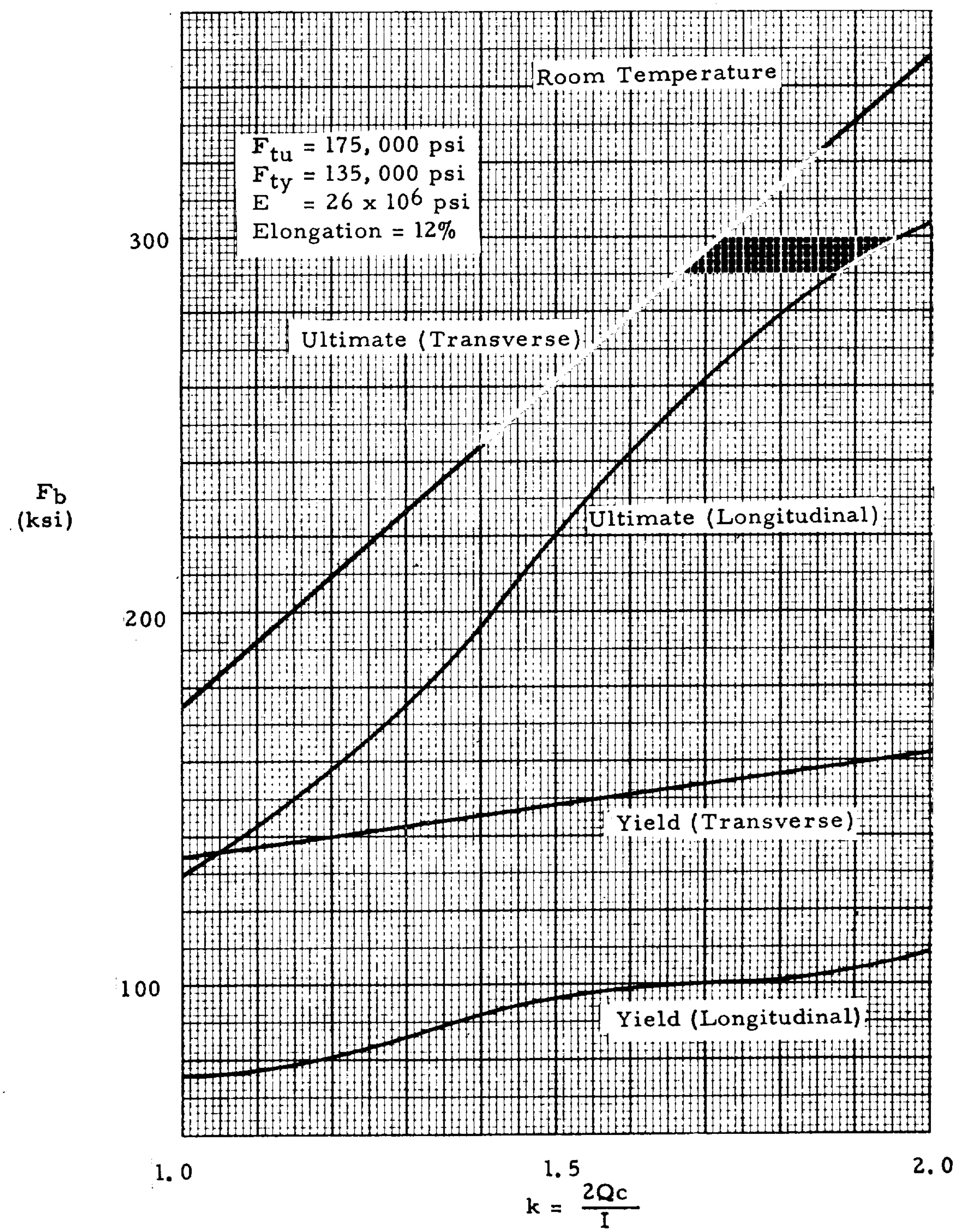


Fig. B4.5.5.1-3 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections 3/4 Hard AISI 301 Stainless Steel Sheet

B4.5.5.1 Stainless Steels-Minimum Properties

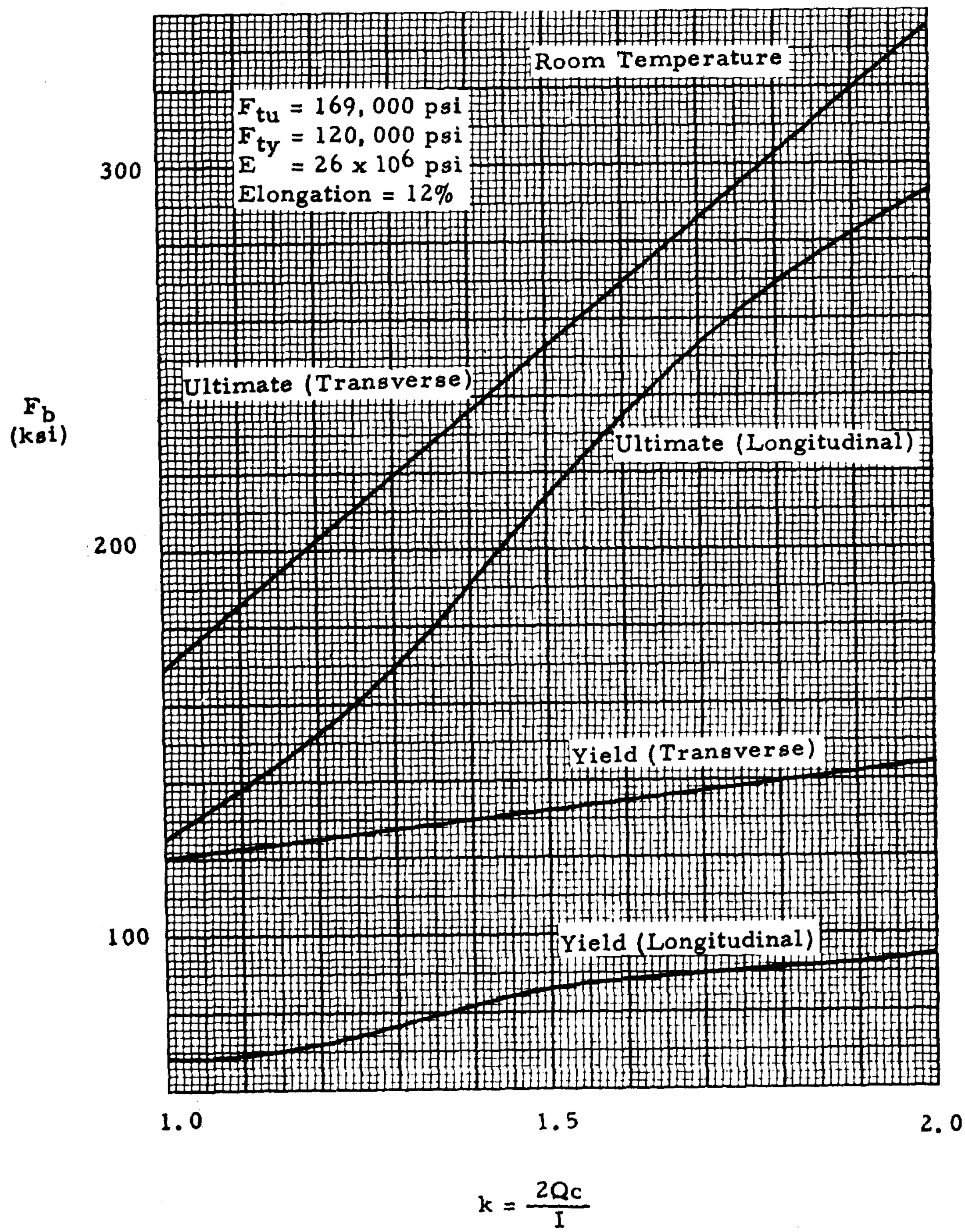


Fig. B4.5.5.1-4 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections Special
3/4 Hard AISI 301 Stainless Steel Sheet

B4.5.5.1 Stainless Steels-Minimum Properties

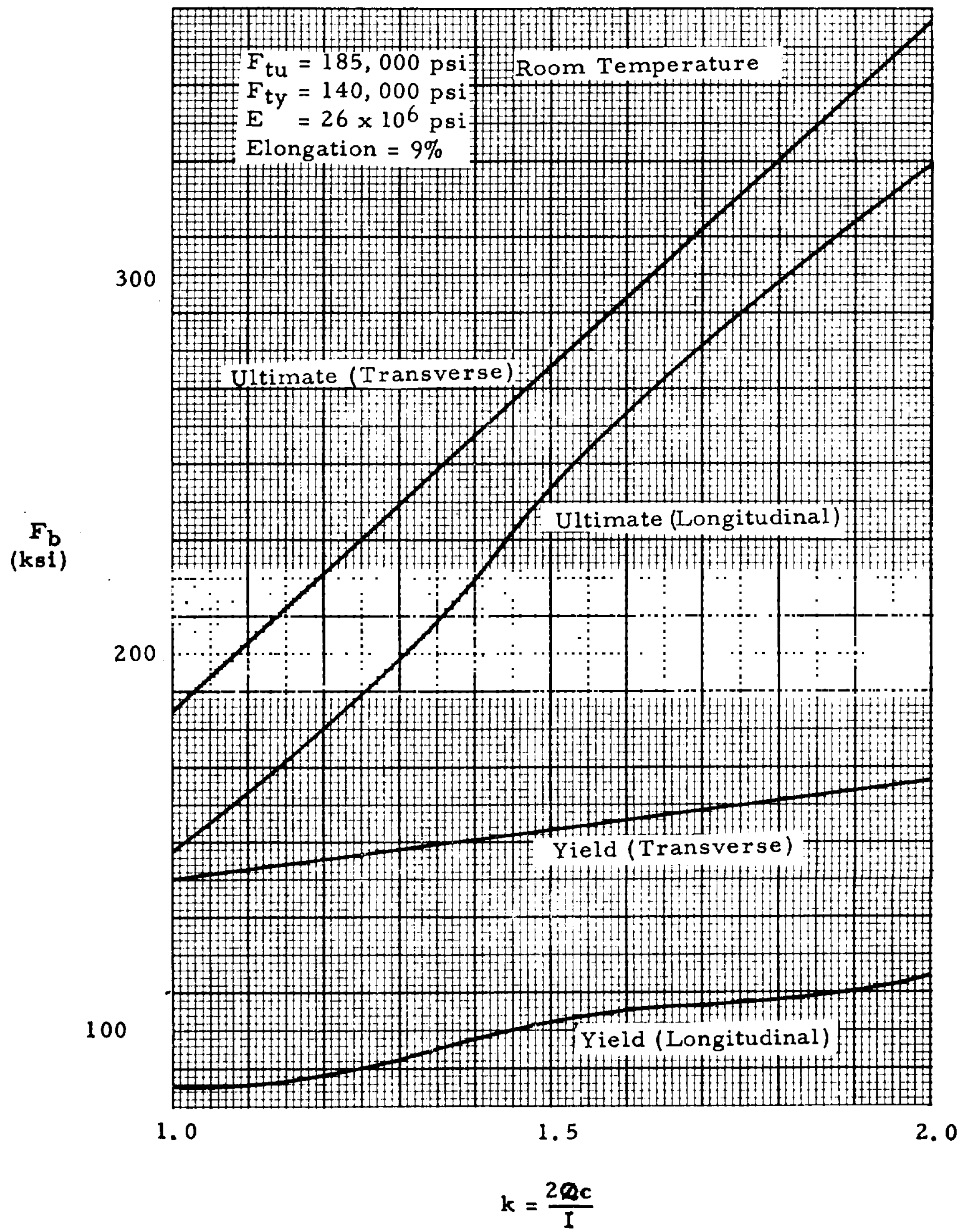


Fig. B4.5.5.1-5 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections Full Hard AISI 301 Stainless Steel Sheet

B4.5.5.1 Stainless Steels-Minimum Properties

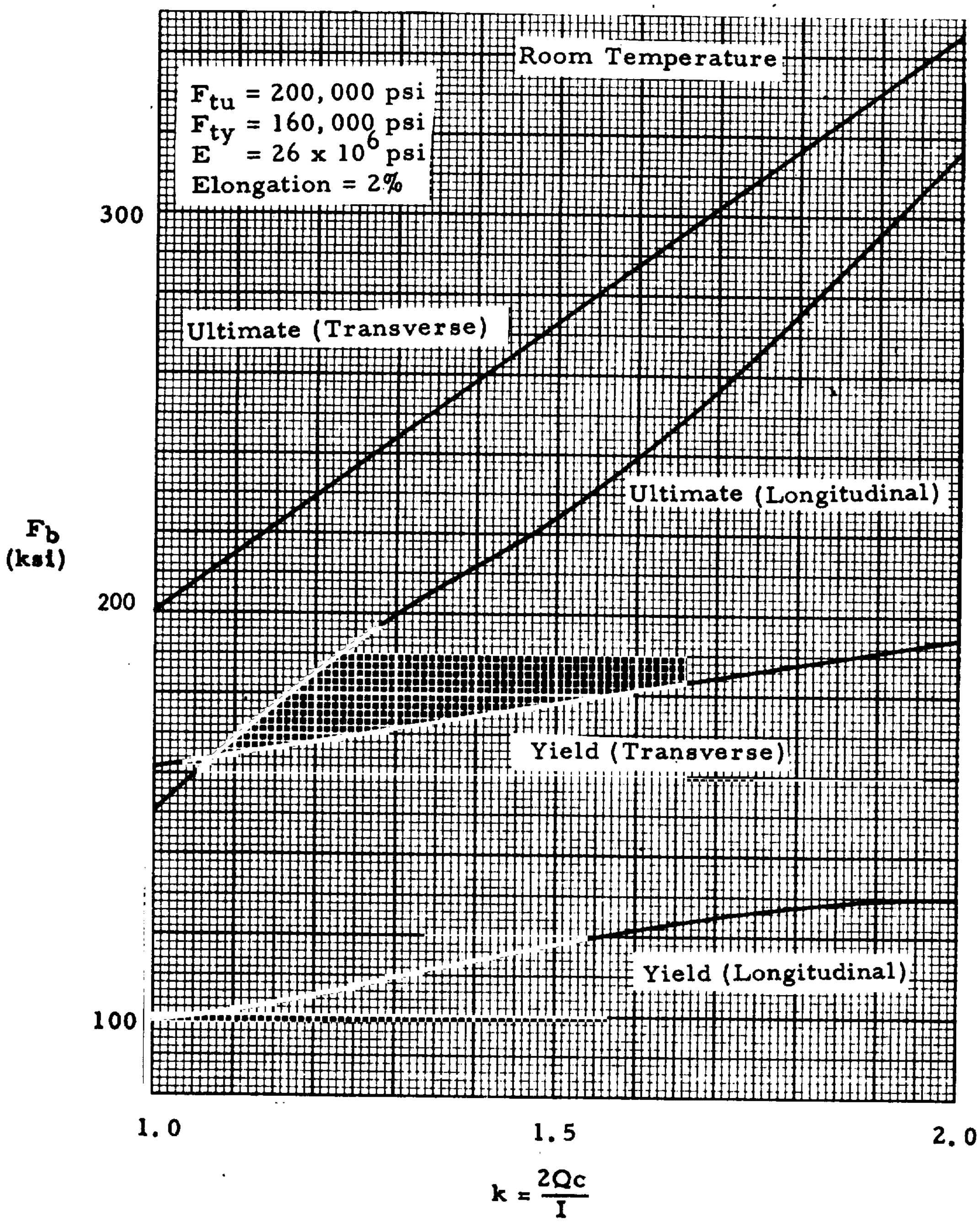


Fig. B4.5.5.1-6 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections Extra Hard AISI 301 Stainless Steel Sheet

B4.5.5.1 Stainless Steels-Minimum Properties

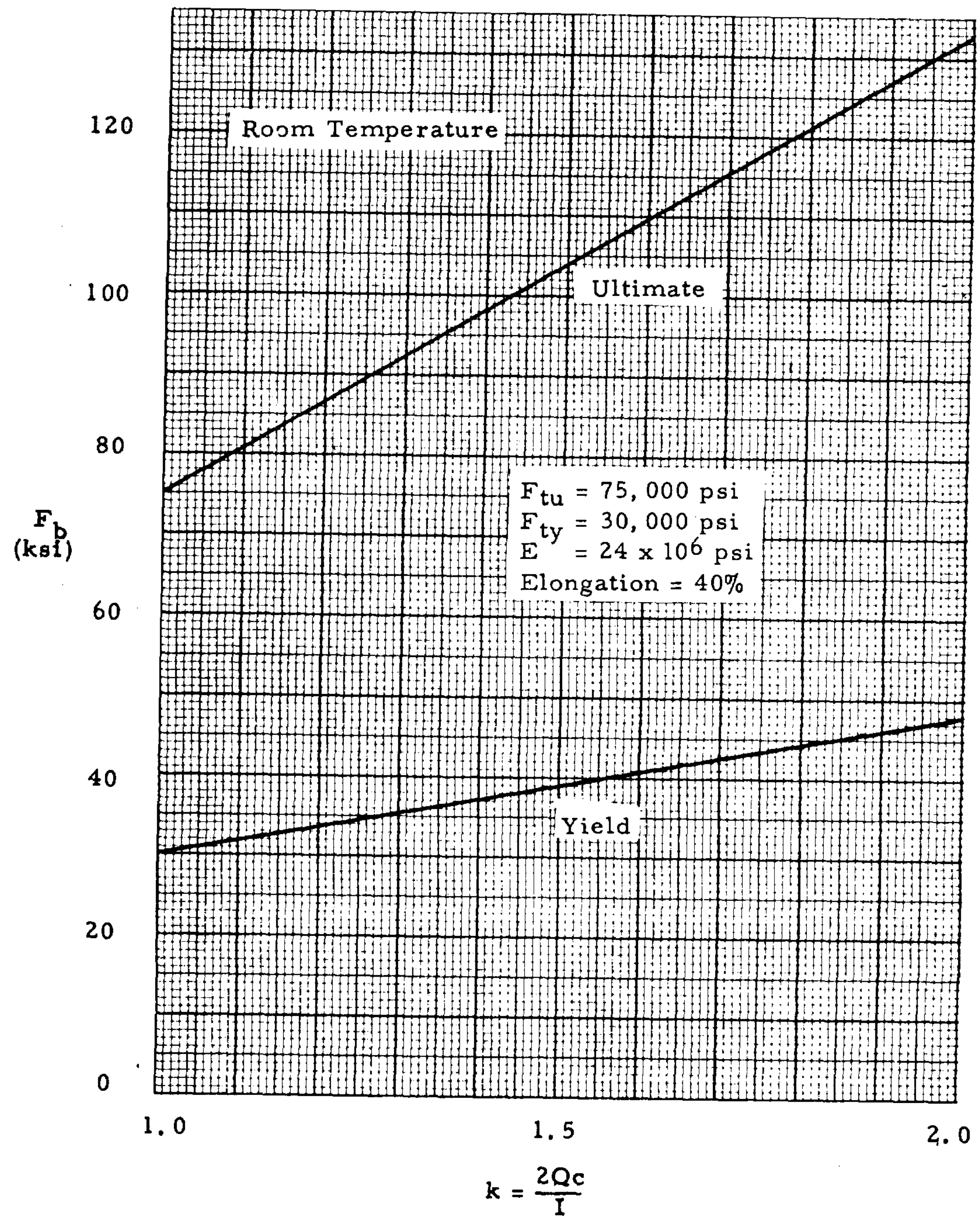


Fig. B4.5.5.1-7 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections Annealed AISI 321 Stainless Steel Sheet

Graph to be furnished when available

Graph to be furnished when available

B4.5.5.1 Stainless Steels-Minimum Properties

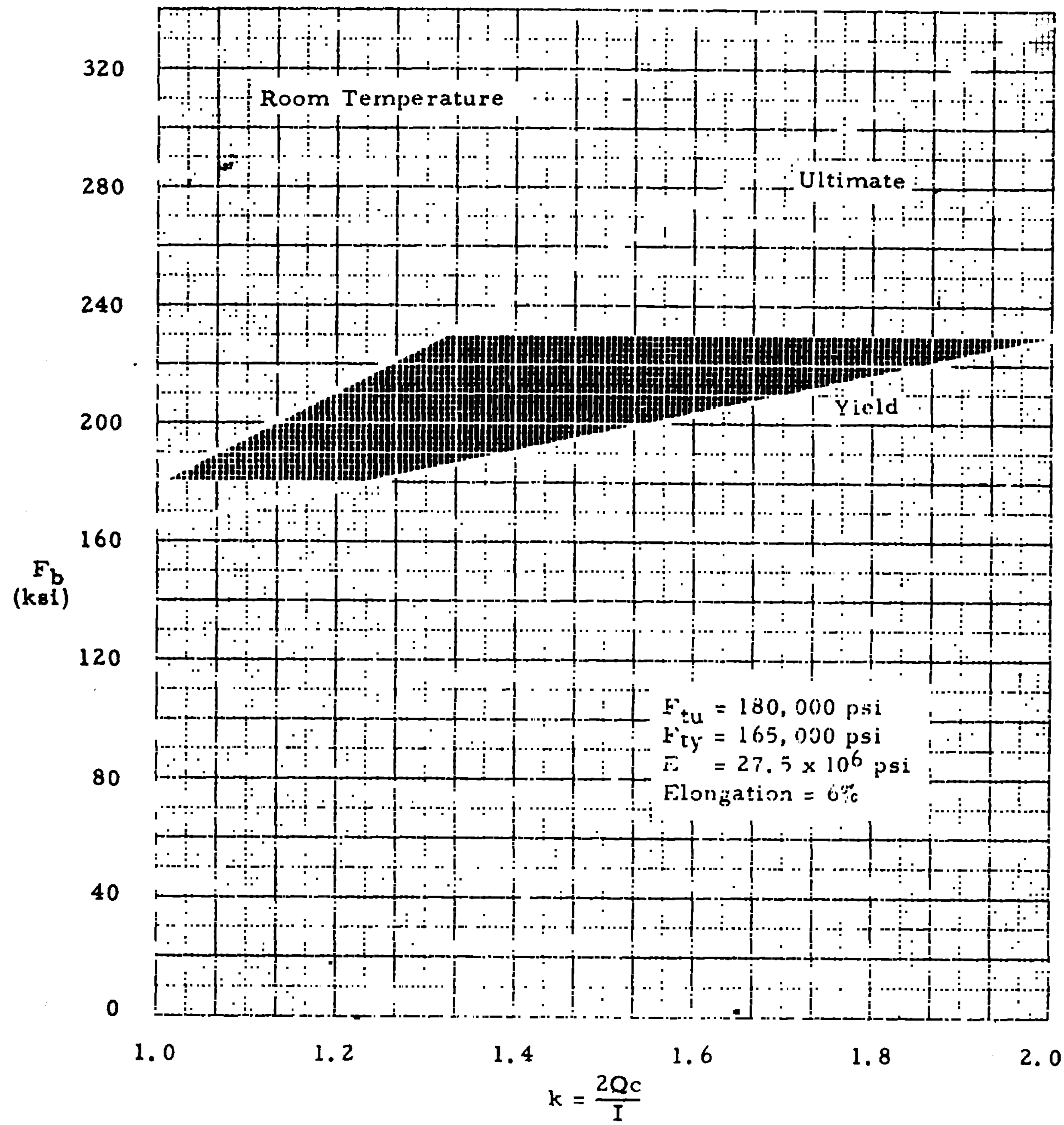


Fig. B4.5.5.1-10 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections 17-4 PH Stainless Steel - Heat Treated - Bars and Forgings

B4.5.5.1 Stainless Steels-Minimum Properties

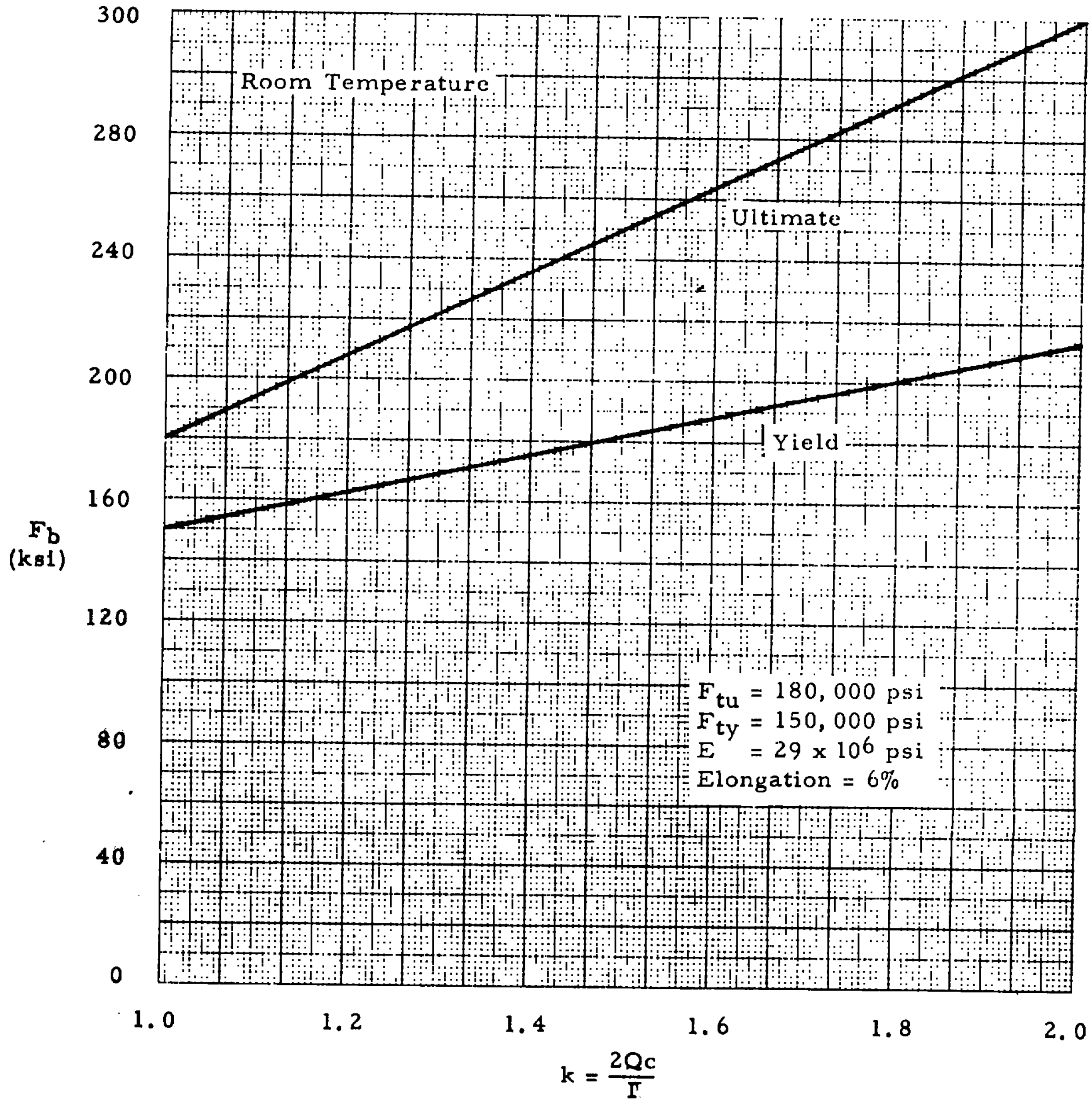


Fig. B4.5.5.1-11 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections 17-7 PH Stainless Steel

B4.5.5.1 Stainless Steels-Minimum Properties

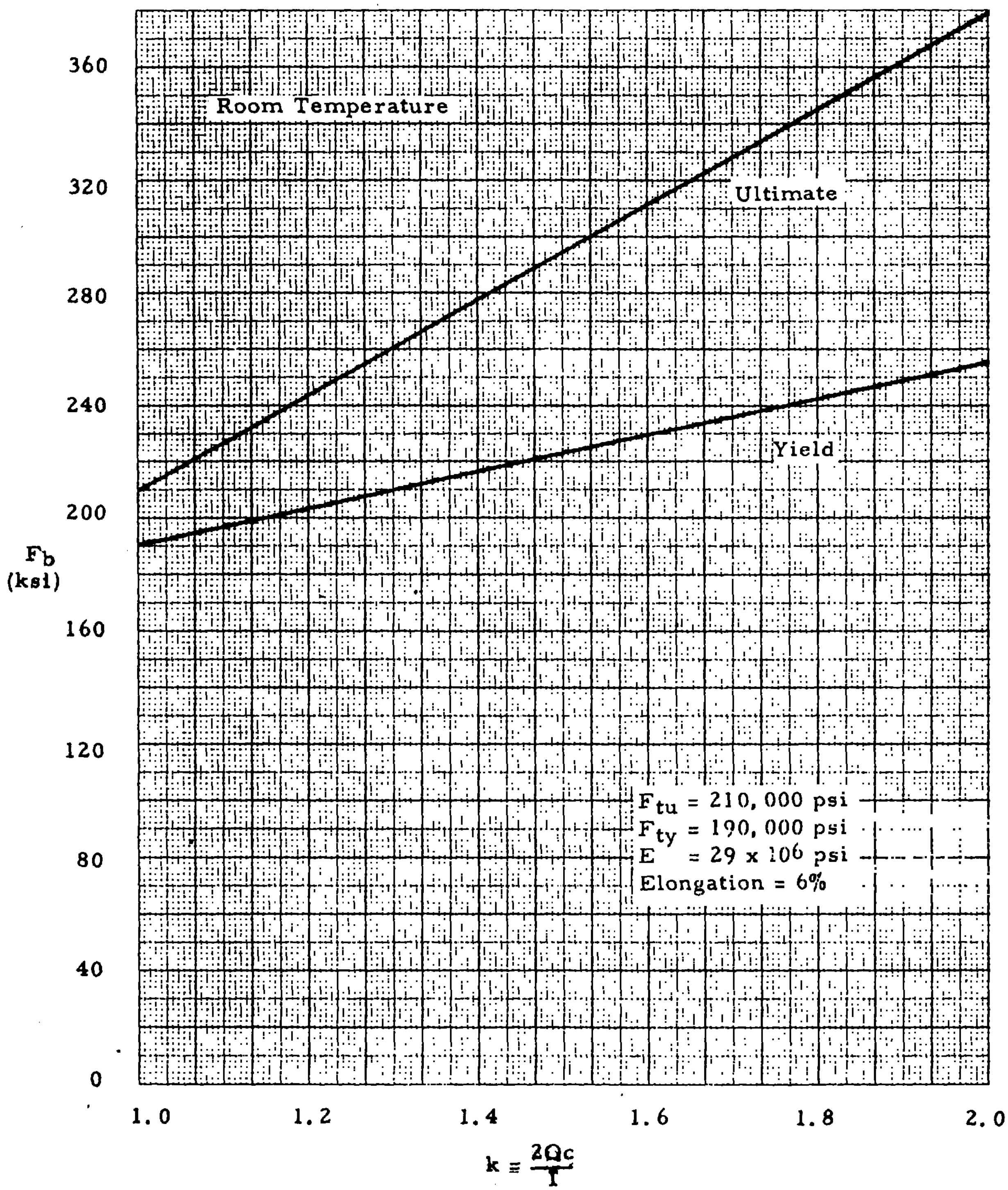


Fig. B4.5.5.1-12 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections 17-7 PH Stainless Steel

B4.5.5.1 Stainless Steels-Minimum Properties

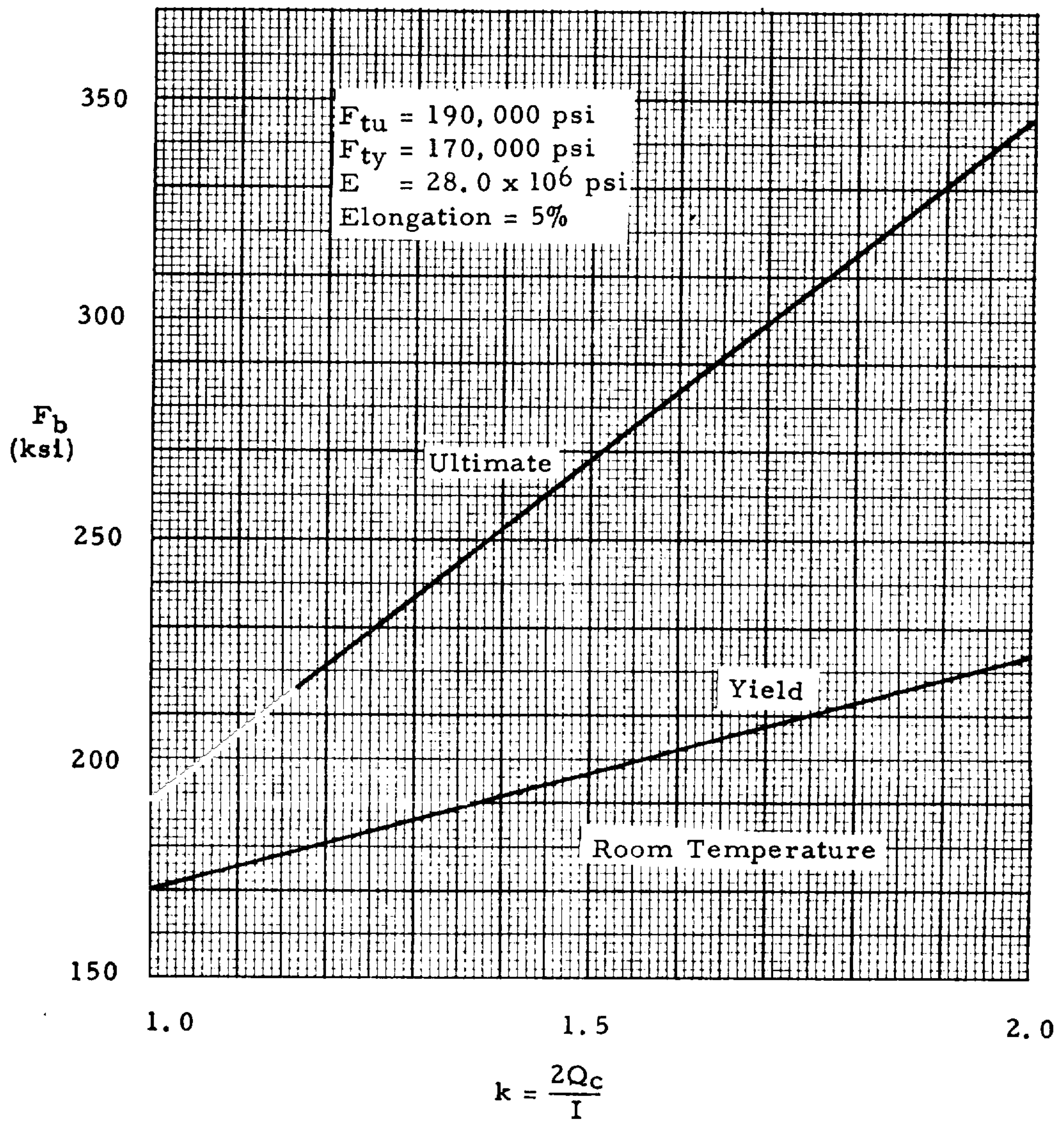


Fig. B4.5.5.1-13 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections PH 15-7 Mo (TH 1050), Stainless Steel Sheet & Strip Thickness 0.020 to 0.185 Inches.

B4.5.5.1 Stainless Steels-Minimum Properties

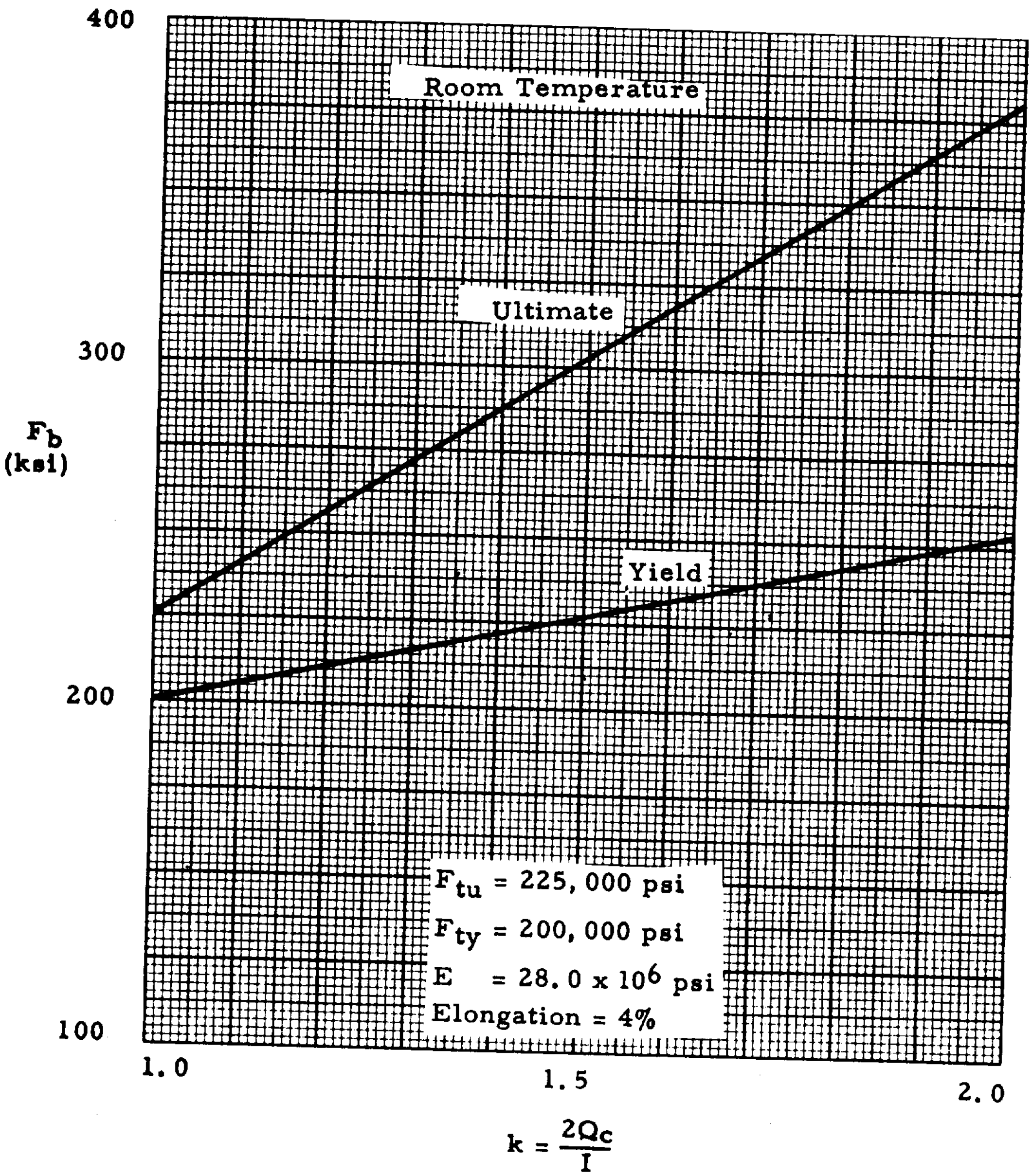


Fig. B4.5.5.1-14 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections PH 15-7 Mo (RH 950) Stainless Steel Sheet & Strip Thickness 0.020 to 0.187 Inches

B4.5.5.1 Stainless Steels-Minimum Properties

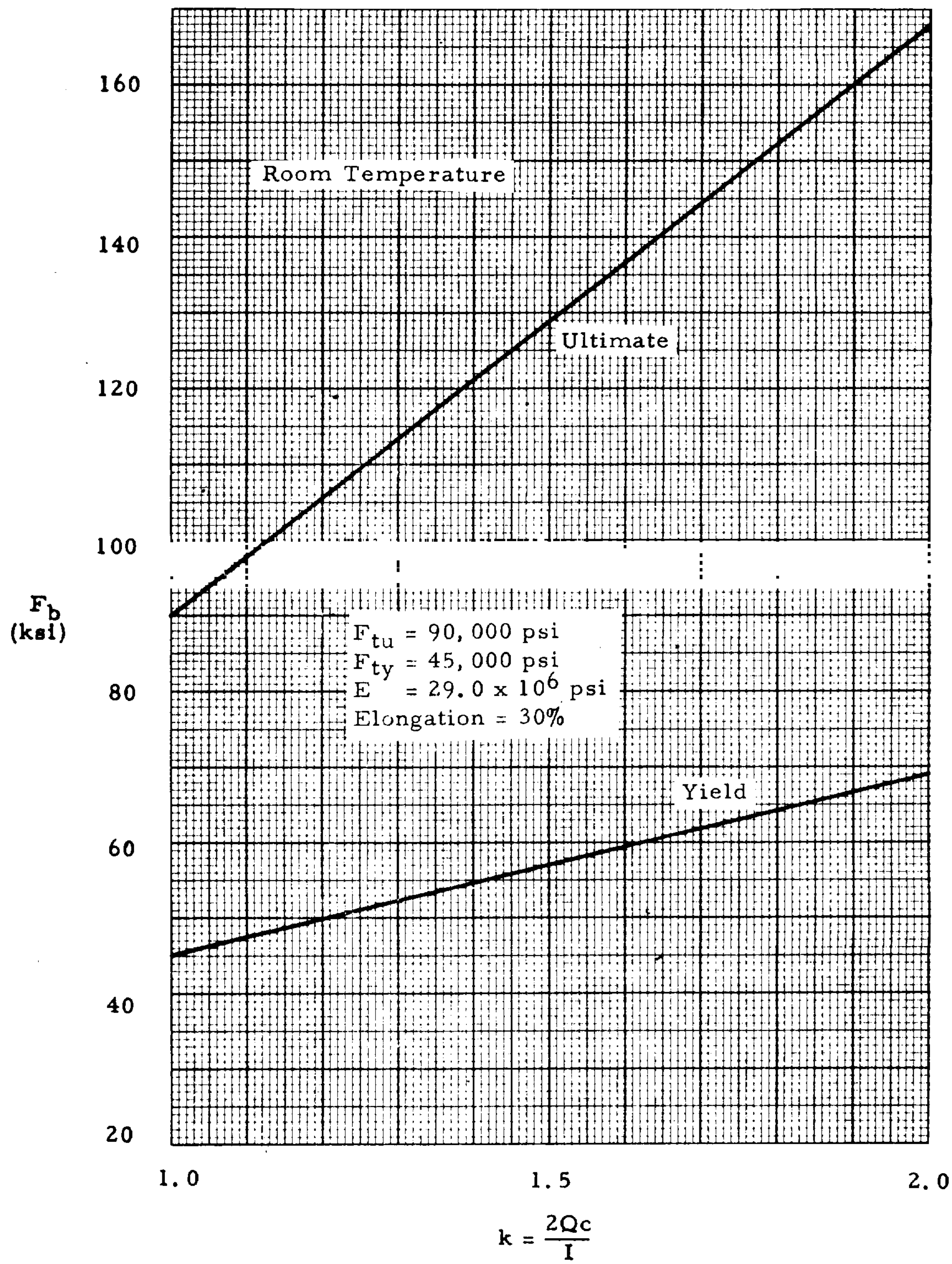


Fig. B4.5.5.1-15 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections 19-9 DL (AMS 5526) & 19-9 DX (AMS 5538) Stainless Steel

B4.5.5.1 Stainless Steels-Minimum Properties

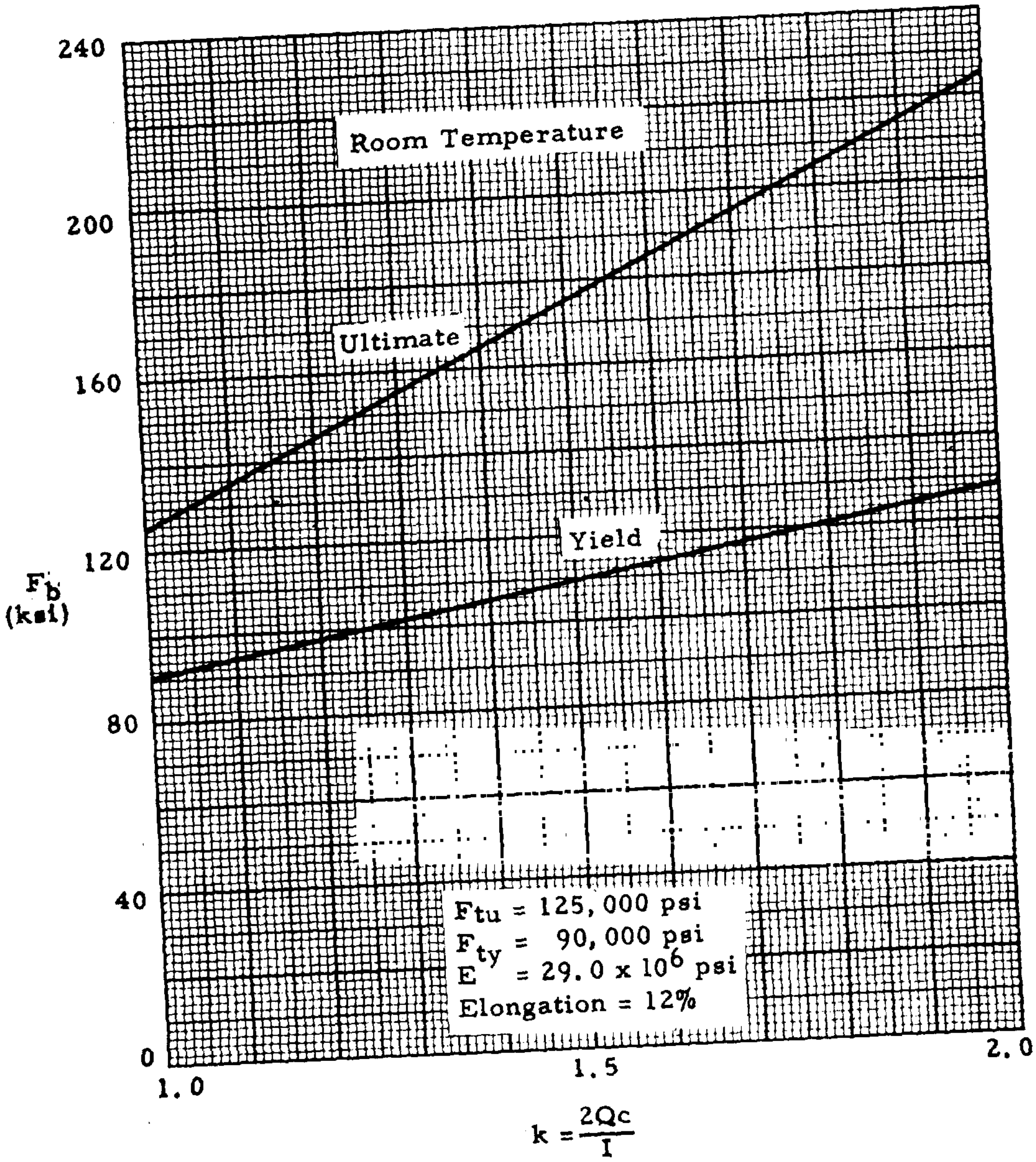


Fig. B4.5.5.1-16 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections 19-9DL (AMS 5527) & 19-9 DX (AMS 5539) Stainless Steel

B4.5.6.1 Stainless Steels-Minimum Properties

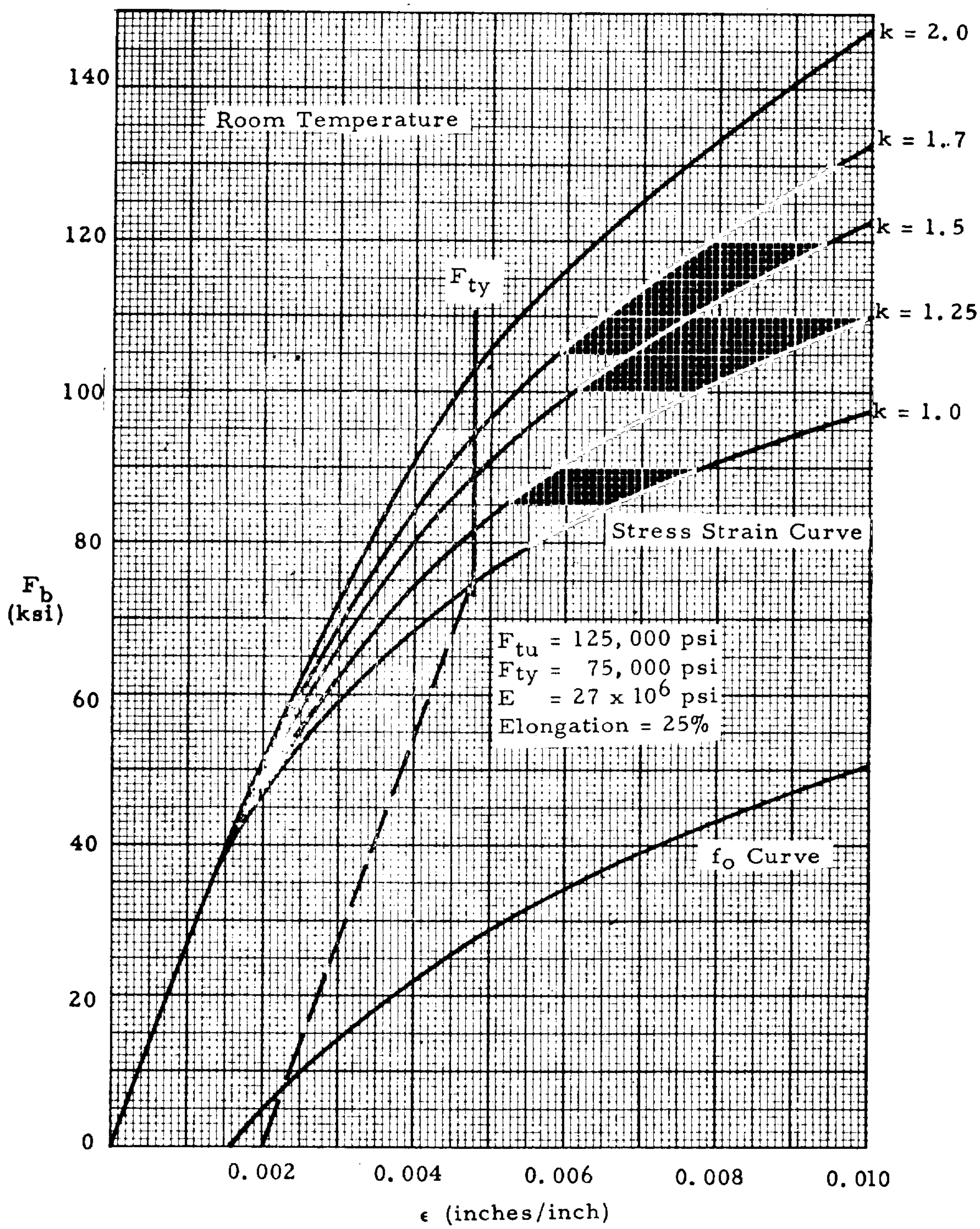


Fig. B4.5.6.1-1 Minimum Plastic Bending Curves 1/4 Hard AISI 301 Stainless Steel Sheet-for Tension or Transverse Compression and Stress Relieved Material-for Tension or Compression

B4.5.6.1 <u>Stainless Steels-Minimum Properties</u>

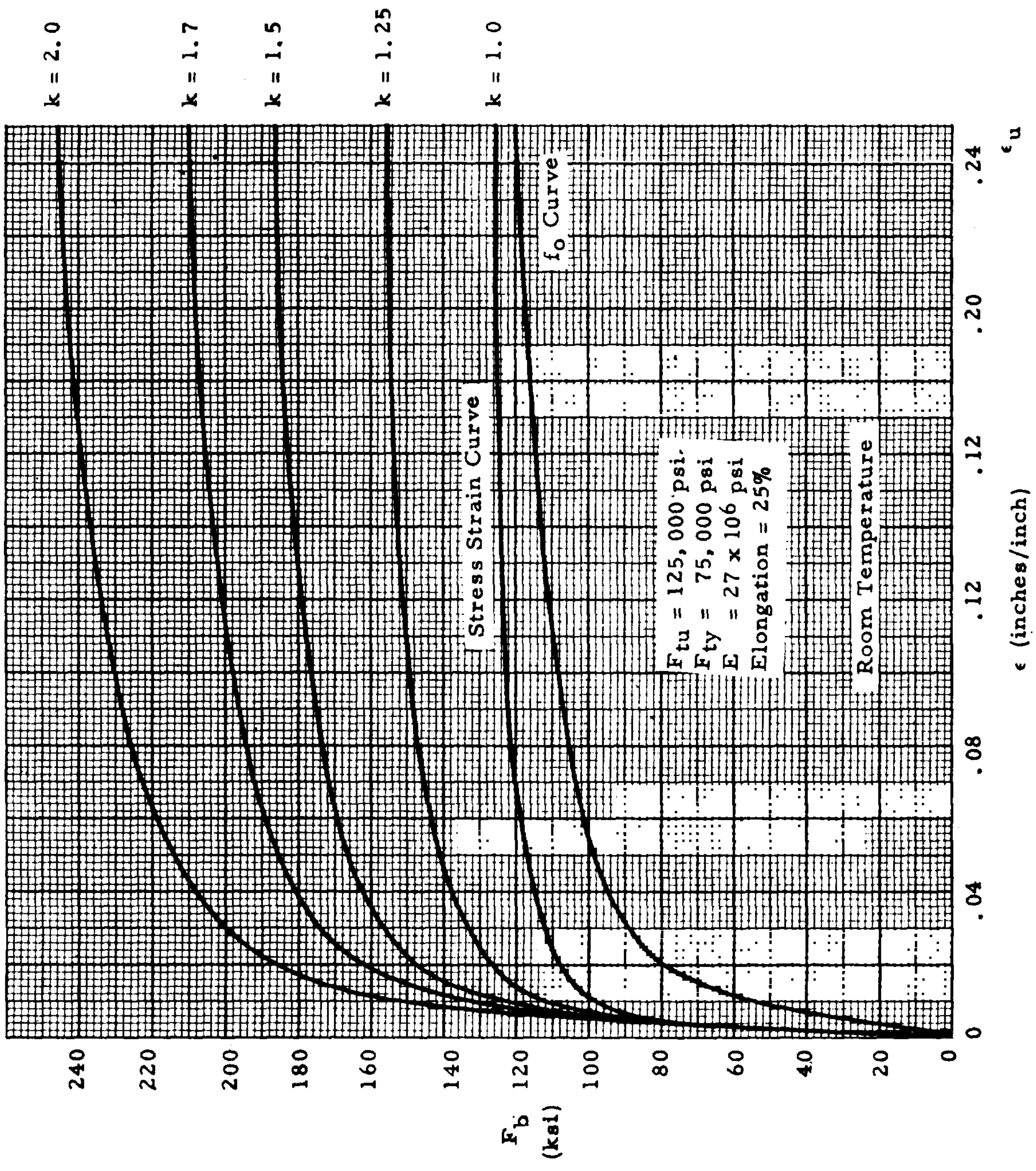


Fig. B4.5.6.1-2 Minimum Plastic Bending Curves 1/4 Hard AISI 301 Stainless Steel Sheet-for Tension or Transverse Compression and Stress Relieved Material-for Tension or Compression

B4.5.6.1 Stainless Steels-Minimum Properties

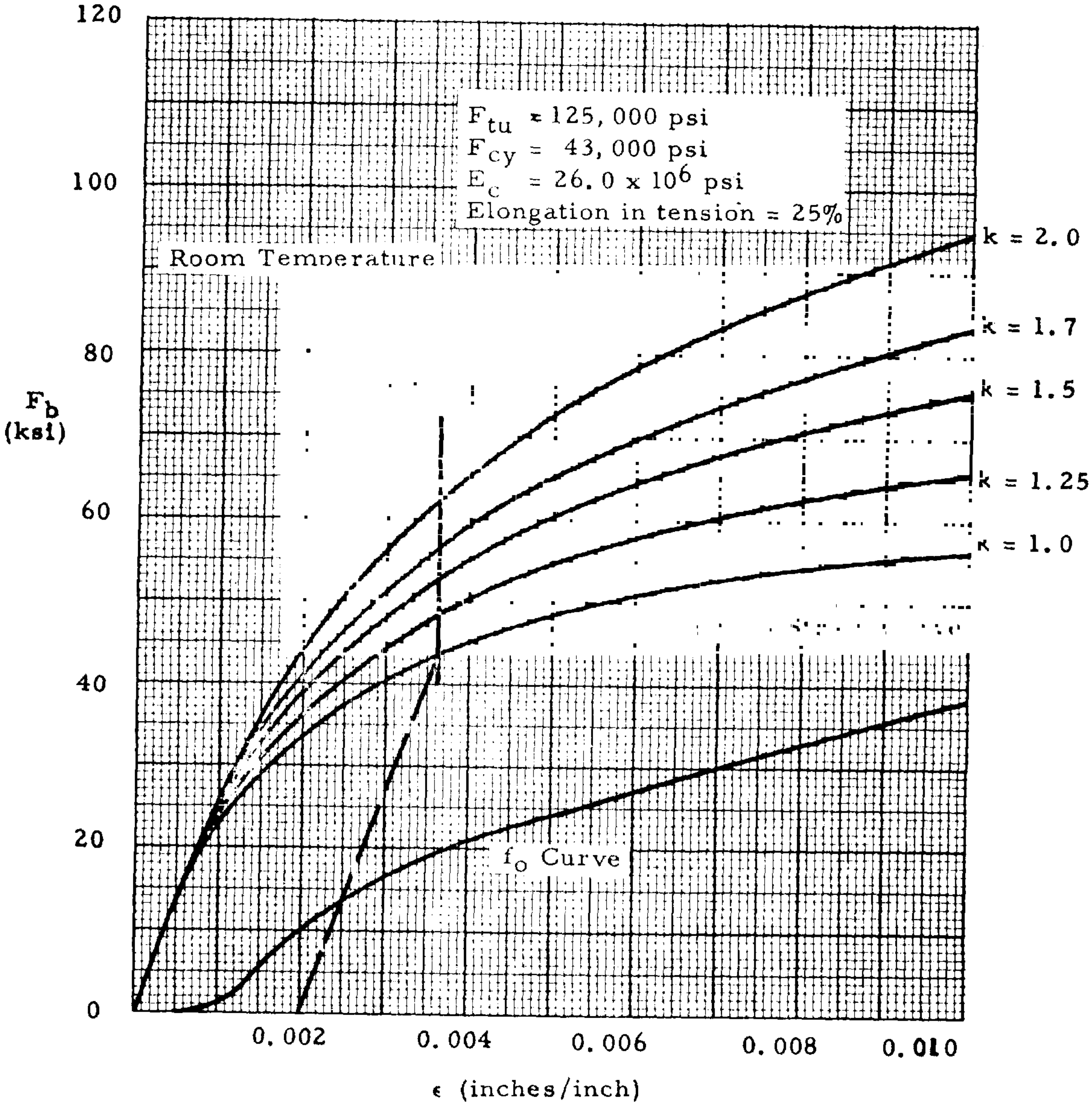


Fig. B4.5.6.1-3 Minimum Plastic Bending Curves 1/4 Hard AISI 301 Stainless Steel Sheet - for Longitudinal Compression

B4.5.6.1 Stainless Steels-Minimum Properties

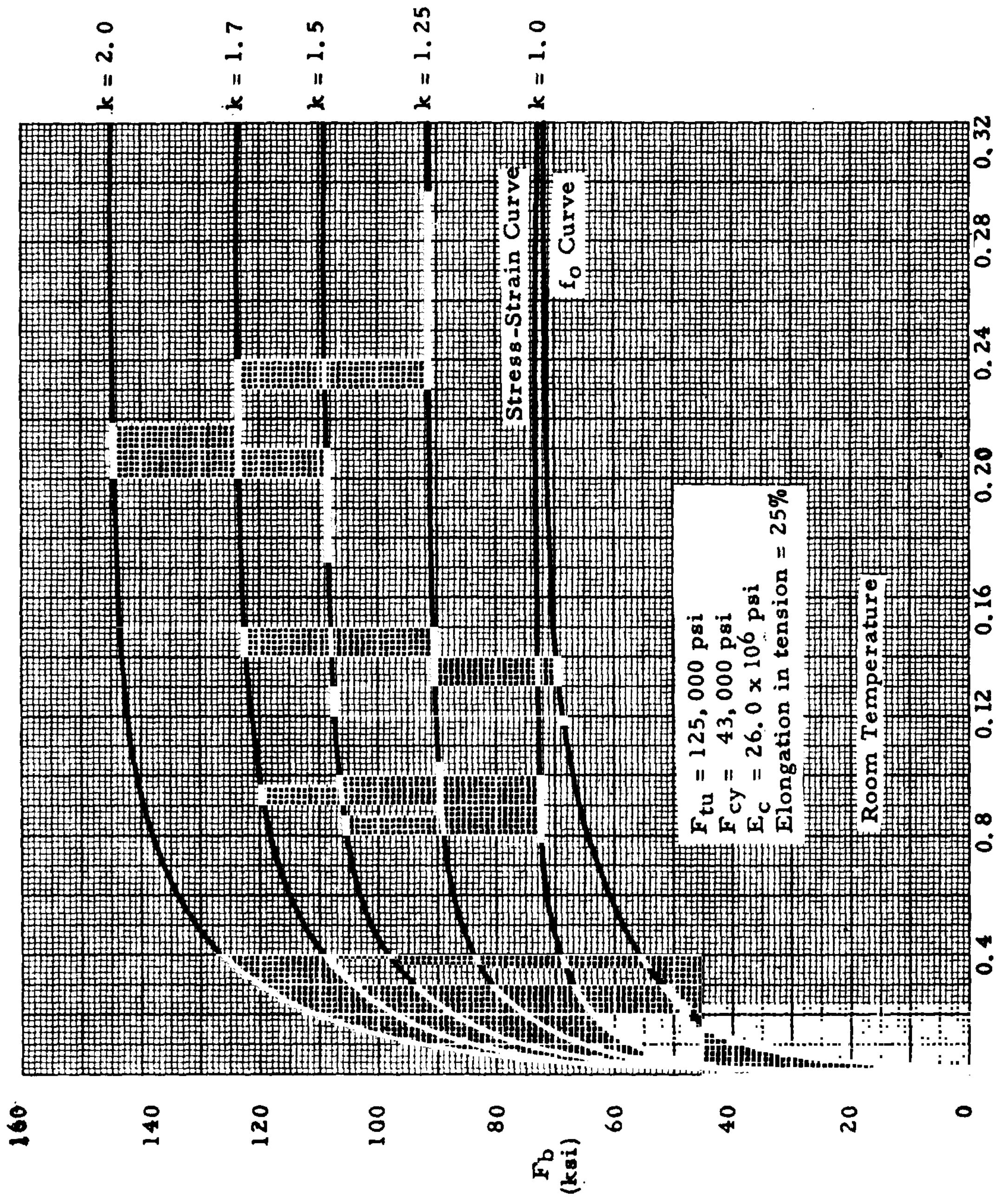


Fig. B4.5.6.1-4 Minimum Plastic Bending Curves 1/4 Hard AISI 301 Stainless Steel Sheet - for Longitudinal Compression

B4.5.6.1 Stainless Steels-Minimum Properties

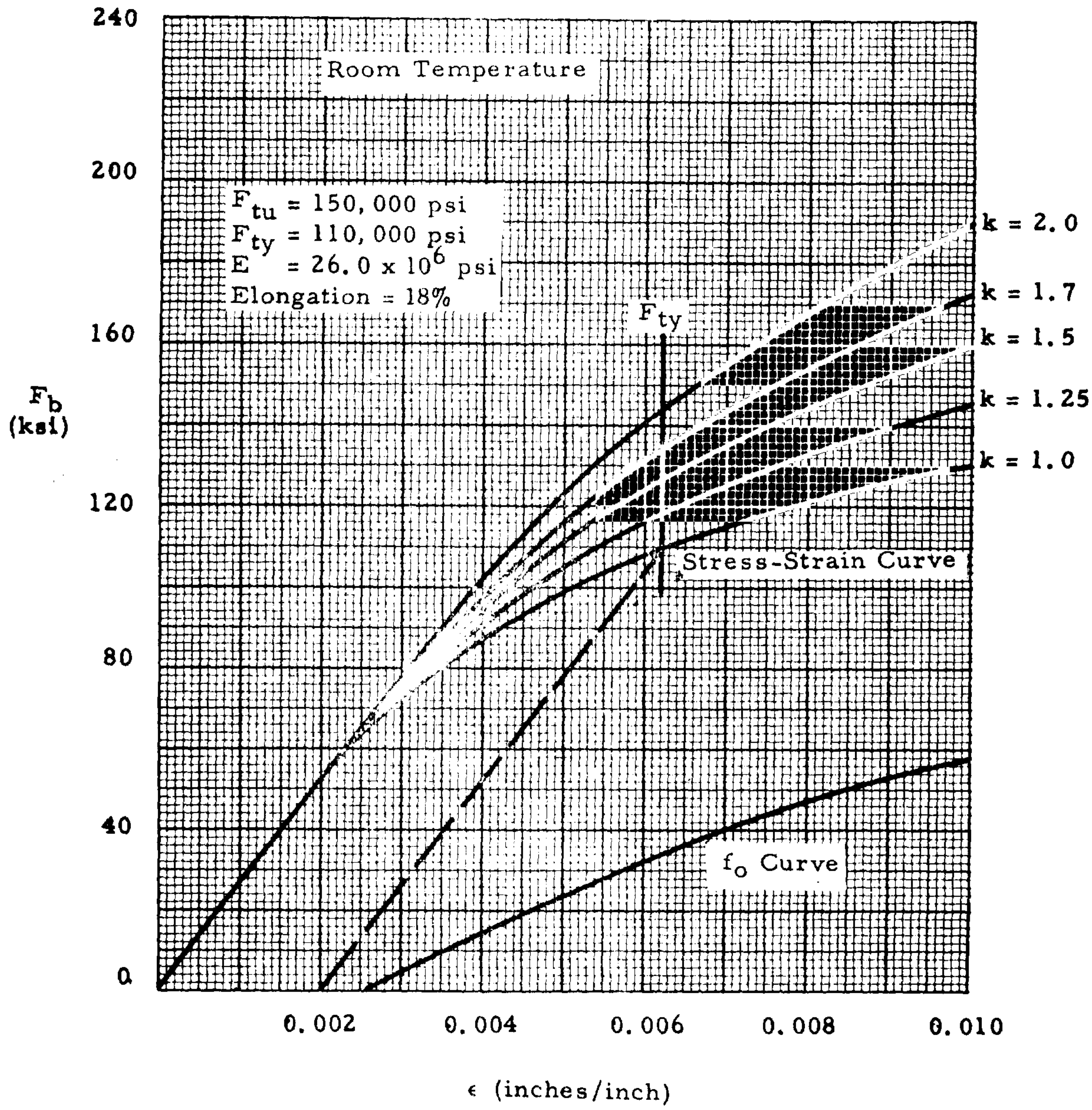


Fig. B4.5.6.1-5 Minimum Plastic Bending Curves 1/2 Hard AISI 301 Stainless Steel Sheet-for Tension or Transverse Compression and Stress Relieved Material-for Tension or Transverse Compression

B4.5.6.1 Stainless Steels-Minimum Properties

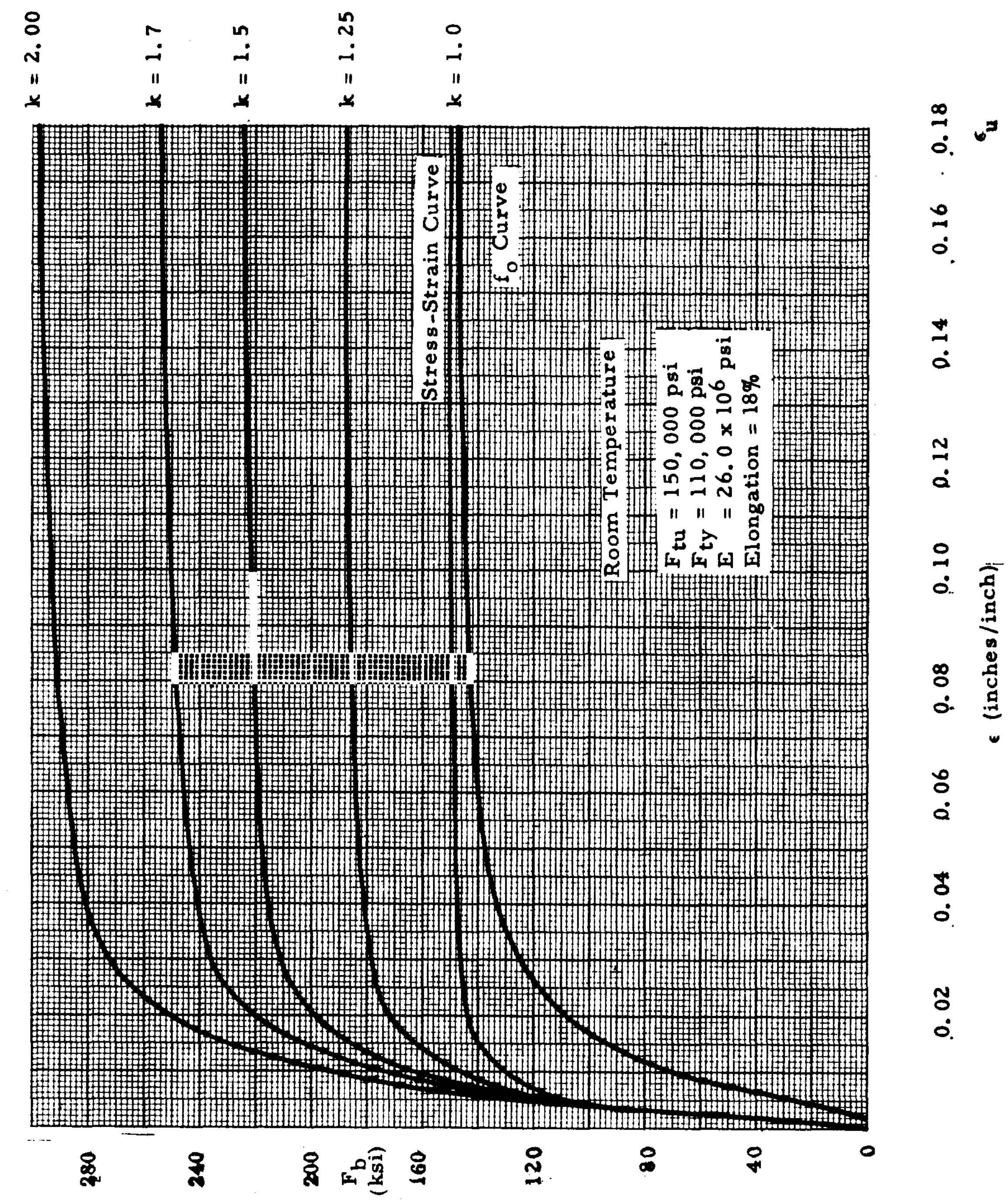


Fig. B4.5.6.1-6 Minimum Plastic Bending Curves 1/2 Hard AISI 301 Stainless Steel Sheet-for Tension or Transverse Compression and Stress Relieved Material-for Tension or Transverse Compression

B4.5.6.1 Stainless Steels-Minimum Properties

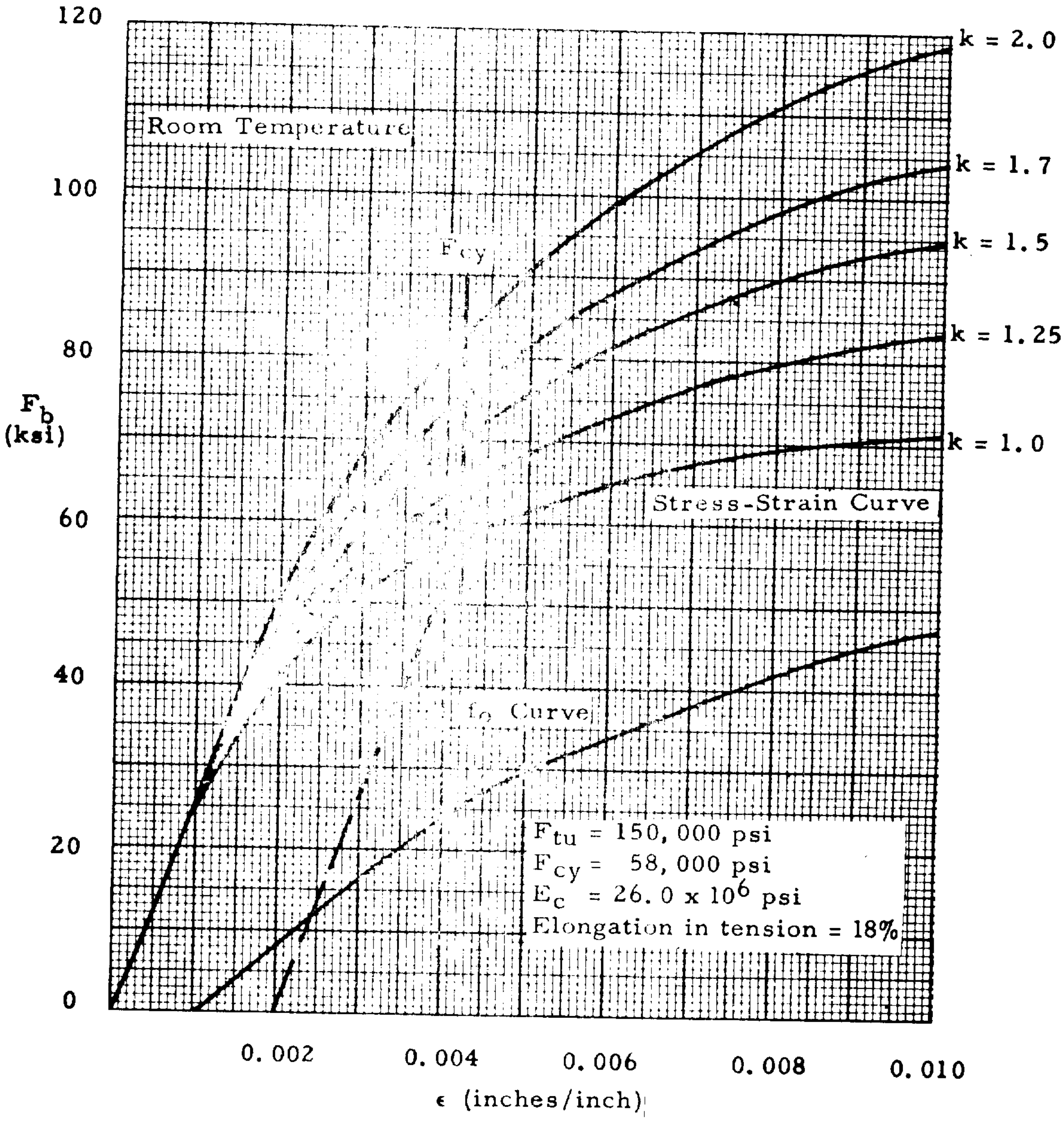


Fig. B4.5.6.1-7 Minimum Plastic Bending Curves 1/2 Hard AISI 301 Stainless Steel Sheet-for Longitudinal Compression

B4.5.6.1 Stainless Steels-Minimum Properties

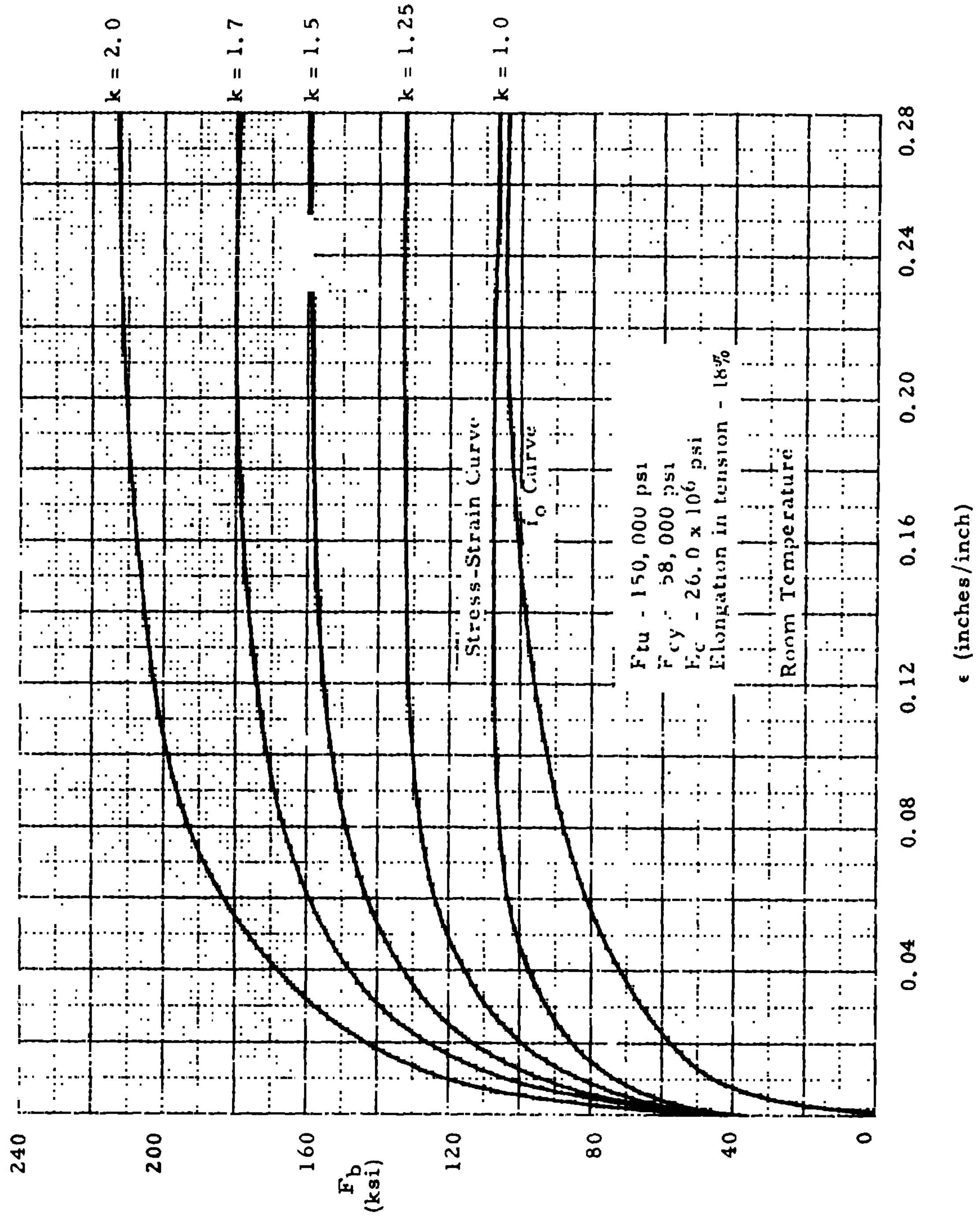


Fig. B4.5.6.1-8 Minimum Plastic Bending Curves 1/2 Hard AISI 301 Stainless Steel Sheet - for Longitudinal Compression

B4.5.6.1 Stainless Steels-Minimum Properties

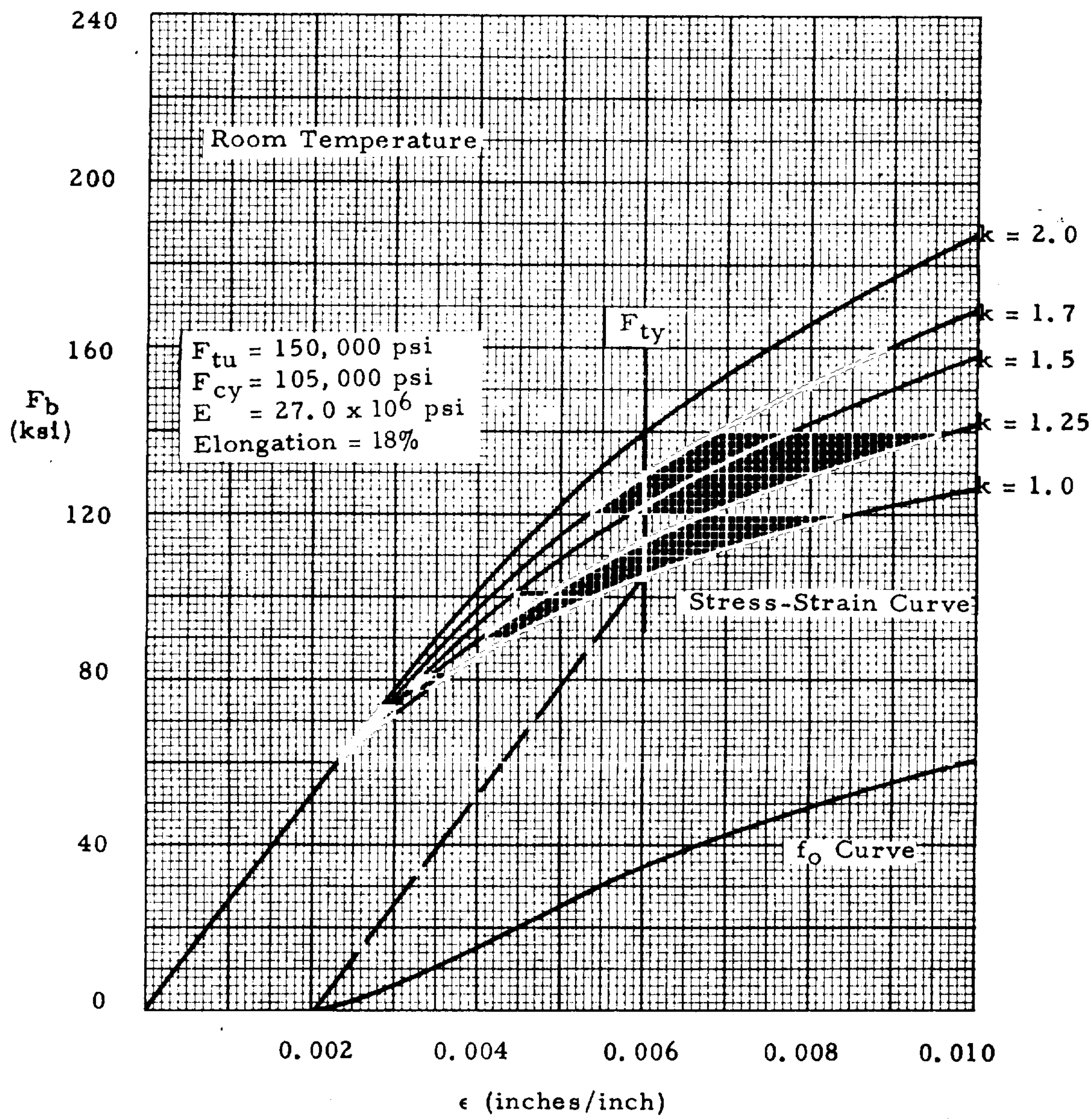


Fig. B4.5.6.1-9 Minimum Plastic Bending Curves Stress Relieved 1/2 Hard AISI 301 Stainless Steel Sheet - for Longitudinal Compression

B4.5.6.1 Stainless Steels-Minimum Properties

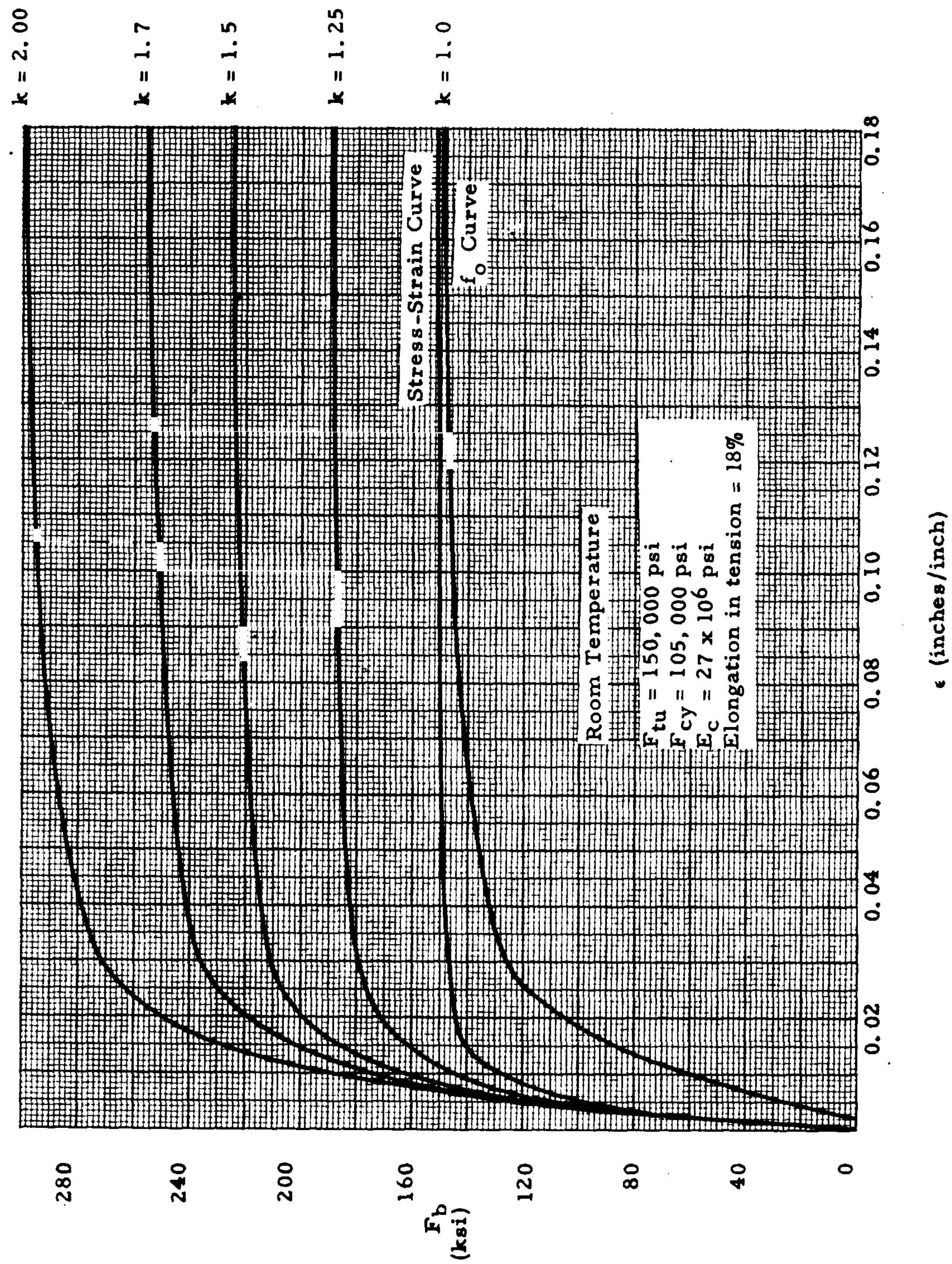


Fig. B4.5.6.1-10 Minimum Plastic Bending Curves Stress Relieved 1/2 Hard AISI 301 Stainless Steel Sheet-for Longitudinal Compression

B4.5.6.1 Stainless Steels-Minimum Properties

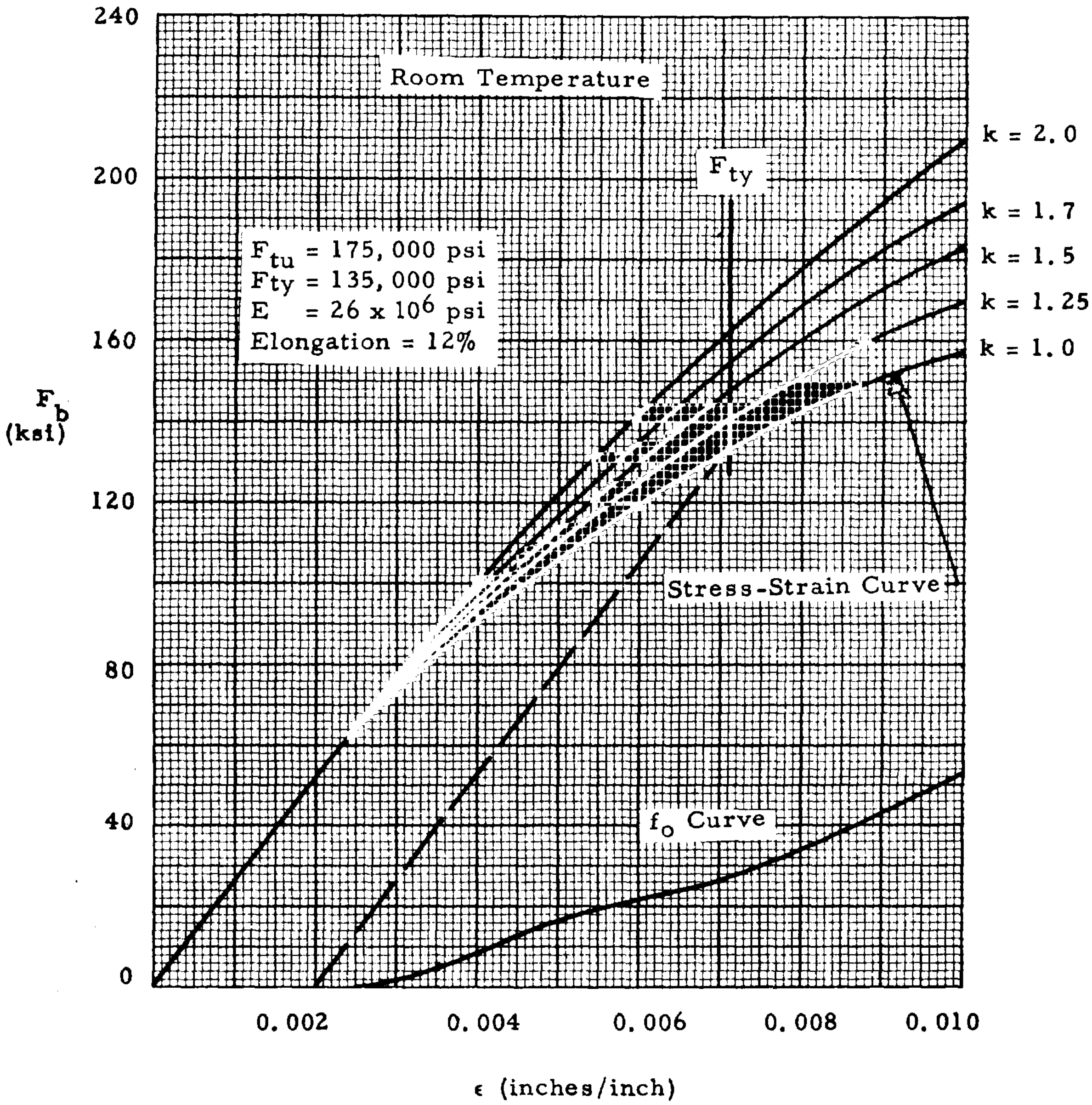


Fig. B4.5.6.1-11 Minimum Plastic Bending Curves 3/4 Hard AISI 301 Stainless Steel Sheet - for Tension or Transverse Compression and Stress Relieved Material - for Tension or Transverse Compression

B4.5.6.1 Stainless Steels-Minimum Properties

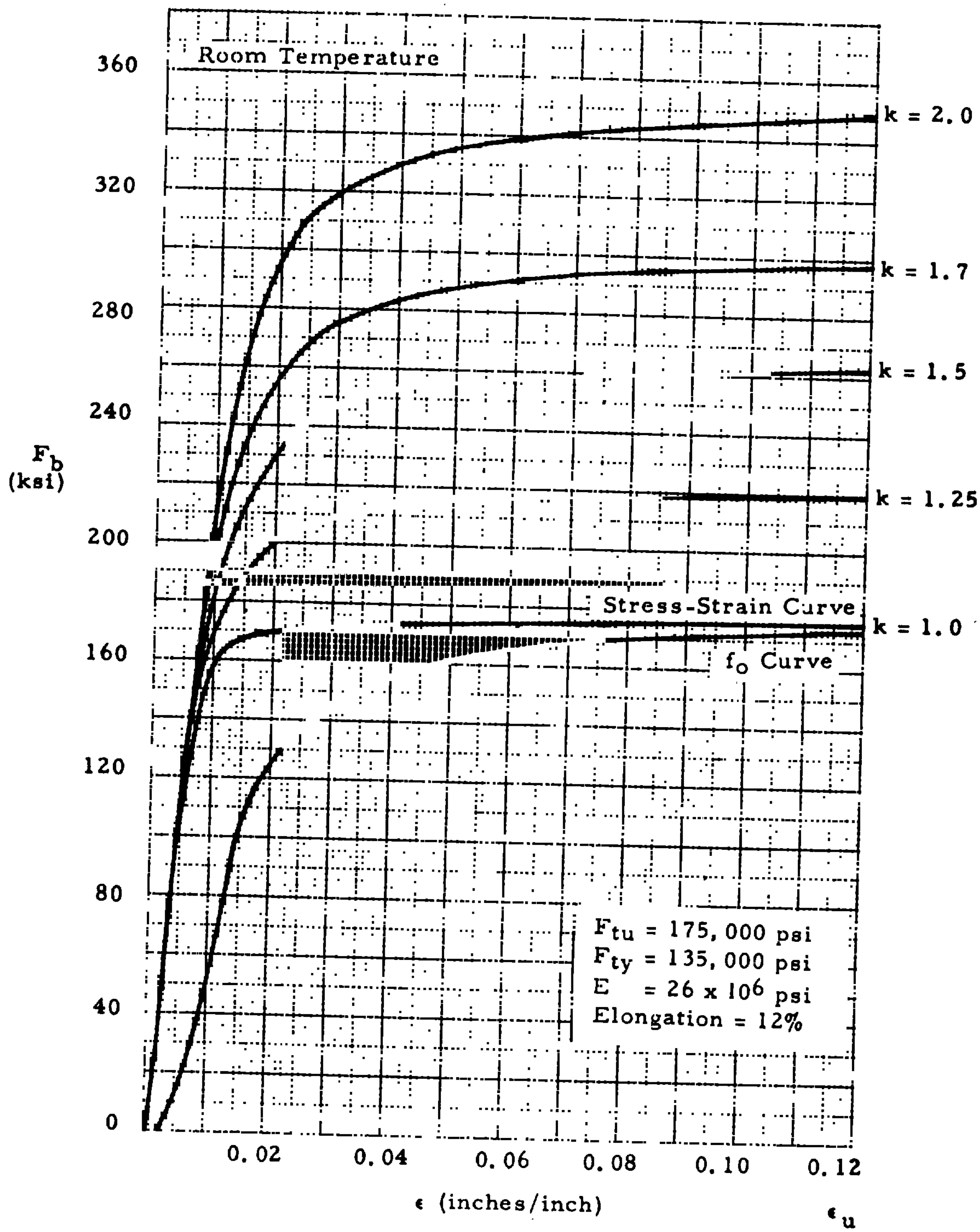


Fig. B4.5.6.1-12 Minimum Plastic Bending Curves 3/4 Hard AISI 301 Stainless Steel Sheet - for Tension or Transverse Compression and Stress Relieved Material - for Tension or Transverse Compression

B4.5.6.1 Stainless Steels-Minimum Properties

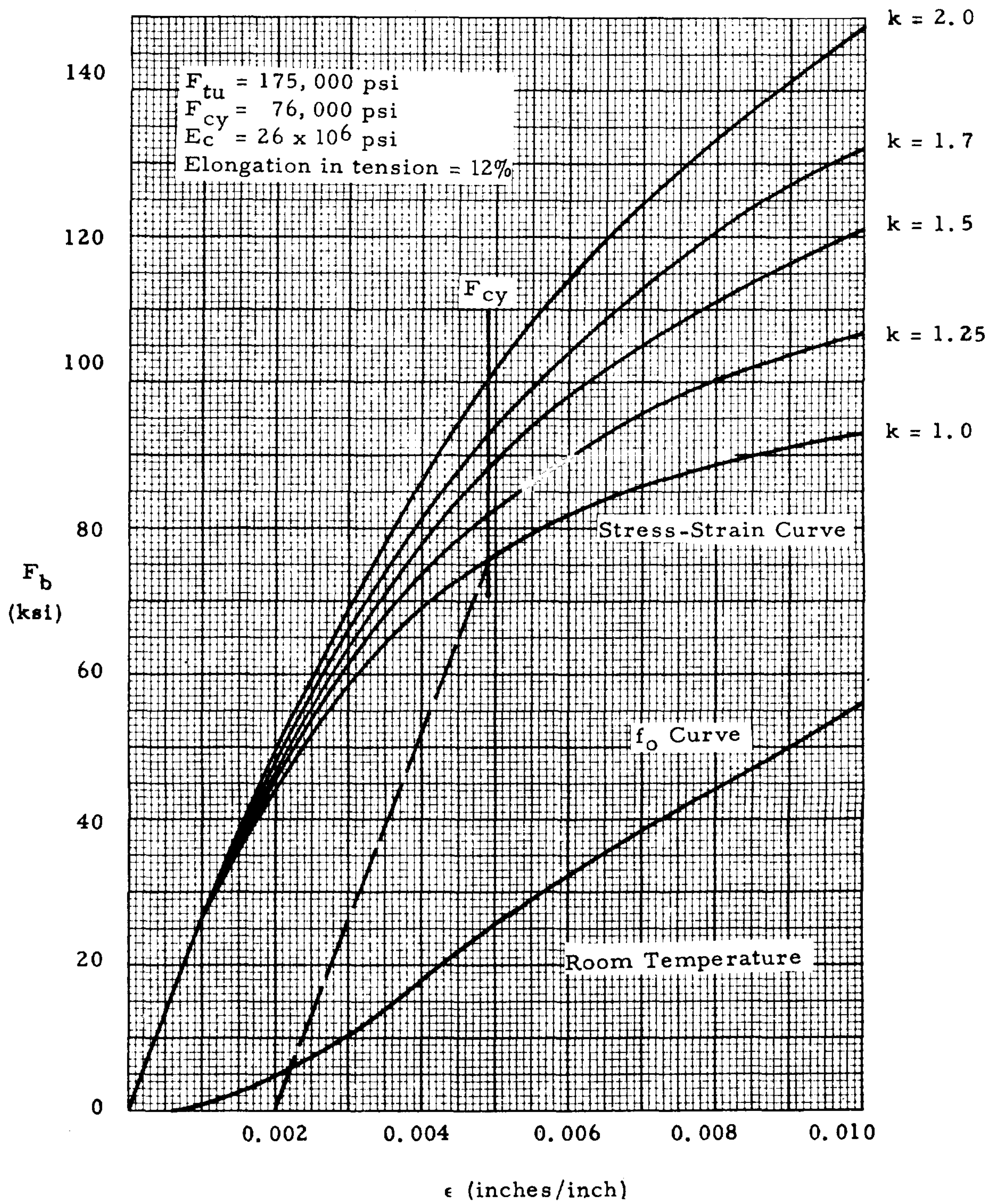


Fig. B4.5.6.1-13 Minimum Plastic Bending Curves 3/4 Hard AISI 301 Stainless Steel Sheet for Longitudinal Compression

B4.5.6.1 Stainless Steels-Minimum Properties

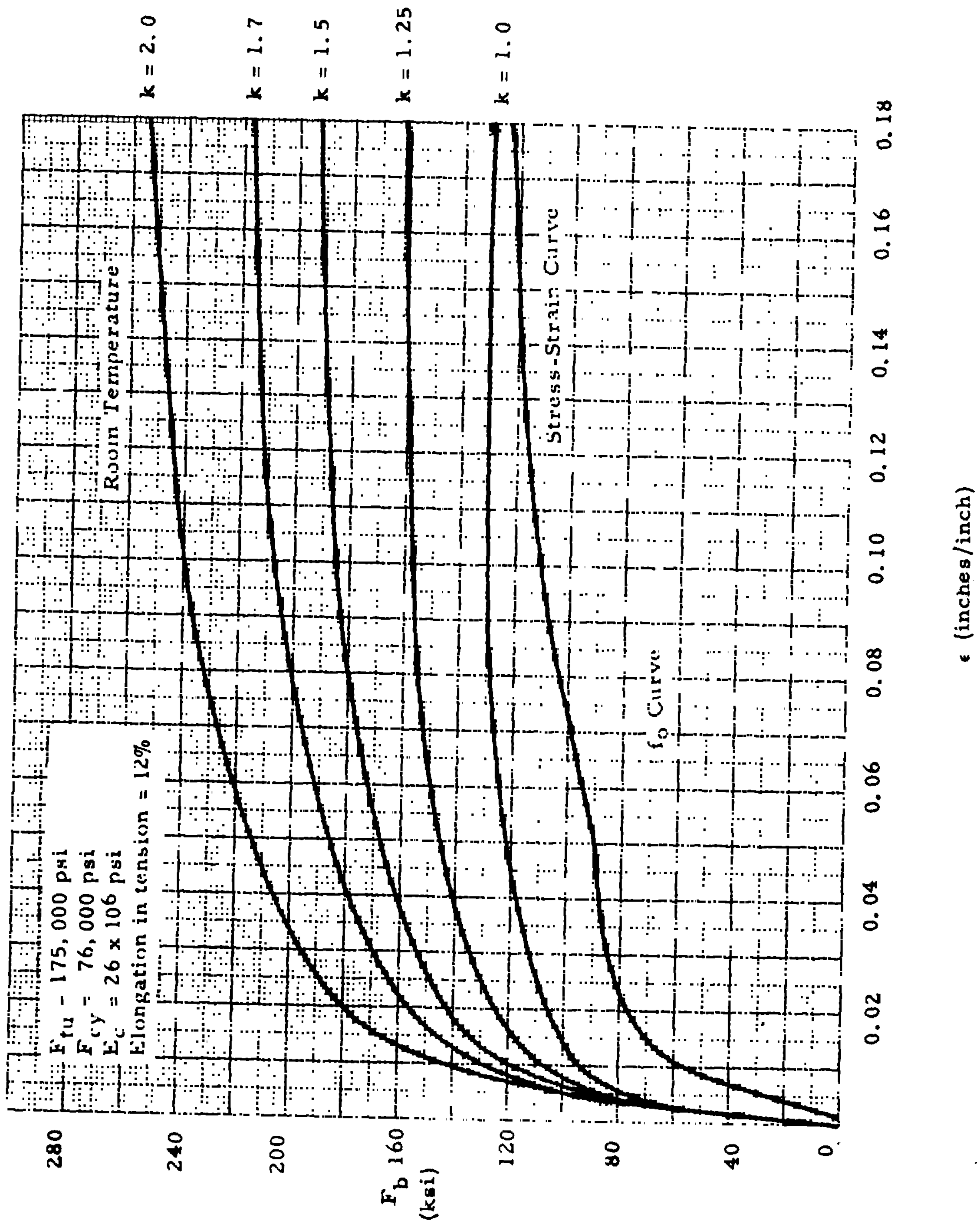


Fig. B4.5.6.1-14 Minimum Plastic Bending Curves 3/4 Hard AISI 301 Stainless Steel Sheet for Longitudinal Compression

B4.5.6.1 Stainless Steels-Minimum Properties

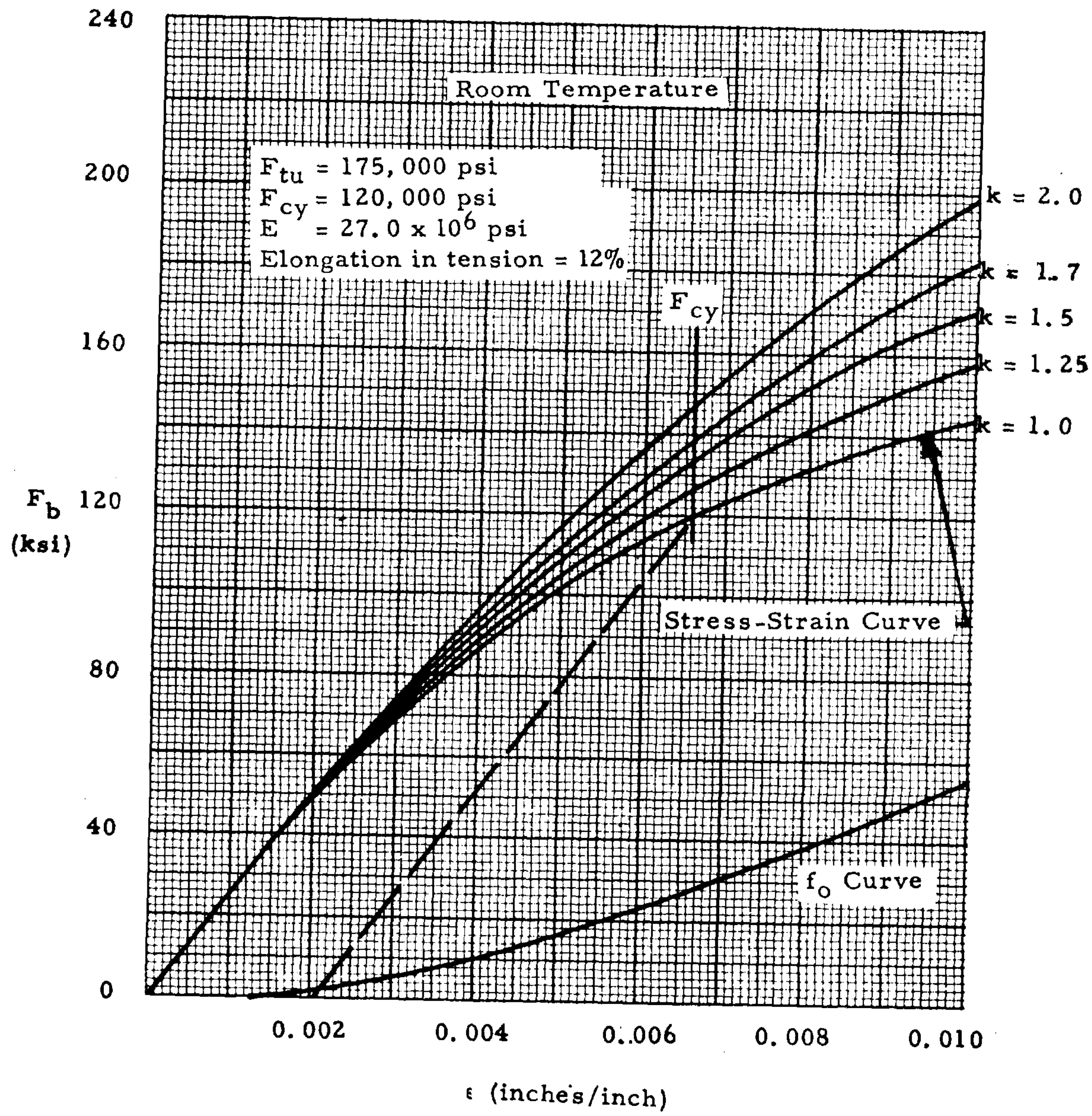


Fig. B4.5.6.1-15 Minimum Plastic Bending Curves Stress Relieved 3/4 Hard AISI 301 Stainless Steel Sheet for Longitudinal Compression

B4.5.6.1 Stainless Steels-Minimum Properties

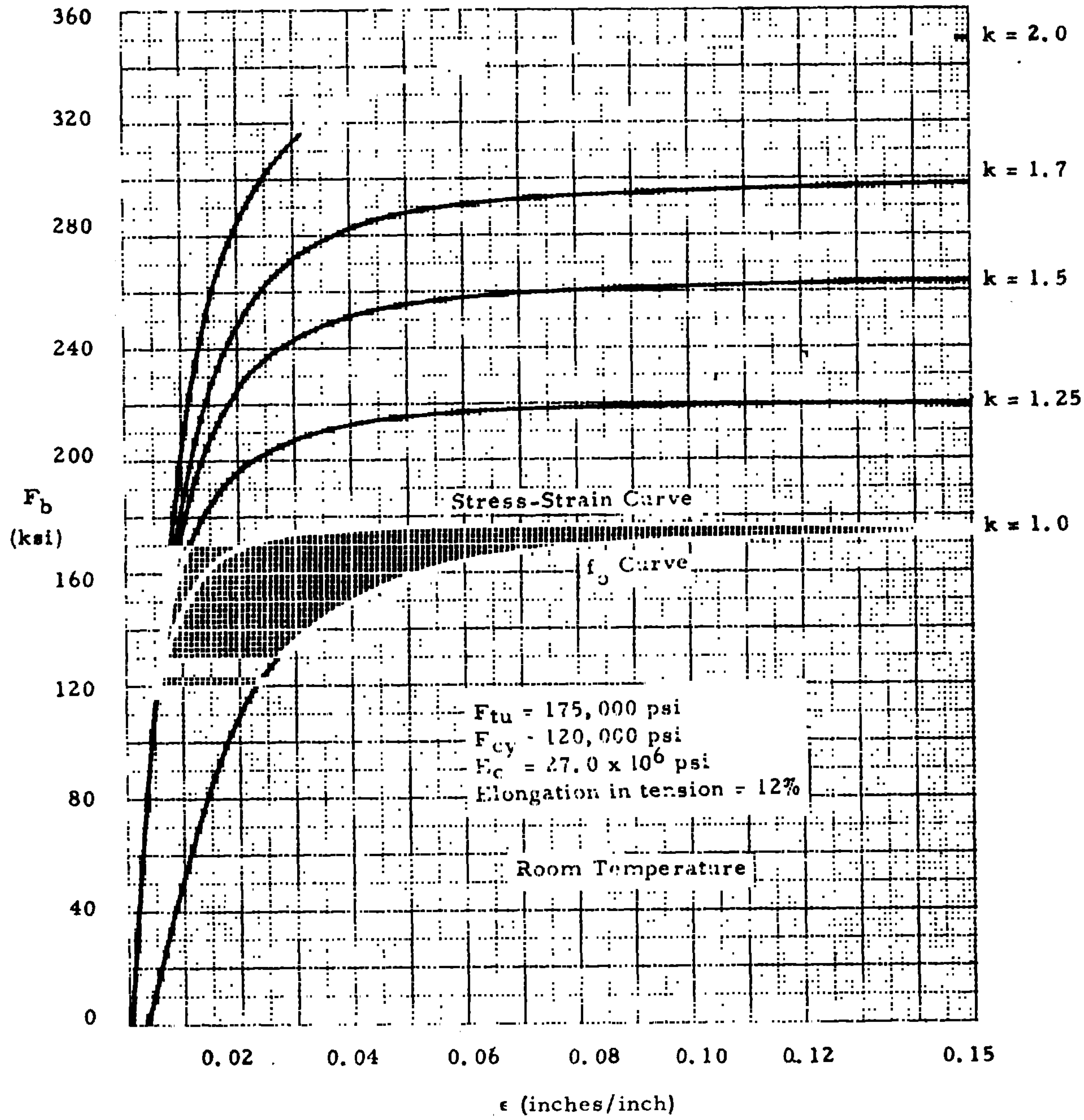


Fig. B4.5.6.1-16 Minimum Plastic Bending Curves Stress Relieved 3/4 Hard AISI 301 Stainless Steel Sheet for Longitudinal Compression

B4.5.6.1 Stainless Steels-Minimum Properties

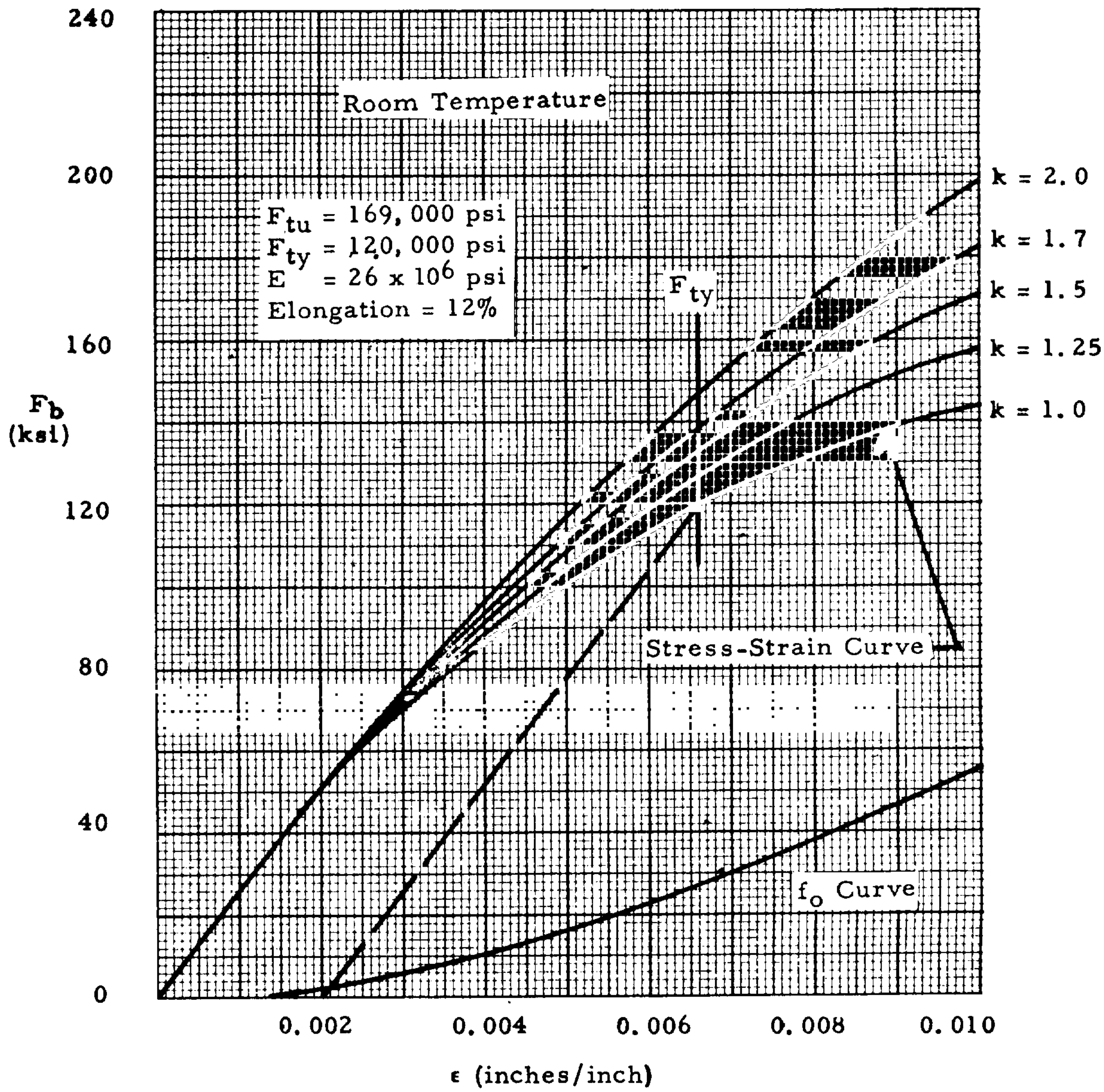


Fig. B4.5.6.1-17 Minimum Plastic Bending Curves Special 3/4 Hard AISI 301 Stainless Steel Sheet for Tension or Transverse Compression

B4.5.6.1 Stainless Steels-Minimum Properties

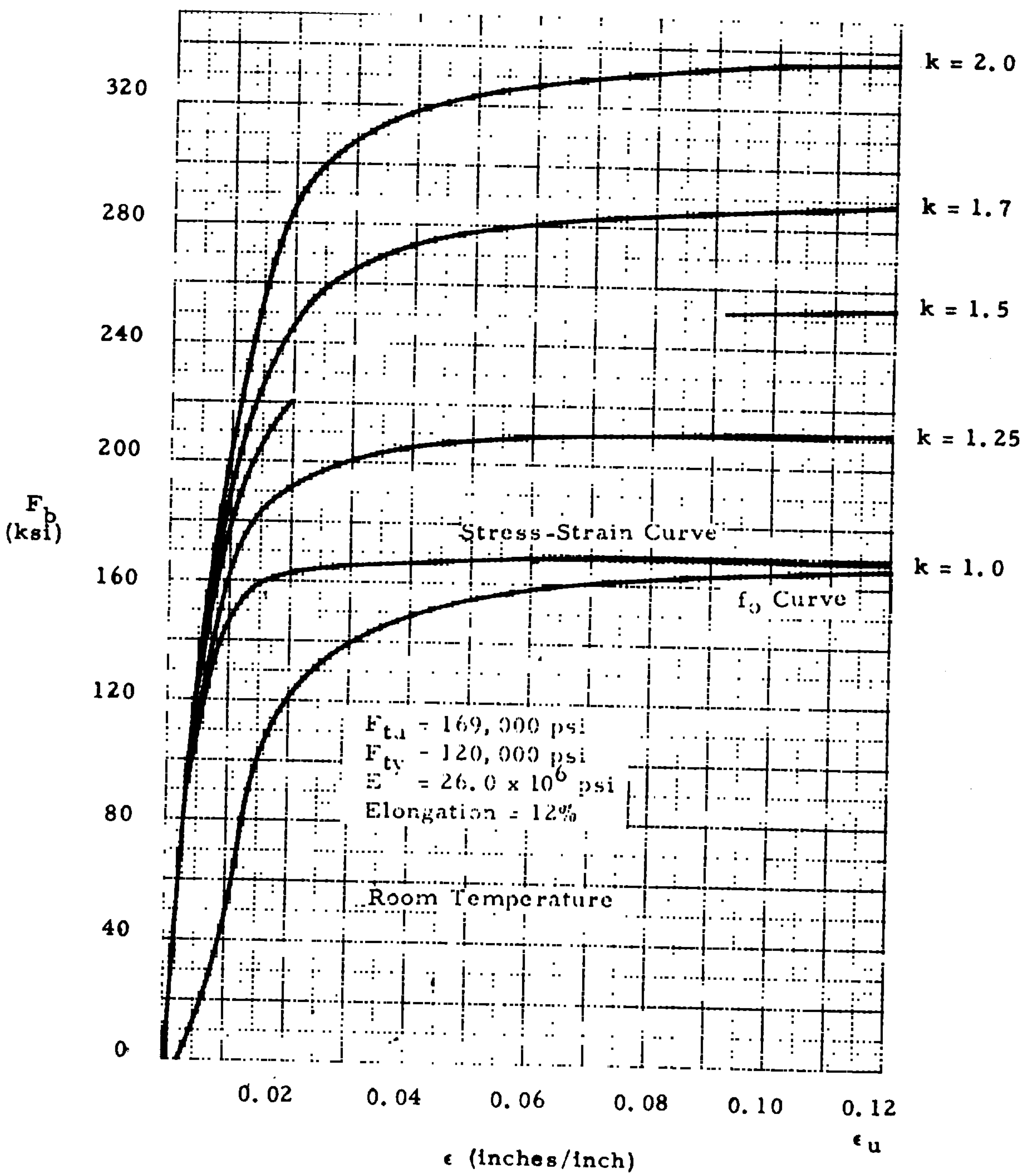


Fig. B4.5.6.1-18 Minimum Plastic Bending Curves · Special 3/4 Hard AISI 301 Stainless Steel Sheet for Tension or Transverse Compression

B4.5.6.1 Stainless Steels-Minimum Properties

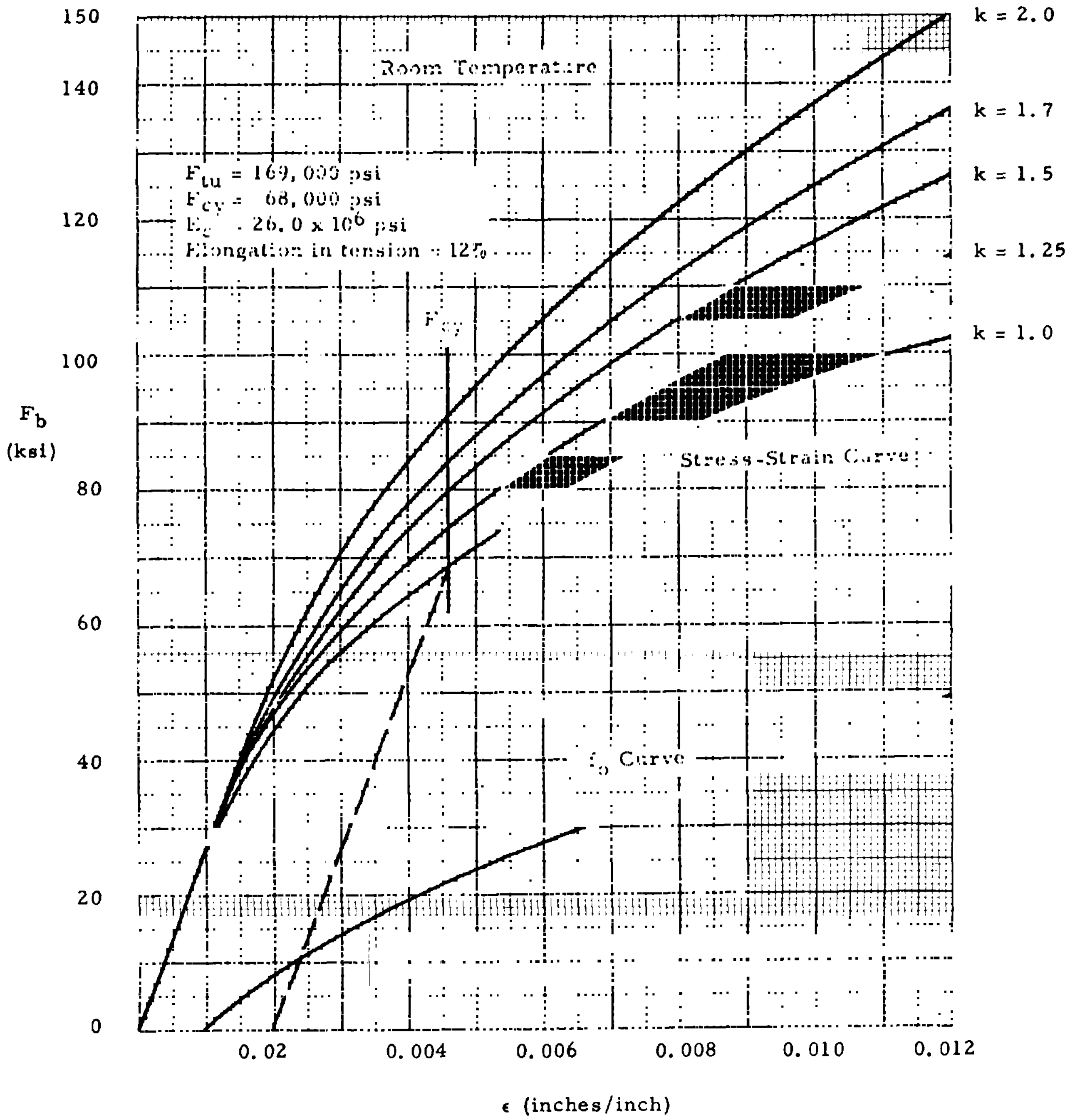


Fig. B4.5.6.1-19 Minimum Plastic Bending Curves
Special 3/4 Hard AISI 301 Stainless
Steel Sheet for Longitudinal Compression

B4.5.6.1 Stainless Steels-Minimum Properties

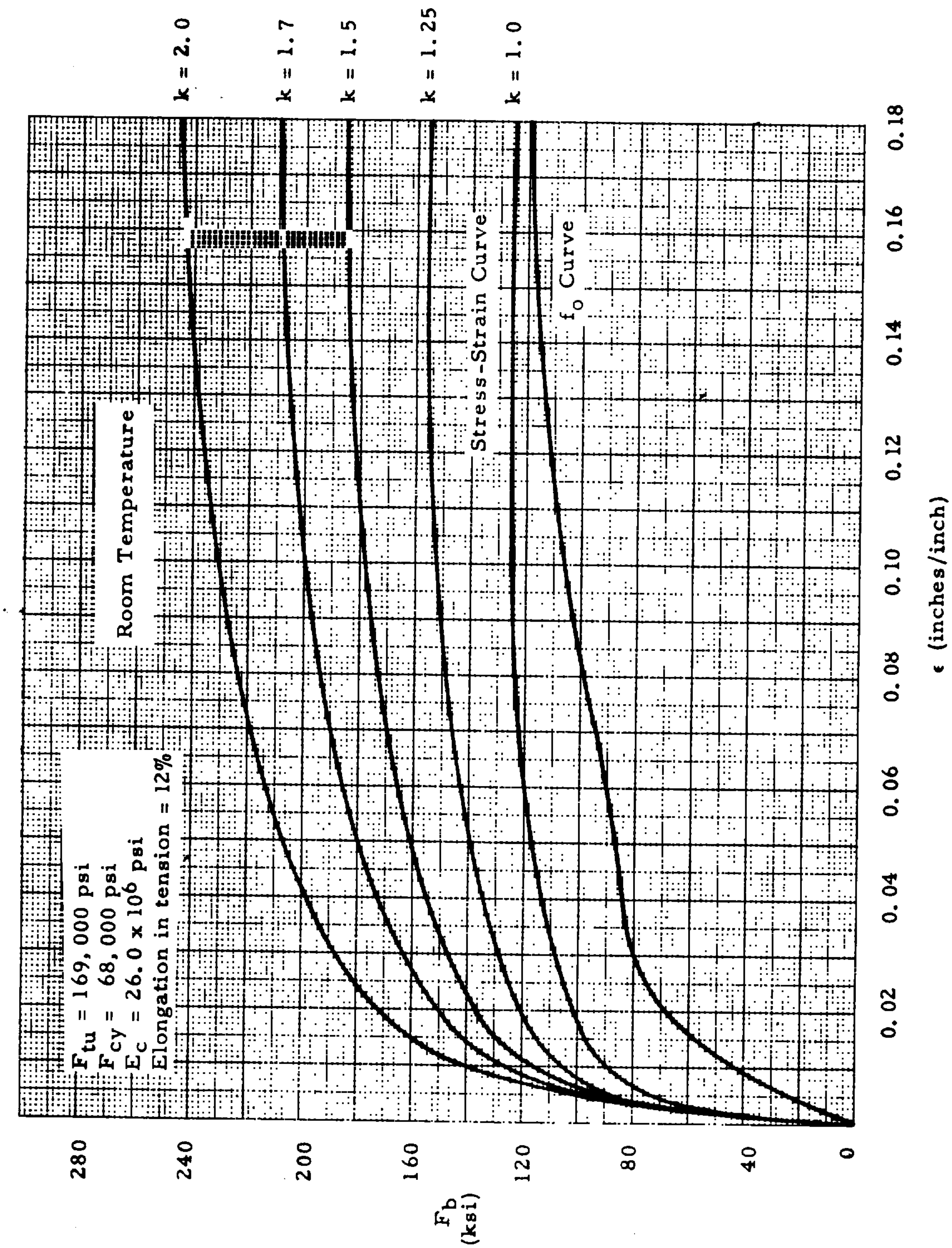


Fig. B4.5.6.1-20 Minimum Plastic Bending Curves
Special 3/4 Hard AISI 301 Stainless
Steel Sheet for Longitudinal Compression

B4.5.6.1 Stainless Steels-Minimum Properties

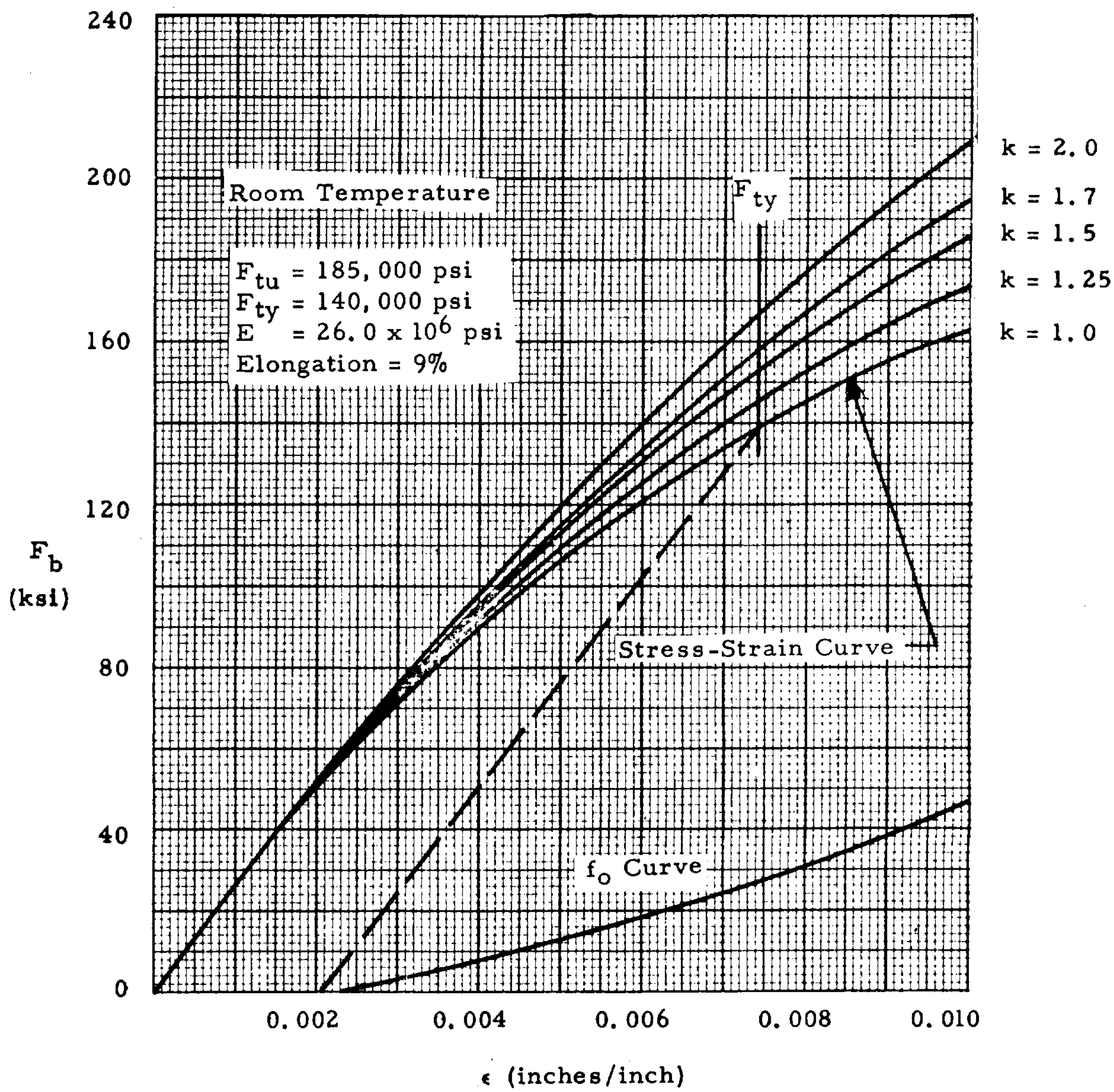


Fig. B4.5.6.1-21 Minimum Plastic Bending Curves Full Hard AISI 301 Stainless Steel Sheet - for Tension or Transverse Compression and Stress Relieved Material - for Tension or Compression

B4.5.6.1 Stainless Steel-Minimum Properties

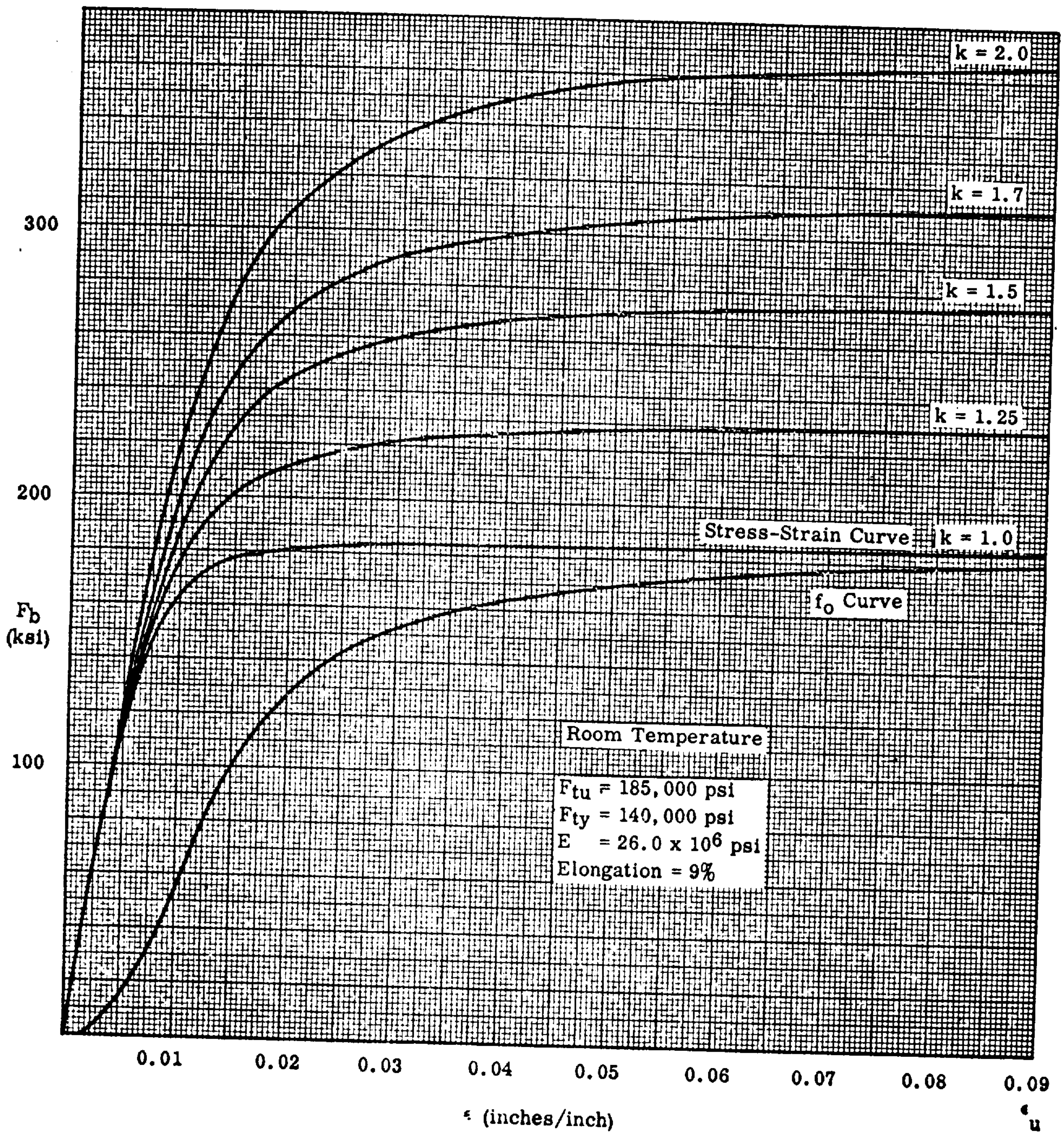


Fig. B4.5.6.1-22 Minimum Plastic Bending Curves Full Hard AISI 301 Stainless Steel Sheet-for Tension or Transverse Compression and Stress Relieved Material - for Tension or Compression

B4.5.6.1 Stainless Steels-Minimum Properties

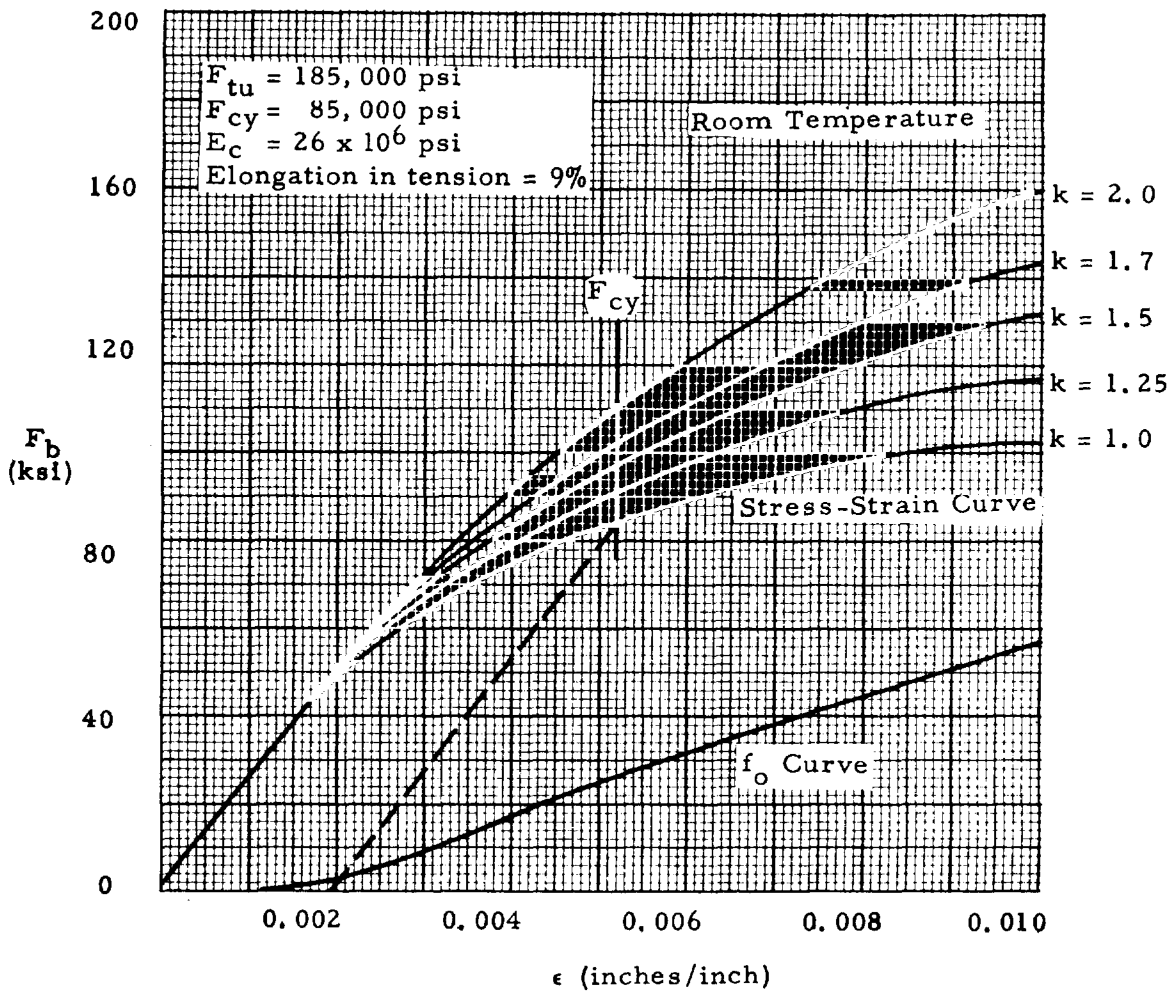


Fig. B4.5.6.1-23 Minimum Plastic Bending Curves Full Hard AISI 301 Stainless Steel Sheet for Longitudinal Compression

B4.5.6.1 Stainless Steels-Minimum Properties

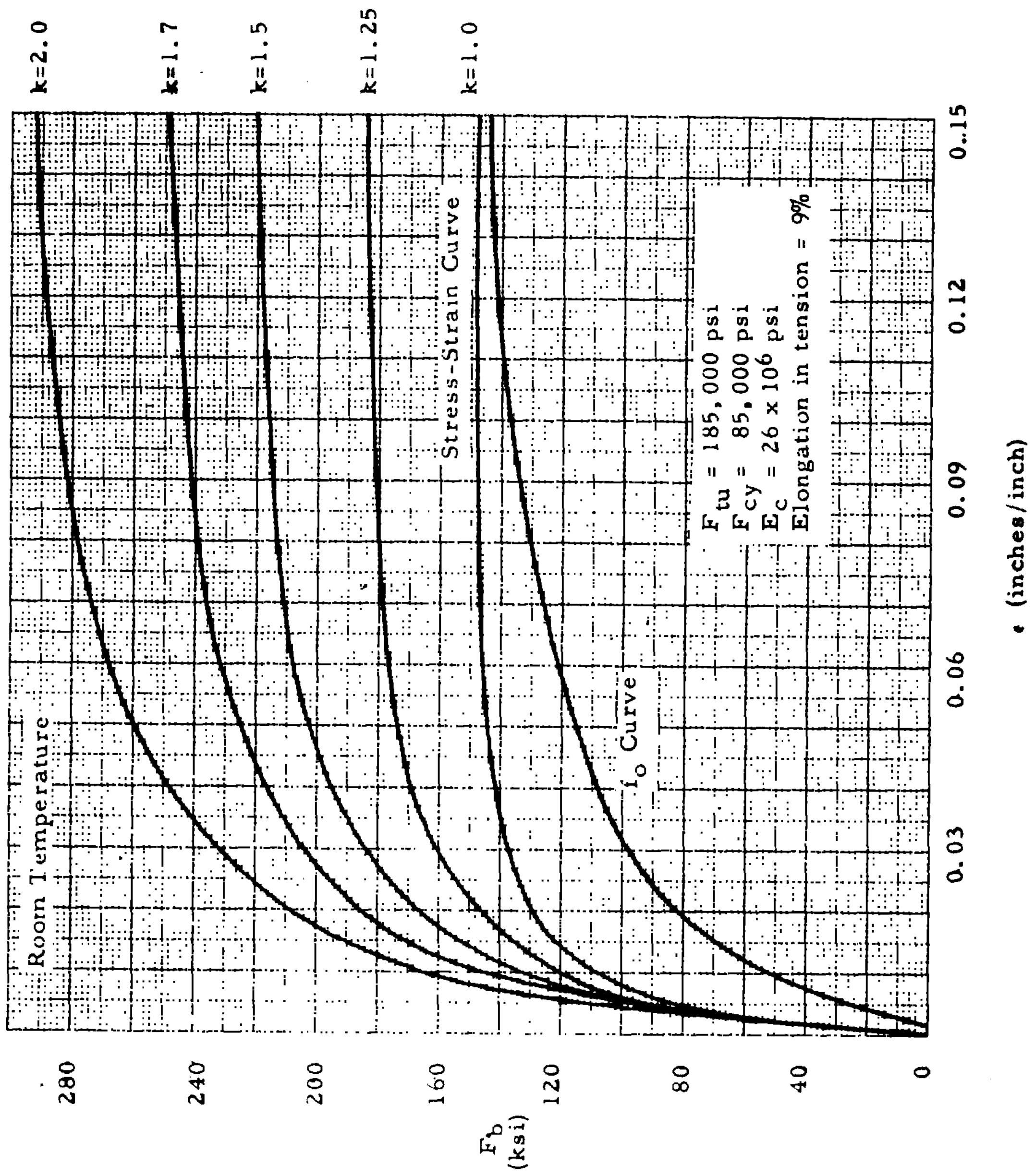


Fig. B4.5.6.1-24 Minimum Plastic Bending Curves Full Hard AISI 301 Stainless Steel Sheet for Longitudinal Compression

B4.5.6.1 Stainless Steels-Minimum Properties

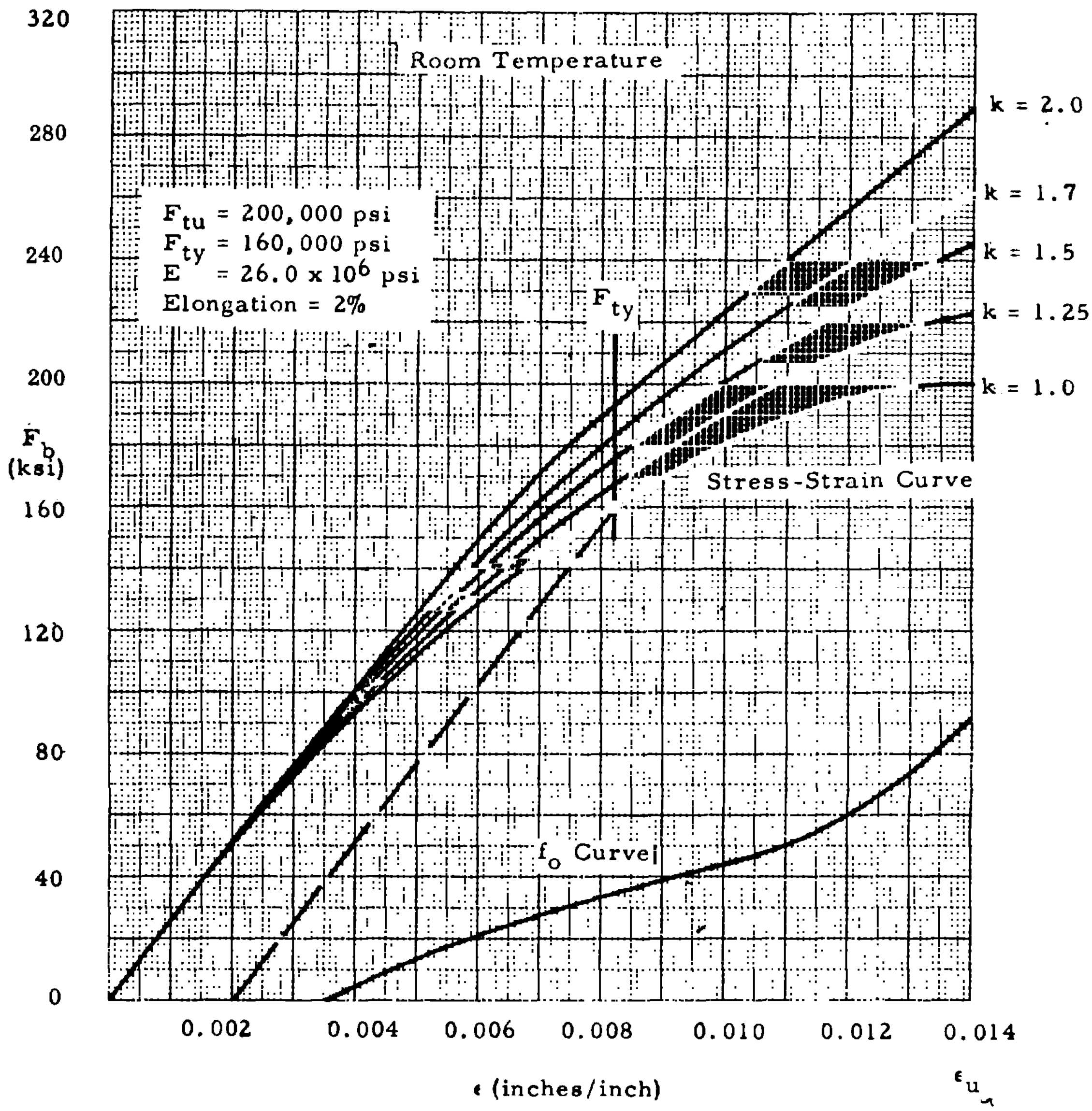


Fig. B4.5.6.1-25 Minimum Plastic Bending Curves Extra Hard AISI 301 Stainless Steel Sheet - for Tension or Transverse Compression and Stress Relieved Material - for Tension or Compression

B4.5.6.1 Stainless Steels-Minimum Properties

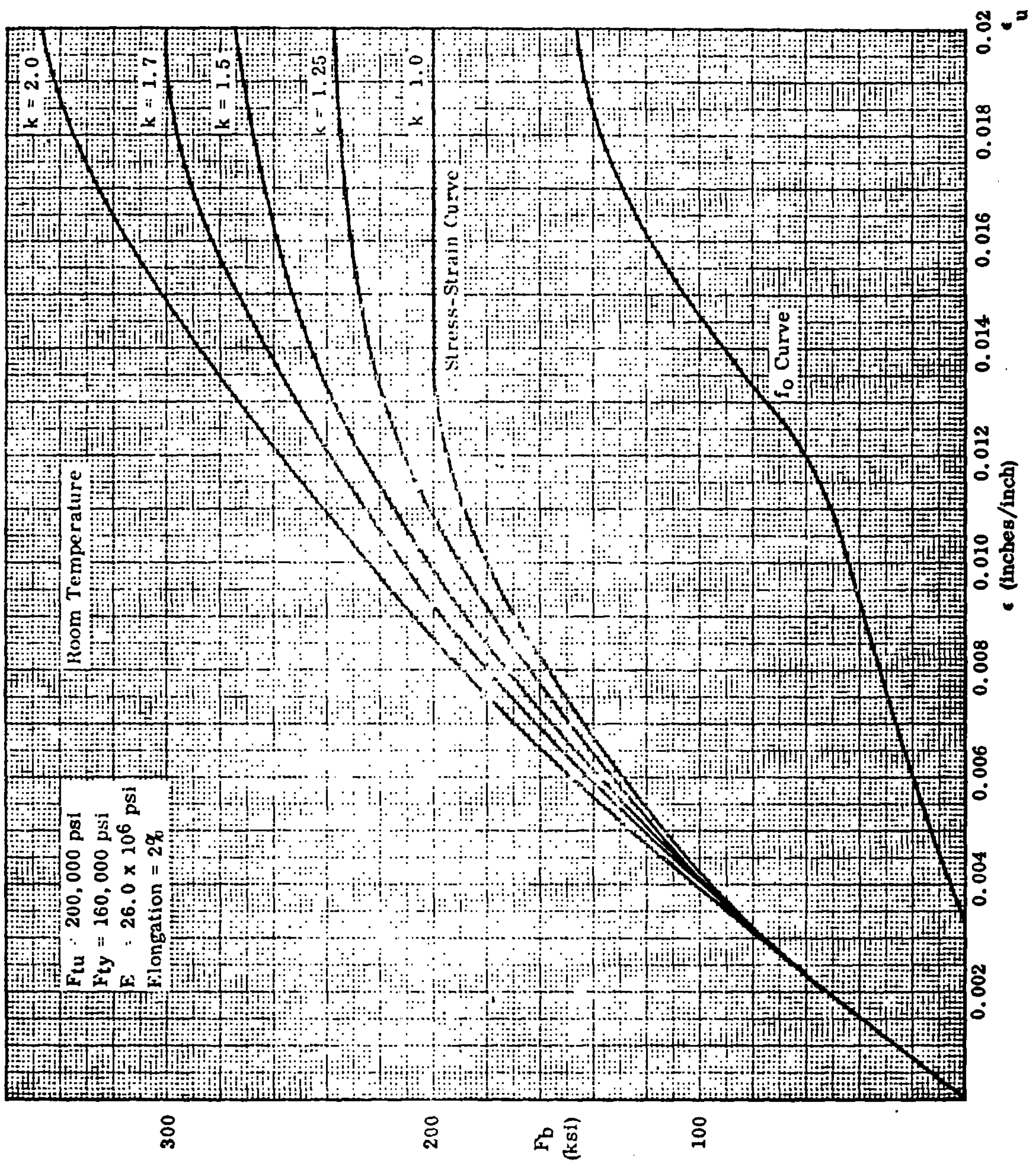


Fig. B4.5.6.1-26 Minimum Plastic Bending Curves Extra Hard AISI 301 Stainless Steel Sheet-for Tension or Transverse Compression and Stress Relieved Material - for Tension or Compression

B4.5.6.1 Stainless Steels-Minimum Properties

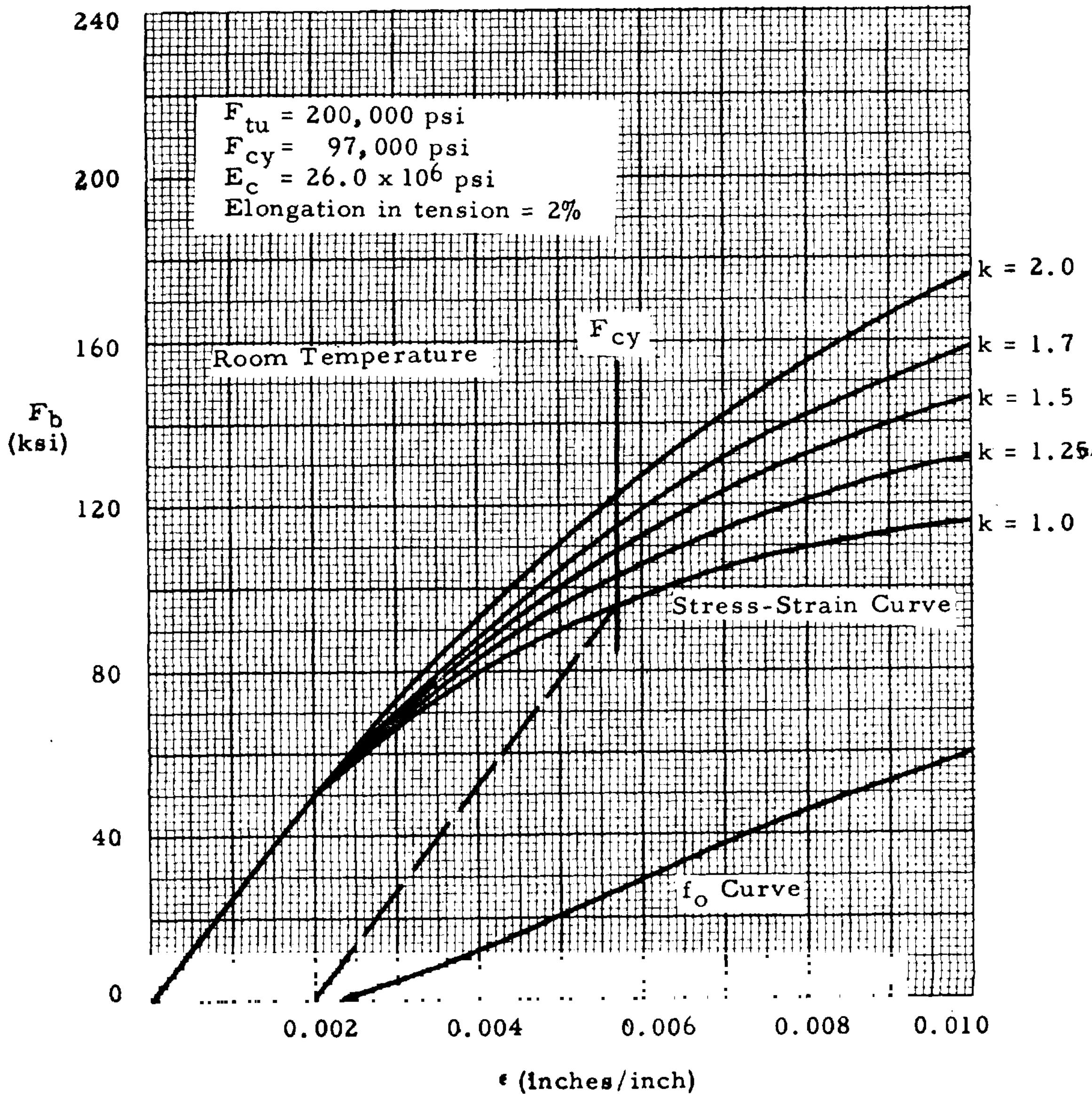


Fig. B4.5.6.1-27 Minimum Plastic Bending Curves Extra Hard AISI 301 Stainless Steel Sheet for Longitudinal Compression

B4.5.6.1 Stainless Steels-Minimum Properties

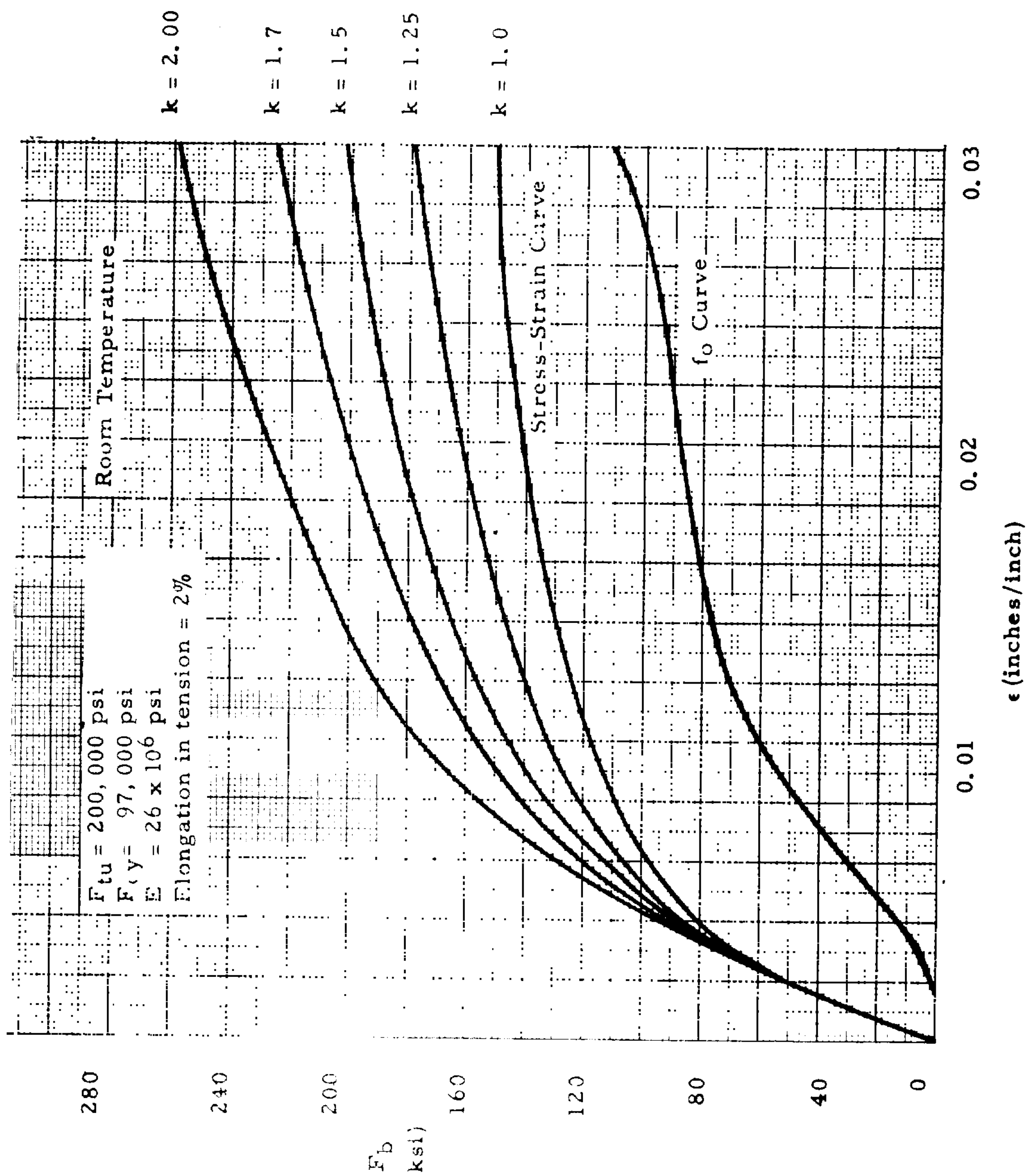


Fig. B4.5.6.1-28 Minimum Plastic Bending Curves Extra Hard AISI 301 Stainless Steel Sheet - for Longitudinal Compression

B4.5.6.1 Stainless Steels-Minimum Properties

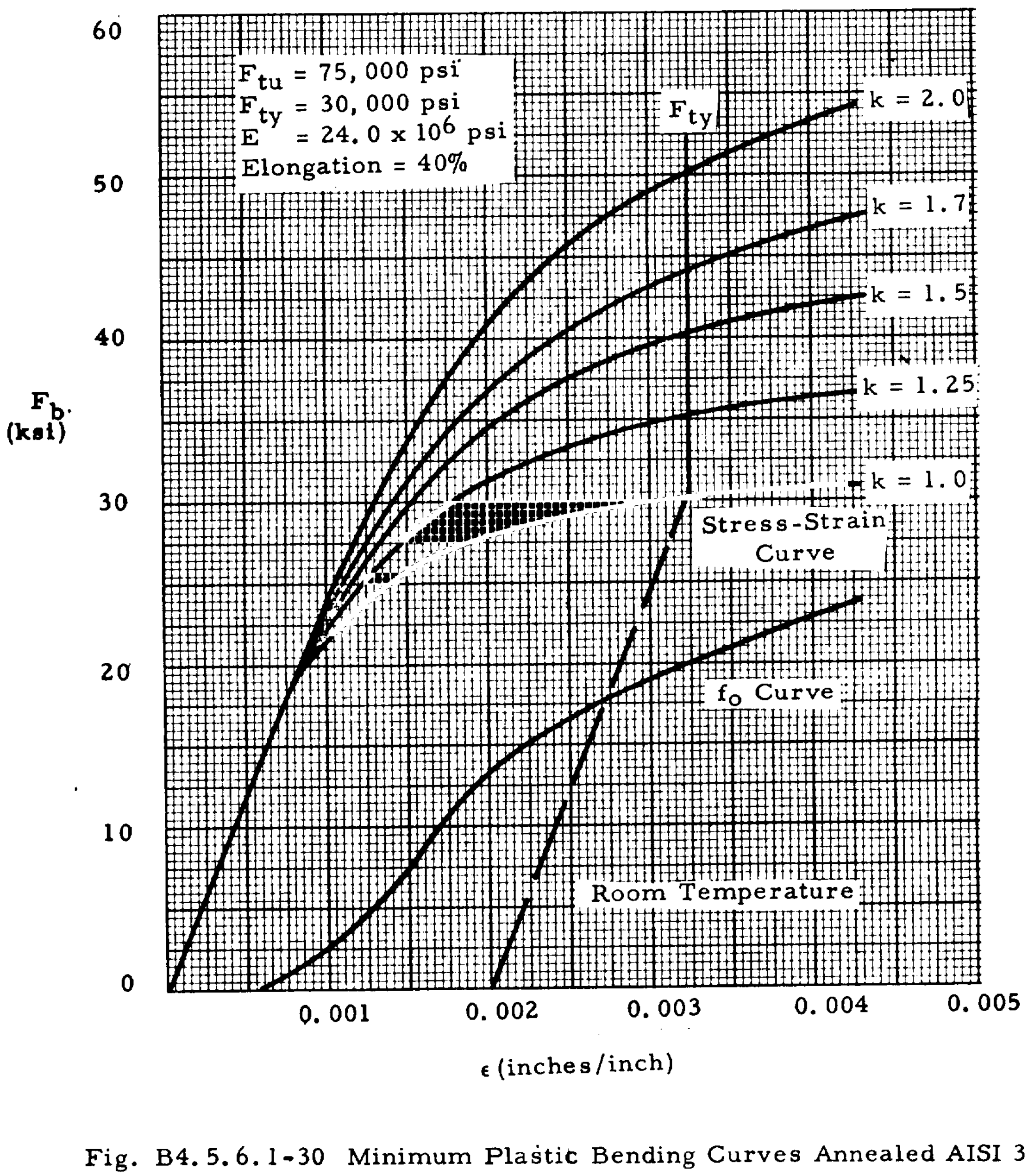


Fig. B4.5.6.1-30 Minimum Plastic Bending Curves Annealed AISI 321 Stainless Steel Sheet

