AEROELASTIC STABILITY OF WIND TURBINE BLADES

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ABSTRACT

The second-degree nonlinear aeroelastic equations for a flexible, twisted, nonuniform wind turbine blade are developed using Hamilton's principle. The derivation of these equations has its basis in the geometric nonlinear theory of elasticity. These equations with periodic coefficients are suitable for determining the aeroelastic stability and response of large wind turbine blades. Methods for solving these equations are discussed.

INTRODUCTION

The recent renewed efforts in wind power machines is due to their prospective use as an alternate energy source. As a result of these efforts several wind turbine projects have been initiated by NASA Lewis Research Center as a part of ERDA's overall wind energy program. To make the wind energy cost effective, progressively larger wind turbines ranging from 100 kW with a rotor diameter of 125 feet to 2000 kW with a rotor diameter of 200 to 300 feet are being considered, and these wind turbines are now in conceptual design. However, as the blade flexibility increases, susceptibility to aeroelastic instability increases. Also, the efficient construction and operation of wind turbines require that the vibratory loads and stresses on the rotor as well as on the combined rotor-tower system be reduced to the lowest possible levels. Thus, the aeroelastic and structural dynamic considerations have a direct bearing on the manufacture, life, and operation of these large wind turbine systems. Although the structural dynamic and aeroelastic technology used to develop rotory wing aircraft appears to be adequate for the development of wind power machines, this technology has to be transformed from aircraft applications to wind power applications, and additional studies have to be conducted to determine the effects of the parameters peculiar to wind power machines on the aeroelastic and structural dynamic behaviour.

The aeroelastic considerations that are common to both the wind turbine blade and the helicopter blade are flap-lag-torsion, flap-torsion, flap-lag instabilities and torsional divergence. The wind velocity gradients due to earth's boundary layer and the gravity loads in the case of wind turbine rotor and the forward velocity in the case of helicopter rotor lead to timewise periodic coefficients in the equations of motion. Several previous studies have considered the helicopter blade and developed the nonlinear aeroelastic equations of motion. More recently reference 1 derived a set of nonlinear aeroelastic equations and compared them to some of the recent equations available in the literature. These comparisons indicated several descrepancies with the results of reference 1, particularly in the nonlinear terms. The reasons for these descrepancies were explained in reference 1. For wind turbine blades reference 2 presented a set of nonlinear aeroelastic equations. An examination of these equations reveals that the reference 2 fails to recover several nonlinear elastic and aerodynamic terms which are of the same order as those retained. The reasons for this failure are the use of an incorrect torsional curvature and the linearization of the resultant transformation matrix between the undeformed and deformed blade coordinates while developing nonlinear equations of motion. In view of this, it is felt that a comprehensive development of the nonlinear aeroelastic equations of motion of wind turbine elastic blade is required. The purpose of this paper is to derive such a set of equations.

The derivation of the nonlinear elastic and dynamic forces follows essentially along the lines of reference 1. In this reference the pretwist of the blades was combined with the elastic twist, following the common practice in the helicopter blade literature. In the present paper, however, the pretwist together with the control inputs will be introduced before the elastic deformations, and a brief summary of the development will be presented. A formal NASA report which is under preparation will provide the details of the development. Methods for solving these equations will be discussed.

MATHEMATICAL MODEL AND COORDINATE SYSTEMS

The mathematical model chosen to represent the wind turbine blade consists of a straight, slender, variable twisted, nonuniform elastic blade. The elastic axis, the mass axis, and the tension axis are taken to be noncoincident; the elastic axis and the feathering axis are assumed coincident with the quarter-chord of the blade. The generalized aerodynamic forces are calculated from strip theory based on a quasi-steady approximation.

Several orthogonal coordinate systems will be employed in the derivation of equations of motion; those which are common to both the dynamic and aerodynamic aspects of the derivation are shown in figures 1 to 3. The axis system $X_I Y_I Z_I$ is fixed in an inertial frame with the origin at the centerline of the hub. The axis system XYZ is obtained by rotating about Z_I axis by the angle $\Psi = \Omega t$ and then about the negative Y_Ω axis by an angle β_{pc}, the angle of built-in coning. All the deformations of the blade are referenced to the XYZ system. The η and ζ axes with the origin at the elastic axis of the cross section are principal axes and are inclined to the Y and Z axes by an amount equal to the geometric pitch angle. The geometric pitch angle is given by $\theta = \theta_{pt} + \theta_c$ where θ_{pt} is the built-in twist angle (pretwist) and θ_c is the collective pitch angle.

