TERMINAL AREA GUIDANCE ALONG CURVED PATHS - A STOCHASTIC CONTROL APPROACH*

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ABSTRACT

In this paper, stochastic control theory is applied to the problem of designing a digital flight compensator for terminal guidance along a helical flight path as a prelude to landing. The development of aircraft, wind, and measurement models is discussed along with a control scheme consisting of feedback gains multiplying estimate of the aircraft and wind states obtained from a Kalman one-step predictor. Preliminary results are presented which indicate that the compensator performs satisfactorily in the presence of both steady winds and gusts.

1. INTRODUCTION

During the past few decades, the number of problems associated with airport traffic has risen dramatically. Among the more pressing areas of concern are high noise levels near airports, fuel conservation, and weather-induced delays, diversions, or closures. In order to alleviate some of these problems, NASA and the FAA have jointly initiated a long range research effort, the Terminal Configured Vehicle (TCV) program (ref. 1). Among the objectives of the TCV program are increased capability for zero-visibility operation, reduced air delays and route time, avoidance of sensitive areas, and reduced noise source intensity. These objectives can be met, at least partially, through the development of precise automatic control along steep, curved approach paths.

A prerequisite to such precise automatic control is the development of improved ground-based navigation and guidance systems along with improved airborne control systems. The ground based improvements in terminal area navigation and guidance will be provided by the Microwave Landing System (MLS). The MLS will periodically provide accurate range, elevation, and azimuth information to the on-board control system.

The purpose of this paper is to present an application of stochastic control theory to the problem of designing an airborne control system that uses the MLS data for terminal area guidance of a Boeing 737 along a helical flight path as a prelude to landing. First, a system model is presented consisting of an aircraft model, wind model, and measurement model. Next, the digital compensator design is presented. Finally, a digital simulation showing the system response using the above compensator is presented.

2. THE SYSTEM MODELS

In this chapter the aircraft, wind, and measurement models are developed.

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2.1 The Aircraft Model

The notation in this section follows closely that in Etkin (ref. 2). Thus, F_I and F_E denote inertial and Earth reference frames, respectively. If $\overline{V}$ is the mass center velocity of an aircraft, $\overline{V}^I$ and $\overline{V}^E$ denote the vector quantity $\overline{V}$ measured with respect to F_I and F_E, respectively. Also $\overline{V}^E{}_V$ is the vector quantity $\overline{V}^E$ expressed in the vertical reference frame (F_V) coordinates.

The basic nonlinear aircraft model consists of the vector force and moment equations, a set of kinematic constraints, inertial velocity equations, and a set of actuator equations. The force equation is $\overline{f} = m\overline{a}_c$, where $\overline{f}$ is the external force and $\overline{a}_c$ is the mass center inertial acceleration. Assuming the Earth is an inertial system and is locally flat, $\overline{a}_c = \dot{\overline{V}}^E$ (the dot notation means time differentiation), so that $\overline{a}_{c_B} = \dot{\overline{V}}^E{}_B + \overline{\omega}^B{}_B \times \overline{V}^E{}_B$, where $\overline{\omega}^B{}_B = [p\ q\ r]'$ is the angular velocity of F_B and $\overline{V}^E{}_B$ is the mass center inertial velocity, both expressed along the body axes. The prime means transpose.

Let $\overline{W}$ denote the wind velocity. Also, let F_A be an atmosphere-fixed reference frame and $\overline{V}$ the mass center velocity with respect to F_A. Then $\overline{V}^E{}_B = L_{Bw}\overline{V}_w + \overline{W}_B$, where L_{Bw} transforms wind-axes components into body-axis components (ref. 2). Hence,

$$\overline{a}_{c_B} = \frac{d}{dt}(L_{Bw}\overline{V}_w + \overline{W}_B) + \overline{\omega}^B{}_B \times (L_{Bw}\overline{V}_w + \overline{W}_B) \quad (2.1\text{-}1)$$