B4.5.6.1 **Stainless Steels-Minimum Properties**

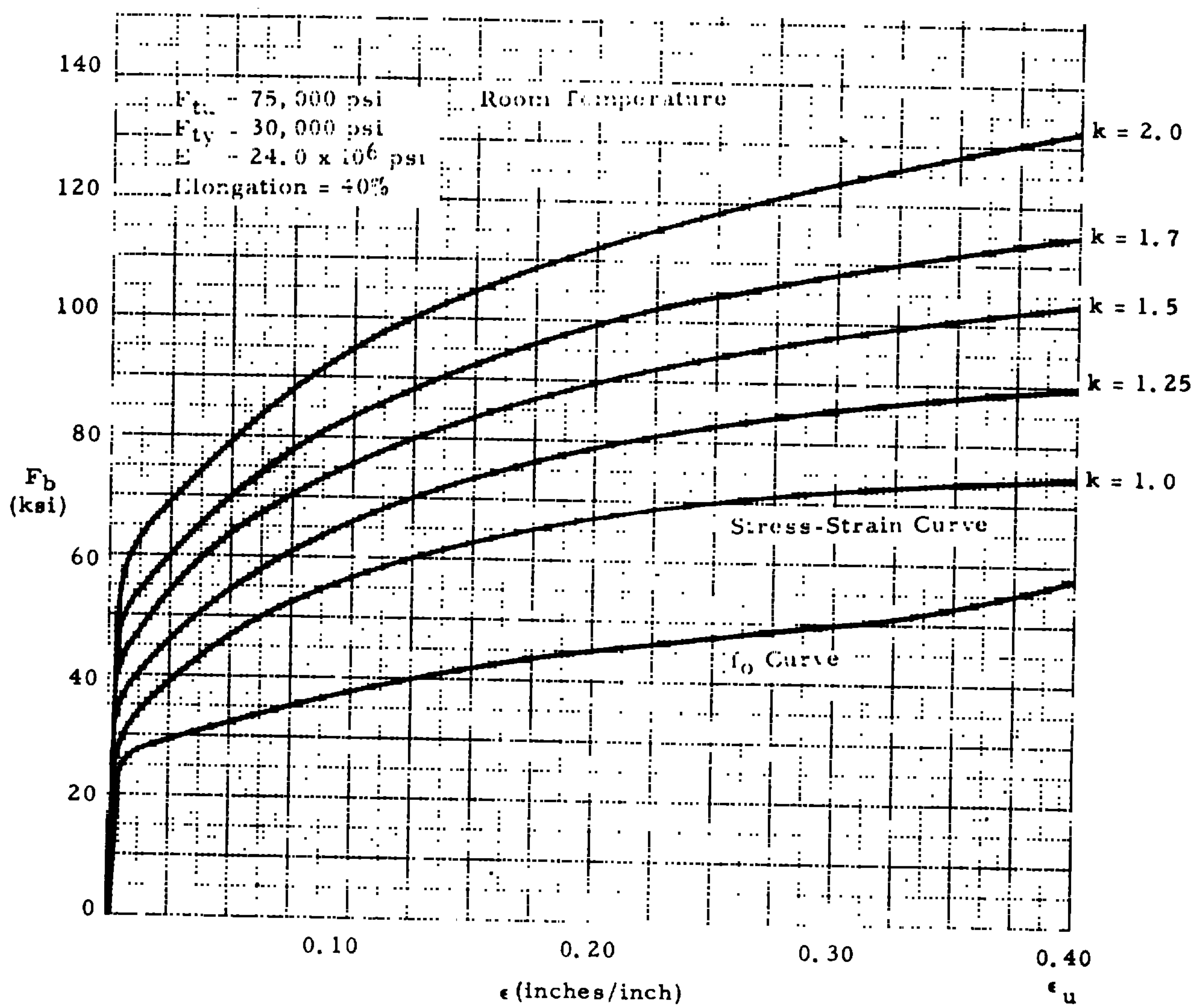


Fig. B4.5.6.1-31 Minimum Plastic Bending Curves Annealed AISI 321 Stainless Steel Sheet

Graph to be furnished when available

Graph to be furnished when available

B4.5.6.1 **Stainless Steels-Minimum Properties**

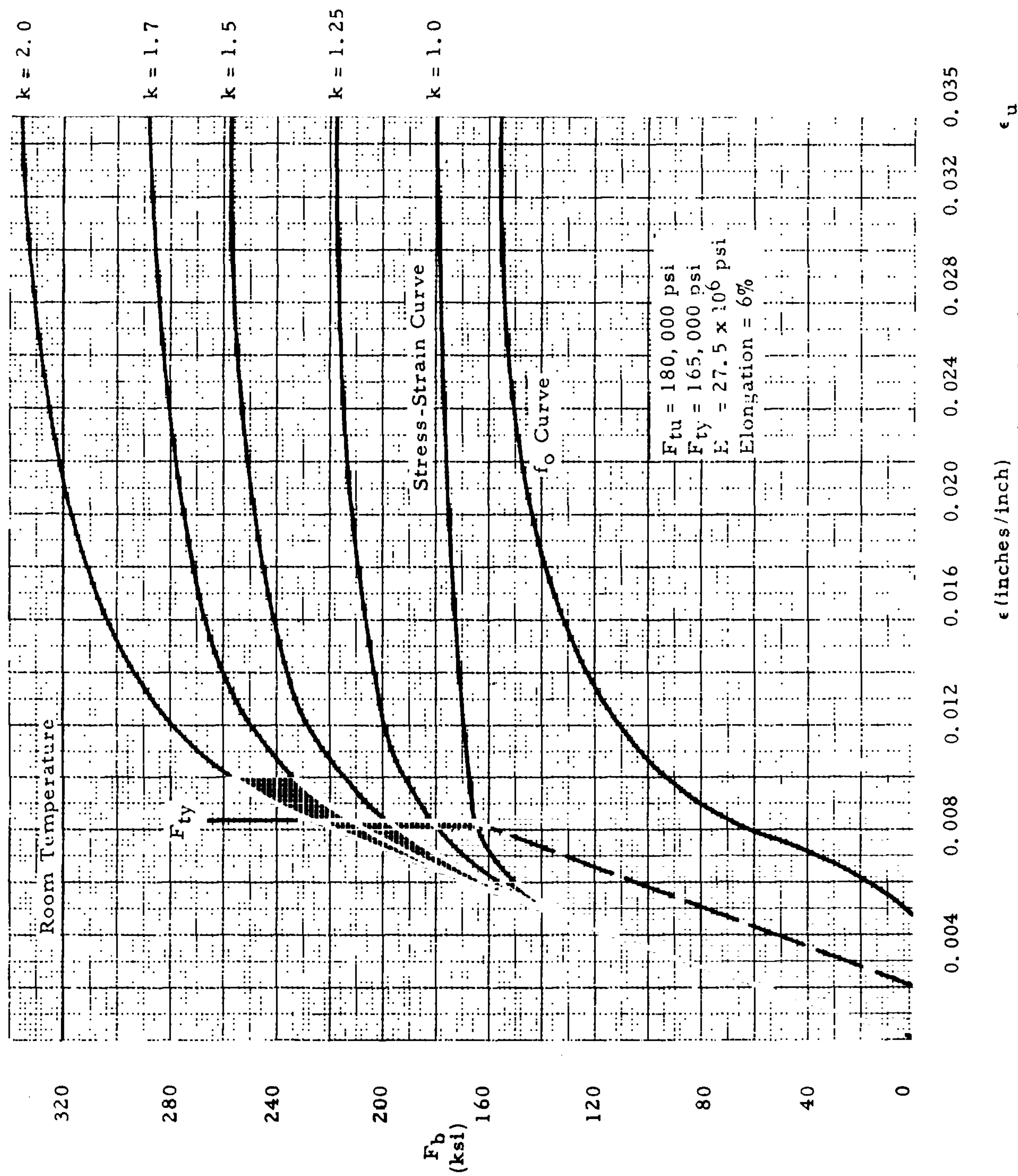


Fig. B4.5.6.1-36 Minimum Plastic Bending Curves 17-4 PH Stainless Steel Heat Treated Bars and Forgings

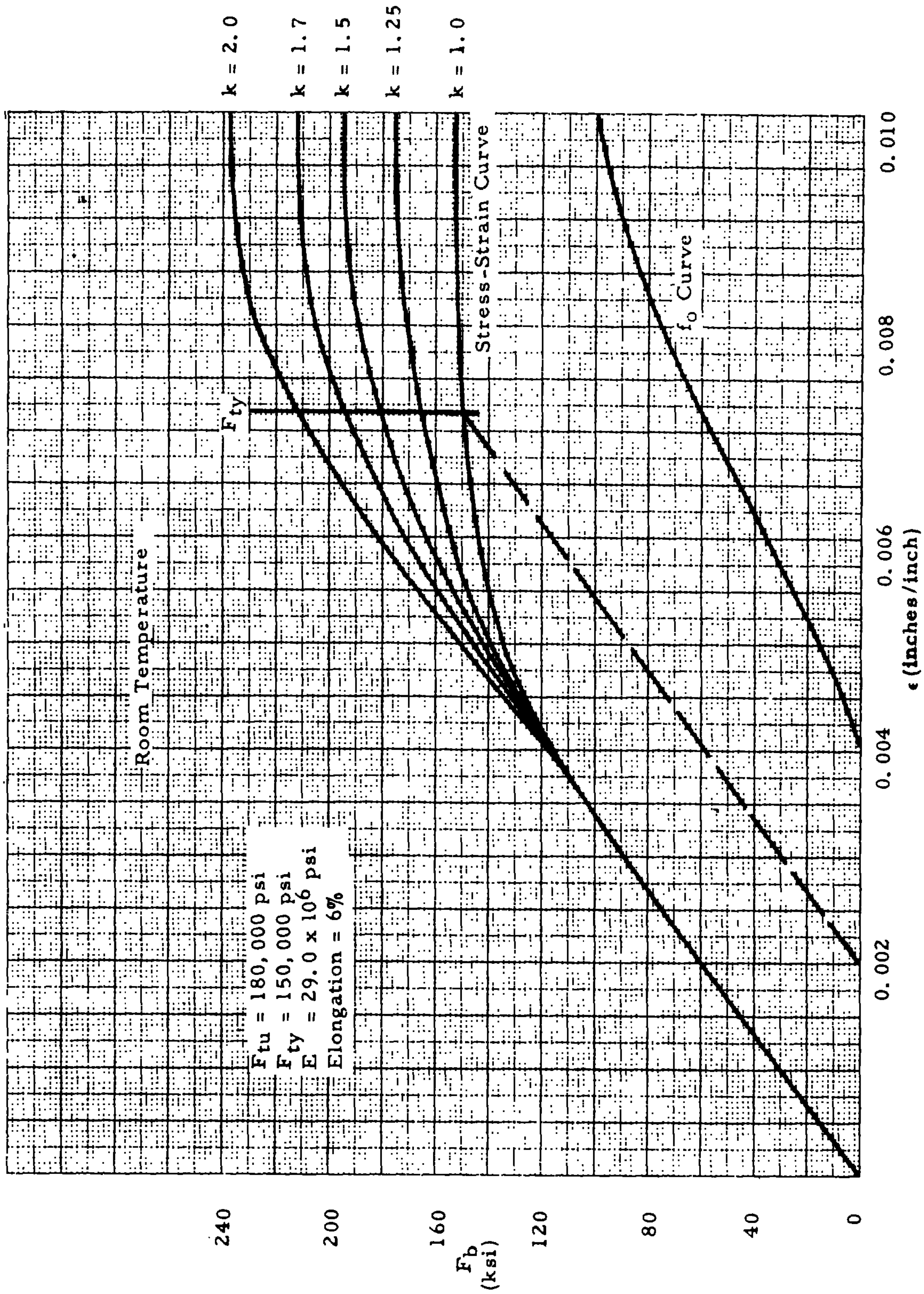


Fig. B4.5.6.1-38 Minimum Plastic Bending Curves 17-7 PH Stainless Steel

B4.5.6.1 **Stainless Steels-Minimum Properties**

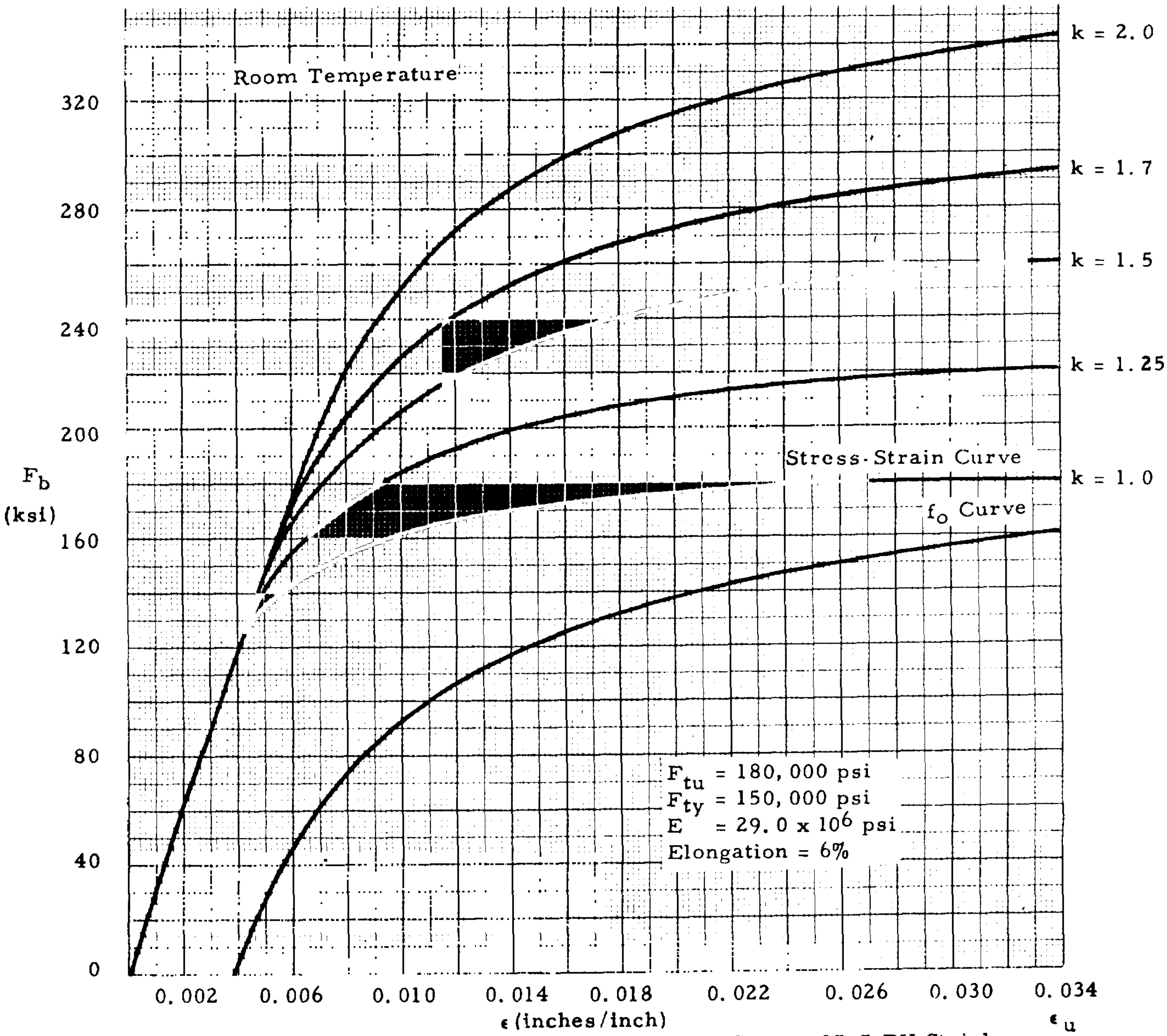


Fig. B4.5.6.1-39 Minimum Plastic Bending Curves 17-7 PH Stainless Steel

B4.5.6.1 **Stainless Steels-Minimum Properties**

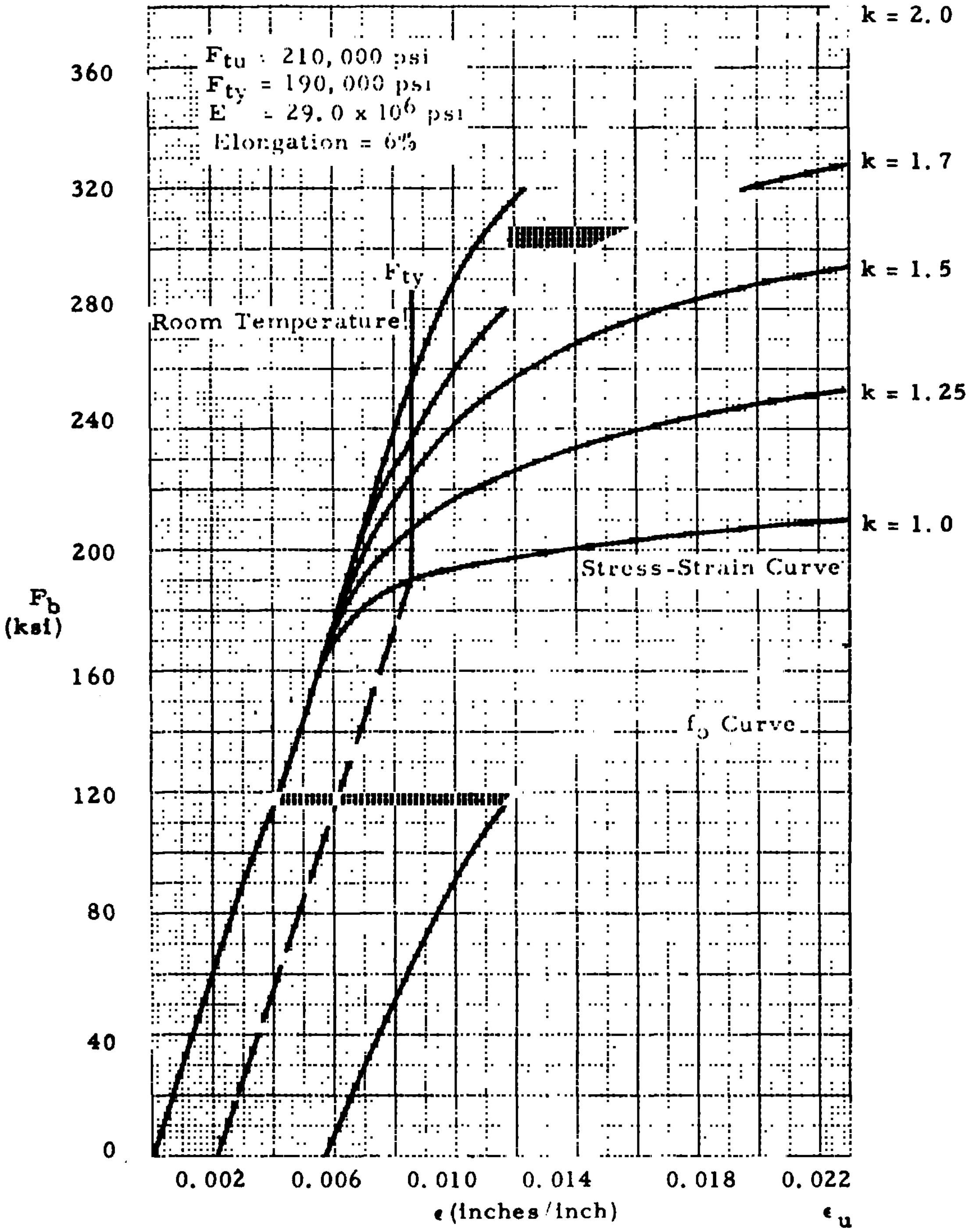


Fig. B4.5.6.1-40 Minimum Plastic Bending Curves 17-7 PH Stainless Steel

B4.5.6.1 Stainless Steels-Minimum Properties

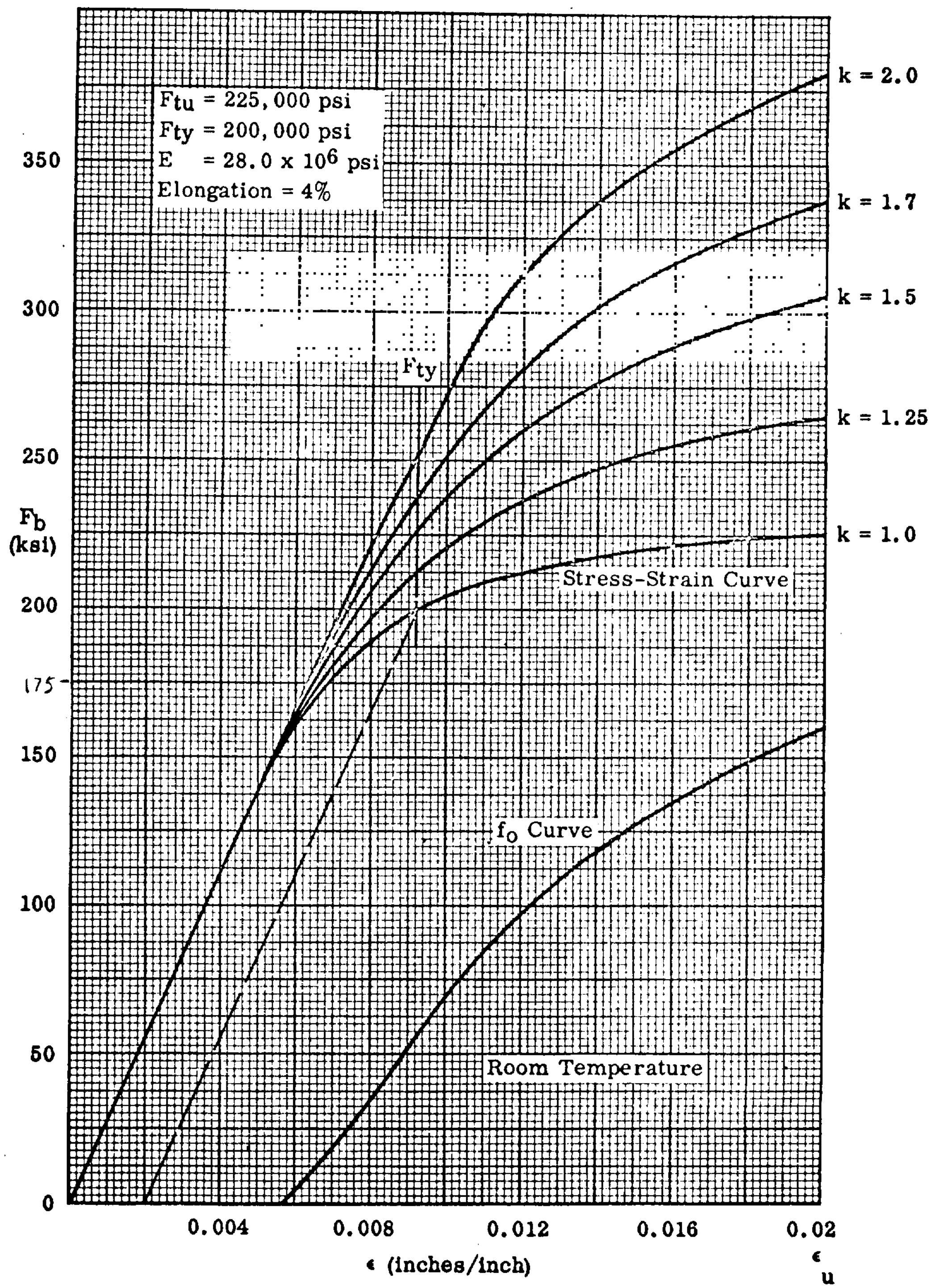


Fig. B4.5.6.1-43 Minimum Plastic Bending Curves for PH 15-7 Mo (RH 950) Stainless Steel Sheet & Strip Thickness 0.020 to 0.187 Inches

Graph to be furnished when available

Graph to be furnished when available

B4.5.6.1 Stainless Steels-Minimum Properties

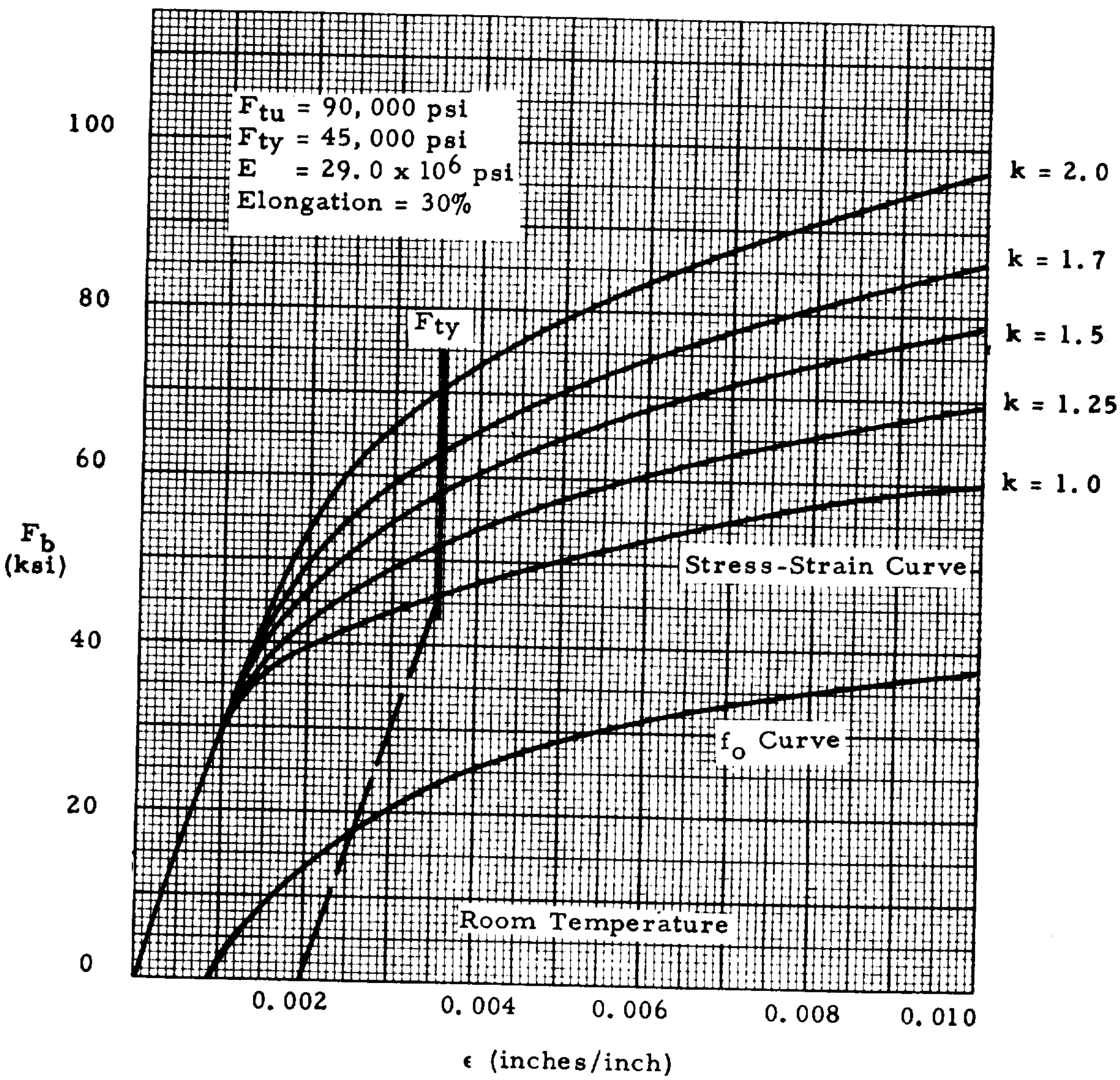


Fig. B4.5.6.1-46 Minimum Plastic Bending Curves for 19-9DL (AMS 5526) & 19-9DX (AMS 5538) Stainless Steel

B4.5.6.1 Stainless Steels-Minimum Properties

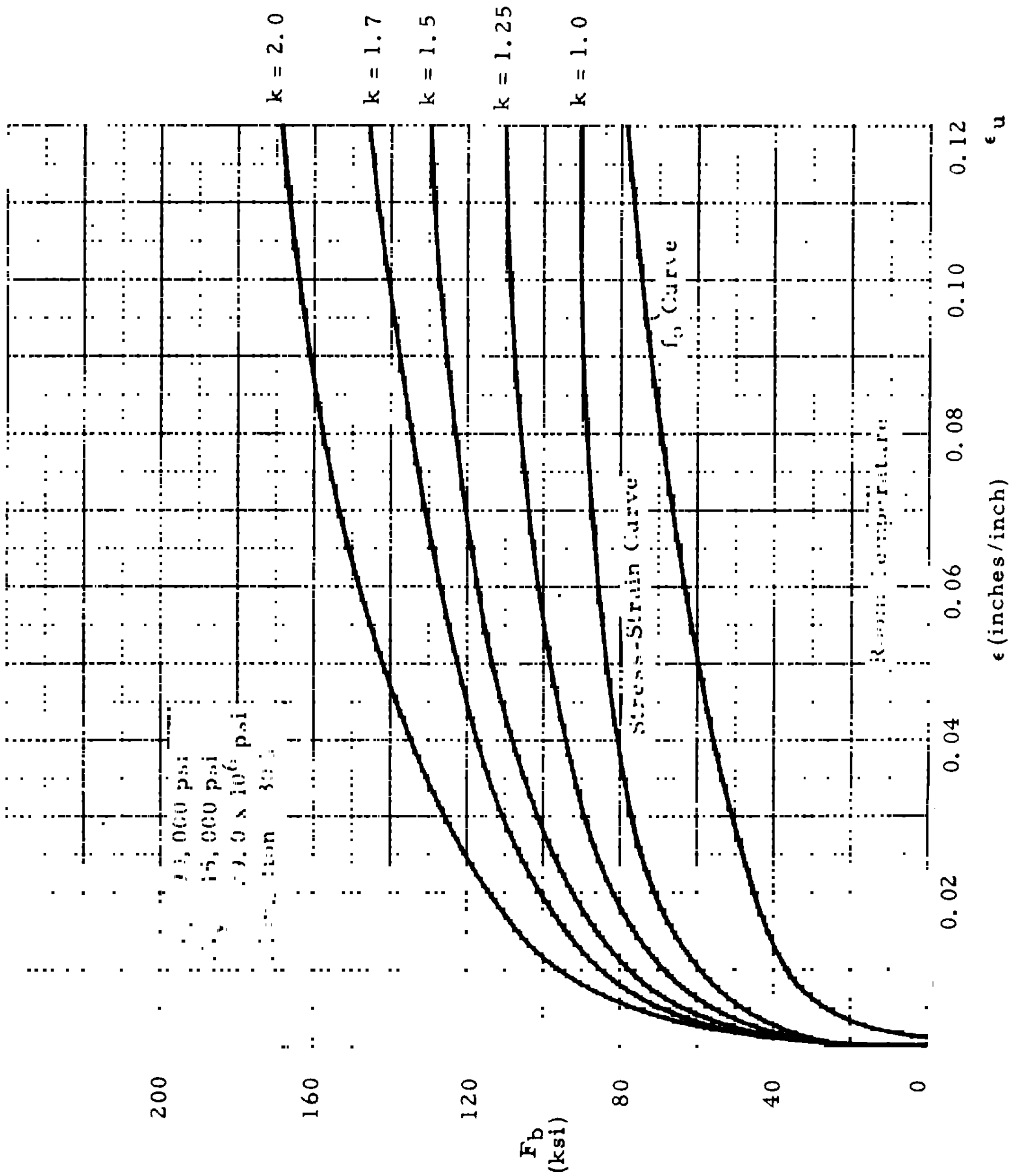


Fig. B4.5.6.1-47 Minimum Plastic Bending Curves 19-9DL (AMS 5526) & 19-9DX (AMS 5538) Stainless Steel

B4.5.6.1 Stainless Steels-Minimum Properties

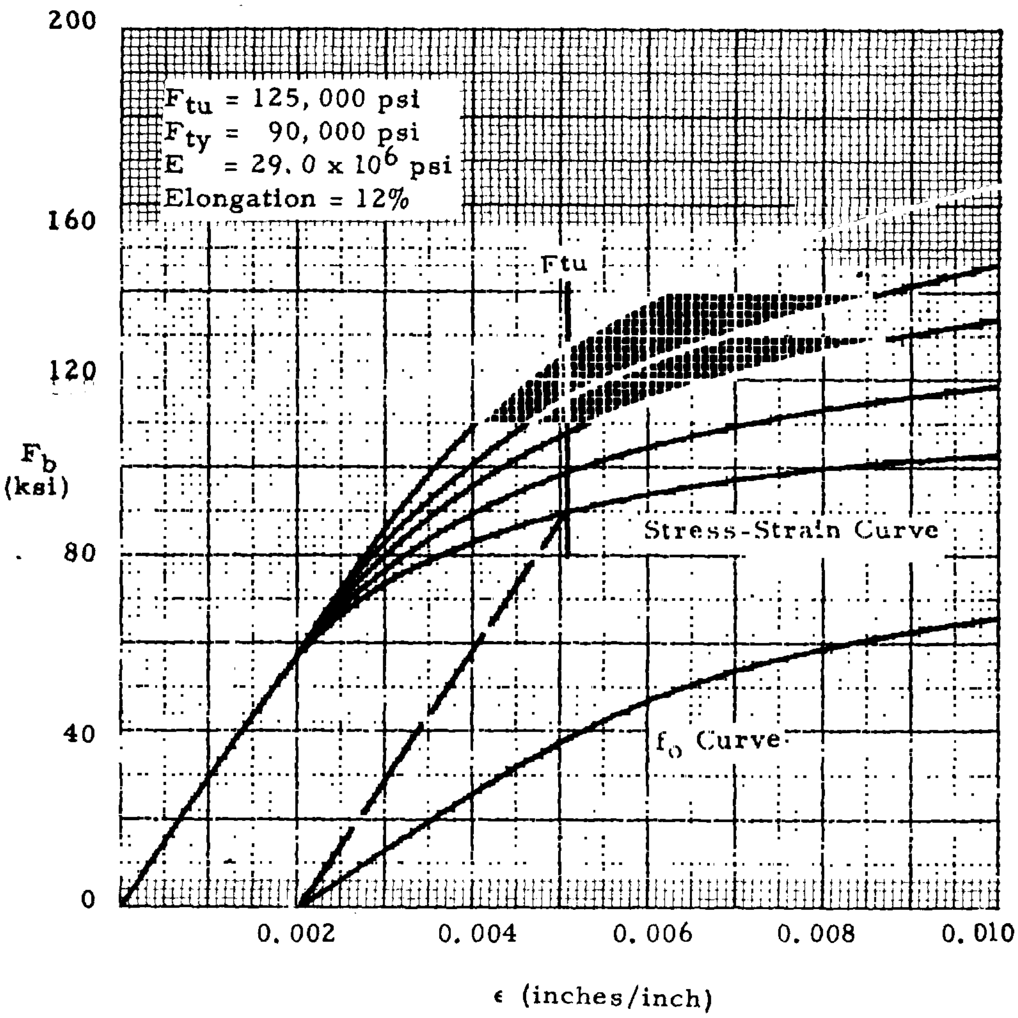


Fig. B4.5.6.1-48 Minimum Plastic Bending Curves 19-9DL (AMS 5527) & 19-9DX (AMS 5539) Stainless Steel

B4.5.6.1 <u>Stainless Steels-Minimum Properties</u>

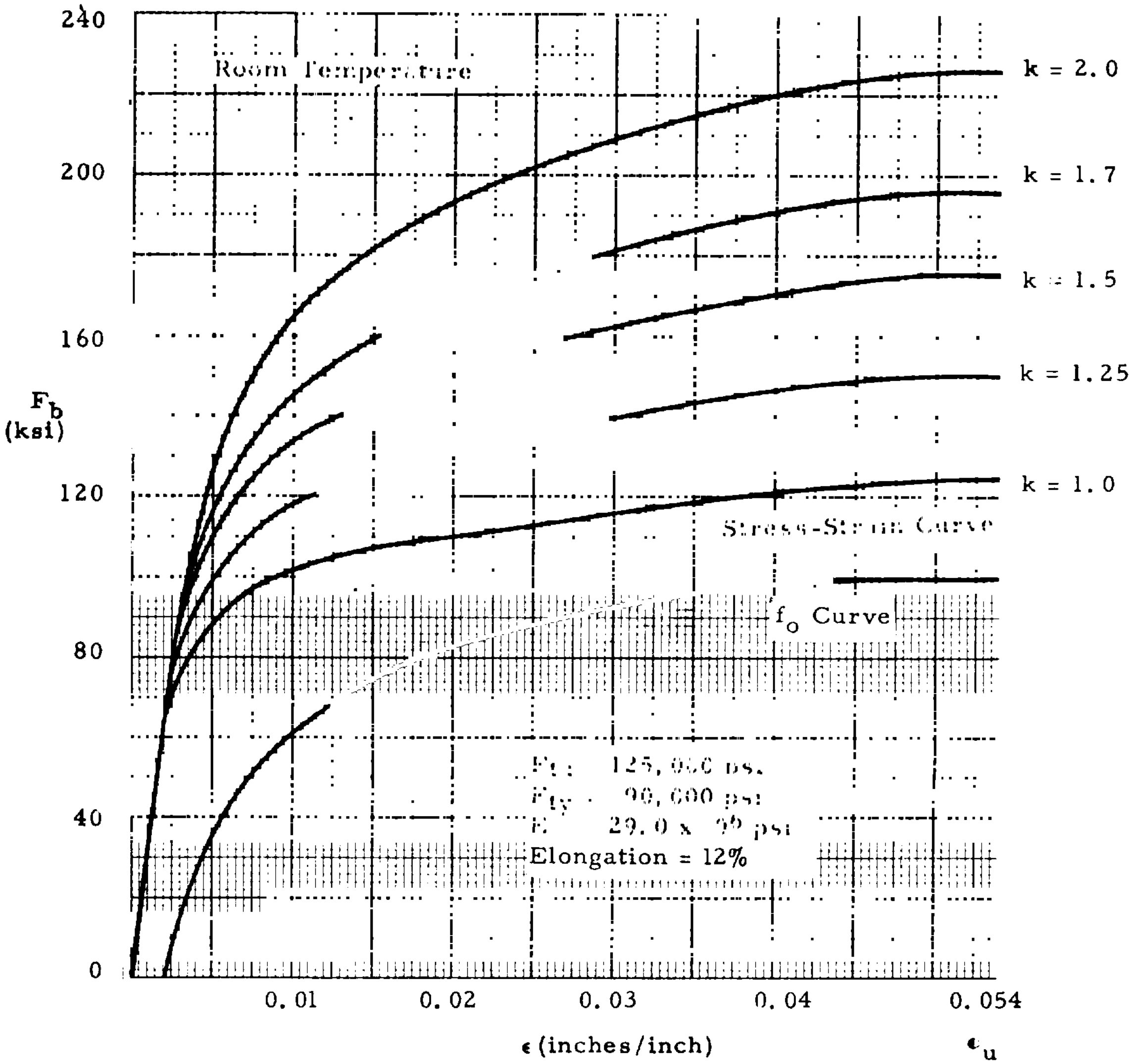


Fig. B4.5.6.1-49 Minimum Plastic Bending Curves 19-9DL (AMS 5527) & 19-9DX (AMS 5539) Stainless Steel

B4.5.5.2 Low Carbon and Alloy Steels-Minimum Properties

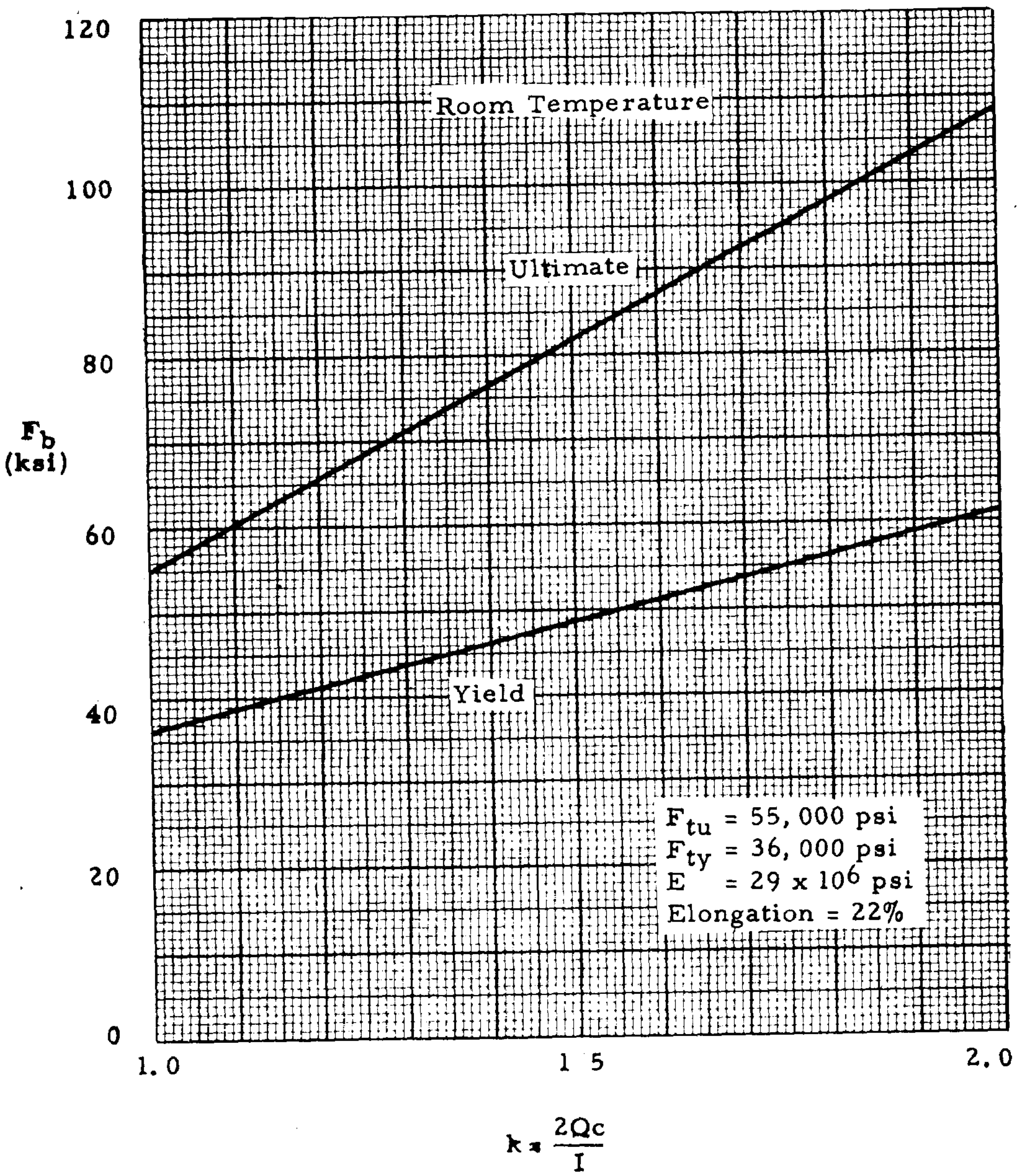


Fig. B4.5.5.2-1 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections, Carbon Steel AISI 1023-1025

B4.5.5.2 Low Carbon and Alloy Steels-Minimum Properties

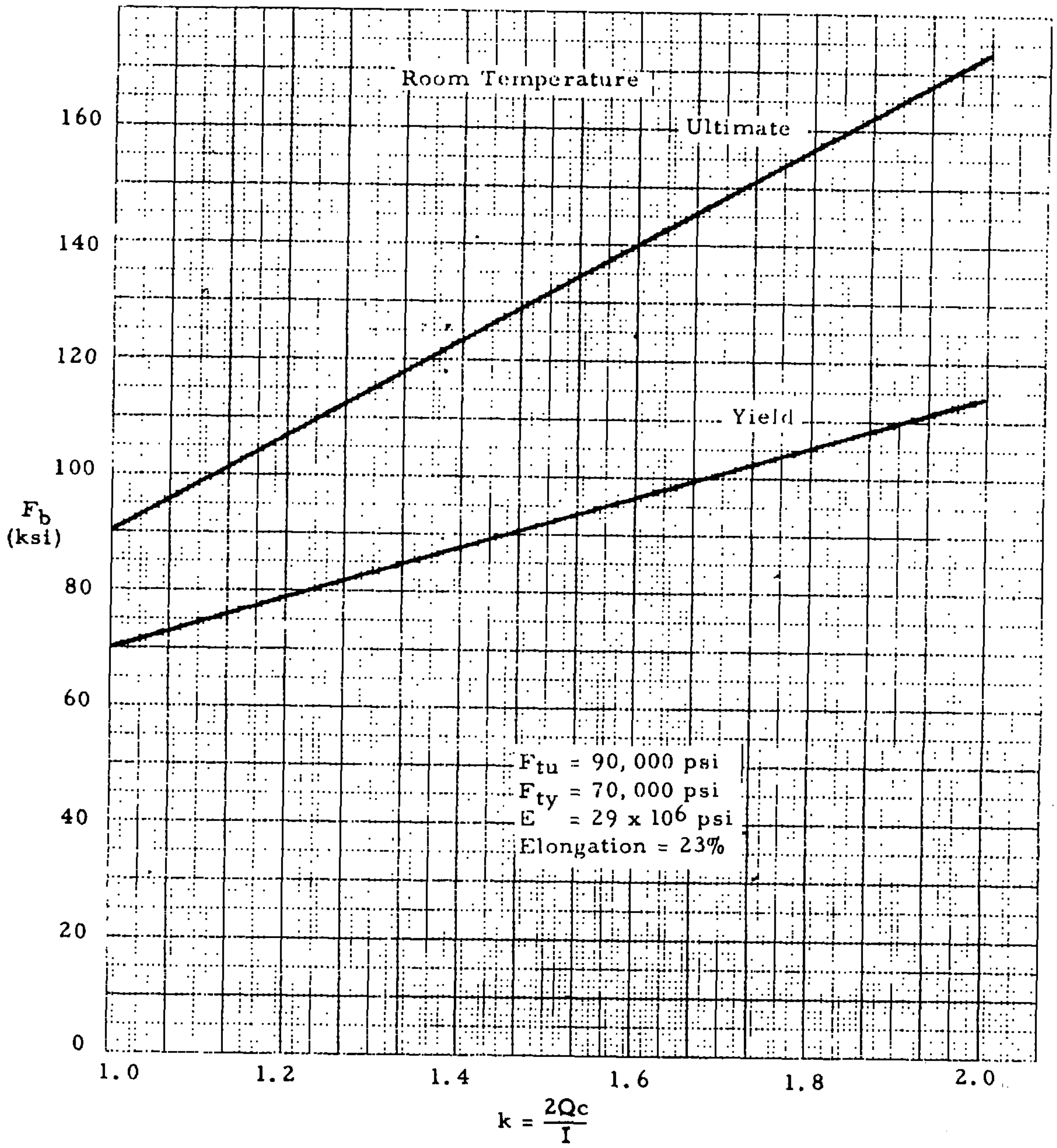


Fig. B4.5.5.2-2 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections AISI Alloy Steel, Normalized, > 0.188 Thick

B4.5.5.2 Low Carbon and Alloy Steels-Minimum Properties

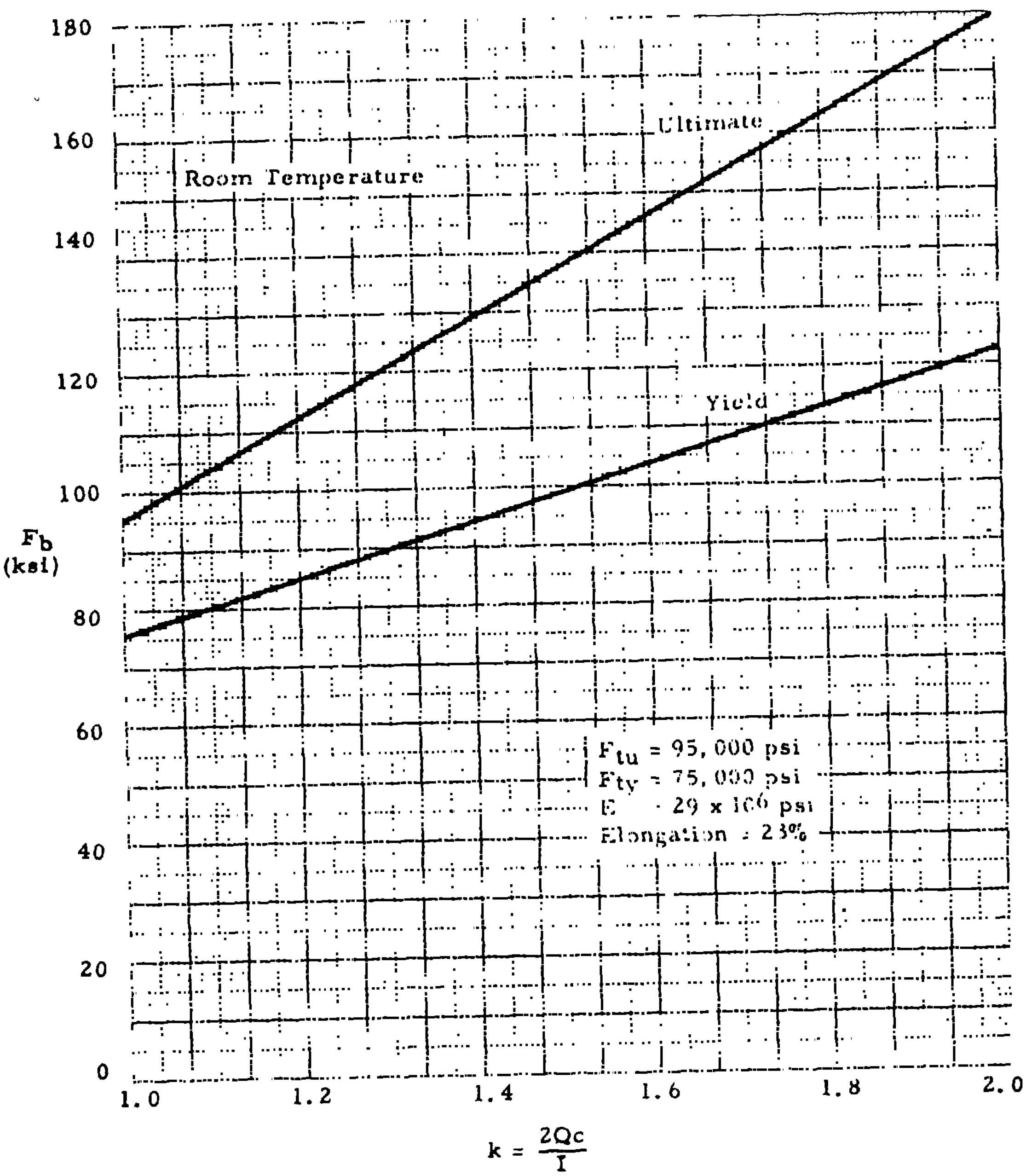


Fig. B4.5.5.2-3 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections AISI Alloy Steel, Normalized, $\leq$ 0.188 Thick

B4.5.5.2 Low Carbon and Alloy Steels-Minimum Properties

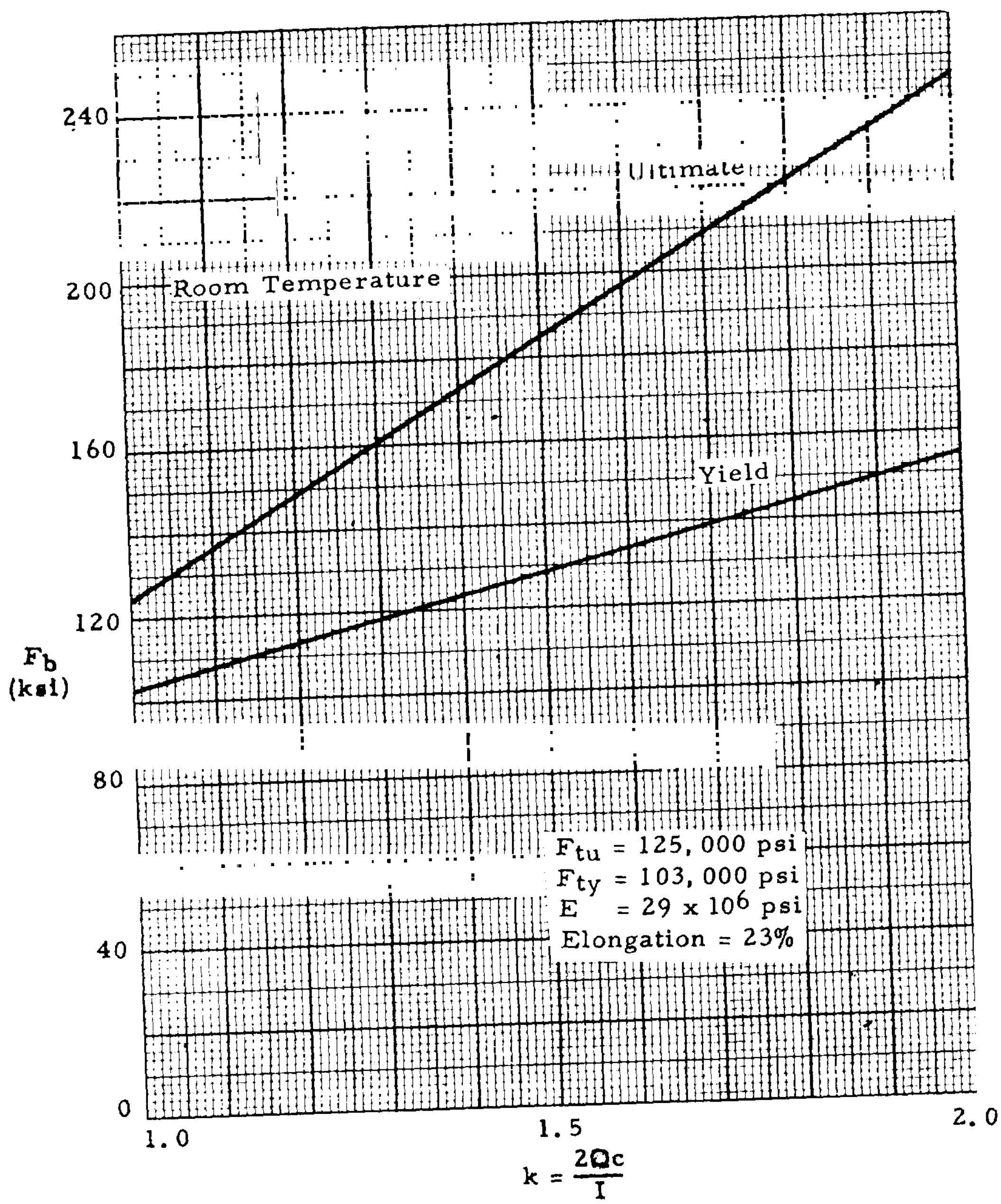


Fig. B4.5.5.2-4 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections AISI Alloy Steel, Heat Treated

B4.5.5.2 Low Carbon and Alloy Steels-Minimum Properties

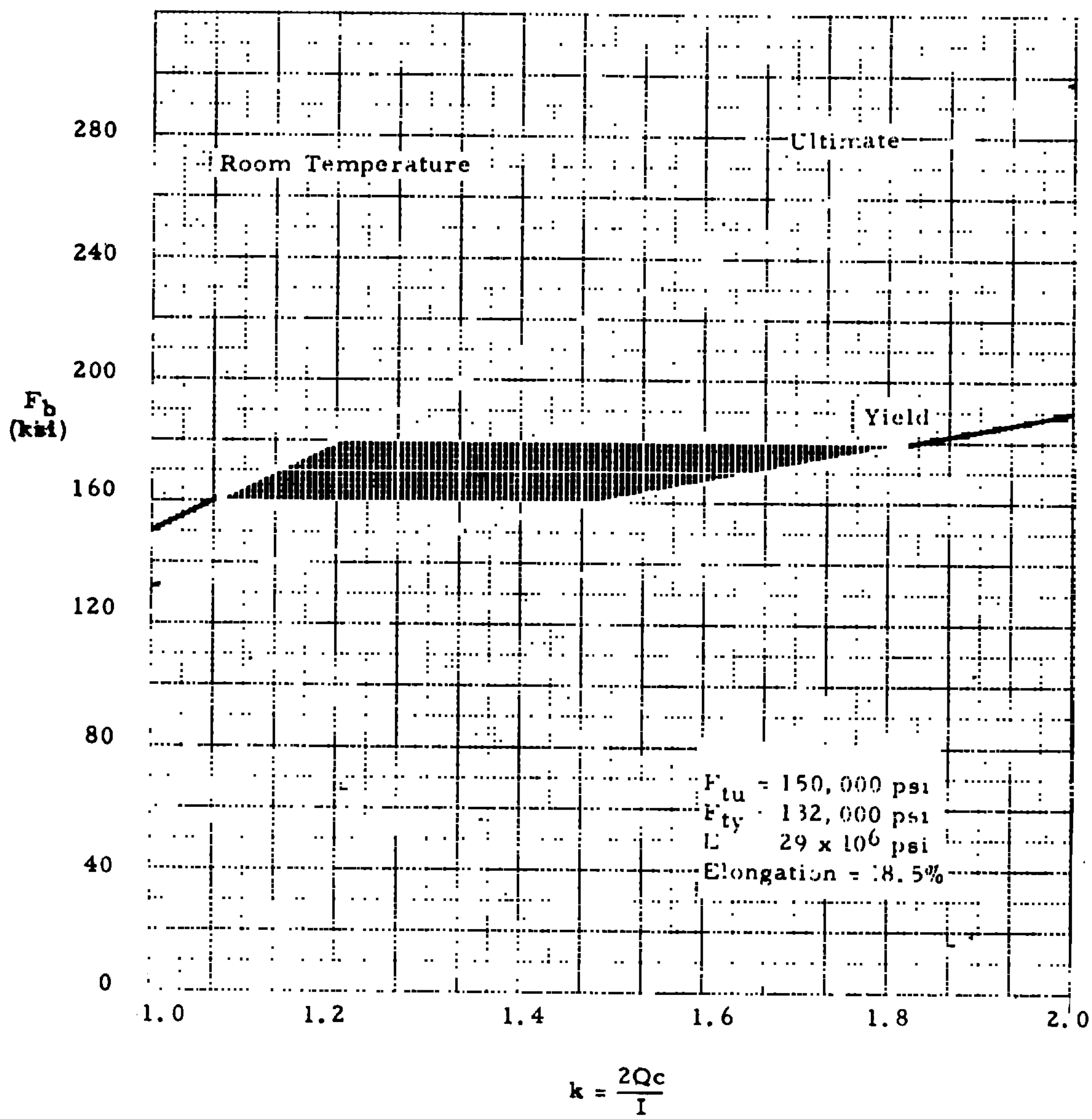


Fig. B4.5.5.2-5 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections AISI Alloy Steel, Heat Treated

B4.5.5.2 Low Carbon and Alloy Steels-Minimum Properties

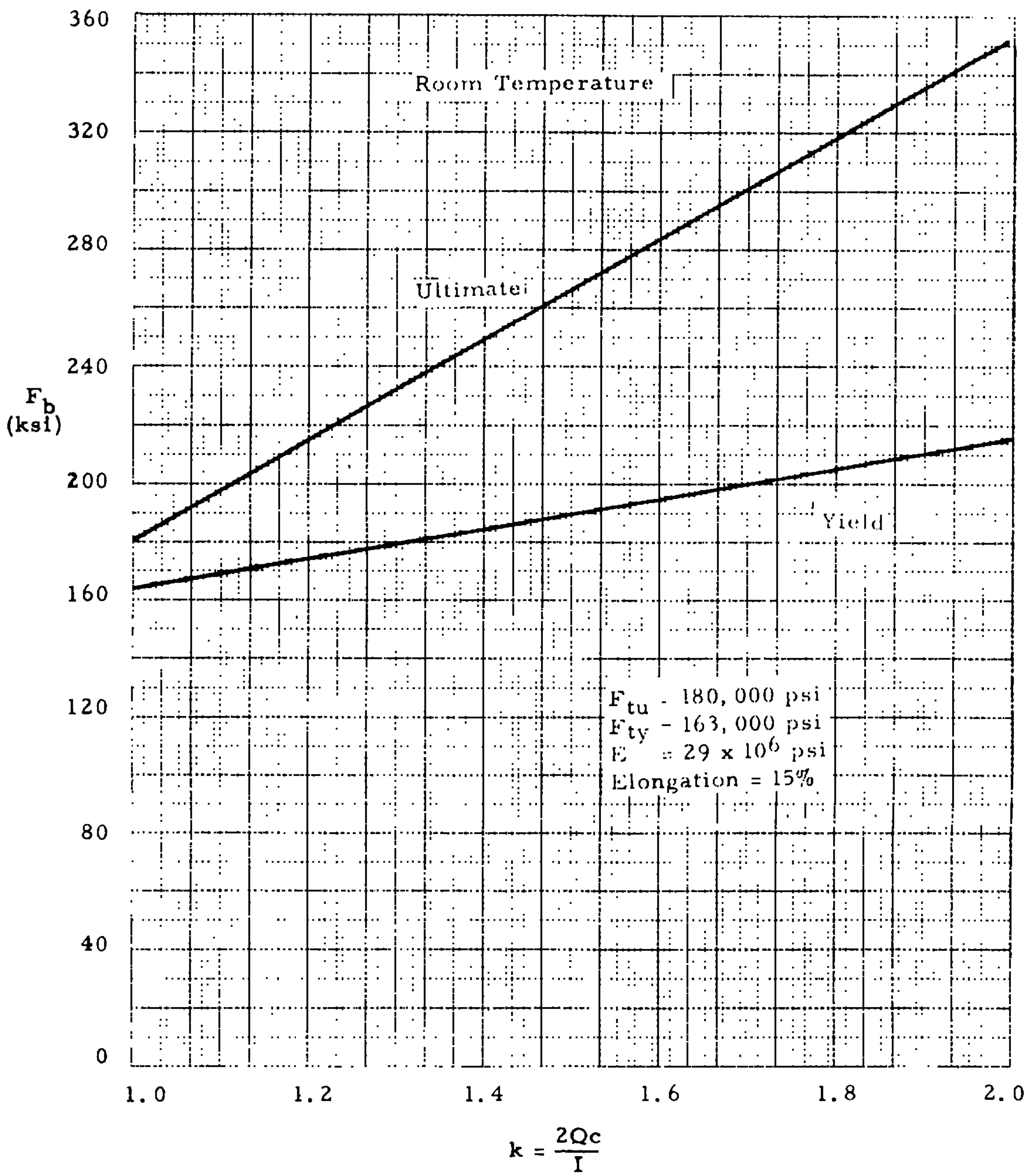


Fig. B4.5.5.2-6 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections AISI Alloy Steel, Heat Treated

B4.5.5.2 Low Carbon and Alloy Steels-Minimum Properties

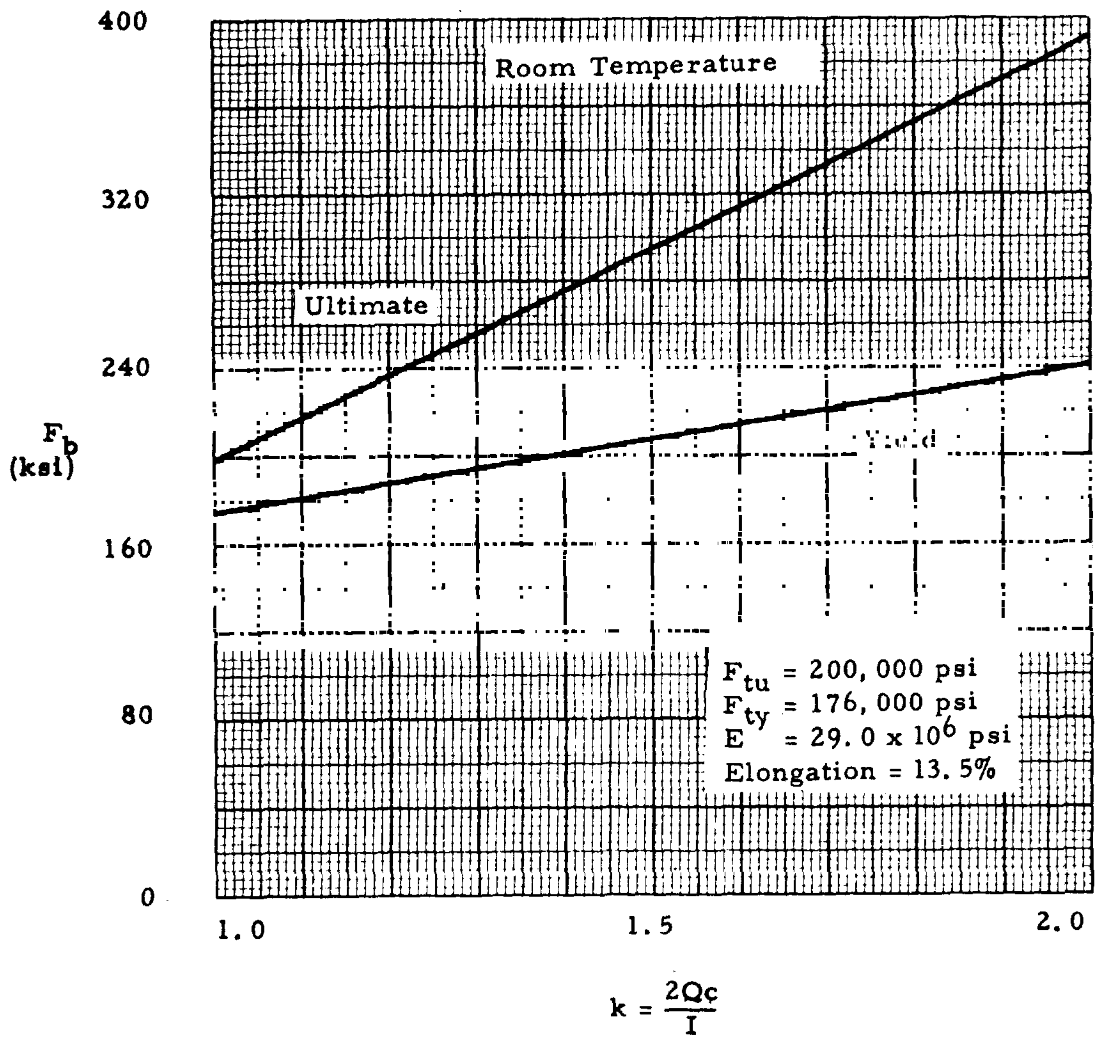


Fig. B4.5.5.2-7 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections AISI Alloy Steel, Heat Treated

B4.5.6.2 Low Carbon and Alloy Steels-Minimum Properties

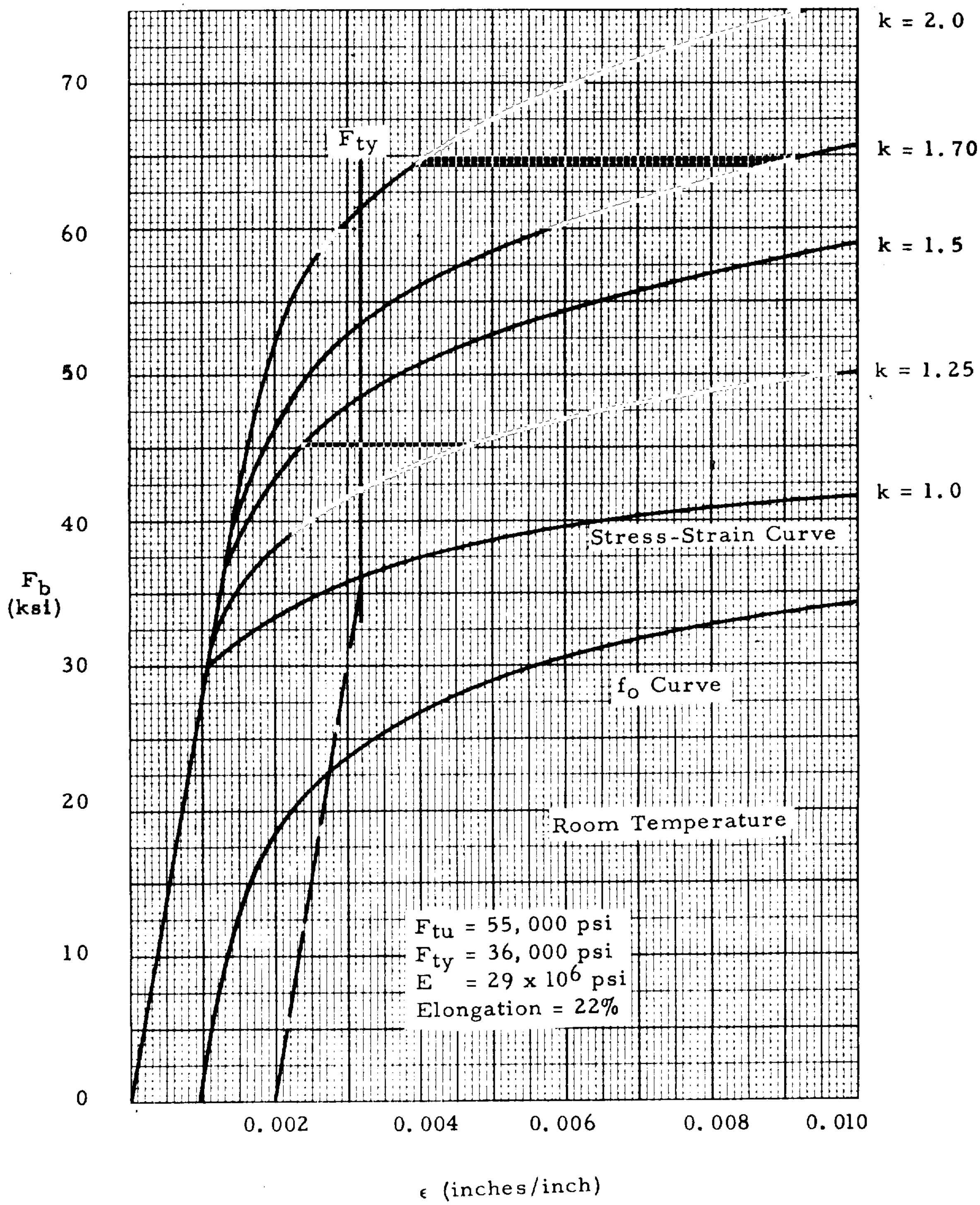


Fig. B4.5.6.2-1 Minimum Plastic Bending Curves Carbon Steel AISI 1023-1025

B4.5.6.2 Low Carbon and Alloy Steels-Minimum Properties

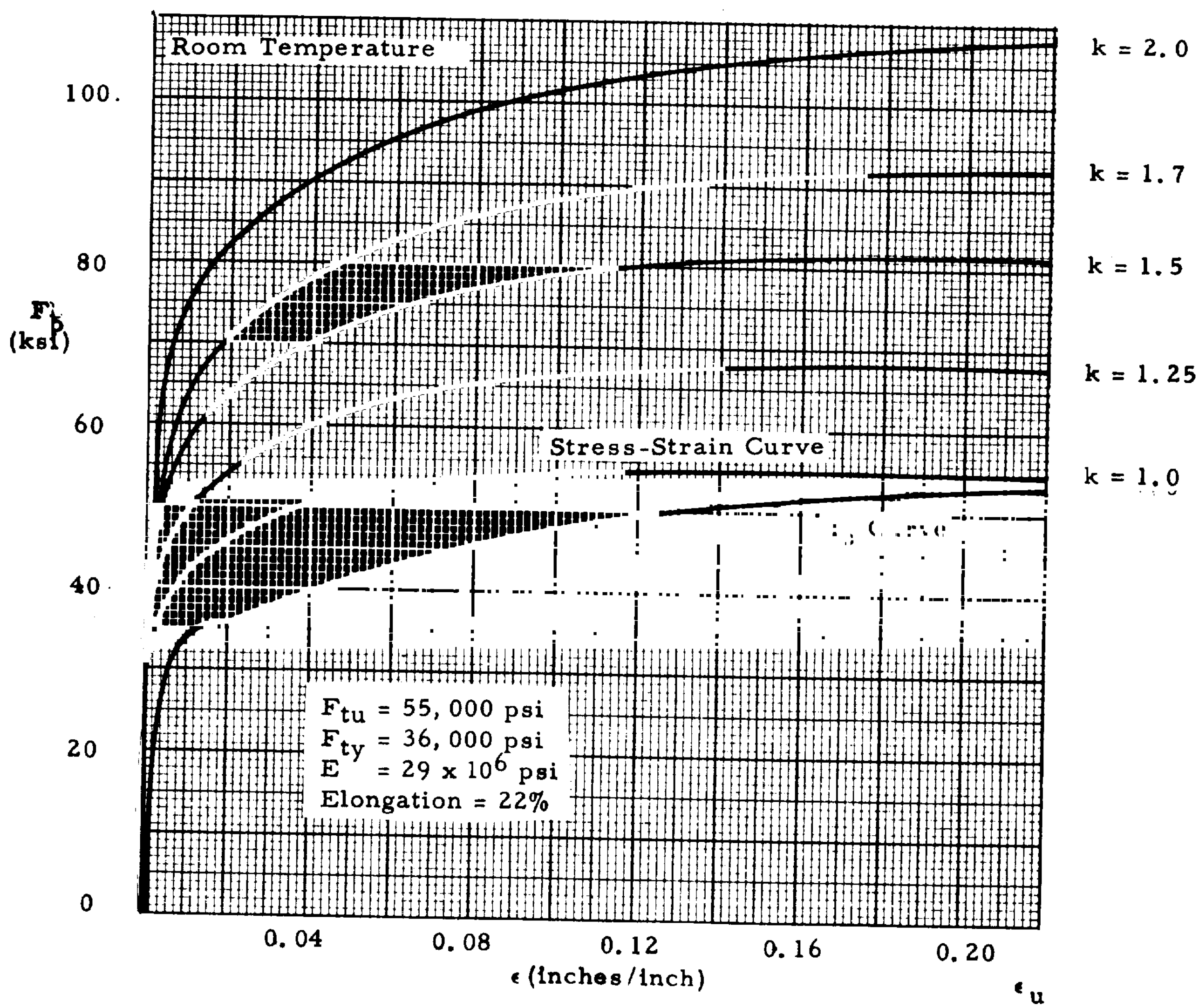


Fig. B4.5.6.2-2 Minimum Plastic Bending Curves Carbon Steel AISI 1023-1025

B4.5.6.2 Low Carbon and Alloy Steels-Minimum Properties

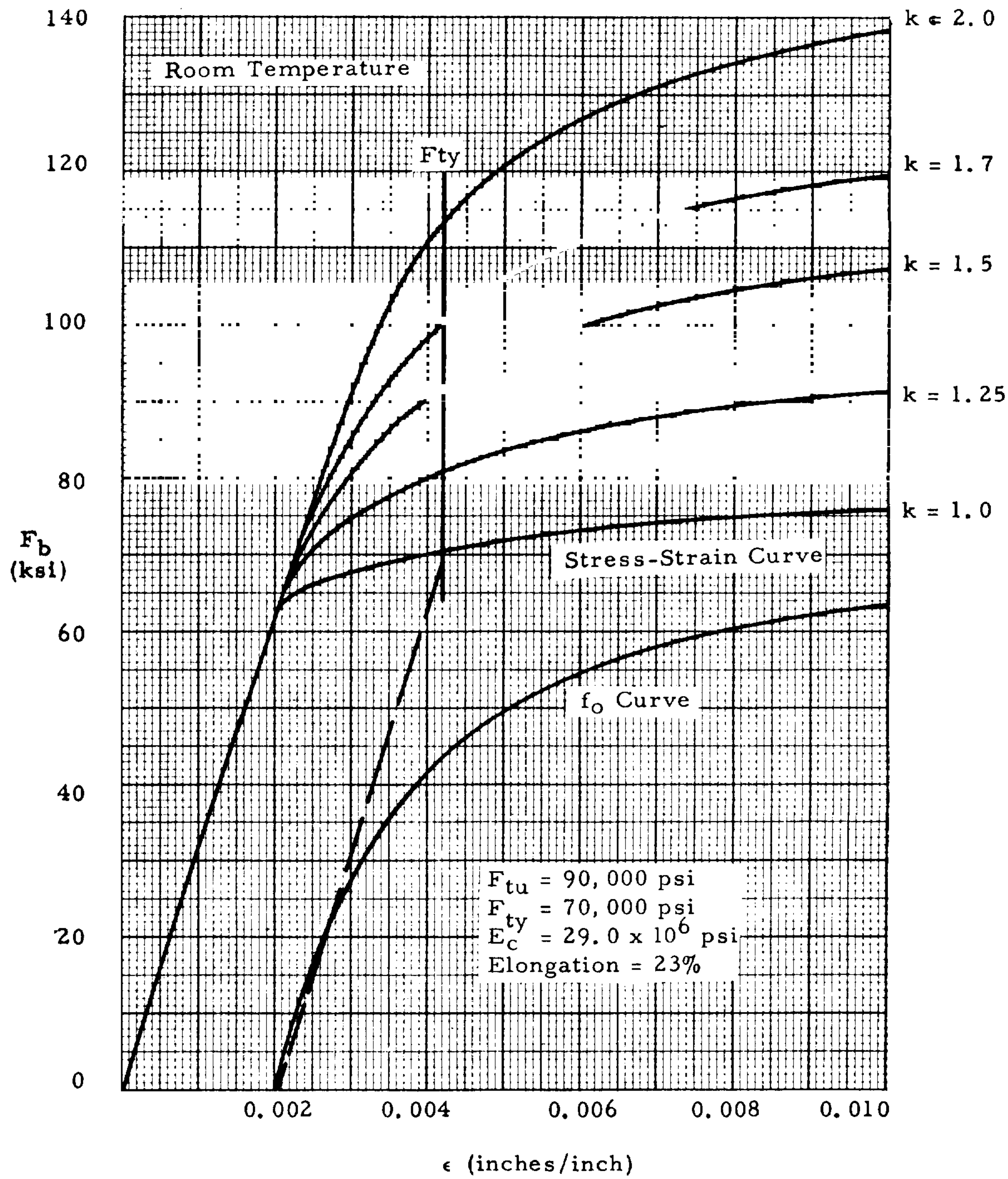


Fig. B4.5.6.2-3 Minimum Plastic Bending Curves AISI Alloy Steel, Normalized, > 0.188 In. Thick

B4.5.6.2 Low Carbon and Alloy Steels-Minimum Properties

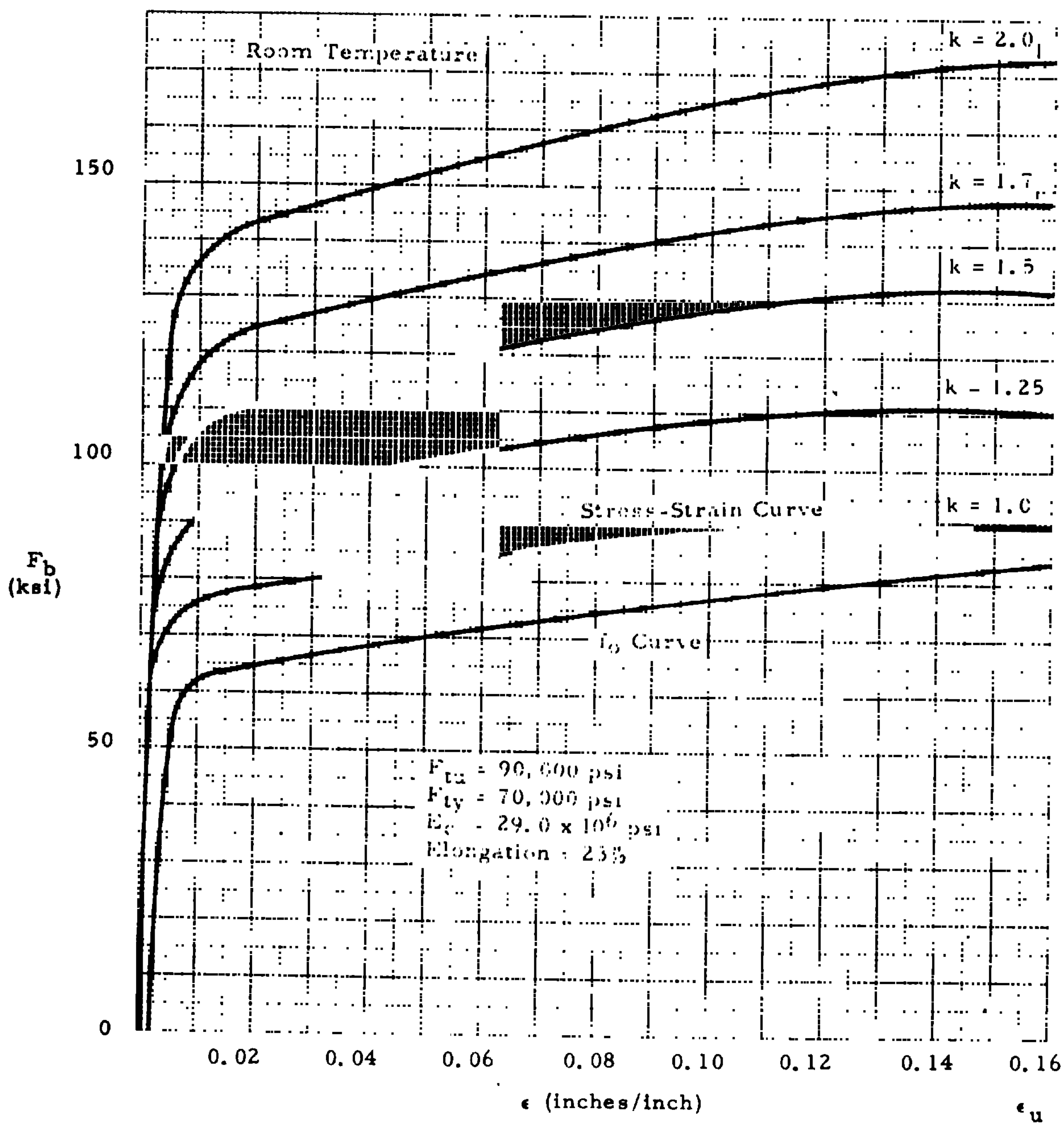


Fig. B4.5.6.2-4 Minimum Plastic Bending Curves AISI Alloy Steel, Normalized, Thickness > 0.188 In.

B4.5.6.2 Low Carbon and Alloy Steels-Minimum Properties

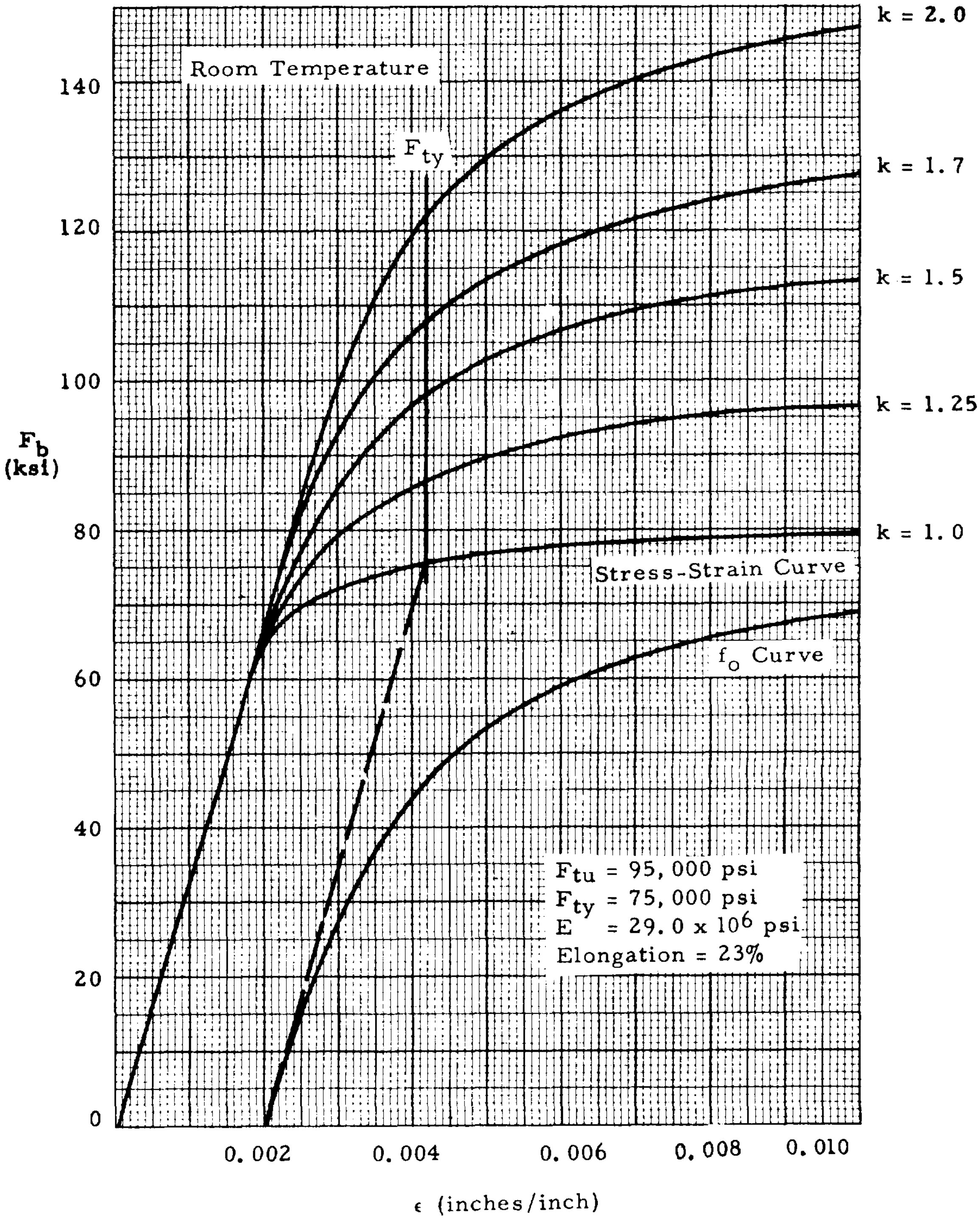


Fig. B4.5.6.2-5 Minimum Plastic Bending Curves AISI Alloy Steel, Normalized, Thickness ≤ 0.188 In.

B4.5.6.2 Low Carbon and Alloy Steels-Minimum Properties

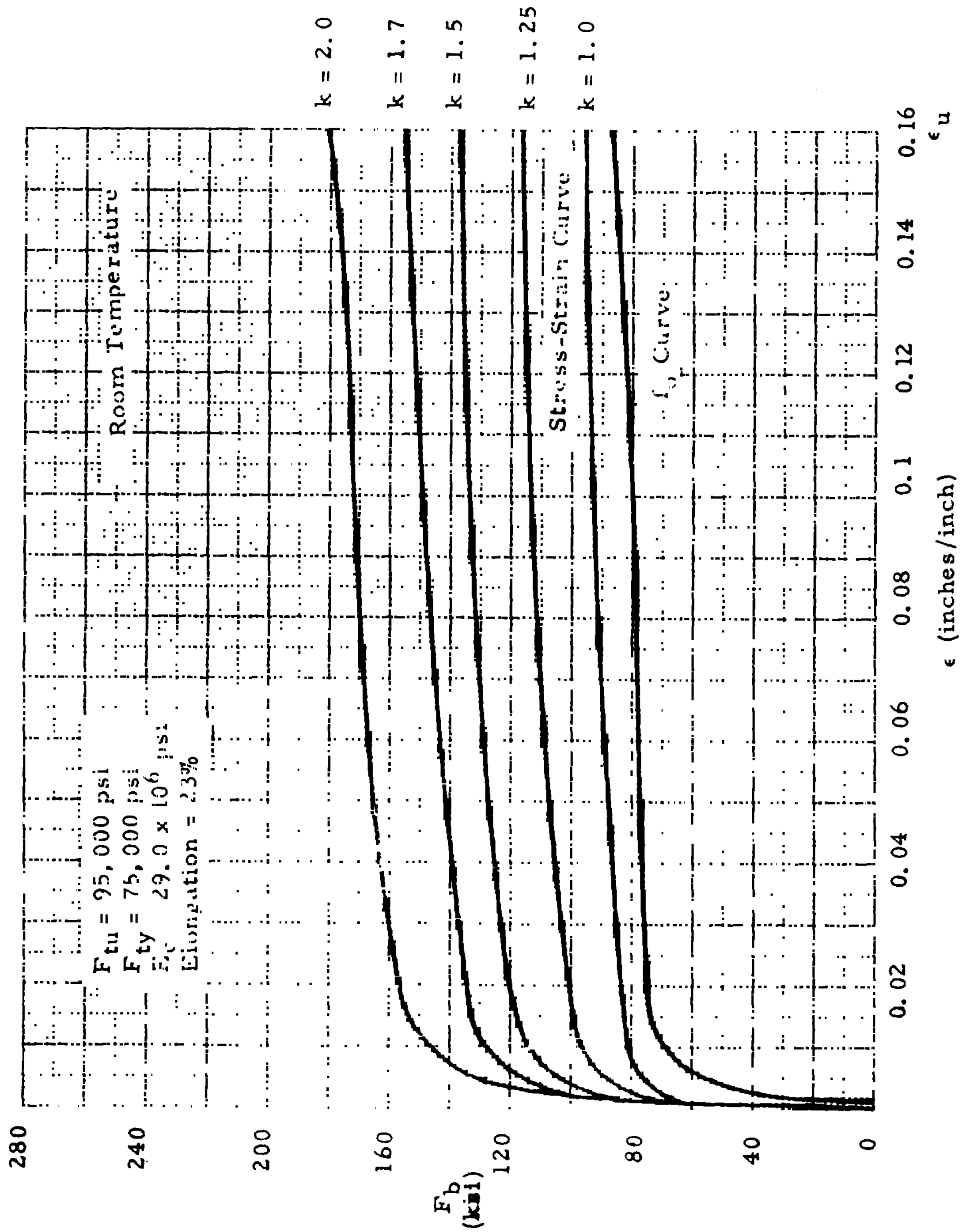


Fig. B4.5.6.2-6 Minimum Plastic Bending Curves AISI Alloy Steel, Normalized, Thickness ≤ 0.188 In.

B4.5.6.2 Low Carbon and Alloy Steels-Minimum Properties

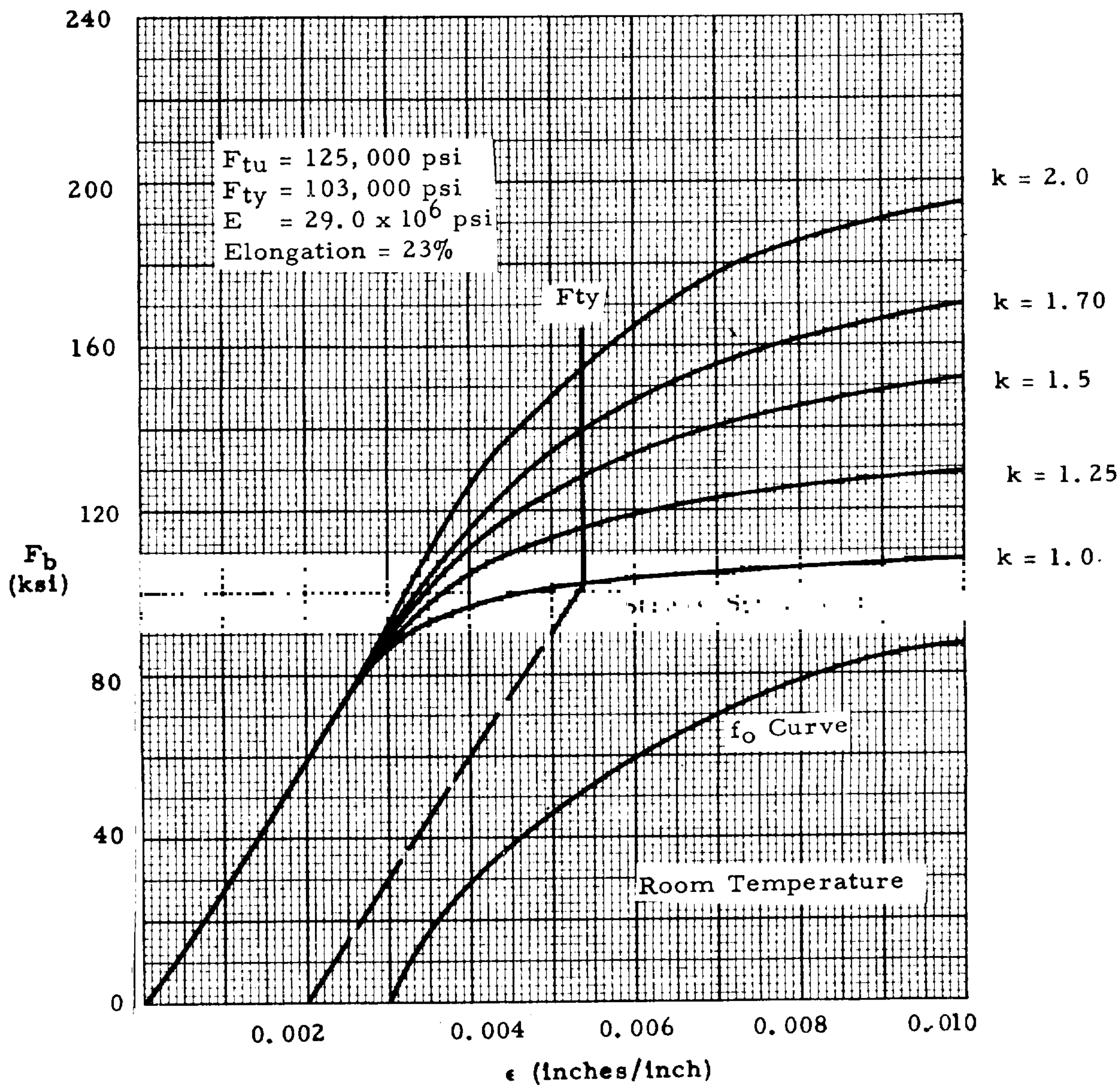


Fig. B4.5.6.2-7 Minimum Plastic Bending Curves AISI Alloy Steel, Heat Treated

B4.5.6.2 Low Carbon and Alloy Steels-Minimum Properties

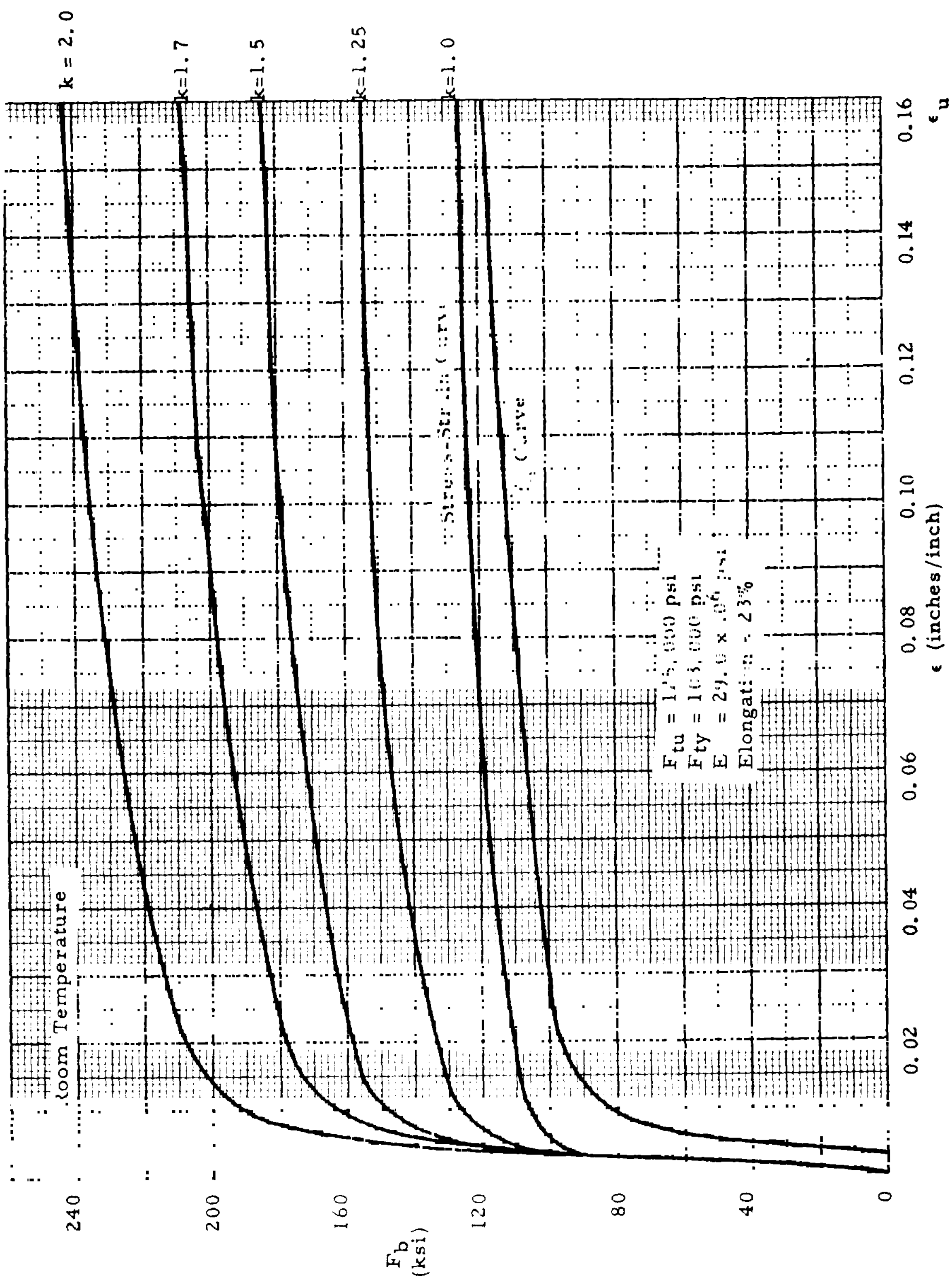


Fig. B4.5.6.2-8 Minimum Plastic Bending Curves AISI Alloy Steel, Heat Treated

Graph to be furnished when available

B4.5.6.2 Low Carbon and Alloy Steels-Minimum Properties

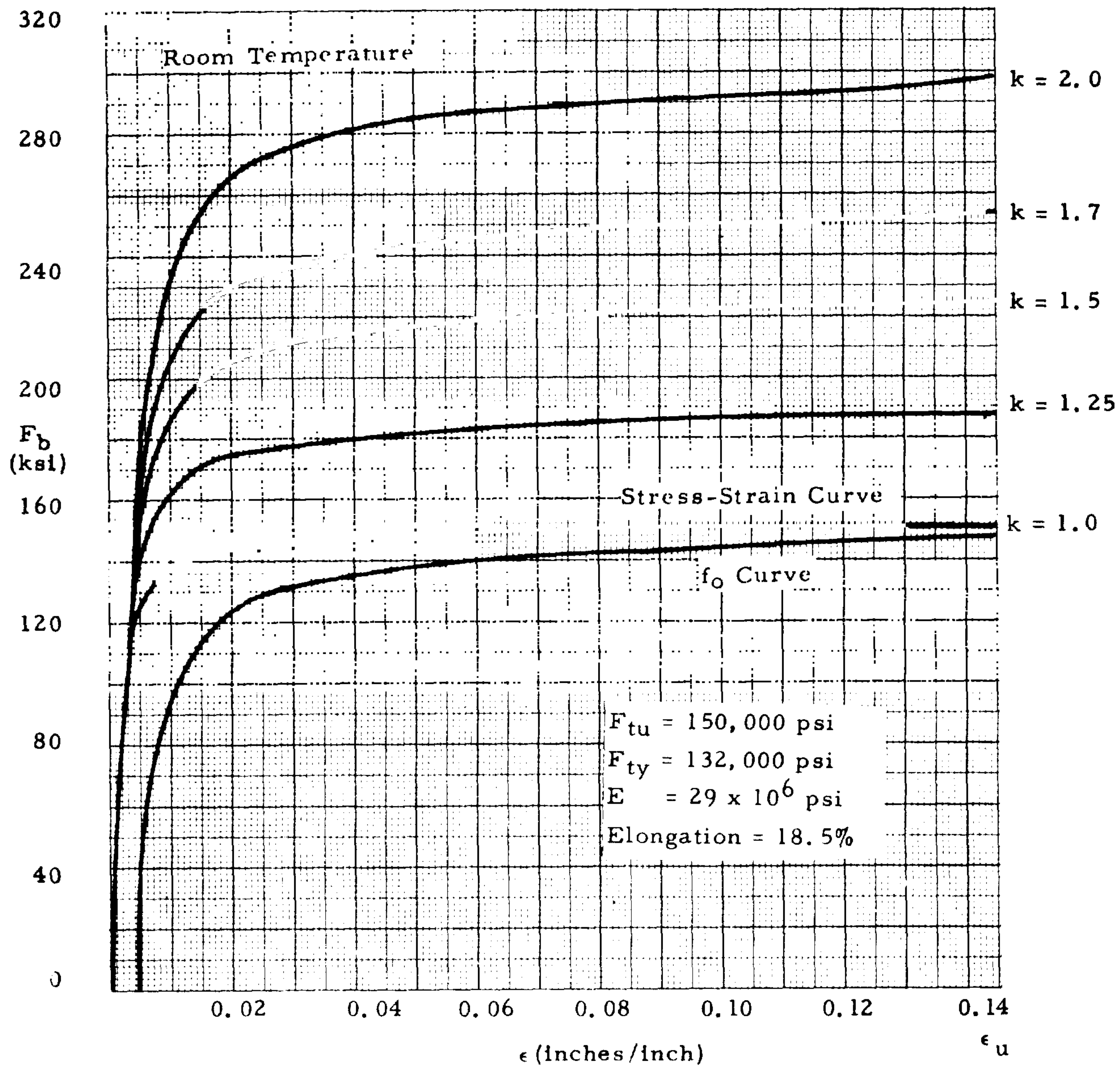


Fig. B4.5.6.2-10 Minimum Plastic Bending Curves AISI Alloy Steel, Heat Treated

B4.5.6.2 Low Carbon and Alloy Steels-Minimum Properties

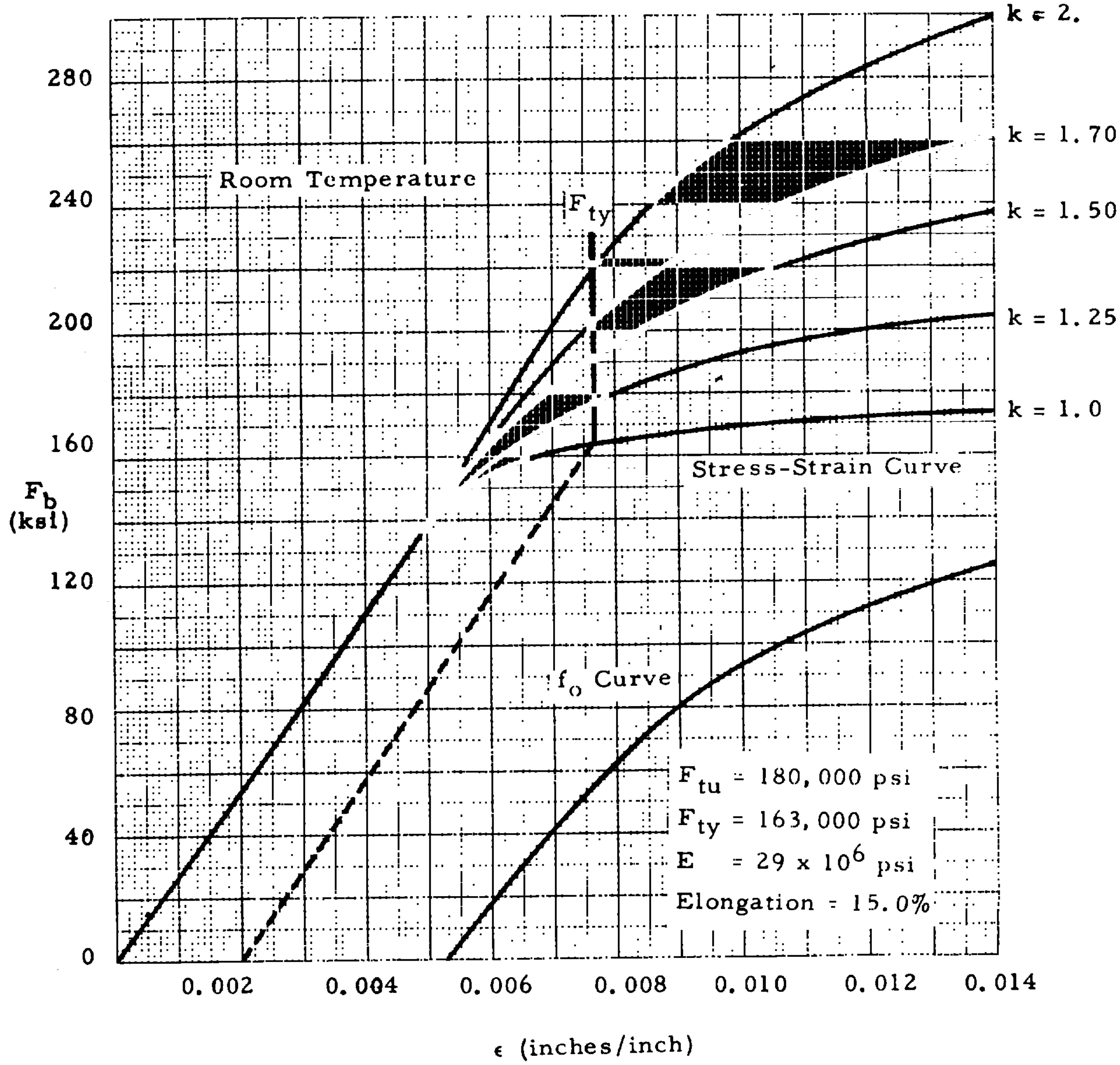


Fig. B4.5.6.2-11 Minimum Plastic Bending Curves AISI Alloy Steel, Heat Treated

B4.5.6.2 Low Carbon and Alloy Steels-Minimum Properties

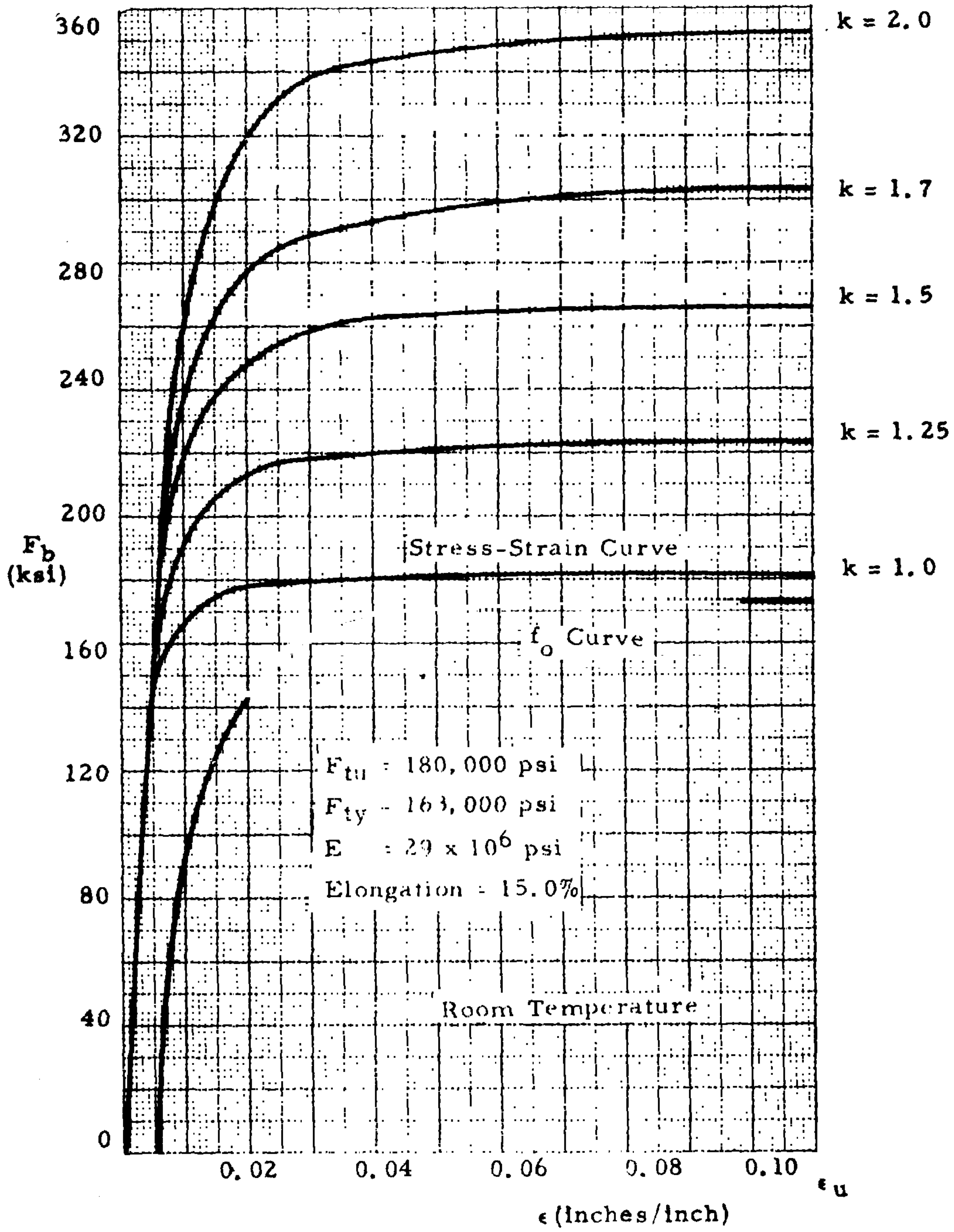


Fig. B4.5.6.2-12 Minimum Plastic Bending Curves AISI Alloy Steel, Heat Treated

B4.5.6.2 Low Carbon and Alloy Steels-Minimum Properties

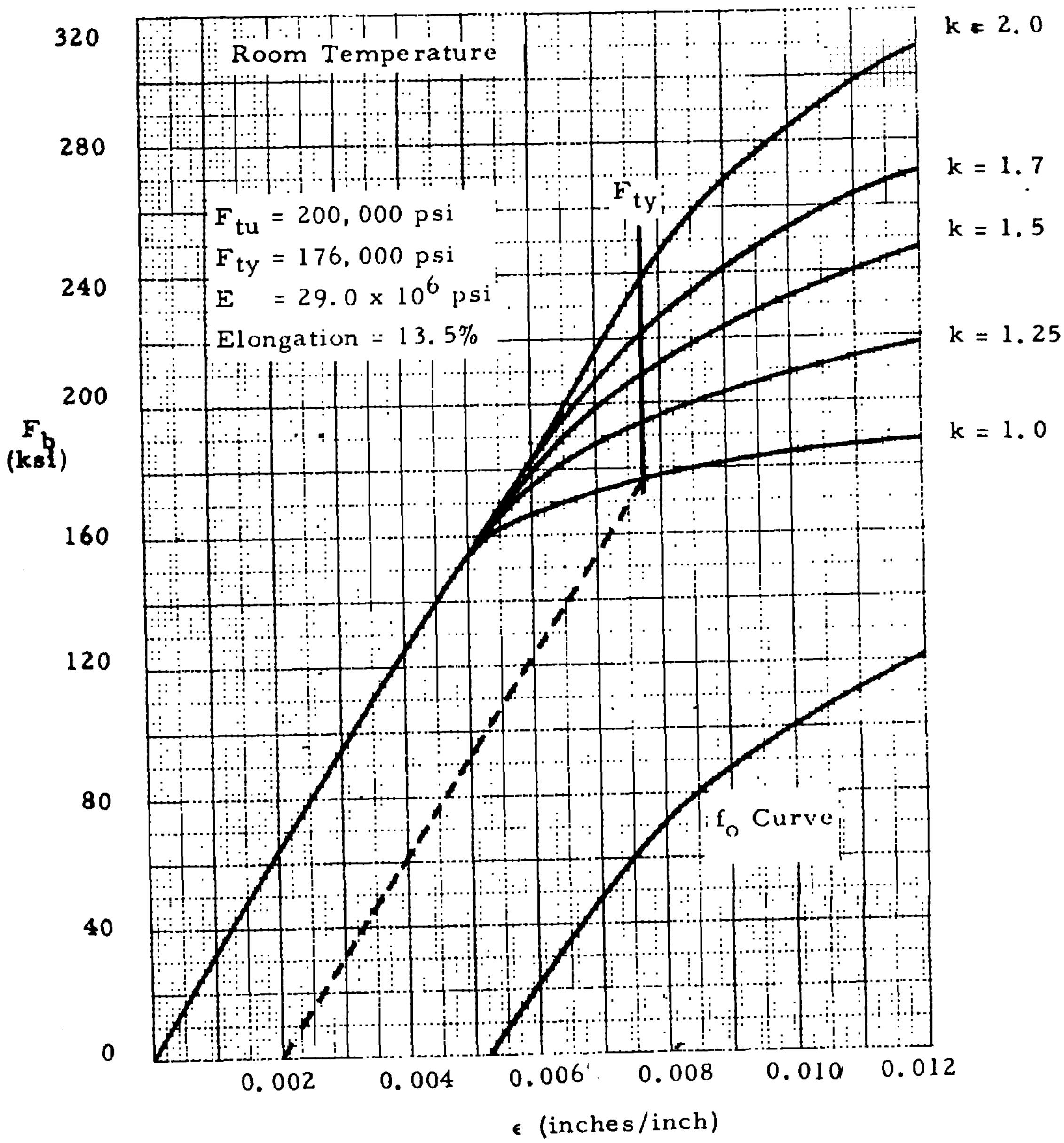


Fig. B4.5.6.2-13 Minimum Plastic Bending Curves AISI Alloy Steel, Heat Treated

B4.5.6.2 <u>Low Carbon and Alloy Steels-Minimum Properties</u>

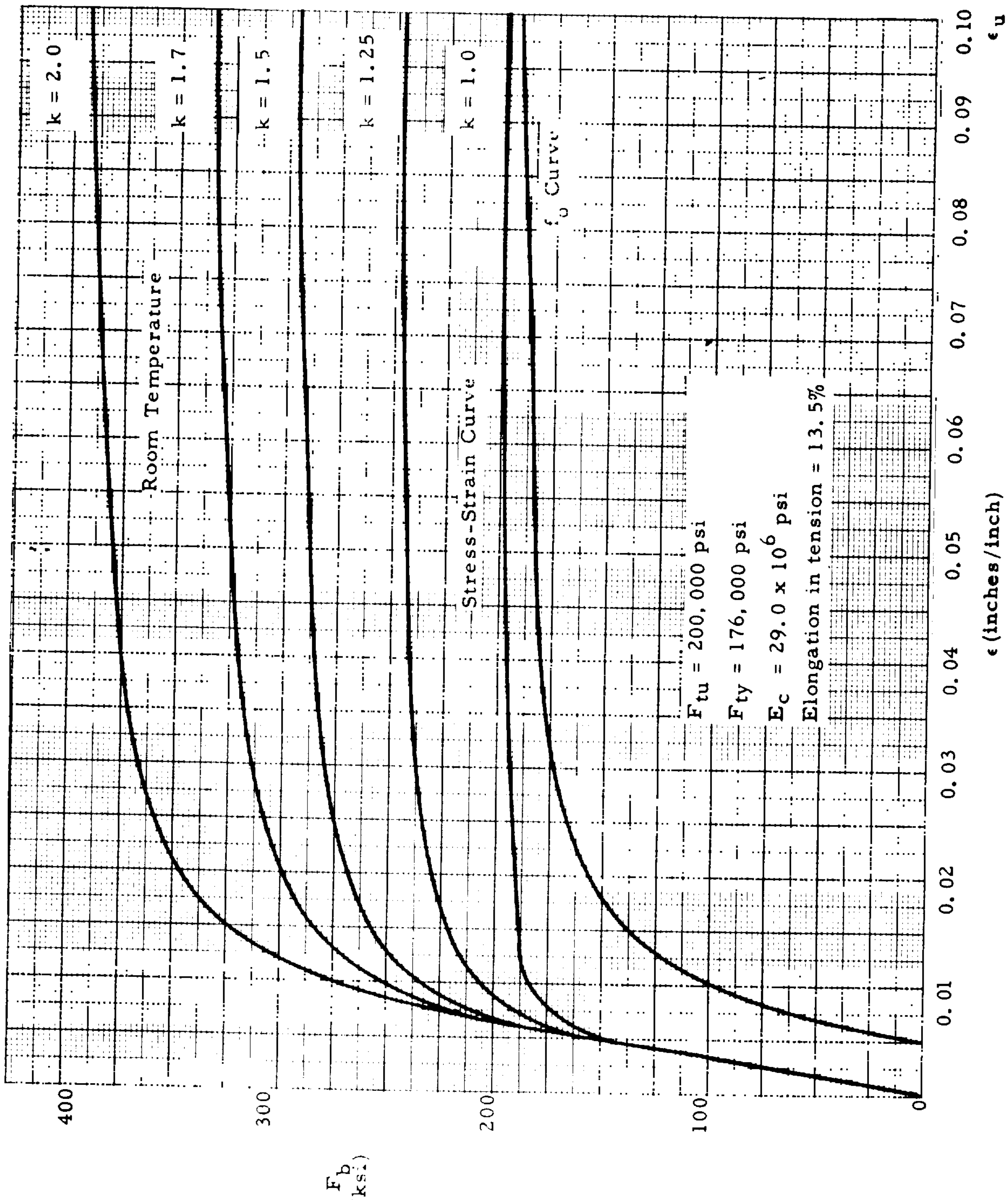


Fig. B4.5.6.2-14 Minimum Plastic Bending Curves AISI Alloy Steel, Heat Treated

B4.5.5.3 Heat Resistant Alloys-Minimum Properties

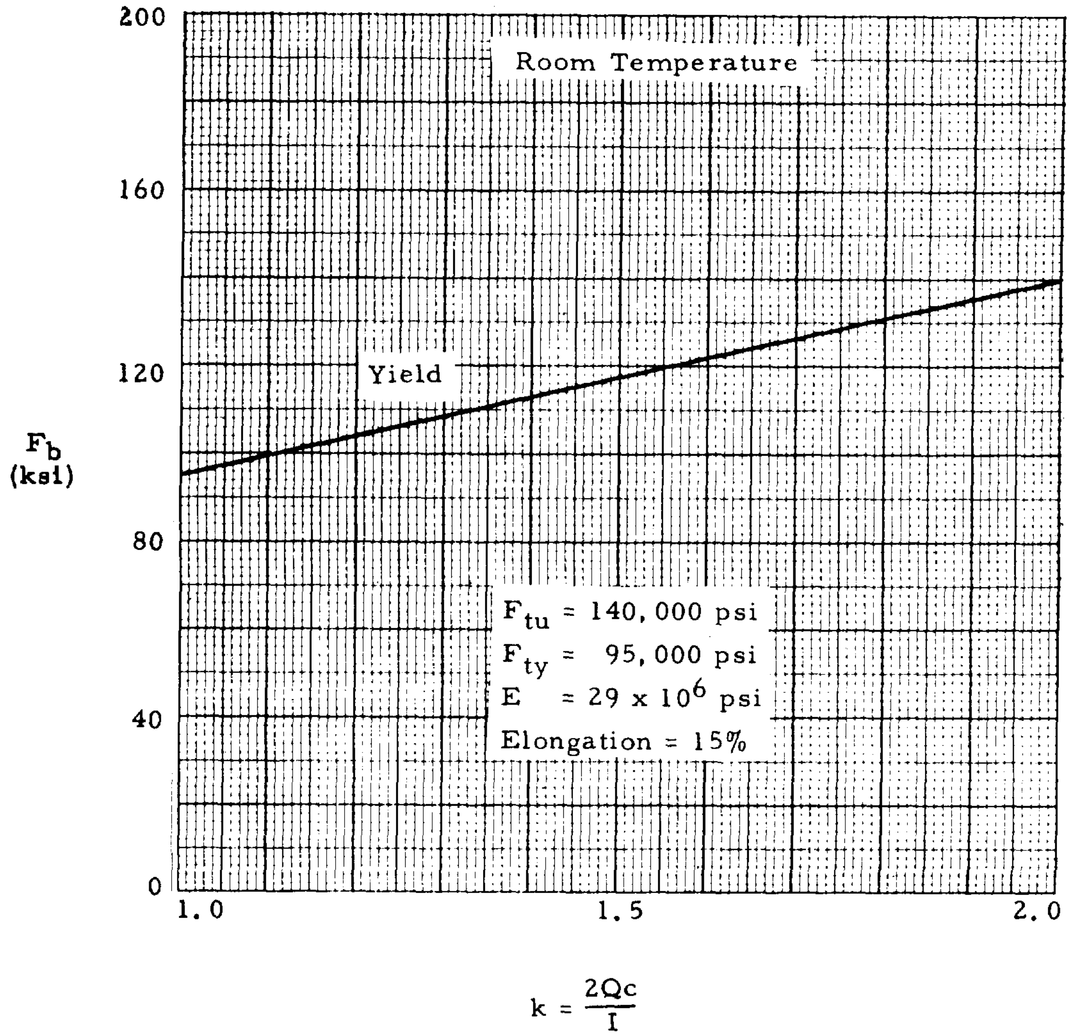


Fig. B4.5.5.3-1 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections A-286 Alloy, Heat Treated

B4.5.5.3 Heat Resistant Alloys-Minimum Properties

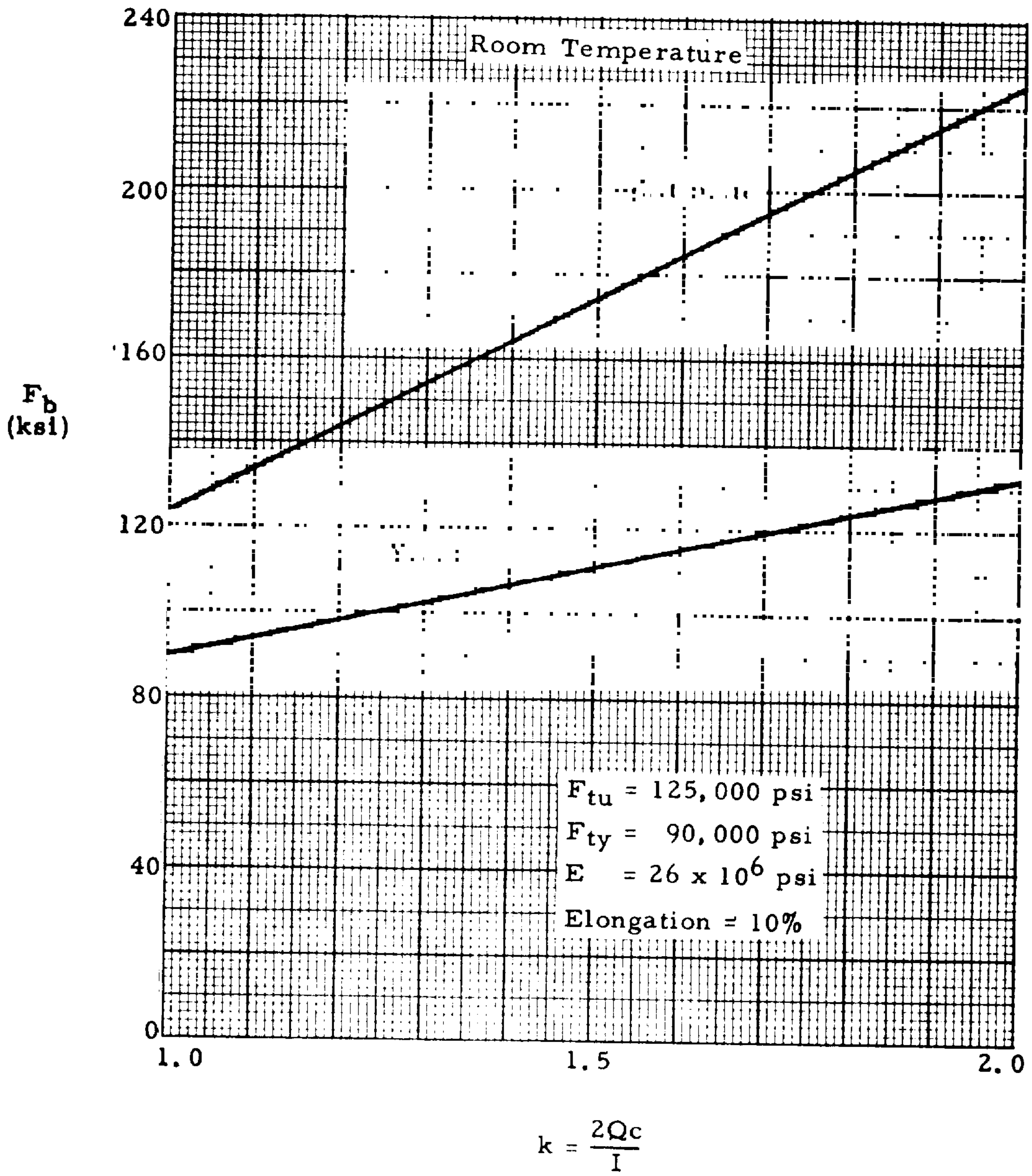


Fig. B4.5.5.3-2 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections Age Hardened, K-Monel Alloy Sheet

B4.5.5.3 **Heat Resistant Alloys-Minimum Properties**

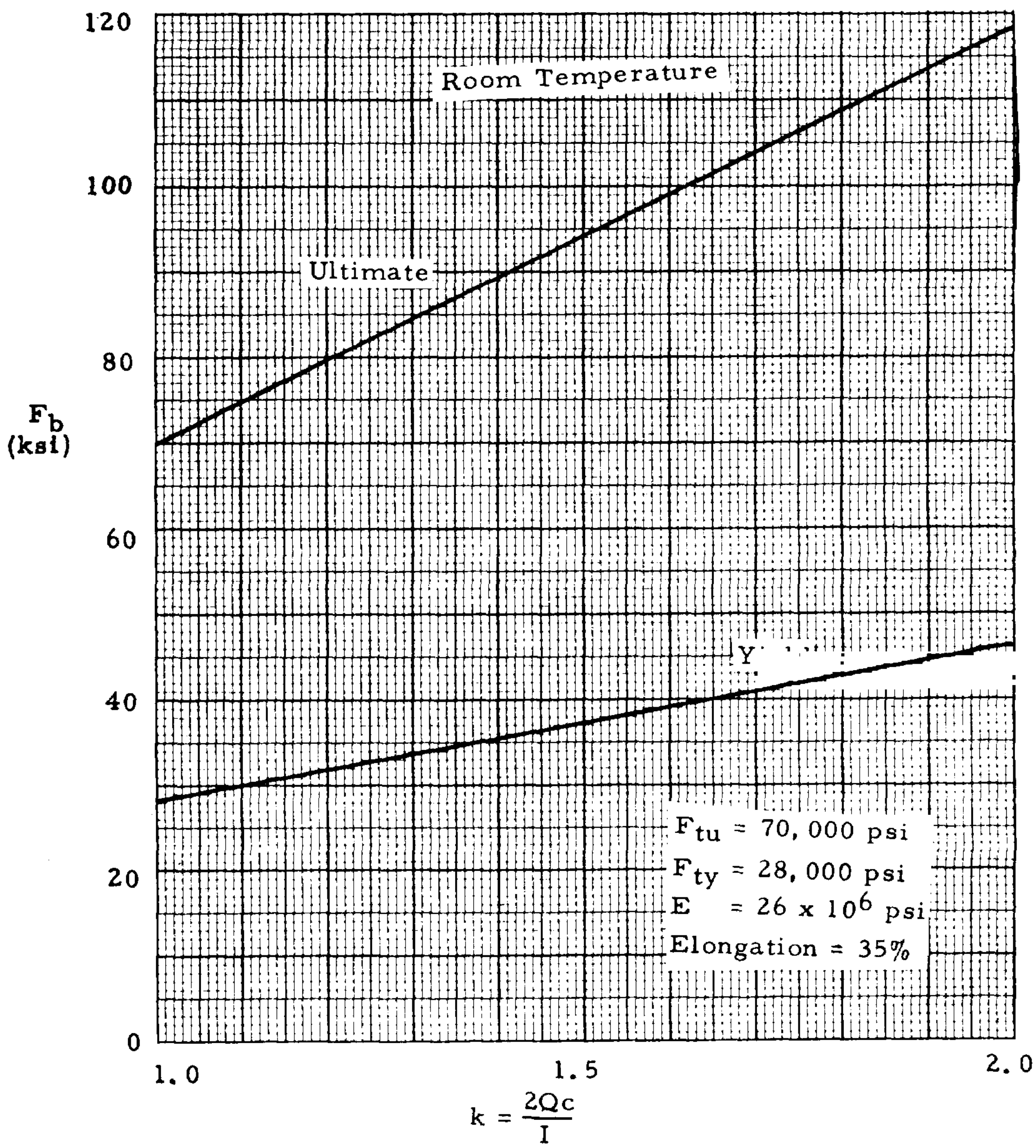


Fig. B4.5.5.3-3 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections Monel Alloy-Cold Rolled, Annealed Sheet

Graph to be furnished when available

B4.5.6.3 Corrosion Resistant Metals-Minimum Properties

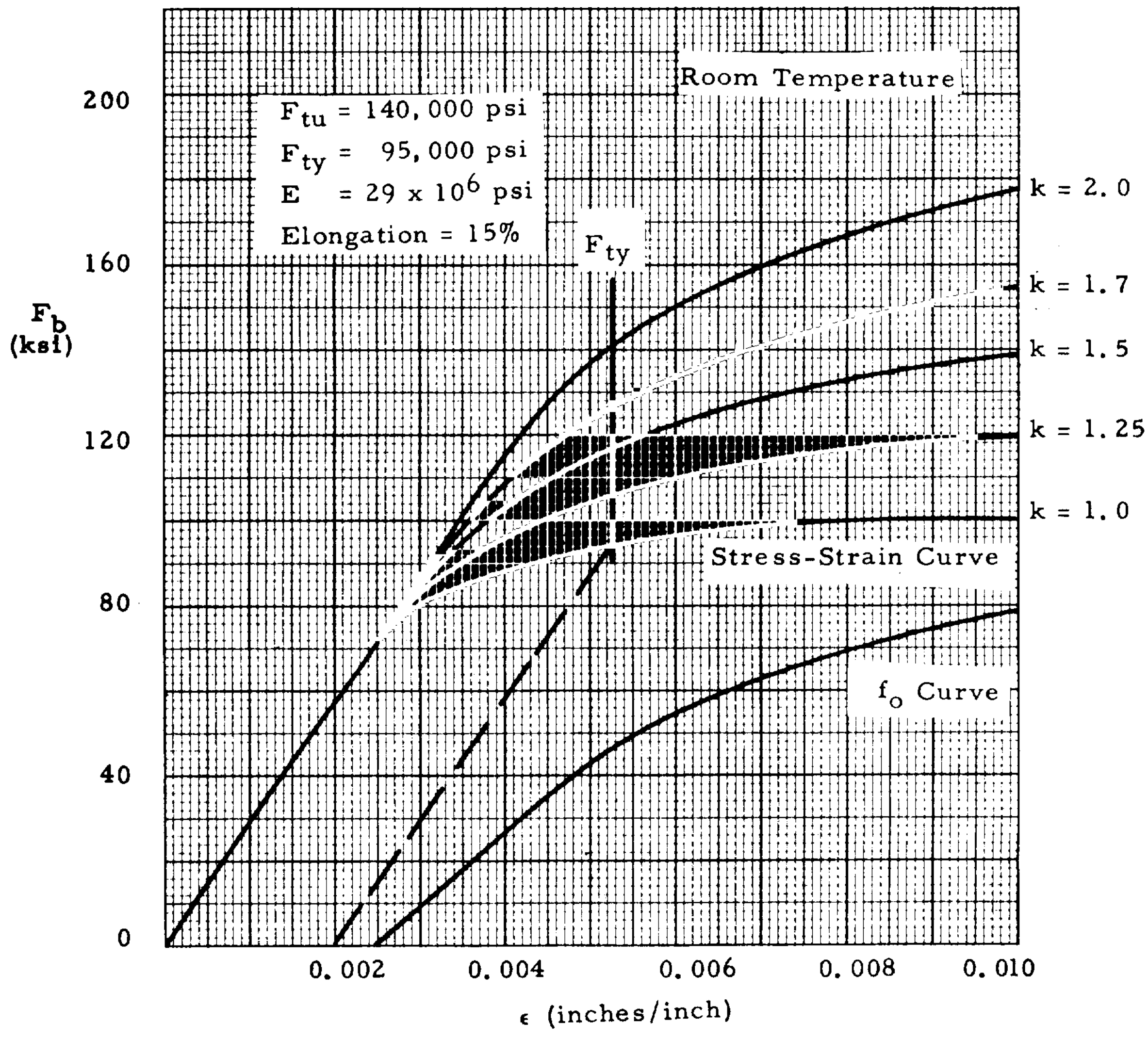


Fig. B4.5.6.3-1 Minimum Plastic Bending Curves A-286 Alloy, Heat Treated

Graph to be furnished when available

Graph to be furnished when available

B4.5.6.3 Corrosion Resistant Metals-Minimum Properties

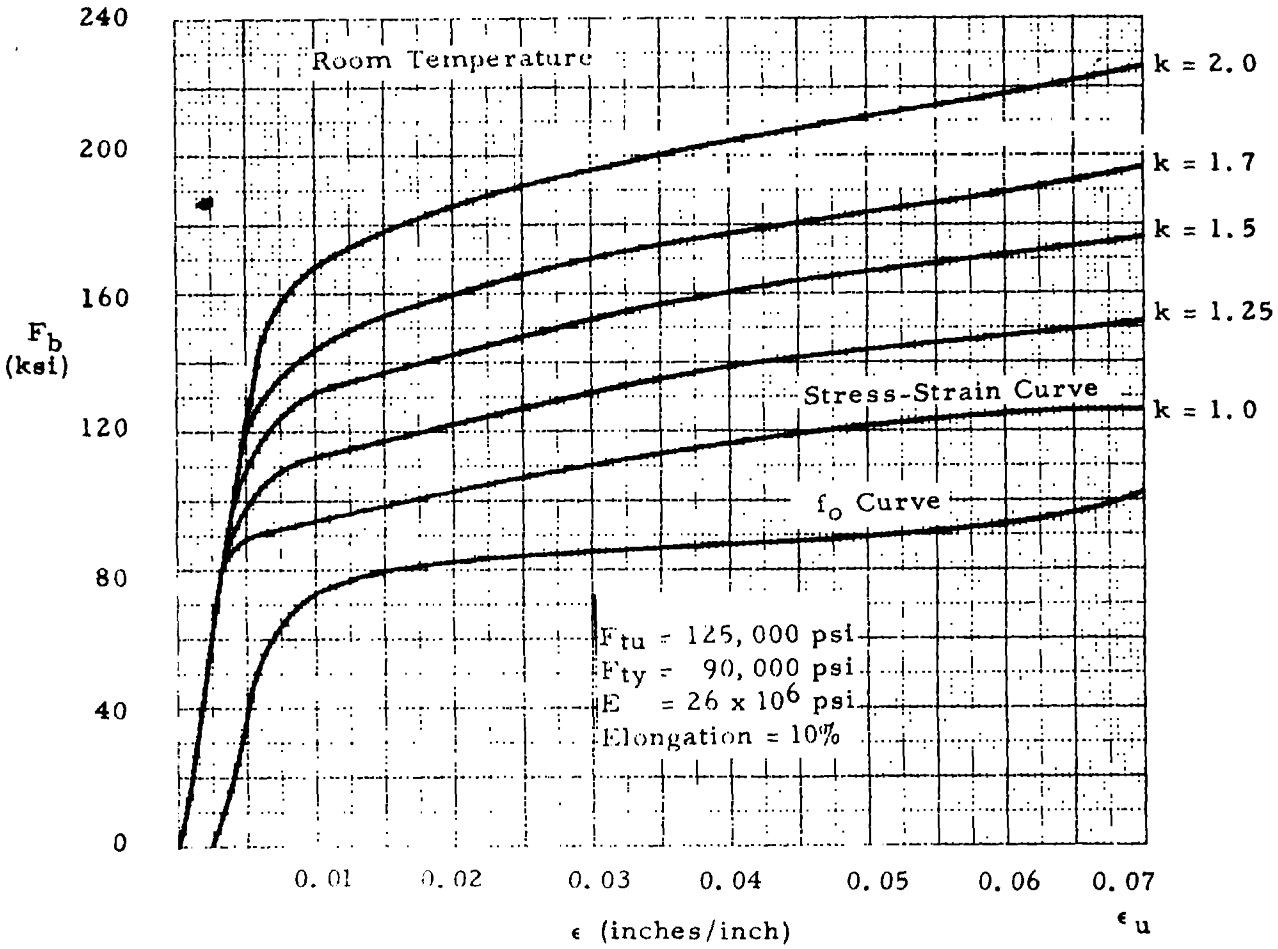


Fig. B4.5.6.3-4 Minimum Plastic Bending Curves for Age Hardened K-Monel Alloy Sheet

B4.5.6.3 Corrosion Resistant Metals-Minimum Properties

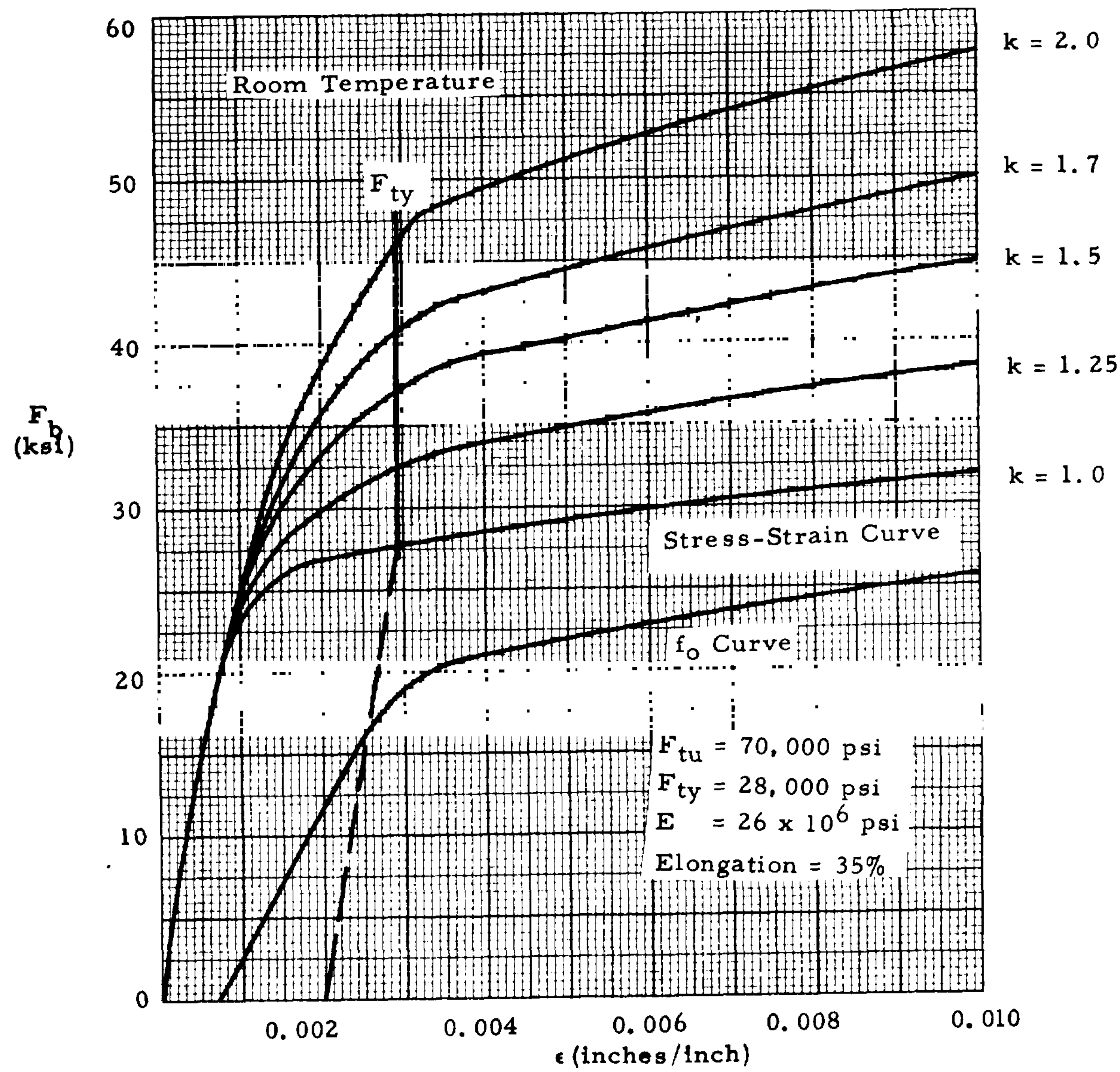


Fig. B4.5.6.3-5 Minimum Plastic Bending Curves Monel Alloy Cold Rolled, Annealed Sheet

B4.5.6.3 Corrosion Resistant Metals-Minimum Properties

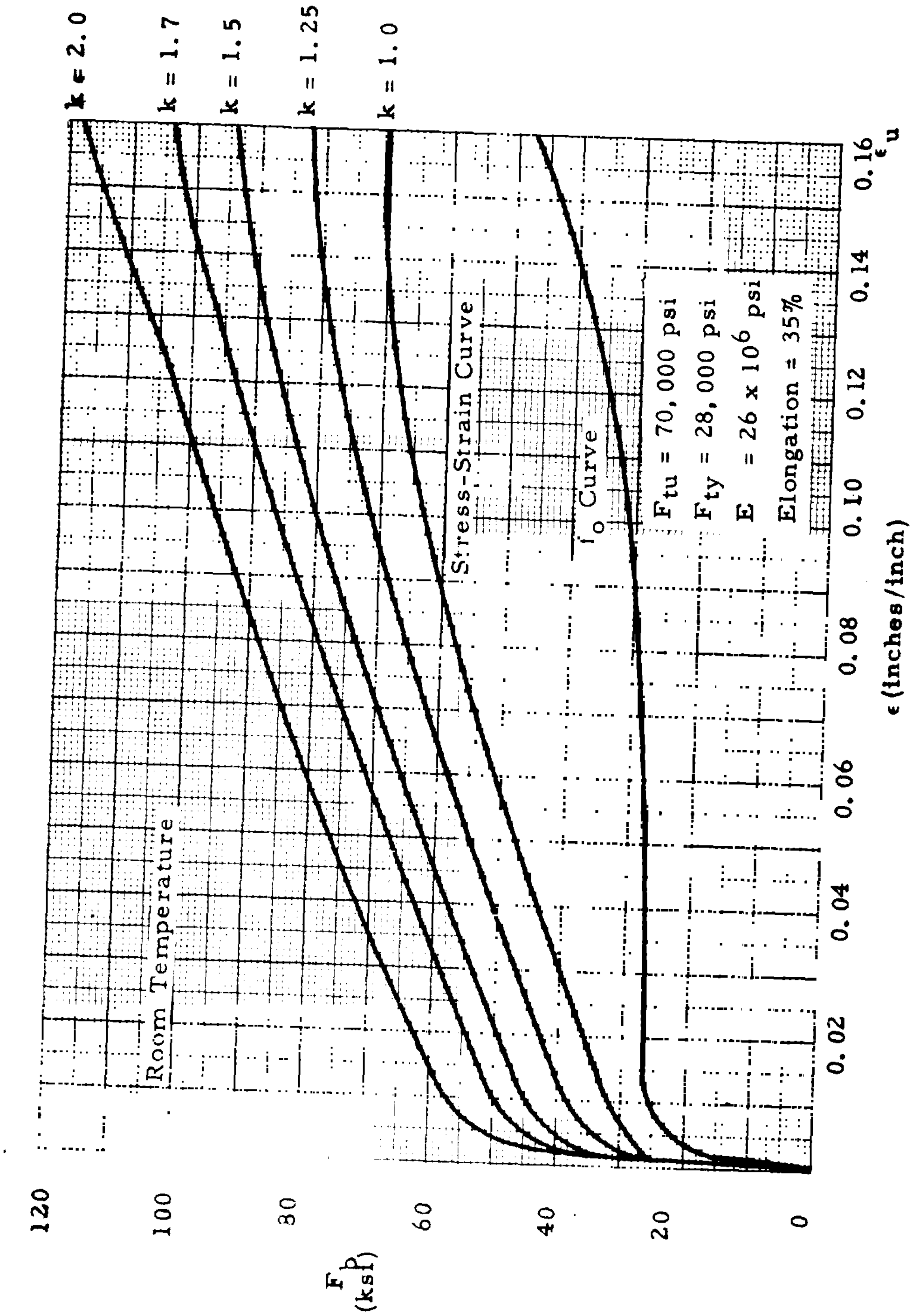


Fig. B4.5.6.3-6 Minimum Plastic Bending Curves Monel Alloy Cold Rolled, Annealed Sheet

B4.5.5.4 Titanium-Minimum Properties

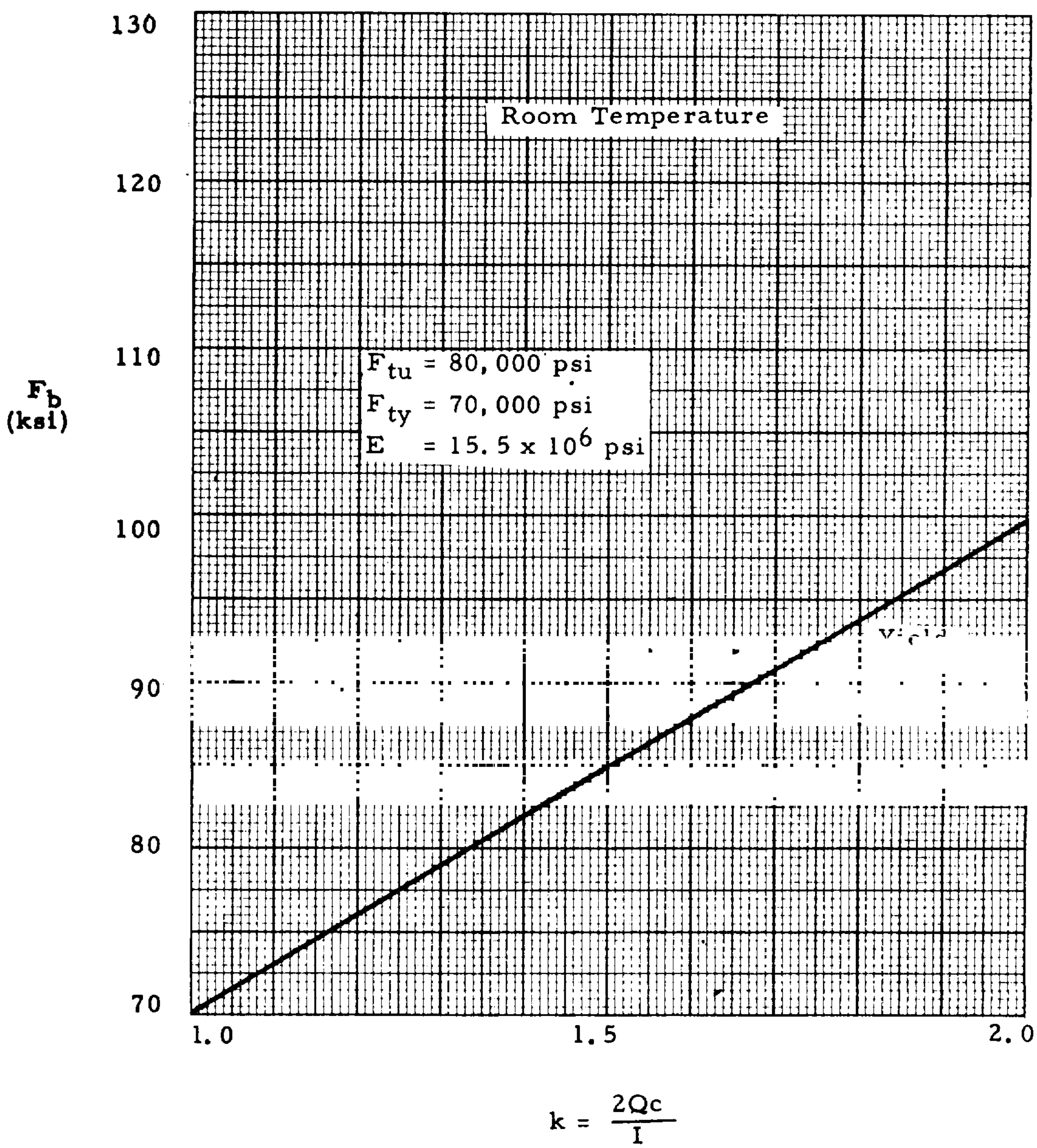


Fig. B4.5.5.4-1 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections Commercially Pure Annealed Titanium

B4.5.5.4 Titanium-Minimum Properties

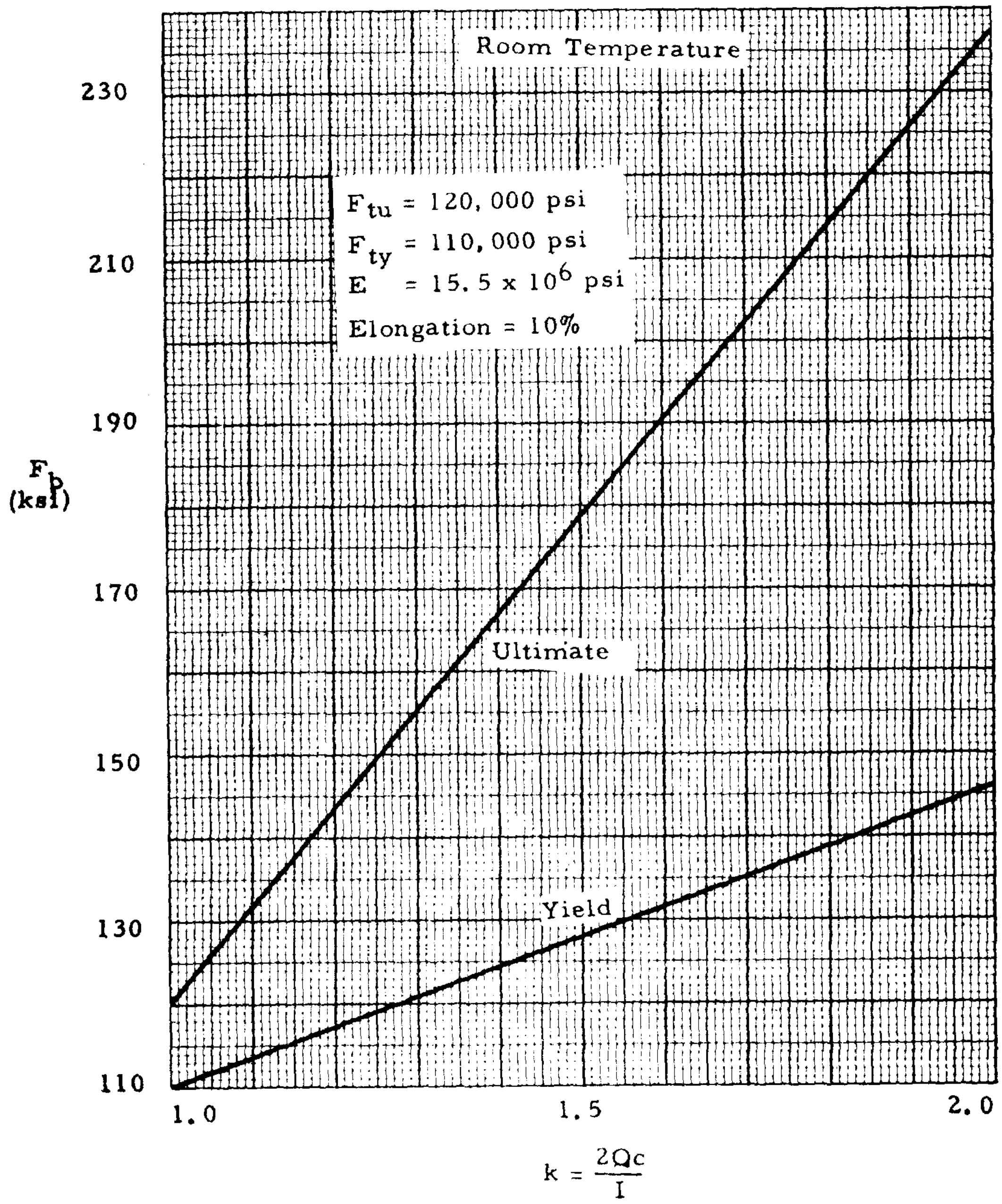


Fig. B4.5.5.4-2 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections Ti-8Mn Titanium Alloy

B4.5.5.4 Titanium-Minimum Properties

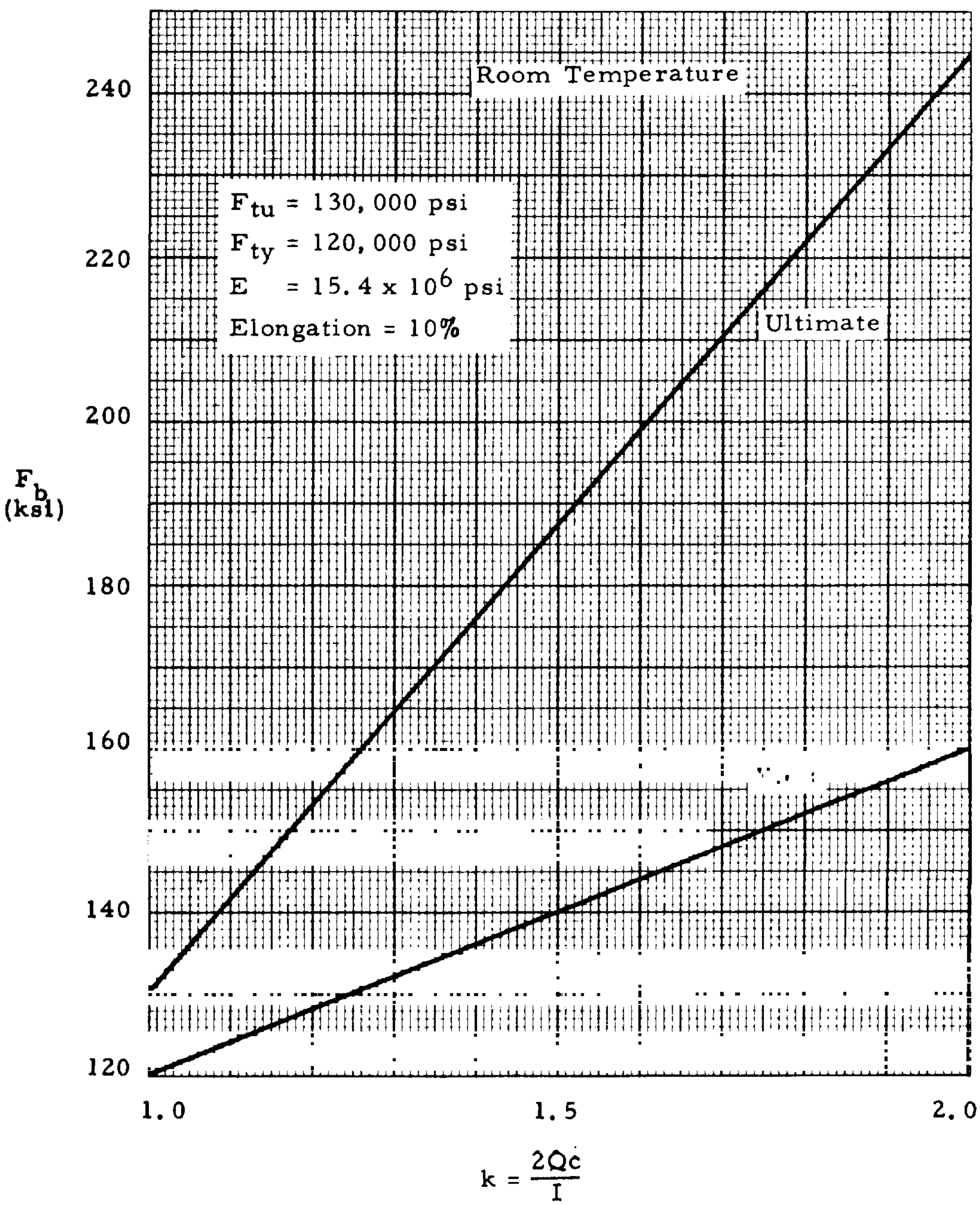


Fig. B4.5.5.4-3 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections Ti-6Al-4V Titanium Alloy

B4.5.5.4 Titanium-Minimum Properties

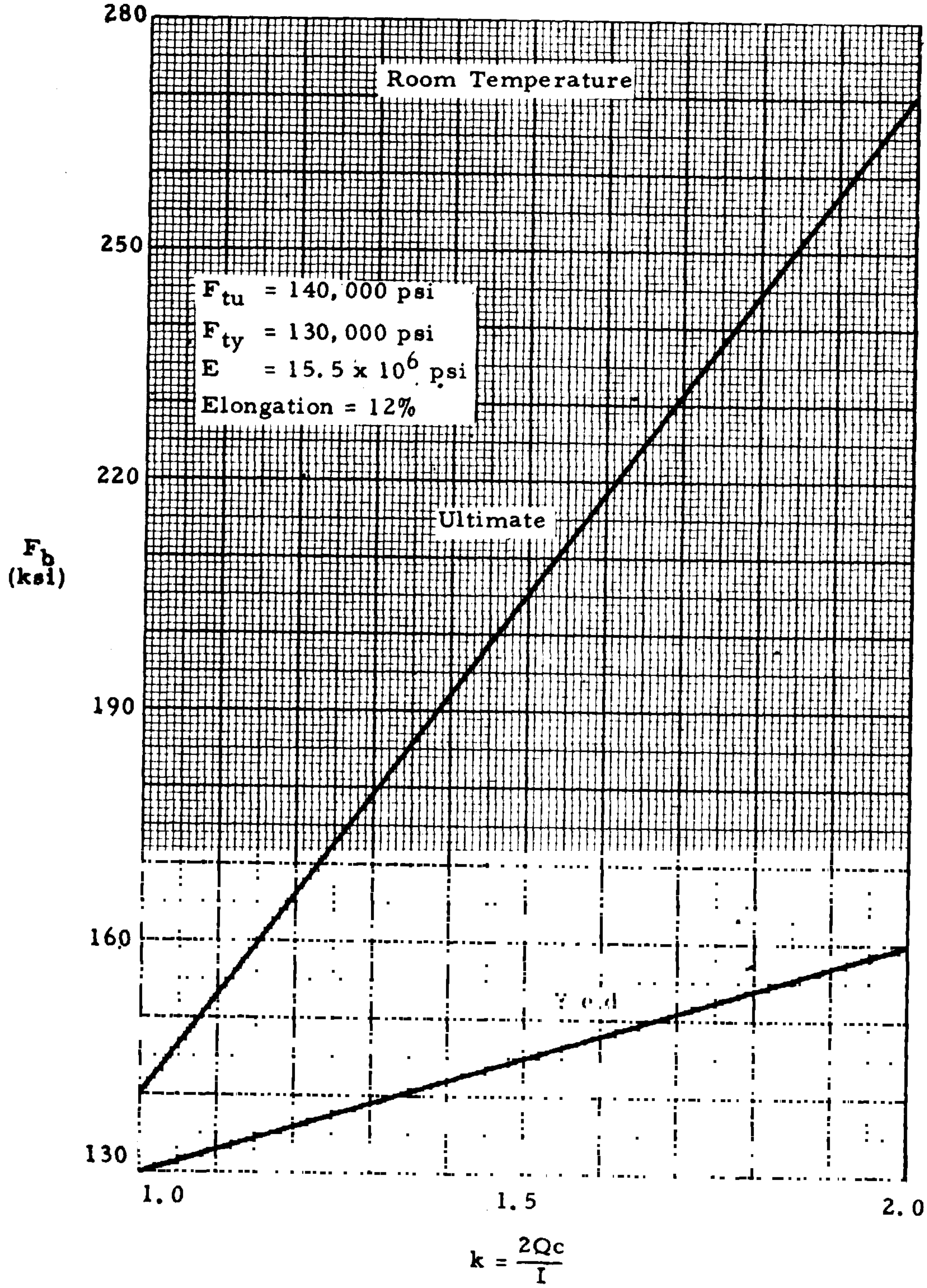


Fig. B4.5.5.4-4 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections Ti-4Mn-4Al Titanium Alloy

B4.5.6.4 **Titanium-Minimum Properties**

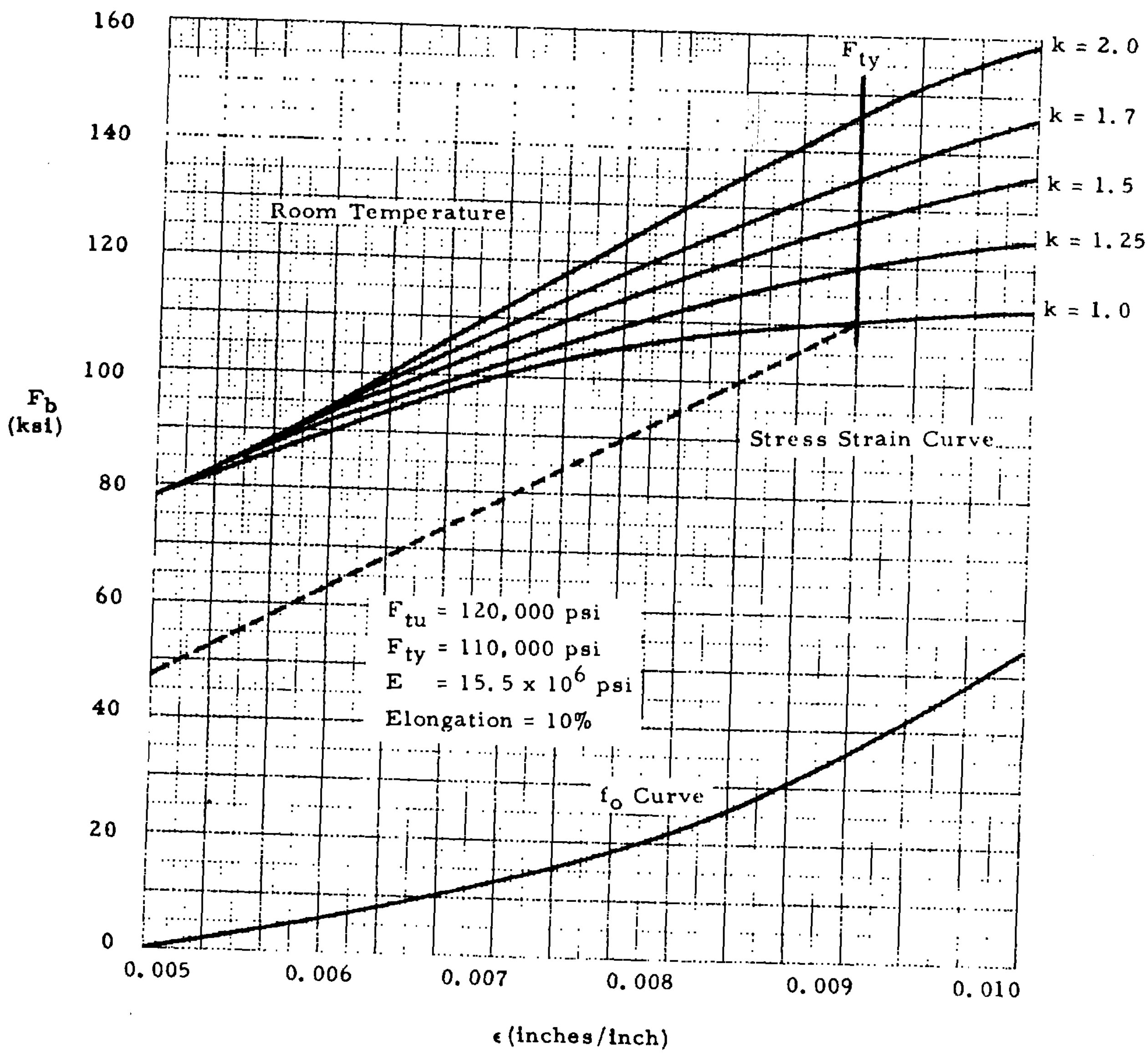


Fig. B4.5.6.4-1 Minimum Plastic Bending Curves for Ti-8Mn Titanium Alloy

B4.5.6.4 Titanium-Minimum Properties

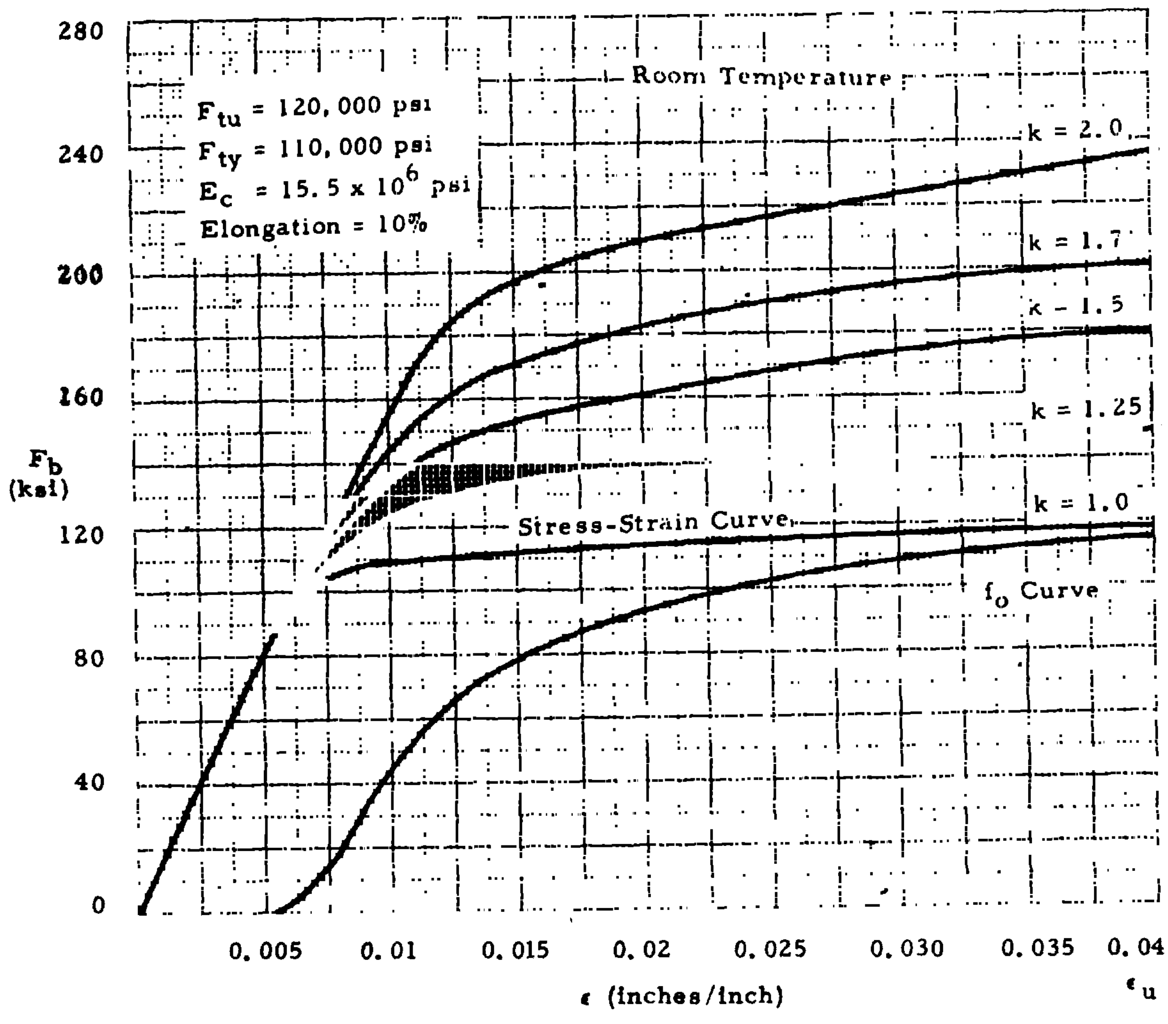


Fig. B4.5.6.4-2 Minimum Plastic Bending Curves Ti-8Mn Titanium Alloy

B4.5.6.4 Titanium-Minimum Properties

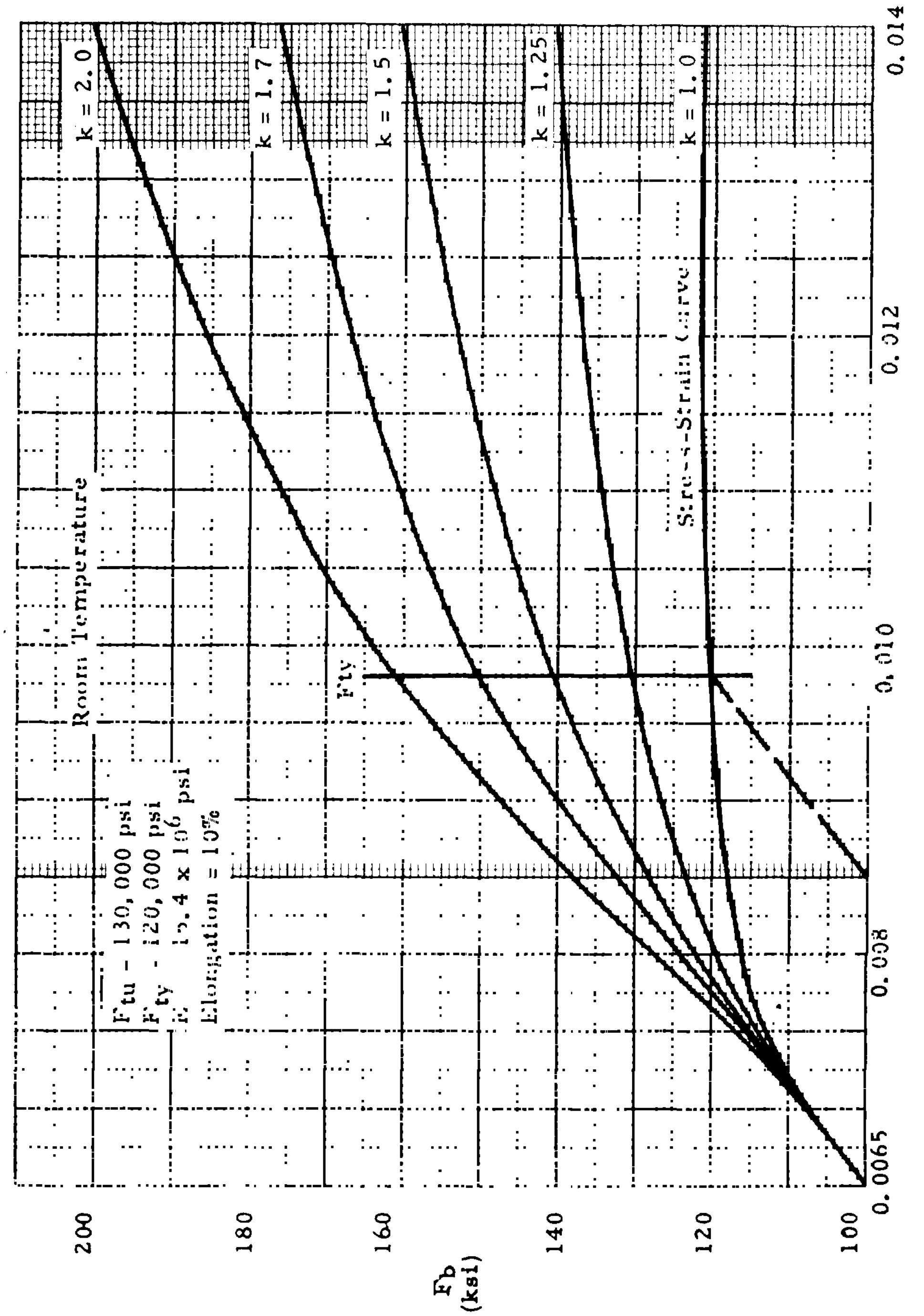


Fig. B4.5.6.4-3 Minimum Plastic Bending Curves for Ti-6Al-4V Titanium Alloy

B4.5.6.4 Titanium-Minimum Properties

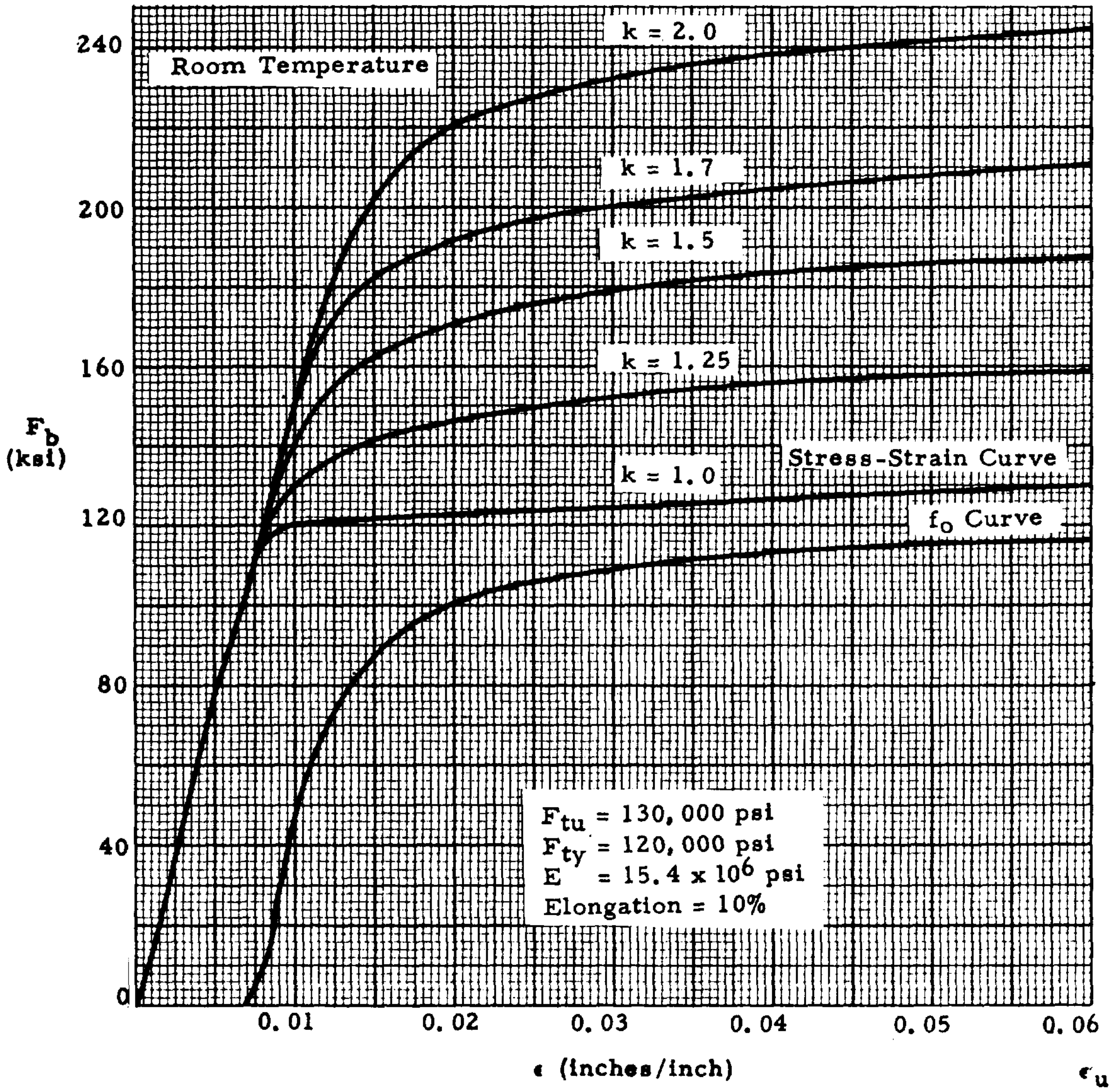


Fig. B4.5.6.4-4 Minimum Plastic Bending Curves Ti-6Al-4V Titanium Alloy

Graph to be furnished when avialable

B4.5.6.4 Titanium-Minimum Properties

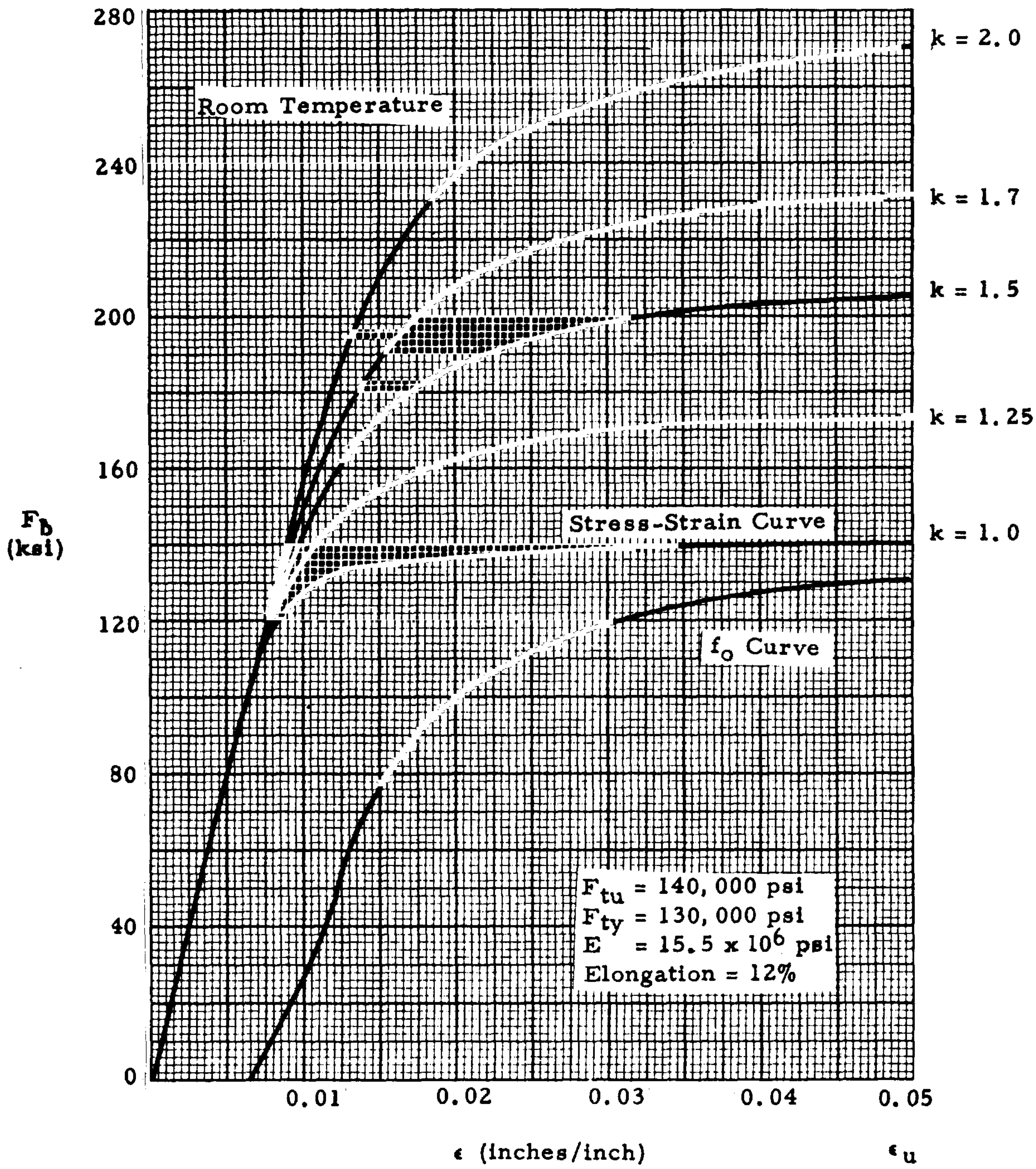


Fig. B4.5.6.4-6 Minimum Plastic Bending Curves for Ti-4Mn-4Al Titanium Alloy

Graph to be furnished when available

B4.5.5.5 Aluminum-Minimum Properties

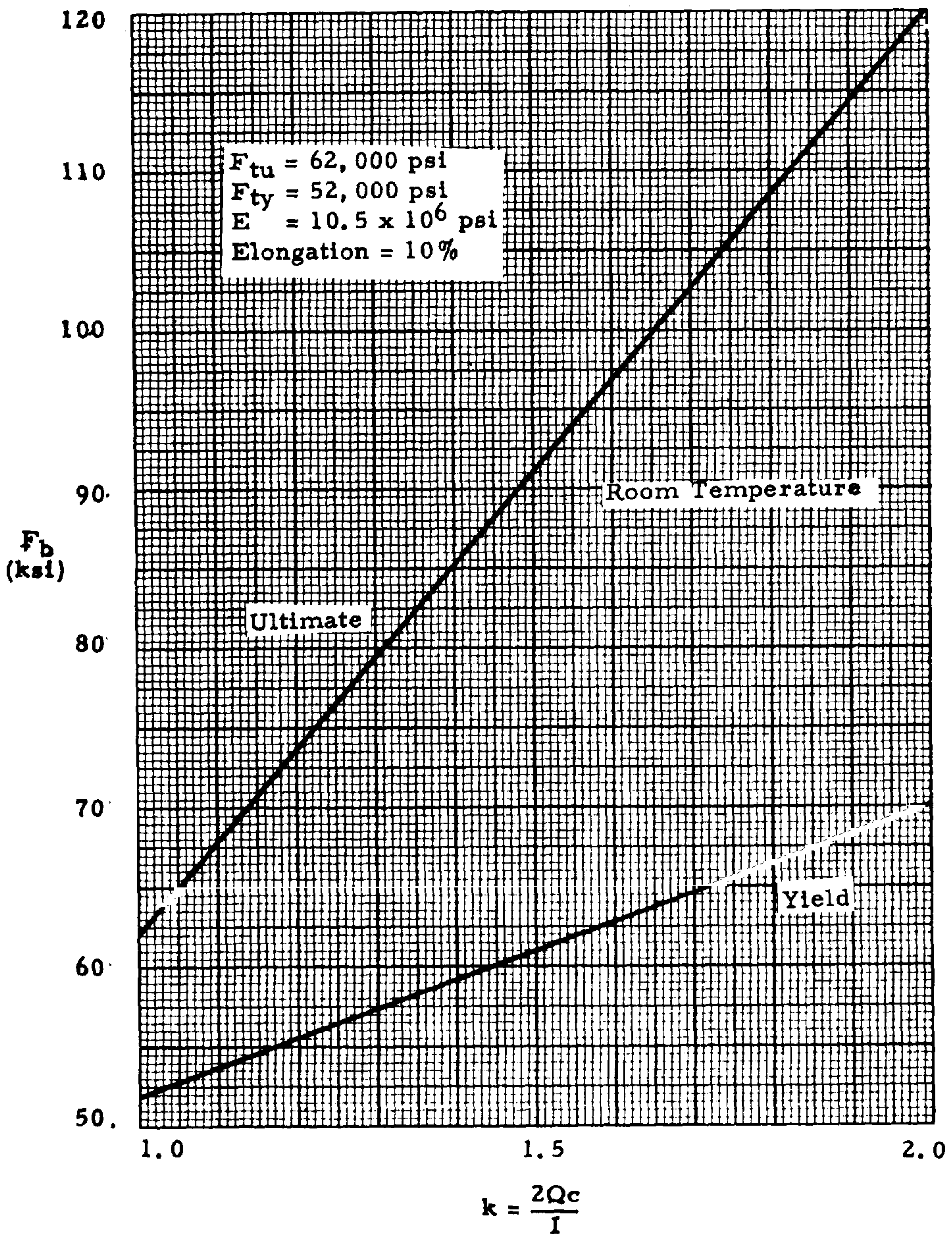


Fig. B4.5.5.5-2 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections 2014-T6 Aluminum Alloy Forgings, (Transverse) Thickness ≤ 4 In.

