SHOCK WAVE-TURBULENT BOUNDARY LAYER INTERACTIONS IN TRANSONIC FLOW*

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SUMMARY

The method of matched asymptotic expansions is used in analyzing the structure of the interaction region formed when a shock wave impinges on a turbulent flat plate boundary layer in transonic flow. Solutions in outer regions, governed by inviscid flow equations, lead to relations for the wall pressure distribution. Solutions in the inner regions, governed by equations in which Reynolds and/or viscous stresses are included, lead to a relation for the wall shear stress. Solutions for the wall pressure distribution are reviewed for both oblique and normal incoming shock waves. Solutions for the wall shear stress are discussed.

SYMBOLS

$\bar{a}^*$	critical velocity of sound, cm/sec (ft/sec)
$\bar{L}$	distance from leading edge of flat plate to shock impingement point, cm (in)
M	Mach number
P	pressure, dimensionless with critical pressure in external flow
R_e	Reynolds number based on critical conditions in external flow and $\bar{L}$
U, V	dimensionless velocity components in the X, Y directions, equations (5a, 5b)
u, v	velocity perturbations
$u_{01}(y)$	variable part of dimensionless velocity in velocity defect layer in the undisturbed boundary layer.
u_τ	dimensionless friction velocity, equation (1c)
X, Y	dimensionless coordinates; $Y = \bar{Y}/\bar{L}$, $X = (\bar{X} - \bar{L})/\bar{L}$

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x,y	stretched coordinates
α	inverse of Kármán constant $\alpha = (0.41)^{-1}$
β	angle made by shock wave with the vertical
β_0	constant of order one
γ	ratio of specific heats, C_p/C_v
δ	order of thickness of undisturbed boundary layer at shock impingement point, dimensionless with $\overline{L}$
δ_*	distance from wall to sonic line in undisturbed flow, dimensionless with $\overline{L}$; $\delta_* = \delta e^{-\beta_0/\alpha} e^{-\epsilon/\alpha u_\tau}$
Δ	order of extent of outer inviscid flow region in X direction, dimensionless with $\overline{L}$; $\Delta = (\gamma+1)^{1/2} \epsilon^{1/2} \delta$
Δ_*	order of extent of inner inviscid flow region in X direction, dimensionless with $\overline{L}$; $\Delta_* = \delta_* u_\tau^{1/2}$
ϵ	parameter, $\epsilon = U_e - 1$
τ_w	wall shear stress, dimensionless with undisturbed flow wall shear stress at shock impingement point

Subscripts

e	external
w	wall
u	upstream of interaction, in undisturbed flow

INTRODUCTION

Shock wave-boundary layer interaction is an important problem in both external and internal flows. At transonic speeds, asymptotic methods have proven successful in dealing with the problem when the boundary layer is laminar (refs. 1,2) and are now being used in analyzing the interaction region in turbulent boundary layers. In general, these analyses are based on two parameters, ϵ and u_τ, where ϵ measures the difference between the flow velocity and the critical sonic velocity in the flow external to the boundary layer, and where u_τ is a dimensionless friction velocity which measures the change in velocity in the velocity defect region of the undisturbed boundary layer. Thus, if overbars denote dimensional quantities

$$U_e = \overline{U}_e/\overline{a}_e^* = 1 + \epsilon \quad ; \quad M_e = 1 + \frac{\gamma+1}{2}\epsilon + \cdots \tag{1a,b}$$

$$u_\tau = \left(\frac{\bar{\tau}_{wu}}{\bar{\rho}_{wu}} \frac{T_e}{T_{wu}} \right)^{1/2} \Big/ \bar{a}_e^* \quad ; \quad U_u = 1 + \epsilon + u_\tau u_{01}(y) + \cdots \qquad (1c,d)$$