The external force is $\overline{f}_B = \overline{A}_B + \overline{T}_B + m\overline{g}_B$, where $\overline{A}_B$ is the aerodynamic force, $\overline{T}_B$ is the thrust, and $m\overline{g}_B$ is the gravity force. Hence, the vector force equation is

$$\frac{d}{dt}(L_{Bw}\overline{V}_w + \overline{W}_B) + \overline{\omega}^B{}_B \times (L_{Bw}\overline{V}_w + \overline{W}_B) = \frac{1}{m}\overline{A}_B + \frac{1}{m}\overline{T}_B + \overline{g}_B \quad (2.1\text{-}2)$$

The vector moment equation is $\overline{G} = \dot{\overline{h}}$, where $\overline{G}$ is the external moment and $\overline{h}$ is the angular momentum. Assuming $\dot{I} = 0$ and neglected and elastic components of $\overline{h}$, $\overline{h}_B = I\,\overline{\omega}^B{}_B$, where I denotes the body-axes moments of inertia. Thus,

$$\overline{G}_B = I\,\dot{\overline{\omega}}^B{}_B + \overline{\omega}^B{}_B \times I\,\overline{\omega}^B{}_B \quad (2.1\text{-}3)$$

The external moment $\overline{G}_B$ includes the effects of the gust angular velocities p_g, q_g, and r_g. These effects show up explicitly in the perturbation model.

The scalar components of (2.1-2) and (2.1-3) involve both the body-axes Euler angles ϕ, θ, and ψ (bank, pitch, and heading angles, respectively) and the body-axes rates p, q, and r. The two sets of variables are related by

$$\begin{aligned}\dot{\phi} &= p + q\sin\phi\tan\theta + r\cos\phi\tan\theta \\ \dot{\theta} &= q\cos\phi - r\sin\phi \\ \dot{\psi} &= (q\sin\phi + r\cos\phi)\sec\theta\end{aligned} \quad (2.1\text{-}4)$$

The Earth position is described in cylindrical coordinates with origin at the helix center at ground level. The position of the origin is assumed to be known with respect to the MLS origin. The rates of the Earth position coordinates (helix radius R, helix angle ν, and altitude h) are given by

$$\begin{aligned}\dot{R} &= V\cos\gamma\cos(\psi_w - \nu) + W_x\cos\nu + W_y\sin\nu \\ \dot{\nu} &= \frac{V}{R}\cos\gamma\sin(\psi_w - \nu) - \frac{W_x}{R}\sin\nu + \frac{W_y}{R}\cos\nu \\ \dot{h} &= V\sin\gamma - W_z\end{aligned} \quad (2.1\text{-}5)$$

where γ and ψ_w are wind-axes elevation and heading angles and W_x, W_y, and W_z are coordinates of $\overline{W}$ in the reference frame F_E.

The remaining equations result from modeling the thrust throttle and stabilizer actuator systems. The thrust-throttle relation is modeled as a first-order lag with a time constant of 0.5 second. Because this relation is a linearization about nominal values, the equation is given with the development of the perturbation model. In addition, throttle and stabilizer rates are commanded inputs, so that throttle and stabilizer positions are state variables.

The nonlinear aircraft model consists of the force equation (2.1-2), the moment equation (2.1-3), the kinematic constraints (2.1-4), the inertial velocity equations (2.1-5), and the actuator relations. The model can be written in usual state variable form as a single, nonlinear vector equation

$$\dot{X} = f(X, U, W, \dot{W}) \tag{2.1-6}$$

where $X = [V\ \beta\ \alpha\ p\ q\ r\ \phi\ \theta\ \psi\ R\ \nu\ h\ T\ \pi\ \mathit{s}]'$ is the total state vector, $U = [\dot{\pi}\ \dot{\mathit{s}}\ \delta_e\ \delta_r\ \delta_a\ \delta_{sp}]'$ is the commanded input vector, $W = [u_g\ v_g\ w_g\ p_g\ q_g\ r_g\ W_R\ W_T]'$ is the wind vector, and where β is the sideslip angle, α is the angle of attack, T is the thrust, π is throttle setting, s is stabilizer, δ_e is the elevator, δ_r is the rudder, δ_a is the aileron, δ_{sp} is the spoiler, u_g, v_g, and w_g are translational gust velocities, p_g, q_g, and r_g are rotational gust velocities, and W_R and W_T are radial and tangential components of the steady wind, respectively.