B4.5.5.5 Aluminum-Minimum Properties

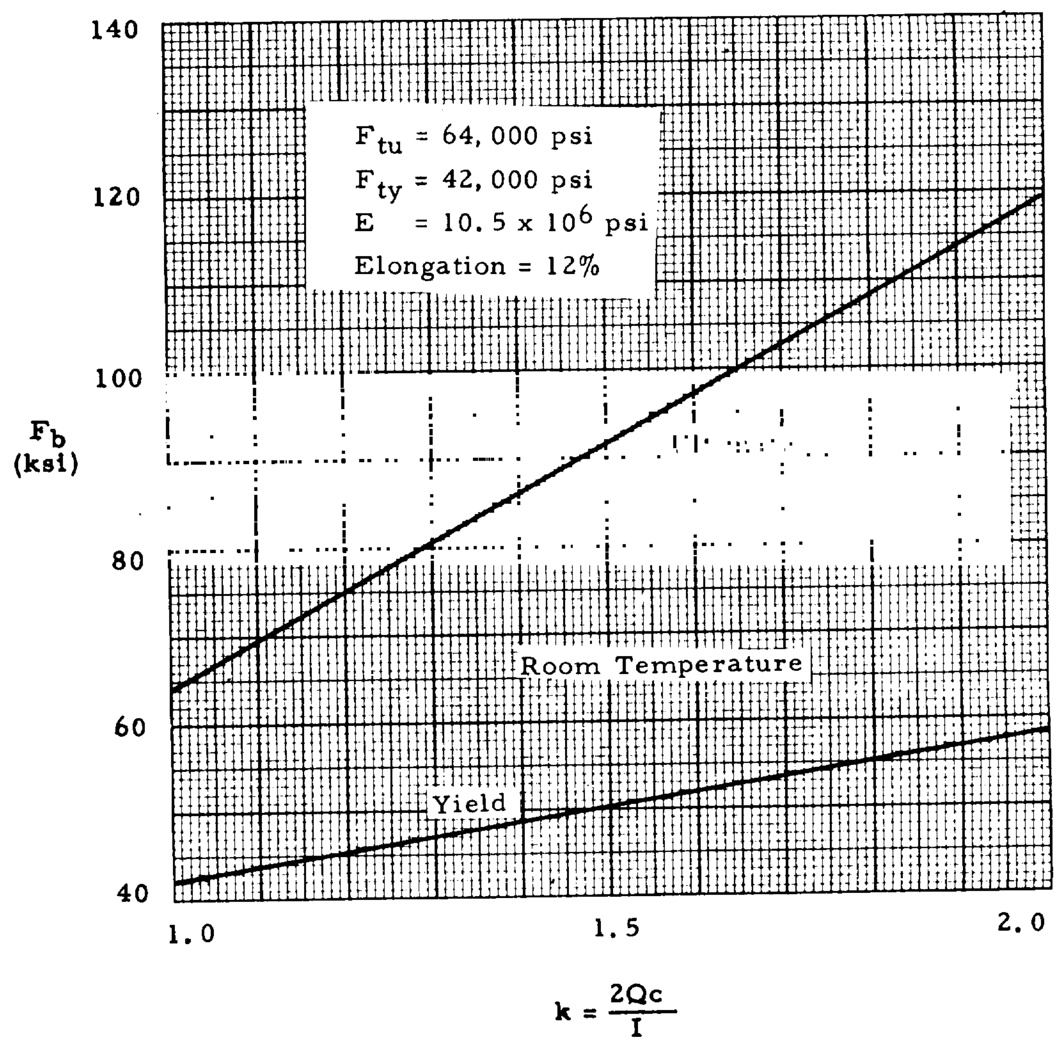


Fig. B4.5.5.5-3 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections 2024-T3 Alloy Sheet & Plate - Heat Treated. Thickness $\leq$ 0.250 In.

B4.5.5.5 Aluminum-Minimum Properties

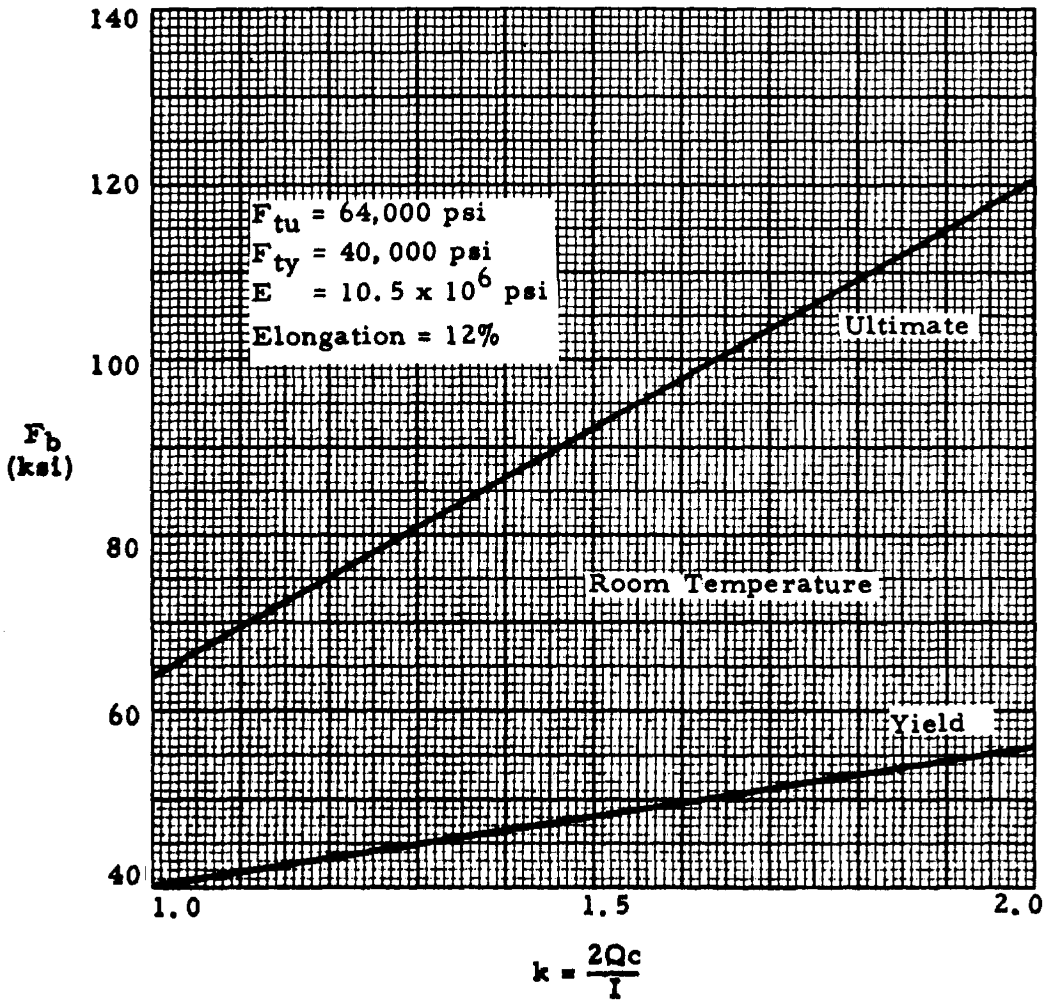


Fig. B4.5.5.5-4 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections 2024-T3 & T4 Aluminum Alloy Sheet & Plate - Heat Treated. Thickness $\leq$ 0.50 In.

B4.5.5.5 Aluminum-Minimum Properties

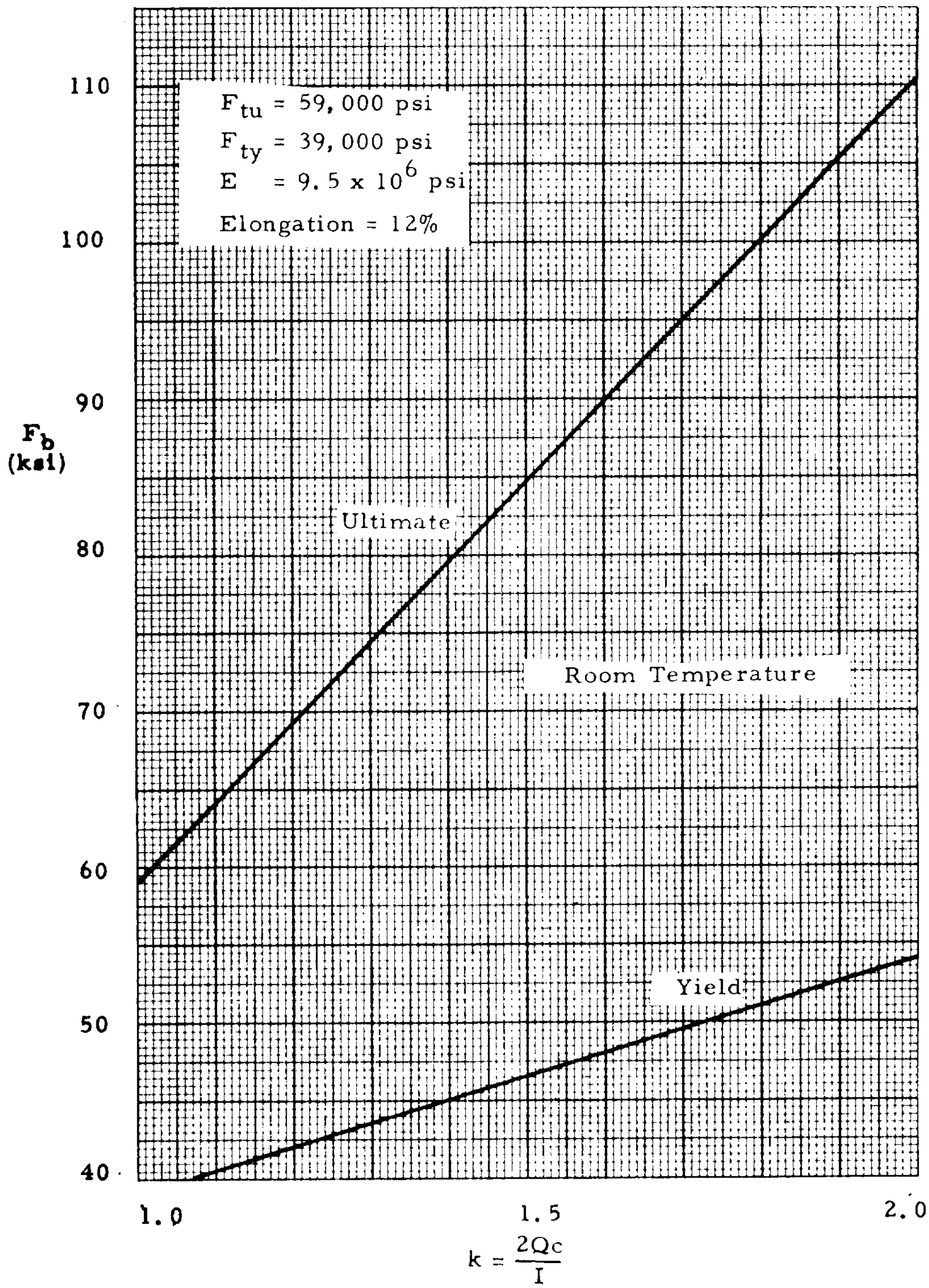


Fig. B4.5.5.5-5 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections 2024-T3 Aluminum Alloy Clad Sheet & Plate - Heat Treated. Thickness 0.010 to 0.062 In.

B4.5.5.5 Aluminum-Minimum Properties

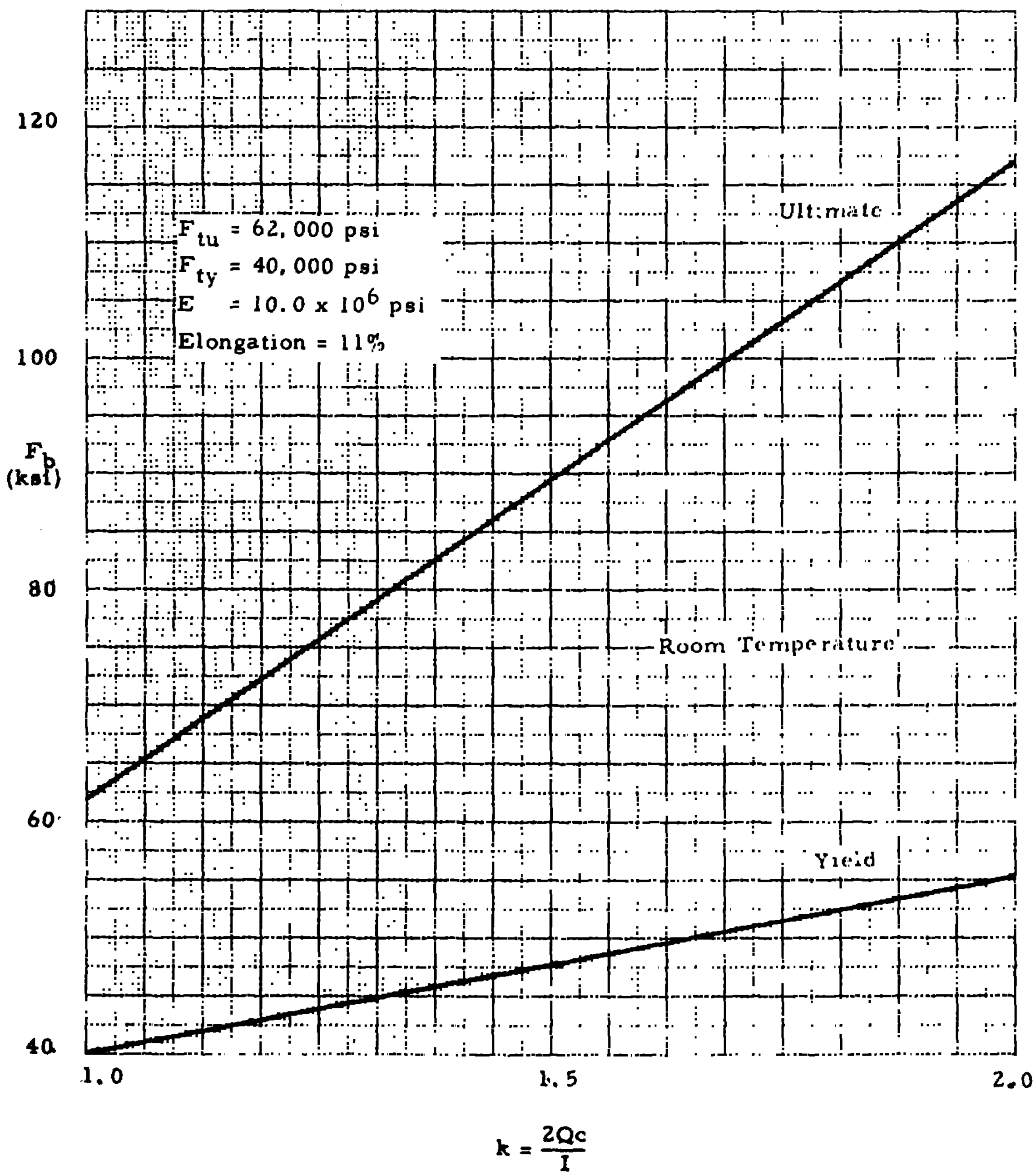


Fig. B4.5.5.5-6 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections 2024-T4 Aluminum Alloy Clad Sheet & Plate - Heat Treated. Thickness 0.25 to 0.50 In.

B4.5.5.5 **Aluminum-Minimum Properties**

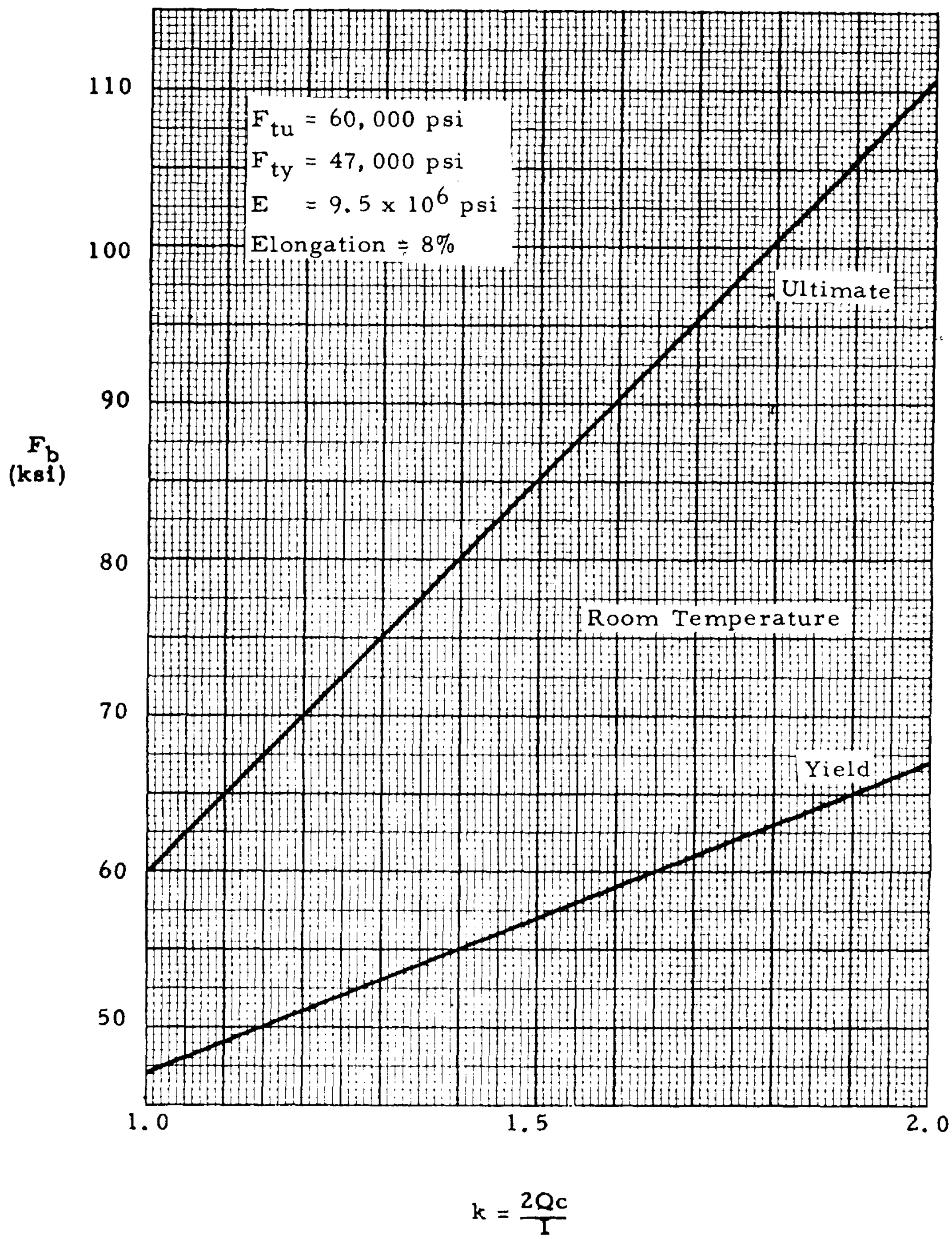


Fig. B4.5.5.5-7 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections 2024-T6 Aluminum Alloy Clad Sheet - Heat Treated & Aged. Thickness < 0.064 In.

B4.5.5.5 Aluminum-Minimum Properties

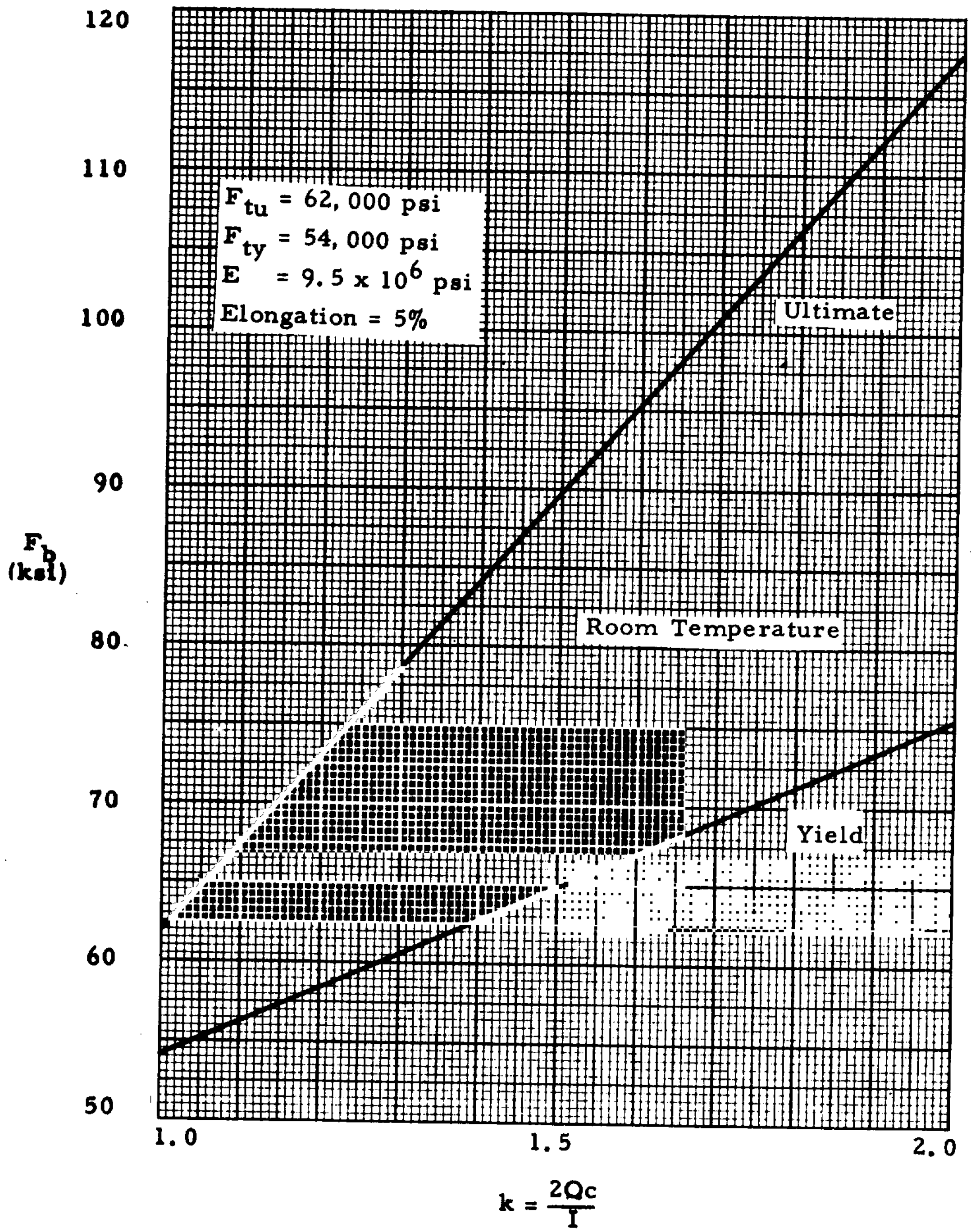


Fig. B4.5.5.5-8 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections 2024-T81 Aluminim Alloy Clad Sheet - Heat Treated, Cold Worked & Aged Thickness < 0.064 In.

B4.5.5.5 Aluminum-Minimum Properties

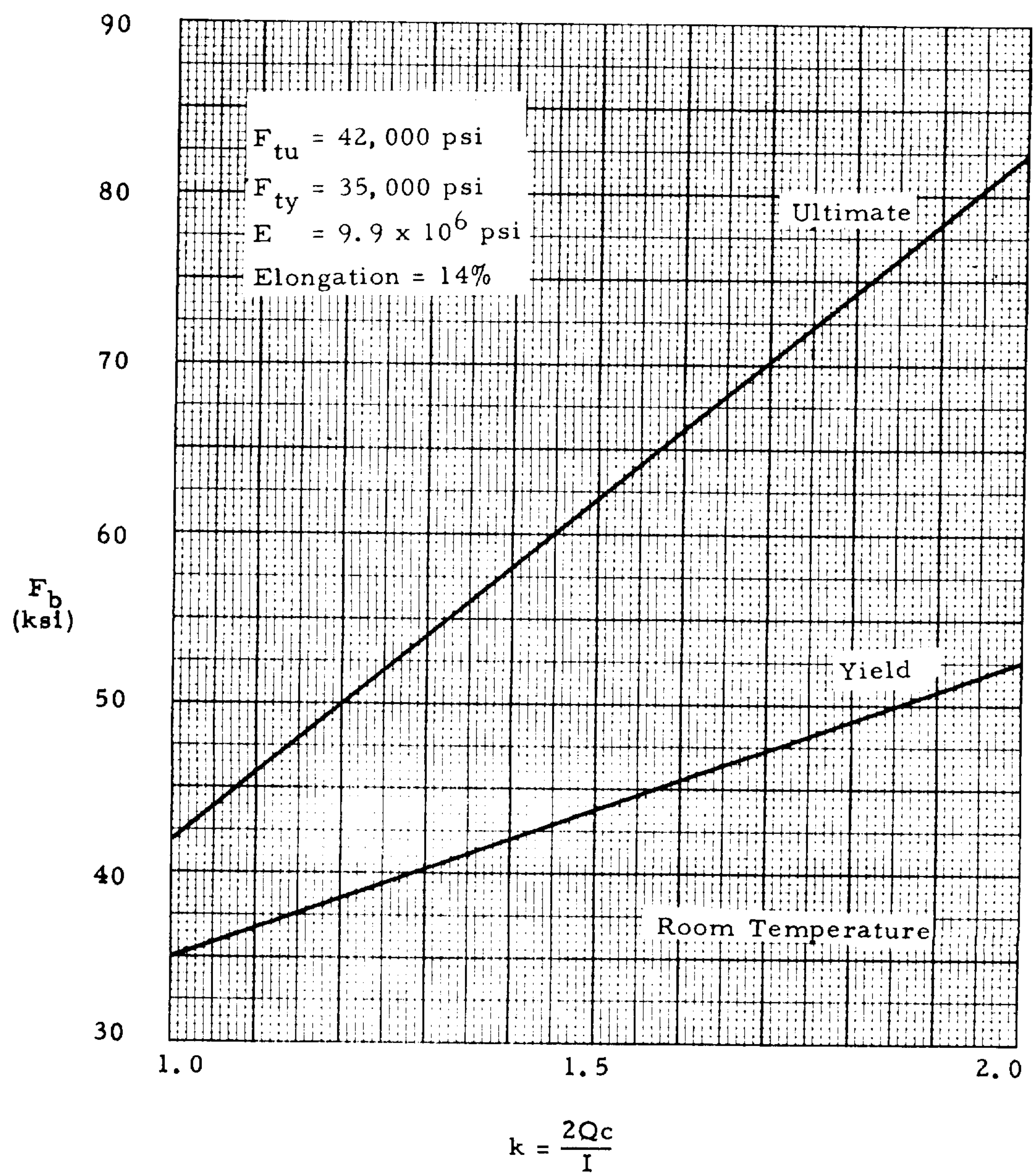


Fig. B4.5.5.5-9 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections 6061-T6 Aluminum Alloy Sheet - Heat Treated & Aged. Thickness ≥ 0.020 In.

B4.5.5.5 Aluminum-Minimum Properties

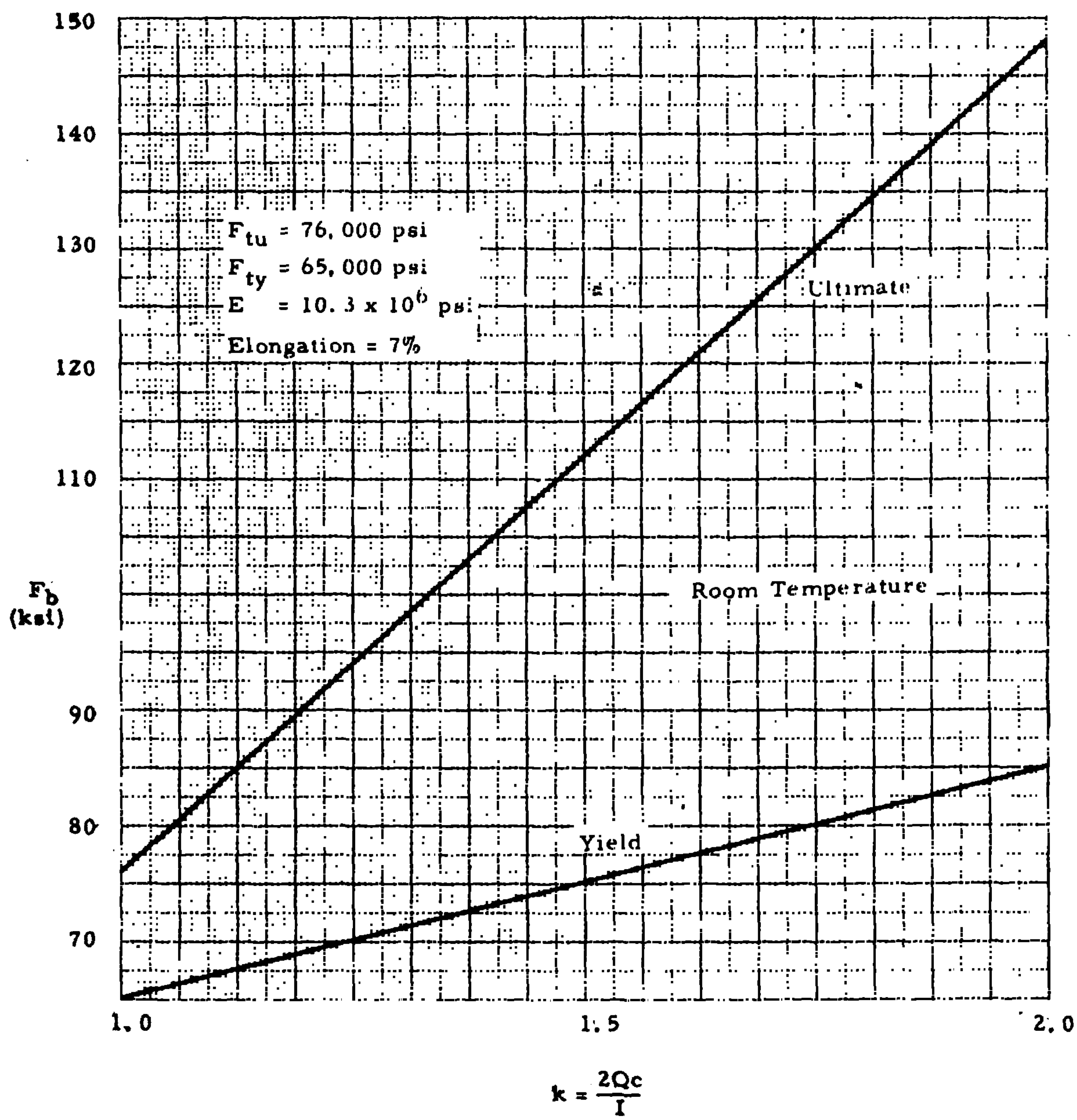


Fig. B4.5.5.5-10 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections 7075-T6 Aluminum Alloy Bare Sheet & Plate. Thickness ≤ .039 In.

B4.5.5.5 Aluminum-Minimum Properties

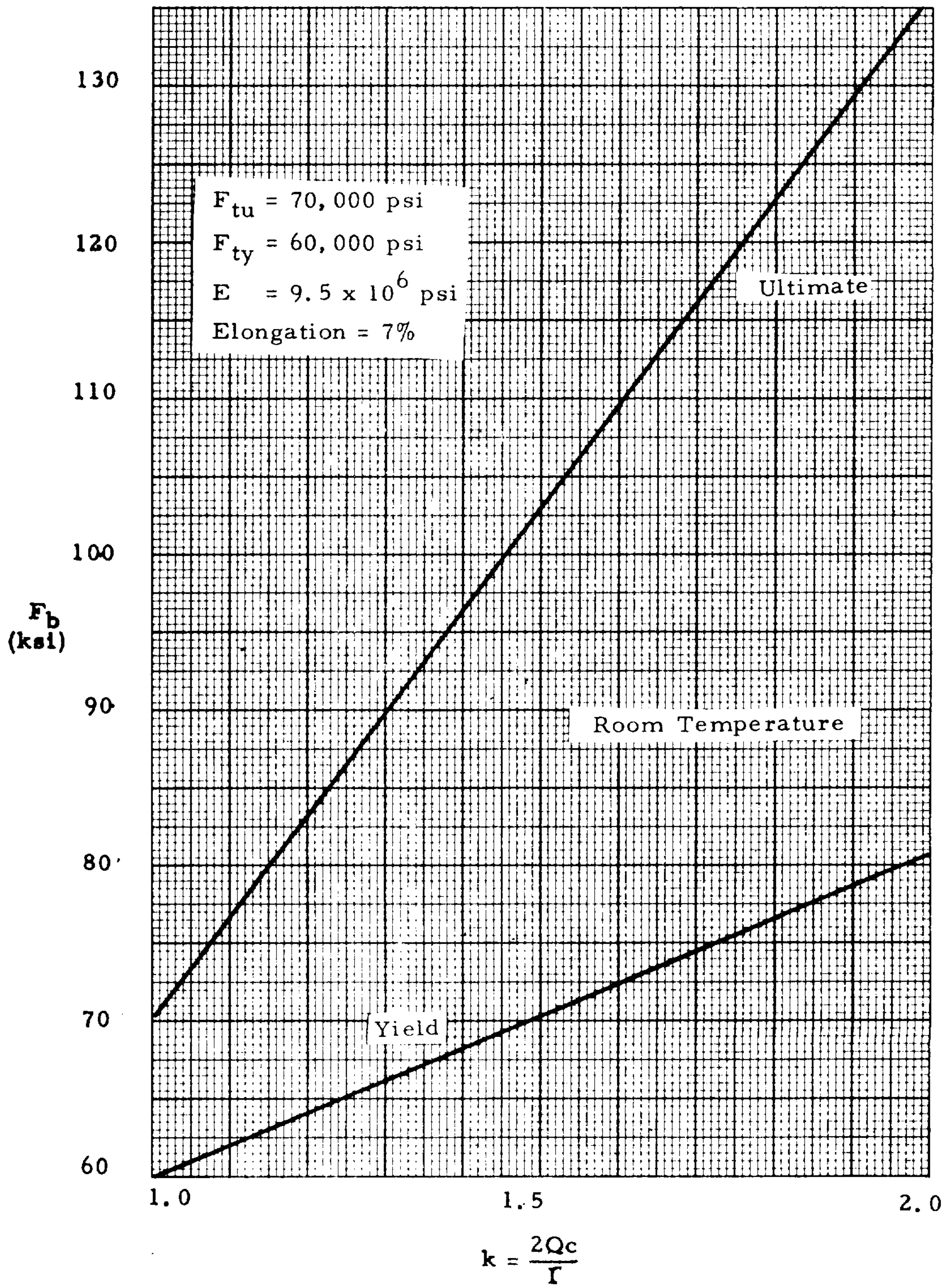


Fig. B4.5.5.5-11 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections 7075-T6 Aluminum Alloy Clad Sheet & Plate. Thickness ≤ .039 In.

B4.5.5.5 Aluminum-Minimum Properties

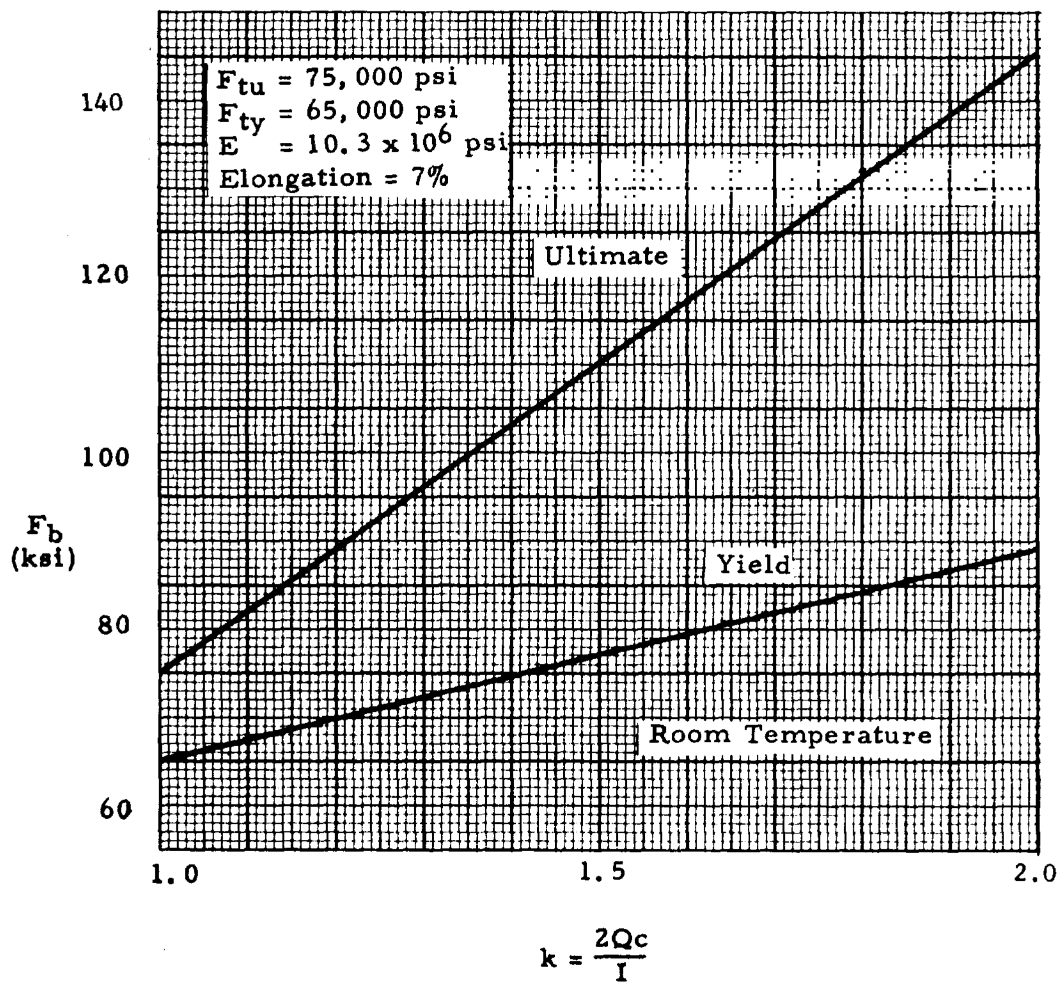


Fig. B4.5.5.5-12 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections 7075-T6 Aluminum Alloy Extrusions. Thickness ≤ 0.25 in.

B4.5.5.5 Aluminum-Minimum Properties

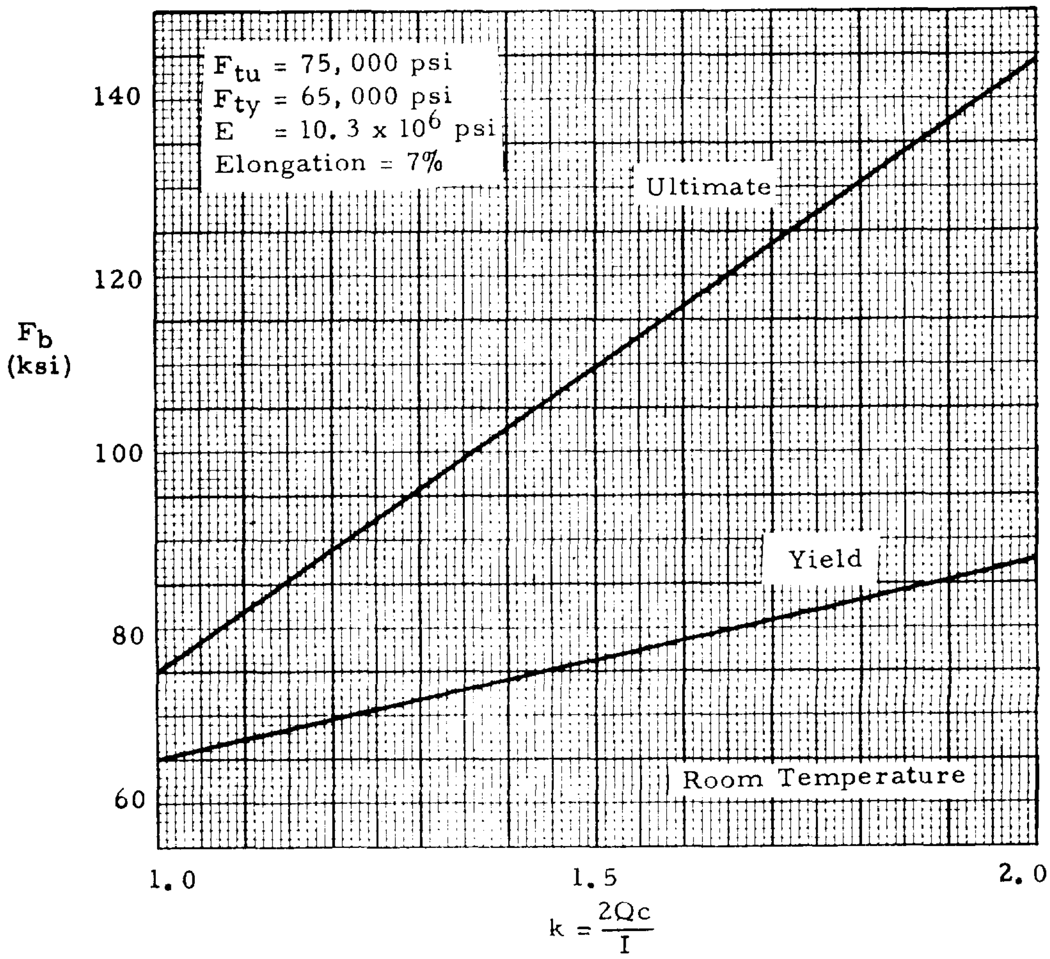


Fig. B4.5.5.5-13 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections 7075-T6 Aluminum Alloy Die Forgings. Thickness ≤ 3 in.

B4.5.5.5 Aluminum-Minimum Properties

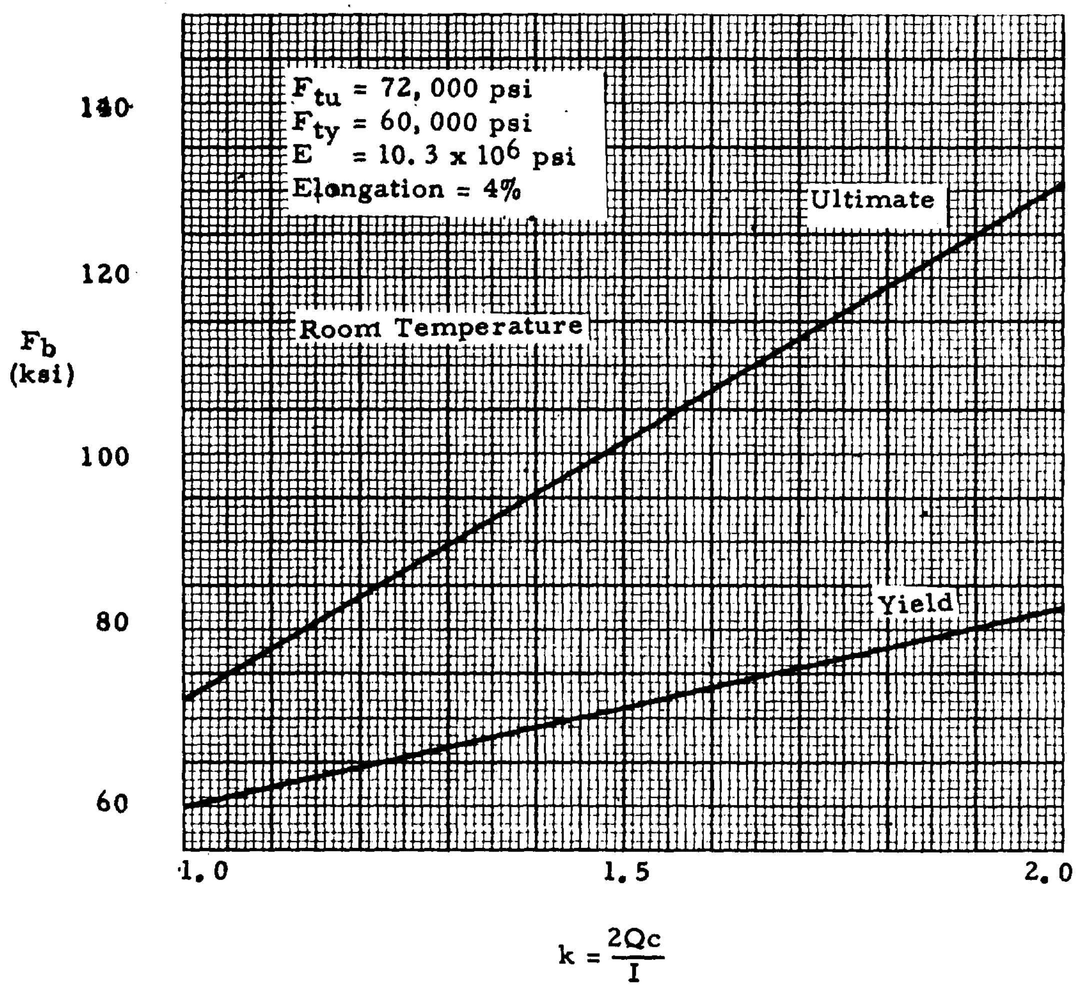


Fig. B4.5.5.5-14 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections 7075-T6 Aluminum Alloy Hand Forgings Area $\leq$ 16 $In.^2$

B4.5.5.5 Aluminum-Minimum Properties

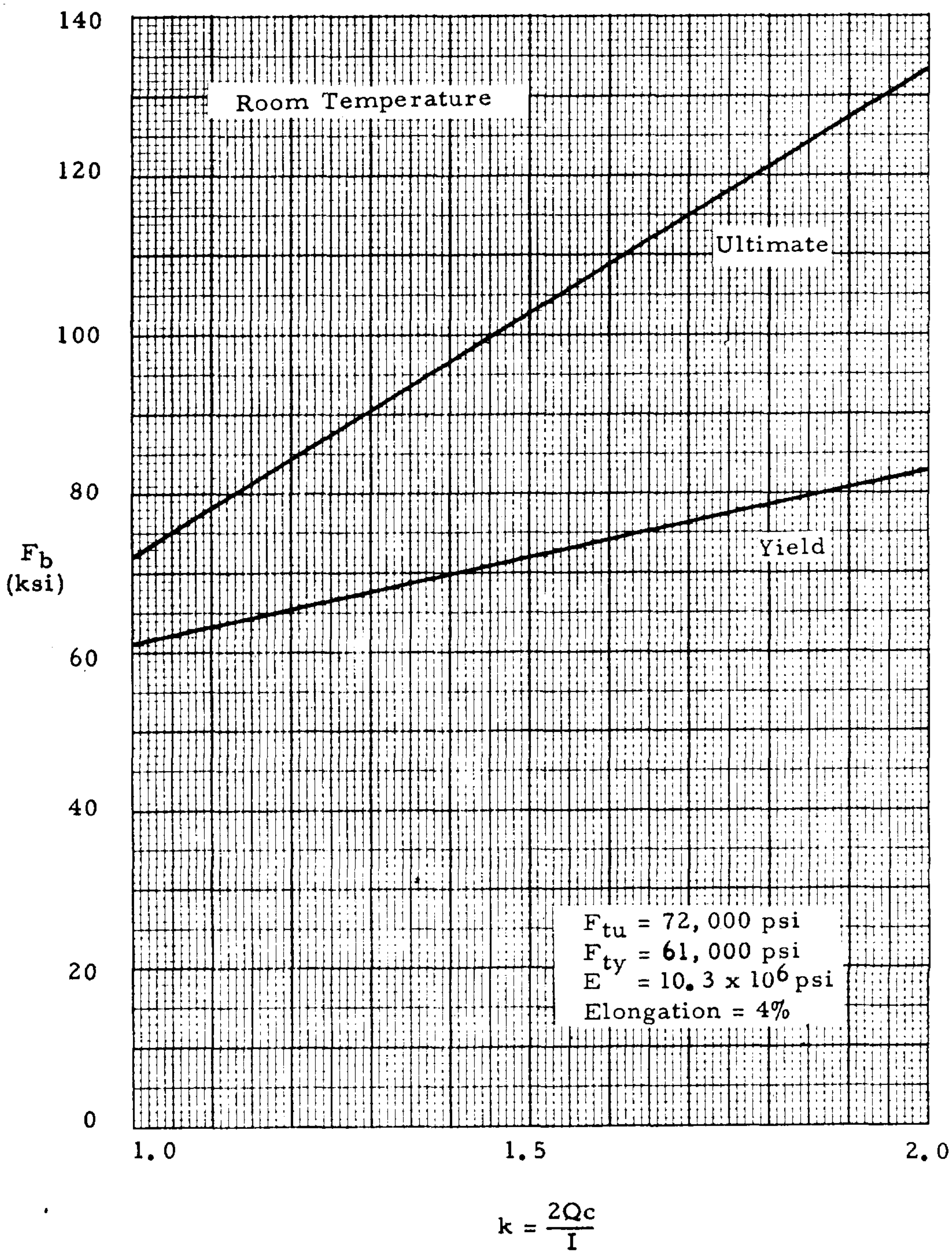


Fig. B4.5.5.5-15 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections 7079-T6 Aluminum Alloy Die Forgings (Transverse). Thickness ≤ 6.0 In.

B4.5.5.5 Aluminum-Minimum Properties

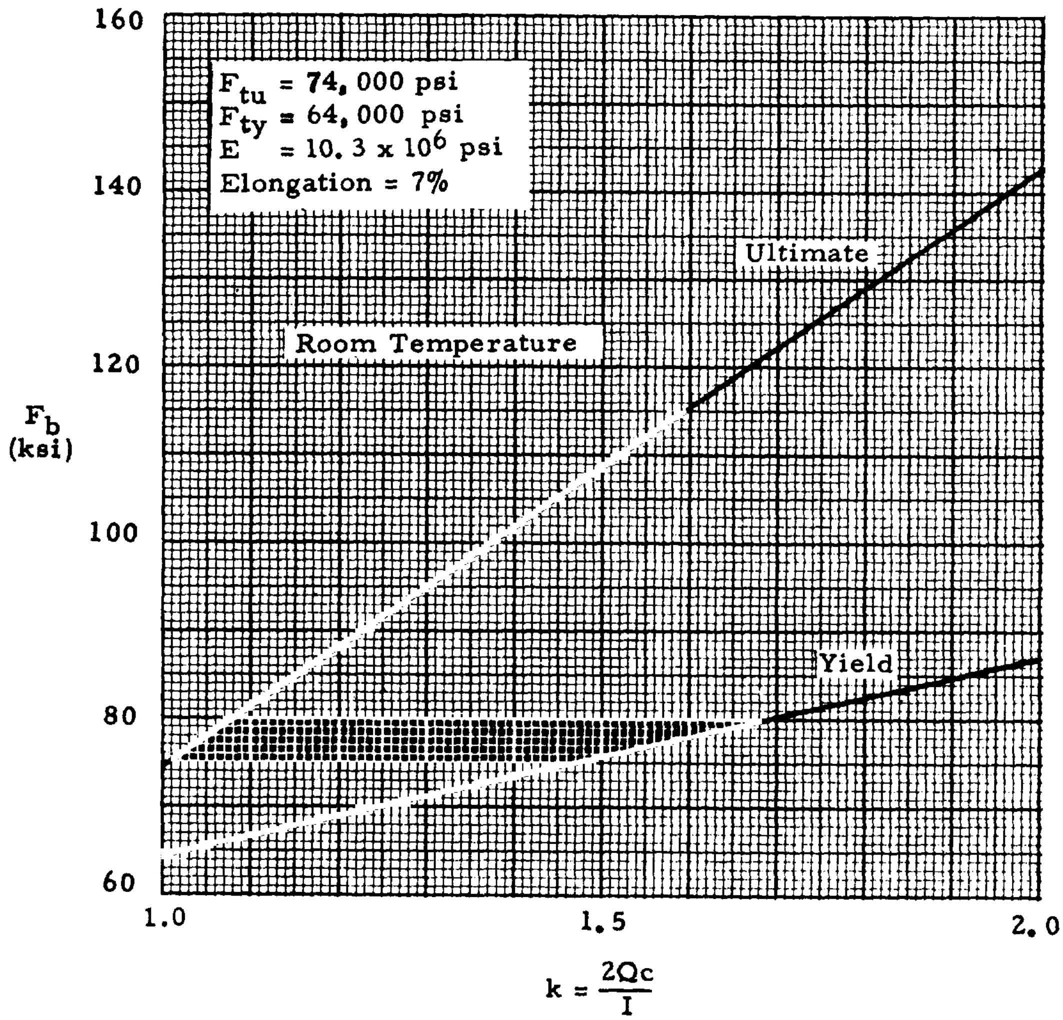


Fig. B4.5.5.5-16 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections 7079-T6 Aluminum Alloy Die Forgings (Longitudinal) Thickness ≤ 6.0 in.

B4.5.5.5 Aluminum-Minimum Properties

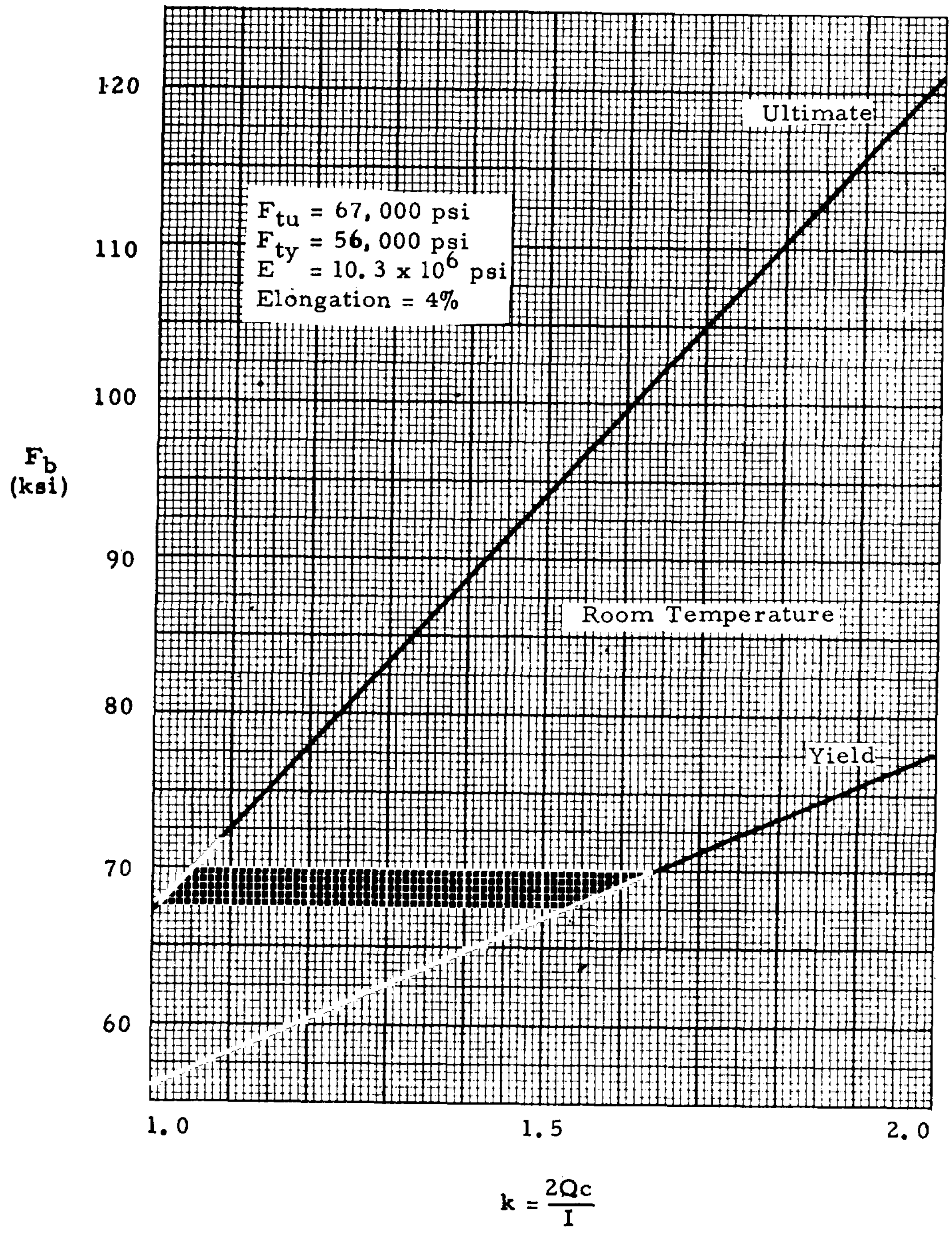


Fig. B4.5.5.5-17 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections 7079-T6 Aluminum Alloy Hand Forgings (Short Transverse) Thickness $\leq$ 6.0 In.

B4.5.5.5 Aluminum-Minimum Properties

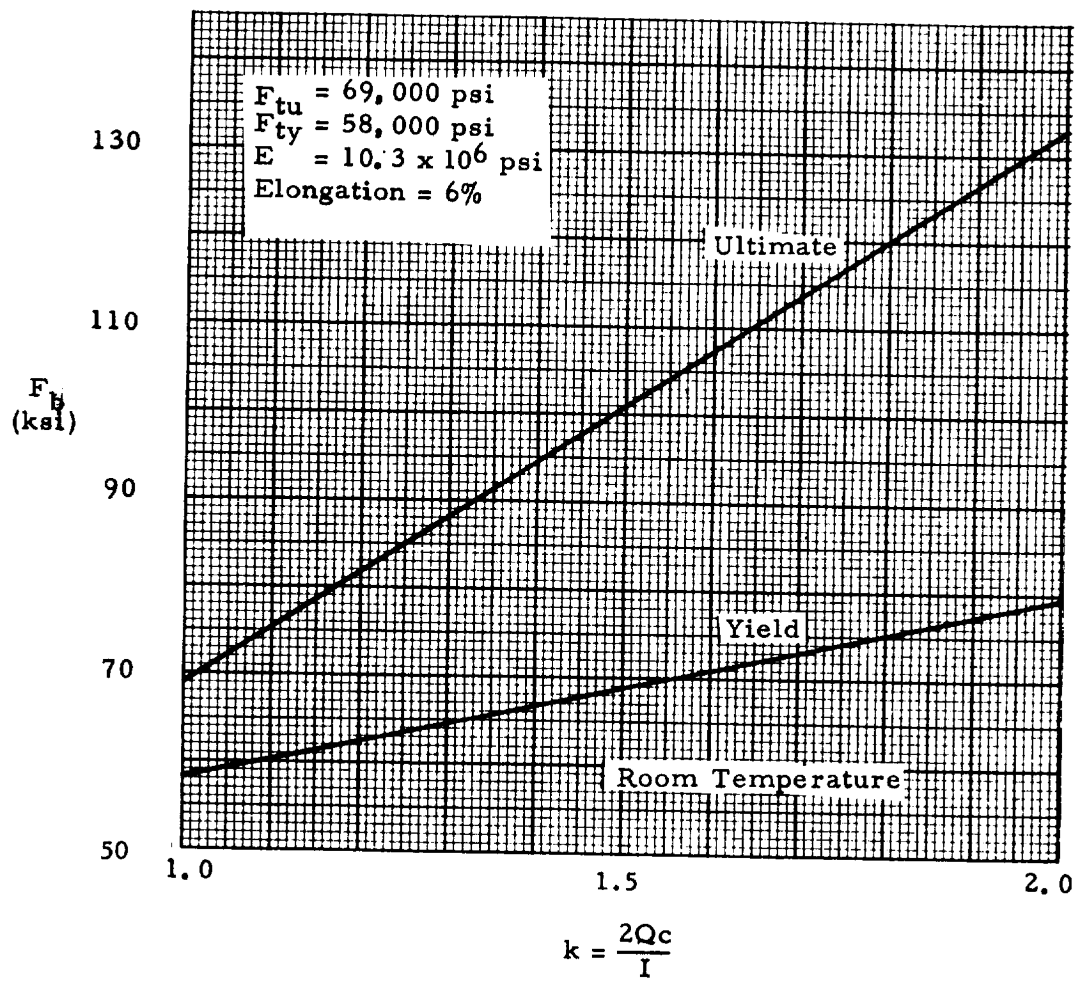


Fig. B4.5.5.5-18 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections 7079-T6 Aluminum Alloy Hand Forgings-(Long Transverse) Thickness ≤ 6 in.

B4.5.5.5 Aluminum - Minimum Properties

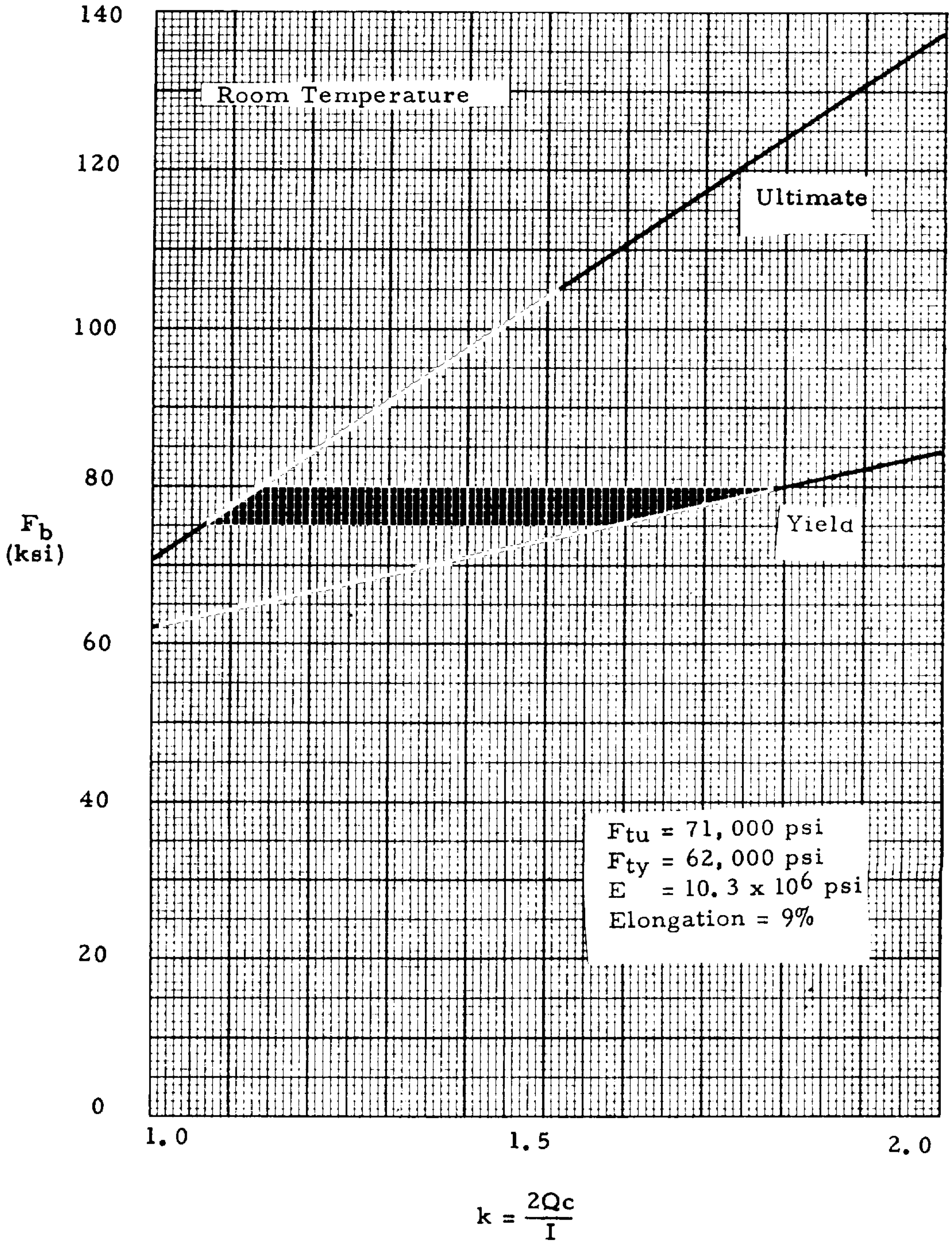


Fig. B4.5.5.5-19 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections 7079-T6 Aluminum Alloy Hand Forgings - (Longitudinal). Thickness ≤ 6 In.

B4.5.6.5 Aluminum-Minimum Properties

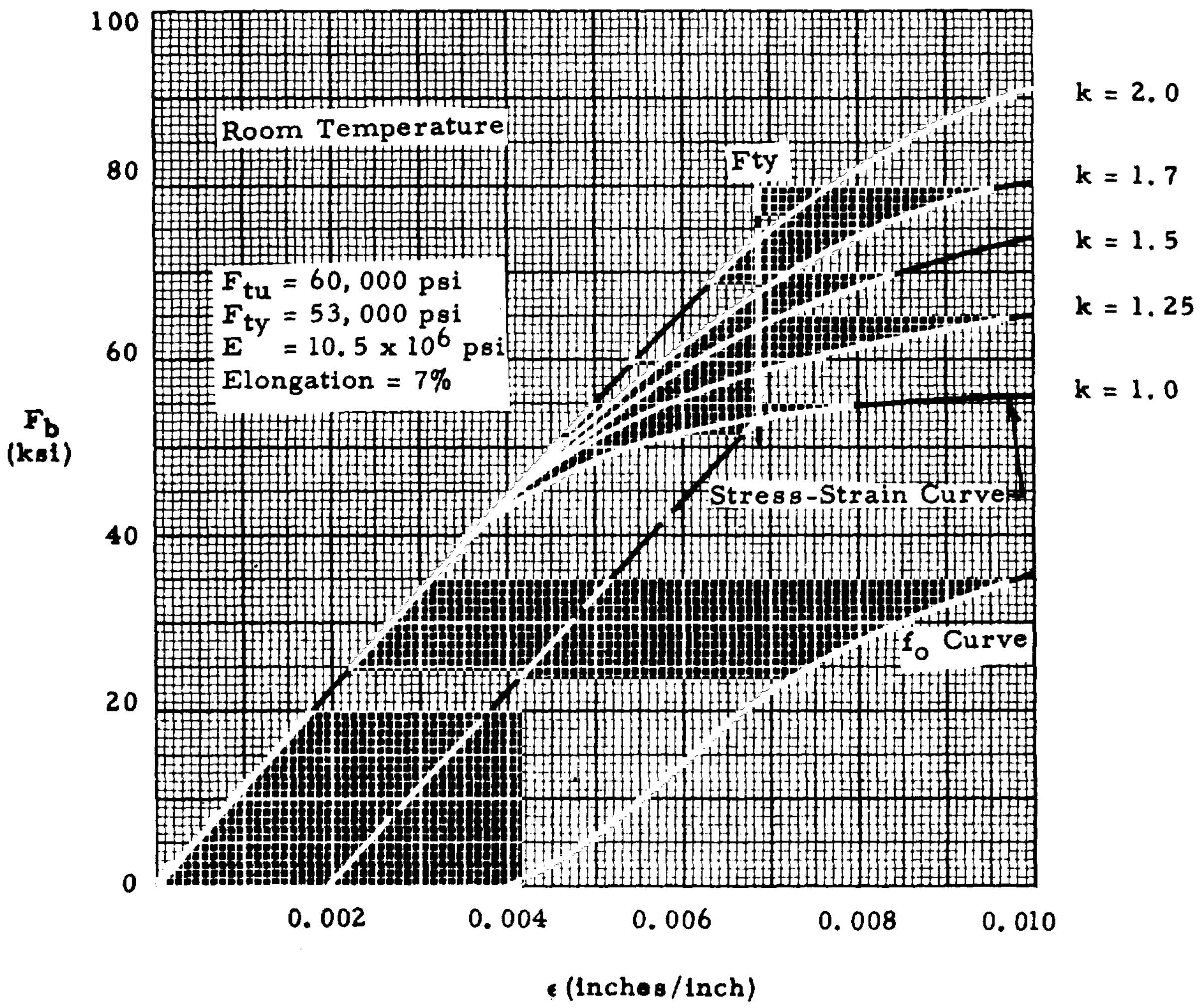


Fig. B4.5.6.5-1 Minimum Plastic Bending Curves 2014-T6 Aluminum Alloy Extrusions. Thickness ≤ .499 in.

Graph to be furnished when available

B4.5.6.5 Aluminum-Minimum Properties

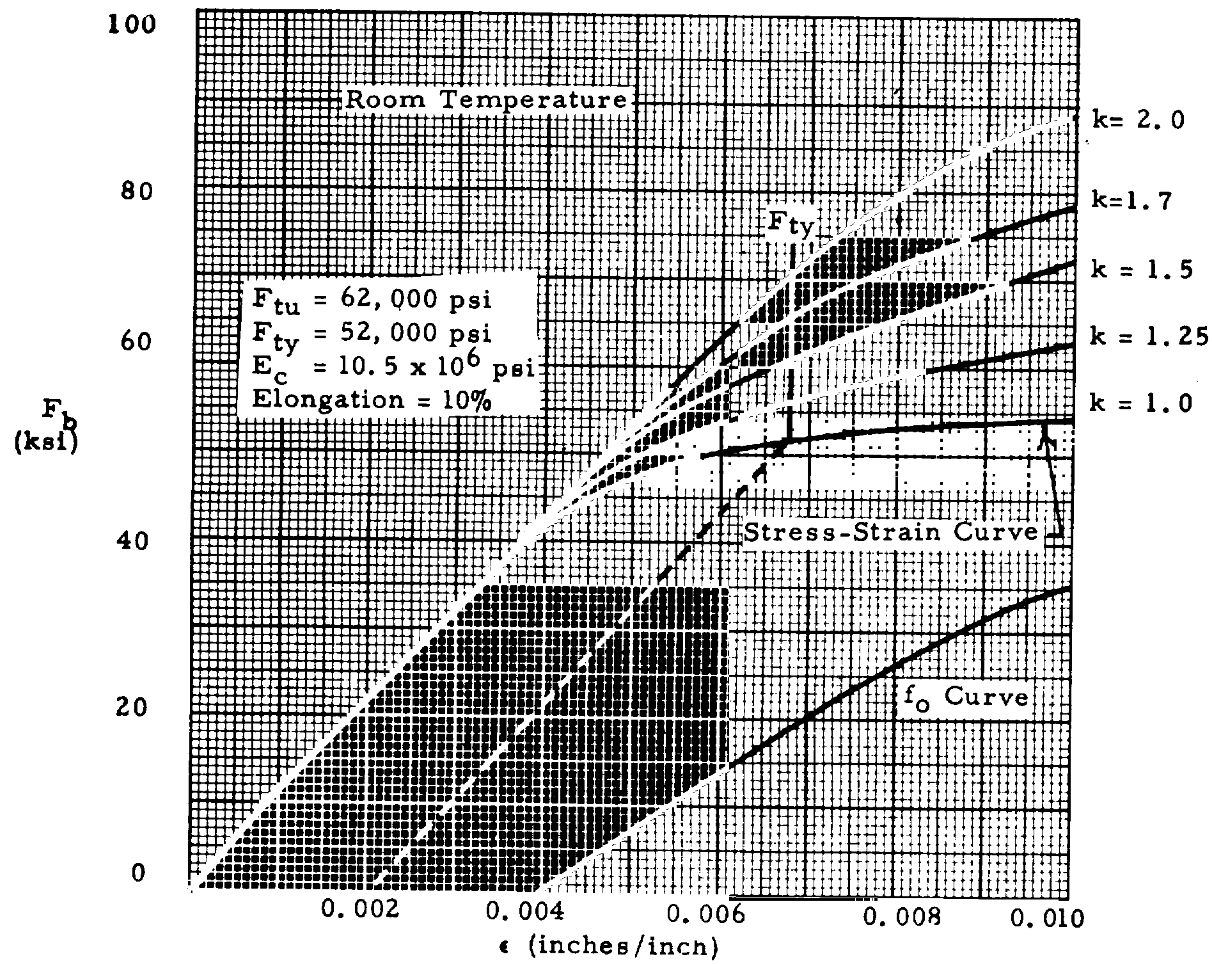


Fig. B4.5.6.5-3 Minimum Plastic Bending Curves 2014-T6 Aluminum Alloy Die Forgings. Thickness ≤ 4 in.

B4.5.6.5 Aluminum-Minimum Properties

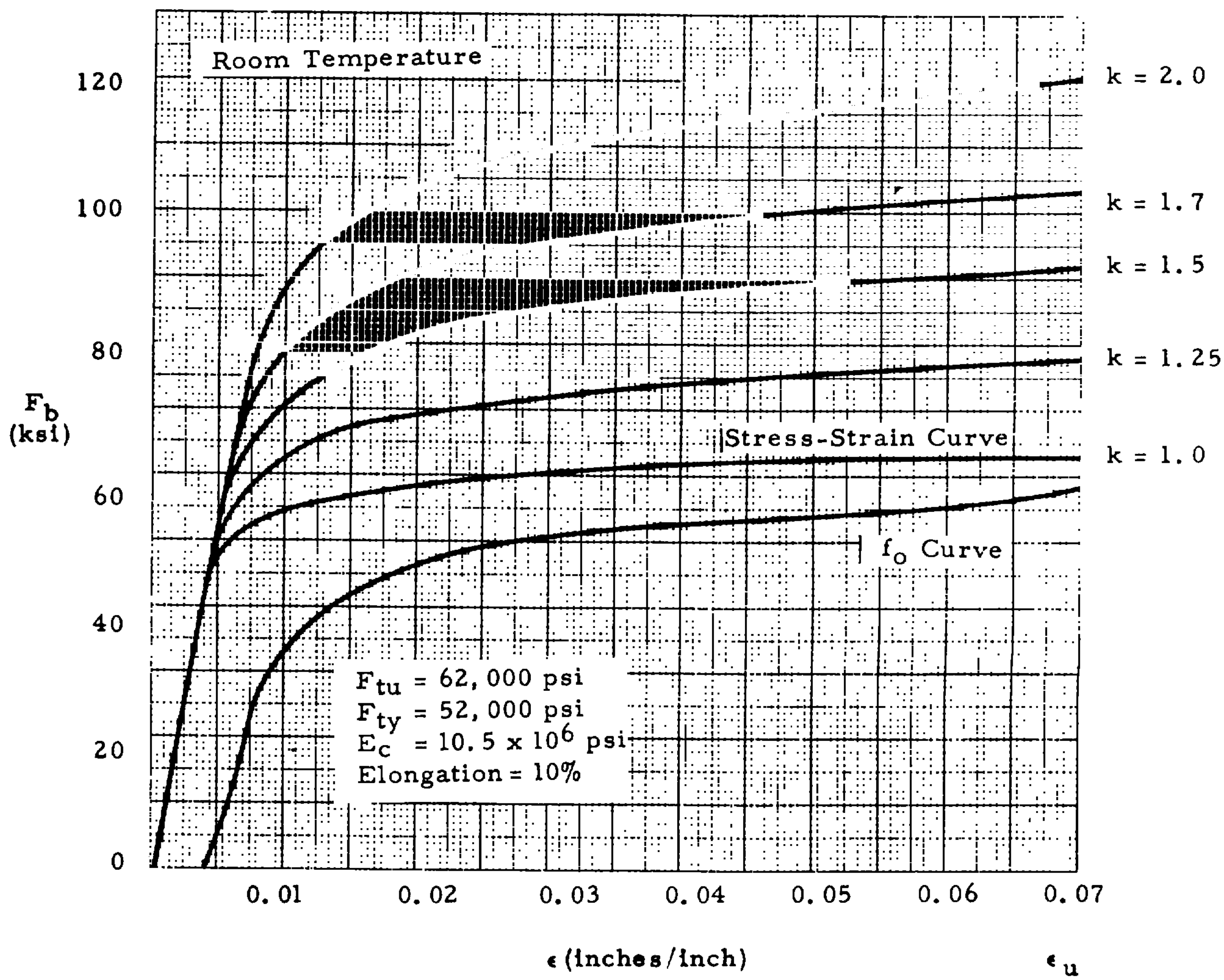


Fig. B4.5.6.5-4 Minimum Plastic Bending Curves 2014-T6 Aluminum Alloy Die Forgings. Thickness ≤ 4 in.

B4.5.6.5 Aluminum-Minimum Properties

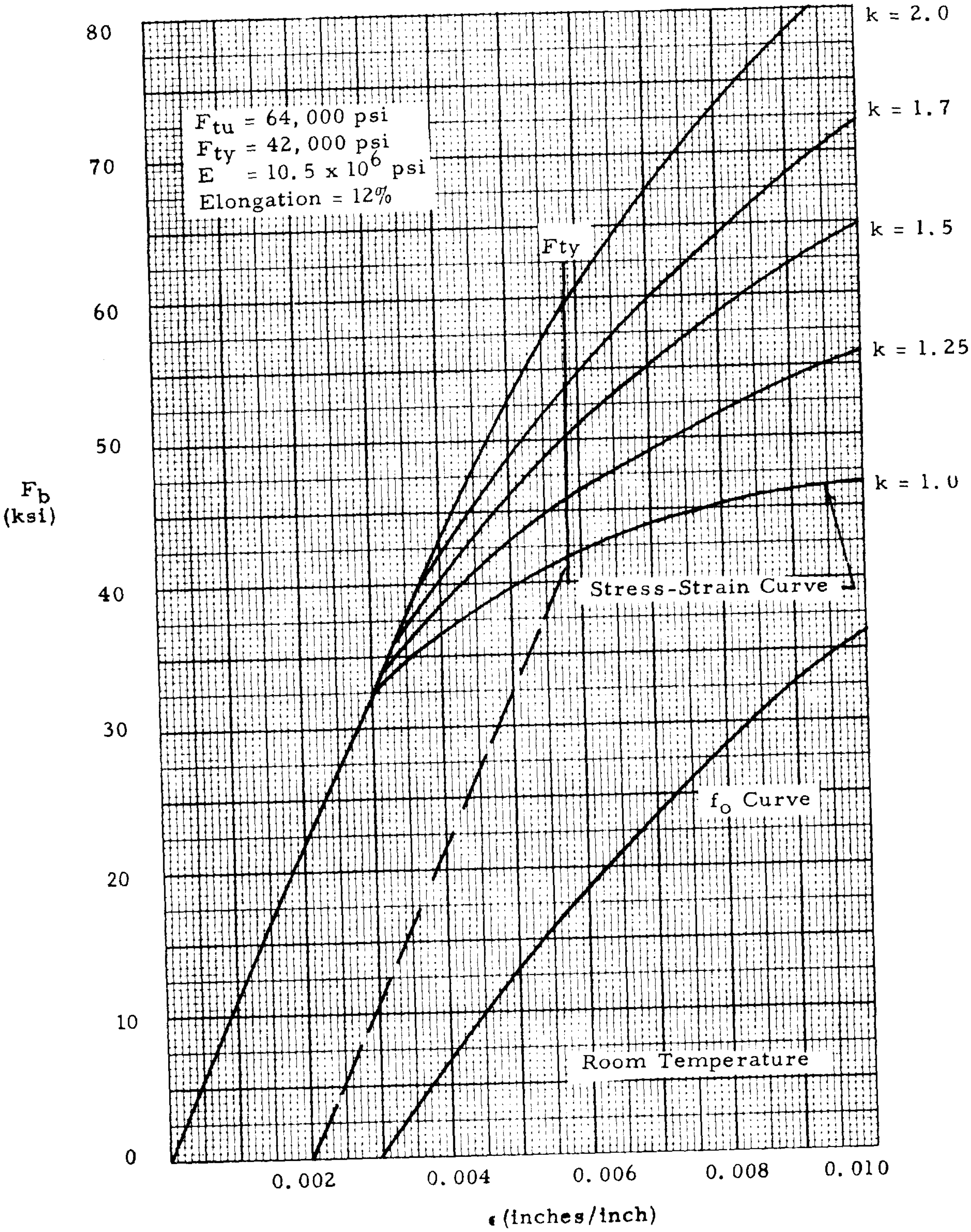


Fig. B4.5.6.5-5 Minimum Plastic Bending Curves 2024-T3 Aluminum Alloy Sheet & Plate - Heat Treated. Thickness ≤ 0.250 Inches

B4.5.6.5 Aluminum-Minimum Properties

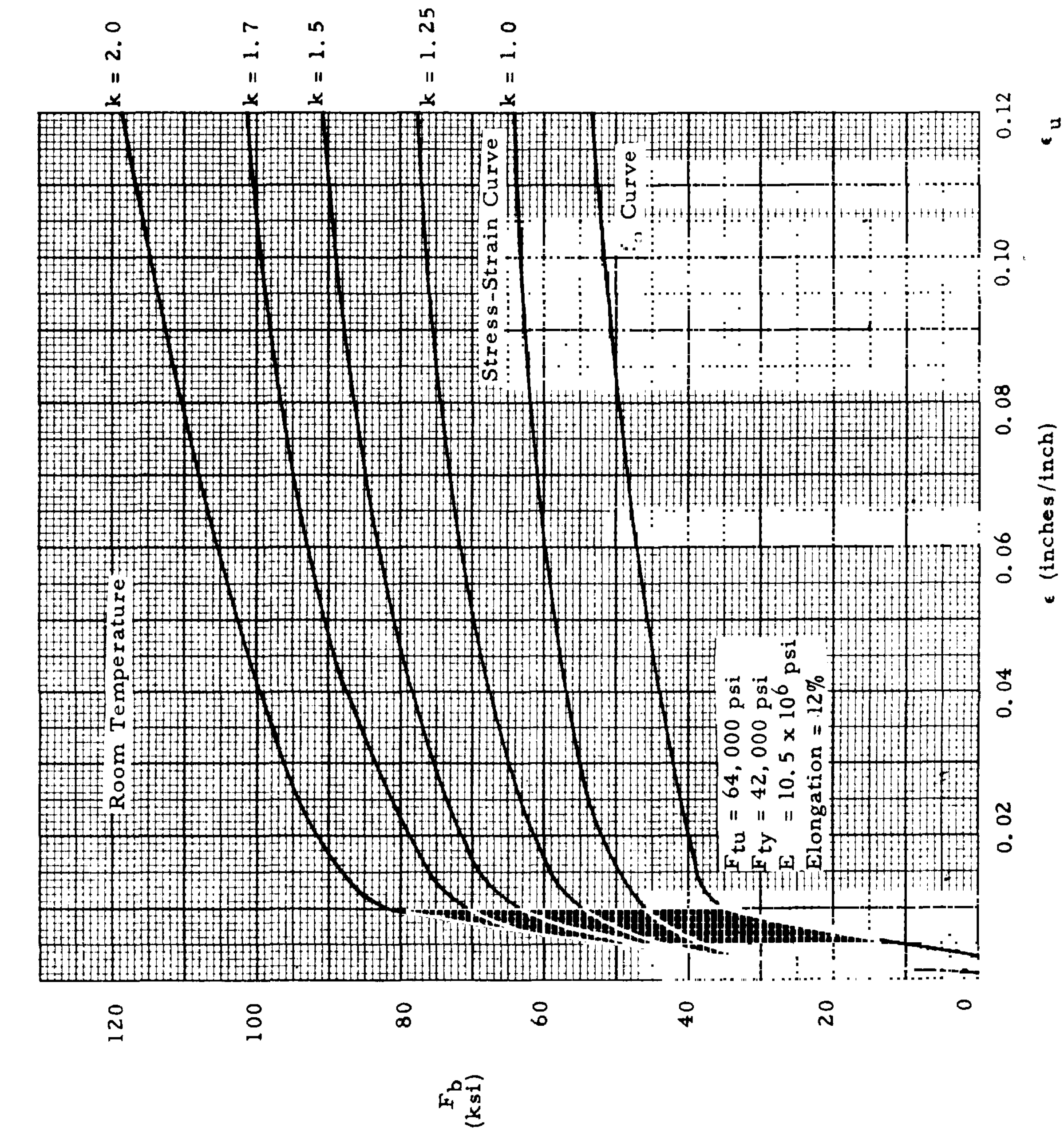


Fig. B4.5.6.5-6 Minimum Plastic Bending Curves 2024-T3 Aluminum Alloy Sheet and Plate - Heat Treated. Thickness ≤ 0.250 Inches

Graph to be furnished when available

B4.5.6.5 Aluminum-Minimum Properties

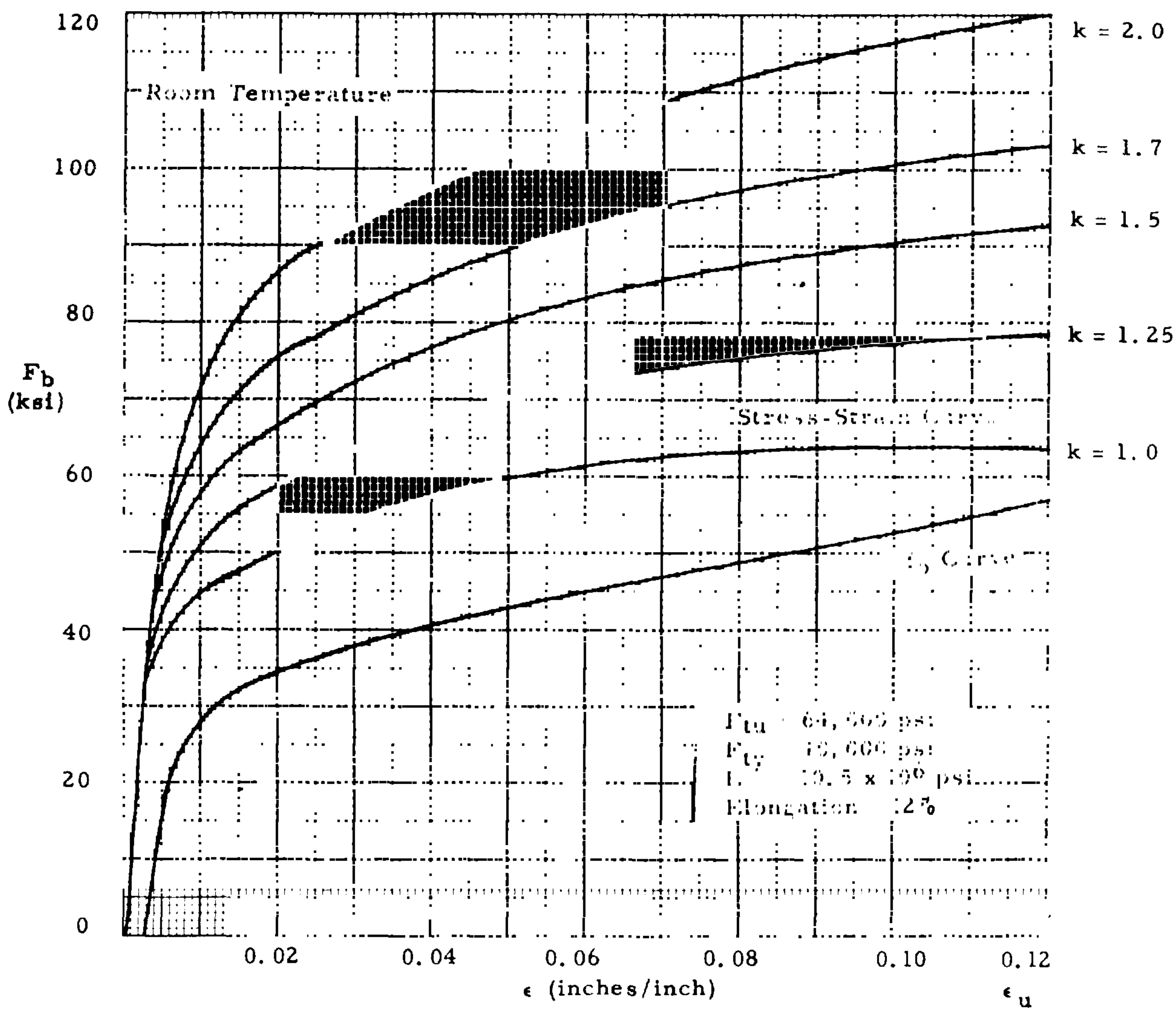


Fig. B4.5.6.5-8 Minimum Plastic Bending Curves for 2024-T3 & T4 Aluminum Alloy-Heat Treated-Sheet & Plate. Thickness ≤ 0.50 Inches

B4.5.6.5 Aluminum-Minimum Properties

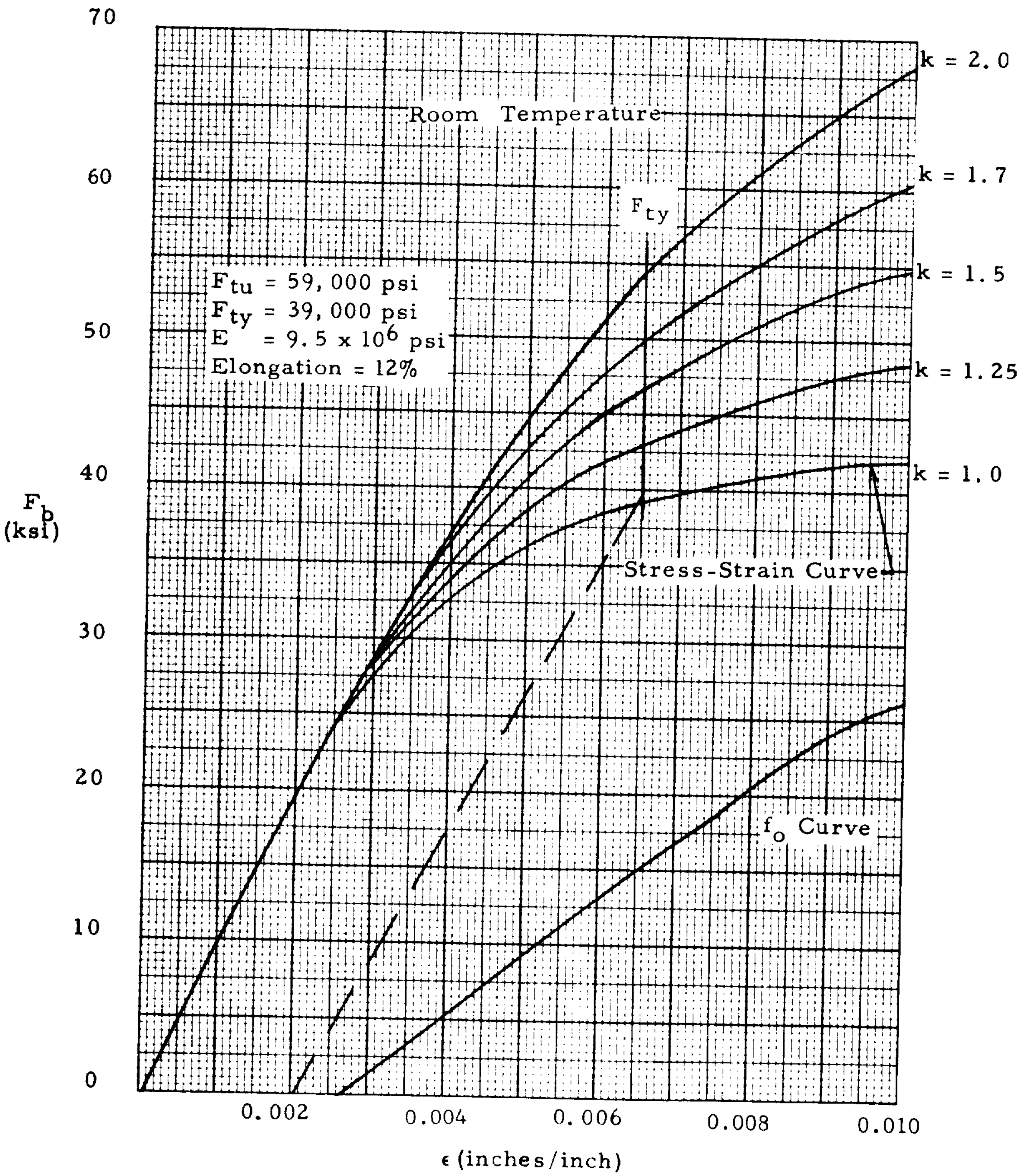


Fig. B4.5.6.5-9 Minimum Plastic Bending Curves 2024-T3 Aluminum Alloy Clad Sheet & Plate - Heat Treated. Thickness 0.010 to 0.062 in.

B4.5.6.5 Aluminum-Minimum Properties

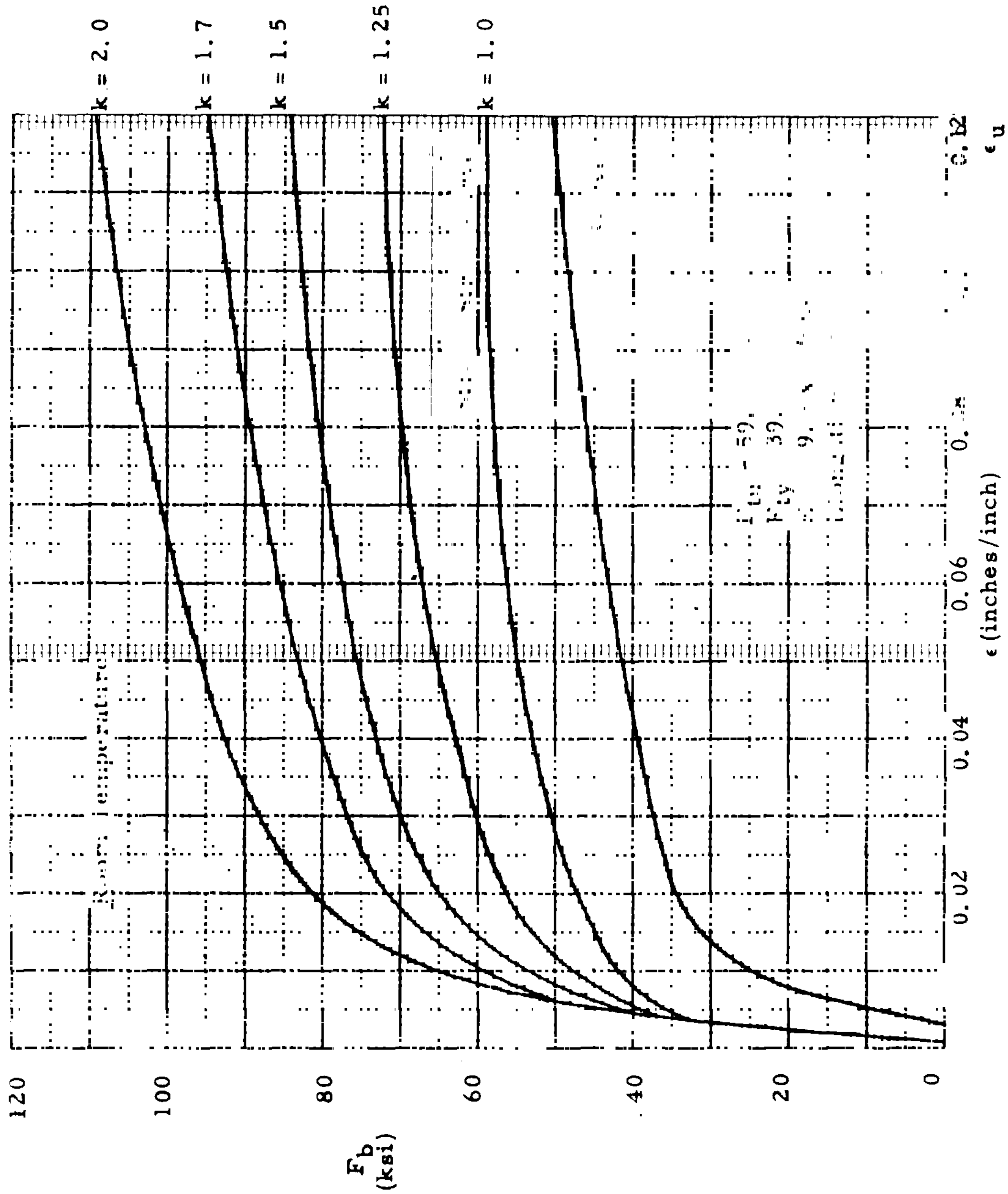


Fig. B4.5.6.5-10 Minimum Plastic Bending Curves 2024-T3 Aluminum Alloy Clad Sheet and Plate-Heat Treated. Thickness 0.010 to 0.062 in.

B4.5.6.5 Aluminum-Minimum Properties

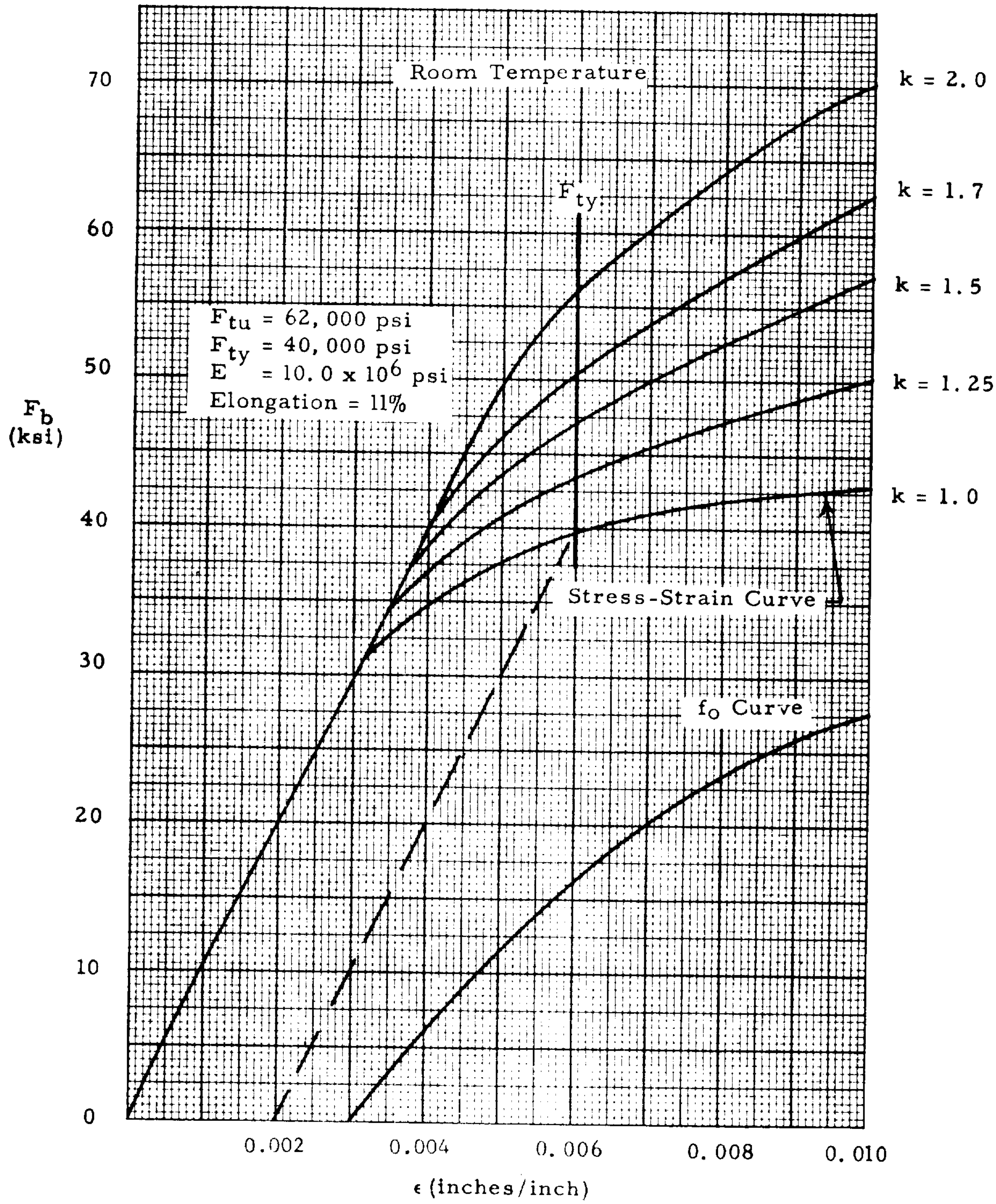


Fig. B4.5.6.5-11 Minimum Plastic Bending Curve 2024-T4 Aluminum Alloy Clad Sheet & Plate - Heat Treated Thickness 0.25 to 0.50 in.

B4.5.6.5 Aluminum-Minimum Properties

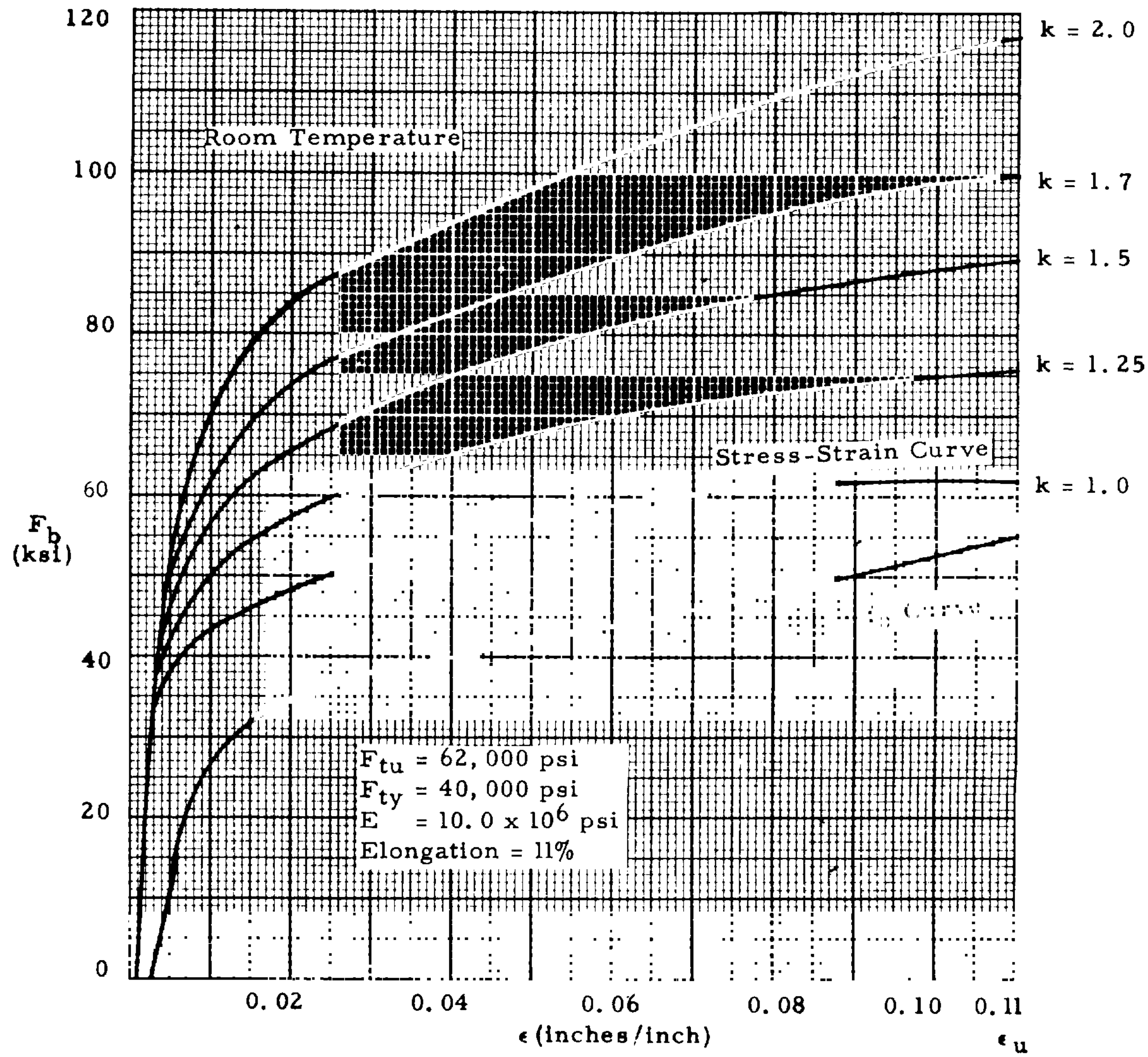


Fig. B4.5.6.5-12 Minimum Plastic Bending Curves 2024-T4 Aluminum Alloy Clad Sheet and Plate - Heat Treated Thickness 0.25 to 0.50 in.

B4.5.6.5 Aluminum-Minimum Properties

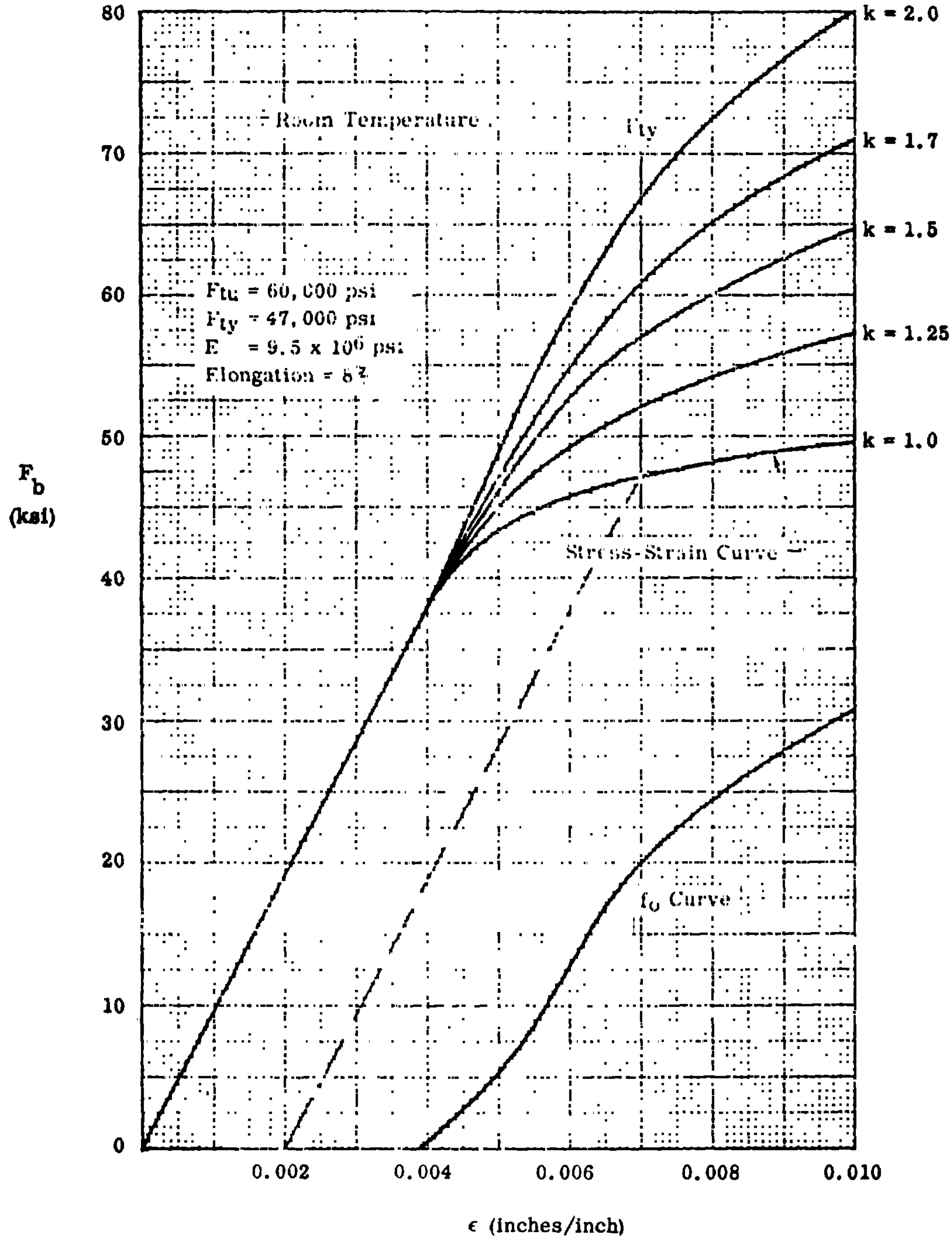


Fig. B4.5.6.5-13 Minimum Plastic Bending Curves 2024-T6 Aluminum Alloy Clad Sheet - Heat Treated & Aged. Thickness < 0.064 Inches

B4.5.6.5 Aluminum-Minimum Properties

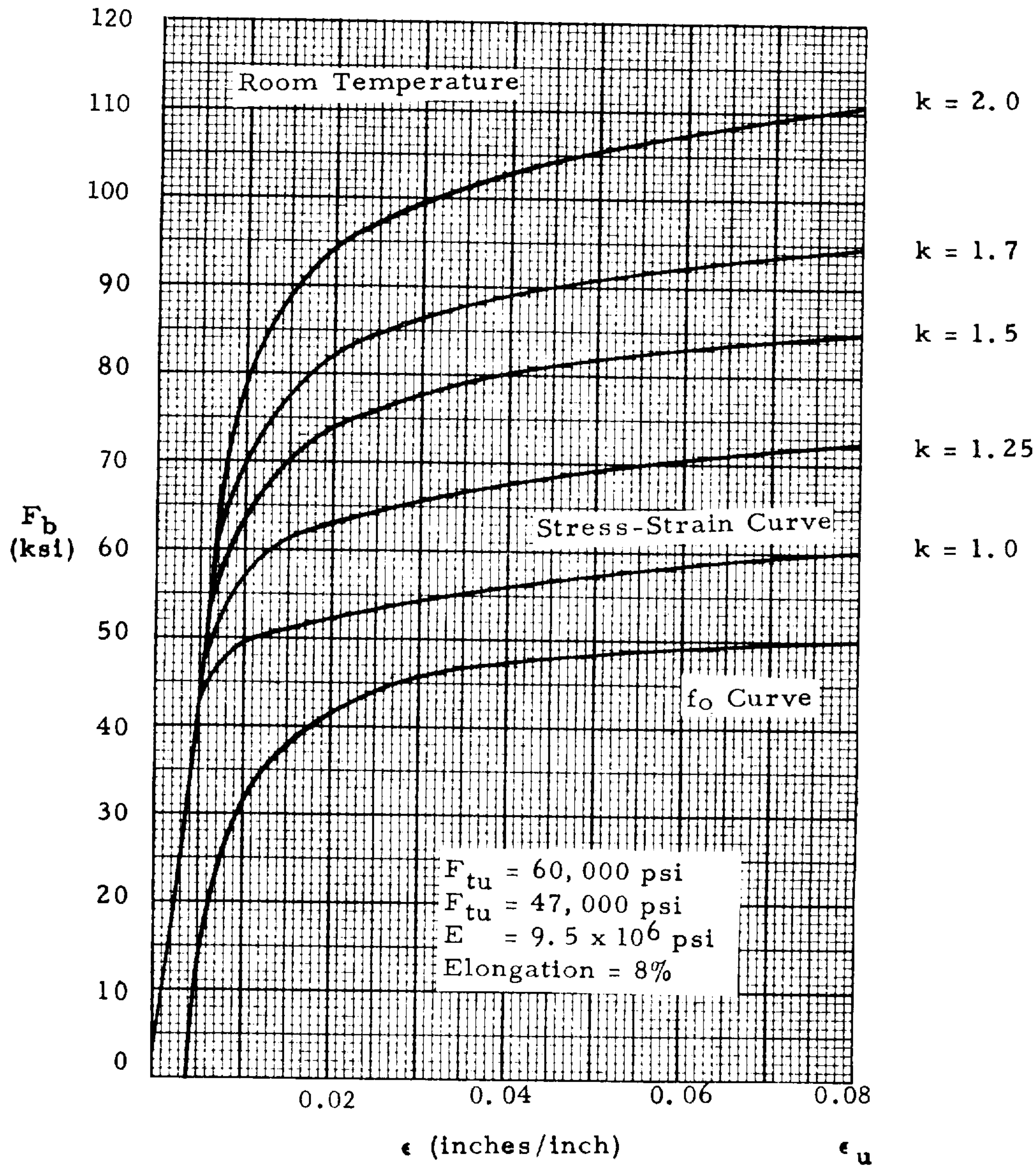


Fig. B4.5.6.5-14 Minimum Plastic Bending Curves 2024-T6 Aluminum Alloy Clad Sheet-Heat Treated and Aged Thickness < 0.064 in.

Graph to be furnished when available

B4.5.6.5 Aluminum-Minimum Properties

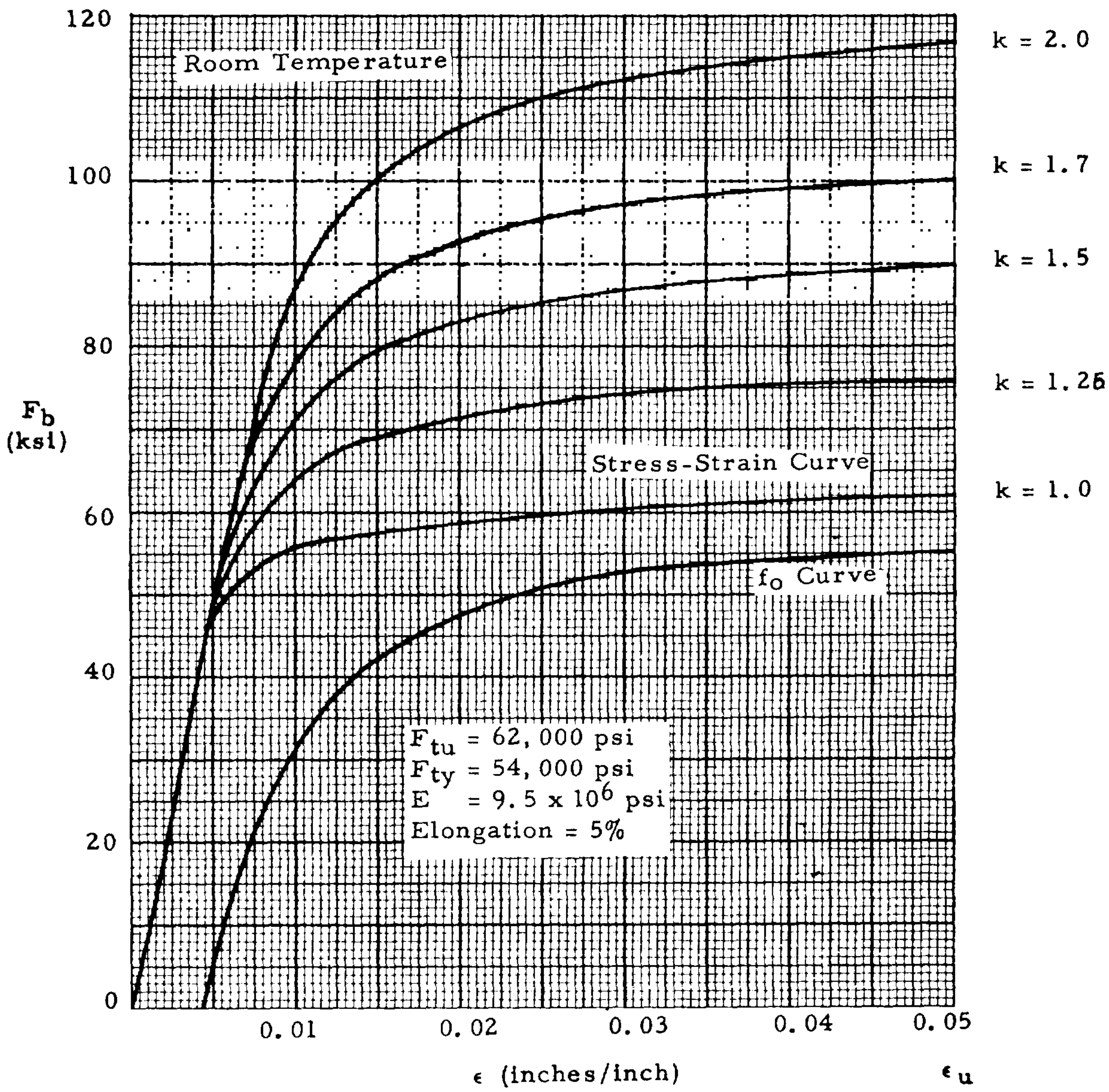


Fig. B4.5.6.5-16 Minimum Plastic Bending Curves 2024-T81 Aluminum Alloy Clad Sheet-Heat Treat, Cold Worked and Aged Thickness < 0.064 in.

B4.5.6.5 Aluminum-Minimum Properties

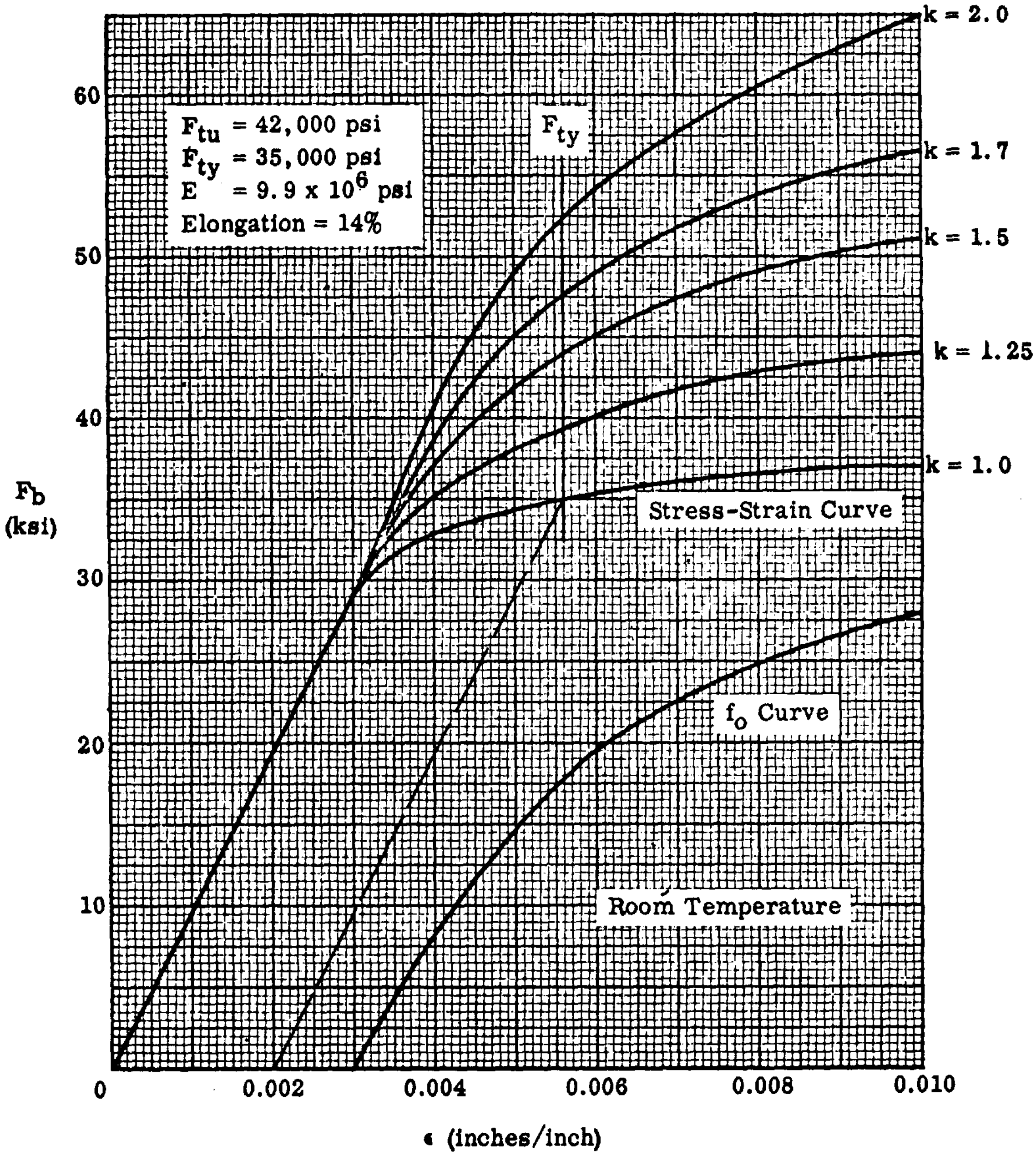


Fig. B4.5.6.5-17 Minimum Plastic Bending Curves 6061-T6 Aluminum Alloy Sheet Heat Treated & Aged. Thickness ≥ 0.020 in.

B4.5.6.5 Aluminum-Minimum Properties

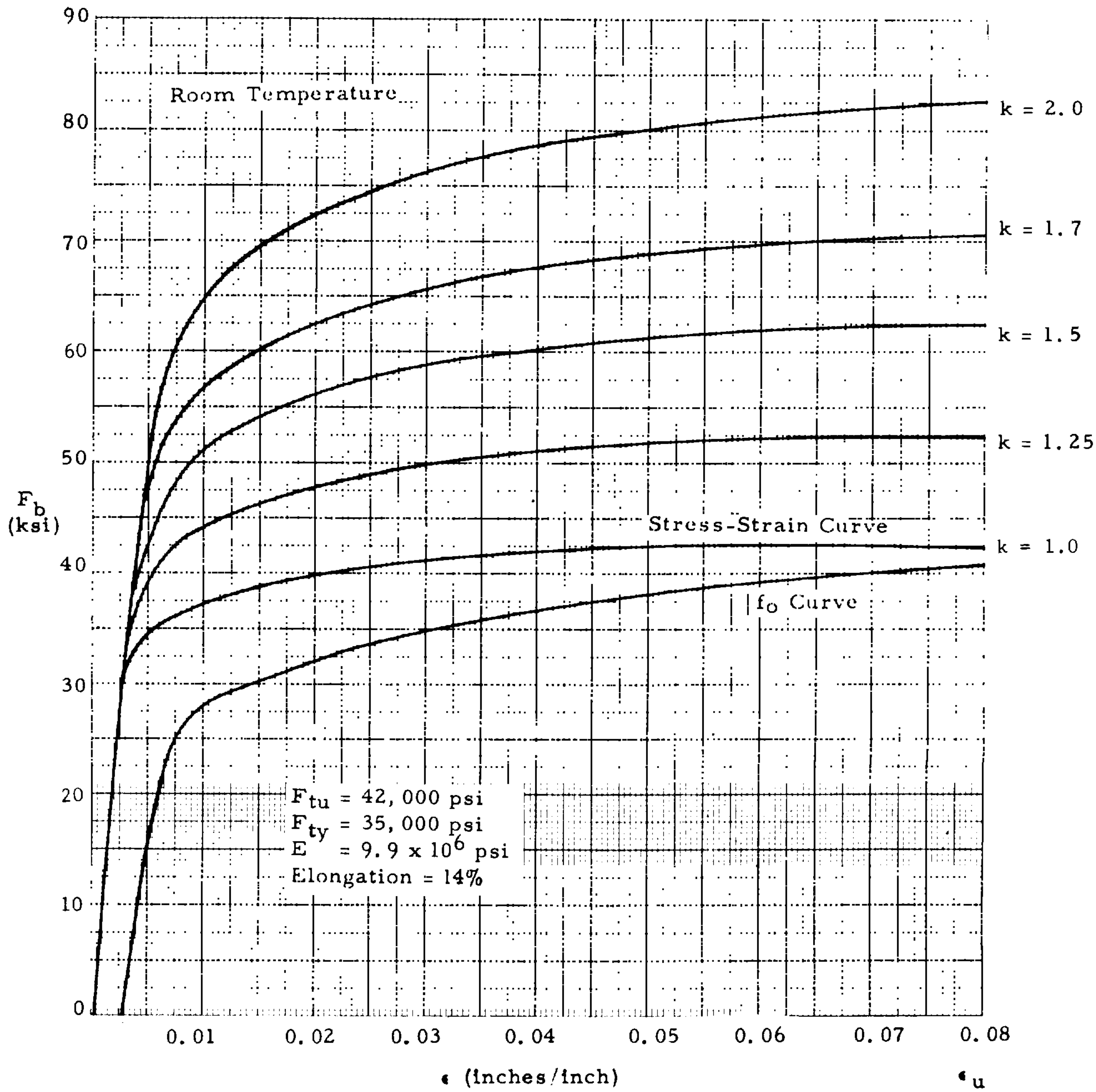


Fig. B4.5.6.5-18 Minimum Plastic Bending Curves 6061-T6 Aluminum Alloy Sheet - Heat Treated & Aged Thickness ≥ 0.020 in.

B4.5.6.5 Aluminum-Minimum Properties

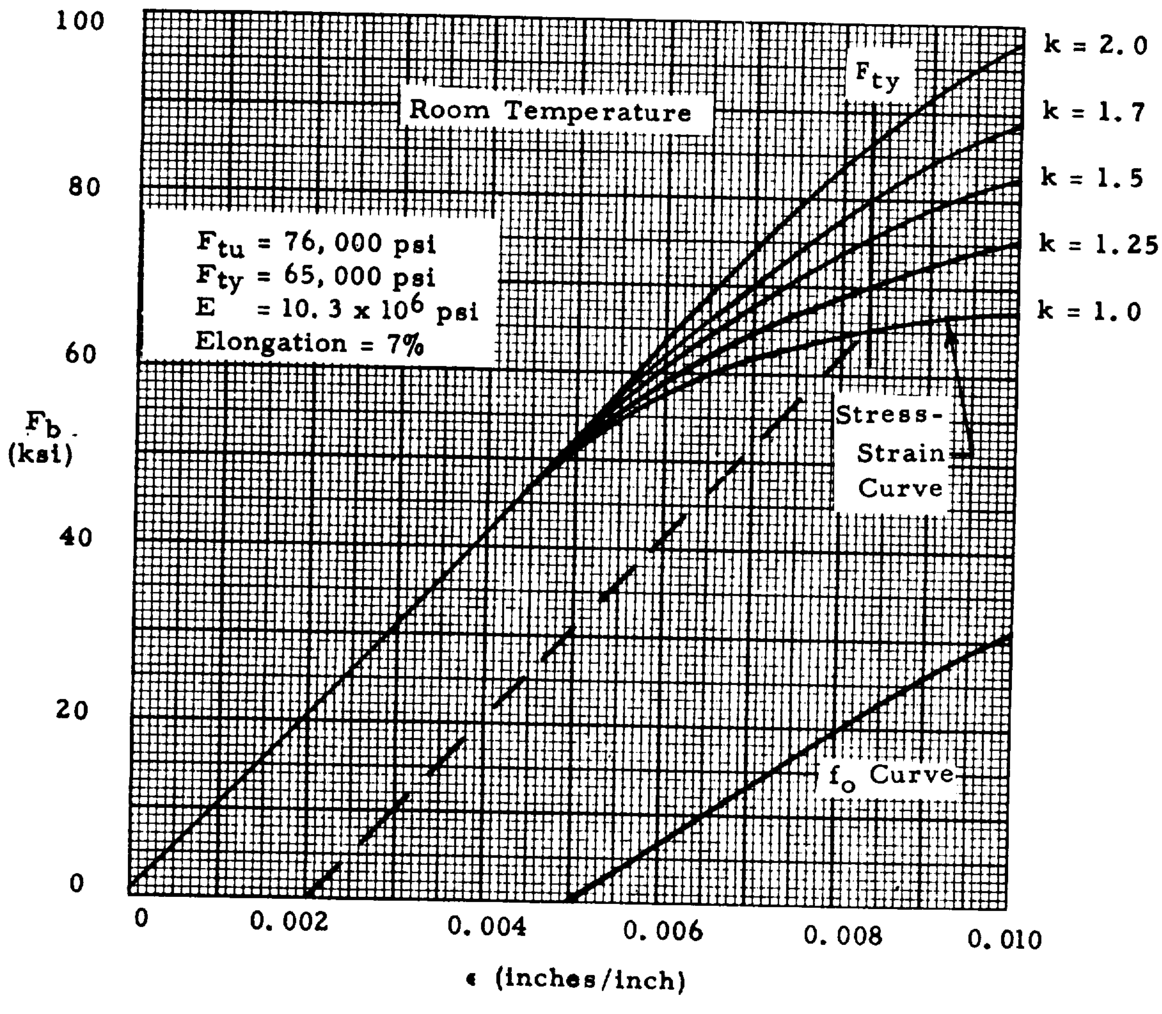


Fig. B4.5.6.5-19 Minimum Plastic Bending Curves 7075-T6 Aluminum Alloy Bare Sheet and Plate. Thickness ≤ .039 in.

B4.5.6.5 Aluminum-Minimum Properties

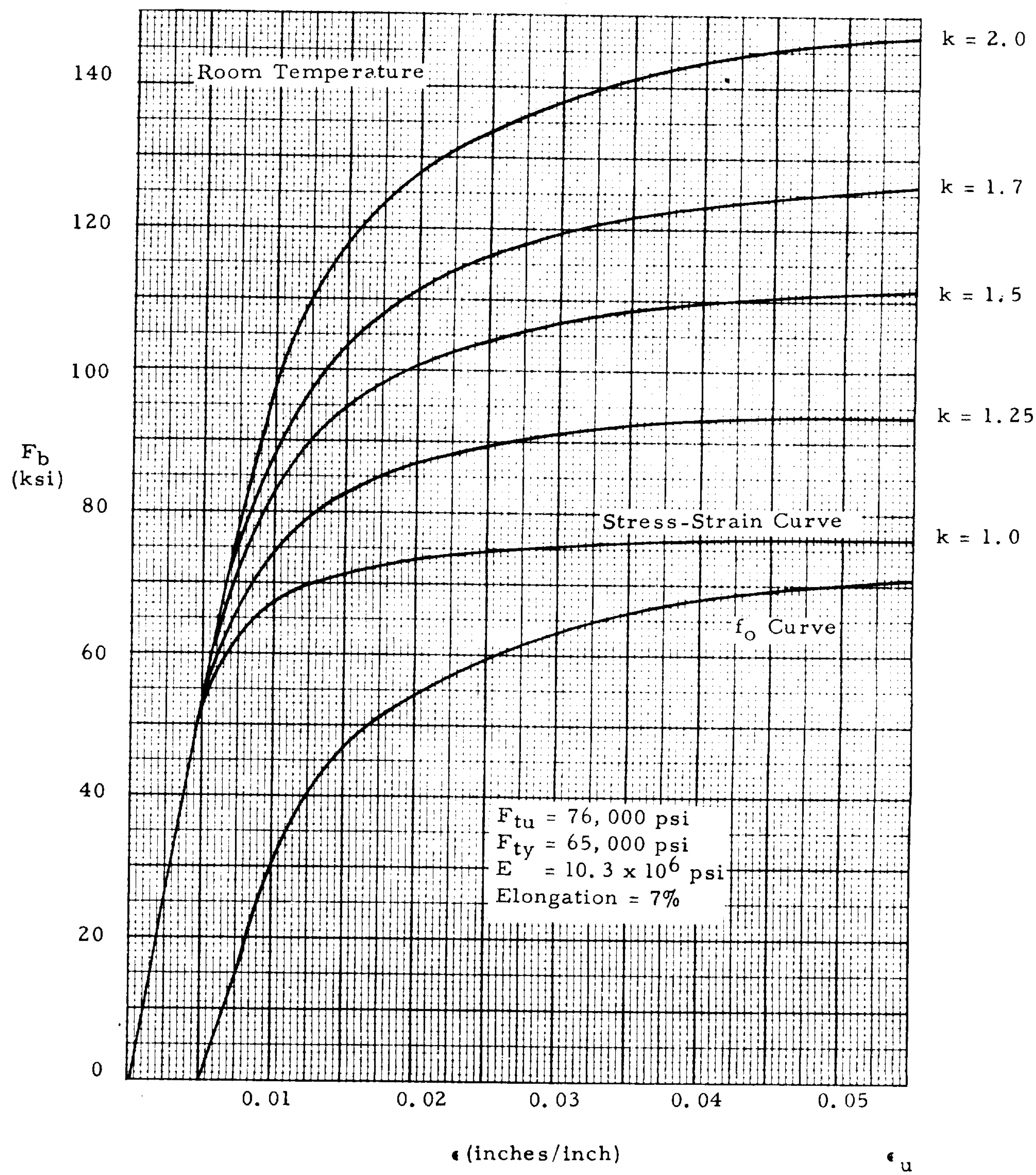


Fig. B4.5.6.5-20 Minimum Plastic Bending Curves 7075-T6 Aluminum Alloy Bare Sheet & Plate Thickness ≤ .039 in.

B4.5.6.5 Aluminum-Minimum Properties

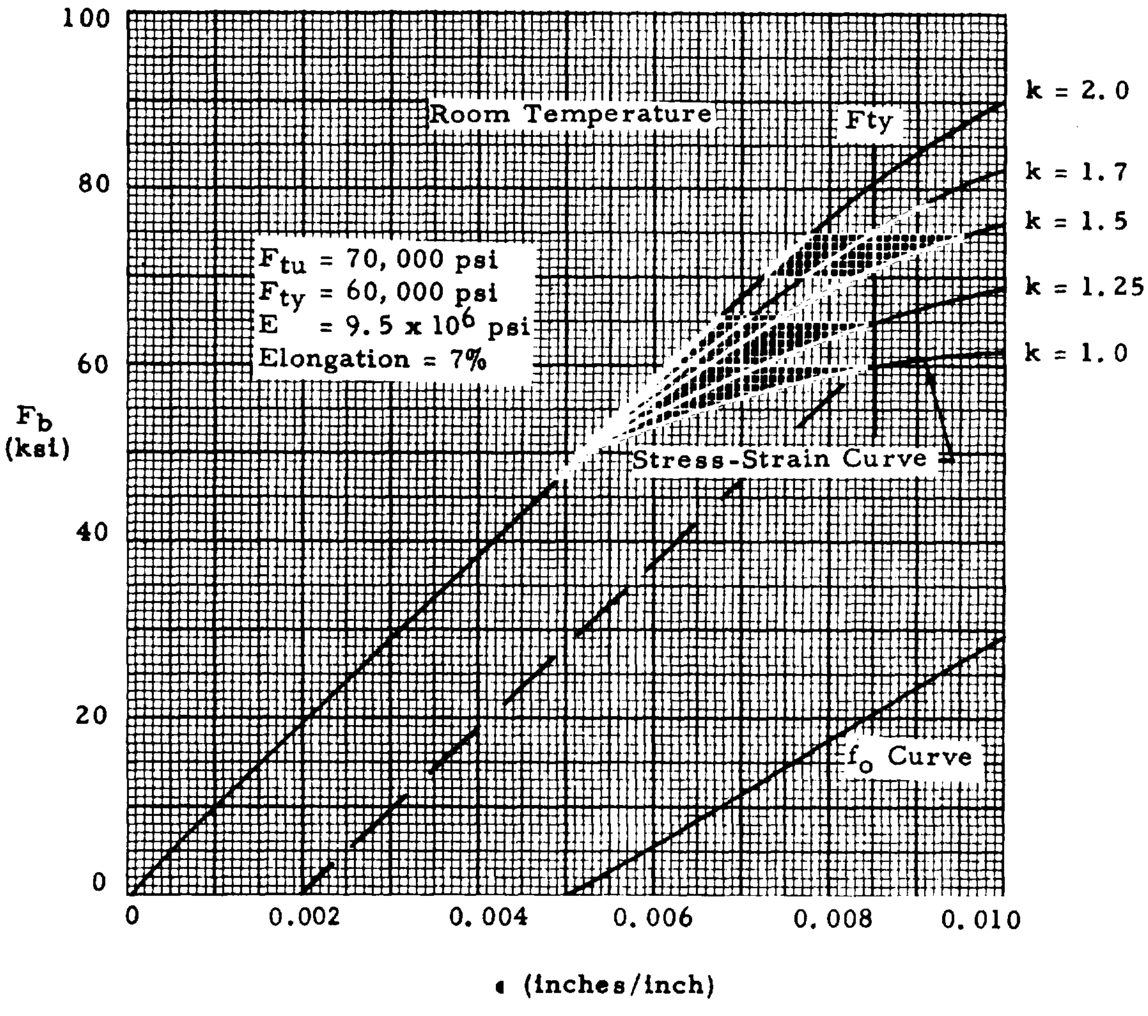


Fig. B4.5.6.5-21 Minimum Plastic Bending Curves 7075-T6 Aluminum Alloy Clad Sheet & Plate. Thickness ≤ 0.039 in.

B4.5.6.5 Aluminum-Minimum Properties

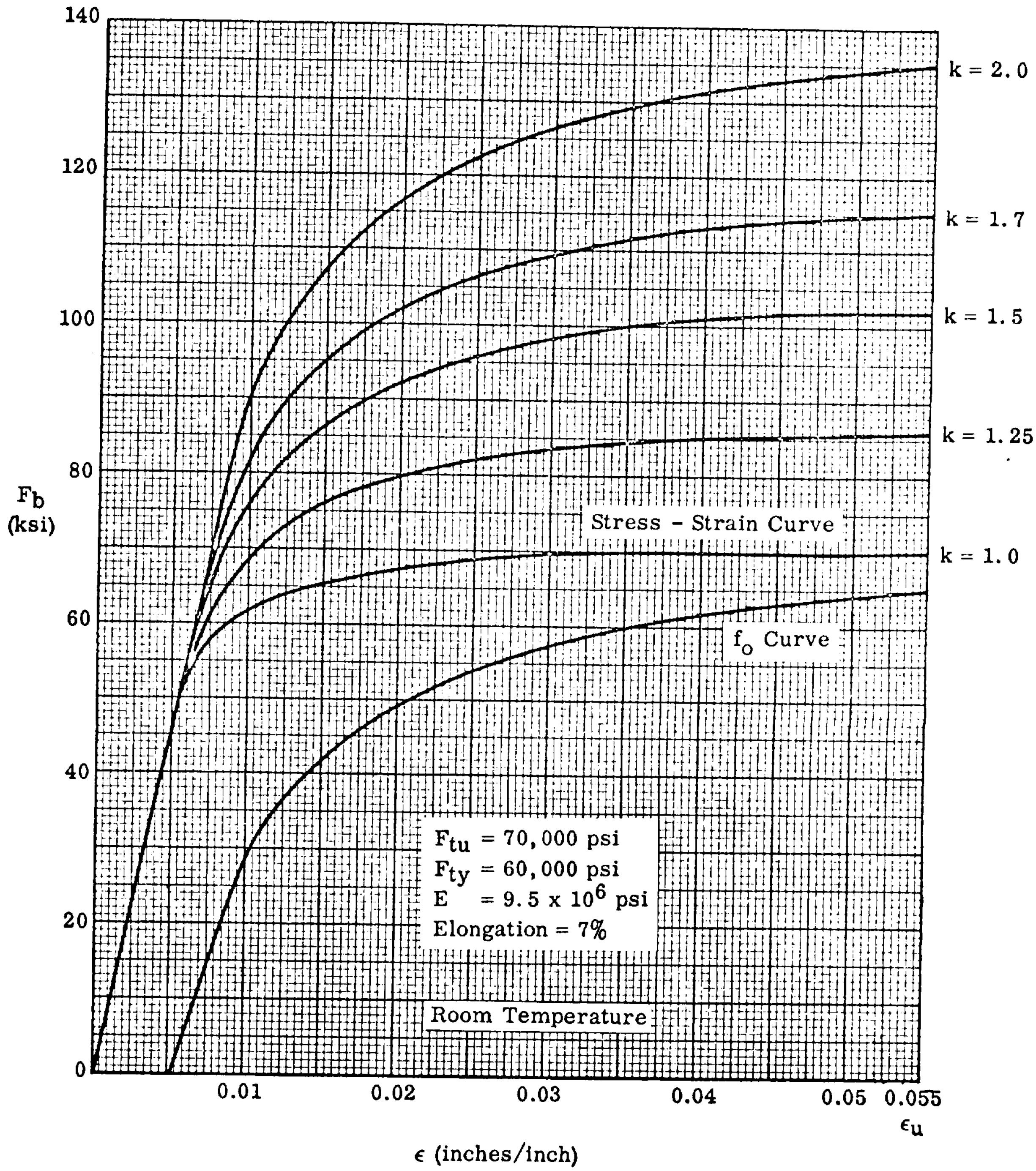


Fig. B4.5.6.5-22 Minimum Plastic Bending Curves 7075-T6 Aluminum Alloy Clad Sheet & Plate Thickness ≤ 0.39 in.

B4.5.6.5 Aluminum-Minimum Properties

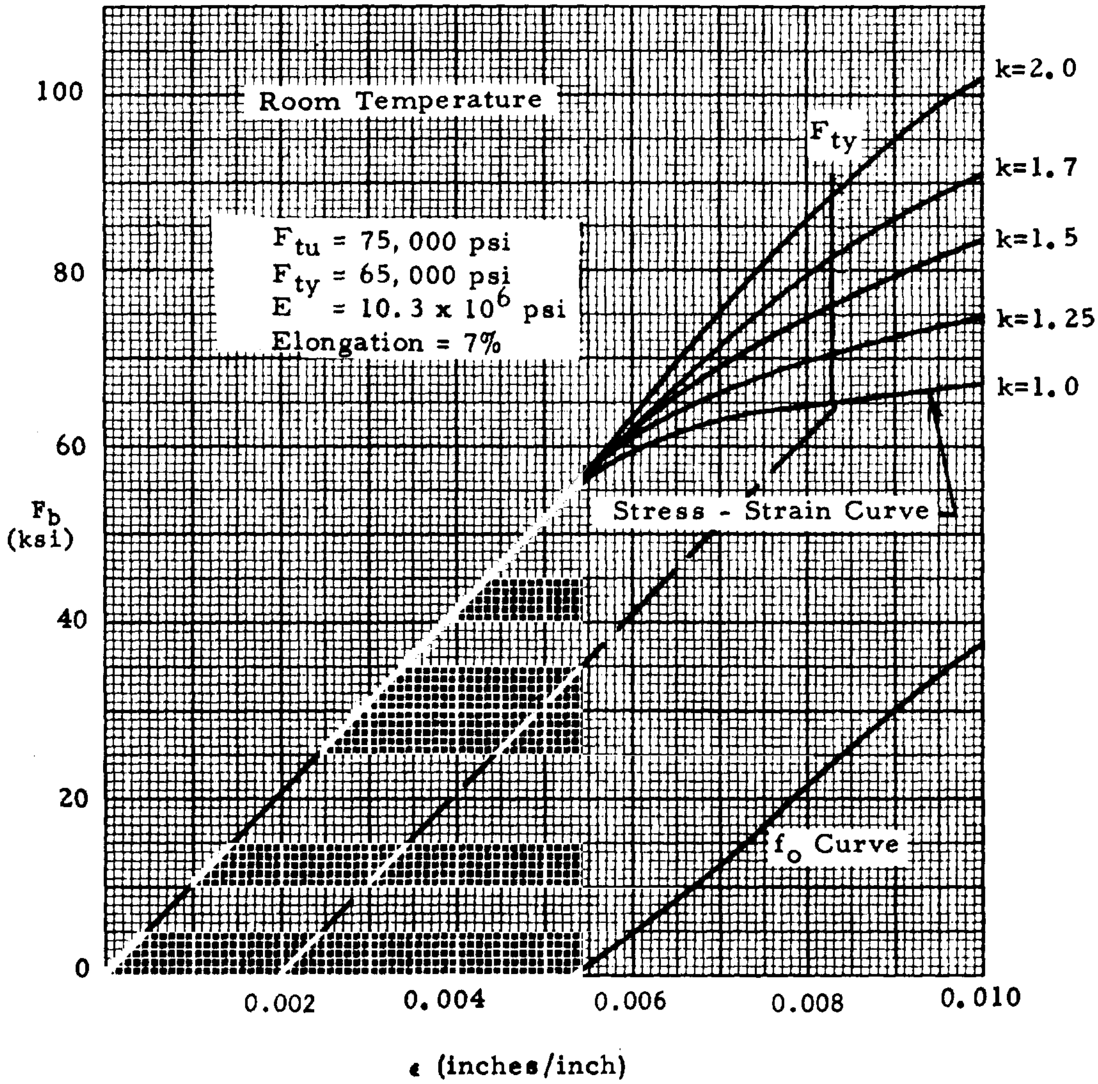


Fig. B4.5.6.5-23 Minimum Plastic Bending Curves 7075-T6 Aluminum Alloy Extrusions. Thickness ≤ 0.25 in.