For transonic flow, then $\epsilon \ll 1$, and u_τ is defined such that for $R_e \gg 1$, $u_\tau \ll 1$, and the various problems considered may be characterized by the relative orders of ϵ and u_τ. The first work done on the turbulent boundary layer problem was done by Adamson and Feo (ref. 3) who considered the case where the impinging shock is weak ($u_\tau^2 \ll \epsilon \ll u_\tau$) and oblique, and Melnik and Grossman (ref. 4) who considered the case where the shock is stronger ($\epsilon = O(u_\tau)$) and normal. In both papers, it was concluded that in order for the interaction to cause separation, it was necessary that $\epsilon = O(1)$. As a first step in this direction, the condition $\epsilon \gg u_\tau$ was discussed for the oblique wave case by Adamson (ref. 5) and for the normal wave case by Adamson and Messiter (ref. 6). In each of these papers it was pointed out, first, that the problem could be divided into two parts consisting of the flow structure in the outer inviscid flow regions and that in the inner near wall regions where Reynolds and viscous stresses had to be taken into consideration, and, second, that the wall pressure distribution could be derived from solutions in the inviscid flow regions, without having to find solutions in the near wall regions. In each case, attention was focused on the inviscid flow regions for near separation conditions.

The present paper is concerned with two objectives. First, it is desired to complete the picture of the shock structure in the oblique wave case, only the outer part of this structure having been given in reference 5, and to show the form of the pressure distribution inferred from such a structure. Second, it is desired to compare this structure and pressure distribution with their counterparts in the normal wave case, and discuss briefly the effects of the very different pressure distributions on the conditions for incipient separation. The presentation is aided by a brief review of the structure of the interaction region in the normal wave case, and this is given in the next section. It should be noted that in the following, parameters are defined such that $\delta = u_\tau$, and that the asymptotic relationship between u_τ and R_e is $u_\tau = b_0 (\alpha \ln R_e)^{-1} + \cdots$, where for $\gamma = 1.4$, $b_0 = 0.94$.

STRUCTURE OF THE INTERACTION REGION - NORMAL SHOCK

The structure of the interaction region for the impinging normal shock case is sketched in figure 1. It should be noted that the sketch is not to scale, and that there are two near wall regions, the so-called Reynolds stress sublayer region and the wall region (ref. 6).

As seen in figure 1, there are two inviscid flow regions. The outer region is scaled by the thickness of the boundary layer, δ, in the Y direction and by the order of the extent of the overall interaction region, Δ, in the X direction. To this scale, one sees a normal shock penetrating the boundary layer, represented by the velocity defect region. In the inner inviscid flow region, scaled by δ_* and Δ_* in the Y and X directions, respectively, (δ_* is the order of the distance from the wall to the sonic line in the undisturbed flow) the upstream influence of the interaction, manifested by a deceleration of the fluid, causes compression waves emanating from the sonic line to coalesce and form a shock wave. This shock wave becomes more and more nearly normal, and as $Y/\delta_* = y^* \to \infty$, connects with the shock wave seen in the outer inviscid flow region.

The wall pressure distribution is found in the limit as the appropriate Y variable (i.e., in either the inner or outer inviscid region) goes to zero. The solution valid everywhere in the interaction region except at the inception of the interaction, where waves are coalescing and the wall pressure first begins to rise from its undisturbed value, is

$$P_w = 1 + \gamma\epsilon + u_\tau \frac{4\gamma x}{\pi} \int_0^\infty \frac{u_{01}(\eta)}{(x^2 + \eta^2)} \, d\eta + \cdots \tag{2a}$$

$$x = X/(\gamma+1)^{1/2} \epsilon^{1/2} u_\tau \tag{2b}$$

Since the asymptotic form for $u_{01}(y)$ as $y \to 0$ is $u_{01} \sim \alpha \ln y + \beta_0$, one can show that as $x \to 0$, P_w reduces to

$$P_w = 1 + \gamma\epsilon - u_\tau 2\gamma\alpha \ln x + \cdots \qquad x << 1 \tag{3}$$

A typical wall pressure distribution is shown in figure 2, for $\epsilon = 0.167$ ($M_e = 1.20$) and $u_\tau = 0.028$ ($R_e = 10^6$), using Coles' form (ref. 7) for u_{01},

$$u_{01} = \alpha \ln y - \frac{\alpha}{2}(1 + \cos\pi y) \qquad y < 1 \tag{4a}$$

$$= 0 \qquad y > 1 \tag{4b}$$

The important points to note are that there are two inviscid flow regions, with upstream influence existing only in the inner region which is exponentially small compared to the outer region (i.e., calculate δ_*/δ and Δ_*/Δ) and that the wall pressure increases monotonically. This point is borne out, for example, by the experimental measurements for flow in tubes described in reference 8.