The perturbation model consists of the first-order terms in a Taylor series expansion of (2.1-6) about a descending helical equilibrium. The equilibrium was determined under a zero wind condition with the aircraft flying a "truly banked" turn (ref. 2) for an airspeed of 64 m/sec (120 knots), bank angle of 15° and angle of elevation of -3°, using data for the Boeing 737 from the TCV program. The coefficients in the perturbation model were computed by evaluating the appropriate partial derivatives at the equilibrium. The coefficients of p_g, q_g, and r_g in the moment equations were computed by evaluating the partials of the aerodynamic terms in (2.1-3) with respect to p, q, and r, respectively. Finally, the thrust-throttle relation is $\delta\dot{T} = -0.5\ \delta T + 313.33\ \delta\pi$, where the thrust is in pounds, the throttle setting in degrees, and the "δ" indicates a perturbation value. The perturbation model in usual state variable form is

$$\dot{x} = Ax + Bu + D_{01}w + D_{02}\,\dot{w} \tag{2.1-7}$$

where x, u, and w are the perturbation counterparts of the total vectors X, U, and W in (2.1-6).

2.2 The Wind Model

As seen in section 2.1, the wind vector in the aircraft model consists of three translational gust velocities u_g, v_g, and w_g; three rotational gust velocities p_g, q_g, and r_g; and two steady wind components W_R and W_T. The gust velocities, all of which are components along the body-axes, are modeled as having the Dryden spectra and are produced for simulation and filter design purposes by a linear system processing white noise. As an example of the linear system design, consider the gust velocity u_g, normalized by the equilibrium airspeed V_e. The power density spectrum of the normalized u_g is $\phi_u(\omega) = (2L_u\sigma_u^2/V_e^3)/(1 + (L_u\omega/V_e^2)$, where σ_u is the rms gust velocity, L_u is a turbulence scale factor, and ω is the frequency variable in rad/sec. Now, if a linear system with transfer function $H(j\omega) = 1/(1 + j\omega\ L_u/V_e)$ is subjected to a white noise input with variance $\sigma^2 = (2L_u\sigma_u 2/V_e^3)$, the output is a random process with the spectrum ϕ_u (ω) (ref. 3). A system with the required transfer is described in state-variable form by $\dot{x}_{w1} = (-V_e/L_u)\ x_{w1} + (V_e/L_u)\ \zeta_1$, $w_1 = x_{w1}$, where ζ_1 is a mean zero white noise process with variance $(2L_u\sigma_u^2/V_e^3)$, x_{w1} is a state variable, and w_1 is the output having the required spectrum. The remaining gust velocities are generated in a similar manner.

As indicated in the previous section, W_R and W_T are the radial and tangential components of the steady wind, which are related to the north wind W_N and east wind W_E by the spiral angle ν:

$$
\begin{aligned}
W_R &= W_N \cos\nu + W_E \sin\nu \\
W_T &= W_N \sin\nu - W_E \cos\nu
\end{aligned}
\tag{2.2-1}
$$

Thus, $\dot{W}_R = -\dot{\nu}\ W_T$ and $\dot{W}_T = \dot{\nu}\ W_R$. For the simulation purposes, north and east winds are selected and W_R and W_T are computed using equations (2.2-1). For the filter design, because a constant coefficient wind model is desired and the equilibrium wind is zero, $\dot{W}_R$ and $\dot{W}_T$ are approximated by

$$
\begin{aligned}
\dot{W}_R &= -\ \dot{\nu}_e\ W_T \\
\dot{W}_T &= \ \ \dot{\nu}_e\ W_R
\end{aligned}
\tag{2.2-2}
$$

where $\dot{\nu}_e$ is the equilibrium spiral angle rate, which is constant and equal to $V_e \cos\gamma_e/R_e$.

