

EXPERIMENT DESIGN FOR PILOT IDENTIFICATION
IN COMPENSATORY TRACKING TASKS

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SUMMARY

A design criterion for input functions in laboratory tracking tasks resulting in efficient parameter estimation is formulated. The criterion is that the statistical correlations between pairs of parameters be reduced in order to minimize the problem of non-uniqueness in the extraction process. The effectiveness of the method is demonstrated for a lower order dynamic system.

INTRODUCTION

The art of human operator modeling has progressed considerably since the pioneering work of Sheridan (ref. 1), Elkind et. al. (ref. 2), and McRuer et. al. (ref. 3). Many of the accepted transfer functions for the human operator in compensatory tracking tasks are given in reference 4. The model structure for the human operator is not known as precisely for such systems as the aircraft, for example. There are large variations from one subject to another and from run to run. In most cases, the pilot model includes such physical parameters as a static gain, an effective time delay made up of transport delays, and high frequency neuromuscular lags or leads and turns which represent the low frequency characteristics of the neuromuscular system dynamics. In addition, a remnant function is generally included in the model to account for pilot anomalies and unsteady behavior. It has been pointed out by various authors (refs. 5-6) that large remnants and the quasi-predictable nature of the inputs cause difficulties in the extraction of the physical parameters in the pilot model.

Early attempts to determine the best set of parameters, in particular pilot models utilizing pilot response data, relied upon analog matching techniques (ref. 7). More recent techniques have applied Kalman filtering methods (ref. 8), maximum likelihood methods (ref. 9), or Newton-Raphson methods (ref. 9) to the system identification problem.

One of the practical difficulties in the extraction of system parameters from experimental data is the non-uniqueness in the solution for the parameter values (ref. 10). In the case of human operator dynamics, the non-uniqueness problem is amplified by the high correlations which can exist between the pilot's effective time delay and the time lags associated with the equalization characteristics discussed by McRuer et. al. (ref. 3). Normally, the time delay is either assumed known or represented by a Pade form which places it in the role of either lead or lag constants (ref. 9).

This paper addresses the task of reducing the non-uniqueness and possibly the effect of the remnant by proper design of the forcing function or disturbance used in laboratory designed tracking tasks.

MODELS

The performance of a human operator in many tracking tasks can be modeled adequately by a quasi-linear describing function. The describing function model consists of a transfer function $Y_p(s)$ and a remnant $\eta(t)$ as shown in figure 1 by the block diagram of a typical compensatory tracking task.

One of the more generalized transfer functions for the compensatory control tasks is discussed in reference 4 and is written as

$$Y_p(s) = K_p e^{-s\tau} \frac{(T_L s + 1)}{(T_I s + 1)} \times \frac{(T_k s + 1)}{(T_k' s + 1)(T_{N1} s + 1) \left[\left(\frac{s}{\omega_N} \right)^2 + \frac{2\xi_N}{\omega_N} s + 1 \right]} \quad (1)$$

In such a model, the parameters K_p , τ , T_L , T_I , T_k , T_k' , T_{N1} , ω_N , and ξ_N are generally poorly determined and are improved upon by using ω_N either analog matching techniques or parameter estimation methods in conjunction with pilot response data.

Depending upon the plant to be controlled, some of the parameters in equation (1) can be eliminated from consideration. For purposes of illustration of how the problem can be put into

a state space formulation for which the methods of modern estimation algorithms can be utilized, the following special case will be considered:

$$Y_c(s) = \frac{K}{s^2} \quad (2)$$

$$Y_p(s) = \frac{K_1(T_3s+1)e^{-s\tau}}{(T_1s+1)(T_2s+1)} \quad (3)$$

In addition, it is assumed that the remnant function is the result of white noise $w_p(t)$ filtered through a second-order linear filter according to

$$\ddot{\eta}(t) + \alpha_1 \dot{\eta}(t) + \alpha_2 \eta(t) = w_p(t) \quad (4)$$