STRUCTURE OF THE INTERACTION REGION - OBLIQUE SHOCK

The shock wave structure for the oblique shock wave case is sketched in figure 3. Again, it is important to note that this sketch is not to scale, and that the shock occurs in regions in which the governing equations are those for inviscid, rotational flow. Viscous and Reynolds stresses must be taken into account in the Reynolds stress sublayer region and wall region which exist between the inviscid flow region and the wall.

As seen in figure 3, the oblique shock structure has many of the features of the normal shock structure deep in the boundary layer, with a more complicated outer structure. The analysis proceeds by considering first, the region far from the wall, and then proceeding toward the wall. In this regard, the upstream influence from the interaction region is confined to a very thin region, just as in the normal shock case. Thus, in analyzing the regions outside the inner inviscid region, one need not consider any waves arising from lower regions and affecting the incoming shock shape. Instead, as mentioned in reference 5, the incoming oblique wave shape may be found by considering a wave of known strength as it penetrates a shear layer represented by the boundary layer. Thus, if one writes the expansions for the velocity components as

$$U = \frac{\overline{U}}{\overline{a}_e^*} = 1 + \epsilon + u_\tau u_{01} + \epsilon u_1 + \cdots = 1 + \epsilon u + \cdots \tag{5a}$$

$$V = \frac{\overline{V}}{\overline{a}_e^*} = \epsilon^{3/2} v + \cdots \tag{5b}$$

where u_1 is the perturbation due to the shock, and if the local angle made by the shock with the vertical, β_s, is expanded as follows,

$$\beta_s = \epsilon^{1/2}\beta + \cdots \tag{6}$$

Then, using the shock-wave relations, one can show that β and $u_u = 1 + u_\tau u_{01}/\epsilon$, the value of u upstream of the shock, are related as follows (ref. 5).

$$\left[\frac{3\beta}{\sqrt{\gamma+1}} + 2\left(\frac{2\beta^2}{\gamma+1} - u_u\right)^{1/2}\right]^{3/5}\left[\frac{\beta}{\sqrt{\gamma+1}} - \left(\frac{2\beta^2}{\gamma+1} - u_u\right)^{1/2}\right]^{2/5} = c \tag{7}$$

where c is a constant. The value of c in equation (7) is calculated by inserting the known value of β for the wave in the flow external to the boundary layer, where $u_u = 1$.

Unlike the normal shock wave case, it does not appear possible to consider one outer inviscid flow region in which the whole outer shock structure is contained; evidently the shock structure is too complicated for this. Instead, a series of regions is considered, each one being deeper in the boundary layer, and each one having smaller and smaller characteristic lengths in both the X and Y directions. In an asymptotic sense, each of these regions corresponds to a limit process such that $u_\tau \to 0$ with an intermediate variable $Y/\eta(u_\tau)$ held fixed where $\eta/u_\tau \to 0$, but $\eta/\delta_* \to \infty$ as $u_\tau \to 0$. To the scale of each of these regions the shock wave structure appears as shown in figure 4a, that is, the solution is that for a regular reflection with the angles associated with each of the shock waves changing as the boundary layer is penetrated. If u_r denotes the value of u (eq. (5a)) downstream of the reflected shock for a given region, with given values of β and u_u, then

$$u_r = \frac{\beta^2}{\gamma+1} - u_u + \frac{\beta}{\sqrt{\gamma+1}}\sqrt{\frac{5\beta^2}{(\gamma+1)} - 4u_u} \tag{8}$$

and the corresponding pressure is,

$$P = 1 + \epsilon\,\gamma\,(-1 + u_u - u_r) + \cdots \tag{9}$$

Now, in each region or limit process description the incoming velocity is uniform to order ϵ. Therefore, the velocity downstream of the reflected wave and thus the pressure there is uniform to order ϵ. Furthermore, for any regions between the one in question and the wall, with the same characteristic X dimension, $\partial P/\partial y = 0$ to order ϵ, so the pressure given in equation (9) is the wall pressure corresponding to the characteristic X dimension in question. Now from equations (7) through (9), one can show that $du_r/du_u < 0$ so that $dP/du_u > 0$. Thus, as one considers regions deeper and deeper in the boundary layer, i.e., as Y and in particular X decrease, the pressure downstream of the reflected shock increases. Hence, as one approaches the shock along the wall, from downstream of the shock, the wall pressure increases.