B4.5.6.5 Aluminum-Minimum Properties

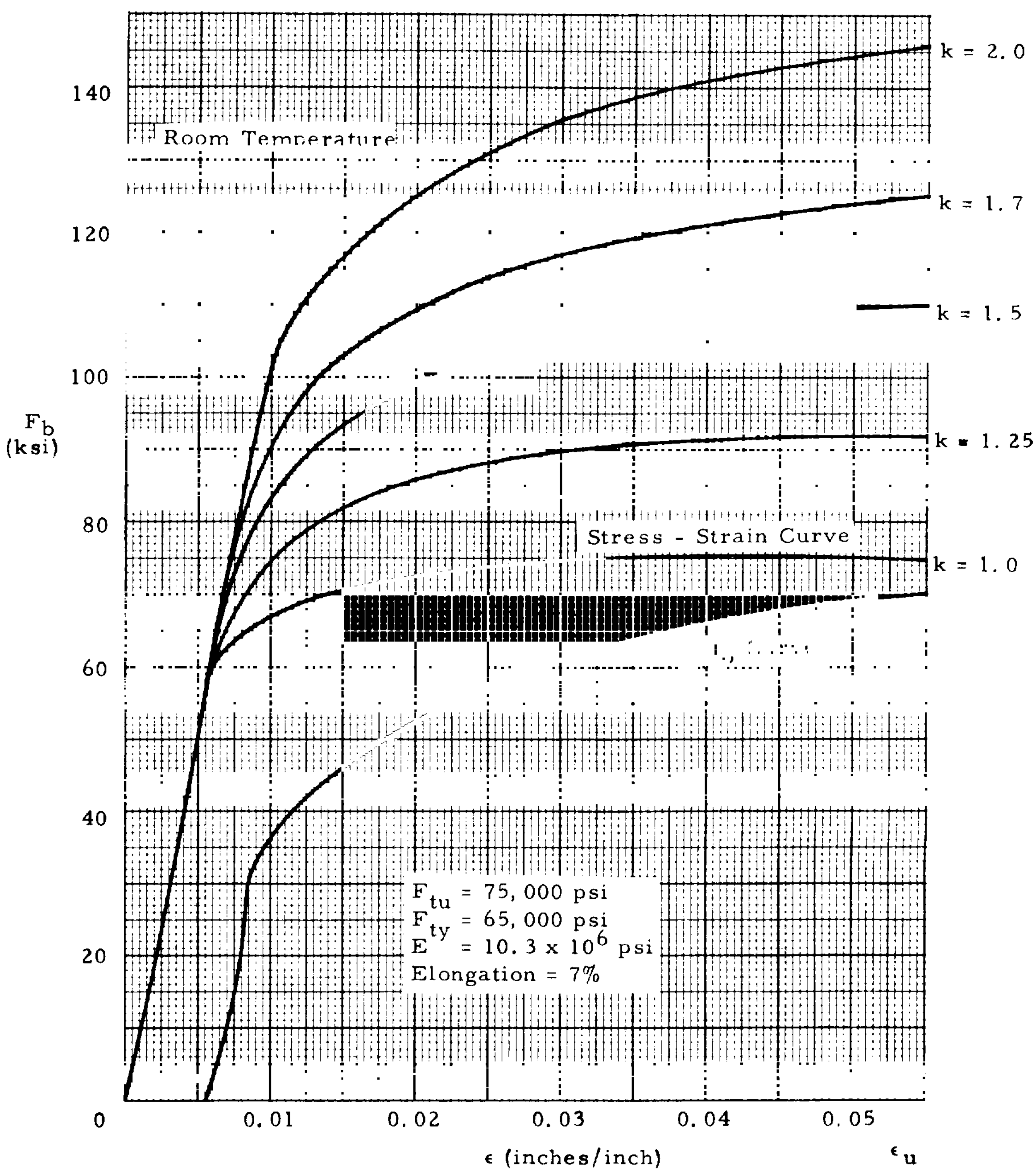


Fig. B4.5.6.5-24 Minimum Plastic Bending Curves 7075-T6 Aluminum Alloy Extrusions. Thickness ≤ 0.25 in.

Graph to be furnished when available

B4.5.6.5 Aluminum-Minimum Properties

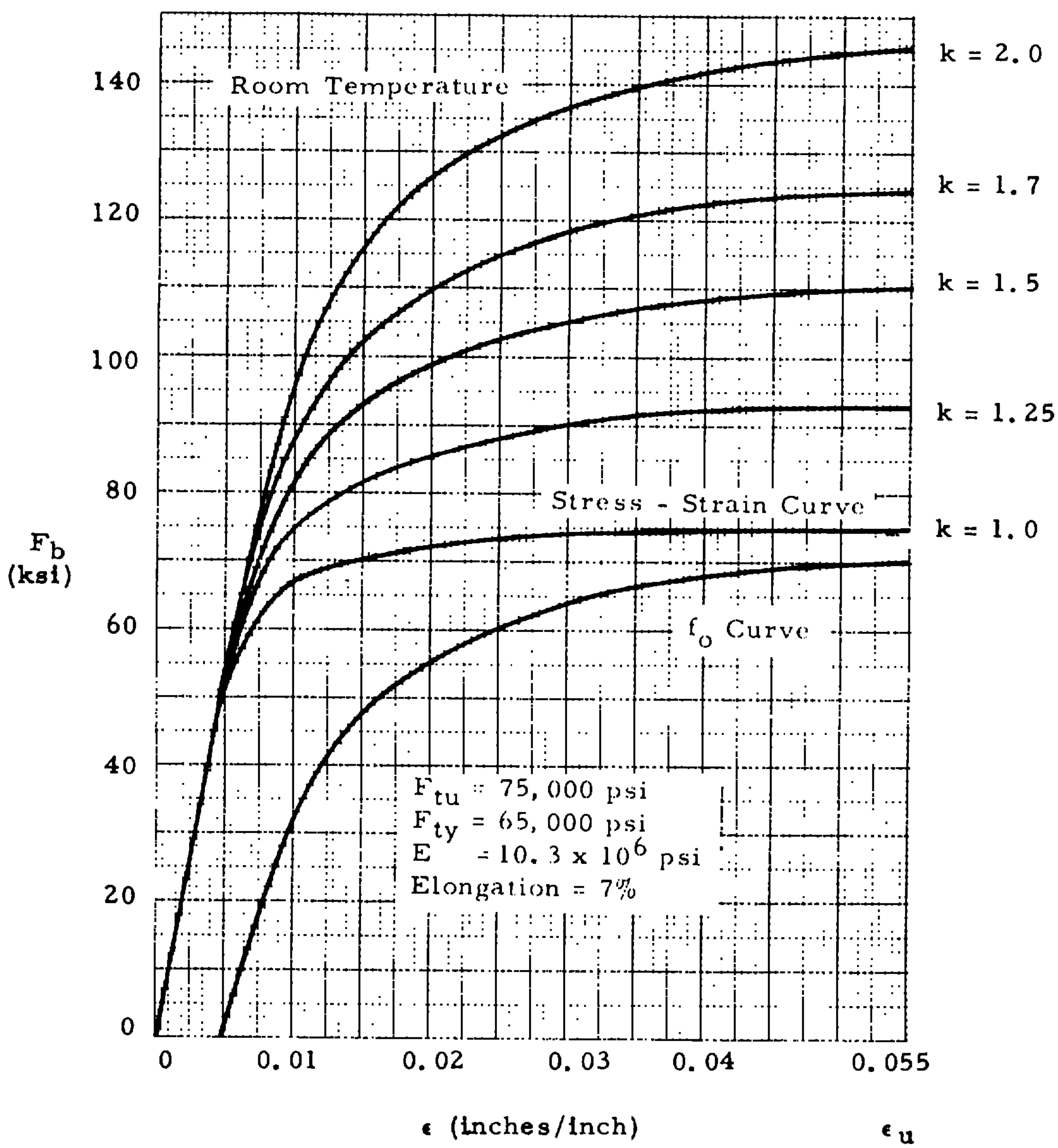


Fig. B4.5.6.5-26 Minimum Plastic Bending Curves 7075-T6 Aluminum Alloy Die Forgings. Thickness ≤ 2 in.

Graph to be furnished when available

B4.5.6.5 Aluminum-Minimum Properties

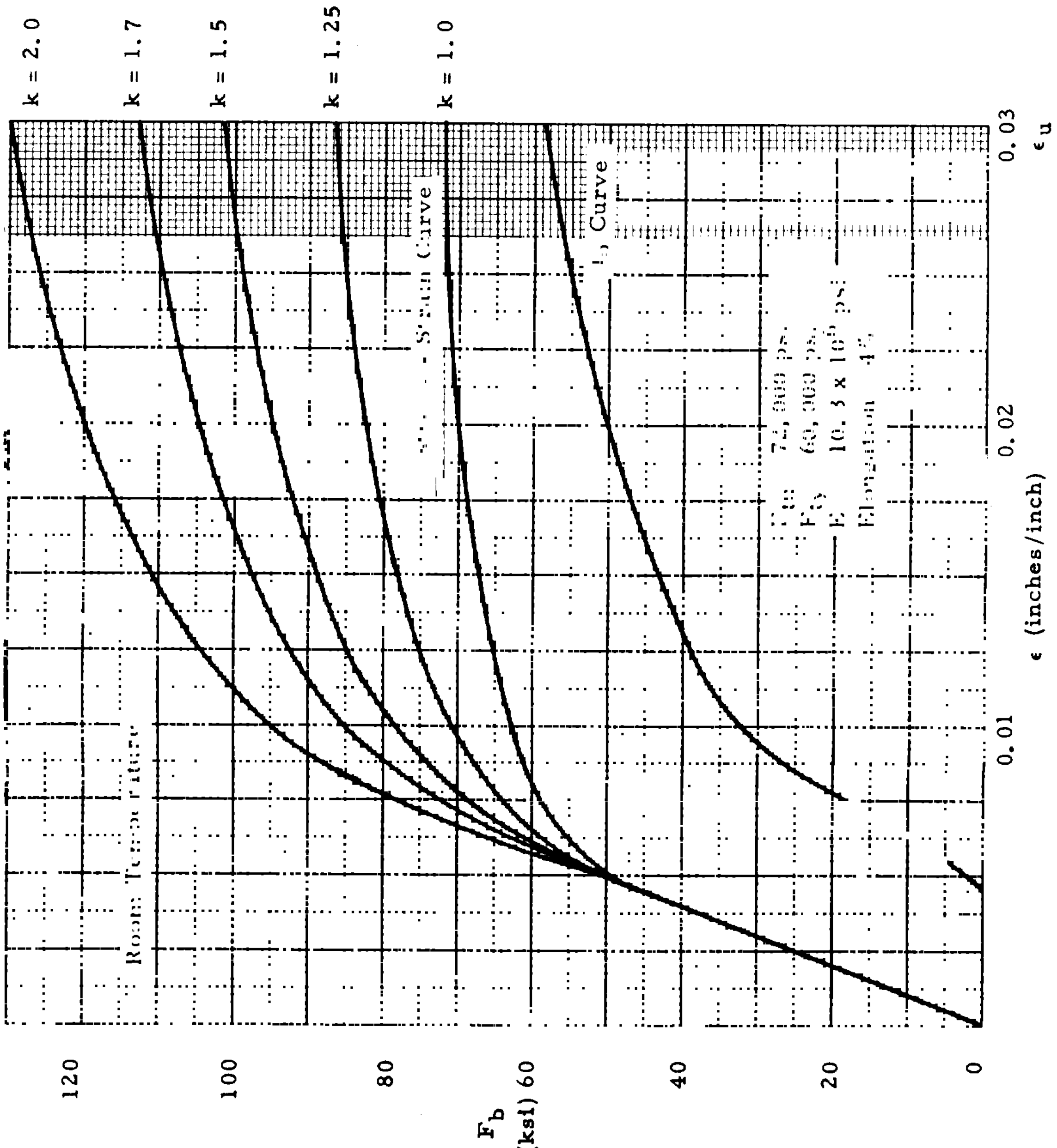


Fig. B4.5.6.5-28 Minimum Plastic Bending Curves 7075-T6 Aluminum Alloy Hand Forgings Area $\leq$ 16 in.^{2}

B4.5.6.5 Aluminum-Minimum Properties

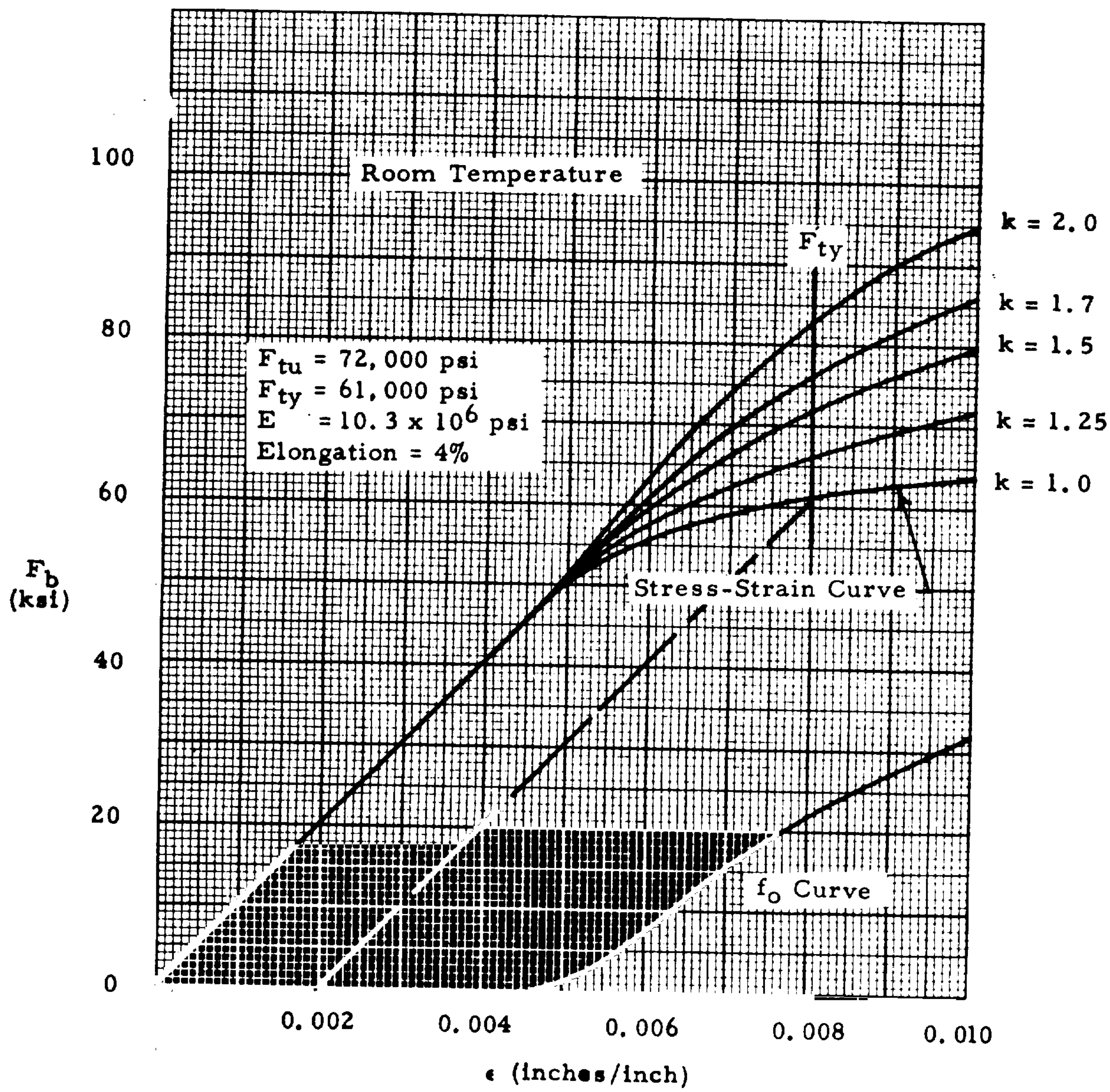


Fig. B4.5.6.5-29 Minimum Plastic Bending Curves 7079-T6 Aluminum Alloy Die Forgings. (Transverse) Thickness ≤ 6.0 in.

Graph to be furnished when available

B4.5.6.5 Aluminum-Minimum Properties

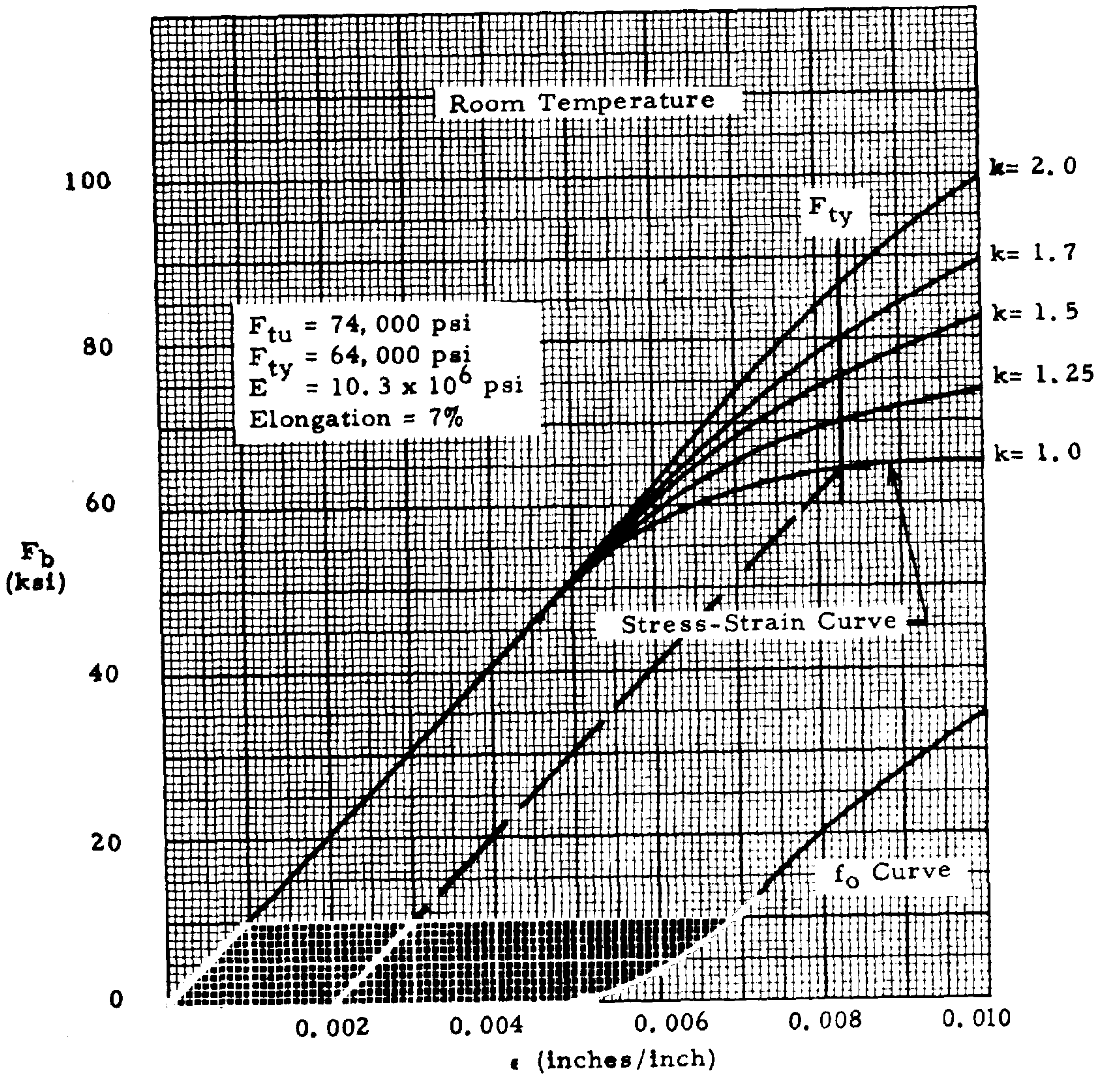


Fig. B4.5.6.5-31 Minimum Plastic Bending Curves 7079-T6 Aluminum Alloy Die Forgings (Longitudinal) Thickness ≤ 6.0 in.

B4.5.6.5 Aluminum-Minimum Properties

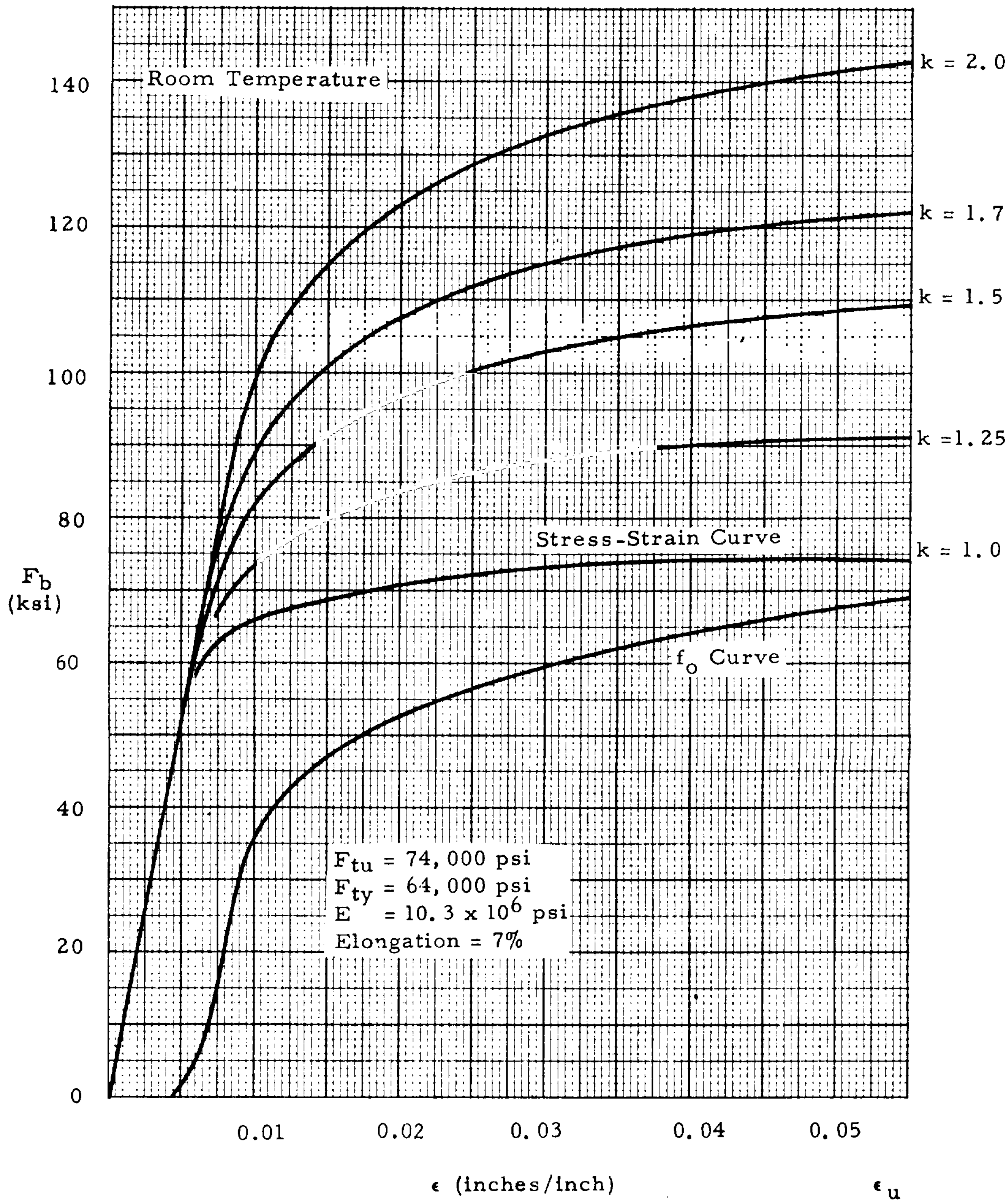


Fig. B4.5.6.5-32 Minimum Plastic Bending Curves 7079-T6 Aluminum Alloy Die Forgings (Longitudinal) Thickness ≤ 6.0 in.

Graph to be furnished when available

B4.5.6.5 Aluminum-Minimum Properties

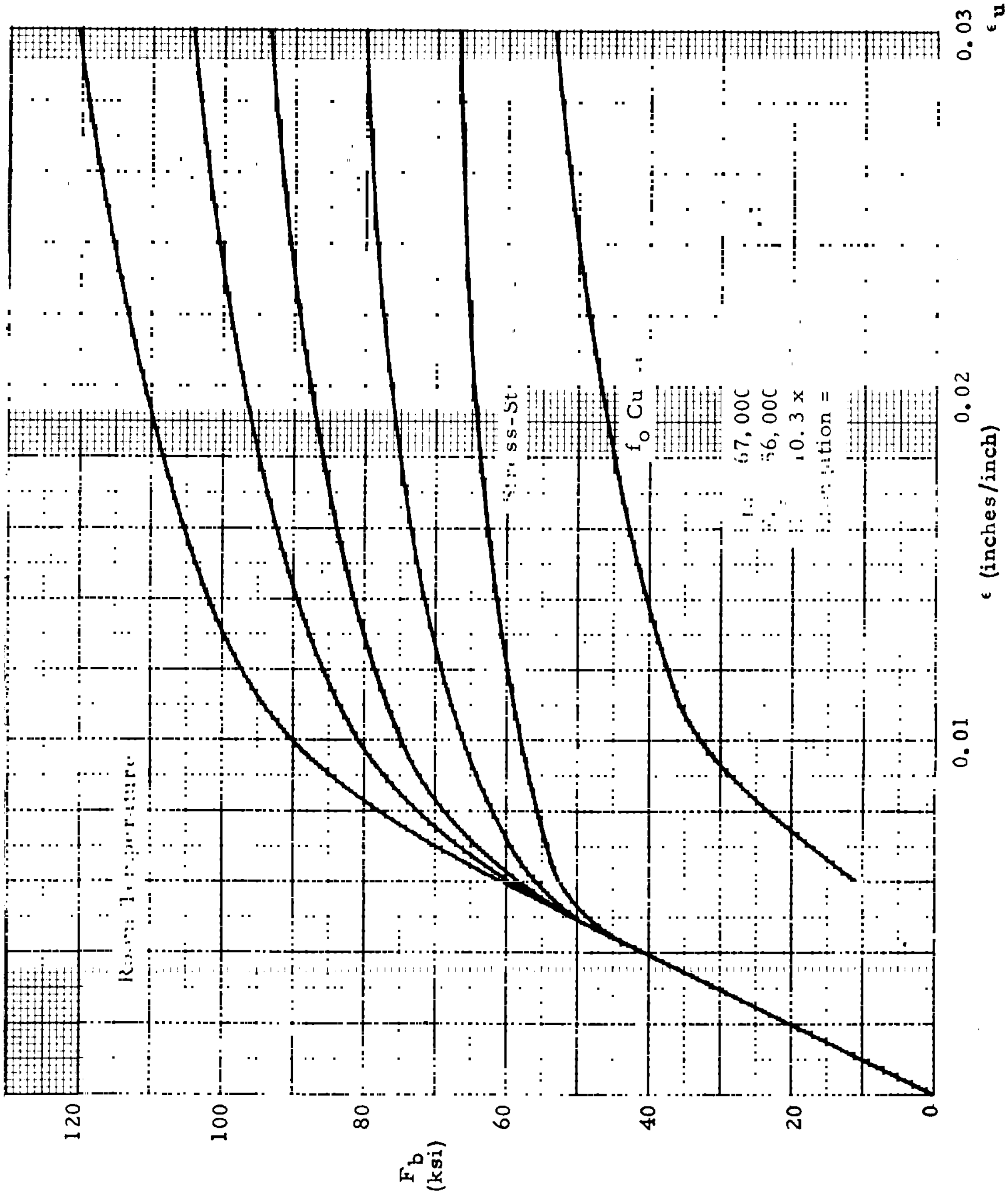


Fig. B4.5.6.5-34 Minimum Plastic Bending Curves 7079-T6 Aluminum Alloy Hand Forgings (Short Transverse) Thickness ≤ 6.0 in.

B4.5.6.5 Aluminum-Minimum Properties

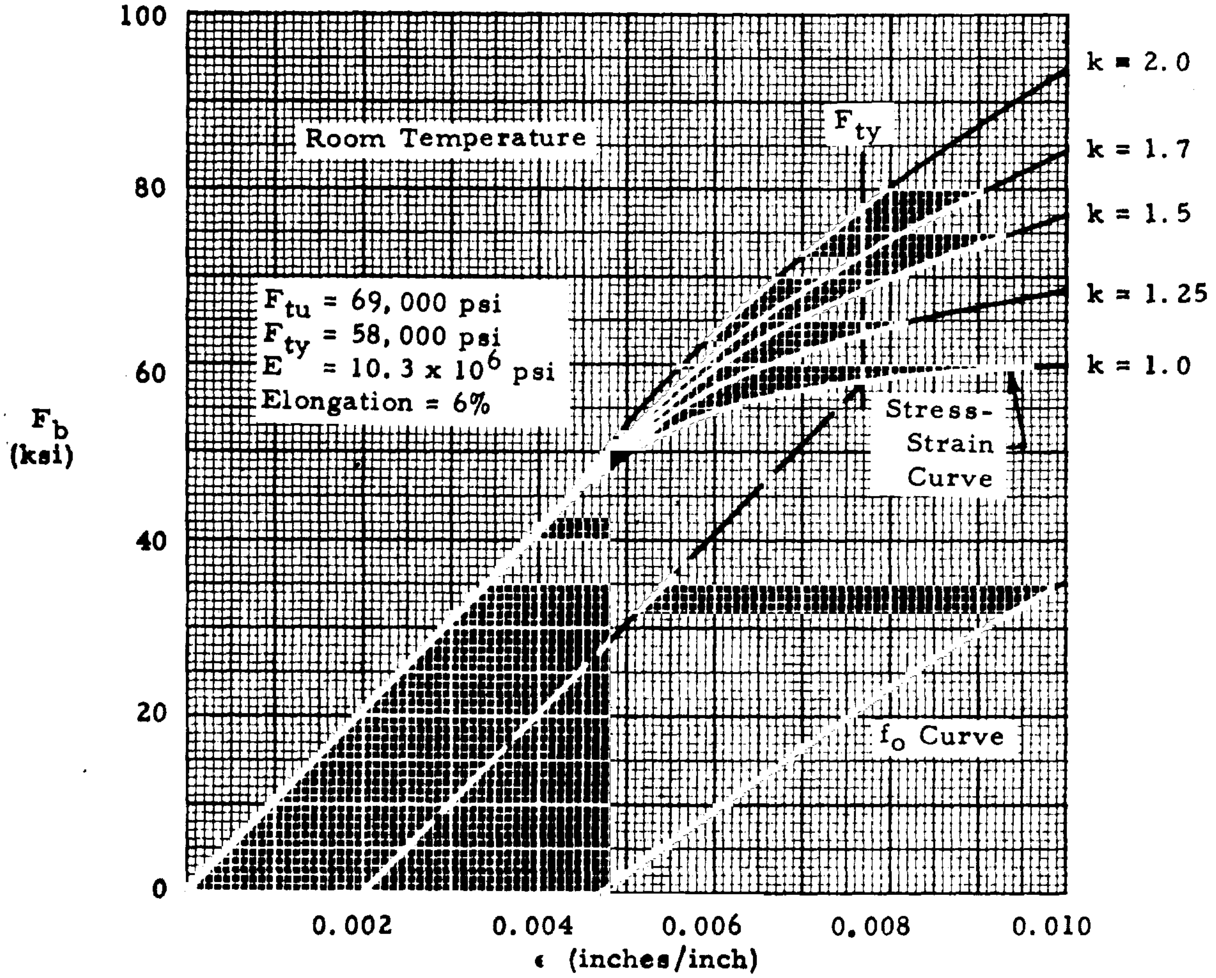


Fig. B4.5.6.5-35 Minimum Plastic Bending Curves 7079-T6 Aluminum Alloy Hand Forgings (Long Transverse) Thickness ≤ 6 in.

B4.5.6.5 Aluminum-Minimum Properties

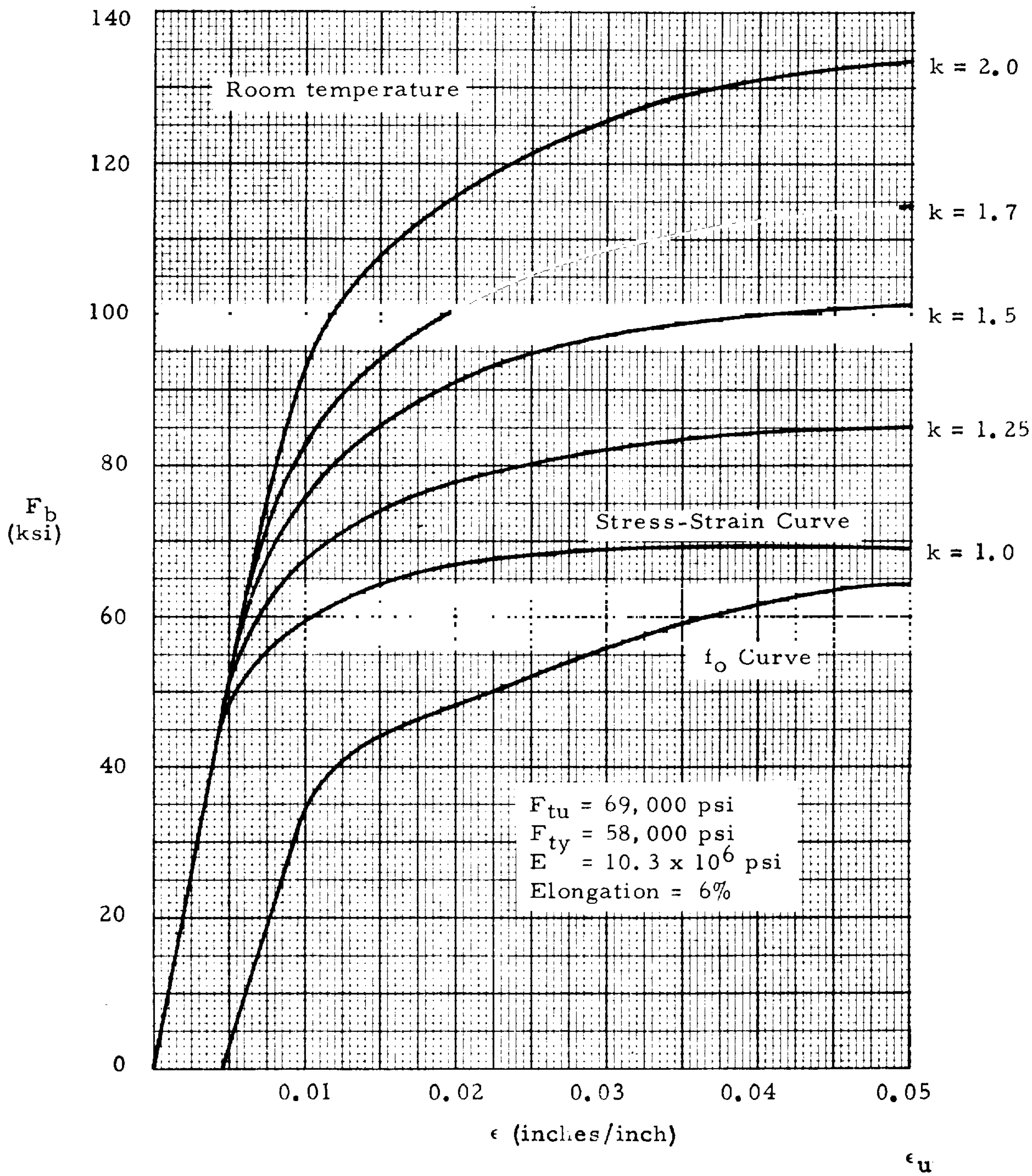


Fig. B4.5.6.5-36 Minimum Plastic Bending Curves 7079-T6 Aluminum Alloy Hand Forgings (Long Transverse) Thickness ≤ 6 in.

Graph to be furnished when available

B4.5.6.5 Aluminum-Minimum Properties

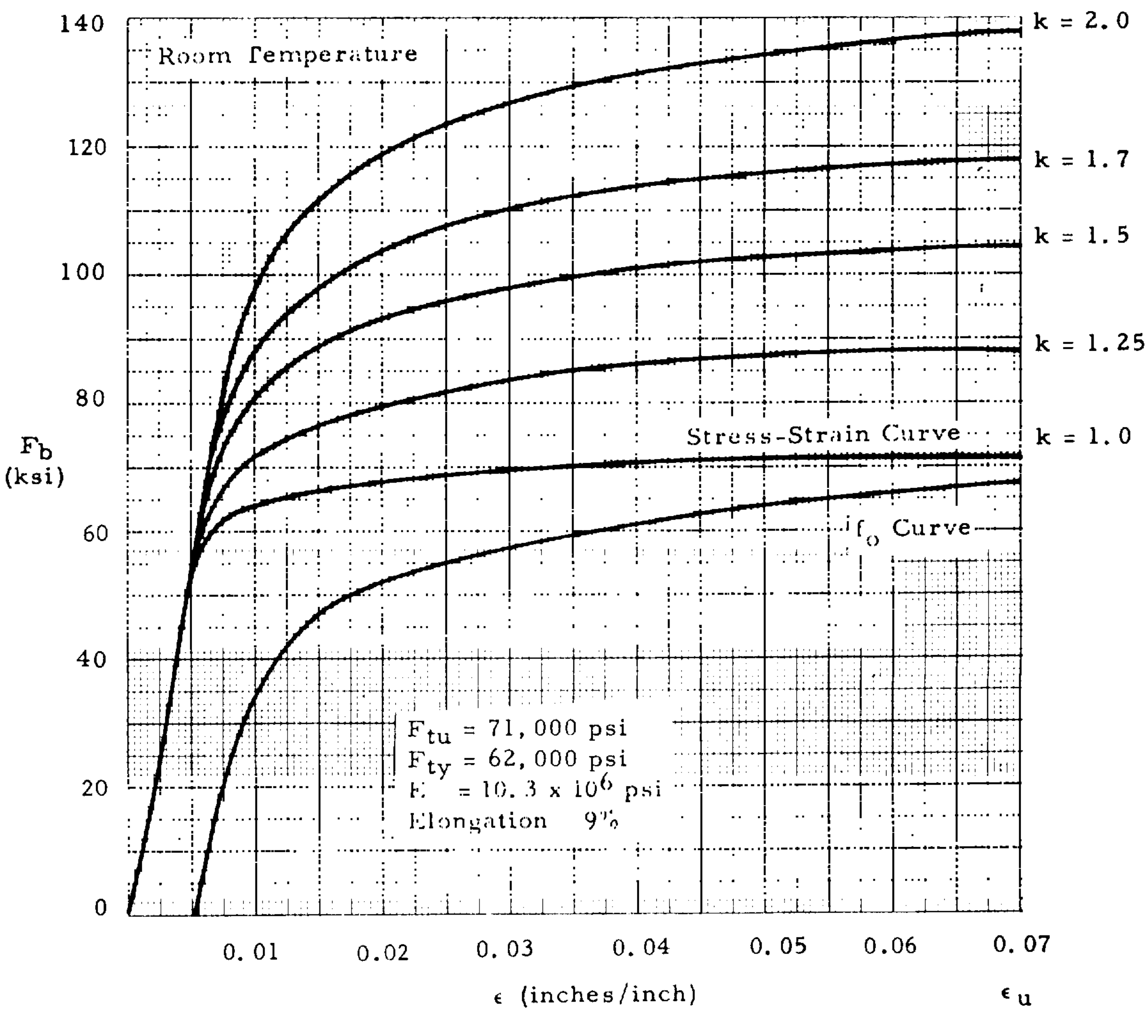


Fig. B4.5.6.5-38 Minimum Plastic Bending Curves 7079-T6 Aluminum Alloy Hand Forgings (Longitudinal) Thickness $\leq$ 6.0 in.

Graph to be furnished when available

B4.5.5.6 Magnesium-Minimum Properties

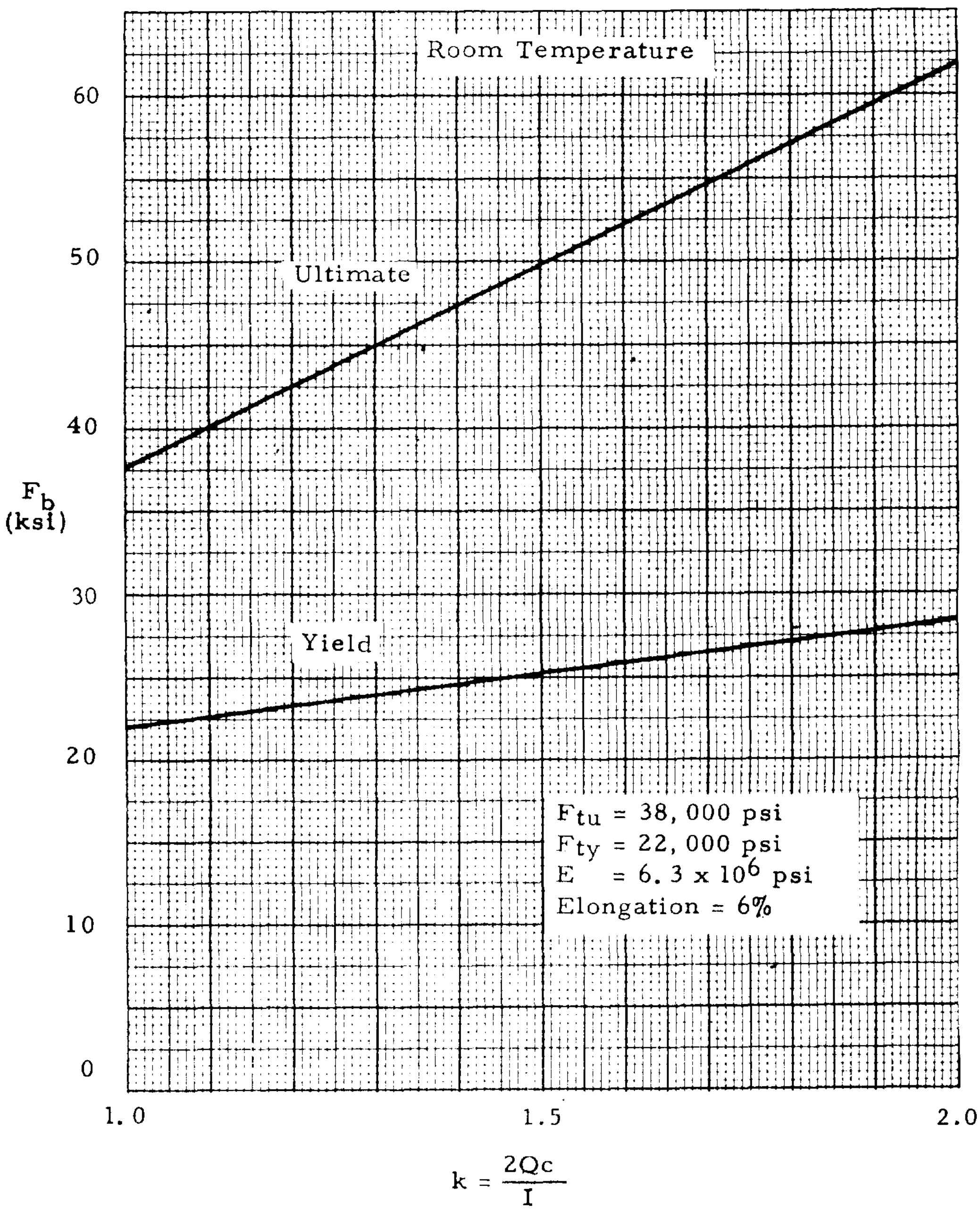


Fig. B4.5.5.6-1 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections AZ61A Magnesium Alloy Forgings (Longitudinal)

B4.5.5.6 Magnesium-Minimum Properties

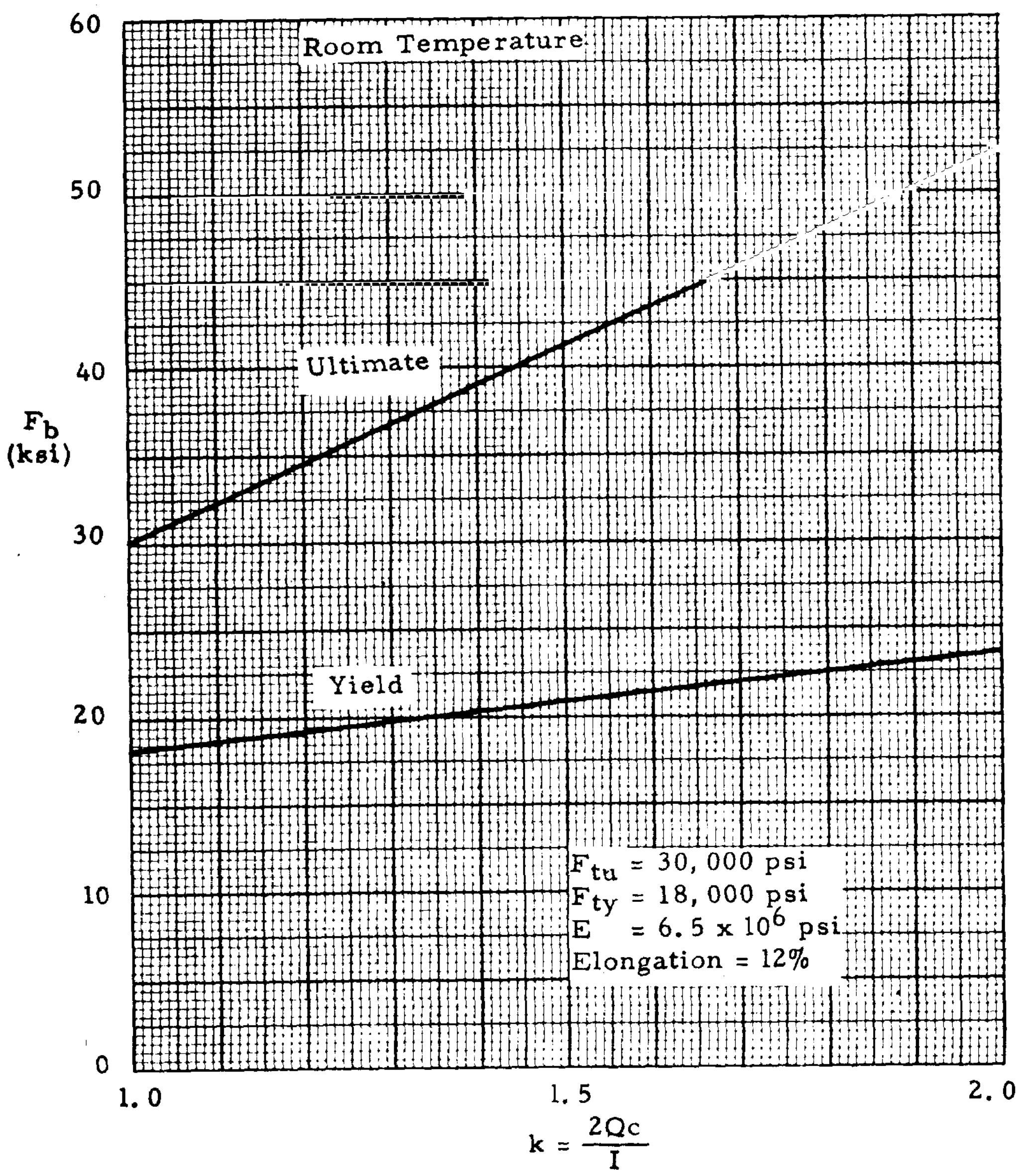


Fig. B4.5.5.6-2 Minimum Bending Modulus of Rupture for Symmetrical Sections HK31A-O Magnesium Alloy Sheet. 0.016 ≥ Thickness ≤ 0.250 in.

Graph to be furnished when available

Graph to be furnished when available

Graph to be furnished when available

B4.5.6.6 Magnesium-Minimum Properties

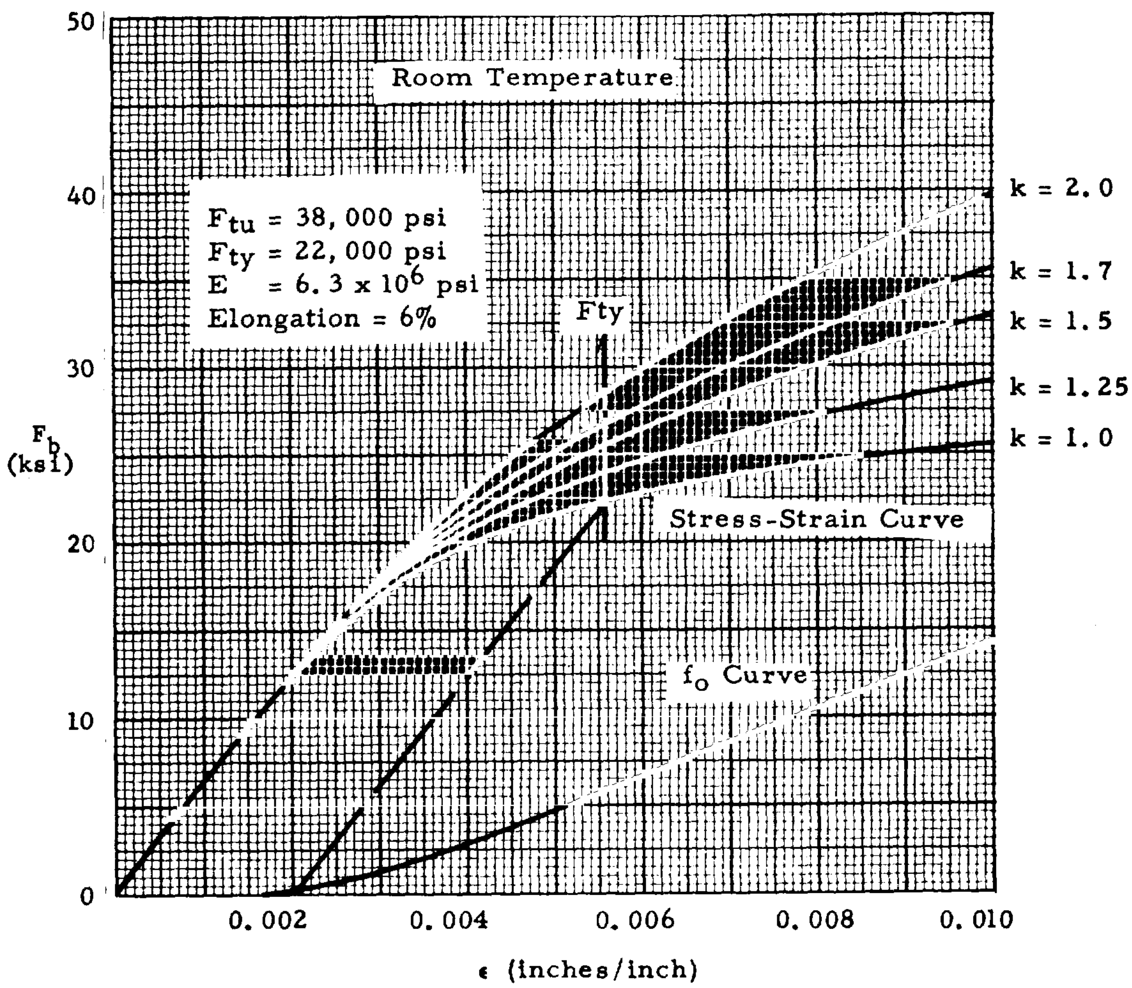


Fig. B4.5.6.6-3 Minimum Plastic Bending Curves AZ61A Magnesium Alloy Forgings (Longitudinal)

B4.5.6.6 Magnesium-Minimum Properties

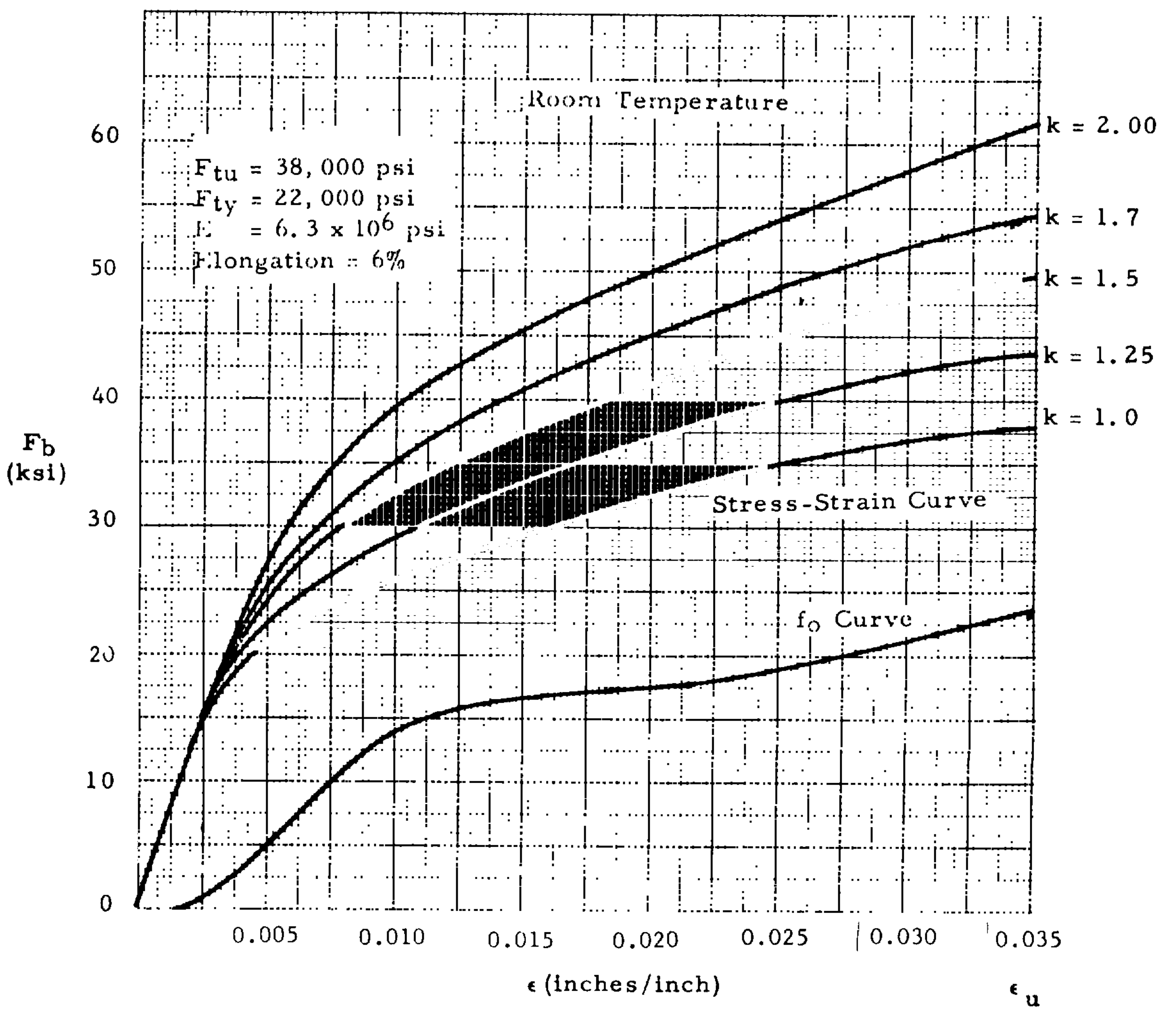


Fig. B4.5.6.6-4 Minimum Plastic Bending Curves AZ61A Magnesium Alloy Forgings (Longitudinal)

B4.5.6.6 Magnesium-Minimum Properties

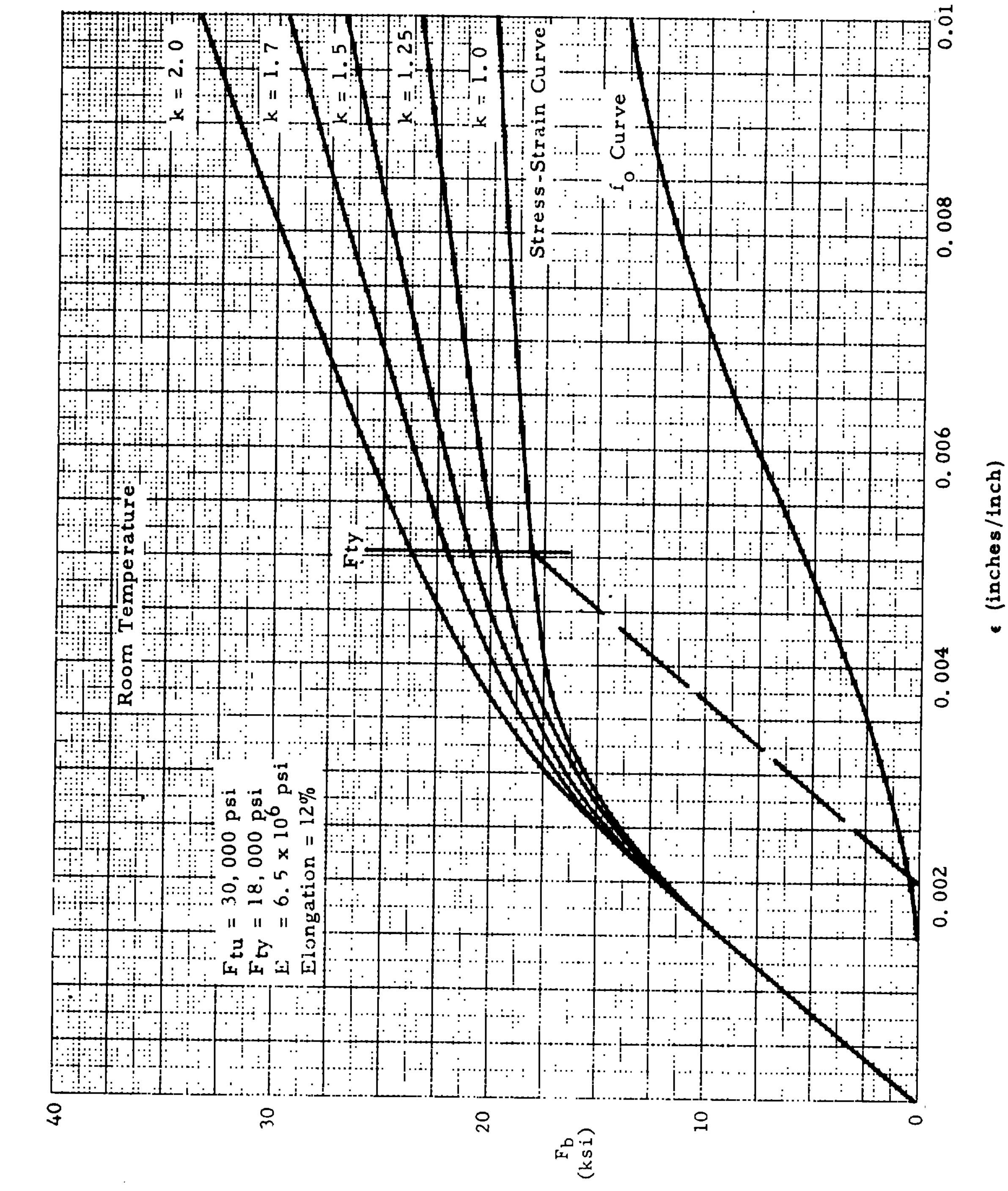


Fig. B4.5.6.6-5 Minimum Plastic Bending Curves HK31A-O Magnesium Alloy Sheet. Thickness ≤ .250

B4.5.6.6 Magnesium-Minimum Properties

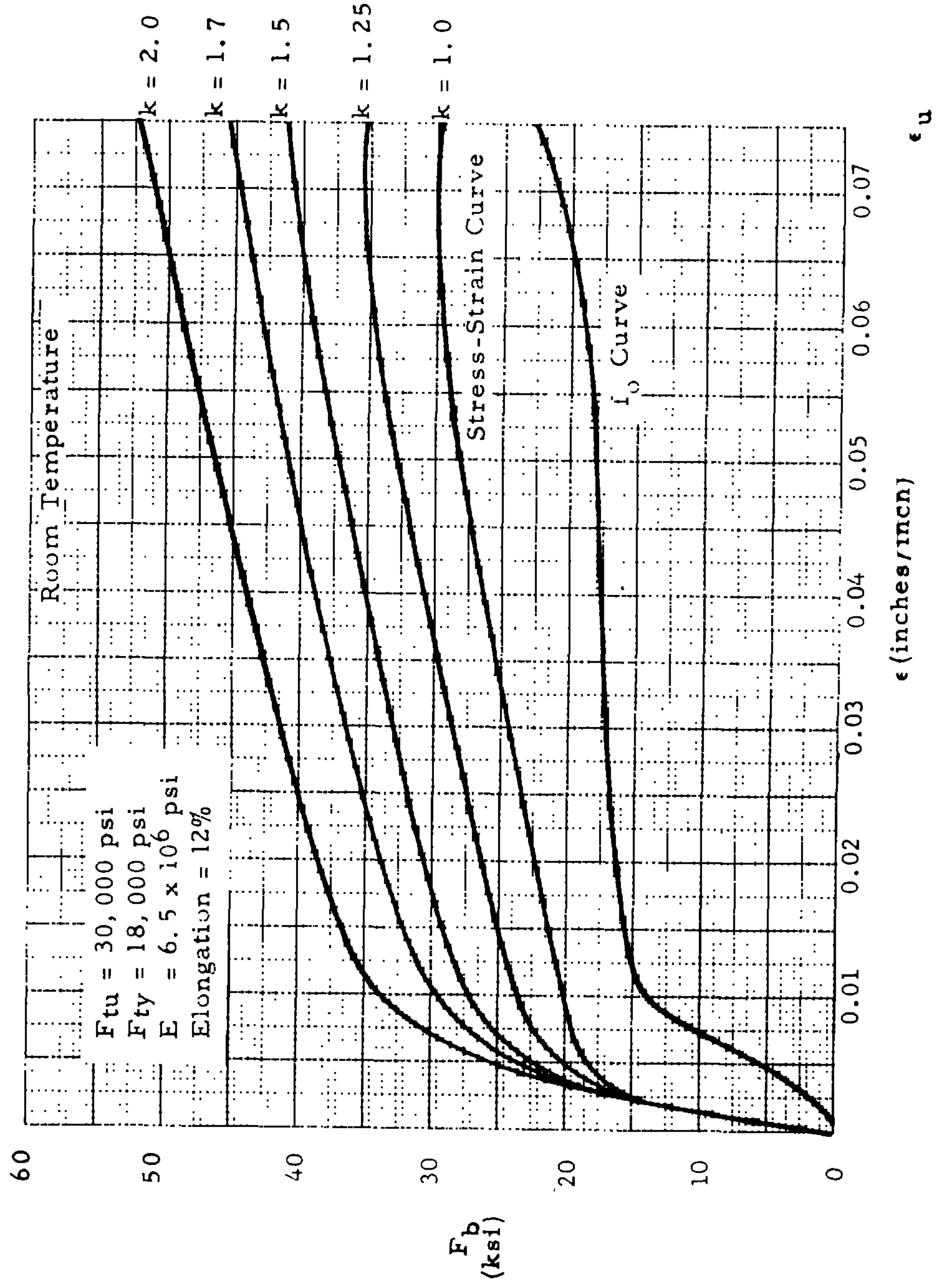


Fig. B4.5.6.6-6 Minimum Plastic Bending Curves for HK31A-O Magnesium Alloy Sheet. Thickness ≥ 016 and ≤ .250

Graph to be furnished when available

Graph to be furnished when available

Graph to be furnished when available

Graph to be furnished when available

B4.5.6.6 Magnesium-Minimum Properties

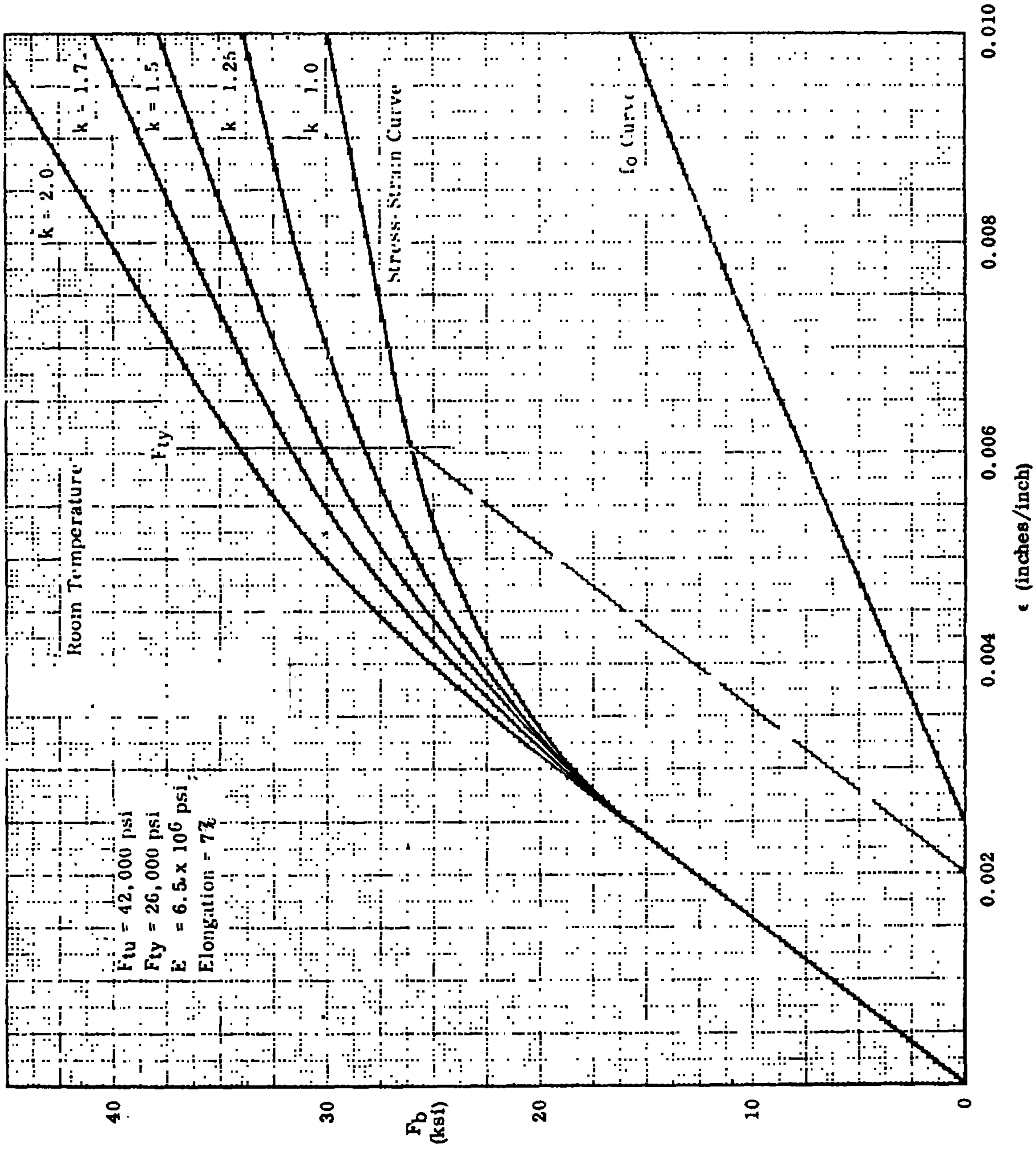


Fig. B4.5.6.6-11 Minimum Plastic Bending Curves ZK60A Magnesium Alloy Forgings (Longitudinal)

B4.5.6.6 Magnesium-Minimum Properties

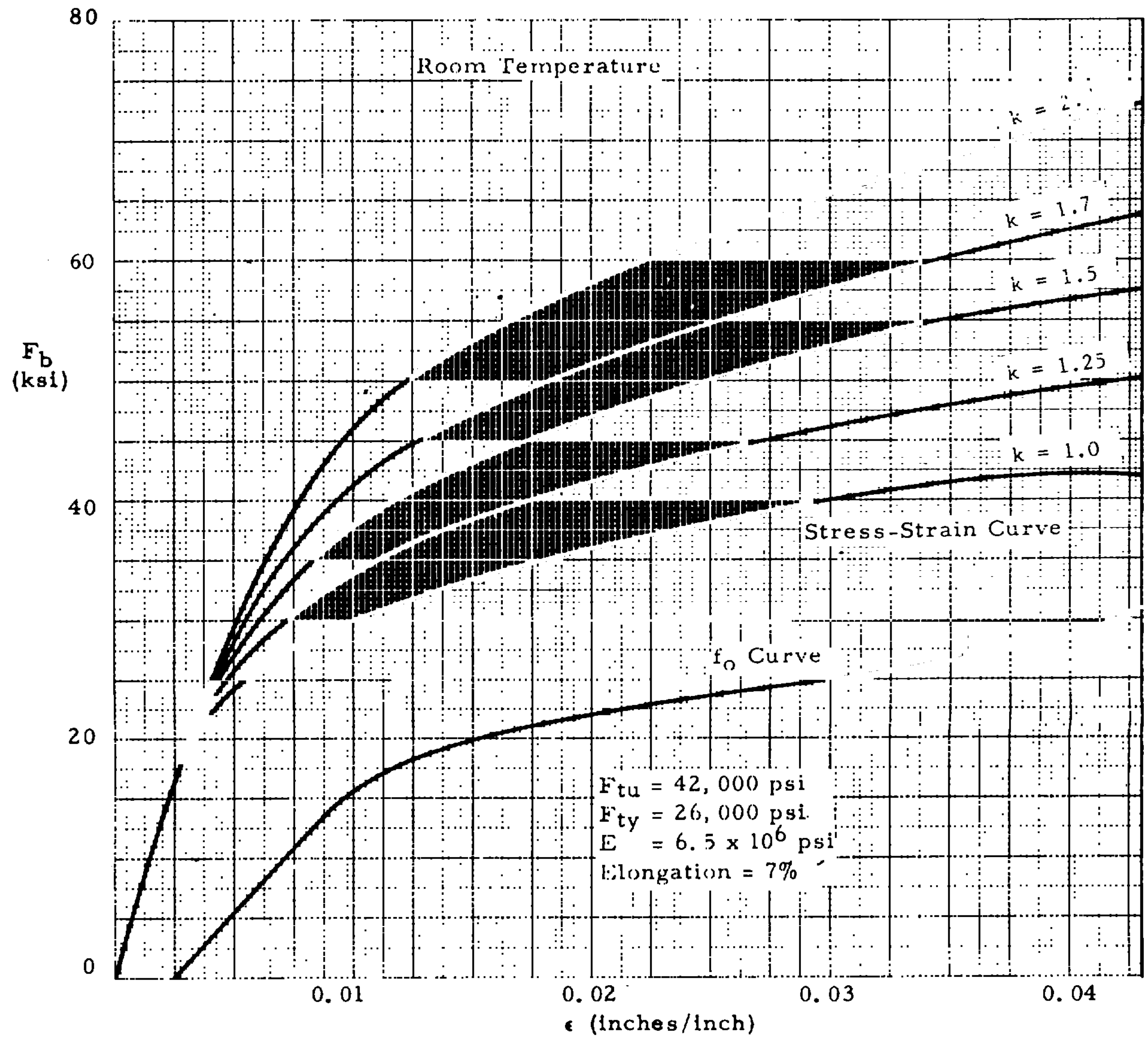


Fig. B4.5.6.6-12 Minimum Plastic Bending Curves ZK60A Magnesium Alloy Forgings (Longitudinal)

B4.5.7 Elastic-Plastic Energy Theory for Bending

B4.5.7.1 General

The Elastic-Plastic Energy Theory is defined as an extension of the Elastic Energy Theory into the plastic range of a material. This section will consider only energy due to bending stresses which may be in the elastic or plastic range, or both. Plastic bending curves found in Section B4.5.6 will be required. Deflections of statically determinate structures due to bending with any or all fiber stresses in the plastic range can be readily determined. Partially or completely plastic statically indeterminate structures can also be solved by procedures similar to those used in the Elastic Energy Theory. Other elastic theories could have been extended as well to include plastic bending effects but the Elastic Energy Theory was chosen due to its simplicity and common usage.

Elastic theories are accurate only if no part of a structure is stressed beyond the proportional limit of a material (no plastic strain). In structures designed to stress levels beyond the proportional limit, the error of an elastic deflection analysis is dependent on the amount of plastic strain involved. In some cases this error may be as much as 100% or more. Therefore when deflection is a limiting factor and plastic strains are involved, an analysis such as the Elastic-Plastic Energy Theory should be used.

B4.5.7.2 Discussion of Margin of Safety

In calculating the margin of safety at yield or ultimate, deflections must be considered as well as loads and/or stress levels, etc.; e.g., although a positive margin of safety for a structural element may be shown on the basis of loads, excessive deflections may occur. If the deflections are then the most critical design condition, the margin of safety becomes

$$\text{M.S.} = \frac{\text{Permissible Load}}{(\text{Safety Factor})\,(\text{Applied Load})} - 1 \qquad (4.5.7.2\text{-}1)$$

where the Permissible Load is the calculated load corresponding to the maximum permissible deflection. Equation (4.5.7.5-11) may be used in obtaining a permissible load level for a maximum permissible deflection by a trial and error process.

B4.5.7.3 Assumptions and Conditions

1. Energy is conserved; i.e., the external work due to a virtual load moving through a real deflection is equal to the internal strain energy developed during that deflection.

2. Plane sections remain plane; i.e., the strain is linearly distributed across any cross-section.

3. Poisson's ratio effects are negligible.

4. The deformations are of a magnitude so small as to not materially affect the geometric relations of various parts of a structure to one another.

B4.5.7.4 Definitions

dA - cross-sectional area of an infinitesimal volume, dV.

c - distance from the neutral axis to the extreme fibers of a cross-section.

δ - real deformation of an infinitesimal volume, dV, in the x-direction.

Δ - real vertical deflection of a beam at the point of application of a virtual load.

ϵ_b - total (elastic plus plastic) strain of an infinitesimal volume, dV, in the x-direction.

$\epsilon_{b_{max}}$ - extreme fiber strain of a cross-section.

F_v - virtual normal force acting on dA.

m - virtual bending moment in a beam due to the application of a virtual load.

W_e - external work equal to a virtual load moving through a real deflection.

W_i - internal strain energy equal to a summation of internal virtual forces times their real deflections.

Q - virtual unit load.

f_{b_v} - virtual bending stress on dA due to m.

B4.5.7.5 Deflection of Statically Determinate Beams

Consider the infinitesimal volume dV of Figure B4.5.7.5-1(a) and (b)

$$f_{b_v} = \frac{my}{I} \quad (4.5.7.5\text{-}1)$$

$$F_v = \text{stress} \times \text{area} = f_{b_v}\, dA$$

$$= \frac{my}{I}\, dA \quad (4.5.7.5\text{-}2)$$

$$\delta = \epsilon_b\, dx \quad (4.5.7.5\text{-}3)$$

Since plane sections remain plane,

$$\epsilon_b = \epsilon_{b_{max}} \frac{y}{c} \quad (4.5.7.5\text{-}4)$$

and

$$\delta = \epsilon_{b_{max}} \frac{y}{c}\, dx \quad (4.5.7.5\text{-}5)$$

By definition,

$$W_e = Q\,\Delta \quad (4.5.7.5\text{-}6)$$

$$W_i = \Sigma\, F_v\, \delta \quad (4.5.7.5\text{-}7)$$

Since energy is conserved,

$$W_e = W_i\,, \quad (4.5.7.5\text{-}8)$$

or

$$Q\,\Delta = \Sigma\, F_v\, \delta \quad (4.5.7.5\text{-}9)$$

Substituting Equations (4.5.7.5-2) and (4.5.7.5-5) into Equation (4.5.7.5-9) since Q is equal to unity, B4.5.7.5 Deflection of Statically Determinate Beams (Cont'd)

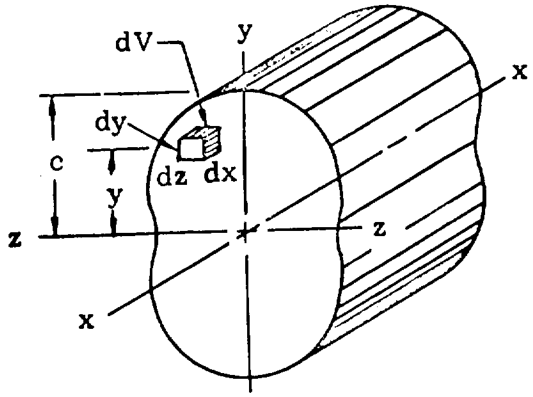
$$\Delta = \int_A \int_0^L \left(\frac{my}{I} \, dA \right) \left(\epsilon_{b_{max}} \; \frac{y}{c} \, dx \right)$$

$$= \int_A \int_0^L \frac{y^2 \, dA}{I} \; \frac{m \epsilon_{b_{max}}}{c} \, dx \, , \qquad (4.5.7.5\text{-}10)$$

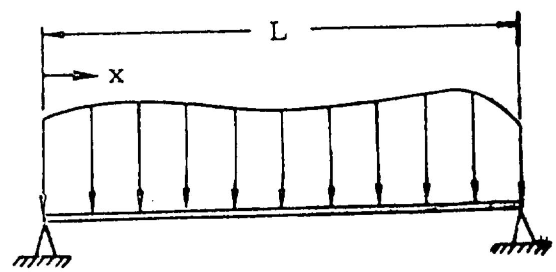
But, by definition, $I = \int_A y^2 \, dA$,

$$\therefore \Delta = \int_0^L \frac{m \, \epsilon_{b_{max}}}{c} \, dx \qquad (4.5.7.5\text{-}11)$$

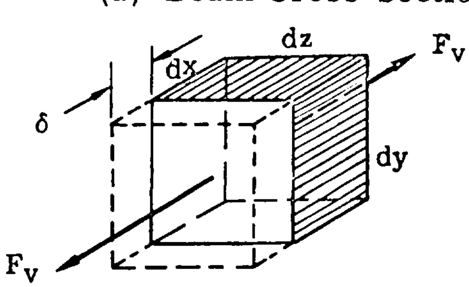
Equation (4.5.8.5-11) can now be solved graphically to find Δ. $\epsilon_{b_{max}}$ can be determined from a plastic bending curve for the applicable material as shown in Figure B4.5.7.5-1(c). Enter the F_b (modulus of rupture) scale with Mc/I and move horizontally across to the plastic bending curve for the specific cross-section; this intersection locates the corresponding $\epsilon_{b_{max}}$ on the ϵ (strain) scale. For beams with varying cross-section, c may be a variable.



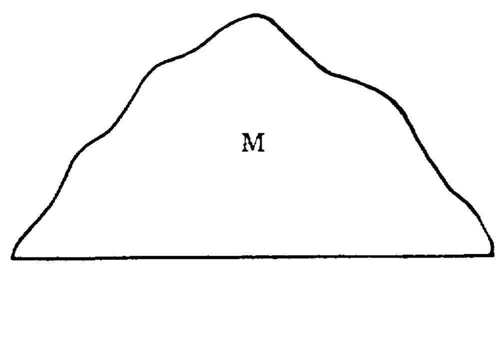
(a) Beam Cross-Section



(d) Real Load Diagram



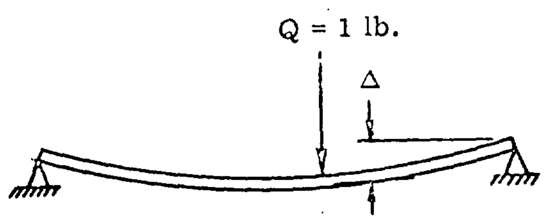
(b) Differential Volume, dV



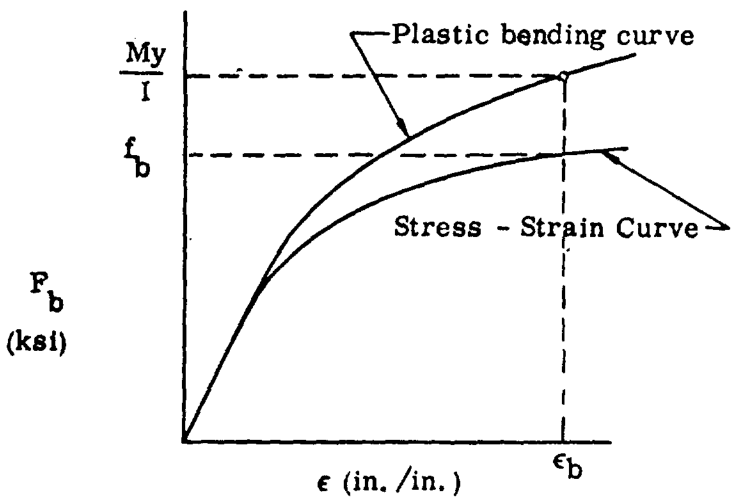
(e) Real Moment Diagram

Figure B4.5.7.5-1

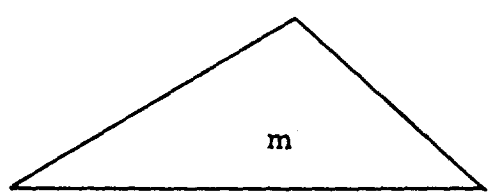
B4.5.7.5 Deflection of Statically Determinate Beams (Cont'd)



(f) Virtual Load Diagram
(Real Deflection Shown)



(c) Plastic Bending Diagram for Beam Cross-Section



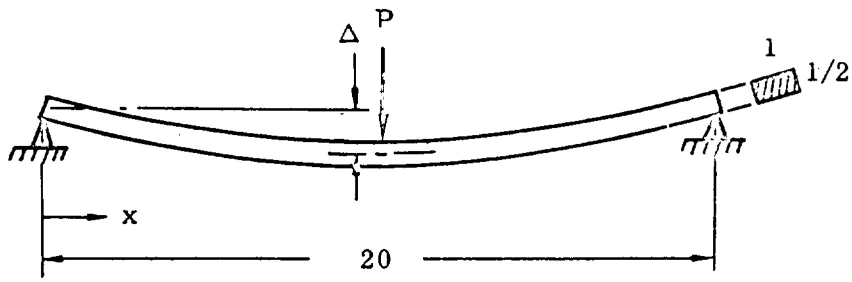
(g) Virtual Moment Diagram

Figure B4.5.7.5-1 (Cont'd)

B4.5.7.6 Example problem

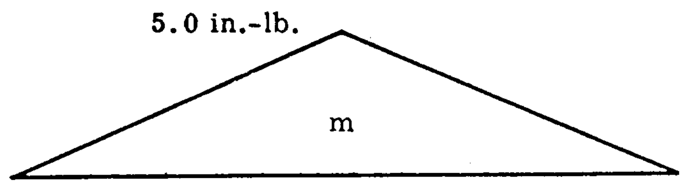
A rectangular beam, simply supported at the ends, is subjected to a concentrated vertical load at its center. Find the vertical deflection of the beam at its center.

Material: 2024-T3 Aluminum Alloy Plate
Beam dimensions: 1/2 in. x 1 in. x 20 in.
Load: P = 400 lb

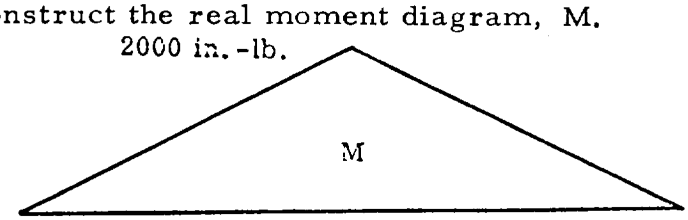


Procedure:

1. Apply a virtual unit load, Q, at the beam center and construct the virtual moment diagram, m.



2. Construct the real moment diagram, M.



3. Calculate the real bending stress, F_b, for each value of x by:

$$F_b = \frac{Mc}{I}$$

4. Enter the plastic bending curves on page 218 (at k=1.5 for a rectangular cross-section) with each value of F_b from Step 3 and determine $\epsilon_{b_{max}}$ for each value of x.

5. Multiply the value of m by the corresponding value of $\epsilon_{b_{max}}$ to obtain a value of $m\,\epsilon_{b_{max}}$ for each value of x.

B4.5.7.6 Example problem (Cont'd)

6. Tabulate the results of the previous steps (see Table B4.5.8.6-1).

7. Construct a plot of $m \epsilon_{b_{max}}$ vs. x and determine the area under the curve. This area represents $\int_0^L m \epsilon_{b_{max}} dx$. (See Figure B4.5.8.6-1)

8. Calculate Δ:

$$\Delta = \int_0^L \frac{m \epsilon_{b_{max}}}{c} dx \qquad \text{Ref eq (4.5.7.5-11)}$$

$c = 0.250$ in.

$$\int_0^L m \epsilon_{b_{max}} dx = 0.165 \qquad \left(\text{by graphical integration}\right)$$

$$\therefore \Delta = \frac{0.165}{0.250} = \underline{\underline{0.661 \text{ in.}}}$$

[By an elastic analysis, was found to be 0.6104 in. which is 7.7% in error. Partially plastic fiber stresses existed only over the middle 8 inches in this example. The elastic analysis would be considerable more in error for higher loadings and /or beams in which the plastic stresses exist over greater lengths; e.g., beams of constant moment, etc.]

B4.5.7.6 Example problem (Cont'd)

P = 400 lb.					
x, in.	m in.-lb.	M in.-lb.	Mc/I, ksi	$\epsilon_{b\,max}$, in./in. x 10^{-3}	$m\,\epsilon_{b\,max}$, in.-lb. x 10^{-3}
0	0	0	0	0	0
1	.5	200	4.8	.4570	.2285
2	1.0	400	9.6	.9140	.9140
3	1.5	600	14.4	1.3710	2.0565
4	2.0	800	19.2	1.8280	3.6560
5	2.5	1000	24.0	2.2850	5.7125
6	3.0	1200	28.8	2.7420	8.2260
7	3.5	1400	33.6	3.14	10.9900
8	4.0	1600	38.4	3.80	15.2000
9	4.5	1800	43.2	4.64	20.8800
10	5.0	2000	48.0	5.83	29.1500
11	4.5	1800	43.2	4.64	20.8800
12	4.0	1600	38.4	3.80	15.2000
13	3.5	1400	33.6	3.14	10.9900
14	3.0	1200	28.8	2.7420	8.2260
15	2.5	1000	24.0	2.2850	5.7125
16	2.0	800	19.2	1.8280	3.656
17	1.5	600	14.4	1.3710	2.0565
18	1.0	400	9.6	.9140	.9140
19	.5	200	4.8	.4570	.2285
20	0	0	0	0	0

Table B4.5.7.6-1

B4.5.7.6 Example problem (Cont'd)

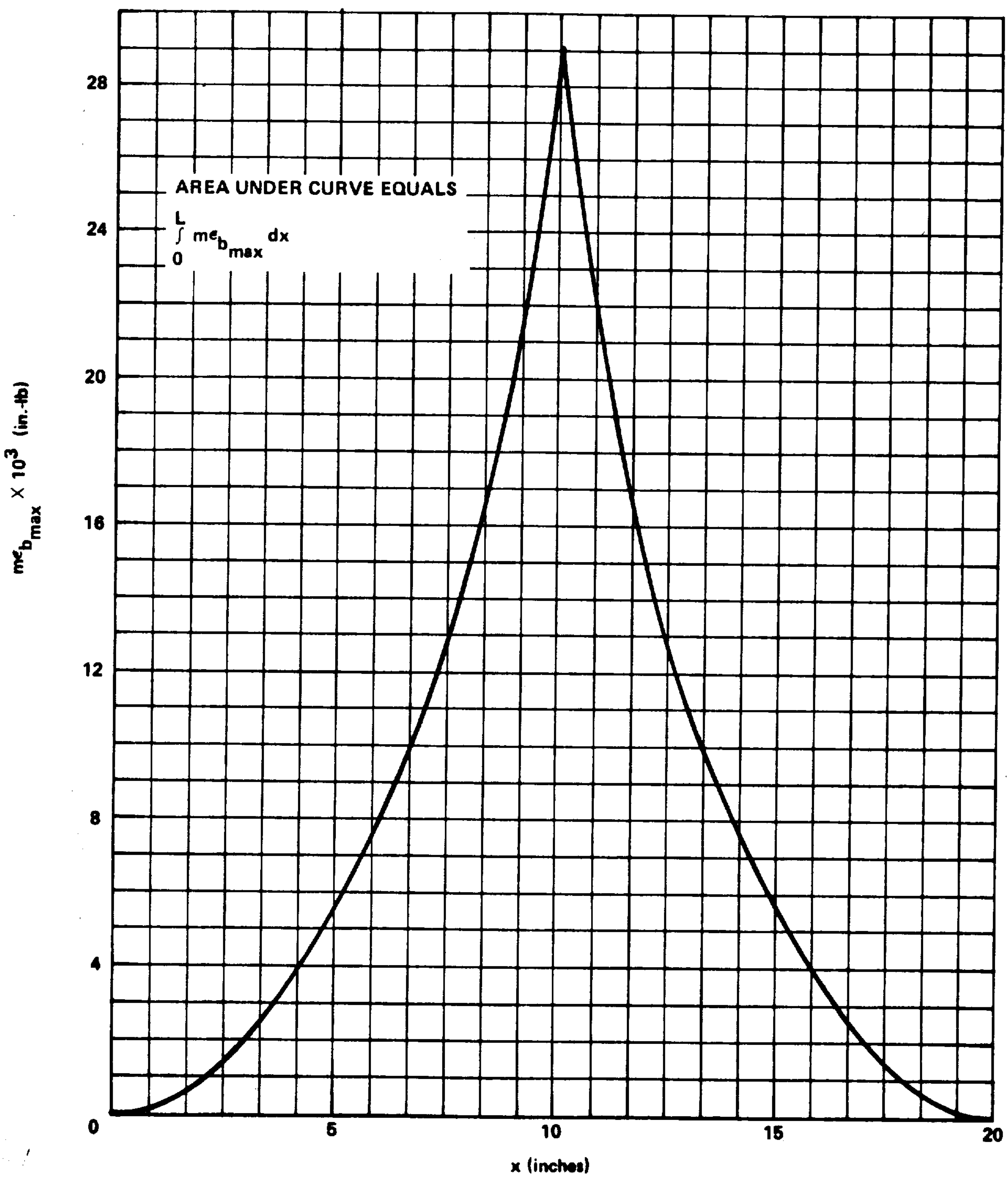


Figure B4.5.7.6-1

SECTION B4.6

BEAMS UNDER AXIAL LOAD

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

B4.6.0 BEAM-COLUMNS

B4.6.1 Introduction

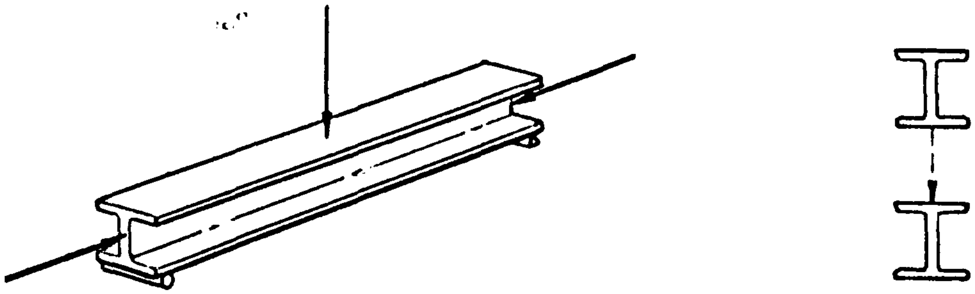
Beam-columns are structural members which are subjected simultaneously to axial and bending loads. The bending may arise from transverse loads, couples applied at any point on the beam, surface shear loads, or from end moments resulting from eccentricity of the axial load at one end (or at both ends) of the member.

There are many problems associated with the analysis of beam-columns. For example, individual beam and column effects cannot be superimposed since they are interdependent. Initial curvature, distortion, inelastic effects, and restraint conditions all affect the deformational characteristics and are important factors. Both strength and overall instability need to be considered. In cases where lateral support is lacking, lateral instability (torsional instability) must be considered. If the elements of the section are relatively thin, and the beam-column is relatively short, local buckling or crippling may occur.

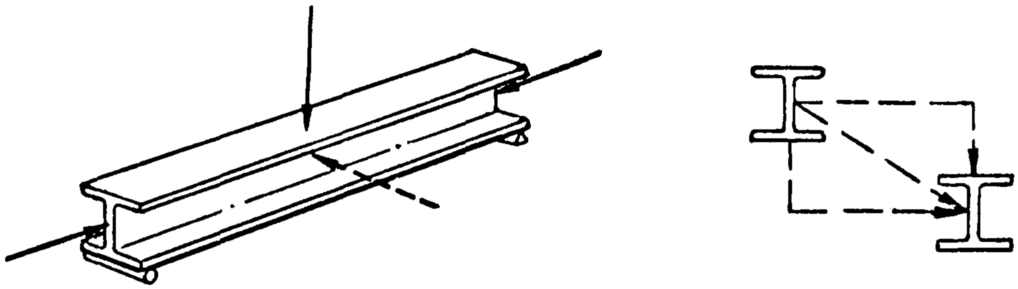
The analysis of shear-web beams is distinctly different from the analysis of simple beams. As a consequence, the effects of axial loads on shear-web beams are beyond the scope of this section; they are, however, considered in the section on shear beams.

It has been shown that beam-columns can have one of three types of overall response (References 1 and 2). These are shown in Figure 1 as indicated below:

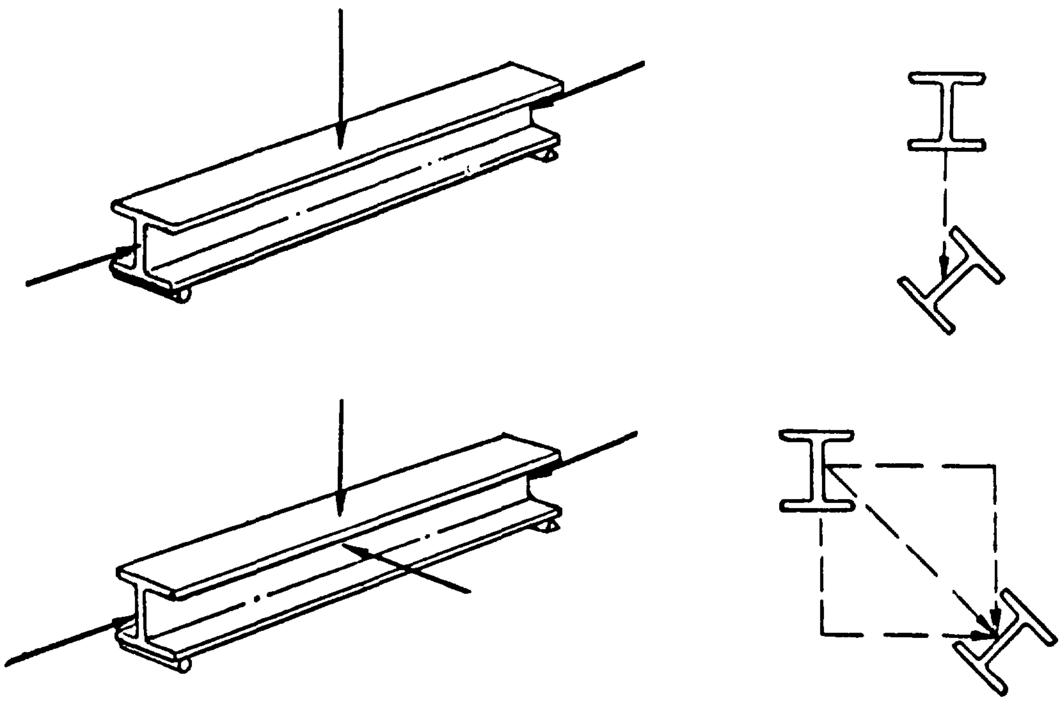
(a) In-Plane Response



(a) IN-PLANE RESPONSE



(b) BIAXIAL IN-PLANE RESPONSE



(c) COMBINED BENDING AND TWISTING RESPONSE

FIGURE 1. BEAM-COLUMN RESPONSES

(b) Biaxial In-Plane Response

(c) Combined Bending and Twisting Response

The particular response for a given member is dependent mainly upon five conditions. These are:

(a) Cross Section Shape

(b) Span Length

(c) Amount of Intermediate Lateral (Torsional) Support

(d) Restraint Conditions at Boundaries

(e) Loading

The analysis procedures employed for these responses will be discussed in the following paragraphs.

In general, the analysis and design procedures for beam-columns with in-plane response are well developed for both the elastic and inelastic stress ranges. However, procedures for biaxial in-plane response and combined bending and twisting response are limited, especially in the inelastic stress range.

One of the more powerful techniques employed in the analysis of beam-columns is referred to as "interaction equations;" these equations are based on experimental data, and they require that the strength of the member as a column and the strength as a beam be determined separately. These strengths are then expressed in terms of stress ratios (applied load/strength) and incorporated in the interaction equation which expresses the effects of the combined loading. Interaction equations will be used in the following discussion.

B4.6.2 Notation

A	Area of Cross Section (in.^{2})
r	Torsion Warping Constant (in.^{6})
d	Eccentricity (in.)
E	Modulus of Elasticity (lbs/in.^{2})
E_t	Tangent Modulus of Elasticity (lbs/in.^{2})
f	Stress (lbs/in.^{2})
f_{cb}	Nominal Extreme Fiber Stress at Lateral Buckling (lbs/in.^{2})
G	Elastic Shear Modulus (lbs/in.^{2})
h	Distance between Centers of Flanges (in.)
I	Moment of Inertia (in.^{4})
I_y	Moment of Inertia about Minor Axis (in.^{4})
K	Uniform Torsion Constant (in.^{4})
j	$\frac{EI}{P}$ (see Tables 1, 2, and 3)
L	Length of Member (in.)
L'	Effective Length (in.)
M	Bending Moment (in.-lb)
M_x, M_y	Principal Axes Bending Moments (in.-lb)
M_{actual}	Bending Moment due to Both Bending and Axial Loads (in.-lb)
M_e, $(M_x)_e$, $(M_y)_e$	Bending Moment at the Elastic Stress Limit or Yield Point (in.-lb)
$(M_x)_u$, $(M_y)_u$	Bending Moment at the Inelastic Stress Limit (in.-lb)

M_1, M_2	Applied Moments at Each End of Beam-Column Loaded with Unequal End Moments (in.-lb)
M_{eq}	Equivalent Moment for Beam-Column Loaded with Unequal End Moments (in.-lb)
M.S.	Margin of Safety
P	Applied Axial Load (lbs)
P_e	Critical Column Buckling Load in the Elastic Stress Range (Euler Load); $P_e = (\pi^2 EI_{min})/(L')^2$
$(P_x)_e$, $(P_y)_e$	Critical Column Buckling Loads in the Elastic Stress Range for Each Principal Axis (lbs)
P_u	Critical Column Buckling Load in the Inelastic Stress Range (lbs)
U	L/j (see Tables 1, 2, and 3)
W	Transverse Load (lb)
w	Transverse Unit Load (lb per linear in.)
Y	Deflection (in.)
Z_x, Z_y	Section Moduli about Principal Axes (in.^{3})
θ	Slope of Beam (radians) to Horizontal, Positive when Upward to the Right

B4.6.3 In-Plane Response

In-plane response, as mentioned herein, refers to beam-columns which, when subjected to combined axial and bending loads acting in one plane, respond substantially without twisting in the same plane, as shown in Figure 1a. This response usually occurs when adequate lateral (torsional) support is provided, or when torsionally stiff sections are used. Beam-columns which respond in this manner have been investigated extensively for both the elastic stress range and the inelastic stress range; these are discussed separately below.

B4.6.3.1 Elastic Analysis

The elastic analysis for the strength of beam-columns is based on the assumption that failure occurs when the computed value of the combined axial and bending stress in the most highly stressed fiber reaches the yield point or yield strength of the material. This definition, in a sense, does not consider the danger of buckling; as such, it is not correct in the limiting case of a pure column. Initial imperfections are always present, and contribute to the response. Thus, if these imperfections are considered, the definition above remains valid. Further discussion of this concept is given by Massonnet (Ref. 2).

B4.6.3.2 Inelastic Analysis

The maximum bending strength of a beam actually is higher than the hypothetical elastic limit strength. As the applied bending moment increases above the yield moment, yielding penetrates into the cross section as shown in Figure 2. By comparison of Figures 2b and 2d, it can be seen

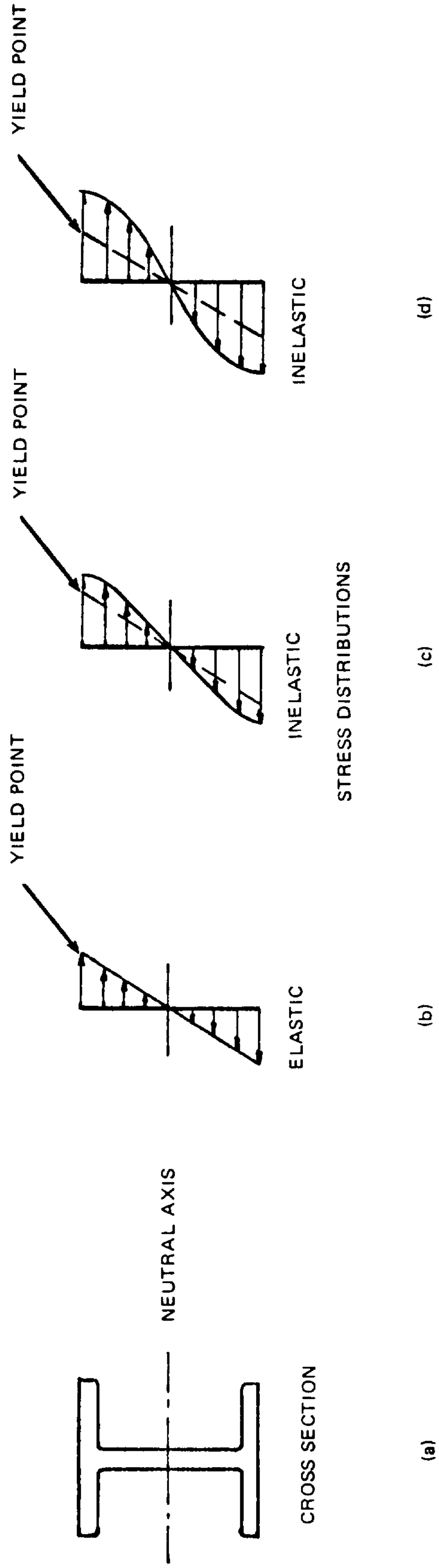


FIGURE 2. RESERVE STRENGTH

that a greater strength is obtained by considering the inelastic stress distribution. Analysis procedures which utilize this extra, or reserve, strength are known as inelastic analyses; this extra strength has been shown to vary, and to depend on three factors (Ref. 2).

(a) The M_{max}/P Ratio - The reserve strength is almost zero for the case of pure buckling (M = 0) and tends, as M_{max}/P increases, toward the value that is associated with pure bending.

(b) The Shape of the Cross Section - For example, the reserve strength is smaller for an I-section than it is for a rectangular section.

(c) The Nature of the Metal - For example, the reserve strength is much higher for material A than it is for material B in Figure 3. (The stress-strain diagram for material A does not have a flat portion, as does the diagram for material B).

Several failure criteria have been used in the inelastic analysis for the strength of beam-columns (Refs. 3, 4, 5, and 6). Basically, the criterion used has been one of the following:

(a) Maximum Stress Criterion - Failure occurs at some prescribed maximum stress level in the inelastic range.

(b) Maximum Strain Criterion - Failure occurs at some prescribed maximum strain level in the inelastic range.

(c) Ultimate Load Criterion - Failure occurs at some ultimate, or collapse, load which utilizes the inelastic behavior of the material.

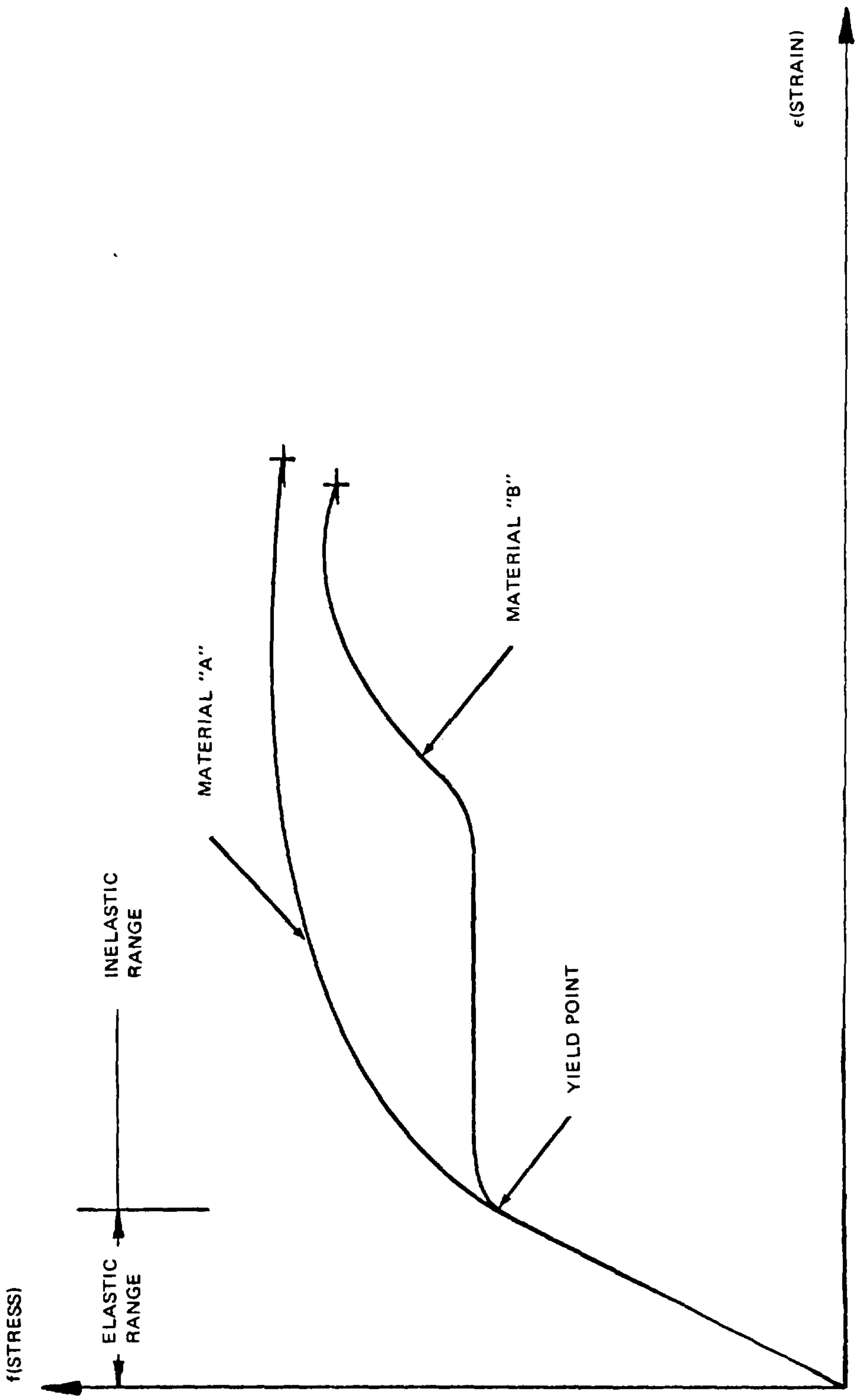


FIGURE 3. RESERVE STRENGTH, MATERIAL VARIABLE

B4.6.4 Biaxial In-Plane Response

This section considers the strength analysis methods of torsionally stiff beam-columns which are subjected to applied bending forces that cause either bending about the major principal axis only or bending about both principal axes (Fig. 1b). The beam-columns are free to deflect in all directions, without twisting. This type of response can occur as a result of one of the following two conditions:

(a) Primary Biaxial Bending

(b) Primary Bending in the Strong Direction

Case 1 is discussed in the following paragraphs.

Little information is available for Case 2. However, it has been predicted that short members with large bending forces will respond essentially in the plane of the applied forces and develop as much strength as if they were restrained from deflecting in the weak direction. On the other hand, long slender members with small bending forces buckle in a direction normal to the plane of the bending forces and develop essentially the strength of the member loaded concentrically, that is, the bending forces cause no loss of strength. Intermediate length members may respond about both principal axes simultaneously, and the strength will then be less than the strength for responding exclusively in one direction or the other.

B4.6.4.1 Elastic Analysis

In the elastic range, when the limiting stress failure criterion is used, the determination of the stresses due to bending about the two principal axes can be made independently, as there is no coupling of the flexural actions. Thus, although few test data are available, practical procedures for the prediction of strength for such cases are based on a knowledge of in-plane response. Hence, elastic solutions for the case of in-plane response can be extended to include biaxial in-plane response; but it has been found (Ref. 1) that the limiting stress criterion is even more conservative for cases of response about both axes than it is for cases of response about one axis. Austin (Ref. 1) extended one form of the interaction equation for in-plane response to include biaxial in-plane response.

B4.6.4.2 Inelastic Analysis

Austin (1) states that there have been no precise theoretical studies based on the von Kármán theory of the strength of beam-columns which fail by bending about both principal axes without twisting. From a practical viewpoint, Austin has extended one form of the interaction equation for in-plane response to biaxial in-plane response for the inelastic range. Little test information is available for this phenomenon. In general, it appears that methods used to predict the inelastic in-plane response, can be extended to predict the inelastic biaxial in-plane response.

B4.6.5 Combined Bending and Twisting Response

Structural members of thin-walled open cross section, when subjected to combined axial and bending forces, may respond by combined bending and twisting (Figure 1c). This phenomenon is commonly called torsional-flexural buckling or buckling by torsion-bending. The twisting action is a result of the low torsional stiffness of members with open cross section such as I, channel or angle. It should be noted that in the discussion which follows it is assumed that the open section members are not subjected to directly applied torsional couples, such as arise when the line of action of transverse loading does not pass through the shear center (Figure 4). The shear center is defined as the point through which the shear force must pass if the member is to bend without twisting.

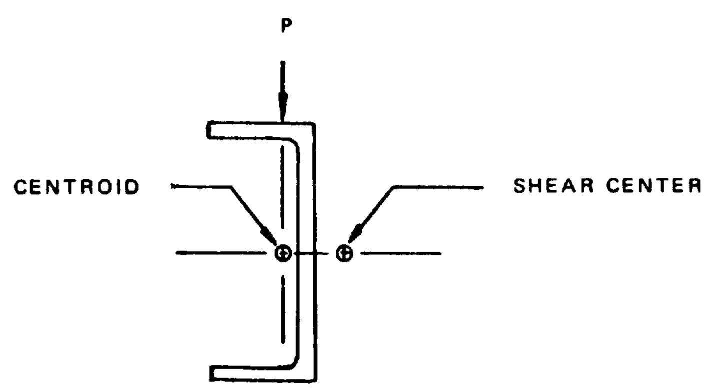


FIGURE 4. SHEAR CENTER

Columns of open cross section will respond by combined bending and twisting under any of the following three conditions:

(a) Axial compression and moments acting to cause bending about both principal axes;

(b) Axial compression and moments acting to cause bending only about the major principal axis when the moment of inertia about the major axis is much greater than about the minor axis; for example, an I-section member with axial compression and end moments acting to cause bending in the plane of the web; and

(c) Axial compression and moments acting to cause bending in a plane parallel to the plane of either principal axis, when the principal axis does not contain the shear center as well as the centroid, as may occur for a channel, tee, or an angle section.

Little information is available on the strength and behavior of members subjected to the first and third conditions (Ref. 1). The greatest amount of study has been devoted to the second case, primarily for I-section members (Refs. 4, 6, 7, 8, 9, 10,and 11). The second case is discussed in the following paragraphs.

B4.6.5.1 Elastic Analysis

As mentioned above, most of the elastic analyses for beam-columns subjected to combined bending and twisting have been limited to uniform I-section members. It has been found that the behavior is similar to the lateral buckling action of an I-beam subjected to transverse forces only. However, exact formulas for critical loads for torsional-flexural buckling are complex (Refs. 4, 6, and 11), therefore, interaction equations have been used. These interaction equations have been shown to agree closely with available data. An interaction equation has been proposed by Hill, Hartman, and Clark (Ref. 10) for aluminum beam-columns; this equation has been verified by Massonnet (Ref. 2) for steel beam columns.

In a theoretical study, Salvadori (Ref. 12) found that the interaction equation used by Hill, Hartman, and Clark gave safe predictions for the combinations of axial compression and bending which produce elastic buckling by torsion-bending. Salvadori considered members whose ends were free to rotate in the plane of the web, but were elastically restrained with respect to rotation in the planes of the flanges.

Little analytical work has been done on the elastic response of tapered members under combined bending and axial load. However, Butler and Anderson (Ref. 13) have performed tests on tapered steel beam-columns, and compared the test results to Salvadori's interaction curves. The outcome of the comparison suggests that Salvadori's curves can be applied to tapered as well as uniform beam-columns. Also, analytical studies of "solid" tapered members have been made by Gatewood (Ref. 14), and the interaction curves obtained are essentially independent of the degree of taper and are closely approximated by the results of Salvadori.

B4.6.5.2 Inelastic Analysis

Again, the majority of studies in the inelastic range have been devoted to I-section members. Hill and Clark (Ref. 9) have shown that the interaction equation used in elastic analysis can also be extended for use in the inelastic range by using the tangent modulus concept. Massonet has also extended the elastic interaction equation for steel into the plastic domain. The results of this extension were compared with results of their tests of steel I-section columns and found to be in excellent agreement for oblique eccentric loading, moment at one end only, and equal and opposite end moments (Ref. 1). Galambos (Ref. 15) presents a thorough study of the inelastic lateral buckling of beams which may be applied in the interaction equation.

B4.6.6 Recommended Practices

Detailed and comprehensive stress analysis shall be performed to ensure efficiency and integrity of the member or structure; and the design shall comply with the particular structure or vehicle requirements, e.g., reliability. In general, the methods of analysis shall follow those given herein. In utilizing these methods, minimum weight designs shall be given prime consideration and the member shall be so designed that

(a) There shall occur no instabilities resulting in collapse of the member from the application of the design loads; and

(b) Deformations resulting from limit loads shall not be so large as to impair the function of the member or nearby components, or so large as to produce undesirable changes in the loading distribution.

The immediate problem in the design of a beam-column is the choice of a suitable cross section to withstand the combined axial and bending loads. Because of the number of variables, direct choice of section is not usually feasible, except for the selection of a shape, such as round, rectangular, I-section, etc. Thus, in general, successive trials must be made to determine the safest and most economical section. The preliminary selection of the cross section at any station along the member may, in many cases, be based on the elementary formula for stress

$$f = \frac{P}{A} \pm \frac{M_x}{Z_x} \pm \frac{M_y}{Z_y} , \qquad (1)$$

where the interaction of the transverse and axial loads is neglected. Naturally, this selection must be improved by a refined analysis.

In choosing an optimum shape, particular attention should be given to the degree of lateral restraint or end restraint which will or can be provided. For example, if little or no lateral restraint is provided, then a torsionally stiff section such as a box or tubular section should be used. Also, when the type of response is not known, all three conditions

(a) In-plane response

(b) Biaxial in-plane response

(c) Combined bending and twisting response

should be analyzed to determine the critical response.

If the analyst or designer has the choice of elastic or inelastic analysis procedures, the following factors should be considered in making the choice.

(a) Member function

(b) Material

(c) Deflection limitations - Deflections may be excessive in the inelastic range

(d) Thermal conditions - Little information is available on thermal effects in the inelastic range

(e) Dynamic conditions - Little information is available for inelastic effects due to dynamic loadings

(f) Reliability

(g) Analysis procedures available for the method of analysis and types of loadings considered.

The recommended practices and procedures for the analysis of beam-columns are discussed in the paragraphs which follow. The analysis and design for local buckling and crippling should be in accordance with the methods presented in Section C1. In complying with the references and recommendations cited above, the designer should keep abreast of current structural research and development. With this approach, optimum methods and designs should be achieved.

B4.6.6.1 In-Plane Response

B4.6.6.1.1 Elastic Analysis

Tensile Axial Loads - The strength of beam-columns subjected to combined flexure and tensile axial loads has been investigated for many commonly encountered cases. In Tables B4.6.1 and B4.6.3 results are tabulated for some of these cases. In general, adequate solutions and methods of analysis for other beam-column loading conditions are available in several references (16, 17, 18, 19 and 20).

Compressive Axial Loads - Many exact solutions have been obtained for the case of beam-columns subjected to combined flexure and compressive axial loads. Results are given in Tables 2 and 3 for some of the more commonly encountered conditions. A number of these and other conditions are discussed below.

Beam columns with intermediate supports have been investigated. Probably the three moment equations (Refs. 11, 18, 20, and 21) are the most powerful method of analysis for this case. A variety of conditions is covered in this technique; for example:

(a) Any type of transverse loads can be included

(b) Span lengths may vary

(c) The moment of inertia may vary from span to span

(d) The effects of rigid supports not in a straight line

(e) The effects of intermediate spring supports (Ref. 11 p. 23)

(f) The effects of uniformly distributed axial loads (Ref. 21).

Niles and Newell (Ref. 18) present tabulated results for many of the cases listed above. Another analytical method which can be used for this problem consists of a solution adapted to matrix form. Saunders (Refs. 22, 23) demonstrates the application of the "transfer matrix" technique to the analysis of nonuniform beam-columns on multi-supports.

Methods and solutions for beam-columns on an elastic foundation can be found in several references (6, 17, 24, and 25). Hetényi (Ref. 17) is a particularly good source for formulation and solution of the differential equations for this problem. He considers both axial tensile and axial compressive loads. See Table B.4.6.3 for illustrations of some of these cases.

For the cases of tapered and stepped members, numerical methods have been shown to give good results with a savings in time and labor. Newmark's method (Ref. 26) is particularly applicable to beam-columns of variable cross section and can be extended to include many commonly encountered loading conditions. Basically, the method is a numerical integration by a sequence of successive approximations. Salvadori, Baron (Ref. 27) define finite difference numerical methods which can be used for the analysis of beam-columns with many loading conditions.

Other conditions not covered in Tables B.4.6.2 and B.4.6.3 can be found in references (4, 11, 16, 18, 19, 26, 28, 29, and 30). The analytical methods which can be adapted to the solution of problems which have not been investigated are available in several references (4, 11, 16, 18, 19, 20, 26, and 29).

The abundance of exact solutions and methods of solution for the elastic analysis of beam-columns which respond in-plane reduces the need for using interaction equations. However, the interaction equation can be, and is,used in many instances. The following simple straight line equation is the basis for several interaction equations,

$$\frac{P}{P_e} + \frac{M_{actual}}{M_e} = 1. \qquad (2)$$

Since M_{actual} is the moment resulting from both the axial and transverse loads, it can be difficult to obtain for complicated conditions. However, for members which satisfy all of the following requirements:

(a) Must be simply supported

(b) Must have uniform cross section

(c) May be subjected to any combination of bending forces producing maximum moment at or near the center of the span

it has been shown (Refs. 1, 2, 10, and 31) that a good approximation of the actual bending moment is given by

$$M_{actual} = \frac{M_x}{1 - P/(P_x)_e} \qquad (3)$$

Here $(P_x)_e$ is the Euler elastic critical load in the plane of the applied moment and M_x is the maximum moment, not considering the moment due to the axial load interacting with the deflections. For the condition of moment due to interaction of the axial load with deflection, Eq. (2) becomes

$$\frac{P}{P_e} + \frac{M_x}{(M_x)_e \left\{1 - P/(P_x)_e\right\}} = 1 \qquad (4)$$

and the corresponding margin of safety is given by

$$M.S. = \left[\frac{1}{\dfrac{P}{P_e} + \dfrac{M_x}{(M_x)_e \left\{1 - P/(P_x)_e\right\}}}\right] - 1. \qquad (5)$$

For eccentrically loaded members, where d is the eccentricity, equal at both ends, Eq. (2) takes the following form

$$\frac{P}{P_e} + \frac{Pd}{(M_x)_e \left\{1 - P/(P_x)_e\right\}} = 1. \qquad (6)$$

The margin of safety for this case can be determined analogously to Eq. (5).

The interaction equation may be written in terms of an equivalent moment, M_{eq}, for a beam-column subjected to unequal end moments as shown in Figure 5.

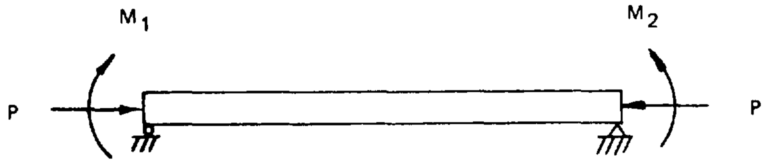


FIGURE 5. UNEQUAL END MOMENTS

Equation (2) becomes

$$\frac{P}{P_e} + \frac{M_{eq}}{(M_x)_e \{1 - P/(P_x)_e\}} = 1 \qquad (7)$$

where a good approximation for M_{eq} as given by Austin (1) is

$$\frac{M_{eq}}{M_1} = 0.6 + 0.4 \frac{M_2}{M_1} \quad \text{for } 1.0 \geq \frac{M_2}{M_1} \geq -0.5 \qquad (8)$$

and

$$\frac{M_{eq}}{M_1} = 0.4 \text{ for } -0.5 \geq \frac{M_2}{M_1} \geq -1.0. \qquad (9)$$

Accurate interaction formulas which are simple and general have not been developed for beam-columns with other than simple supports; for example, cases where each end can be free, hinged, fixed, or elastically restrained both with respect to rotation and translation. However, it is conservative to use Eq. (4) with $M_x = (M_x)_{max}$, where $(M_x)_{max}$ is the maximum moment in the member and is determined by an ordinary structural analysis without regard to the effects of axial load. In utilizing this method, the effective length concept should be employed in determining the Euler load.

B4.6.6.1.2 Inelastic Analysis

A practical interaction formula for predicting the strength of metal beam-columns under combined compressive axial loads and bending, which respond in-plane and in the inelastic region, has been shown (Refs. 1, 2, 7, 10,and 31) to be

$$\frac{P}{P_u} + \frac{M_x}{(M_x)_u \{1 - P/(P_x)_e\}} = 1. \qquad (10)$$

where P_u is the strength of the member as a column in the inelastic range. The value of P_u can be found by the tangent modulus method (Refs. 4 11), Johnson's modified parabolas (Refs. 29, 31), or by methods presented in the section on columns. As before, M_x is the moment due to transverse bending without the axial load, and $(M_x)_u$ is the ultimate moment which the section can inelastically withstand. In determining the ultimate moment, $(M_x)_u$, for the interaction equation above, one of the three methods presented in paragraph 4.4.3.2,

(a) Trapezoidal Stress Distribution

(b) Plastic Design

(c) Double Elastic Moduli

can be used. The trapezoidal stress distribution method developed by Cozzone (Refs. 5, 29) is widely used in the aerospace industry in structural design. It is especially adaptable to metals which behave like aluminum alloys. In general, the trapezoidal stress distribution method will be preferable; however, there are certain cases where one of the other methods may prove superior. For example, the plastic design method (Refs. 3, 16 and 32) has been primarily developed for steels.

Obviously, interaction equation 10 is an extension of the one presented in the previous section (Eq. 4). As a consequence, it is subject to the same limitations, i.e., simple supports, uniform sections, and equal end-moments. However, for beam-columns with unequal end-moments (Fig. 10), the equivalent moment (M_{eq}) as defined by equations 8 and 9 may be substituted for M_x in equation 10. Additional investigation is required to determine the limits of applicability of this interaction equation for other boundary and loading conditions.

In employing the inelastic method, excessive deflections may occur. In general, except for the case of plastic design, there are few or no methods for calculating the resulting deflections. Thus, if there is a prespecified deflection limitation the inelastic method may not be adequate.

Inelastic analysis procedures for combined tensile axial loads and bending have not been developed. However, previous elastic analytical methods can probably be extended to cover inelastic behavior.

B4.6.6.2 Biaxial In-Plane Response

Beam-columns which are torsionally stiff and free to deflect in all directions may have a biaxial in-plane response condition (Fig. 1) resulting from either of two loading conditions. These two conditions consist of compressive axial load and

(a) Primary Bending in the Strong Direction; or

(b) Biaxial Bending.

For case 1, primary bending in the strong direction, the member is always designed so that it will have only in-plane response in the strong direction. This is accomplished by checking the buckling value for the weak direction, and, if necessary providing adequate intermediate supports or stiffness for that direction. For the condition where a biaxial inplane response does occur, Austin (Ref. 1) and Massonnet (Ref. 2) both provide an excellent discussion.

It can be seen that only biaxial in-plane response for case 2 is of primary practical interest. Therefore, it is discussed in the following paragraphs.

B4.6.6.2.1 Elastic Analysis

"The determination of the stresses due to bending about the two principal axes can be made independently as there is no coupling of the flexural actions in the elastic range" (Ref. 1). Thus, solutions for elastic in-plane response problems (Paragraph 4.4.6.1.1) can be extended to include bending about both principal axes by simple superposition of results.

For beam-columns which are subjected to axial compression and bending moments about both principal axes, Austin has also stated that the interaction equation for in-plane response, Equation 4, can be modified to take into account the biaxial loading. Thus Equation 4 becomes

$$\frac{P}{P_e} + \frac{M_x}{(M_x)_e \{1 - P/(P_x)_e\}} + \frac{M_y}{(M_y)_e \{1 - P/(P_y)_e\}} = 1. \quad (11)$$

Equation 11 is subject to the same restrictions as Equation 4. Also, the equivalent moment concept as discussed in Paragraph 4.4.6.1.1 can be utilized for unequal end moments.

B4.6.6.2.2 Inelastic Analysis

The recommended procedure for inelastic analysis is an extension of the interaction equation from in-plane response to include biaxial in-plane response. Equation 10 then is

$$\frac{P}{P_u} + \frac{M_x}{(M_x)_u\{1 - P/(P_x)_e\}} + \frac{M_y}{(M_y)_u\{1 - P/(P_y)_e\}} = 1. \quad (12)$$

Using this interaction equation, the same restrictions discussed in Paragraph 4.4.6.1.2 will also apply here.

4.4.6.3 Combined Bending and Twisting Response

The methods summarized in the preceding sections are applicable to problems of in-plane response, and should be applied only to beam-columns which are restrained against twisting by adequate bracing or to beam-columns which possess a high torsional rigidity.

A torsionally weak beam-column of open section such as the wide-flange, tee, or angle is apt to twist as well as bend during the response. The various possibilities wherein twist may be involved are summarized as follows:

(a) If the shear center axis and centroidal axis are not coincident, the member may respond by a combination of twisting and bending, with the tendency toward twist failure increasing for very thin-walled, torsionally weak, short column sections.

(b) If the shear center axis and the centroidal axis are coincident, as in the case of the I- or Z- shapes, buckling by pure twist may occur without bending.

The recommended formulae given herein for both elastic and inelastic analysis are in the form of interaction equations and are applicable only to cases with bending in the strong direction. These equations have a simple form, are convenient to use, are accurate, and have a wide scope of application. If a theoretical solution is desired, References 4, 6, and 11 contain analytical investigations of torsional-flexural response for many common sections and loadings. However, most of the work has been done for axial compression and moments acting to cause only bending about the major principal axis when the moment of inertia about the major axis is much greater than about the minor axis (e.g., I-section). It has been proposed by Austin that the interaction equations can be extended to include primary bending about both axes. However, there are few data, experimental or analytical, available to verify this.

B4.6.6.3.1 Elastic Analysis

For the elastic analysis of doubly symmetric I-section members subjected to primary bending in the plane of the web the following interaction equation is recommended:

$$\frac{P}{P_e} + \frac{M_x}{f_{cb}\, Z_x \{1 - P/(P_x)_e\}} = 1. \qquad (13)$$

The value of f_{cb} is the nominal extreme fiber stress at lateral buckling for a member subjected to a uniform moment causing bending in the plane of the web. The value of f_{cb} is given by

$$f_{cb} = \frac{\pi^2 E I_y h}{2 Z_x L^2} \sqrt{1 + \frac{KGL^2}{\pi^2 E r}} \qquad (14)$$

A complete study of this lateral buckling is given by Clark and Hill in Reference 8.

When the maximum moment is not at or near the center of the span, the interaction equation may be excessively conservative. This is particularly true when end moments are of opposite sign and the maximum end moment is used for M_x. However, the interaction equation cited above can be used if an equivalent uniform moment is calculated and substituted for M_x. The recommended expression for the determination of the equivalent uniform moment as given by Massonnet (Ref. 2) is

$$M_{eq} = \sqrt{0.3 \quad (M_1)^2 + (M_2)^2 \ + \ 0.4 \ M_1 M_2} \quad . \qquad (15)$$

Massonnet also states that the interaction equation is not necessarily limited to doubly symmetric I-shaped sections, but can be used for all shapes provided that the proper effective lengths, equivalent moments, and appropriate expressions for f_{cb} are adopted. However, these extensions should be applied with discretion, as little work has been done to support this.

Relatively few studies have been conducted on the elastic stability of nonuniform or tapered beam columns which fail by combined bending and twisting. Butler, Anderson (Ref. 13), and Gatewood (Ref. 14) have investigated tapered members, and their results indicate that the previous interaction equation may be used. However, additional tests under a variety of loading conditions will be necessary to establish this possibility conclusively.

B4.6.6.3.2 Inelastic Analysis

It has been predicted by various sources (Refs. 1, 2, 10 and 31) that the interaction equation recommended for elastic analysis will also be adequate for inelastic analysis provided that the tangent modulus is used. For this case, Equation 14 becomes

$$f_{cb} = \frac{\pi^2 E_t I_y h}{2 Z_x L^2} \sqrt{1 + \frac{KGL^2}{\pi^2 E\, r}} \quad . \tag{16}$$

and the associated interaction equation is

$$\frac{P}{P_u} + \frac{M_x}{f_{cb}\, Z_x\, \{1 - P/(P_x)_e\}} = 1. \tag{17}$$

Galambos (Ref. 15) should be consulted for further study of the inelastic lateral buckling value to be used in the interaction equation.

Table B.4.6.1-1

COMBINED LOADING - TENSION FLEXURE BEAMS

Notation: M = bending moment (in.-lb) due to the combined loading, positive when clockwise, negative when counterclockwise; M_1 and M_2 are applied external couples (in.-lb.) positive when acting as shown; Y = deflection (in.), positive when upward, negative when downward; θ = slope of beam (radians) to horizontal, positive when upward to the right; $j = \sqrt{\frac{EI}{P}}$ where E = modulus of elasticity, I = moment of inertia ($in.^4$) of cross section about the horizontal central axis, P = axial load (lb.); U = L/j; W = transverse load (lb.); w = transverse unit load (lb. per linear in.). All dimensions are in inches, all forces in pounds, all angles in radians.

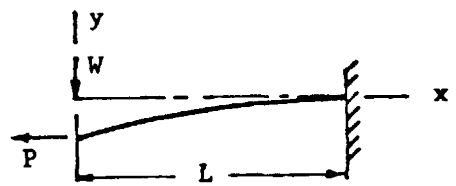
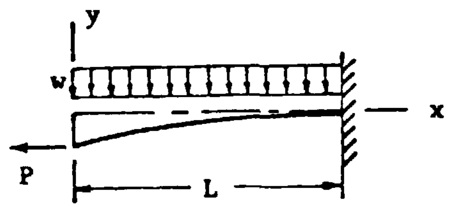
Manner of loading and support	Formulas for maximum bending moment, maximum deflection, end slope, and constraining moments
A. 	$\text{Max } M = -Wj \tanh U$ at $x = L$ $\text{Max } Y = -\frac{W}{P}(L - j \tanh U)$ at $x = 0$
B. 	$\text{Max } M = -wj\left[L \tanh U - j(1 - \text{sech } U)\right]$ at $x = L$ $\text{Max } Y = -\frac{wj}{P}\left[j\left(1 - \frac{U^2}{2} - \text{sech } U\right) - L(\tanh U - U)\right]$ at $x = 0$

Table B.4.6.1-2

COMBINED LOADING - TENSION FLEXURE BEAMS

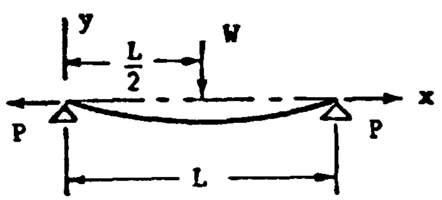
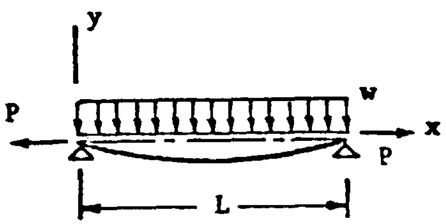
Manner of loading and support	Formulas for maximum bending moment, maximum deflection, end slope, and constraining moments.
C. 	Max $M = \frac{1}{2} Wj \tanh \frac{U}{2}$ at $x = \frac{L}{2}$ Max $Y = -\frac{W}{P}\left(\frac{L}{4} - \frac{1}{2} \tanh \frac{U}{2}\right)$ at $x = \frac{L}{2}$
D. 	Max $M = w(j)^2 \left(1 - \operatorname{sech} \frac{U}{2}\right)$ Max $Y = -\frac{w}{P}\left[\frac{L^2}{8} - (j)^2 \left(1 - \operatorname{sech} \frac{U}{2}\right)\right]$

Table B.4.6.1-3

COMBINED LOADING - TENSION FLEXURE BEAMS

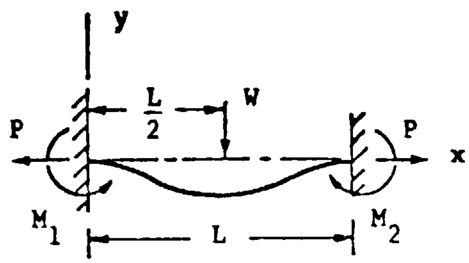
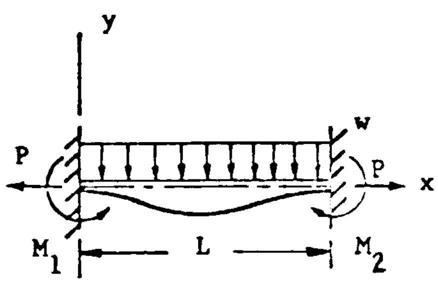
Manner of loading and support	Formulas for maximum bending moment, maximum deflection, end slope, and constraining moments.
E. 	$M_1 = M_2 = \frac{Wj}{2}\left(\frac{\cosh\frac{U}{2} - 1}{\sinh\frac{U}{2}}\right)$ $\text{Max} + M = \frac{Wj}{2}\left(\frac{1 - \cosh\frac{U}{2}}{\sinh\frac{U}{2}\cosh\frac{U}{2}} + \tanh\frac{U}{2}\right) \text{ at } x = \frac{L}{2}$ $\text{Max } Y = -\frac{Wj}{2P}\left[\frac{U}{2} - \tanh\frac{U}{2} - \frac{\left(1 - \cosh\frac{U}{2}\right)^2}{\sinh\frac{U}{2}\cosh\frac{U}{2}}\right]$
F. 	$M_1 = M_2 = w(j)^2\left(\frac{\frac{U}{2} - \tanh\frac{U}{2}}{\tanh\frac{U}{2}}\right)$ $\text{Max} + M = w(j)^2\left(1 - \frac{\frac{U}{2}}{\sinh\frac{U}{2}}\right) \text{ at } x = \frac{L}{2}$ $\text{Max } Y = -\frac{w(j)^2}{8P}\left[\frac{4U\left(1 - \cosh\frac{U}{2}\right)}{\sinh\frac{U}{2}} + U^2\right] \quad \text{at } x = \frac{L}{2}$

Table B.4.6.2-1 Beam Column Formulas

Notation: M = bending moment (in.-lb.) due to the combined loading, positive when clockwise, negative when counterclockwise; M_1 and M_2 are applied external couples (in.-lb.) positive when acting as shown; Y = deflection (in.), positive when upward, negative when downward; θ = slope of beam (radians) to horizontal, positive when upward to the right; $j = \sqrt{\frac{EI}{P}}$ where E = modulus of elasticity, I = moment of inertia (in.^{4}) of cross section about the horizontal central axis, P = axial load (lb.); U = L/j; W = transverse load (lb.); w = transverse unit load (lb. per linear in.). All dimensions are in inches, all forces in pounds, all angles in radians.

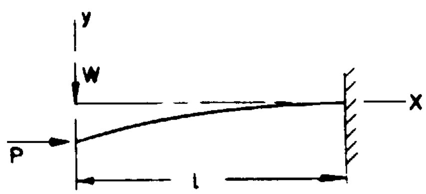
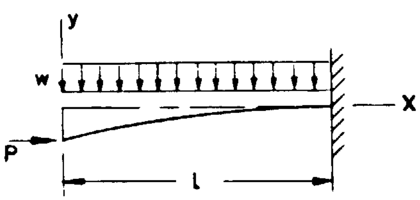
Manner of loading and support	Formulas for maximum bending moment, maximum deflection, end slope, and constraining moments
1. 	$\text{Max } M = -Wj \tan U$ at $x = L$ $\text{Max } Y = -\frac{W}{P}(j \tan U - L)$ at $x = 0$ $\theta = \frac{W}{P}\left(\frac{1 - \cos U}{\cos U}\right)$ at $x = 0$
2. 	$\text{Max } M = -wj\left[j(1 - \sec U) + L \tan U\right]$ at $x = L$ $\text{Max } Y = -\frac{wj}{P}\left[j\left(1 + \frac{1}{2}U^2 - \sec U\right) + L(\tan U - U)\right]$ at $x = 0$ $\theta = \frac{w}{P}\left[\frac{L}{\cos U} - j\left(\frac{1 - \cos 2U}{\sin 2U}\right)\right]$

Table B.4.6.2-2 Beam Column Formulas

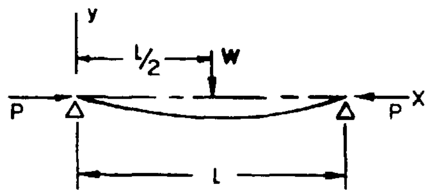
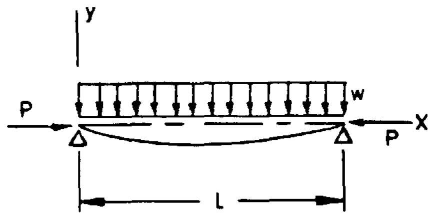
Manner of loading and support	Formulas for maximum bending moment, maximum deflection, end slope, and constraining moments
3. 	$\text{Max } M = \frac{1}{2} Wj \tan \frac{U}{2}$ at $x = \frac{L}{2}$ $\text{Max } Y = -\frac{1}{2}\frac{Wj}{P}\left(\tan \frac{U}{2} - \frac{U}{2}\right)$ at $x = \frac{L}{2}$ $\theta = -\frac{W}{2P}\left(\frac{1 - \cos \frac{U}{2}}{\cos \frac{U}{2}}\right)$ at $x = 0$
4. 	$\text{Max } M = w\,(j)^2 \left(\sec \frac{U}{2} - 1\right)$ at $x = \frac{L}{2}$ $\text{Max } Y = -\frac{w(j)^2}{P}\left(\sec \frac{U}{2} - 1 - \frac{U^2}{8}\right)$ at $x = \frac{L}{2}$ $\theta = -\frac{wj}{P}\left[-\frac{1}{2}U + \frac{1 - \cos U}{\sin U}\right]$ at $x = 0$

Table B.4.6.2-3 Beam Column Formulas

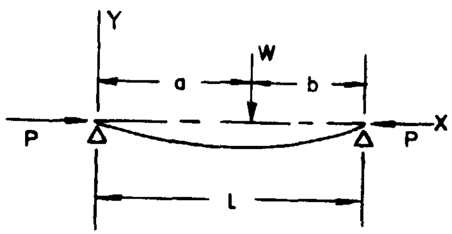
Manner of loading and support	Formulas for maximum bending moment, maximum deflection, end slope and constraining moments
5. 	Moment equation: x = 0 to x = a: $M = \frac{Wj \sin\frac{b}{j} \sin\frac{x}{j}}{\sin U}$ Max M at $x = \frac{\pi j}{2}$ if $\frac{\pi j}{2} < a$ Moment equation: x = a to x = L: $M = \frac{Wj \sin\frac{a}{j} \sin\frac{L-x}{j}}{\sin U}$ Max M at $x = L - \frac{\pi j}{2}$ if $\left(L - \frac{\pi j}{2}\right) > a$ Max M is at x = a if $\frac{\pi j}{2} > a$ and $\left(L - \frac{\pi j}{2}\right) < a$ Deflection equation: x = 0 to x = a: $Y = \frac{Wj}{P}\left(\frac{\sin\frac{b}{j} \sin\frac{x}{j}}{\sin U} - \frac{bx}{Lj}\right)$ Deflection equation: x = a to x = L: $Y = \frac{Wj}{P}\left[\frac{\sin\frac{a}{j} \sin\frac{L-x}{j}}{\sin U} - \frac{a(L-x)}{Lj}\right]$ $\theta = -\frac{W}{P}\left(\frac{b}{L} + \frac{\sin\frac{a}{j}}{\tan U} - \cos\frac{a}{j}\right)$ at x = 0 $\theta = \frac{W}{P}\left(\frac{a}{L} - \frac{\sin\frac{a}{j}}{\sin U}\right)$ at x = L

Table B.4.6.2-4 Beam Column Formulas

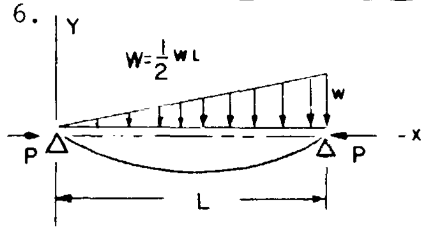
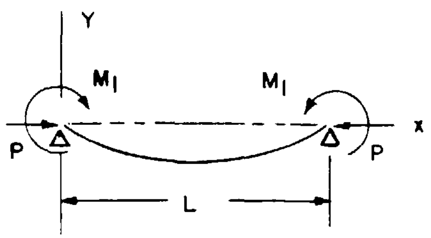
Manner of loading and support	Formulas for maximum bending moment, maximum deflection, end slope, and constraining moments.
6. 	Moment equation: $x = 0$ to $x = L$; $M = w\,(j)^2\left(\frac{\sin\frac{x}{j}}{\sin U} - \frac{x}{L}\right)$ Max M at $x = j\ \text{arc cos}\left(\frac{\sin U}{U}\right)$ Deflection equation: $x=0$ to $x=L$; $y = -\frac{w}{P}\left(\frac{x^3}{6L} + \frac{(j)^2\sin\frac{x}{j}}{\sin U} - \frac{jx}{U} - \frac{Lx}{6}\right)$ $\theta = -\frac{w}{P}\left(\frac{j}{\sin U} - \frac{j}{U} - \frac{L}{6}\right)$ at $x=0$ $\quad \theta = \frac{w}{P}\left(-\frac{j}{\tan U} + \frac{j}{U} - \frac{L}{3}\right)$ at $x = L$
7. 	Max $M = M_1 \sec\frac{U}{2}$ at $x = \frac{L}{2}$ Max $Y = -\frac{M_1}{P}\left(\frac{1 - \cos\frac{U}{2}}{\cos\frac{U}{2}}\right)$ at $x = \frac{L}{2}$ $\theta = -\frac{M_1}{Pj}\tan\frac{U}{2}$ at $x = 0$

Table B.4.6.2-5 Beam Column Formulas

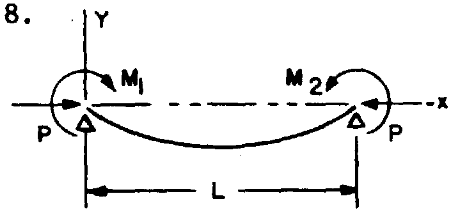
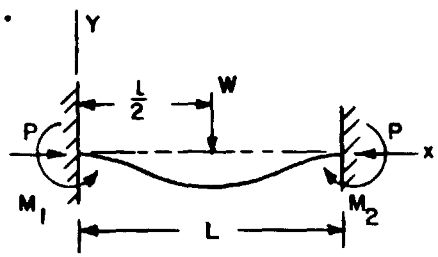
Manner of loading and support	Formulas for maximum bending moment, maximum deflection, end slope, and constraining moments.
8. 	Moment equation: x=0 to x=L: $M = \left(\frac{M_2 - M_1 \cos U}{\sin U}\right) \sin \frac{x}{j} + M_1 \cos \frac{x}{j}$ Max M at $x = j \text{ arc tan}\left(\frac{M_2 - M_1 \cos U}{M_1 \sin U}\right)$ Deflection equation: x=0 to x=L: $Y = \frac{1}{P}\left[M_1 + (M_2 - M_1)\frac{x}{L}\right.$ $\left. - (M_2 - M_1 \cos U) \frac{\sin \frac{x}{j}}{\sin U} - M_1 \cos \frac{x}{j}\right]$ $\theta = \frac{1}{P}\left(\frac{M_2 - M_1}{L} - \frac{M_2 - M_1 \cos U}{j \sin U}\right)$ at $x = 0$ $\theta = \frac{1}{P}\left(\frac{M_2 - M_1}{L} - \frac{M_2 - M_1 \cos U}{j \sin U} \cos U + \frac{M_1}{j} \sin U\right)$ at $x = L$
9. 	$M_1 = M_2 = \frac{1}{2} Wj\left(\frac{1 - \cos \frac{U}{2}}{\sin \frac{U}{2}}\right)$ at $x = \frac{L}{2}$ $M = \frac{1}{2} Wj \left(\tan \frac{U}{2} - \frac{1 - \cos \frac{U}{2}}{\sin \frac{U}{2} \cos \frac{U}{2}}\right)$ Max $Y = - \frac{Wj}{2P}\left[\tan \frac{U}{2} - \frac{U}{2} - \frac{\left(1 - \cos \frac{U}{2}\right)^2}{\sin \frac{U}{2} \cos \frac{U}{2}}\right]$

Table B.4.6.2-6 Beam Column Formulas

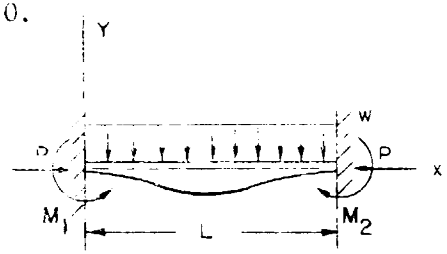
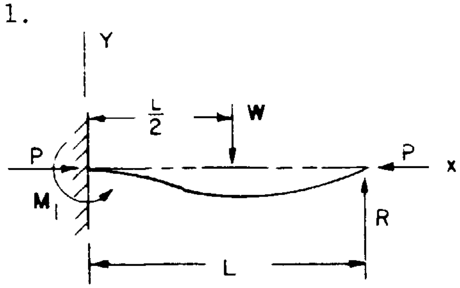
Type of loading and support	Formulas for maximum bending moment, maximum deflection, end slope, and constraining moments.
10. 	$M_1 = M_2 = w\,(j)^2\left(1 - \dfrac{\frac{U}{2}}{\tan\frac{U}{2}}\right)$ at $x = \frac{L}{2}$, $M = w\,(j)^2\left(\dfrac{\frac{U}{2}}{\sin\frac{U}{2}} - 1\right)$ $\text{Max } Y = -\dfrac{w(j)^2}{P}\left[-\left(1 - \dfrac{\frac{U}{2}}{\tan\frac{U}{2}}\right)\left(\dfrac{1 - \cos\frac{U}{2}}{\cos\frac{U}{2}}\right) + \sec\frac{U}{2} - \frac{U^2}{8} - 1\right]$
11. 	$\text{Max } M = M_1 = \dfrac{WL}{2}\left[\dfrac{j \tan U\,(\sec\frac{U}{2} - 1)}{j \tan U - L}\right]$ $R = \dfrac{W}{2} - \dfrac{M_1}{L}$ Moment equation: $x = \frac{L}{2}$ to $x = L$ $M = M_1\left(\dfrac{\sin\frac{x}{j}}{\tan U} - \cos\frac{x}{j}\right) + Wj\left(\sin\frac{U}{2}\cos\frac{x}{j} - \dfrac{\sin\frac{U}{2}\,\sin\frac{x}{j}}{\tan U}\right)$ Deflection equation: $x = \frac{L}{2}$ to $x = L$; $Y = -\dfrac{1}{P}\left[M_1\left(1 - \dfrac{x}{L} + \dfrac{\sin\frac{x}{j}}{\tan U} - \cos\frac{x}{j}\right) - Wj\left(\dfrac{L-x}{2j} + \dfrac{\sin\frac{U}{2}\,\sin\frac{x}{j}}{\tan U} - \sin\frac{U}{2}\cos\frac{x}{j}\right)\right]$

Table B.4.6.2-7 Beam Column Formulas

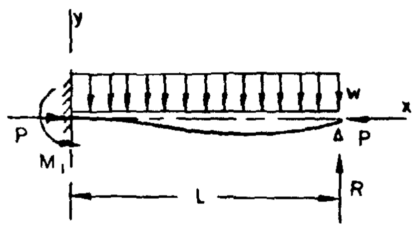
Manner of loading and support	Formulas for maximum bending moment, maximum deflection, end slope, and constraining moments.
12. 	$M_{ax}\ M = M_1 = wLj\left[\frac{\tan U\left(\tan\frac{U}{2} - \frac{U}{2}\right)}{\tan U - U}\right]$ $R = \frac{wL}{2} - \frac{M_1}{L}$ Moment equation: x=0 to x=L; $M = M_1\left(\cot U \sin\frac{x}{j} - \cos\frac{x}{j}\right)$ $+ w(j)^2\left[\frac{\sin\frac{x}{j}}{\sin U}(1-\cos U) + \cos\frac{x}{j} - 1\right]$ Deflection equation: x=0 to x=L; $Y = -\frac{1}{P}\left[M_1\left(1 - \frac{x}{L} + \cot U \sin\frac{x}{j} - \cos\frac{x}{j}\right)\right.$ $\left. - w(j)^2\left(\cot U \sin\frac{x}{j} - \frac{\sin\frac{x}{j}}{\sin U} - \cos\frac{x}{j} + \frac{Lx - x^2}{2(j)^2} + 1\right)\right]$
13. Same as Case I (cantilever with end load) except that P is tensile.	$M_{ax}\ M = -Wj \tanh U$ at x = L $M_{ax}\ Y = -\frac{W}{P}(L - j\tanh U)$ at x = 0
14. Same as Case 2 (cantilever with uniform load) except that P is tensile.	$M_{ax}\ M = -wj\left[L\tanh U - j(1 - \text{sech}\ U)\right]$ at x = L $M_{ax}\ Y = -\frac{wj}{P}\left[j\left(1 - \frac{U^2}{2} - \text{sech}\ U\right) - L(\tanh U - U)\right]$ at x = 0

Table B.4.6.2-8 Beam Column Formulas

Manner of loading and support	Formulas for maximum bending moment, maximum deflection, end slope, and constraining moments.
15. Same as Case 3 (end supports, center load) except that P is tensile.	$\text{Max } M = \frac{1}{2} Wj \tanh \frac{U}{2} \text{ at } x = \frac{L}{2}$ $\text{Max } Y = -\frac{W}{P}\left(\frac{L}{4} - \frac{j}{2} \tanh \frac{U}{2}\right) \text{ at } x = \frac{L}{2}$
16. Same as Case 4 (end supports, uniform load) except that P is tensile.	$\text{Max } M = w(j)^2 \left(1 - \text{sech} \frac{U}{2}\right)$ $\text{Max } Y = -\frac{w}{P}\left[\frac{L^2}{8} - (j)^2 \left(1 - \text{sech} \frac{U}{2}\right)\right]$
17. Same as Case 9 (fixed ends, center load) except that P is tensile.	$M_1 = M_2 = \frac{Wj}{2}\left(\frac{\cosh \frac{U}{2} - 1}{\sinh \frac{U}{2}}\right)$ $\text{Max } + M = \frac{Wj}{2}\left(\frac{1 - \cosh \frac{U}{2}}{\sinh \frac{U}{2} \cosh \frac{U}{2}} + \tanh \frac{U}{2}\right) \text{at } x = \frac{L}{2}$ $\text{Max } Y = -\frac{Wj}{2P}\left[\frac{U}{2} - \tanh \frac{U}{2} - \frac{\left(1 - \cosh \frac{U}{2}\right)^2}{\sinh \frac{U}{2} \cosh \frac{U}{2}}\right]$

Table B.4.6.3-1

REFERENCE LISTING FOR OTHER BEAM-COLUMN LOADING CONDITIONS

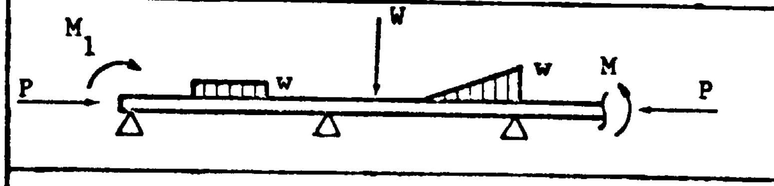
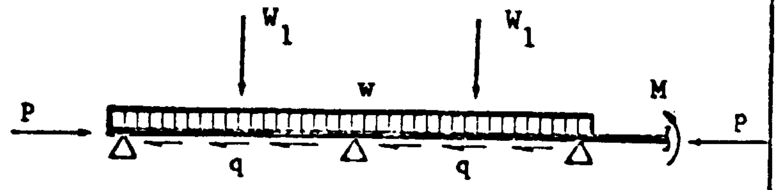
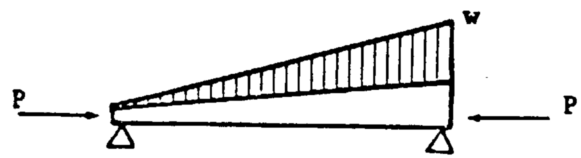
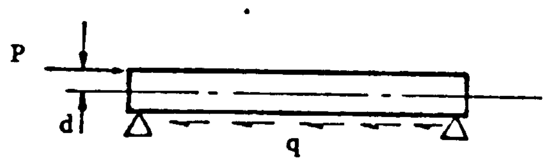
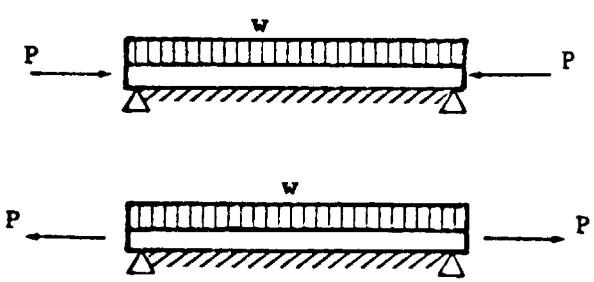
LOADING CONDITION	DESCRIPTION	REF.
	General Loads on Multi-Span Supports	22
	Uniformly Distributed Axial Load on Three or More Supports	18
	Ends Hinged, Tapered Beam-Column with Triangular Load (Applicable also to any general cross section and loading)	21
	Ends Pinned with An Offset Longitudinal Compressive Load Balanced by a Uniform Shear Flow	31
	Ends Pinned with Elastic Foundations Tensile and Compressive Axial Load	13

Table B.4.[illegible].3-2

REFERENCE LISTING FOR OTHER BEAM-COLUMN LOADING CONDITIONS

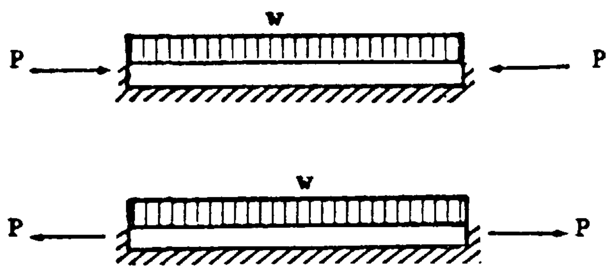
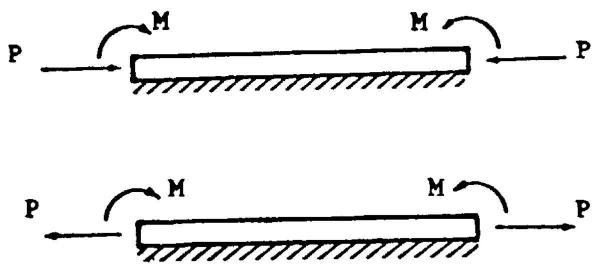
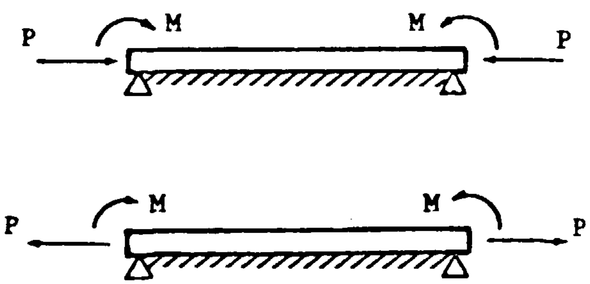
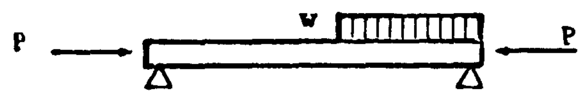
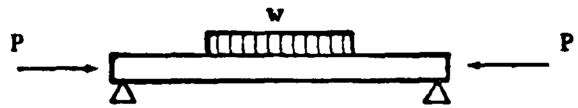
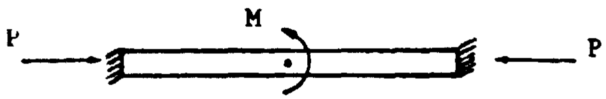
LOADING CONDITION	DESCRIPTION	REF.
	Ends Fixed with Elastic Foundations Tensile and Compressive Axial Load	13
	Ends Free with Elastic Foundation and Equal Concentrated End Moments Tensile and Compressive Axial Load	13
	Ends Pinned with Elastic Foundation and Equal Concentrated End Moments Tensile and Compressive Axial Load	13

Table B.4.6.3-3

REFERENCE LISTING FOR OTHER BEAM-COLUMN LOADING CONDITIONS

LOADING CONDITION	DESCRIPTION	REF.
	Ends Pinned with Local Uniformly Distributed Transverse Load	13
	Ends Pinned with Partial Uniformly Distributed Transverse Load	22
	Ends Fixed with Applied Moment at Center	31
	Ends Pinned with Triangular Load	31

REFERENCES:

1. Austin, W. J.: Strength and Design of Metal Beam-Columns, Proceedings, ASCE, April 1961, page 1802.

2. Massonnet, C.: Stability Considerations in the Design of Steel Columns, Proceedings, ASCE, ST7, September 1959.

3. Beedle, L. S.: Plastic Design of Steel Frames, John Wiley and Sons, Inc., New York, 1958.

4. Bleich, F.: Buckling Strength of Metal Structures, McGraw-Hill Book Co., Inc., New York, 1952.

5. Cozzone, J.: Bending in the Plastic Range, Journal Aeronautical Sciences, May 1943.

6. Seeley, F. and Smith, J.: Advanced Mechanics of Materials, John Wiley and Sons, Inc., New York, 1957.

7. Clark, J. W.: Eccentrically Loaded Aluminum Columns, Transactions, ASCE, Vol. 120, 1955, page 1116.

8. Clark, J. W., and Hill, A. N.,: Lateral Buckling of Beams, Proceedings, ASCE, Vol. 86, No. ST7, July, 1960.

9. Hill, H. N. and Clark, J. W.: Lateral Buckling of Eccentrically Loaded I- and H-Section Columns, Proceedings, First National Congress of Applied Mechanics, ASME, 1952.

10. Hill, H. N., Hartmann, E. C., and Clark, J. W.: Design of Aluminum Alloy Beam-Columns, Transactions, ASCE, Vol. 121, 1956.

11. Timoshenko, S. and Gere, J. M.: Theory of Elastic Stability, Second edition, McGraw-Hill Book Company, Inc., New York, 1961.

12. Salvadori, M. G.: Lateral Buckling of Eccentrically Loaded I-Columns, Transactions, ASCE Vol. 121, 1956.

13. Butler, D. J. and Anderson, G. B.: The Elastic Buckling of Tapered Beam-Columns, Welding Research Journal Supplement, January 1963.

14. Gatewood, B. E.: Buckling Loads for Beams of Variable Cross Section Under Combined Loads, Journal of the Aeronautical Sciences, Vol. 22, 1955.

15. Galambos, T. V.: Inelastic Lateral Buckling of Beams, Proceedings, ASCE, October, 1963.

16. Bresler, Boris and Lin, T. Y.: Design of Steel Structures, John Wiley and Sons, Inc., New York, 1960.

17. Hetényi, M.: Beams on Elastic Foundation, the University of Michigan Press, Ann Arbor, Michigan, 1955.

18. Niles, A. S. and Newell, J. S.: Airplane Structures, John Wiley and Sons, Inc., Volume II, third edition, New York, 1949.

19. Roark, R. J.: Formulas for Stress and Strain, Third Edition, McGraw-Hill Book Company, Inc., New York, 1954.

20. Timoshenko, S.: Strength of Materials Part II, Advanced Theory and Problems, 3rd ed., D. Van Nostrand Co., Inc., Princeton, New Jersey, 1956.

21. Laird, W. and Bryson, A.: An Analysis of Continuous Beam-Columns with Uniformly Distributed Axial Load, Proc. of 3rd U. S. National Congress of Applied Mechanics, ASME, N.Y., 1958.

22. Saunders, H.: Beam-Column of Nonuniform Sections by Matrix Methods, Journal of the Aerospace Sciences, September 1961.