Putting together the gust system equations and the steady wind equations (2.2-2) yields a time-invariant, linear wind model of the form $\dot{x}_w = A_w\ x_w + B_w\zeta$ and $w = C_w\ x_w$, where ζ is the white noise vector generating the gusts, x_w is the state vector of the wind model, and w is the wind vector used in the perturbation model. In order to use x_w in the perturbation model equations, $w = C_w\ x_w$ and $\dot{w} = C_w\ A_w\ x_w + C_w\ B_w\ \zeta$ are substituted into equation (2.1-7) to give a combined aircraft/wind model:

$$
\begin{aligned}
\dot{x} &= Ax + Bu + D_0\ x_w + D_1\ \zeta \\
\dot{x}_w &= A_w\ x_w + B_w\ \zeta
\end{aligned}
\tag{2.2-3}
$$

where $D_0 = D_{01}\ C_w + D_{02}\ C_w\ A_w$ and $D_1 = D_{02}\ C_w\ B_w$

2.3 The Measurement Model

Measurements available for control purposes consist of the MLS data (range, azimuth, and elevation) and a number of on-board sensor readings. The total measurement vector is $Y = [\mathcal{R}\ Az\ E\ell\ p\ q\ r\ \phi\ \theta\ \psi\ V\ h_b\ \dot{h}_b\ \ddot{x}_B\ \ddot{y}_B\ \ddot{z}_B]'$, where $\mathcal{R}$, Az, $E\ell$ are the MLS data; p, q, r are angular velocities from rate gyros; ϕ, θ, ψ are bank angle, pitch, and heading from position gyros; V is an airspeed indicator reading, h_b and $\dot{h}_b$ are barometric altimeter and vertical speed indicator readings; and $\ddot{x}_B$, $\ddot{y}_B$, $\ddot{z}_B$ are body-mounted accelerometer readings.

The total measurements are computed for simulation purposes by computing the total state variables as equilibrium values plus increments and expressing the measurements in terms of the states. In order to compute the MLS data, it is assumed that the $\mathcal{R}$, Az, and $E\ell$ are measured with respect to a common origin and that the helix center is known with respect to that origin. If the ground coordinates of the helix center with respect the MLS origin are (x_o, y_o), then

$$r = \sqrt{(x_o + R\cos\nu)^2 + (y_o + R\sin\nu)^2 + h^2}$$

$$Az = \tan^{-1}\left[\frac{y_o + R\sin\nu}{x_o + R\cos\nu}\right]$$

$$E\ell = \tan^{-1}\left[\frac{h}{(x_o + R\cos\nu)^2 + (y_o + R\sin\nu)^2}\right]$$

where R, ν, and h are coordinates 10, 11, and 12 of the total state vector.

The fourth through eleventh measurements are the same as total states. Also $\dot{h}_b = V_e \sin\gamma_e + \dot{x}_{12}$, where the derivative $\dot{x}_{12}$ is computed from equation 2.2-3). Finally, expressions for the accelerometer readings are obtained by writing out the scalar components of the acceleration $\bar{a}_{c_B}$ from equation (2.1-1).

In the simulation, the measurements are generated using the above relations along with random noise effects. Except for the airspeed and vertical speed indicators, the noise is an additive, white, mean zero Gaussian process with standard deviation as shown in Table 1 (ref. 4 and 5). The airspeed and vertical speed indicator noises are multiplicative, where the indicated measurement is obtained by multiplying the actual measurement by a normal random variable of mean 1 and standard deviation as given in Table 1.

The incremental measurements to be used with the perturbation model are calculated by subtracting the equilibrium measurement values from the total measurements discussed above except for the first three incremental measurements. For these first three, total helix radius R, helix angle ν, and altitude h are computed from range, azimuth, and elevation using the following equations:

$$R = \sqrt{(r\cos E\ell\cos Az - x_o)^2 + (r\cos E\ell\sin Az - y_o)^2}$$

$$\gamma = \tan^{-1}\left[\frac{r\cos E\ell\sin Az - y_o}{r\cos E\ell\sin Az - x_o}\right]$$

$$h = r\sin E\ell$$

Then the equilibrium radius, angle, and altitude are subtracted to generate the incremental measurements.