As successive regions, each deeper in the boundary layer, are considered, the structure shown in figure 4a is found until the limiting conditions for which a regular reflection is possible are reached. From equation (8), this is seen to occur when β has decreased such that

$$\beta^2 = 4(\gamma+1)u_u/5 \tag{10}$$

As this condition is reached, the shock structure evidently takes on the Guderley Mach stem configuration (ref. 9). To the scale of the region in which this flow configuration is found, the shock structure, then, is that seen in figure 4b. That is, as the wall is approached, the incoming shock (Mach stem) becomes a normal shock.

Finally, beneath the Mach stem region is the inner inviscid region, the same region found in the normal shock wave case, in which compression waves emanating from the sonic line coalesce to form a shock which becomes more and more nearly normal as it moves away from the wall until it merges with the normal shock in the Mach stem region.

The wall pressures associated with the regular reflection and the normal shock problems have been calculated. Although the joining of these two parts of the solution (through the Guderley Mach stem region) is more difficult, the general shape of the wall pressure distribution seems clear. Thus, just as in the normal shock wave case, as one moves downstream toward the Mach stem region and the shock becomes stronger, the wall pressure increases. On the other hand, from the outer inviscid regions discussed above, it was shown that as one moves upstream toward the Mach stem region, the wall pressure increases also. Therefore, the wall pressure distribution must, as shown in figure 5, go through a maximum in the Mach stem region, this maximum pressure being the pressure behind the normal part of the wave (Mach stem) shown in figure 4b. This form for the pressure distribution is also apparent in experimental results as shown, for example, in reference 10. In these experiments the external flow was supersonic but since the flow near the wall is transonic, the general features of the structure of the interaction region shown here must apply.

DISCUSSION

The structure of the shock wave in the interaction region is seen to be much more complicated in the oblique shock case (fig. 3) than in the normal shock case (fig. 1) and this is reflected in the fact that, in general, solutions are much more difficult to obtain in the oblique than in the normal shock case. The differences in the wall pressure distributions in the two cases are illustrated by comparing figures 2 and 5. In the normal shock wave case, the pressure increases monotonically, while in the oblique shock wave case, the pressure goes through a maximum and decreases then to its final value. Now, in the inviscid flow regions, the pressure perturbation is directly proportional to the negative of the perturbation in U, and as the wall is approached, the solution for U is matched with a corresponding solution from the Reynolds stress sublayer region which is in turn matched with a solution in the wall region. The result is that as the pressure increases, the velocity near the wall decreases, such that $\partial U/\partial Y$ near the wall decreases and hence the wall shear stress, τ_w, decreases. In fact, τ_w can be written directly in terms of the pressure perturbations (refs. 3 and 4). Therefore, in the normal-shock wave case, τ_w decreases monotonically to its lowest value, while in the oblique wave case, it goes through a minimum and rises to its final value. This situation is expected to hold up to separation, i.e., for the case of incipient separation. Hence, the point at which τ_w first goes to zero (incipient separation) will occur within the interaction region for the oblique shock case, and at the far downstream limit of the interaction region in the normal shock case. In the event that separation occurs, the results discussed above do not hold, of course; in that case, the displacement of the incoming flow by the separated bubble causes a change in the outer shock structure, and thus in the pressure distributions.