23. Saunders, H: Matrix Analysis of a Nonuniform Beam-Column on Multi-Supports, AIAA Journal, April 1963.

24. Lee, S., Wang, T. and Kao, J.: Continuous Beam-Columns on Elastic Foundation, Proceedings, ASCE EM2, April 1961.

25. Sundara, K. and Anantharamu, S.: Finite Beam-Columns on Elastic Foundation, Proceedings, ASCE, EM6, December 1963.

26. Newmark, N. M.: Numerical Procedure for Computing Deflections, Moments, and Buckling Loads, Transactions, ASCE, Vol. 108, 1943.

27. Salvadori, M. G. and Baron, M. L.: Numerical Methods in Engineering, Prentice-Hall, Inc., New Jersey, 1962.

28. Woodberry, R. F. H.: Analysis of Beam-Columns, Design News, April 1959.

29. Bruhn, E. F.: Analysis and Design of Flight Vehicle Structures, Tri-State Offset Co., Cincinnati, Ohio, 1965.

30. Griffel, W.: Method for Solving Beam-Column Problems, Product Engineering, September 14, 1964.

31. Guide to Design Criteria for Metal Compression Members, Column Research Council, Engrg, Foundation, 1960.

32. Plastic Design in Steel, American Institute of Steel Construction, Inc., New York, 1959.

SECTION B4.7

LATERAL BUCKLING OF BEAMS

TABLE OF CONTENTS

B4.7 LATERAL BUCKLING OF BEAMS

4.7.1 INTRODUCTION

A beam of general cross section which is bent in the plane of greatest flexural rigidity may buckle in the plane perpendicular to the plane of greatest flexural rigidity at a certain critical value of the load. Concern for lateral buckling is more significant in the design of beams without lateral support when the flexural rigidity of the beam in the plane of bending is large in comparison with the lateral bending rigidity.

Consider the beam with two planes of symmetry shown in Figure 4.7-1. This beam is assumed to be subjected to arbitrary loads acting perpendicular to the xz plane. By assuming that a small lateral deflection occurs under the action of these loads, the critical value of load can be obtained from the differential equations of equilibrium for the deflected beam (Ref. 1).

Beams with various cross sections and particular cases of loading and boundary conditions will be considered in this section.

4.7.1.1 General Cross Section

The general expression for the elastic buckling strength of beams can be expressed by the following equation (Ref. 3).

$$f_{cr} = \frac{C_1 \pi^2 E I_y}{S_c (KL)^2} \left[C_2 g + C_3 k + \sqrt{(C_2 g + C_3 k)^2 + \frac{C_w}{I_y} \left(1 + \frac{GJ (KL)^2}{\pi^2 E C_w} \right)} \right] \quad (1)$$

where:

f_{cr} = critical stress for lateral buckling

E = modulus of elasticity, lb/in.^{2}

I_y = modulus of inertia of beam cross section about the y axis, in.^{4}

L = distance between points of support against lateral bending and twisting, in.

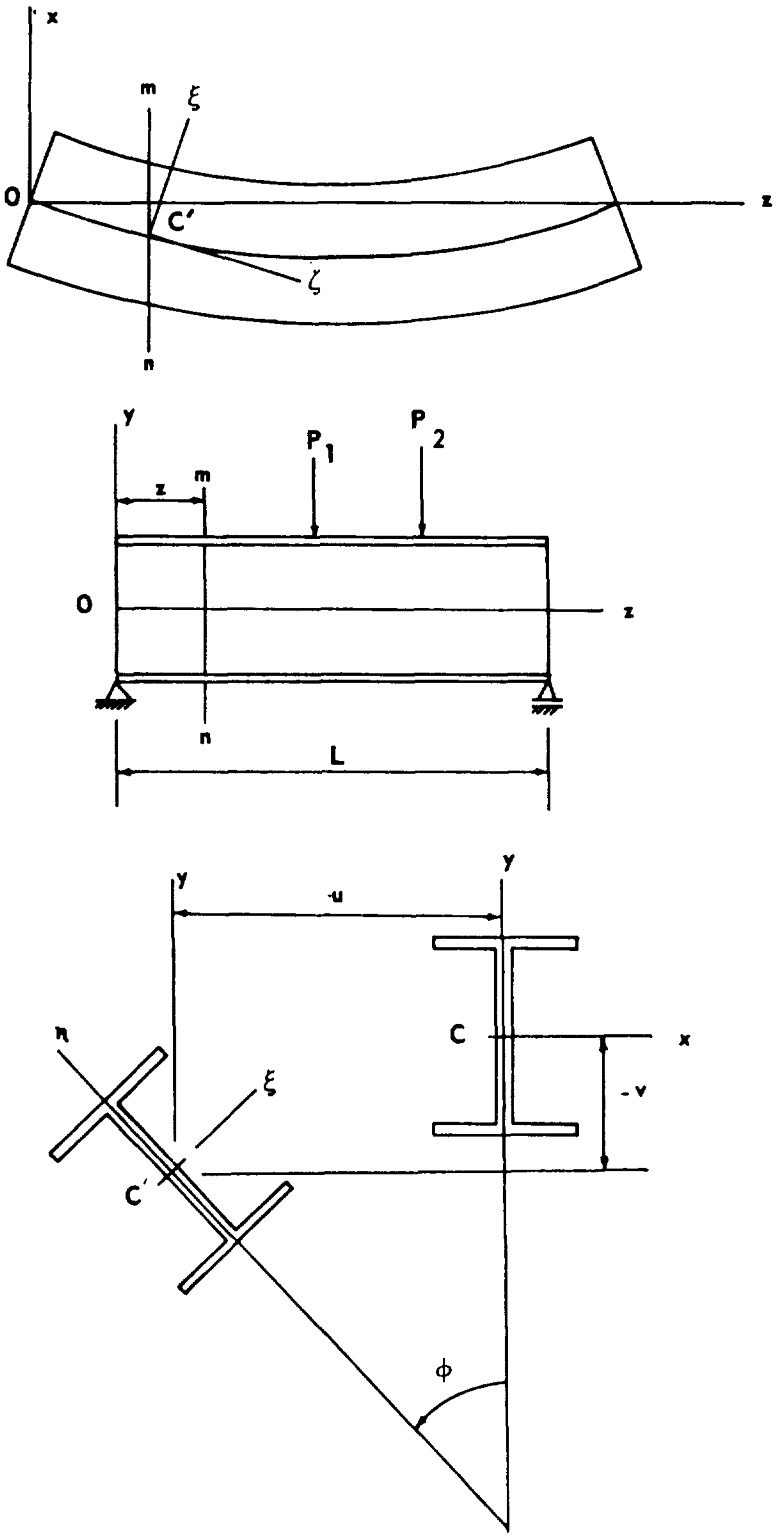


FIGURE 4.7-1 LATERAL BUCKLING

C_w = torsion warping constant, $in.^6$

g = distance from shear center to point of application of transverse load (positive when load is below shear center and negative otherwise), in.

G = shear modulus of elasticity, $lb/in.^2$

J = torsion constant, $in.^4$

S_c = section modulus for stress in compression flange, $in.^3$

$$k = e + \frac{1}{2I_x} \int_A (x^2 + y^2)\, dA, \text{ in.}$$

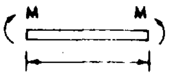
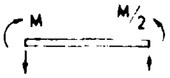
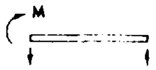
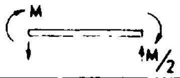
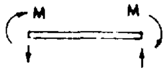
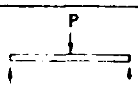
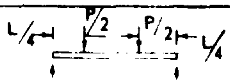
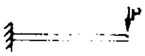
e = distance from shear center to centroid, positive if between centroid and compression flange, in.

C_1, C_2, C_3, K = constants which depend mainly on conditions of loading and support for the beam (Table 4.7-1).

In the equation above, it is assumed that the lines of action of the loads pass through the shear center and the centroid, and that the loads attach to the beam in such a manner that their lines of action remain parallel to their initial directions as the beam deflects. It is also assumed that the shear center lies on a principal axis through the centroid.

The coefficients C_1, C_2, C_3, and K are derived in Reference 3. They depend mainly on the conditions of loading and support for the beam. The values of C_1, C_2, C_3, and K given in Table 4.7-1 have been obtained from Reference 3.

Table 4.7-1. Values of Coefficients in Formula for Elastic Buckling Strength of Beams

CASE NO.	LOADING	RESTRAINT	VALUE OF COEFFICIENTS K	C_1	C_2	C_3
1		SIMPLE SUPPORT	1.0	1.0	–	1.0
		FIXED	0.5	1.0	–	1.0
2		SIMPLE SUPPORT	1.0	1.31	–	
		FIXED	0.5	1.30	–	
3		SIMPLE SUPPORT	1.0	1.77	–	6.5
		FIXED	0.5	1.78	-	
4		SIMPLE SUPPORT	1.0	2.33	–	
		FIXED	0.5	2.29	–	
5		SIMPLE SUPPORT	1.0	2.56	–	
		FIXED	0.5	2.23	-	
6		SIMPLE SUPPORT	1.0	1.13	0.45	
		FIXED	0.5	0.97	0.29	
7		SIMPLE SUPPORT	1.0	1.30	1.55	
		FIXED	0.5	0.86	0.82	
8		SIMPLE SUPPORT	1.0	1.35	0.55	2.5
		FIXED	0.5	1.07	0.42	
9		SIMPLE SUPPORT	1.0	1.70	1.42	
		FIXED	0.5	1.04	0.84	
10		SIMPLE SUPPORT	1.0	1.04	0.42	
		FIXED				
	CANTILEVER BEAMS					
11		WARPING RESTRAINED AT SUPPORTED END	1.0	1.28	0.64	
12		WARPING RESTRAINED AT SUPPORTED END	1.0	2.05		

4.7.2 SYMMETRICAL SECTIONS

For sections that are symmetrical about the horizontal axis or about a point (channels, zee sections, etc.), the quantity k in equation (1) is equal to zero. The expression for elastic buckling strength can then be written

$$f_{cr} = \frac{C_1 \pi^2 E I_y}{S_c (KL)^2} \left[C_2 g + \sqrt{(C_2 g)^2 + \frac{C_w}{I_y}\left(1 + \frac{GJ (KL)^2}{\pi^2 E C_w}\right)} \right] \qquad (2)$$

Values of C_1, C_2, and K can be obtained from Table 4.7-1.

4.7.2.1 I-Beams

Given below are solutions for particular cases of load and boundary conditions for I-beams. For cases not considered below, equation (2) should be used.

I. Pure Bending

If an I-beam is subjected to couples M_o at the ends, the critical value of the moment M_o is

$$(M_o)_{cr} = \frac{\pi}{L} \sqrt{E I_y \, GJ \left(1 + \frac{E C_w \pi^2}{G J L^2}\right)} \quad . \qquad (3)$$

This expression can be represented in the form

$$(M_o)_{cr} = K_1 \frac{\sqrt{E I_y GJ}}{L} \qquad (4)$$

where

$$K_1 = \pi \sqrt{1 + \frac{E C_w \pi^2}{G J L^2}} \quad .$$

Values of K_1 are given in Table 4.7-2.

Table 4.7-2. Values of the Factor K_1 for I-Beams in Pure Bending

$\frac{L^2GJ}{EC_w}$	0	0.1	1	2	4	6	8	10	12
K_1	∞	31.4	10.36	7.66	5.85	5.11	4.70	4.43	4.24
$\frac{L^2GJ}{EC_w}$	16	20	24	28	32	36	40	100	∞
K_1	4.00	3.83	3.73	3.66	3.59	3.55	3.51	3.29	π

II. <u>Cantilever Beam, Load at End</u>

If a cantilever beam is subjected to a force applied at the centroid of the end cross section, the critical value of the load P is

$$P_{cr} = K_2 \frac{\sqrt{EI_y GJ}}{L^2} \qquad (5)$$

where

$$K_2 = \frac{4.013}{\left(1 - \sqrt{\frac{EC_w}{L^2GJ}}\right)^2}$$

For values of $\frac{L^2GJ}{EC_w}$ greater than 0.1, values of K_2 are given in Table 4.7-3. For values of $\frac{L^2GJ}{EC_w}$ less than 0.1, see Reference 1, page 258, for values of K_2.

Table 4.7-3. Values of the Factor K_2 for Cantilever Beams of I-Section

$\frac{L^2GJ}{EC_w}$	0.1	1	2	3	4	6	8
K_2	44.3	15.7	12.2	10.7	9.76	8.69	8.03
$\frac{L^2GJ}{EC_w}$	10	12	14	16	24	32	40
K_2	7.58	7.20	6.96	6.73	6.19	5.87	5.64

III. <u>Simply Supported Beam, Load at Middle</u>

If a simply supported I-beam is subjected to a load P applied at the centroid of the middle cross section, the critical value of the load P is

$$P_{cr} = K_3 \frac{\sqrt{EI_y GJ}}{L^2} \quad . \tag{6}$$

Values of K_3 obtained from Reference 1, page 264, are given in Table 4.7-4(a)

Table 4.7-4(a). Values of K_3 for Simply Supported I-Beams With Concentrated Load at Middle

Load Applied At	L^2GJ/EC_w						
	0.4	4	8	16	24	32	48
Upper Flange	51.5	20.1	16.9	15.4	15.0	14.9	14.8
Centroid	86.4	31.9	25.6	21.8	20.3	19.6	18.8
Lower Flange	147	50.0	38.2	30.3	27.4	25.4	23.[illegible]
Load Applied At	L^2GJ/EC_w						
	64	80	96	160	240	320	400
Upper Flange	15.0	15.0	15.1	15.3	15.3	15.6	15.8
Centroid	18.3	18.1	17.9	17.5	17.4	17.2	17.2
Lower Flange	22.4	21.7	21.1	20.0	19.3	19.0	18.7

If lateral support is provided at the middle of the beam, values of K_3 are given in Table 4.7-4(b).

Table 4.7-4(b). Values of the Factor K_3 for Lateral Support at Middle

$\frac{L^2GJ}{EC_w}$	0.4	4	8	16	32	96	128	200	400
K_3	466	154	114	86.4	69.2	54.5	52.4	49.8	47.4

If lateral support is provided at both ends of the beam, values of K_3 are given in Table 4.7-4(c).

Table 4.7-4(c). Values of the Factor K_3 for Lateral Support at Ends

$\frac{L^2GJ}{EC_w}$	0.4	4	8	16	24	32	64	128	200	320
K_2	268	88.8	65.5	50.2	43.6	40.2	34.1	30.7	29.4	28.4

IV. Simply Supported Beam, Uniform Load

If a simply supported I-beam is subjected to a uniform load q, the critical value of this load can be expressed in the form

$$(ql)_{cr} = K_4 \frac{\sqrt{EI_y GJ}}{L^2} \tag{7}$$

Values of K_4 obtained from Reference 1, page 267, are given in Table 4.7-5(a).

Table 4.7-5(a). Values of K_4 for Simply Supported I-Beams with Uniform Load

Load Applied At	$L^2\ GJ/EC_w$						
	0.4	4	8	16	24	32	48
Upper Flange	92.9	36.3	30.4	27.5	26.6	26.1	25.9
Centroid	143.0	53.0	42.6	36.3	33.8	32.6	31.5
Lower Flange	223.	77.4	59.6	48.0	43.6	40.5	37.8
Load Applied At	**$L^2\ GJ/EC_w$**						
	64	80	128	200	280	360	400
Upper Flange	25.9	25.8	26.0	26.4	26.5	26.6	26.7
Centroid	30.5	30.1	29.4	29.0	28.8	28.6	28.6
Lower Flange	36.4	35.1	33.3	32.1	31.3	31.0	30.7

If the beam has lateral support at the middle, K_4 is given by Table 4.7-5(b).

Table 4.7-5(b). Values of K_4 with Lateral Support at Middle

Load Applied At	$L^2\ GJ/EC_w$							
	0.4	4	8	16	64	96	128	200
Upper Flange	587	194	145	112	91.5	73.9	71.6	69.0
Centroid	673	221	164	126	101.	79.5	76.4	72.8
Lower Flange	774	251	185	142	112	85.7	81.7	76.9

If the beam has lateral support at both ends of the beam, K_4 is given by Table 4.7-5(c).

Table 4.7-5(c). Values of K_4 with Lateral Support at Ends

$\frac{L^2GJ}{EC_w}$	0.4	4	8	16	32	96	128	200	400
K_4	488	161	119	91.3	73.0	58.0	55.8	53.5	51.2

4.7.2.2 Rectangular Beams

For a beam of rectangular section of width b and height h, the warping rigidity C_w can be taken as zero; therefore, equation (2) becomes

$$f_{cr} = \frac{C_1 \pi^2 EI_y}{S_c (KL)^2} \left[C_2 g + \sqrt{(C_2 g)^2 + \frac{GJ(KL)^2}{\pi^2 EI_y}} \right] . \qquad (8)$$

If the load is applied at the centroid, g = 0; therefore,

$$f_{cr} = \frac{C_1 \pi^2 \sqrt{EI_y GJ}}{S_c KL} . \qquad (9)$$

By taking $G = \frac{3}{8} E$, $J = 0.31\, hb^3$, $I = \frac{1}{12} hb^3$, and $S_c = \frac{bh^2}{6}$

$$f_{cr} = \frac{1.86\, C_1}{K} \frac{Eb^2}{Lh} \qquad (10)$$

or

$$f_{cr} = K_f \frac{Eb^2}{Lh} \qquad (11)$$

where

$$K_f = \frac{1.86\, C_1}{K} .$$

Values of K_f are given in Figure 4.7-2 and Table 4.7-6 for several load cases. For cases not available in Table 4.7-6 and Figure 4.7-2, refer to Table 4.7-1 for values of C_1 and K for use in equation 10.

Equation 8 must be used for loads not applied at the centroid for any of the given cases.

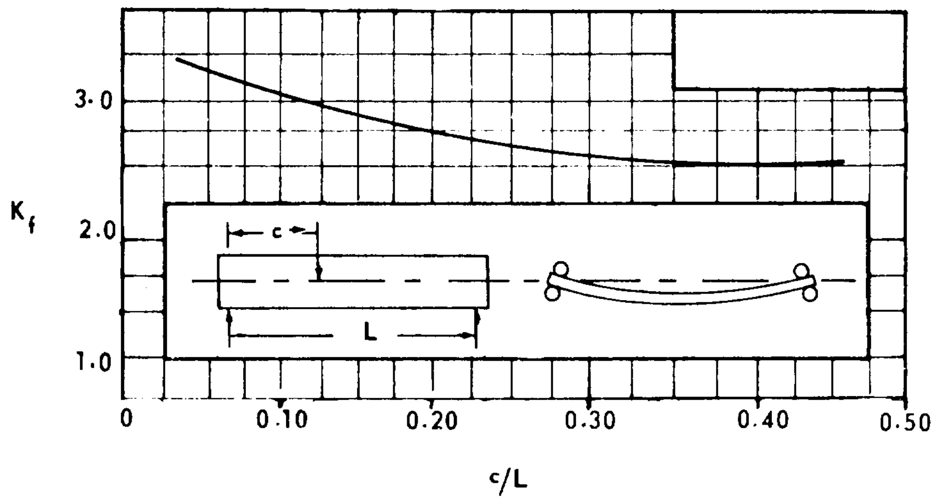
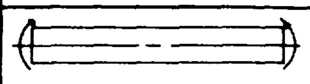
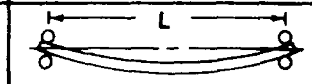
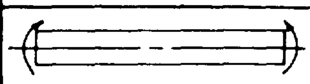
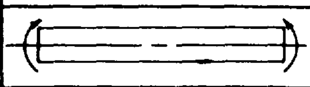
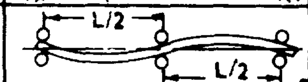
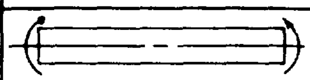
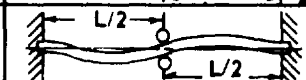
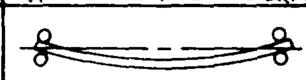
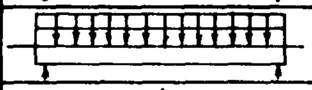
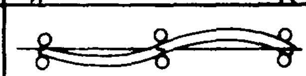
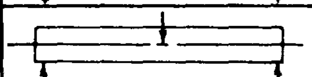
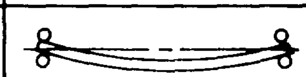
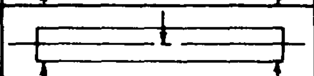
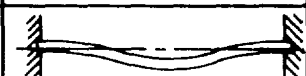
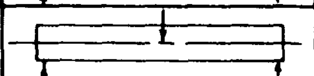
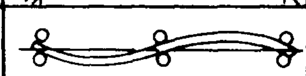
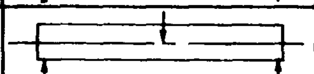
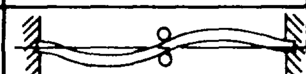
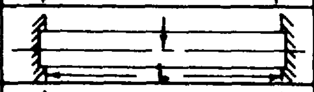
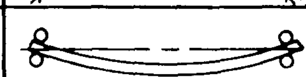
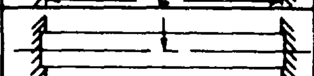
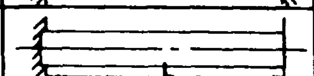
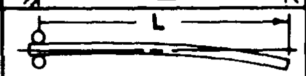
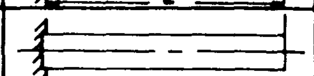
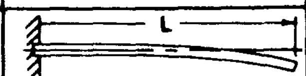
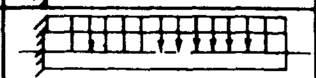
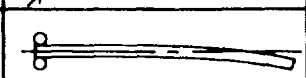
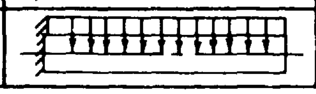
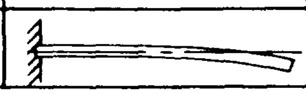


FIGURE 4.7-2 CONSTANTS FOR DETERMINING THE LATERAL STABILITY OF DEEP RECTANGULAR BEAMS

Table 4.7-6. Constants for Determining the Lateral Stability of Rectangular Beams

Case	Side View	Top View	K_f
1			1.86
2			3.71
3			3.71
4			5.45
5			2.09
6			3.61
7			4.87
8			2.50
9			3.82
10			6.57
11			7.74
12			3.13
13			3.48
14			2.37
15			2.37
16			3.80
17			3.80

4.7.3 UNSYMMETRICAL I-SECTIONS

For I-beams symmetrical about the vertical axis, but unsymmetrical about the horizontal axis and subjected to uniform bending moment, the following approximate equation for the elastic buckling stress should be used (Ref. 3).

$$f_{cr} = \frac{\pi^2 EI_y}{S_c (KL)^2} \left[e + \sqrt{e^2 + \frac{C_w}{I_y} \left(1 + \frac{GJ(KL)^2}{\pi^2 EC_w} \right)} \right] \qquad (8)$$

4.7.4 SPECIAL CONDITIONS

4.7.4.1 Oblique Loads

The case of a beam subjected to a uniform bending moment that does not lie in one of the principal planes of the cross section is discussed in References 4 and 5. Reference 5 shows that the equation for the critical moment takes the form of equation 1 with $C_1 = C_3 = 1.0$, $C_2 = 0$. The quantity I_y is replaced by the expression $I_{\bar{y}} I_{\bar{x}} / I_x$, in which $\bar{y}$ and $\bar{x}$ denote principal axes and the x axis is the axis normal to the plane of bending.

4.7.4.2 Nonuniform Cross Section

A concise solution for the lateral buckling strength of a tapered rectangular beam, subjected to constant bending moment and simply supported at the ends, is presented in Reference 6. Tapered cantilever I-beams have been investigated experimentally in Reference 7.

4.7.4.3 Special End Conditions

Solutions have been obtained (Ref. 8) for the buckling strength of I-beams under a load (either uniform or a concentrated load at the center) acting perpendicular to the principal plane having maximum bending rigidity and with various degrees of restraint against rotation of the beam about either plane. Each type of restraint was considered to vary between zero and complete fixity. In all cases, the beams were considered to be fixed at the ends against rotation about a longitudinal axis perpendicular to the plane of the cross section.

Frequently, a cantilever beam is simply the overhanging end of a beam that extends over two or more supports. In this case, the supported end of the cantilever beam may not be fixed against lateral bending of the beam flanges but some restraint is supplied by continuity at the support. In such cases, a conservative estimate of the buckling strength can be made by considering the warping constant, C_w, to be zero in the buckling formula.

If a beam is continuous beyond one or both supports, the end conditions for any one span are generally between the cases of complete fixity and simple support covered in Table 1. The effect of continuity has been discussed in References 9 and 10.

4.7.4.4 Inelastic Buckling

It is explained in Reference 11 that it is possible to obtain a lower limit to the theoretical buckling stress in the inelastic range by substituting the tangent modulus, E_t, corresponding to the maximum stress in the beam for the elastic modulus, E, in the elastic buckling formula. Tests on aluminum alloy beams show that this method gives a close approximation to the experimental buckling stress when the bending moment is constant along the length (Ref. 12 and 13). Tests of aluminum alloy beams subjected to unequal end moments, with the ratio of the moment at one end to the moment at the other end varying from 1.0 to -1.0, resulted in experimental critical stresses varying from 8 percent below to 39 percent above the values computed by the tangent modulus method.

4.7.5 REFERENCES

1. Timoshenko, S. P., and Gere, J. M: Theory of Elastic Stability. McGraw-Hill Book Company, Inc., New York, 1961.

2. Structural Design Manual, Northrop Aircraft, Inc.

3. Clark, J. W., and Hill, H. N: Lateral Buckling of Beams. Proceedings of the American Society of Civil Engineers, Journal of the Structural Division, July, 1960.

4. Goodier, J. N: Flexural-Torsional Buckling of Bars of Open Section. Bulletin No. 28, Cornell University Engineering Experiment Station, 1942.

5. McCalley, R. C., Jr: Discussion of Paper by H. N. Hill, E. C. Hartmann, and J. W. Clark. Transactions of the ASCE, Vol. 121, 1956, p. 15.

6. Lee, L. H. N: On the Lateral Buckling of a Tapered Narrow Rectangular Beam. ASME Journal of Applied Mechanics, September, 1959, p. 457.

7. Krefeld, W. J., Butler, D. J., and Anderson, G. B: Welding Cantilever Wedge Beams. The Welding Journal Research Supplement, March, 1959.

8. Austin, W. J., Yegian, S., and Tung, T. P: Lateral Buckling of Elastically End-Restrained I-Beams. Separate No. 673, Proceedings of the ASCE, Vol. 81, 1955.

9. Salvadori, M. G: Lateral Buckling of I-Beams. Transactions of the ASCE, Vol. 120, 1955, p. 1165.

10. Nylander, H: Torsion, Bending and Lateral Buckling of I-Beams. Bulletin No. 22, Division of Building Statics and Structural Engineering, Royal Institute of Technology, Stockholm, Sweden, 1956.

11. Bleich, F: Buckling Strength of Metal Structures. McGraw-Hill Book Company, Inc., 1952, pp. 55 and 165.

REFERENCES (Concluded):

12. Dumont, C., and Hill, H. N: Lateral Stability of Equal-Flanged Aluminum Alloy I-Beams Subjected to Pure Bending. T. N. 770, NACA, 1940.

13. Clark, J. W., and Jombock, J. R: Lateral Buckling of I-Beams Subjected to Unequal End Moments. Paper No. 1291, Journal of the Engineering Mechanics Division, Proceedings of the ASCE, Vol. 83, July, 1957.

SECTION B4.8

SHEAR BEAMS

TABLE OF CONTENTS

TABLE OF CONTENTS (Concluded)

DEFINITIONS OF SYMBOLS

t — thickness of web plate

t_u — thickness of attached stiffener leg

b — spacing of intermediate stiffeners, or width of unstiffened web plate

b_c — clear web plate distance between stiffeners

d_c — clear depth of web plate

α_e — effective aspect ratio;

$= b/d_c$ for unstiffened web plates, or web plates reinforced by single-sided stiffeners;

$= b_c/d_c$ for web plates reinforced by double-sided stiffeners

D — flexural rigidity of unit width of plate $= Et^3/12(1-u^2)$

E — Young's modulus

μ — Poisson's ratio

I — moment of inertia of stiffener about base of stiffener (next to web)

$\gamma = \dfrac{EI}{Db}$

γ_L — limiting value of γ

K_s — critical shear-stress coefficient

K_L — limiting value of K_s

η_s, η_B — plasticity coefficients which account for the reduction of modulus of elasticity for stress above the elastic limit; within the elastic range, $\eta = 1$

A_f — area of tension or compression flange

DEFINITIONS OF SYMBOLS (Continued)

C_r rivet factor $= \frac{p - D_r}{p}$

D_r rivet diameter

f_b applied bending stress

f_s applied web shear stress

$F_{s_{cr}}$ critical (or initial) buckling stress in shear

M applied bending moment

p rivet spacing

q applied web shear flow

S, V applied transverse shear on beam

A parameter used for type of shear beam selection

h height of beam between centroids of flanges

$B_T = EI_T$, flexural rigidity of transverse stiffeners

$\gamma_T = B_T/Db$, nondimensional flexural rigidity parameter for transverse stiffeners

C_T torsional rigidity of transverse stiffeners

$B_L = EI_L$, flexural rigidity of longitudinal stiffeners

$\gamma_L = \frac{B_L}{Dd_c}$, nondimensional stiffeners parameter for longitudinal stiffeners

C_L torsional rigidity of longitudinal stiffeners

$\Gamma_L = C_L/Dd_c$

DEFINITIONS OF SYMBOLS (Concluded)

Γ_T = C_T/Dd_c

G modulus of rigidity

K_{LH} limiting value of K_s for web reinforced by vertical stiffeners and a central horizontal stiffener

m distance from edge of plate to longitudinal stiffener

$F_{B_{cr}}$ critical (or initial) buckling stress in bending

I_o stiffener moment of inertia for longitudinal stiffener

4.8.0 SHEAR BEAMS

The analysis and design of a metal beam composed of flange members riveted or welded to web members are common problems in aerospace structural design.

Shear beams denote a particularly efficient type of beam. In shear beams the moment resistance is provided by the flanges, which are concentrated near the extreme fibers, and the shear resistance is provided by the thin web connecting the tension and compression caps.

The analysis and design of shear beams as structural components are generally based upon the web response to the applied shear loads. If buckling of the web is inhibited within the design ultimate load, the beam is known as a shear-resistant beam. If, however, the web is allowed to buckle after some application of load causing shear to be resisted in part by tension-field action, the beam is known as a tension-field beam.

The type of shear beam most suitable for a particular design application may depend on many factors. One of the most common factors is based on economy of weight. H. Wagner [1] offers the following criterion, based upon the economy of weight:

$$A = \frac{\sqrt{V}}{h} ,$$

where

V = shear load, lb

and

h = depth of web, in.,

with the recommendation that when $A < 7$ the tension-field web is best, and when $A > 11$ the shear-resistant web is best. When $7 < A < 11$, there is little choice between the two; factors other than weight will then determine the type of web to be used.

The criterion above should not be adhered to rigidly, however, because new data and design techniques have become available that have resulted in reduced weight designs for shear-resistant beams.

Shear-resistant beams and tension-field beams will be discussed in Paragraphs 4.8.1 and 4.8.2 respectively.

4.8.1 PLANE-STIFFENED SHEAR RESISTANT BEAMS

As previously stated, a shear beam whose web is so designed that it does not buckle under the applied loads is referred to as a shear-resistant beam. The analysis of this beam is primarily one of stability. That is, with the exception of the tension flanges, the web, the compression flange, and the stiffeners are all designed from a stability standpoint rather than from a material allowable stress standpoint.

The stability of the shear web can always be increased by increasing its thickness, but such a design will not always be economical with respect to the weight of the material used. A more economical solution is obtained by keeping the thickness of the plate as small as possible (just thick enough to fulfill strength requirements) and increasing the stability by introducing stiffeners. The weight of such stiffeners will usually be less than the additional weight introduced by an adequate increase in the thickness of the plate.

The design criteria of shear-resistant beams may be stated as follows:

1. Local buckling of the web between the stiffeners under combined shear, bending, axial, and transverse stresses must not occur.
2. Elements of the transverse stiffeners must not buckle locally under the transverse stresses.
3. Elements of the flange must not buckle locally under the longitudinal stresses.

If the criteria above are not met, the procedures of analysis that follow are not applicable.

Design analysis techniques for shear resistant beams are given in the following paragraphs.

4.8.1.1 Stability of Web Panel

The critical buckling stress, $F_{s_{cr}}$, of a web panel of height d_c, width b, and thickness t, is given by

$$\frac{F_{s_{cr}}}{\eta_s} = \frac{K_s \pi^2 E}{12(1-\mu^2)} \left(\frac{t}{d_c}\right)^2 \qquad \text{for } d_c \leq b \qquad (1.a)$$

or

$$\frac{F_{s_{cr}}}{\eta_s} = \frac{K_s \pi^2 E}{12(1-\mu^2)} \left(\frac{t}{b}\right)^2 \qquad \text{for } b < d_c \quad , \qquad (1.b)$$

where K_s is a function of the aspect ratio, d_c/b, and the edge restraint offered by the stiffeners and flanges.

Figure 1 shows how the value of the critical shear stress coefficient, K_s, increases with decreasing values of the aspect ratio, b/d_c ($d_c \leq b$), for a number of different edge conditions. This figure indicates quite clearly why vertical stiffeners are so effective in increasing the buckling stress of rectangular webs.

In view of the importance of obtaining the correct design of intermediate stiffeners, a number of theoretical and experimental investigations have been made to determine the relationship between the size and spacing of intermediate stiffeners, and the buckling stress of the stiffened web plate. These investigations have also included the effects of stiffener torsional rigidity, stiffener thickness and rivet location, central longitudinal stiffness, and single-sided or double-sided stiffness. The procedure for the design and analysis of these effects will be given below.

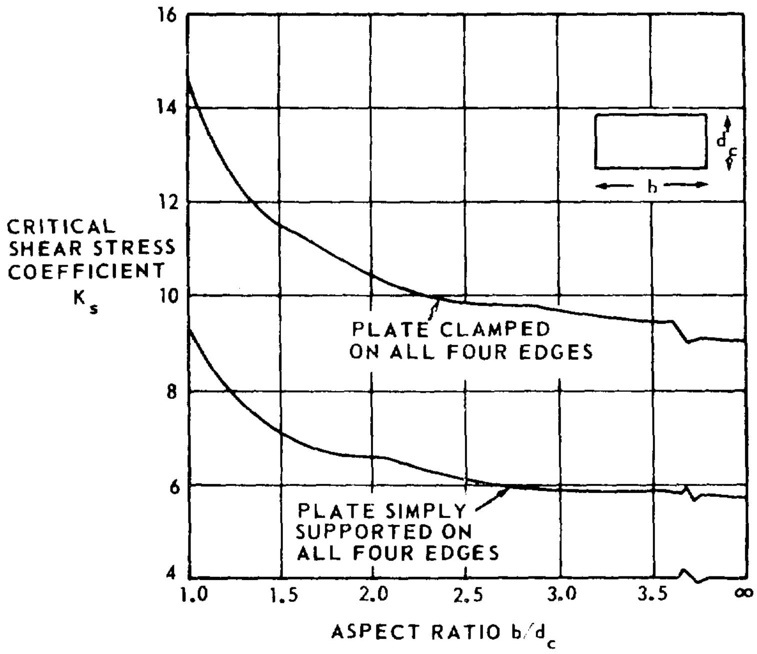


FIGURE 1. K_S VERSUS ASPECT RATIO FOR DIFFERENT EDGE RESTRAINTS

I. Transverse Stiffeners — Flexural Rigidity Only

Theoretical work has been performed [2-4] to ascertain the relationship between K_S and the nondimensional parameter γ $(= \frac{EI}{Db})$ for various aspect ratios and boundary conditions. Figure 2 is a typical plot showing the relationship of these parameters. It will be noted that points of discontinuity occur on the K_S/γ curves. These points of discontinuity denote where changes in the buckle pattern occur. It can also be seen that when a certain value of γ is reached, there is no appreciable increase in K_s. This value of γ is called γ_L or limiting value of γ, since higher values would result in an inefficient design.

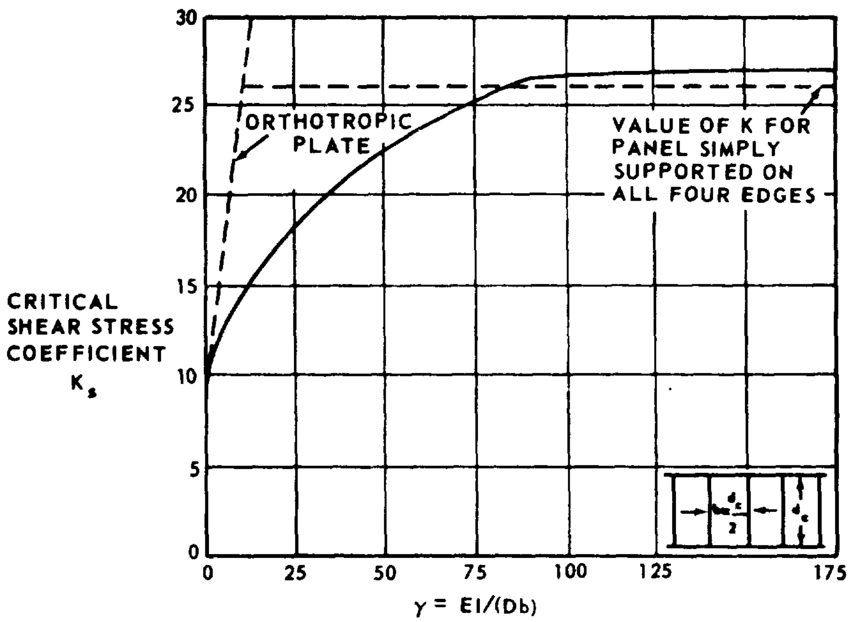


FIGURE 2. THE THEORETICAL K/γ RELATIONSHIPS DERIVED BY STEIN AND FRALICH FOR INFINITELY LONG PLATES SIMPLY SUPPORTED AT THE EDGES AND REINFORCED BY STIFFENERS $0.5d_c$ APART.

For design purposes, the following relationships should be used. However, it should be noted that these relationships are valid only for stiffeners whose thickness is equal to or greater than the thickness of the web plate.

$$\gamma_L = 27.75 (\alpha_e)^{-2} - 7.5 \text{ for double-sided stiffeners,} \quad (2.a)$$

$$\gamma_L = 21.5 (\alpha_e)^{-2} - 7.5 \text{ for single-sided stiffeners,} \quad (2.b)$$

$$K_L = 7.0 + 5.6\ (\alpha_e)^{-2} \text{ for both single- and double-sided stiffeners,} \quad (3)$$

where

α_e = b_c/d_c for double-sided stiffeners $(b_c \geq d_c)$ and

α_e = b/d_c for single-sided stiffeners $(b \geq d_c)$.

II. Transverse Stiffeners — Effect of Stiffener Thickness and Rivet Location

Investigations have been carried out to investigate fully the various parameters that affect the behavior of single-sided stiffeners having attached legs thinner than the web-plate [5]. The parameters t_u/t and c/t were studied in these investigations to evaluate their effect on K_s, where t, t_u, and c are as shown in Figure 3. It was shown that the primary influence on K_s was the value c and that the t_u/t variation had little effect on K_s. Figure 4 shows the variation of K_s with the parameter $(t_u/t)(t/c)^{1/2}$. From the figure it can be seen that for the stiffener to provide a value of K_s equal to K_L, it is necessary that

$$(t_u/t)(t/c)^{1/2} \geq 0.27 . \quad (4)$$

This equation can be used to determine the position of the rivet for fully effective stiffeners

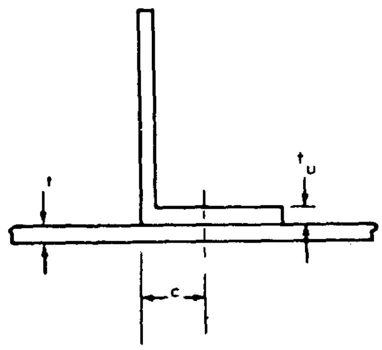


FIGURE 3. VALUES OF t, t_u, AND c

for various values of t_u/t. For values of $(t_u/t)(t/c)^{1/2}$ less than 0.27 the value of K_s should be reduced as shown in Figure 4.

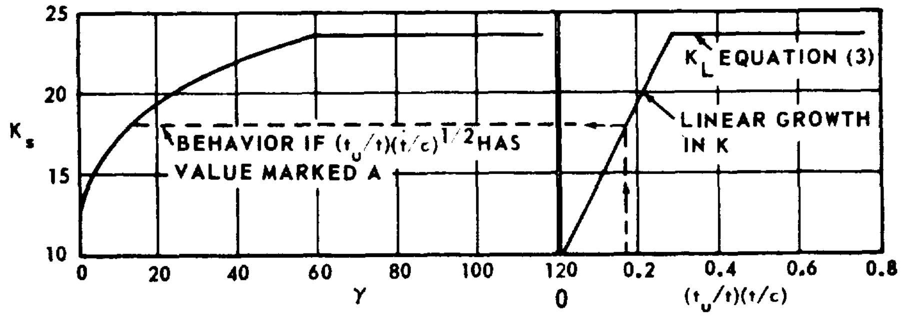


FIGURE 4. VARIATION OF K_s WITH THE STIFFENER THICKNESS AND RIVET LOCATION PARAMETER

III. Transverse Stiffeners — Flexural and Torsional Rigidity

Theoretical results have been obtained [4, 6-8] that provide relationships between K_s and the flexural rigidity of the stiffeners for various values of the ratio of torsional rigidity to flexural rigidity for simply supported or clamped longitudinal edges. It was assumed that the stiffeners were symmetrically disposed about the midplane of the web plate as shown in Figure 5.

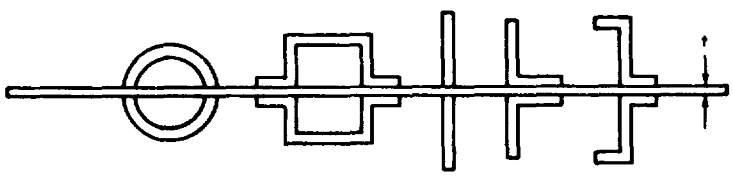


FIGURE 5. VARIOUS SHAPES OF STIFFENERS

Figures 6 and 7 give K_s/γ relationships for b = d and b = d/2 with the longitudinal edges simply supported. Figures 8 and 9 give K_s/γ relationships for b = d and b = d/2 with the longitudinal edges clamped. The maximum value of C_T/B_T plotted is for a closed circular tube. This value is 0.769.

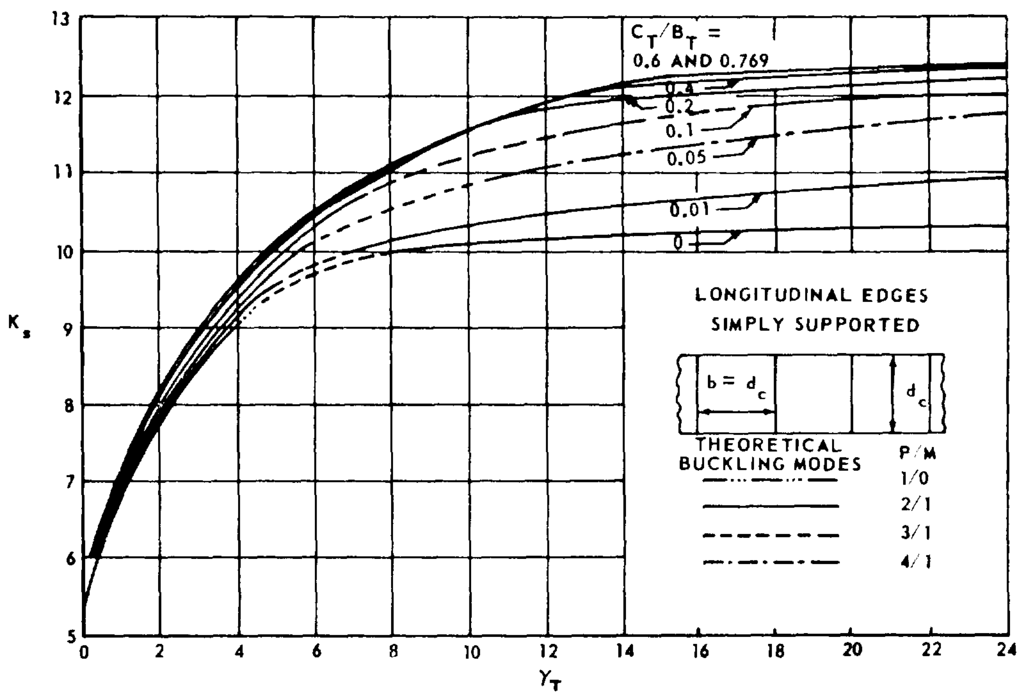


FIGURE 6. K_s VERSUS γ_T RELATIONSHIPS FOR b = d_c

From these figures it is shown that very significant increases in the buckling resistance, K_s, are obtainable by using closed-section stiffeners in place of the open-section stiffeners so frequently used. For example, by using a thin-walled circular tube for the stiffeners (C_T/B_T = 0.769) the gain in K_L

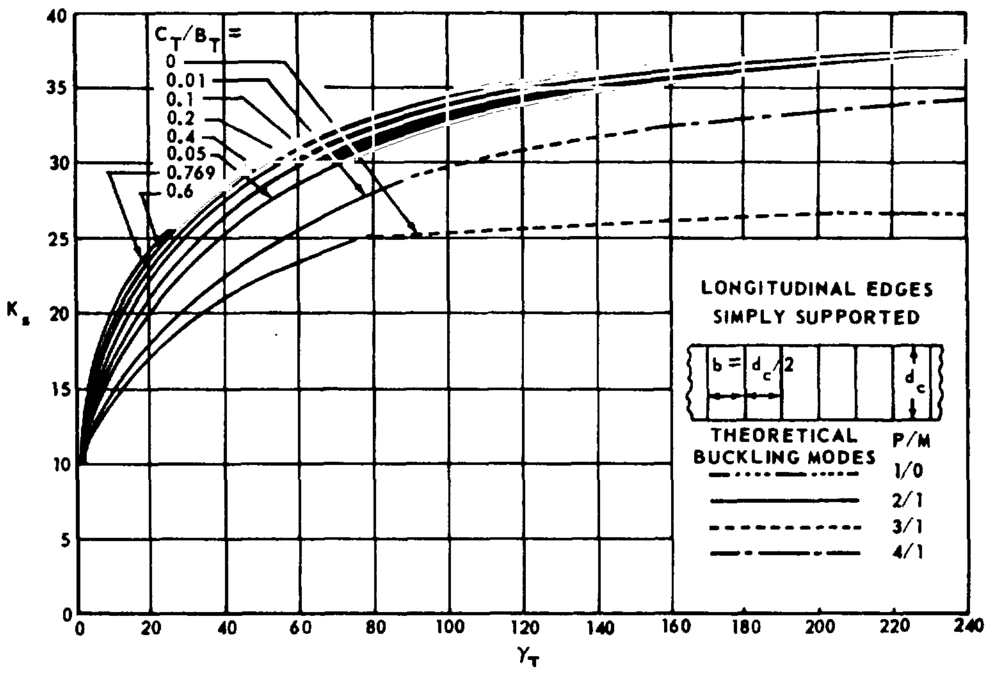


FIGURE 7. K_S VERSUS γ_T RELATIONSHIPS FOR $b = d_c/2$

is 25 percent and 60 percent for $\alpha = 1$ and $\alpha = 2$, respectively, when the longitudinal edges are simply supported. For the case of clamped edges, the gains are 13 percent and 43 percent for $\alpha = 1$ and $\alpha = 2$, respectively. Thus, if a minimum weight design is desired, consideration should be given to closed-section stiffeners.

IV. Transverse and Central Longitudinal Stiffeners

The use of deep beams with webs having a high depth-to-thickness ratio, may make it desirable to employ both vertical and horizontal stiffening. When

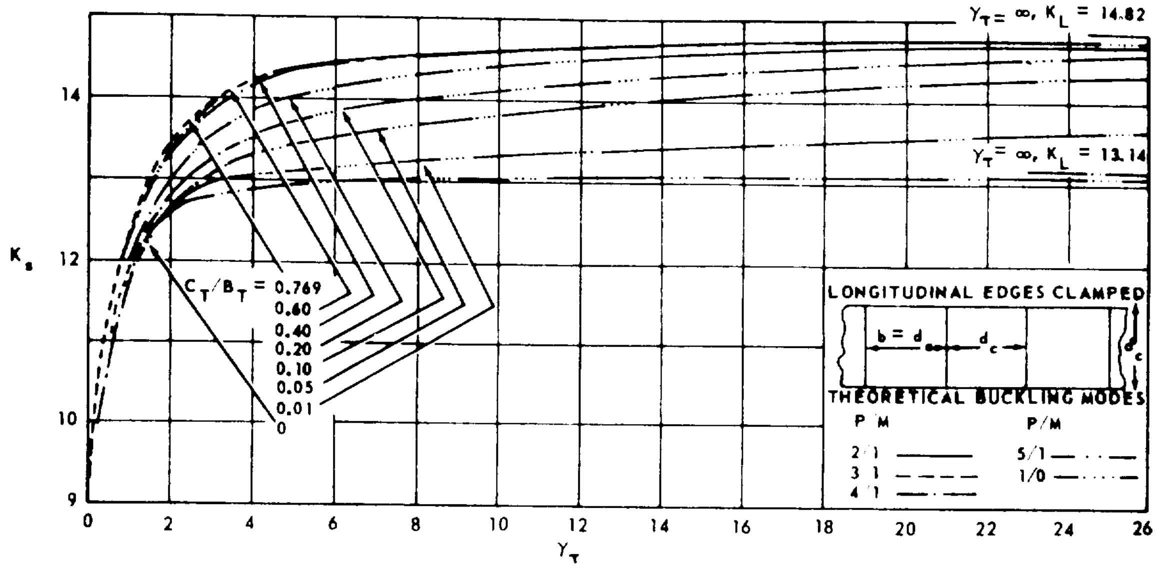


FIGURE 8. K, γ_T RELATIONSHIPS; ASPECT RATIO α = 1.0

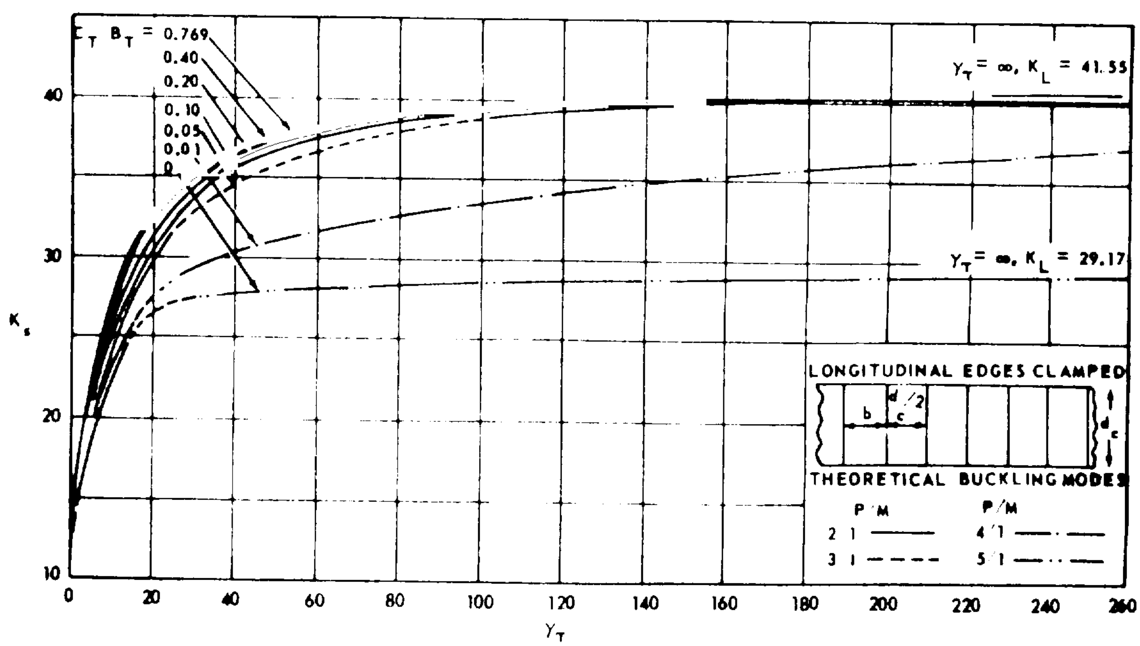


FIGURE 9. K, γ_T RELATIONSHIPS; ASPECT RATIO α = 2.0

a web is subjected to shear, the most effective position for a single horizontal stiffener is at middepth. This combination of vertical and horizontal stiffening can result in more economical designs than are possible when only vertical stiffeners are employed. For example, the weight of stiffening required to achieve a given buckling stress with horizontal and vertical stiffening can be as little as 50 percent of the weight required when only vertical stiffeners are used.

If neither of the transverse or central longitudinal stiffeners has torsional rigidity and the vertical stiffeners have a rigidity equal to or greater than EI_{LV}, then the value of γ_{LH} necessary to produce the limiting value of K_{LH} is given by

$$\gamma_{LH} = 11.25\ (b/d_c)^2 \tag{5}$$

and

$$K_{LH} = 29 + 4.5(b/d_c)^{-2} \quad . \tag{6}$$

Additional weight savings can be achieved if torsionally strong stiffeners are used in either the transverse or longitudinal direction. For example, studies in Reference 6 have shown gains in the value of K_L (which is proportional to the weight of the web) of up to 25 and 60 percent for α equal to one and two, respectively, by using closed-section stiffeners in the transverse direction only. Investigators have given parameter studies on the following in References 4 and 9:

1. Transverse and longitudinal stiffeners of closed tubular cross section.
2. Transverse stiffeners of closed tubular cross section; longitudinal stiffener possessing only flexural rigidity.

3. Transverse stiffeners possessing only flexural rigidity, the longitudinal stiffeners being of closed tubular cross section.

Figure 10 enables one to make an assessment of the benefits that result from using torsionally strong stiffeners when a central longitudinal stiffener is used in conjunction with a system of equally spaced transverse stiffeners. When $\alpha \leq 1$, it is evident that little increase is obtained in the buckling resistance by using torsionally strong transverse stiffeners, but that a considerable increase in buckling resistance will be obtained by using a torsionally strong longitudinal stiffener. However, as α increases (that is, as the transverse stiffeners are

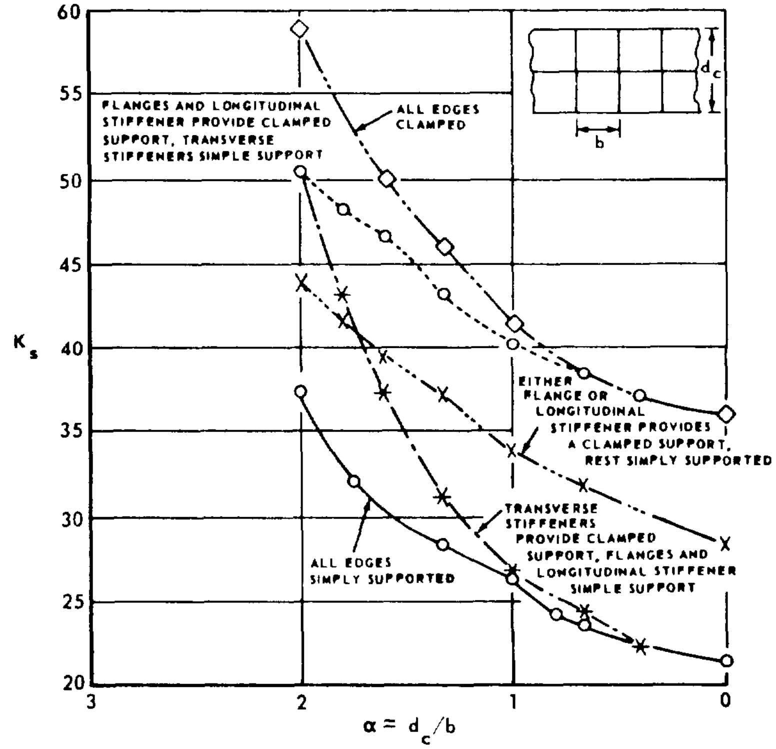


FIGURE 10. K_S VERSUS ASPECT RATIO FOR VARIOUS EDGE RESTRAINTS

more closely spaced), the increase in K resulting from the use of torsionally strong stiffeners becomes more significant; until, when $\alpha = 1.7$, the buckling resistance obtained with torsionally strong transverse stiffeners and a longitudinal stiffener without torsional rigidity is equal to that obtained with a torsionally strong longitudinal stiffener and transverse stiffeners possessing only flexural rigidity. Thus, it will be readily seen that is is necessary to know the relationships between K_s and the stiffener properties for the cases enumerated above. Figures 11, 12, and 13 give typical relationships between K_s and γ_T for b = d and different positions of the torsionally strong stiffener. Curves for other values of the aspect ratio can be obtained from Reference 4. The points

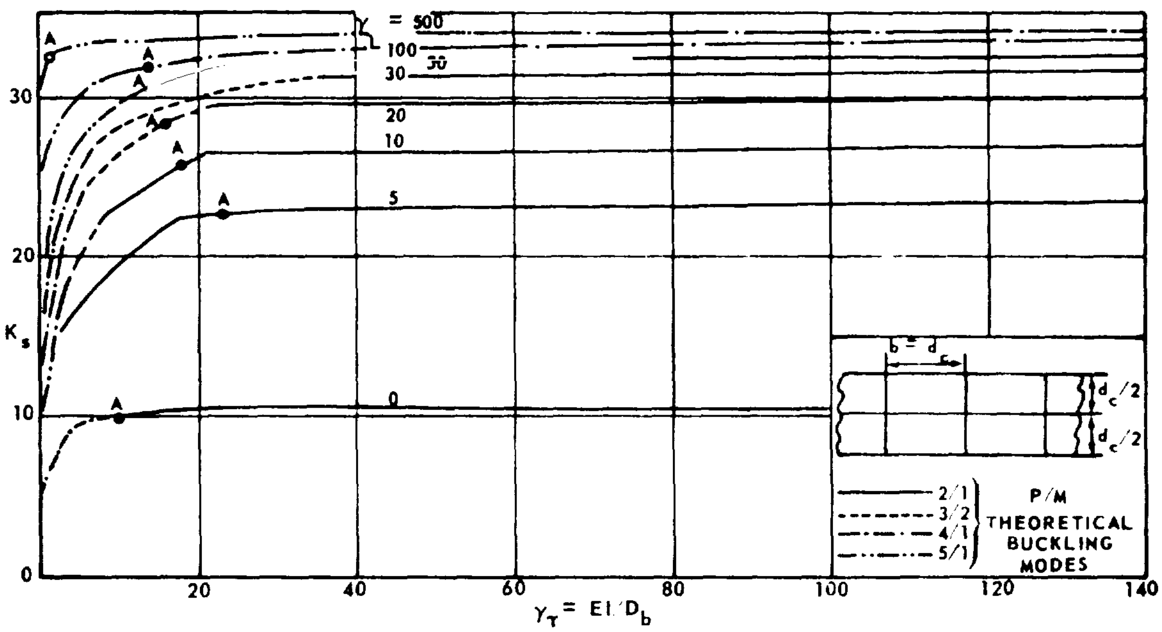


FIGURE 11. K_s, γ_T RELATIONSHIPS; LONGITUDINAL STIFFENER WITH TORSION ($C_L = 0.769\ B_L$); TRANSVERSE STIFFENERS WITH ONLY FLEXURAL RIGIDITY ($C_T = 0$); ASPECT RATIO $\alpha = 1.0$

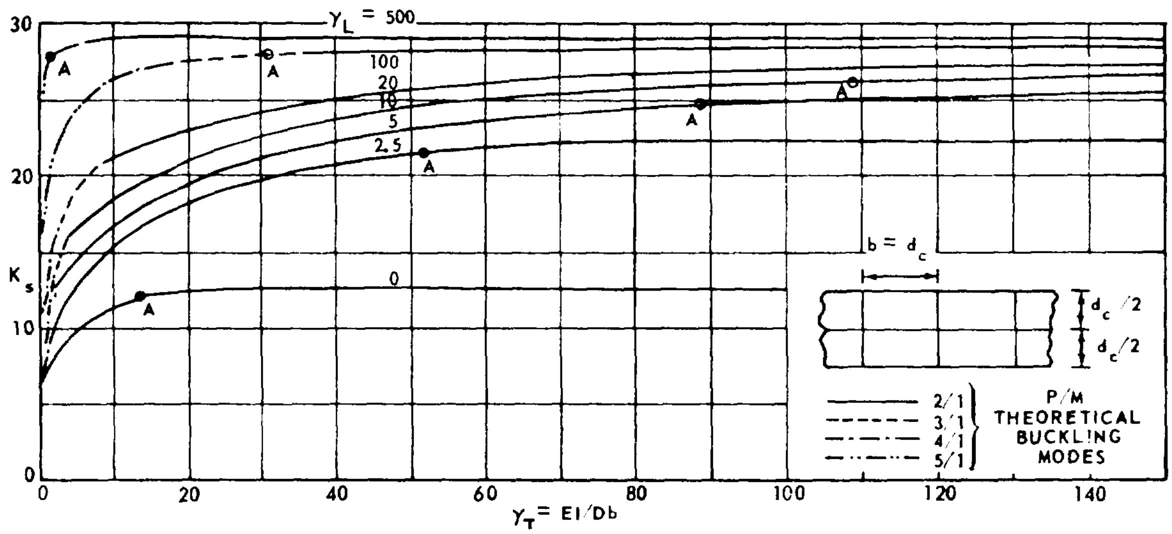


FIGURE 12. K_s, γ_T RELATIONSHIPS; LONGITUDINAL EDGES SIMPLY SUPPORTED; TRANSVERSE STIFFENERS WITH TORSIONAL RIGIDITY (C_T = 0.769 B_T); LONGITUDINAL STIFFENERS WITH ONLY FLEXURAL RIGIDITY (C_L = 0); ASPECT RATIO α = 1.0

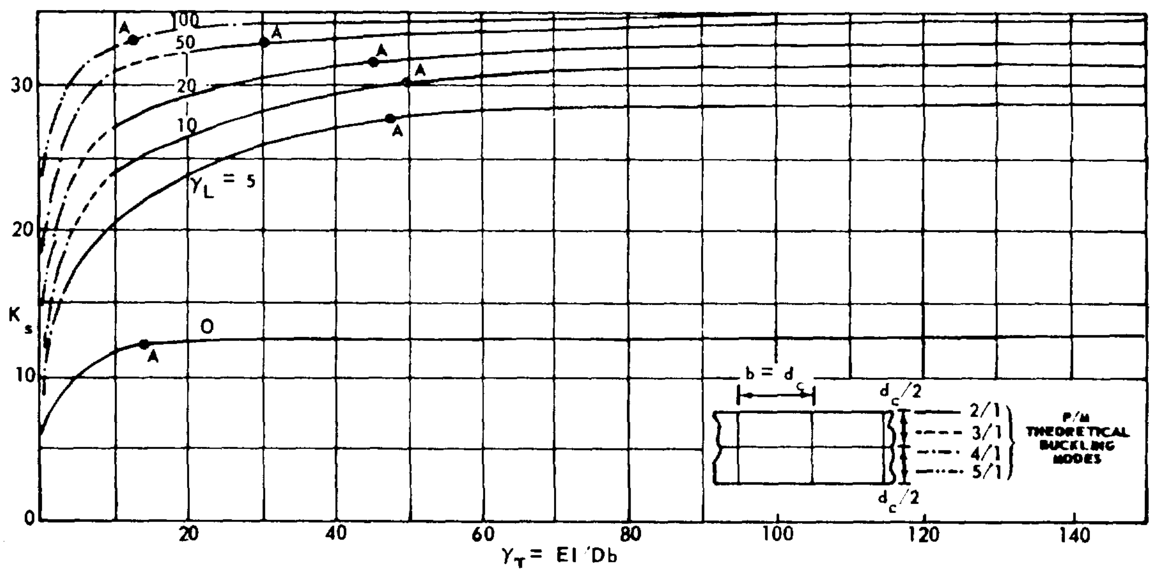


FIGURE 13. K_s, γ_T RELATIONSHIPS; SIMPLY SUPPORTED LONGITUDINAL EDGES; ALL STIFFENERS WITH TORSIONAL RIGIDITY (C_T = 0.769 B_T, C_L = 0.769 B_L); ASPECT RATIO α = 1.0

marked A on the curves of Figures 11, 12, and 13 are values of γ_T which would give 95 percent of the limiting value of K_s. This is suggested as a good cutoff point for an efficient design. If greater values of γ_T are chosen, only a small increase in the value of K_s would occur. Thus, the extra increase in γ_T would not be very beneficial from a weight standpoint.

V. Longitudinally Stiffened Web Plates in Longitudinal Compression

In deep beams, it is often economical to stiffen the web plate by longitudinal stiffeners in locations where the longitudinal compressive stresses resulting from bending are high. Two positions of the stiffener will be considered here: (1) The stiffener located at the longitudinal center line of the web, that is, at the neutral axis (Fig. 14a) and (2) the stiffener located in the compressive region at a distance from the edge of the plate (Fig. 14b). In case 1, the stiffener itself does not carry compressive stresses.

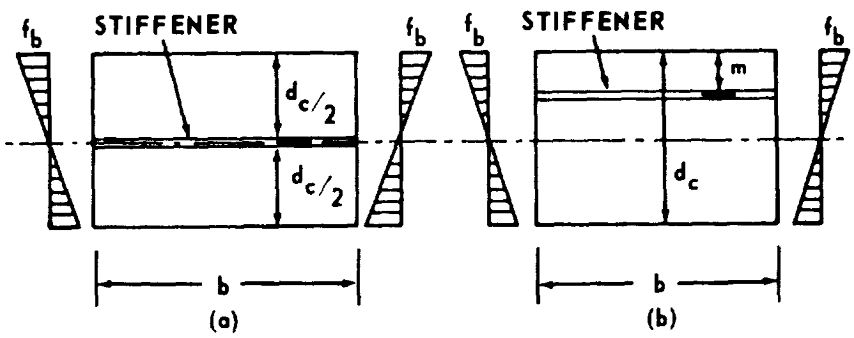


FIGURE 14. POSITIONS OF LONGITUDINAL STIFFENERS

Adding this longitudinal stiffener results in another structural part and more assembly cost; therefore, such construction is not widely used although it is a structural arrangement that will save structural weight under certain conditions of beam depth, span, and external loading.

A. Stiffener at the Centerline

For a stiffener at the centerline, the largest practical value of the stiffener moment of inertia is

$$\frac{I_o}{\eta_B} = 0.92 t^3 d_c \qquad (7)$$

With this value of I_o, the critical bending stress $F_{B_{cr}}$ is

$$\frac{F_{B_{cr}}}{\eta_B} = \frac{35.6\,\pi^2 E}{12(1-\mu^2)} \left(\frac{t}{d_c}\right)^2 \quad \text{for } \alpha \geq 2/3 \quad . \qquad (8)$$

For values of $F_{B_{cr}}$ for stiffener moment of inertia less than I_o, see Reference 10.

B. Stiffeners Located Between Compression Edge and Neutral Axis

The increase in buckling strength that can be obtained by a stiffener at the centerline of the web amounts to only 50 percent of the unstiffened plate in the inelastic range. Stiffeners at the centerline are therefore not very effective in improving the stability of web plates in case of pure bending stresses.

For a stiffener spaced at a distance $m = d_c/4$, the critical buckling stress is given by

$$\frac{F_{B_{cr}}}{\eta_B} = \frac{101\,\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{d_c}\right)^2 \qquad (\alpha \geq 0.4), \qquad (9)$$

if $\gamma \geq \gamma_o$.

Where $\gamma = \dfrac{EI}{\eta_B D d_c}$,

$$\gamma_o = \frac{EI_o}{Dd_c} = (12.6 + 50\,\delta)\,\alpha^2 - 3.4\,\alpha^3 \qquad (\alpha \leq 1.6) \text{ and}$$

$$\delta = \frac{A}{\eta d_c t}\ , \quad \alpha = b/d_c\ .$$

Comparison of the results obtained above for a stiffener at the centerline of the plate with the results obtained for a plate stiffened in the compression region shows that the reinforcement in the latter case is much more effective.

Limited numerical results have been obtained for plates reinforced by a longitudinal stiffener located at a distance $m = d_c/5$ from the compression edge of the plate. This information is plotted in Figures 15 and 16. The largest value of the buckling strength of the plate stiffener system corresponds to $K = 129$ and is larger than in the case of a stiffener located at the distance $d_c/4$ from the compression edge.

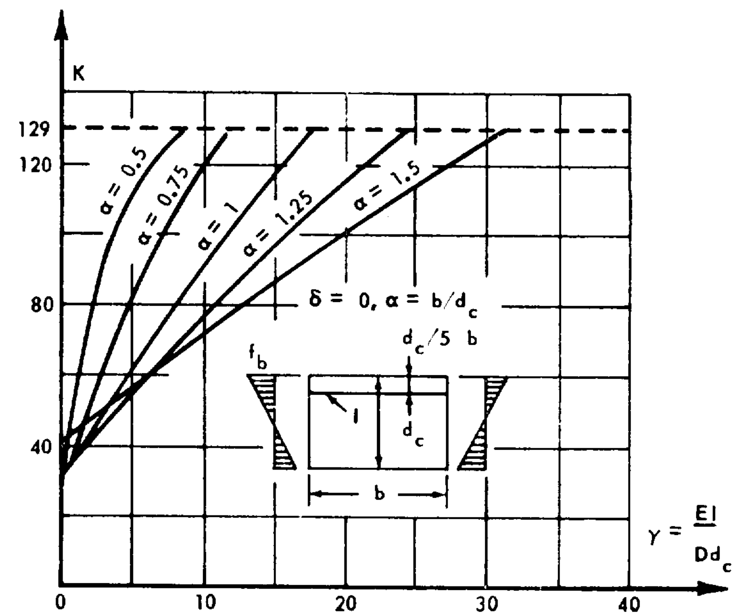


FIGURE 15. K VERSUS γ FOR VARIOUS VALUES OF α, $\delta = 0$

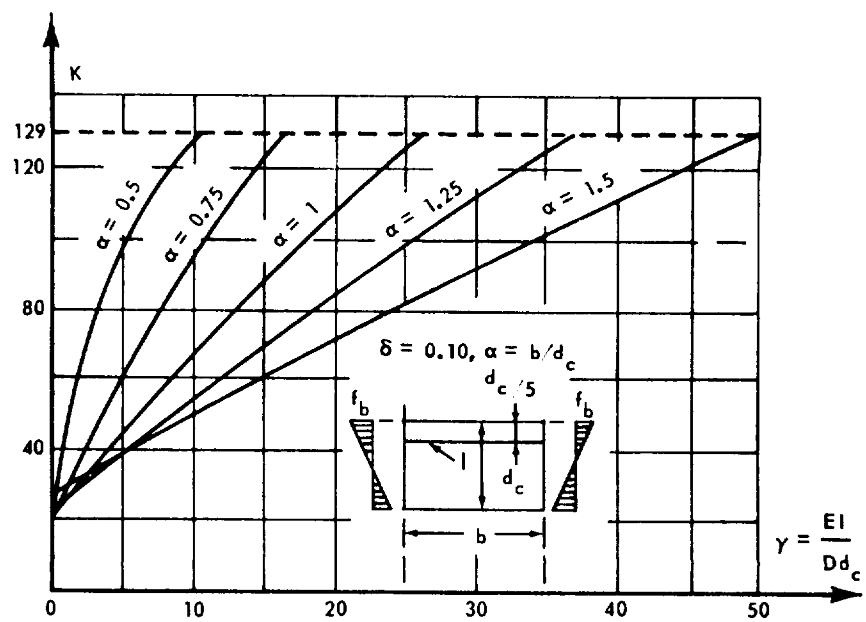


FIGURE 16. K VERSUS γ FOR VARIOUS VALUES OF α, $\delta = 0.10$

VI. Combined Stresses

The above mentioned bending, shear, and possibly axial and transverse stresses that act upon the web should be interacted by the following equations.

$$R_s^2 + R_{c_L} \leqq 1 \tag{10}$$

$$R_s^2 + R_{c_T} \leqq 1 \tag{11}$$

$$R_s^2 + R_B^2 \leqq 1 \tag{12}$$

$$R_B^{1.75} + R_{c_L} \leqq 1 \tag{13}$$

where

$$R_s = \frac{f_s}{F_{s_{cr}}}, \quad R_B = \frac{f_b}{F_{B_{cr}}}, \quad R_c = \frac{f_c}{F_{C_{cr}}},$$

and the subscript, L, indicates longitudinal and the subscript, T, indicates transverse. The critical values of $F_{C_{cr}}$ should be obtained from Section C2.1.1.

If the interaction equations above are not satisfied, an iteration of the design must be performed.

4.8.1.2 Flange Design

The beam flanges are designed for tensile and compressive normal forces. The ultimate allowable stress for the tension flange is equal to F_{tu} of the material, reduced by the attachment efficiency factor. For riveted or bolted connections, the efficiency factor is the ratio of the net area to the gross area of the cap.

Compression flanges should be designed for column stability. The loads on the compression flange can be a combination of normal force, longitudinal shear at the web-flange connection, and transverse forces. Specific analysis techniques are available in Section B4.4.0.

4.8.1.3 Rivet Design

I. Web-to-Stiffener

Although no exact information is available on the strength required of the attachment of the stiffeners to the web, the data in Table B4.8-I are recommended.

Table B4.8-I. Rivets: Web-to-Stiffener

Web Thickness (in.)	Rivet Size	Rivet Spacing (in.)
0.025	AD 3	1.00
0.032	AD 4	1.25
0.040	AD 4	1.10
0.051	AD 4	1.00
0.064	AD 4	0.90
0.072	AD 5	1.10
0.081	AD 5	1.00
0.091	AD 5	0.90
0.102	DD 6	1.10
0.125	DD 6	1.00
0.156	DD 6	0.90
0.188	DD 8	1.00

II. Stiffeners-to-Flange

No information is available on the strength required of the attachment of the stiffeners to the flange. It is recommended that one rivet the next size larger than that used in the attachment of the stiffeners to the web or two rivets the same size be used whenever possible.

III. Web-to-Flange

The rivet size and spacing should be designed so that the rivet allowable (bearing or shear) divided by $q \times p$, the applied web shear flow times the rivet spacing, gives the proper margin of safety. For a good design and to avoid undue stress concentration, the rivet factor, C_r, should not be less than 0.6.

4.8.1.4 Design Approach

The design of stiffened shear-resistant beams is a trial-and-error method. Assuming q, b, h, and E are known, the first step is to assume a reasonable value of t and compute $f_s = q/t$. The problem is to find the moment of inertia of a stiffener required to develop an initial buckling stress, $F_{s_{cr}}$, in the web greater than f_s by the desired margin of safety. The procedure is to choose the desired $F_{s_{cr}}$, and $F_{s_{cr}}/\eta_s$ if required. K_s is then found from equation (1) as a function of $F_{s_{cr}}/\eta$ and d/t or h/t. Then, depending on the type of stiffening arrangement used, the required I of the stiffener is obtained from Paragraph 4.8.1.1. Particular attention should be given to stiffener properties that provide an efficient design. For a minimum-weight design, consideration should be given to longitudinal stiffeners and/or torsionally strong stiffeners as discussed in Paragraph 4.8.1.1. Attention should also be given to stiffener thickness and rivet location as discussed in Paragraph 4.8.1.1-II.

4.8.1.5 Stress Analysis Procedure

The stress analysis procedure for the web of a stiffened shear resistant beam is straightforward and easy to apply. Since q, b, h, E, t, f_s, and I are all known, the first step is to obtain K_s from the appropriate curve in Paragraph 4.8.1.1, depending upon the aspect ratio, torsional rigidity, stiffener thickness, etc. Then $F_{s_{cr}}$ can be obtained from equation (1). If necessary, values of η_s can be obtained from Section C2.0. Then the margin of safety for the web is

$$M.S. = \frac{F_{s_{cr}}}{f_s} - 1. \qquad (14)$$

4.8.1.6 Other Types of Web Design

The web designs discussed previously require a large number of parts (stiffeners) to achieve lightness. To keep manufacturing costs low, the number of parts must be kept to a minimum. The problem is one of weight trade-off versus manufacturing expense.

Three types of shear-resistant, nonbuckling webs are frequently used in design to save the expense of stiffeners. Actually, the web in most cases is as light as, or lighter than, a web with separate stiffeners. There is a general limitation, however, in that a stiffener must be provided wherever a significant load is introduced into the beam. The web types are:

I. Web with formed vertical beads at a minimum spacing

II. Web with round lightening holes having 45 degree formed flanges at various spacing

III. Web with round lightening holes having formed beaded flanges and vertical formed beads between holes.

The webs with holes, II and III, also provide built-in access space for the many hydraulic and electrical lines that are sometimes required.

Procedures for the design of these beams should be obtained from Reference 11.

4.8.2 PLANE TENSION-FIELD BEAMS

If web buckling occurs after some application of load, the shear load beyond buckling is resisted in part by pure tension-field action of the web, and in part by shear-resistant action of the web (Fig. 17). This action of the web is defined as an incomplete tension field, or partial tension field.

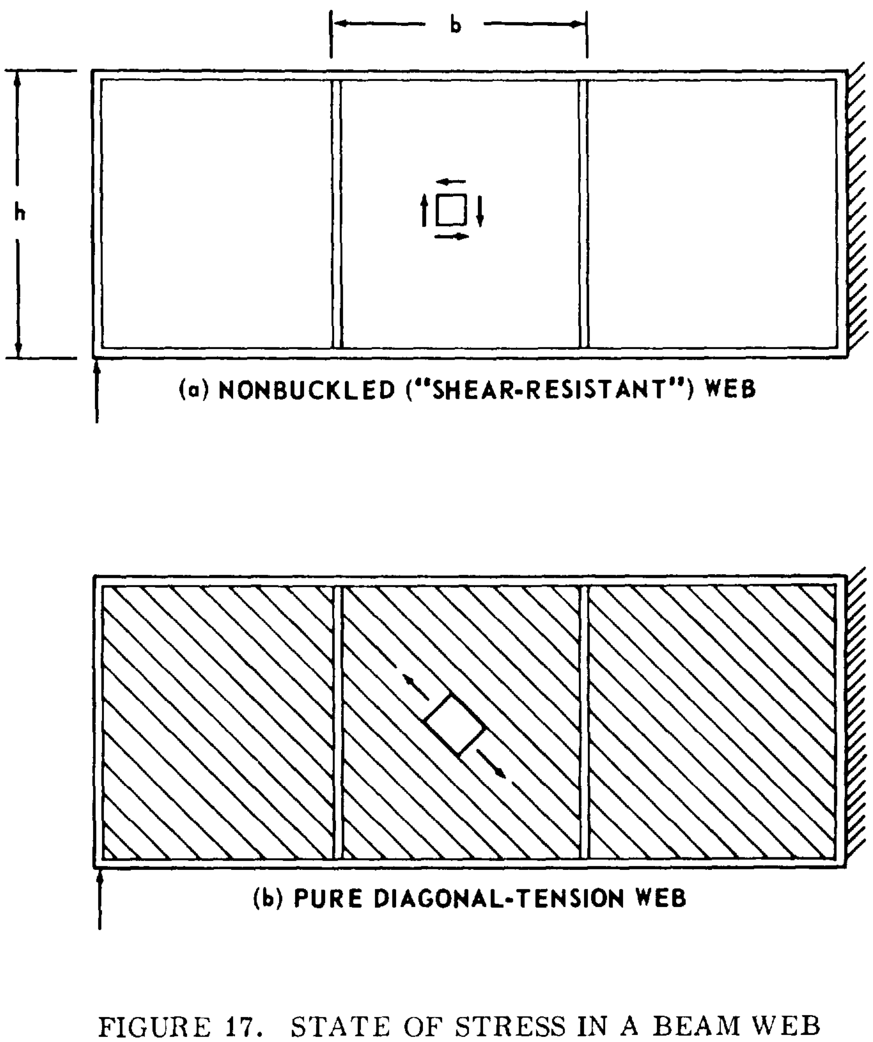


FIGURE 17. STATE OF STRESS IN A BEAM WEB

The theory of pure tension-field beams was published by Wagner [1]. In this theory, after initial web buckling, the total shear load is resisted by pure tension-field action of the web. Such behavior is essentially nonexistent in practice, and will not be discussed here.

Kuhn, Peterson, and Levin [12], developed a semiempirical analysis for partial tension-field beams. Correlation with experimental results indicates that the analysis is conservative for the beams within the range of beams tested. The experimental verification for the analysis was restricted to 2024S-T and 7075S-T aluminum alloy beams. As a consequence, the Kuhn analysis is limited to beams of these alloys. Extension of the analysis to include other alloys is not documented, and the designer must exercise considerable caution in attempting an extension of Kuhn's work in the analysis of beams fabricated from other alloys.

4.8.2.1 General Limitations and Symbols

The methods of analysis and design given herein are believed to furnish reasonable assurance of conservative strength predictions, provided that normal design practices and proportions are used. The most important points are:

I. The uprights should not be "too thin"; keep $t_u/t > 0.6$.

II. The upright spacing should be in the range $0.2 < b/h < 1.0$.

III. The method of analysis presented here is applicable only to beams with webs in the range $60 < h/t < 1500$.

When $h/t < 115$, the portal frames effect and the effect of unsymmetrical flanges must be taken into account by using Reference 13.

SYMBOLS (in addition to those given in the front matter)

A_f	area of tension or compression flange
A_u	actual area of upright (stiffener)
A_{ue}	effective area of upright
C_1, C_2, C_3	stress concentration factors
F_{co}	column yield stress (the column stress at $\frac{L'}{\rho} = 0$)
F_c	allowable column stress
F_{max}	ultimate allowable compressive stress for natural crippling
F_o	ultimate allowable compressive stress for forced crippling
F_s	ultimate allowable web shear stress
I_f	average moment of inertia of beam flanges
I_s	required moment of inertia of upright about its base
I_u	moment of inertia of upright about its base
M_{sb}	secondary bending moment in the flange
P	applied shear load
P_u	upright end load
e	distance of upright centroid to web
f_{cent}	compressive stress at centroidal axis of upright
f_f	compressive stress in flange because of the distributed vertical component of the diagonal tension
f_u	average lengthwise compressive stress in upright

f_{sb}	secondary bending stress in flange because of the distributed vertical component of the diagonal tension
$f_{u_{max}}$	maximum compressive stress in upright
k	diagonal tension factor
b	spacing of uprights
h	effective depth of beam centroid of compression flange to centroid of tension flange
q_r	rivet shear load, web-to-flange and web splices
α	angle of diagonal tension
ρ	radius of gyration of upright with respect to its centroidal axis parallel to web (no portion of web to be included)
k_{ss}	critical shear stress coefficient
R_d, R_h	restraint coefficients
α_{PDT}	angle of diagonal tension for pure tension field beam
N	applied transverse load
Q'	static moment of cross section
h_u	height of stiffener
L_e	effective stiffener length
ω	load-per-inch acting normal to end bay stiffener

4.8.2.2 Analysis of Web

The web shear flow can be closely approximated by

$$q = \frac{V}{h} . \tag{15}$$

From this, it follows that the web shear stress is

$$f_s = q/t \quad . \tag{16}$$

The critical buckling stress of the web is given by

$$\frac{F_{s_{cr}}}{\eta} = k_{ss} \frac{\pi^2 E}{12(1-\mu^2)} \left(\frac{t}{b}\right)^2 \left[R_h + 1/2(R_d - R_h)\left(\frac{b}{d_c}\right)^3\right] \qquad b < d_c \tag{17.a}$$

and

$$\frac{F_{s_{cr}}}{\eta} = k_{ss} \frac{\pi^2 E}{12(1-\mu^2)} \left(\frac{t}{d_c}\right)^2 \left[R_d + 1/2(R_h - R_d)\left(\frac{d_c}{b}\right)^3\right] \qquad b > d_c . \tag{17.b}$$

The value of k_{ss} is obtained from Figure 18. The values R_h and R_d, the restraint coefficients, are given in Figure 19. Figure 20 provides $F_{s_{cr}}$ for the case $\eta \neq 1$. When R_h is very small, the value of the critical shear stress calculated from the equation above may be less than the value computed disregarding the presence of stiffeners. In this case, the stiffeners are disregarded and the latter value is used.

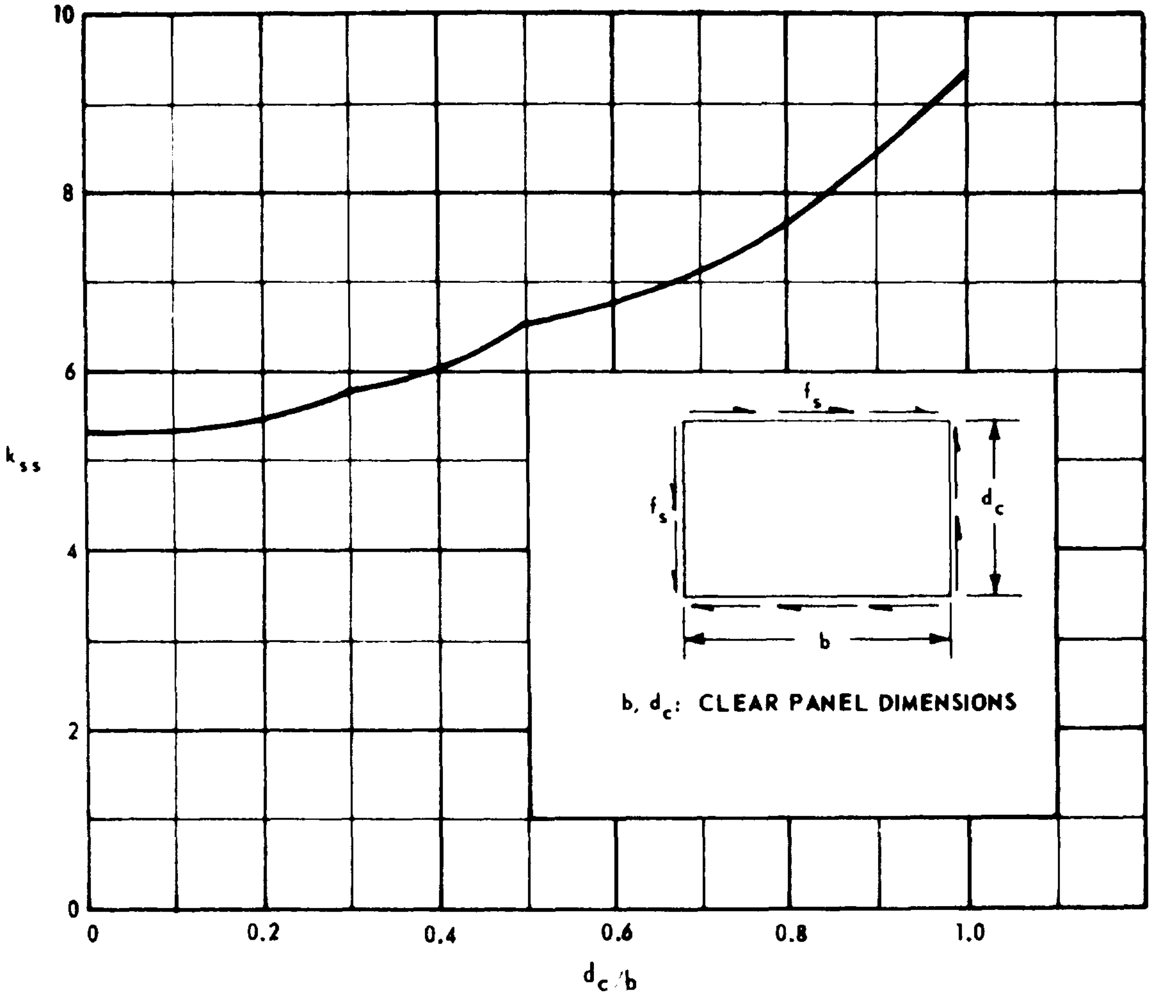


FIGURE 18. k_{ss} VERSUS d_c/b

The loading ratio, $f_s/F_{s_{cr}}$, is used to determine the tension field factor, k.

It may be calculated by

$$k = \tanh\left(0.5 \log_{10} f_s/F_{s_{cr}}\right) \qquad f_s > F_{s_{cr}} , \tag{18}$$

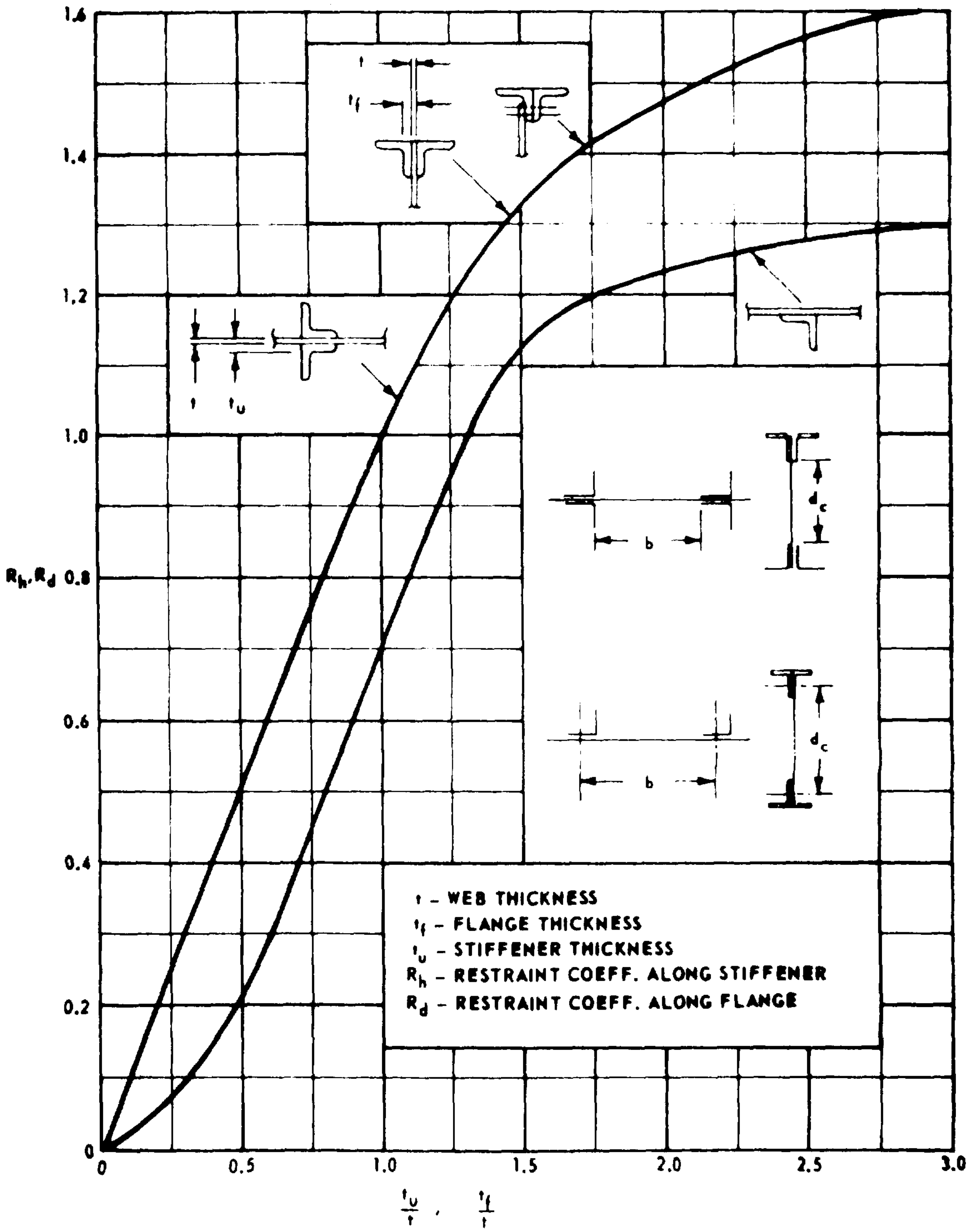


FIGURE 19. EDGE RESTRAINT COEFFICIENTS FOR WEB BUCKLING STRESS

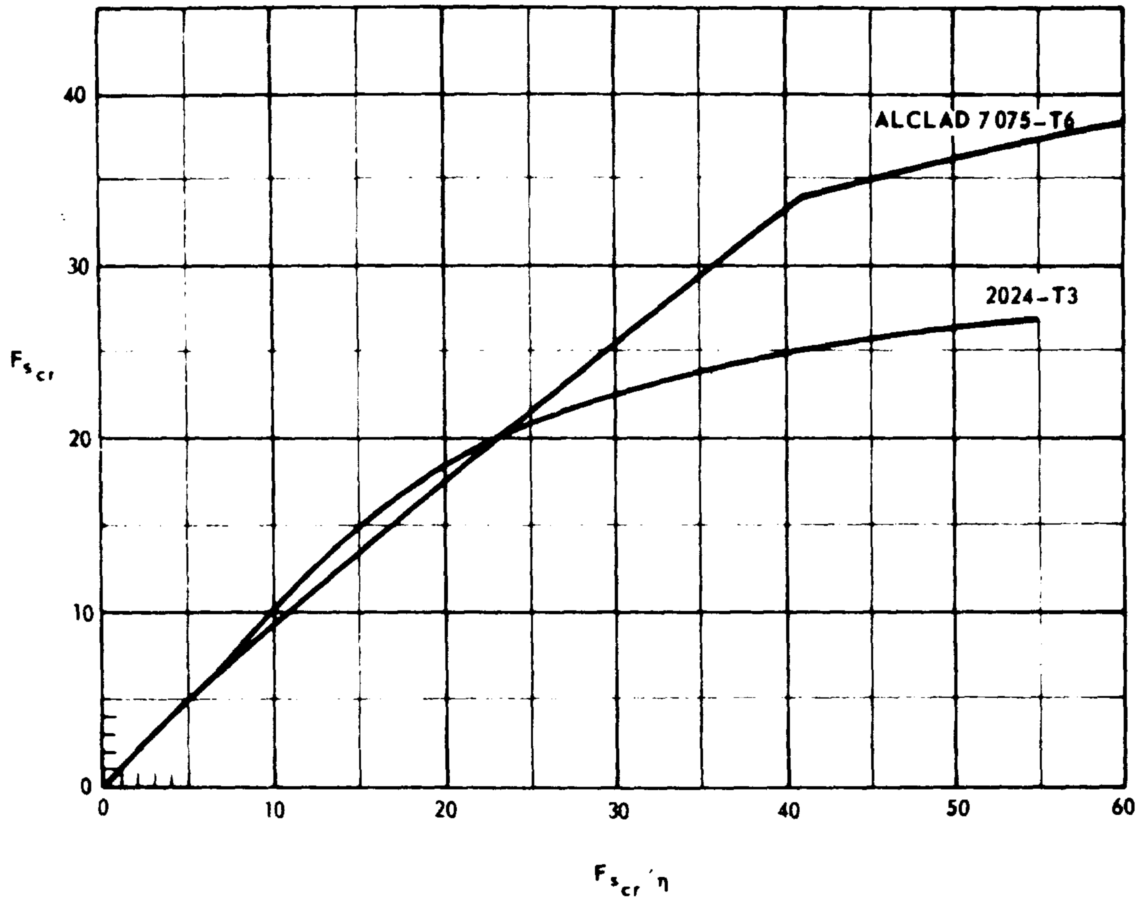


FIGURE 20. VALUES OF $F_{s_{cr}}$ WHEN $\eta \neq 1$

or it may read from Figure 21. For values of $f_s \leq F_{s_{cr}}$, the web is in the unbuckled state.

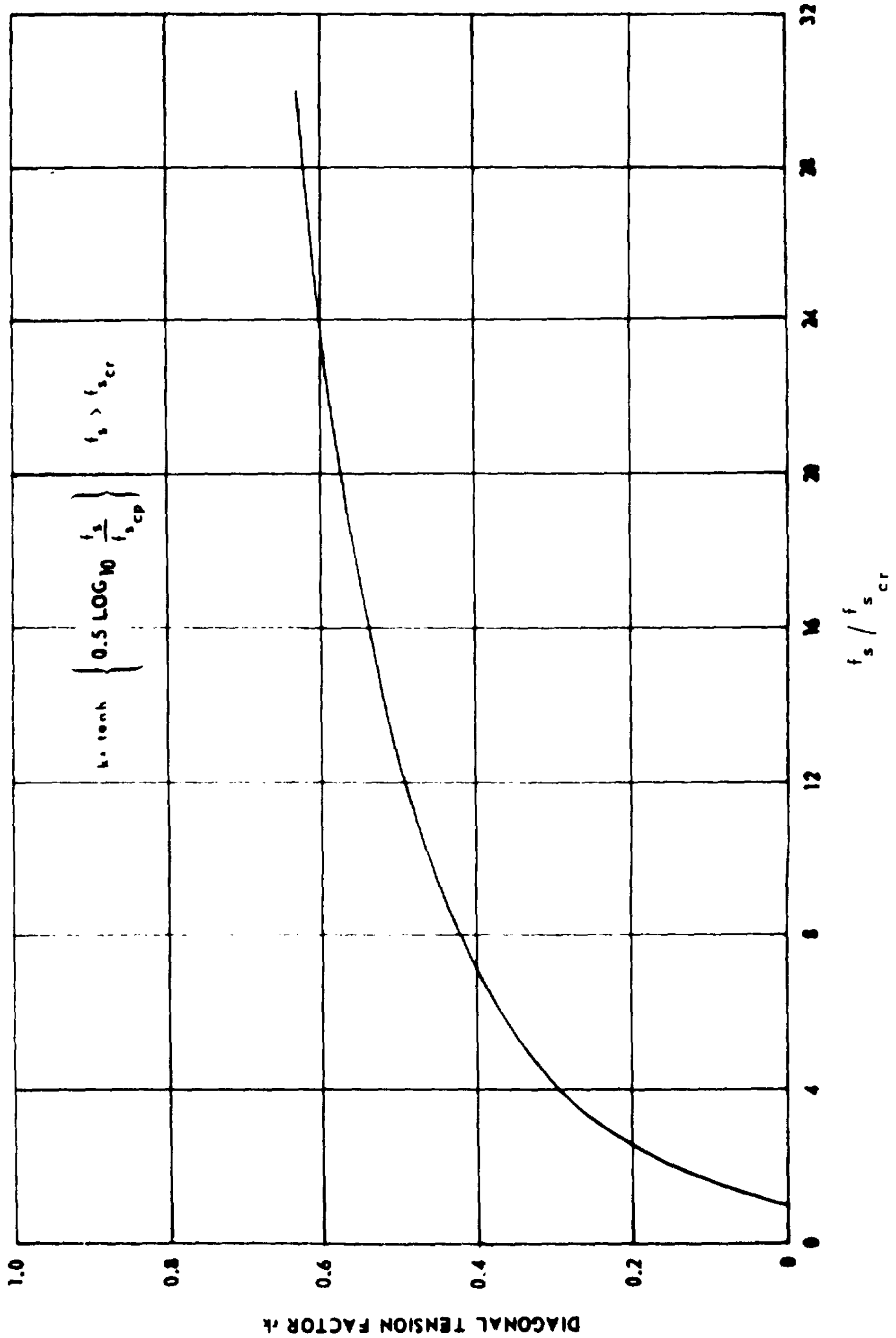


FIGURE 21. TENSION FIELD FACTOR VERSUS LOADING RATIO

The angle of the diagonal tension is then obtained from Figure 22, which shows the variation of tan α as a function of k and tb/A_{ue}. For double stiffeners, A_{ue} is equal to the cross-sectional area of the stiffeners. For single stiffeners,

$$A_{ue} = \frac{A_u}{1 + (e/\rho)^2} \quad . \tag{19}$$

It is recommended that the diagonal tension factor at ultimate load be limited to a maximum value,

$$k_{max} = 0.78 - (t - 0.012)^{1/2} \quad , \tag{20}$$

to avoid excessive wrinkling and permanent set at limit load, thereby inviting fatigue failure.

The average web shearing stress, f_s, may be appreciably smaller than the maximum web stress. The maximum web stress is given by

$$f_{s_{max}} = f_s (1 + k^2C_1)(1 + kC_2) \quad , \tag{21}$$

where C_1 and C_2 are empirical coefficients obtained from Figure 23. C_1 is a correction factor accounting for α differing from 45 degrees. C_2 is the stress-concentration factor arising from the flange flexibility.

The web allowable stress, $F_{s_{all}}$, is given in Figure 24 as a function of k and α_{PDT}, the angle that the buckles would assume if the web could reach the

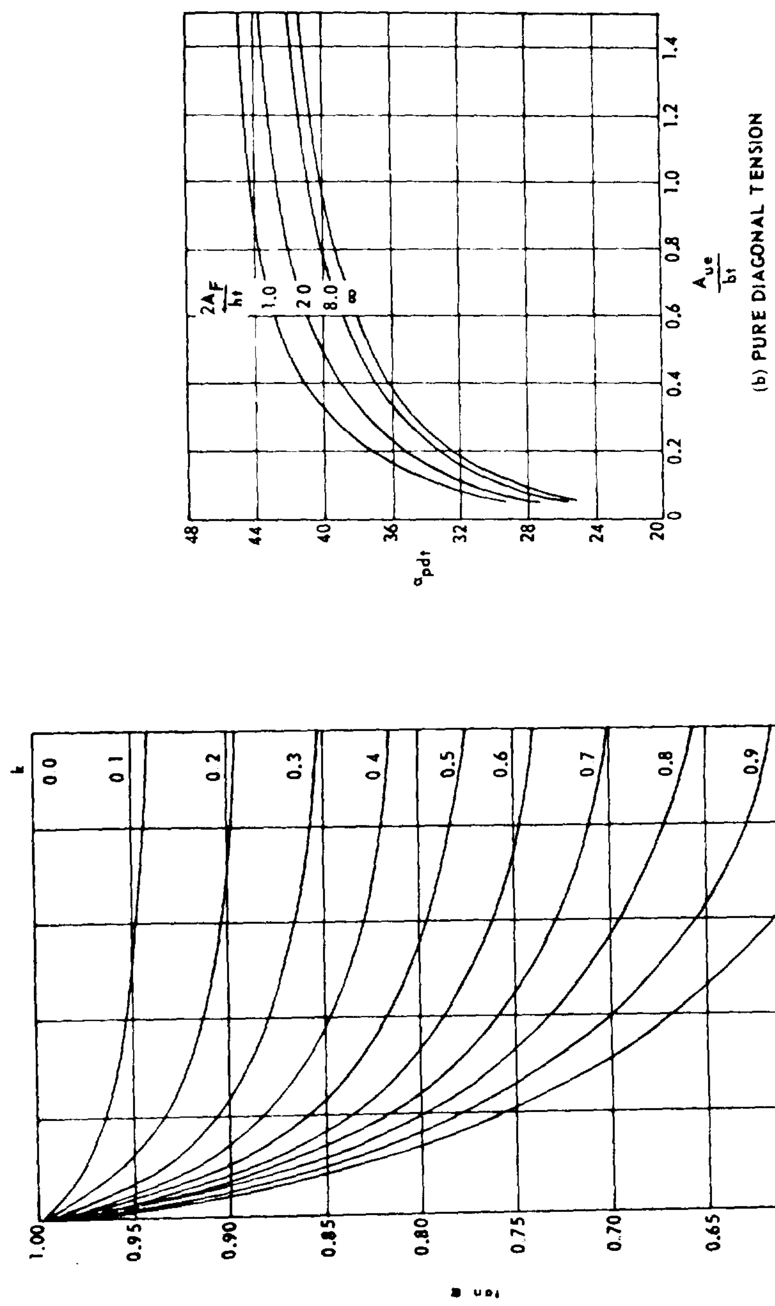


FIGURE 22. ANGLE OF DIAGONAL TENSION

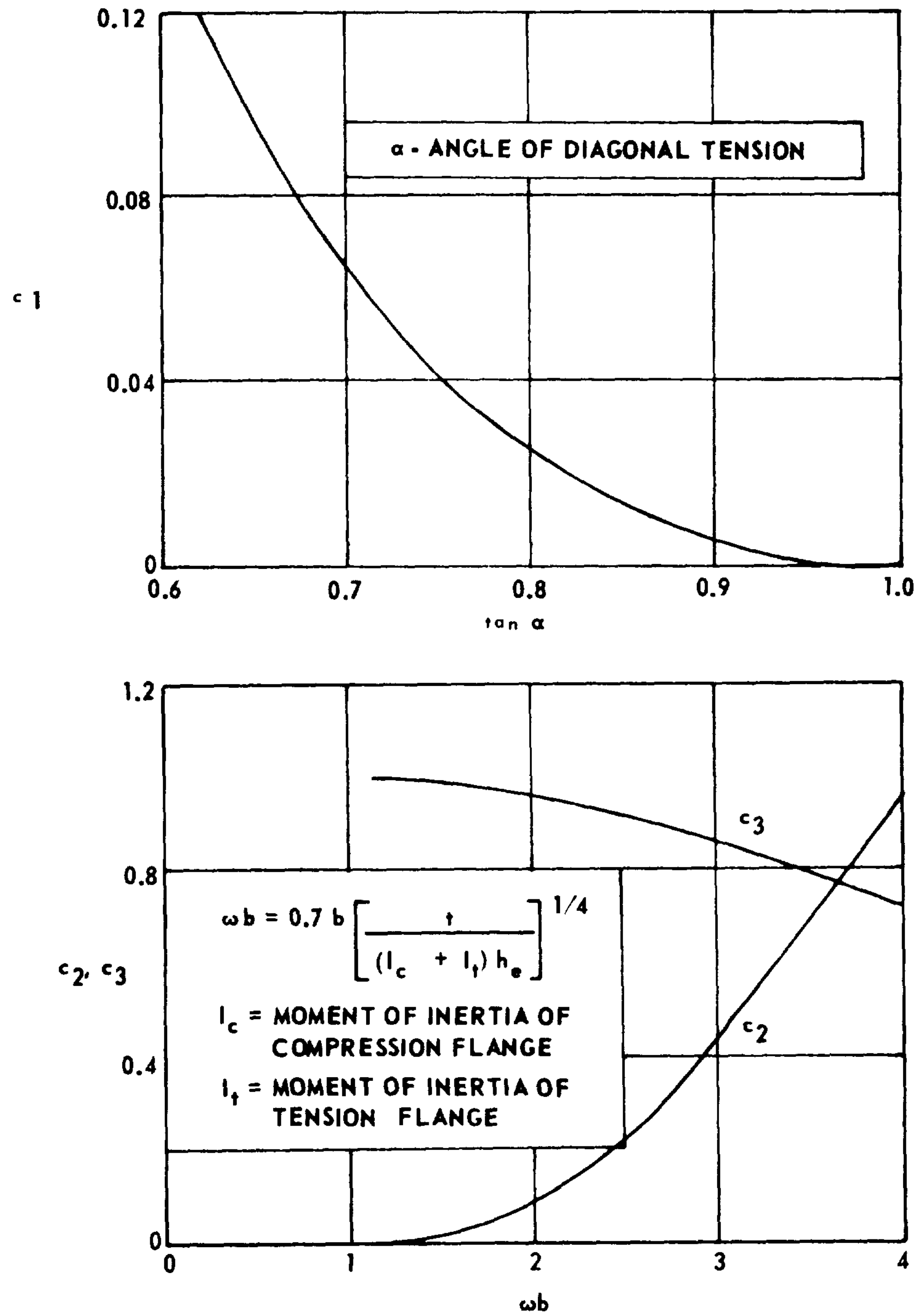
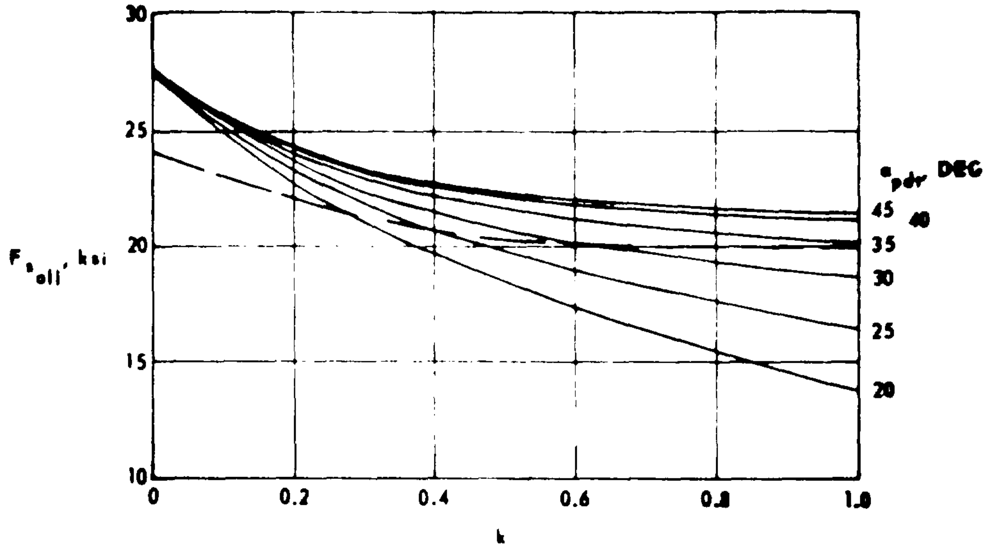
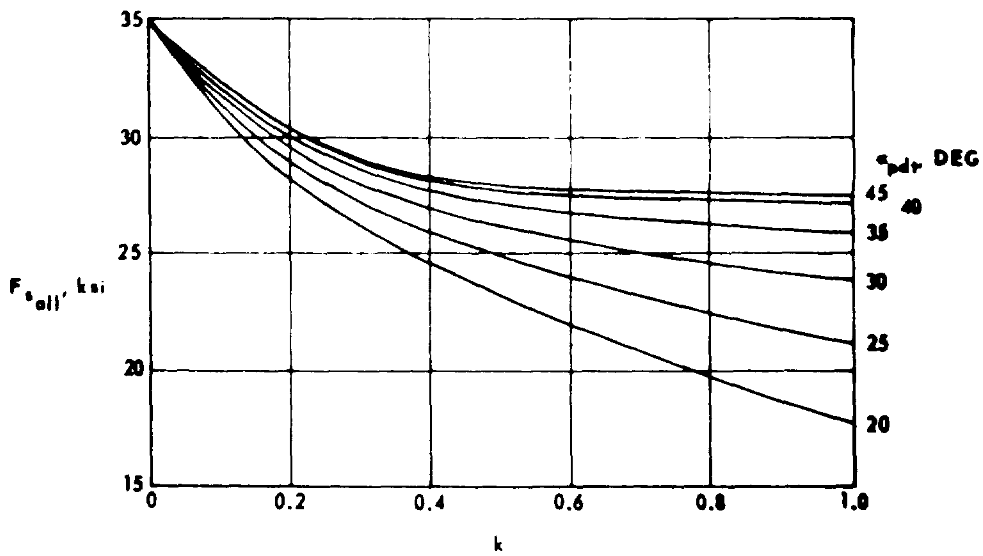


FIGURE 23. EMPIRICAL COEFFICIENTS FOR MAXIMUM SHEAR STRESS IN WEB AND FOR SECONDARY BENDING MOMENT IN FLANGES



(a) 2024-T3 ALUMINUM ALLOY. $f_{t_{ult}}$ = 62 ksi
DASHED LINE IS ALLOWABLE YIELD STRESS



(b) ALCLAD 7075-T6 ALUMINUM ALLOY. $f_{t_{ult}}$ = 72 ksi

FIGURE 24. BASIC ALLOWABLE VALUES OF $f_{s_{max}}$

state of pure diagonal tension without rupturing. The values of $F_{s_{all}}$ have been established by tests and may be called "basic allowable." For different connections, they are applied as follows:

I. Bolts, just snug, heavy washers under bolt heads, or web plates sandwiched between flange angles; use basic allowables.

II. Bolts, just snug, bolt heads bearing directly on sheet; reduce basic allowables 10 percent.

III. Rivets, assumed to be tight; increase basic allowables 10 percent.

IV. Rivets, assumed to be loosened in service; use basic allowables.

The allowable stresses given are valid if the allowable bearing stresses on the sheet or rivets are not exceeded. They are not valid for countersunk rivets. For webs of unusual dimensions arranged unsymmetrically with respect to the flange, use Figure 25 to obtain $F_{s_{all}}$.

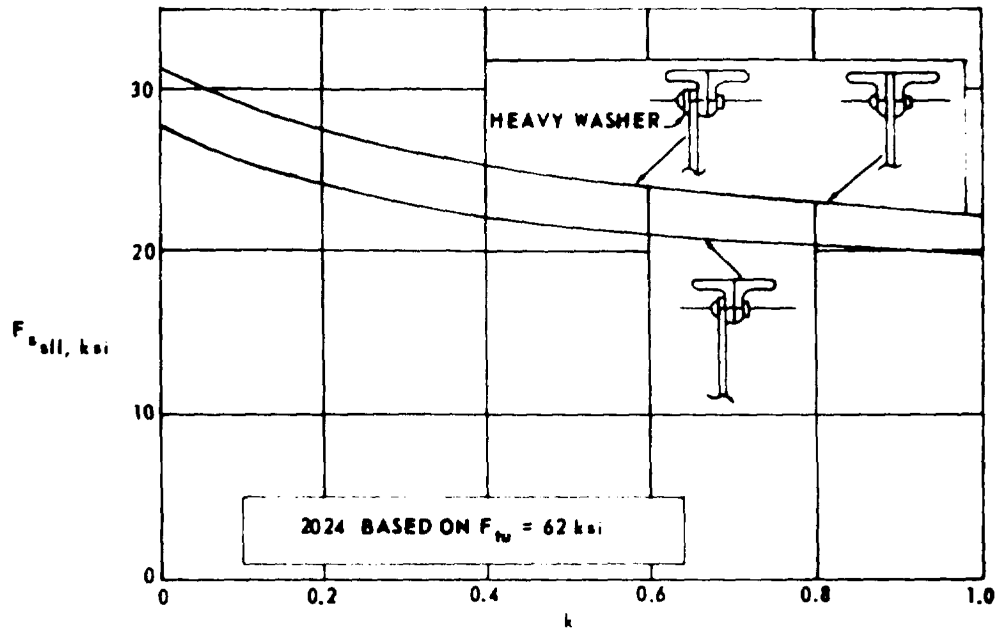
4.8.2.3 Analysis of Stiffeners

Stiffener loads result from the web diagonal tension and the transverse load not carried by the web. The stiffener load is given by

$$P_u = ktbf_s \tan\alpha + Nb \left[1 - \frac{F_{s_{cr}} tb}{f_s (td + A_u)} \right] , \tag{22}$$

where N is positive for a transverse compressive load. This load is resisted by the stiffener and the effective web. The effective width of the web working with the stiffener may be assumed to be given by

$$\frac{b_e}{b} = 0.5(1-k) . \tag{23}$$



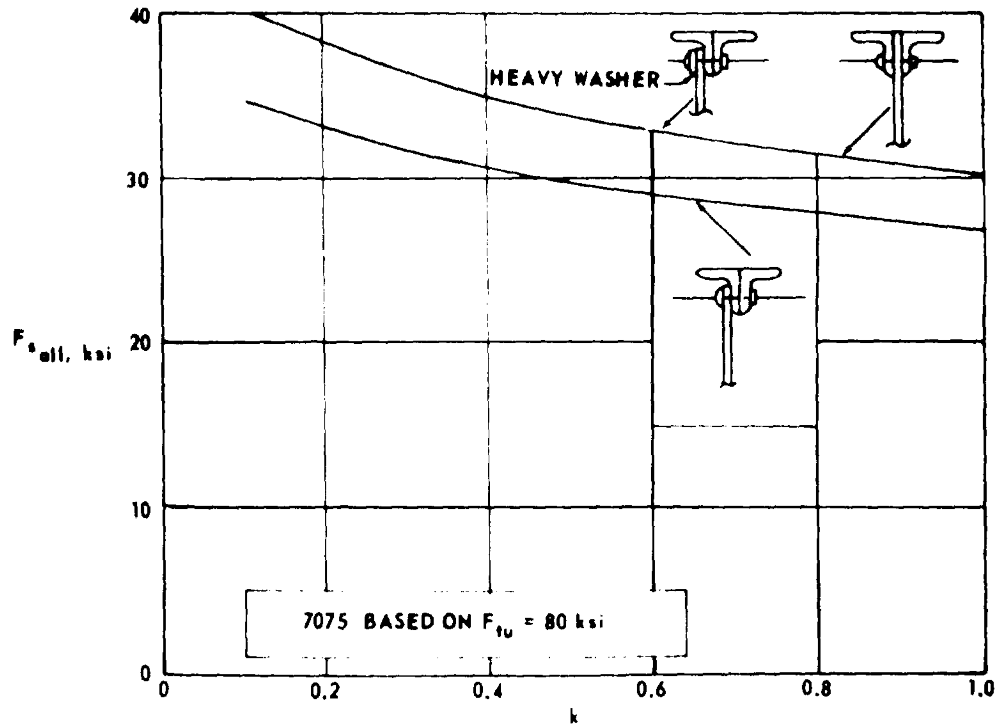


FIGURE 25. ALLOWABLE WEB STRESSES FOR 2024 AND 7075 AT ROOM TEMPERATURE

The average stiffener stress is then

$$f_u = \frac{P_u}{A_{ue} + 0.5(1-k)tb} \quad . \tag{24}$$

The maximum compressive stress in the stiffener occurs near the neutral axis of the beam. The ratio of the maximum stiffener stress to the average stiffener stress, $f_{u_{max}}/f_u$, is obtained from Figure 26.

Stiffeners may fail by column action or by local crippling. Column failure by true elastic instability is possible only in (symmetrical) double stiffeners.

A single stiffener is an eccentrically loaded compression member whose failing stress is a function of the web and stiffener properties. To guard against excessive bowing and column stress, the following must be adhered to:

I. The stress f_u must not exceed the column yield stress.

II. The average stress over the column cross section, $f_{cent} = f_u A_{ue}/A_u$, must not exceed the allowable stress for a column with the slenderness ratio $h_u/2\rho$.

The effective column length for double stiffeners is

$$L_e = \frac{h_u}{\sqrt{1 + k^2(3-2\,b/h_u)}} \qquad b < 1.5h \tag{25}$$

$$L_e = h_u \qquad b \geq 1.5h \quad . \tag{26}$$

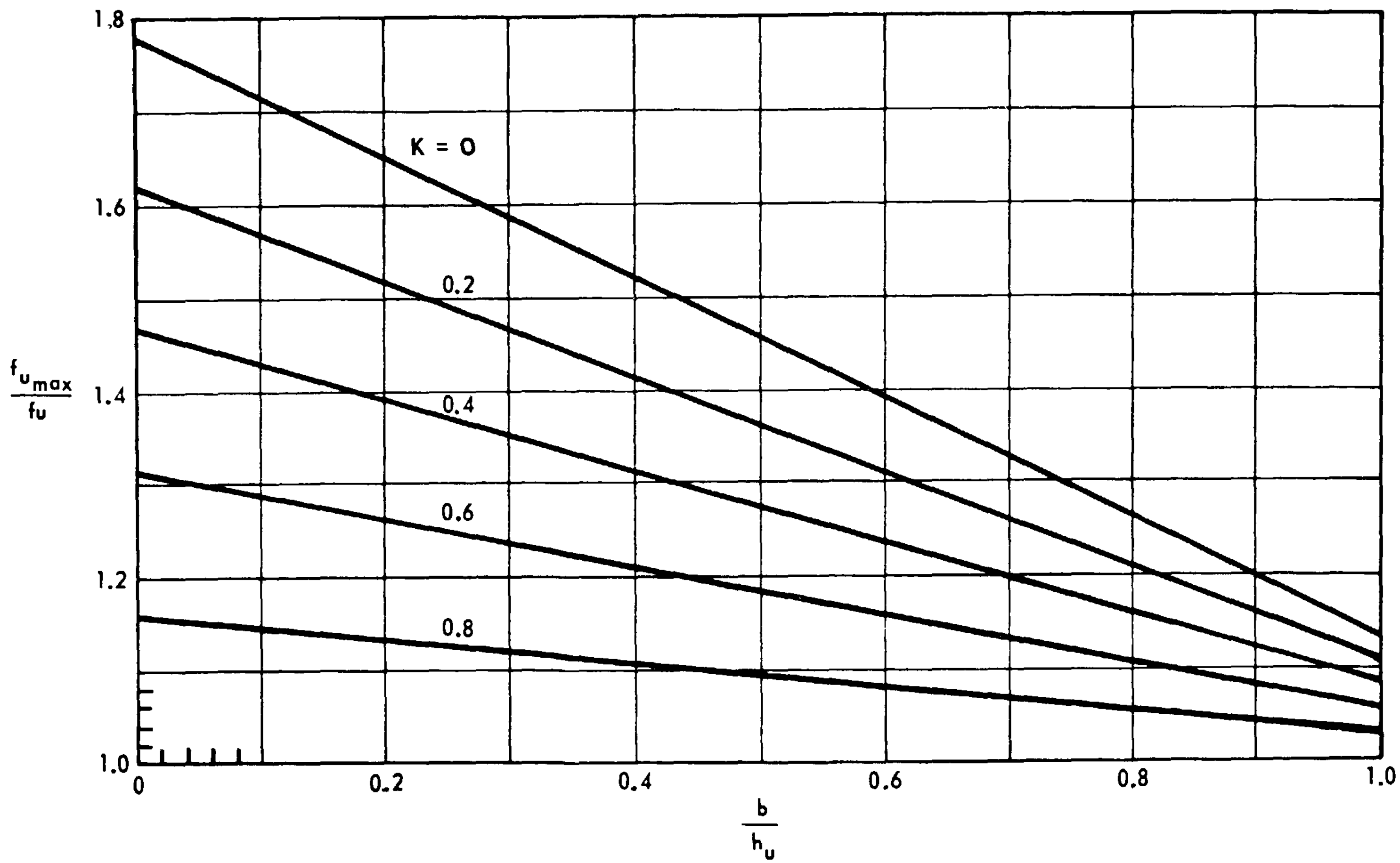


FIGURE 26. RATIO OF MAXIMUM STRESS TO AVERAGE STRESS IN WEB STIFFENER

To avoid column failure of double stiffeners, the average stress, f_u, should be less than the allowable stress taken from the column curve for solid sections of the stiffener material, with the slenderness ratio L_e/ρ.

Forced crippling of stiffeners must be considered. In this mode of failure, the attached leg of the stiffener is deformed by being forced to adapt itself to the web shear wrinkles.

The allowable forced crippling stress is given by the empirical equation

$$F_o = Ck^{2/3}\left(\frac{t_u}{t}\right)^{1/3} , \qquad (27)$$

where C is a constant as follows:

	Single Stiffener	Double Stiffener
2024-T (Bare)	C = 26.0	21.0
7075-T (Bare)	C = 32.5	26.0

Nomographs for F_o are given in Figure 27. If F_o exceeds the material proportional limit, a plasticity factor, η, equal to E_{sec}/E_c is used in the equation above.

Torsional stability of single stiffeners is provided by meeting the following criteria:

$$(f_s - F_{s_{cr}})h_e t = 0.23E\left(\frac{J_1 th^2}{b}\right)^{1/3} , \qquad (28)$$

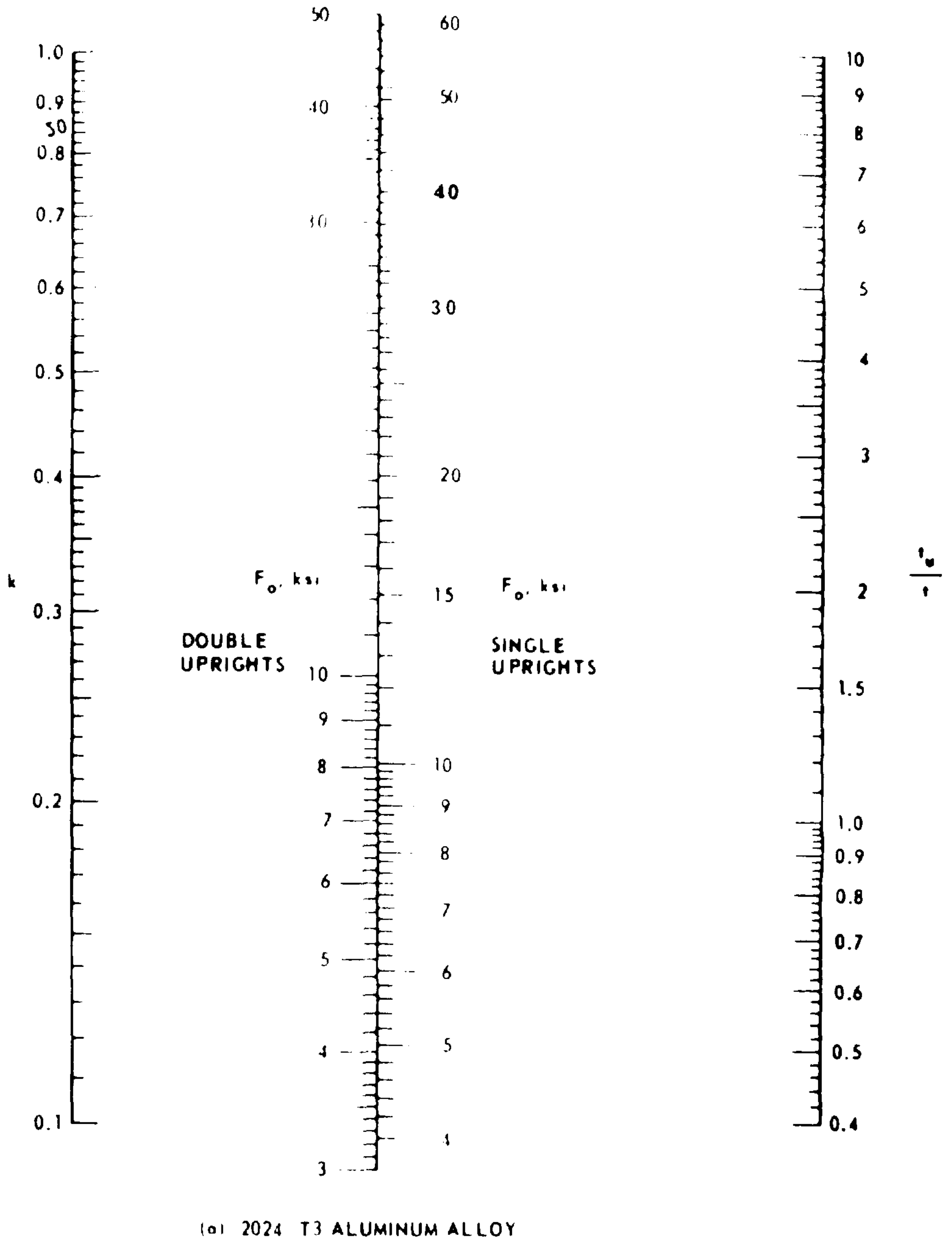


FIGURE 27. NOMOGRAM FOR ALLOWABLE UPRIGHT STRESS (FORCED CRIPPLING)

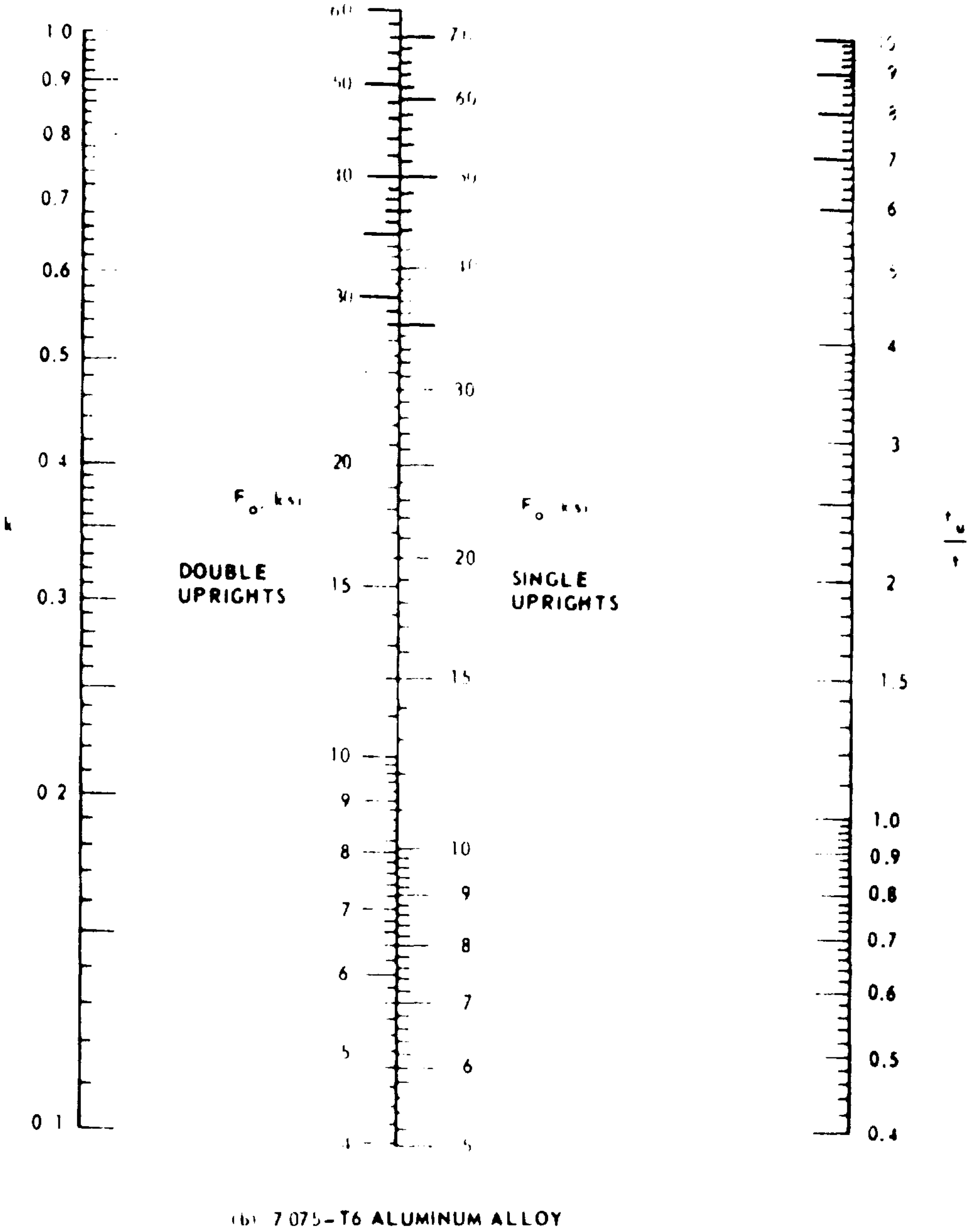


FIGURE 27. (Concluded)

where

$(f_s - F_{s_{cr}}) h_e t$ = total web shear load above buckling which can be carried before the stiffener cripples

J_1 = the effective polar moment of inertia of the stiffener 1/3 (developed width) t_u^3. This applies for formed sheet stiffeners.

To prevent a forced crippling type of failure when the upright resists an external compressive load in addition to the compressive load resulting from diagonal tension, an interaction equation such as the following must be used to evaluate this effect:

$$\left(\frac{f_{u_{max}}}{F_o}\right)^{1.5} + \left(\frac{f_{ce}}{F_{ce}}\right) = 1 \quad , \qquad (29)$$

where $f_{u_{max}}$ and F_o are the maximum upright compressive stress and the allowable forced crippling stress, respectively, for diagonal tension acting alone, and f_{ce} and F_{ce} are the actual and allowable compressive stress, respectively, resulting from external compressive stress acting alone.

An effective area of web plus upright may be used in computing f_{ce}. The allowable crippling stress, F_{max}, may be used for F_{ce}.

The effect of the external load should also be investigated with respect to column failure of the upright. To prevent column failure under combined loading the following criteria should be fulfilled:

$$f_u + f_{ce} \le F_{co} \tag{30}$$

and

$$f_{cent} + f_{ce} \le F_c \quad . \tag{31}$$

4.8.2.4 Analysis of Flange

The flange stress is the result of the superposition of three individual stresses: (1) primary bending stresses, (2) axial compression because of the flange-parallel component of the web diagonal tension, and (3) secondary bending stresses because of the stiffener-parallel component of the web diagonal tension.

The primary bending stresses are given by

$$f_{prim} = \frac{M_c}{I_f} \left[1 - \frac{F_{s_{cr}}}{f_s} \left(1 - \frac{I_f}{I}\right) \right] \quad , \tag{32}$$

where

I = moment of inertia of section

and

I_f = moment of inertia of section (web neglected).

The total axial load because of the flange-parallel component of the web diagonal tension and applied axial load is

$$P_{axial} = khtf_s \cot\alpha + P_a \left[1 - \frac{F_{s_{cr}}}{f_s} \left(\frac{th}{A}\right) \right] \quad . \tag{33}$$

P_a is positive for compressive axial load. The axial flange stress is then

$$f_{axial} = \frac{P_{axial}}{A_c + A_t + 0.5(1-k)th} \quad , \tag{34}$$

where A_c and A_t are the area of the compression and tension flange.

The secondary bending stress is given by

$$f_{sec} = M_{sec}\left(\frac{C}{I}\right)_f \tag{35}$$

where

$$M_{sec} = C_3 \frac{P_u b}{12} \quad \text{(over stiffener)}$$

and

$$M_{sec} = C_3 \frac{P_u b}{24} \quad \text{(midway between stiffeners)} \quad .$$

C_3 is an empirical stress concentration factor, given in Figure 23.

The allowable stress for the compression flange can be found by the methods of Section C1.0. The allowable tension stress for a tension flange is given by F_{tu} of the material, modified by the attachment efficiency factor.

4.8.2.5 Analysis of Rivets

Web-to-Flange: The flange-web shear flow at the line of attachment is

$$q = \frac{V}{h'} (1 + 0.414k) \quad , \tag{36}$$

where h' = beam depth between attachment line of flange web.

Web-to-Stiffener: The stiffener-web rivets, for double stiffeners, must develop sufficient longitudinal shear strength to make the two stiffeners act as a unit until column failure occurs. The shear strength should be

$$q = \frac{2 F_{cy} Q}{b_s L_e} \quad , \tag{37}$$

where b_s = outstanding stiffener flange width.

The stiffener-web connectors must carry a tension component as follows:

$$N' = 0.15t F_{tu} \text{ (double stiffener)} \tag{38}$$

and

$$N' = 0.22t F_{tu} \text{ (single stiffener)} \quad . \tag{39}$$

The interaction of shear and tension in the connectors is given in Reference 11.

Stiffener-to-Flange: The stiffener-to-flange connectors are designed with the empirical relationship

$$P_u = f_u A_{ue} \quad , \tag{40}$$

which gives the load in the stiffener. The connection must transfer this load into the cap.

4.8.2.6 Analysis of End of Beam

The previous discussion has been concerned with the "interior" bays of a beam. The vertical stiffeners in these areas are subject, primarily, only to axial compression loads, as presented. The outer, or "end bay," is a special case. Since the diagonal tension effect results in an inward pull on the end stiffener, it produces bending in it, as well as the usual compressive axial load. Obviously, the end stiffener must be considerably heavier than the others, or at least supported by additional members to reduce the stresses resulting from bending.

The component of the running-load-per-inch that produces bending in such edge members is given by the formulas

$$\omega = kq \tan \alpha \quad (41)$$

for edge members parallel to the neutral axis (stringers) and

$$\omega = kq \cot \alpha \quad (42)$$

for members normal to the neutral axis (stiffeners). The longer the unsupported length of the edge member subjected to ω, the greater will be the bending moment it must carry.

There are, in general, three ways of dealing with the edge member subjected to bending, the object being to keep the weight down.

I. Simply "beef-up" or strengthen the edge member so it can carry all of its loads. (This is inefficient for long unsupported lengths.)

II. Increase the thickness of the end bay panel either to make it nonbuckling or to reduce k, and thereby reduce the running load producing bending in the edge member. (This is usually inefficient for large panels.)

III. Provide additional members (stiffeners) to support the edge member and thereby reduce its bending moment because of ω. (This requires additional parts.)

Actually, a combination of these methods might be best.

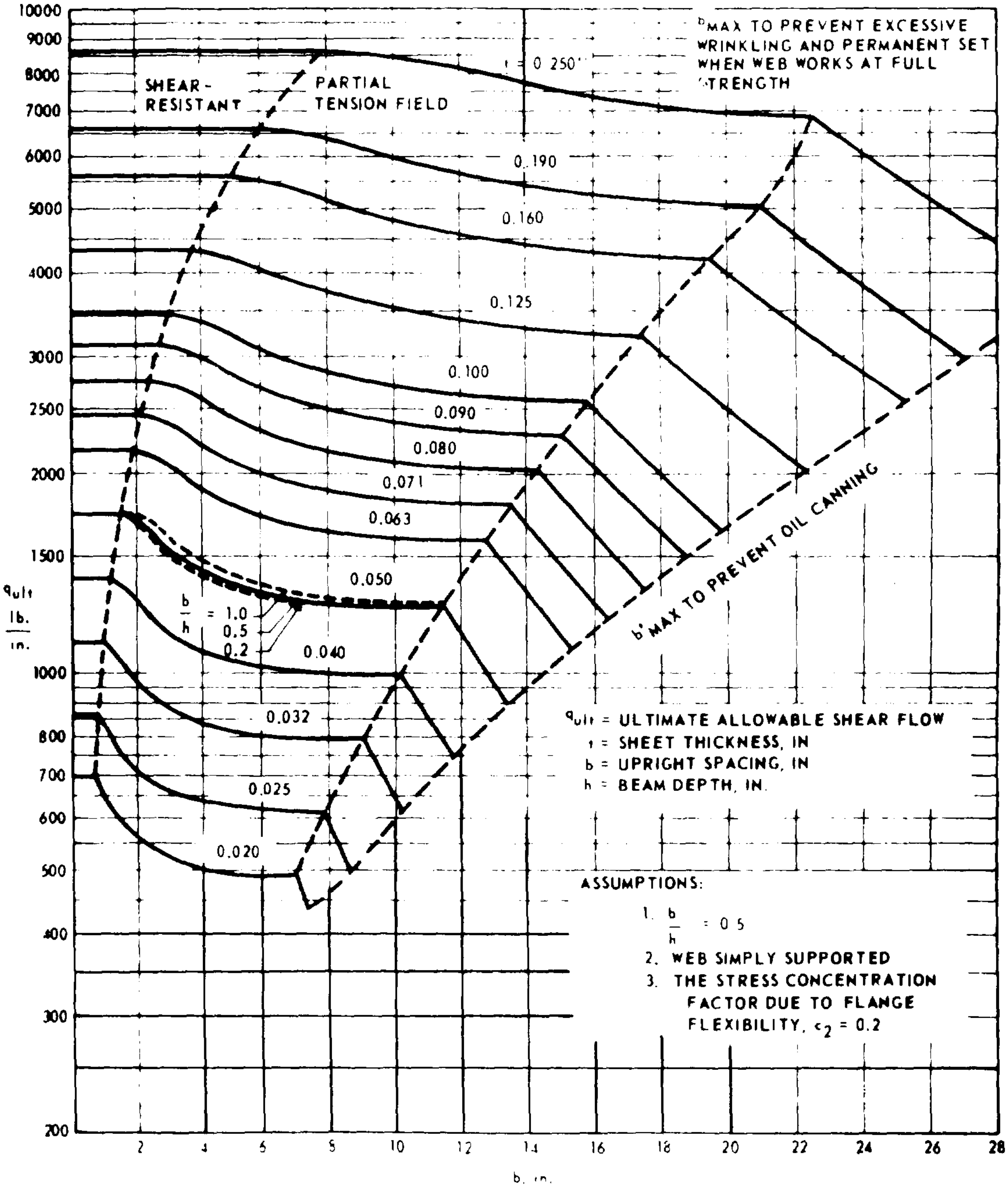
4.8.2.7 Beam Design

This paragraph presents methods to facilitate efficient preliminary design of tension field beams.

Allowable Shear Flow: Figure 28 gives the ultimate allowable shear flow, q, for 7075S-T6 Alclad sheet as a function of the sheet thickness, t, and the stiffener spacing, b. The dashed line on the left is the approximate boundary between shear-resistant and tension-field beams when the sheet is loaded to full strength. The central dashed line is a limitation on b_{max}, the maximum stiffener spacing, in order to minimize the possibility of excessive wrinkling and permanent set when the web works at full strength. Stiffener spacings greater than b_{max} can be used, if necessary, by using shear flow availables which conform to the limitation on the value of k_{max} given in Paragraph 4.8.2.2. The dashed line at the right establishes b'_{max}, the absolute maximum stiffener spacing in order to prevent "oil canning."

One of the assumptions made in the construction of Figure 28 is that the aspect ratio $b/h = 0.5$. Varying b/h has only a small effect on the curves, as can be seen from the curve for 0.050 sheet, where additional curves for $b/h = 0.2$ and $b/h = 1.0$ are plotted. The relationship for other sheet gages is approximately the same.

Stiffener Area Estimation: Figure 29 presents the stiffener area to web area ratio plotted as a function of b/h and $\sqrt{q/h}$, the square root of the structural index. This index is a measure of the loading intensity on the beam. These curves are to be used only as a means of roughly approximating the required area of stiffener for preliminary design and preliminary weight



NOTE: FOR BARE 7075S-T6, MULTIPLY BY 1.07

FIGURE 28. ULTIMATE ALLOWABLE SHEAR FLOW ALCLAD 7075S-T6 SHEET

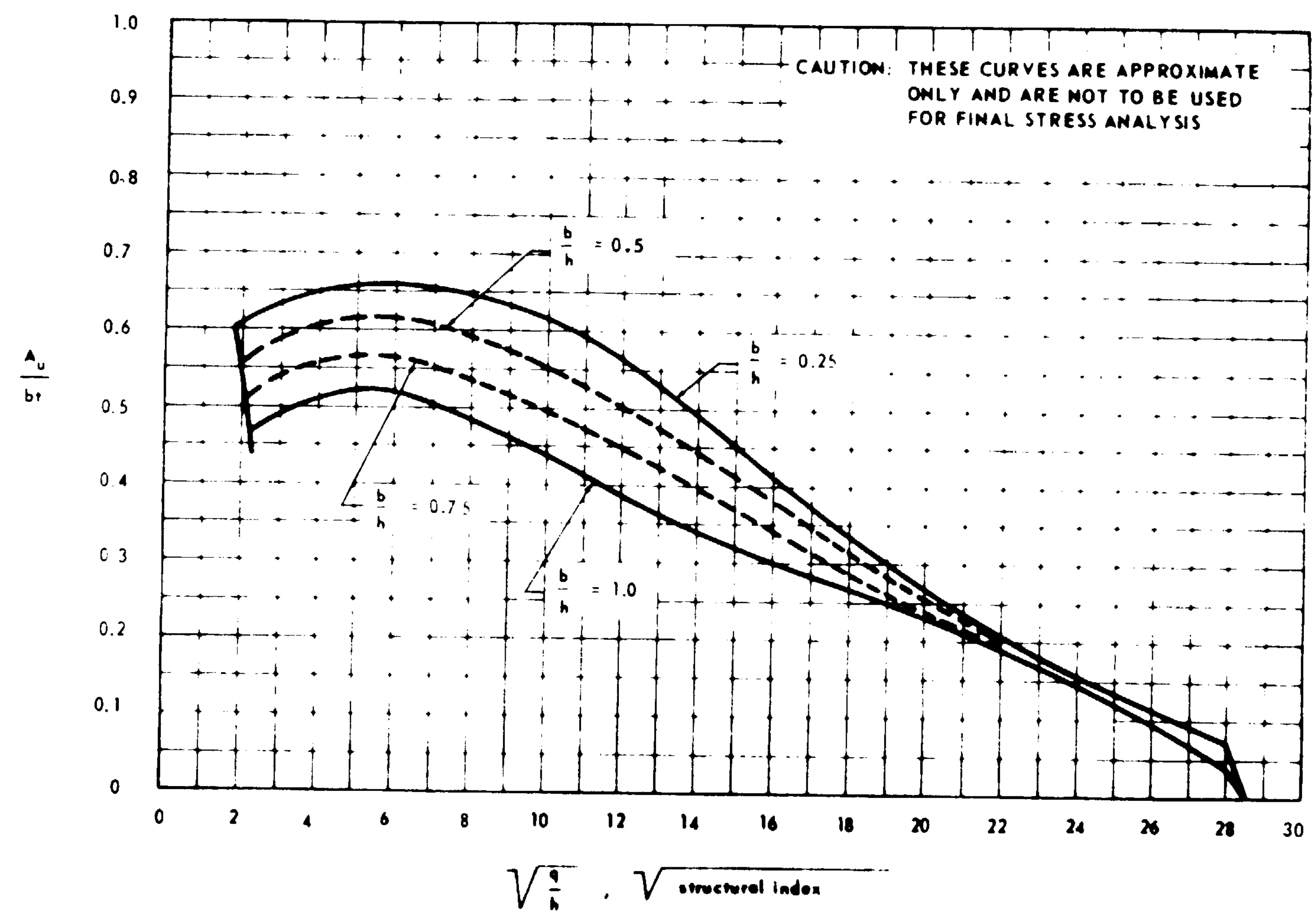


FIGURE 29. CHART FOR ESTIMATING EFFICIENT STIFFENER SIZE ALCLAD 7075S-T6 WEB 70707S-T6 SINGLE ANGLE STIFFENER

estimation. If the stiffener design is limited to standard sections, the required area might be larger than that given in Figure 29 since a zero margin of safety cannot always be obtained. The curves are for 7075S-T single-angle stiffeners. Curves for double stiffeners or 2024S-T material, similar to Figure 29, can be found in Reference 12.

Design Method: The preliminary design of 7075S-T web-stiffener can be arrived at in the following manner:

1. The design shear flow, q, and the depth of beam, h, are usually known. This fixes $\sqrt{q/h}$, the square root of the structural index.
2. The stiffener spacing, b, is often determined by considerations not under the control of the designer. If such is the case, inspection of Figure 29 shows that a stiffener spacing, b, equal to the beam depth, h, is desirable for minimum weight design of the web-stiffener system. However, it is possible that wide stiffener spacing might induce excessive secondary bending in the flange. In general, b/h ratios from 0.5 to around 0.8 are commonly used for tension-field beams.
3. The required web thickness, t, can be obtained from Figure 28 since q and b are known. This figure can also be used to check the stiffener against the maximum allowable spacing, b_{max}.
4. Estimate the required value A_u/bt with the aid of Figure 29.
5. Compute the approximate cross-sectional area of stiffener as follows:

$$A_u = \left(\frac{A_u}{bt}\right) bt \ .$$

6. Choose a stiffener with the proper area. Unless the beam is very deep, or unless there are other design considerations, a single-angle stiffener is an efficient design. Also, a stocky, equal-legged angle gives greater resistance against forced crippling, which is usually the dominant mode of failure.

REFERENCES

1. Wagner, H., "Flat Sheet Metal Girders with Very Thin Metal Web," Parts I-III, NACA TMS 604-606, 1931.

2. Cook, I. T., and Rockey, K. C., "Shear Buckling of Clamped and Simply Supported Infinitely Long Plates Reinforced by Transverse Stiffeners," Aeronautical Quarterly, Vol. XIII, February 1962, p. 41.

3. Cook, I. T., and Rockey, K. C., "Shear Buckling of Rectangular Plates with Mixed Boundary Conditions," Aeronautical Quarterly, Vol. XIV, November 1963, pp. 349-356.

4. Rockey, K. C., and Cook, I. T., "Shear Buckling of Orthogonally Stiffened Infinitely Long Simply Supported Plates — Stiffeners Having Torsional and Flexural Rigidities," Aeronautical Quarterly, February 1969, p. 75.

5. Rockey, K. C., "Influence of Stiffener Thickness and Rivet Postition upon the Effectiveness of Single-Sided Stiffeners on Shear Webs," Aeronautical Quarterly, February 1964, p. 97.

6. Rockey, K. C., and Cook, I. T., "Shear Buckling of Clamped and Simply Supported Infinitely Long Plates Reinforced by Closed-Section Transverse Stiffeners," Aeronautical Quarterly, Vol. XIII, August 1962, p. 212.

7. Rockey, K. C., and Cook, I. T., "Influence of the Torsional Rigidity of Transverse Stiffeners upon the Shear Buckling of Stiffened Plates," Aeronautical Quarterly, Vol. XV, May 1964, p. 198.

8. Rockey, K. C., and Cook, I. T., "Shear Buckling of Clamped Infinitely Long Plates — Influence of Torsional Rigidity of Transverse Stiffeners," Aeronautical Quarterly, February 1965, p. 92.

9. Rockey, K. C., and Cook, I. T., "Shear Buckling of Clamped and Simply Supported Infinitely Long Plates by Transverse Stiffeners and a Central Longitudinal Stiffener," Aeronautical Quarterly, Vol. XIII, May, 1962, pp. 95-114.

10. Bleich, F., Buckling Strength of Metal Structures, McGraw-Hill Book Company, Inc., New York, 1952.

11. Bruhn, E. F., Analysis and Design of Flight Vehicle Structures, Tri-state Offset Company, Cincinnati, Ohio, 1965.

12. Kuhn, P., Peterson, J. P., and Levin, L. R., "A Summary of Diagonal Tension," Parts I and II, NACA TN2661 and TN2662, 1952.

13. Levin, L. R., "Strength Analysis of Stiffened Thick Beam Webs with Ratios of Web Depth to Web Thickness of Approximately 60," NACA TN2930, May 1953.

BIBLIOGRAPHY

Rockey, K. C., "The Design of Intermediate Vertical Stiffeners on Web Plates Subjected to Shear," Aeronautical Quarterly, Vol. VII, November, 1956.

Stein, M., and Fralich, R. W., "Critical Shear Stress of Infinitely Long, Simply Supported Plate with Transverse Stiffeners," NACA TN1851, April 1949.

Structural Design Manual, Northrop Aircraft, Inc.

SECTION B5

FRAMES

TABLE OF CONTENTS

B 5.0.0 FRAMES

This section deals with specialized methods of analyzing statically indeterminate structures. The procedures of analyzing rigid bents and circular rings are given in detail in Section B 5.1.0 and B 5.2.0 respectively.

A sample problem to illustrate the methods and procedures is given in each section.

B 5.1.0 Analysis of Statically Indeterminate Frames by the Method of Moment-distribution.

Moment-distribution is a convenient method of reducing statically indeterminate structures to a problem in statics. Moment-distribution does not involve the solution of simultaneous equations, but consists of a series of converging cycles that may be terminated at the degree of precision required by the problem.

B 5.1.1 Discussion of the Method of Moment-distribution.

The method of moment-distribution requires a knowledge of moment-area theorems and slope-deflection equations. The five basic factors involved in the method of moment-distribution are: fixed-end moments, stiffness factors, distribution factors, distributed moments, and carry-over moments.

Only structures comprised of prismatic members with no joint translation are considered in this article. All members are assumed to be elastic.

The fixed-end moments are obtained through the use of a general "end-moment" equation derived in the development of the slope-deflection method. (See Fig. B 5.1.1-1)

$$M_{AB} = \frac{2EI}{L}\left(2\theta_A + \theta_B - 3\psi_{AB}\right) + \frac{2}{L^2}\left[\left(\Omega o\right)_A - 2\left(\Omega o\right)_B\right] \quad \ldots\ldots\ldots \quad (1)$$

$$M_{BA} = \frac{2EI}{L}\left(2\theta_B + \theta_A - 3\psi_{AB}\right) + \frac{2}{L^2}\left[2\left(\Omega o\right)_A - \left(\Omega o\right)_B\right] \quad \ldots\ldots\ldots \quad (2)$$

where

M_{AB} is the moment acting on the "A" end of member AB.
M_{BA} is the moment acting on the "B" end of member AB.
E is the modulus of elasticity.
I is the moment of inertia.
L is the length of member AB.
θ is the rotation of the tangent to the elastic curve at the end of a member and is positive for clockwise rotation.
ψ is the rotation of the chord joining the ends of the elastic curve referred to the original direction of the member and is positive for clockwise rotation.
$(\Omega o)_A$ is the static moment about a vertical axis through "A" of the area under the M_o portion of the bending-moment diagram shown on Fig. B 5.1.1-1.
$(\Omega o)_B$ is the static moment about an axis through "B".

B 5.1.1 Discussion of the Method of Moment-distribution (Cont'd)

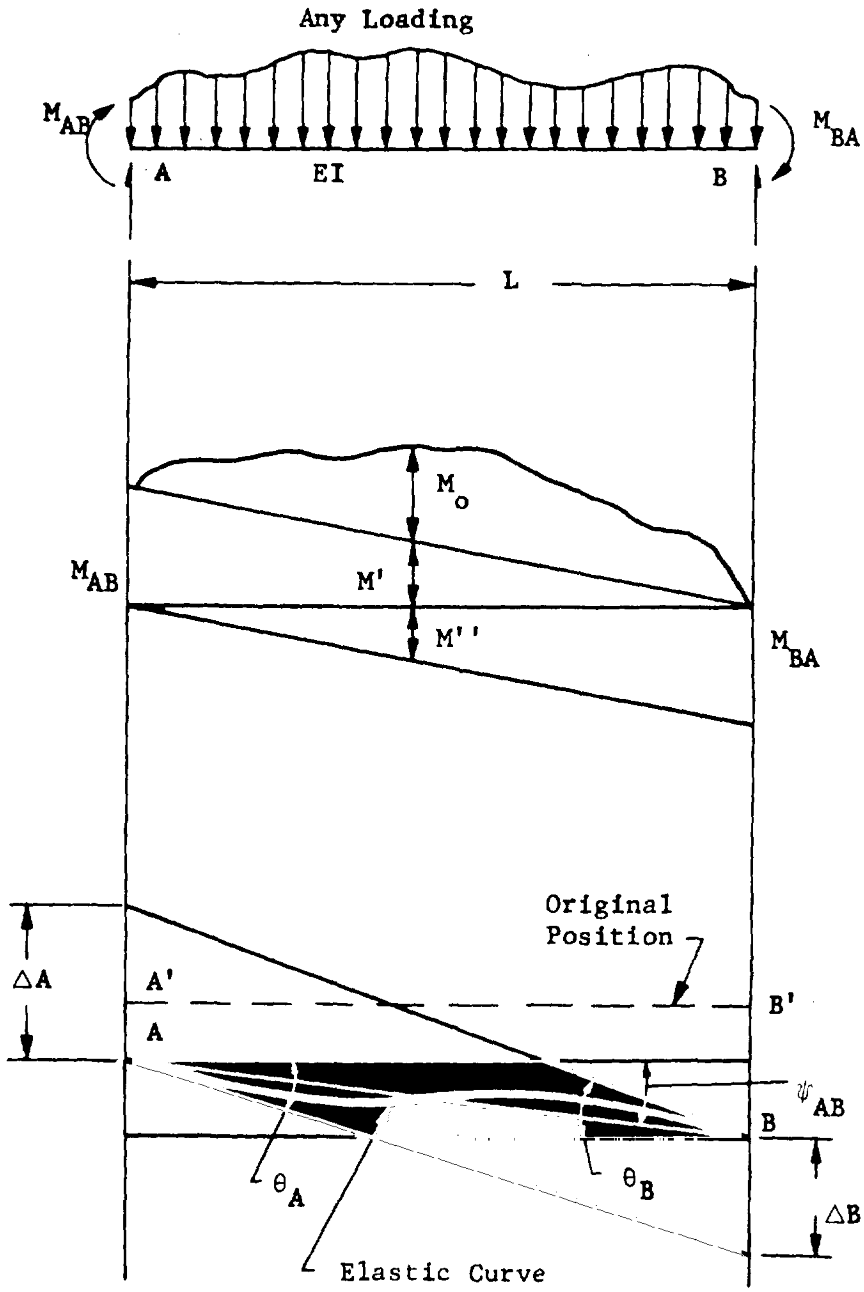


Fig. B 5.1.1-1

B 5.1.1 Discussion of the Method of Moment-distribution (Cont'd)

If θ_A, θ_B and ψ_{AB} are all equal to zero, then both ends of the member are completely fixed against rotation or translation and the member is called a fixed-end beam. The last terms of Eqs. (1) and (2) are therefore equal to the so-called "fixed end moments". Denoting fixed end moments as FEM and setting θ_A, θ_B and ψ_{AB} equal to zero.

$$FEM_{AB} = \frac{2}{L^2}\left[(\Omega o)_A - 2\,(\Omega o)_B \right] \quad \text{.................. (3)}$$

$$FEM_{BA} = \frac{2}{L^2}\left[2(\Omega o)_A - (\Omega o)_B \right] \quad \text{...................... (4)}$$

Fixed end moments for various type of loading are given in Table B 5.1.1-1

Equations (3) and (4) are summarized by one general equation by calling the near end of a member "N" and the far end "F". Also let

$$K_{NF} = \text{stiffness factor for member NF} = \frac{I_{NF}}{L_{NF}} \quad \text{.............. (5)}$$

then the fundamental slope deflection equation is

$$M_{NF} = 2EK_{NF}\,(2\theta_N + \theta_F - 3\psi_{NF}) + FEM_{NF} \quad \text{............... (6)}$$

The conditions to be met at a joint are: (1) that the angle of rotation be the same for the ends of all members that are rigidly connected at that joint and (2) that the algebraic sum of all moments is zero. The method of moment-distribution renders to zero by iteration any unbalance in moment at a joint to satisfy the latter condition.

To distribute the unbalanced moment mentioned above, a distribution factor DF is required. This factor represents the relative portion of the unbalanced moment which is reacted by a member and for any bar bm, DF_{bm} is given by

$$DF_{bm} = \frac{K_{bm}}{\sum_b K} \quad \text{.. (7)}$$

where the summation is meant to include all members meeting at joint b.

The distributed moment in any bar bm is then

$$M_{bm} = -DF_{bm}M \quad \text{..................................... (8)}$$

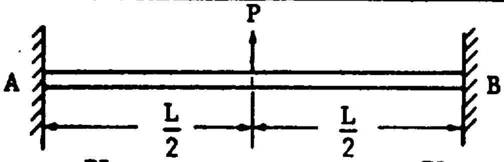
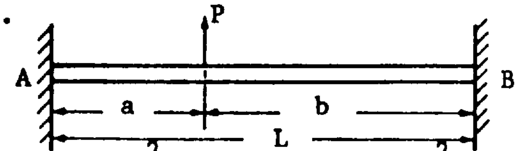
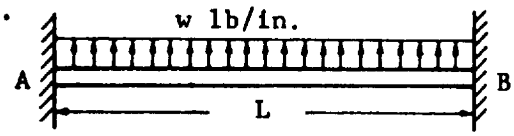
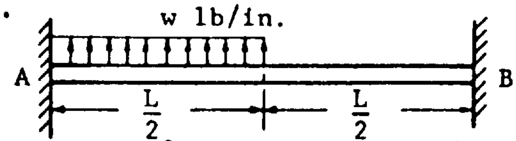
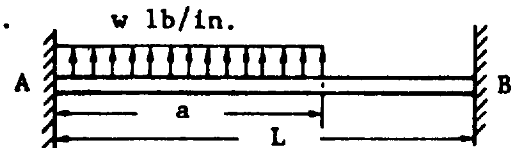
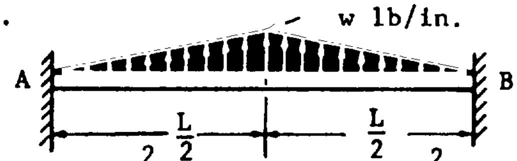
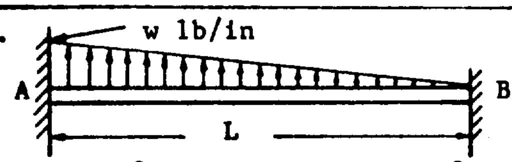
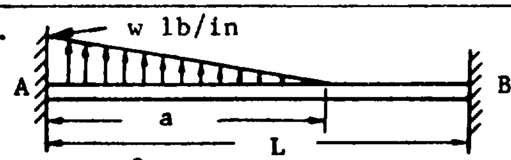
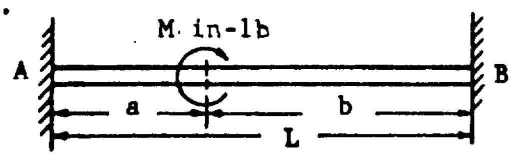
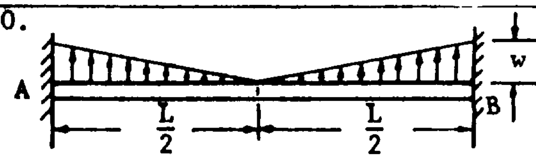
Equation (8) may be interpreted as follows:

"The distributed moment developed at the 'b' end of member 'bm' as joint 'b' is unlocked and allowed to rotate under an unbalanced moment 'M' is equal to the distribution factor DF_{bm} times the unbalan moment 'M' with the sign reversed."

B 5.1.1 Discussion of the Method of Moment-distribution (Cont'd)

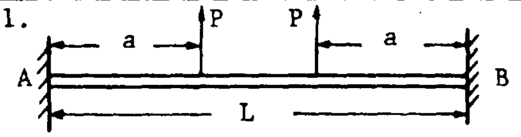
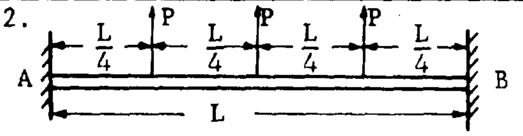
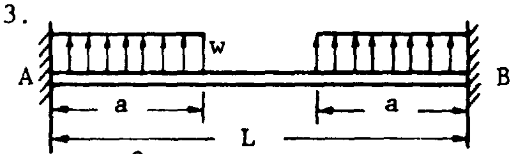
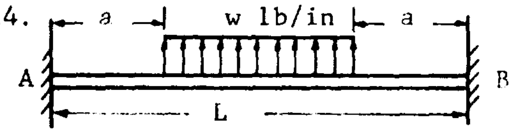
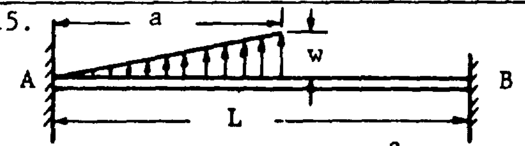
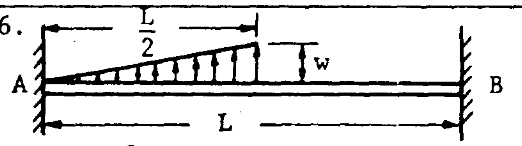
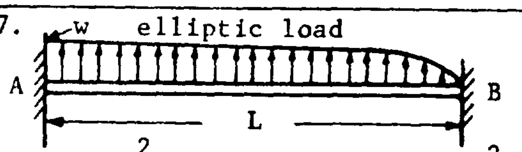
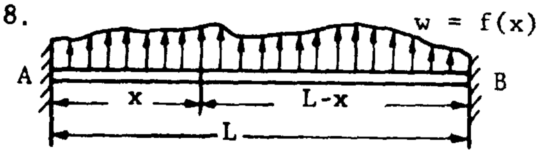
Table B 5.1.1-1 Fixed-end Moments for Beams

Notation: P = load (lb); w = unit load (lb per linear in.).
M = bending moment (in.-lb) positive when clockwise.