The fifteen incremental measurements are linear functions of x and x_w. Hence, the incremental measurement vector y can be written as $y = C_x + C_w X_w + v$, where v is a noise term whose coordinates are assumed to be white, mean zero Gaussian processes with standard deviations that are the same as for the total measurements except for the noise terms in y_1, y_2, y_3, y_{10}, and y_{12} (see Table 1).

3. THE CONTROL SYSTEM DESIGN

Using the aircraft, wind, and measurement models presented in the previous chapter, the total system model is

$$\dot{x}(t) = Ax(t) + Bu(t) + D_o w(t) + D_1\zeta(t) \tag{3-1}$$

$$\dot{w}(t) = A_w w(t) + B_w\zeta(t) \tag{3-2}$$

$$y(t) + Cx(t) + C_w w(t) + v(t) \tag{3-3}$$

where x, u, w and y are the state, control, wind disturbance, and measurement vectors resp., v(t) is a white, Gaussian vector of measurement noise and $\zeta(t)$ is a white, Gaussian noise vector that drives the wind system and corrupts the aircraft system. The matrices A, B, D_o, D_1, A_w, B_w, C and C_w are time-invariant with appropriate dimensions.

The problem of designing a feedback compensator is posed as the usual linear, stochastic regulator problem. For the regulator problem, a quadratic cost functional of the form

$$J = \frac{1}{2} E \{\int_{t_o}^{t_f} [x'(t) Qx(t) + u'(t) Ru(t)]dt\} \tag{3-4}$$

is used, where E is the expectation operator, the prime denotes the transpose, and both Q and R are positive definite, time-invariant weighting matrices. The problem can now be stated as follows. Given the linear system of equations (3-1 to 3-3), find a control u such that the cost functional J of equation (3-4) is minimized.

The first step in solving this problem is to transform the system of equations (3-1 to 3-4) into their discrete-time equivalents. This is done for several reasons. First, a digital compensator is desired since the on-board computer is digital and any control algorithm must be compatible with a digital system. Secondly, the MLS data is only provided periodically. Therefore, the measurement system is inherently a discrete-time one. Finally, a digital simulation is used to test each design. Therefore, the discrete-time equivalent difference equations make the simulation very easy to implement on a digital computer.

The equivalents are obtained by integrating the system differential equations and cost functional over each sampling period (ref. 6).

If we restrict u(t) to be constant over the sampling period, the resultant discrete-time equations are

$$x_{k+1} = \phi x_k + \Gamma_2 w_k + \Gamma_1 u_k + \xi_k \tag{3-5}$$

$$w_{k+1} = \phi_w w_k + \eta_k \tag{3-6}$$

where the subscript k denotes the k^{th} sample and $[\xi_k' \mid \eta_k']'$ is a mean zero, white noise sequence whose variance depends on the variance of the continuous noise vector $\zeta(t)$.

The discrete-time equivalent measurement equation can be obtained directly from the continuous time equation:

$$y = Cx_k + C_w w_k + v_k \tag{3-7}$$

Finally, the discrete-time equivalent of the cost functional (3-4) can be written as a sum of n integrals. The resultant expression for the cost functional becomes:

$$J = \frac{1}{2} E \{\sum_{k=o}^{n} x_{k+1}' \hat{Q} x_{k+1} + 2x_{k+1}' \hat{N} w_{k+1} + 2x_k' \hat{M} u_k + u_k' \hat{R} u_k\} \tag{3-8}$$

Note that, since the original system and cost matrices are time-invariant, so are the discrete-time system and cost matrices.

The problem can now be restated as follows. Find a control sequence u_k which minimizes the cost functional J in equation (3-8) subject to the constraints that the state equations (3-5, 6, 7) must be satisfied and that $\{u_k\}$ must explicitly depend only on the past measurements $y_k = \{y_o, y_1, \ldots, y_{k-1}\}$, where v_k is a zero-mean, Gaussian, white noise sequence independent of $[\xi_k' \mid \eta_k']'$.