From the block diagram

$$T_1 T_2 \ddot{u}(t) + (T_1 + T_2) \dot{u}(t) + u(t) = K_1 [T_3 \dot{e}(t-\tau) + e(t-\tau)] \quad (5)$$

$$\ddot{c}(t) = K \delta(t) \quad (6)$$

Equations (4)-(5) can be written in terms of the state variables

$$\begin{aligned} y(t) &= (y_1, y_2, y_3, y_4, y_5, y_6)^T \\ &= [c(t), \dot{c}(t), u(t), \frac{K_1 T_3}{T_1} r(t) + u(t+\tau) + T_2 \dot{u}(t+\tau), \\ &\quad \eta(t), \dot{\eta}(t)]^T \end{aligned} \quad (7)$$

as

$$\dot{y}(t) = F_1 y(t) + F_2 y(t-\tau) + b(t) + d(t) \quad (8)$$

where

$$F_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & K & 0 & K & 0 \\ 0 & 0 & \frac{1}{T_2} & 0 & 0 & 0 \\ \frac{K_1}{T_1} & \frac{K_1 T_3}{T_2} & 0 & \frac{1}{T_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\alpha_2 & -\alpha_1 \end{bmatrix} \quad (9)$$

$$F_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{T_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (10)$$

and
$$b(t) = [0, 0, 0, \frac{K_1}{T_1} r(t), 0, 0]^T \quad (11)$$

$$d(t) = [0, 0, 0, 0, 0, w_p(t)]^T \quad (12)$$

The measurements used in the estimation procedure are generally such quantities as the plant output $c(t)$, the stick output $\delta(t)$, etc. These quantities are written as a vector

$$z(t) = h(y) + v(t) \quad (13)$$

where $v(t)$ is assumed to be a white noise process with known statistics.

DESIGN METHOD

The design of optimal inputs for non-time delay differential systems has been investigated by numerous authors. Goodwin (ref. 11) designed an input which minimized the covariance of the error estimate. Mehra (ref. 12) designed an input to maximize the sensitivity of the system output to the system parameters. The philosophy adopted in the present work is to design an input to minimize directly the correlation between certain pairs of parameters.

The problem is stated in terms of a differential-difference equation as given in equation (8) restated as

$$\dot{y}(t) = f(t, y(t), y(t-\tau), p) + u(t) + w(t) \quad (14)$$

with initial values

$$y(t) = 0, t_0 - \tau \leq t \leq t_0$$

τ is the constant time delay and p is the unknown parameter vector. The vector $u(t)$ is to be chosen optimally based on a design criterion and $w(t)$ is the white noise process related to the remnant.

The system measurement from which p is estimated is taken as the system state vector $y(t)$. The noise in the measurement is assumed a Gaussian white noise process with zero mean and covariance R .

In addition to the state equation expressed by equation (14) it will prove useful to introduce additional state variables defined by the elements of the sensitivity matrix $A(t)$ and the time dependent covariance matrix of the error in the estimate $C(t)$ defined respectively according to

$$\dot{A}(t) = \frac{\partial f}{\partial x(t)} A(t) + \frac{\partial f}{\partial x(t-\tau)} A(t-\tau) + \frac{\partial f}{\partial p} \quad (15)$$

$$A(t) = 0, \quad t_0 - \tau \leq t \leq t_0$$

and

$$\dot{C}(t) = -C(t) A^T(t) R^{-1} A(t) C(t) \quad (16)$$

$$C(t_f) = \left[\int_{t_0}^{t_f} A^T(t) R^{-1} A(t) dt \right]^{-1} \quad (17)$$