1.  $M_A = \frac{PL}{8}$ $M_B = -\frac{PL}{8}$	2.  $M_A = \frac{Pab^2}{L^2}$ $M_B = -\frac{Pa^2b}{L^2}$
3.  $M_A = \frac{wL^2}{12}$ $M_B = -\frac{wL^2}{12}$	4.  $M_A = \frac{11wL^2}{192}$ $M_B = -\frac{5wL^2}{192}$
5.  $M_A = \frac{wa^2}{12L^2}(6L^2-8aL+3a^2)$ $M_B = -\frac{wa^2}{12L^2}(4aL-3a^2)$	6.  $M_A = \frac{5wL^2}{96}$ $M_B = -\frac{5wL^2}{96}$
7.  $M_A = \frac{wL^2}{20}$ $M_B = -\frac{wL^2}{30}$	8.  $M_A = \frac{wa^2}{60L^2}(10L^2-10aL+3a^2)$ $M_B = -\frac{wa^3}{60L^2}(5L-3a)$
9.  $M_A = \frac{Mb}{L}(3\frac{a}{L} - 1)$, $M_B = -\frac{Ma}{L}(3\frac{b}{L} - 1)$	10.  $M_A = \frac{wL^2}{32}$ $M_B = -\frac{wL^2}{32}$

B 5.1.1 Discussion of the Method of Moment-distribution (Cont'd)

Table B 5.1.1-1 Fixed-end Moments for Beams (Cont'd)

11.  $M_A = Pa(1 - \frac{a}{L})$ $\quad M_B = -M_A$	12.  $M_A = \frac{15PL}{48}$ $\quad M_B = -M_A$
13.  $M_A = \frac{wa^2}{6L}(3L-2a)$ $\quad M_B = -M_A$	14.  $M_A = \frac{w}{12L}(L^3 - a^2L + 4a^3)$ $\quad M_B = -M_A$
15.  $M_A = \frac{wa^2}{30}(10 - 15\frac{a}{L} + 6\frac{a^2}{L^2})$ $M_B = \frac{wa^3}{20L^2}(5L-4a)$	16.  $M_A = \frac{wL^2}{30}$ $\quad M_B = -\frac{3wL^2}{160}$
17.  $M_A = \frac{wL^2}{13.52}$ $\quad M_B = -\frac{wL^2}{15.86}$	18.  $M_A = \frac{1}{L^2}\int_0^L x(L-x)^2 f(x)dx$ $M_B = \frac{-1}{L^2}\int_0^L x^2(L-x)f(x)dx$

B 5.1.1 Discussion of the Method of Moment-distribution (Cont'd)

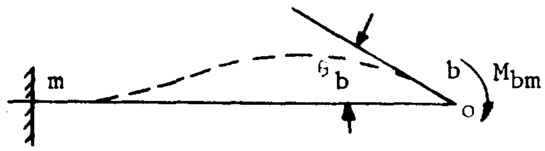


Fig. B 5.1.1-2

The "carry-over" moment is obtained by applying Eq. (6) considering $\theta_m = \psi_{bm} = o$ (see Fig. B 5.1.1-2). The carry-over moment is equal to one-half of its corresponding distributed moment and has the same sign.

$M_{bm} = 4EK_{bm}\theta_b$ and $M_{mb} = 2EK_{bm}\theta_b$

Hence,

$$M_{mb} = \frac{1}{2} M_{bm} \qquad \text{... (9)}$$

The sign convention adopted for this work deviates from the usual convention used in elementary beam analysis as found in Sec. B 4.1.1 of this manual. The positive sense for moments has been adopted from the convention used in slope-deflection equations; namely, moments acting clockwise on the ends of a member are positive. (See Fig. B 5.1.1-3)

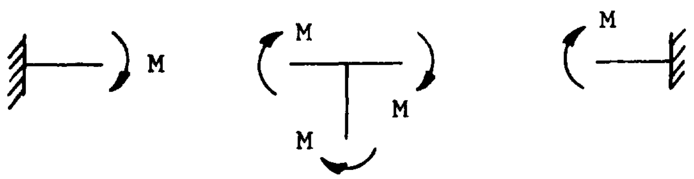


Fig. B 5.1.1-3 Positive sense for bending moments

The following procedure is established for the process of moment-distribution analysis:

1. Compute the stiffness factor for each member and record as shown in example problem one.

B 5.1.1 <u>Discussion of the Method of Moment-distribution (Cont'd)</u>

2. Compute the distribution factor for each member at each joint and record as shown in example problem one.

3. Compute the fixed-end moments for each loaded span and record with proper signs as shown in example problem one.

4. Balance the moments at a joint by multiplying the unbalanced moment by the distributor factor, changing sign, and recording the balancing moment below the fixed-end moment. The unbalanced moment is the algebraic sum of the fixed-end moments of a joint.

5. Draw a horizontal line below the balancing moment. The algebraic sum of all moments at any joint above the horizontal line must be zero.

6. Record the carry-over moment at the opposite ends of the member. Carry-over moments have the same sign as the corresponding balancing moments and are one-half their magnitude.

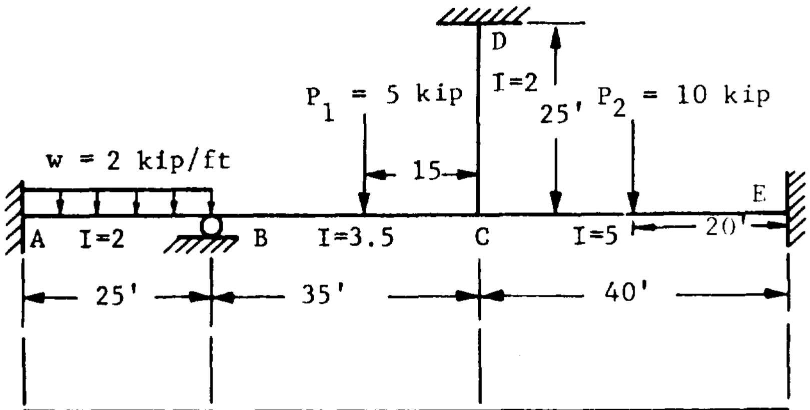
7. Move to a new joint and repeat the process for the balance and carry-over of moments for as many cycles as desired to meet the accuracy required by the problem. The unbalanced moment for each cycle will be the algebraic sum of the moments at the joint recorded below the last horizontal line.

8. Obtain the final moment at the end of each member as the algebraic sum of all moments tabulated at this point. The total of the final moments for all members at any joint must be zero.

9. Reactions, vertical shear, and bending moments of the member may be found through statics by utilizing the above mentioned final moments (step 8).

B 5.1.2 Sample Problem.

PROBLEM #1. Compute the end moments and draw the shear and bending-moment diagrams for this frame.



	AB	BA	BC	DC	CB	CE	CD	EC
K		.08	.1		.1	.125	.08	
DF		.44	.56		.33	.41	.26	
FEM	104	-104	+18	0	-25	+50	0	-50
		+38	+48		-8	-10	-7	
	+19		-4	-3	+24			-5
		+2	+2		-8	-10	-6	
	+1		-4	-3				-5
		+2	+2					
Σ	+124	-62	+62	-6	-17	+30	-13	-60

Fig. B 5.1.2-1

B 5.1.2 Sample Problem (Cont'd)

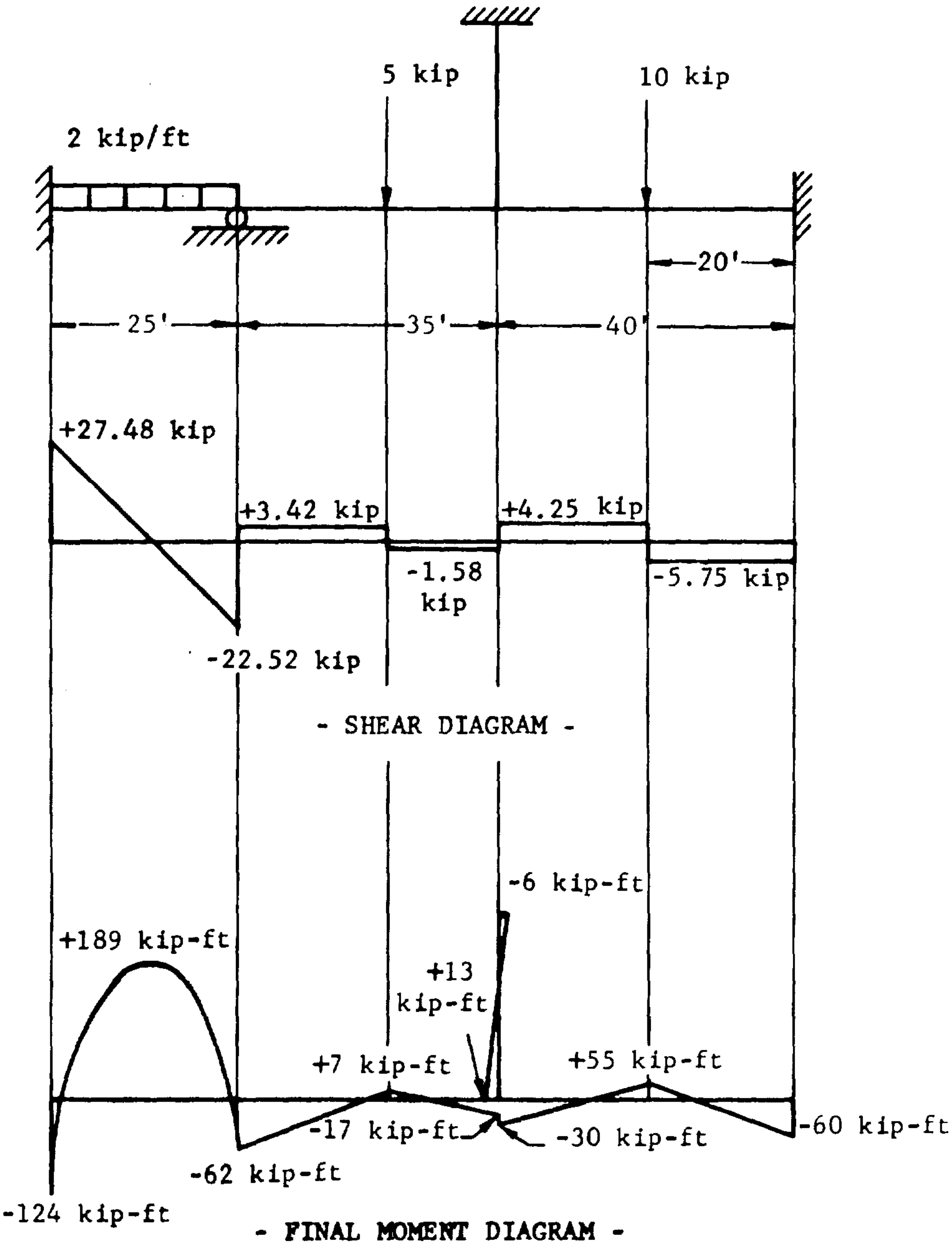


Fig. B 5.1.2-2

B 5.1.3 Application of Moment-distribution to Advanced Problems.

In article B 5.1.1, "Discussion of moment-distribution", the basic principles of moment-distribution were founded. Moment-distribution may be applied to complex structures involving joint translation, settlement of supports, non-prismatic members, symmetrical and unsymmetrical bents and other involved structures. For information on the technique of solving such structures see the references listed in Section B 5.0.0. Methods for accelerating the convergence and other short-cuts may also be obtained in these references.

B 5.1.4 Particular Solution of Bents and Semicircular Arches

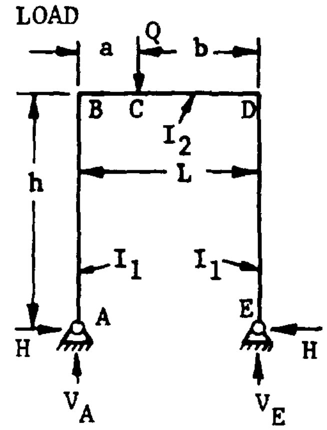
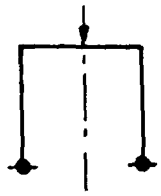
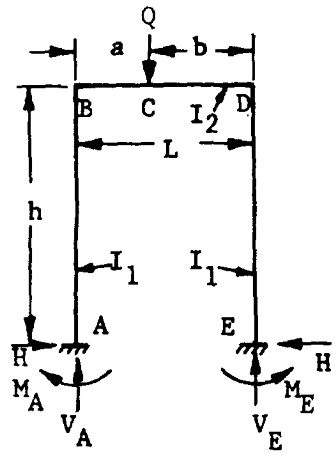
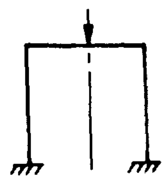
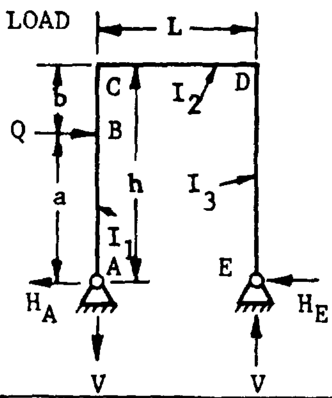
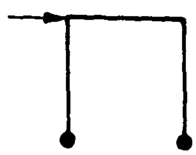
In the tables that follow, formulas for computing reactions are given for several load cases. In all cases constraining moments, reaction, and applied loads are positive when acting as shown and

$$K = \frac{I_2 h}{I_1 L} \quad \text{for cases 1 through 18}$$

$$K = \frac{I_1 S_2}{I_2 S_1} \quad \text{for cases 19 through 28}$$

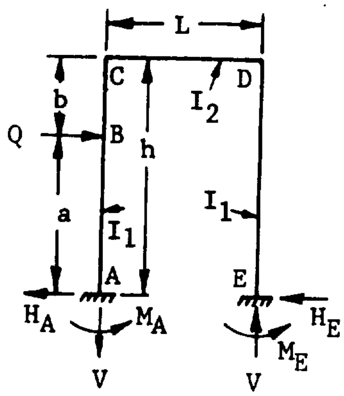
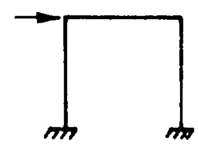
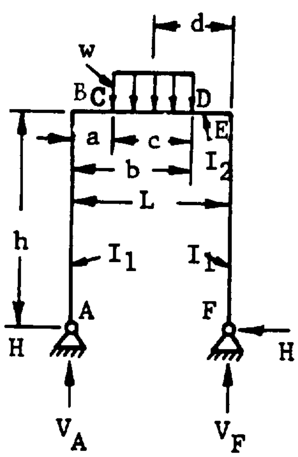
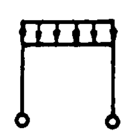
B 5.1.4 <u>Particular Solution of Bents and Semicircular Arches (Cont'd)</u>

Table B 5.1.4-1 Reactions and Constraining Moments in Two Legged Rectangular Bents

1. VERT. CONCENTRATED LOAD 	$V_A = \frac{Qb}{L}$ $\quad V_E = Q - V_A$ $H = \frac{3Qab}{2Lh(2K + 3)}$ <u>FOR SPECIAL CASE:</u> $a = b = \frac{L}{2}$ $V_A = V_E = \frac{Q}{2}$ $H = \frac{3QL}{8h(2K + 3)}$ 
2. VERT. CONCENTRATED LOAD 	$V_A = \frac{Qb}{L}\left[1 + \frac{a(b - a)}{L^2(6K + 1)}\right]$ $\quad V_E = Q - V_A$ $H = \frac{3Qab}{2Lh(K + 2)}$ $M_A = \frac{Qab}{L}\left[\frac{1}{2(K + 2)} - \frac{(b - a)}{2L(6K + 1)}\right]$ $M_E = \frac{Qab}{L}\left[\frac{1}{2(K + 2)} + \frac{(b - a)}{2L(6K + 1)}\right]$ <u>FOR SPECIAL CASE:</u> $a = b = \frac{L}{2}$ $V_A = V_E = \frac{Q}{2}$ $\quad M_A = M_E = \frac{QL}{8(K + 2)}$ 
3. HORIZ. CONCENTRATED LOAD 	$V = \frac{Qa}{L}$ $\quad H_A = Q - H_E$ $H_E = \frac{Qa}{2h}\left[\frac{bK(a + h)}{h^2(2K + 3)} + 1\right]$ FOR SPECIAL CASE: $b = 0,\ a = h$ $V = \frac{Qh}{L}$ $H_E = H_A = \frac{Q}{2}$ 

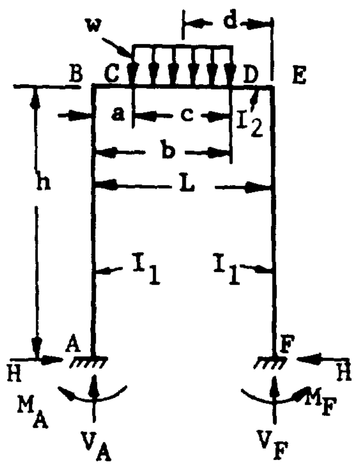
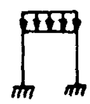
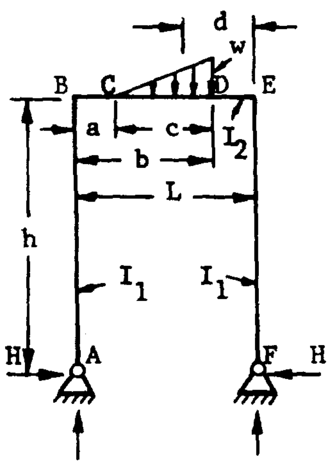
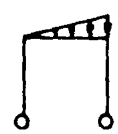
B 5.1.4 Particular Solution of Bents and Semicircular Arches (Cont'd)

Table B 5.1.4-1 Reactions and Constraining Moments in Two Legged Rectangular Bents (Cont'd)

4. HORIZ. CONCENTRATED LOAD  	$V = \frac{3Qa^2K}{Lh(6K + 1)}$ $\quad H_A = Q - H_E$ $H_E = \frac{Qab}{2h^2}\left[\frac{h}{b} - \frac{h + b + K(b - a)}{h(K + 2)}\right]$ $M_A = \frac{Qa}{2h}\left[\frac{b(h + b + bK)}{h(K + 2)} + h - \frac{3aK}{(6K + 1)}\right]$ $M_E = \frac{Qa}{2h}\left[\frac{-b(h + b + bK)}{h(K + 2)} + h - \frac{3aK}{(6K + 1)}\right]$ FOR SPECIAL CASE: $b = 0,\ a = h$ $V = \frac{3QhK}{L(6K + 1)}$ $H_A = H_E = \frac{Q}{2}$ $M_A = M_E = \frac{Qh(3K + 1)}{2(6K + 1)}$ 
5. VERT. UNIFORM RUNNING LOAD  $d = L - \frac{a}{2} - \frac{b}{2}$	$V_A = \frac{wcd}{L}$ $V_F = wc - V_A = \frac{wc}{L}\left(a + \frac{c}{2}\right) = wc\left(1 - \frac{d}{L}\right)$ $H = \frac{3}{2h}\left[\frac{X_1 + X_2}{2K + 3}\right] = \frac{3wc}{24Lh(2K + 3)}\left[12dL - 12d^2 - c^2\right]$ where: $X_1 = -\frac{wc}{24L}\left[24\frac{d^3}{L} - 6\frac{bc^2}{L} + 3\frac{c^2}{L} + 4c^2 - 24d^2\right]$ $X_2 = \frac{wc}{24L}\left[24\frac{d^3}{L} - 6\frac{bc^2}{L} + 3\frac{c^3}{L} + 2c^2 - 48d^2 + 24dL\right]$ FOR SPECIAL CASE: $a = 0,\ c = b = L,\ d = \frac{L}{2}$ $V_A = V_F = \frac{wL}{2}$ $H = \frac{wL^2}{4h(2K + 3)}$ 

B 5.1.4 <u>Particular Solution of Bents and Semicircular Arches (Cont'd)</u>

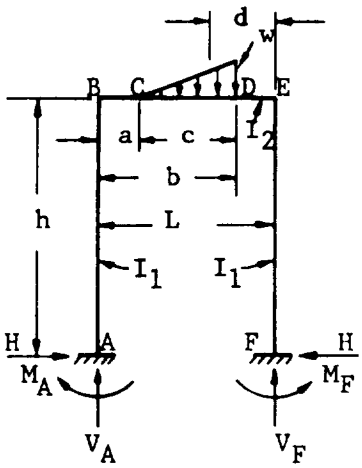
Table B 5.1.4-1 Reactions and Constraining Moments in Two Legged Rectangular Bents (Cont'd)

6. VERT. UNIFORM RUNNING LOAD  $d = L - \frac{a}{2} - \frac{b}{2}$	$V_A = \frac{wcd}{L} + \frac{X_1 - X_2}{L(6K + 1)}$ X_1 and X_2 are given in case 5 $V_F = wc - V_A$ $H = \frac{3(X_1 + X_2)}{2h(K + 2)}$ $\quad M_A = \frac{X_1 + X_2}{2(K + 2)} - \frac{X_1 - X_2}{2(6K + 1)}$ $M_F = \frac{X_1 + X_2}{2(K + 2)} + \frac{X_1 - X_2}{2(6K + 1)}$ <u>FOR SPECIAL CASE:</u> $a = 0,\ c = b = L,\ d = \frac{L}{2}$ $V_A = V_F = \frac{wL}{2}$ $H = \frac{wL^2}{4h(K + 2)}$ $\quad M_A = M_F = \frac{wL^2}{12(K + 2)}$ 
7. VERT. TRIANGULAR RUNNING LOAD  $d = L - \frac{a}{3} - \frac{2b}{3}$	$V_A = \frac{wcd}{2L}$ $V_F = \frac{wc}{2} - V_A = \frac{wc}{2L}\left(a + \frac{2c}{3}\right)$ $H = \frac{3}{2h}\left[\frac{X_3 + X_4}{2K + 3}\right] = \frac{3wc}{4Lh(2K + 3)}\left[dL - \frac{c^2}{18} - d^2\right]$ WHERE: $X_3 = -\frac{wc}{2L}\left[\frac{d^3}{L} + \frac{c^2}{9} + \frac{51c^3}{810L} + \frac{c^2b}{6L} - d^2\right]$ $X_4 = \frac{wc}{2L}\left[\frac{d^3}{L} + \frac{c^2}{18} + \frac{51c^3}{810L} - \frac{c^2b}{6L} - 2d^2 + dL\right]$ <u>FOR SPECIAL CASE:</u> $a=o,\ c=b=L,\ d = \frac{L}{3}$ $V = \frac{wL}{6}$ $H = \frac{wL^2}{8h(2K + 3)}$ 

B 5.1.4 Particular Solution of Bents and Semicircular Arches (Cont'd)

Table B 5.1.4-1 Reactions and Constraining Moments in Two Legged Rectangular Bents (Cont'd)

8. VERT. TRIANGULAR RUNNING LOAD



$$d = L - \frac{a}{3} - \frac{2b}{3}$$

$$V_A = \frac{wcd}{2L} + \frac{X_3 - X_4}{L(6K + 1)}$$

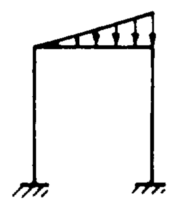
X_3 and X_4 are given in case 7

$$V_F = \frac{wc}{2} - V_A \qquad H = \frac{3(X_3 + X_4)}{2h(K + 2)}$$

$$M_A = \frac{X_3 + X_4}{2(K + 2)} - \frac{X_3 - X_4}{2(6K + 1)}$$

$$M_F = \frac{X_3 + X_4}{2(K + 2)} + \frac{X_3 - X_4}{2(6K + 1)}$$

FOR SPECIAL CASE: $a = o$, $c = b = L$, $d = \frac{L}{3}$



$$V_A = \frac{wL}{6}\left[1 - \frac{1}{10(6K + 1)}\right]$$

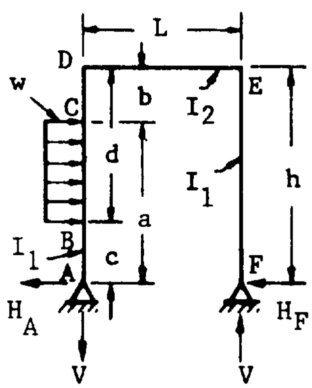
$$V_F = \frac{wL}{3}\left[1 + \frac{1}{20(6K + 1)}\right]$$

$$H = \frac{wL^2}{8h(K + 2)}$$

$$M_A = \frac{wL^2}{120}\left[\frac{5}{K + 2} + \frac{1}{6K + 1}\right]$$

$$M_F = \frac{wL^2}{120}\left[\frac{5}{K + 2} - \frac{1}{6K + 1}\right]$$

9. HORIZ. UNIFORM RUNNING LOAD

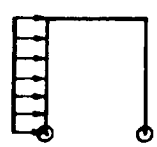


$$V = \frac{w(a^2 - c^2)}{2L} \qquad H_A = w(a - c) - H_F$$

$$H_F = \frac{w(a^2 - c^2)}{4h} + \frac{K\left[w(a^2 - c^2)(2h^2 - a^2 - c^2)\right]}{8h^3(2K + 3)}$$

FOR SPECIAL CASE: $c = o$, $b = o$, $a = d = h$

$$V = \frac{wh^2}{2L}$$



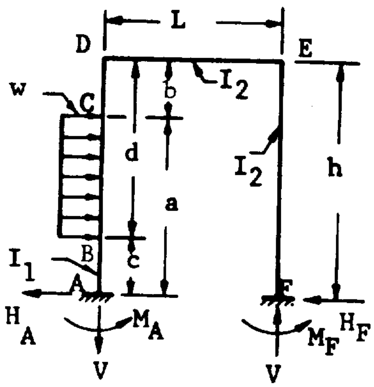
$$H_A = wh - H_F$$

$$H_F = \frac{wh}{4}\left[1 + \frac{K}{2(2K + 3)}\right]$$

B 5.1.4 Particular Solution of Bents and Semicircular Arches (Cont'd)

Table B 5.1.4-1 Reactions and Constraining Moments in Two Legged Rectangular Bents (Cont'd)

10. HORIZ. UNIFORM RUNNING LOAD



$$V = \frac{w(a^2 - c^2)}{2L} - \frac{M_A}{L} - \frac{M_F}{L}$$

$$H_A = w(a - c) - H_F$$

$$H_F = \frac{w(a^2 - c^2)}{4h} - \frac{X_5}{2h} + \frac{X_6(K - 1)}{2h(K + 2)}$$

WHERE:

$$X_5 = \frac{w}{12h^2}\left[d^3(4h - 3d) - b^3(4h - 3b)\right]$$

$$X_6 = \frac{w}{12h^2}\left[a^3(4h - 3a) - c^3(4h - 3c)\right]$$

$$M_A = \frac{(3K + 1)\left[\frac{w(a^2 - c^2)}{2} - X_5\right]}{2(6K + 1)} + \frac{X_6}{2}\left[\frac{1}{K + 2} + \frac{3K}{6K + 1}\right] + X_5$$

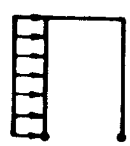
$$M_F = \frac{(3K + 1)\left[\frac{w(a^2 - c^2)}{2} - X_5\right]}{2(6K + 1)} - \frac{X_6}{2}\left[\frac{1}{K + 2} - \frac{3K}{6K + 1}\right]$$

FOR SPECIAL CASE: $c = o$, $b = o$, $a = d = h$:

$$V = \frac{wh^2K}{L(6K + 1)} \qquad H_A = wh - H_F$$

$$H_F = \frac{wh(2K + 3)}{8(K + 2)} \qquad M_F = \frac{wh^2}{24}\left[\frac{18K + 5}{6K + 1} - \frac{1}{K + 2}\right]$$

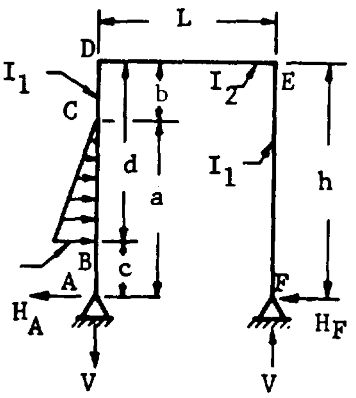
$$M_A = \frac{wh^2}{24}\left[\frac{30K + 7}{6K + 1} + \frac{1}{K + 2}\right]$$



B 5.1.4 Particular Solution of Bents and Semicircular Arches (Cont'd)

Table B 5.1.4-1 Reactions and Constraining Moments in Two Legged Rectangular Bents (Cont'd)

11. HORIZ. TRIANGULAR RUNNING LOAD



$$V = \frac{w}{6L}\left(a^2 + ac - 2c^2\right) \qquad H_A = \frac{w(a - c)}{2} - H_F$$

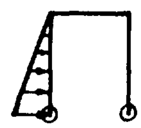
$$H_F = \frac{VL}{2h} + \frac{KX_7}{(2K + 3)h}$$

WHERE:

$$X_7 = \frac{w}{120h^2(d-b)}\left[3(4d^5+b^5) - 15h(3d^4+b^4) + 20h^2(2d^3+b^3) - 15bd^2(2h-d)^2\right]$$

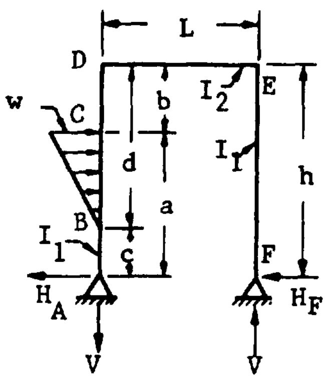
FOR SPECIAL CASE: b=c=o, a=d=h:

$$V = \frac{wh^2}{6L} \qquad H_A = \frac{wh}{2} - H_F$$



$$H_F = \frac{wh}{12}\left[1 + \frac{7K}{10(2K + 3)}\right]$$

12. HORIZ. TRIANGULAR RUNNING LOAD



$$V = \frac{w}{6L}(2a + c)(a - c)$$

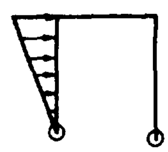
$$H_A = \frac{w(a - c)}{2} - H_F \qquad H_F = \frac{VL}{2h} + \frac{KX_{10}}{h(2K + 3)}$$

WHERE:

$$X_{10} = \frac{w}{120h^2(a-c)}\left[-30h^2c\,(a^2-c^2) + 20h^2(a^3-c^3) + 15c(a^4-c^4) - 12(a^5-c^5)\right]$$

FOR SPECIAL CASE: b=c=o, a=d=h:

$$V = \frac{wh^2}{3L}$$



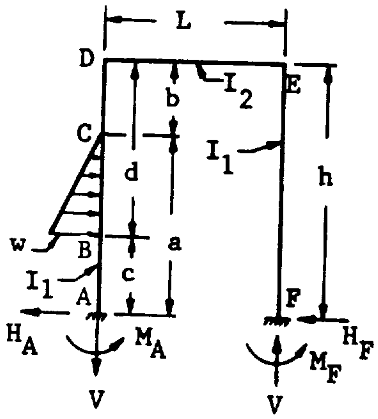
$$H_A = \frac{wh}{2} - H_F$$

$$H_F = \frac{wh}{10}\left[\frac{4K + 5}{2K + 3}\right]$$

B 5.1.4 Particular Solution of Bents and Semicircular Arches (Cont'd)

Table B 5.1.4-1 Reactions and Constraining Moments in Two Legged Rectangular Bents (Cont'd)

13. HORIZ. TRIANGULAR RUNNING LOAD



$$V = \frac{w(a^2 + ac - 2c^2)}{6L} - \frac{M_A}{L} - \frac{M_F}{L}$$

$$H_A = \frac{w(a - c)}{2} - H_F$$

$$H_F = \frac{w(a^2 + ac - 2c^2)}{12h} - \frac{X_8}{2h} + \frac{X_9(K-1)}{2h(K+2)}$$

WHERE:

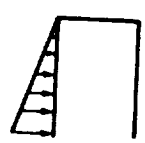
$$X_8 = \frac{w}{60h^2(d-b)}\left[15(h+b)(d^4-b^4) - 12(d^5-b^5) - 20bh(d^3-b^3)\right]$$

$$X_9 = \frac{w}{60h^2(d-b)}\left[10d^2h^2(2d-3b) + 10bh(4d^3 + b^2h - b^3) - d^4(30h+15b) + 12d^5 + 3b^5\right]$$

$$M_A = \frac{(3K + 1)\left[\frac{w(a^2 + ac - 2c^2)}{6} - X_8\right]}{2(6K + 1)} + \frac{X_9}{2}\left[\frac{1}{K + 2} + \frac{3K}{6K + 1}\right] + X_8$$

$$M_F = \frac{(3K + 1)\left[\frac{w(a^2 + ac - 2c^2)}{6}\right] - X_8}{2(6K + 1)} - \frac{X_8}{2}\left[\frac{1}{K + 2} - \frac{3K}{6K + 1}\right]$$

FOR SPECIAL CASE: $b=c=o$, $a=d=h$



$$V = \frac{wh^2K}{4L(6K + 1)} \qquad H_A = \frac{wh}{2} - H_F$$

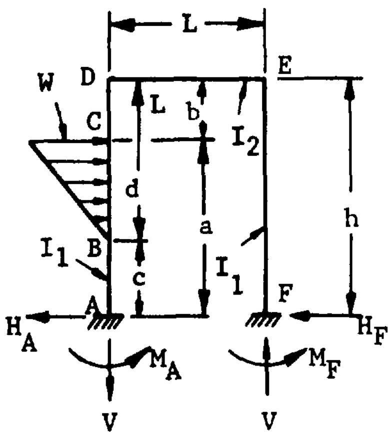
$$H_F = \frac{wh(3K + 4)}{40(K + 2)} \qquad M_F = \frac{wh^2}{60}\left[\frac{27K+7}{2(6K+1)} - \frac{1}{K+2}\right]$$

$$M_A = \frac{wh^2}{60}\left[\frac{27K + 7}{2(6K + 1)} + \frac{3K + 7}{K + 2}\right]$$

B 5.1.4 Particular Solution of Bents and Semicircular Arches (Cont'd)

Table B 5.1.4-1 Reactions and Constraining Moments in Two Legged Rectangular Bents (Cont'd)

14. HORIZ. TRIANGULAR RUNNING LOAD



$$V = \frac{w(2a + c)(a - c)}{6L} - \frac{M_A}{L} - \frac{M_F}{L}$$

$$H_A = \frac{w(a - c)}{2} - H_F$$

$$H_F = \frac{w(2a^2 - ac - c^2)}{12h} - \frac{X_{11}}{2h} + \frac{X_{12}(K - 1)}{2h(K + 2)}$$

where:

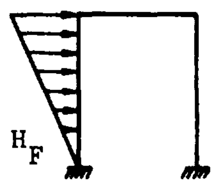
$$X_{11} = \frac{w}{60h^2(d - b)} \left[5hd^4 - 3d^5 - 20hdb^3 - 12b^4(d + h) \right]$$

$$X_{12} = \frac{w}{60h^2(a - c)} \left[15(h + c)(a^4 - c^4) - 12(a^5 - c^5) - 20ch(a^3 - c^3) \right]$$

$$M_A = \frac{\left[3K+1\right]\left[\frac{w(2a^2 - ac - c^2)}{6} - X_{11}\right]}{2(6K + 1)} + \frac{X_{12}}{2}\left[\frac{1}{K + 2} + \frac{3K}{6K + 1}\right] + X_{11}$$

$$M_F = \frac{\left[3K+1\right]\left[\frac{w(2a^2 - ac - c^2)}{6} - X_{11}\right]}{2(6K + 1)} - \frac{X_{22}}{2}\left[\frac{1}{K + 2} - \frac{3K}{6K + 1}\right]$$

FOR SPECIAL CASE: $b = c = o$, $a = d = h$



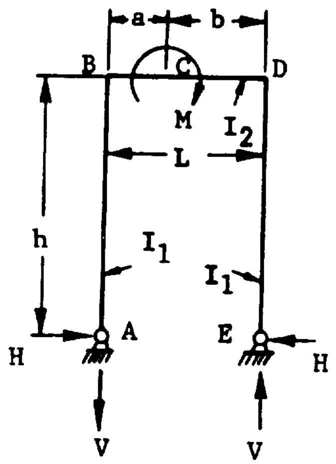
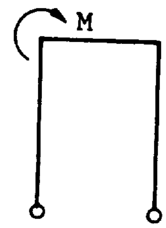
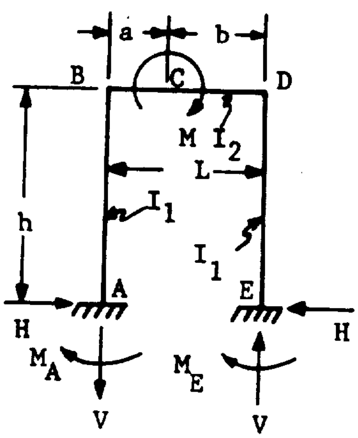
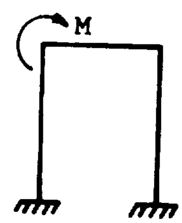
$$V = \frac{3Kwh^2}{4L(6K + 1)} \qquad H_A = \frac{wh}{2} - H_F$$

$$H_F = \frac{wh(7K + 11)}{40(K + 2)} \qquad M_A = \frac{wh^2}{120}\left[\frac{87K+22}{6K+1} + \frac{3}{K+2}\right]$$

$$M_F = \frac{wh^2}{40}\left[\frac{21K + 6}{6K + 1} - \frac{1}{K + 2}\right]$$

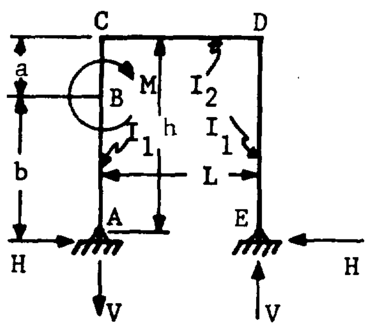
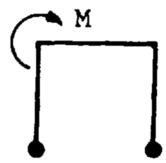
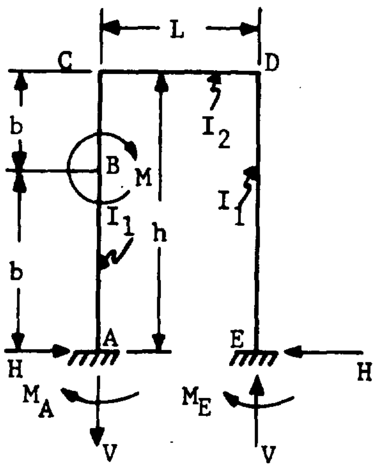
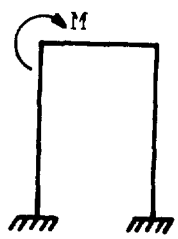
B 5.1.4 Particular Solution of Bents and Semicircular Arches (Cont'd)

Table B 5.1.4-1 Reactions and Constraining Moments in Two Legged Rectangular Bents (Cont'd)

15. MOMENT ON HORIZ. SPAN  	$V = \frac{M}{L}$ $H = \frac{3(b - L/2)M}{Lh(2K + 3)}$ FOR SPECIAL CASE: a=o, b=L $V = \frac{M}{L}$ $H = \frac{3M}{2h(2K + 3)}$ 
16. MOMENT ON HORIZ. SPAN  	$V = \frac{6(ab + L^2K)M}{L^3(6K + 1)}$ $H = \frac{3(b - a)M}{2Lh(K + 2)}$ $M_A = M\left[\frac{6ab(K+2) - L\left[a(7K+3) - b(5K-1)\right]}{2L^2(K+2)(6K+1)}\right]$ $M_E = VL - M - M_A$ FOR SPECIAL CASE: a=o, b=L $V = \frac{6KM}{L(1 + 6K)}$ $H = \frac{3M}{2h(K + 2)}$ $M_A = \frac{(5K - 1)M}{2(K + 2)(6K + 1)}$ 

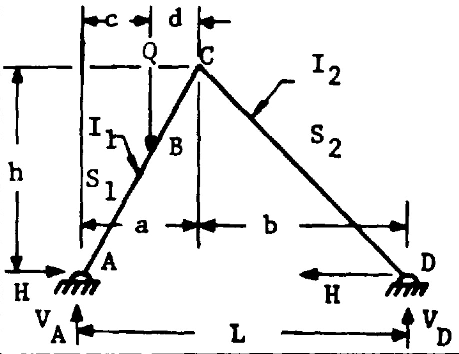
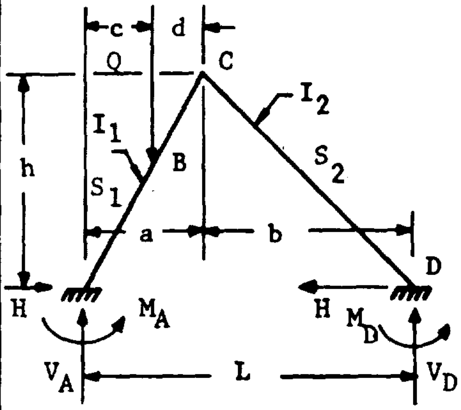
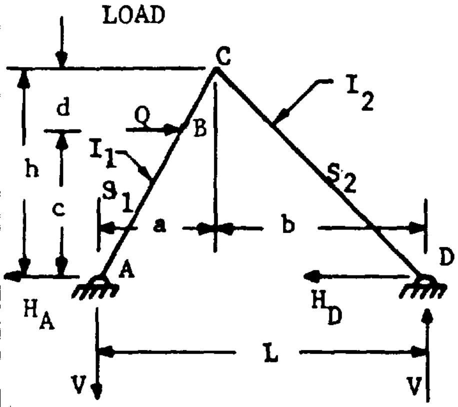
B 5.1.4 Particular Solution of Bents and Semicircular Arches (Cont'd)

Table B 5.1.4-1 Reactions and Constraining Moments in Two Legged Rectangular Bents (Cont'd)

17. MOMENT ON SIDE SPAN 	$V = \frac{M}{L}$ $H = \frac{3[K(2ab+a^2) + h^2]M}{2h^3(2K+3)}$ FOR SPECIAL CASE: a=o, b=h $V = \frac{M}{L}$ $H = \frac{3M}{2h(2K+3)}$ 
18. MOMENT ON SIDE SPAN 	$V = \frac{6bKM}{hL(6K+1)}$ $H = \frac{3bM[2a(K+1) + b]}{2h^3(K+2)}$ $M_A = \frac{-M}{2h^2(K+2)(6K+1)} [4a^2+2ab+b^2+K(26a^2-5b^2) + 6aK^2(2a-b)]$ $M_E = VL - M - M_A$ FOR SPECIAL CASE: a=o; b=h $V = \frac{6KM}{L(6K+1)}$ $H = \frac{3M}{2h(K+2)}$ $M_A = \frac{M(5K-1)}{2(K+2)(6K+1)}$ 

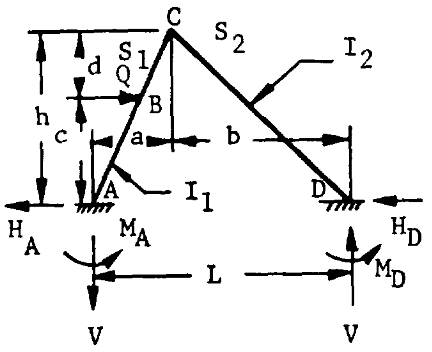
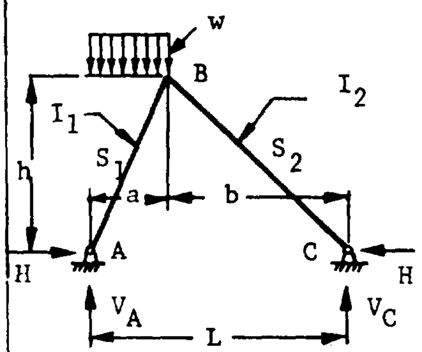
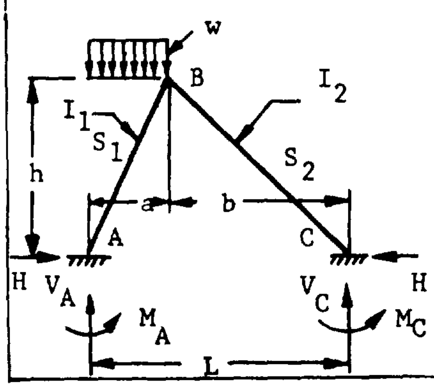
B 5.1.4 Particular Solution of Bents and Semicircular Arches (Cont'd)

Table B 5.1.4-2 Reactions and Constraining Moments in Triangular Bents (Cont'd)

19. VERT. CONCENTRATED LOAD 	$V_A = Q - V_D$ $V_D = \frac{Qc}{L}$ $H = \frac{Qc}{h}\left[\frac{b}{L} + \frac{d(a+c)}{2a^2(K+1)}\right]$
20. VERT. CONCENTRATED LOAD 	$V_A = Q - V_D$ $V_D = \frac{Qc}{L}\left[1 - \frac{d(a+d)}{2a^2}\right]$ $H = \frac{Qcb}{Lh} + \frac{Qcd}{6La^2h(K+1)}\left\{\left[-b(3K+4) - 2L\right]\left[(a+d)\right] + 2(2L+b)(a+c) + 3ac\right\}$ $M_A = \frac{Qcd}{6a^2(K+1)}\left[(a+d)(3K+4) - 2(a+c)\right]$ $M_D = \frac{Qc^2d}{2a^2(K+1)}$
21. HORIZ. CONCENTRATED LOAD 	$V = \frac{Qc}{L}$ $H_A = Q - H_D$ $H_D = \frac{Qc}{h}\left[\frac{b}{L} + \frac{d(h+c)}{2h^2(K+1)}\right]$

B 5.1.4 Particular Solution of Bents and Semicircular Arches (Cont'd)

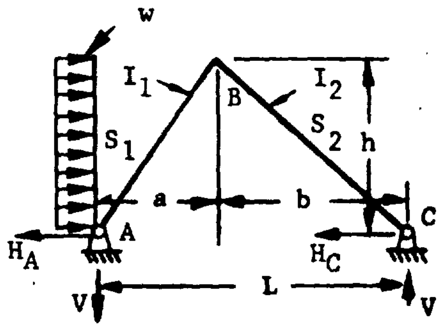
Table B 5.1.4-2 Reactions and Constraining Moments in Triangular Bents (Cont'd)

22. HORIZ. CONCENTRATED LOAD  	$V = \frac{Qc}{L}\left[1 - \frac{d(h+d)}{2h^2}\right]$; $H_A = Q - H_D$ $H_D = \frac{Qc}{Lh}\left\{b + \frac{d}{6h^2(K+1)}\left[(h+d)(-b[3K+4] - 2L) + 2(2L+b)(h+e) + 3ac\right]\right\}$ $M_A = \frac{Qcd}{6h^2(K + 1)}\left[(h+d)(3K+4) - 2(h+c)\right]$ $M_D = \frac{Qcd}{6h^2(K + 1)}(h + 2c + d)$
23. VERTICAL UNIFORM RUNNING LOAD  	$V_A = wa\left[1 - \frac{a}{2L}\right]$ $V_C = \frac{wa^2}{2L}$ $H = \frac{wa^2}{8h}\left[\frac{4b}{L} + \frac{1}{1 + K}\right]$
24. VERTICAL UNIFORM RUNNING LOAD  	$V_A = wa\left[1 - \frac{3a}{8L}\right]$: $V_C = \frac{3wa^2}{8L}$ $H = \frac{wa^2}{24Lh(K + 1)}\left[b(10 + 9K) + 2L + a\right]$ $M_A = \frac{wa^2(3K + 2)}{24(K + 1)}$ $M_c = \frac{wa^2}{24(K + 1)}$

B 5.1.4 Particular Solution of Bents and Semicircular Arches (Cont'd)

Table B 5.1.4-2 Reactions and Constraining Moments in Triangular Bents (Cont'd)

25. HORIZ. UNIFORM RUNNING LOAD

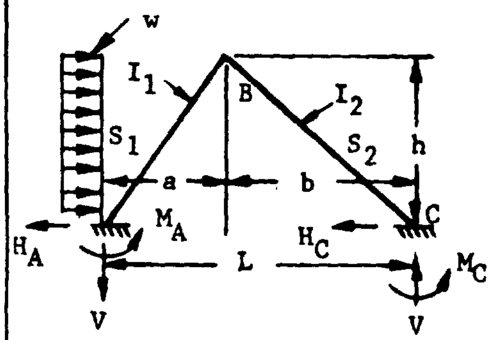


$$V = \frac{wh^2}{2L}$$

$$H_A = wh - H_c$$

$$H_C = \frac{wh}{8}\left[\frac{4b}{L} + \frac{1}{K+1}\right]$$

26. HORIZ. UNIFORM RUNNING LOAD



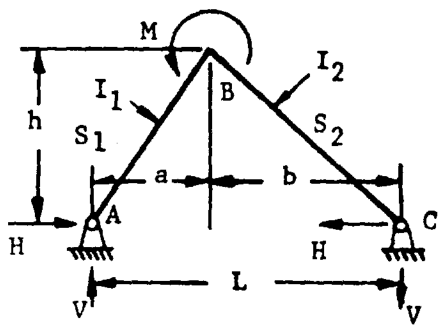
$$V = \frac{3wh^2}{8L}$$

$$M_A = wh - M_C$$

$$M_C = \frac{wh}{8L(K+1)}\left[b(3K+4) + a\right]$$

$$M_A = \frac{wh^2(3K+2)}{24(K+1)} \qquad M_C = \frac{wh^2}{24(K+1)}$$

27. APPLIED MOMENT AT APEX

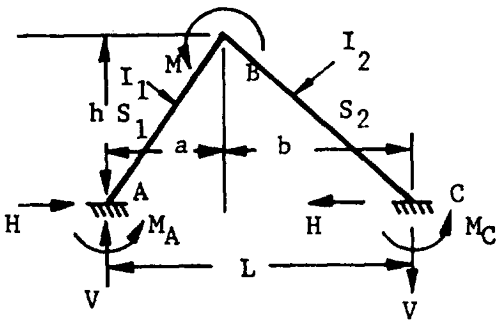
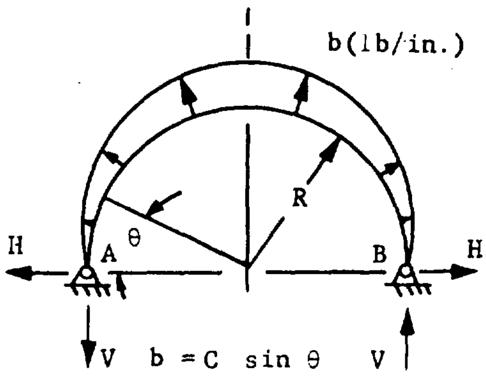


$$V = \frac{M}{L}$$

$$H = \frac{M}{hL}\left[\frac{a - bK}{K+1}\right]$$

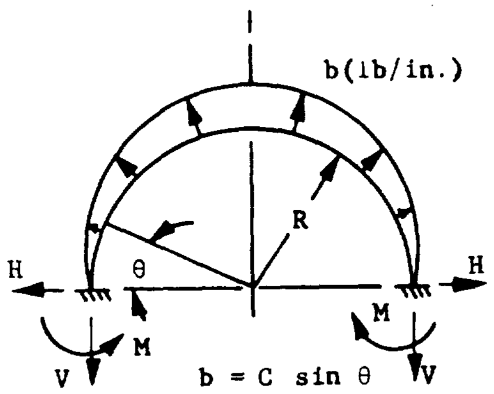
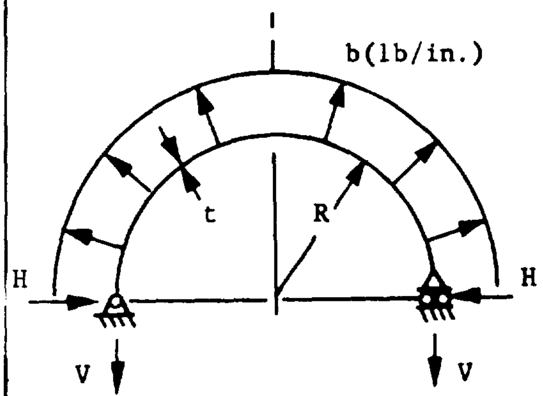
B 5.1.4 Particular Solution of Bents and Semicircular Arches (Cont'd)

Table B 5.1.4-3 Reactions and Constraining Moments in Triangular Bents and Semicircular Frames or Arches (Cont'd)

28. APPLIED MOMENT AT APEX 	$V = \frac{3M}{2L}$ $H = \frac{3M(a - bK)}{2hL(K + 1)}$ $M_A = \frac{KM}{2(K + 1)}$ $M_C = \frac{M}{2(K + 1)}$
29. SINUSOIDAL NORMAL PRESSURE 	$V = \frac{C\pi R}{4}$ $H = \frac{CR}{4}$ $M_\theta = \frac{CR^2}{4}\left[(\pi-2\theta)\cos\theta - \pi + 3\sin\theta\right]$ (Positive moment acts clockwise on section ahead.)

B 5.1.4 Particular Solution of Bents and Semicircular Arches (Cont'd)

Table B 5.1.4 Reactions and Constraining Moments in Semicircular Frames or Arches (Cont'd)

30. SINUSOIDAL NORMAL PRESSURE 	$V = \frac{C\pi R}{4}$ $H = \frac{CR}{4}\left[\frac{3\pi^2 - 32}{8 - \pi^2}\right] = .31974CR$ $M = \frac{CR^2}{4}\left[\frac{\pi^3 - 10\pi}{8 - \pi^2}\right] = .05478CR^2$ $M_\theta = CR^2\left[.81974 \sin\theta - .84018 + \frac{\cos\theta}{2}\left(\frac{\pi}{2} - \theta\right)\right]$ (Positive moment acts clockwise on section ahead.)
31. UNIFORM NORMAL PRESSURE 	M = 0 at all points since pin points permit a uniform hoop tension. T, where: T = V = bR H = 0

B5.2.0 Analysis of Arbitrary Ring by Tabular Method

The following analysis is a corrected version of work taken originally from Reference 6.

Rings or frames with ends built in, elastically restrained, or pinned may be analyzed by the procedure outlined in this section. The procedure is given in tabular form (Table B5.2.0-1) with added notes to define the sequence to be followed.

A sample problem is given in Sec. B5.2.1 to illustrate the procedure. This sample problem considers the energy due to direct and shear loads. The effects of shear flow are presented as a supplement to the sample problem.

Procedure

I. Procedure to Obtain Initial Geometric and Elastic Data

(1) Set out the neutral axis of the ring between the end points T and O. (In a complete ring T and O coincide.)

(2) Divide the neutral axis into a number of segments which are conveniently but not necessarily of equal length Δs. Ten to twenty segments will usually give sufficient accuracy. Mark the joints and central points of the segments. In symmetrical rings a complete segment should lie each side of the axis of symmetry.

(3) Calculate the values $\Delta s/EI$, $\Delta s/EA$, $\Delta s/GA'$ at the segment centers.

(4) For a built-in ring, calculate a and b from equation (1) below which defines the elastic center C and the axes X' and Y' (See Figures B5.2.0-1 and -2.)

For a pinned-end ring, no translation of axes from O is necessary. Use the X and Y axes as they are defined in Figures B5.2.0-3 and -4,

(5) For a built-in ring obtain the co-ordinates x', y', and the angles ψ' at the segment centers. For a pinned ring obtain x, y, and ψ directly. In symmetrical rings only half the ring need be considered.

B 5.2.0 Analysis of Arbitrary Ring by Tabular Method (Cont'd)

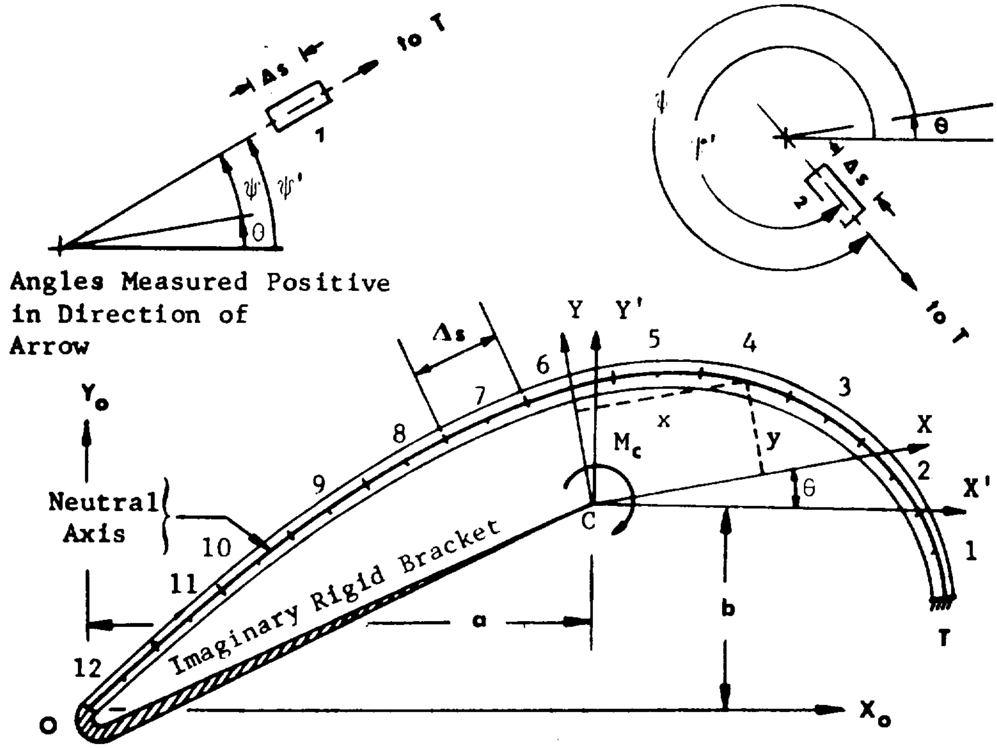


Fig. B5.2.0-1 General Ring with Built-In Ends

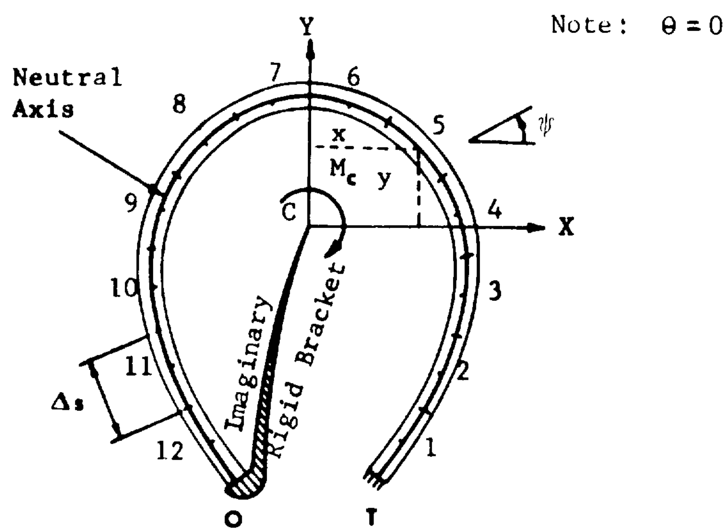


Fig. B5.2.0-2 Symmetrical Built-In Ring

B 5.2.0 Analysis of Arbitrary Ring by Tabular Method (Cont'd)

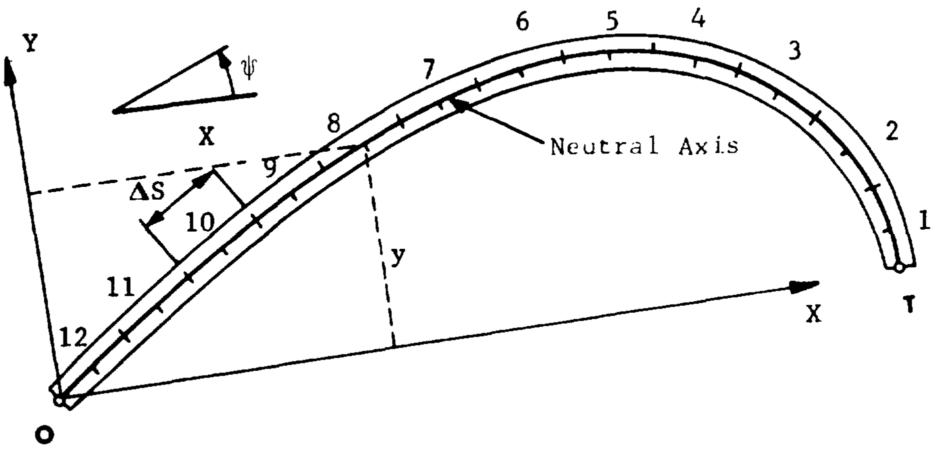


Fig. B 5.2.0-3 General Ring with Pinned End

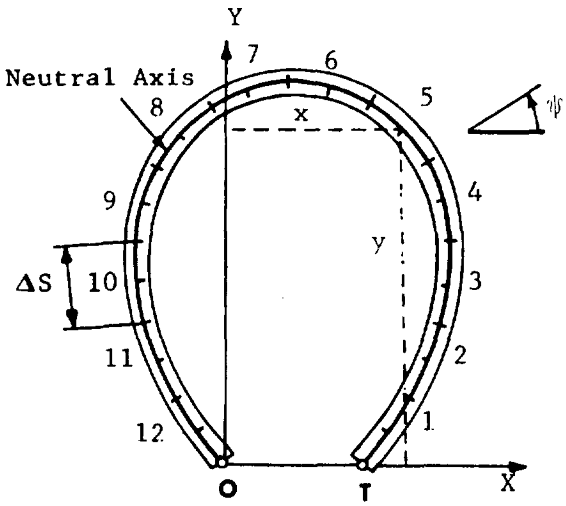


Fig. B 5.2.0-4 Symmetrical Pinned-End Ring

(6) Note that ψ, θ, and ψ' are measured as shown in Figure B5.2.0-1. The angle must originate at the end coming from O, and be measured counterclockwise to the end going to T.

For the slope of the tangent to the neutral axis of the element, use the slope at the midspan of the element.

NOTE: $\Delta s/EA$ and $\Delta s/GA'$ are not usually required, since in most cases the thrust and shear energies are negligible compared with the bending energy.

General Notes

I. Limits of Application

The method may be applied to any ring or curved beam in which the deflections are linear functions of the loads and in which the elementary formula connecting curvature and bending moment holds.

II. Calculations

(1) The advantage of choosing X, Y, and M_c as redundant reactions is due to their orthogonality. Thus, one can calculate them directly without having to solve simultaneous equations.

(2) The calculations are shown in tabular form using finite segments. Graphical integration may also be used and in exceptional cases an analytical method may be applied.

(3) The referenced table is set out for an arbitrary built-in ring, but is also applicable when the supports T and O "give" elastically (See General Note III, 6.) In all other cases the size of the table is reduced but the form is essentially the same. A considerable reduction in the numerical work is possible when the ring is symmetrical (See notes IV and VI.)

(4) In practice certain steps of the calculation may be conveniently done with additional columns, e.g., for x and y one requires x' cos θ, etc.

(5) Numerical Accuracy. Although the data in columns 1-6 and 24-26 will not exceed three-figure accuracy it is necessary to retain four or five figures in the remaining columns in order that the final results shall be correct to three figures.

III. Arbitrary Built-In Ring (Fig. B 5.2.0-1)

Equations required and the stage at which they are used are as follows:

(1) $$a = \frac{\sum \frac{x_o \Delta s}{EI}}{\sum \frac{\Delta s}{EI}}$$

$$b = \frac{\sum \frac{y_o \Delta s}{EI}}{\sum \frac{\Delta s}{EI}}$$

(2) After column 12,

$$\tan 2\theta = \frac{2\Sigma x' y' \frac{\Delta s}{EI} - \Sigma \sin 2\phi' \left(\frac{\Delta s}{EA} - \frac{\Delta s}{GA'}\right)}{\Sigma (x'^2 - y'^2) \frac{\Delta s}{EI} - \Sigma \cos 2\phi' \left(\frac{\Delta s}{EA} - \frac{\Delta s}{GA'}\right)}$$

$x = x' \cos \theta + y' \sin \theta$; $y = y' \cos \theta - x' \sin \theta$.

(3) M_o, N_o, and S_o are calculated for the ring built in at T and free at o.

(4) After column 33,

$$\dot{X} = - \frac{-\Sigma M_o y \frac{\Delta s}{EI} + \Sigma N_o \cos \psi \frac{\Delta s}{EA} + \Sigma S_o \sin \psi \frac{\Delta s}{GA'}}{\Sigma y^2 \frac{\Delta s}{EI} + \Sigma \cos^2 \psi \frac{\Delta s}{EA} + \Sigma \sin^2 \psi \frac{\Delta s}{GA'}}$$

B 5.2.0 Analysis of Arbitrary Ring by Tabular Method (Cont'd)

$$Y = - \frac{\Sigma M_o x \frac{\Delta s}{EI} + \Sigma N_o \sin \psi \frac{\Delta s}{EA} - \Sigma S_o \cos \psi \frac{\Delta s}{GA'}}{\Sigma x^2 \frac{\Delta s}{EI} + \Sigma \sin^2 \psi \frac{\Delta s}{EA} + \Sigma \cos^2 \psi \frac{\Delta s}{GA'}}$$

$$M_c = - \frac{\Sigma M_o \frac{\Delta s}{EI}}{\Sigma \frac{\Delta s}{EI}}$$

(5) After column 39,

$$M = M_c - Xy + Yx + M_o$$

$$N = N_o + X \cos \psi + Y \sin \psi$$

$$S = S_o + X \sin \psi - Y \cos \psi$$

(6) Effect of Elastic Supports. The elastic characteristics at the supports T and O are represented by the two sets of three coefficients, k_{Tm}, k_{Tn}, k_{Ts}; k_{Om}, k_{On}, k_{Os}. Thus k_{Tm} is the moment required to rotate T through one radian, k_{Tn} is the force required to move T in a direction defined by ψ_T' through unit distance, and k_{Ts} is the force required to move T in a direction normal to ψ_T' through unit distance. At each support the rotation and the two displacements are assumed elastically orthogonal, i.e., when a force is applied to the support T in the direction ψ_T', there is no rotation and no movement normal to ψ_T'.

For the purposes of the calculations the support flexibilities may be represented by two additional segments considered concentrated at T and O, of which the "elastic weights" are respectively:

$$\frac{\Delta s}{EI} = \frac{1}{k_{Tm}}, \frac{\Delta s}{EA} = \frac{1}{k_{Tn}}, \frac{\Delta s}{GA'} = \frac{1}{k_{Ts}} ;$$

$$\frac{\Delta s}{EI} = \frac{1}{k_{Om}}, \frac{\Delta s}{EA} = \frac{1}{k_{On}}, \frac{\Delta s}{GA'} = \frac{1}{k_{Os}}$$

B 5.2.0 Analysis of Arbitrary Ring by Tabular Method (Cont'd)

Note that ψ'_T and ψ'_0 need not necessarily be the values of ψ' at T and O. The values above and coordinates are entered in rows T and O of the table.

IV. Symmetrical Built-In Ring (Fig. B 5.2.0-2)

(1) CY is the line of symmetry and columns 4-12 are not required as X, Y, and ψ are calculated directly.

(2) Any loading may be analyzed as a symmetrical and an anti-symmetrical loading.

(3) For symmetrical loading Y is statically determinate, being half the total load in the direction YC and should hence be included in the external loading. Thus in the table, columns 18, 20, 23, 28, 30, 33, 35, 38, and 39 are not required, and in the expressions for M, N, S terms involving Y are omitted. X and M_c are determined from the formulas in III (4).

(4) For antisymmetrical loading X and M_c are statically determinate being half the total load in the direction XC and equal and opposite to half the moment of the external load about C respectively; they should hence be included in the external loading. Thus, in the table, columns 19, 21, 22, 27, 29, 31, 32, 34, 36, and 37 are not required, and in the expressions for M, N, S the terms involving M_c and X are omitted. Y is determined from the formula in III (4).

(5) Note that only half the ring need be considered in the table, i.e., elements 7-12 and support O. For symmetrical loading M and N are symmetrical and S antisymmetrical. For anti-symmetrical loading M and N are antisymmetrical and S symmetrical. The summations are, of course, only to be taken over half the ring.

(6) When the supports have symmetrical elastic characteristics the method of this section still applies by introducing an additional segment at O, as described in III (6).

V. Arbitrary Pinned Ring (Fig. B 5.2.0-3)

(1) Columns 4, 12, 18, 20, 23, 27, 28, 30, 33, 35, 38, and 39 are not required.

(2) Y is statically determinate and should hence be included in the external loading.

(3) X is determined from the formula in III (4).

B5.2.0 Analysis of Arbitrary Ring by Tabular Method (Cont'd)

(4) The formulas for M, N, and S are:

$M = M_o - Xy$; $N = N_o + X \cos \psi$; $S = S_o + X \sin \psi$.

(5) Effect of Elastic Supports. In this case the procedure of III (6) should be applied, noting that the terms k_{Tm}, k_{Om} do not exist.

VI. Symmetrical Pinned Ring (Fig. B5.2.0-4)

(1) Any loading may be analyzed as a symmetrical and an antisymmetrical loading.

(2) For symmetrical loading, the load Y at O is obviously half the total load in the direction Y, and only half the ring need be considered in the table, i.e., elements 7-12 and support O. Otherwise, the procedure is the same as in the general case.

(3) For antisymmetrical loading X is statically determinate, being half the total load in the direction X. Thus, in this case the problem is solved purely by statics.

(4) When the supports have symmetrical elastic characteristics the method of this section still applies. For the symmetrical loading an additional segment should be introduced at O, as described in IV (6) and V (5).

Notation

A = cross-sectional area
A' = effective area of cross section for shear stiffness
C = elastic center = centroid of elastic weights $\Delta s/EI$
E = Young's modulus
G = shear modulus
I = moment of inertia of cross section
k = elastic constants of supports T and O (see note III,6)
M_c = moment at elastic center in fixed ring
M_o, N_o, S_o = bending moment, direct load, and shear load in ring supported at T, due to external loading and any of the reactions X, Y, and M_c, which may be statically determined (e.g., in a pinned ring Y is always statically determined). M_o is positive for compression in outer fibers. N_o is positive for compression and S_o is positive when acting outwards on the right-hand side of a section (Subscripts do not refer to point "O")
M, N, S = total bending moment, direct load, and shear load at any section. Sign convention as for M_o, N_o, S_o
O = origin, or left-hand end point of ring
T = terminus, or right-hand end point of ring or cross-sectional area of ring
Δs = length of segment
x_o, y_o = co-ordinates referred to arbitrary orthogonal axes OX', OY' of built-in ring
ψ' = angle defining slope of neutral axis of built-in ring, with respect to CX'
x, y = co-ordinates referred to geometric and elastic orthogonal axes CX, CY of built-in ring; or, co-ordinates referred to orthogonal axes OX, OY of pinned ring, where OX passes through T
θ = angle between CX' and CX
ψ = angle defining slope of neutral axis for built-in ring, with respect to CX ($\psi = \psi' - \theta$)
X, Y = reactions in directions CX, CY at elastic center for built-in ring, or in directions OX, OY for pinned ring
x',y' = coordinates referred to geometric and elastic orthogonal axes CX' and CY' of built-in ring

TABLE B 5.2.0-1

Segments or Point	1	2	3	4	5	6	7	8	9	10	11	12
	$\frac{\Delta s}{EI}$	$\frac{\Delta s}{EA}$	$\frac{\Delta s}{GA'}$	x'	y'	ψ'	sin $2\psi'$	cos $2\psi'$	$x'y'\frac{\Delta s}{EI}$	$[x'^2 - y'^2]\frac{\Delta s}{EI}$	$\sin 2\psi'\left[\frac{\Delta s}{EA} - \frac{\Delta s}{GA'}\right]$	$\cos 2\psi'\left[\frac{\Delta s}{EA} - \frac{\Delta s}{GA'}\right]$
	Columns 1 to 8 give properties of ring. Operations in this row described in terms of the column numbers								4x5x1	(4^2-5^2) x1	(2-3) x7	(2-3) x8
T												
1												
2												
3												
		Can usually be omitted (see cols. 20-23)									Can usually be omitted (see cols. 20-23)	
10												
11												
12												
0												
	Σ	The symbol Σ under a column denotes that the column should be summed							Σ	Σ	Σ	Σ

TABLE B 5.2.0-1 (Cont'd)

	13	14	15	16	17	18	19	20	21	22	23
Segments or Point	x	y	ψ	sin ψ	cos ψ	$x^2 \frac{\Delta s}{EI}$	$y^2 \frac{\Delta s}{EI}$	$\sin^2 \psi \frac{\Delta s}{EA}$	$\cos^2 \psi \frac{\Delta s}{EA}$	$\sin^2 \psi \frac{\Delta s}{GA'}$	$\cos^2 \psi \frac{\Delta s}{GA'}$
	See Note III (2)		6-6			13^2 x1	14^2x1	16^2x2	17^2x2	16^2x3	17^2x3
T											
1											
2											
3											
								Omit these columns when energy due to direct and shear loads is negligible.			
10											
11											
12											
0											
						Σ	Σ	Σ	Σ	Σ	Σ

TABLE B 5.2.0-1 (Cont'd)

	24	25	26	27	28	29	30	31	32	33
Segments or Point	M_o	N_o	S_o	$M_o \frac{\Delta s}{EI}$	$M_o x \frac{\Delta s}{EI}$	$M_o y \frac{\Delta s}{EI}$	$N_o \sin \psi \frac{\Delta s}{EA}$	$N_o \cos \psi \frac{\Delta s}{EA}$	$S_o \sin \psi \frac{\Delta s}{GA'}$	$S_o \cos \psi \frac{\Delta s}{GA'}$
	See Note III (3)			24x1	24x13x1	24x14x1	25x16x2	25x17x2	26x16x3	26x17x3
T										
1										
2										
3										
							Omit these columns when energy due to direct and shear loads is negligible			
10										
11										
12										
0										
				Σ	Σ	Σ	Σ	Σ	Σ	Σ

TABLE B5.2.0-1 (Concluded)

	34	35	36	37	38	39	40	41	42
Segments or Point	Xy	Yx	X sin ψ	X cos ψ	Y sin ψ	Y cos ψ	M	N	S
	Xx14	See Note III (5) Yx13	Xx16	Xx17	Yx16	Yx17	M_c+24 -34+35	25+37 +38	26+36 -39

B5.2.1 Sample Problem

The frame to be analyzed in this sample problem is the symmetrical built-in ring illustrated in Figure 5.2.1-1. The ring is divided into 24 segments for this problem. The section properties at the center of each segment are used as average values for the entire segment. A typical cross section of the ring is shown in Figure 5.2.1-2.

The necessary calculations to determine the magnitudes of M, N, and S at each segment are shown in tabular form on pages 42 through 46. The results have been plotted on graphs, which are shown on pages 47 through 49.

The problem data, statement, and conditions follow.

Given:

$P = 100$ lb	$R_a = 47$ in.
$Q = 100$ lb	$E = 10 \times 10^6$ psi
$M = 1000$ in.-lb	$G = 3.85 \times 10^6$ psi
$R_o = 50$ in.	$A = h \times 1$
	$A' = 5/6\ A$ (See Ref. 7)

Problem:

Determine the bending moment M, direct load N, and shear load S for the given conditions by use of Sec. B5.2.0.

B5.2.1 Sample Problem (Cont'd)

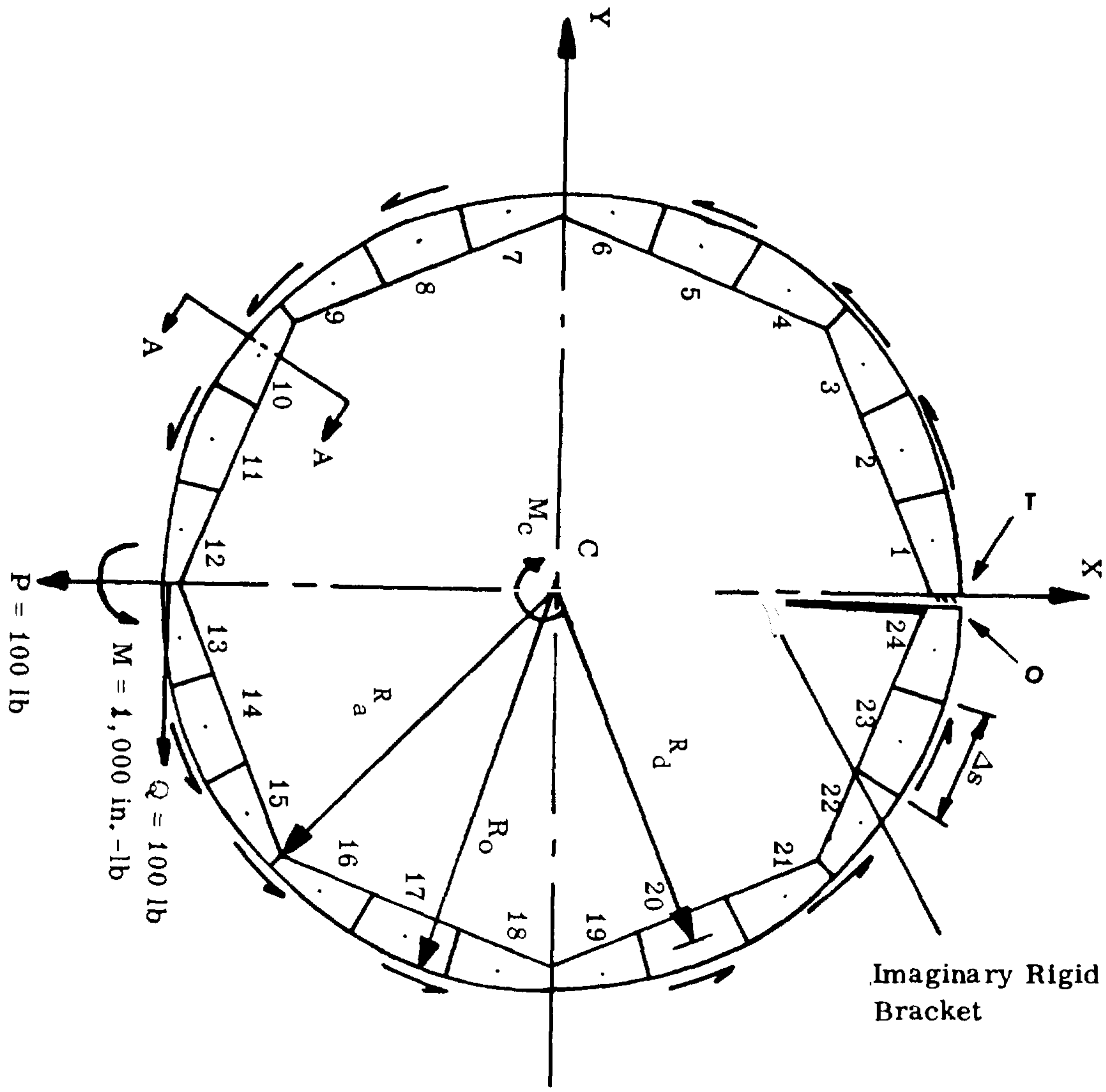


Figure B5.2.1-1 Frame for Sample Problem

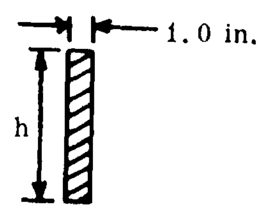


Figure B5.2.1-2 Typical Frame Cross Section

B5.2.1 Sample Problem (Cont'd)

	1	2	3	4	5	6	7	8
Segment or Points	$\frac{\Delta s}{EI}$	$\frac{\Delta s}{EA}$	$\frac{\Delta s}{GA'}$	x'	y'	ψ'	sin $2\psi'$	cos $2\psi'$
	Operations in this row described in terms of column numbers							
T	10^{-6}	10^{-6}	10^{-6}					
1	.11608	.24631	.76774					
2	.05156	.18591	.57946					
3	.11608	.24631	.76774					
4	.11608	.24631	.76774					
5	.05156	.18591	.57946					
6	.11608	.24631	.76774					
7	.11608	.24631	.76774					
8	.05156	.18591	.57946					
9	.11608	.24631	.76774					
10	.11608	.24631	.76774					
11	.05156	.18591	.57946					
12	.11608	.24631	.76774	Columns 4 through 12 not required for a symmetrical built-in ring;				
13	.11608	.24631	.76774					
14	.05156	.18591	.57946					
15	.11608	.24631	.76774					
16	.11608	.24631	.76774					
17	.05156	.18591	.57946					
18	.11608	.24631	.76774					
19	.11608	.24631	.76774					
20	.05156	.18591	.57946					
21	.11608	.24631	.76774					
22	.11608	.24631	.76774					
23	.05156	.18591	.57946					
24	.11608	.24631	.76774					
0								
Σ	2.26979	Sum of Columns						

B5.2.1 Sample Problem (Cont'd)

9	10	11	12	13	14	15	16	17
$x'y'\frac{\Delta s}{EI}$	$\left[x'^2 - y'^2\right]\frac{\Delta s}{EI}$	$\sin 2\psi'\left[\frac{\Delta s}{EA} - \frac{\Delta s}{GA'}\right]$	$\cos 2\psi'\left[\frac{\Delta s}{EA} - \frac{\Delta s}{GA'}\right]$	x	y	ψ	$\sin\psi$	$\cos\psi$
4×5×1	(4^2-5^2) ×1	(2-3) ×7	(2-3) ×8					
				See Note IV(1)				
				47.070	6.197	277.5	-.99144	.13051
				43.155	17.875	292.5	-.92387	.38267
				37.666	28.902	307.5	-.79334	.60875
	Columns 4 through 12 not required for a symmetrical built-in ring.			28.902	37.666	322.5	-.60875	.79334
				17.875	43.155	337.5	-.38268	.92387
				6.197	47.070	352.5	-.13051	.99144
				- 6.197	47.070	7.5	.13051	.99144
				-17.875	43.155	22.5	.38267	.92387
				-28.902	37.666	37.5	.60875	.79334
				-37.666	28.902	52.5	.79334	.60875
				-43.155	17.875	67.5	.92387	.38267
				-47.070	6.197	82.5	.99144	.13051
				-47.070	-6.197	97.5	.99144	-.13051
				-43.155	-17.875	112.5	.92387	-.38267
				-37.666	-28.902	127.5	.79334	-.60875
				-28.902	-37.666	142.5	.60875	-.79334
				-17.875	-43.155	157.5	.38267	-.92387
				- 6.197	-47.070	172.5	.13051	-.99144
				6.197	-47.070	187.5	-.13051	-.99144
				17.875	-43.155	202.5	-.38268	-.92387
				28.902	-37.666	217.5	-.60875	-.79334
				37.666	-28.902	232.5	-.79334	-.60875
				43.155	-17.875	247.5	-.92387	-.38267
				47.070	- 6.197	262.5	-.99144	-.13051

B5. 2. 1 Sample Problem (Cont'd)

18	19	20	21	22	23	24	25	26
$x^2 \frac{\Delta s}{EI}$	$y^2 \frac{\Delta s}{EI}$	$\sin^2 \psi \frac{\Delta s}{EA}$	$\cos^2 \psi \frac{\Delta s}{EA}$	$\sin^2 \psi \frac{\Delta s}{GA'}$	$\cos^2 \psi \frac{\Delta s}{GA'}$	M_o	N_o	S_o
$13^2 \times 1$	$14^2 \times 1$	$16^2 \times 2$	$17^2 \times 2$	$16^2 \times 3$	$17^2 \times 3$	See Note III(3) *		
10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}			
257.19	4.46	.24212	.00420	.75466	.01308	-11.7	.246	.320
96.02	16.47	.15868	.02722	.49459	.08485	3.9	2.356	.962
164.69	96.96	.15503	.09128	.48322	.28451	16.2	1.661	1.441
96.96	164.69	.09128	.15503	.28451	.48322	22.7	-1.996	.957
16.47	96.02	.02722	.15868	.08485	.49459	-1.7	-8.366	-1.251
4.46	257.19	.00420	.24212	.01308	.75466	-63.0	-16.810	-5.799
4.46	257.19	.00420	.24212	.01308	.75466	-211.2	-26.356	-13.065
16.47	96.02	.02722	.15868	.08485	.49459	-500.6	-35.790	-23.116
96.96	164.69	.09128	.15503	.28451	.48322	-880.3	-43.784	-35.664
164.69	96.96	.15503	.09128	.48322	.28451	-1457.8	-49.024	-50.058
96.02	16.47	.15868	.02722	.49459	.08485	-2259.2	-50.361	-65.318
257.19	4.46	.24212	.00420	.75466	.01308	-3169.8	-46.935	-80.198
257.19	4.46	.24212	.00420	.75466	.01308	-2527.4	47.798	18.909
96.02	16.47	.15868	.02722	.49459	.08485	-2244.8	29.656	27.523
164.69	96.96	.15503	.09128	.48322	.28451	-1920.2	12.470	31.835
96.96	164.69	.09128	.15503	.28451	.48322	-1535.4	-2.385	32.319
16.47	96.02	.02722	.15868	.08485	.49459	-1166.6	-13.896	29.736
4.46	257.19	.00420	.24212	.01308	.75466	-814.6	-21.484	25.028
4.46	257.19	.00420	.24212	.01308	.75466	-532.4	-25.028	19.205
16.47	96.02	.02722	.15868	.08485	.49459	-337.5	-24.849	13.224
96.96	164.69	.09128	.15503	.28451	.48322	-171.4	-21.647	7.890
164.69	96.96	.15503	.09128	.48322	.28451	-81.4	-16.406	3.772
96.02	16.47	.15868	.02722	.49459	.08485	-41.4	-10.278	1.151
257.19	4.46	.24212	.00420	.75466	.01308	-11.2	-4.448	0.000
2543.16	2543.16	2.7141	2.7141	8.45964	8.45964			

*Calculations on pages 50 through 53

B5.2.1 Sample Problem (Cont'd)

27	28	29	30	31	32	33	34	35
$M_o \frac{S}{EI}$	$M_o x \frac{S}{EI}$	$M_o y \frac{S}{EI}$	$N_o \sin\psi \frac{S}{EA}$	$N_o \cos\psi \frac{S}{EA}$	$S_o \sin\psi \frac{S}{GA'}$	$S_o \cos\psi \frac{S}{GA'}$	Xy	Yx
							See Note III(5)	
24x1	24x13x1	24x14x1	25x16x2	25x17x2	26x16x3	26x17x3	Xx14	Yx13
10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}		
-1.35	-63.7	-8.4	-.06000	.00791	-.24357	.03206	32.403	-1193.318
.20	8.7	3.6	-.40466	.16761	-.51500	.21332	93.465	-1094.065
1.88	70.7	54.2	-.32457	.24905	-.87768	.67347	151.123	-954.908
2.64	76.2	99.4	.29928	-.39003	-.44727	.58289	196.948	-732.723
-.09	-1.6	-3.7	.59519	-1.4369	.27741	-.66972	225.649	-453.167
-7.31	-45.3	-344.1	.54037	-4.1050	.58105	-4.4140	246.120	-157.106
-24.52	152.0	-1154.2	-.84724	-6.4362	-1.3091	-9.9447	246.120	157.106
-25.81	461.4	-1113.9	-2.5462	-6.1472	-5.1259	-12.375	225.649	453.167
-102.19	2953.4	-3849.0	-6.5650	-8.5568	-16.668	-21.722	196.948	732.723
-169.22	6374.0	-4890.0	-9.5797	-7.3507	-30.489	-23.395	151.123	954.908
-116.48	5026.7	-2082.1	-8.6498	-3.5828	-34.968	-14.484	93.465	1094.065
-367.97	17320.4	-2280.2	-11.462	-1.5088	-61.044	-8.036	32.403	1193.318
-293.39	13810.1	1818.2	11.672	-1.5365	14.393	-1.895	-32.403	1193.318
-115.74	4994.7	2068.9	5.094	-2.1098	14.734	-6.103	-93.465	1094.065
-222.90	8395.8	6442.3	2.437	1.8698	19.390	-14.878	-151.123	954.908
-178.23	5151.2	6713.2	-.3576	.46605	15.105	-19.685	-196.948	732.723
-60.14	1075.1	2595.6	-.9886	2.3867	6.594	-15.919	-255.649	453.167
-94.56	586.0	4450.9	-.6906	5.2464	2.508	-19.051	-246.120	157.106
-61.80	-382.9	2908.8	.8046	6.1119	-1.924	-14.618	-246.120	-157.106
-17.40	-311.0	750.9	1.7679	4.2679	-2.932	-7.079	-225.649	-453.167
-19.89	-575.0	749.3	3.2458	4.2300	-3.688	-4.806	-196.948	-732.723
-9.45	-355.8	273.0	3.2059	2.4599	-2.267	-1.763	-151.123	-954.908
-2.13	-92.1	38.1	1.7654	.7312	-.6162	-.25523	-93.465	-1094.07
-1.30	-61.3	8.1	1.0862	.143	0.0000	0.00000	-32.403	-1193.32
-1887.15	64567.7	13248.0	-9.9626	-18.563	-89.533	-199.59		

B5.2.1 Sample Problem (Cont'd)

36	37	38	39	40	41	42
X sin ψ	X cos ψ	Y sin ψ	Y cos ψ	M	N	S
See Note III (5)				M_c + 24 − 34 + 35	25 + 37 + 38	26 + 36 − 39
X × 16	X × 17	Y × 16	Y × 17			
-5.184	.682	25.135	-3.309	-406.0	26.063	-1.555
-4.831	2.001	23.422	-9.702	-352.3	27.779	5.833
-4.148	3.183	20.113	-15.433	-258.4	24.957	12.726
-3.183	4.148	15.433	-20.113	-75.5	17.585	17.887
-2.001	4.831	9.702	-23.422	150.9	6.167	20.170
-.682	5.184	3.309	-25.135	365.2	-8.317	18.654
.682	5.184	-3.309	-25.135	531.2	-24.481	12.752
2.001	4.831	-9.702	-23.422	558.4	-40.661	2.307
3.183	4.148	-15.433	-20.113	486.9	-55.069	-12.368
4.148	3.183	-20.113	-15.433	177.4	-65.954	-30.477
4.831	2.001	-23.422	-9.702	-427.2	-71.782	-50.785
5.184	.682	-25.135	-3.309	-1177.5	-71.388	-71.705
5.184	-.682	-25.135	3.309	-470.3	21.981	20.784
4.831	-2.001	-23.422	9.702	-225.8	4.233	22.652
4.148	-3.183	-20.113	15.433	17.2	-10.826	20.550
3.183	-4.148	-15.433	20.113	225.6	-21.966	15.389
2.001	-4.831	-9.702	23.422	343.6	-28.429	8.315
.682	-5.184	-3.309	25.135	420.0	-29.997	.575
-.682	-5.184	3.309	25.135	388.0	-26.903	-6.612
-2.001	-4.831	9.702	23.422	266.3	-19.978	-12.199
-3.183	-4.148	15.433	20.113	124.2	-10.362	-15.406
-4.148	-3.183	20.113	15.433	-53.8	.524	-15.809
-4.831	-2.001	23.422	9.702	-210.6	11.143	-13.382
-5.184	-.682	25.135	3.309	-340.7	20.005	-8.493

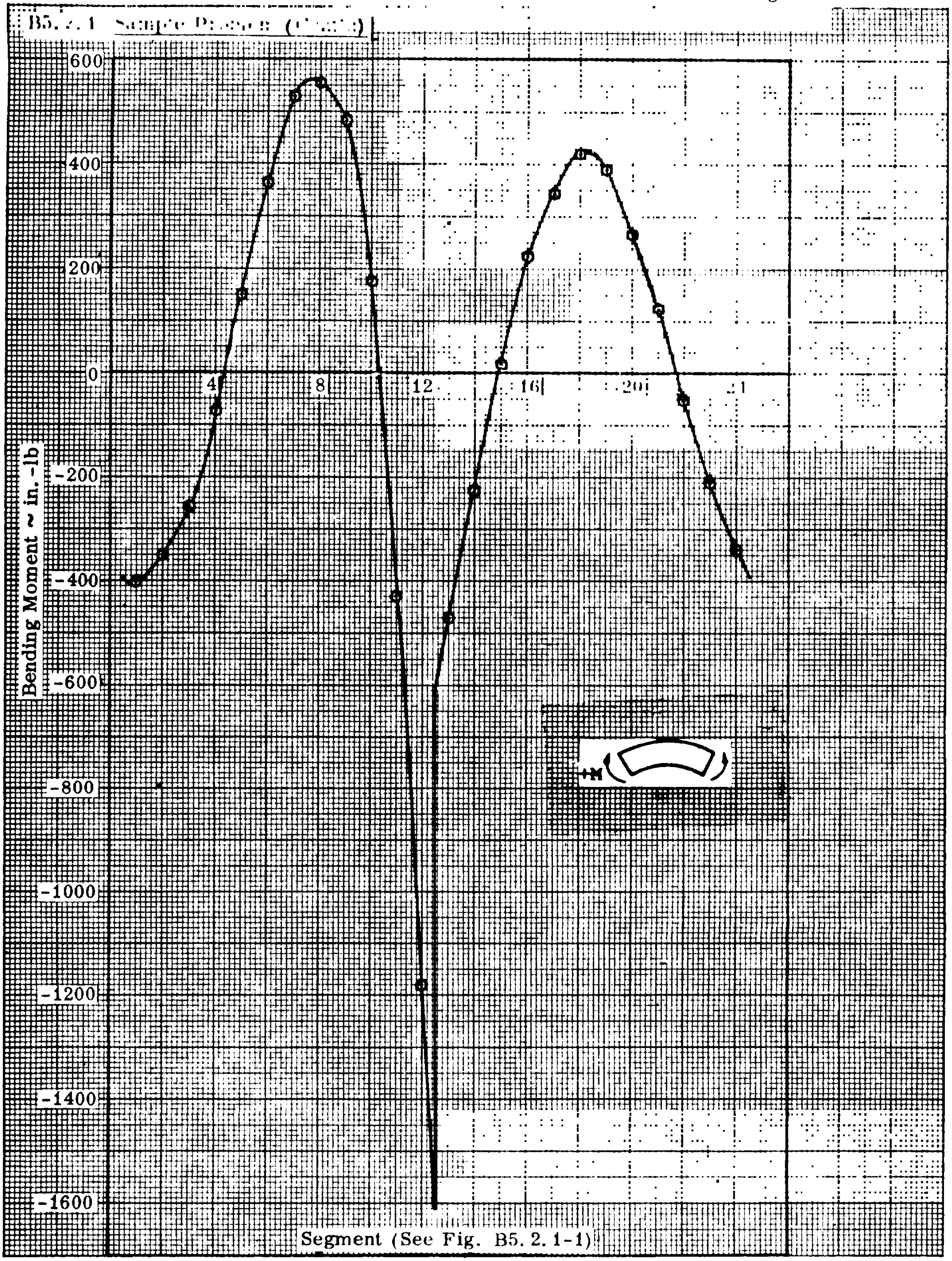


Figure B5.2.1-3 Bending Moment Diagram of Sample Problem

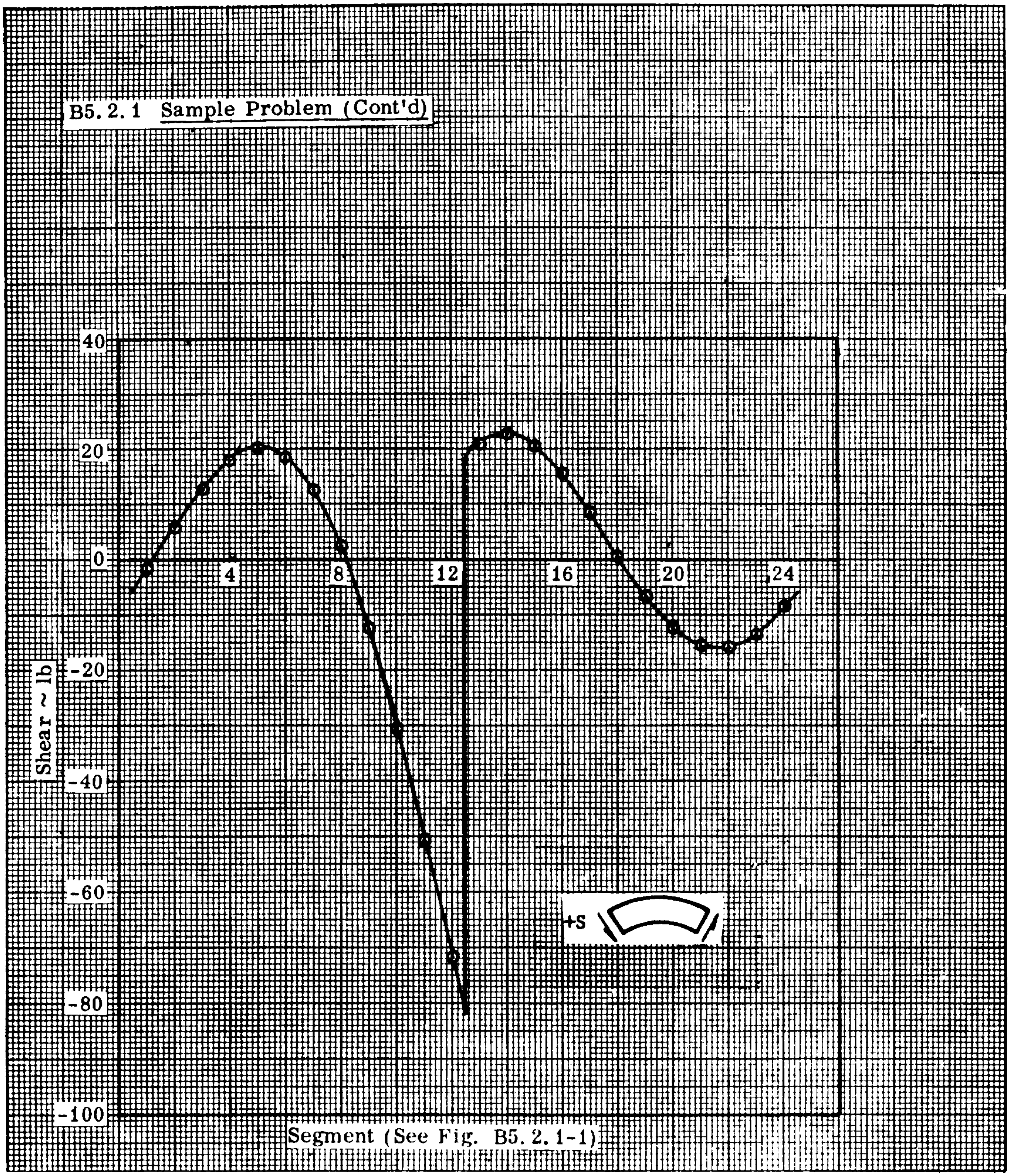


Figure B5.2.1-4 Shear Diagram of Sample Problem

B5.2.1 Sample Problem (Cont'd)

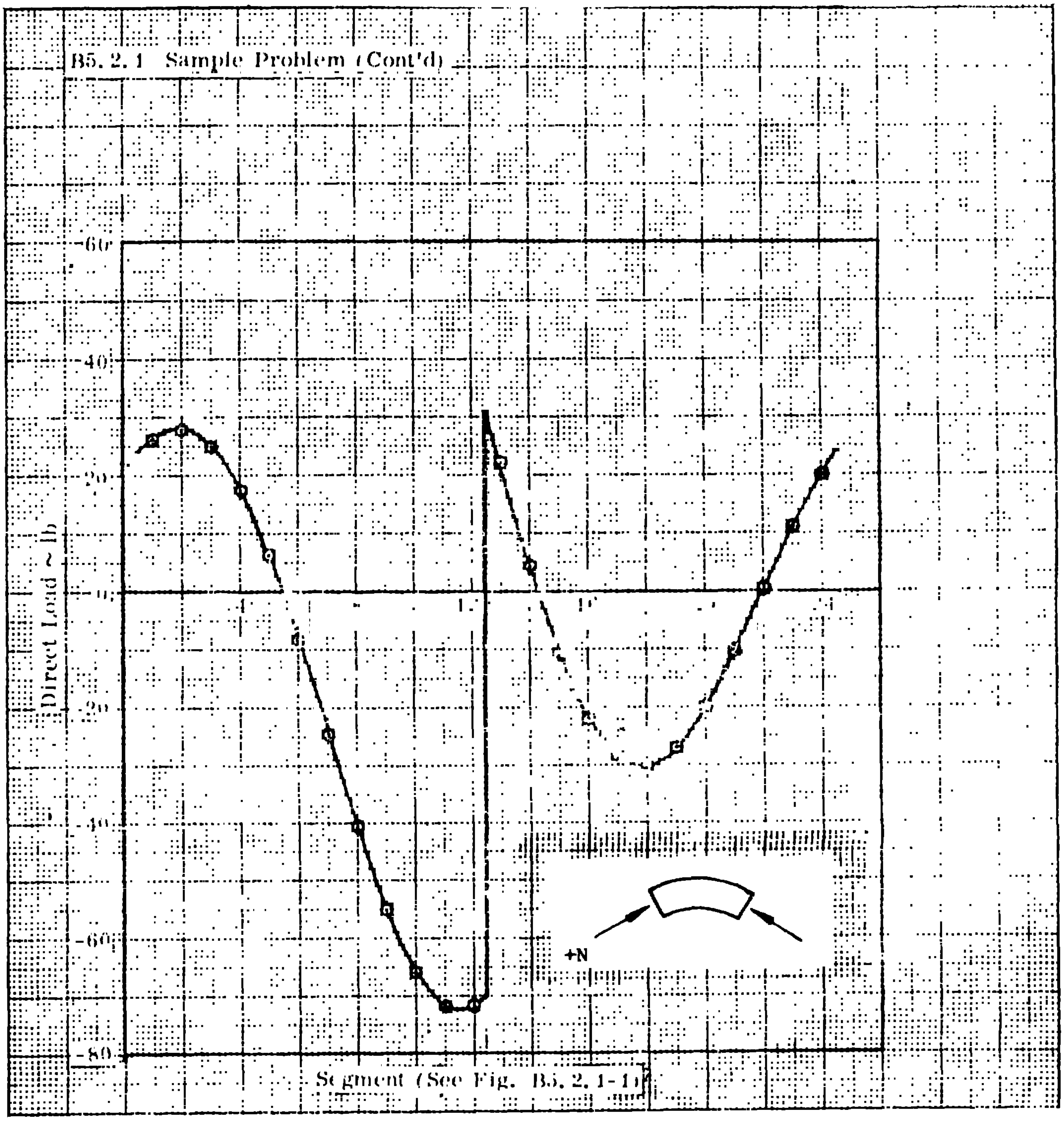


Figure B5.2.1-5 Direct Load Diagram of Sample Problem

B5.2.1 Sample Problem (Cont'd)

The values for M_o, N_o, and S_o are calculated and shown in the tables on pages 52 and 53. The necessary formulas for computing the shear flows are shown below. In these equations, counterclockwise shear flow is considered positive.

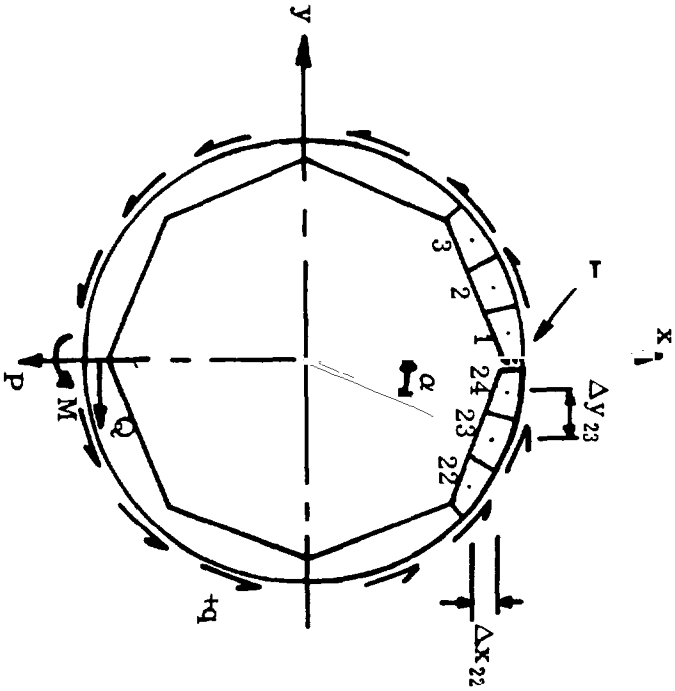


Figure B5.2.1-6 Symbols and Sign Conventions for Computation of M_o, N_o and S_o

$$q_{P_\alpha} = \frac{P}{\pi R_o} \sin \alpha \qquad q_{M\alpha} = \frac{-M}{2\pi R_o^2}$$

$$q_{Q\alpha} = \frac{Q}{\pi R_o} \sin(\alpha + 90°) - \frac{48.5\,Q}{2\pi R_o^2} \qquad q_\alpha = q_{P_\alpha} + q_{Q\alpha} + q_{M\alpha}$$

In order to perform the calculations for M_o, N_o, and S_o, the shear force acting on each segment must be known. This shear force is obtained by multiplying the average shear flow acting on the segment by the length over which it acts.

B5.2.1 Sample Problem (Concluded)

This average shear flow is determined by using Simpson's Rule. For the typical segment in Figure B5.2.1-7, this average shear flow is:

$$q_{avg.} = \frac{1}{6} (q'_\alpha + 4q''_\alpha + q'''_\alpha)$$

where:

q'_α = Shear flow at left end of the segment.

q''_α = Shear flow at center of the segment.

q'''_α = Shear flow at right end of the segment.

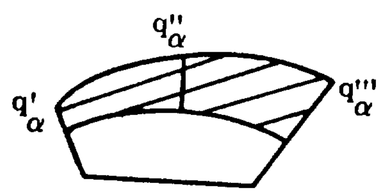


Figure B5.2.1-7 Shear Flow Acting on a Typical Segment

The formulas used for computing the values in the tables on pages 52 and 53 are listed under the tables. The index (n) refers to the column number and its range is indicated where necessary. The formulas for S_o, N_o, and M_o are applicable for all values of n, therefore, the index was omitted.

B5.2.1 Sample Problem (Cont'd)

Seg.	α	D = R_d	h	Δx	Δy	$q_{avg.}$	F	H
	deg.							
24	7.5	47.477	5.046	.000	.000	.340	4.448	4.410
23	22.5	46.711	6.578	3.915	11.678	.457	5.982	9.937
22	37.5	47.477	5.046	5.489	11.027	.518	6.776	15.312
21	52.5	47.477	5.046	8.764	8.764	.518	6.776	19.437
20	67.5	46.711	6.578	11.027	5.489	.457	5.982	21.727
19	82.5	47.477	5.046	11.678	3.915	.340	4.448	22.307
18	97.5	47.477	5.046	12.394	.000	.174	2.279	22.010
17	112.5	46.711	6.578	11.678	-3.915	-.029	-.378	22.154
16	127.5	47.477	5.046	11.027	-5.489	-.255	-3.341	24.188
15	142.5	47.477	5.046	8.764	-8.764	-.490	-6.409	29.273
14	157.5	46.711	6.578	5.489	-11.027	-.716	-9.372	37.931
13	172.5	47.477	5.046	3.915	-11.678	-.919	-12.029	49.857
12	187.5	47.477	5.046	.000	-12.394	-1.085	-14.198	63.934
11	202.5	46.711	6.578	-3.915	-11.678	-1.202	-15.732	78.468
10	217.5	47.477	5.046	-5.489	-11.027	-1.262	-16.526	91.579
9	232.5	47.477	5.046	-8.764	-8.764	-1.262	-16.526	101.640
8	247.5	46.711	6.578	-11.027	-5.489	-1.202	-15.732	107.660
7	262.5	47.477	5.046	-11.678	-3.915	-1.085	-14.198	109.513
6	277.5	47.477	5.046	-12.394	.000	-.919	-12.029	107.943
5	292.5	46.711	6.578	-11.678	3.915	-.716	-9.372	104.357
4	307.5	47.477	5.046	-11.027	5.489	-.490	-6.409	100.455
3	322.5	47.477	5.046	-8.764	8.764	-.255	-3.341	97.805
2	337.5	46.711	6.578	-5.489	11.027	-.029	-.378	97.456
1	352.5	47.477	5.046	-3.915	11.678	.174	2.279	99.715

$1 \le n \le 23$

$$\Delta x_n = D_{n+1} \cos \alpha_{n+1} - D_n \cos \alpha_n$$

$$\Delta y_n = D_n \sin \alpha_n - D_{n+1} \sin \alpha_{n+1}$$

$$F_n = (q_{avg_n})(\Delta s_n)$$

$1 \le n \le 23$

$$H_n = H_{n+1} + F_n \cos \alpha_n$$

$$V_n = V_{n+1} + F_n \sin \alpha_n$$

$$M_n = H_{n+1} \Delta x_n - V_{n+1} \Delta y_n + M_{n+1} + F_n \frac{h}{2}$$

For n = 24:

$$\Delta x_n = \Delta y_n = 0, \quad F_n = (q_{avg_n})(\Delta s_n), \quad H_n = F_n \cos \alpha_n, \quad V_n = F_n \sin \alpha_n, \quad M_n = F_n \frac{h}{2}$$

B5.2.1 Sample Problem (Cont'd)

Seg.	V	M	H'	V'	M'	S_o	N_o	M_o
24	.581	11.2	4.410	.581	11.2	.000	-4.448	-11.2
23	2.870	41.4	9.937	2.870	41.4	1.151	-10.278	-41.4
22	6.995	81.4	15.312	6.995	81.4	3.772	-16.406	-81.4
21	12.370	171.4	19.437	12.370	171.4	7.890	-21.647	-171.4
20	17.897	337.5	21.727	17.897	337.5	13.224	-24.849	-337.5
19	22.307	532.4	22.307	22.307	532.4	19.205	-25.028	-532.4
18	24.567	814.6	22.010	24.567	814.6	25.028	-21.484	-814.6
17	24.217	1166.6	22.154	24.217	1166.6	29.736	-13.896	-1166.6
16	21.567	1535.4	24.188	21.567	1535.4	32.319	-2.385	-1535.4
15	17.665	1920.2	29.273	17.665	1920.2	31.835	12.470	-1920.2
14	14.079	2244.8	37.931	14.079	2244.8	27.523	29.656	-2244.8
13	12.509	2527.4	49.857	12.508	2527.4	18.909	47.798	-2527.4
12	14.362	2646.6	-36.066	-85.638	3169.8	-80.198	-46.935	-3169.8
11	20.382	2512.3	-21.532	-79.618	2259.2	-65.318	-50.361	-2259.2
10	30.442	2264.6	-8.421	-69.558	1457.8	-50.058	-49.024	-1457.8
9	43.553	1687.1	1.640	-56.447	880.3	-35.664	-43.784	-880.3
8	58.088	753.7	7.660	-41.912	500.6	-23.116	-35.790	-500.6
7	72.164	-312.0	9.513	-27.836	211.2	-13.065	-26.356	-211.2
6	84.090	-1699.6	7.943	-15.910	63.0	-5.799	-16.810	-63.0
5	92.749	-3320.3	4.357	-7.251	1.7	-1.251	-8.366	-1.7
4	97.834	-4996.3	.455	-2.166	-22.7	.957	-1.996	22.7
3	99.868	-6742.5	-2.195	-.132	-16.2	1.441	1.661	16.2
2	100.012	-8381.9	-2.544	.012	-3.9	.962	2.356	3.9
1	99.715	-9925.7	-.285	-.285	11.7	.320	.246	-11.7

$13 \leq n \leq 24$

$H'_n = H_n$

$V'_n = V_n$

$M'_n = M_n$

$1 \leq n \leq 12$

$H'_n = H_n - Q$

$V'_n = V_n - P$

$M'_n = M_n - P[D_n \sin(\alpha_n - 180°)] + Q[48.5 - D_n \cos(\alpha_n - 180°)] + M$

$S_o = -V' \cos\alpha + H' \sin\alpha, \quad N_o = -V' \sin\alpha - H' \cos\alpha, \quad M_o = -M'$

B5.0.0 - FRAMES

References

1. Timoshenko, S., Theory of Structures, McGraw-Hill Book Company, Inc., New York, 1945.

2. Sutherland, H. and Bowman, H. L., Structural Theory, Fourth Edition, John Wiley & Sons, Inc., New York, 1954.

3. Wilbur, J. B. and Norris, C. H., Elementary Structural Analysis, First Edition, McGraw-Hill Book Co., Inc., New York, 1948.

4. Grinter, L. E., Theory of Modern Steel Structures, Vol. II, The Macmillan Co., New York, 1949.

5. Perry, D. J., Aircraft Structures, McGraw-Hill Book Co., Inc., New York, 1950.

6. Argyris, J. H., Dunne, P. C., Tye, W., et al., Structural Principles and Data, Fourth Edition, The New Era Publishing Co., Ltd., London, No date.

7. Roarke, R. J., Formulas for Stress and Strain, p 1120, Third Edition, McGraw-Hill Book Co., Inc., New York, 1954.

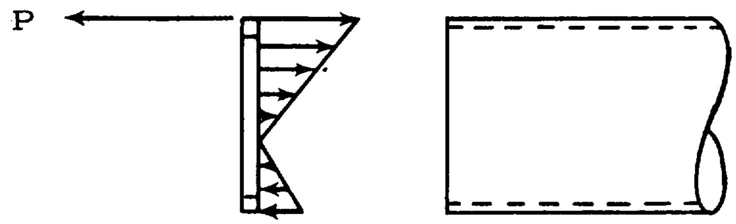
SECTION B6

RINGS

TABLE OF CONTENTS

B 6.0.0 Rings

Charts and tables are presented to facilitate the analysis of rings and ring-supported shells. Section B 6.1.0 deals with rings that are rigid with respect to the resisting medium, i.e., for an out-of-plane loading, the free body of a ring supported by a thin shell is as follows:



Only bending is considered in the deflection curves for the in-plane load cases given on pages 9 through 54. Refer to page 56.1 to include the effects of shear and normal forces.

Section B 6.2.0 deals with circular cylindrical shells supported by "flexible" rings. The choice between the use of this section over section B 6.1.0 for any given problem is left to the analyst. Experience should be gained on both methods.

B 6.1.0 Rigid Rings

In general, four basic loadings are required to define all loads on a ring loaded in the plane of the ring. They are:

1. Loading by a single radial force
2. Loading by a single tangential force
3. Loading by a single moment
4. Loading by a distributed load.

Special cases for out-of-plane loadings are considered in Section B 6.1.2.

The procedure for calculating the Bending Moments of the basic in-plane loading is briefly reviewed in the following discussion. Many other loading conditions may be analyzed by using these basic cases by applying the principal of superposition.

The general equation for the transverse force (shearing force on a cross-section) is

$$S = \frac{dm}{ds} = \frac{dm}{r\,d\phi} \qquad \text{.................................} \qquad (1)$$

The general equation for the axial force is derived as follows:

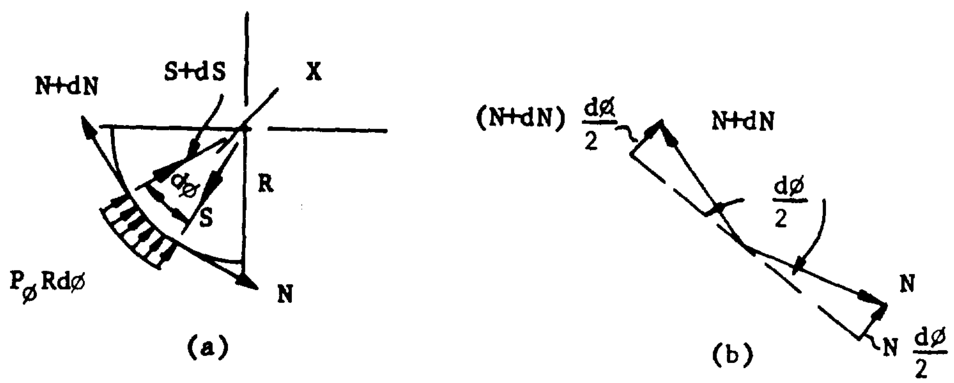


Fig. B 6.1.0-1

B 6.1.0 Rigid Rings (Cont'd)

$$\Sigma F_x = 0 = -S + (S + dS) + (N + dN)\frac{d\phi}{2} + N\frac{d\phi}{2} + P_\phi R d\phi$$

$$= dS + N d\phi + dN\frac{d\phi}{2} + P_\phi R d\phi$$

Neglecting the second order term, $\left(dN\frac{d\phi}{2}\right)$

$$N = -\frac{dS}{d\phi} - P_\phi R \quad \ldots\ldots \quad (2)$$

The second term ($P_\phi R$) occurs in the case of a distributed load.

The procedure to obtain an expression for the bending moment is illustrated by the following specific case:

Load

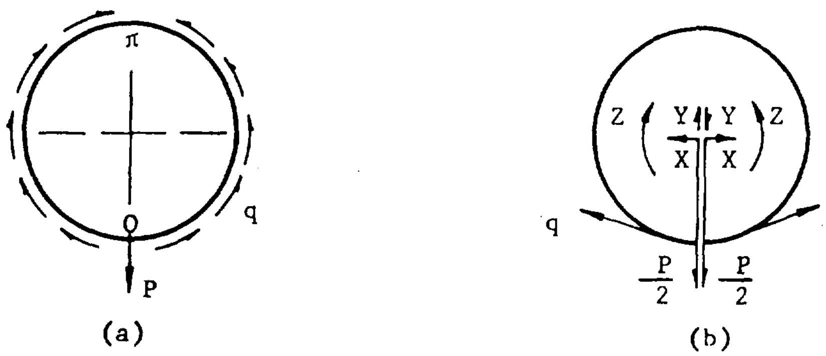


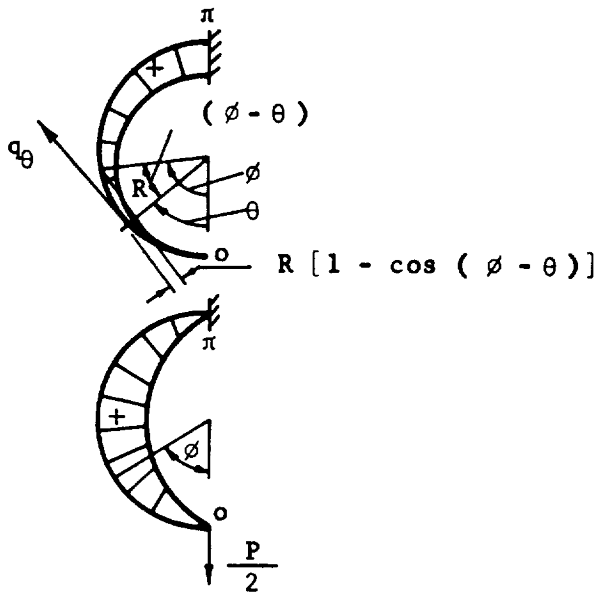
Fig. B 6.1.0-2

Because of symmetry Y = 0

The shear flow distribution is obtained from q = SQ/I, where Q is the static moment of half the ring, S is the shearing force, and I is the moment of inertia of the section.

$$q_\phi = \frac{S}{\pi R^3}\int_0^\phi R d\phi \; R\cos\phi = \frac{S\sin\phi}{\pi R} = \frac{P\sin\phi}{\pi R}$$

B 6.1.0 Rigid Rings (Cont'd)



$$dM_q = q_\phi \, R d\theta \;\; R \left[1 - \cos(\phi - \theta)\right]$$

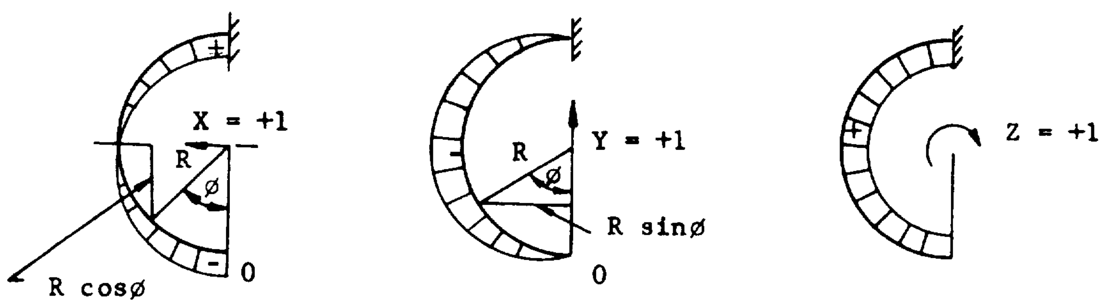
$$= \frac{P}{\pi R} \sin\theta \;\; R^2 d\theta \left[1 - \cos(\phi - \theta)\right]$$

$$M_{q_\theta} = \frac{PR}{\pi} \int_0^{\phi} \sin\theta \left[1 - \cos(\phi - \theta)\right] d\theta$$

$$= \frac{PR}{\pi} \left(1 - \cos\phi - \frac{\phi \sin \phi}{2}\right)$$

$$M_{p_\phi} = \frac{P}{2} R \sin\phi$$

Fig. B 6.1.0-3



$M_x = -R \cos\phi$ $\quad$ $M_y = -R \sin\phi$ $\quad$ $M_z = +1$

Fig.B 6.1.0-4

B 6.1.0 Rigid Rings (Cont'd)

The displacement "i" due to load system "k" = $\delta_{ik} = \int \frac{M_i M_k ds}{EI}$

$$EI \quad \delta_{xx} = \int_0^{\pi} (-R \cos \phi)^2 \, Rd\phi = \frac{\pi R^3}{2}$$

$$EI \quad \delta_{yy} = \int_0^{\pi} (-R \sin \phi)^2 \, Rd\phi = \frac{\pi R^3}{2}$$

$$EI \quad \delta_{zz} = \int_0^{\pi} (+1)^2 \, Rd\phi = \pi R$$

Deflections due to unit loads shown in Fig. B 6.1.0-4

Displacements due to applied forces and reactions

$$EI \quad \delta_{xo} = \int (M_p + M_q) M_x ds \qquad \text{Because of symmetry } M_p M_x = 0$$

$$= \int_0^{\pi} \left(\frac{1}{\pi} - \frac{\cos\phi}{\pi} - \frac{\phi \sin \phi}{2\pi} \right) PR \, (-R \cos\phi) \, Rd\phi = \frac{3PR^3}{8}$$

$$EI \quad \delta_{zo} = \int (M_p + M_q) M_z d\phi$$

$$= \int_0^{\pi} PR \left(\frac{\sin\phi}{2} + \frac{1}{\pi} - \frac{\cos\phi}{\pi} - \frac{\phi \sin\phi}{2\pi} \right) (+1) \, Rd\phi = \frac{3PR^2}{2}$$

$$X = - \frac{\delta_{xo}}{\delta_{xx}} = - \frac{3P}{4\pi}$$

$$Z = - \frac{\delta_{zo}}{\delta_{zz}} = - \frac{3PR}{2\pi}$$

Redundant obtained by equating deflections.

By superposition

$$M = M_p + M_q + XM_x + ZM_z$$

$$= \frac{PR \sin \phi}{2} + \left(\frac{1}{\pi} - \frac{\cos \phi}{\pi} - \frac{\phi \sin \phi}{2\pi} \right) PR + \left(- \frac{3P}{4\pi} \right) \left(-R \cos \phi \right)$$

$$+ \left(- \frac{3PR}{2\pi} \right) (+1) = \left[(\pi - \phi) \sin \phi - \frac{\cos \phi}{2} - 1 \right] \frac{PR}{2\pi}$$

$$S = \frac{dM}{Rd\phi} = \left[- \frac{\sin \phi}{2} + (\pi - \phi) \cos \phi \right] \frac{P}{2\pi}$$

$$N = - \frac{dS}{d\phi} = \left[(\pi - \phi) \sin\phi + \frac{3 \cos \phi}{2} \right] \frac{P}{2\pi}$$

B 6.1.0 Rigid Rings (Cont'd)

Sign Convention

Moments which produce tension on the inner fibers are positive.

Transverse forces which act upward to the left of the cut are positive.

Axial forces which produce tension in the frame are positive.

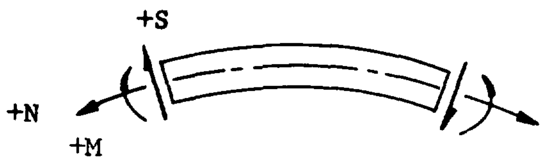


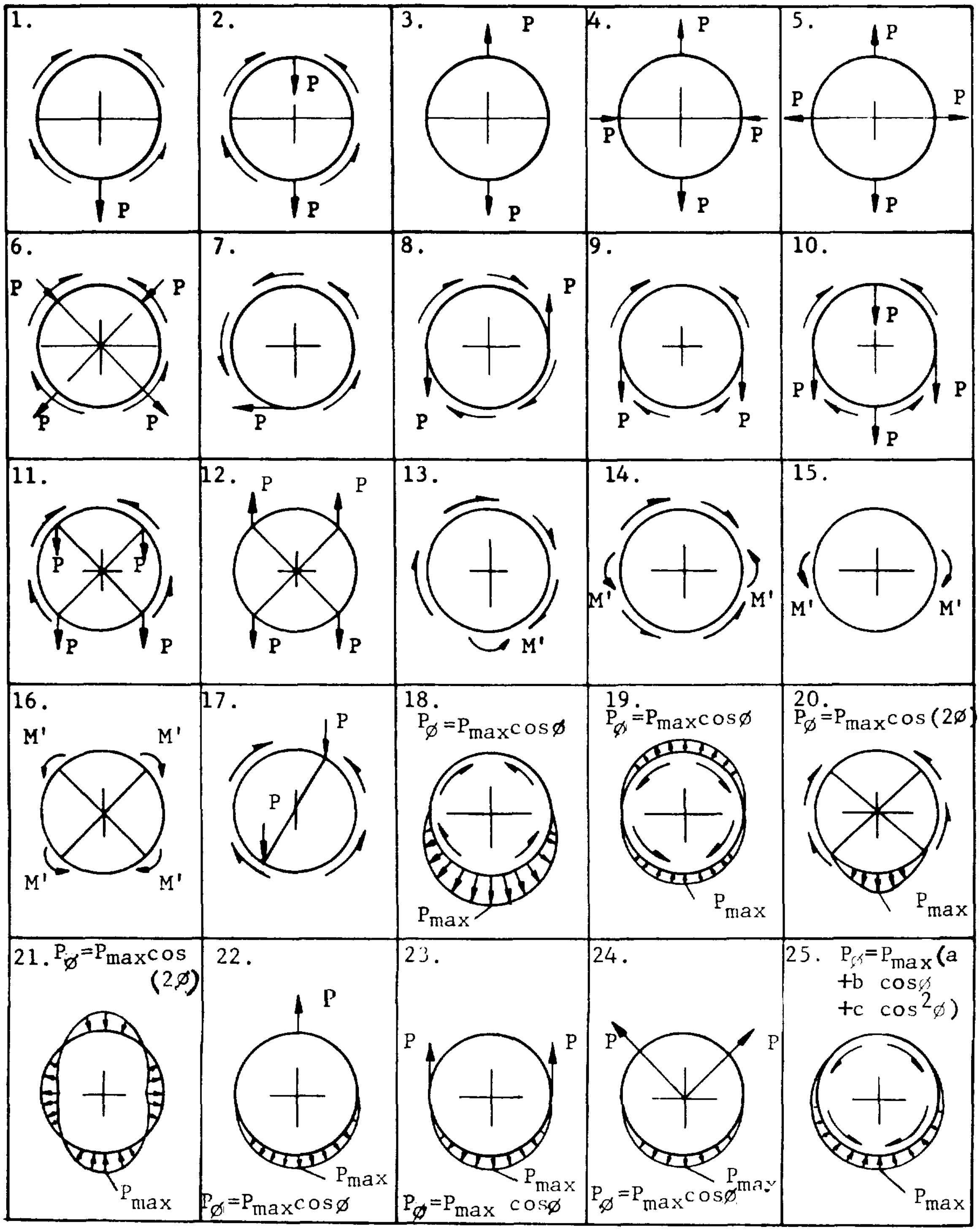
Fig. B 6.1.0-5 Positive Sign Convention

B 6.1.1 <u>In-Plane Load Cases</u>

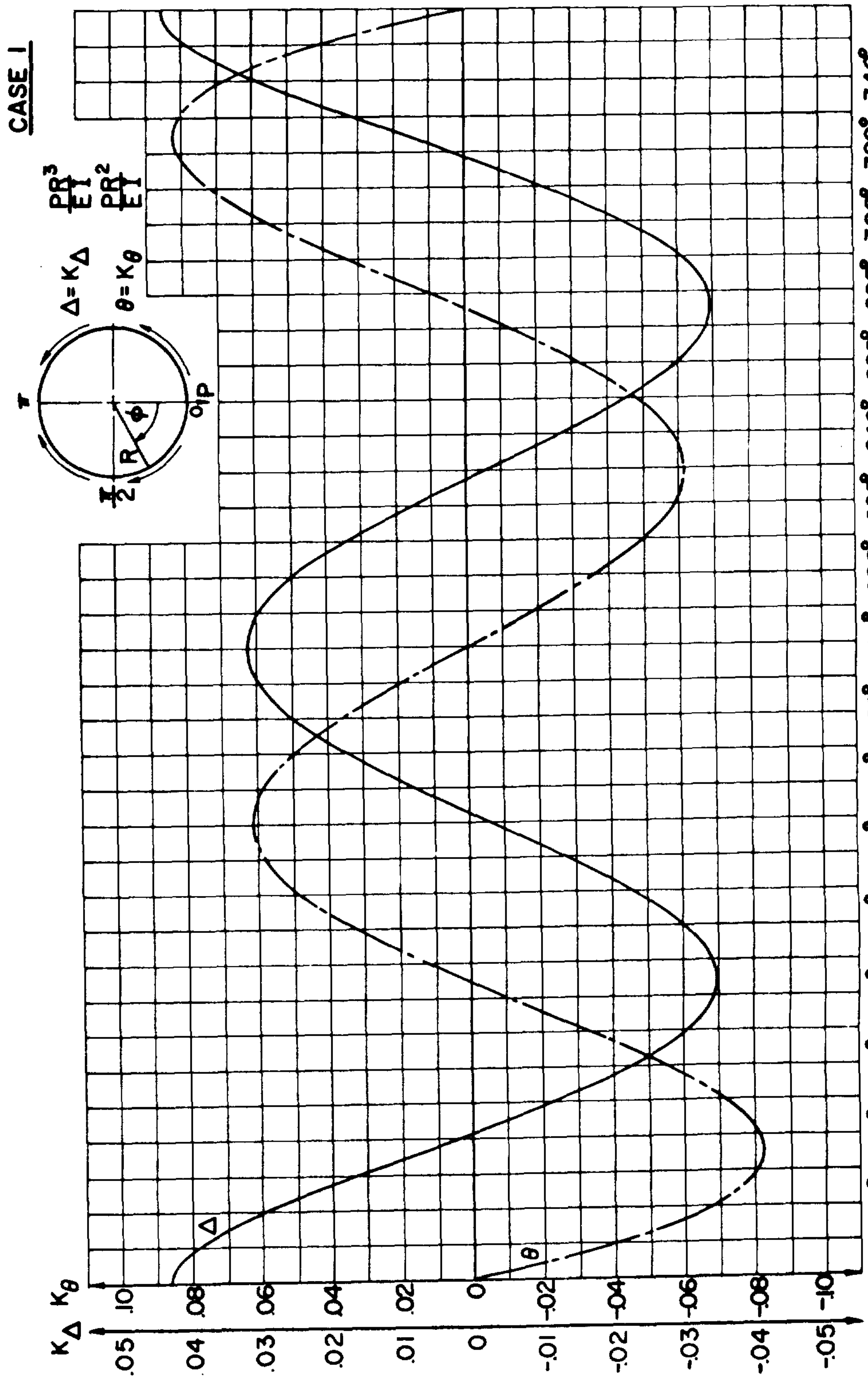
Coefficients to obtain slope, deflection, bending moment, shear, and axial force along with equations for these values are given for some of the frequently occuring load cases.

B 6.1.1 In-Plane Load Cases (Cont'd)

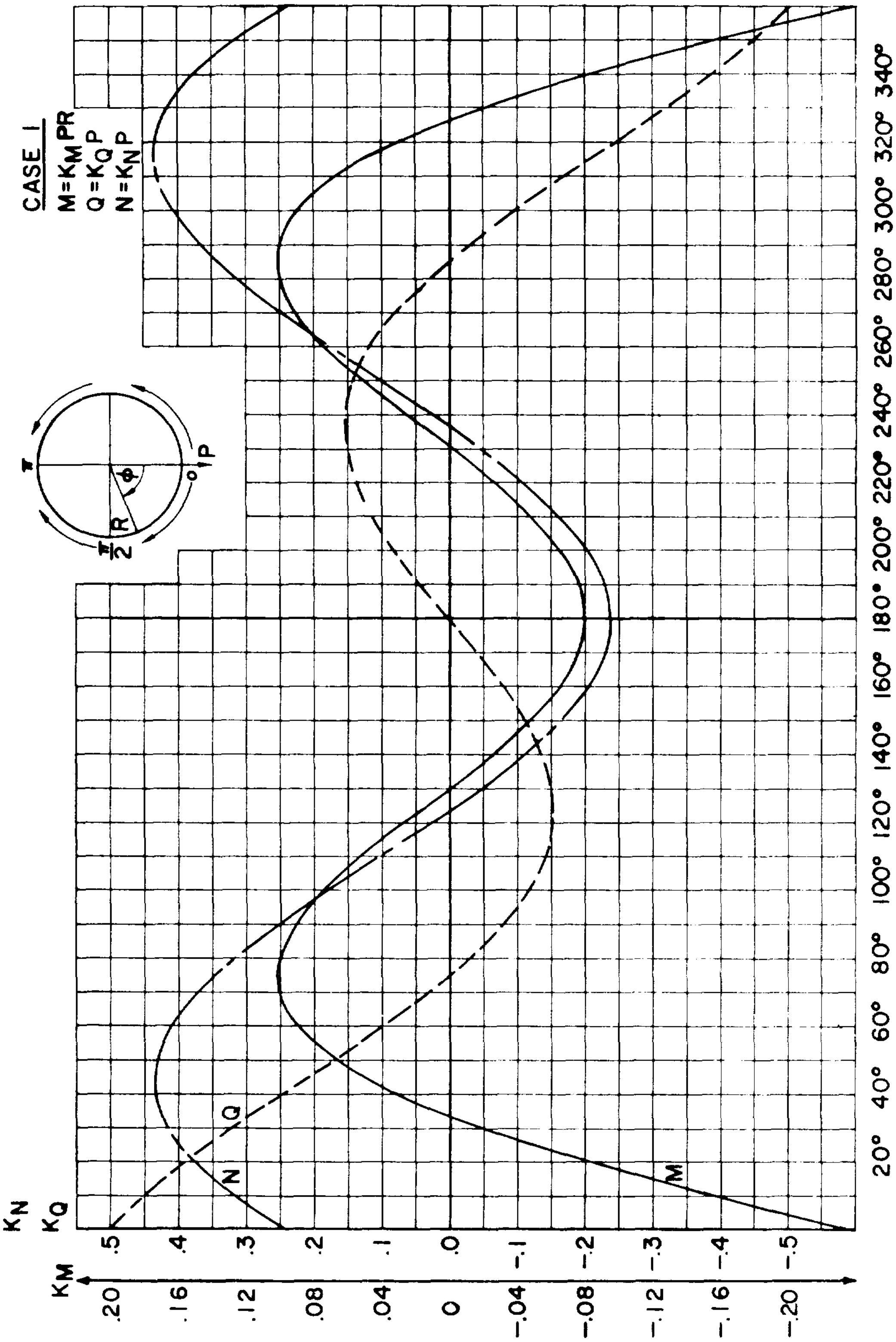
Index



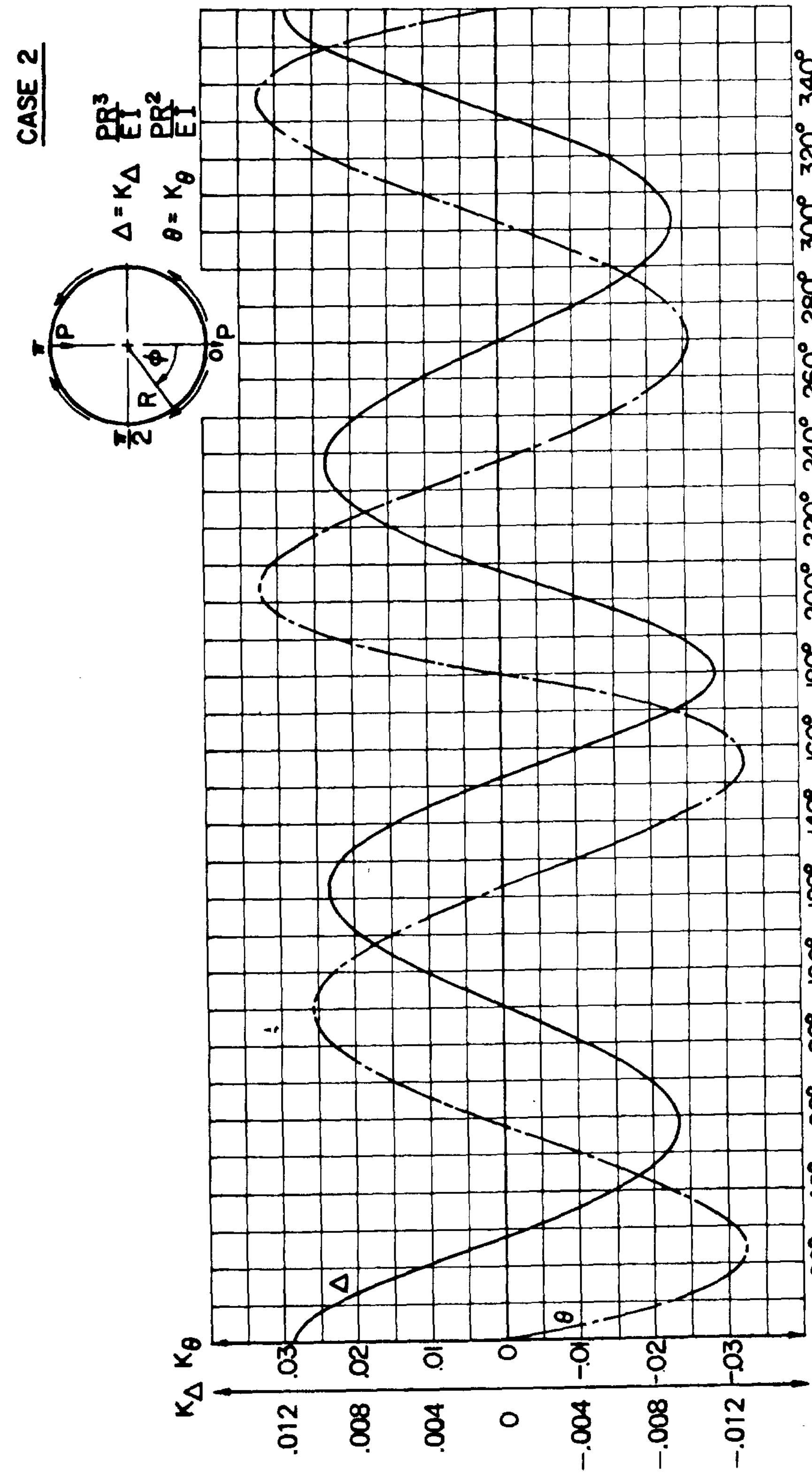
B 6.1.1 In-Plane Load Cases (Cont'd)



B 6.1.1 In-Plane Load Cases (Cont'd)



B 6.1.1 In-Plane Load Cases (Cont'd)



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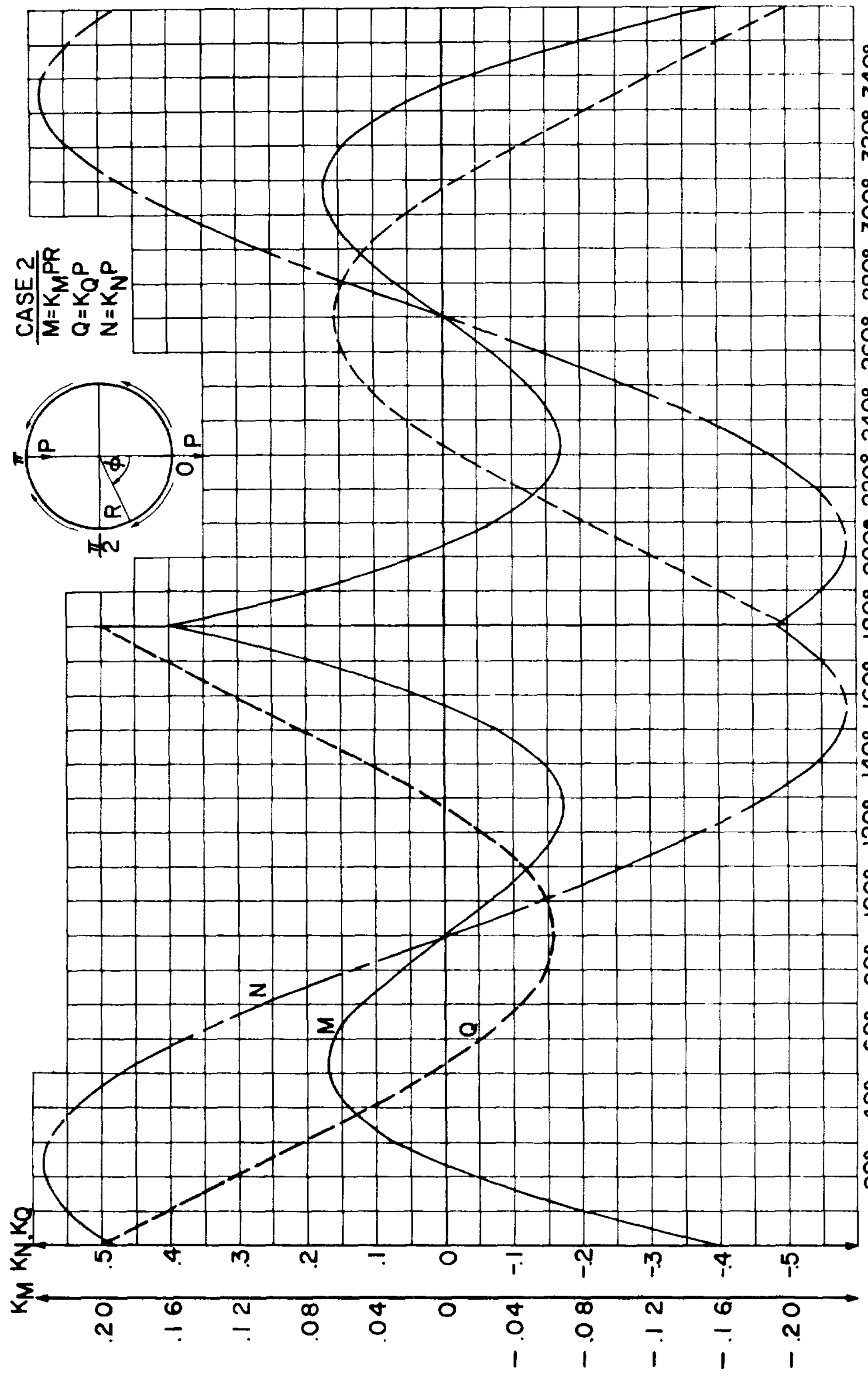

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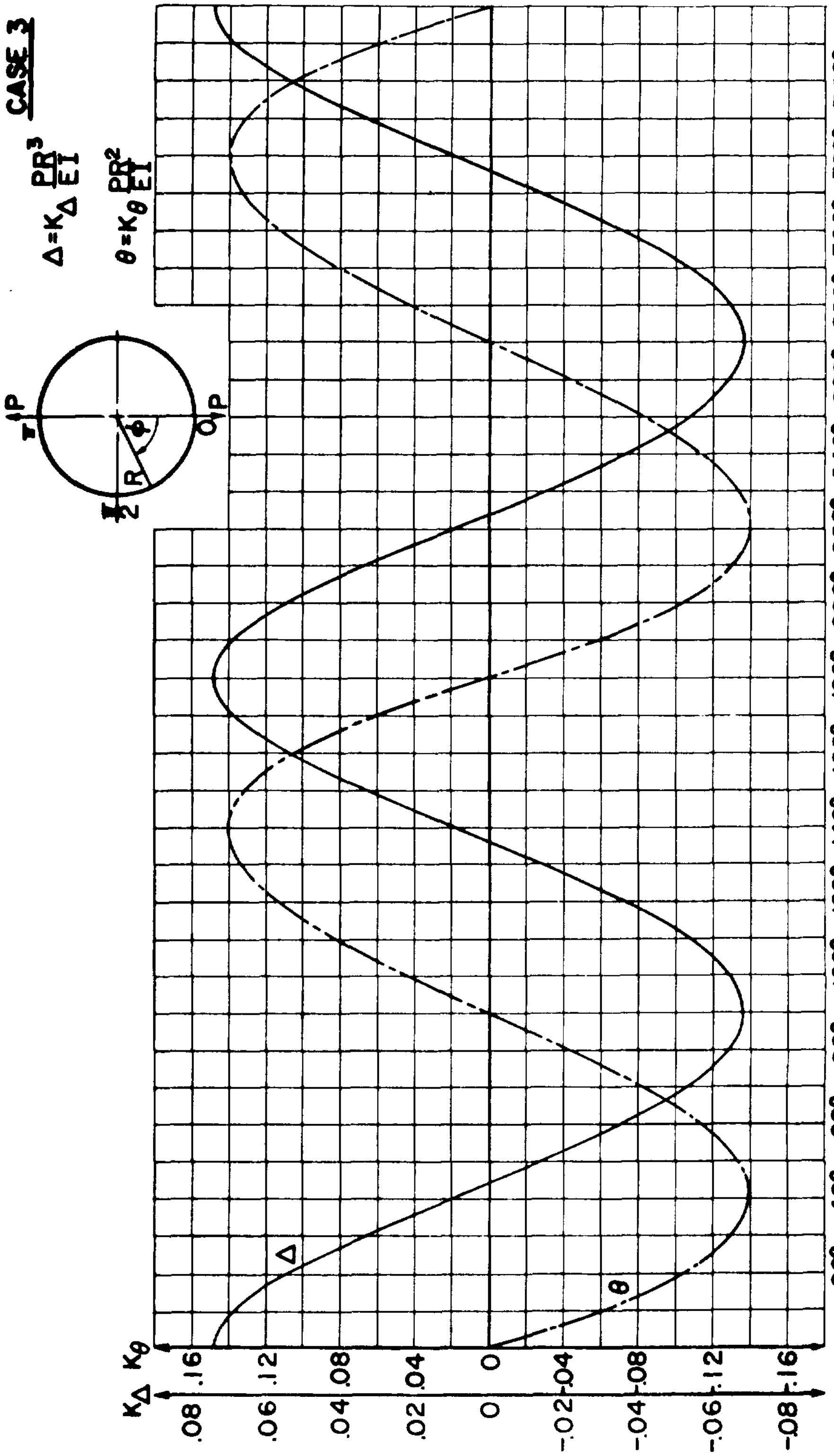

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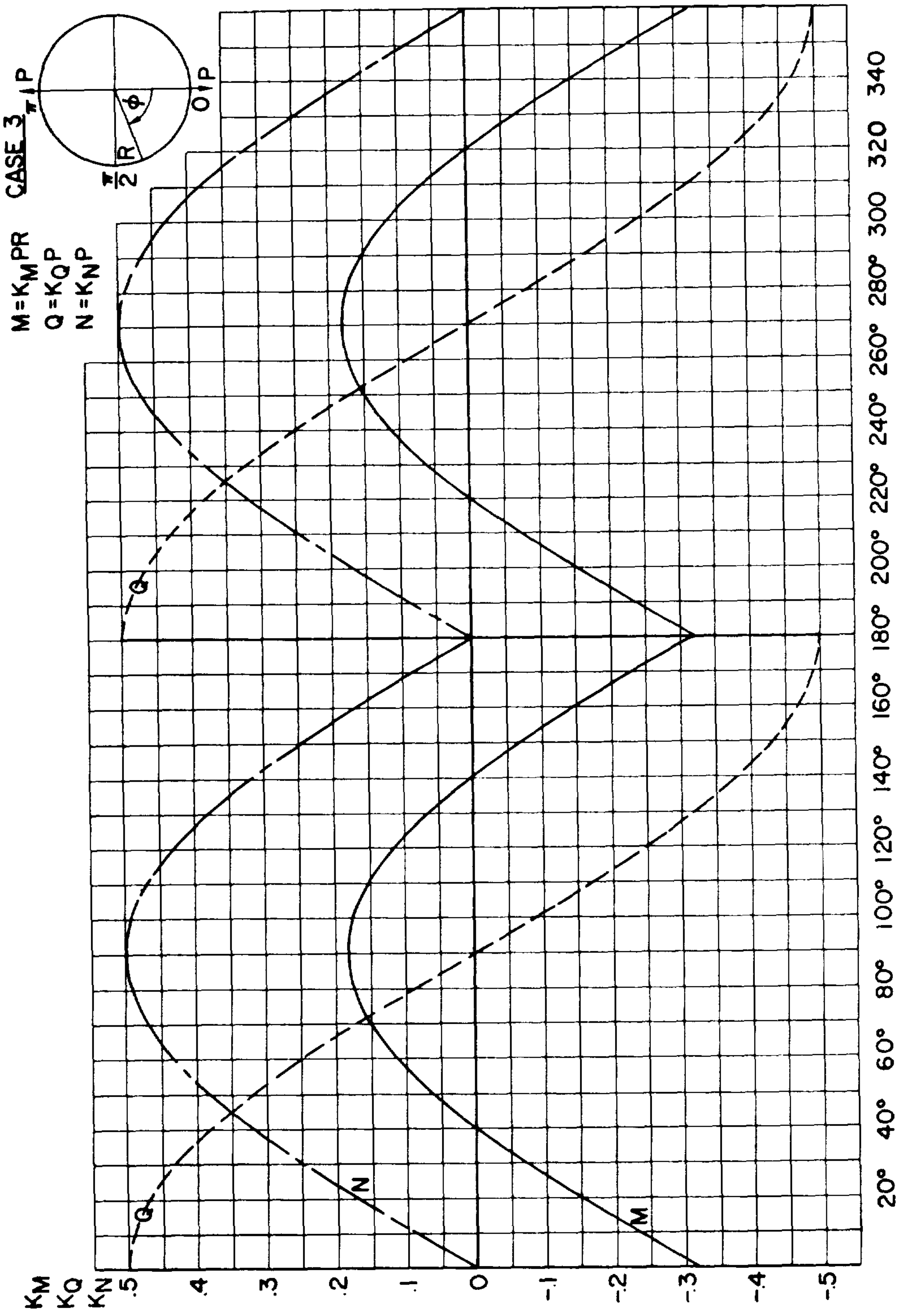

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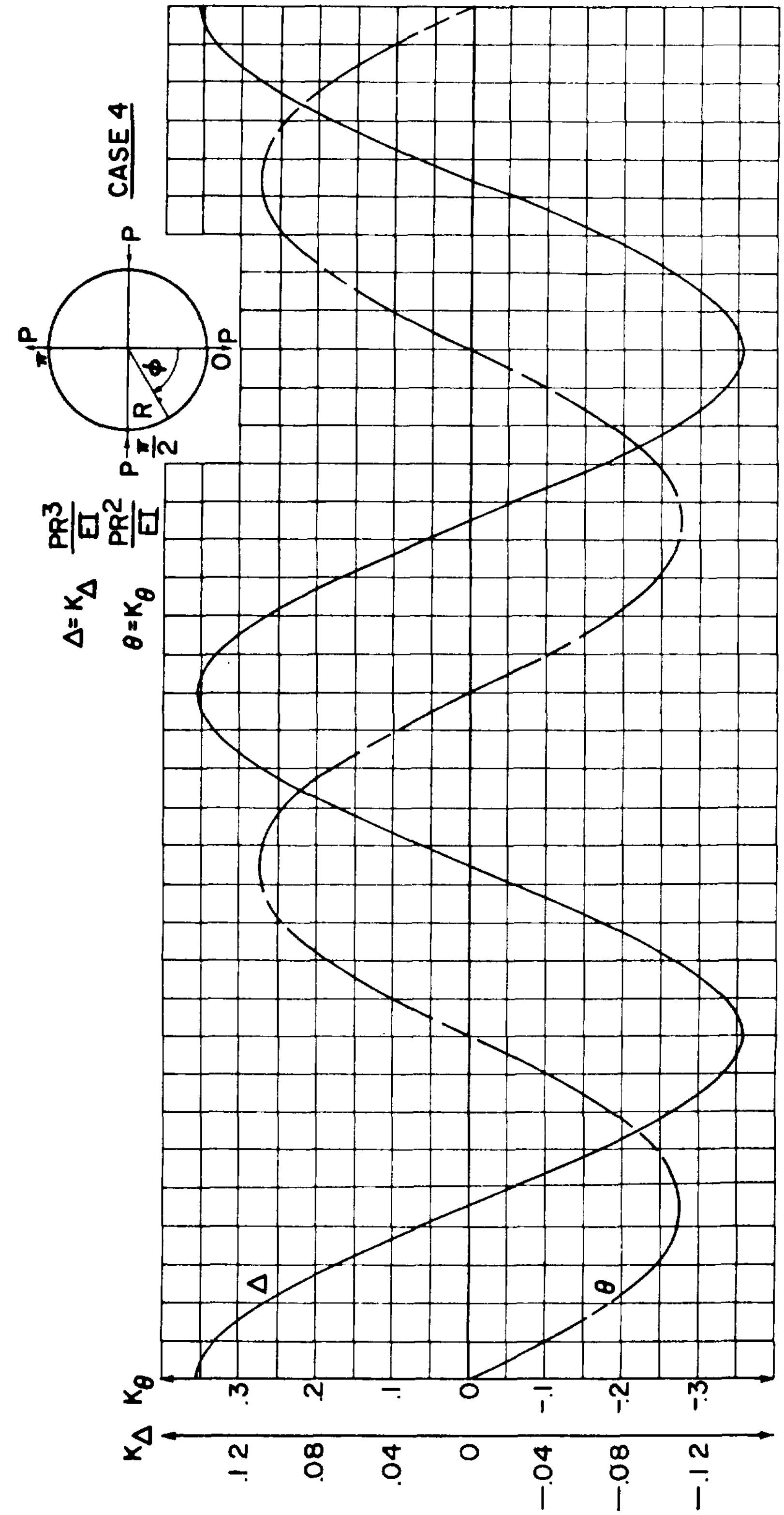

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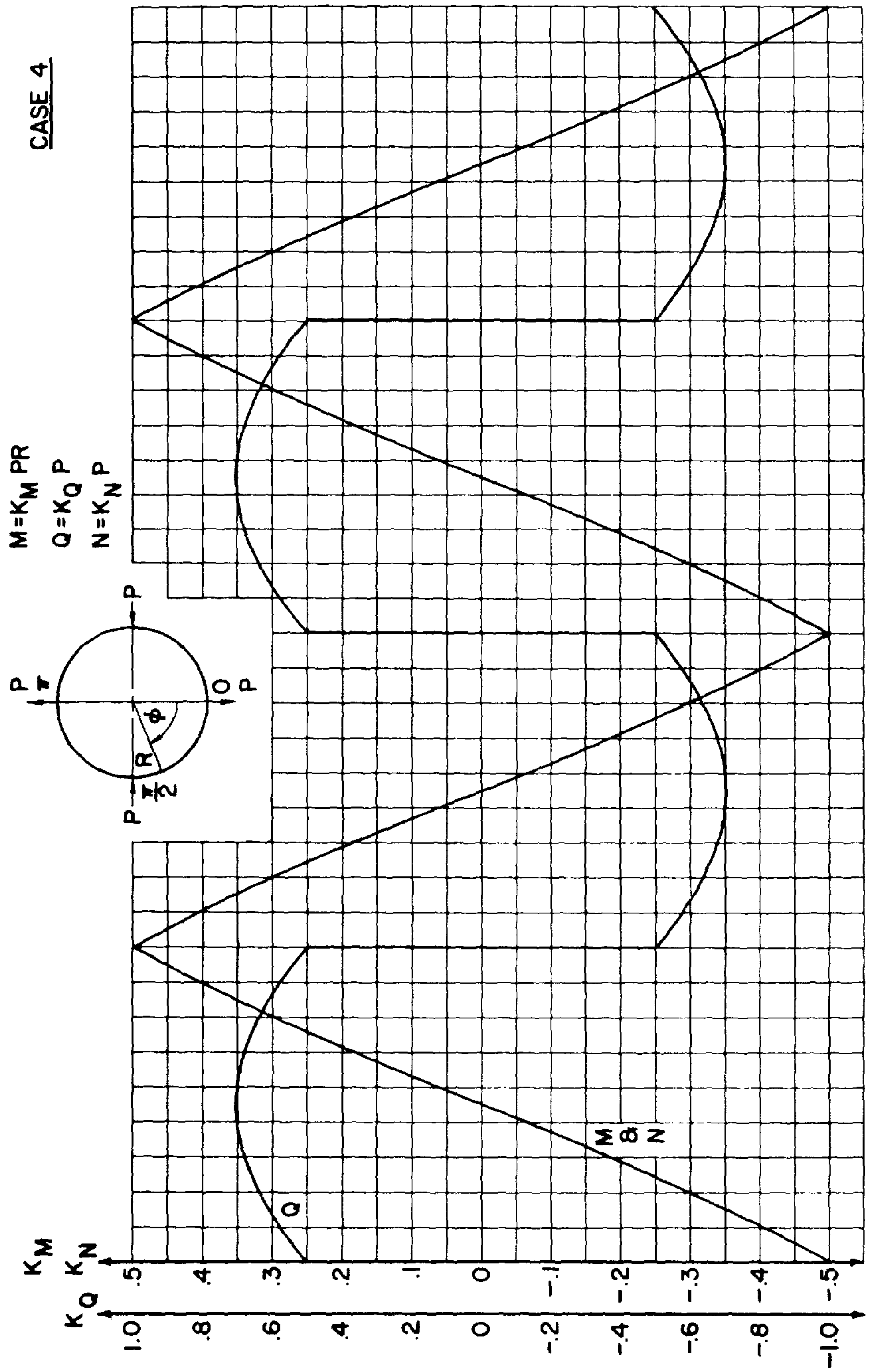

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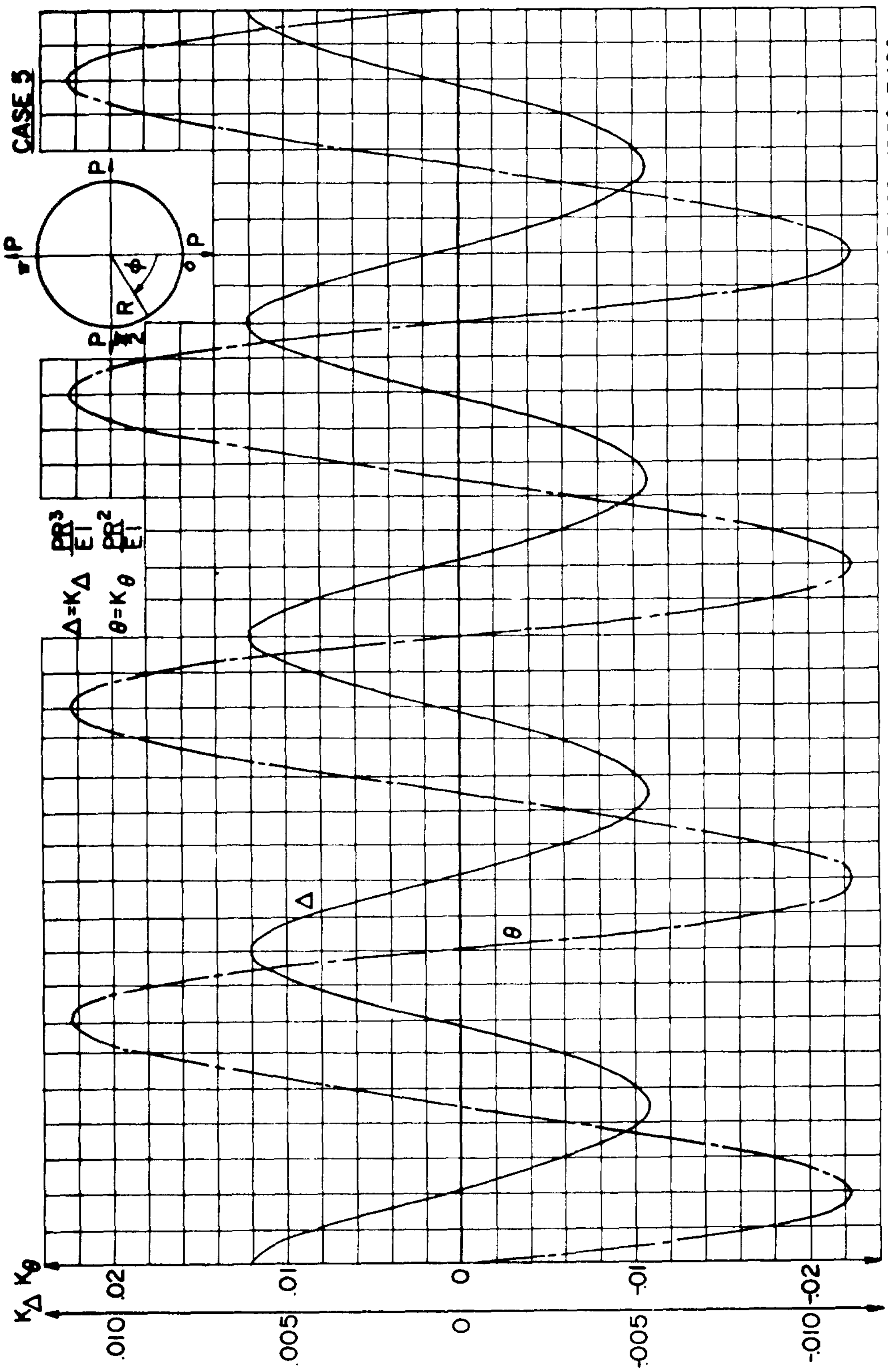