The generalized coordinates defining the configuration of the deformed blade are shown in figure 3. The deformations u, v, w, and ϕ both displace and rotate the $x\eta\zeta$ coordinate system to $x_3y_3z_3$ where x_3 axis is tangent to the deformed elastic axis.

DEVELOPMENT OF EQUATIONS OF MOTION

The equations of motion are derived using Hamilton's principle

$$\int_{t_0}^{t_1} (\delta T - \delta V + \delta W)dt = 0 \tag{1}$$

where

$$\delta W = \delta W_D + \delta W_A \tag{2}$$

In equation (1), T is the kinetic energy, V is the strain energy, and δW is the virtual work done by all the nonconservative damping and aerodynamic forces. These are given by

$$V = \frac{1}{2}\int_0^R \iint_A (\sigma_{xx}\,\gamma_{xx} + \sigma_{x\eta}\gamma_{x\eta} + \sigma_{x\zeta}\,\gamma_{x\zeta})d\eta\, d\zeta\, dx \tag{3}$$

$$T = \frac{1}{2}\int_0^R \iint_A \rho\,\frac{d\overline{r_1}}{dt}\cdot\frac{d\overline{r_1}}{dt}\, d\eta\, d\zeta\, dx \tag{4}$$

$$\delta W_D = -\int_0^R E^* \iint_A \dot{\gamma}_{xx}\,\delta\gamma_{xx}\, d\eta\, d\zeta\, dx$$

$$-\int_0^R G^* \iint_A (\dot{\gamma}_{x\eta}\,\delta\gamma_{x\eta} + \dot{\gamma}_{x\zeta}\,\delta\gamma_{x\zeta})d\eta\, d\zeta\, dx \tag{5}$$

$$\delta W_A = \int_0^R (A_u\delta u + A_v\delta v + A_w\delta w + M_\phi\delta\phi)dx \tag{6}$$

For the strain energy, the stresses are proportional to strains $\sigma_{xx} = E\gamma_{xx}$; $\sigma_{x\eta} = G\gamma_{x\eta}$, and $\sigma_{x\zeta} = G\gamma_{x\zeta}$. In the kinetic energy, $d\overline{r}_1/dt$ is the velocity vector of an arbitrary point in the blade. The coefficients E^* and G^* account for internal damping of the material in tension and shear. The loads A_u, A_v, A_w, and M_ϕ are of aerodynamic origin. To develop the explicit expressions for strains and the aerodynamic loads, the expressions for curvatures ω_{x_3}, ω_{y_3}, and ω_{z_3} and the transformation matrix, [T], between coordinate axes systems $x\eta\zeta$ and $x_3y_3z_3$ are needed. Imposing the geometric pitch rotation before the elastic deformation and following the procedures of reference 1, the second-degree expressions for curvatures and transformation matrix are given by

$$\left.\begin{aligned}
\omega_{x_3} &= \phi' + \theta'_{pt}\left(1 - \frac{v'^2}{2} - \frac{w'^2}{2}\right) - (v'\cos\theta + w'\sin\theta)(-v'\sin\theta + w'\cos\theta)' \\
\omega_{y_3} &= -w''(\cos\theta - \phi\sin\theta) + v''(\sin\theta + \phi\cos\theta) \\
\omega_{z_3} &= v''(\cos\theta - \phi\sin\theta) + w''(\sin\theta + \phi\cos\theta)
\end{aligned}\right\} \quad (7)$$

$$[T] = \begin{bmatrix}
1 - \frac{v'^2}{2} - \frac{w'^2}{2} & v'\cos\theta + w'\sin\theta & -v'\sin\theta + w'\cos\theta \\
\begin{aligned}&-v'(\cos\theta - \phi\sin\theta)\\ &-w'(\sin\theta + \phi\cos\theta)\end{aligned} & 1 - \frac{(v'\cos\theta + w'\sin\theta)^2}{2} - \frac{\phi^2}{2} & \begin{aligned}&\phi - \underline{(v'\cos\theta + w'\sin\theta)}\\ &\cdot\underline{\underline{(-v'\sin\theta + w'\cos\theta)}}\end{aligned} \\
\begin{aligned}&v'(\sin\theta + \phi\cos\theta)\\ &-w'(\cos\theta - \phi)\sin\theta\end{aligned} & -\phi & 1 - \frac{(-v'\sin\theta + w'\cos\theta)^2}{2} - \frac{\phi^2}{2}
\end{bmatrix} \quad (8)$$