With these additional equations, it is possible to incorporate equations (14)-(16) into one differential equation of the form

$$F(t, X(t), X(t-\tau), \dot{X}(t), \dot{X}(t-\tau), u(t), w(t), p) = 0 \quad (18)$$

where the augmented state vector X is defined as

$$X(t) = [x^T(t) \quad C^{(1)T}(t) \quad \dots \quad C^{(m)T}(t) \quad A^{(1)T}(t) \quad \dots \quad A^{(n)T}(t)]^T$$

and

$$C^{(1)}(t) = [c_{11}(t), c_{22}(t), \dots, c_{mm}(t)]^T$$

$$C^{(2)}(t) = [c_{12}(t), c_{23}(t), \dots, c_{m-1,m}(t)]^T$$

$$C^{(m)}(t) = c_{1m}(t)$$

$$A^{(1)}(t) = \left[\frac{\partial x_1}{\partial p_1}(t), \frac{\partial x_1}{\partial p_2}(t), \dots, \frac{\partial x_1}{\partial p_m}(t) \right]^T$$

.....

$$A^{(n)}(t) = \left[\frac{\partial x_n}{\partial p_1}(t), \frac{\partial x_n}{\partial p_2}(t), \dots, \frac{\partial x_n}{\partial p_m}(t) \right]^T$$

The optimal input is designed to minimize the average value of a performance index representing a weighted sum of the squares of the correlation coefficient plus an arbitrary function in the control variable. The correlation coefficient between parameters ρ_i and ρ_j is defined as

$$\rho_{ij} = \sqrt{C_{ij}(t_f) / C_{ii}(t_f) C_{jj}(t_f)}, \quad -1 \leq \rho_{ij} \leq 1$$

The performance index can then be written in conventional form as

$$J(u) = E \{ \psi(X(t_f)) + \int_{t_0}^{t_f} L(X(t), u(t), t) dt \} \quad (19)$$

where the first term represents the weighted sum of the squares of the correlation coefficients. The second term can be chosen according to whatever physical requirement is desired between the state and control vectors.

EXAMPLE WITH LOW ORDER DYNAMICS

As an example of the design concept, consider the scalar system

$$\dot{x}(t) + \frac{1}{\xi} x(t-\tau) = u(t) + w(t), \quad 0 \leq t \leq 4 \quad (20)$$

$$x(t) = 0, \quad -\tau \leq t \leq 0$$

where ξ and τ are the system parameters to be estimated and u and w are the input and white noise functions respectively. The input or control is constrained according to

$$|u| \leq 2$$

and is assumed zero prior to time zero.

The performance index, which is the square of the correlation coefficient between parameters ξ and τ is

$$J(u) = \frac{\int_0^4 a_1(t) a_2(t) dt}{\left(\int_0^4 a_1^2(t) dt \right) \left(\int_0^4 a_2^2(t) dt \right)} \quad (21)$$

The form of the optimal control is a bang-bang control expressed as

$$\dot{u}(t) = 2 \operatorname{sgn} g(\hat{x}(t, \tau), t) \quad (22)$$

where g is the switching function which can be computed from the maximum principle. The optimal estimate of $x(t-\tau)$ denoted

$\hat{x}(t, \tau)$ has been shown by Kwakernaak (ref. 13) to be of the form

$$\hat{x}(t, \tau) = \int_0^t K^0(t, \tau, \sigma) z(\sigma) d\sigma \quad (23)$$

where the Kernel function K^0 is a function of the state equation and noise source.

To continue, a further simplification to the deterministic case will be made. For this case, the sensitivity matrix is the two component row vector

$$A(t) = \left[\frac{\partial x(t)}{\partial \xi}, \frac{\partial x(t)}{\partial \sigma} \right] \quad (24)$$

which satisfy the equations

$$\dot{a}_1(t) + \frac{1}{\xi} a_1(t-\tau) = -\frac{1}{\xi} \dot{x}(t) \quad (25)$$

$$\dot{a}_2(t) = \frac{1}{\xi} \dot{x}(t-\tau)$$

By use of Laplace transform techniques, the solutions to equations (20) and (25) are written as

$$x(t) = \int_0^t h(t-\sigma) u(\sigma) d\sigma \quad (26)$$

$$a_1(t) = -x(t) + \frac{1}{\xi} \int_0^t h(t-\sigma) x(\sigma) d\sigma \quad (27)$$