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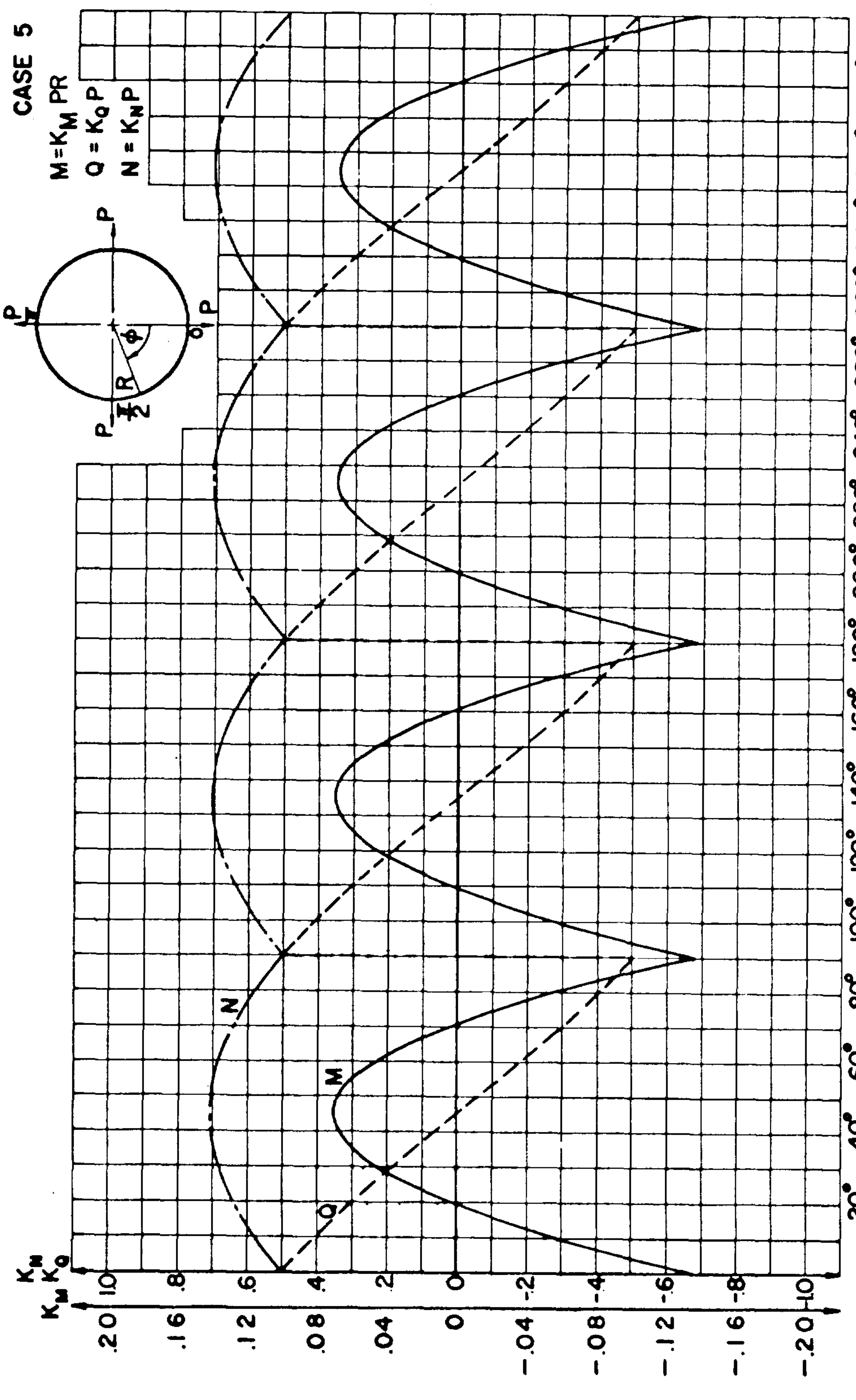
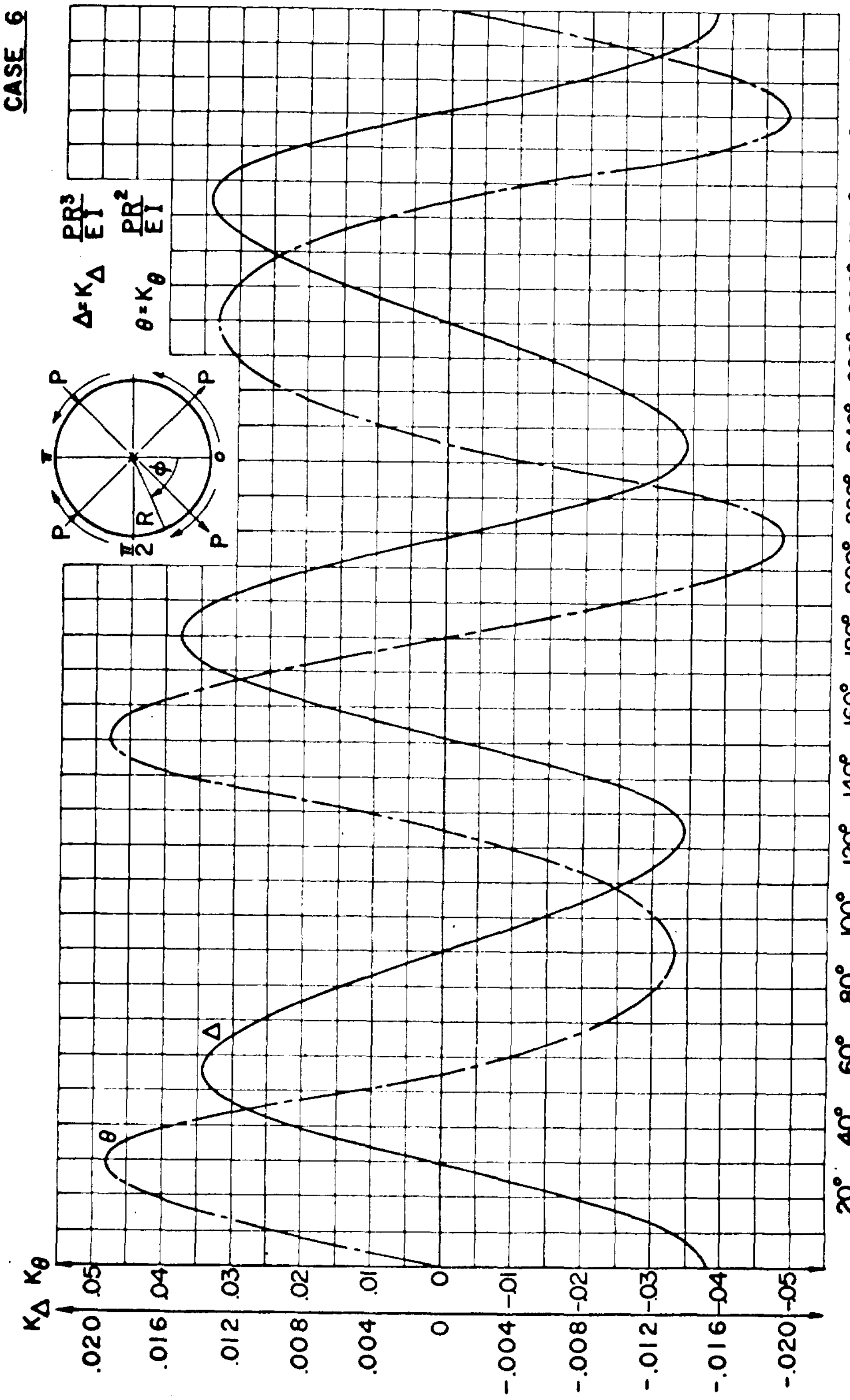


B 6.1.1 In-Plane Load Cases (Cont'd)

CASE 6



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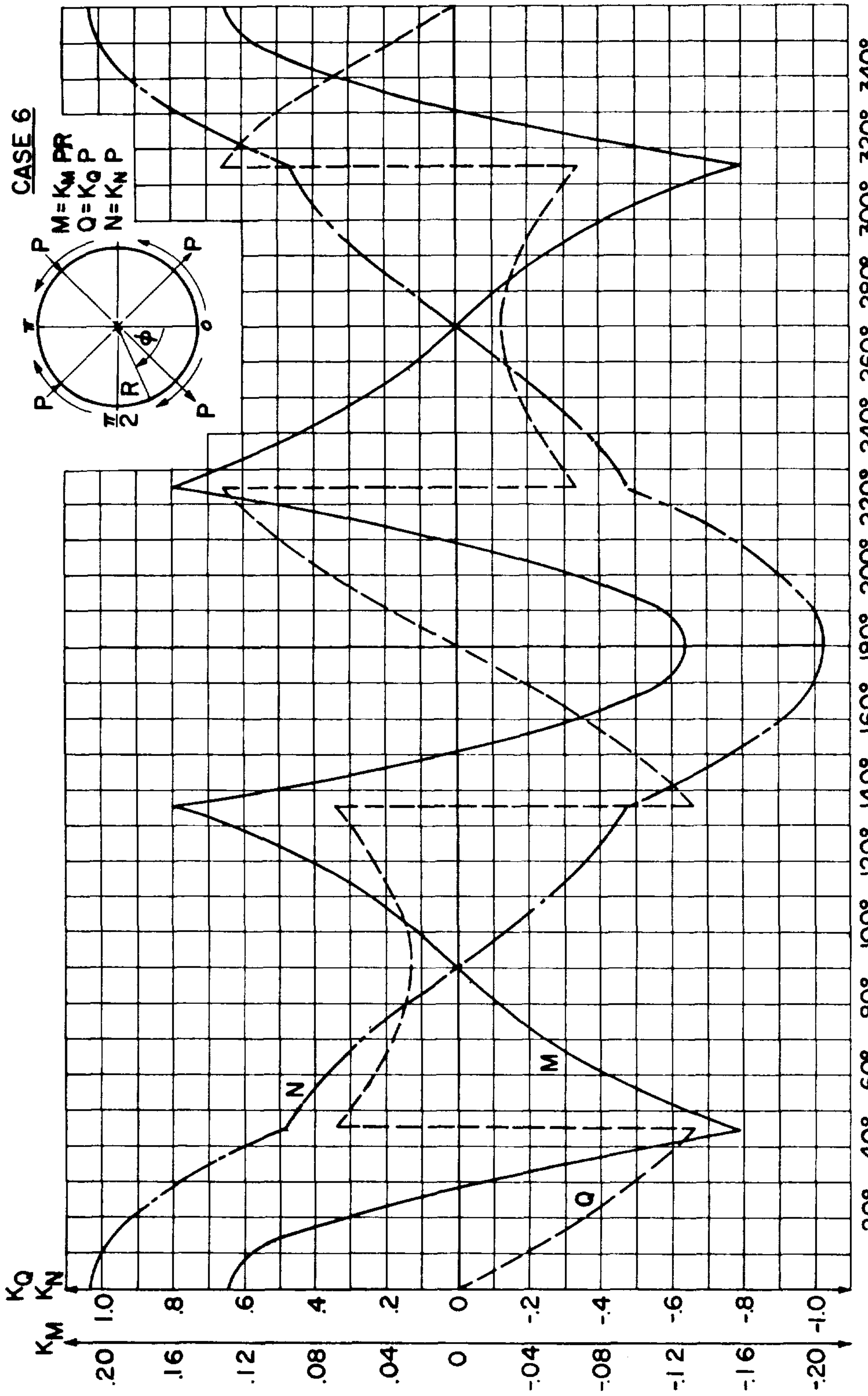

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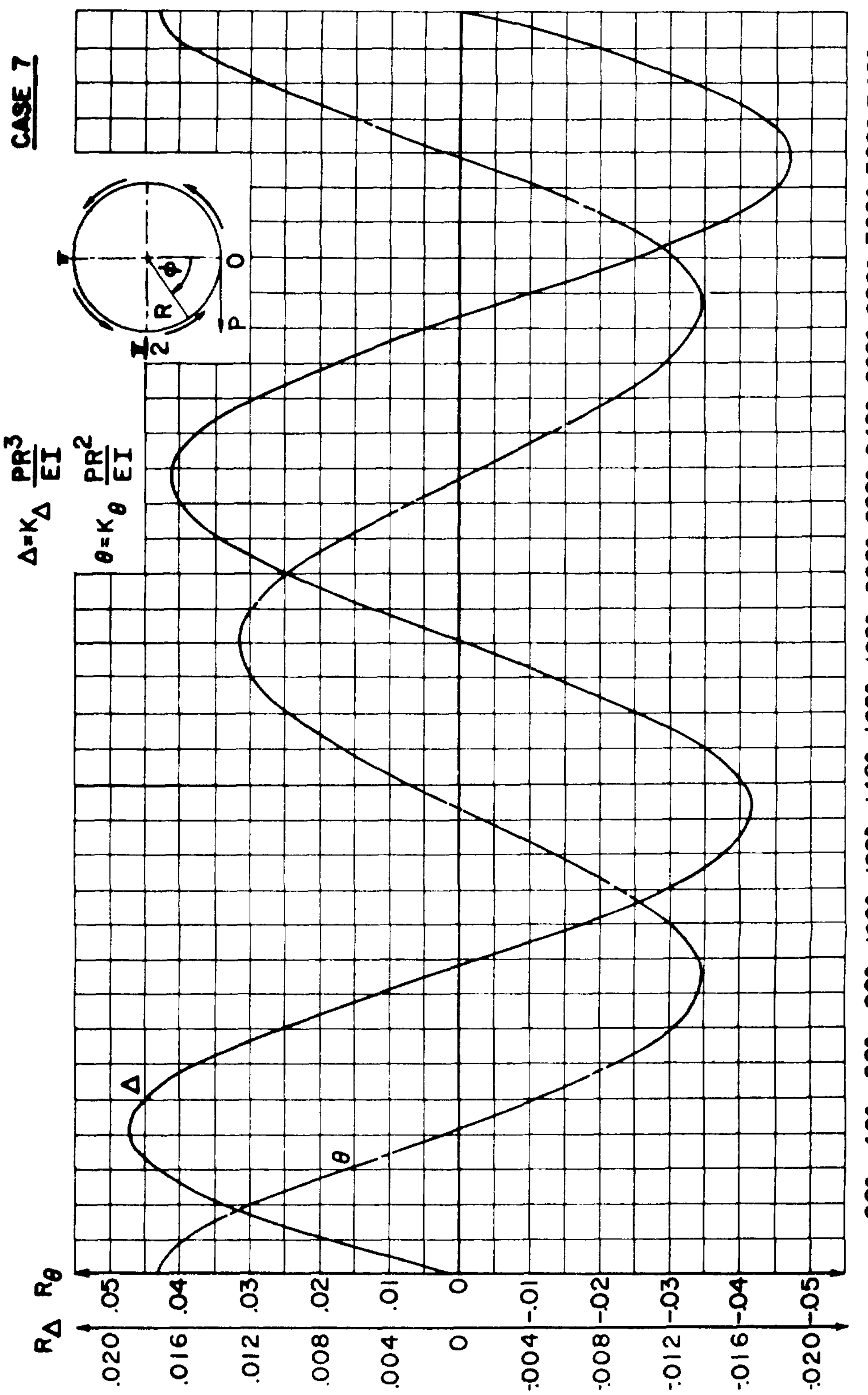

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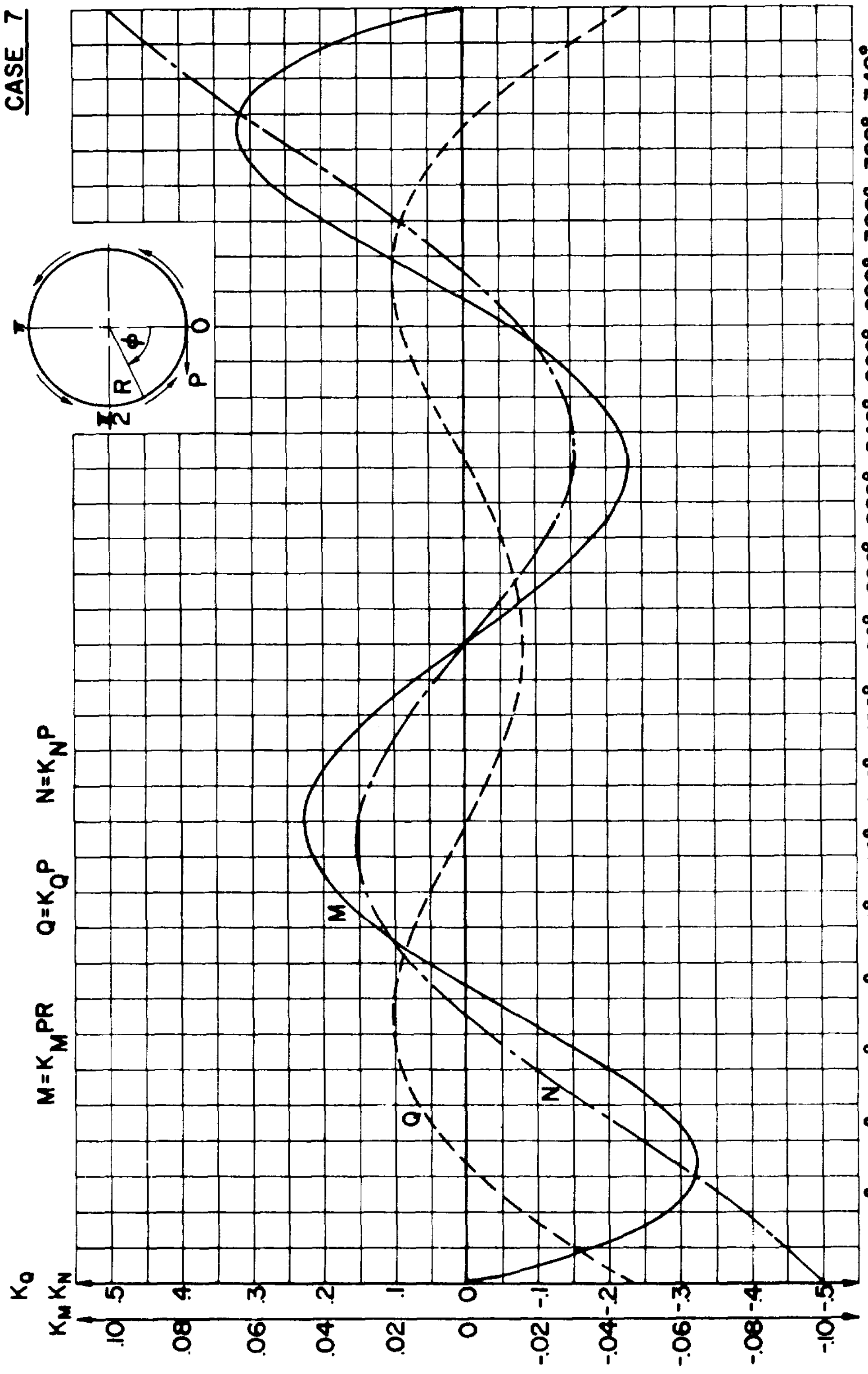

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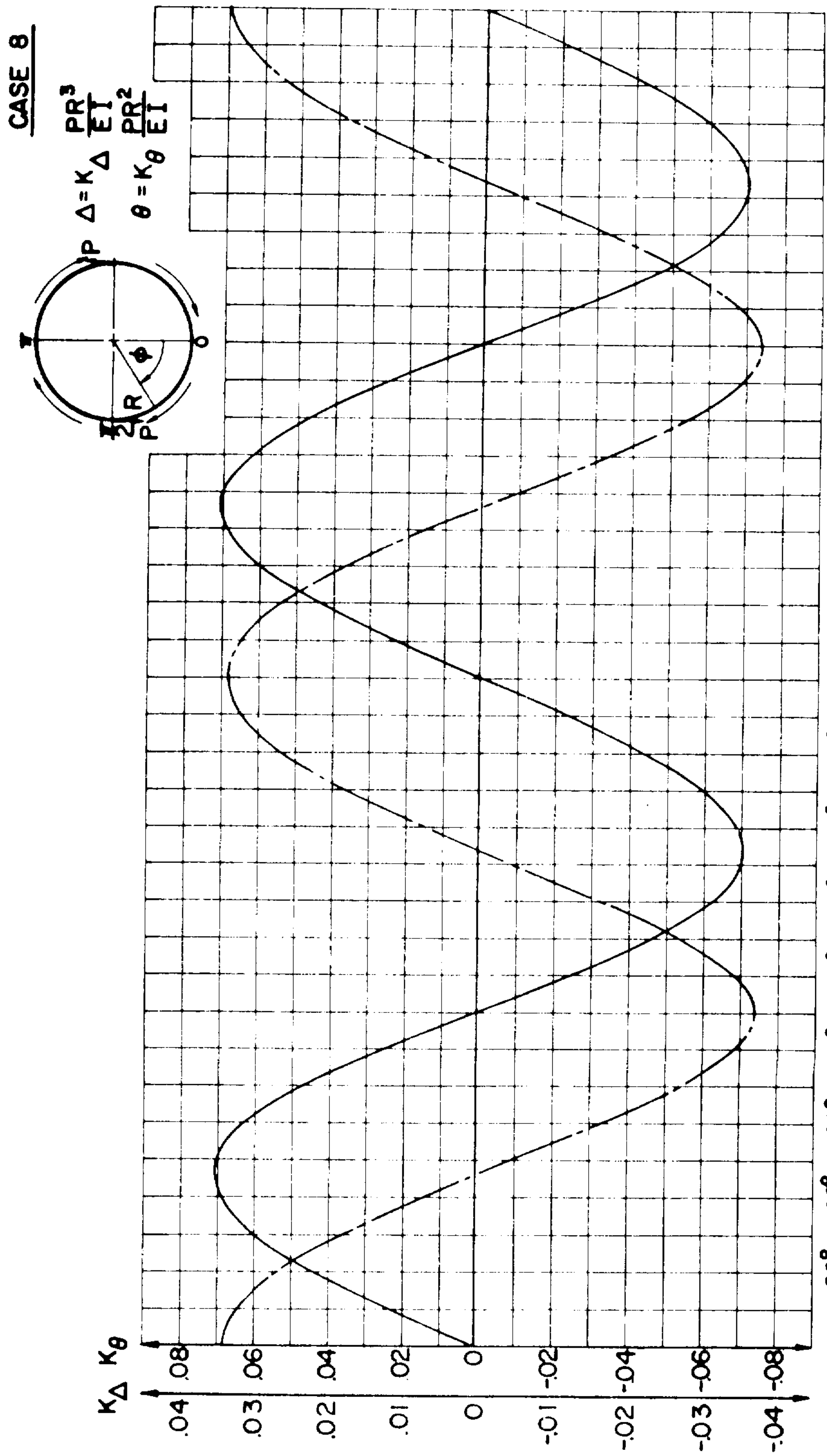

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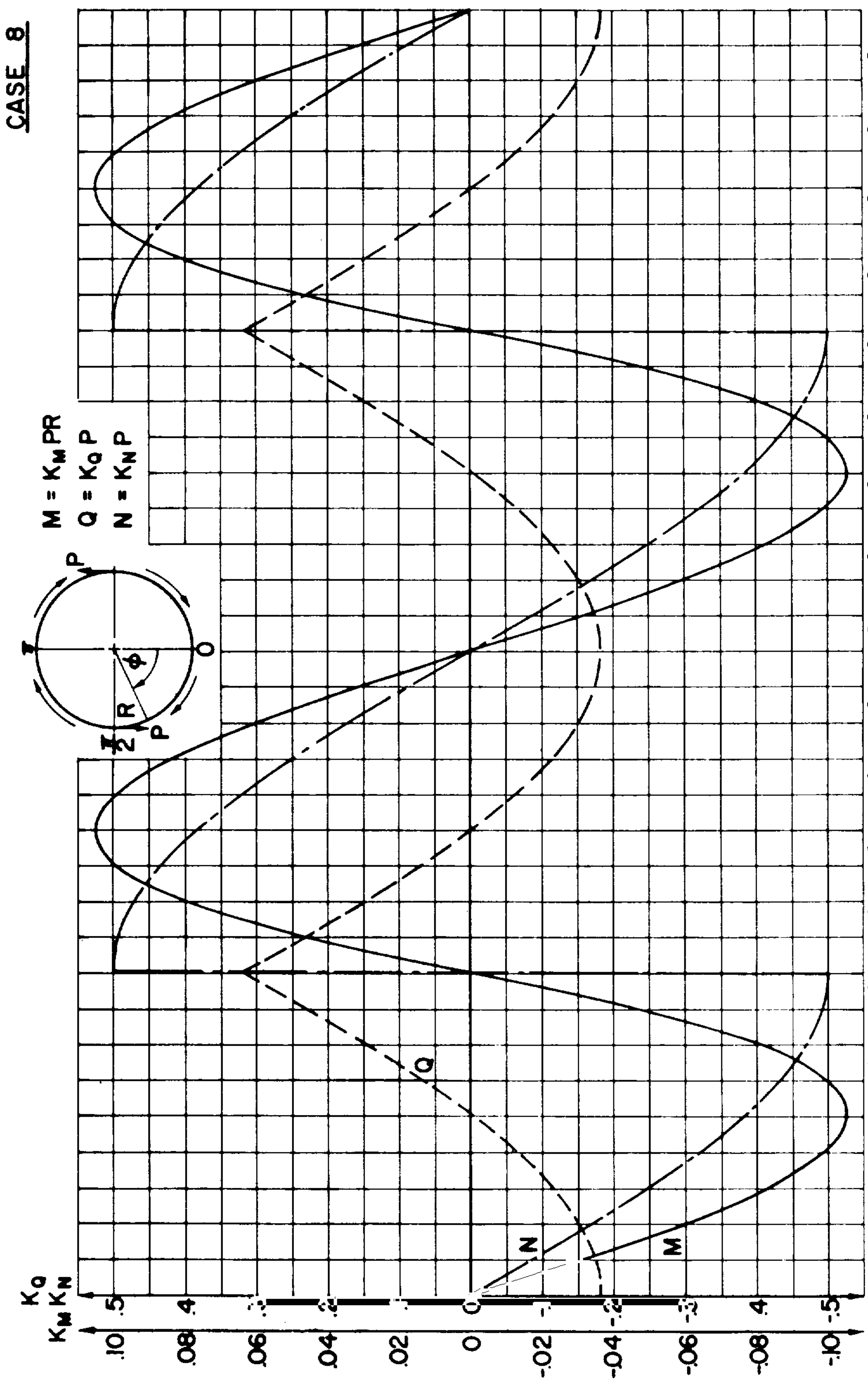

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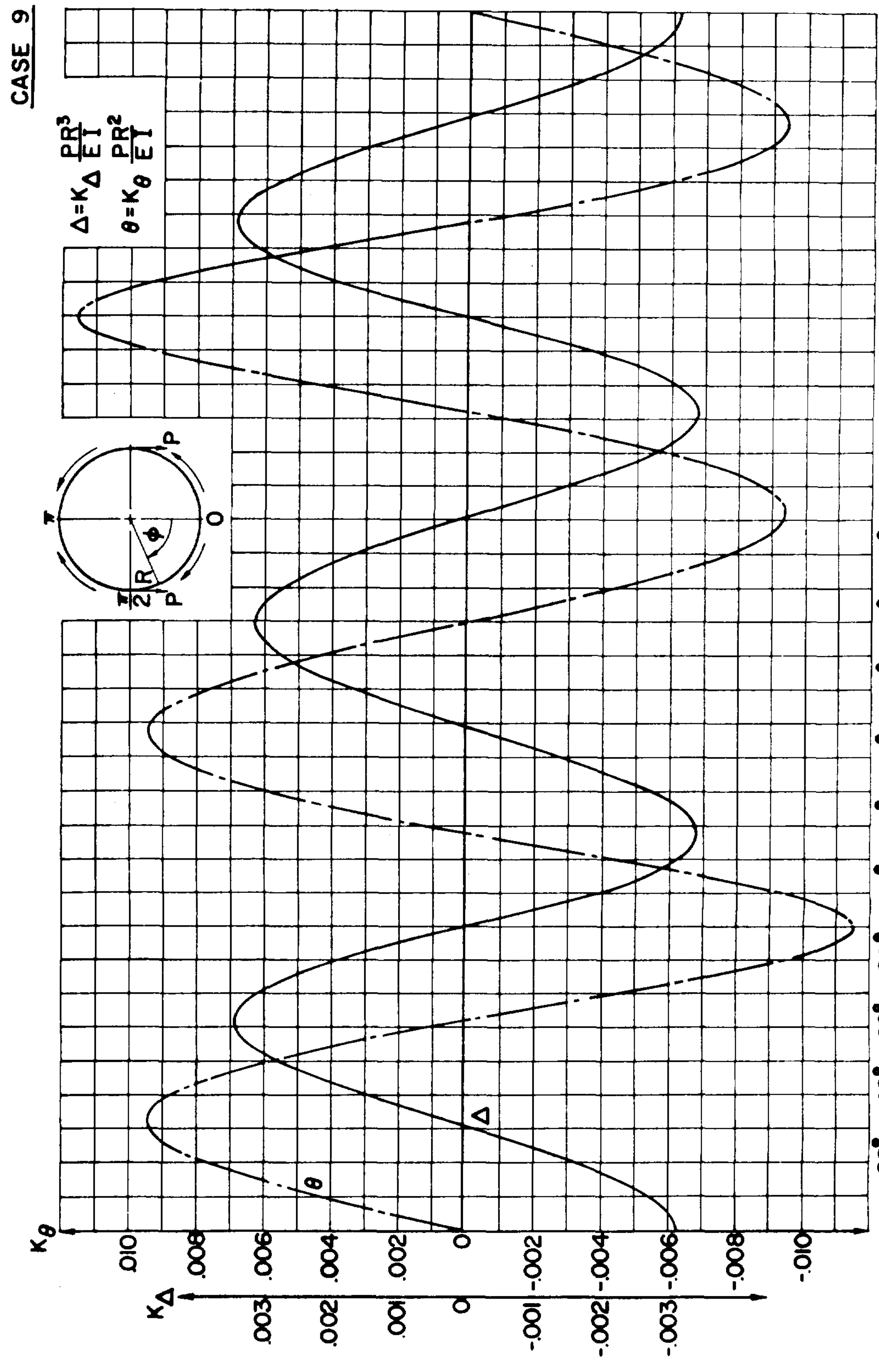
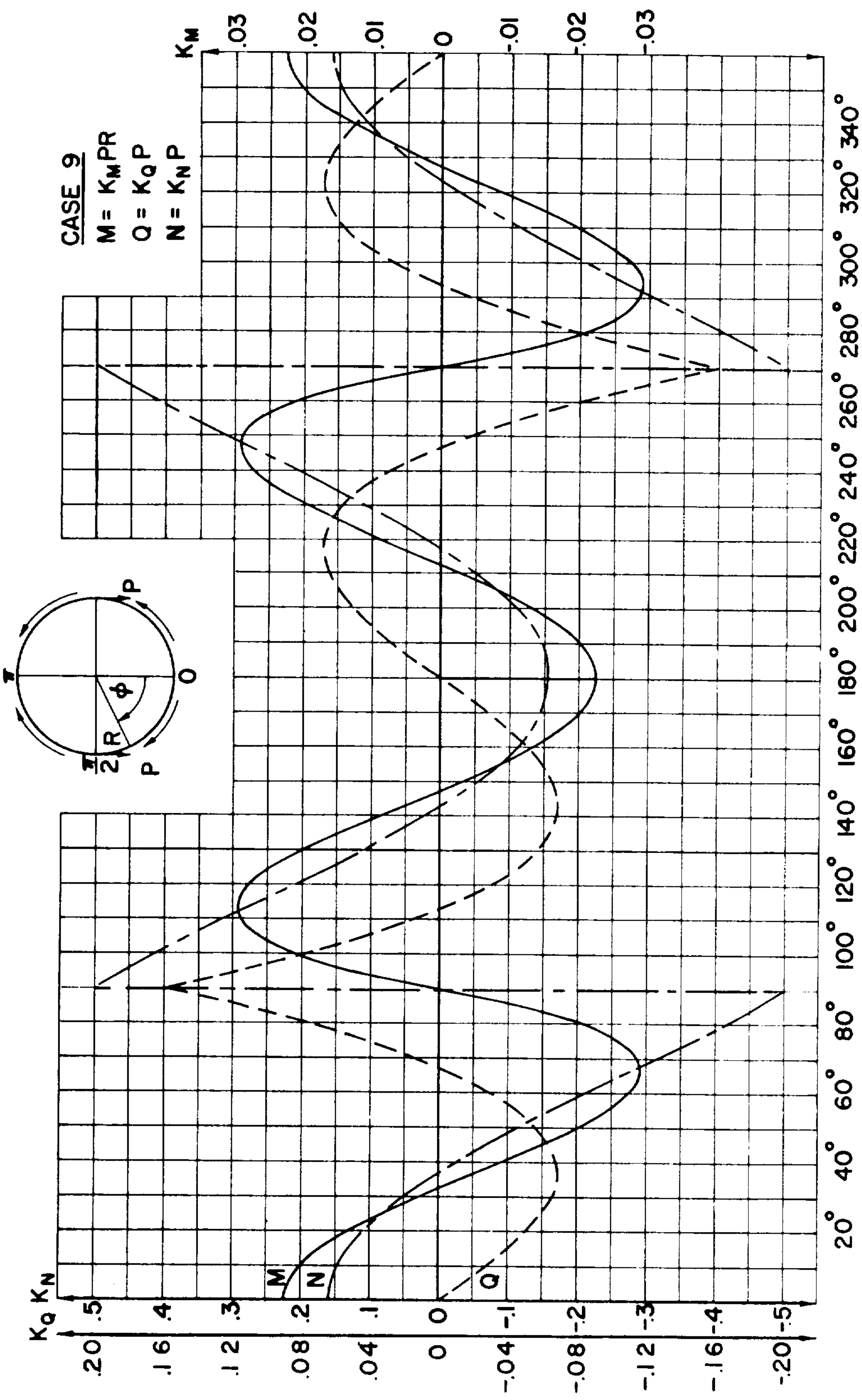


B 6.1.1 <u>In-Plane Load Cases (Cont'd)</u>



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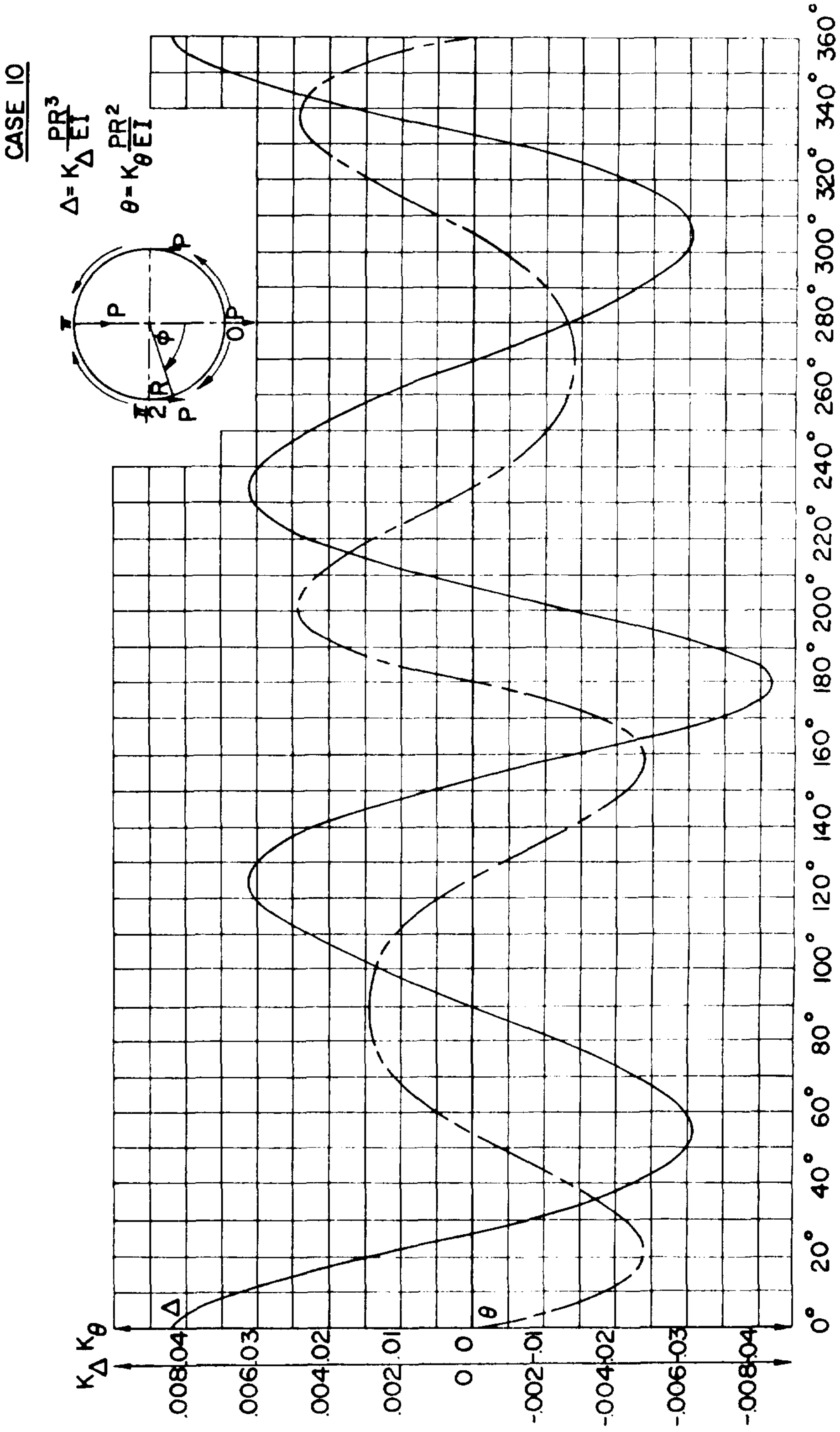

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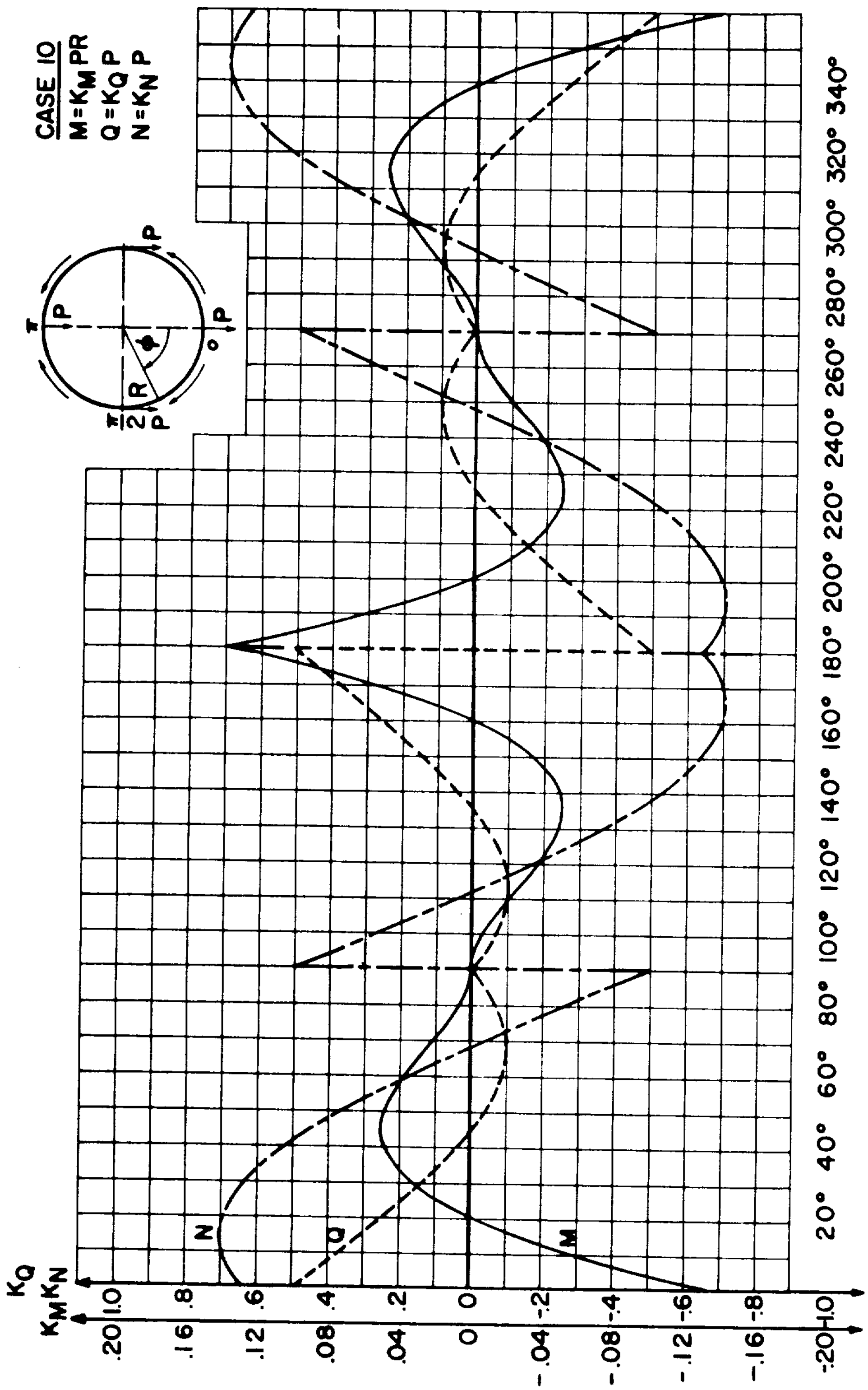

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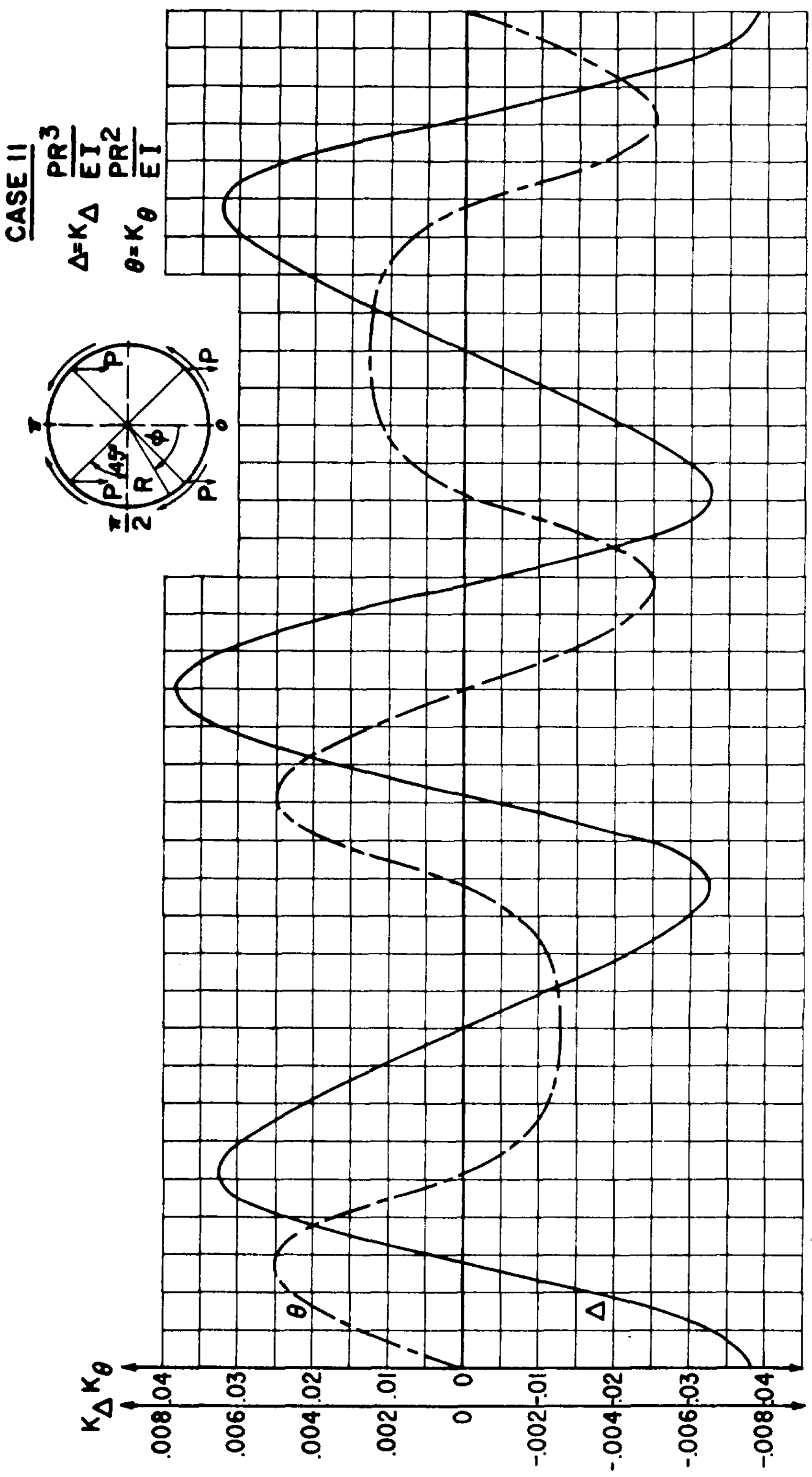

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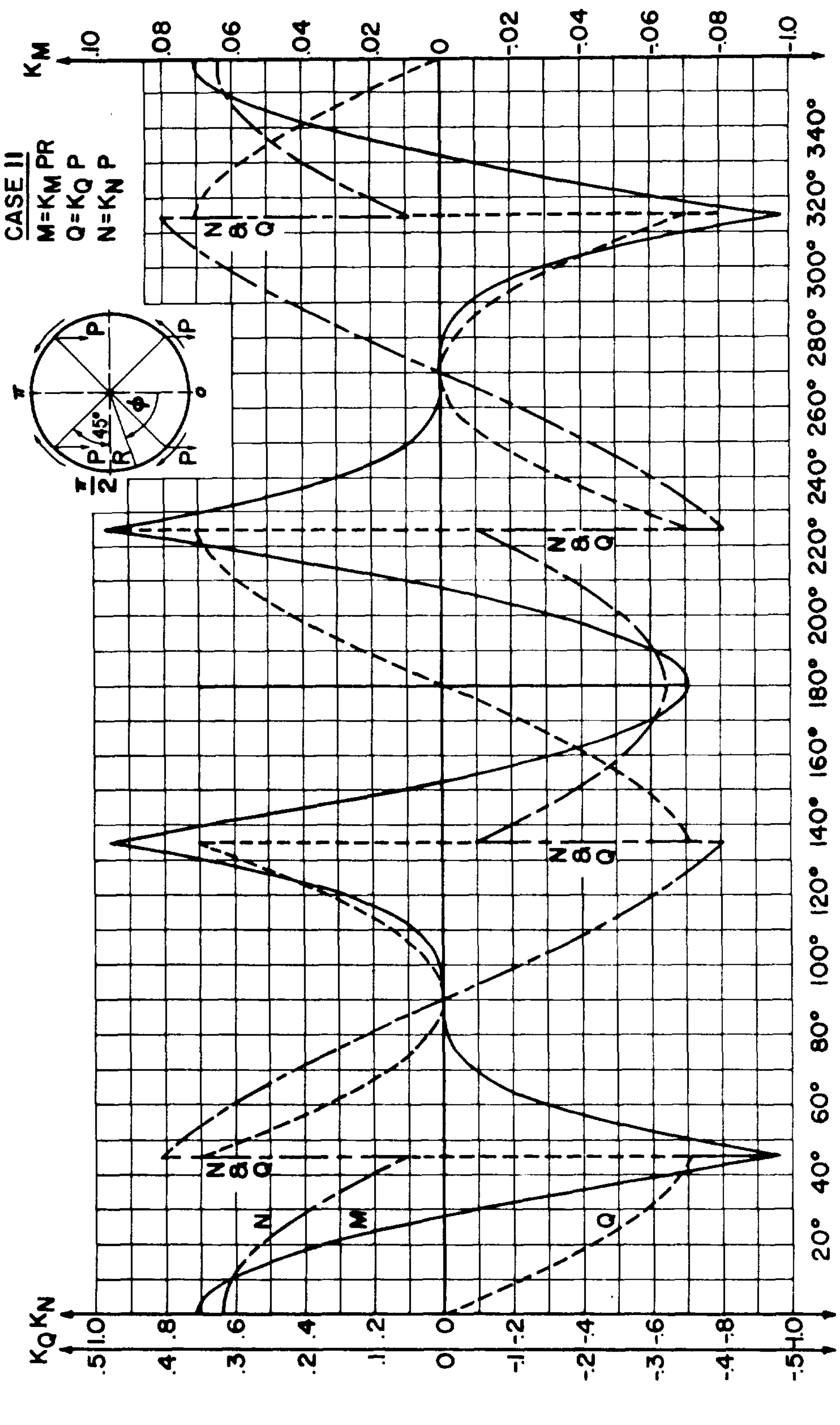

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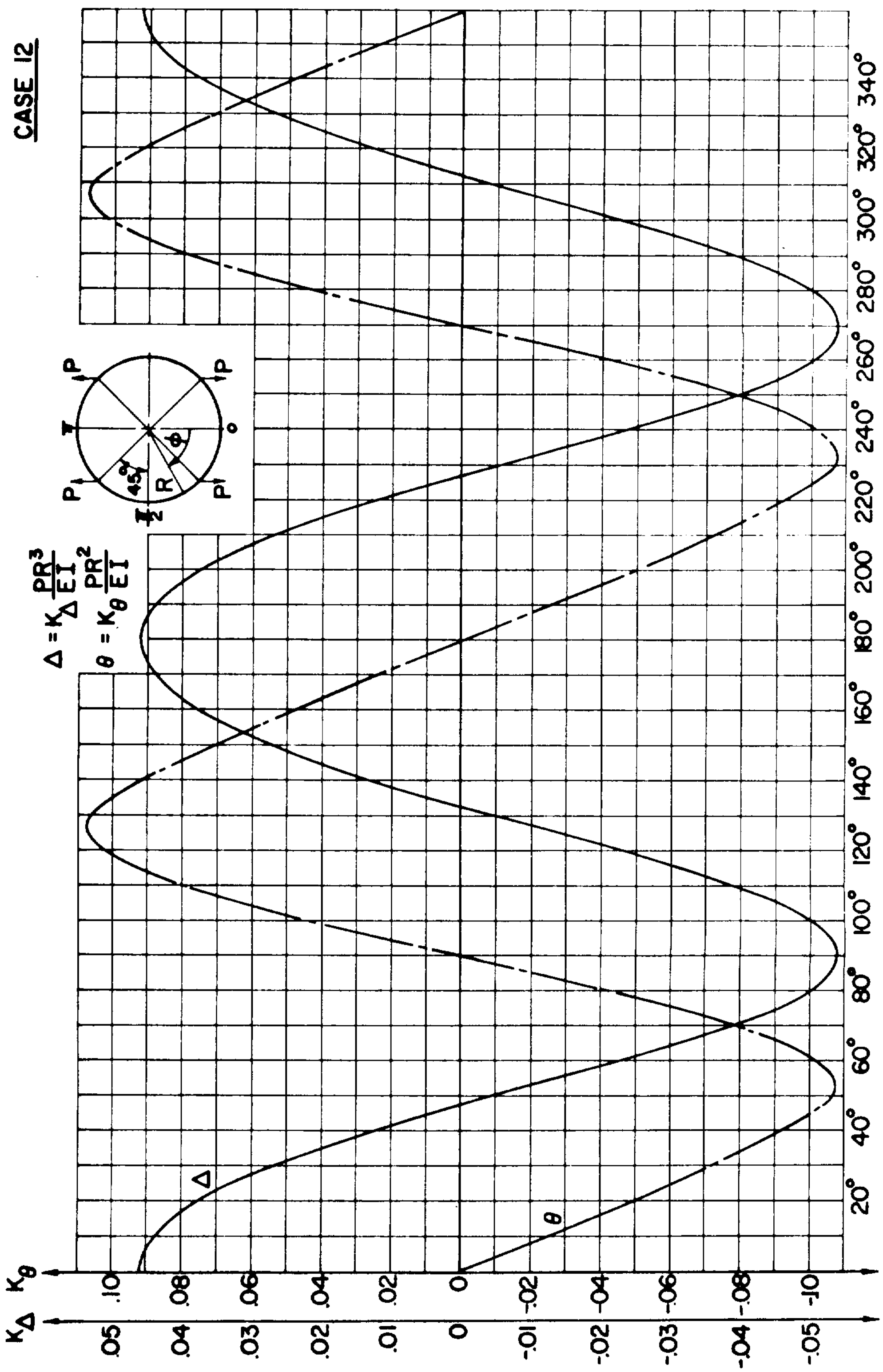

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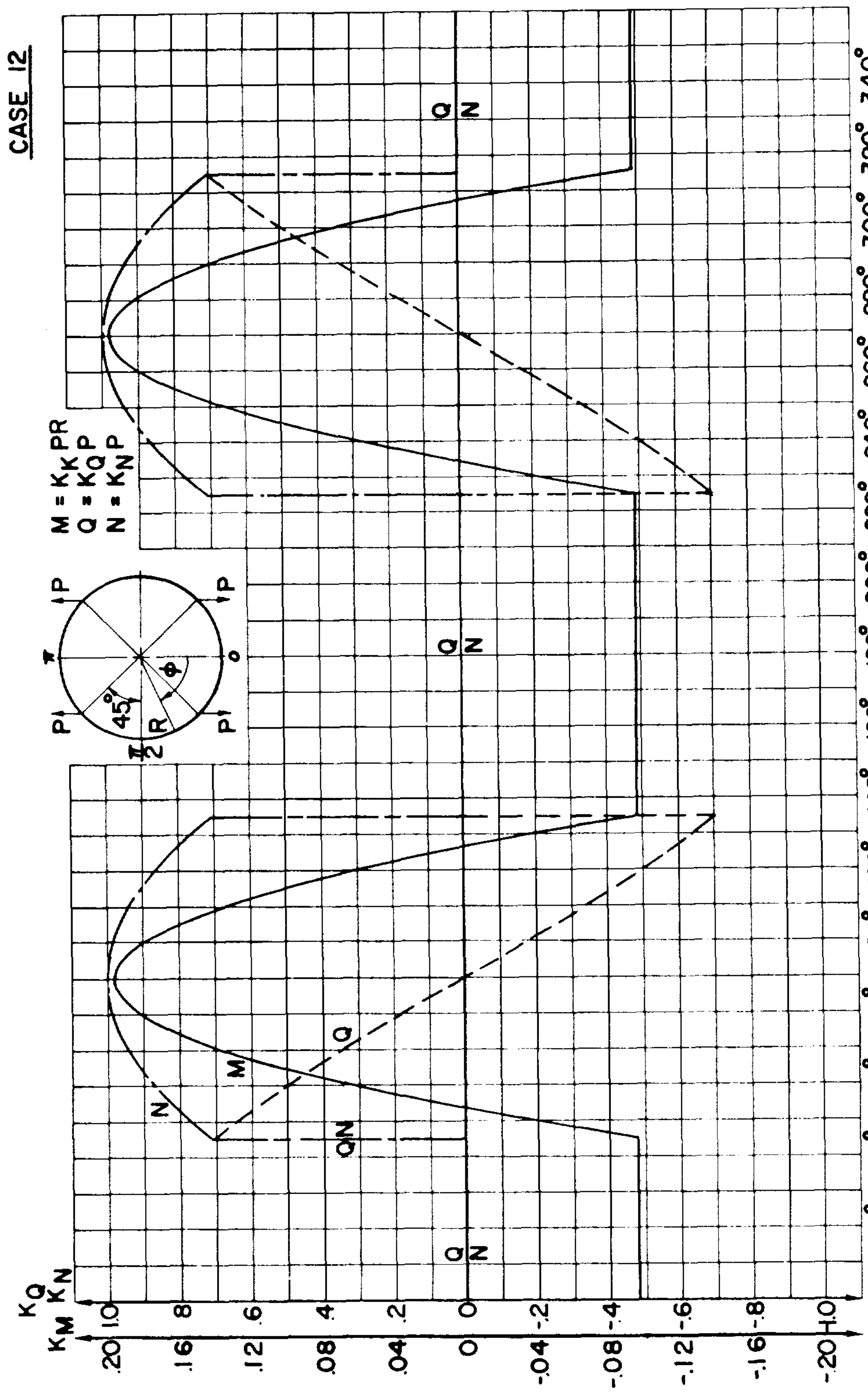

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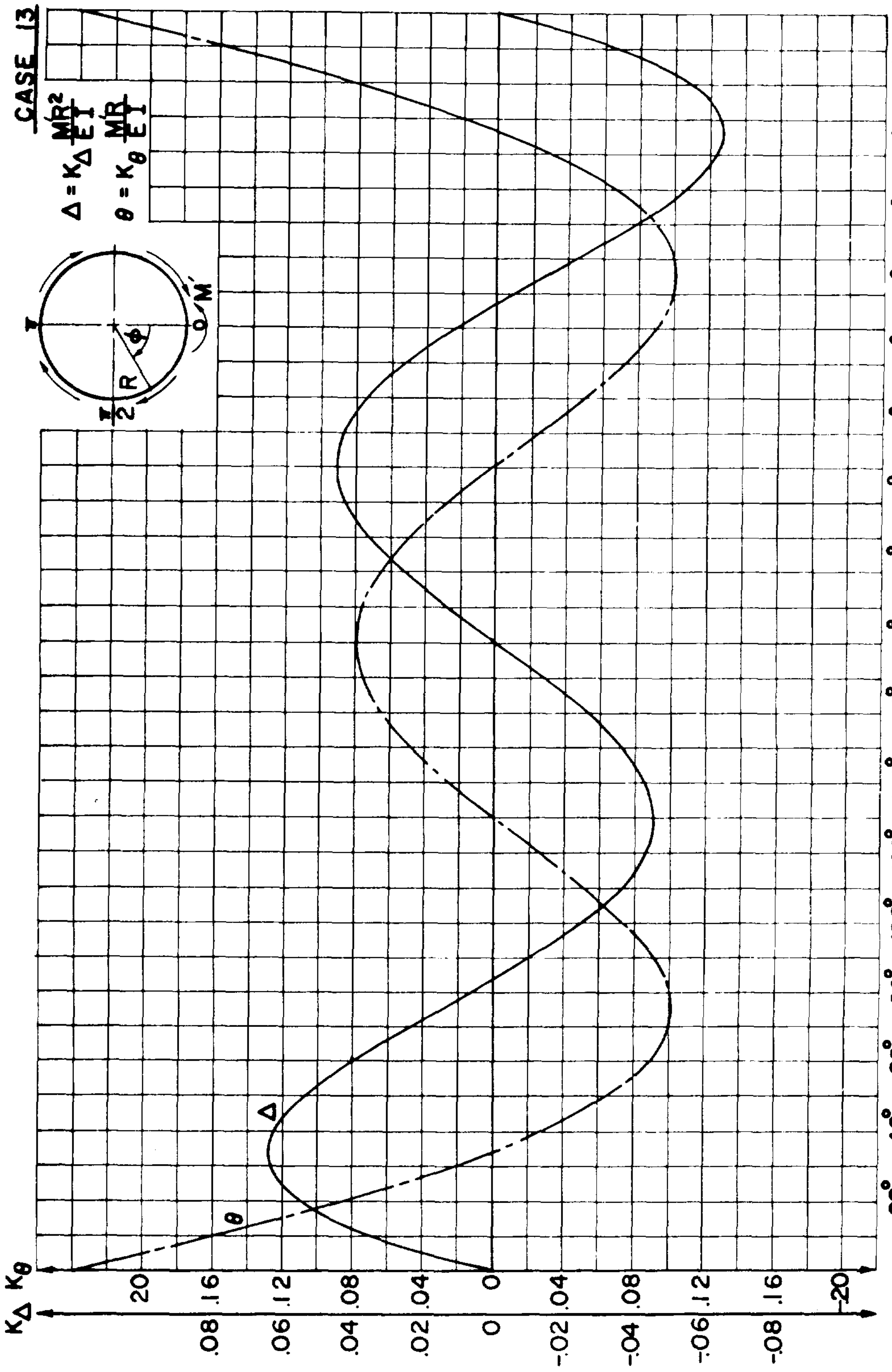

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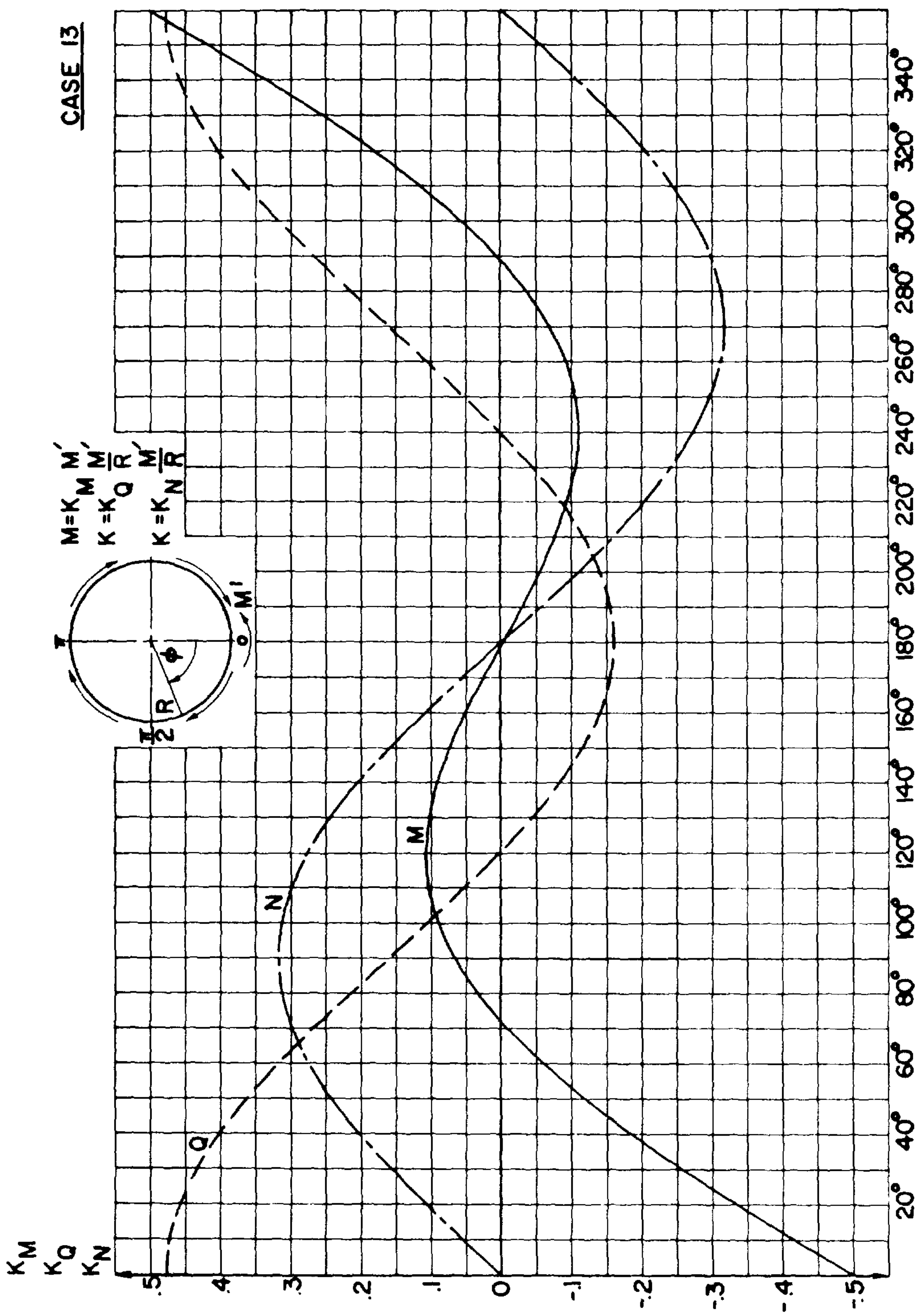

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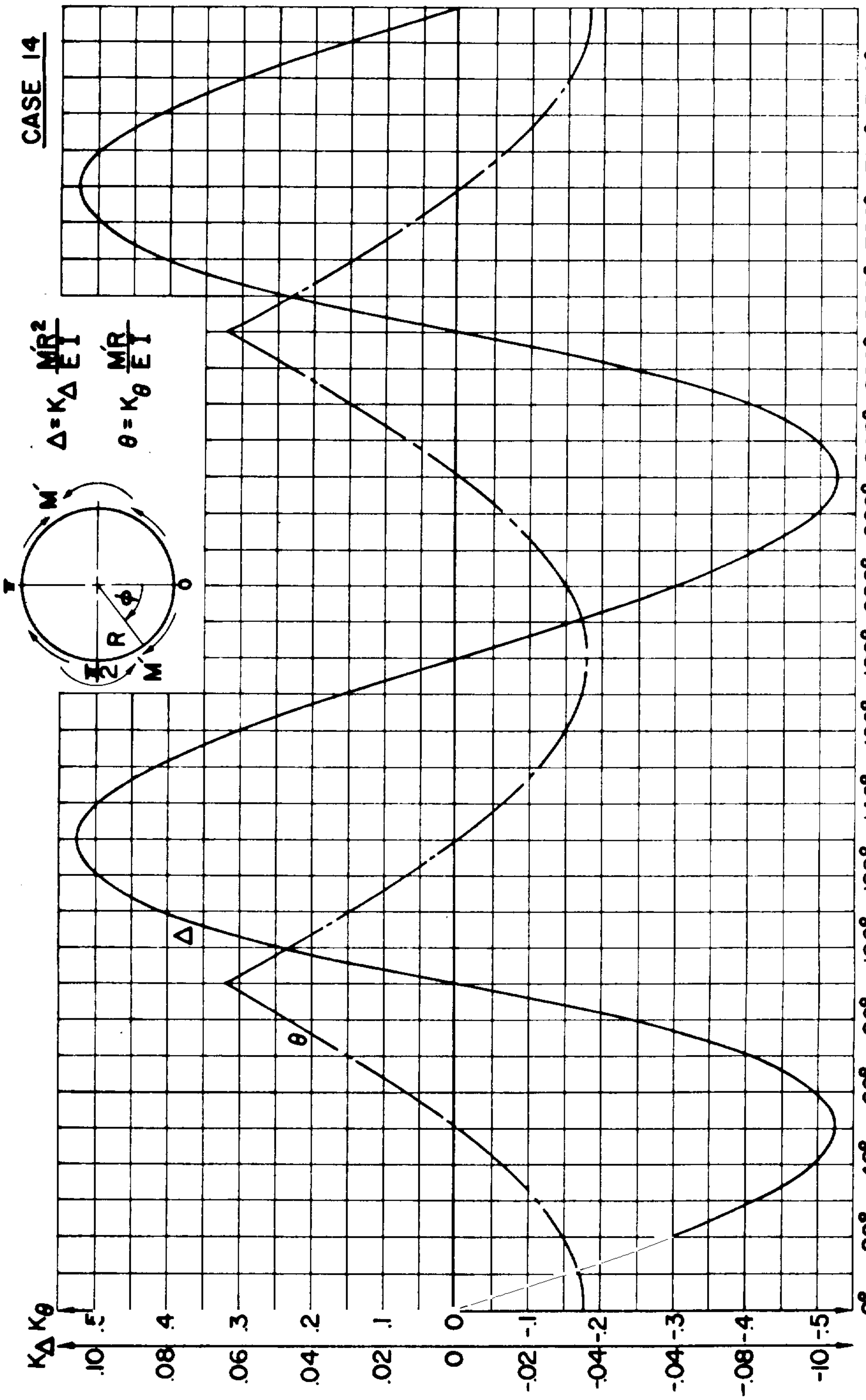
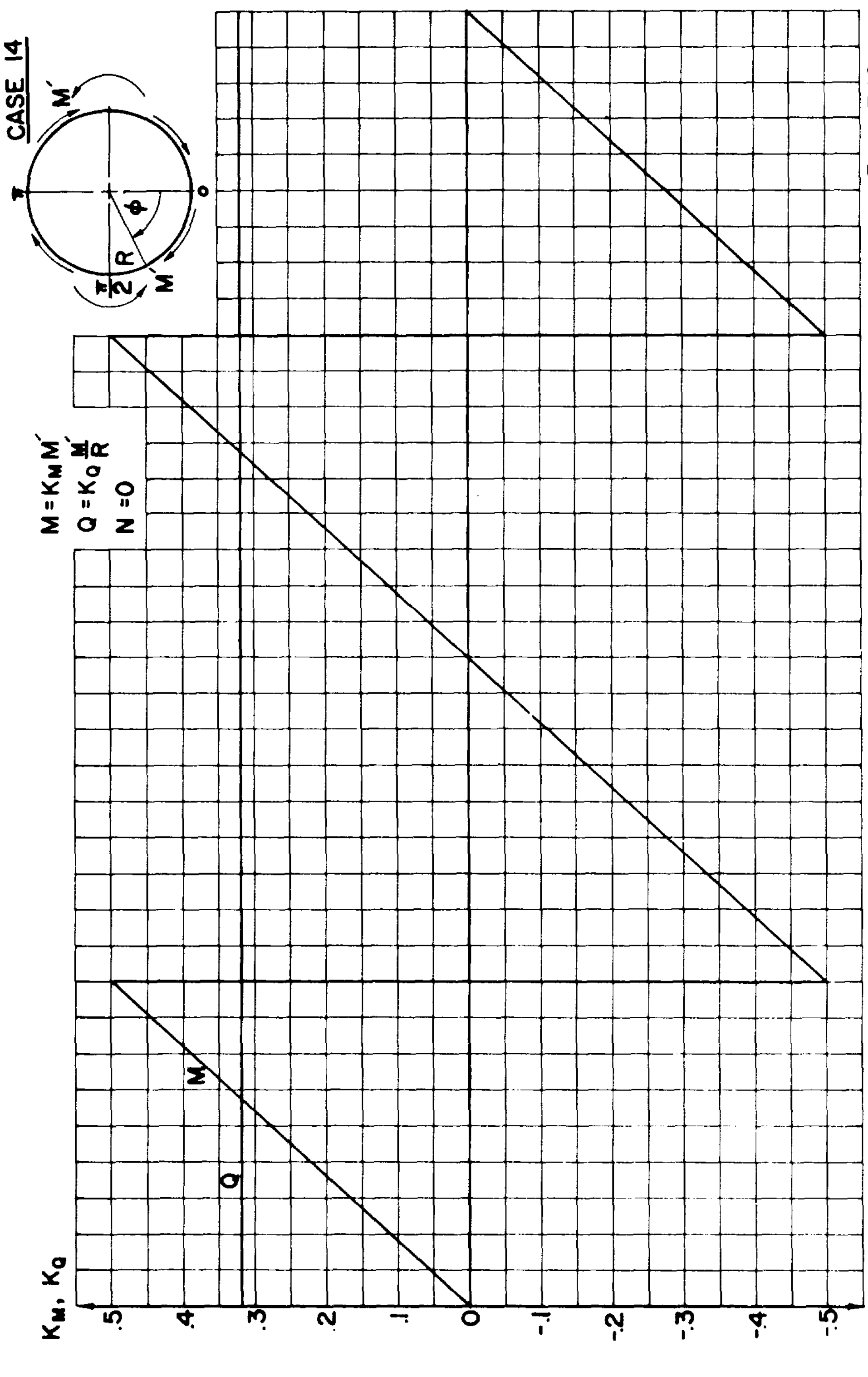
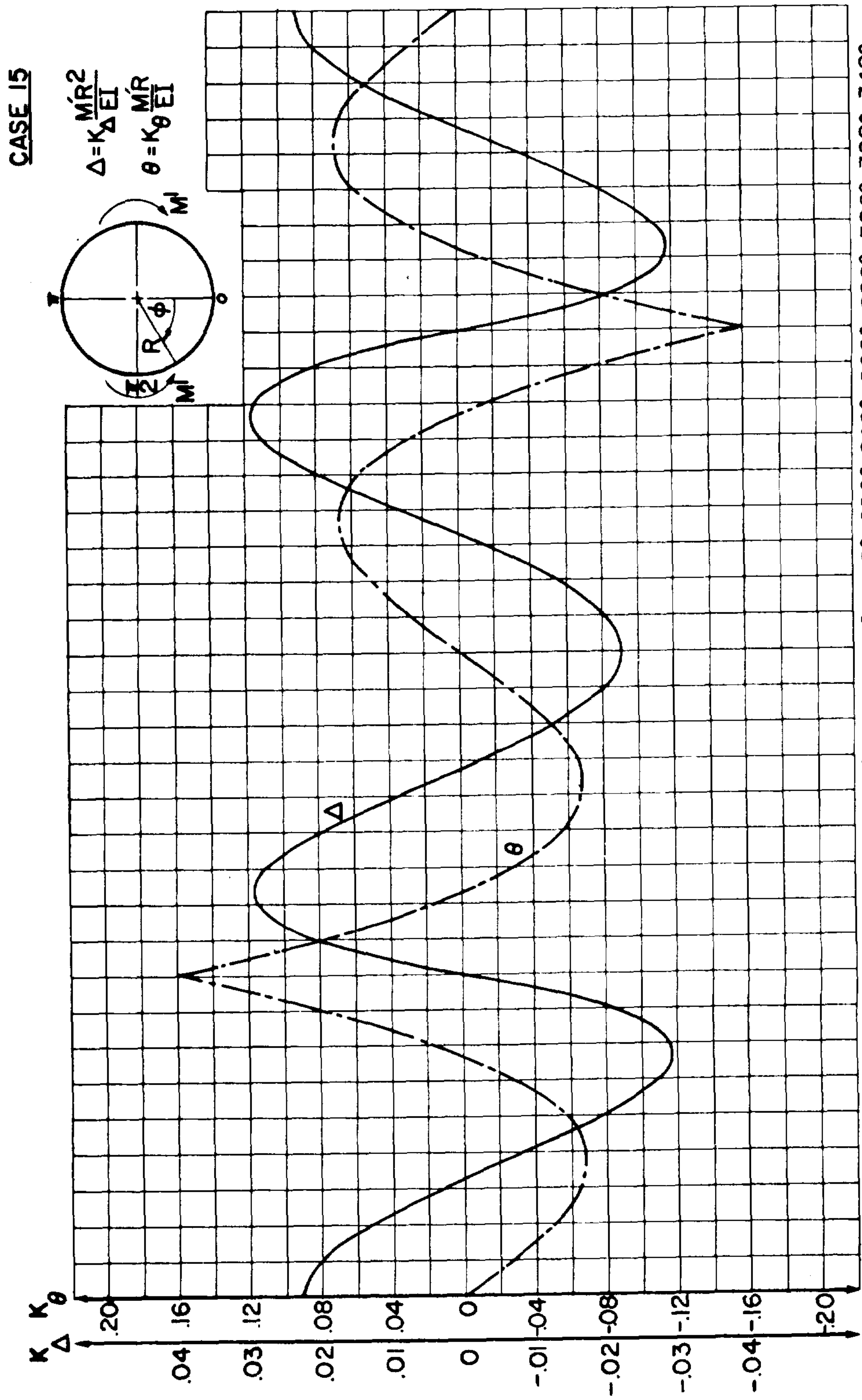


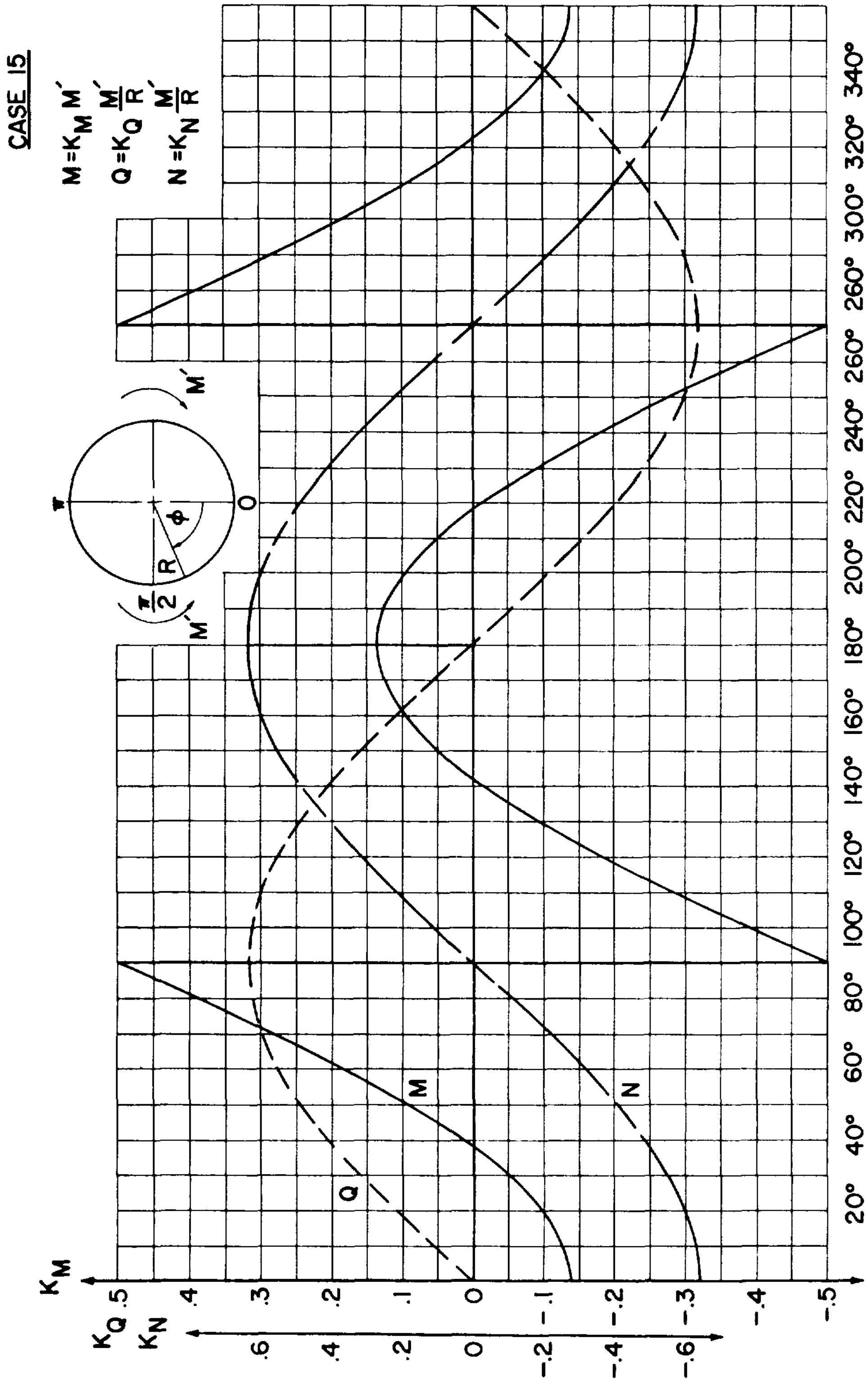
B 6.1.1 In-Plane Load Cases (Cont'd)



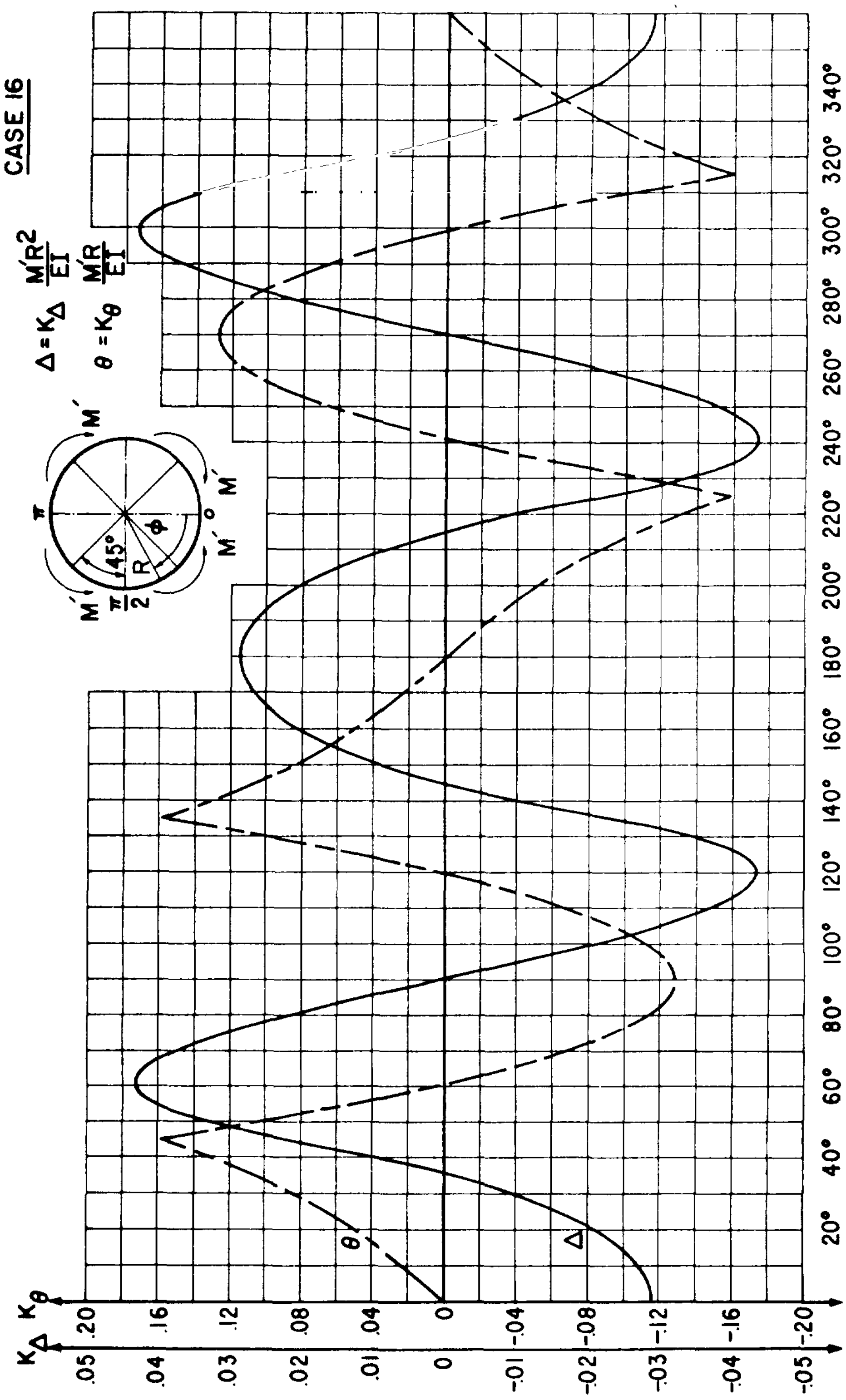
B 6.1.1 In-Plane Load Cases (Cont'd)



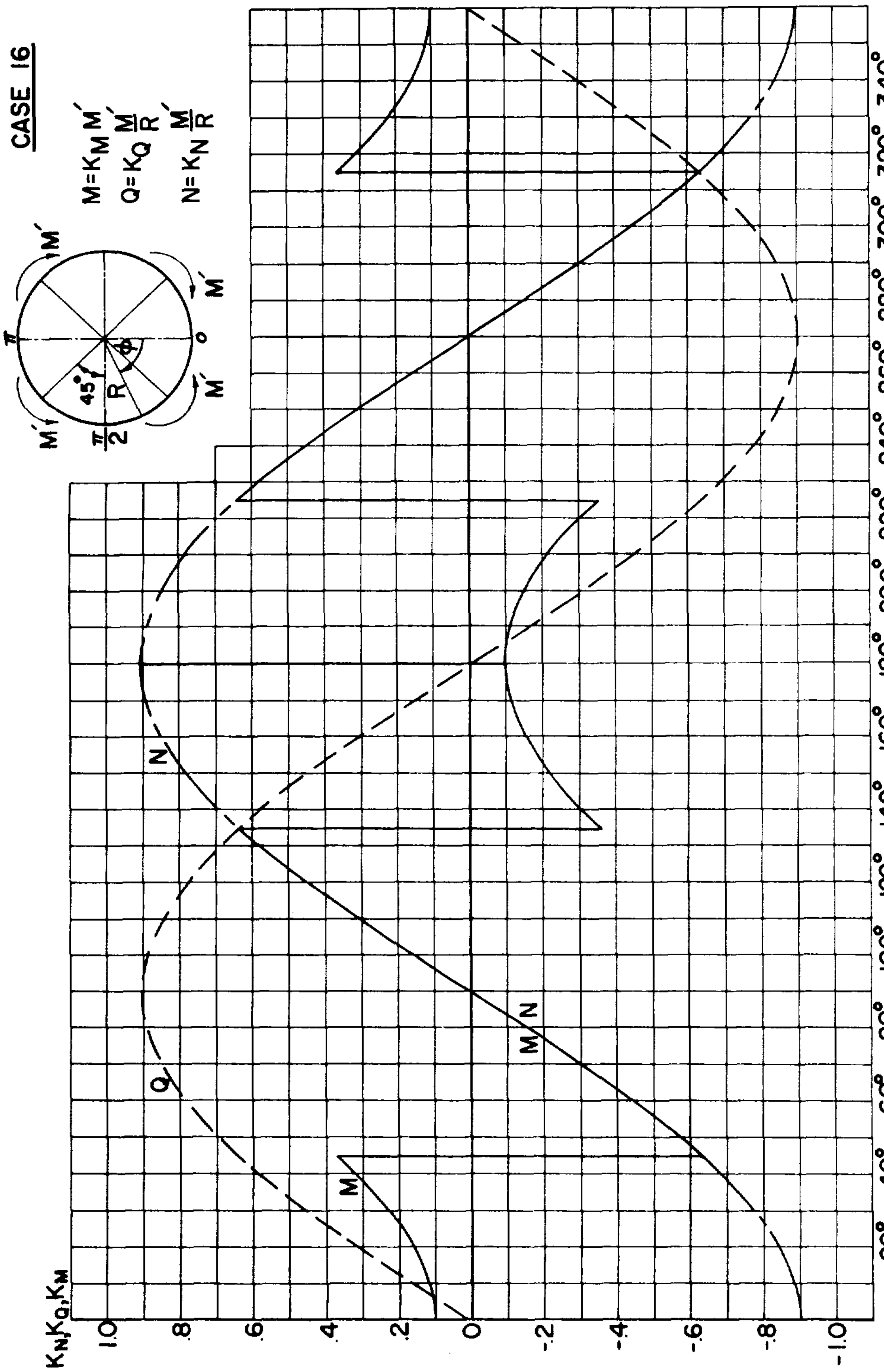
B 6.1.1 In-Plane Load Cases (Cont'd)



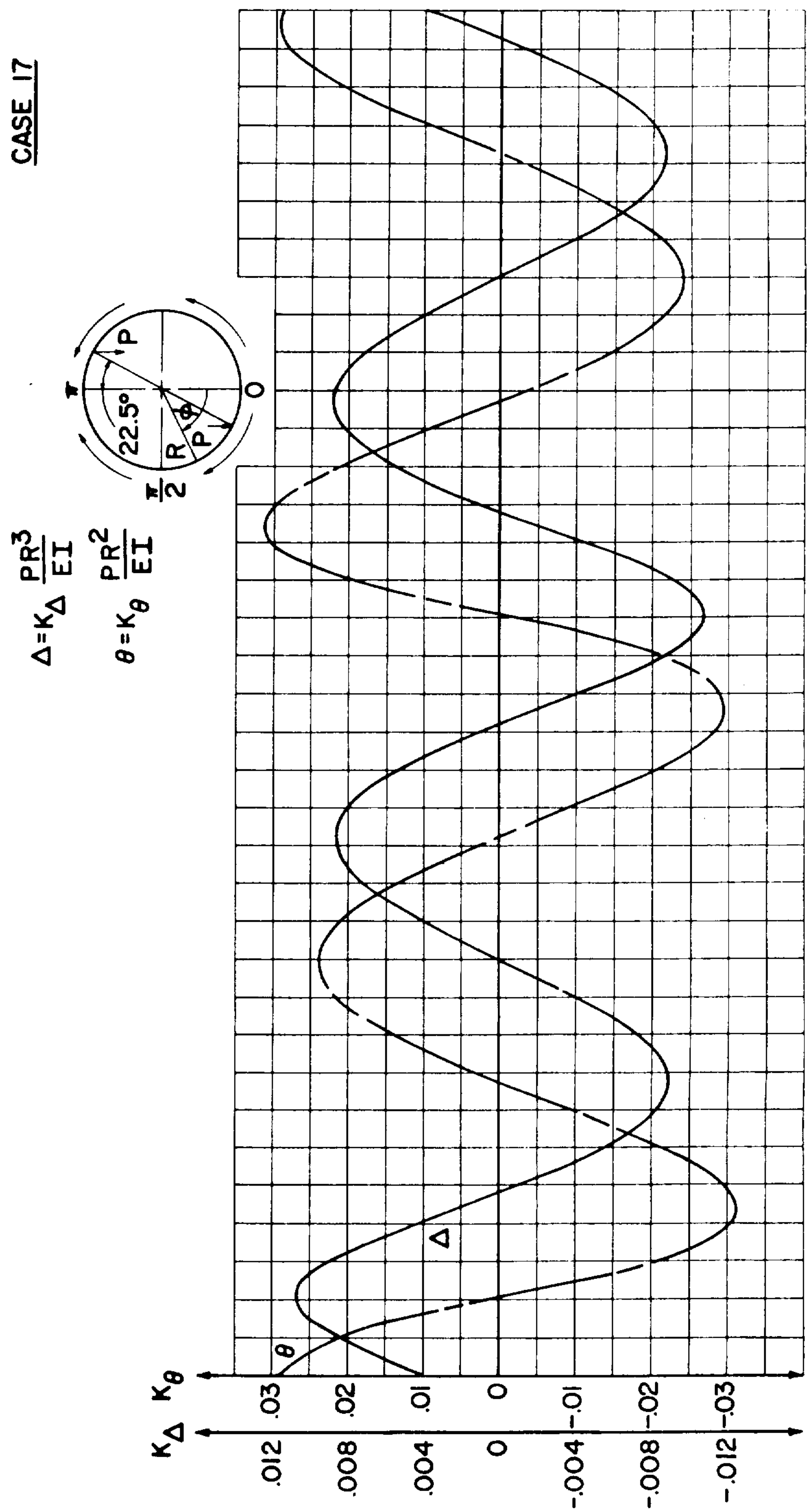
B 6.1.1 In-Plane Load Cases (Cont'd)



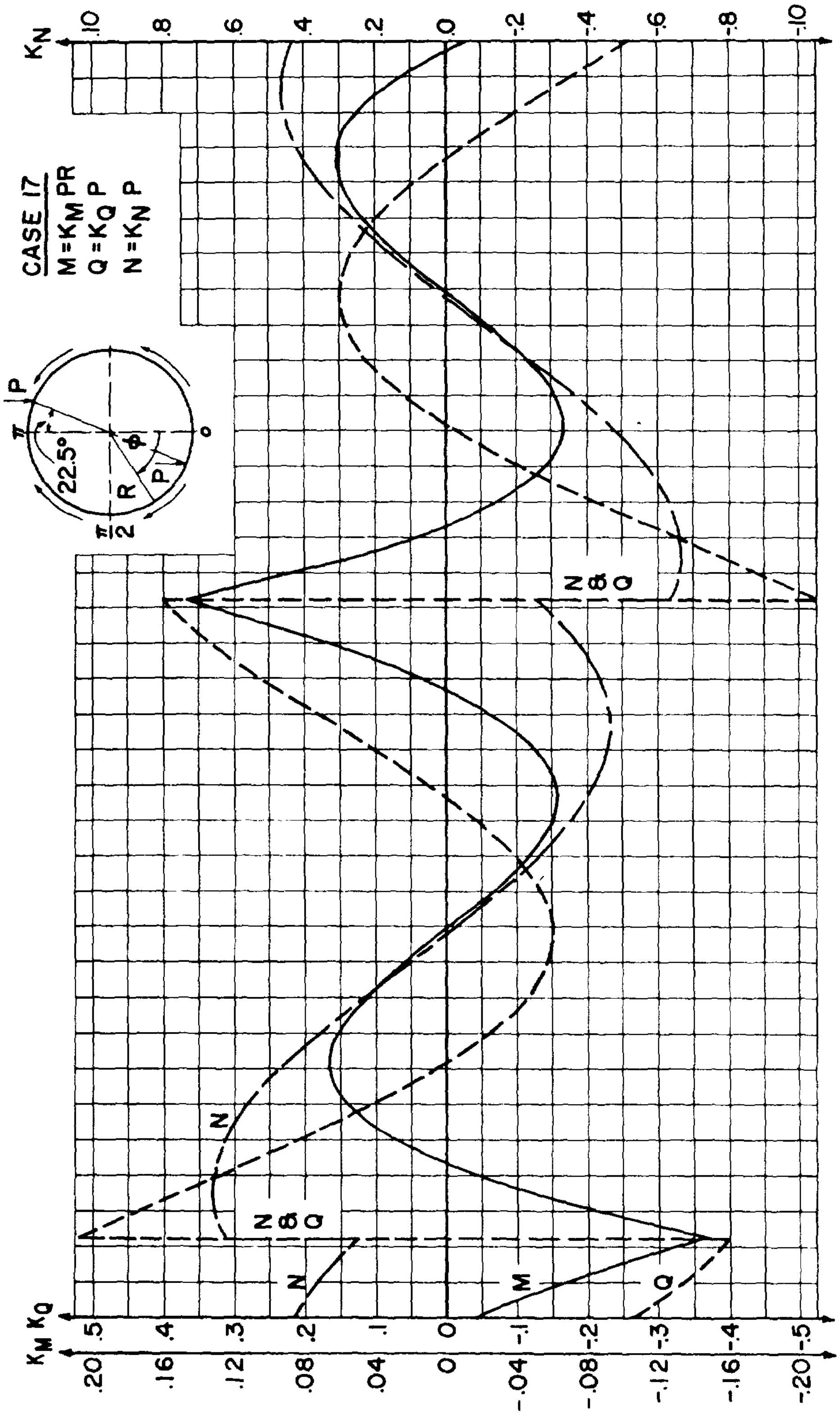
B 6.1.1 In-Plane Load Cases (Cont'd)



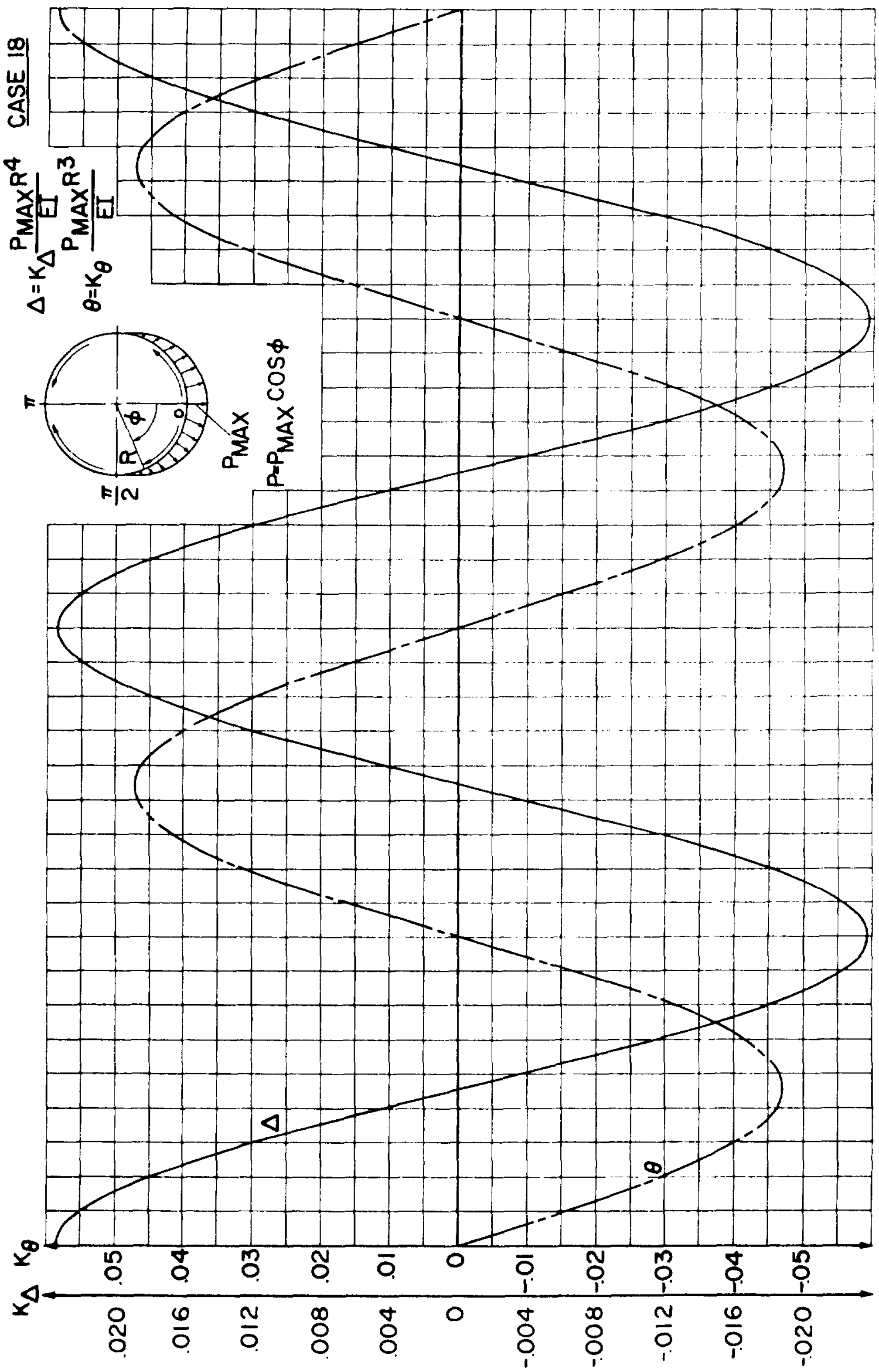
B 6.1.1 In-Plane Load Cases (Cont'd)



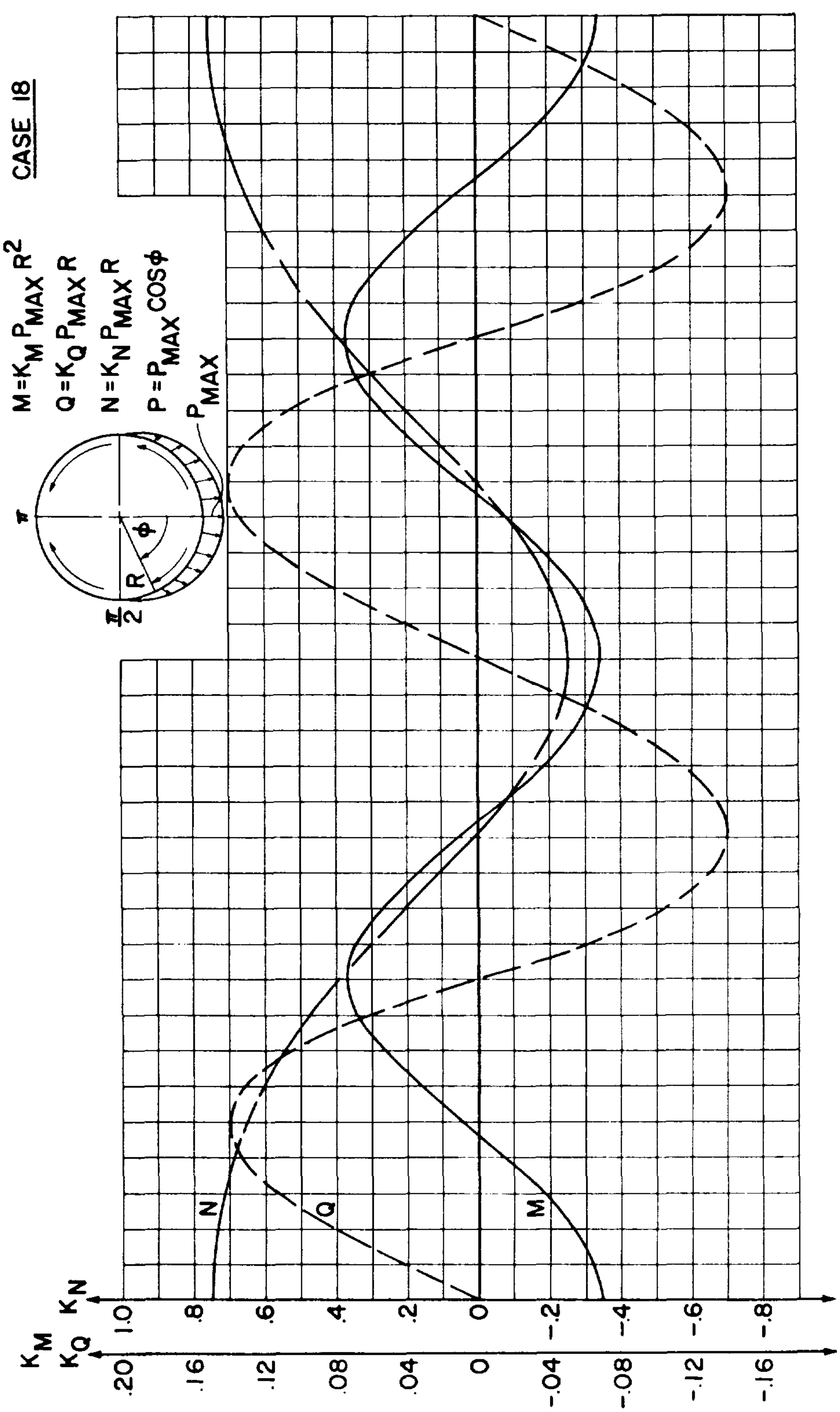
B 6.1.1 In-Plane Load Cases (Cont'd)



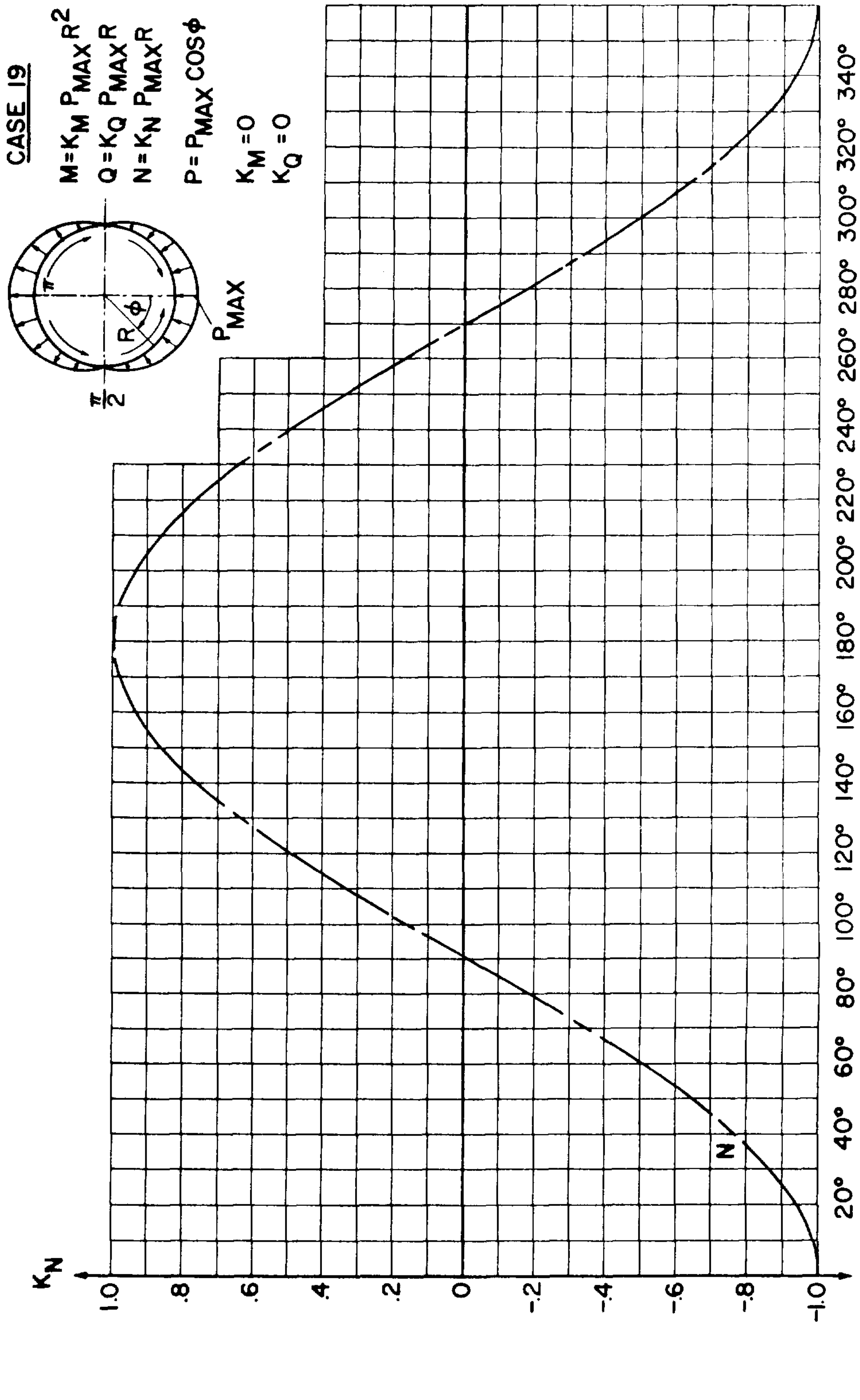
B 6.1.1 In-Plane Load Cases (Cont'd)



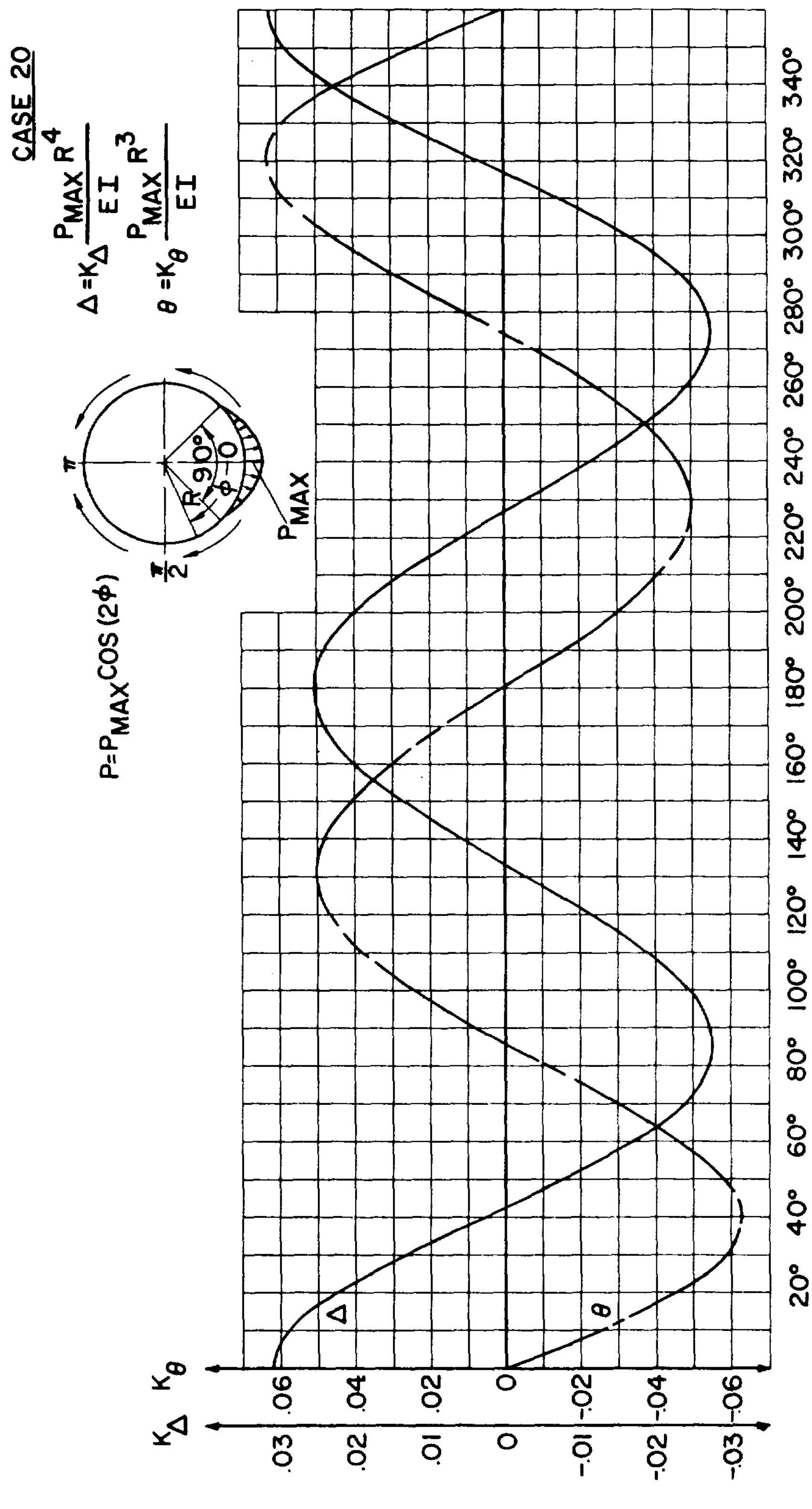
B 6.1.1 In-Plane Load Cases (Cont'd)



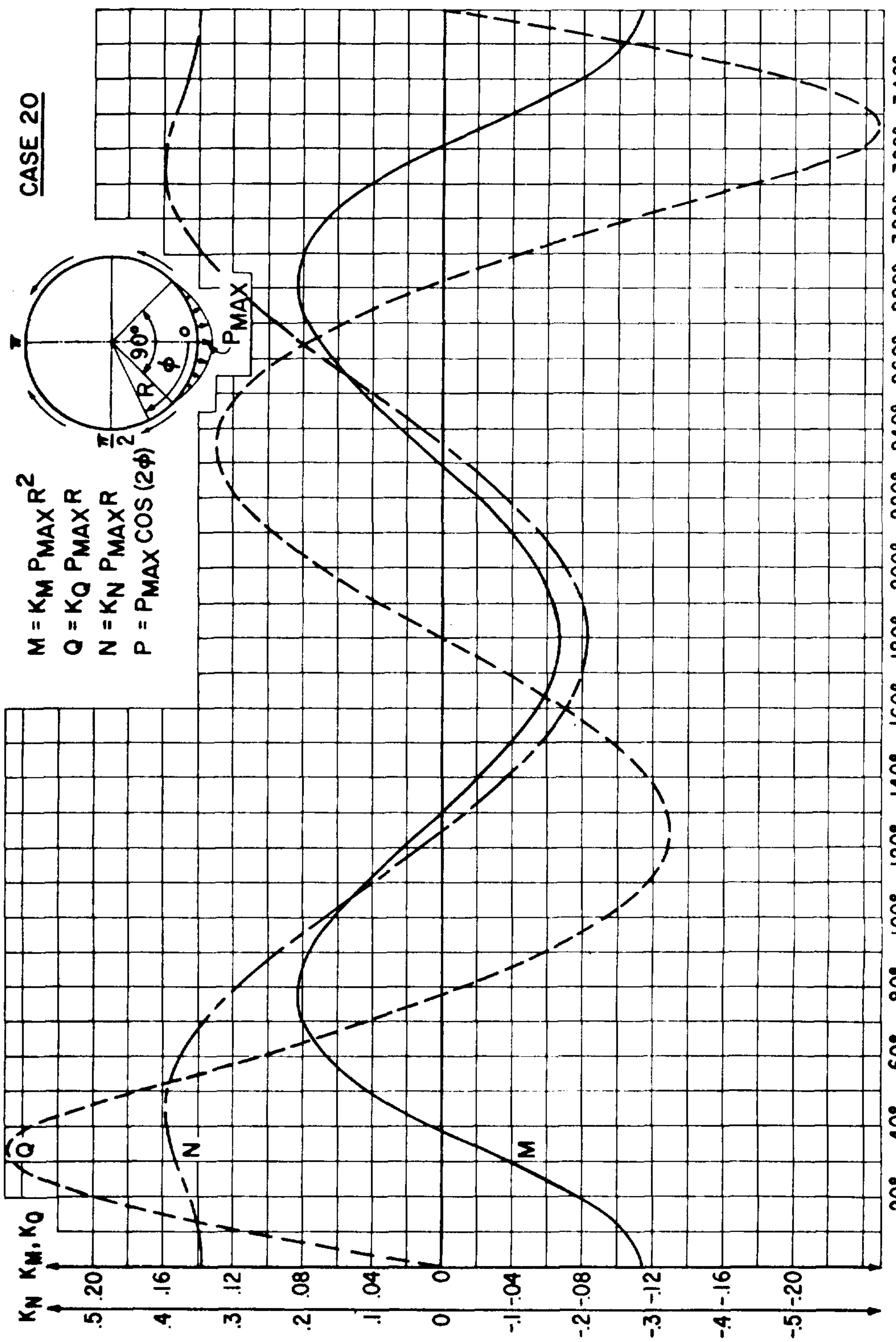
B 6.1.1 In-Plane Load Cases (Cont'd)



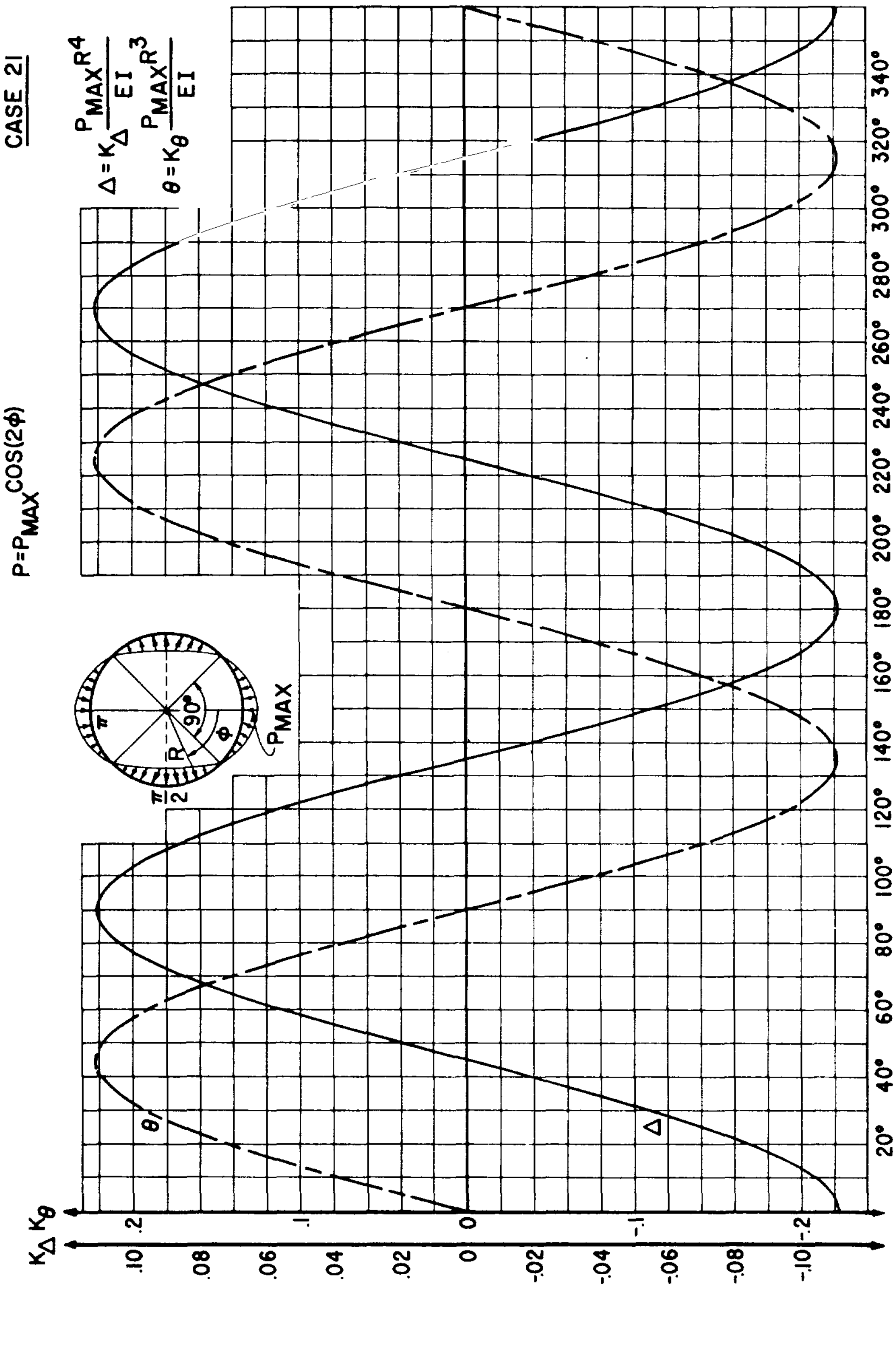
B 6.1.1 In-Plane Load Cases (Cont'd)



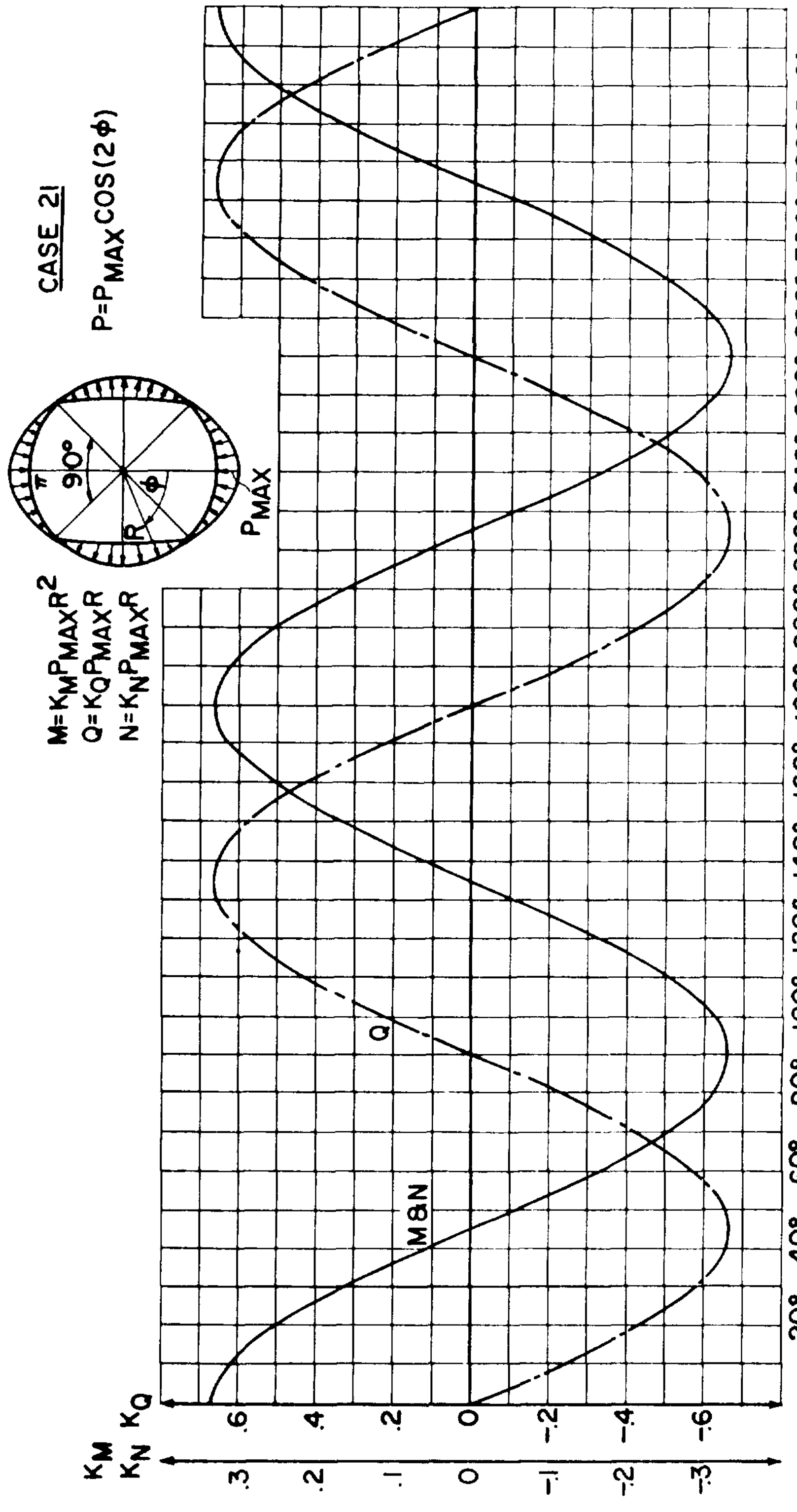
B 6.1.1 In-Plane Load Cases (Cont'd)



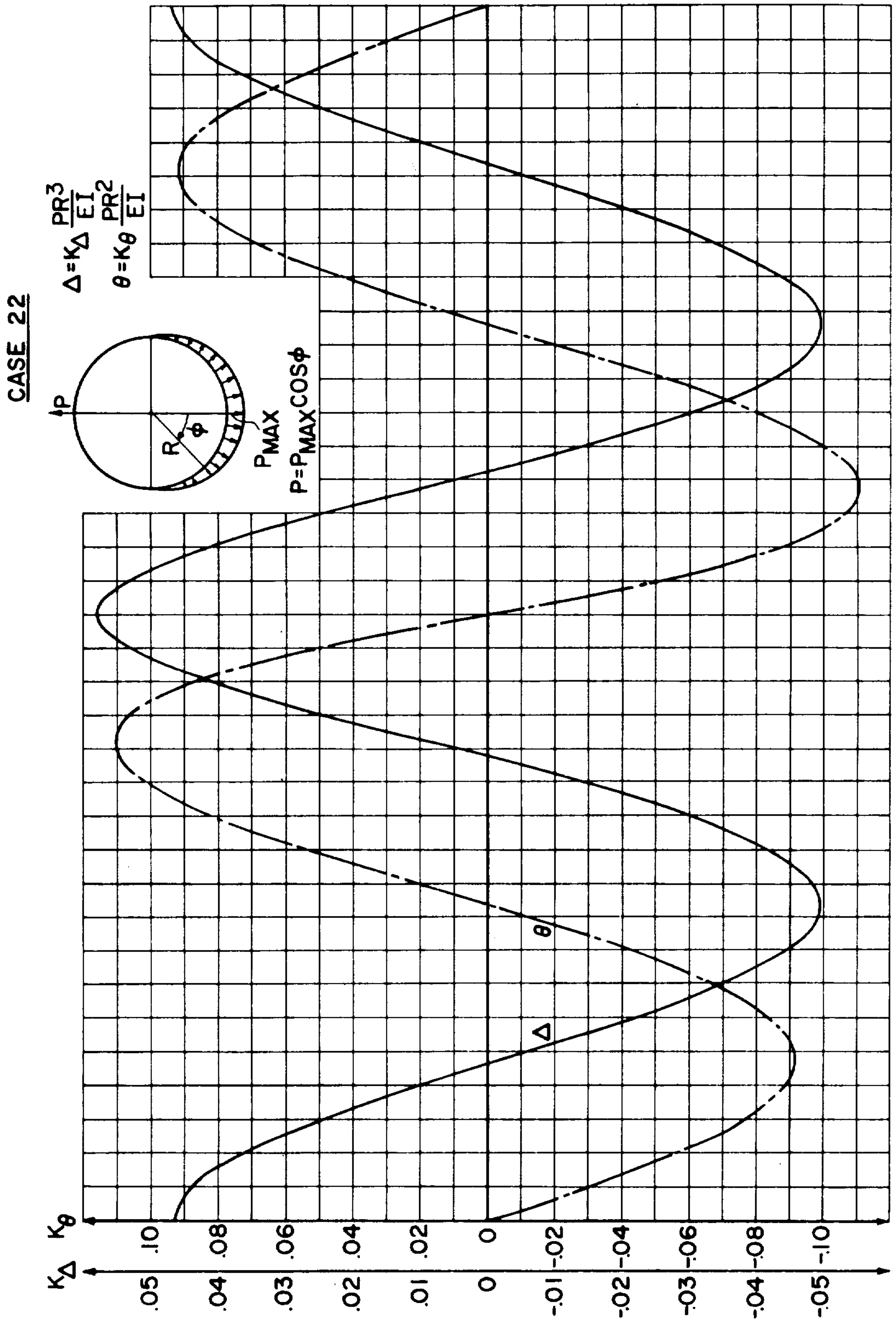
B 6.1.1 In-Plane Load Cases (Cont'd)



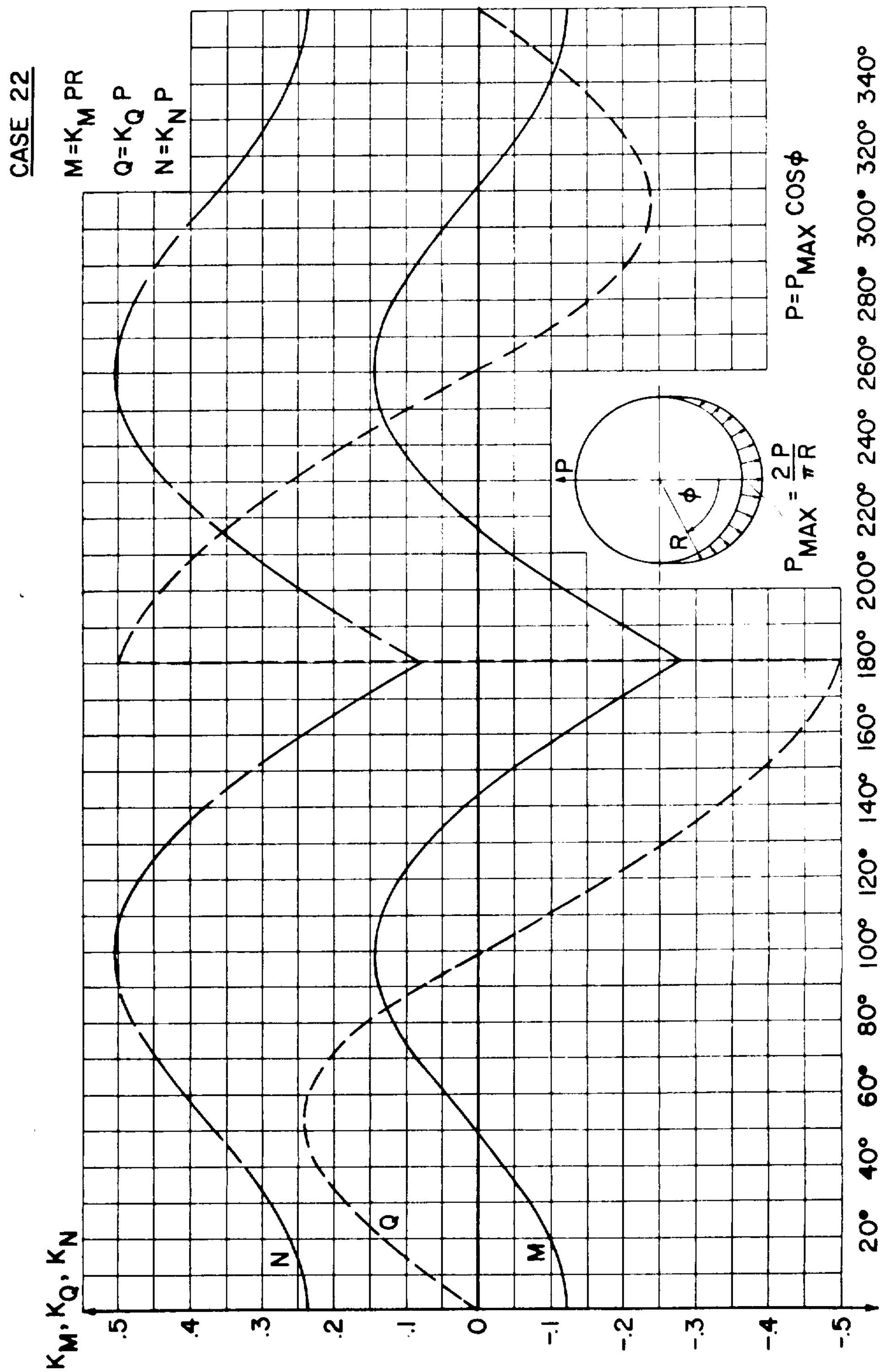
B 6.1.1 <u>In-Plane Load Cases (Cont'd)</u>



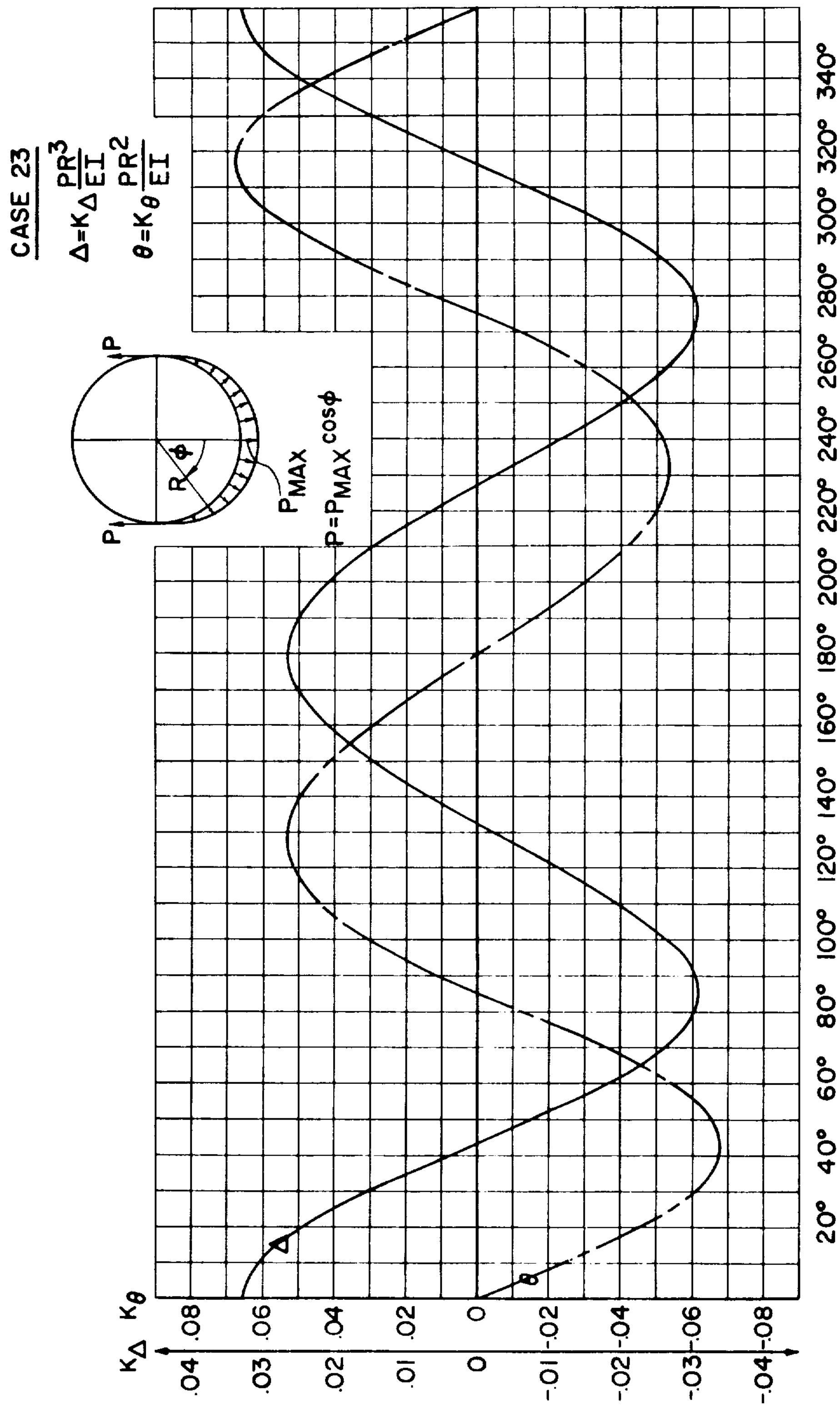
B 6.1.1 In-Plane Load Cases (Cont'd)



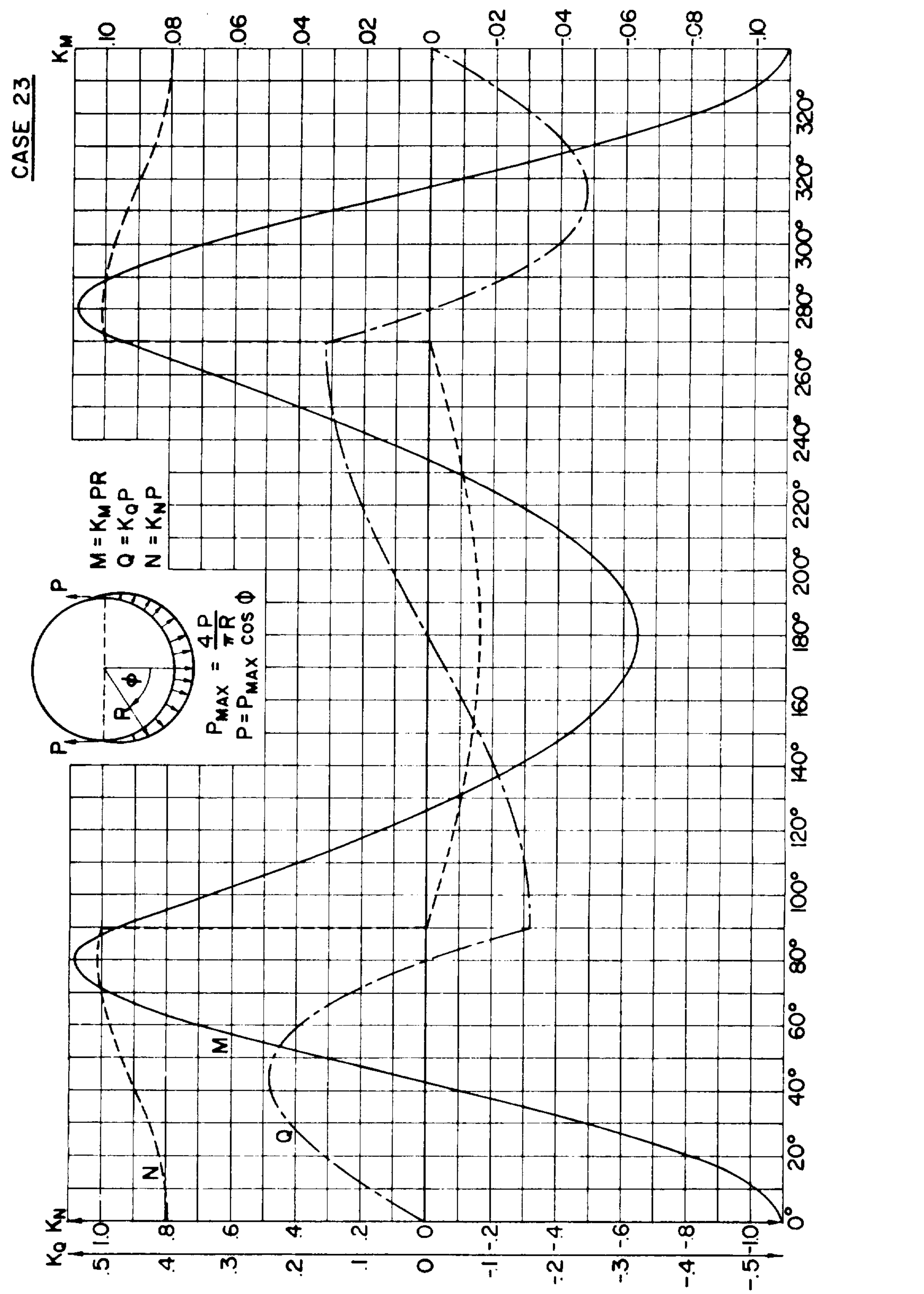
B 6.1.1 In-Plane Load Cases (Cont'd)



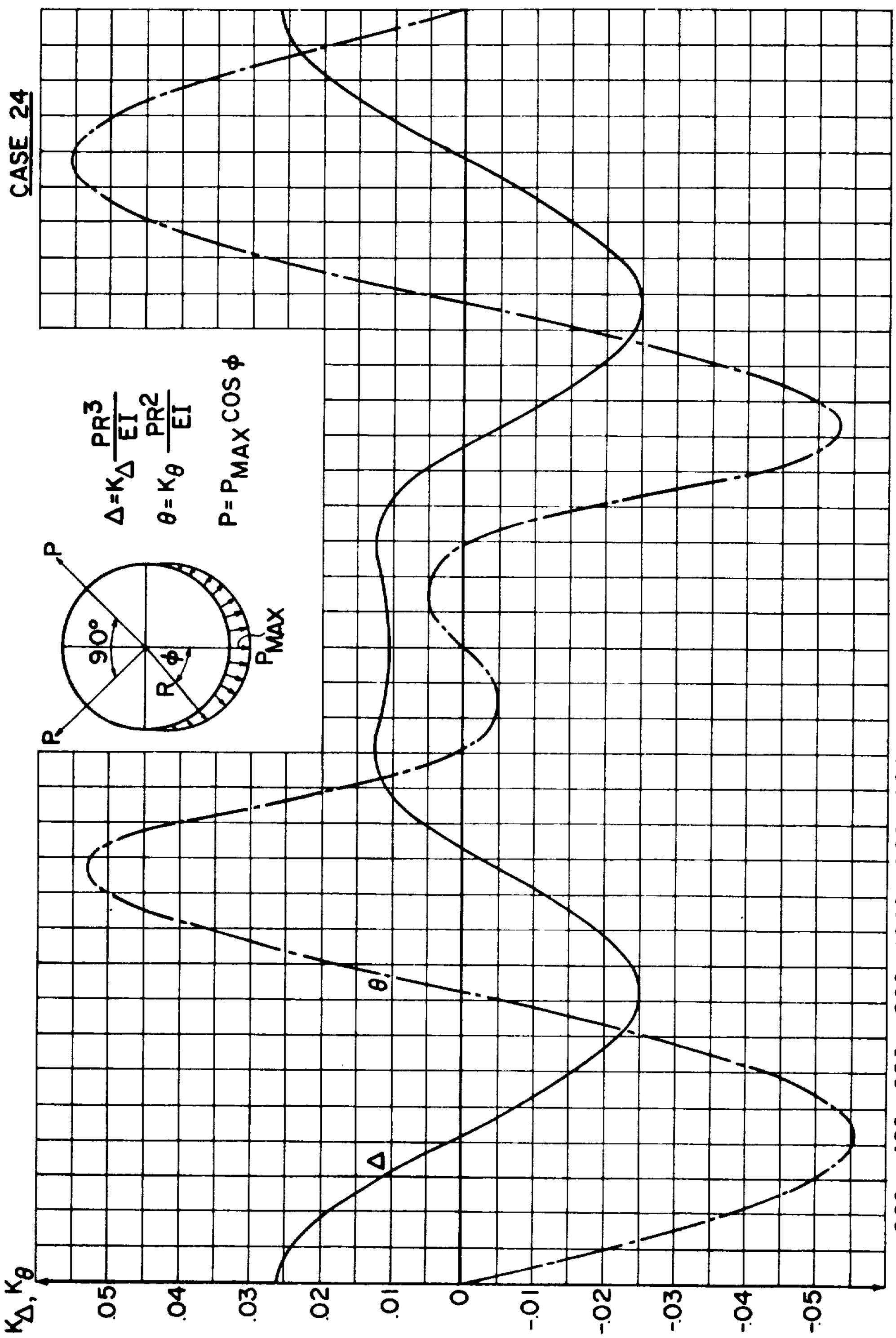
B 6.1.1 In-Plane Load Cases (Cont'd)



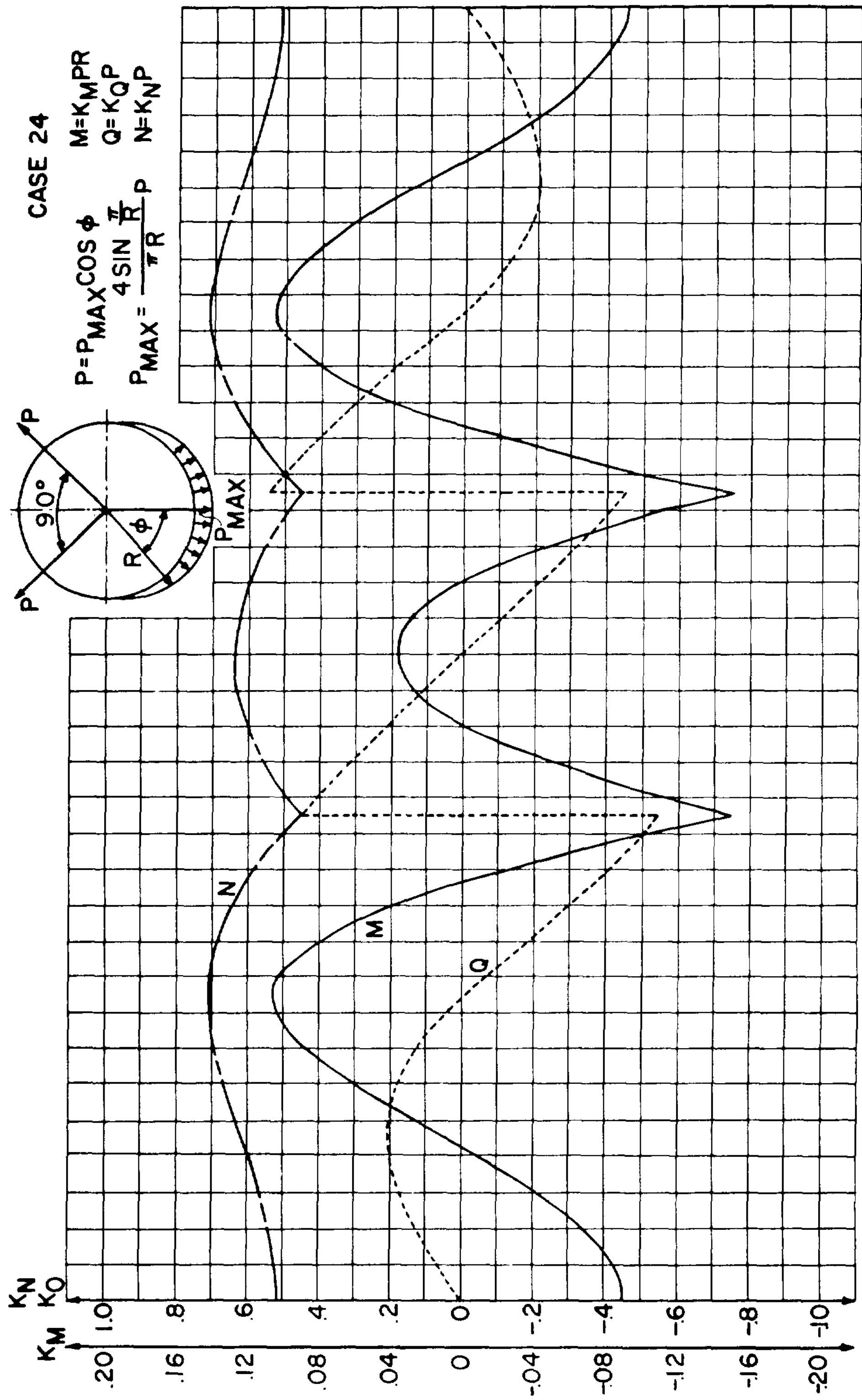
B 6.1.1 In-Plane Load Cases (Cont'd)



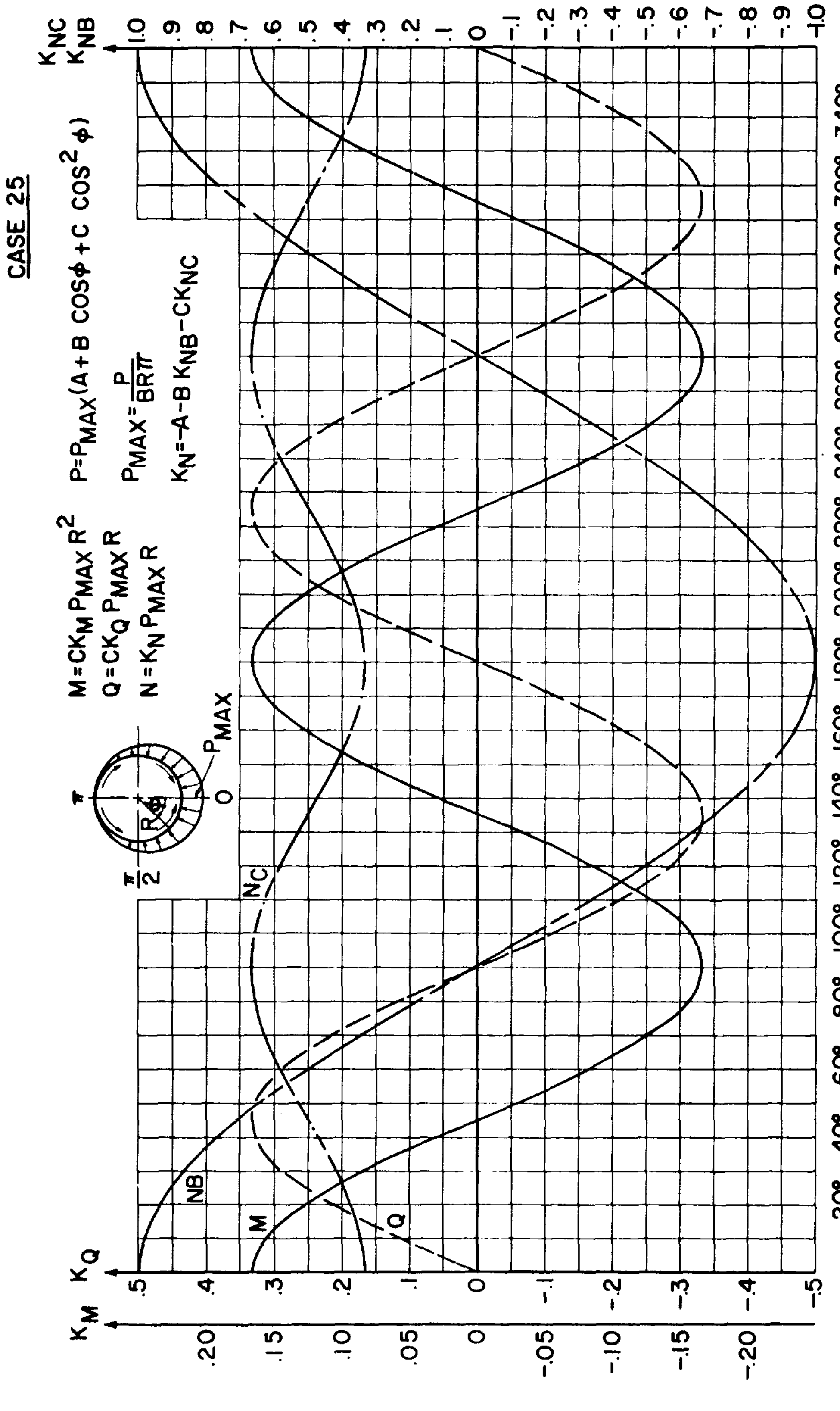
B 6.1.1 In-Plane Load Cases (Cont'd)



B 6.1.1 In-Plane Load Cases (Cont'd)



B 6.1.1 In-Plane Load Cases (Cont'd)



B 6.1.1 In-plane Load Cases (Cont'd)

Deflection curves for the three basic load cases due to shear and normal forces are displayed on the following pages. A shape factor (β) that is to be used with the curves for shear deflection of various cross-sections is tabulated below.

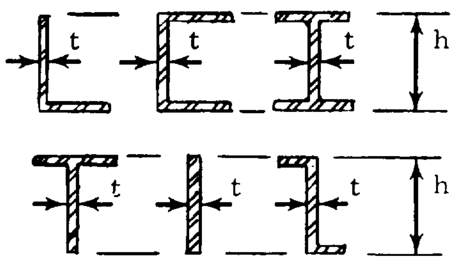
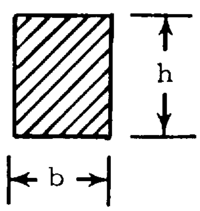
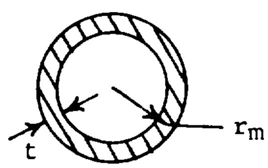
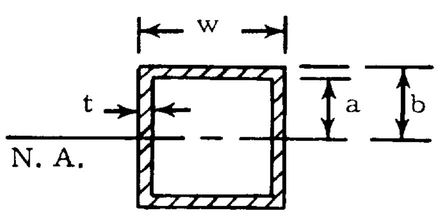
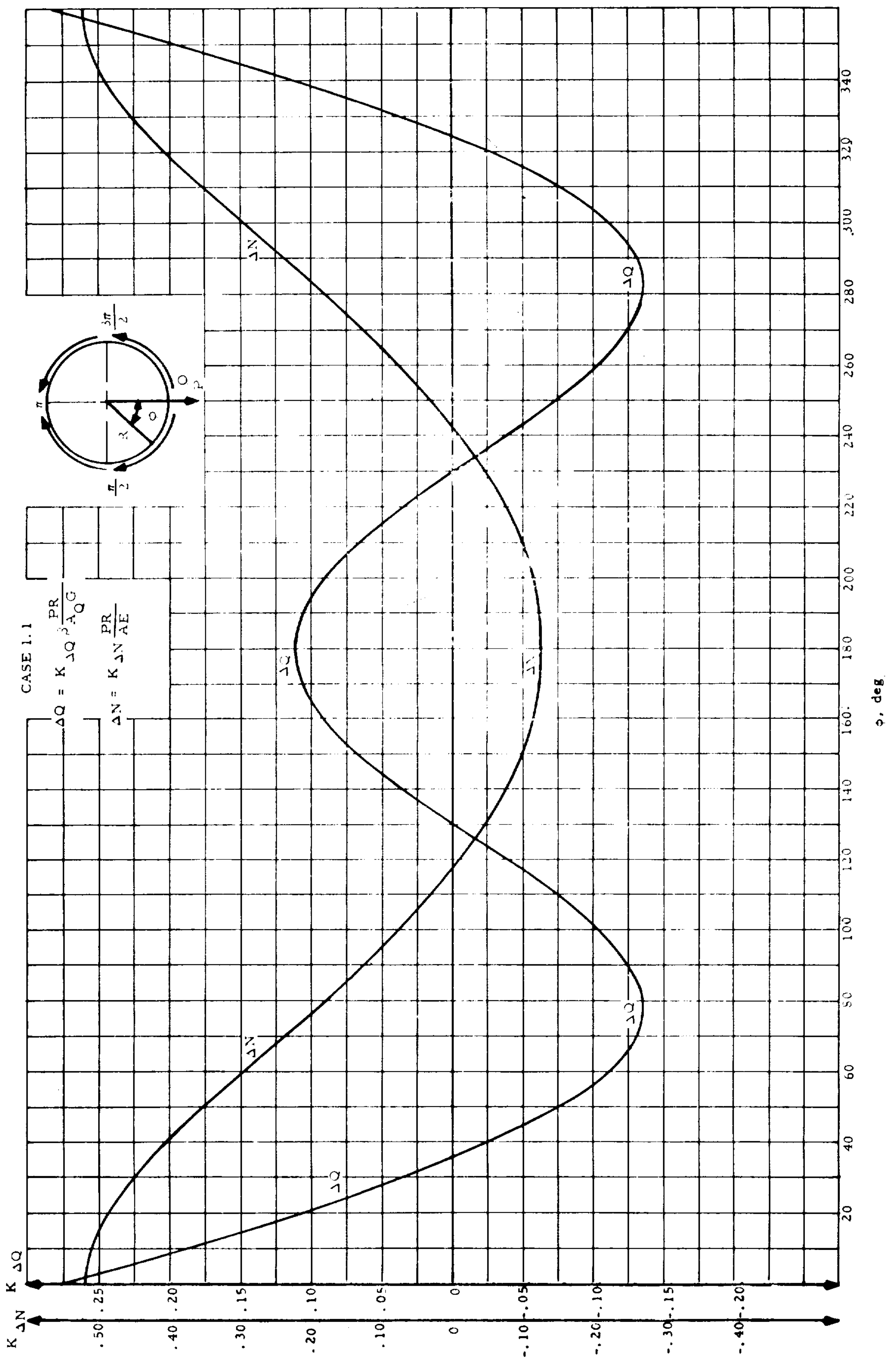
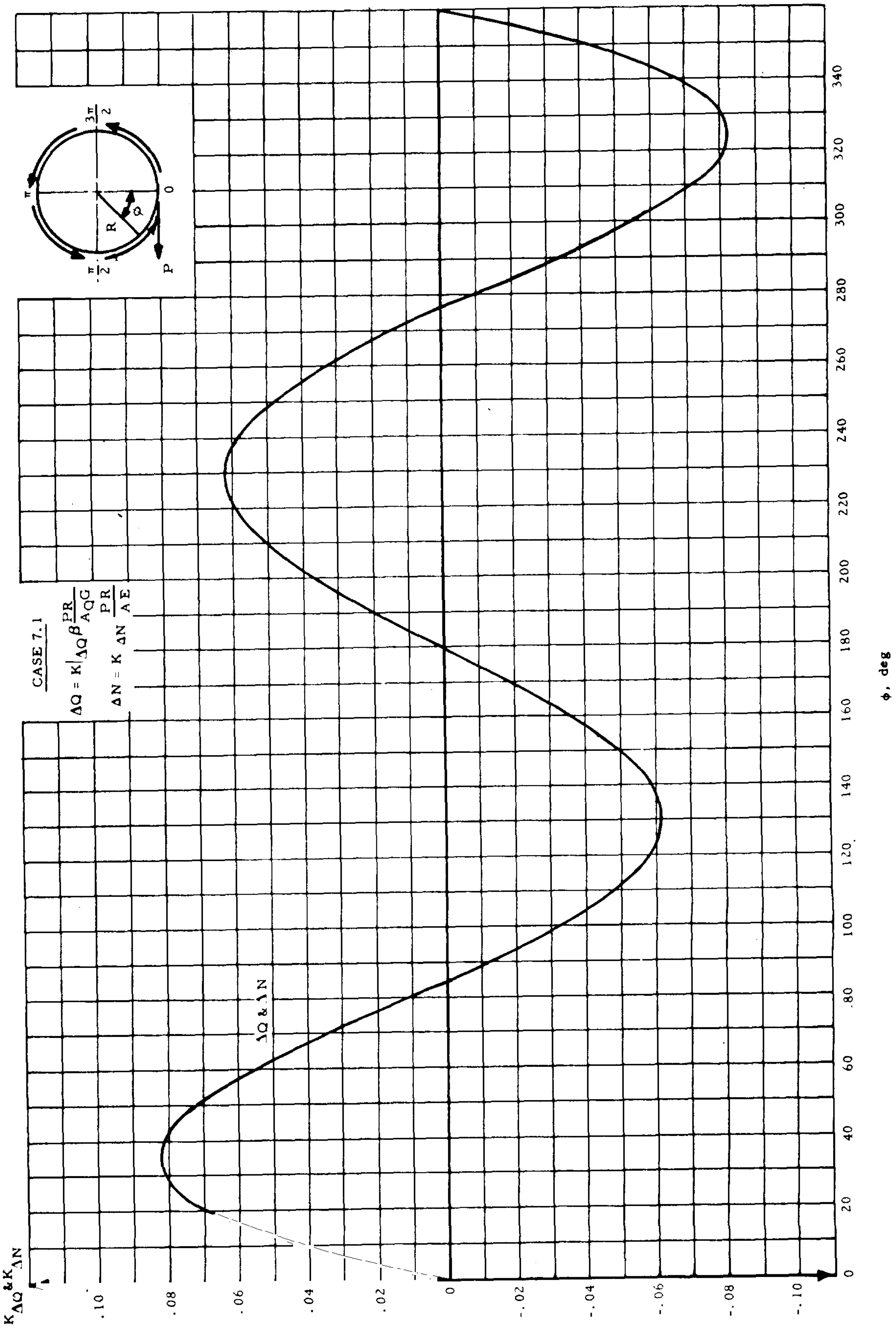
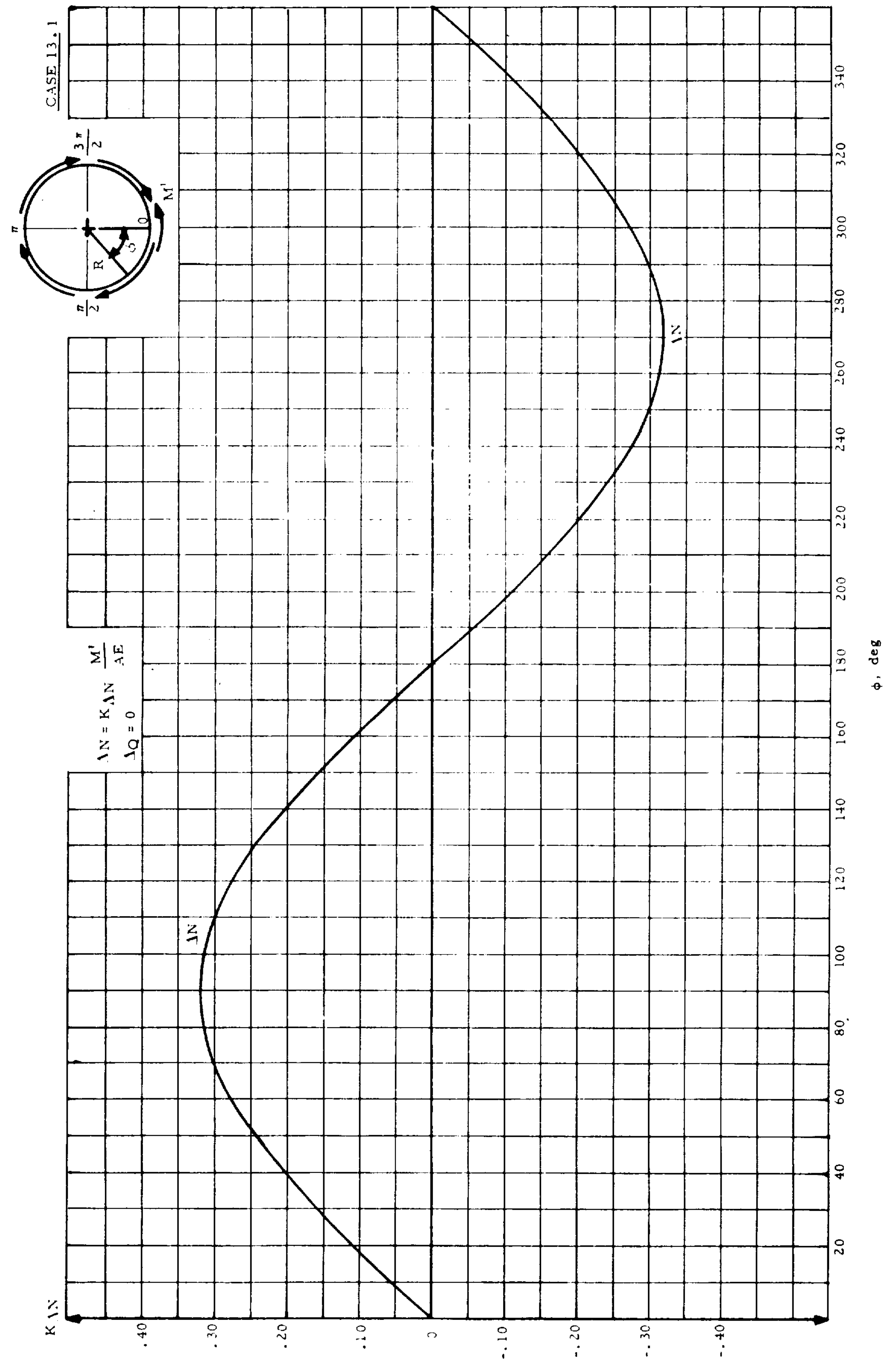
Cross-Section	Shear Area	Shape Factor, β
	Area of Web $A_Q = th$	$\beta = 1.00$
	Entire Area $A_Q = bh$	$\beta = 1.20$ for $b \geq 0.50h$ $\beta = 1.00$ for $b < 0.50h$
	Entire Area $A_Q = 2\pi r_m t$	$\beta = 2.00$
 ρ = radius of gyration with respect to the neutral axis	Entire Area $A_Q = (w)^2 - (2a)(w-2t)$	$\beta = \left[1 + \frac{3(b^2 - a^2)a}{2b^3}\left(\frac{w}{t} - 1\right)\right]\left[\frac{4b^2}{10\rho^2}\right]$ If the flanges are of nonuniform thickness, they may be replaced by an "equivalent" section whose flanges have the same width and area as those of the actual section.

Fig. B6.1.1-1

B 6.1.1 In-Plane Load Cases (Cont'd)




CASE 7.1
ΔQ = K|ΔQ β PR/(A_Q G)
ΔN = K ΔN PR/(AE)
ΔQ & ΔN
K_ΔQ & K_ΔN
.10
.08
.06
.04
.02
0
-.02
-.04
-.06
-.08
-.10
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
320
340
φ, deg
3π/2
π
0
π/2
R
φ
P


CASE 13.1
ΔN = K_ΔN M'/AE
ΔQ = 0
ΔN
K_ΔN
φ, deg
M'
R
.40
.30
.20
.10
0
-.10
-.20
-.30
-.40
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
320
340

B 6.1.2 Out-of-plane Load Cases

Sign Convention

The following sign convention is given to define the positive directions for out-of-plane loads.

Moments which produce tension on the inner fibers are positive. Torque "T" and lateral shear "V" are positive as shown in Fig. B 6.1.2-1.