The above system of equations (3-5, 6, 7, 8) can be augmented to obtain a form very similar to the discrete linear quadratic Gaussian stochastic control problem. However, if the normal method of solution is applied, an important difficulty surfaces. The total system may be unstable and uncontrollable due to an unstable wind system. Therefore, if the augmented system is solved with an unstable wind system, the solution to the Riccati equation diverges due to the presence of unstable and uncontrollable poles. But, under certain conditions the gains will be bounded.

It can be shown (ref. 6) that the solution to the stochastic optimal control problem described previously exists and is given by:

$$u_k^* = - H_k\hat{x}_k - H_{wk}\hat{w}_k$$

$$H_k = \tilde{R}_k^{-1} G_k, \; H_{wk} = \tilde{R}_k^{-1} G_{wk}, \; \tilde{R}_k = \hat{R} + \Gamma_1' P_k\Gamma_1$$

$$G_k = \Gamma_1' P_k\phi + \hat{M}' \quad , \quad G_{wk} = \Gamma_1'(P_{wk} \phi_w + P_k\Gamma_2)$$

$$P_{k-1} = \phi' P_k\phi + \hat{Q} - G_k' R_k^{-1} G_k, \; P_n = \hat{Q}$$

$$P_{wk-1} = (\phi-\Gamma_1\tilde{R}^{-1}G_k)'(P_{wk}\phi_w + P_k\Gamma_2) + \hat{N}, \; P_{wn} = \hat{N}$$

where $\hat{x}_k$ and $\hat{w}_k$ are one-step predicted estimates of x_k and w_k given by:

$$\hat{x}_k = E\,\{x_k|y_{k-1}\} \text{ and } \hat{w}_k = E\,\{w_k|y_{k-1}\}$$

It should be noted that the above gain equations remain valid for any i-step predicted estimate (where i = 0 represents a filtered estimate). The one-step prediction was used here in order to account for computational delays present in the on-board computer.

Also, the equations above are of a recursive nature. Therefore, at each sampling instant a new optimal gain is calculated. To implement this would require storing all the intermediate values of each gain matrix. This, in turn would require a greater amount of storage than is normally available for small, on-board computers. For these reasons, a suboptimal design was used consisting of only the steady state gains obtained when the index on the recursive relations tends to infinity.

It should be emphasized that the optimal control for a system with disturbances consists of two parts. The first part feeds back state estimates multiplied by an optimal gain H_k. This gain is exactly the same as would have been calculated with no disturbance present. The second term feeds back the disturbance estimates multiplied by a gain H_{wk}, which depends on the disturbance.

The next step is to obtain the state and wind estimate. This is accomplished by first augmenting the discrete-time equations (3-5, 6, 7):

$$\begin{bmatrix} x_{k+1} \\ \hline w_{k+1} \end{bmatrix} = \begin{bmatrix} \phi & \Gamma_2 \\ 0 & \phi_w \end{bmatrix} \begin{bmatrix} x_k \\ \hline w_k \end{bmatrix} + \begin{bmatrix} \Gamma_1 \\ \hline 0 \end{bmatrix} u_k + \begin{bmatrix} \xi_k \\ \hline \eta_k \end{bmatrix} + \chi_k$$

$$y_k = [C \,\vdots\, C_w] \begin{bmatrix} w_k \\ \hline w_k \end{bmatrix} + v_k$$

χ_k is a white noise term representing modeling error.

Given the past measurements y_k, it can be shown (ref.7) that the one-step predicted estimates of x_{k+1} and w_{k+1} are given by the equations below:

$$\begin{bmatrix} \hat{x}_{k+1} \\ \hline \hat{w}_{k+1} \end{bmatrix} = \begin{bmatrix} \phi & \Gamma_2 \\ 0 & \phi_w \end{bmatrix} \begin{bmatrix} \hat{x}_k \\ \hline \hat{w}_k \end{bmatrix} + \begin{bmatrix} \Gamma_1 \\ \hline 0 \end{bmatrix} u_k + L_k \left\{ y_k - [C \,\vdots\, C_w] \begin{bmatrix} \hat{x}_k \\ \hline \hat{w}_k \end{bmatrix} \right\}$$