Using equations (7) and (8) and following the procedure of reference 1, the following strain displacement relations can be derived.

$$\begin{aligned}
\gamma_{xx} &= u' + \zeta[-w''(\cos\theta - \phi\sin\theta) + v''(\sin\theta + \phi\cos\theta)] \\
&\quad -\eta[v''(\cos\theta - \phi\sin\theta) + w''(\sin\theta + \phi\cos\theta)] \\
&\quad + (\eta^2 + \zeta^2)\left(\frac{\phi'^2}{2} + \phi'\,\theta'_{pt}\right) \\
\gamma_{x\eta} &= -\zeta[\phi' + \underline{\underline{(v'v'' - w'w'')\cos\theta\sin\theta - v'w''\cos^2\theta}} \\
&\quad \underline{\underline{+w'v''\sin^2\theta + \theta'_{pt}\left(\frac{v'^2}{2}\cos2\theta + v'w'\sin2\theta - \frac{w'^2}{2}\cos2\theta\right)}}] \\
\gamma_{x\zeta} &= \eta[\phi' + \underline{\underline{(v'v'' - w'w'')\cos\theta\sin\theta - v'w''\cos^2\theta + w'v''\sin^2\theta}} \\
&\quad + \underline{\underline{\theta'_{pt}\left(\frac{v'^2}{2}\cos2\theta + v'w'\sin2\theta - \frac{w'^2}{2}\cos2\theta\right)}}]
\end{aligned} \quad (9)$$

The position vector of a general point in the cross section of the deformed blade is given by

$$\overline{r_1} = \begin{Bmatrix} x_1 \\ y_1 \\ z_1 \end{Bmatrix} = \begin{Bmatrix} x + u - U_F \\ v \\ w \end{Bmatrix} + [T]^T \begin{Bmatrix} 0 \\ \eta \\ \zeta \end{Bmatrix} \quad (10)$$

where the axial foreshortening of the elastic axis due to bending is given by

$$U_F = \frac{1}{2} \int_0^x (v'^2 + w'^2)dx \tag{11}$$

The angular velocity of the triad $x\eta\zeta$ is obtained by projecting Ω and it is given by

$$\bar{\omega} = (\Omega\beta_{pc}\,\bar{e}_x + \Omega\theta_{pt}\,\bar{e}_\eta + \Omega\bar{e}_\zeta) \tag{12}$$

The remaining details of the derivation of the equations of motion follow from reference 1. An essential feature of the derivation is the introduction of a mathematical ordering scheme which is compatible with the assumption of a slender beam. This scheme was discussed in reference 1. Adapting the same scheme, the higher terms in the generalized elastic and inertia forces are dropped. The aerodynamic forces derived in reference 1 are modified so as to make them suitable for wind turbine blades. In this modification both the velocity gradients due to the earth's boundary layer and wind gusts are considered. The aerodynamic forces are retained in a general second-degree form because the ordering scheme which is imposed would depend on the order assigned to the inflow ratio, pretwist, collective pitch and other aerodynamic parameters. The final equations of motion of the blade are as follows:

$$\begin{aligned} S_u - Q_{D_u} - I_u &= A_u + F_{Gu} \\ S_v - Q_{D_v} - I_v &= A_v + F_{Gv} \\ S_w - Q_{D_w} - I_w &= A_w \\ S_\phi - Q_{D_\phi} - I_\phi &= A_\phi \end{aligned} \tag{13}$$

In the above equations the generalized elastic forces S_u , S_v , S_w , and S_ϕ , the inertia forces I_u , I_v , I_w , and I_ϕ , the aerodynamic forces A_u , A_v , A_w , and A_ϕ are nonlinear coupled partial differential operators in u , v , w , and ϕ , and the generalized damping forces are linear uncoupled partial differential operators in u, v, w, and ϕ . The parameters F_{Gu} and F_{Gv} account for gravitational effects. Because of space limitations, the details of the development of the equations of motions and the explicit expressions for all the generalized forces are not presented herein. However, these details will be given in a formal NASA report which is under preparation.