$$a_2(t) = h(0)x(t-\tau) + \int_0^t h(t-\sigma) \dot{x}(\sigma-\tau) d\sigma \quad (28)$$

where $h(t)$ is the unit impulse response

$$h(t) = \sum e^{\lambda_i t} [1 - \tau/\xi e^{-\lambda_i \tau}]^{-1} \quad (29)$$

and λ_k is the k th root value of the characteristic equation

$$\xi \lambda + e^{-\lambda \tau} = 0 \quad (30)$$

The effectiveness of the design technique for $\xi = \tau = 1/2$ is shown in figure 2. The correlation $\rho_{\xi\sigma}$ is reduced from a value of 0.60 to 0.09 by means of the design technique.

CONCLUDING REMARKS

An optimal design technique for disturbance functions used in laboratory tracking tasks has been developed. The objective is to reduce statistical correlations between various parameter pairs to effect a more efficient parameter extraction from noisy experimental data. Preliminary calculations indicate the design methods to be effective.

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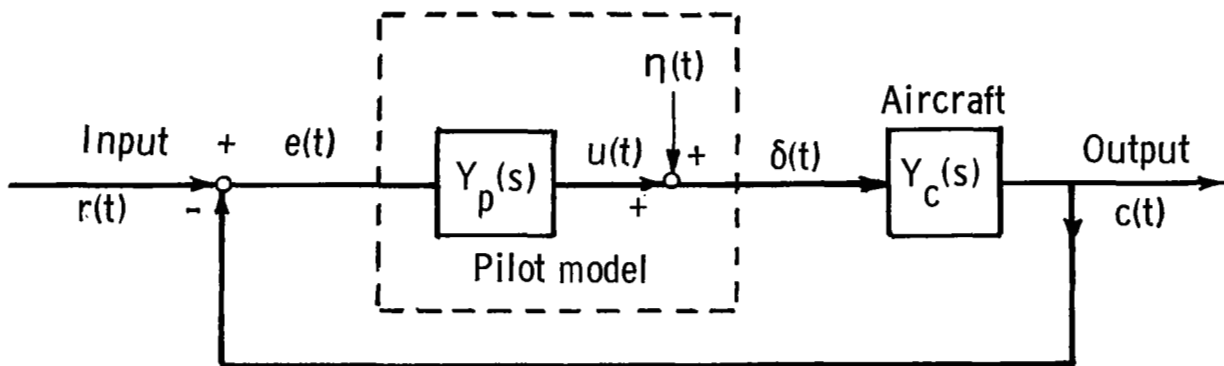


Figure 1.- Block diagram of the compensatory tracking task.

----- Non-optimal input, $J = 0.60$
 ————— Optimal input, $J = 0.09$

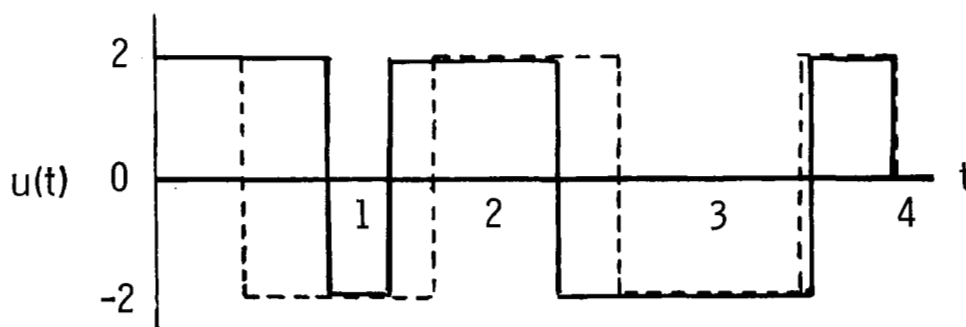


Figure 2.- Control input history.