where $\hat{x}_o = m_o = E\,\{x_o\}$, $\hat{w}_o = 0$, and

$$L_k = \begin{bmatrix} \phi & \Gamma_2 \\ 0 & \phi_w \end{bmatrix} \Sigma_k \, [C \mid C_w]' \, ([C \mid C_w] \, \Sigma_k \, [C \mid C_w]' + \theta_k)^{-1}$$

where $\Sigma_k = E \left\{ \begin{bmatrix} \tilde{x}_k \\ \tilde{w}_k \end{bmatrix} \begin{bmatrix} \tilde{x}_k \\ \tilde{w}_k \end{bmatrix}' \right\}$, $\begin{bmatrix} \tilde{x}_k \\ \tilde{w}_k \end{bmatrix} = \begin{bmatrix} x_k \\ w_k \end{bmatrix} - \begin{bmatrix} \hat{x}_k \\ \hat{w}_k \end{bmatrix}$ is the estimation error, and Σ_k can be found by solving the Riccati type equation:

$$\Sigma_{k+1} = \begin{bmatrix} \phi & \Gamma_2 \\ 0 & \phi_w \end{bmatrix} \{\Sigma_k - \Sigma_k \, [C \mid C_w]' \, ([C \mid C_w] \, \Sigma_k \, [C \mid C_w]' + \theta_k)^{-1} [C \mid C_w] \, \Sigma_k\} \begin{bmatrix} \phi & \Gamma_2 \\ 0 & \phi_w \end{bmatrix}' + \Xi_k$$

where $\Sigma_0 = E \left\{ \begin{bmatrix} x_o \\ w_o \end{bmatrix} \begin{bmatrix} x_o \\ w_o \end{bmatrix}' \right\}$, $\Xi_k \, \delta_{kj} = E \left\{ \begin{bmatrix} \xi_k \\ \eta_k \end{bmatrix} \begin{bmatrix} \xi_k \\ \eta_k \end{bmatrix}' \right\} + E \{\chi_k \; \chi_k'\}$

$$\theta_k \, \delta_{kj} = E \, \{v_k \, v_j'\}$$

and $E \left\{ \nu_k \begin{bmatrix} \xi_j \\ \eta_j \end{bmatrix}' \right\} = E \left\{ \nu_k \begin{bmatrix} x_o \\ w_o \end{bmatrix}' \right\} = E \left\{ \begin{bmatrix} \xi_k \\ \eta_k \end{bmatrix} \begin{bmatrix} x_o \\ w_o \end{bmatrix}' \right\} = 0$ for all k

$j = 0, 1, 2, \ldots.$

It must be noted that, as with the control equations, these equations are recursive. For the same reasons discussed previously, a suboptimal predictor was implemented using only the steady state solutions to the above equations.

Now that the optimal controls have been defined for specific cost matrices the main concern becomes testing to see if the chosen cost matrices lead to an acceptable system response or if they must be modified to achieve this goal. The digital simulation described in the next section will provide the final step of the design procedure satisfying the above testing and modification requirements.

4. SIMULATION RESULTS

The main objective considered in generating these results was to design the control system so that the aircraft position was kept close to the equilibrium in the presence of various steady wind magnitudes. Thus, the objective was to keep the helix radius, altitude, and airspeed perturbations small while allowing some of the other perturbations, such as the altitude variables, to become relatively large.

The first set of results (Table 2) shows how the control system reacts to increasing steady winds. Three simulation runs were made with steady wind velocities of 3.05 m/sec (10 ft/sec), 6.1 m/sec (20 ft/sec), and 12.2 m/sec (40 ft/sec). The rms gust velocity in all runs was 0.61 m/sec (2 ft/sec). The results in Table 2 indicate that, as the steady wind increases, the maximum deviations in the bank angle, heading angle, and helix angle also increase proportionally, while the maximum deviations in helix radius, altitude, and airspeed increase at a much smaller rate.

The results in Table 3 show the importance of the control gain matrix H_w in controlling the aircraft in the presence of winds. Two runs were made with

12.2 m/sec (40 ft/sec) winds, one with H_w used in the control scheme and one without H_w. As the results indicate, the H_w term in the control calculations has a large effect in providing satisfactory control in the presence of winds.