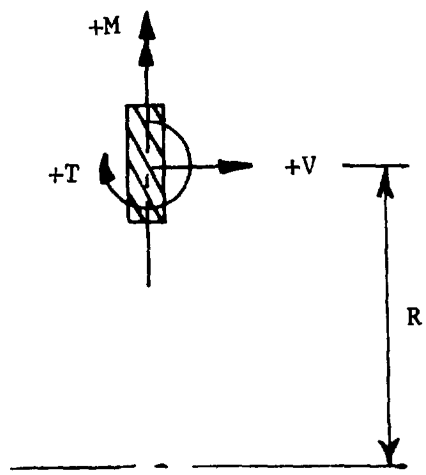


Fig. B 6.1.2-1

B 6.1.2 Out-of-Plane Load Cases (Cont'd)

Index

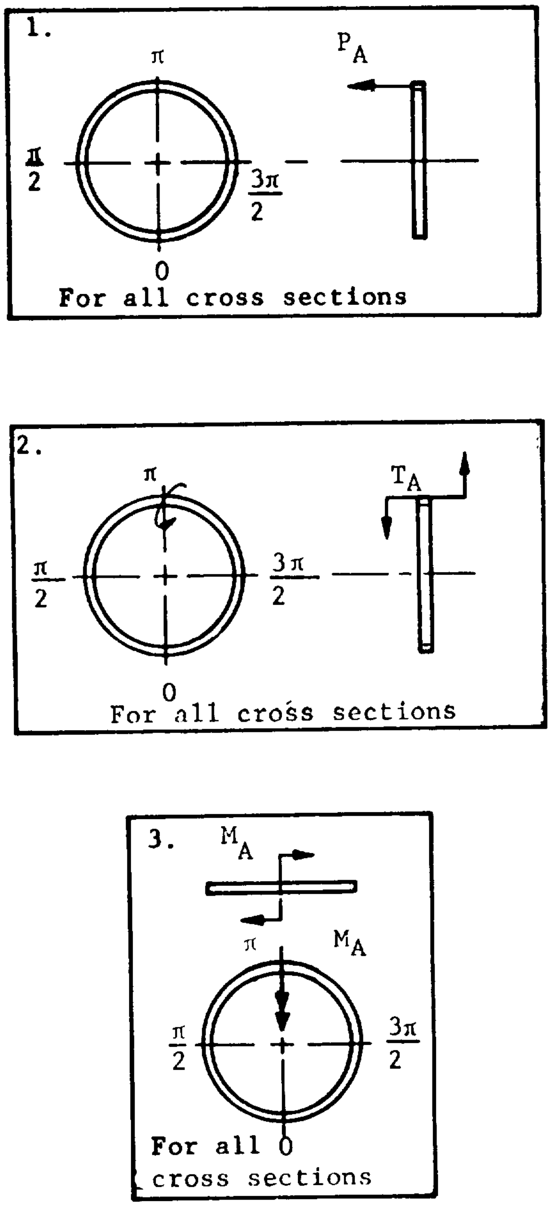
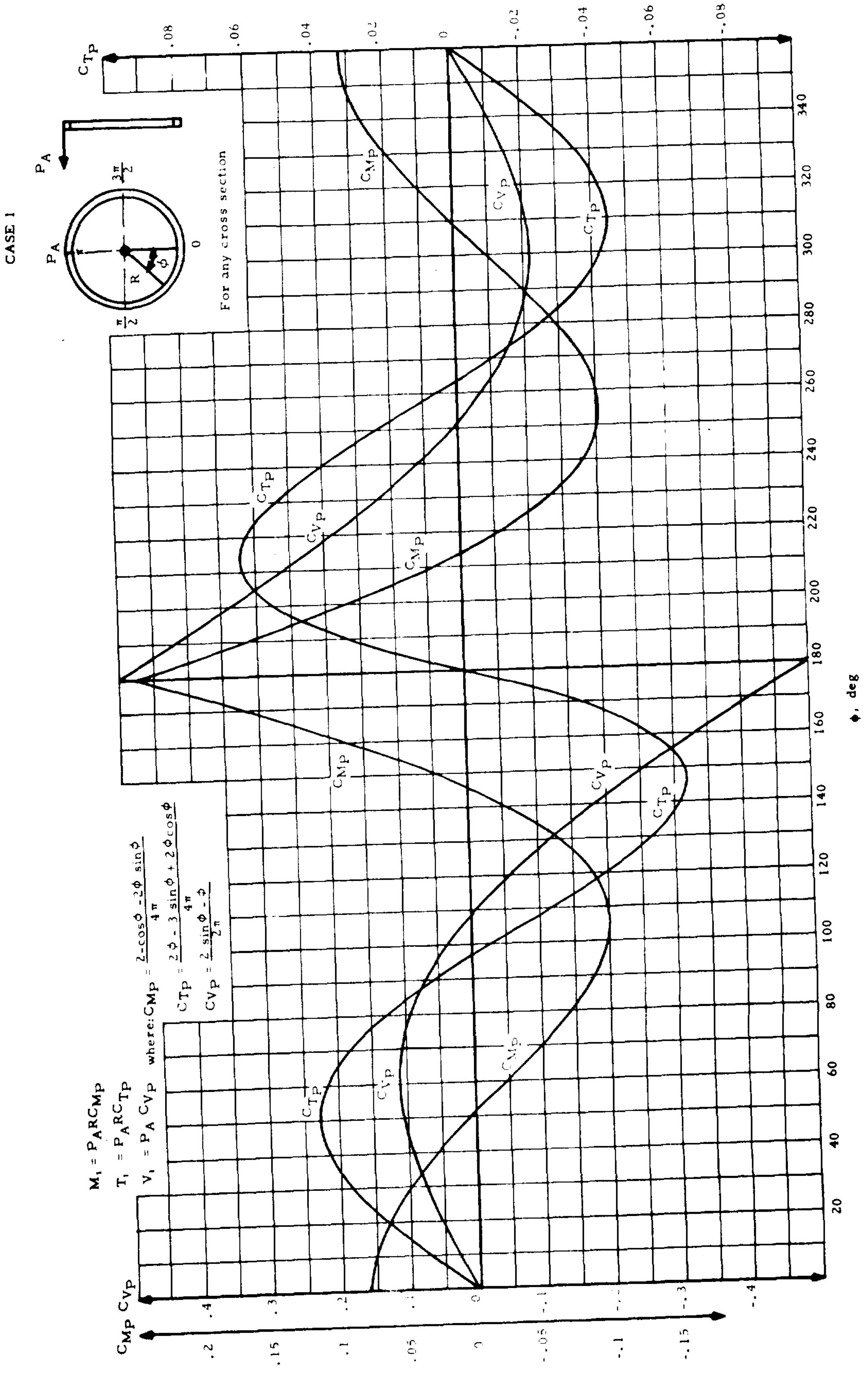
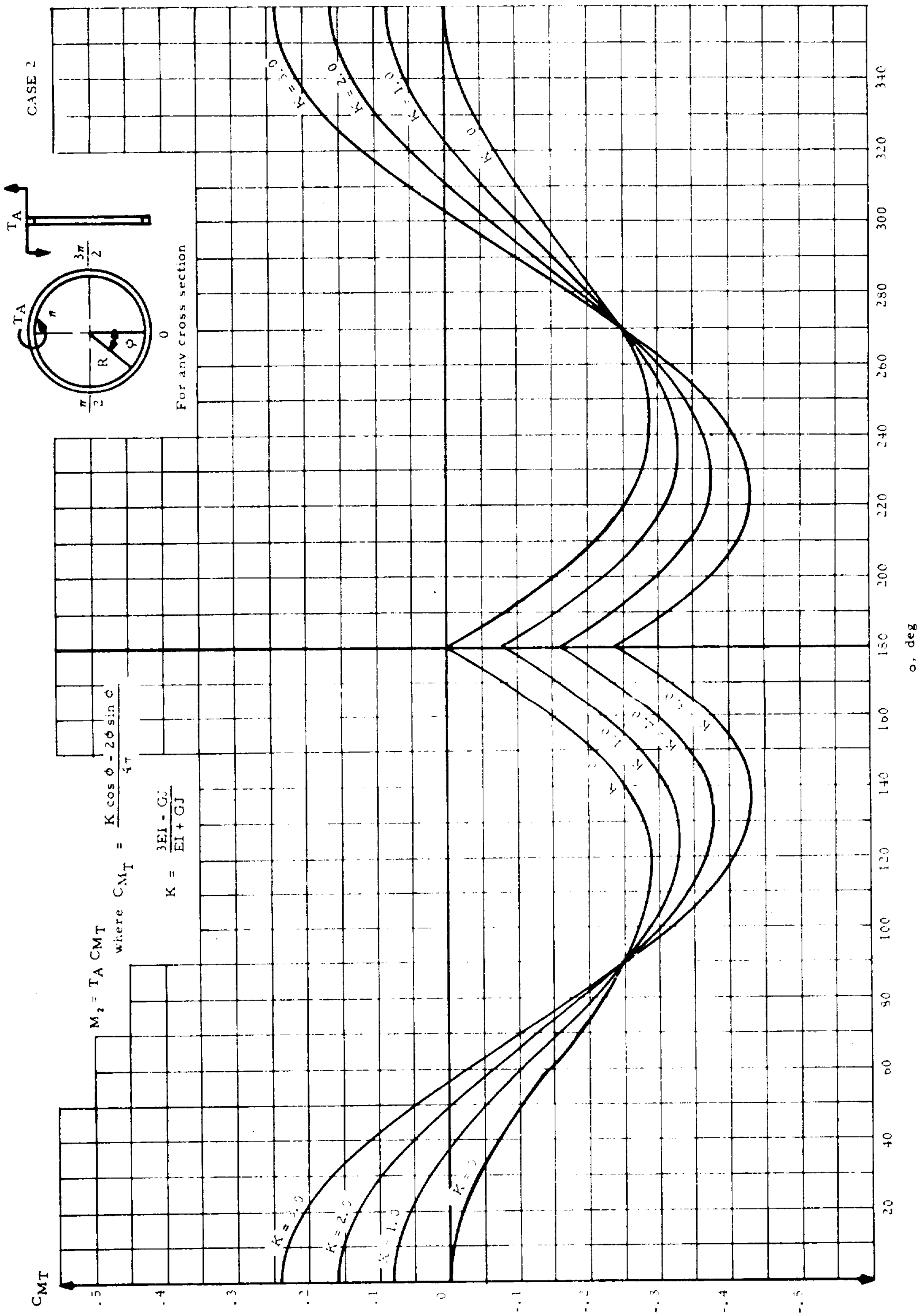
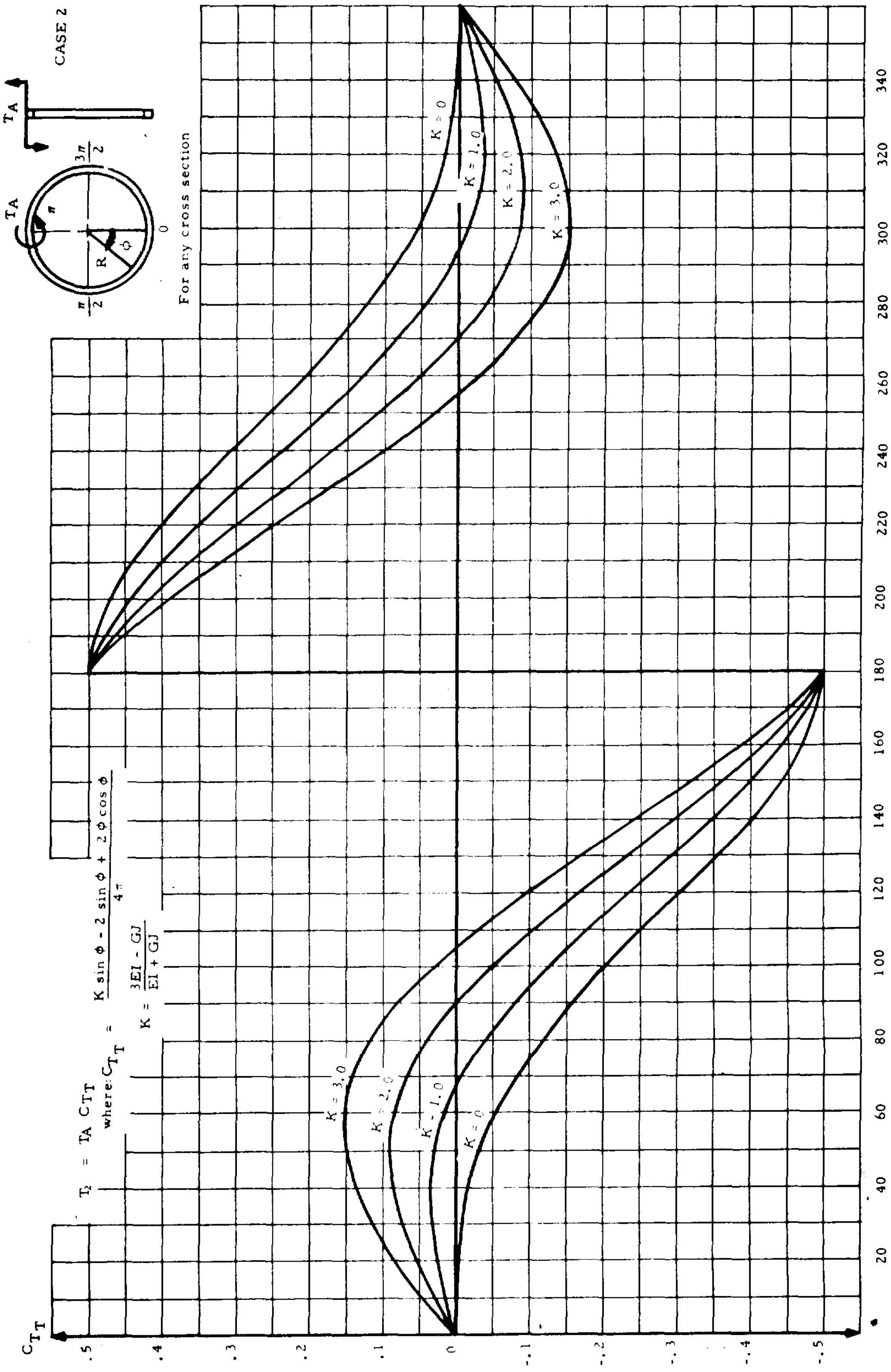


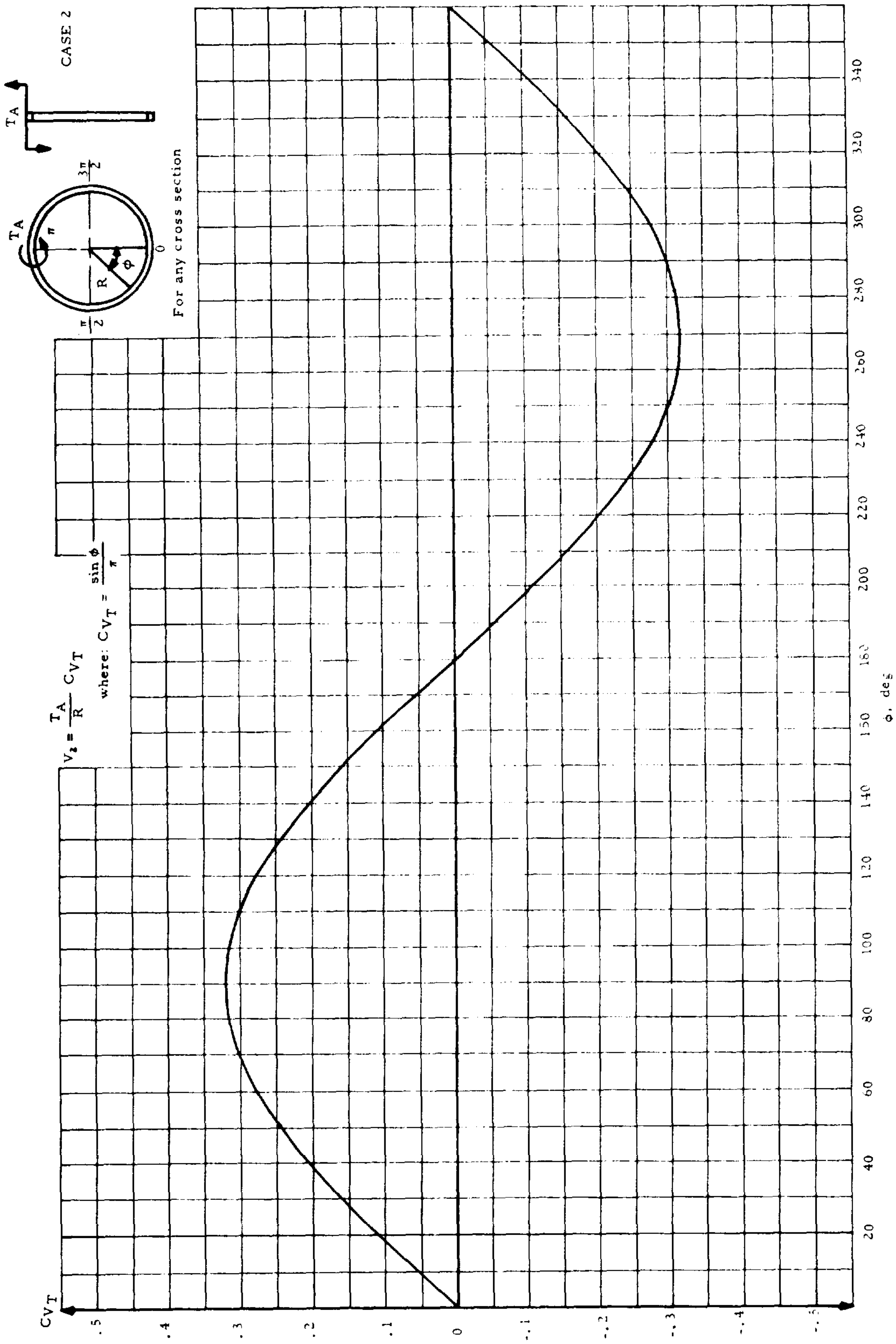
Fig. B 6.1.2-2

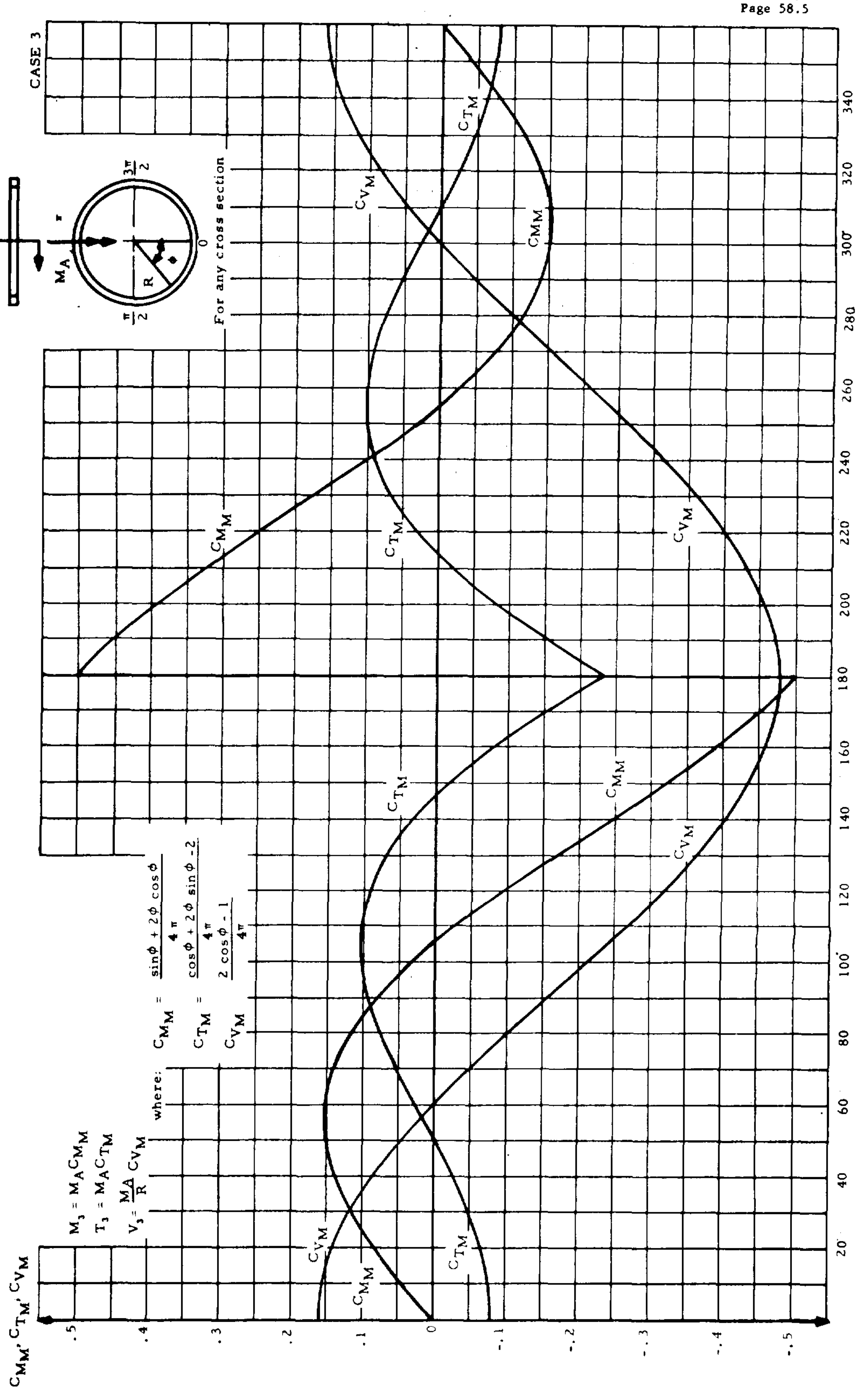
B 6.1.2 Out-of-Plane Load Cases (Cont'd)




CASE 2
For any cross section
$M_2 = T_A C_{M_T}$
where $C_{M_T} = \frac{K \cos \phi - 2\phi \sin \phi}{4\pi}$
$K = \frac{3EI - GJ}{EI + GJ}$
K = 3.0
K = 2.0
K = 1.0
K = 0
φ, deg
C_{M_T}


CASE 2
TA
3π/2
π
π/2
R
φ
0
For any cross section
$T_2 = T_A C_{T_T}$
where: $C_{T_T} = \frac{K \sin\phi - 2\sin\phi + 2\phi\cos\phi}{4\pi}$
$K = \frac{3EI - GJ}{EI + GJ}$
K = 0
K = 1.0
K = 2.0
K = 3.0
C_{T_T}
.5
.4
.3
.2
.1
0
-.1
-.2
-.3
-.4
-.5
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
320
340
φ, deg


CASE 2
TA
For any cross section
$V_z = \frac{T_A}{R} C_{V_T}$
where: $C_{V_T} = \frac{\sin \phi}{\pi}$
C_{V_T}
.5
.4
.3
.2
.1
0
-.1
-.2
-.3
-.4
20
40
60
80
100
120
140
160
200
220
240
260
280
300
320
340
φ, deg


CASE 3
For any cross section
M_A
M_A
R
φ
π
0
π/2
3π/2
$M_3 = M_A C_{M_M}$
$T_3 = M_A C_{T_M}$
$V_3 = \frac{M_A}{R} C_{V_M}$
where:
$C_{M_M} = \frac{\sin\phi + 2\phi\cos\phi}{4\pi}$
$C_{T_M} = \frac{\cos\phi + 2\phi\sin\phi - 2}{4\pi}$
$C_{V_M} = \frac{2\cos\phi - 1}{4\pi}$
$C_{M_M}, C_{T_M}, C_{V_M}$
.5
.4
.3
.2
.1
0
-.1
-.2
-.3
-.4
-.5
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
320
340
φ, deg
C_{V_M}
C_{M_M}
C_{T_M}

B 6.2.0 Analysis of Frame-Reinforced Cylindrical Shells

Tables are presented giving the loads and displacements in a flexible frame supported by a circular cylindrical shell and subjected to concentrated radial tangential, and moment loads. Additional tables give the loads in the shell. The solutions are presented in terms of two basic parameters, one of which is of second-order importance. Procedure for modifying the important parameter to account for certain non-uniform properties of the structure are presented.

Notation

A $\frac{2.25}{\gamma^4}$

B $\left(\frac{L_r}{L_c}\right)^2 / \gamma^2$

E Young's modulus ~ lb/in^2

E_f Young's modulus of unloaded frames ~ lb/in^2

E_o Young's modulus of loaded frame ~ lb/in^2

E_{sk} Young's modulus of skin ~ lb/in^2

e base of natural logarithms

F axial force in loaded frame ~ lb

G shear modulus ~ lb/in^2

I moment of inertia of a typical unloaded frame ~ in^4

I_ℓ moment of inertia of an unloaded frame, distance "ℓ" from the loaded frame ~ in^4

I_o moment of inertia of the loaded frame ~ in^4

i $I/\ell o$ ~ in^3

K_n $$\frac{n\sqrt{n^2-1}}{2\sqrt{3}} \cdot \frac{1 + 2\,\frac{n^2-1}{3}\left(\frac{L_r}{L_c}\right)^2}{\sqrt{1 + \frac{n^2-1}{3}\left(\frac{L_r}{L_c}\right)^2}}$$

ℓ distance from loaded frame to undistorted shell section ~ in

B 6.2.0 Analysis of Frame-Reinforced Cylindrical Shells (Cont'd)

Notation (Cont'd)

L_c characteristic length (see Glossary) = $\frac{r}{\sqrt{6}} \left[\frac{t' r^2}{\ell} \right]^{1/4}$ ~ in

L_r characteristic length (see Glossary) = $\frac{r}{2} \sqrt{\frac{Et'}{Gt}}$ ~ in

ℓ_o frame spacing ~ in

M bending moment in loaded frame ~ in-lb

M_o externally applied concentrated moment ~ in.lb

P_o externally applied radial load ~ lb

p axial load per inch in the shell ~ lb/in

q shear flow in shell ~ lb/in

r radius of skin line ~ in

S transverse shear force in loaded frame ~ lb

s transverse shear per inch in shell ~ lb/in

T_o externally applied tangential load ~ lb

t skin panel thickness ~ in

t' effective skin panel thickness for axial loads ~ in

t_e weighted average of all the bending material (skin and stiffeners) adjacent to the loaded frame, assumed uniformly distributed around the perimeter ~ in.

u axial displacement of shell ~ in.

v tangential displacement of shell ~ in.

w radial displacement of shell ~ in.

x axial co-ordinate of shell ~ in.

γ "beef up" parameter $I_o/2i\ L_c$

γ_ℓ γ for nearby heavy frame

B 6.2.0 Analysis of Frame-Reinforced Cylindrical Shells (Cont'd)

Notations (Cont'd)

θ rotational displacement ~ radians

ϕ polar co-ordinate of frame and shell

Basic Assumptions

In the method of attack with which this section is mainly concerned, a simplified structural model (Fig. B 6.2.0-1) is used to obtain a solution for a uniform shell stretching to infinity on both sides of the loaded frame. Clearly the effects of any frame can be propagated only a finite distance along the shell. In practice, the perturbations from the "elementary beam theory" are, at worst, negligible at some characteristic length "L_c" inches away from the loaded frame. Procedures for modifying the solution to account for discontinuities and non-uniform properties are discussed in the following sections. For the model used, the following assumptions are made:

(1) Concentrated loads are applied to the loaded frame and are reacted an infinite distance away on either one or both sides. The shell extends to infinity on both sides.

(2) The loaded frame has in-plane bending flexibility. It is free to warp out of its plane and to twist. It has no axial or shearing flexibilities. Its moment of inertia for circumferential bending is constant.

(3) The effects of the eccentricity of the skin attachment with respect to the frame neutral axis is ignored for both the loaded and unloaded frames.

(4) The shell consists of skin, longerons, and frames similar to the loaded frame, but possibly with different moments of inertia. The skin and longerons have no bending stiffness. All properties of the shell are uniform.

(5) The longerons are "smeared out" over the circumference giving an equivalent constant thickness, t', (including effective skin), for axial loads.

(6) The shell frames, but not the loaded frame, are "smeared out" in the direction of the shell axis, giving an equivalent moment of inertia per inch, "i", for circumferential bending loads.

B 6.2.0 Analysis of Frame-Reinforced Cylindrical Shells Cont'd)

Basic Assumptions (Cont'd)

The simplified structural model described by the basic assumptions bear only slight resemblance to practical space vehicle shells, however the difference is compensated by, modification of certain parameters as discussed in the following pages.

Glossary of Terminology

Characteristic length - In this section there are two characteristic lengths, defined as follows: L_c is the distance required for the exponential envelope of the lowest order self-equilibrating stress system to decay to 1/e (e ~ base of natural logarithms) of its value at x = o, provided that the skin panels are rigid in shear. L_r is the distance required for the envelope of the lowest order self-equilibrating stress system to decay to 1/e of its value at x = o, provided that the frames are rigid in bending.

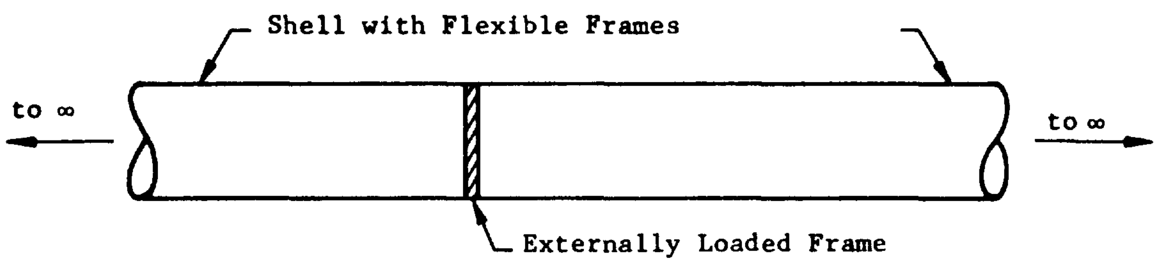


Fig. B 6.2.0-1

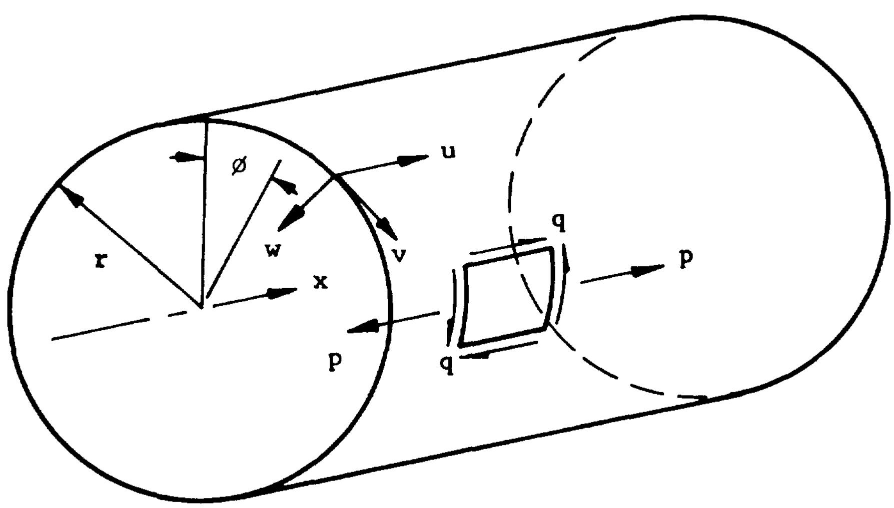


Fig. B 6.2.0-2 Load per inch and displacements in the shell (Loaded frame at x=o)

B 6.2.0 Analysis of Frame-Reinforced Cylindrical Shells (Cont'd)

Evaluation of Parameters L_r, L_o, and γ

Case of uniform shell

In cases where the shell happens to satisfy all the assumptions listed in the previous pages, and in particular, if the skin thickness, stringer area, and shell-frame moment of inertia are uniform in both the axial and circumferential directions, the following formulas may be used:

$$L_c = \frac{r}{\sqrt{6}} \left[\frac{t' r^2}{i} \right]^{1/4} \quad \text{.......................} \quad (1)$$

$$L_r = \frac{r}{2} \sqrt{\frac{E\ t'}{Gt}} \quad \text{.......................} \quad (2)$$

$$\gamma = \frac{I_o}{2iL_c} \quad \text{..................................} \quad (3)$$

Young's modulus for skin, stiffeners and all frames is assumed equal. Coefficients are obtained by use of these parameters (L_c, L_r, γ) in the tables. These coefficients yield the required loads and deformations when substituted into Eqs. 14 thru 21. In non-uniform shells, use the modified parameters indicated in the following equations:

Case of non-uniform shell

(a) In the case that the shell properties, i, t, and t', vary over the surface of the shell to a moderate degree, the following formulas and definitions are appropriate:

$$L_c = \frac{r}{\sqrt{6}} \left[\frac{E_{sk}\ t_e\ r^2}{E_f\ i} \right]^{1/4} \quad \text{.....................} \quad (4)$$

$$L_r = \frac{r}{2} \sqrt{\frac{E_{sk}\ t'}{G\ t}} \quad \text{........................} \quad (5)$$

$$\gamma = \frac{E_o\ I_o}{2E_f\ i\ L_c} \quad \text{...............................} \quad (6)$$

The stiffness factors, Gt, E_{sk}, t_e, and $E_f i$, must be averaged in the neighborhood of the loaded frame. The factors Gt and $E_{sk} i$ shall be averaged over a length of shell extending approximately one-half of a characteristic length from the loaded frame in both directions.

B 6.2.0 Analysis of Frame-Reinforced Cylindrical Shells (Cont'd)

Case of non-uniform shell (Cont'd)

(b) When unloaded frames have unequal moment of inertia or are unequally spaced, the following weighting factor is used for computing $E_f i$:

$$E_f i = (E_f i)_{fwd} + (E_f i)_{aft} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

$$(E_f i)_{fwd} = \frac{1}{L_c} \underset{fwd}{\Sigma} (W E_f I_f) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

$$(E_f i)_{aft} = \frac{1}{L_c} \underset{aft}{\Sigma} (W E_f I_f) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

Where

$$W = 1 - \frac{x}{L_c} \quad \text{for } x < L_c$$

$$= o \quad \text{for } x > L_c$$

(x is measured forward and aft of loaded frame)

The summations in Eqs. (8) and (9) are to be extended over all frames except the loaded frame. The method of calculation gives greater importance to frames closest to the loaded frame and less importance to those farther away. For the case of a single, particular heavy, neighboring frame, or for other neighboring discontinuities such as rigid bulkheads, a free end, or a plane of symmetry, the correction factors to be discussed is applicable. If those corrections are applied, the heavy frame or other discontinuity must be ignored in applying Eqs. (7), (8) and (9). In particular, if the loaded frame is near the end of the shell, the shell must be continued beyond the end, fictitiously, in the summations of Eqs. (7), (8), and (9), as though the shell were symmetric about the loaded frame and extended for a length greater than L_c on both sides of the loaded frame.

The method of calculation indicated in this sub-section exaggerates the effect of frames which are heavier than average when compared with the more accurate method of correction given in the next section. Since L_c depends on $(E_f i)^{1/4}$, an initial estimate of $E_f i$ is required in order to calculate the L_c used in Eqs. (7), (8), and (9).

B 6.2.0 Analysis of Frame-Reinforced Cylindrical Shells (Cont'd)

Corrections to γ , the "Beef-Up" parameter

The general form of the modified "beef-up" parameter, γ^*, is:

$$\gamma^* = \gamma \, . f_a . f_b . f_c . \text{ , etc.,} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

where γ is computed by the methods of the preceding section, and f_a, f_b, and f_c are factors accounting for effects of nearby heavy frames etc.

Modification for different value of L_r/L_c

The value of L_r/L_c used in the graphs are 0.2, 0.4, and 1.0. To account for values of this parameter between 0.2 and 1.0, graphical interpolation should be used. Otherwise, the following formula may be applied.

$$\gamma^* = \gamma \frac{\sqrt{1 + \left[\left(\frac{L_r}{L_c}\right)^*\right]^2}}{1 + 2\left[\left(\frac{L_r}{L_c}\right)^*\right]^2} \; \frac{1 + 2\left[\left(\frac{L_r}{L_c}\right)''\right]^2}{\sqrt{1 + \left[\left(\frac{L_r}{L_c}\right)''\right]^2}} \qquad (11)$$

where $(L_r/L_c)''$ is the value of the parameter for the shell, and $(L_r/L_c)^*$ is the value of the parameter closest to $(L_r/L_c)''$, for which graphs are available.

Modification for finite frame spacing

The modification for finite frame spacing is as follows:

$$\gamma^* = \gamma \left\{ 1 + \frac{\ell_o}{2L_c K_2} \left(1 + \frac{1}{2\,\gamma\, K_2} \right) \left[4 \left(\frac{L_r}{L_c} \right)^2 + \frac{1}{1 + \left(\frac{L_r}{L_c} \right)^2} \right] \right\}$$

where

ℓ_o = distance from loaded frame to adjacent frames

$$K_2 = \frac{1 + 2\left(\frac{L_r}{L_c}\right)^2}{\sqrt{1 + \left(\frac{L_r}{L_c}\right)^2}}$$

B 6.2.0 Analysis of Frame-Reinforced Cylindrical Shells (Cont'd)

Modification for nearby heavy frames and for other similar nearby discontinuities.

The corrections to "γ" in a previous section are not intended to account for discontinuities in circumferential bending stiffness. The form of the correction for these effects is:

$$\gamma^* = \gamma \cdot f(2) \qquad (13)$$

Fig. B 6.2.0-3 shows f(2) plotted for nearby heavy frames and for nearby rigid bulkheads. Fig. B 6.2.0-4 shows f(2) plotted for a finite length of shell terminated in various ways on one side of the loaded frame. The validity of the correction is considered doubtful for $f(2) < 0.25$, due to the importance of higher order stress systems. Figures B 6.2.0-3 and B 6.2.0-4 are for $L_r/L_c = 0.4$, but their variation with L_r/L_c is negligible for conventional shell-frame structures and adequate in other applications for $L_r/L_c < 0.75$. The corrections for nearby planes of symmetry and antisymmetry can be used to solve problems where two similar frames are simultaneously loaded. To illustrate the method the two following examples are given:

Example 1

A frame of moment of inertia 4.0 in^4 that is subjected to concentrated loads is supported in a uniform shell whose characteristic length, L_c, is 200 inches and moment of inertia per unit length, i, is 0.10 $in.^3$. A heavy frame having a moment of inertia 16.0 in^4 is 50 inches to one side of this frame. The loaded frame and shell loads are required.

The parameters needed are:

$$\gamma = \frac{4.0}{2(.1)(200)} = 0.10 \qquad \text{by Eq. (3)}$$

$$\gamma_\ell = \frac{16}{2(.1)\ 200} = 0.40$$

$$\frac{\ell}{L_c} = \frac{50}{200} = 0.25$$

Using γ_ℓ and ℓ/L_c in Fig. B 6.2.0-3 yields f(2) = 0 75

$$\therefore\ \gamma^* = 0.75\ (0.10) = 0.075 \qquad \text{by Eq. (13)}$$

B 6.2.0 Analysis of Frame-Reinforced Cylindrical Shells (Cont'd)

Example 1 (Cont'd)

Use $\gamma = 0.075$ instead of 0.10 in the curves to account for the presence of the heavy frame on the stresses in and near the loaded frame.

Example 2

A shell whose characteristic length, L_c, is 250 inches is supported by a large number of identical frames whose moments of inertia are 2.0 in^4, spaced 24 inches apart. A pair of frames 96 inches apart are subjected to concentrated loads at the same polar angle, ϕ . The two radial loads are of equal magnitude but opposite sign, while the tangential loads are of the same magnitude and sign. The loads in the loaded frames and shell are to be found.

$$i = \frac{I}{\ell_o} = \frac{2}{24} = .0833$$

$$\gamma = \frac{I_o}{2iL_c} = \frac{2}{2(.0833)(250)} = 0.048 \qquad \text{by Eq. (3)}$$

$$\frac{\ell}{L_c} = \frac{48}{250} = 0.192$$

For the tangential loads there is a plane of symmetry midway between the loaded frames, while for the radial loads a plane of anti-symmetry exists at the same place. From Fig. B 6.2.0-4 it is seen that for the radial load stress system, $f(2) = 0.32$, while for the tangential loading $f(2) = 1.75$. Hence, the values of γ^* to be used in the graphs are 0.015 and 0.084, respectively.

Eccentricity between skin line and neutral axis of the loaded frame.

In the three types of perturbation just discussed, it is possible to account for the effects by modifying γ only, since the "elementary - beam-theory" part of the solution is always valid. In the case when the eccentricity between skin line and neutral axis of the loaded frame exists, the "elementary-beam-theory" solution is also affected. This particular aspect is discussed in Appendix E of reference 1.

B 6.2.0 Analysis of Frame-Reinforced Cylindrical Shells (Cont'd)

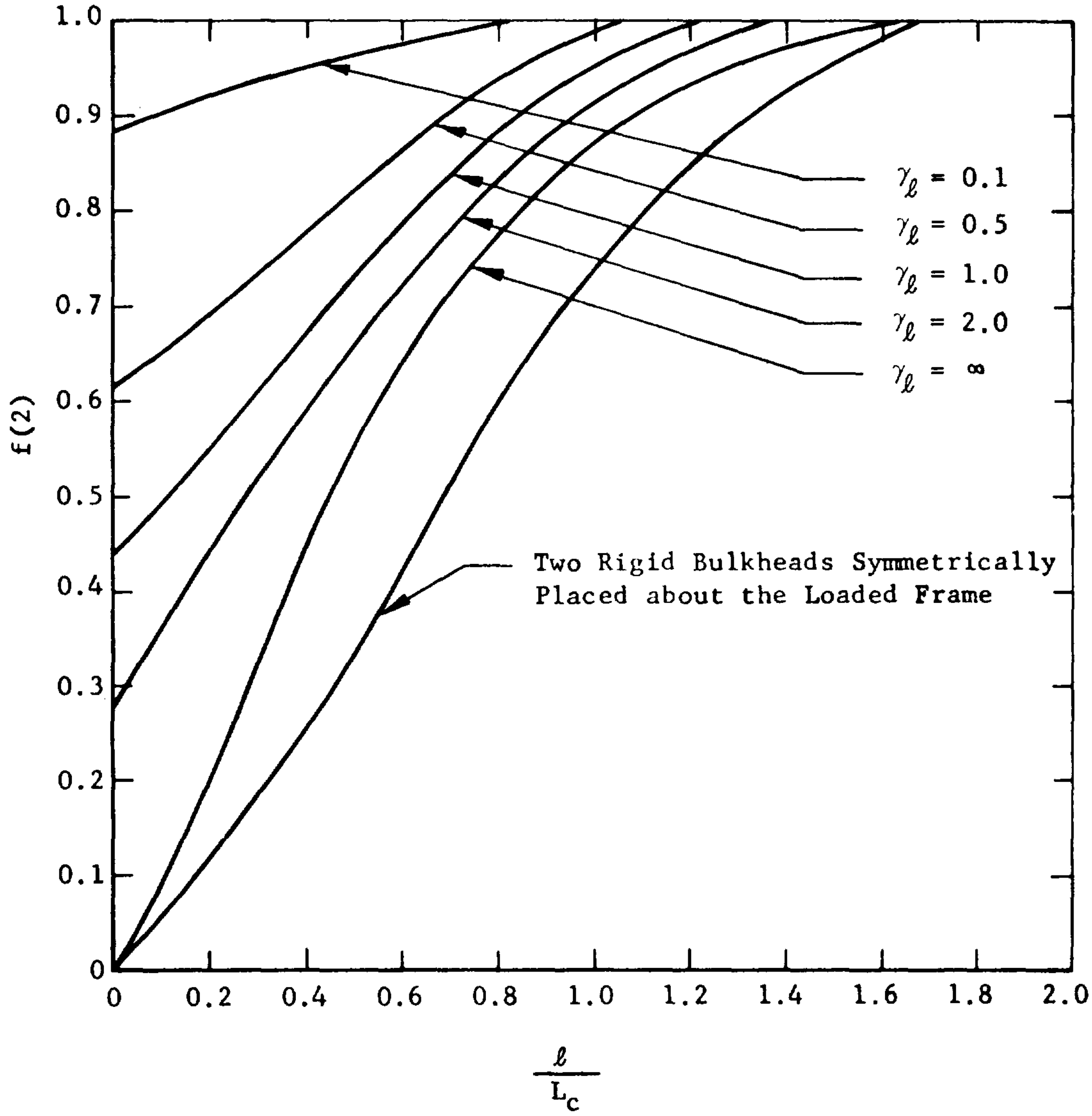


Fig. B 6.2.0-3 A single frame on one side of loaded frame or two rigid bulkheads symmetrically placed about the loaded frame curves of f(2) and f(3). L_r/L_c = 0.4.

B 6.2.0 Analysis of Frame-Reinforced Cylindrical Shells (Cont'd)

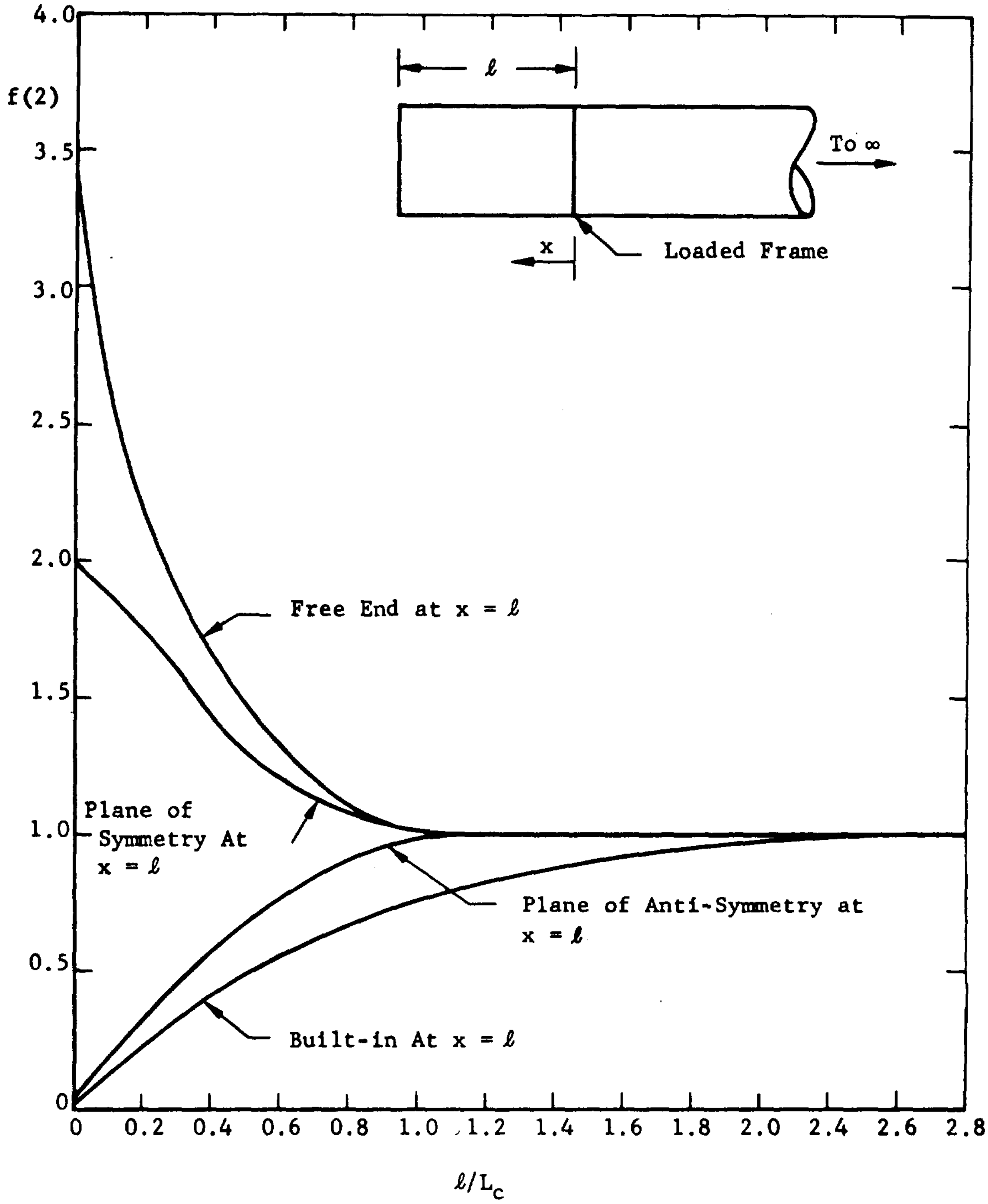


Fig. B 6.2.0-4 Finite length of shell on one side of loaded frame f(2) vs ℓ/L_c for various boundary conditions at $x = \ell$, $L_r/L_c = 0.4$.

B 6.2.1 Calculations by Use of Tables

Eqs. (14) thru (21) are given in this section, by which the effects of a concentrated load or moment on a shell-supported frame may be computed by using the tabulated coefficients. The method of computing γ is indicated in a previous section. These enable the shear flow and axial load at all points in the shell and the internal loads and displacements of the loaded frame to be computed.

The following parts of the overall solution are omitted in the tabulated coefficients:

(1) The "elementary-beam-theory" part of skin shear flow which is calculated from beam theory.

(2) The "elementary-beam-theory" part of the axial load intensity in longerons which should be calculated from beam theory.

(3) The rigid translations and rotation of the loaded frame.

As a consequence of items (1) and (2), shear flow and axial load intensity in the shell, as calculated from the tables, can be added directly to the results of an "engineers bending theory" calculation. The shear flow and axial load distributions given in the tables are assumed to be symmetrical with respect to the loaded frame. In a shell that is unsymmetric about the loaded frame, the shear flows and axial loads are not symmetric about the loaded frame. It is not possible to derive a simple correction for this effect, but the exact solutions indicated in reference 2 are applicable.

Distributed load on a frame

The effect of a distributed load on one frame may be obtained by superimposing the effects of the concentrated loads into which the distributed load can be resolved. The axial load and shear flow in the shell can be obtained for loads on several frames by a similar superposition, since "p" and "q" are tabulated in Ref. 3 NASA TN D402 as a function of x/L_c.

Frames adjacent to the loaded frame

At the present time it is not possible by use of tables to compute the internal forces in frames adjacent to the loaded frame. It is, however, a simple matter to tabulate the frame-bending moment per inch, "m", and the other internal forces as a function of x/L_c. The bending moment in an adjacent frame, due to a force applied at the loaded frame, is then obtained by multiplying "m" at the frame station by I_ℓ/i (see Appendix D of reference 1).

B 6.2.1 Calculations by Use of Tables (Cont'd)

Effect of local reinforcement of the loaded frame

It is not practical to attempt to cover, by a set of tables or charts, the many possible reinforcing patterns that can be used to locally strengthen frames in the region of applied concentrated loads. A solution is presented in Appendix A of reference 3, together with a simple example, to illustrate the numerical procedure. A loaded frame, whose moment of inertia varies around the circumference in any manner can be treated as a frame of constant moment of inertia that is reinforced to produce the actual inertia variation.

Tables

The loads and displacements of the loaded frame and loads in the shell are given in terms of the non-dimensional coefficients of the tables by the formulas below. The tables contained in this section are for M, S, F, p, and q at x = o.

Coefficients for displacements v, w, and γ are tabulated in Ref. 3 along with coefficients for "q" and "p" as a function of x/L_c.

$$q = C_{qp} \frac{P_o}{r} + C_{qt} \frac{T_o}{r} + C_{qm} \frac{M_o}{r^2} \quad \ldots\ldots\ldots\ldots (14)$$

$$p = C_{pp} \frac{P_o}{r} \left(\frac{L_c}{r}\right) + C_{pt} \frac{T_o}{r} \left(\frac{L_c}{r}\right) + C_{pm} \frac{M_o}{r^2} \left(\frac{L_c}{r}\right) \quad (15)$$

$$M = C_{mp} P_o r + C_{mt} T_o r + C_{mm} M_o \quad \ldots\ldots\ldots\ldots (16)$$

$$S = C_{sp} P_o + C_{st} T_o + C_{sm} \frac{M_o}{r} \quad \ldots\ldots\ldots\ldots (17)$$

$$F = C_{fp} P_o + C_{tt} T_o + C_{fm} \frac{M_o}{r} \quad \ldots\ldots\ldots\ldots (18)$$

$$v = C_{vp} P_o \frac{\gamma r^3}{EI_o} + C_{vt} T_o \frac{\gamma r^3}{EI_o} + C_{vm} M_o \frac{\gamma r^2}{EI_o} \quad \ldots\ldots (19)$$

$$W = C_{wp} P_o \frac{\gamma r^3}{EI_o} + C_{wt} T_o \frac{\gamma r^3}{EI_o} + C_{wm} M_o \frac{\gamma r^2}{EI_o} \quad \ldots\ldots\ldots\ldots (20)$$

$$\theta = C_{\theta p} P_o \frac{\gamma r^2}{EI_o} + C_{\theta t} T_o \frac{\gamma r^2}{EI_o} + C_{\theta m} M_o \frac{\gamma r}{EI_o} \quad \ldots\ldots\ldots (21)$$

B 6.2.1 Calculations by Use of Tables (Cont'd)

Sign Convention

Loads, moments, and displacements are positive in the loaded frame as shown in Fig. B 6.2.1-1.

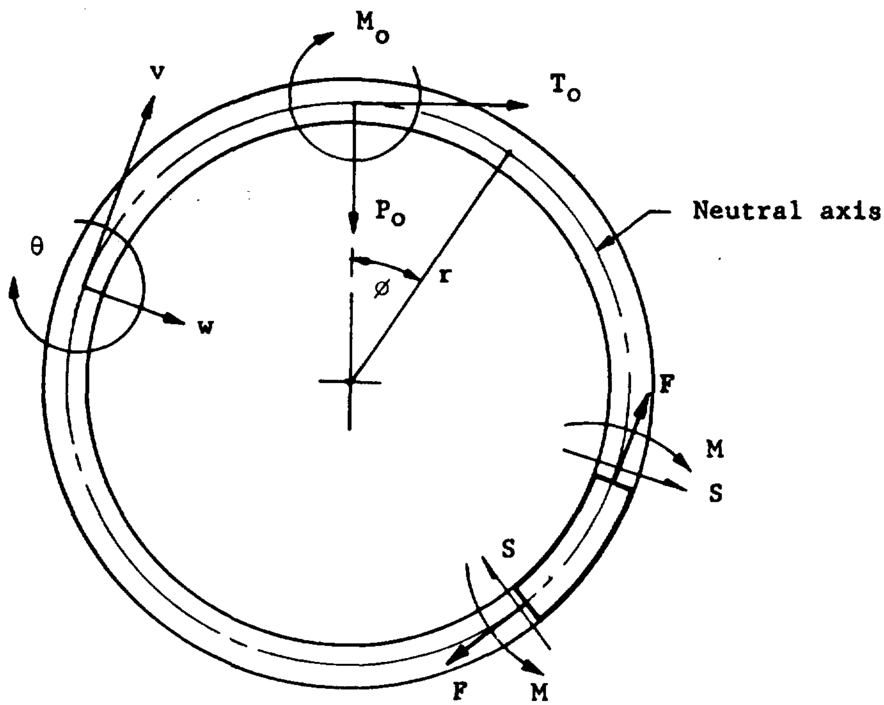


Fig. B 6.2.1-1

B 6.2.1 Calculations by Use of Tables (Cont'd)

Frame Loads		Index of Tables		
	Coefficient	Lr/Lc=.200	Lr/Lc=.400	Lr/Lc=1.000
Bending Moment, M	C_{mp}	B 6.2.1-1	B 6.2.1-5	B 6.2.1-9
	C_{mt}	B 6.2.1-13	B 6.2.1-17	B 6.2.1-21
	C_{mm}	B 6.2.1-25	B 6.2.1-29	B 6.2.1-33
Shear, S	C_{sp}	B 6.2.1-2	B 6.2.1-6	B 6.2.1-10
	C_{st}	B 6.2.1-14	B 6.2.1-18	B 6.2.1-22
	C_{sm}	B 6.2.1-26	B 6.2.1-30	B 6.2.1-34
Axial Load, F	C_{fp}	B 6.2.1-3	B 6.2.1-7	B 6.2.1-11
	C_{ft}	B 6.2.1-15	B 6.2.1-19	B 6.2.1-23
	C_{fm}	B 6.2.1-27	B 6.2.1-31	B 6.2.1-35
Shear Flow, q At Ring	C_{qp}	B 6.2.1-4	B 6.2.1-8	B 6.2.1-12
	C_{qt}	B 6.2.1-16	B 6.2.1-20	B 6.2.1-24
	C_{qm}	B 6.2.1-28	B 6.2.1-32	B 6.2.1-36

TABLE B 6.2.1-1

	C_{mp}			L_r/L_c = .200			X = 0		
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
ϕ °									
0	.0463	.0536	.0646	.0833	.1066	.1221	.1430	.1713	.2072
5	.0141	.0201	.0296	.0465	.0682	.0829	.1030	.1303	.1652
10	.0004	.0036	.0096	.0219	.0396	.0522	.0698	.0946	.1267
15	-.0038	-.0032	-.0007	.0064	.0189	.0286	.0429	.0638	.0918
20	-.0047	-.0054	-.0055	-.0029	.0043	.0109	.0214	.0376	.0605
25	-.0041	-.0056	-.0073	-.0081	-.0057	-.0022	.0043	.0156	.0327
30	-.0033	-.0050	-.0075	-.0108	-.0122	-.0116	-.0089	-.0028	.0083
35	-.0029	-.0045	-.0072	-.0120	-.0164	-.0182	-.0190	-.0176	-.0128
40	-.0027	-.0041	-.0069	-.0123	-.0189	-.0226	-.0264	-.0295	-.0307
45	-.0024	-.0037	-.0064	-.0122	-.0202	-.0254	-.0317	-.0367	-.0454
50	-.0022	-.0034	-.0059	-.0117	-.0206	-.0269	-.0351	-.0455	-.0572
55	-.0022	-.0033	-.0056	-.0112	-.0205	-.0274	-.0371	-.0503	-.0662
60	-.0021	-.0031	-.0053	-.0106	-.0198	-.0272	-.0379	-.0531	-.0726
65	-.0019	-.0029	-.0049	-.0098	-.0189	-.0263	-.0376	-.0543	-.0766
70	-.0018	-.0027	-.0045	-.0091	-.0177	-.0250	-.0365	-.0540	-.0783
75	-.0017	-.0025	-.0042	-.0083	-.0163	-.0234	-.0347	-.0525	-.0779
80	-.0015	-.0022	-.0037	-.0075	-.0148	-.0214	-.0323	-.0499	-.0757
85	-.0013	-.0020	-.0033	-.0066	-.0132	-.0193	-.0294	-.0464	-.0718
90	-.0012	-.0017	-.0029	-.0058	-.0115	-.0169	-.0262	-.0421	-.0665
100	-.0008	-.0012	-.0020	-.0039	-.0080	-.0119	-.0190	-.0317	-.0523
110	-.0005	-.0006	-.0011	-.0021	-.0044	-.0068	-.0113	-.0200	-.0349
120	-.0000	-.0001	-.0001	-.0004	-.0010	-.0018	-.0036	-.0078	-.0158
130	.0003	.0004	.0007	.0013	.0022	.0029	.0036	.0040	.0033
140	.0006	.0009	.0014	.0027	.0051	.0070	.0101	.0147	.0211
150	.0008	.0012	.0020	.0039	.0074	.0104	.0154	.0237	.0362
160	.0009	.0015	.0025	.0048	.0091	.0130	.0194	.0305	.0478
170	.0011	.0017	.0027	.0053	.0102	.0145	.0219	.0347	.0550
180	.0011	.0017	.0028	.0055	.0105	.0150	.0227	.0361	.0575

TABLE B 6.2.1-2

C_{sp}			$L_r/L_c = .200$				X = 0		
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
ϕ^o									
0	-.5000	-.5000	-.5000	-.5000	-.5000	-.5000	-.5000	-.5000	-.5000
5	-.2464	-.2738	-.3066	-.3467	-.3813	-.3989	-.4182	-.4393	-.4610
10	-.0863	-.1199	-.1637	-.2230	-.2787	-.3085	-.3423	-.3803	-.4204
15	-.0223	-.0449	-.0804	-.1374	-.1988	-.2342	-.2762	-.3256	-.3796
20	.0000	-.0111	-.0342	-.0802	-.1381	-.1744	-.2197	-.2755	-.3390
25	.0098	.0049	-.0087	-.0426	-.0927	-.1269	-.1719	-.2301	-.2991
30	.0080	.0074	.0014	-.0205	-.0602	-.0901	-.1323	-.1894	-.2603
35	.0026	.0045	.0038	-.0082	-.0374	-.0623	-.0994	-.1530	-.2227
40	.0025	.0042	.0052	-.0005	-.0208	-.0405	-.0721	-.1205	-.1865
45	.0032	.0041	.0056	.0039	-.0091	-.0236	-.0494	-.0914	-.1519
50	.0007	.0020	.0042	.0056	-.0011	-.0111	-.0307	-.0657	-.1190
55	.0002	.0014	.0037	.0067	.0048	-.0013	-.0152	-.0428	-.0880
60	.0023	.0028	.0044	.0080	.0093	.0065	-.0023	-.0226	-.0590
65	.0020	.0026	.0043	.0084	.0124	.0125	.0084	-.0049	-.0320
70	.0006	.0018	.0038	.0086	.0146	.0171	.0171	.0106	-.0073
75	.0018	.0027	.0045	.0092	.0166	.0208	.0244	.0240	.0153
80	.0028	.0035	.0051	.0098	.0180	.0238	.0303	.0355	.0355
85	.0016	.0027	.0048	.0099	.0189	.0259	.0349	.0451	.0533
90	.0014	.0026	.0048	.0101	.0197	.0274	.0385	.0530	.0687
100	.0025	.0034	.0054	.0106	.0205	.0293	.0431	.0641	.0922
110	.0021	.0032	.0052	.0104	.0203	.0293	.0445	.0694	.1061
120	.0017	.0028	.0048	.0098	.0192	.0280	.0432	.0696	.1107
130	.0024	.0031	.0047	.0090	.0174	.0254	.0396	.0652	.1069
140	.0009	.0019	.0036	.0074	.0148	.0217	.0341	.0570	.0954
150	.0018	.0022	.0033	.0061	.0117	.0172	.0270	.0456	.0774
160	.0004	.0009	.0019	.0040	.0080	.0118	.0187	.0317	.0545
180	0	0	0	0	0	0	0	0	0

TABLE B 6.2.1-3

C_{fp}			L_r/L_c = .200				X = 0		
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
ϕ^o									
0	-3.1241	-2.7540	-2.3243	-1.8128	-1.3837	-1.1695	-.9368	-.6856	-.4302
5	-2.4792	-2.2671	-1.9943	-1.6314	-1.2958	-1.1182	-.9180	-.6942	-.4597
10	-1.1964	-1.2557	-1.2662	-1.1891	-1.0445	-.9445	-.8155	-.6537	-.4679
15	-.3940	-.5499	-.6954	-.7949	-.7955	-.7625	-.6996	-.5999	-.4669
20	-.1715	-.2693	-.3960	-.5335	-.6033	-.6121	-.5966	-.5472	-.4617
25	-.0452	-.0990	-.1944	-.3340	-.4412	-.4787	-.4997	-.4932	-.4517
30	.0683	.0250	-.0532	-.1847	-.3089	-.3648	-.4124	-.4406	-.4382
35	.0355	.0235	-.0153	-.1084	-.2212	-.2818	-.3428	-.3944	-.4231
40	-.0207	-.0085	-.0149	-.0690	-.1610	-.2193	-.2858	-.3527	-.4062
45	.0148	.0157	.0087	-.0318	-.1104	-.1659	-.2351	-.3131	-.3869
50	.0284	.0231	.0159	-.0128	-.0764	-.1264	-.1939	-.2776	-.3661
55	-.0182	-.0109	-.0050	-.0147	-.0590	-.1007	-.1625	-.2466	-.3444
60	-.0167	-.0114	-.0064	-.0107	-.0436	-.0786	-.1348	-.2173	-.3211
65	.0197	.0127	.0074	-.0014	-.0290	-.0591	-.1101	-.1897	-.2963
70	.0022	.0005	-.0007	-.0043	-.0233	-.0473	-.0912	-.1652	-.2710
75	-.0223	-.0164	-.0114	-.0091	-.0204	-.0386	-.0756	-.1427	-.2449
80	.0040	.0018	-.0004	-.0030	-.0132	-.0283	-.0599	-.1206	-.2177
85	.0156	.0099	.0048	.0001	-.0084	-.0204	-.0467	-.1001	-.1902
90	-.0125	-.0091	-.0066	-.0051	-.0083	-.0162	-.0365	-.0816	-.1626
100	.0151	.0102	.0060	.0028	.0002	-.0039	-.0157	-.0460	-.1069
110	-.0151	-.0100	-.0056	-.0014	.0020	.0026	-.0006	-.0153	-.0528
120	.0139	.0102	.0076	.0071	.0098	.0124	.0148	.0130	-.0014
130	-.0071	-.0038	-.0003	.0047	.0118	.0176	.0260	.0365	.0450
140	.0044	.0044	.0054	.0090	.0167	.0241	.0366	.0570	.0856
150	.0052	.0052	.0064	.0107	.0199	.0288	.0446	.0730	.1185
160	-.0059	-.0022	.0022	.0094	.0209	.0314	.0500	.0845	.1428
170	.0140	.0115	.0109	.0143	.0245	.0349	.0544	.0920	.1579
180	-.0103	-.0050	.0008	.0094	.0223	.0339	.0546	.0939	.1627

TABLE B 6.2.1-4

C_{qp}	L_r/L_c = .200						X = 0		
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
ϕ^o									
0	0	0	0	0	0	0	0	0	0
5	6.8999	5.3381	3.7739	2.2737	1.3216	.9481	.6150	.3339	.1206
10	6.6814	5.4895	4.1517	2.7002	1.6671	1.2306	.8224	.4601	.1711
15	2.4739	2.5437	2.3603	1.6606	1.3094	1.0231	.7232	.4273	.1668
20	.5810	.9941	1.2537	1.2436	1.0055	.8322	.6217	.3870	.1582
25	.7800	.8745	.9778	.9864	.8433	.7205	.5574	.3597	.1521
30	.1713	.2726	.4288	.5741	.5809	.5312	.4381	.3000	.1334
35	-.5612	-.3448	-.0862	.1887	.3234	.3387	.3111	.2325	.1106
40	-.1377	-.1158	-.0359	.1134	.2230	.2483	.2420	.1911	.0952
45	.1709	.0721	.0278	.0709	.1456	.1729	.1800	.1513	.0794
50	-.2963	-.2542	-.1992	-.0968	.0138	.0624	.0965	.0994	.0588
55	-.3463	-.2896	-.2340	-.1492	-.0509	-.0011	.0417	.0613	.0422
60	.0732	-.0008	-.0616	-.0832	-.0473	-.0162	.0171	.0383	.0302
65	-.0570	-.0879	-.1158	-.1236	-.0899	-.0594	-.0230	.0079	.0156
70	-.3732	-.3042	-.2486	-.1985	-.1441	-.1077	-.0645	-.0227	.0010
75	-.1369	-.1430	-.1497	-.1522	-.1324	-.1096	-.0762	-.0377	-.0084
80	.0374	-.0236	-.0761	-.1166	-.1223	-.1104	-.0857	-.0506	-.0169
85	-.2538	-.2231	-.1982	-.1793	-.1591	-.1406	-.1113	-.0706	-.0275
90	-.2942	-.2513	-.2155	-.1879	-.1666	-.1497	-.1226	-.0824	-.0350
100	-.0925	-.1122	-.1287	-.1417	-.1449	-.1397	-.1242	-.0925	-.0443
110	-.1401	-.1429	-.1449	-.1461	-.1447	-.1402	-.1277	-.0996	-.0507
120	-.2095	-.1865	-.1671	-.1515	-.1421	-.1363	-.1250	-.1000	-.0528
130	-.0055	-.0420	-.0724	-.0956	-.1065	-.1083	-.1049	-.0886	-.0492
140	-.2383	-.1952	-.1589	-.1301	-.1145	-.1080	-.0992	-.0817	-.0453
150	.0514	.0103	-.0242	-.0507	-.0636	-.0671	-.0674	-.0597	-.0348
160	-.1563	-.1241	-.0970	-.0756	-.0641	-.0597	-.0547	-.0455	-.0253
170	.0282	.0107	-.0040	-.0154	-.0210	-.0226	-.0232	-.0209	-.0125
180	0	0	0	0	0	0	0	0	0

TABLE 6.2.1-5

C_{mp}			L_r/L_c = .400				X = 0		
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
ϕ^o									
0	.0546	.0626	.0744	.0938	.1172	.1325	.1526	.1792	.2116
5	.0206	.0274	.0380	.0558	.0780	.0926	.1120	.1379	.1695
10	.0028	.0071	.0145	.0284	.0472	.0600	.0775	.1012	.1306
15	-.0048	-.0033	.0005	.0096	.0237	.0341	.0487	.0691	.0951
20	-.0073	-.0077	-.0070	-.0026	.0064	.0138	.0250	.0414	.0630
25	-.0071	-.0088	-.0103	-.0101	-.0060	-.0015	.0059	.0178	.0344
30	-.0060	-.0083	-.0112	-.0142	-.0144	-.0129	-.0091	-.0019	.0092
35	-.0049	-.0072	-.0109	-.0160	-.0199	-.0209	-.0207	-.0182	-.0127
40	-.0040	-.0062	-.0100	-.0165	-.0231	-.0264	-.0293	-.0312	-.0312
45	-.0032	-.0052	-.0089	-.0160	-.0247	-.0297	-.0354	-.0413	-.0466
50	-.0027	-.0044	-.0078	-.0151	-.0250	-.0315	-.0394	-.0489	-.0589
55	-.0024	-.0039	-.0069	-.0140	-.0246	-.0319	-.0417	-.0541	-.0683
60	-.0022	-.0034	-.0061	-.0127	-.0235	-.0315	-.0424	-.0572	-.0750
65	-.0020	-.0030	-.0054	-.0115	-.0220	-.0302	-.0420	-.0585	-.0792
70	-.0018	-.0028	-.0048	-.0103	-.0203	-.0285	-.0406	-.0582	-.0810
75	-.0017	-.0025	-.0043	-.0091	-.0184	-.0263	-.0384	-.0565	-.0807
80	-.0015	-.0022	-.0038	-.0080	-.0164	-.0238	-.0356	-.0537	-.0784
85	-.0013	-.0019	-.0033	-.0069	-.0144	-.0212	-.0323	-.0498	-.0744
90	-.0011	-.0017	-.0028	-.0058	-.0123	-.0184	-.0286	-.0451	-.0689
100	-.0007	-.0011	-.0018	-.0038	-.0082	-.0126	-.0204	-.0339	-.0542
110	-.0004	-.0006	-.0009	-.0019	-.0043	-.0069	-.0119	-.0213	-.0361
120	.0000	.0000	.0000	-.0001	-.0005	-.0014	-.0035	-.0082	-.0164
130	.0003	.0005	.0008	.0016	.0028	.0036	.0043	.0045	.0034
140	.0007	.0010	.0016	.0031	.0057	.0079	.0112	.0159	.0218
150	.0009	.0013	.0022	.0043	.0081	.0115	.0169	.0254	.0375
160	.0011	.0016	.0027	.0052	.0099	.0141	.0211	.0326	.0495
170	.0012	.0018	.0029	.0057	.0110	.0157	.0237	.0370	.0570
180	.0012	.0018	.0030	.0059	.0113	.0163	.0245	.0385	.0595

TABLE B 6.2.1-6

	C_{sp}			L_r/L_c = .400			X = 0		
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
ϕ °									
0	-.5000	-.5000	-.5000	-.5000	-.5000	-.5000	-.5000	-.5000	-.5000
5	-.2862	-.3103	-.3382	-.3715	-.3998	-.4141	-.4298	-.4469	-.4644
10	-.1334	-.1664	-.2075	-.2605	-.3084	-.3336	-.3619	-.3936	-.4265
15	-.0508	-.0784	-.1175	-.1744	-.2313	-.2629	-.2997	-.3421	-.3875
20	-.0096	-.0279	-.0585	-.1101	-.1680	-.2023	-.2438	-.2933	-.3479
25	.0100	-.0003	-.0214	-.0636	-.1173	-.1514	-.1944	-.2477	-.3083
30	.0141	.0105	-.0014	-.0322	-.0782	-.1098	-.1516	-.2056	-.2693
35	.0113	.0122	.0078	-.0120	-.0486	-.0763	-.1149	-.1672	-.2311
40	.0093	.0119	.0119	.0009	-.0264	-.0495	-.0836	-.1322	-.1940
45	.0074	.0103	.0128	.0085	-.0102	-.0284	-.0572	-.1007	-.1583
50	.0042	.0074	.0113	.0121	.0012	-.0121	-.0351	-.0724	-.1243
55	.0026	.0054	.0097	.0138	.0092	.0005	-.0163	-.0472	-.0921
60	.0029	.0048	.0087	.0145	.0149	.0102	-.0015	-.0249	-.0619
65	.0023	.0039	.0075	.0142	.0185	.0174	.0109	-.0053	-.0339
70	.0014	.0029	.0063	.0136	.0208	.0227	.0210	.0117	-.0081
75	.0019	.0031	.0060	.0132	.0224	.0267	.0291	.0264	.0154
80	.0025	.0034	.0059	.0128	.0233	.0295	.0355	.0389	.0364
85	.0018	.0029	.0054	.0122	.0236	.0313	.0404	.0492	.0550
90	.0018	.0029	.0053	.0118	.0237	.0325	.0441	.0577	.0710
100	.0024	.0034	.0055	.0114	.0232	.0333	.0482	.0694	.0955
110	.0022	.0032	.0053	.0108	.0221	.0323	.0489	.0747	.1099
120	.0019	.0029	.0050	.0100	.0203	.0302	.0468	.0745	.1147
130	.0021	.0030	.0047	.0091	.0181	.0269	.0425	.0695	.1107
140	.0012	.0021	.0033	.0076	.0152	.0227	.0362	.0605	.0987
150	.0016	.0021	.0032	.0061	.0120	.0178	.0285	.0483	.0801
160	.0006	.0011	.0020	.0041	.0082	.0122	.0196	.0336	.0564
170	.0006	.0008	.0011	.0022	.0042	.0062	.0100	.0172	.0291
180	0	0	0	0	0	0	0	0	0

TABLE B 6.2.1-7

	C_{fp}			L_r/L_c = .400			X = 0		
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
ϕ^o									
0	-2.5833	-2.2692	-1.9132	-1.4981	-1.1540	-.9826	-.7962	-.5944	-.3897
5	-2.1796	-1.9745	-1.7225	-1.4023	-1.1162	-.9672	-.8006	-.6156	-.4238
10	-1.3098	-1.3065	-1.2558	-1.1296	-.9685	-.8695	-.7484	-.6028	-.4422
15	-.6505	-.7536	-.8308	-.8520	-.8017	-.7514	-.6773	-.5753	-.4514
20	-.3303	-.4325	-.5401	-.6297	-.6514	-.6384	-.6039	-.5418	-.4543
25	-.1256	-.2088	-.3175	-.4404	-.5120	-.5286	-.5280	-.5028	-.4513
30	.0134	-.0537	-.1529	-.2867	-.3888	-.4273	-.4541	-.4611	-.4433
35	.0334	-.0013	-.0689	-.1840	-.2930	-.3433	-.3884	-.4205	-.4319
40	.0152	.0080	-.0279	-.1150	-.2180	-.2732	-.3299	-.3811	-.4173
45	.0322	.0293	.0069	-.0607	-.1555	-.2123	-.2764	-.3423	-.3996
50	.0334	.0330	.0212	-.0271	-.1088	-.1635	-.2302	-.3057	-.3796
55	.0027	.0115	.0136	-.0130	-.0773	-.1265	-.1917	-.2719	-.3578
60	-.0017	.0062	.0119	-.0024	-.0525	-.0958	-.1577	-.2397	-.3341
65	.0138	.0143	.0161	.0066	-.0328	-.0705	-.1280	-.2092	-.3088
70	.0027	.0047	.0087	.0065	-.0213	-.0523	-.1036	-.1814	-.2824
75	-.0113	-.0062	.0004	.0038	-.0139	-.0386	-.0830	-.1555	-.2551
80	.0019	.0017	.0037	.0059	-.0065	-.0262	-.0644	-.1307	-.2268
85	.0078	.0052	.0047	.0062	-.0016	-.0167	-.0485	-.1077	-.1981
90	-.0068	-.0049	-.0021	.0024	-.0001	-.0104	-.0356	-.0867	-.1692
100	.0078	.0051	.0035	.0046	.0056	.0015	-.0127	-.0476	-.1111
110	-.0077	-.0049	-.0023	.0019	.0073	.0085	.0042	-.0141	-.0547
120	.0077	.0059	.0049	.0064	.0121	.0162	.0192	.0157	-.0014
130	-.0030	-.0010	.0015	.0058	.0141	.0211	.0305	.0406	.0469
140	.0032	.0036	.0049	.0088	.0178	.0264	.0404	.0614	.0887
150	.0037	.0042	.0058	.0105	.0205	.0304	.0479	.0780	.1227
160	-.0020	.0006	.0040	.0104	.0219	.0330	.0532	.0898	.1478
170	.0085	.0078	.0086	.0133	.0243	.0356	.0569	.0972	.1633
180	-.0042	-.0007	.0035	.0109	.0235	.0354	.0575	.0993	.1684

TABLE B 6.2.1-8

C_{qp}				L_r/L_c = .400			X = 0		
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
ϕ^o									
0	0	0	0	0	0	0	0	0	0
5	4.5167	3.4237	2.3731	1.4049	.8096	.5797	.3761	.2046	.0742
10	4.8735	3.8926	2.8593	1.8075	1.0973	.8055	.5364	.2998	.1115
15	2.6158	2.4071	2.0290	1.4710	.9846	.7551	.5254	.3066	.1186
20	1.2698	1.4050	1.3858	1.1586	.8536	.6825	.4948	.3004	.1203
25	.9842	1.0669	1.0863	.9696	.7586	.6247	.4669	.2923	.1203
30	.3646	.5031	.6291	.6717	.5895	.5094	.3990	.2609	.1115
35	-.2178	-.0112	.2055	.3780	.4094	.3808	.3182	.2202	.0985
40	-.1082	-.0244	.1032	.2464	.3036	.2973	.2609	.1888	.0877
45	.0058	-.0028	.0447	.1448	.2106	.2197	.2043	.1557	.0754
50	-.2519	-.2087	-.1306	-.0042	.0948	.1257	.1361	.1152	.0599
55	-.2785	-.2426	-.1841	-.0793	.0194	.0585	.0831	.0813	.0460
60	-.0536	-.0965	-.1150	-.0804	-.0141	.0199	.0470	.0550	.0342
65	-.1146	-.1384	-.1505	-.1240	-.0638	-.0282	.0054	.0257	.0211
70	-.2757	-.2460	-.2202	-.1765	-.1128	-.0742	-.0342	-.0026	.0081
75	-.1503	-.1593	-.1687	-.1615	-.1229	.0924	-.0561	-.0218	-.0019
80	-.0576	-.0946	-.1285	-.1473	-.1291	-.1062	-.0742	-.0386	-.0111
85	-.2088	-.1952	-.1878	-.1800	-.1551	-.1313	-.0977	-.0576	-.0208
90	-.2300	-.2084	-.1939	-.1836	-.1632	-.1428	-.1118	-.0710	-.0285
100	-.1227	-.1337	-.1443	-.1554	-.1545	-.1446	-.1231	-.0869	-.0394
110	-.1445	-.1458	-.1474	-.1509	-.1509	-.1447	-.1285	-.0962	-.0465
120	-.1750	-.1625	-.1523	-.1458	-.1427	-.1380	-.1256	-.0977	-.0493
130	-.0610	-.0805	-.0962	-.1086	-.1156	-.1160	-.1100	-.0895	-.0471
140	-.1732	-.1500	-.1308	-.1160	-.1090	-.1058	-.0988	-.0806	-.0429
150	-.0111	-.0332	-.0510	-.0645	-.0716	-.0736	-.0723	-.0617	-.0340
160	-.1076	-.0903	-.0760	-.0648	-.0592	-.0572	-.0539	-.0450	-.0246
170	.0016	-.0078	-.0155	-.0212	-.0242	-.0251	-.0251	-.0218	-.0123
180	0	0	0	0	0	0	0	0	0

TABLE B 6.2.1-9

C_{mp}			$L_r/L_c = 1.000$				X = 0		
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
ϕ^o									
0	.0713	.0811	.0951	.1171	.1421	.1574	.1763	.1988	.2222
5	.0349	.0438	.0568	.0777	.1017	.1166	.1350	.1569	.1799
10	.0117	.0183	.0287	.0465	.0678	.0813	.0983	.1187	.1403
15	-.0018	.0021	.0091	.0225	.0400	.0515	.0662	.0834	.1036
20	-.0087	-.0075	-.0040	.0047	.0176	.0267	.0387	.0538	.0701
25	-.0115	-.0123	-.0119	-.0080	.0001	.0064	.0153	.0269	.0398
30	-.0117	-.0141	-.0162	-.0166	-.0133	-.0097	-.0042	.0037	.0129
35	-.0108	-.0141	-.0181	-.0220	-.0230	-.0222	-.0200	-.0160	-.0108
40	-.0094	-.0131	-.0182	-.0249	-.0298	-.0315	-.0324	-.0323	-.0311
45	-.0079	-.0116	-.0173	-.0260	-.0341	-.0380	-.0420	-.0455	-.0482
50	-.0065	-.0100	-.0159	-.0258	-.0364	-.0423	-.0488	-.0557	-.0621
55	-.0053	-.0085	-.0141	-.0246	-.0371	-.0445	-.0533	-.0632	-.0730
60	-.0043	-.0071	-.0123	-.0229	-.0365	-.0451	-.0557	-.0682	-.0809
65	-.0035	-.0059	-.0106	-.0208	-.0350	-.0444	-.0564	-.0708	-.0860
70	-.0029	-.0048	-.0090	-.0185	-.0328	-.0426	-.0555	-.0714	-.0885
75	-.0024	-.0040	-.0075	-.0162	-.0300	-.0399	-.0533	-.0702	-.0887
80	-.0020	-.0032	-.0062	-.0139	-.0269	-.0366	-.0500	-.0674	-.0866
85	-.0016	-.0026	-.0050	-.0116	-.0235	-.0328	-.0459	-.0632	-.0825
90	-.0013	-.0020	-.0039	-.0095	-.0201	-.0287	-.0411	-.0578	-.0768
100	-.0007	-.0011	-.0021	-.0055	-.0131	-.0199	-.0300	-.0443	-.0609
110	-.0002	-.0003	-.0006	-.0021	-.0065	-.0109	-.0180	-.0285	-.0411
120	.0003	.0005	.0007	.0008	-.0005	-.0024	-.0060	-.0118	-.0191
130	.0007	.0011	.0018	.0033	.0048	.0052	.0053	.0045	.0030
140	.0011	.0016	.0027	.0053	.0091	.0118	.0152	.0194	.0237
150	.0014	.0021	.0035	.0068	.0126	.0171	.0234	.0319	.0415
160	.0016	.0024	.0040	.0079	.0151	.0209	.0295	.0413	.0551
170	.0017	.0026	.0043	.0086	.0166	.0233	.0332	.0472	.0636
180	.0018	.0027	.0044	.0088	.0171	.0241	.0345	.0492	.0665

TABLE B 6.2.1-10

	C_{sp}		L_r/L_c = 1.000				X = 0		
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
ϕ^o									
0	-.5000	-.5000	-.5000	-.5000	-.5000	-.5000	-.5000	-.5000	-.5000
5	-.3359	-.3556	-.3778	-.4037	-.4251	-.4357	-.4471	-.4589	-.4700
10	-.2032	-.2338	-.2700	-.3145	-.3526	-.3723	-.3935	-.4159	-.4371
15	-.1117	-.1433	-.1839	-.2374	-.2867	-.3125	-.3413	-.3723	-.4022
20	-.0517	-.0791	-.1174	-.1728	-.2275	-.2574	-.2915	-.3290	-.3657
25	-.0143	-.0352	-.0679	-.1199	-.1756	-.2074	-.2446	-.2865	-.3281
30	.0055	-.0082	-.0331	-.0782	-.1311	-.1630	-.2012	-.2454	-.2901
35	.0141	.0069	-.0099	-.0461	-.0937	-.1239	-.1615	-.2060	-.2521
40	.0174	.0151	.0052	-.0218	-.0625	-.0900	-.1255	-.1687	-.2144
45	.0173	.0183	.0142	-.0041	-.0369	-.0610	-.0931	-.1336	-.1774
50	.0148	.0182	.0186	.0084	-.0164	-.0364	-.0644	-.1010	-.1416
55	.0122	.0167	.0204	.0168	-.0002	-.0158	-.0391	-.0709	-.1071
60	.0103	.0150	.0205	.0222	.0125	.0012	-.0170	-.0433	-.0743
65	.0082	.0129	.0194	.0253	.0220	.0150	.0019	-.0184	-.0435
70	.0063	.0107	.0178	.0266	.0290	.0260	.0181	.0040	-.0148
75	.0053	.0092	.0161	.0269	.0340	.0345	.0316	.0237	.0116
80	.0047	.0080	.0145	.0264	.0372	.0410	.0427	.0409	.0355
85	.0039	.0068	.0129	.0254	.0391	.0457	.0516	.0555	.0568
90	.0034	.0059	.0115	.0241	.0399	.0489	.0585	.0678	.0753
100	.0032	.0050	.0095	.0211	.0392	.0515	.0670	.0854	.1041
110	.0028	.0044	.0079	.0181	.0364	.0504	.0696	.0943	.1216
120	.0025	.0038	.0067	.0153	.0324	.0466	.0674	.0957	.1281
130	.0024	.0035	.0058	.0127	.0277	.0410	.0614	.0902	.1244
140	.0018	.0027	.0047	.0101	.0225	.0341	.0524	.0791	.1114
150	.0016	.0023	.0037	.0077	.0171	.0263	.0412	.0634	.0908
160	.0009	.0015	.0025	.0051	.0115	.0178	.0283	.0442	.0640
170	.0006	.0008	.0013	.0026	.0058	.0090	.0144	.0227	.0331
180	0	0	0	0	0	0	0	0	0

TABLE B 6.2.1-11

C_{f_p}			L_r/L_c = 1.000				X = 0		
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
ϕ^o									
0	-1.9400	-1.6935	-1.4193	-1.1056	-.8503	-.7251	-.5917	-.4532	-.3238
5	-1.7475	-1.5619	-1.3438	-1.0794	-.8532	-.7392	-.6155	-.4850	-.3618
10	-1.2762	-1.2137	-1.1129	-.9562	-.7967	-.7086	-.6082	-.4978	-.3900
15	-.8448	-.8724	-.8677	-.8103	-.7190	-.6594	-.5857	-.4992	-.4107
20	-.5472	-.6122	-.6608	0.6724	-.6371	-.6034	-.5553	-.4931	-.4252
25	-.3183	-.3983	-.4781	-.5399	-.5517	-.5415	-.5179	-.4800	-.4337
30	-.1485	-.2305	-.3245	-.4190	-.4678	-.4777	-.4763	-.4616	-.4366
35	-.0607	-.1268	-.2147	-.3201	-.3922	-.4173	-.4341	-.4398	-.4347
40	-.0174	-.0631	-.1358	-.2392	-.3243	-.3604	-.3919	-.4153	-.4284
45	.0185	-.0139	-.0732	-.1704	-.2624	-.3065	-.3496	-.3882	-.4178
50	.0329	.0129	-.0317	-.1168	-.2090	-.2576	-.3090	-.3598	-.4034
55	.0248	.0180	-.0097	-.0782	-.1645	-.2146	-.2710	-.3307	-.3858
60	.0221	.0220	.0059	-.0476	-.1262	-.1757	-.2347	-.3008	-.3651
65	.0251	.0266	.0174	-.0236	-.0935	-.1410	-.2006	-.2706	-.3416
70	.0164	.0213	.0195	-.0085	-.0675	-.1115	-.1695	-.2408	-.3159
75	.0068	.0142	.0179	.0013	-.0466	-.0860	-.1408	-.2114	-.2882
80	.0090	.0143	.0190	.0095	-.0288	-.0634	-.1142	-.1823	-.2589
85	.0089	.0128	.0180	.0144	-.0147	-.0443	-.0900	-.1541	-.2283
90	.0011	.0061	.0133	.0155	-.0044	-.0285	-.0684	-.1269	-.1968
100	.0051	.0067	.0115	.0181	.0114	-.0028	-.0304	-.0753	-.1324
110	-.0019	.0008	.0062	.0160	.0198	.0147	-.0002	-.0287	-.0684
120	.0044	.0046	.0073	.0161	.0257	.0277	.0245	.0128	-.0072
130	.0001	.0017	.0050	.0143	.0284	.0363	.0436	.0483	.0489
140	.0028	.0037	.0062	.0145	.0305	.0426	.0586	.0779	.0977
150	.0032	.0043	.0067	.0145	.0318	.0469	.0696	.1011	.1377
160	.0010	.0030	.0061	.0142	.0323	.0495	.0771	.1177	.1672
170	.0054	.0060	.0082	.0152	.0332	.0514	.0817	.1279	.1854
180	.0003	.0026	.0062	.0143	.0328	.0515	.0829	.1312	.1915

TABLE B 6.2.1-12

C_{qp}			$L_r/L_c = 1.000$				X = 0		
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
ϕ^o									
0	0	0	0	0	0	0	0	0	0
5	2.3911	1.7706	1.1976	.6909	.3905	.2770	.1778	.0954	.0340
10	2.8901	2.2299	1.5779	.9578	.5636	.4076	.2669	.1461	.0529
15	2.0653	1.7386	1.3439	.8934	.5626	.4195	.2832	.1596	.0593
20	1.4211	1.3108	1.1070	.8040	.5387	.4130	.2865	.1657	.0628
25	1.1162	1.0708	.9505	.7324	.5142	.4028	.2855	.1685	.0650
30	.6514	.7014	.6937	.5926	.4465	.3605	.2631	.1595	.0629
35	.2195	.3460	.4322	.4374	.3633	.3050	.2308	.1443	.0583
40	.1100	.2086	.2973	.3371	.3016	.2612	.2035	.1307	.0539
45	.0457	.1095	.1864	.2444	.2392	.2148	.1731	.1144	.0482
50	-.1338	-.0564	.0398	.1328	.1643	.1584	.1352	.0936	.0407
55	-.1900	-.1289	-.0439	.0540	.1043	.1107	.1015	.0740	.0334
60	-.1221	-.1085	-.0642	.0099	.0611	.0735	.0732	.0566	.0265
65	-.1581	-.1494	-.1142	-.0451	.0129	.0325	.0421	.0372	.0188
70	-.2275	-.2065	-.1675	-.0973	-.0328	-.0068	.0118	.0180	.0109
75	-.1753	-.1774	-.1631	-.1161	-.0601	-.0336	-.0112	.0022	.0042
80	-.1343	-.1524	-.1568	-.1295	-.0827	-.0568	-.0318	-.0124	-.0023
85	-.1929	-.1921	-.1857	-.1566	-.1099	-.0825	-.0536	-.0275	-.0089
90	-.1981	-.1946	-.1896	-.1676	-.1268	-.1004	-.0703	-.0399	-.0146
100	-.1472	-.1565	-.1658	-.1648	-.1416	-.1208	-.0925	-.0582	-.0235
110	-.1493	-.1527	-.1585	-.1615	-.1479	-.1317	-.1060	-.0703	-.0298
120	-.1537	-.1500	-.1497	-.1516	-.1441	-.1320	-.1099	-.0756	-.0330
130	-.0972	-.1059	-.1148	-.1249	-.1263	-.1196	-.1030	-.0731	-.0327
140	-.1315	-.1220	-.1154	-.1137	-.1122	-.1066	-.0929	-.0659	-.0303
150	-.0515	-.0607	-.0687	-.0773	-.0825	-.0809	-.0725	-.0535	-.0247
160	-.0763	-.0690	-.0634	-.0608	-.0607	-.0587	-.0523	-.0386	-.0179
170	-.0157	-.0196	-.0228	-.0261	-.0285	-.0284	-.0259	-.0195	-.0091
180	0	0	0	0	0	0	0	0	0

TABLE B 6.2.1-13

C_{mt}			L_r/L_c = .200				$X = 0$	
γ / ϕ^{o}	.20	.03	.05	.10	.20	.30	.50	1.00	3.00
0	0	0	0	0	0	0	0	0	0
5	-.0025	-.0031	-.0040	-.0056	-.0076	-.0089	-.0107	-.0131	-.0162
10	-.0030	-.0040	-.0056	-.0085	-.0122	-.0147	-.0182	-.0229	-.0289
15	-.0028	-.0040	-.0059	-.0097	-.0147	-.0182	-.0231	-.0298	-.0385
20	-.0024	-.0036	-.0056	-.0098	-.0157	-.0199	-.0258	-.0342	-.0451
25	-.0020	-.0031	-.0051	-.0093	-.0156	-.0202	-.0269	-.0365	-.0491
30	-.0017	-.0026	-.0044	-.0084	-.0148	-.0196	-.0267	-.0370	-.0509
35	-.0015	-.0022	-.0038	-.0074	-.0135	-.0183	-.0255	-.0361	-.0506
40	-.0012	-.0019	-.0037	-.0064	-.0120	-.0165	-.0235	-.0340	-.0487
45	-.0010	-.0015	-.0026	-.0053	-.0102	-.0144	-.0209	-.0310	-.0454
50	-.0008	-.0012	-.0021	-.0043	-.0085	-.0121	-.0180	-.0273	-.0409
55	-.0006	-.0009	-.0015	-.0033	-.0067	-.0097	-.0148	-.0231	-.0355
60	-.0004	-.0006	-.0011	-.0023	-.0049	-.0074	-.0115	-.0186	-.0294
65	-.0002	-.0004	-.0006	-.0014	-.0032	-.0050	-.0082	-.0139	-.0229
70	-.0001	-.0001	-.0002	-.0006	-.0016	-.0028	-.0050	-.0092	-.0161
75	.0001	.0001	.0002	.0002	-.0001	-.0007	-.0019	-.0045	-.0093
80	.0002	.0003	.0005	.0009	.0012	.0013	.0010	-.0001	-.0026
85	.0003	.0005	.0008	.0015	.0025	.0031	.0037	.0042	.0039
90	.0005	.0007	.0011	.0020	.0035	.0047	.0062	.0080	.0099
100	.0006	.0009	.0015	.0029	.0053	.0072	.0101	.0145	.0204
110	.0007	.0011	.0018	.0034	.0063	.0088	.0128	.0190	.0280
120	.0008	.0011	.0019	.0036	.0068	.0096	.0141	.0214	.0324
130	.0007	.0011	.0018	.0035	.0067	.0095	.0141	.0218	.0335
140	.0007	.0010	.0016	.0032	.0061	.0086	.0129	.0201	.0314
150	.0005	.0008	.0013	.0026	.0050	.0071	.0106	.0167	.0263
160	.0004	.0006	.0009	.0018	.0035	.0050	.0075	.0120	.0189
170	.0002	.0003	.0005	.0010	.0018	.0026	.0039	.0062	.0099
180	0	0	0	0	0	0	0	0	0

TABLE B 6.2.1-14

	C_{st}		L_r/L_c = .200				X = 0		
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
$\phi°$									
0	-.0463	-.0536	-.0646	-.0833	-.1066	-.1221	-.1430	-.1713	-.2072
5	-.0141	-.0201	-.0296	-.0465	-.0682	-.0829	-.1030	-.1303	-.1652
10	-.0004	-.0036	-.0096	-.0219	-.0396	-.0522	-.0698	-.0946	-.1267
15	.0038	.0032	.0007	-.0064	-.0189	-.0286	-.0429	-.0638	-.0918
20	.0047	.0054	.0055	.0029	-.0043	-.0109	-.0214	-.0376	-.0605
25	.0041	.0056	.0073	.0081	.0057	.0022	-.0043	-.0156	-.0327
30	.0033	.0050	.0075	.0108	.0122	.0116	.0089	.0027	-.0083
35	.0029	.0045	.0072	.0020	.0164	.0182	.0190	.0176	.0128
40	.0027	.0041	.0069	.0123	.0189	.0226	.0264	.0295	.0307
45	.0024	.0037	.0064	.0122	.0202	.0254	.0317	.0387	.0454
50	.0022	.0034	.0059	.0117	.0206	.0269	.0351	.0455	.0572
55	.0022	.0033	.0056	.0112	.0205	.0274	.0371	.0503	.0662
60	.0021	.0031	.0053	.0106	.0198	.0272	.0379	.0531	.0726
65	.0019	.0029	.0049	.0098	.0189	.0263	.0376	.0543	.0766
70	.0018	.0027	.0045	.0091	.0177	.0250	.0365	.0540	.0783
75	.0017	.0025	.0042	.0083	.0163	.0234	.0347	.0525	.0779
80	.0015	.0022	.0037	.0075	.0148	.0214	.0323	.0499	.0757
85	.0013	.0020	.0033	.0066	.0132	.0193	.0294	.0464	.0718
90	.0012	.0017	.0029	.0058	.0115	.0169	.0262	.0421	.0665
100	.0008	.0012	.0020	.0039	.0080	.0119	.0190	.0317	.0523
110	.0005	.0006	.0011	.0021	.0044	.0068	.0113	.0200	.0349
120	.0000	.0001	.0001	.0004	.0010	.0018	.0036	.0078	.0158
130	-.0003	-.0004	-.0007	-.0013	-.0022	-.0029	-.0036	-.0040	-.0033
140	-.0006	-.0009	-.0014	-.0027	-.0051	-.0070	-.0101	-.0147	-.0211
150	-.0008	-.0012	-.0020	-.0039	-.0074	-.0104	-.0154	-.0237	-.0362
160	-.0010	-.0015	-.0025	-.0048	-.0091	-.0130	-.0194	-.0305	-.0478
170	-.0011	-.0017	-.0027	-.0053	-.0102	-.0145	-.0219	-.0347	-.0550
180	-.0011	-.0017	-.0028	-.0055	-.0105	-.0150	-.0227	-.0361	-.0575

TABLE B 6.2.1-15

C_{f_t}				L_r/L_c = .200				X = 0	
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
ϕ^o									
0	-.5000	-.5000	-.5000	-.5000	-.5000	-.5000	-.5000	-.5000	-.5000
5	-.2464	-.2738	-.3066	-.3467	-.3813	-.3989	-.4182	-.4393	-.4610
10	-.0863	-.1199	-.1637	-.2230	-.2787	-.3085	-.3423	-.3803	-.4204
15	-.0223	-.0449	-.0804	-.1374	-.1988	-.2342	-.2762	-.3256	-.3796
20	.0000	-.0111	-.0342	-.0802	-.1381	-.1744	-.2197	-.2755	-.3390
25	.0098	.0049	-.0087	-.0426	-.0927	-.1269	-.1719	-.2301	-.2991
30	.0080	.0074	.0014	-.0205	-.0602	-.0903	-.1323	-.1894	-.2603
35	.0026	.0045	.0038	-.0082	-.0374	-.0623	-.0994	-.1530	-.2227
40	.0025	.0042	.0052	-.0005	-.0208	-.0405	-.0721	-.1205	-.1865
45	.0032	.0041	.0056	.0039	-.0091	-.0238	-.0494	-.0914	-.1519
50	.0007	.0020	.0042	.0056	-.0011	-.0111	-.0307	-.0657	-.1190
55	.0002	.0014	.0037	.0067	.0048	-.0013	-.0152	-.0428	-.0880
60	.0023	.0028	.0044	.0080	.0093	.0065	-.0023	-.0226	-.0590
65	.0020	.0026	.0043	.0084	.0124	.0125	.0084	-.0049	-.0320
70	.0006	.0018	.0038	.0086	.0146	.0171	.0171	.0106	-.0073
75	.0018	.0027	.0045	.0092	.0166	.0208	.0244	.0240	.0153
80	.0028	.0035	.0051	.0098	.0180	.0238	.0303	.0355	.0355
85	.0016	.0027	.0048	.0099	.0189	.0259	.0349	.0451	.0533
90	.0014	.0026	.0048	.0101	.0197	.0274	.0385	.0530	.0687
100	.0025	.0034	.0054	.0106	.0205	.0293	.0431	.0641	.0922
110	.0021	.0032	.0052	.0104	.0203	.0293	.0445	.0694	.1061
120	.0017	.0028	.0048	.0098	.0192	.0280	.0432	.0696	.1107
130	.0024	.0031	.0047	.0090	.0174	.0254	.0396	.0652	.1069
140	.0009	.0019	.0036	.0074	.0148	.0217	.0341	.0570	.0934
150	.0018	.0022	.0033	.0061	.0117	.0172	.0270	.0456	.0774
160	.0004	.0009	.0019	.0040	.0080	.0118	.0187	.0317	.0545
170	.0007	.0008	.0012	.0022	.0041	.0061	.0096	.0163	.0281
180	0	0	0	0	0	0	0	0	0

TABLE B 6.2.1-16

C_{qt}			L_r/L_c = .400				X = 0		
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
∅°									
0	1.3464	1.1650	.9557	.7093	.5064	.4071	.3012	.1897	.0799
5	1.0085	.9055	.7738	.6008	.4439	.3624	.2723	.1741	.0743
10	.3621	.3933	.4016	.3692	.3057	.2620	.2063	.1378	.0610
15	-.0382	.0401	.1140	.1674	.1739	.1623	.1379	.0986	.0461
20	-.1457	-.0972	-.0339	.0362	.0747	.0824	.0798	.0633	.0320
25	-.2033	-.1771	-.1303	-.0609	-.0061	.0145	.0282	.0306	.0184
30	-.2532	-.2324	-.1946	-.1304	-.0691	-.0408	-.0157	.0015	.0058
35	-.2291	-.2239	-.2059	-.1618	-.1076	-.0781	-.0480	-.0216	-.0048
40	-.1925	-.1993	-.1975	-.1732	-.1305	-.1032	-.0718	-.0399	-.0137
45	-.2007	-.2018	-.1997	-.1823	-.1470	-.1219	-.0904	-.0549	-.0214
50	-.1972	-.1952	-.1928	-.1814	-.1540	-.1321	-.1025	-.0659	-.0274
55	-.1629	-.1671	-.1712	-.1691	-.1516	-.1342	-.1082	-.0727	-.0318
60	-.1519	-.1550	-.1586	-.1591	-.1473	-.1335	-.1107	-.0771	-.0349
65	-.1576	-.1546	-.1530	-.1511	-.1418	-.1304	-.1106	-.0791	-.0370
70	-.1360	-.1356	-.1359	-.1364	-.1312	-.1229	-.1067	-.0784	-.0377
75	-.1105	-.1139	-.1171	-.1204	-.1188	-.1131	-.1003	-.0757	-.0373
80	-.1100	-.1092	-.1089	-.1094	-.1080	-.1038	-.0934	-.0719	-.0362
85	-.1019	-.0994	-.0975	-.0968	-.0959	-.0929	-.0848	-.0666	-.0343
90	-.0739	-.0759	-.0777	-.0799	-.0812	-.0799	-.0744	-.0598	-.0315
100	-.0599	-.0576	-.0559	-.0553	-.0560	-.0559	-.0536	-.0448	-.0247
110	-.0178	-.0204	-.0229	-.0255	-.0283	-.0299	-.0305	-.0275	-.0162
120	-.0069	-.0051	-.0039	-.0037	-.0054	-.0071	-.0092	-.0104	-.0072
130	.0264	.0248	.0232	.0210	.0180	.0154	.0115	.0065	.0019
140	.0404	.0406	.0404	.0392	.0365	.0338	.0291	.0212	.0101
150	.0561	.0563	.0561	.0549	.0520	.0491	.0437	.0336	.0171
160	.0734	.0718	.0701	.0677	.0641	.0608	.0547	.0430	.0225
170	.0708	.0722	.0731	.0727	.0700	.0670	.0609	.0485	.0257
180	.0853	.0830	.0806	.0776	.0737	.0702	.0637	.0507	.0270

TABLE B 6.2.1-17

C_{mt}			L_r/L_c = .400				X = 0		
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
ϕ^o									
0	0	0	0	0	0	0	0	0	0
5	-.0031	-.0038	-.0048	-.0065	-.0085	-.0098	-.0115	-.0138	-.0166
10	-.0041	-.0052	-.0070	-.0101	-.0139	-.0164	-.0197	-.0242	-.0297
15	-.0039	-.0053	-.0076	-.0117	-.0169	-.0204	-.0252	-.0316	-.0395
20	-.0034	-.0048	-.0073	-.0119	-.0182	-.0225	-.0284	-.0364	-.0464
25	-.0027	-.0041	-.0065	-.0113	-.0182	-.0230	-.0297	-.0389	-.0506
30	-.0022	-.0033	-.0056	-.0103	-.0172	-.0223	-.0295	-.0396	-.0525
35	-.0017	-.0027	-.0046	-.0089	-.0157	-.0208	-.0282	-.0387	-.0523
40	-.0013	-.0021	-.0037	-.0075	-.0138	-.0188	-.0260	-.0365	-.0504
45	-.0010	-.0016	-.0029	-.0061	-.0117	-.0163	-.0232	-.0334	-.0469
50	-.0007	-.0012	-.0021	-.0047	-.0096	-.0136	-.0199	-.0294	-.0423
55	-.0005	-.0008	-.0015	-.0035	-.0074	-.0109	-.0163	-.0249	-.0368
60	-.0003	-.0005	-.0009	-.0023	-.0053	-.0081	-.0127	-.0200	-.0305
65	-.0001	-.0002	-.0004	-.0012	-.0033	-.0054	-.0090	-.0150	-.0237
70	.0000	.0000	.0000	-.0003	-.0015	-.0028	-.0054	-.0099	-.0167
75	.0002	.0003	.0004	.0006	.0002	-.0004	-.0019	-.0048	-.0096
80	.0003	.0005	.0008	.0013	.0018	.0018	.0013	.0000	-.0027
85	.0004	.0007	.0011	.0020	.0031	.0037	.0043	.0045	.0040
90	.0006	.0008	.0013	.0025	.0043	.0055	.0070	.0087	.0102
100	.0007	.0011	.0017	.0034	.0061	.0082	.0112	.0156	.0210
110	.0008	.0012	.0020	.0039	.0071	.0099	.0141	.0204	.0290
120	.0008	.0013	.0021	.0040	.0076	.0106	.0154	.0230	.0336
130	.0008	.0012	.0020	.0039	.0074	.0104	.0153	.0233	.0347
140	.0007	.0011	.0018	.0035	.0066	.0094	.0140	.0215	.0325
150	.0006	.0009	.0014	.0028	.0054	.0077	.0115	.0179	.0272
160	.0004	.0006	.0010	.0020	.0038	.0054	.0082	.0128	.0196
170	.0002	.0003	.0005	.0010	.0020	.0028	.0042	.0066	.0102
180	0	0	0	0	0	0	0	0	0

TABLE B 6.2.1-18

C_{st}	$L_r/L_c = .400$						X = 0		
γ / ϕ^o	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
0	-.0546	-.0626	-.0744	-.0938	-.1172	-.1325	-.1526	-.1792	-.2116
5	-.0206	-.0274	-.0380	-.0558	-.0780	-.0926	-.1120	-.1379	-.1695
10	-.0028	-.0071	-.0145	-.0284	-.0472	-.0600	-.0775	-.1012	-.1306
15	.0048	.0033	-.0005	-.0096	-.0237	-.0341	-.0487	-.0691	-.0951
20	.0073	.0077	.0070	.0026	-.0064	-.0138	-.0250	-.0414	-.0630
25	.0071	.0088	.0103	.0101	.0060	.0015	-.0059	-.0178	-.0344
30	.0060	.0083	.0112	.0142	.0144	.0129	.0091	.0019	-.0092
35	.0049	.0072	.0109	.0160	.0199	.0209	.0207	.0182	.0127
40	.0040	.0062	.0100	.0165	.0231	.0264	.0293	.0312	.0312
45	.0032	.0052	.0089	.0160	.0247	.0297	.0354	.0413	.0466
50	.0027	.0044	.0073	.0151	.0250	.0315	.0394	.0489	.0589
55	.0024	.0039	.0069	.0140	.0246	.0319	.0417	.0541	.0683
60	.0022	.0034	.0061	.0127	.0235	.0315	.0424	.0572	.0750
65	.0020	.0030	.0054	.0115	.0220	.0302	.0420	.0585	.0792
70	.0018	.0028	.0048	.0103	.0203	.0285	.0406	.0582	.0810
75	.0017	.0025	.0043	.0091	.0184	.0263	.0384	.0565	.0807
80	.0015	.0022	.0038	.0080	.0164	.0238	.0356	.0537	.0784
85	.0013	.0019	.0033	.0069	.0144	.0212	.0323	.0498	.0744
90	.0011	.0017	.0028	.0058	.0123	.0184	.0286	.0451	.0689
100	.0007	.0011	.0018	.0038	.0082	.0126	.0204	.0339	.0542
110	.0004	.0006	.0009	.0019	.0043	.0069	.0119	.0213	.0361
120	.0000	.0000	.0000	.0001	.0005	.0014	.0035	.0082	.0164
130	-.0003	-.0005	-.0008	-.0016	-.0028	-.0036	-.0043	-.0045	-.0034
140	-.0007	-.0010	-.0016	-.0031	-.0057	-.0079	-.0112	-.0159	-.0218
150	-.0009	-.0013	-.0022	-.0043	-.0081	-.0115	-.0169	-.0254	-.0375
160	-.0011	-.0016	-.0027	-.0052	-.0099	-.0141	-.0211	-.0326	-.0495
170	-.0012	-.0018	-.0029	-.0057	-.0110	-.0157	-.0237	-.0370	-.0570
180	-.0012	-.0018	-.0030	-.0059	-.0113	-.0163	-.0245	-.0385	-.0595

TABLE B 6.2.1-19

	C_{ft}			L_r/L_c = .400			X = 0		
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
ϕ^o									
0	-.5000	-.5000	-.5000	-.5000	-.5000	-.5000	-.5000	-.5000	-.5000
5	-.2862	-.3103	-.3382	-.3715	-.3998	-.4141	-.4298	-.4469	-.4644
10	-.1334	-.1664	-.2075	-.2605	-.3084	-.3336	-.3619	-.3936	-.4265
15	-.0508	-.0784	-.1175	-.1744	-.2313	-.2629	-.2997	-.3421	-.3875
20	-.0096	-.0279	-.0585	-.1101	-.1680	-.2023	-.2438	-.2933	-.3479
25	.0100	-.0003	-.0214	-.0636	-.1173	-.1514	-.1944	-.2477	-.3083
30	.0141	.0105	-.0014	-.0322	-.0782	-.1098	-.1516	-.2056	-.2693
35	.0113	.0122	.0078	-.0120	-.0486	-.0763	-.1149	-.1672	-.2311
40	.0093	.0119	.0119	.0009	-.0264	-.0495	-.0836	-.1322	-.1940
45	.0074	.0103	.0128	.0085	-.0102	-.0284	-.0572	-.1007	-.1583
50	.0042	.0074	.0113	.0121	.0012	-.0121	-.0351	-.0724	-.1243
55	.0026	.0054	.0097	.0138	.0092	.0005	-.0168	-.0472	-.0921
60	.0029	.0048	.0087	.0145	.0149	.0102	-.0015	-.0249	-.0619
65	.0023	.0039	.0075	.0142	.0185	.0174	.0109	-.0053	-.0339
70	.0014	.0029	.0063	.0136	.0208	.0227	.0210	.0117	-.0081
75	.0019	.0031	.0060	.0132	.0224	.0267	.0291	.0264	.0154
80	.0025	.0034	.0059	.0128	.0233	.0295	.0355	.0389	.0364
85	.0018	.0029	.0054	.0122	.0236	.0313	.0404	.0492	.0550
90	.0018	.0029	.0053	.0118	.0237	.0325	.0441	.0577	.0710
100	.0024	.0034	.0055	.0114	.0232	.0333	.0482	.0694	.0955
110	.0022	.0032	.0053	.0108	.0221	.0323	.0489	.0747	.1099
120	.0019	.0029	.0050	.0100	.0203	.0302	.0468	.0745	.1147
130	.0021	.0030	.0047	.0091	.0181	.0269	.0425	.0695	.1107
140	.0012	.0021	.0038	.0076	.0152	.0227	.0362	.0605	.0987
150	.0006	.0021	.0032	.0061	.0120	.0178	.0285	.0483	.0801
160	.0006	.0011	.0020	.0041	.0082	.0122	.0196	.0336	.0564
170	.0006	.0008	.0011	.0022	.0042	.0062	.0100	.0172	.0291
180	0	0	0	0	0	0	0	0	0

TABLE B 6.2.1-20

C_{qt}			L_r/L_c = .400				X = 0		
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
ϕ^o									
0	1.0802	.9271	.7551	.5572	.3969	.3188	.2357	.1481	.0619
5	.8619	.7629	.6421	.4909	.3590	.2918	.2182	.1386	.0585
10	.4200	.4205	.3988	.3427	.2715	.2285	.1766	.1157	.0501
15	.0895	.1419	.1824	.1975	.1794	.1594	.1297	.0889	.0399
20	-.0676	-.0167	.0374	.0844	.0998	.0970	.0853	.0625	.0295
25	-.1646	-.1238	-.0702	-.0087	.0292	.0397	.0432	.0365	.0190
30	-.2271	-.1947	-.1465	-.0812	-.0302	-.0102	.0051	.0122	.0088
35	-.2291	-.2129	-.1809	-.1260	-.0734	-.0488	-.0261	-.0088	-.0004
40	-.2111	-.2086	-.1925	-.1523	-.1040	-.0781	-.0512	-.0266	-.0086
45	-.2098	-.2094	-.2000	-.1698	-.1267	-.1008	-.0716	-.0417	-.0156
50	-.2000	-.2006	-.1964	-.1759	-.1400	-.1159	-.0865	-.0535	-.0215
55	-.1735	-.1785	-.1811	-.1714	-.1445	-.1236	-.0959	-.0620	-.0281
60	-.1594	-.1640	-.1681	-.1644	-.1447	-.1270	-.1015	-.0679	-.0296
65	-.1547	-.1555	-.1576	-.1559	-.1415	-.1267	-.1039	-.0715	-.0320
70	-.1363	-.1378	-.1408	-.1424	-.1335	-.1221	-.1025	-.0724	-.0333
75	-.1160	-.1189	-.1231	-.1272	-.1230	-.1146	-.0985	-.0713	-.0336
80	-.1089	-.1092	-.1109	-.1141	-.1122	-.1061	-.0928	-.0687	-.0330
85	-.0980	-.0970	-.0974	-.1000	-.0998	-.0957	-.0853	-.0645	-.0316
90	-.0767	-.0780	-.0799	-.0837	-.0857	-.0836	-.0761	-.0588	-.0294
100	-.0562	-.0551	-.0546	-.0562	-.0589	-.0590	-.0558	-.0451	-.0235
110	-.0215	-.0230	-.0244	-.0270	-.0309	-.0329	-.0332	-.0287	-.0159
120	-.0038	-.0029	-.0025	-.0032	-.0063	-.0088	-.0114	-.0119	-.0075
130	.0244	.0235	.0224	.0206	.0171	.0140	.0096	.0047	.0010
140	.0411	.0410	.0407	.0395	.0363	.0331	.0278	.0195	.0089
150	.0568	.0568	.0565	.0552	.0521	.0488	.0427	.0320	.0156
160	.0715	.0705	.0693	.0674	.0640	.0605	.0539	.0414	.0208
170	.0735	.0742	.0743	.0734	.0705	.0672	.0605	.0471	.0240
180	.0823	.0809	.0793	.0771	.0735	.0700	.0631	.0492	.0251

TABLE B 6.2.1-21

C_{mt}			L_r/L_c = 1.000				X = 0		
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
ϕ^{o}									
0	0	0	0	0	0	0	0	0	0
5	-.0045	-.0054	-.0066	-.0084	-.0106	-.0119	-.0136	-.0155	-.0175
10	-.0065	-.0080	-.0102	-.0138	-.0179	-.0205	-.0237	-.0275	-.0315
15	-.0069	-.0088	-.0118	-.0168	-.0226	-.0263	-.0308	-.0363	-.0421
20	-.0064	-.0086	-.0120	-.0179	-.0251	-.0296	-.0354	-.0423	-.0497
25	-.0055	-.0077	-.0113	-.0177	-.0258	-.0311	-.0377	-.0458	-.0544
30	-.0044	-.0065	-.0100	-.0166	-.0252	-.0309	-.0382	-.0471	-.0567
35	-.0034	-.0053	-.0085	-.0149	-.0236	-.0295	-.0371	-.0466	-.0568
40	-.0026	-.0041	-.0069	-.0129	-.0213	-.0271	-.0348	-.0444	-.0550
45	-.0018	-.0030	-.0054	-.0106	-.0185	-.0241	-.0315	-.0410	-.0514
50	-.0012	-.0020	-.0039	-.0084	-.0154	-.0205	-.0275	-.0366	-.0466
55	-.0007	-.0012	-.0026	-.0062	-.0122	-.0167	-.0231	-.0314	-.0407
60	-.0002	-.0006	-.0014	-.0041	-.0090	-.0128	-.0183	-.0256	-.0339
65	.0001	.0000	-.0004	-.0022	-.0058	-.0089	-.0134	-.0196	-.0266
70	.0004	.0005	.0004	-.0005	-.0029	-.0051	-.0085	-.0133	-.0190
75	.0006	.0009	.0011	.0011	-.0001	-.0015	-.0038	-.0071	-.0113
80	.0008	.0012	.0017	.0024	.0024	.0019	.0008	-.0011	-.0036
85	.0010	.0014	.0022	.0035	.0046	.0049	.0050	.0046	.0038
90	.0011	.0016	.0026	.0044	.0065	.0076	.0088	.0099	.0108
100	.0013	.0019	.0031	.0057	.0094	.0118	.0150	.0188	.0229
110	.0013	.0021	.0033	.0064	.0111	.0145	.0192	.0252	.0318
120	.0013	.0020	.0033	.0065	.0117	.0157	.0213	.0287	.0371
130	.0012	.0019	.0031	.0061	.0113	.0154	.0213	.0293	.0385
140	.0011	.0016	.0027	.0053	.0101	.0139	.0195	.0272	.0361
150	.0009	.0013	.0022	.0043	.0082	.0114	.0161	.0227	.0304
160	.0006	.0009	.0015	.0030	.0057	.0080	.0115	.0163	.0219
170	.0003	.0005	.0008	.0015	.0030	.0042	.0060	.0085	.0114
180	0	0	0	0	0	0	0	0	0

TABLE B 6.2.1-22

C_{st}			$L_r/L_c = 1.000$				X = 0		
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
ø°									
0	-.0713	-.0811	-.0951	-.1171	-.1421	-.1574	-.1763	-.1988	-.2222
5	-.0349	-.0438	-.0568	-.0777	-.1017	-.1166	-.1350	-.1569	-.1799
10	-.0117	-.0183	-.0287	-.0465	-.0678	-.0813	-.0983	-.1187	-.1403
15	.0018	-.0021	-.0091	-.0225	-.0400	-.0515	-.0662	-.0843	-.1036
20	.0087	.0075	.0040	-.0047	-.0176	-.0267	-.0387	-.0538	-.0701
25	.0115	.0123	.0119	.0080	-.0001	-.0064	-.0153	-.0269	-.0398
30	.0117	.0141	.0162	.0166	.0133	.0097	.0042	-.0037	-.0129
35	.0108	.0141	.0181	.0220	.0230	.0222	.0200	.0160	.0108
40	.0094	.0131	.0182	.0249	.0298	.0315	.0324	.0323	.0311
45	.0079	.0116	.0173	.0260	.0341	.0380	.0420	.0455	.0482
50	.0065	.0010	.0159	.0258	.0364	.0423	.0488	.0557	.0621
55	.0053	.0085	.0141	.0246	.0371	.0445	.0533	.0632	.0730
60	.0043	.0071	.0123	.0229	.0365	.0451	.0557	.0682	.0809
65	.0035	.0059	.0106	.0208	.0350	.0444	.0564	.0708	.0860
70	.0029	.0048	.0090	.0185	.0328	.0426	.0555	.0714	.0885
75	.0024	.0040	.0075	.0162	.0300	.0399	.0533	.0702	.0887
80	.0020	.0032	.0062	.0139	.0269	.0366	.0500	.0674	.0866
85	.0016	.0026	.0050	.0116	.0235	.0328	.0459	.0632	.0825
90	.0013	.0020	.0039	.0095	.0201	.0287	.0411	.0578	.0768
100	.0007	.0011	.0021	.0055	.0131	.0199	.0300	.0443	.0609
110	.0002	.0003	.0006	.0021	.0065	.0109	.0180	.0285	.0411
120	-.0003	-.0005	-.0007	-.0008	.0005	.0024	.0060	.0118	.0191
130	-.0007	-.0011	-.0018	-.0033	-.0048	-.0052	-.0053	-.0045	-.0030
140	-.0011	-.0016	-.0027	-.0053	-.0091	-.0118	-.0152	-.0194	-.0237
150	-.0014	-.0021	-.0035	-.0068	-.0126	-.0171	-.0234	-.0319	-.0415
160	-.0016	-.0024	-.0040	-.0079	-.0151	-.0209	-.0295	-.0413	-.0551
170	-.0017	-.0026	-.0043	-.0086	-.0166	-.0233	-.0332	-.0472	-.0636
180	-.0018	-.0027	-.0044	-.0088	-.0171	-.0241	-.0345	-.0492	-.0665

TABLE B 6.2.1-23

C_{ft}				L_r/L_c = 1.000				X = 0	
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
ϕ^o									
0	-.5000	-.5000	-.5000	-.5000	-.5000	-.5000	-.5000	-.5000	-.5000
5	-.3359	-.3556	-.3778	-.4037	-.4251	-.4357	-.4471	-.4589	-.4700
10	-.2032	-.2338	-.2700	-.3145	-.3528	-.3723	-.3935	-.4159	-.4371
15	-.1117	-.1433	-.1839	-.2374	-.2867	-.3125	-.3413	-.3723	-.4022
20	-.0517	-.0791	-.1174	-.1728	-.2275	-.2574	-.2915	-.3290	-.3656
25	-.0143	-.0352	-.0679	-.1199	-.1756	-.2074	-.2446	-.2865	-.3281
30	.0055	-.0082	-.0332	-.0782	-.1311	-.1630	-.2012	-.2454	-.2901
35	.0141	.0069	-.0099	-.0461	-.0937	-.1239	-.1615	-.2060	-.2521
40	.0174	.0151	.0052	-.0218	-.0625	-.0900	-.1255	-.1687	-.2144
45	.0173	.0183	.0142	-.0041	-.0369	-.0610	-.0931	-.1336	-.1774
50	.0148	.0182	.0186	.0084	-.0164	-.0364	-.0644	-.1010	-.1416
55	.0122	.0167	.0204	.0168	-.0002	-.0158	-.0391	-.0709	-.1071
60	.0103	.0150	.0205	.0222	.0125	.0012	-.0170	-.0433	-.0743
65	.0082	.0129	.0194	.0253	.0220	.0150	.0019	-.0184	-.0435
70	.0063	.0107	.0178	.0266	.0290	.0260	.0181	.0040	-.0148
75	.0053	.0092	.0161	.0269	.0340	.0345	.0316	.0237	.0116
80	.0047	.0080	.0145	.0264	.0372	.0410	.0427	.0409	.0355
85	.0039	.0068	.0129	.0254	.0391	.0457	.0516	.0555	.0568
90	.0034	.0059	.0115	.0241	.0399	.0489	.0585	.0678	.0753
100	.0032	.0050	.0095	.0211	.0392	.0515	.0670	.0854	.1041
110	.0028	.0044	.0079	.0181	.0364	.0504	.0696	.0943	.1216
120	.0025	.0038	.0067	.0153	.0324	.0466	.0674	.0957	.1281
130	.0024	.0035	.0058	.0127	.0277	.0410	.0614	.0902	.1244
140	.0018	.0027	.0047	.0101	.0225	.0341	.0524	.0791	.1114
150	.0016	.0023	.0037	.0077	.0171	.0263	.0412	.0634	.0908
160	.0009	.0015	.0025	.0051	.0115	.0178	.0283	.0442	.0640
170	.0006	.0008	.0013	.0026	.0058	.0090	.0144	.0227	.0331
180	0	0	0	0	0	0	0	0	0

TABLE B 6.2.1-24

C_{qt}			L_r/L_c = 1.000			X = 0			
γ \ ϕ^o	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
0	.7669	.6485	.5185	.3726	.2574	.2025	.1453	.0872	.0343
5	.6531	.5647	.4622	.3404	.2393	.1897	.1371	.0829	.0327
10	.4076	.3797	.3345	.2650	.1959	.1587	.1170	.0719	.0288
15	.1882	.2039	.2051	.1831	.1462	.1221	.0927	.0585	.0239
20	.0401	.0732	.0993	.1094	.0982	.0859	.0678	.0443	.0185
25	-.0704	-.0308	.0093	.0421	.0521	.0501	.0428	.0296	.0129
30	-.1490	-.1092	-.0633	-.0162	.0099	.0165	.0187	.0152	.0073
35	-.1855	-.1536	-.1116	-.0609	-.0254	-.0124	-.0029	.0020	.0020
40	-.1975	-.1765	-.1427	-.0943	-.0543	-.0370	-.0218	-.0100	-.0029
45	-.2053	-.1910	-.1642	-.1199	-.0780	-.0579	-.0383	-.0208	-.0073
50	-.2016	-.1933	-.1740	-.1363	-.9560	-.0742	-.0518	-.0299	-.0112
55	-.1859	-.1841	-.1731	-.1441	-.1071	-.0858	-.0620	-.0371	-.0145
60	-.1724	-.1737	-.1683	-.1468	-.1143	-.0939	-.0696	-.0428	-.0171
65	-.1611	-.1631	-.1608	-.1454	-.1176	-.0985	-.0747	-.0469	-.0191
70	-.1437	-.1471	-.1482	-.1390	-.1156	-.0996	-.0770	-.0493	-.0203
75	-.1254	-.1298	-.1335	-.1295	-.1125	-.0977	-.0770	-.0502	-.0210
80	-.1127	-.1160	-.1198	-.1189	-.1063	-.0938	-.0751	-.0497	-.0211
85	-.0987	-.1011	-.1049	-.1064	-.0979	-.0877	-.0714	-.0480	-.0206
90	-.0808	-.0837	-.0882	-.0921	-.0874	-.0797	-.0659	-.0450	-.0196
100	-.0549	-.0558	-.0587	-.0637	-.0642	-.0605	-.0517	-.0364	-.0162
110	-.0243	-.0257	-.0285	-.0342	-.0383	-.0379	-.0341	-.0250	-.0115
120	-.0020	-.0021	-.0033	-.0076	-.0131	-.0151	-.0153	-.0123	-.0060
130	.0230	.0224	.0211	.0172	.0109	.0072	.0036	-.0009	-.0002
140	.0415	.0413	.0406	.0378	.0317	.0269	.0207	.0131	.0054
150	.0573	.0572	.0567	.0544	.0487	.0434	.0352	.0237	.0102
160	.0703	.0697	.0689	.0669	.0614	.0557	.0462	.0318	.0139
170	.0753	.0755	.0752	.0739	.0689	.0631	.0529	.0368	.0163
180	.0803	.0796	.0787	.0769	.0717	.0659	.0554	.0386	.0171

TABLE B 6.2.1-25

	C_{mm}			L_r/L_c = .200				X = 0	
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
ϕ^o									
0	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
5	.2439	.2708	.3026	.3411	.3737	.3900	.4075	.4262	.4448
10	.0833	.1159	.1581	.2145	.2665	.2937	.3241	.3574	.3915
15	.0195	.0409	.0745	.1278	.1841	.2160	.2532	.2958	.3411
20	-.0024	.0075	.0285	.0704	.1224	.1545	.1939	.2414	.2940
25	-.0119	-.0080	.0036	.0334	.0771	.1067	.1450	.1937	.2500
30	-.0098	-.0100	-.0059	.0121	.0455	.0707	.1056	.1524	.2094
35	-.0040	-.0067	-.0075	.0008	.0239	.0440	.0740	.1169	.1721
40	-.0037	-.0060	-.0083	-.0059	.0089	.0240	.0486	.0864	.1378
45	-.0042	-.0056	-.0081	-.0092	-.0012	.0094	.0285	.0604	.1065
50	-.0015	-.0032	-.0062	-.0098	-.0074	-.0010	.0128	.0384	.0781
55	-.0008	-.0023	-.0052	-.0100	-.0114	-.0084	.0004	.0197	.0525
60	-.0027	-.0034	-.0055	-.0102	-.0142	-.0139	-.0093	.0040	.0296
65	-.0022	-.0030	-.0049	-.0098	-.0156	-.0175	-.0166	-.0091	.0091
70	-.0007	-.0019	-.0041	-.0092	-.0162	-.0199	-.0221	-.0198	-.0088
75	-.0017	-.0026	-.0044	-.0091	-.0167	-.0215	-.0263	-.0286	-.0245
80	-.0026	-.0032	-.0046	-.0090	-.0168	-.0225	-.0293	-.0356	-.0380
85	-.0013	-.0022	-.0040	-.0084	-.0165	-.0228	-.0312	-.0410	-.0494
90	-.0010	-.0020	-.0038	-.0080	-.0161	-.0228	-.0324	-.0450	-.0587
100	-.0019	-.0025	-.0039	-.0077	-.0152	-.0221	-.0330	-.0497	-.0718
110	-.0014	-.0021	-.0035	-.0070	-.0140	-.0205	-.0317	-.0504	-.0781
120	-.0009	-.0016	-.0030	-.0061	-.0124	-.0184	-.0291	-.0481	-.0783
130	-.0016	-.0020	-.0029	-.0055	-.0107	-.0160	-.0256	-.0435	-.0733
140	-.0002	-.0009	-.0019	-.0043	-.0087	-.0131	-.0212	-.0369	-.0640
150	-.0013	-.0014	-.0019	-.0035	-.0068	-.0101	-.0164	-.0289	-.0511
160	.0000	-.0004	-.0009	-.0022	-.0045	-.0068	-.0111	-.0198	-.0356
170	-.0005	-.0005	-.0007	-.0012	-.0023	-.0035	-.0056	-.0101	-.0182
180	0	0	0	0	0	0	0	0	0

TABLE B 6.2.1-26

	C_{sm}		L_r/L_c = .200				X = 0		
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
ϕ^o									
0	-3.1704	-2.8075	-2.3889	-1.3961	-1.4903	-1.2916	-1.0799	-.8569	-.6374
5	-2.4933	-2.2873	-2.0239	-1.6779	-1.3640	-1.2011	-1.0209	-.8245	-.6249
10	-1.1968	-1.2592	-1.2758	-1.2111	-1.0840	-.9967	-.8853	-.7483	-.5946
15	-.3902	-.5468	-.6947	-.8013	-.8144	-.7911	-.7425	-.6638	-.5588
20	-.1669	-.2639	-.3905	-.5306	-.6077	-.6230	-.6179	-.5848	-.5222
25	-.0411	-.0934	-.1871	-.3259	-.4355	-.4766	-.5041	-.5088	-.4844
30	.0716	.0299	-.0457	-.1740	-.2967	-.3532	-.4035	-.4379	-.4464
35	.0383	.0279	-.0081	-.0964	-.2048	-.2636	-.3239	-.3768	-.4103
40	-.0180	-.0044	-.0080	-.0566	-.1421	-.1967	-.2594	-.3232	-.3755
45	.0172	.0194	.0150	-.0196	-.0902	-.1405	-.2034	-.2744	-.3414
50	.0306	.0266	.0213	-.0010	-.0557	-.0995	-.1588	-.2320	-.3089
55	-.0160	-.0076	.0006	-.0035	-.0386	-.0733	-.1254	-.1963	-.2732
60	-.0146	-.0082	-.0011	-.0002	-.0237	-.0514	-.0969	-.1642	-.2484
65	.0216	.0156	.0123	.0035	-.0102	-.0328	-.0725	-.1354	-.2198
70	.0040	.0032	.0033	.0048	-.0056	-.0222	-.0548	-.1112	-.1927
75	-.0206	-.0133	-.0073	-.0008	-.0041	-.0153	-.0409	-.0902	-.1669
80	.0055	.0040	.0034	.0045	.0016	-.0069	-.0277	-.0707	-.1420
85	.0169	.0119	.0031	.0067	.0048	-.0011	-.0173	-.0537	-.1184
90	-.0113	-.0073	-.0037	.0007	.0032	.0007	-.0103	-.0395	-.0961
100	.0159	.0114	.0080	.0067	.0082	.0080	.0033	-.0142	-.0546
110	-.0147	-.0094	-.0045	.0007	.0064	.0094	.0108	.0047	-.0180
120	.0139	.0103	.0073	.0074	.0107	.0141	.0185	.0208	.0144
130	-.0074	-.0042	-.0009	.0034	.0095	.0147	.0224	.0325	.0417
140	.0038	.0035	.0040	.0063	.0117	.0171	.0265	.0422	.0645
150	.0044	.0040	.0044	.0068	.0125	.0184	.0292	.0493	.0823
160	-.0069	-.0037	-.0003	.0046	.0118	.0185	.0306	.0540	.0950
170	.0128	.0099	.0081	.0090	.0143	.0204	.0325	.0573	.1028
180	-.0114	-.0067	-.0020	.0039	.0119	.0189	.0319	.0577	.1052

TABLE B 6.2.1-27

C_{fm}	$L_r/L_c = .200$						X = 0		
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
ϕ^o									
0	0	0	0	0	0	0	0	0	0
5	-13.8275	-10.7039	-7.5756	-4.5751	-2.6709	-1.9239	-1.2578	-.6955	-.2691
10	-13.4182	-11.0342	-8.3587	-5.4556	-3.3895	-2.5165	-1.7000	-.9755	-.3975
15	-5.0301	-5.1699	-4.8030	-3.8036	-2.7011	-2.1287	-1.5289	-.9369	-.4161
20	-1.2709	-2.0970	-2.6162	-2.5961	-2.1198	-1.7732	-1.3523	-.8830	-.4253
25	-1.6945	-1.8835	-2.0901	-2.1072	-1.8211	-1.5756	-1.2492	-.8539	-.4387
30	-.5018	-.7044	-1.0167	-1.3074	-1.3210	-1.2216	-1.0354	-.7591	-.4259
35	.9398	.5070	-.0102	-.5600	-.8293	-.8600	-.8049	-.6476	-.4038
40	.0709	.0269	-.1329	-.4314	-.6506	-.7012	-.6886	-.5867	-.3951
45	-.5668	-.3693	-.2806	-.3669	-.5163	-.5709	-.5851	-.5276	-.3839
50	.3487	.2645	.1546	-.0503	-.2714	-.3686	-.4368	-.4426	-.3613
55	.4319	.3185	.2072	.0377	-.1590	-.2586	-.3441	-.3834	-.3451
60	-.4221	-.2741	-.1525	-.1092	-.1811	-.2434	-.3098	-.3523	-.3361
65	-.1745	-.1127	-.0568	-.0413	-.1087	-.1697	-.2425	-.3042	-.3197
70	.4473	.3093	.1982	.0979	-.0110	-.0837	-.1702	-.2537	-.3010
75	-.0336	-.0216	-.0080	-.0030	-.0427	-.0884	-.1550	-.2321	-.2906
80	-.3882	-.2663	-.1613	-.0803	-.0688	-.0927	-.1421	-.2123	-.2796
85	.1905	.1292	.0794	.0414	.0012	-.0358	-.0946	-.1759	-.2621
90	.2702	.1844	.1126	.0575	.0149	-.0188	-.0732	-.1534	-.2483
100	-.1285	-.0891	-.0561	-.0301	-.0236	-.0341	-.0650	-.1284	-.2249
110	-.0190	-.0134	-.0094	-.0070	-.0097	-.0188	-.0437	-.0999	-.1978
120	.1433	.0974	.0585	.0272	.0086	-.0030	-.0257	-.0756	-.1700
130	-.2329	-.1599	-.0990	-.0526	-.0309	-.0272	-.0340	-.0667	-.1455
140	.2719	.1858	.1131	.0556	.0244	.0114	-.0061	-.0412	-.1141
150	-.2619	-.1797	-.1108	-.0578	-.0320	-.0250	-.0242	-.0398	-.0895
160	.2038	.1394	.0851	.0423	.0194	.0106	.0005	-.0178	-.0573
170	-.1116	-.0766	-.0472	-.0245	-.0134	-.0101	-.0090	-.0134	-.0303
180	0	0	0	0	0	0	0	0	0

TABLE B 6.2.1-28

C_{qm}	$L_r/L_c = .200$						X = 0		
γ / ϕ^o	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
0	-97.7290	-74.4364	-51.6332	-30.3642	-17.2652	-12.2413	-7.8356	-4.1889	-1.4893
5	-41.7926	-34.0444	-25.4518	-16.2837	-9.8887	-7.2309	-4.7779	-2.6374	-.9661
10	40.2106	26.5155	14.7833	5.9408	1.9823	.9005	.2249	-.0799	-.0984
15	42.0065	31.0848	20.2264	10.4161	4.9552	3.1130	1.6913	.7197	.1872
20	2.8684	5.1413	5.4306	3.9087	2.2096	1.4673	.8248	.3489	.0828
25	-.5785	2.1281	3.5464	3.3068	2.2199	1.6092	1.0069	.4945	.1494
30	11.7559	9.6871	7.7161	5.3971	3.4144	2.4917	1.6068	.8383	.2800
35	1.2469	1.7724	2.3110	2.4197	1.9284	1.5390	1.0773	.6052	.2152
40	-7.9488	-5.0669	-2.3947	-.2853	.4982	.5901	.5280	.3514	.1408
45	2.0650	1.4856	1.2581	1.2770	1.1823	1.0331	.7994	.4997	.1979
50	5.0499	3.4313	2.2455	1.5479	1.2076	1.0236	.7873	.4990	.2028
55	-4.3005	-3.0045	-1.8179	-.6911	-.0276	.1653	.2573	.2332	.1173
60	-2.9426	-2.0933	-1.3235	-.5619	-.0555	.1126	.2073	.2037	.1088
65	4.5897	3.0818	1.8342	.9697	.6422	.5471	.4486	.3173	.1466
70	.4406	.2655	.1117	.0524	.1224	.1709	.2022	.1849	.1012
75	-4.4378	-3.0739	-1.9354	-1.0138	-.4589	-.2426	-.0658	.0406	.0508
80	1.2554	.8245	.4588	.1911	.1172	.1218	.1356	.1309	.0780
85	3.2939	2.2316	1.3312	.6320	.3181	.2394	.1905	.1479	.0802
90	-2.6982	-1.8694	-1.1800	-.6454	-.3409	-.2145	-.0965	-.0060	.0251
100	3.1751	2.1480	1.2849	.6090	.2751	.1787	.1178	.0826	.0464
110	-3.3286	-2.3047	-1.4447	-.7765	-.4300	-.3035	-.1879	-.0864	-.0184
120	2.6050	1.7598	1.0489	.4929	.2079	.1170	.0543	.0220	.0098
130	-1.7893	-1.2520	-.7998	-.4472	-.2669	-.2037	-.1458	-.0883	-.0341
140	.4320	.2718	.1362	.0294	-.0266	-.0447	-.0547	-.0504	-.0276
150	.6222	.3999	.2137	.0684	-.0071	-.0325	-.0504	-.0537	-.0335
160	-1.8151	-1.2679	-.8087	-.4512	-.2690	-.2081	-.1577	-.1113	-.0562
170	2.3464	1.5815	.9388	.4373	.1790	.0912	.0214	-.0237	-.0292
180	-2.7488	-1.9078	-1.2014	-.6510	-.3698	-.2758	-.1996	-.1351	-.0671

TABLE B 6.2.1-29

C_{mm}			L_r/L_c = .400				X = 0		
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
ϕ^o									
0	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
5	.2831	.3065	.3334	.3651	.3914	.4043	.4183	.4331	.4478
10	.1294	.1612	.2005	.2504	.2946	.3172	.3422	.3694	.3968
15	.0469	.0731	.1099	.1627	.2144	.2425	.2745	.3105	.3480
20	.0063	.0231	.0512	.0982	.1498	.1798	.2154	.2569	.3015
25	-.0127	-.0038	.0149	.0522	.0991	.1284	.1647	.2088	.2577
30	-.0162	-.0138	-.0042	.0219	.0609	.0875	.1220	.1660	.2168
35	-.0130	-.0149	-.0124	.0031	.0329	.0555	.0867	.1285	.1788
40	-.0107	-.0140	-.0156	-.0084	.0125	.0307	.0576	.0957	.1436
45	-.0084	-.0119	-.0156	-.0145	-.0015	.0121	.0340	.0673	.1114
50	-.0050	-.0085	-.0134	-.0168	-.0108	-.0016	.0152	.0430	.0820
55	-.0032	-.0062	-.0112	-.0172	-.0166	-.0114	.0004	.0223	.0554
60	-.0032	-.0053	-.0096	-.0167	-.0202	-.0183	-.0111	.0049	.0315
65	-.0024	-.0041	-.0079	-.0155	-.0218	-.0228	-.0199	-.0096	.0101
70	-.0013	-.0029	-.0063	-.0139	-.0223	-.0255	-.0263	-.0216	-.0087
75	-.0017	-.0028	-.0056	-.0126	-.0221	-.0271	-.0310	-.0312	-.0250
80	-.0021	-.0029	-.0051	-.0115	-.0215	-.0277	-.0342	-.0339	-.0391
85	-.0014	-.0023	-.0043	-.0102	-.0205	-.0276	-.0361	-.0448	-.0510
90	-.0012	-.0021	-.0040	-.0093	-.0194	-.0270	-.0371	-.0491	-.0608
100	-.0016	-.0023	-.0037	-.0080	-.0172	-.0251	-.0370	-.0538	-.0744
110	-.0014	-.0020	-.0033	-.0069	-.0149	-.0225	-.0349	-.0543	-.0809
120	-.0010	-.0017	-.0029	-.0060	-.0128	-.0196	-.0314	-.0515	-.0812
130	-.0013	-.0018	-.0027	-.0052	-.0108	-.0165	-.0272	-.0463	-.0760
140	-.0005	-.0011	-.0020	-.0042	-.0087	-.0133	-.0223	-.0391	-.0663
150	-.0010	-.0012	-.0018	-.0033	-.0066	-.0101	-.0170	-.0305	-.0529
160	-.0002	-.0005	-.0010	-.0022	-.0044	-.0068	-.0115	-.0208	-.0368
170	-.0004	-.0004	-.0006	-.0011	-.0022	-.0034	-.0058	-.0106	-.0189
180	0	0	0	0	0	0	0	0	0

TABLE B 6.2.1-30

C_{sm}				L_r/L_c = .400				X = 0	
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
ϕ^o									
0	-2.6379	-2.3318	-1.9876	-1.5919	-1.2712	-1.1151	-.9488	-.7736	-.6013
5	-2.2001	-2.0020	-1.7604	-1.4581	-1.1942	-1.0598	-.9126	-.7534	-.5933
10	-1.3126	-1.3136	-1.2703	-1.1581	-1.0157	-.9295	-.8259	-.7040	-.5728
15	-.6457	-.7503	-.8313	-.8616	-.8254	-.7854	-.7260	-.6444	-.5465
20	-.3231	-.4248	-.5332	-.6270	-.6578	-.6523	-.6289	-.5832	-.5173
25	-.1184	-.2000	-.3072	-.4303	-.5060	-.5271	-.5340	-.5206	-.4856
30	.0193	-.0455	-.1418	-.2725	-.3744	-.4145	-.4450	-.4592	-.4524
35	.0383	.0060	-.0581	-.1679	-.2731	-.3223	-.3677	-.4024	-.4192
40	.0192	.0142	-.0179	-.0985	-.1949	-.2468	-.3006	-.3499	-.3861
45	.0354	.0345	.0158	-.0447	-.1308	-.1826	-.2410	-.3009	-.3530
50	.0361	.0374	.0290	-.0120	-.0838	-.1320	-.1908	-.2568	-.3207
55	.0052	.0153	.0205	.0010	-.0527	-.0946	-.1500	-.2178	-.2895
60	.0005	.0097	.0180	.0104	-.0290	-.0644	-.1153	-.1825	-.2591
65	.0158	.0173	.0215	.0181	-.0108	-.0403	-.0860	-.1508	-.2296
70	.0045	.0075	.0135	.0168	-.0010	-.0239	-.0630	-.1232	-.2014
75	-.0096	-.0037	.0046	.0129	.0045	-.0123	-.0446	-.0989	-.1744
80	.0034	.0039	.0074	.0139	.0099	-.0023	-.0288	-.0771	-.1484
85	.0091	.0072	.0080	.0131	.0127	.0045	-.0163	-.0579	-.1237
90	-.0057	-.0032	.0007	.0082	.0122	.0080	-.0071	-.0416	-.1003
100	.0085	.0062	.0053	.0084	.0138	.0142	.0077	-.0137	-.0569
110	-.0073	-.0044	-.0014	.0038	.0116	.0154	.0161	.0072	-.0186
120	.0077	.0059	.0049	.0064	.0126	.0176	.0227	.0239	.0150
130	-.0034	-.0015	.0006	.0042	.0113	.0175	.0262	.0361	.0435
140	.0025	.0026	.0033	.0058	.0120	.0185	.0292	.0457	.0670
150	.0028	.0029	.0036	.0062	.0124	.0189	.0311	.0526	.0853
160	-.0030	-.0010	.0014	.0053	.0121	.0189	.0321	.0572	.0984
170	.0073	.0060	.0057	.0076	.0133	.0199	.0332	.0601	.1064
180	-.0054	-.0026	.0005	.0050	.0121	.0191	.0330	.0608	.1089

TABLE B 6.2.1-31

C_{fm}				L_r/L_c = .400			X = 0		
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
ϕ^o									
0	0	0	0	0	0	0	0	0	0
5	-9.0612	-6.8751	-4.7739	-2.8376	-1.6469	-1.1871	-.7799	-.4370	-.1762
10	-9.8022	-7.8404	-5.7738	-3.6703	-2.2499	-1.6663	-1.1281	-.6548	-.2783
15	-5.3140	-4.8966	-4.1404	-3.0243	-2.0515	-1.5926	-1.1332	-.6955	-.3195
20	-2.6485	-2.9189	-2.8804	-2.4261	-1.8161	-1.4739	-1.0985	-.7097	-.3495
25	-2.1029	-2.2682	-2.3070	-2.0737	-1.6518	-1.3838	-1.0684	-.7191	-.3752
30	-.8884	-1.1654	-1.4174	-1.5025	-1.3381	-1.1780	-.9571	-.6809	-.3822
35	.2531	-.1602	-.5935	-.9386	-1.0014	-.9441	-.8189	-.6230	-.3796
40	.0117	-.1559	-.4110	-.6975	-.8119	-.7992	-.7263	-.5822	-.3799
45	-.2367	-.2195	-.3144	-.5146	-.6462	-.6645	-.6337	-.5365	-.3758
50	.2599	.1735	.0174	-.2354	-.4334	-.4952	-.5161	-.4743	-.3636
55	.2962	.2245	.1075	-.1022	-.2996	-.3777	-.4269	-.4233	-.3527
60	-.1686	-.0826	-.0458	-.1148	-.2474	-.3154	-.3696	-.3857	-.3441
65	-.0594	-.0118	.0125	-.0405	-.1609	-.2321	-.2993	-.3400	-.3307
70	.2523	.1928	.1413	.0539	-.0736	-.1507	-.2308	-.2940	-.3154
75	-.0068	.0112	.0298	.0154	-.0617	-.1226	-.1953	-.2639	-.3037
80	-.1984	-.1243	-.0566	-.0189	-.0552	-.1011	-.1652	-.2362	-.2913
85	.1005	.0734	.0585	.0429	-.0069	-.0545	-.1216	-.2019	-.2755
90	.1416	.0985	.0694	.0488	.0082	-.0327	-.0947	-.1763	-.2613
100	-.0681	-.0462	-.0248	-.0026	-.0044	-.0243	-.0673	-.1396	-.2348
110	-.0101	-.0075	-.0044	.0028	.0027	-.0097	-.0420	-.1067	-.2062
120	.0742	.0494	.0290	.0159	.0097	.0004	-.0244	-.0803	-.1771
130	-.1218	-.0829	-.0514	-.0266	-.0126	-.0118	-.0238	-.0648	-.1497
140	.1417	.0954	.0569	.0273	.0134	.0069	-.0070	-.0434	-.1187
150	-.1370	-.0929	-.0571	-.0302	-.0160	-.0120	-.0146	-.0358	-.0911
160	.1064	.0717	.0431	.0206	.0095	.0055	-.0012	-.0190	-.0596
170	-.0584	-.0396	-.0243	-.0128	-.0070	-.0051	-.0052	-.0116	-.0306
180	0	0	0	0	0	0	0	0	0

TABLE B 6.2.1-32

C_{qm}			L_r/L_c = .400				X = 0		
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
ϕ^o									
0	-62.1528	-46.3842	-31.5480	-18.2282	-10.2682	-7.2619	-4.6437	-2.4849	-.8858
5	-30.0038	-23.7304	-17.2080	-10.6842	-6.3692	-4.6271	-3.0428	-1.6754	-.6134
10	19.1109	11.5482	5.5868	1.5718	.0654	-.2490	-.3649	-.3128	-.1526
15	25.0324	17.4571	10.5683	4.9268	2.0886	1.2067	.5730	.1873	.0233
20	6.1706	5.6958	4.3340	2.4420	1.1474	.6816	.3233	.0958	.0031
25	3.6888	4.2100	3.8175	2.5915	1.4597	.9794	.5626	.2483	.0647
30	8.9015	7.5251	5.8716	3.8263	2.2428	1.5726	.9705	.4823	.1530
35	2.2920	2.6892	2.7776	2.3028	1.5709	1.1743	.7718	.4086	.1379
40	-3.4110	-1.5650	-.0631	.7956	.8480	.7225	.5297	.3081	.1128
45	1.2677	1.2710	1.3908	1.3967	1.1408	.9323	.6746	.3963	.1492
50	2.6000	1.9523	1.5795	1.3423	1.0762	.8906	.6592	.3995	.1559
55	-2.3734	-1.5494	-.7206	.0362	.3662	.4083	.3715	.2621	.1143
60	-1.7017	-1.1773	-.6211	-.0429	.2650	.3257	.3180	.2380	.1092
65	2.2585	1.4671	.9048	.6225	.5377	.4889	.4077	.2824	.1256
70	.1360	.0268	-.0130	.0807	.2086	.2485	.2520	.2015	.0992
75	-2.3966	-1.6838	-1.0745	-.5123	-.1398	-.0050	.0867	.1136	.0694
80	.5871	.3399	.1448	.0634	.1067	.1396	.1596	.1435	.0781
85	1.6735	1.0854	.5987	.2682	.1751	.1670	.1620	.1372	.0739
90	-1.4478	-1.0169	-.6725	-.3869	-.1809	-.0869	-.0041	.0460	.0409
100	1.6180	1.0633	.5998	.2455	.1013	.0756	.0693	.0658	.0411
110	-1.7763	-1.2274	-.7822	-.4489	-.2648	-.1861	-.1076	-.0388	-.0010
120	1.3239	.8695	.4944	.2001	.0544	.0162	-.0007	.0004	.0046
130	-.9743	-.6848	-.4476	-.2709	-.1835	-.1486	-.1101	-.0663	-.0246
140	.1891	.1023	.0308	-.0268	-.0600	-.0691	-.0693	-.0553	-.0271
150	.2857	.1670	.0697	-.0059	-.0481	-.0624	-.0697	-.0625	-.0346
160	-.9856	-.6922	-.4515	-.2681	-.1784	-.1498	-.1248	-.0950	-.0491
170	1.1873	.7766	.4396	.1815	.0481	.0014	-.0353	-.0530	-.0374
180	-1.4738	-1.0224	-.6521	-.3690	-.2280	-.1829	-.1465	-.1096	-.0571

TABLE B 6.2.1-33

	C_{mm}		L_r/L_c = 1.000				X = 0		
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
δ^{o}									
0	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
5	.3314	.3502	.3713	.3953	.4145	.4238	.4335	.4434	.4525
10	.1968	.2258	.2598	.3007	.3349	.3518	.3698	.3864	.4057
15	.1049	.1345	.1721	.2207	.2640	.2863	.3105	.3360	.3601
20	.0454	.0705	.1054	.1549	.2024	.2277	.2561	.2867	.3160
25	.0089	.0276	.0566	.1022	.1497	.1764	.2069	.2407	.2737
30	-.0099	.0017	.0231	.0616	.1059	.1321	.1631	.1983	.2334
35	-.0175	-.0122	.0014	.0312	.0701	.0945	.1244	.1595	.1953
40	-.0199	-.0191	-.0121	.0090	.0412	.0629	.0907	.1243	.1595
45	-.0191	-.0213	-.0196	-.0066	.0184	.0369	.0616	.0926	.1260
50	-.0160	-.0202	-.0225	-.0167	.0010	.0159	.0369	.0644	.0950
55	-.0129	-.0180	-.0229	-.0229	-.0120	-.0009	.0160	.0395	.0664
60	-.0105	-.0156	-.0219	-.0263	-.0214	-.0140	-.0013	.0177	.0404
65	-.0081	-.0129	-.0199	-.0274	-.0278	-.0239	-.0153	-.0012	.0168
70	-.0059	-.0102	-.0173	-.0271	-.0319	-.0310	-.0266	-.0173	-.0042
75	-.0048	-.0083	-.0150	-.0259	-.0341	-.0360	-.0353	-.0308	-.0229
80	-.0040	-.0068	-.0128	-.0241	-.0349	-.0392	-.0419	-.0420	-.0391
85	-.0029	-.0053	-.0107	-.0219	-.0345	-.0408	-.0466	-.0509	-.0530
90	-.0023	-.0043	-.0089	-.0196	-.0334	-.0413	-.0497	-.0579	-.0645
100	-.0019	-.0031	-.0064	-.0154	-.0298	-.0397	-.0520	-.0665	-.0812
110	-.0015	-.0024	-.0046	-.0118	-.0253	-.0359	-.0504	-.0692	-.0898
120	-.0012	-.0018	-.0034	-.0088	-.0207	-.0309	-.0461	-.0670	-.0910
130	-.0012	-.0016	-.0027	-.0066	-.0164	-.0256	-.0400	-.0609	-.0859
140	-.0007	-.0011	-.0020	-.0048	-.0124	-.0202	-.0329	-.0519	-.0753
150	-.0008	-.0010	-.0016	-.0034	-.0089	-.0149	-.0251	-.0407	-.0604
160	-.0003	-.0006	-.0010	-.0021	-.0057	-.0098	-.0168	-.0280	-.0421
170	-.0003	-.0004	-.0005	-.0011	-.0028	-.0048	-.0085	-.0142	-.0216
180	0	0	0	0	0	0	0	0	0

TABLE B 6.2.1-34

C_{sm}			$L_r/L_c = 1.000$			$X = 0$			
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
ϕ^o									
0	-2.0112	-1.7745	-1.5144	-1.2227	-.9924	-.8826	-.7680	-.6519	-.5460
5	-1.7824	-1.6057	-1.4006	-1.1571	-.9549	-.8557	-.7504	-.6420	-.5417
10	-1.2879	-1.2320	-1.1416	-1.0027	-.8645	-.7900	-.7065	-.6165	-.5302
15	-.8430	-.8744	-.8768	-.8328	-.7589	-.7109	-.6520	-.5835	-.5143
20	-.5385	-.6047	-.6569	-.6771	-.6547	-.6300	-.5940	-.5468	-.4953
25	-.3068	-.3860	-.4662	-.5318	-.5518	-.5479	-.5332	-.5069	-.4735
30	-.1368	-.2164	-.3083	-.4023	-.4545	-.4680	-.4722	-.4653	-.4495
35	-.0499	-.1127	-.1966	-.2981	-.3691	-.3951	-.4142	-.4238	-.4240
40	-.0079	-.0500	-.1176	-.2143	-.2945	-.3289	-.3594	-.3829	-.3973
45	.0264	-.0023	-.0559	-.1444	-.2283	-.2684	-.3077	-.3427	-.3696
50	.0394	.0228	-.0158	-.0911	-.1726	-.2153	-.2602	-.3041	-.3413
55	.0301	.0265	.0044	-.0535	-.1275	-.1701	-.2177	-.2675	-.3128
60	.0264	.0291	.0183	-.0247	-.0897	-.1306	-.1790	-.2327	-.2842
65	.0286	.0325	.0280	-.0028	-.0585	-.0966	-.1443	-.1998	-.2556
70	.0193	.0262	.0284	.0100	-.0348	-.0689	-.1140	-.1694	-.2273
75	.0092	.0181	.0254	.0175	-.0166	-.0461	-.0876	-.1412	-.1996
80	.0109	.0175	.0251	.0234	-.0019	-.0268	-.0642	-.1149	-.1723
85	.0105	.0153	.0229	.0260	.0088	-.0114	-.0441	-.0909	-.1457
90	.0024	.0082	.0172	.0250	.0157	.0002	-.0273	-.0691	-.1201
100	.0058	.0077	.0136	.0236	.0245	.0170	-.0004	-.0310	-.0715
110	-.0017	.0011	.0067	.0181	.0263	.0256	.0179	-.0002	-.0273
120	.0040	.0041	.0065	.0152	.0262	.0301	.0305	.0246	.0120
130	-.0006	.0006	.0032	.0111	.0236	.0310	.0383	.0438	.0458
140	.0018	.0021	.0035	.0092	.0214	.0308	.0433	.0585	.0740
150	.0019	.0022	.0032	.0077	.0192	.0298	.0461	.0692	.0962
160	-.0006	.0006	.0021	.0063	.0172	.0286	.0476	.0764	.1121
170	.0037	.0034	.0038	.0066	.0166	.0281	.0484	.0807	.1218
180	-.0015	.0000	.0017	.0054	.0157	.0275	.0484	.0820	.1250

TABLE B 6.2.1-35

	C_{fm}			L_r/L_c = 1.000			X = 0		
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
ϕ^{o}									
0	0	0	0	0	0	0	0	0	0
5	-4.8099	-3.5690	-2.4230	-1.4096	-.8087	-.5818	-.3834	-.2186	-.0957
10	-5.8355	-4.5150	-3.2111	-1.9708	-1.1825	-.8704	-.5890	-.3474	-.1611
15	-4.2130	-3.5595	-2.7701	-1.8701	-1.2076	-.9213	-.6487	-.4016	-.2009
20	-2.9511	-2.7305	-2.3228	-1.7169	-1.1863	-.9349	-.6819	-.4402	-.2345
25	-2.3889	-2.2761	-2.0355	-1.5994	-1.1629	-.9400	-.7056	-.4715	-.2645
30	-1.4619	-1.5620	-1.5465	-1.3444	-1.0521	-.8801	-.6854	-.4781	-.2849
35	-.6217	-.8746	-1.0471	-1.0573	-.9092	-.7925	-.6441	-.4713	-.2992
40	-.4247	-.6219	-.7992	-.8787	-.8078	-.7270	-.6116	-.4660	-.3123
45	-.3165	-.4441	-.5978	-.7140	-.7035	-.6547	-.5712	-.4539	-.3215
50	.0238	-.1310	-.3234	-.5094	-.5725	-.5607	-.5143	-.4310	-.3253
55	.1192	-.0029	-.1729	-.3687	-.4692	-.4822	-.4637	-.4087	-.3275
60	-.0314	-.0587	-.1473	-.2955	-.3979	-.4227	-.4221	-.3888	-.3286
65	.0276	.0103	-.0601	-.1983	-.3143	-.3536	-.3728	-.3630	-.3261
70	.1560	.1139	.0358	-.1045	-.2335	-.2856	-.3226	-.3350	-.3210
75	.0432	.0473	.0188	-.0752	-.1873	-.2403	-.2851	-.3119	-.3158
80	-.0448	-.0086	.0001	-.0545	-.1482	-.1999	-.2499	-.2886	-.3089
85	.0686	.0670	.0544	-.0039	-.0973	-.1523	-.2100	-.2620	-.2993
90	.0779	.0709	.0610	.0168	-.0647	-.1176	-.1778	-.2385	-.8891
100	-.0191	-.0005	.0182	.0160	-.0303	-.0719	-.1285	-.1971	-.2665
110	-.0005	.0064	.0180	.0238	-.0033	-.0358	-.0871	-.1584	-.2395
120	.0317	.0243	.0237	.0275	.0125	-.0117	-.0558	-.1245	-.2097
130	-.0495	-.0321	-.0142	.0060	.0088	-.0047	-.0378	-.0976	-.1784
140	.0583	.0393	.0262	.0228	.0198	.0086	-.0189	-.0709	-.1440
150	-.0561	-.0377	-.0217	-.0045	.0058	.0026	-.0141	-.0522	-.1098
160	.0437	.0292	.0179	.0127	.0125	.0085	-.0042	-.0317	-.0732
170	-.0239	-.0161	-.0097	-.0031	.0016	.0015	-.0034	-.0163	-.0371
180	0	0	0	0	0	0	0	0	0

TABLE B 6.2.1-36

C_{qm}				$L_r/L_c = 1.000$			X = 0		
γ	.02	.03	.05	.10	.20	.30	.50	1.00	3.00
ϕ^o									
0	-31.7536	-23.1673	-15.3880	-8.6690	-4.7950	-3.3572	-2.1254	-1.1238	-.3945
5	-17.3813	-13.2664	-9.2573	-5.5108	-3.1804	-2.2751	-1.4713	-.7945	-.2841
10	5.4898	2.7088	.7776	-.2647	-.4778	-.4507	-.3635	-.2345	-.0958
15	10.5519	6.8034	3.7189	1.4750	.4932	.2270	.0611	-.0137	-.0197
20	4.3776	3.1981	1.9501	.8392	.2789	.1171	.0155	-.0264	-.0210
25	3.8961	3.1934	2.2354	1.1908	.5517	.3318	.1654	.0593	.0109
30	5.9630	4.7197	3.3307	1.8966	.9885	.6535	.3788	.1769	.0534
35	2.8636	2.6501	2.1608	1.4126	.8125	.5626	.3428	.1692	.0543
40	.0075	.6560	.9600	.8644	.5864	.4335	.2815	.1481	.0504
45	1.4429	1.4980	1.4191	1.1099	.7410	.5534	.3670	.1992	.0704
50	1.6132	1.4836	1.3352	1.0542	.7298	.5579	.3804	.2130	.0777
55	-.7033	-.1903	.2395	.4707	.4416	.3731	.2772	.1671	.0646
60	-.6161	-.2349	.1258	.3677	.3841	.3380	.2612	.1635	.0652
65	.9018	.7029	.6127	.5607	.4674	.3934	.2971	.1843	.0736
70	-.0213	.0260	.1382	.2707	.3036	.2812	.2301	.1524	.0693
75	-1.0856	-.7279	-.3703	-.0338	.1293	.1598	.1556	.1156	.0521
80	.1333	.0683	.0656	.1403	.1932	.1955	.1738	.1240	.0550
85	.5879	.3548	.2110	.1747	.1869	.1833	.1620	.1164	.0522
90	-.6825	-.5034	-.3270	-.1275	.0127	.0584	.0815	.0736	.0373
100	.5932	.3512	.1670	.0736	.0727	.0804	.0818	.0667	.0329
110	-.7865	-.5678	-.3917	-.2333	-.1134	-.0609	-.0170	.0086	.0104
120	.4923	.2955	.1312	.0163	-.0118	-.0081	.0011	.0082	.0067
130	-.4488	-.3315	-.2423	-.1753	-.1236	-.0948	-.0633	-.0333	-.0110
140	.0303	-.0078	-.0431	-.0750	-.0831	-.0772	-.0632	-.0417	-.0174
150	.0696	.0195	-.0238	-.0631	-.0828	-.0837	-.0754	-.0549	-.0249
160	-.4516	-.3294	-.2323	-.1654	-.1369	-.1243	-.1057	-.0753	-.0341
170	.4398	.2684	.1270	.0175	-.0465	-.0666	-.0746	-.0631	-.0316
180	-.6521	-.4636	-.3120	-.2032	-.1574	-.1414	-.1206	-.0872	-.0401

References

1. MacNeal, Richard H., and John A. Bailie, Analysis of Frame-Reinforced Cylindrical Shells, Part I - Basic Theory. NASA TN D-400, 1960.

2. MacNeal, Richard H., and John A. Bailie, Analysis of Frame-Reinforced Cylindrical Shells, Part II - Discontinuities of Circumferential-Bending Stiffness in the Axial Directions. NASA TN D-401, 1960.

3. MacNeal, Richard H., and John A. Bailie, Analysis of Frame-Reinforced Cylindrical Shells, Part III - Applications. NASA TN D-402, 1960.