METHODS OF SOLUTION

There are three methods to solve the equations of motion derived above. These are: (1) Galerkin's method[3] and Floquet-Liapunov theory[4]; (2) Integrating Matrix method[5] and Floquet-Liapunov theory; and (3) Approximate method[6] in conjunction with multiblade coordinates. Any one of these methods can be used to solve the above equations of motion. However, the choice depends on the system parameters.

The first two methods are independent of the number of blades whereas the third method is dependent upon the number of blades on the rotor since the multiblade coordinate transformations are functions of the number of blades. These multi-blade coordinate transformations have been developed and applied to rotors with polar symmetry, i.e., rotors with three or more blades. More recently, a similar transformation has been developed for rotors with only two blades in reference 7.

The first two methods involve a numerical integration of the equations of motion whereas the third method does not. But the validity of the third method depends on the parameters of the system in addition to the number of blades. Several studies have been conducted in the literature to determine the validity of the third method for rotors with three or more blades. Based on the results of these studies and the experience of the writer, it appears that the third method can be applied to wind turbine rotors with three or more blades in the preliminary analyses. However, for rotors with two blades the applicability of the transformation given in reference 7, and the validity of the third method which uses this transformation need further research.

CONCLUSIONS

A set of nonlinear second-degree coupled axial-flap-lag-torsional equations of motion for a single, flexible, twisted, nonuniform wind turbine blade were presented. Methods for solving these equations were discussed.

REFERENCES

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3. Bisplinghoff, R. L.; and Ashley, H.: Principles of Aeroelasticity. John Wiley and Sons, Inc., New York, 1962.

4. Hammond, C. E.: An Application of Floquet Theory to Prediction of Mechanical Instability. Journal of the American Helicopter Society, Vol. 19, No. 4, October 1974.

5. White, W. F., Jr.; and Malatino, R. E.: A Numerical Method for Determining the Natural Vibration Characteristics of Rotating Nonuniform Cantilever Blades. NASA TM X-72751, October 1975.

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DISCUSSION

Q. Can you identify any reasons why earlier publications have not included your underlined terms especially if you contend that some are of the order of magnitude as the usually accepted terms?

A. Most of the earlier publications were unable to include these underlined terms because of a partial linearization of the resultant rotational transformation matrix between the coordinates of the deformed and the undeformed blade or the use of an incorrect expression for the torsional curvature.

Q. Hodges and Friedmann had a long controversy on omitted terms. They finally agreed. How is your system different?

A. Examining the latest references of both Hodges and Friedmann, it is clear that they are not in complete agreement. Hodges partially linearized the transformation matrix between the coordinates of the deformed and the undeformed blade in his dissertation and hence obtained an incorrect expression for the torsional curvature. He tried to correct this in subsequent publications. In so doing, the torsional curvature was improperly identified. Friedmann also partially linearized the resultant transformation matrix while developing the nonlinear equations of motion in most of his publications. More discussion on these terms was presented in the cited reference 1.

Both Hodges and Friedmann in their publications combined the pretwist with the elastic twist of the blades. In the present development since the derivation of the nonlinear equations of motion involves finite rotations, the sequence of which is important, the pretwist together with the control inputs are introduced before the elastic deformations.

C. It is believed that prior researchers in rotory wing aeroelasticity, e.g., Daughaday, DuWaldt, Pizialli and others have in fact developed nonlinear terms you refer to.

A. I am aware of some of the earlier publications by Daughaday, DuWaldt, and Piziall i. In these publications, they have not addressed the nonlinear terms reported in the subject paper.