5. CONCLUDING REMARKS

A linear, time-invariant perturbation model of an aircraft flying a descending helix in the presence of winds was developed. An automatic control system was designed through an application of the linear-quadratic-Gaussian discrete-time regulator theory.

Using a performance criteria including maintaining a circular ground track of correct radius as well as maintaining proper altitude and airspeed, preliminary simulation results have shown that the control system can perform satisfactorily in the presence of steady winds without excessively large deviations in other variables, such as bank and pitch angles.

REFERENCES

1. J. P. Reeder, R. T. Taylor, and T. M. Walsh, "New Design and Operating Techniques for Improved Terminal Area Compatibility," Society of Automotive Engineers Air Transportation Meeting, Dallas, Texas, April 30 - May 2, 1974.

2. B. Etkin, Dynamics of Atmospheric Flight. New York: John Wiley & Sons, Inc., 1972.

3. A. Papoulis, Probability, Random Variables, and Stochastic Processes. New York: McGraw-Hill Book Company, 1965.

4. W. C. Hoffman, W. M. Hollister, and R. W. Simpson, "Functional Error Analysis and Modeling for ATC Systems Concepts Evaluation," Aerospace Systems, Inc., Report No. DOT-TSC-212-72-1, May, 1972.

5. W. F. Hodge and W. H. Bryant, "Monte Carlo Analysis of Inacuracies in Estimated Aircraft Parameters Caused by Unmodeled Flight Instrumentation Errors," NASA TN D-7712, Feb., 1975.

6. N. Halyo and R. H. Foulkes, "On the Quadratic Sampled-Data Regulator With Unstable Random Disturbances," Proceedings, 1974 IEEE Systems, Man, and Cybernetics Conference, Dallas, 1974.

7. J. S. Meditch, Stochastic Optimal Linear Estimation and Control, New York: McGraw-Hill Book Company, 1969.

TABLE 1. MEASUREMENT NOISE STANDARD DEVIATIONS

MEASUREMENT	NOISE STANDARD DEVIATION
Range	.305 m (1 foot)
Azimuth	0.41×10^{-3} rad
Elevation	0.61×10^{-3} rad
Helix Radius	1.52 m (5 feet)
Helix Angle	10^{-3} rad
Altitude	3.05 m (10 feet)
Rate gyros (p, q, r)	0.1 deg/sec
Bank gyro	0.5 deg
Pitch gyro	0.15 deg
Heading gyro	1. deg
Airspeed (total)	.02
Airspeed (incremental)	.02 V_e
Barometric altimeter	8.38 m (27.5 feet)
Vertical speed indicator (total)	.05
Vertical speed indicator (incremental)	.05 $\dot{h}_{b_e}$
Accelerometer ($\ddot{x}_B$, $\ddot{z}_B$)	4.91 cm/sec^2 (.161 ft/sec^2)
Accelerometer ($\ddot{y}_B$)	.491 cm/sec^2 (.016 ft/sec^2)

TABLE 2. EFFECT OF STEADY WINDS

Steady Wind Magnitude, m/sec (ft/sec)	3.05 (10)	6.1 (20)	12.2 (40)
Maximum Perturbation Magnitude			
Bank angle, deg	2.0	3.1	6.7
Heading angle, deg	9.1	16.3	31.4
Helix angle, deg	5.2	9.7	19.5
Airspeed, m/sec (ft/sec)	1.7 (5.6)	1.4 (4.5)	2.0 (6.6)
Helix radius, m (ft)	5.6 (18.3)	7.3 (24.0)	11.0 (36.0)
Altitude, m (ft)	7.3 (24.0)	8.1 (26.7)	10.0 (32.8)

TABLE 3. EFFECT OF CONTROL GAINS H_W

Maximum Perturbation Magnitude	with H_W	without H_W
Airspeed, m/sec (ft/sec)	2.1 (6.8)	3.2 (10.4)
Helix radius, m (ft)	11.3 (37.0)	81.7 (268.1)
Altitude, m (ft)	10.3 (33.8)	16.0 (52.4)