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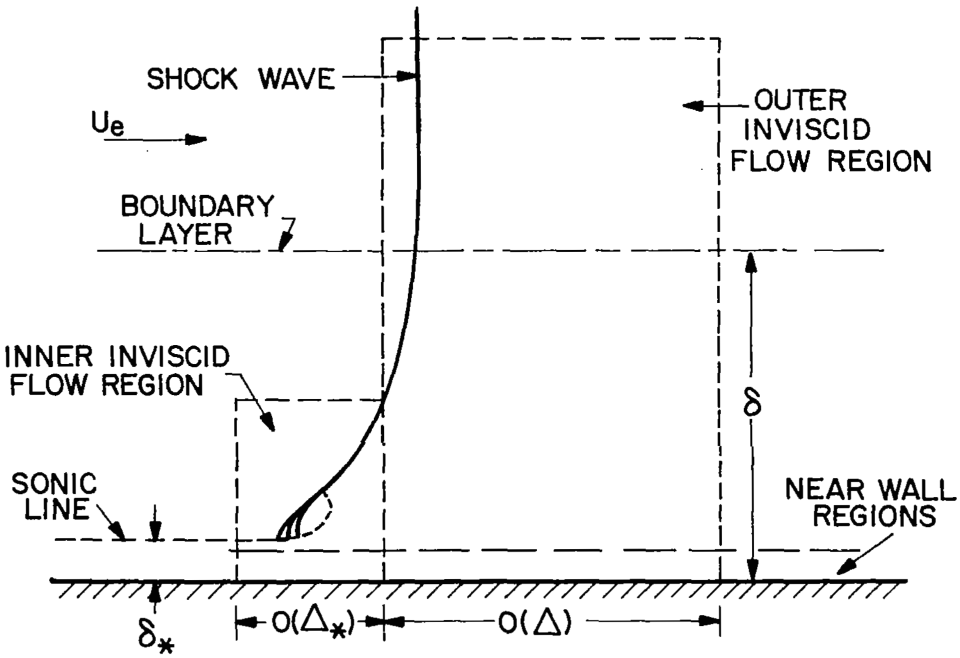


Figure 1.- Sketch of structure of interaction region for case where incoming wave is normal.

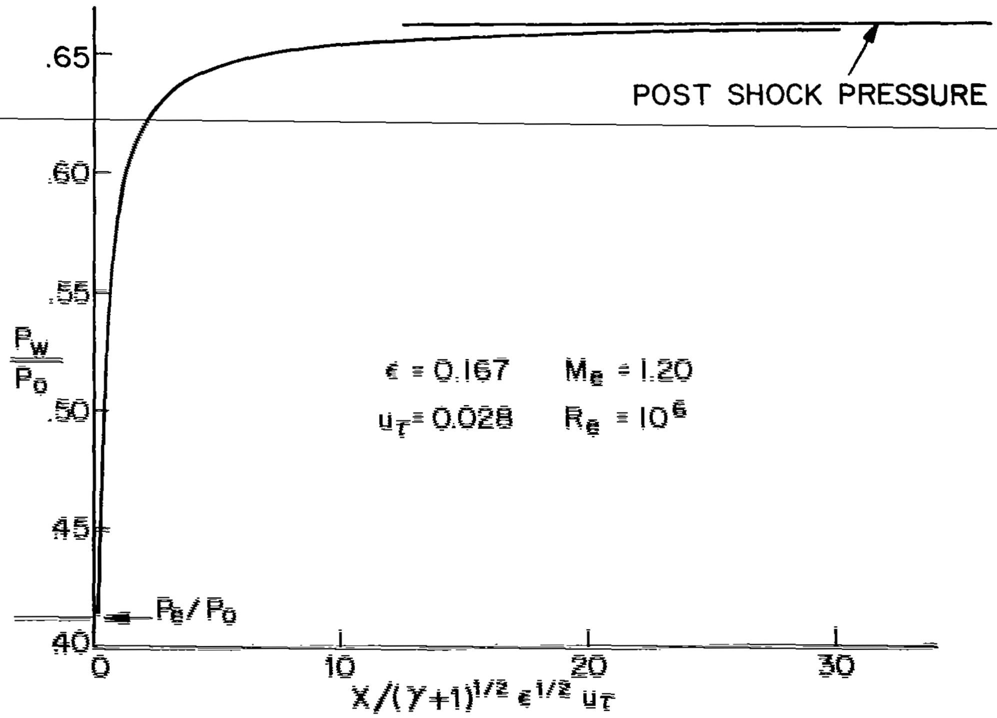


Figure 2.- Calculated wall pressure distribution for normal shock wave interacting with a turbulent flat plate boundary layer with $M_e = 1.20$ and $R_e = 10^6$.