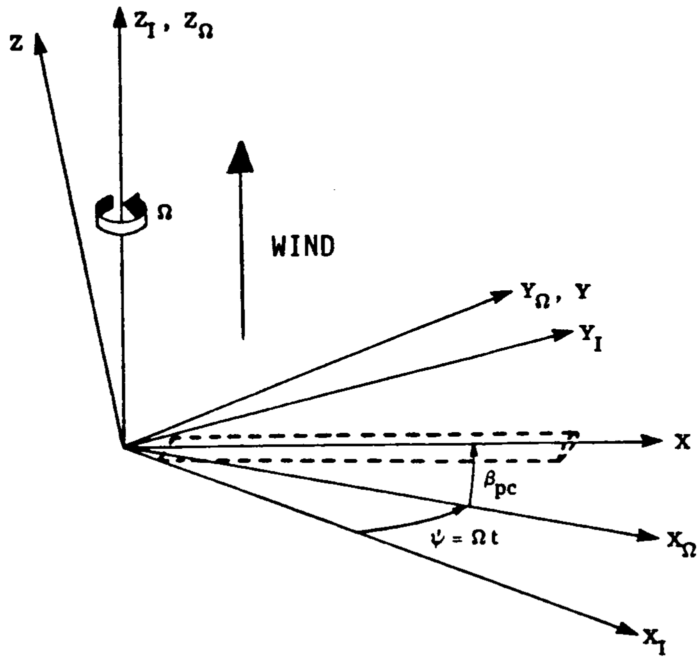

Figure 1. - Coordinate systems of undeformed blade. (Section pitch angle, Θ, not shown.)

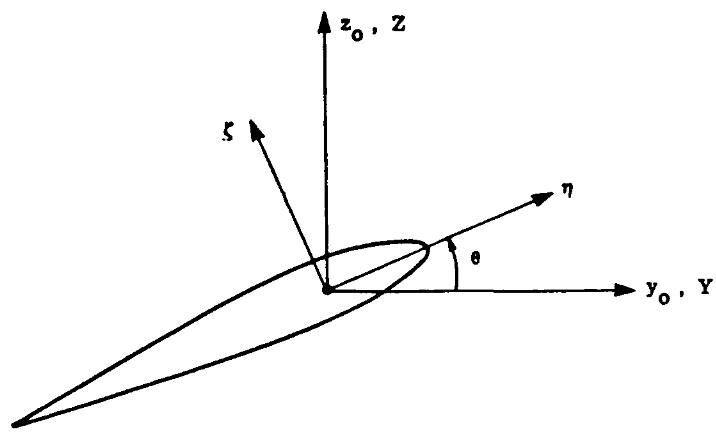

Figure 2. - Coordinate systems of blade cross section.

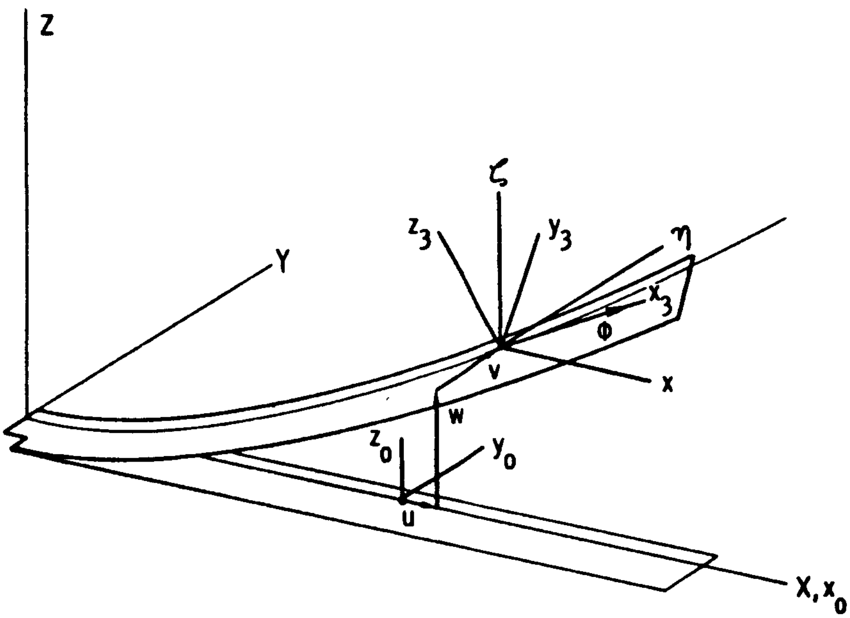

Figure 3. - Schematic representation of undeformed and deformed blade. (Section pitch angle, Θ, not shown.)