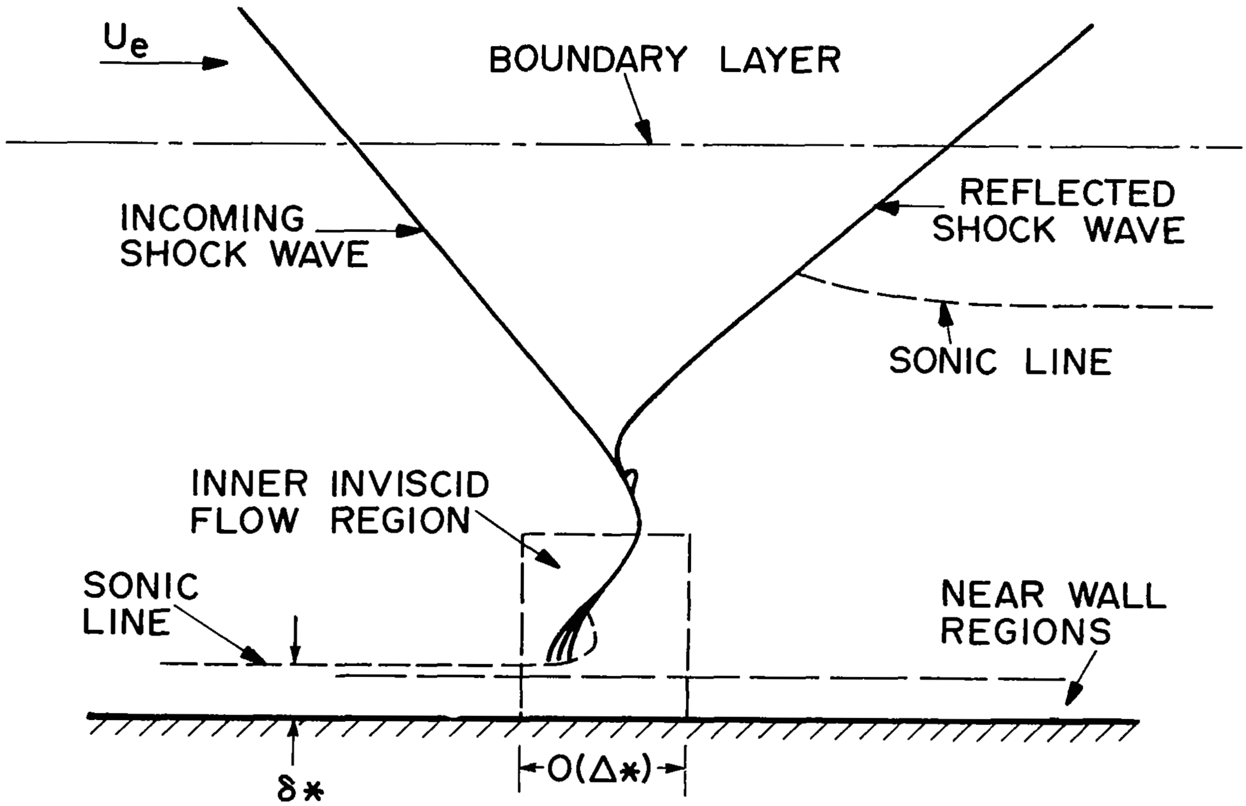
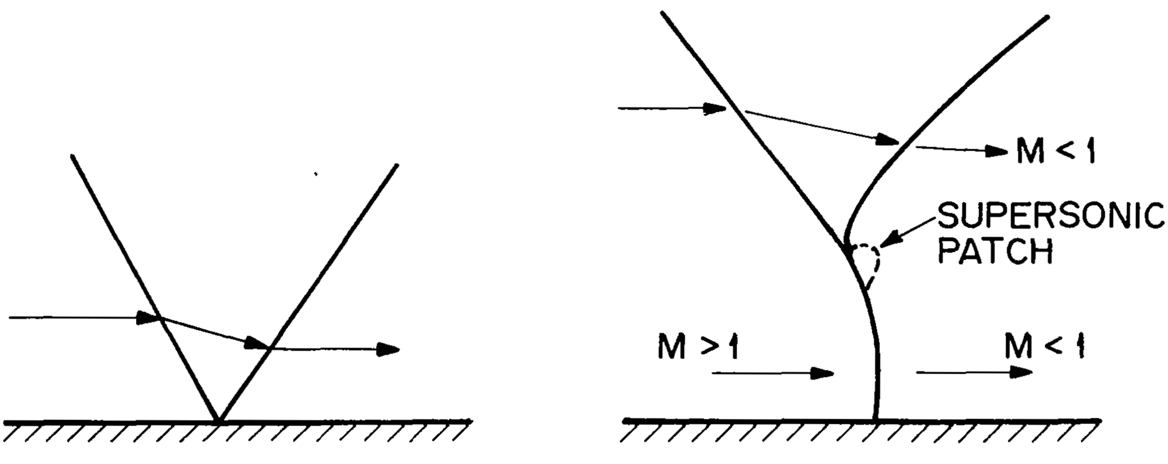


Figure 3.- Sketch of structure of interaction region for case where incoming wave is oblique.



(a) Regular reflection.

(b) Guderley Mach stem configuration.

Figure 4.- Sketch of shock structure in intermediate inviscid flow regions for case where incoming wave is oblique.

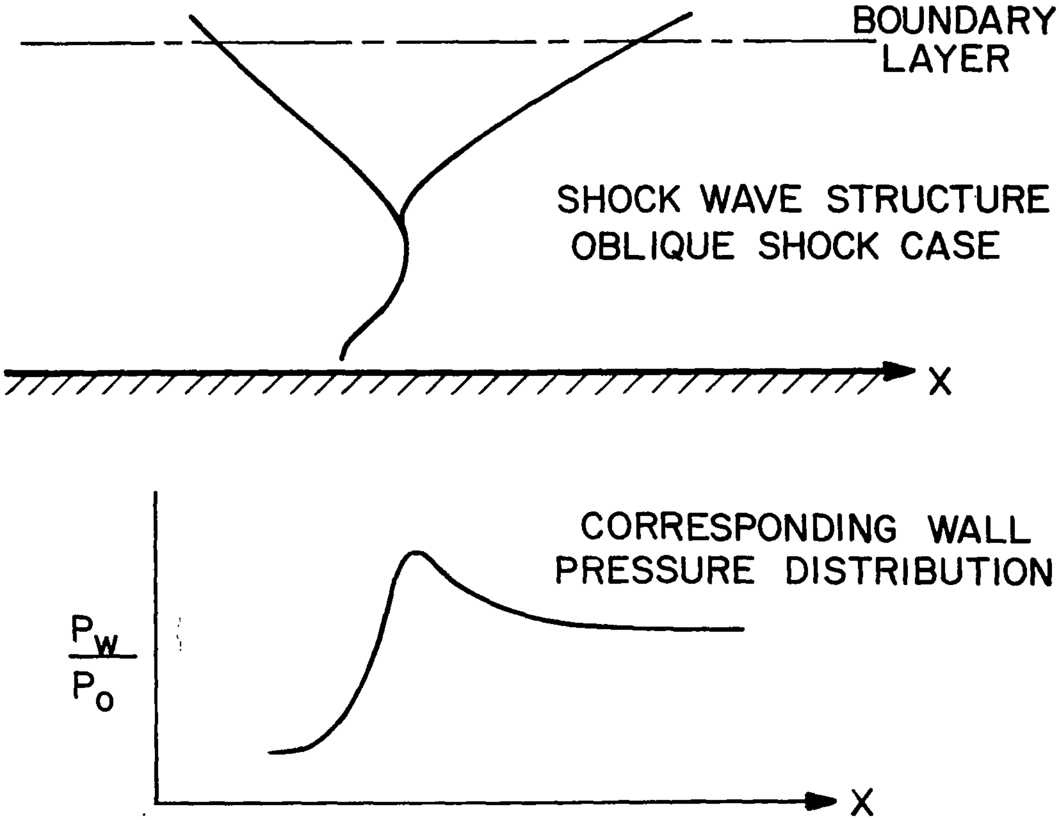


Figure 5.- Sketch of wall pressure distribution corresponding to indicated shock wave structure for case where incoming wave is oblique.